Focused recovery with the curvelet transform

joint work with
Deli Wang
(visitor from Jilin university)
and
Gilles Hennenfent

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Focal transform with curvelets

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Focused recovery

Non-data-adaptive Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) derives from ***sparsity*** of seismic data.

Berkhout and Verschuur's ***data-adaptive*** Focal transform derives from ***focusing*** of seismic data by the major primaries.

Both approaches entail the ***inversion*** of a linear operator.

Combination of the two yields

- improved focusing => more sparsity
- curvelet sparsity => better focusing

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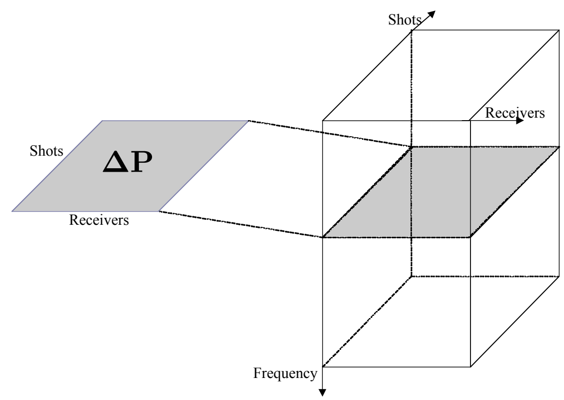
Primary operator



Frequency slice from data matrix with dominant primaries.

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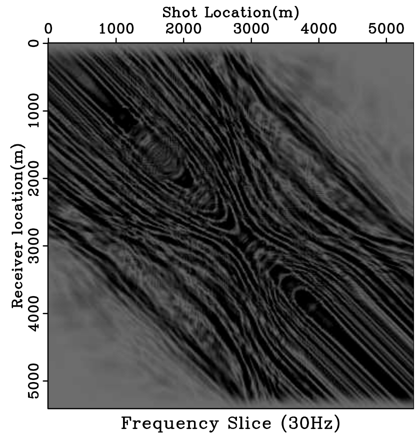
Primary operator



Primary operator

Primaries to first-order multiples:

$$\mathbf{\Delta p} \mapsto \mathbf{m}^1 = (\mathbf{\Delta P}\boldsymbol{\mathcal{A}} *_{t,x} \mathbf{\Delta p})$$

First-order multiples into primaries:

$$\mathbf{m}^1 \mapsto \mathbf{\Delta p} \approx (\mathbf{\Delta P}\boldsymbol{\mathcal{A}} \otimes_{t,x} \mathbf{\Delta p})$$

with the acquisition matrix

$$\boldsymbol{\mathcal{A}} = \left(\boldsymbol{\mathcal{S}}^\dagger \mathbf{R} \boldsymbol{\mathcal{D}}^\dagger\right)$$

"inverting" for source and receiver wavelet wavelets geometry and surface reflectivity.

Curvelet-based Focal transform

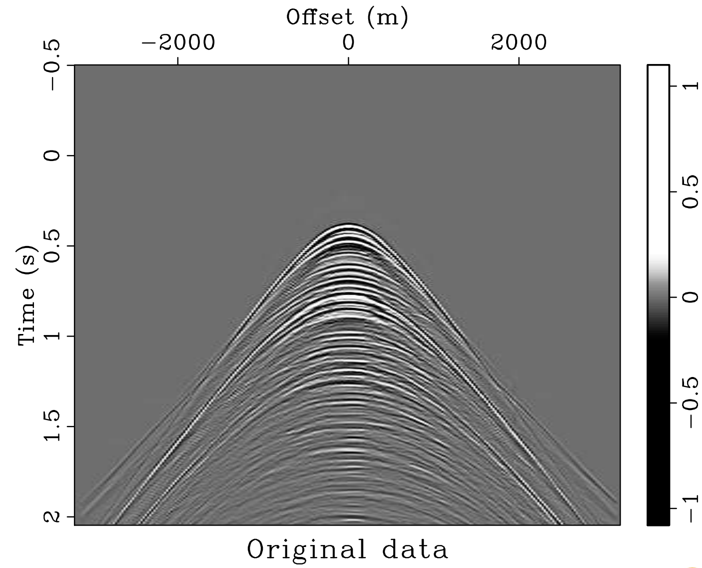
Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

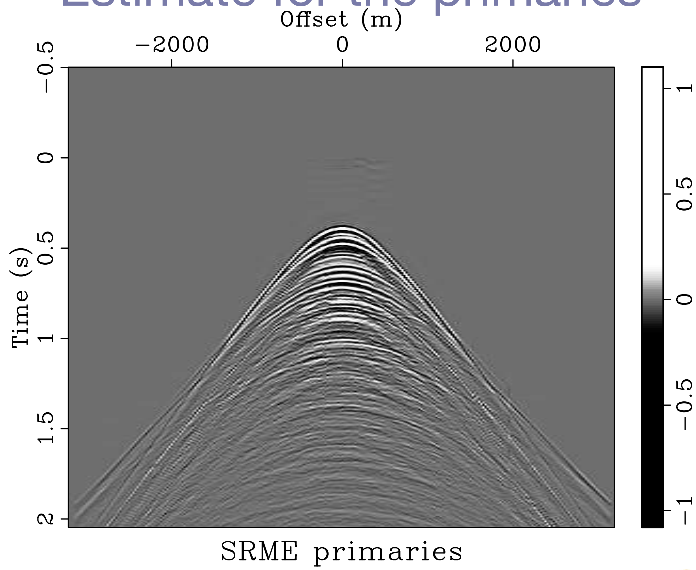
with

$$\begin{aligned} \mathbf{A} &:= \mathbf{\Delta P C}^T \quad \text{and } \mathbf{\Delta P} := \mathbf{F}^H \text{block diag}\{\Delta\}\mathbf{F} \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data.} \end{aligned}$$

Total data

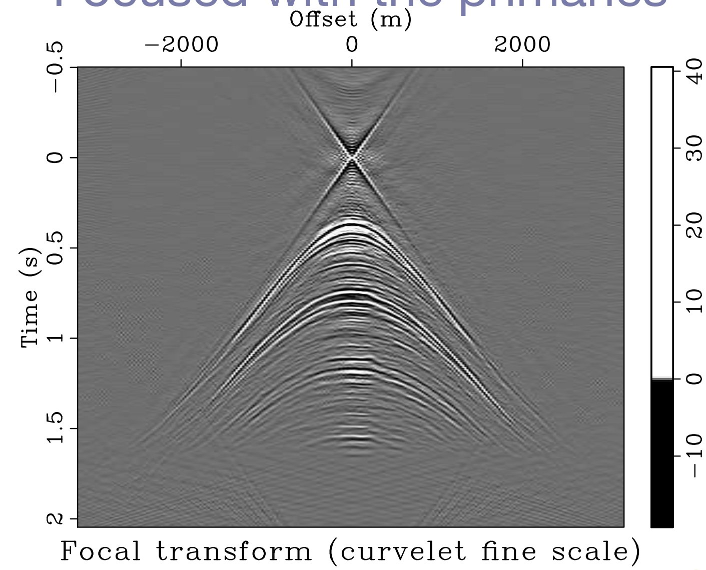


Estimate for the primaries



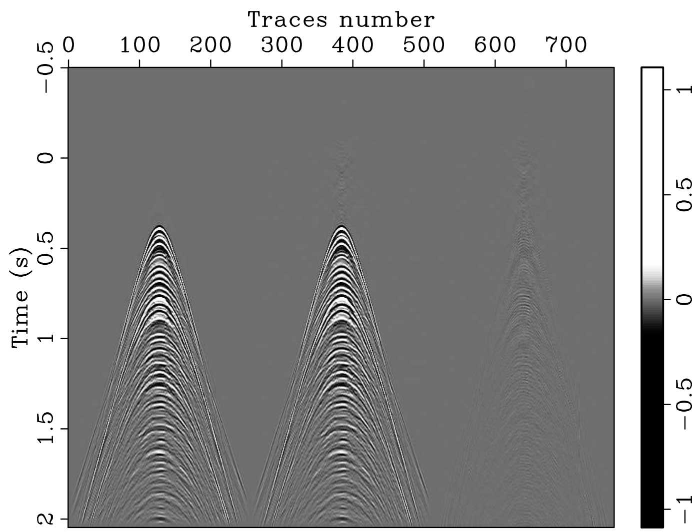
SRME primaries

Focused with the primaries



Focal transform (curvelet fine scale)

Difference



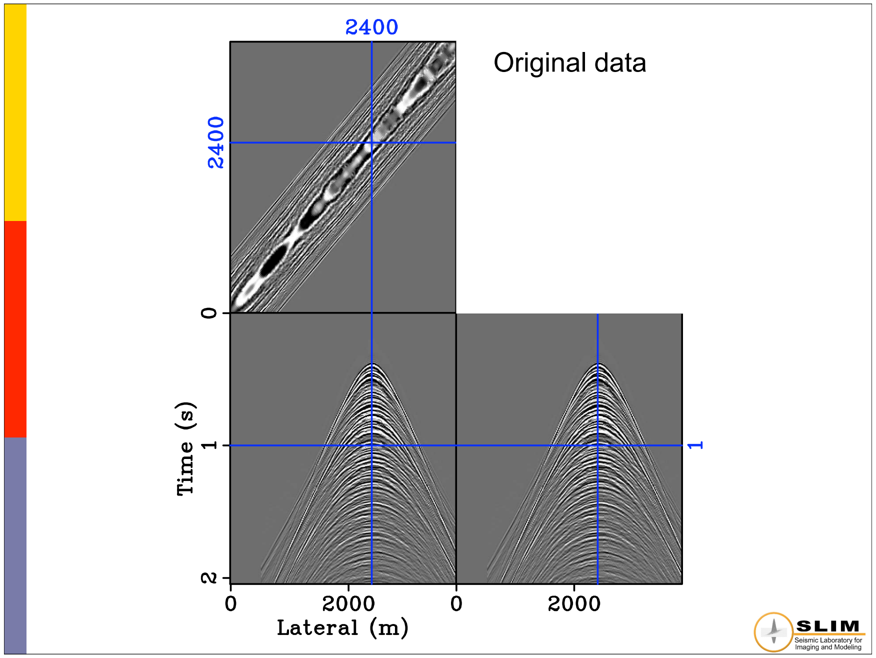
Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \le \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{S}^T &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{P}(:) \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$


Original data
2400
2400
0
1
2
Time (s)
0
2000
0
2000
Lateral (m)
1
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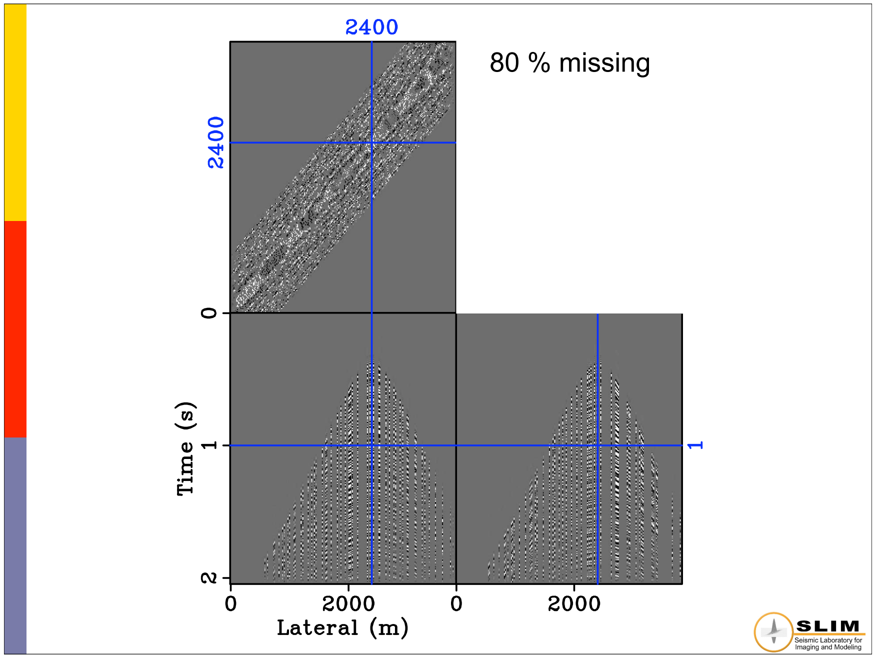
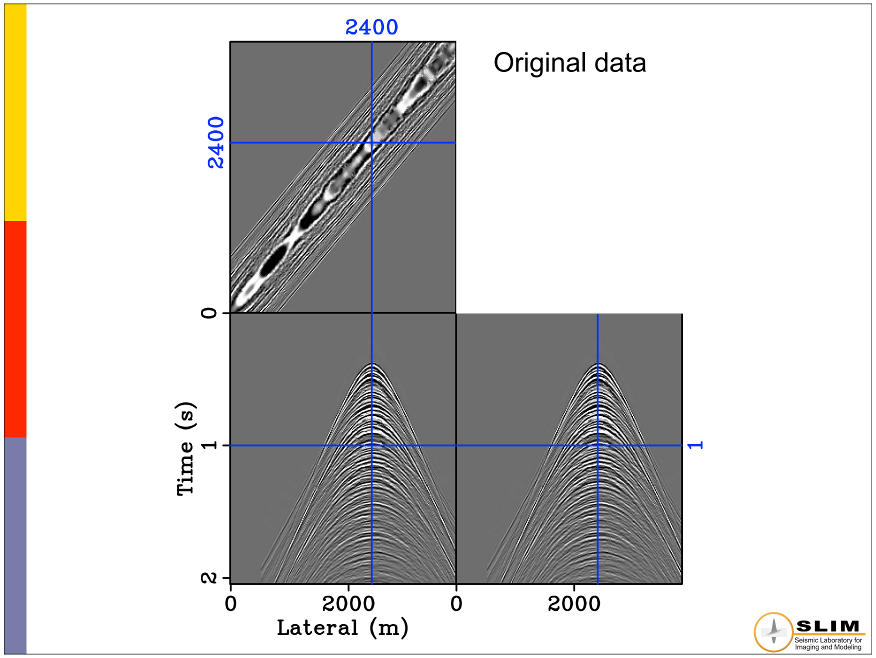
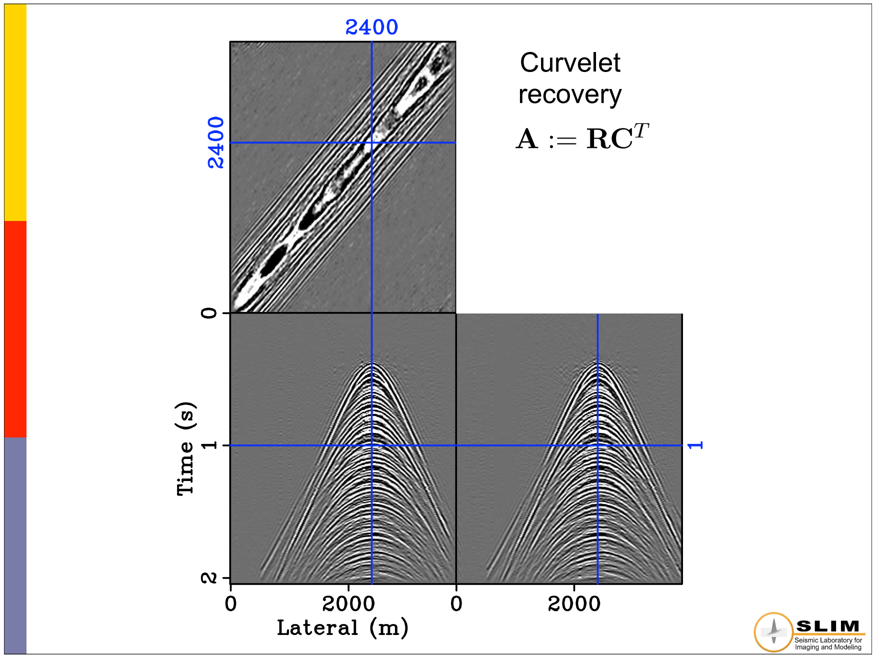
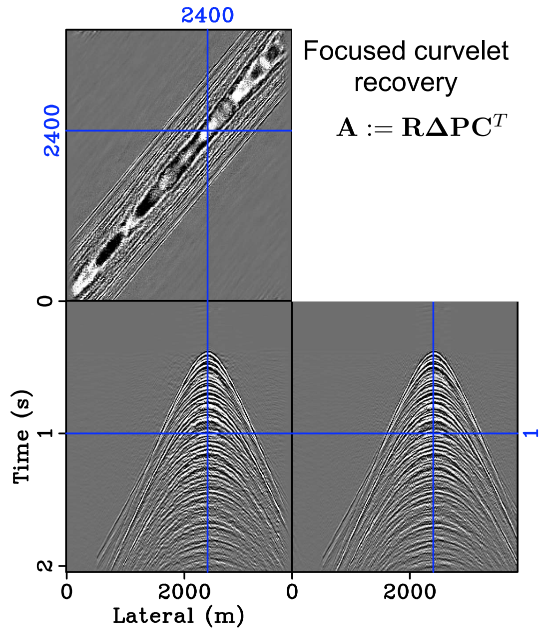

80 % missing
2400
2400
0
1
2
Time (s)
0
2000
0
2000
Lateral (m)
1
SLIM
Seismic Laboratory for Imaging and Modeling

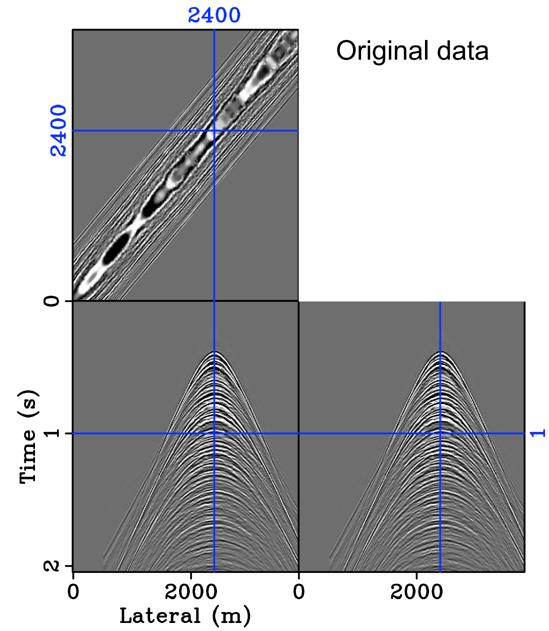

Original data
2400
2400
0
1
2
Time (s)
0
2000
0
2000
Lateral (m)
1
SLIM
Seismic Laboratory for Imaging and Modeling


Curvelet recovery
$\mathbf{A} := \mathbf{R}\mathbf{C}^T$
2400
2400
0
1
2
Time (s)
0
2000
0
2000
Lateral (m)
1
SLIM
Seismic Laboratory for Imaging and Modeling

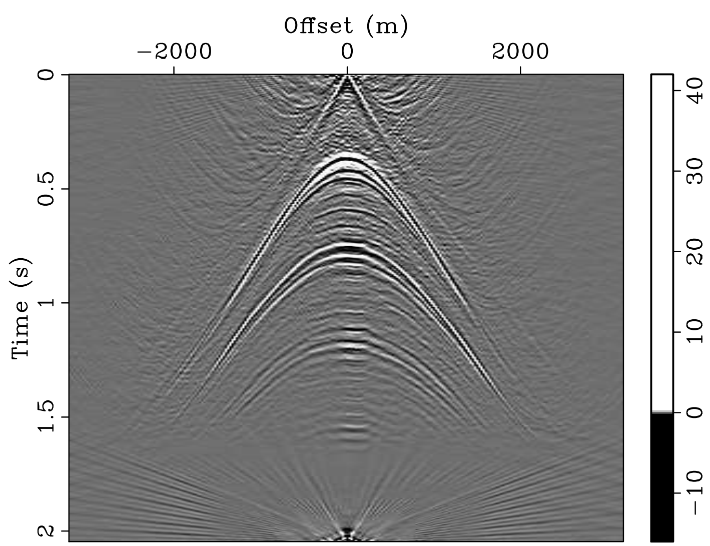


Focused curvelet recovery

$$\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$$



Original data



Recovered Data in Focal Domain

Observations

Focal transform

- allows for incorporation of a priori information
- is reminiscent of an imaging operator
- works because of addition compression and incoherence
- leads to an improved recovery

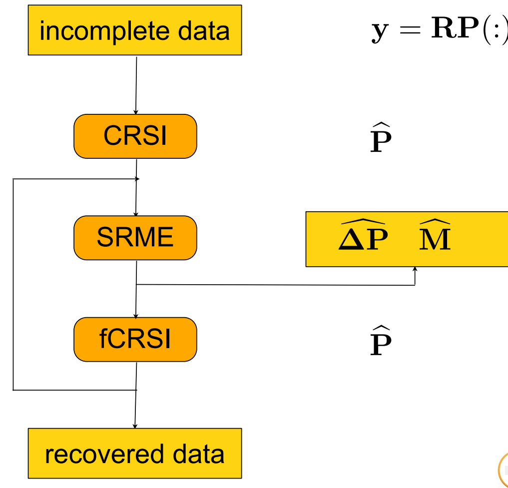
Outlook

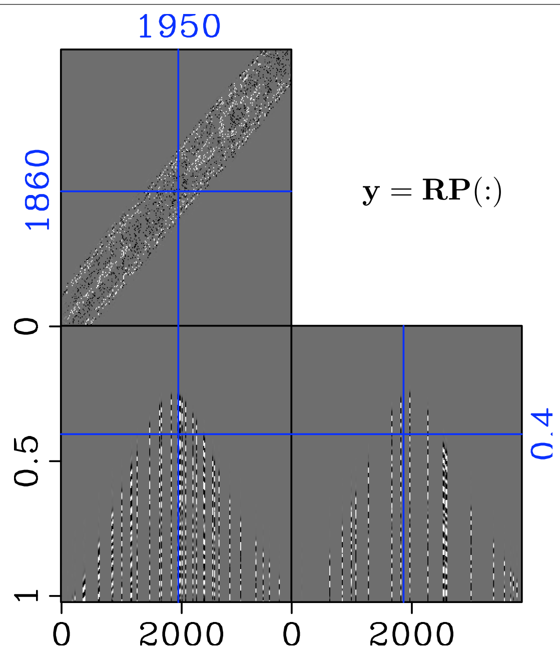
- Restriction corresponds to a compression of the operator
- Opens the way to migration-based recovery
- or a more “blue sky” approach of compressive wavefield extrapolation

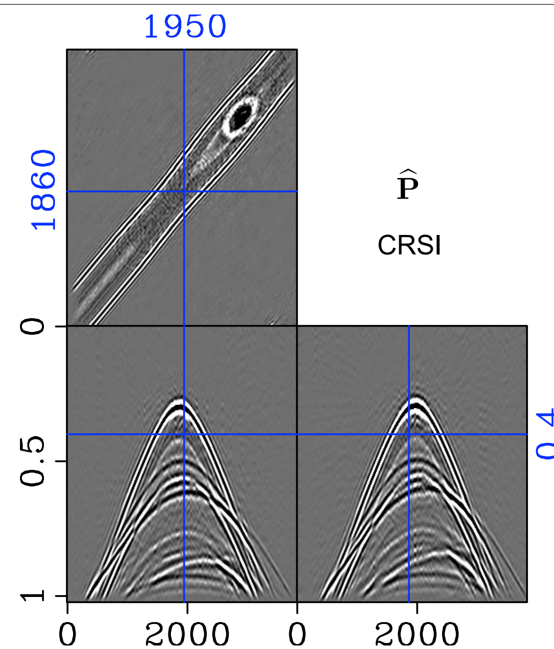
Nonlinear primary-multiple prediction

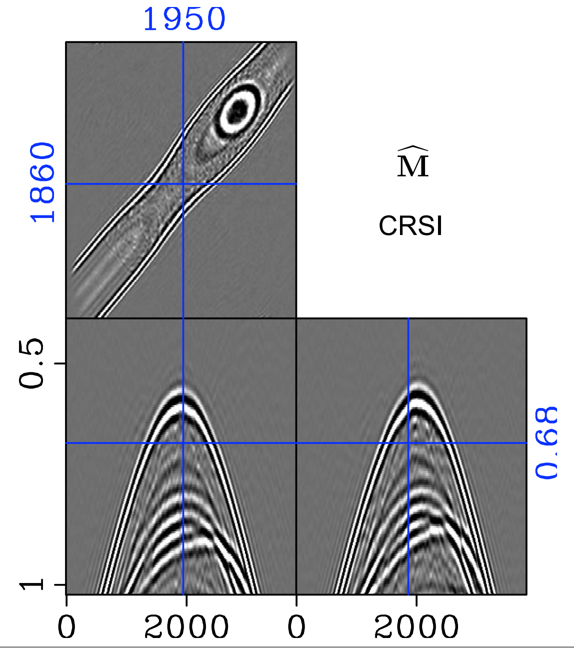
joint work with Deli Wang (visitor from Jilin university) and Eric Verschuur

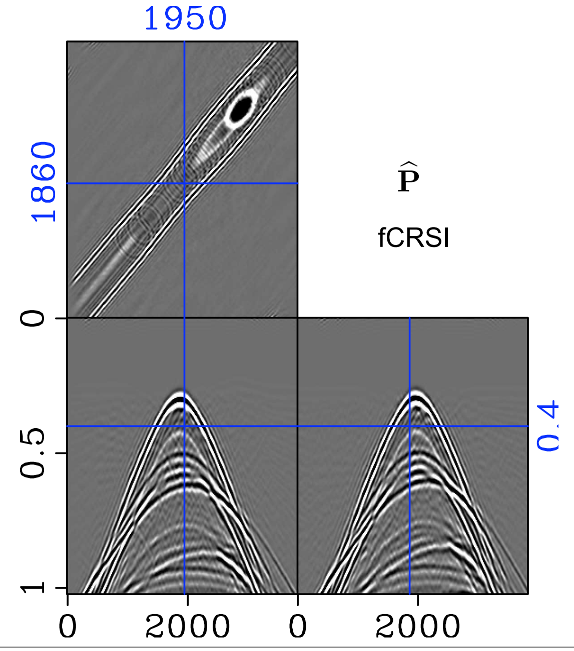
Multiple prediction with fCRSI

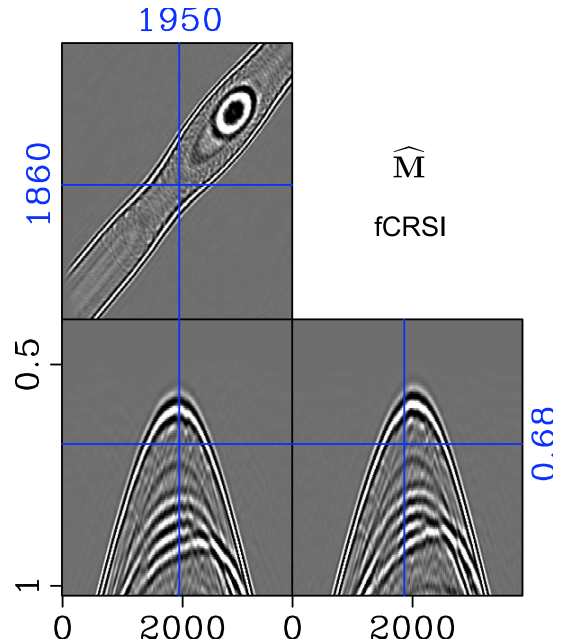




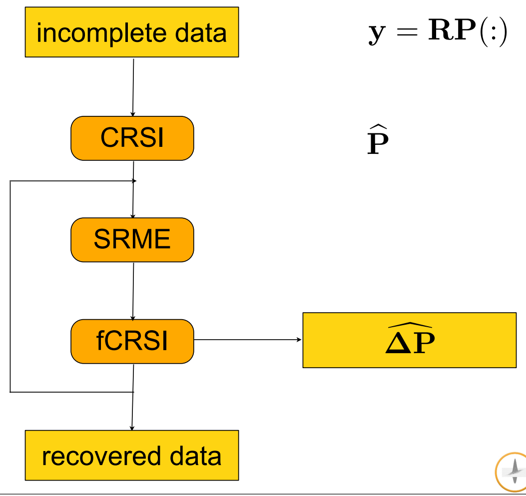








Primary prediction with fCRSI



$\mathbf{y} = \mathbf{RP}(:)$

$\widehat{\mathbf{P}}$

Curvelet-based Focal transform

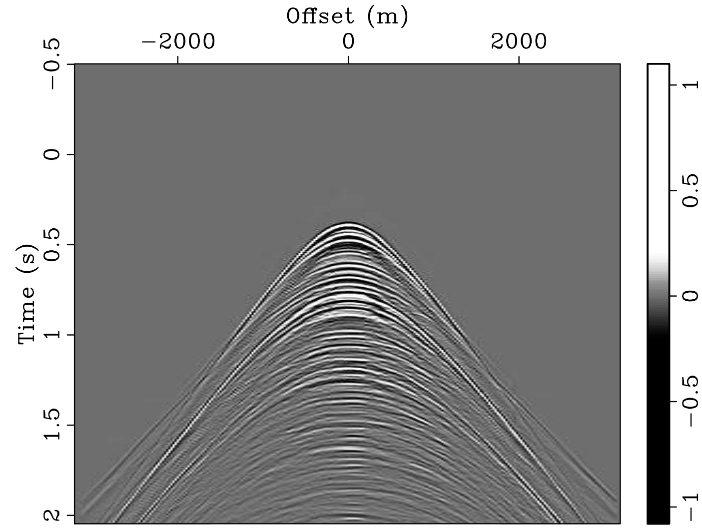
Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

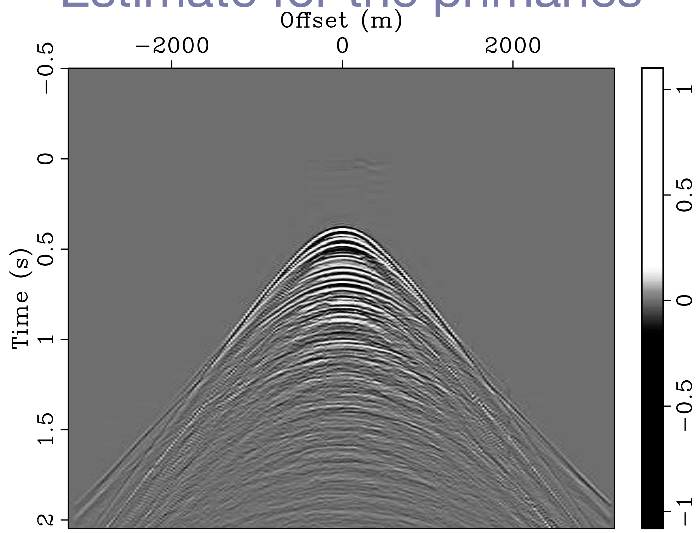
$$\begin{aligned} \mathbf{A} &:= \mathbf{\Delta P C}^T \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{focused data.} \end{aligned}$$

Total data



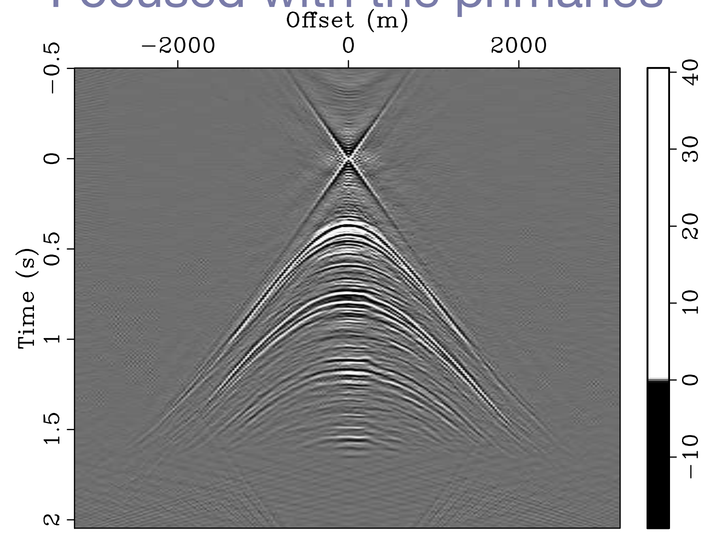
$\widehat{\mathbf{P}}$

Estimate for the primaries



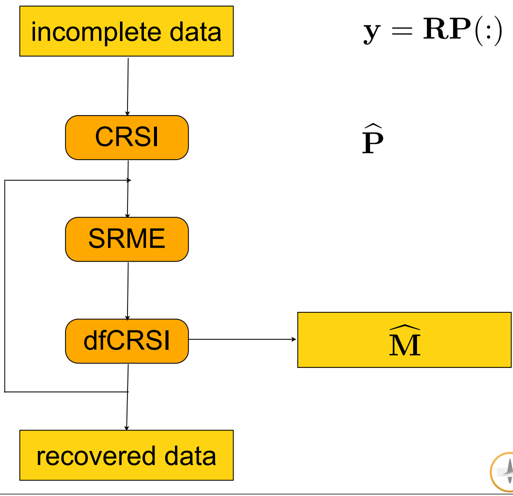
$\widehat{\mathbf{\Delta P}}$

Focused with the primaries



$\tilde{\mathbf{f}}$

Multiple prediction with dfCRSI



$\mathbf{y} = \mathbf{RP}(:)$

$\widehat{\mathbf{P}}$

Curvelet-based deFocal transform

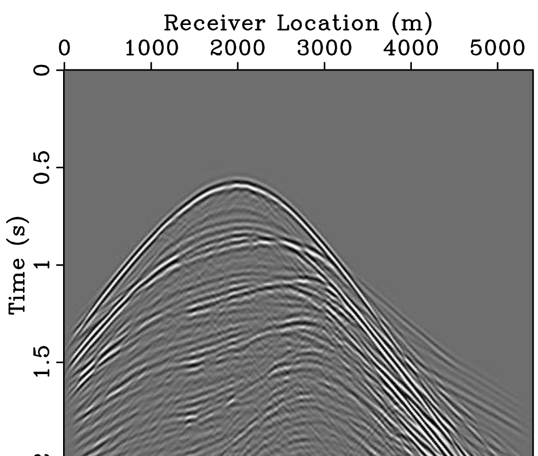
Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{Ax} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

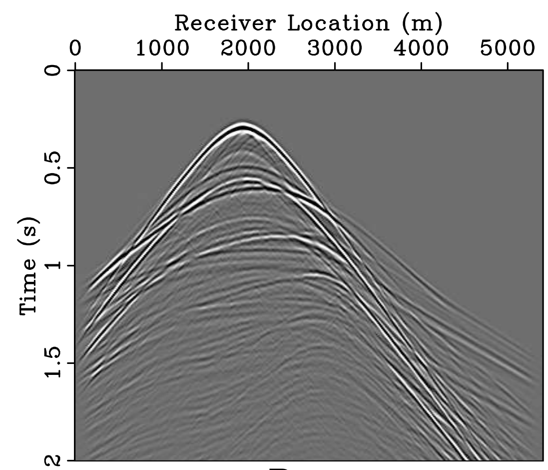
$$\begin{aligned} \mathbf{A} &:= \boldsymbol{\Delta}\mathbf{P}^T\mathbf{C}^T \\ \mathbf{S} &:= \mathbf{C} \\ \mathbf{y} &= \mathbf{P}(:) \\ \mathbf{P} &= \text{total data} \\ \tilde{\mathbf{f}} &= \text{defocussed data.} \end{aligned}$$

SRME predicted multiples



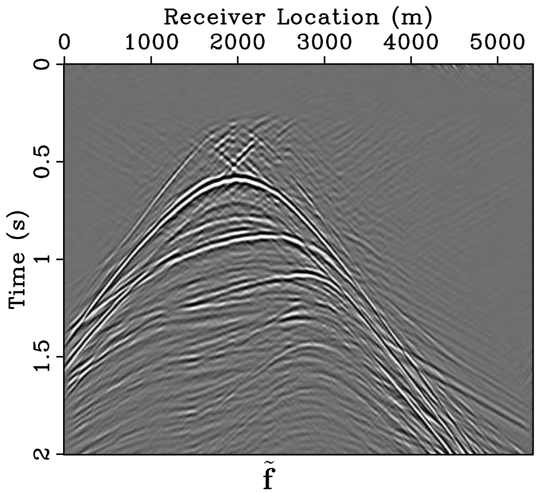
$\widehat{\mathbf{P}} = \mathbf{PP}$

Original data

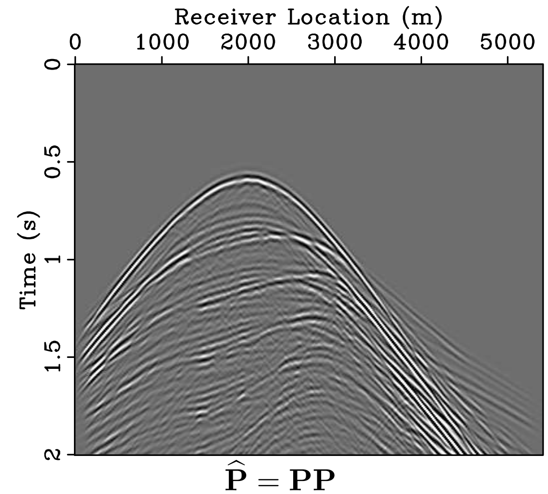


$\mathbf{P}$

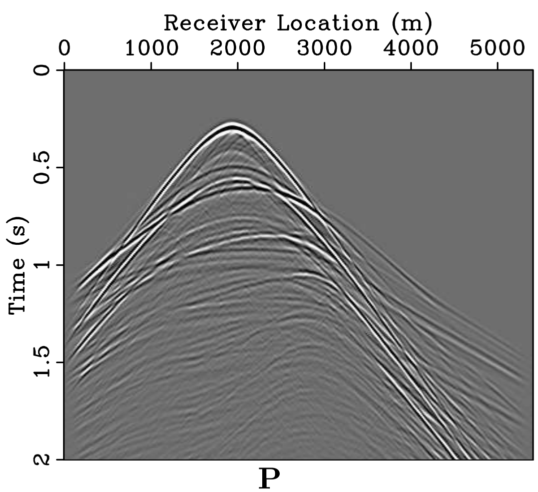
Multiple estimate by dfCRSI



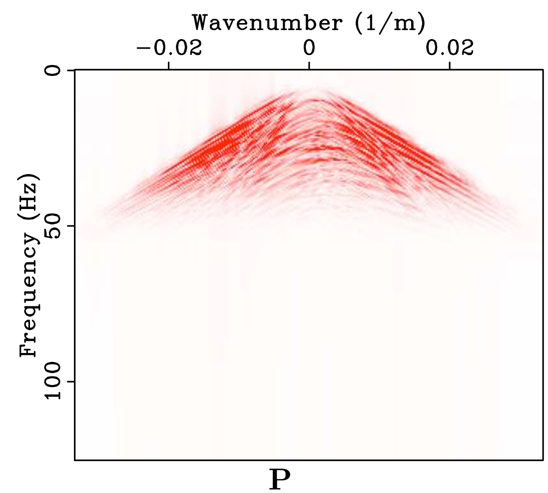
SRME predicted multiples



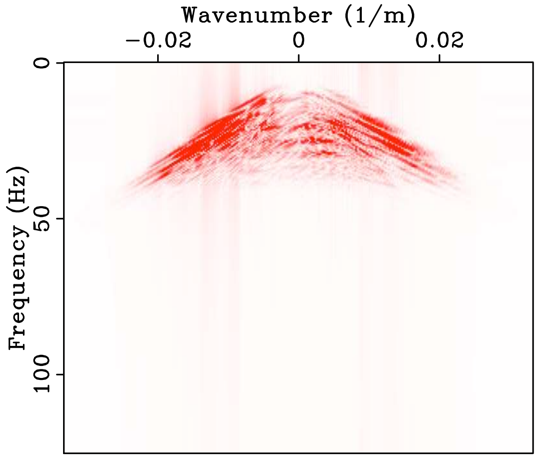
Original data



Original data

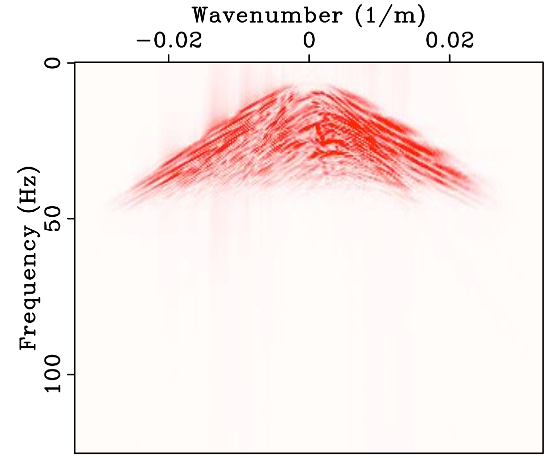


SRME predicted multiples



$\widehat{\mathbf{P}} = \mathbf{P}\mathbf{P}$

Multiple estimate by dfCRSI



$\tilde{\mathbf{f}}$

Observations

Daisy-chaining CRSI with fCRSI leads to an improved recovery.

Focused multiples are “deconvolved” w.r.t. source signature and directivity pattern.

Multiple prediction obtained by inverting the “correlation” with the primaries operator

- have a broader frequency band & better amplitudes
- suffer from cross terms due to remnant multiples in the primary operator

Extensions

part of SINBAD II

CRSI

Quantitatively determine

- how random is random enough
- acquisition strategies that favor successful recovery
- predictions for accuracy for the recovery error given an acquisitions
- design of acquisition strategies given a desired resolution and recovery accuracy

Impose reciprocity on the recovered matrix

- exploit symmetry relations
- use

$$\mathbf{P} = \frac{1}{2}(\mathbf{P} + \mathbf{P}^T) + \frac{1}{2}(\mathbf{P} - \mathbf{P}^T)$$

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Motivation

Focal transform is like a migration towards the sources.

The same as imaging a point reflector with a directivity pattern.

Use the redundancy in prestack imaged data.

Migration and differential semblance imaging conditions that penalize

- defocusing
- non-smoothness in angle/ray-parameter

Combine these two focusing in the curvelet domain => multiscale & multidirectional focussing …

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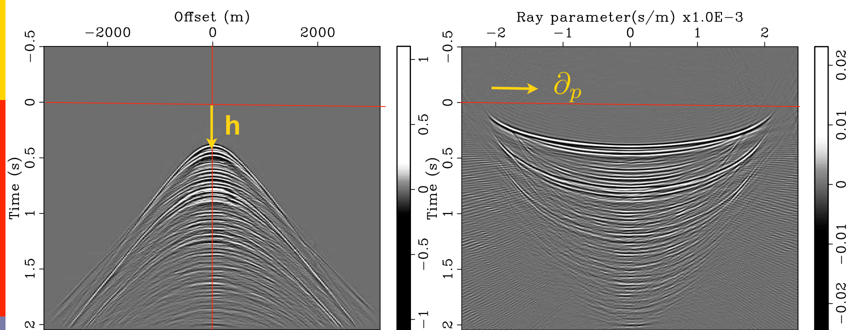
Example unfocused



Focusing in phase space

- penalize distance away from focal point (0,0)
- penalize smoothness along line (0,p)

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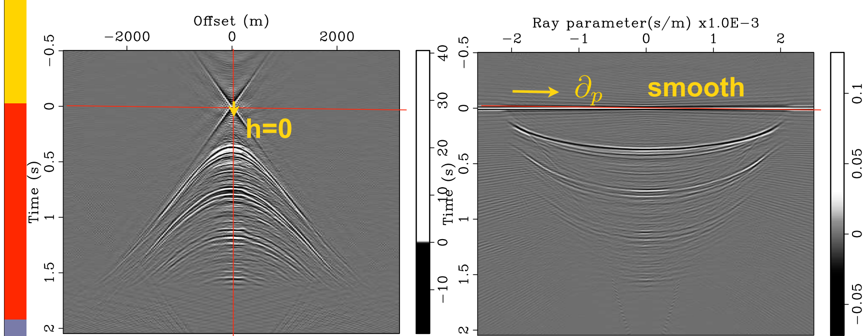
Example focused



Minimize

$$J_h(\tilde{\mathbf{f}}) = \|\mathbf{h}\tilde{\mathbf{f}}\|_2 \quad \text{and} \quad J_\theta(\widehat{\tilde{\mathbf{f}}}) = \|\partial_p \widehat{\tilde{\mathbf{f}}}\|_2$$

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Focusing

Curvelet domain is parameterized by (x, y, θ).

Joint focusing in ***space*** and ***angle*** can be implemented in the *curvelet* domain.

$$\mathbf{P}: \begin{cases} \tilde{\mathbf{x}} = \min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \tilde{\mathbf{f}} = \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T\tilde{\mathbf{x}} \end{cases}$$

where **W** penalizes ***defocusing ...***

Alternative to curvelet domain matched filtering.

Applications

- primary-multiple separation in the focal or inverse data domain (multiples do not focus ...)
- focusing as part of imaging

Focal transform

- allows for incorporation of a priori information
- leads to an improved recovery

DeFocal transform

- enhances frequency content of the multiples
- contains Xterms related to remnant multiples

Focusing

- combined with wavefield extrapolation will enhance the recovery
- will lead to an alternative non-adaptive primary-multiple separation scheme

Outlook

- Restriction <=> compression of the operators
- Opens the way to migration-based recovery
- or a more "blue sky" approach of compressive wavefield extrapolation