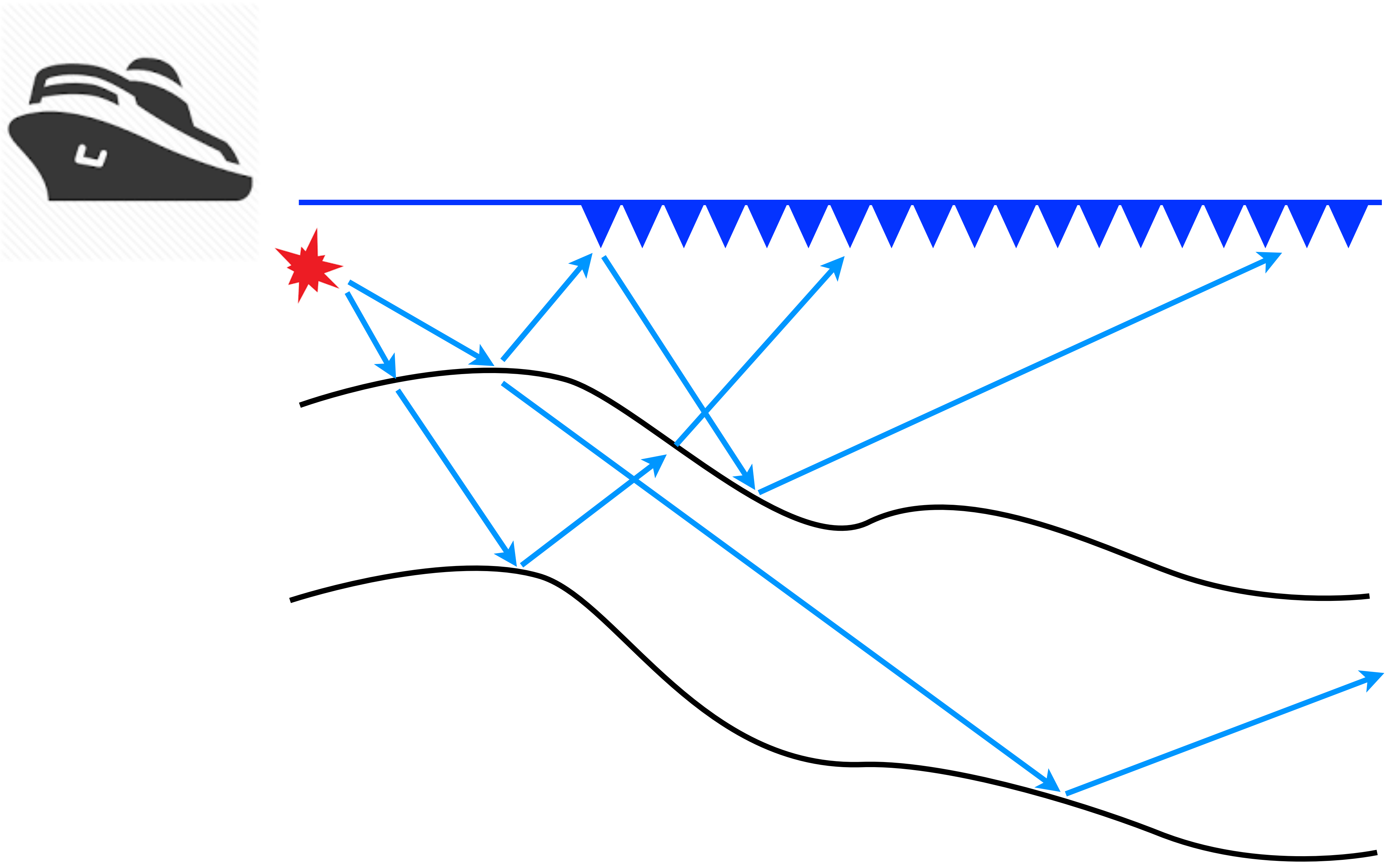
Fast imaging with surface-related **multiples**

Ning Tu supervised by Prof. Felix J. Herrmann

SLIM
University of British Columbia

Seismic data acquisition and imaging



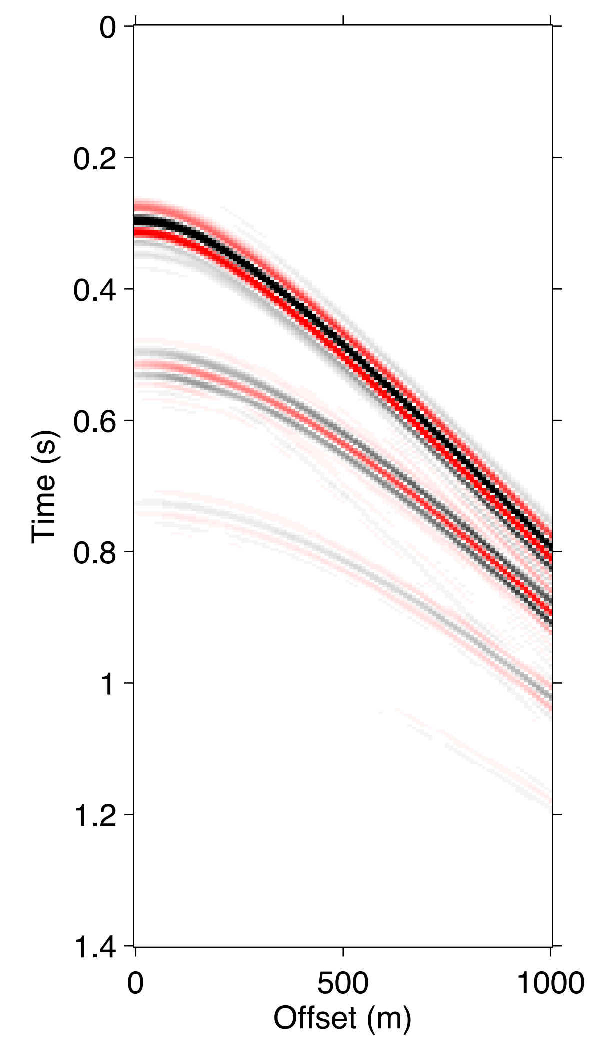
sea surface

ocean bottom

a subsurface reflector

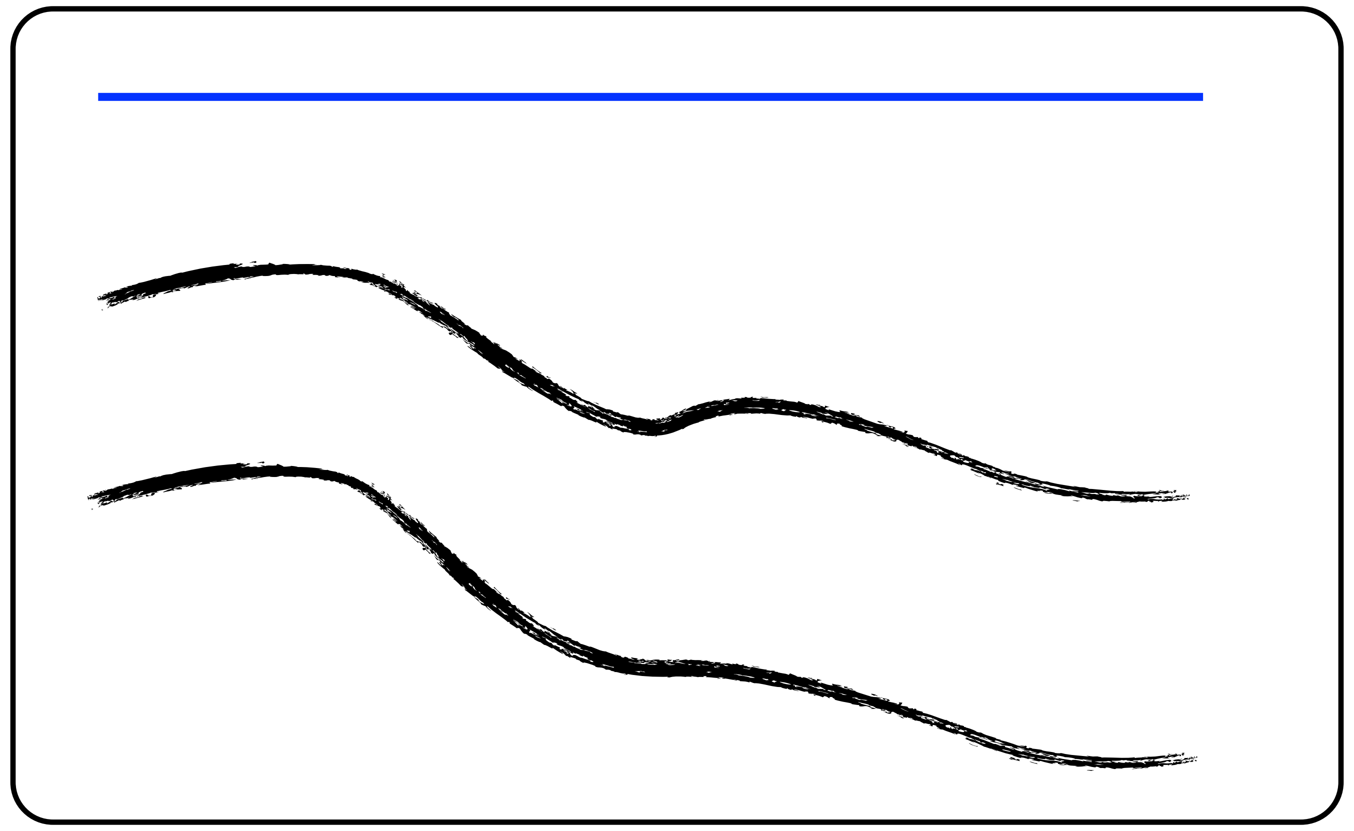
Earth model

acquisition & data processing



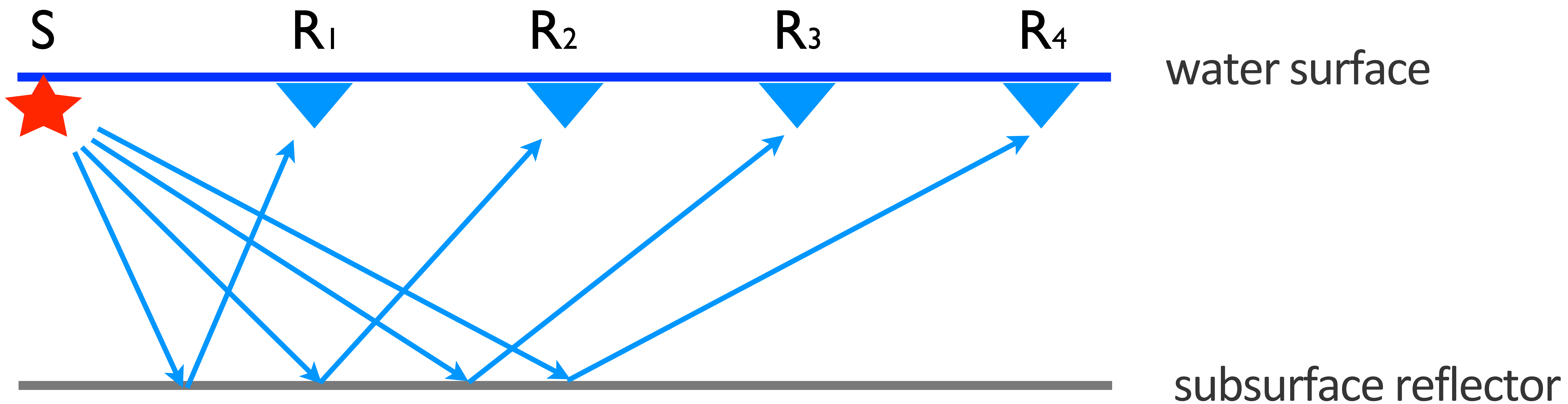
reflection data

imaging



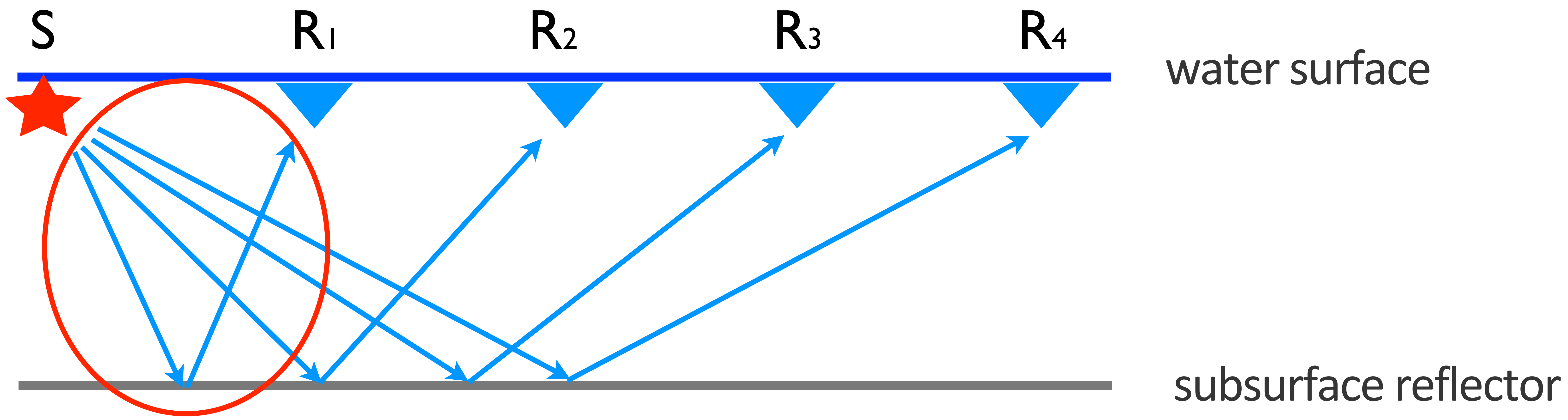
image

Surface-related multiples: the **origin**



The ***primary*** reflections

Surface-related multiples: the **origin**



The ***primary*** reflections

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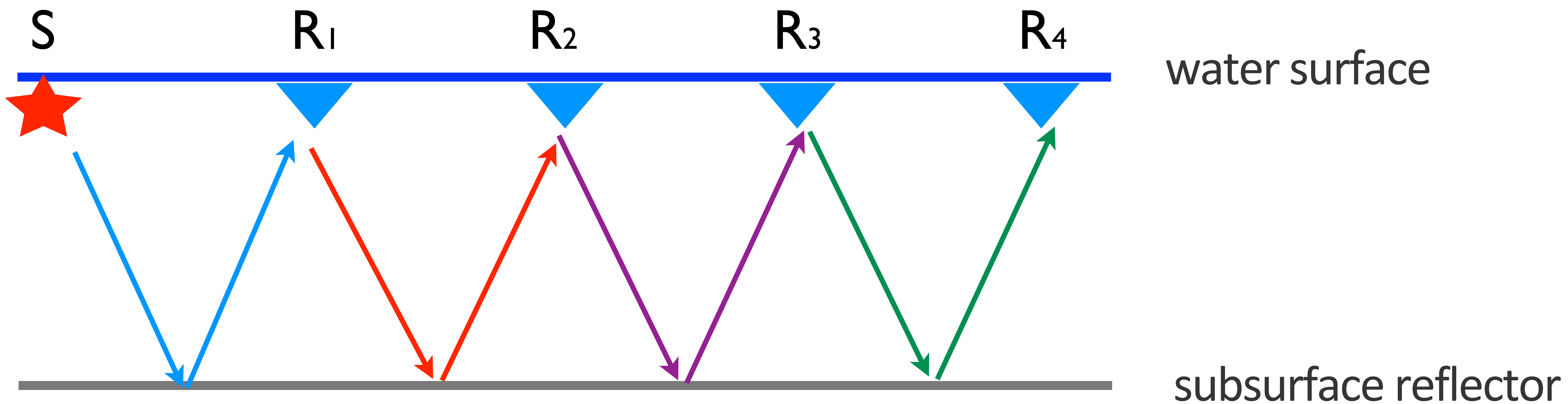
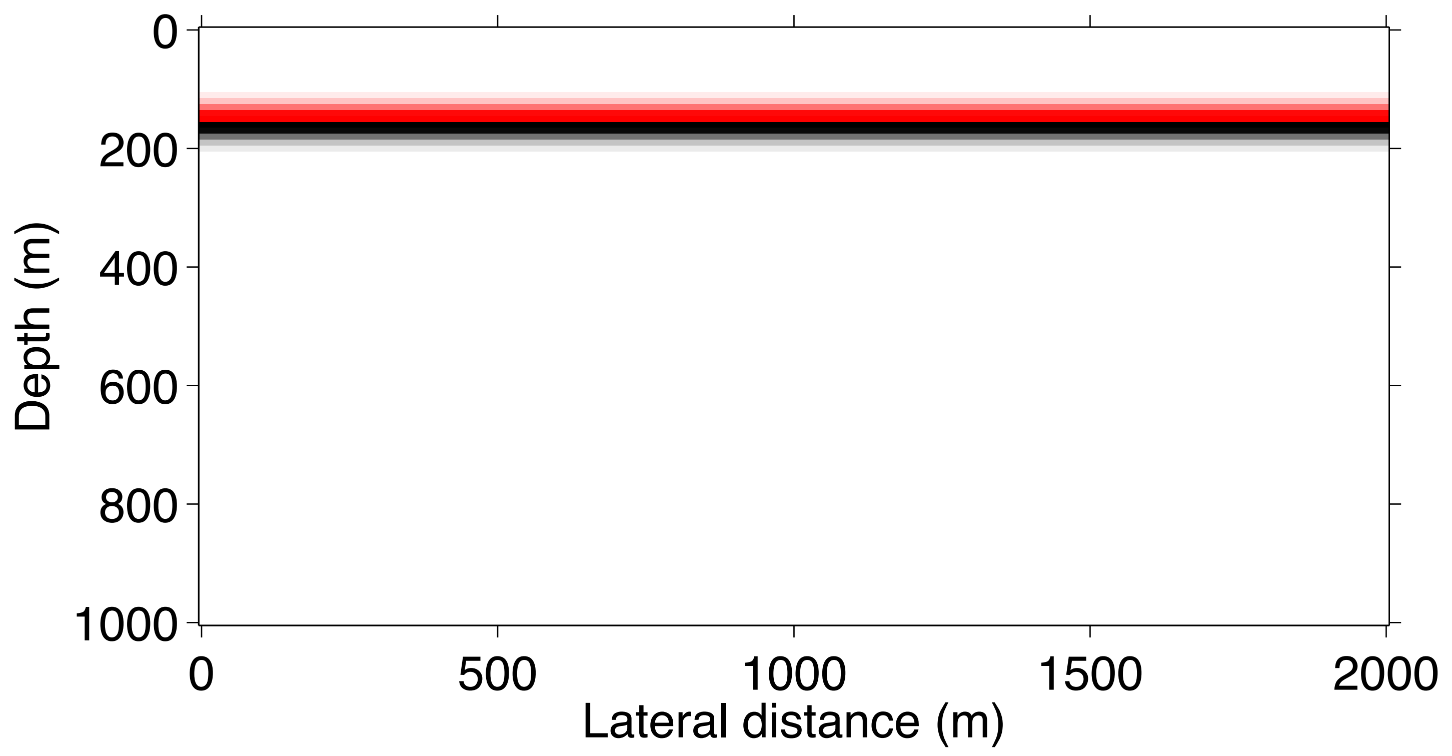


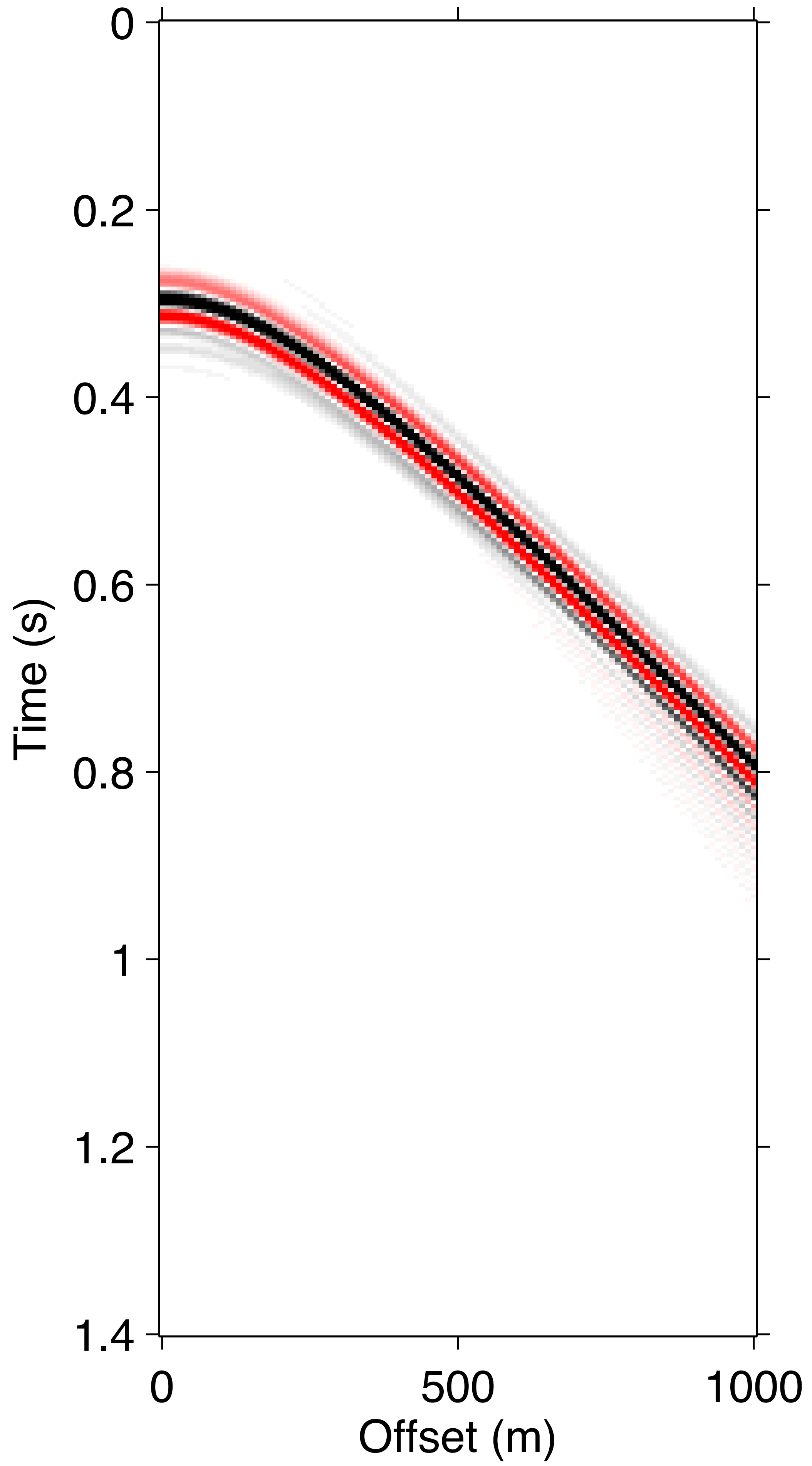
The ***primary*** + its 1st/2nd/3rd-order ***surface-related multiples*** reflections

Surface-related multiples: the **problem**

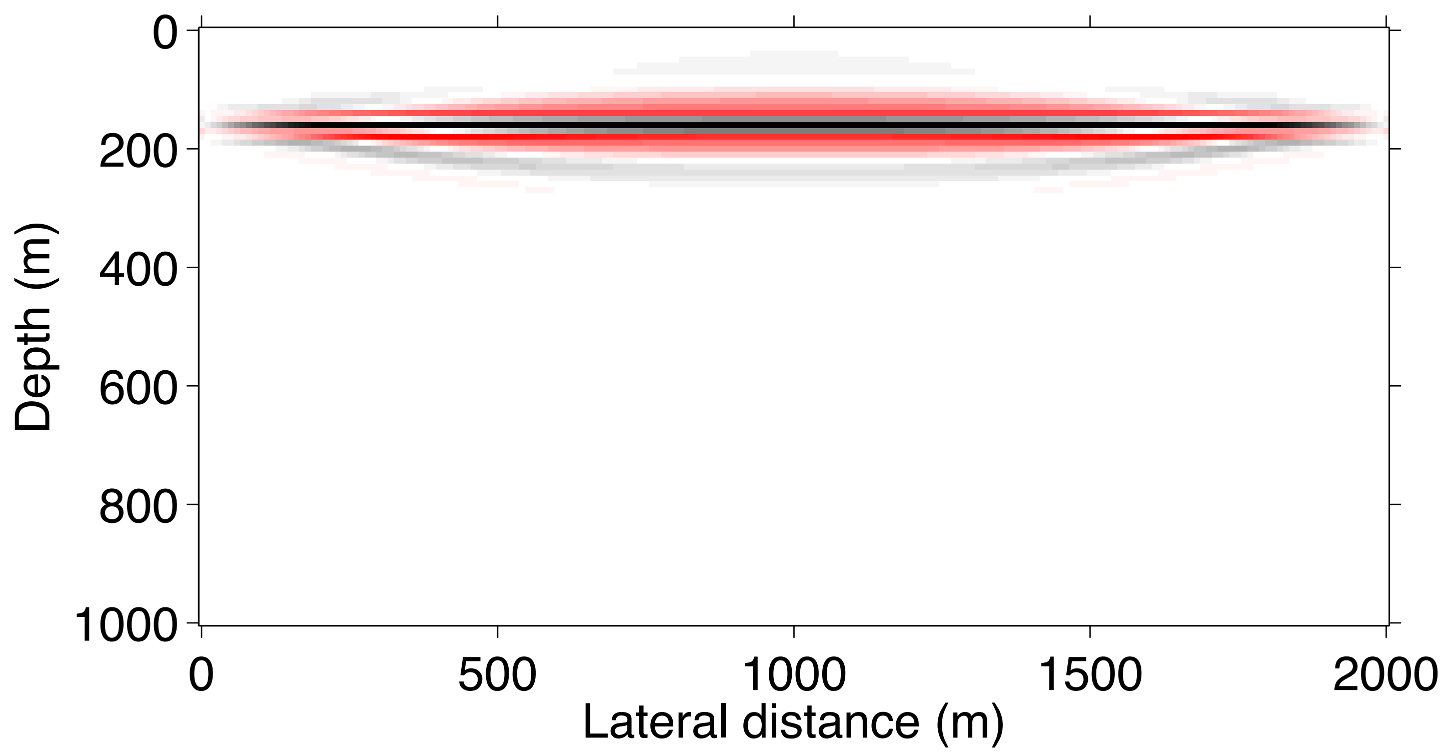


True model

"primary-only" data acquisition →

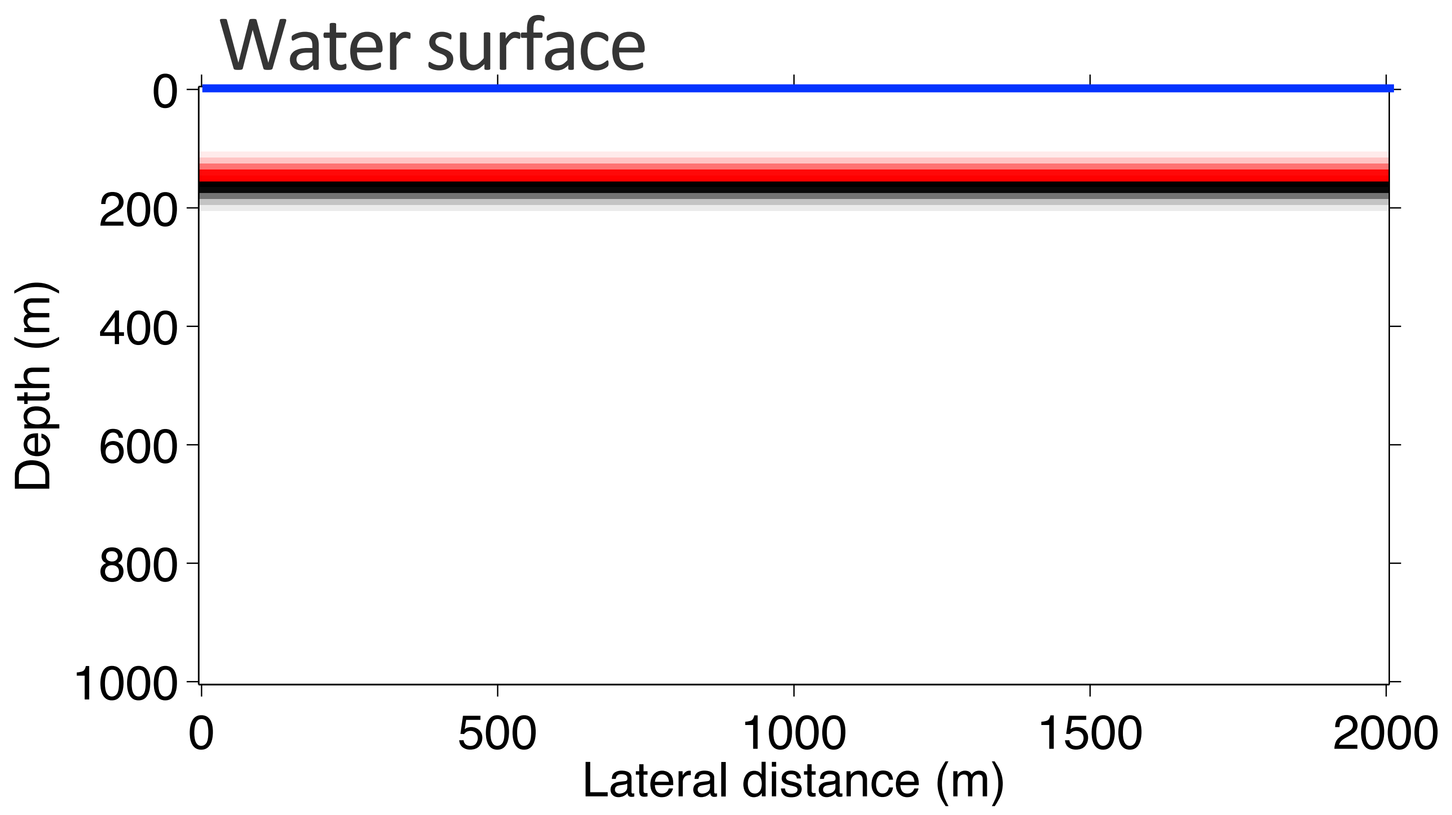


← conventional imaging



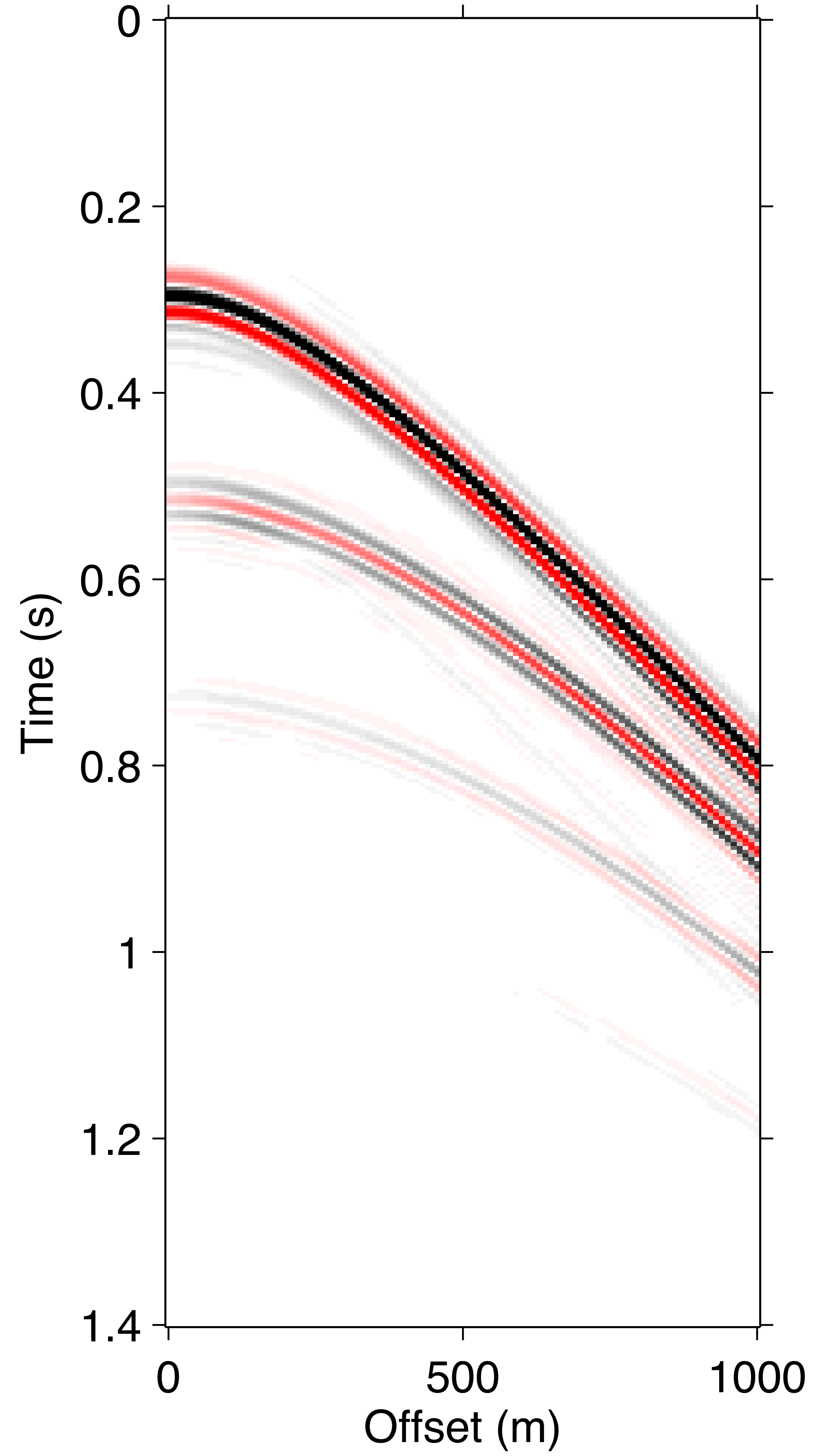
Image

Surface-related multiples: the **problem**

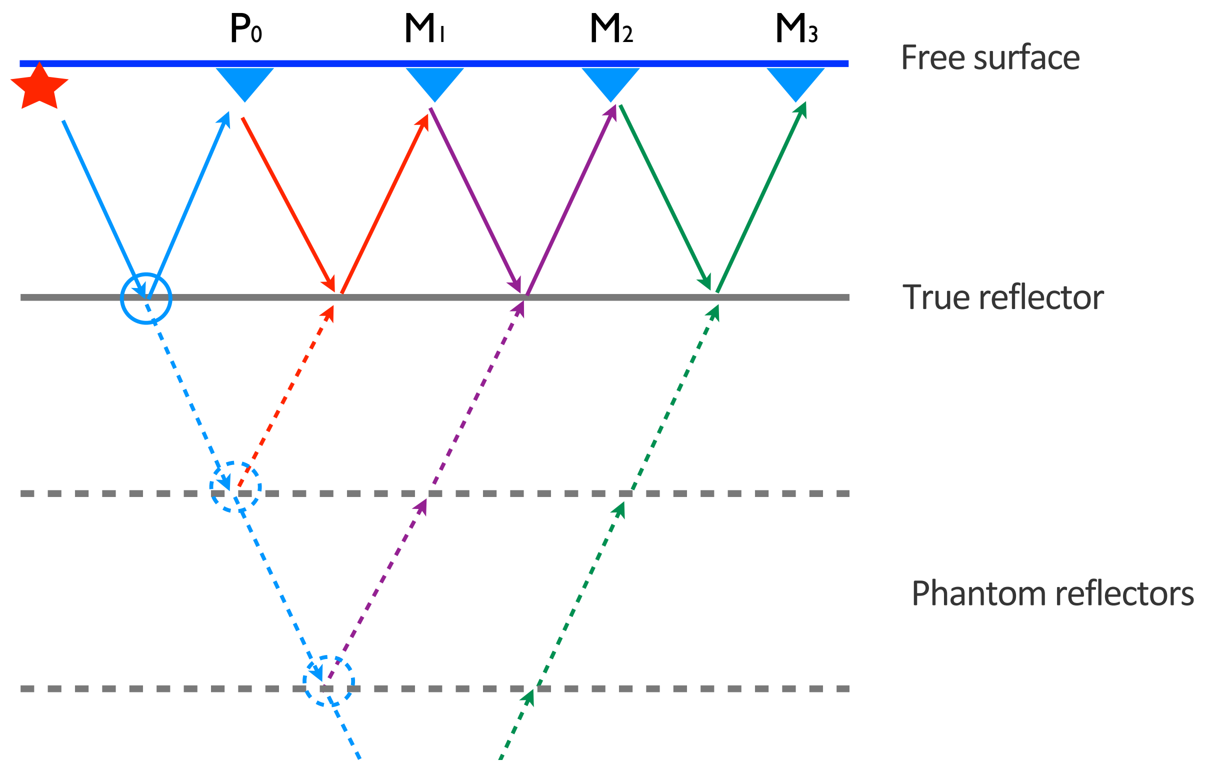


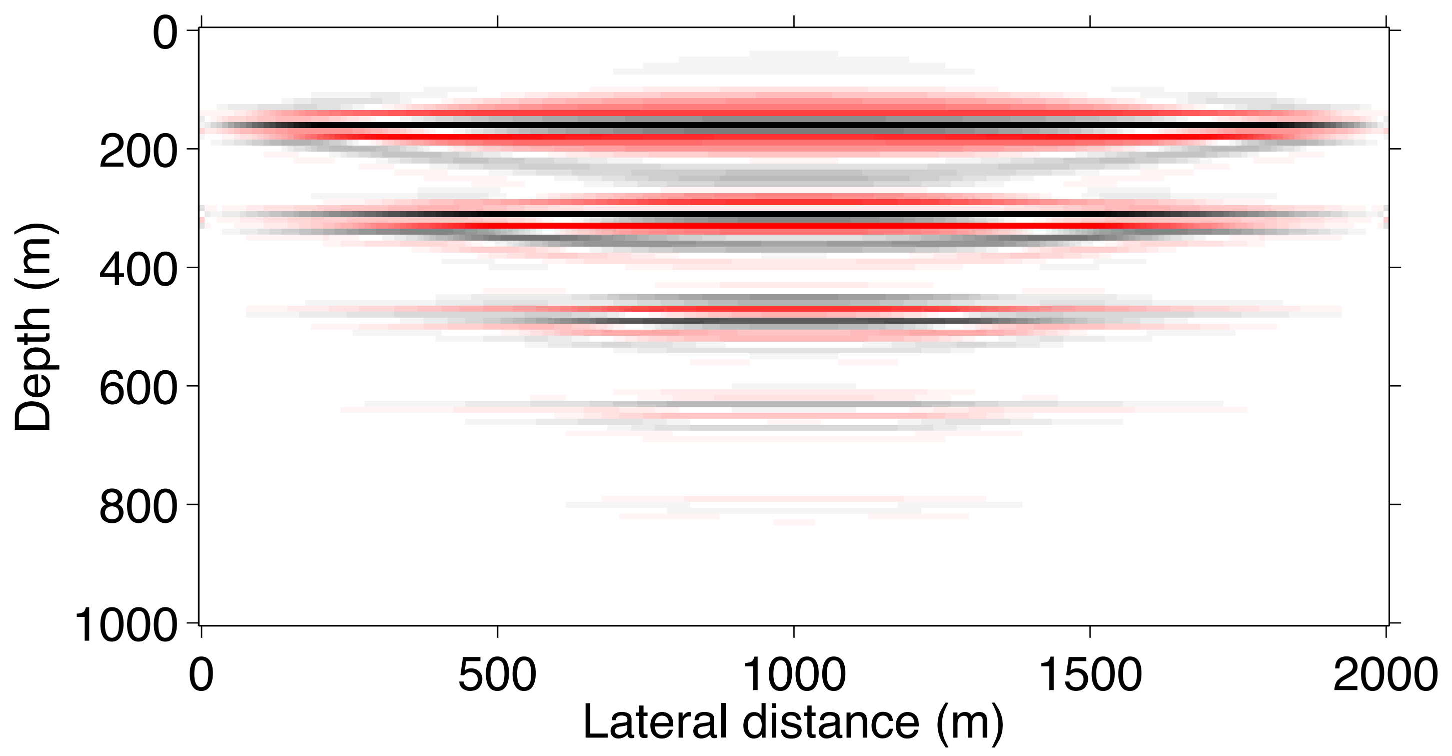
True model

data acquisition ***with multiples***



conventional imaging

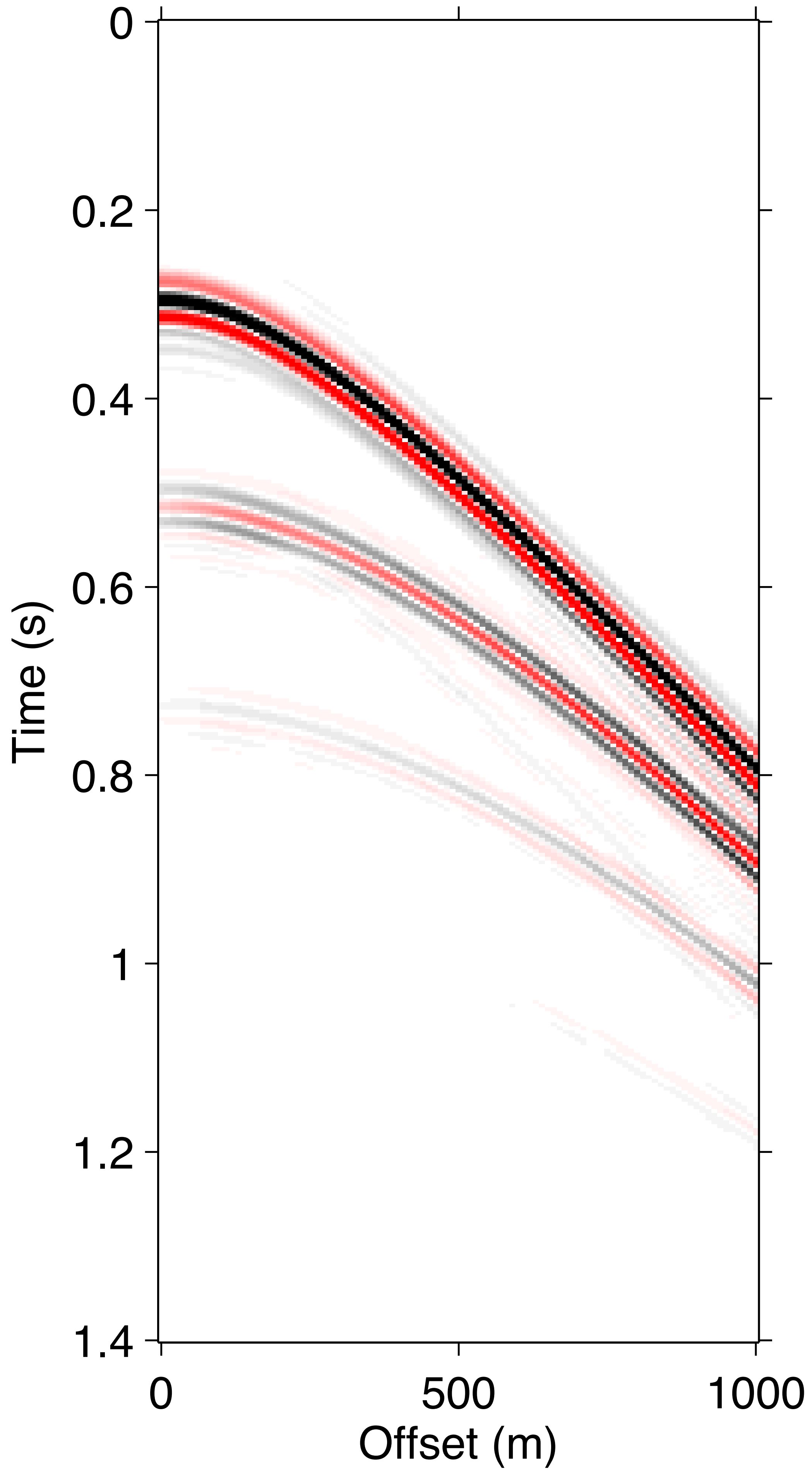


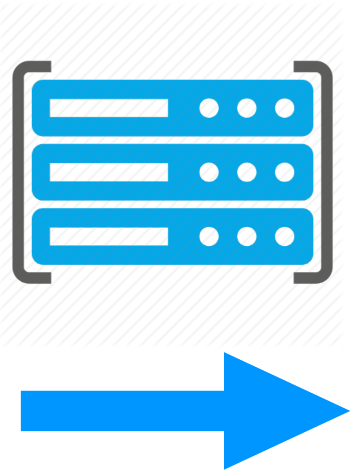


Image

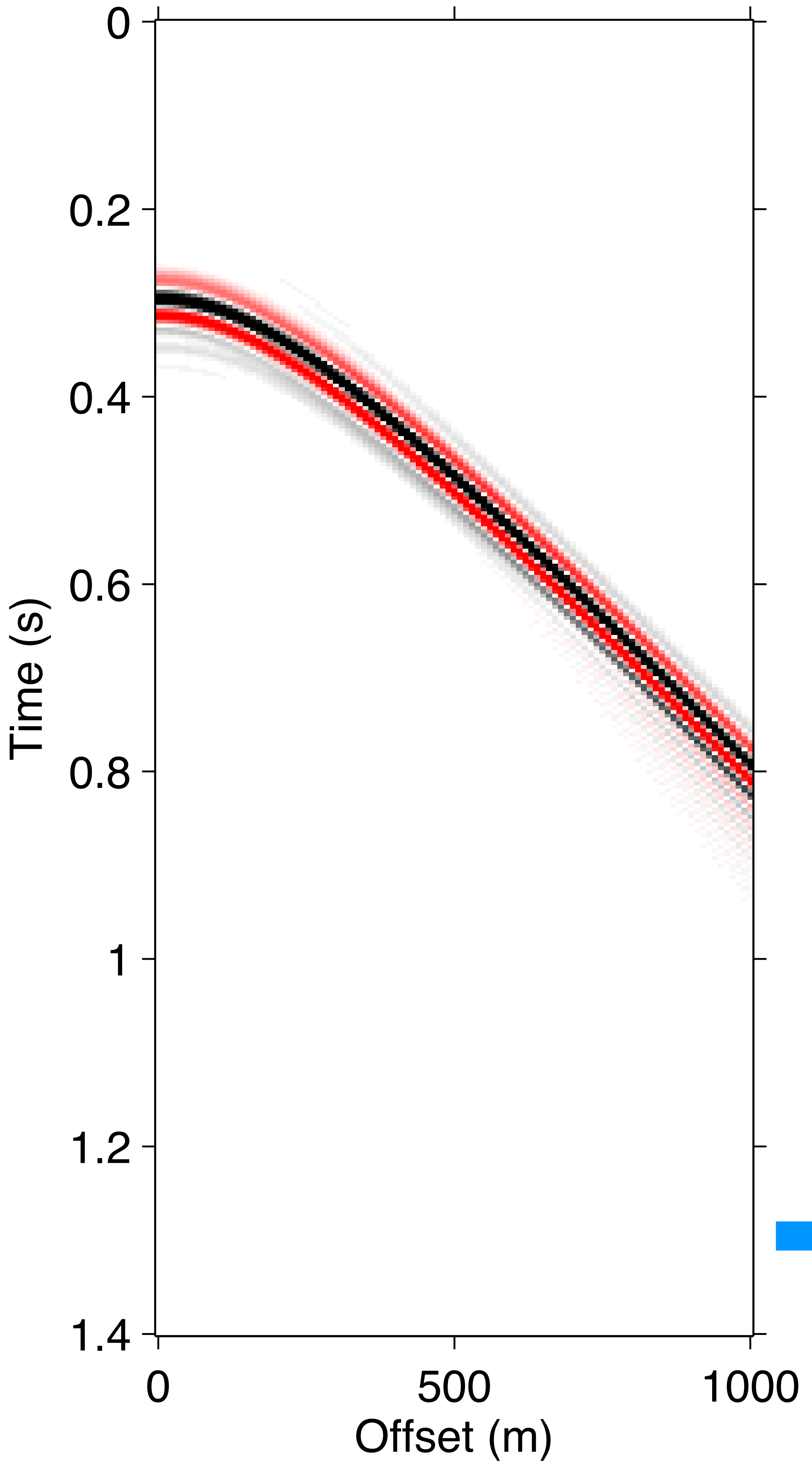
Existing data space solutions - demultiple

[Assumption: dense source/receiver sampling]





$\mathcal{O}(n^3)$



+

$\mathcal{O}(n^3)$

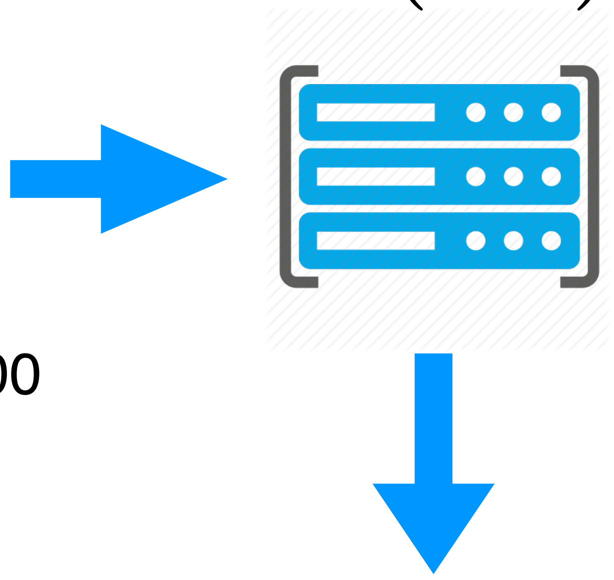
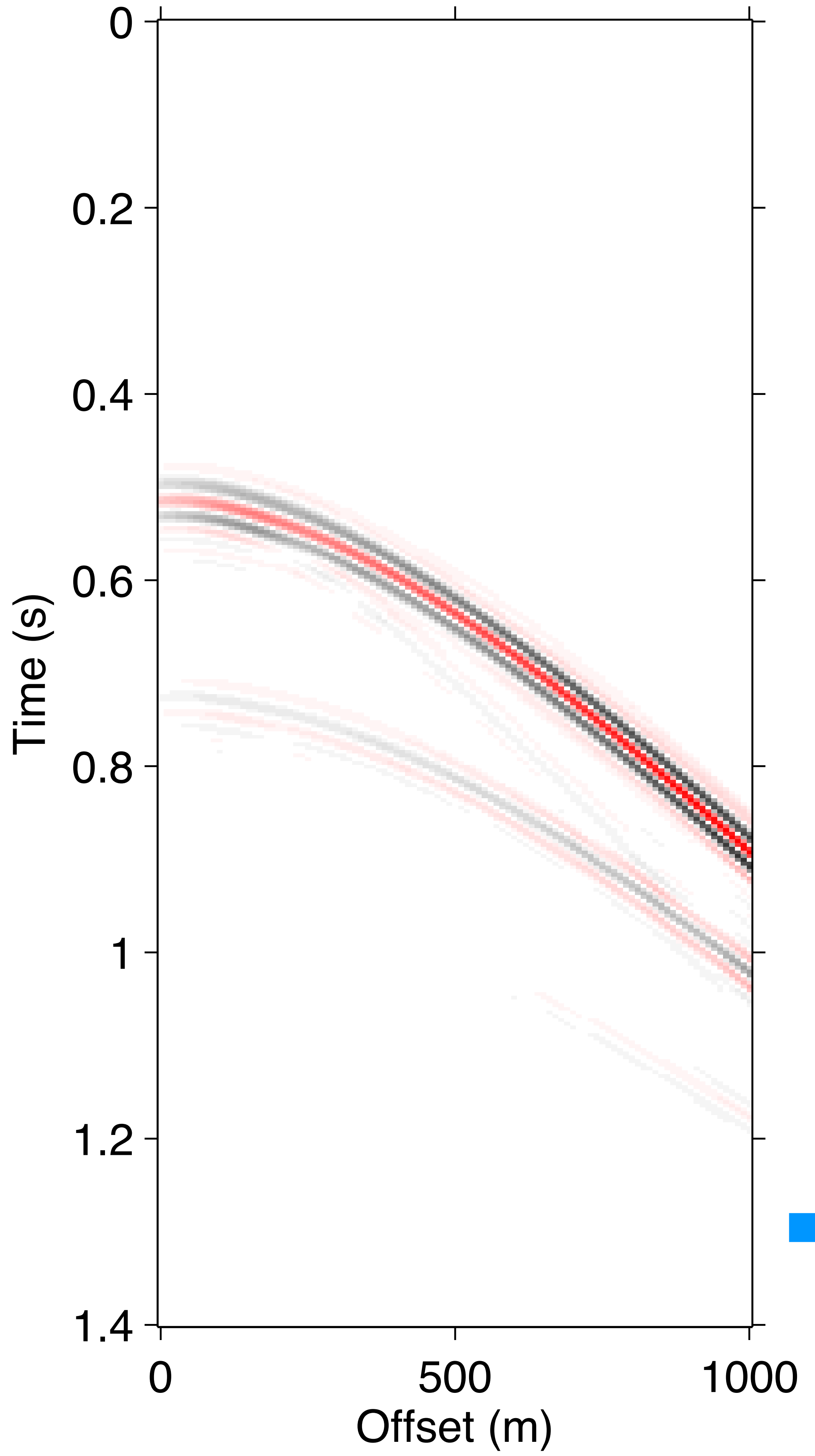
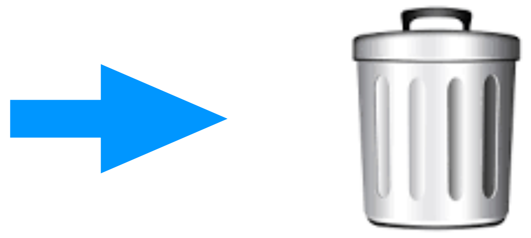
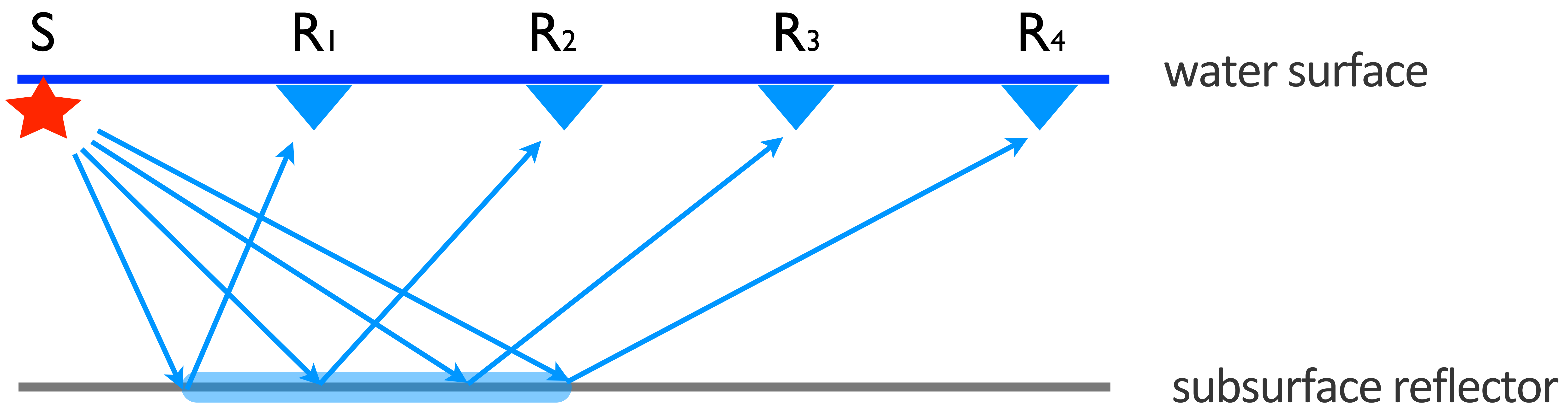


Image of primaries

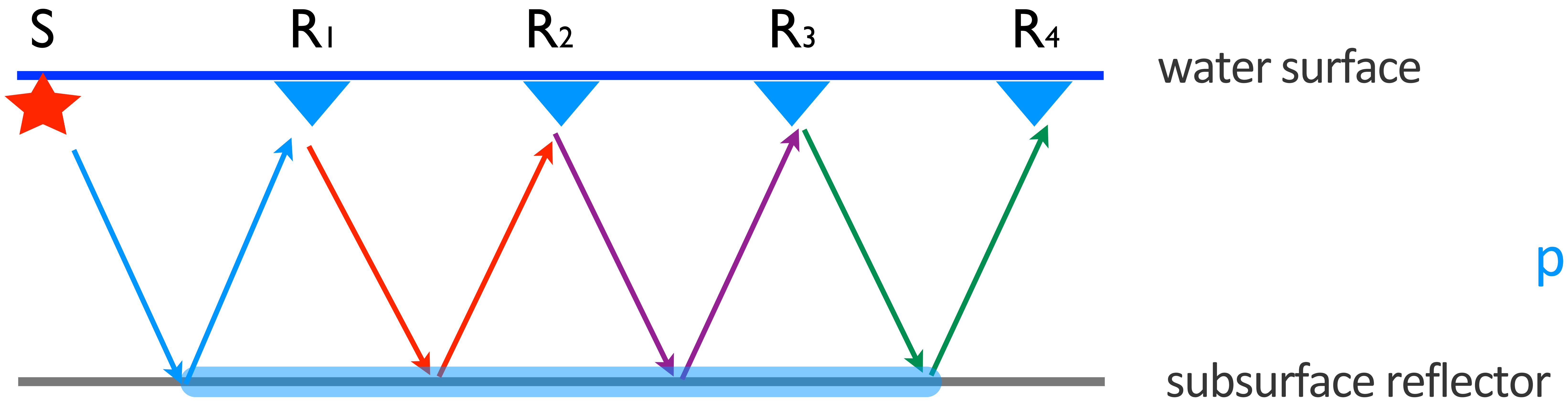




Model space **solutions** - imaging with multiples



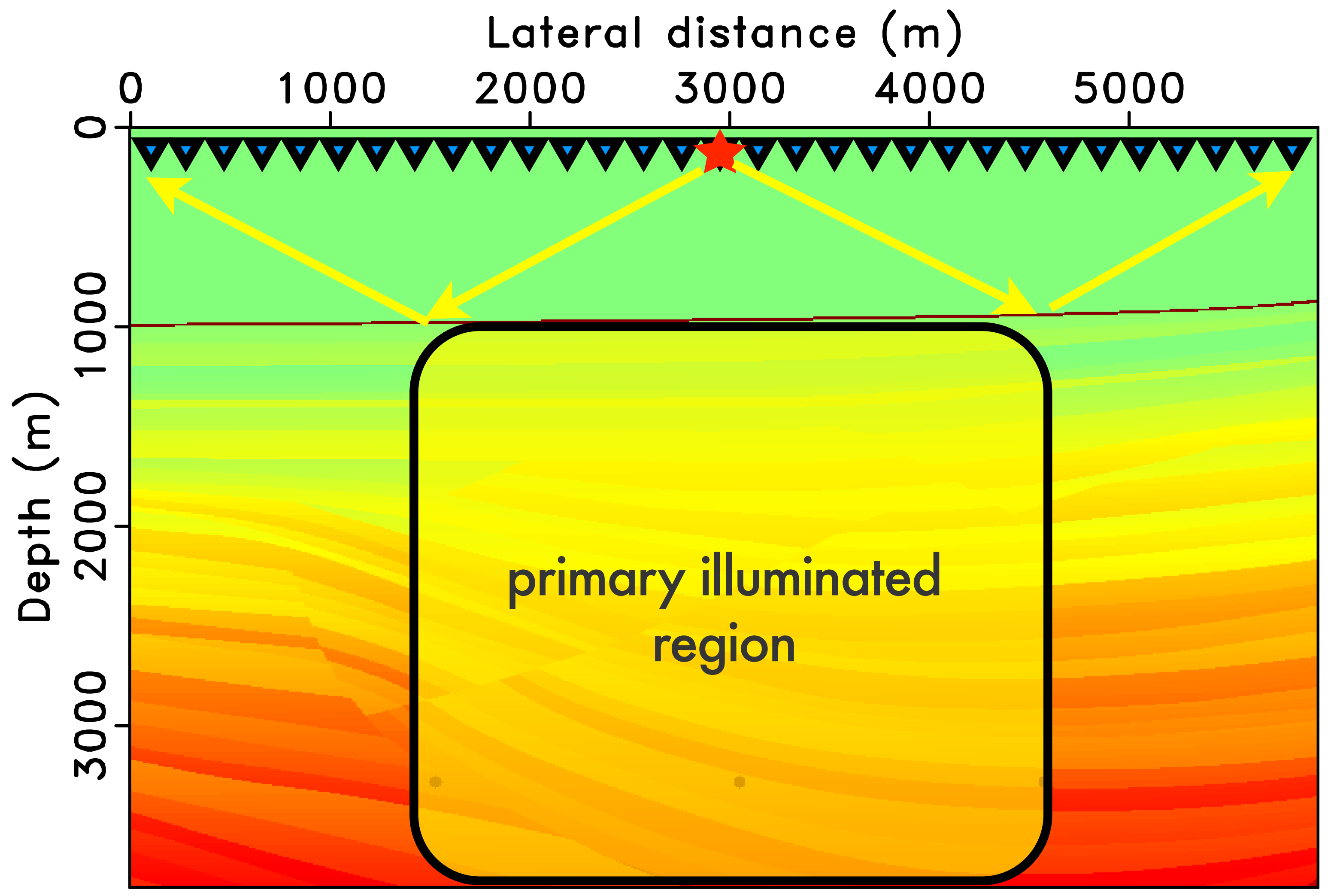
Illumination by
primaries



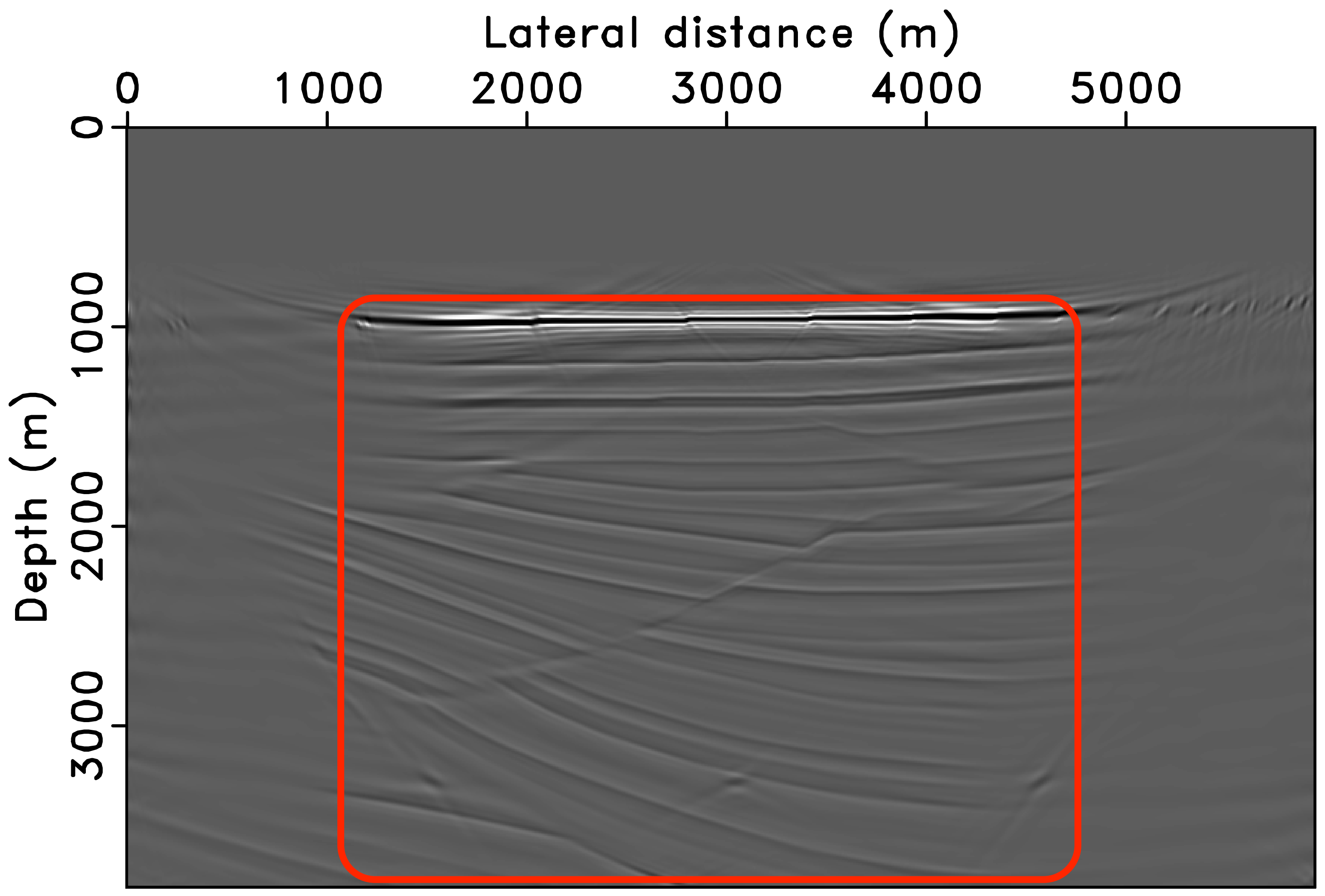
Illumination by
primaries+***multiples***

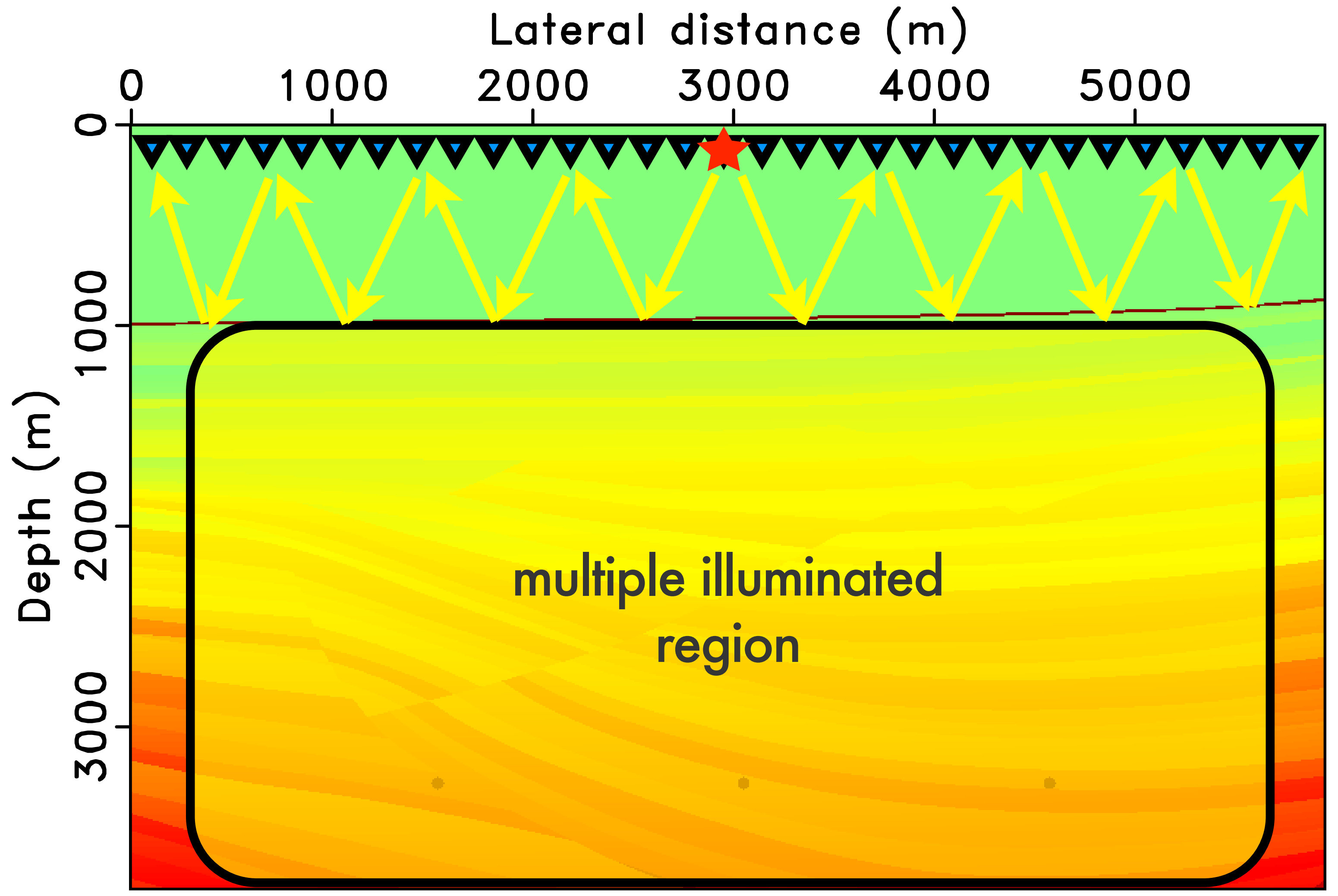
Model space solutions - imaging with multiples

Chapter 6



Numerical example





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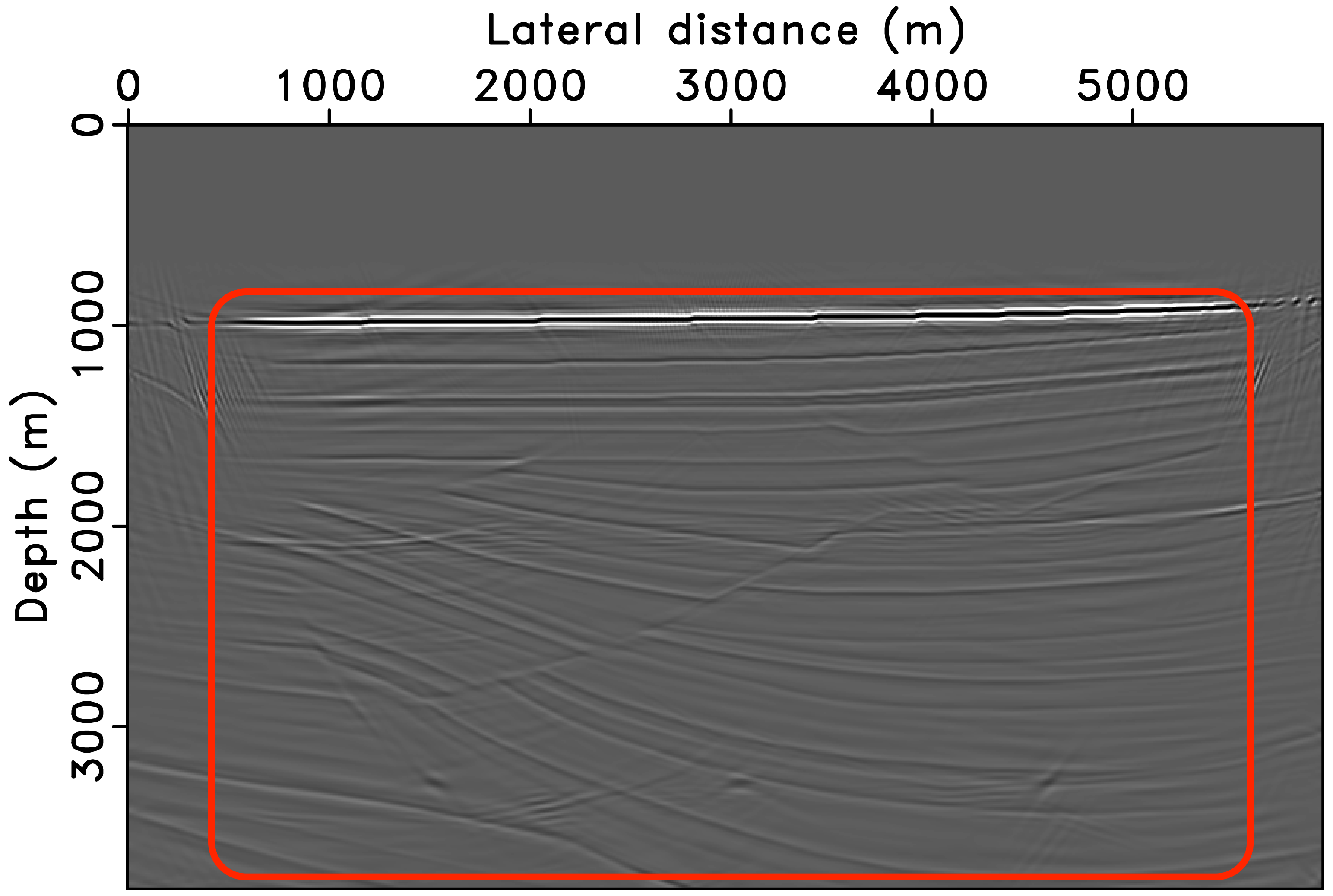
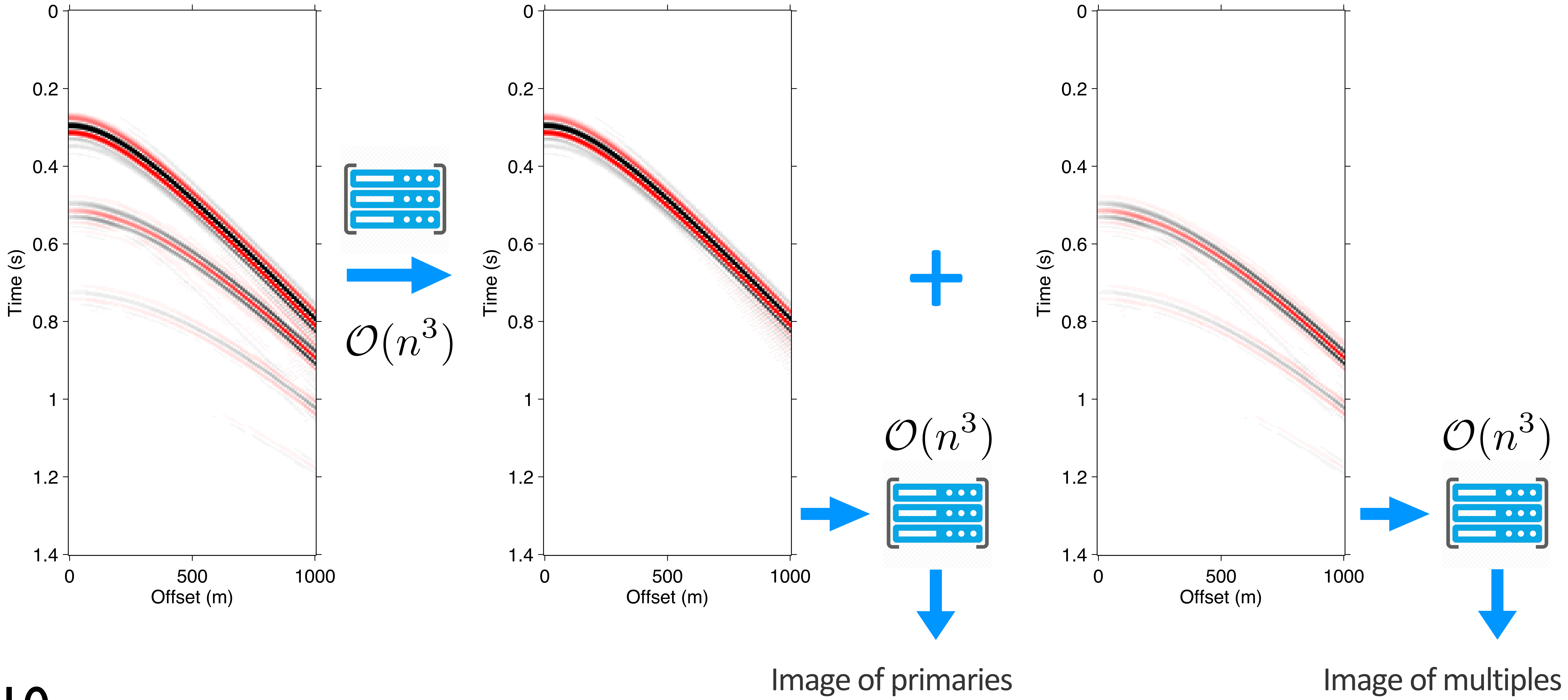


Model space **solutions** - the reality

[1] reliance on primary/multiple separation



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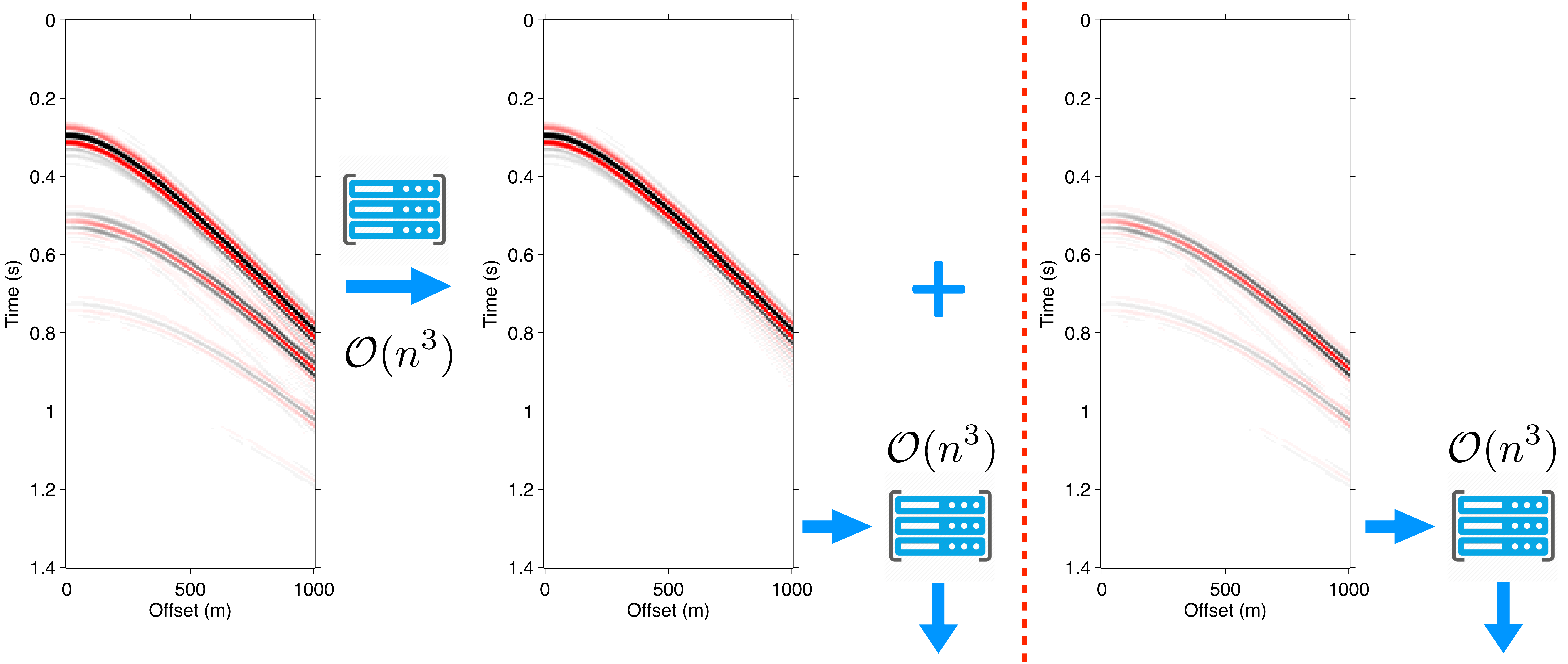


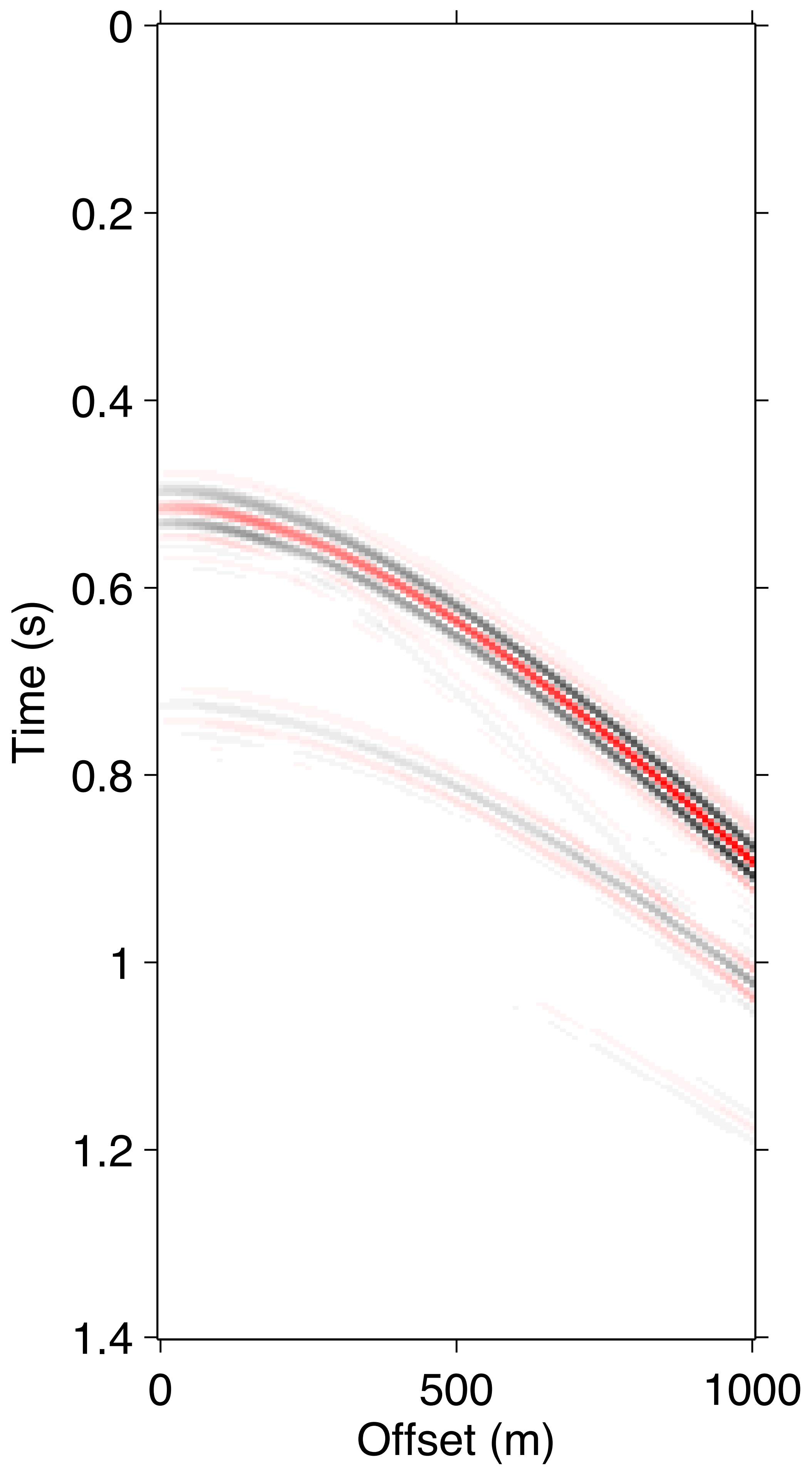
Image of primaries

Image of multiples

Model space solutions - the reality

[2] improper handling of multiples during imaging --- cross-correlation imaging

Chapter 2
Liu et al., 2011



Cross-correlation imaging by injecting data as source

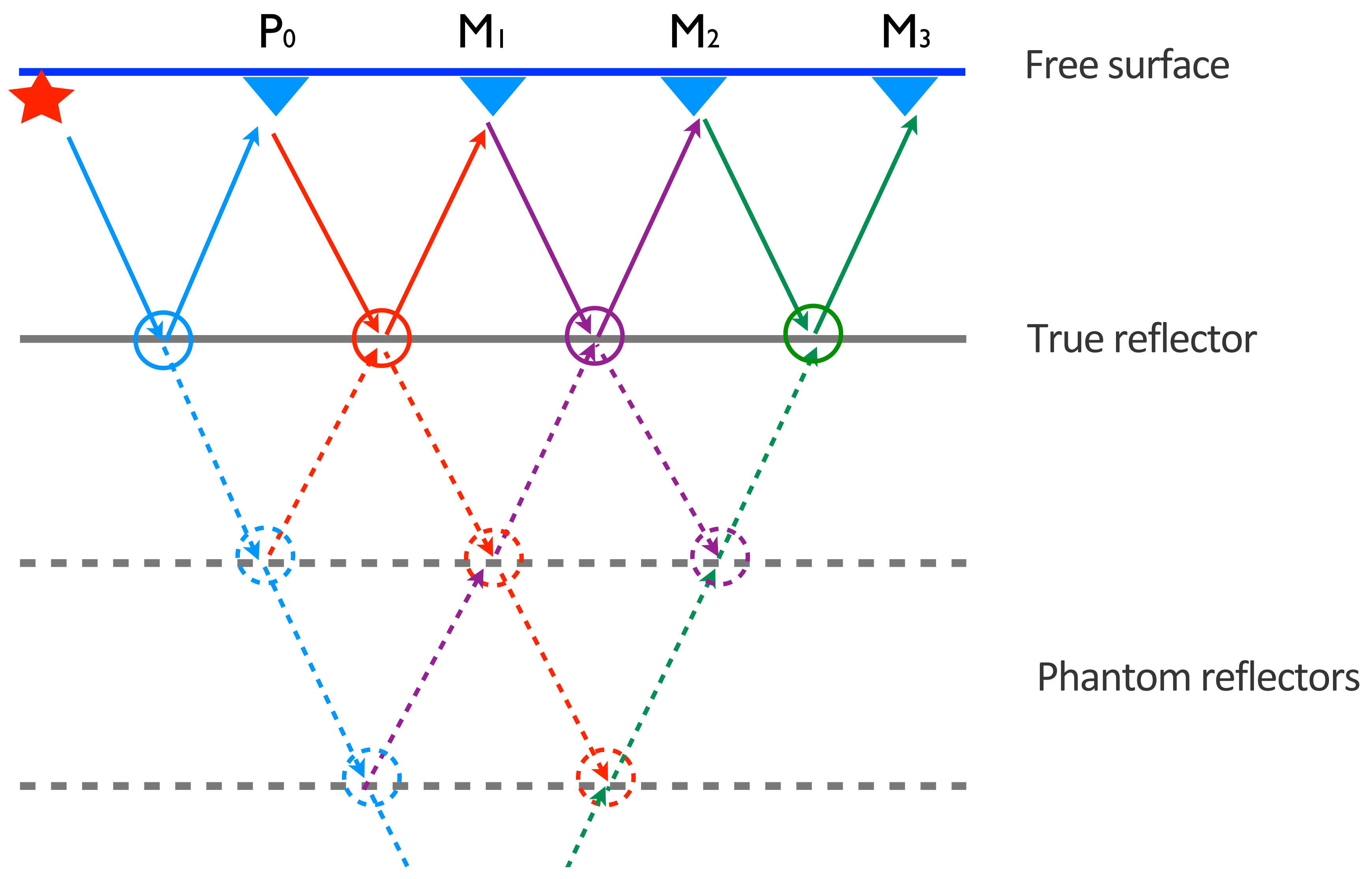
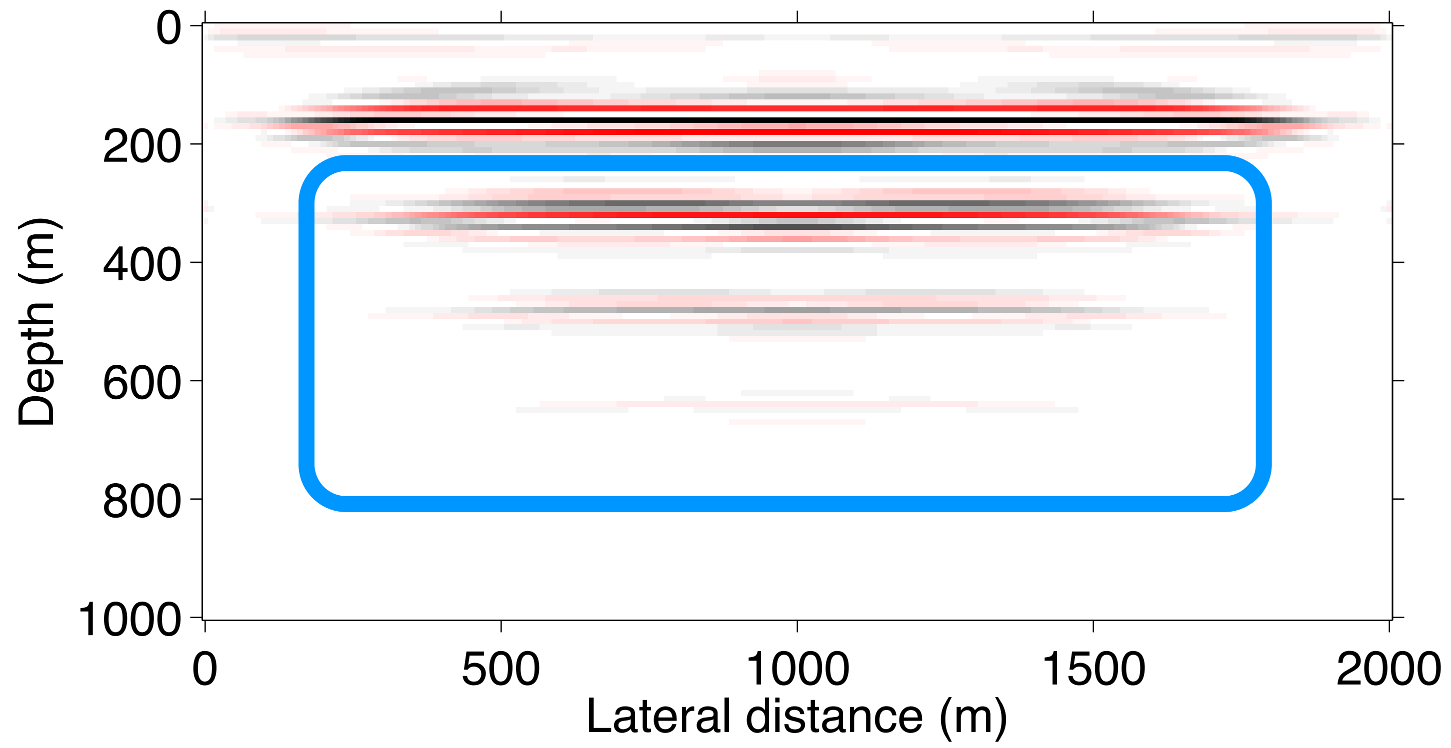
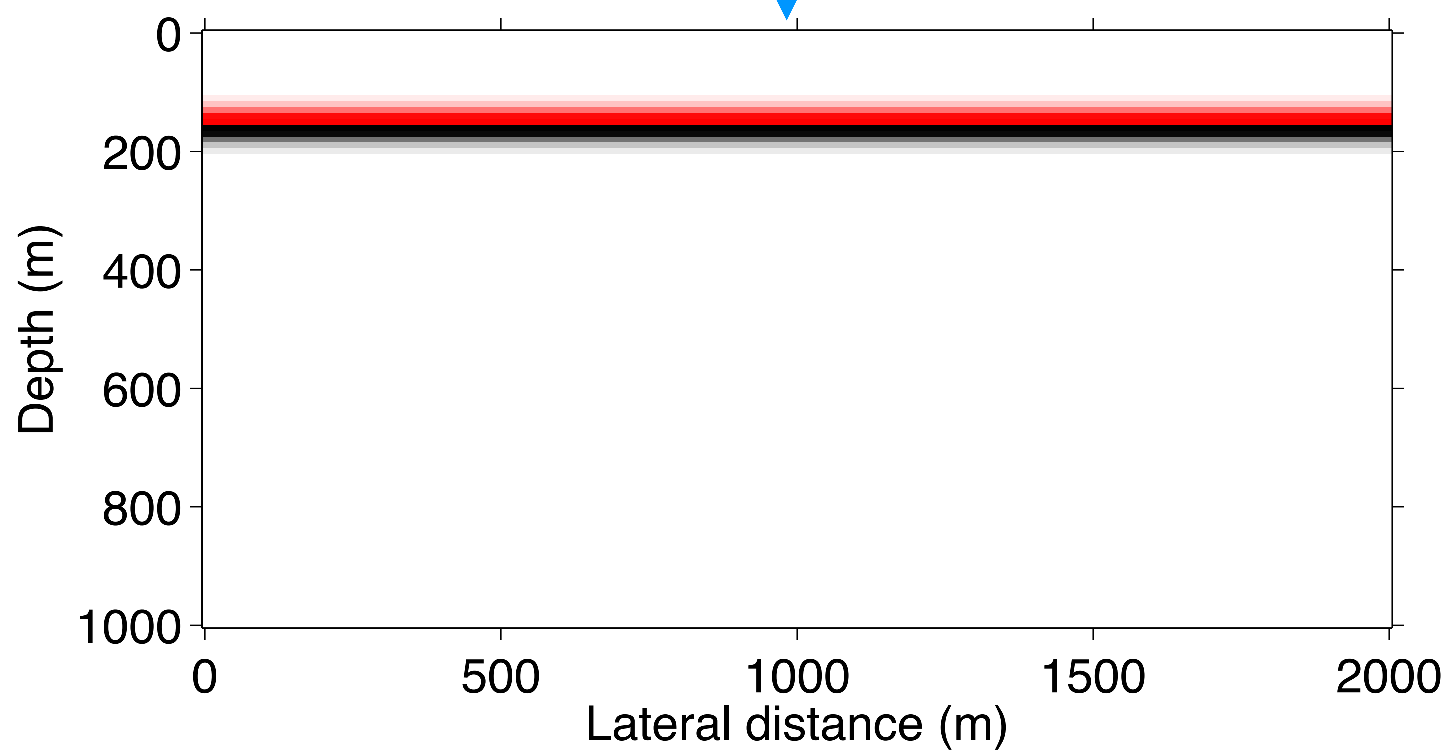


Image of multiples



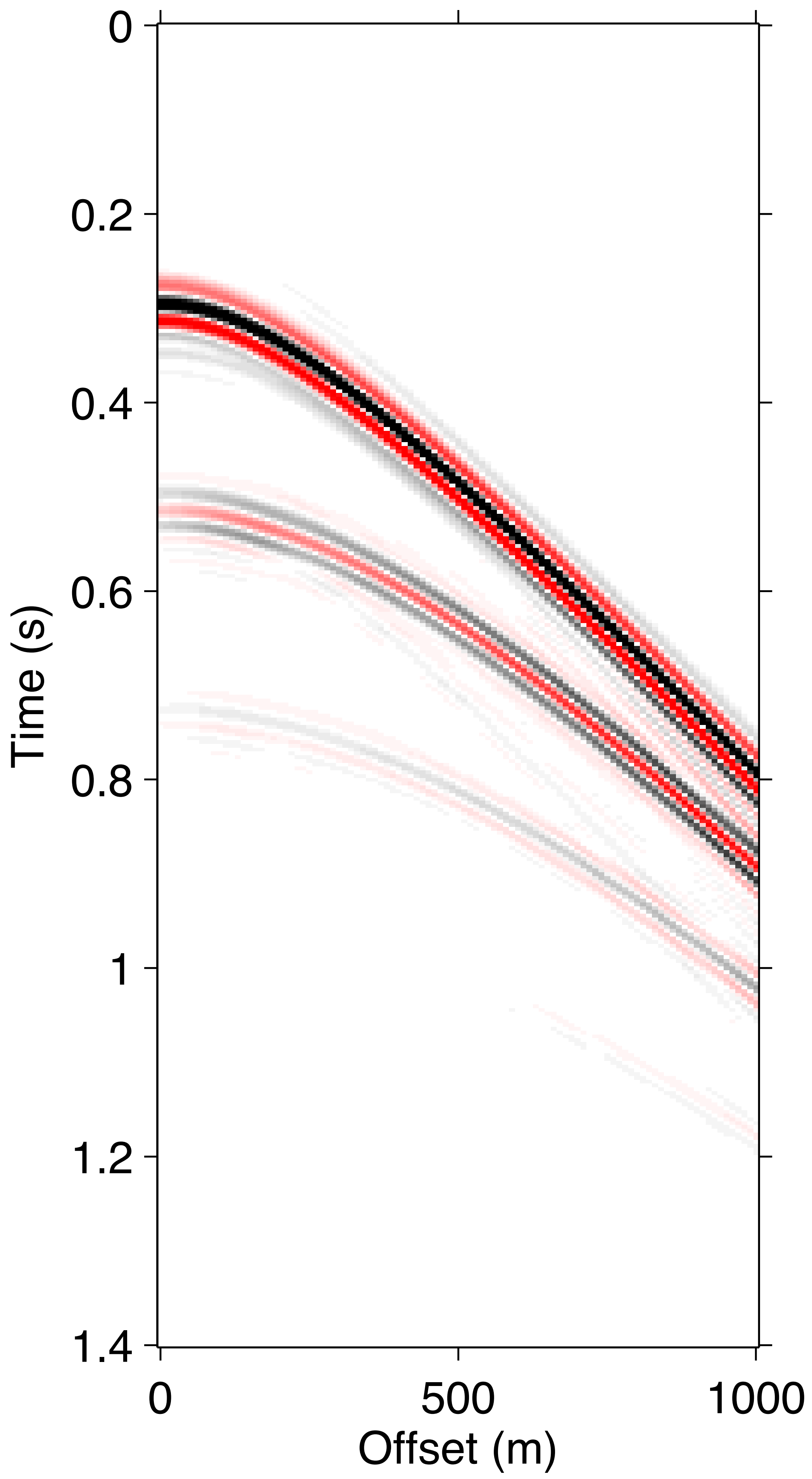


True model

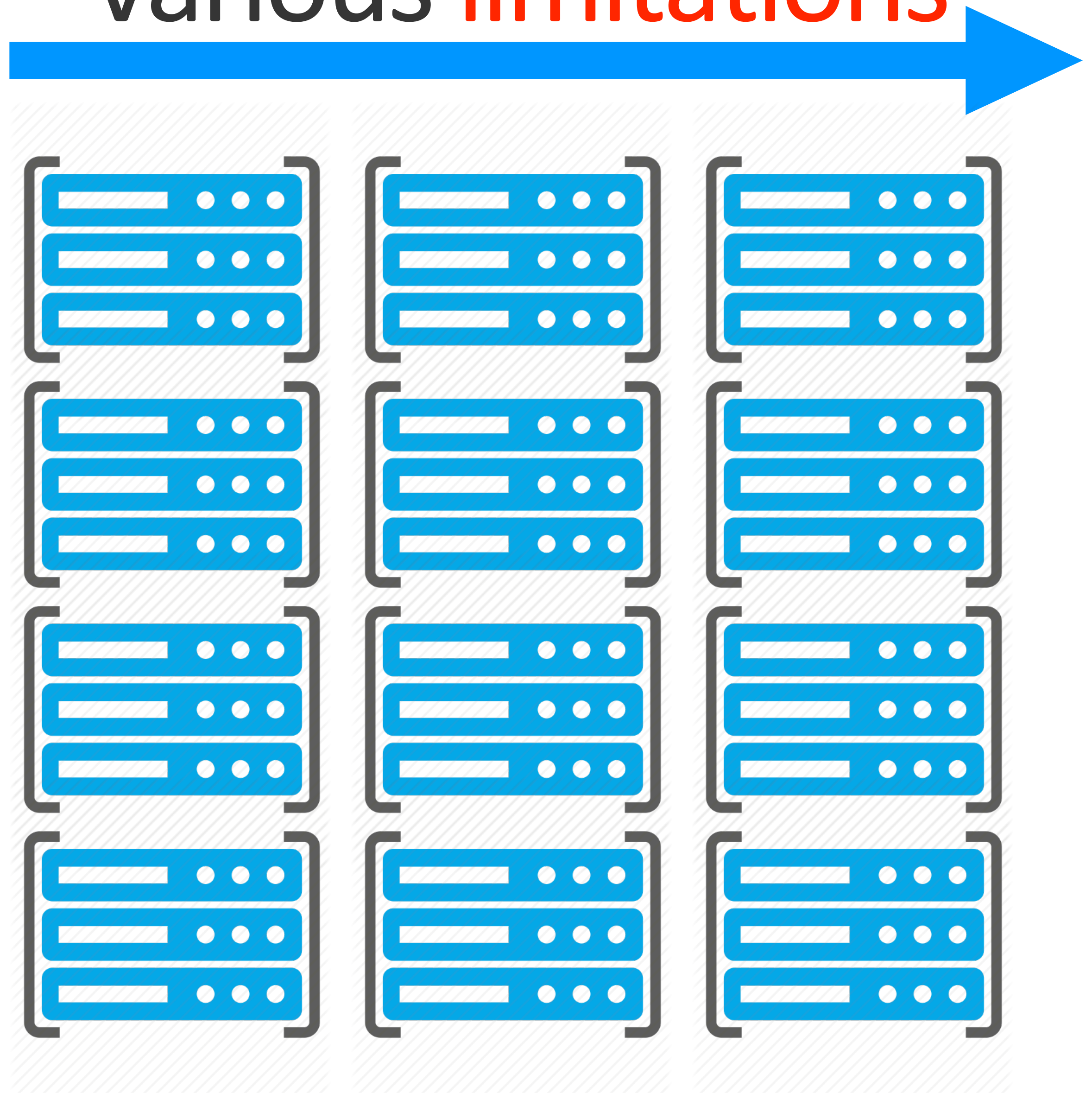
Model space solutions - the reality

[3] expensive computation by iterative inversion

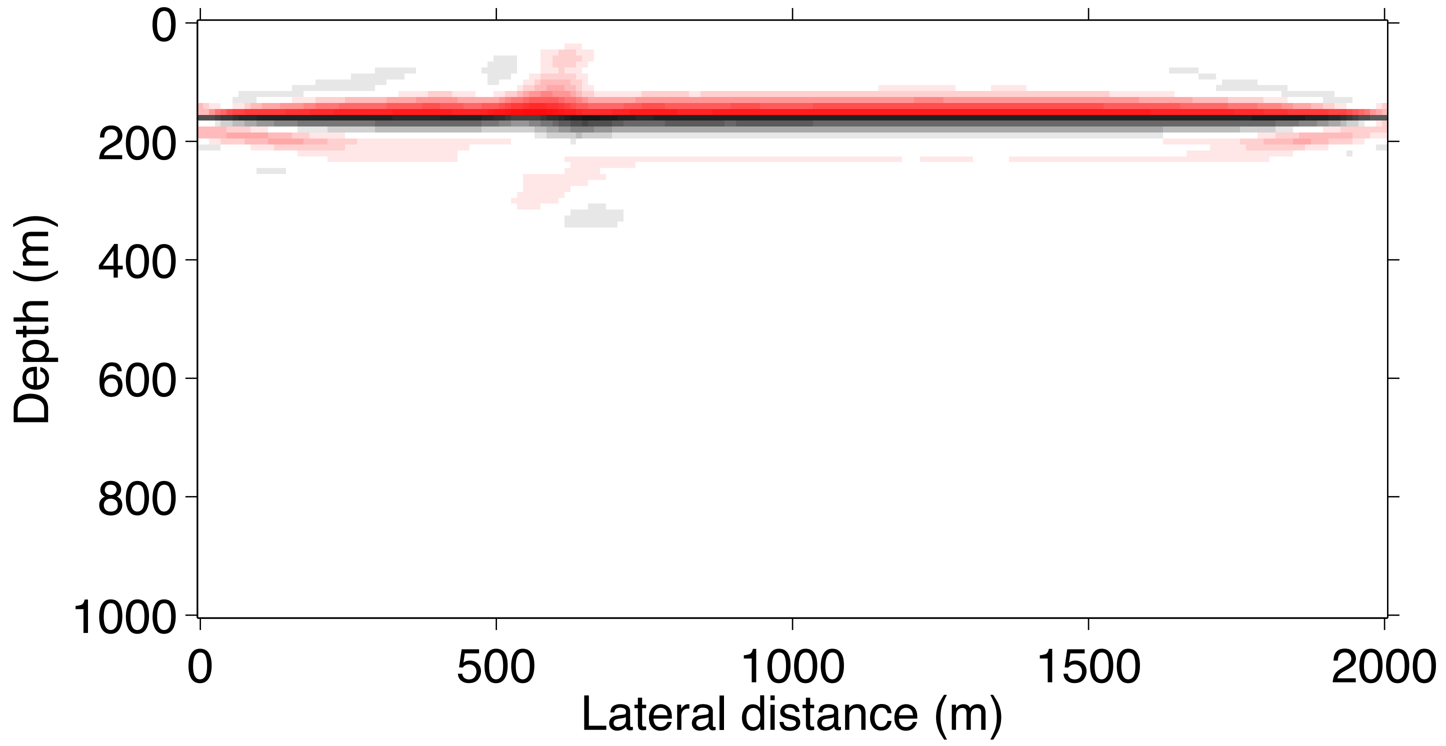
Verschuur and Berkhout 2011
Wong et al. 2014



Iterative inversions w/ various limitations



$$\mathcal{O}(n^3 m),\ m = \#iter$$



Model space solutions - the reality

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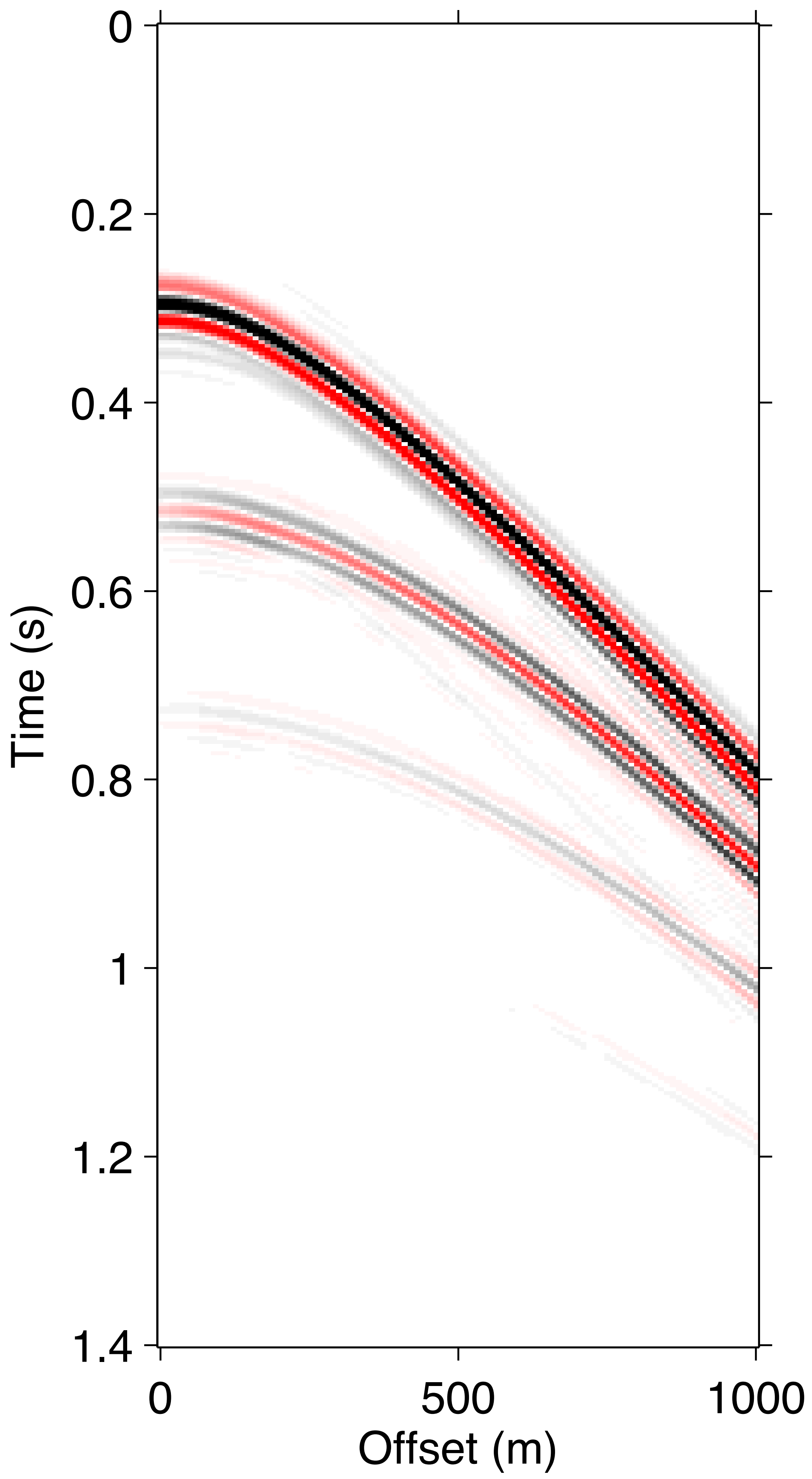
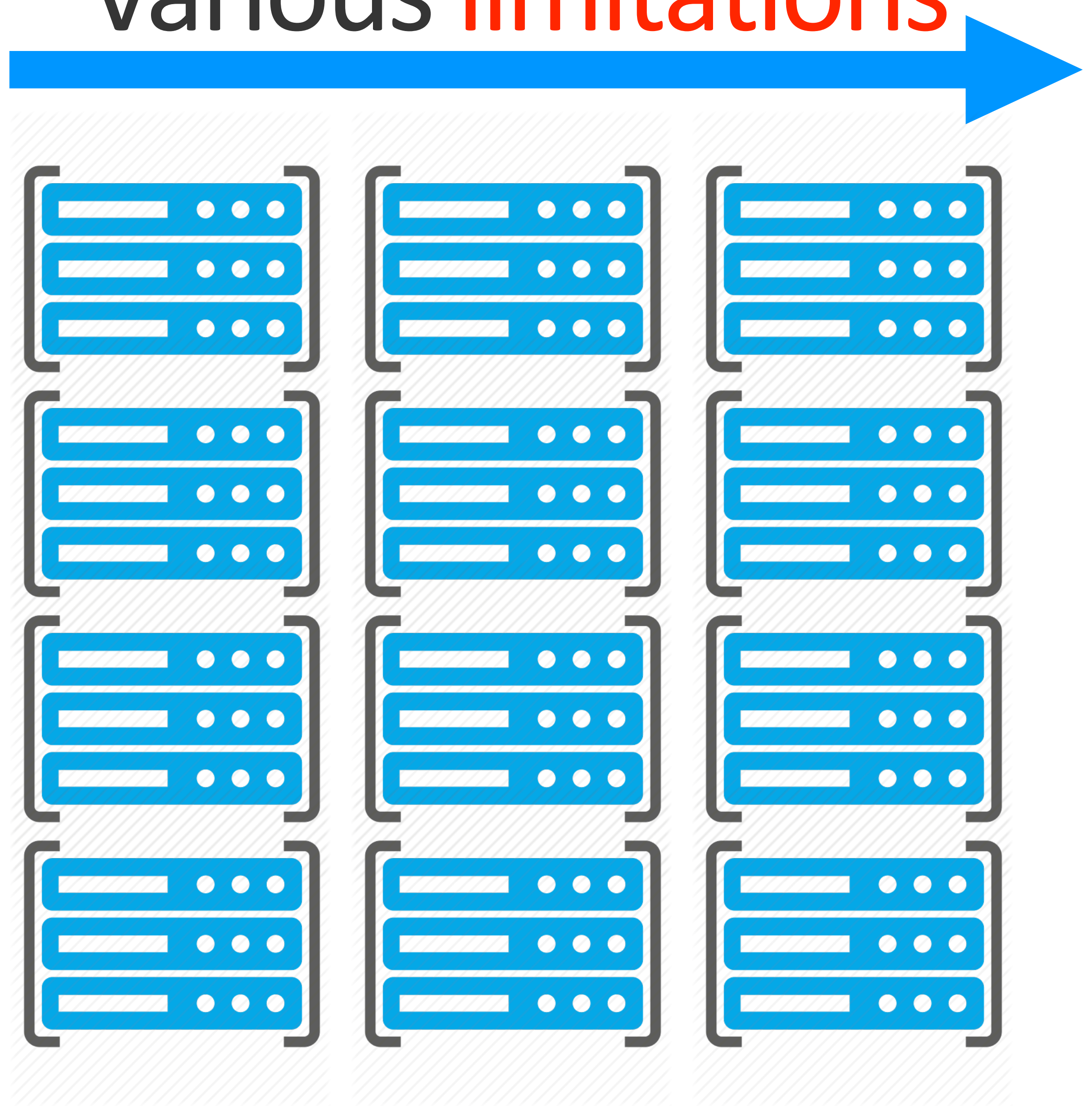


selected orders of multiples, prior knowledge on the source wavelet...

Iterative inversions w/ various limitations



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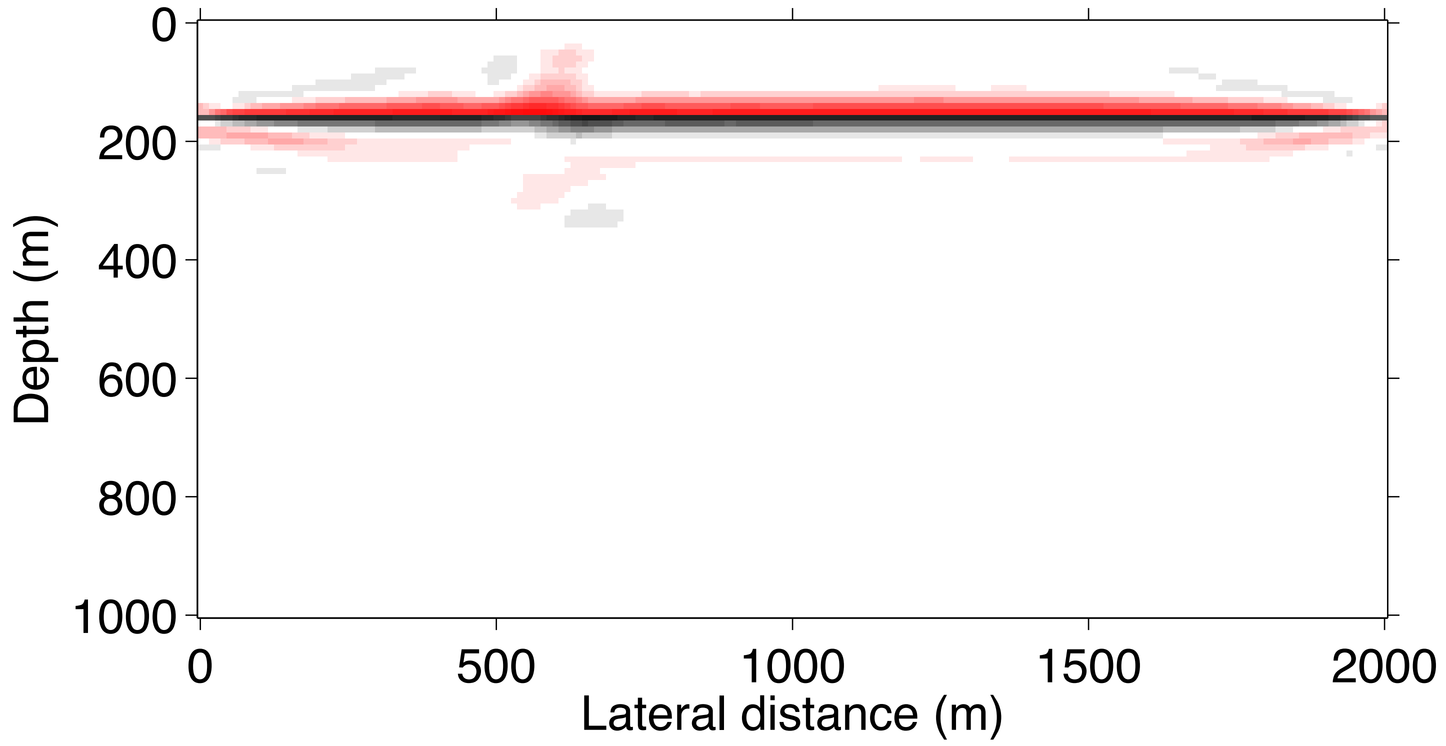
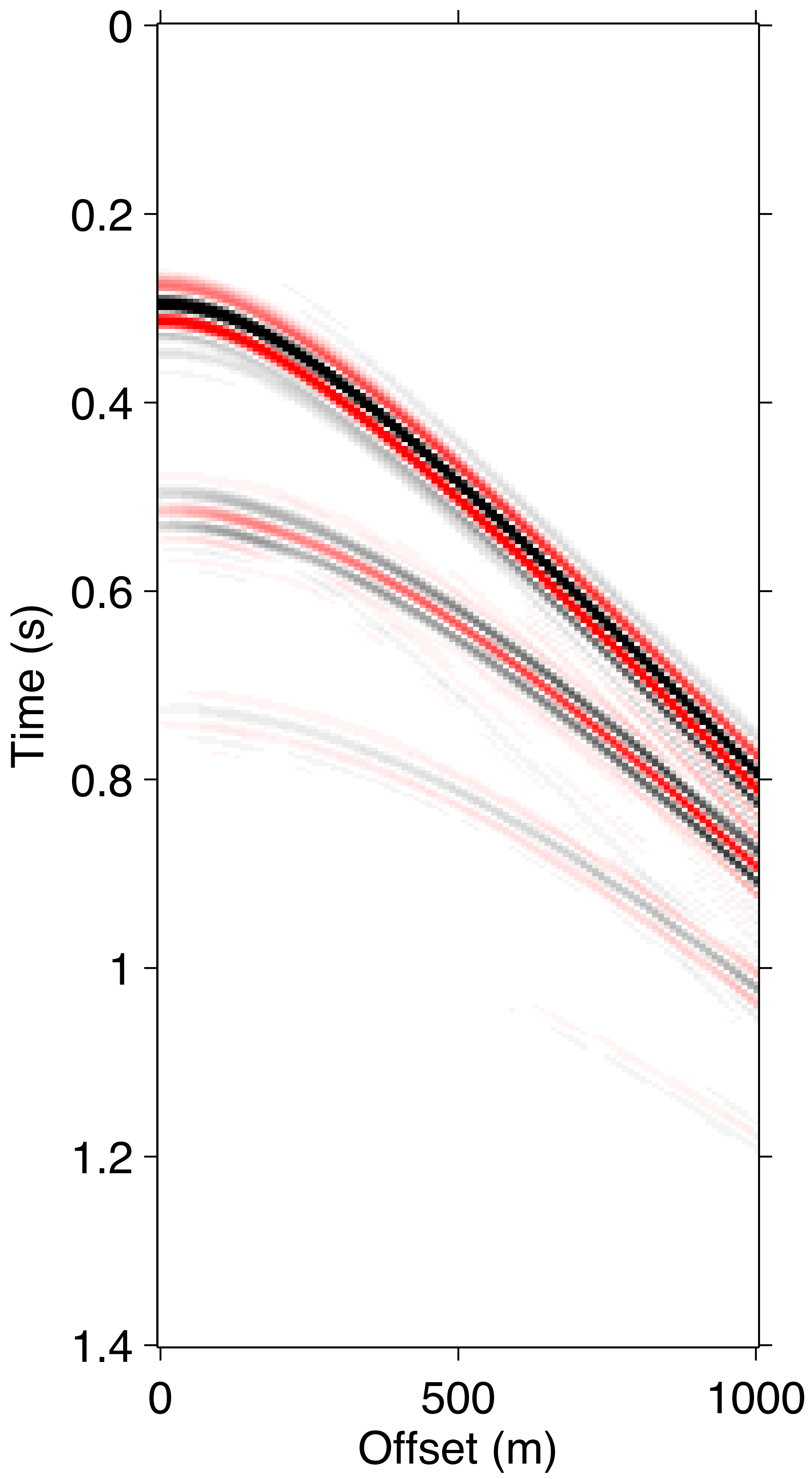


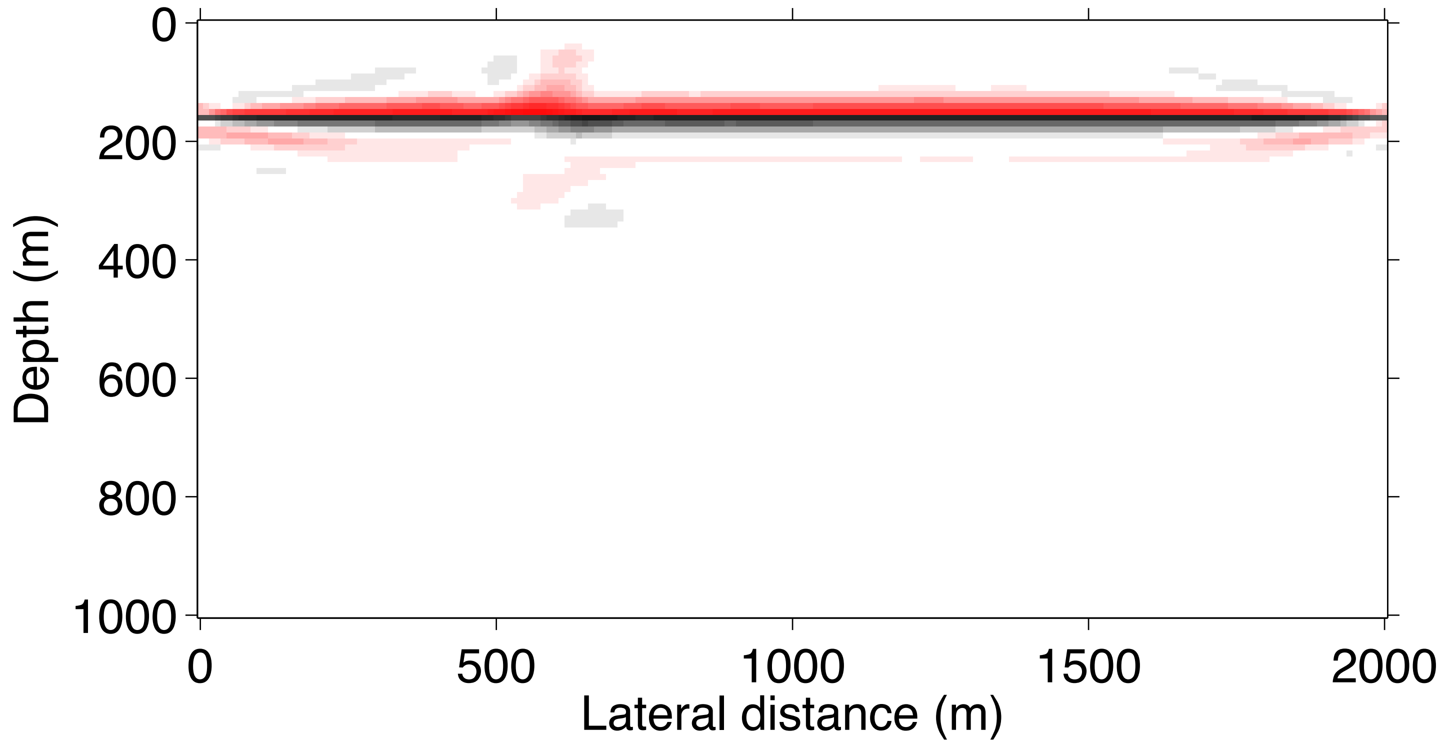
Objective & contribution of this work

Assumption: dense receiver sampling, fixed receivers, knowledge of a background model



Ning Tu's PhD thesis

$\mathcal{O}(n^3)$



Method

Physics at the water surface

[The Surface-Related Multiple Elimination (SRME) relation]

Verschuur et al., 1992

$$\mathbf{P}_i = \mathbf{X}_i(\mathbf{S}_i + \mathbf{R}_i\mathbf{P}_i)$$

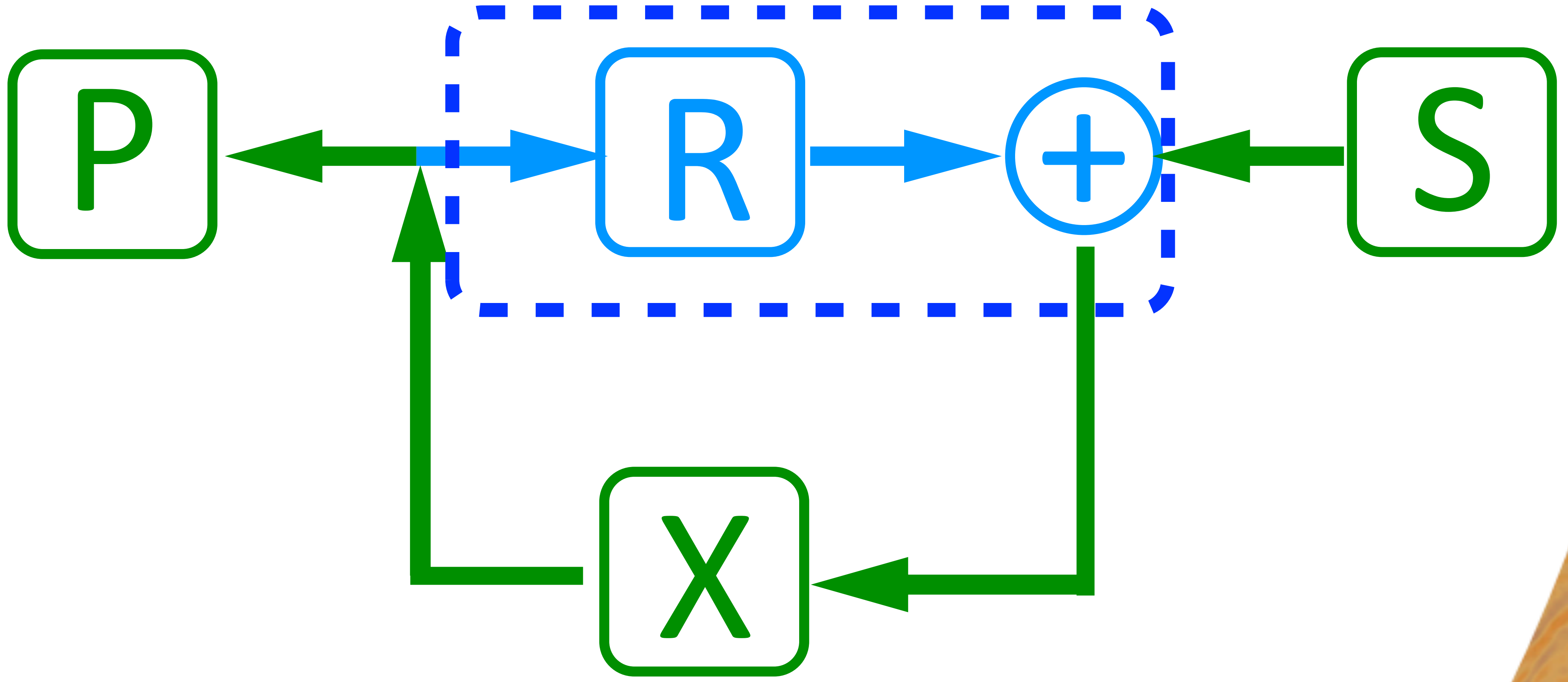
$\mathbf{P}$: pressure wavefield: primaries + multiples

$\mathbf{X}$: surface-**free** Green's function

$\mathbf{S}$: source wavefield $= w_i\mathbf{I}$

$\mathbf{R}$: surface reflectivity $\approx -\mathbf{I}$

i : frequency index $1 \cdots n_f$



Representation using linearized modelling

Linearized modelling of the surface-free Green's function:

$$\mathbf{X}_i \approx \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*\mathbf{I})$$

$\nabla\mathcal{F}_i$: linearized modelling

$\mathbf{m}_0$: background model

$\delta\mathbf{m}$: model perturbations

$\mathbf{I}$: impulsive source array

$\mathbf{D}_s^*$: source injection operator

Eliminating dense matrix-matrix products

[SRME relation & wave-equation solver]

Combined with water-surface physics:

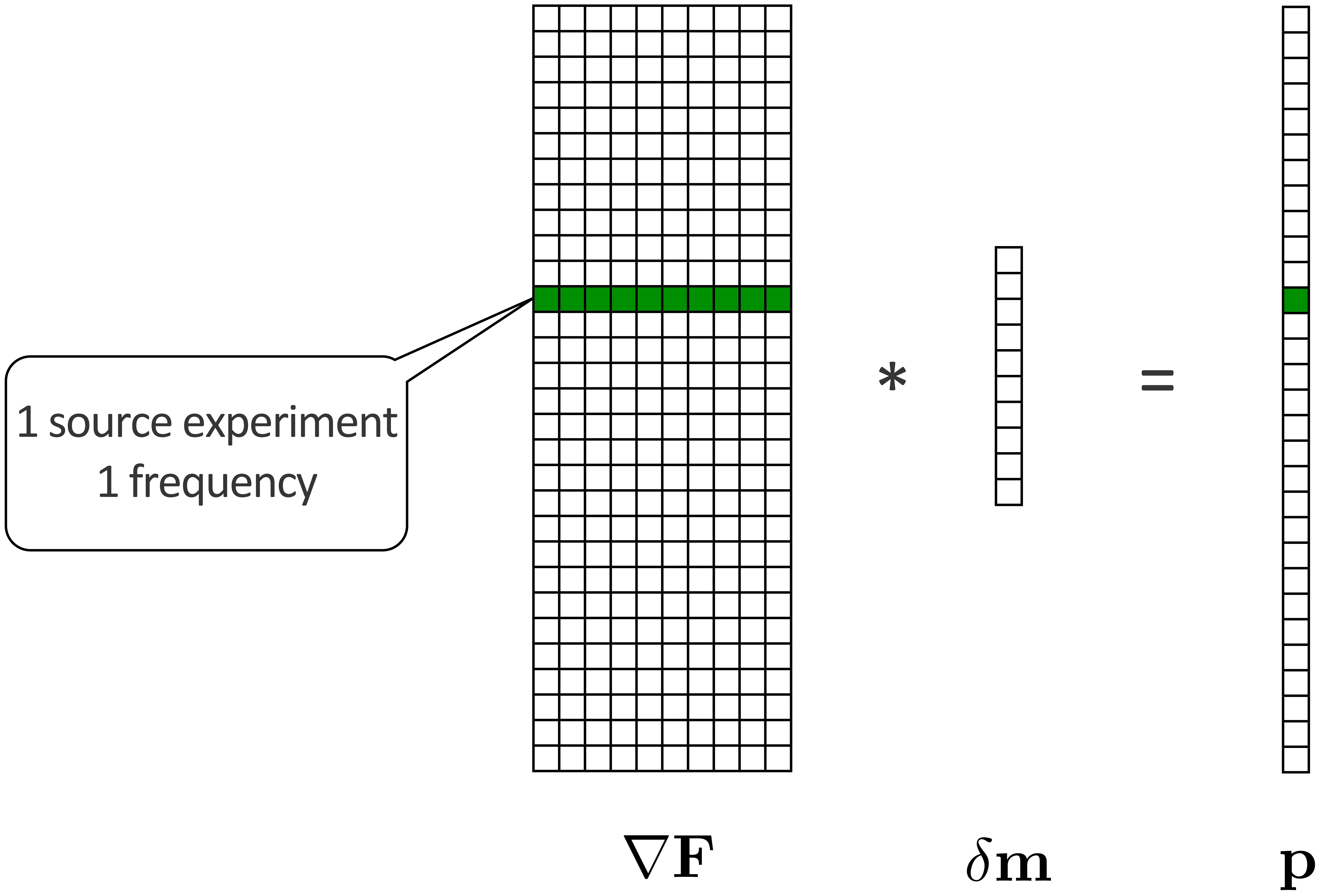
$$\mathbf{P}_i \approx \underbrace{\nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*\mathbf{I}}^{\mathbf{X}_i})(\mathbf{S}_i - \mathbf{P}_i) \longrightarrow \text{Dense matrix products}$$

$$= \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*(\mathbf{S}_i - \mathbf{P}_i)) \longrightarrow \text{Wave-equation solves with total downgoing data}$$

In canonical form of linear system of equations

[**Ax=b** where **A** is a matrix, **x** and **b** are vectors]

$$
\begin{aligned}
\mathbf{p}_i &= \mathrm{vec}(\mathbf{P}_i) \\
&\approx \nabla\mathbf{F}_i[\mathbf{m}_0; \mathbf{S}_i - \mathbf{P}_i]\delta\mathbf{m}
\end{aligned}
$$

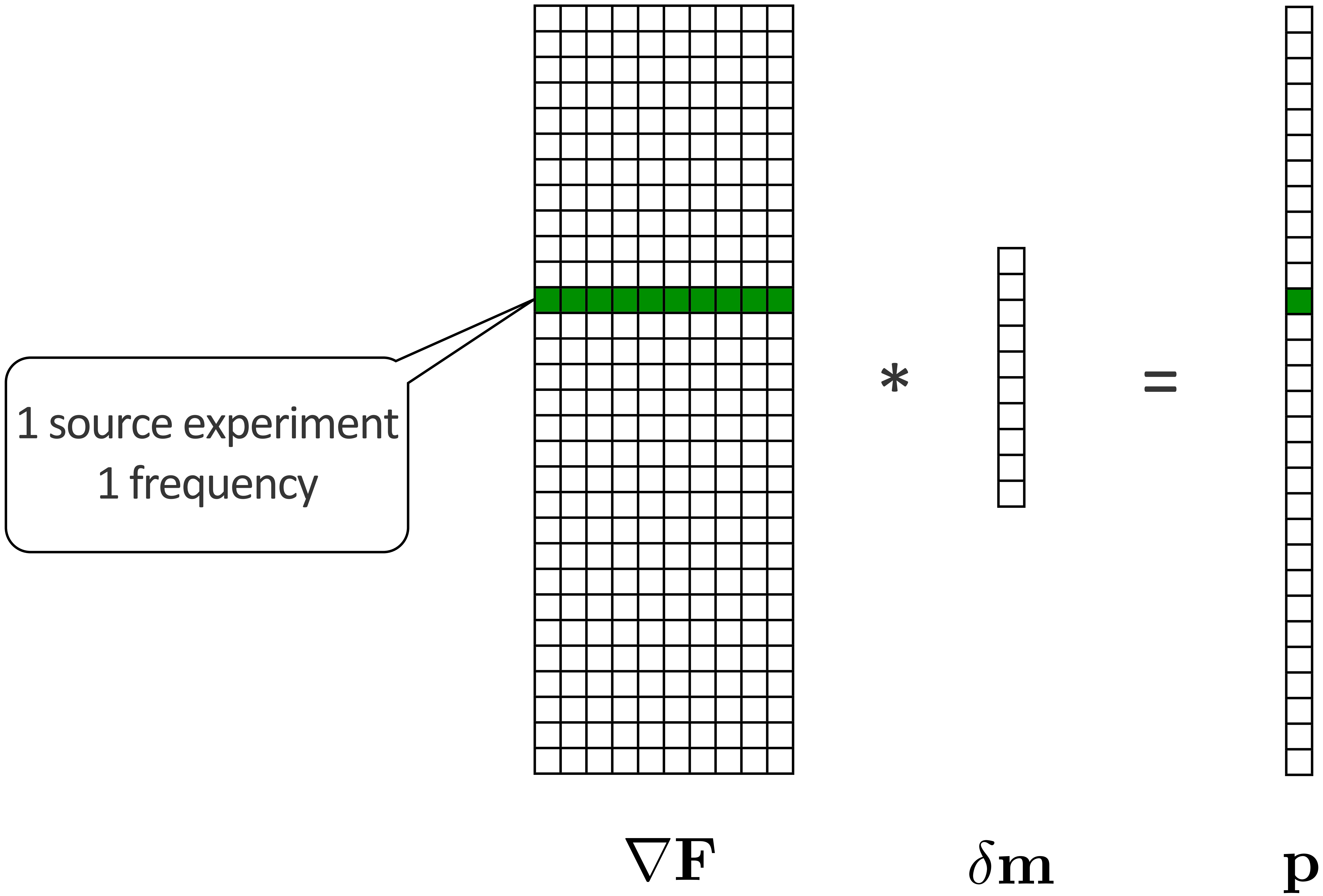


In canonical form of linear system of equations

[Ax=b where A is a matrix, x and b are vectors]

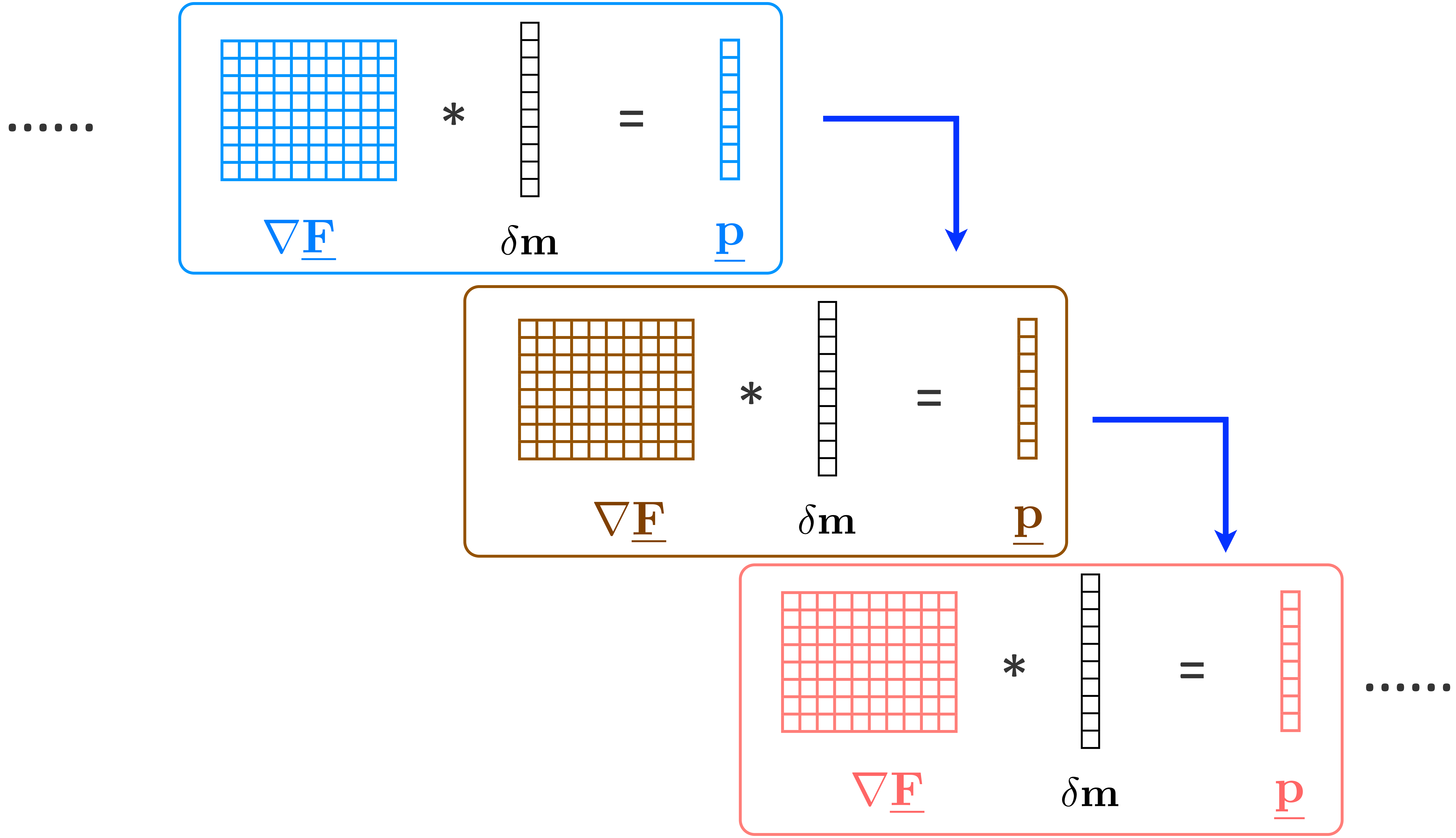
$$\begin{aligned}\mathbf{p}_i &= \mathrm{vec}(\mathbf{P}_i)\\ &\approx \nabla\mathbf{F}_i[\mathbf{m}_0;\mathbf{S}_i - \mathbf{P}_i]\delta\mathbf{m}\end{aligned}$$

Example:
200 sources, 200 frequencies,
200 receivers, 500*500 model
(1) operator has **$2*10^{12}$** entries
(2) conventional imaging involves **80,000** wave-equations solves
(3) iterative inversion w/ 50 iter. involves **8,000,000** wave-equation solves.



Dimensionality reduction w/ rerandomizaion

Chapter 2, 3



Dimensionality reduction w/ rerandomizaion

Chapter 2, 3

......

$$\nabla\underline{\mathbf{F}} * \delta\mathbf{m} = \underline{\mathbf{p}}$$

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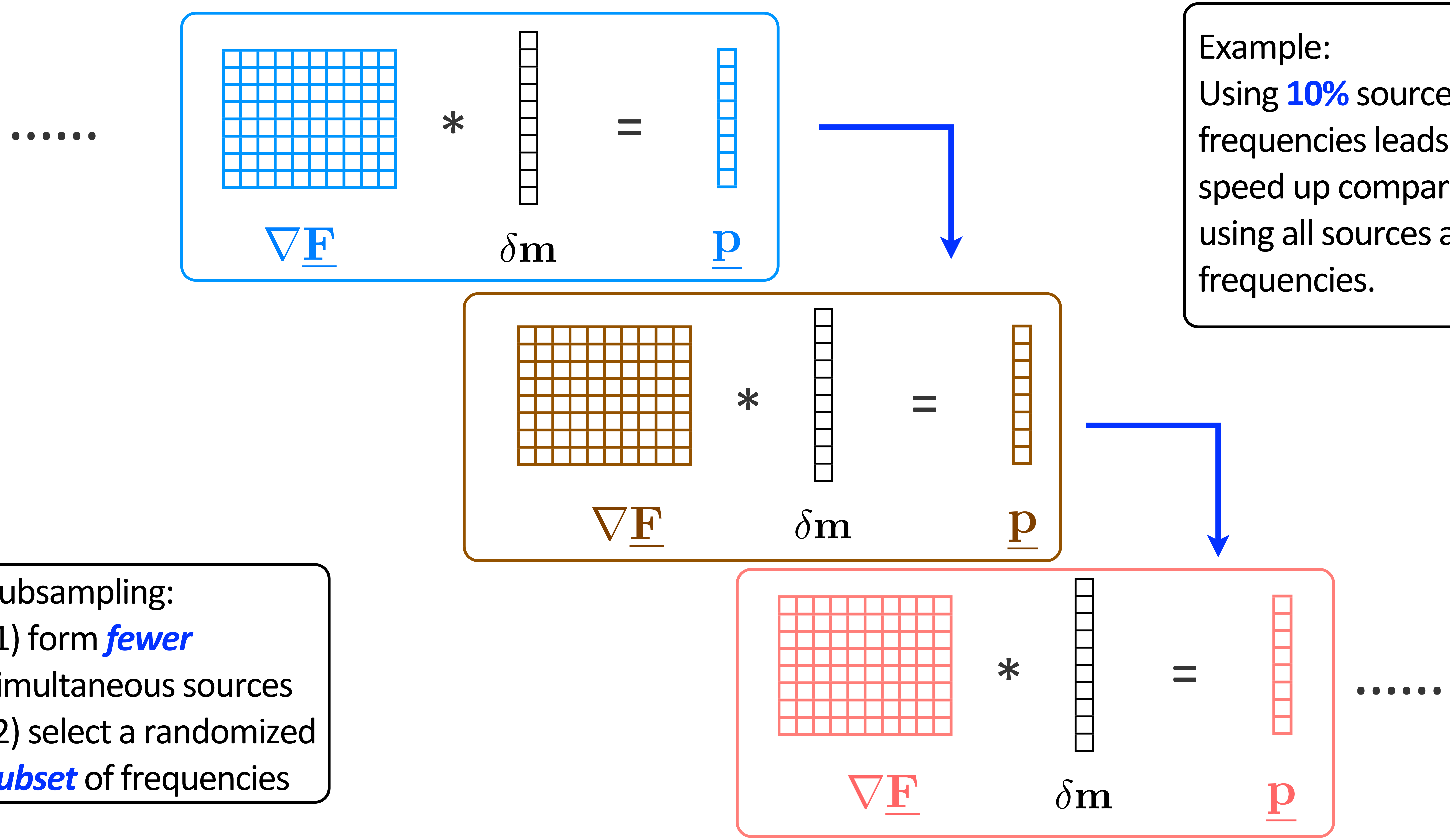
$$\nabla\underline{\mathbf{F}} * \delta\mathbf{m} = \underline{\mathbf{p}}$$

......

Subsampling:
(1) form ***fewer*** simultaneous sources
(2) select a randomized ***subset*** of frequencies

Dimensionality reduction w/ rerandomizaion

Chapter 2, 3



Example:
Using **10%** sources, **10%** frequencies leads to **100X** speed up compared with using all sources and frequencies.

Subsampling:
(1) form ***fewer*** simultaneous sources
(2) select a randomized ***subset*** of frequencies

Sparsity promoting imaging

van den Berg and Friedlander, 2008
Herrmann et al., 2008
Appendix A.1

Formulation of the l^{th} subproblem:

$$\underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i\in\Omega_l} \|\underline{\mathbf{p}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{S}}_i - \underline{\mathbf{P}}_i]\underbrace{\mathbf{C}^*\mathbf{x}}_{\delta\mathbf{m}}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau_l$$

Sparsity constraint τ_l is computed automatically using SPGl1.

$\mathbf{C}$: curvelet transform

$\underline{\cdot}$: wavefields with subsampled source experiments

Ω : randomized frequency subset

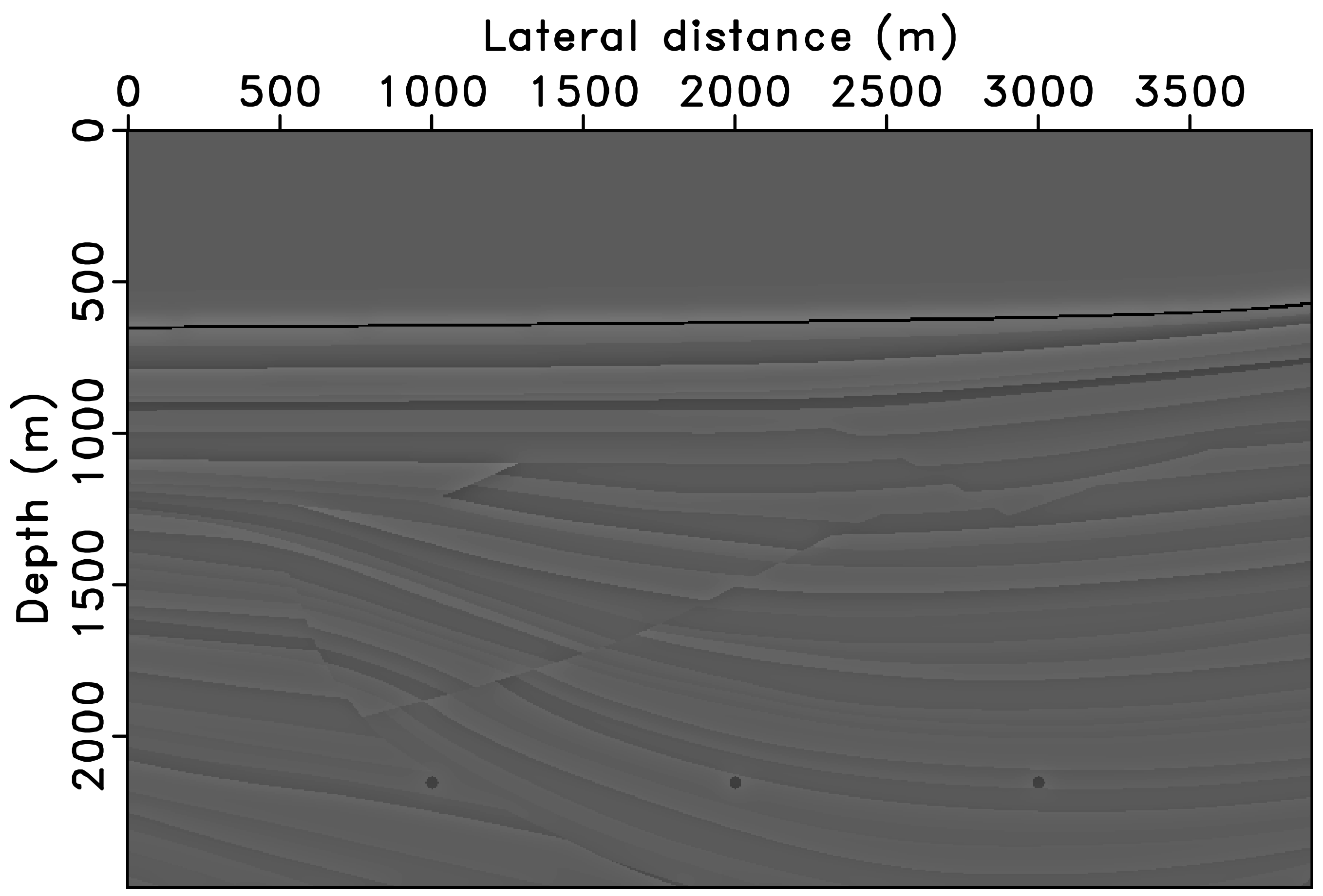
$\underline{\mathbf{S}}_i = \mathbf{S}_i\underline{\mathbf{E}}_l$

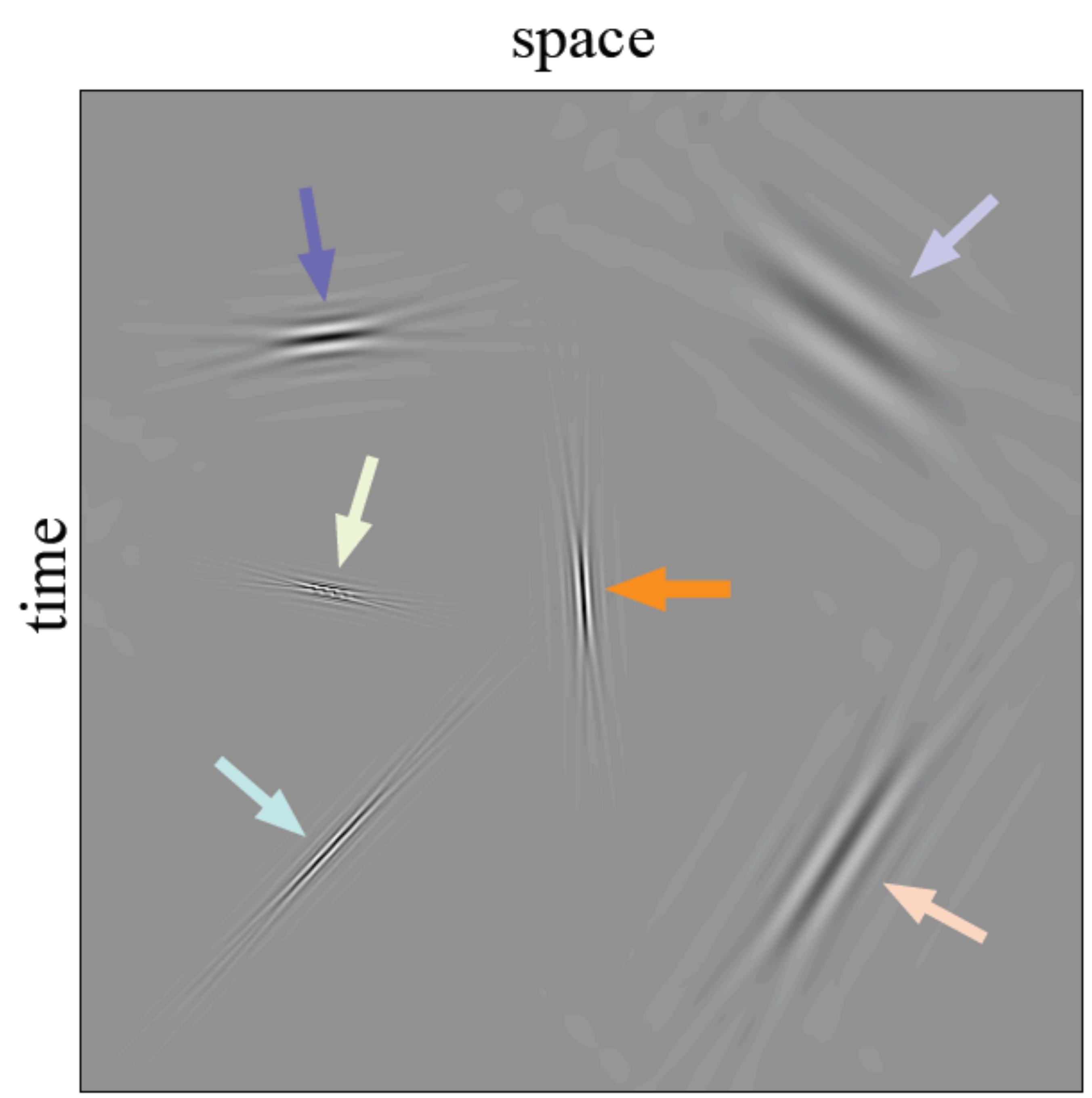
$\underline{\mathbf{P}}_i = \mathbf{P}_i\underline{\mathbf{E}}_l$

$\mathbf{E}_l$: encoding operator

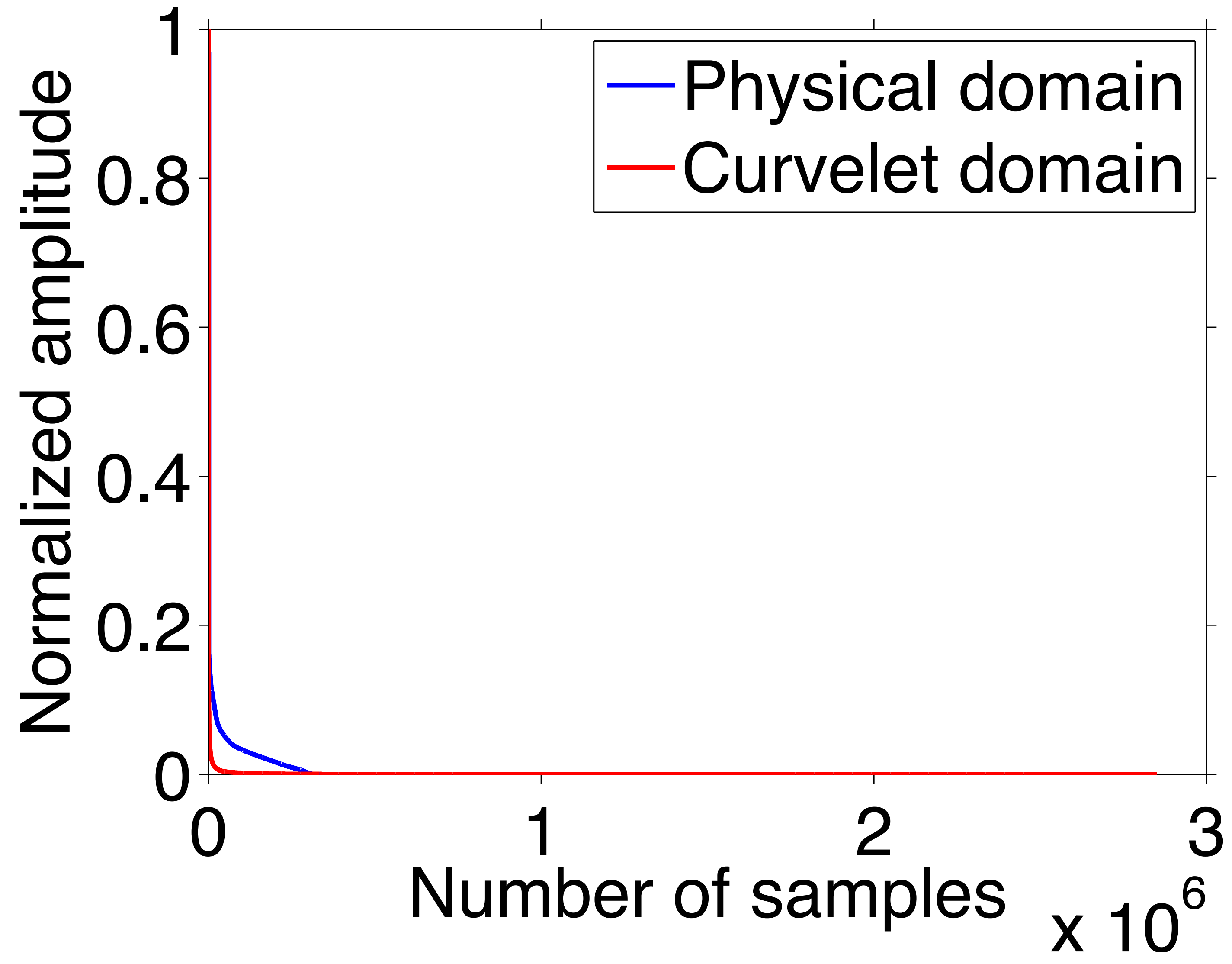
Curvelet domain sparsity

Candès and Donoho, 2004
Candès et al., 2006
Herrmann and Hennenfent , 2008





Curvelet “atoms”



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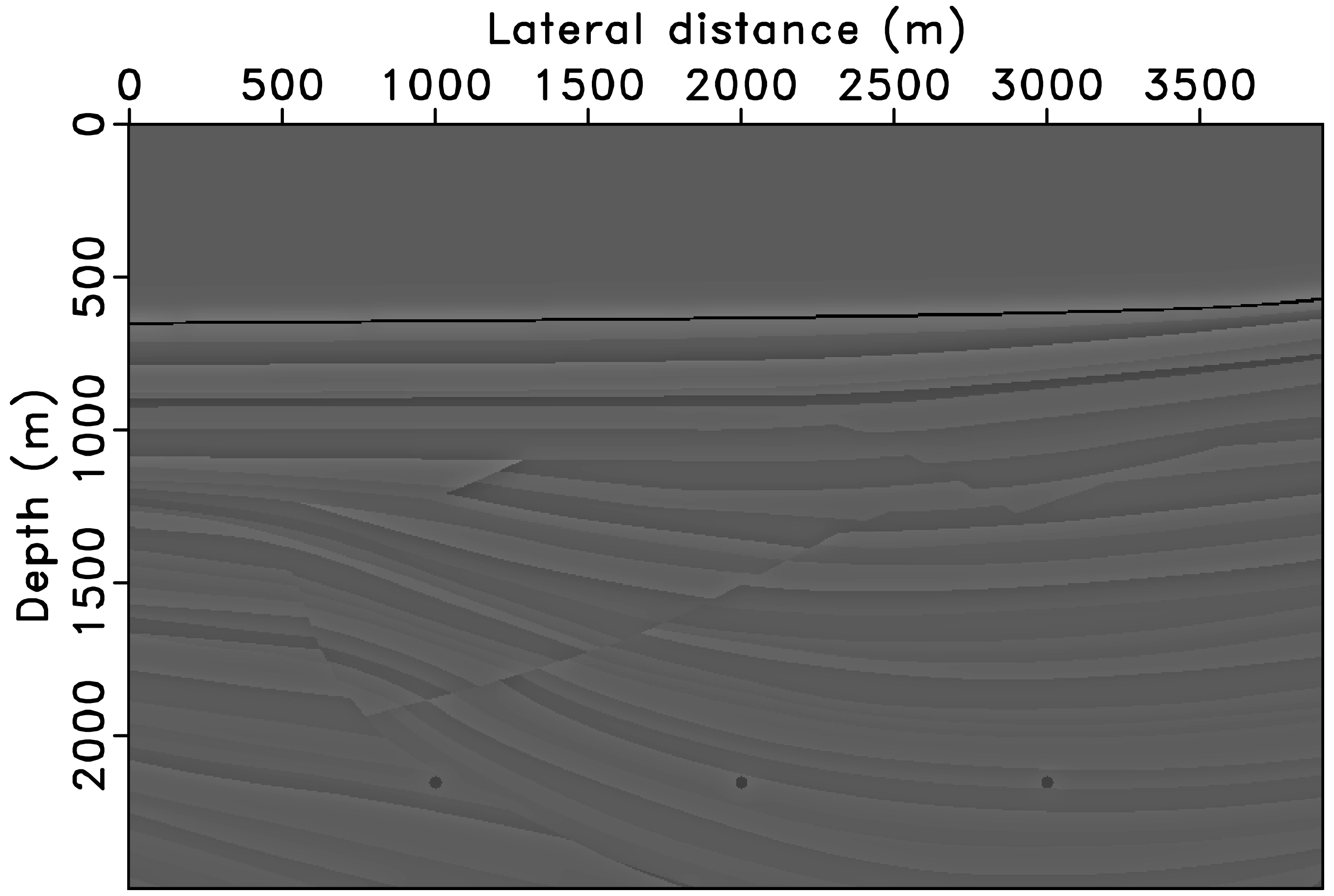

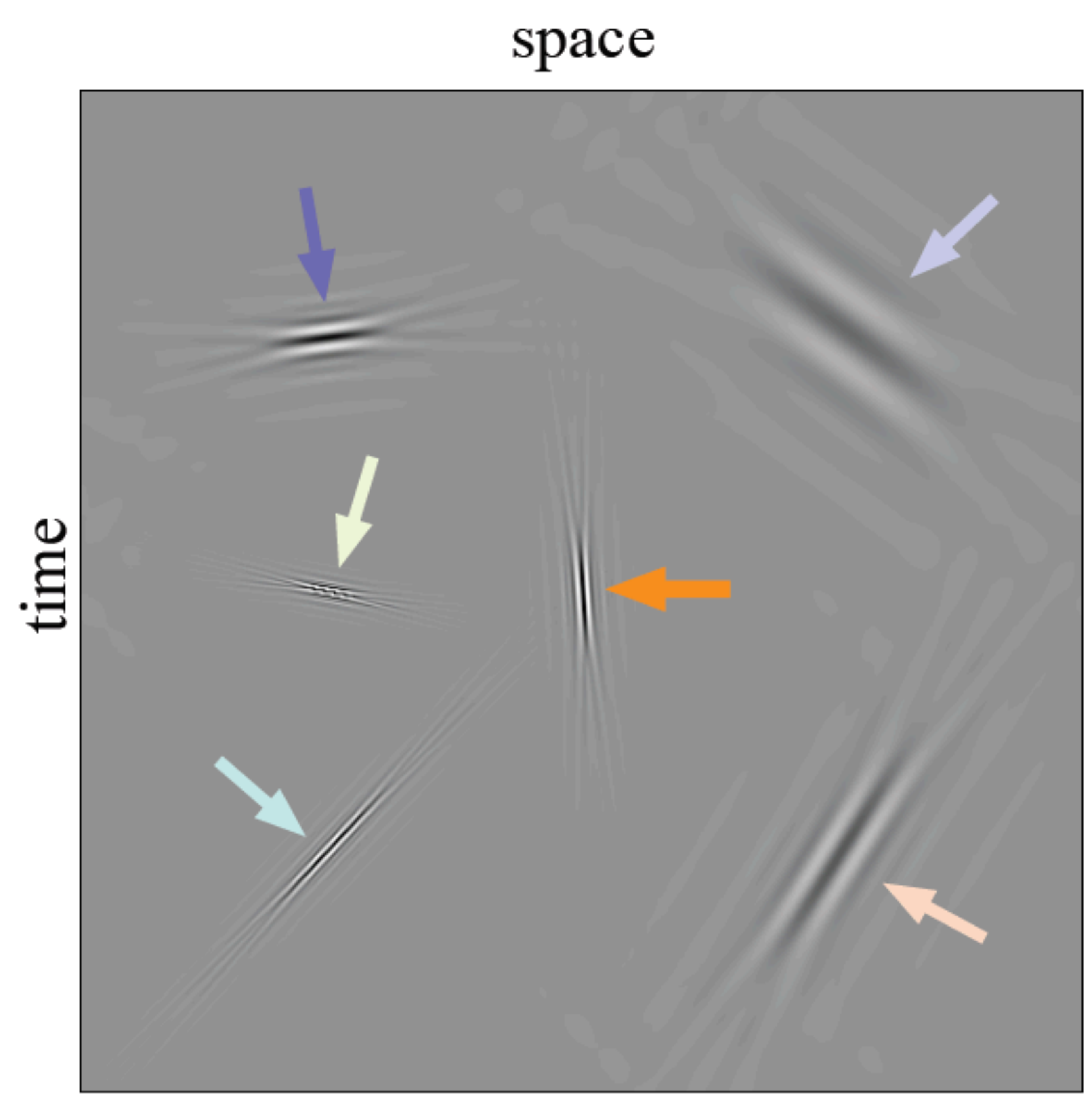

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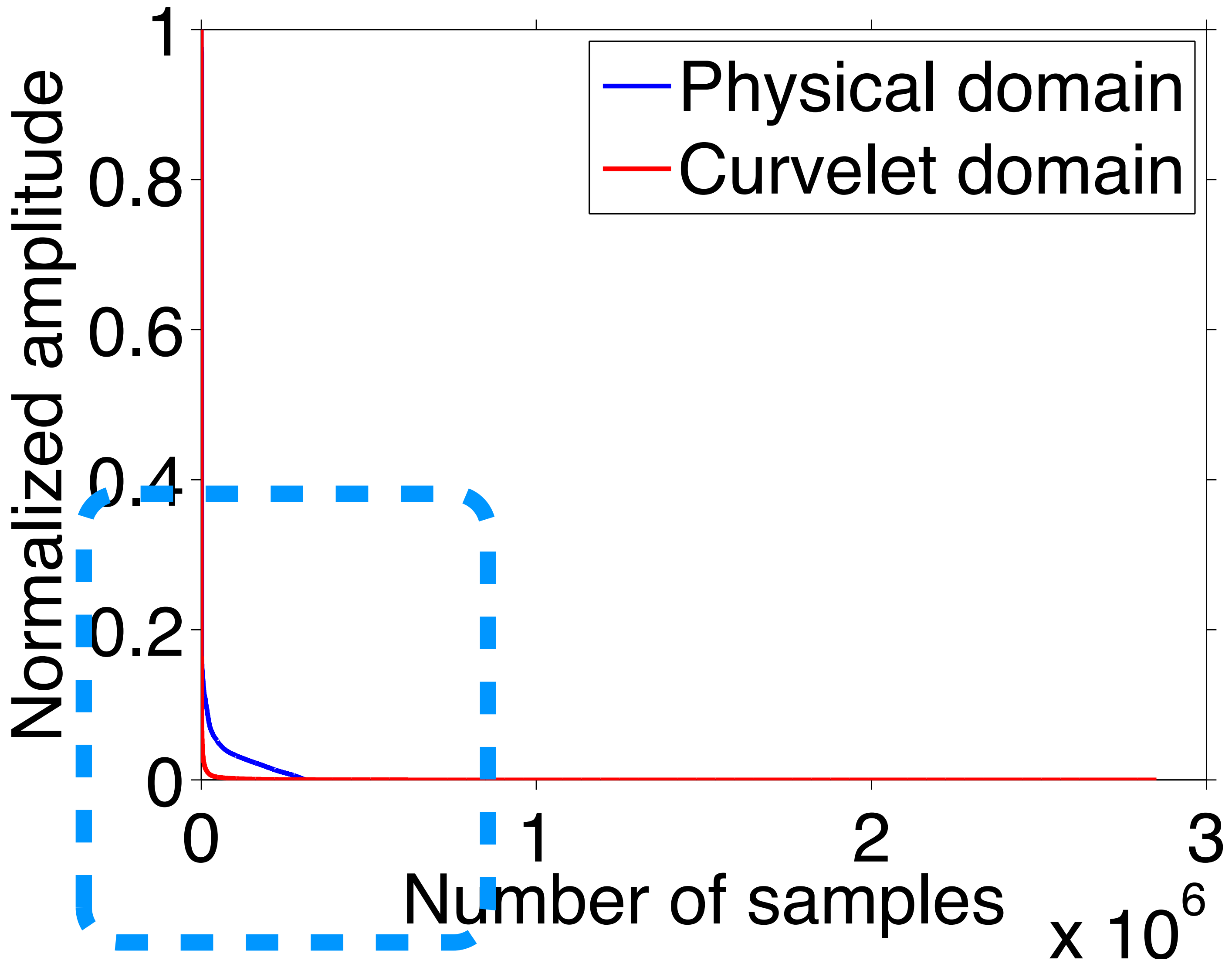

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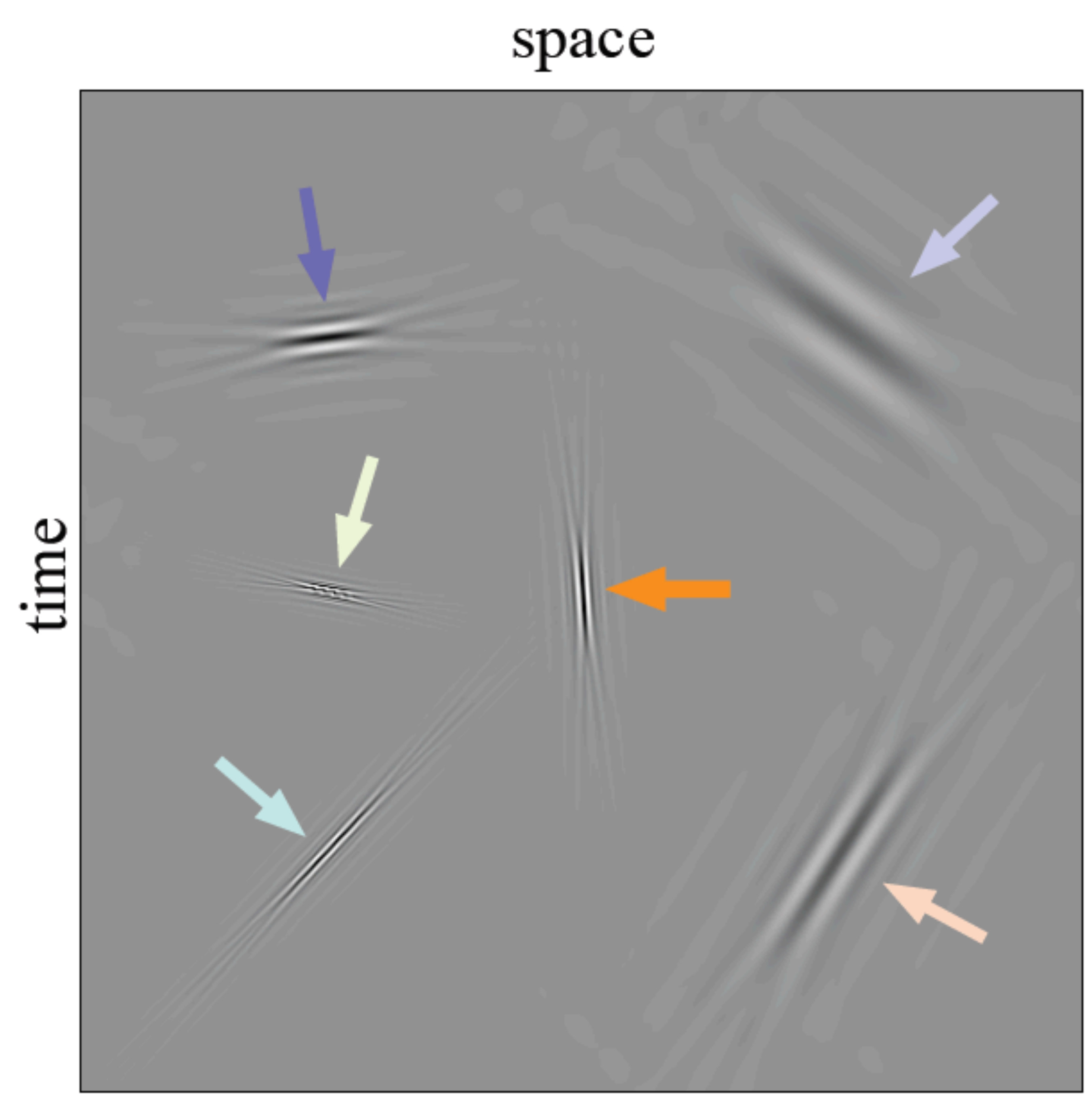

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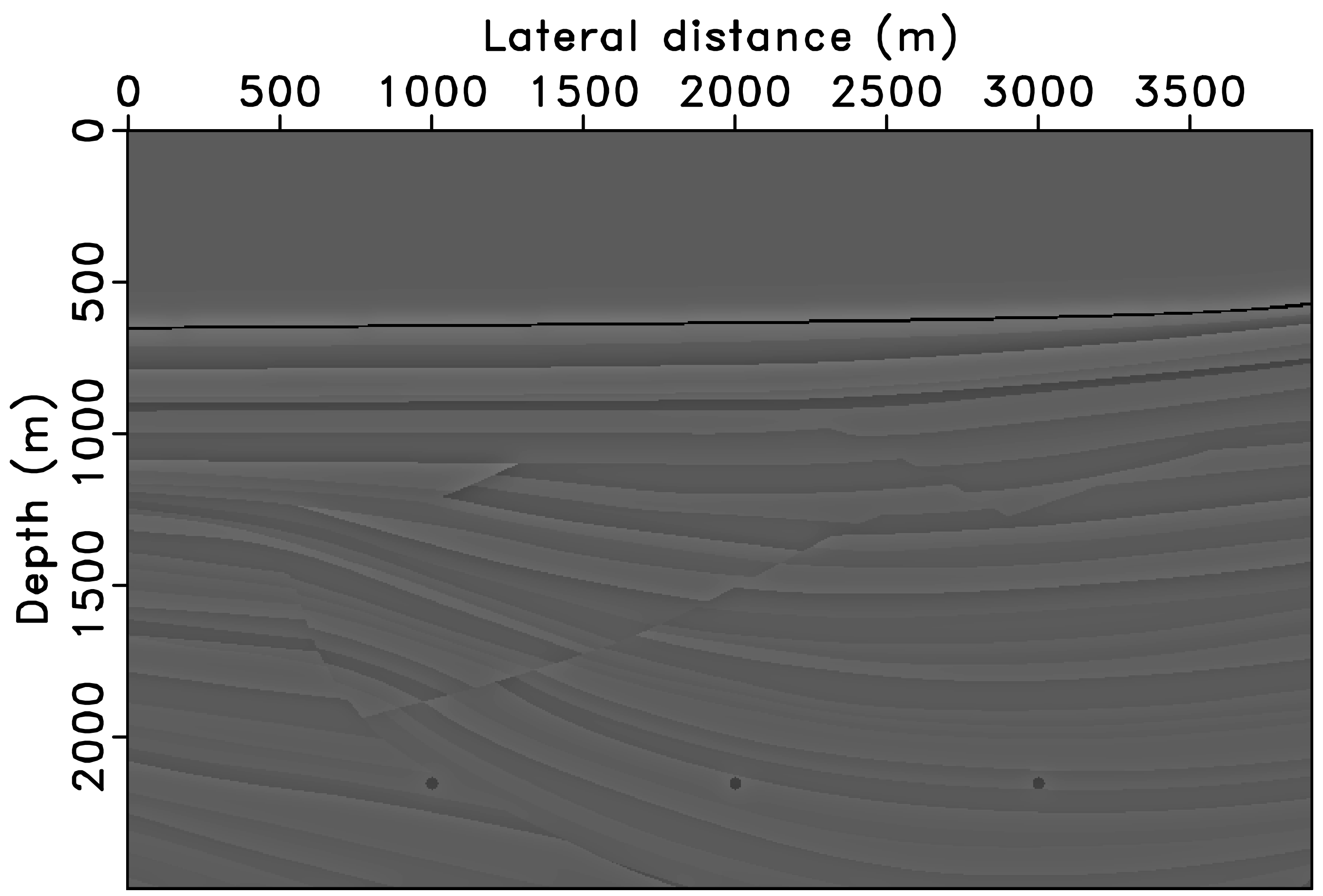
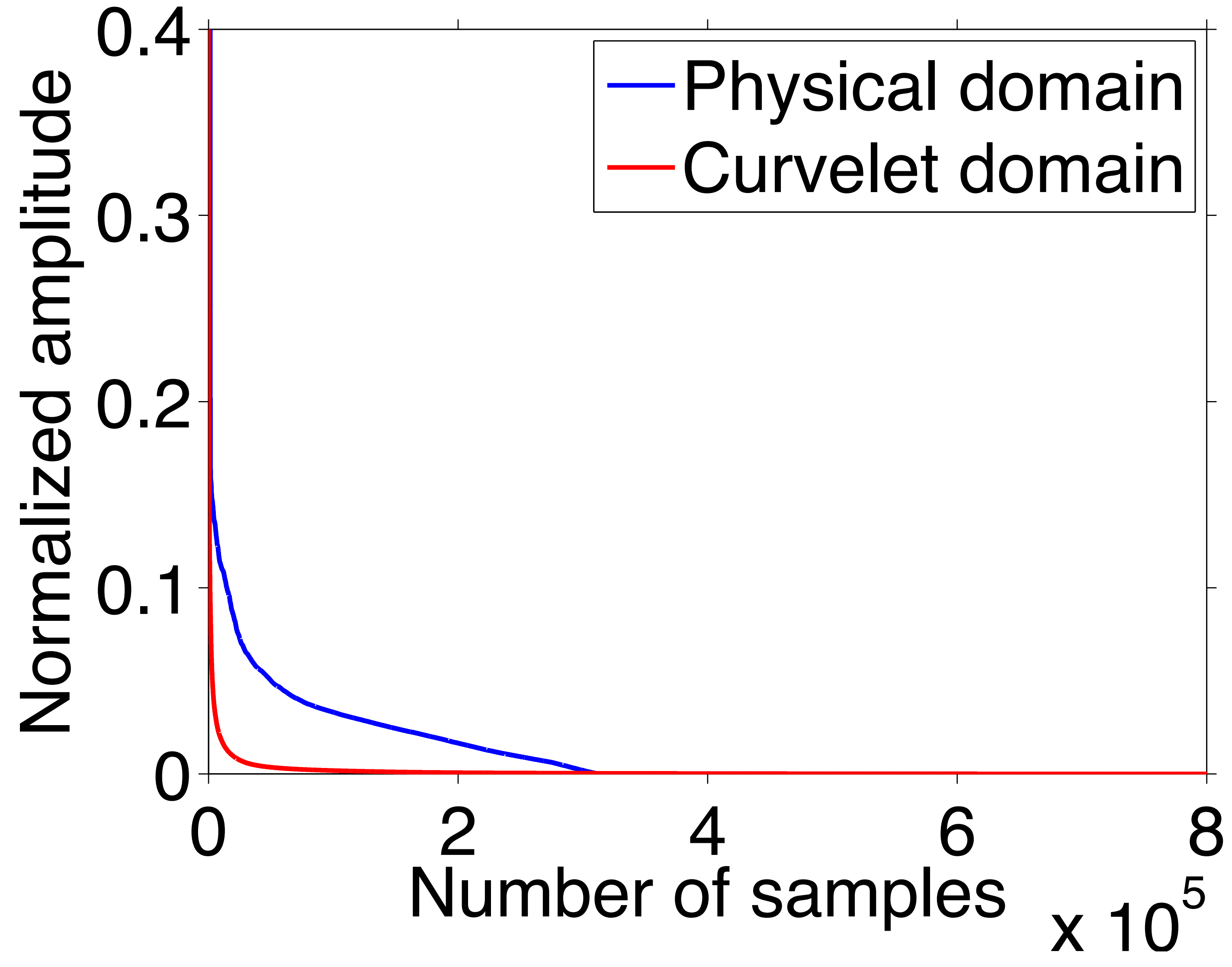
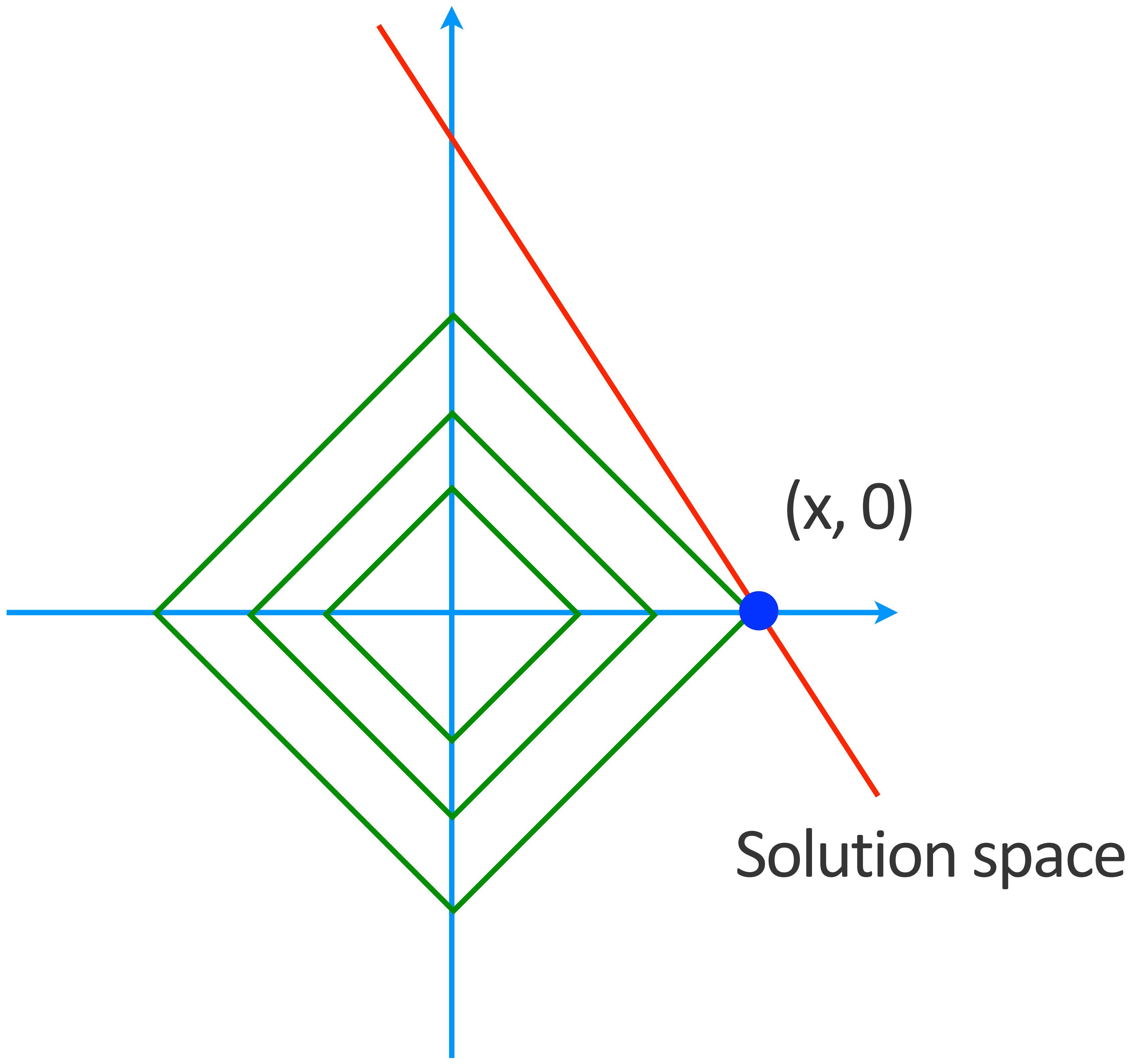
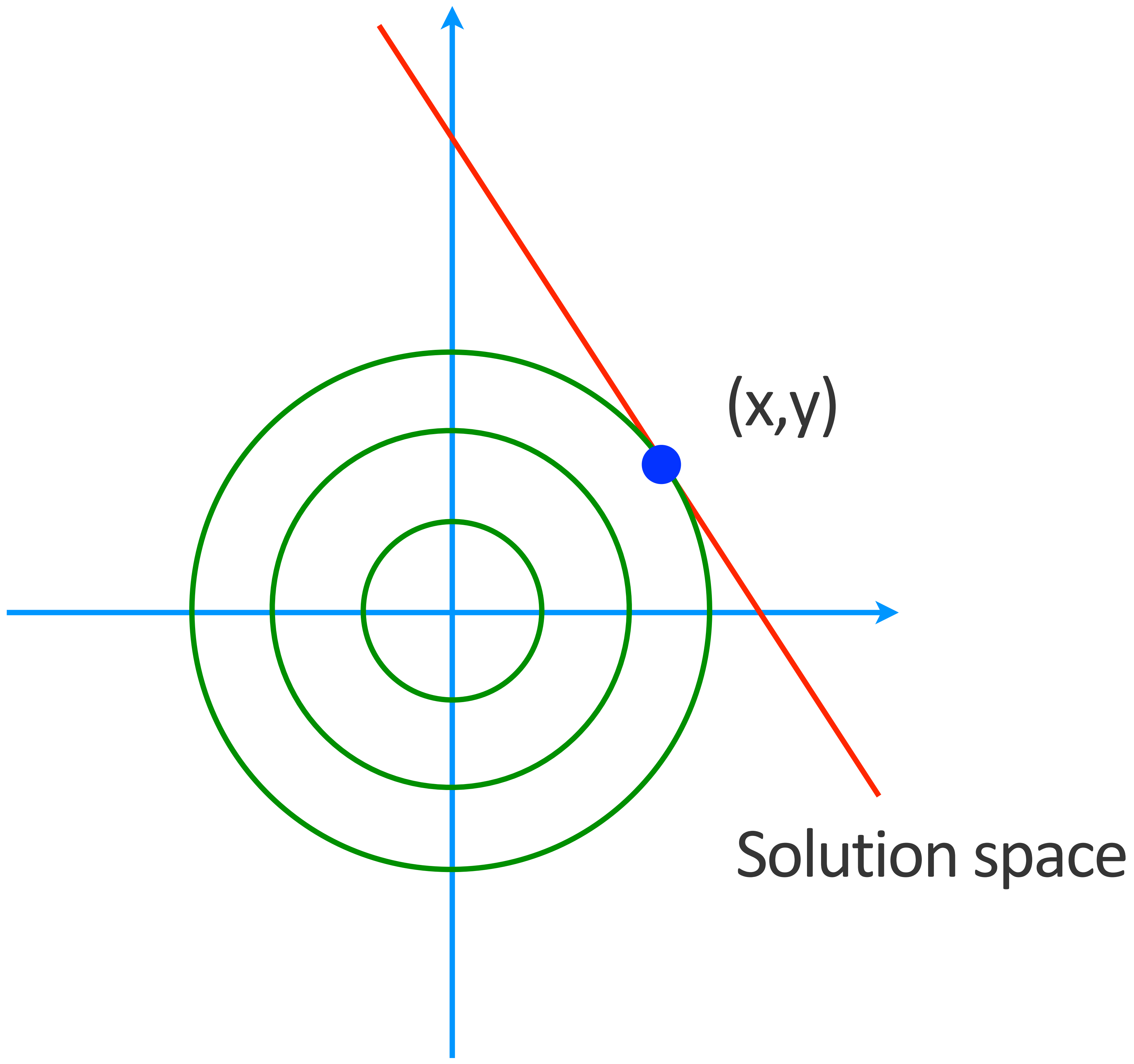




L1 vs L2 sparsity



L1



L2

Source estimation

Chapter 5

- The joint inversion formulation requires the knowledge of the source wavelet:

$$\mathbf{S}_i = w_i \mathbf{I} \longrightarrow \underline{\mathbf{S}}_i = w_i \underline{\mathbf{E}}$$

- In each iteration, once $\mathbf{x}$ is updated, w_i is also updated:

$$\tilde{w}_i(\mathbf{x}) = \frac{< \underline{\mathbf{p}}_i - \nabla \mathbf{F}_i[\mathbf{m}_0, -\underline{\mathbf{P}}_i]\mathbf{C}^*\mathbf{x}, \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{E}}]\mathbf{C}^*\mathbf{x} >}{< \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{E}}]\mathbf{C}^*\mathbf{x}, \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{E}}]\mathbf{C}^*\mathbf{x} >}$$

Removal of scaling ambiguity using multiples

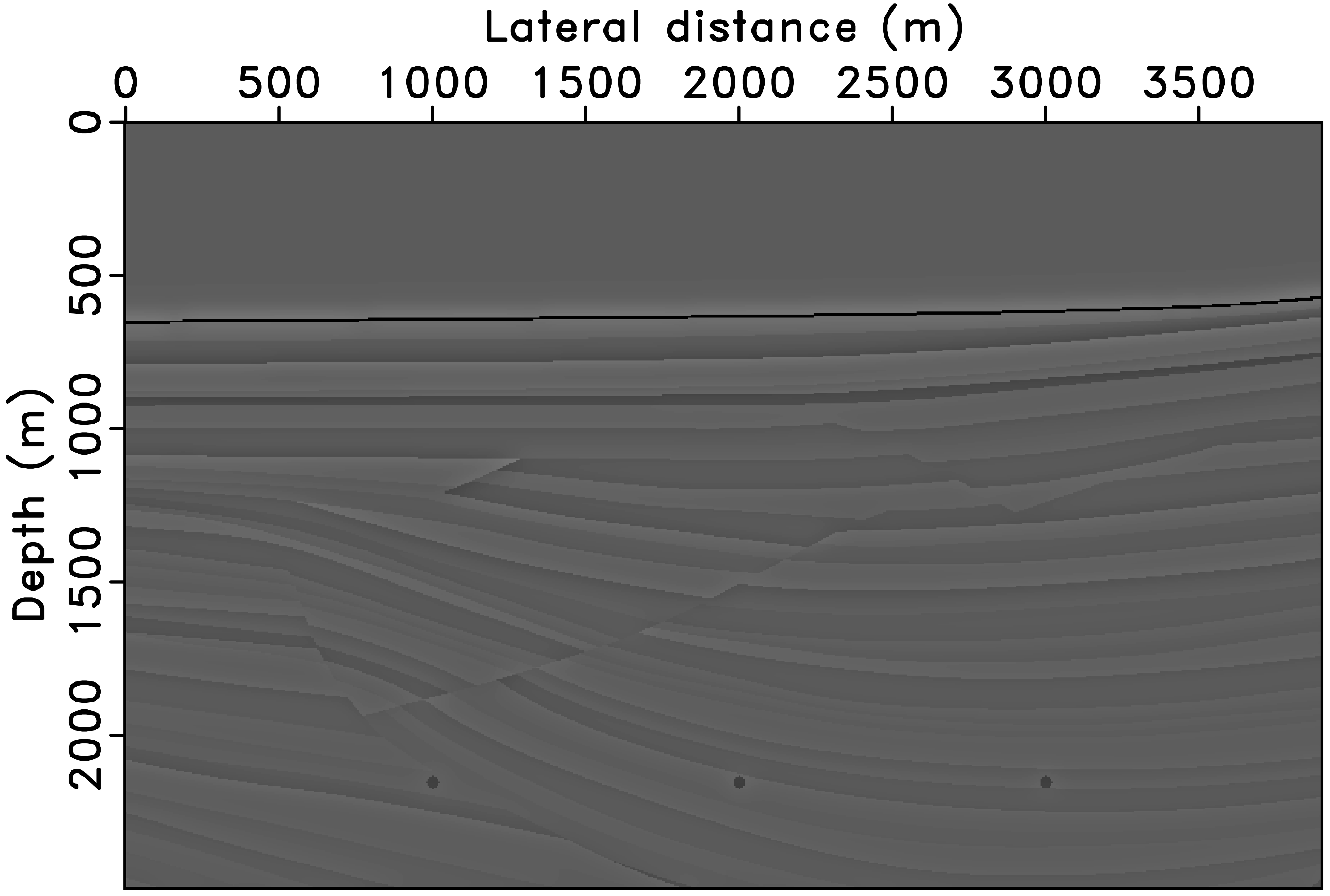
With ***primary***-only data:

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i\in\Omega} \|\underline{\mathbf{p}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, w_i\underline{\mathbf{E}}]\mathbf{C}^*\mathbf{x}\|_2^2 \quad \longrightarrow \quad f(\mathbf{x}, \boldsymbol{w}) = f(\alpha\mathbf{x}, \frac{1}{\alpha}\boldsymbol{w})$$

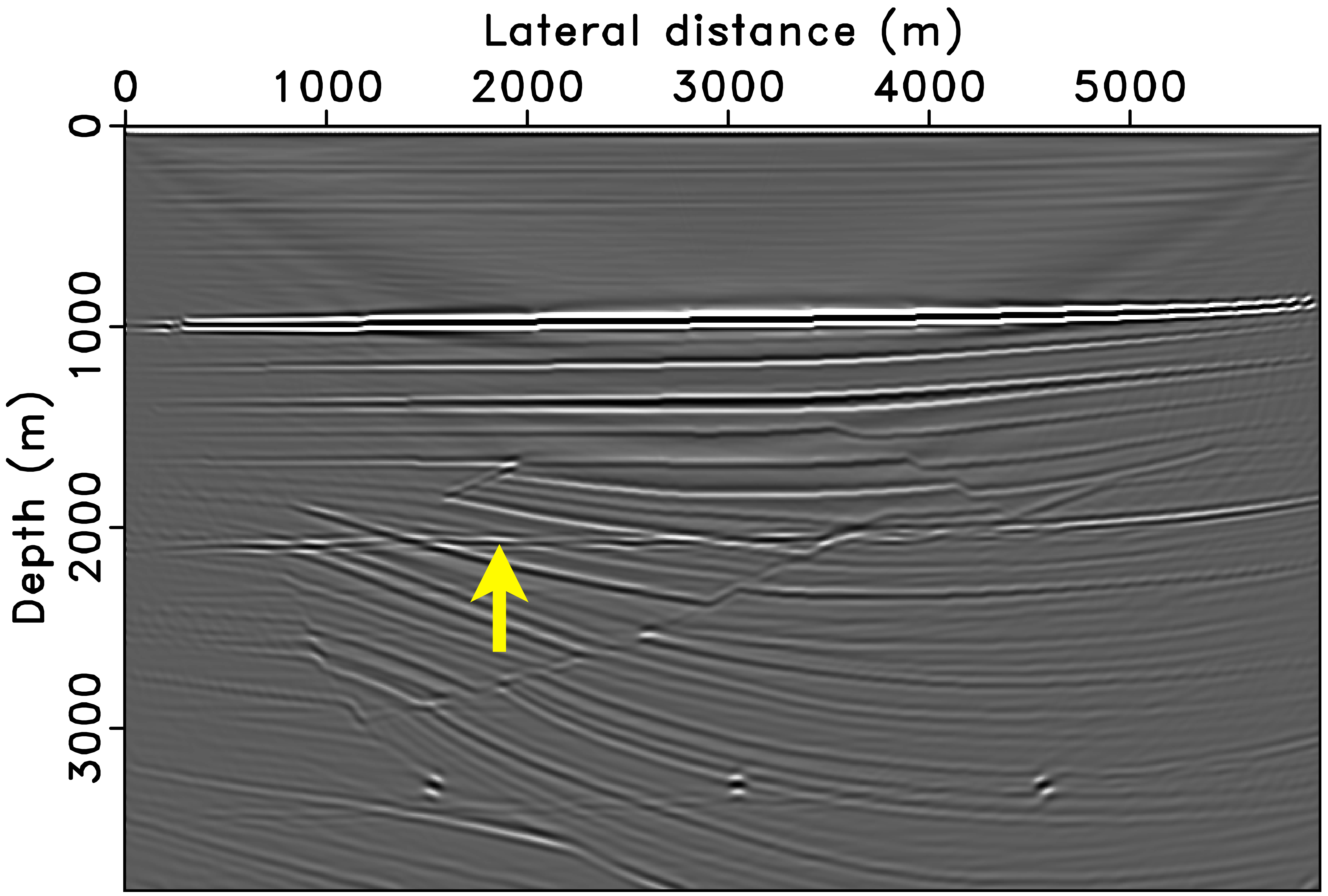
With ***multiples***:

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i\in\Omega} \|\underline{\mathbf{p}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, w_i\underline{\mathbf{E}} - \underline{\mathbf{P}}_i]\mathbf{C}^*\mathbf{x}\|_2^2 \quad \longrightarrow \quad f(\mathbf{x}, \boldsymbol{w}) \neq f(\alpha\mathbf{x}, \frac{1}{\alpha}\boldsymbol{w})$$

Synthetic example

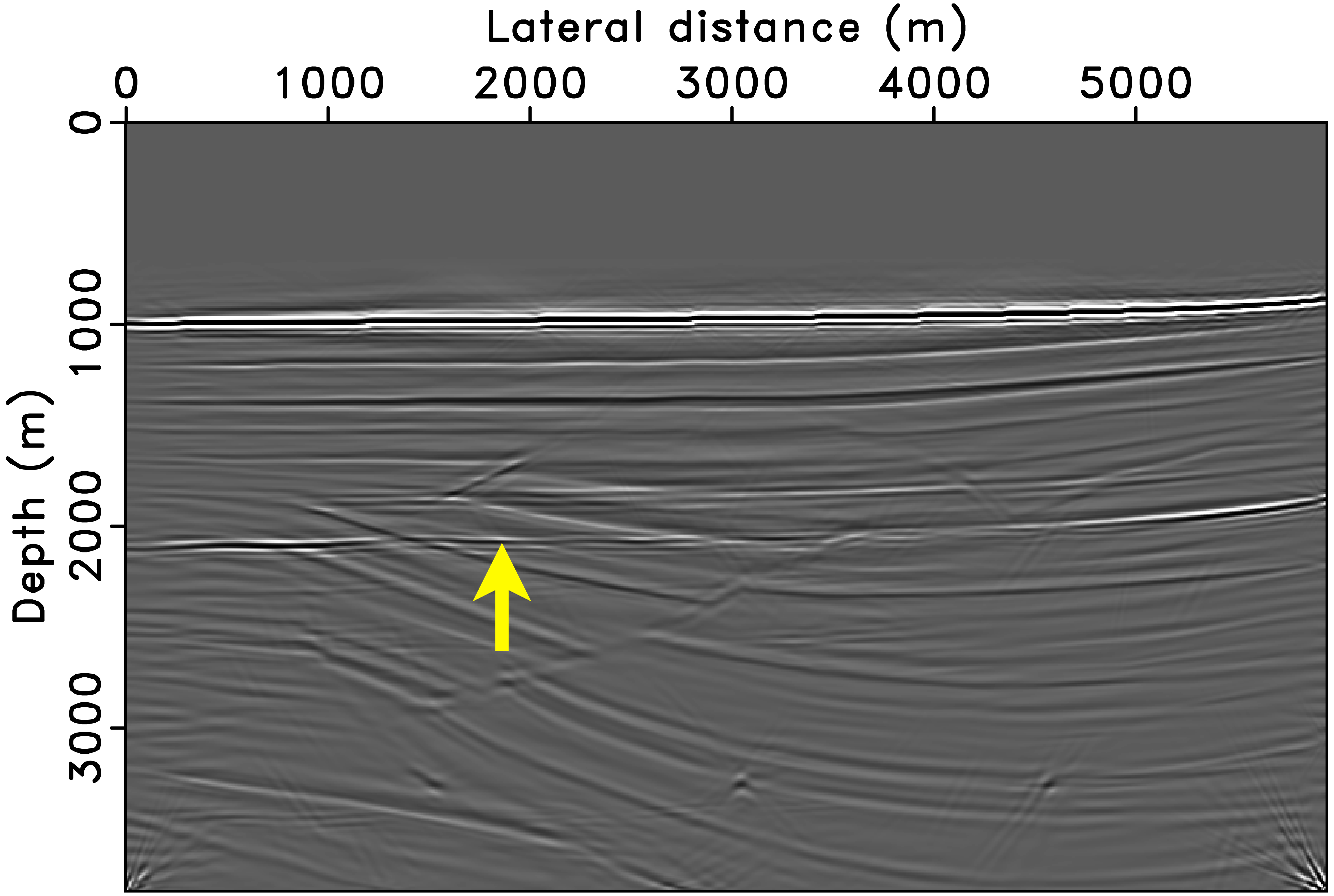


True perturbations



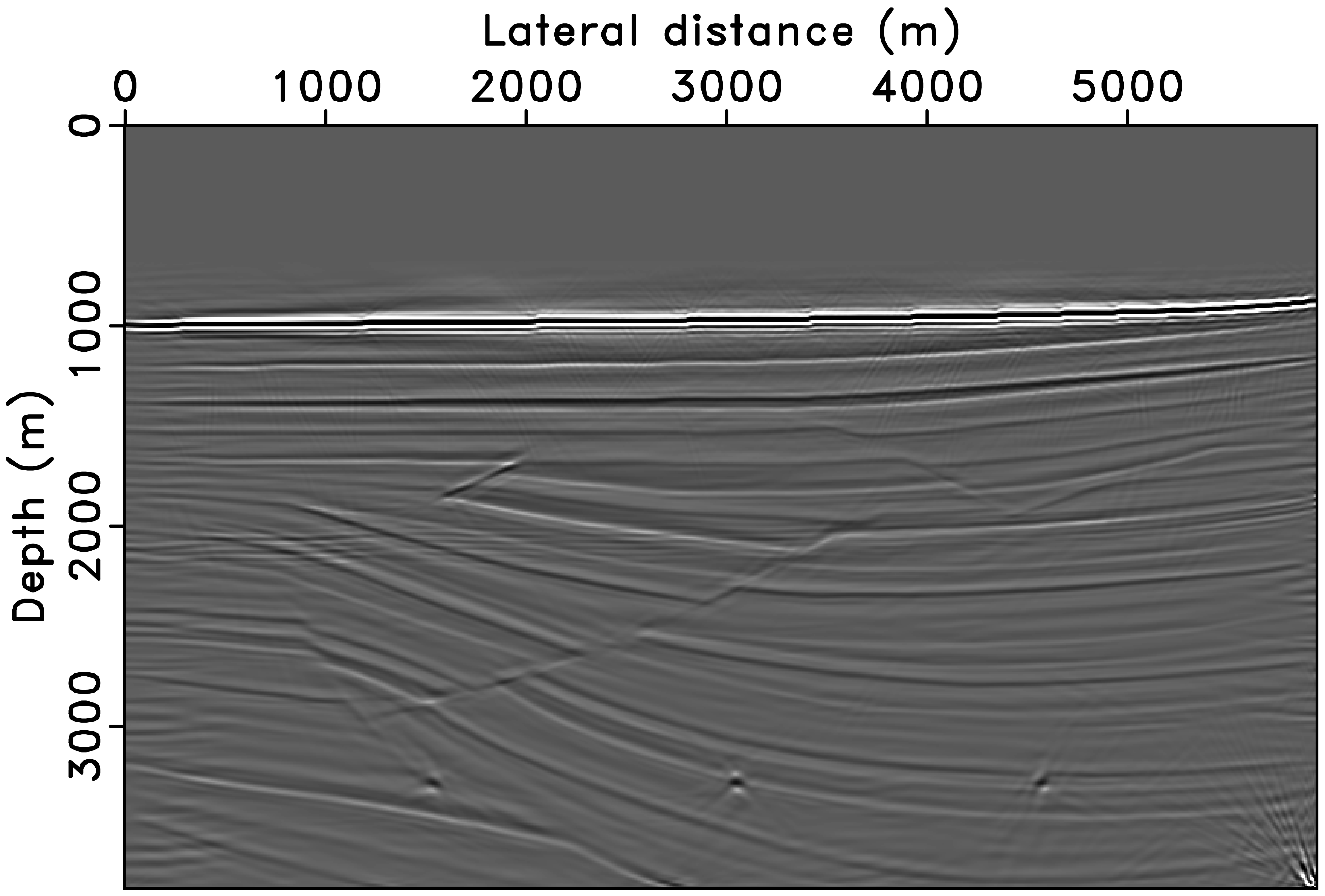
Cross-correlation image
w/ **total downgoing** data as areal source

simulation cost ~**1 conventional imaging** of all data



Fast inversion w/ sparsity promotion
w/o accounting for multiples

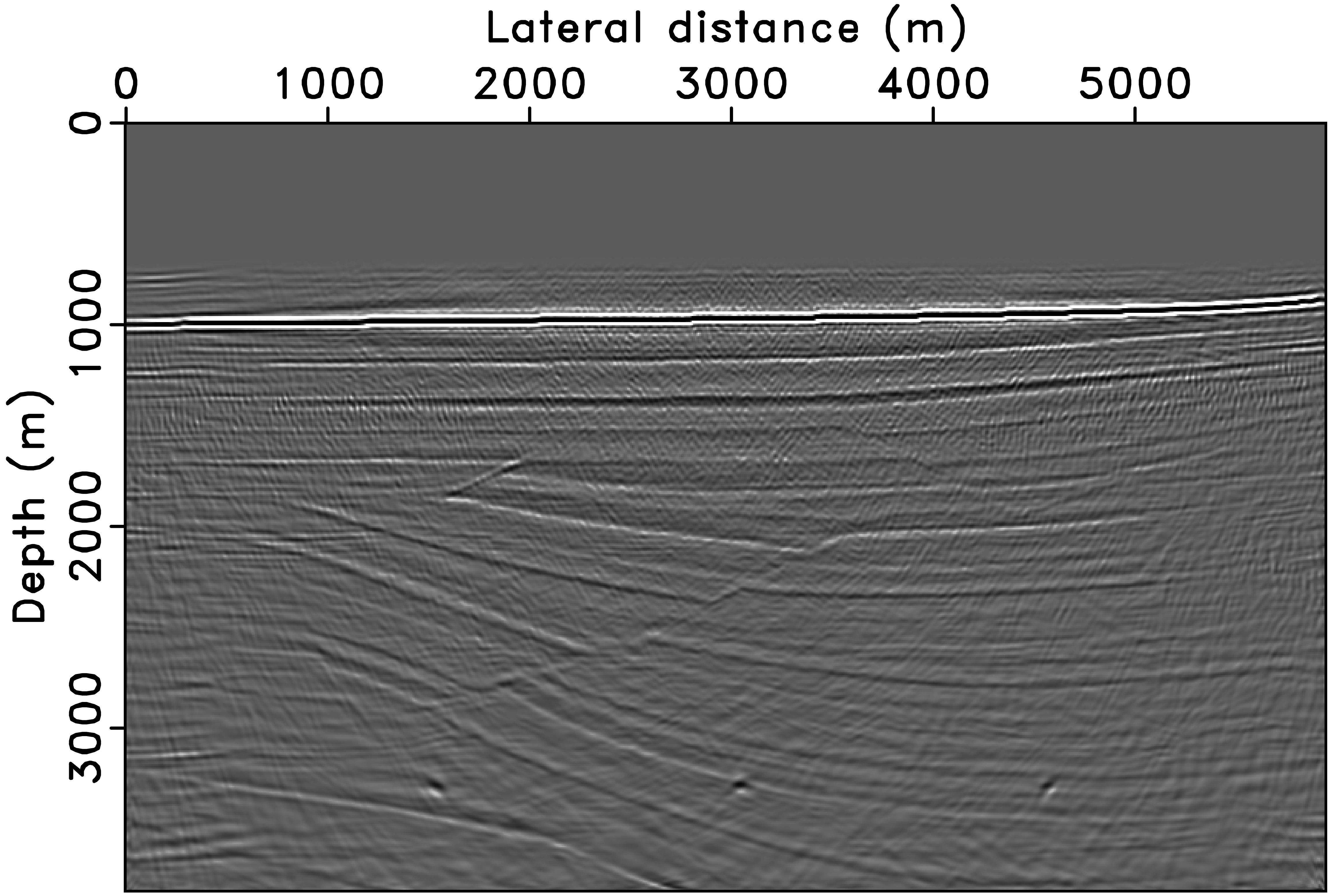
simulation cost ~**1 conventional imaging** of all data



Fast inversion w/ sparsity promotion

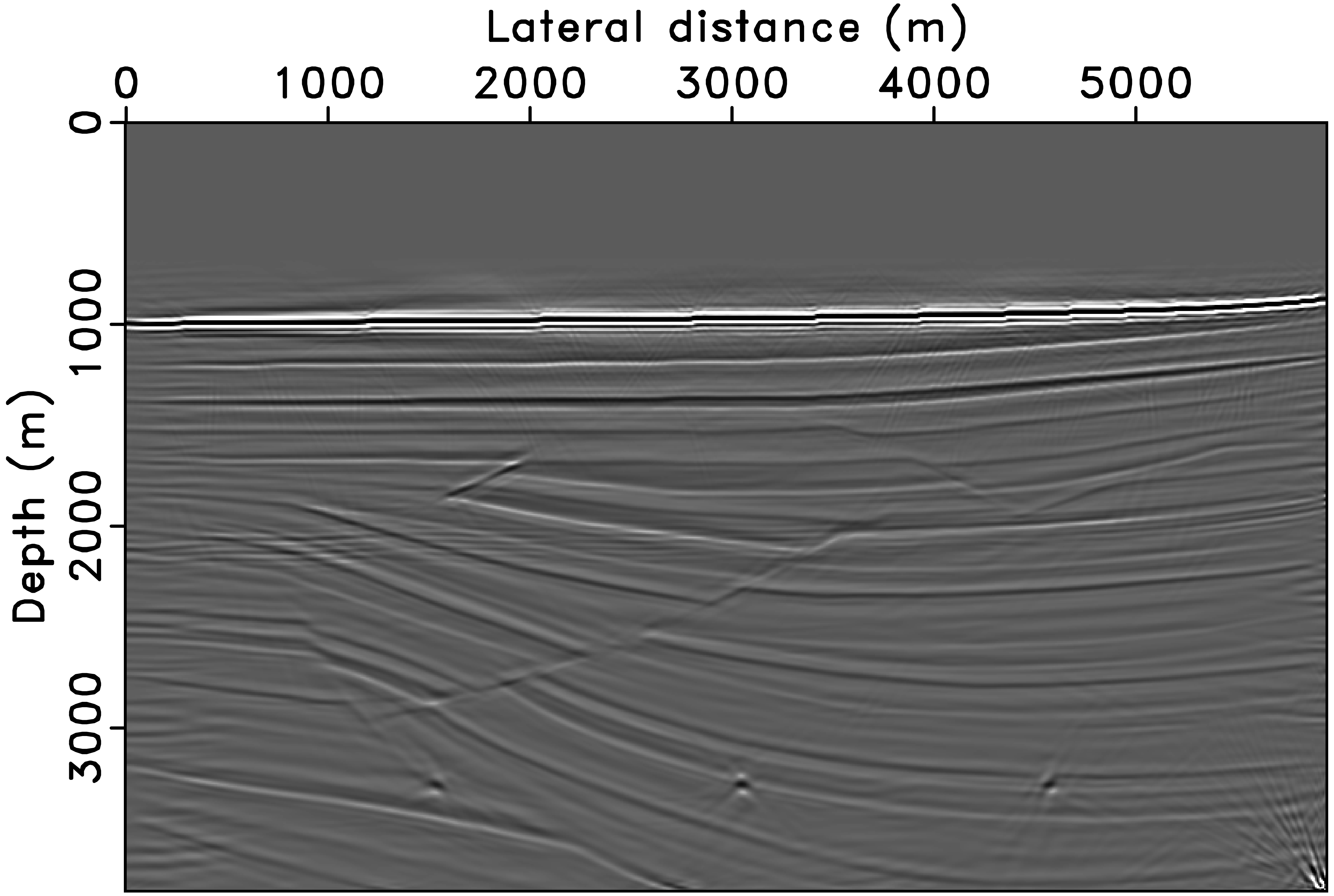
w/ accounting for multiples

simulation cost ~**1 conventional imaging** of all data



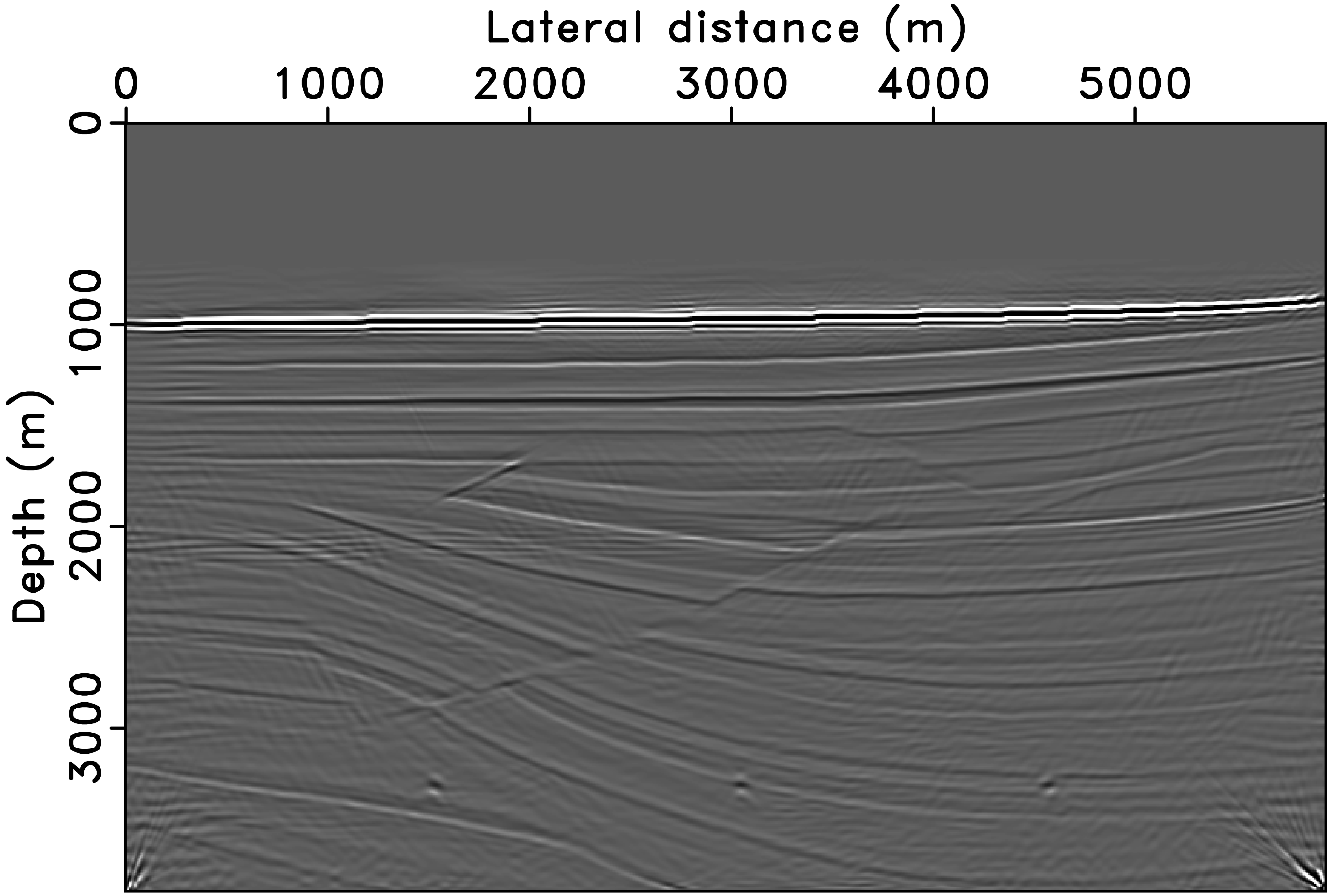
same as the previous image
except **without** rerandomization

simulation cost ~**1 conventional imaging** of all data



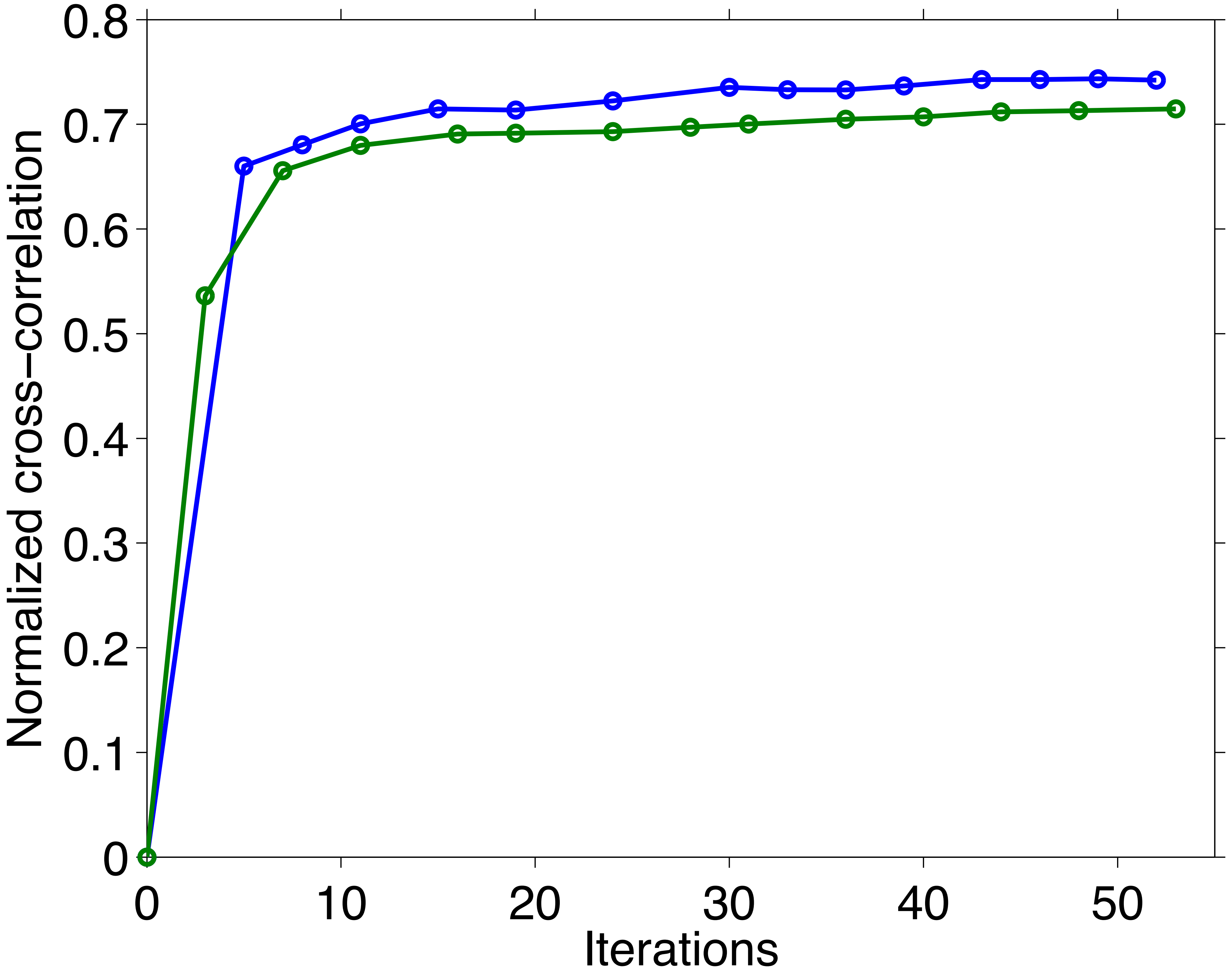
Fast inversion w/ sparsity promotion
w/ accounting for multiples

simulation cost ~**1.5 conventional imaging** of all data



Fast inversion w/ sparsity promotion
w/ accounting for multiples

Convergence analysis



Normalized cross-correlation:

$$\mathrm{NCC}(\mathbf{v}_1, \mathbf{v}_2) = \frac{< \mathbf{v}_1, \mathbf{v}_2 >}{\|\mathbf{v}_1\|_2 \|\mathbf{v}_2\|_2}$$

NCC between true model and inverted model:

blue: inversion w/ ***true*** source wavelet

green: inversion w/ source ***estimation***

Conclusions

- Multiples, if properly used, ***increase*** subsurface illumination.
- Multiples can be properly imaged by iterative ***inversion***.
- **Contributions** of this work:
  - The otherwise prohibitively expensive inversion w/ multiples can now be carried out ***efficiently***.
  - Joint inversion of primaries and ***all orders*** of multiples is ***feasible*** via source estimation.

Future work and further extension

Simplified implementation of sparse inversion

Future work and further extension

Simplified implementation of sparse inversion

```
r = b_sub - A_sub*x;
g = A_sub'*r;
rnorm = norm(r,2);
gnorm = norm(g,2);
steplength = (rnorm/gnorm)^2;
z = z+steplength*g;
x = sign(z).*max(0,abs(z)-lambda);
```

LBP

Future work and further extension

Simplified implementation of sparse inversion

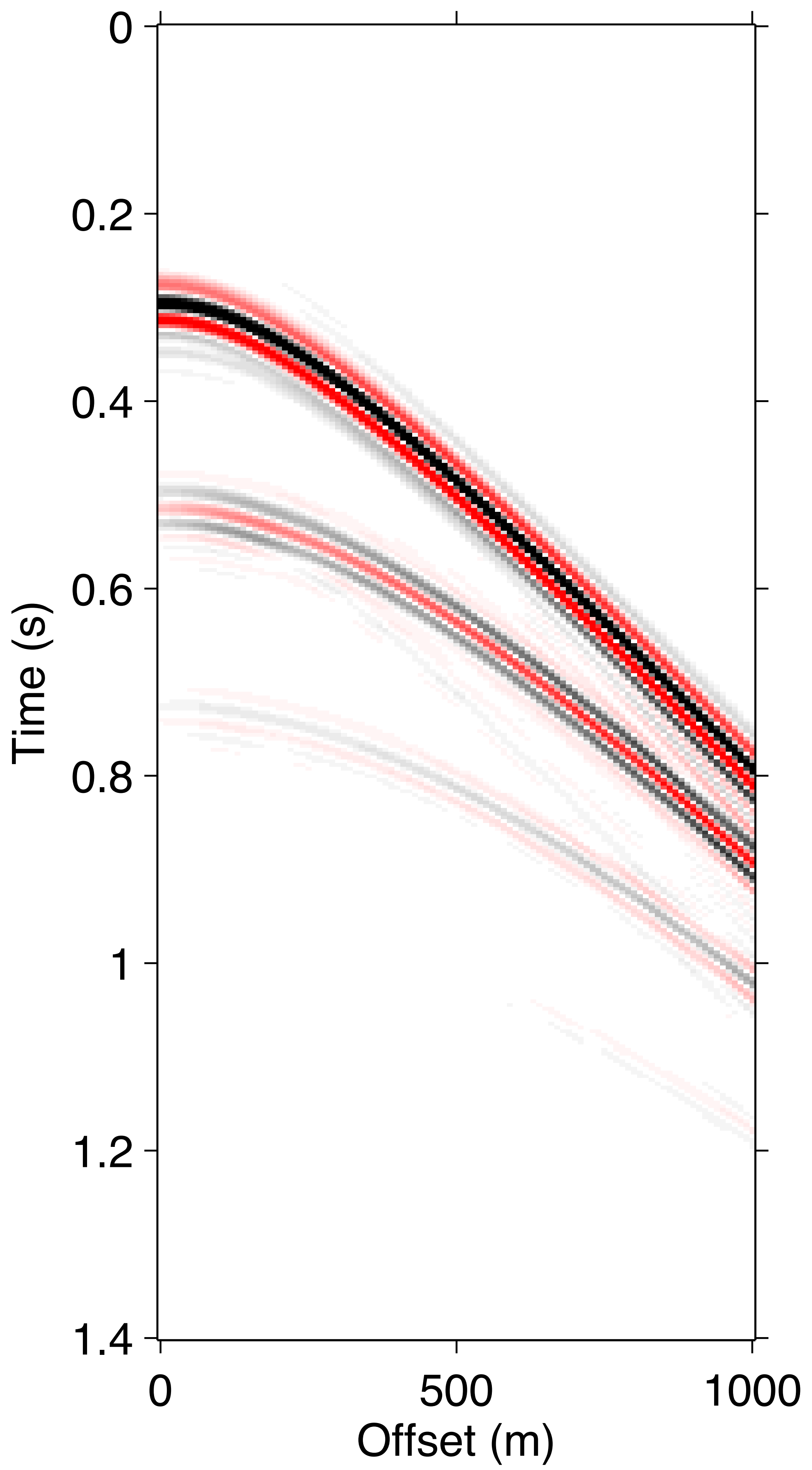
```
r = b_sub - A_sub*x;
g = A_sub'*r;
rnorm = norm(r,2);
gnorm = norm(g,2);
steplength = (rnorm/gnorm)^2;
z = z+steplength*g;
x = sign(z).*max(0,abs(z)-lambda);
```

LBP

```
function [x,r,g,info] = spgl1( A, b, tau, sigma, x, options )

m = length(b);

%----------------------------------------------------------------------
% Check arguments.
%----------------------------------------------------------------------
if ~exist('options','var'), options = []; end
if ~exist('x','var'), x = []; end
if ~exist('sigma','var'), sigma = []; end
if ~exist('tau','var'), tau = []; end

if nargin < 2 || isempty(b) || isempty(A)
   error('At least two arguments are required');
elseif isempty(tau) && isempty(sigma)
   tau = 0;
   sigma = 0;
   singleTau = false;
elseif isempty(sigma) % && ~isempty(tau)  <-- implied
   singleTau = true;
else
   if isempty(tau)
      tau = 0;
   end
   singleTau = false;
end

%----------------------------------------------------------------------
% Grab input options and set defaults where needed.
%----------------------------------------------------------------------
defaultopts = spgSetParms(...
'fid'        ,      1 , ... % File ID for output
'verbosity'  ,      2 , ... % Verbosity level
'iterations' ,   10*m , ... % Max number of iterations
'nPrevVals'  ,      3 , ... % Number previous func values for linesearch
'bpTol'      ,  1e-06 , ... % Tolerance for basis pursuit solution
'optTol'     ,  1e-04 , ... % Optimality tolerance
'decTol'     ,  1e-04 , ... % Req'd rel. change in primal obj. for Newton
'stepMin'    ,  1e-16 , ... % Minimum spectral step
'stepMax'    ,  1e+05 , ... % Maximum spectral step
'rootMethod' ,      2 , ... % Root finding method: 2=quad,1=linear (not used).
'activeSetIt',    Inf , ... % Exit with EXIT_ACTIVE_SET if nnz same for # its.
'subspaceMin',      0 , ... % Use subspace minimization
'iscomplex'  ,    NaN , ... % Flag set to indicate complex problem
'maxMatvec'  ,    Inf , ... % Maximum matrix-vector multiplies allowed
'weights'    ,      1 , ... % Weights W in ||Wx||_1
'Kaczmarz'   ,      0 , ... % Toggles whether Kaczmarz mode is on (experimental)
'KaczScale'  ,      1 , ... % Scaling factor for Tau when using Kaczmarz-type submatrices
'quitPareto' ,      0 , ... % Exits when pareto curve is reached
'minPareto'  ,      3 , ... % If quitPareto is on, the minimum number of iterations before checking for quitPareto conditions
'lineSrchIt' ,      1 , ... % Maximum number of line search iterations for spgLineCurvy, originally 10 ...
'feasSrchIt' ,  10000 , ... % Maximum number of feasible direction line search iteraitons, originally 10 ...
'ignorePErr' ,      0 , ... % Ignores projections error by issuing a warning instead of an error ...
'project'    , @NormL1_project , ...
'primal_norm', @NormL1_primal  , ...
'dual_norm'  , @NormL1_dual      ...
   );
options = spgSetParms(defaultopts, options);

fid           = options.fid;
logLevel      = options.verbosity;
maxIts        = options.iterations;
nPrevVals     = options.nPrevVals;
bpTol         = options.bpTol;
optTol        = options.optTol;
decTol        = options.decTol;
stepMin       = options.stepMin;
stepMax       = options.stepMax;
activeSetIt   = options.activeSetIt;
subspaceMin   = options.subspaceMin;
maxMatvec     = max(3,options.maxMatvec);
weights       = options.weights;
quitPareto    = options.quitPareto;
minPareto     = options.minPareto;
Kaczmarz      = options.Kaczmarz;
lineSrchIt    = options.lineSrchIt;
feasSrchIt    = options.feasSrchIt;
ignorePErr    = options.ignorePErr;

% maxLineErrors TEMPORARILY DISABLED to prevent very large scaling issue to throw off algorithm
maxLineErrors = Inf;     % Maximum number of line-search failures.
pivTol        = 1e-12;  % Threshold for significant Newton step.

%----------------------------------------------------------------------
% Initialize local variables.
%----------------------------------------------------------------------
iter          = 0;  itnTotLSQR = 0; % Total SPGL1 and LSQR iterations.
nProdA        = 0;  nProdAt    = 0;
lastFv        = -inf(nPrevVals,1);  % Last m function values.
nLineTot      = 0;                  % Total no. of linesearch steps.
printTau      = false;
nNewton       = 0;
bNorm         = norm(b,2);
stat          = false;
timeProject   = 0;
timeMatProd   = 0;
nnzIter       = 0;                  % No. of its with fixed pattern.
nnzIdx        = [];                 % Active-set indicator.
subspace      = false;              % Flag if did subspace min in current itn.
stepG         = 1;                  % Step length for projected gradient.
testUpdateTau = 0;                  % Previous step did not update tau

% Determine initial x, vector length n, and see if problem is complex
```

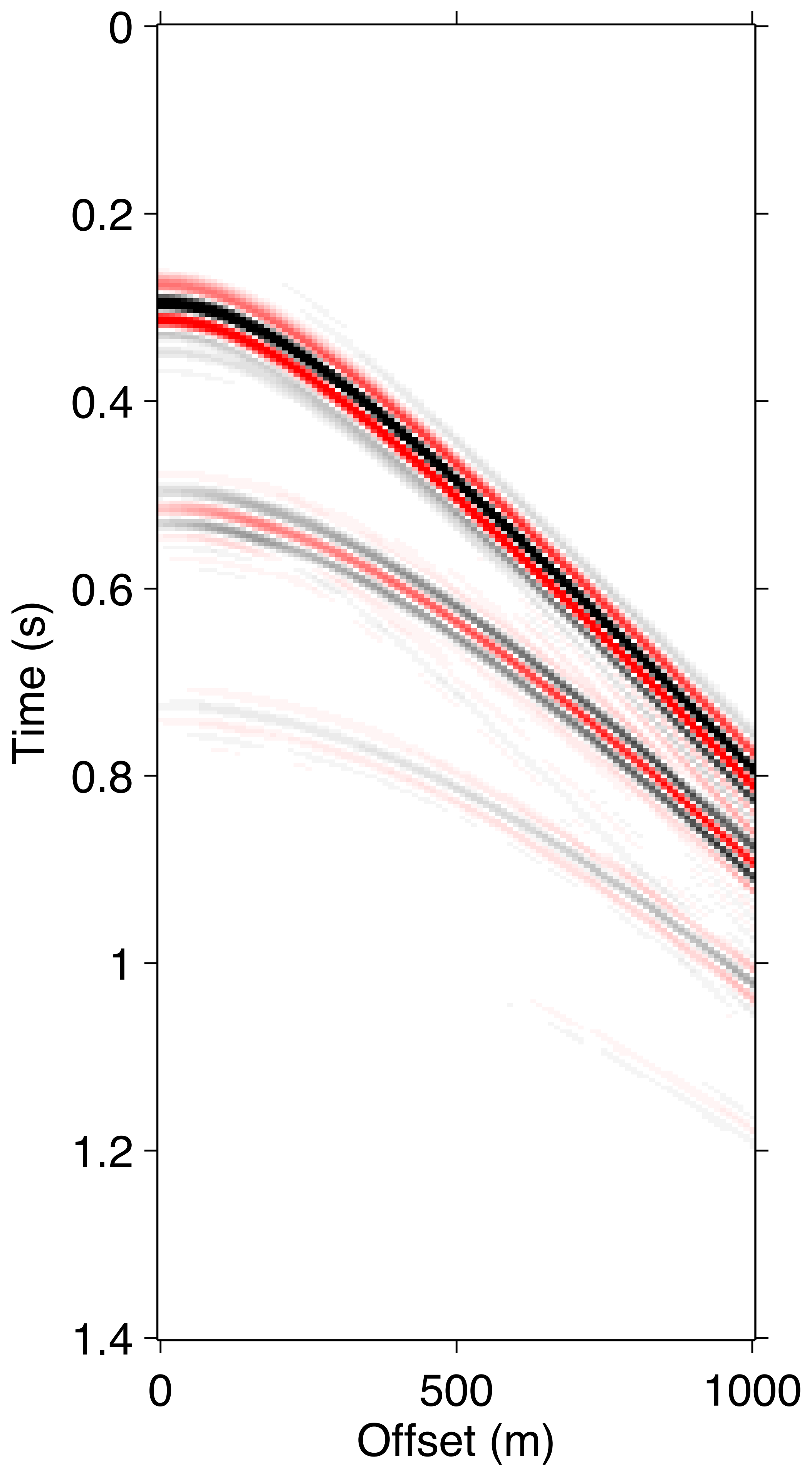
SPGl1

Future work and further extension

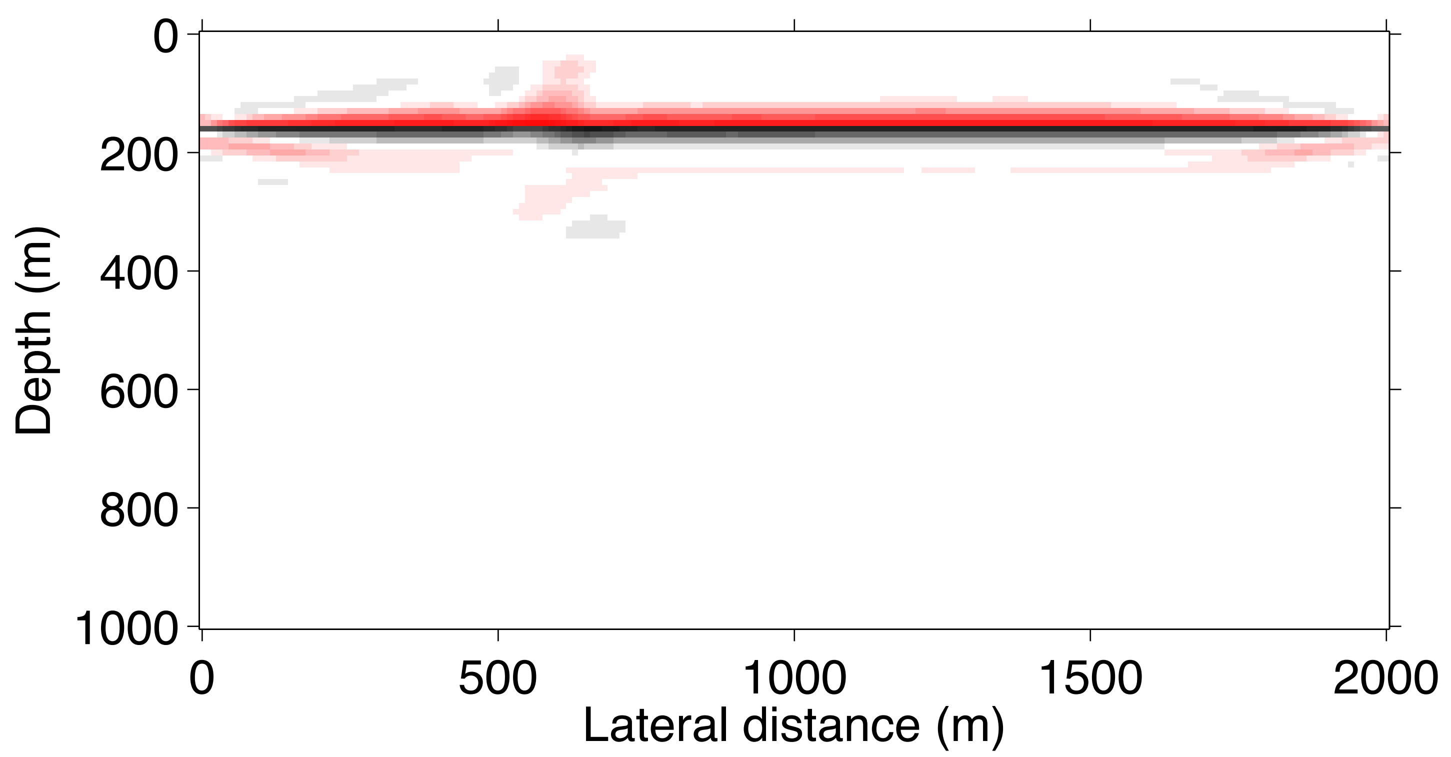


Future work and further extension

Chapter 8

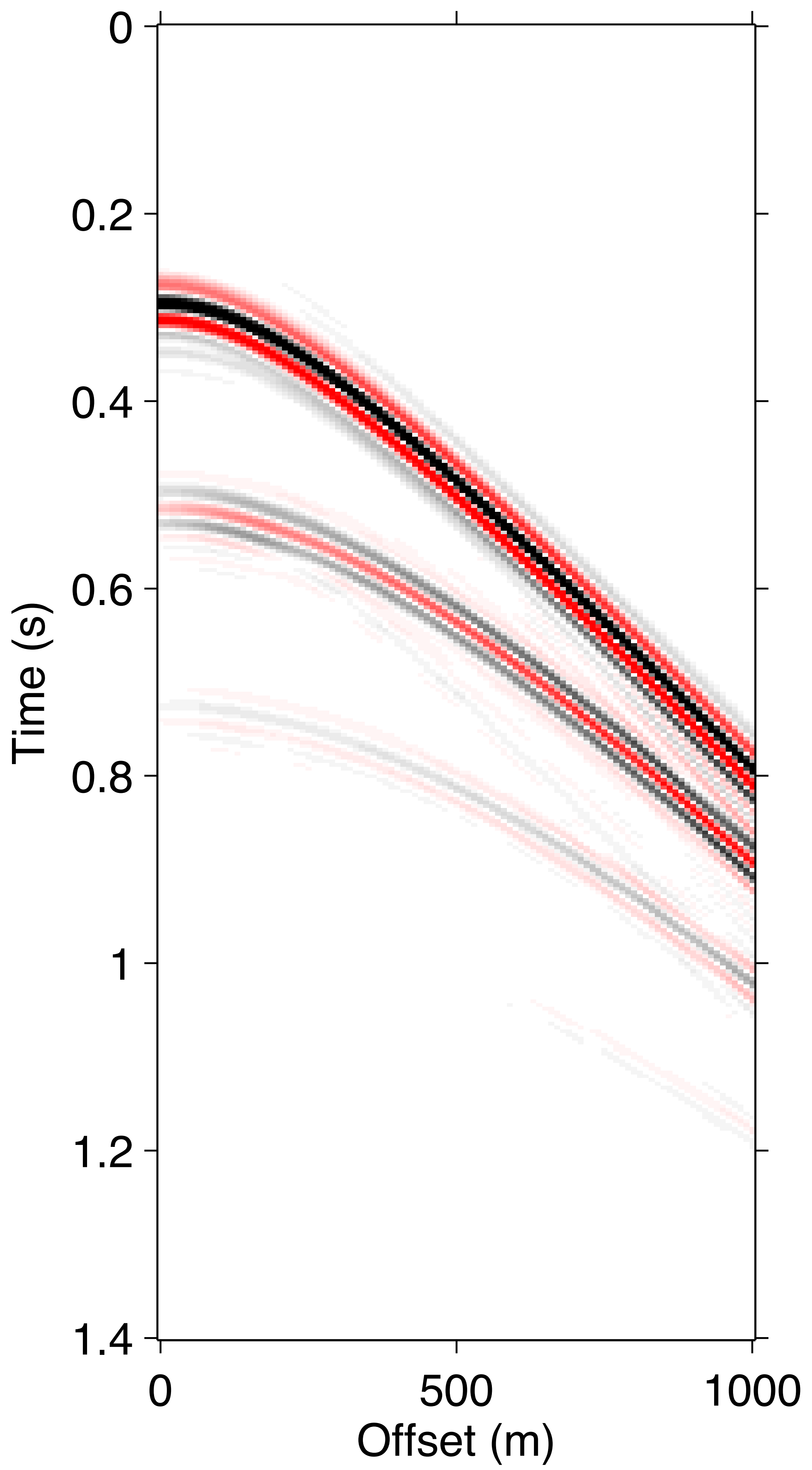


Imaging

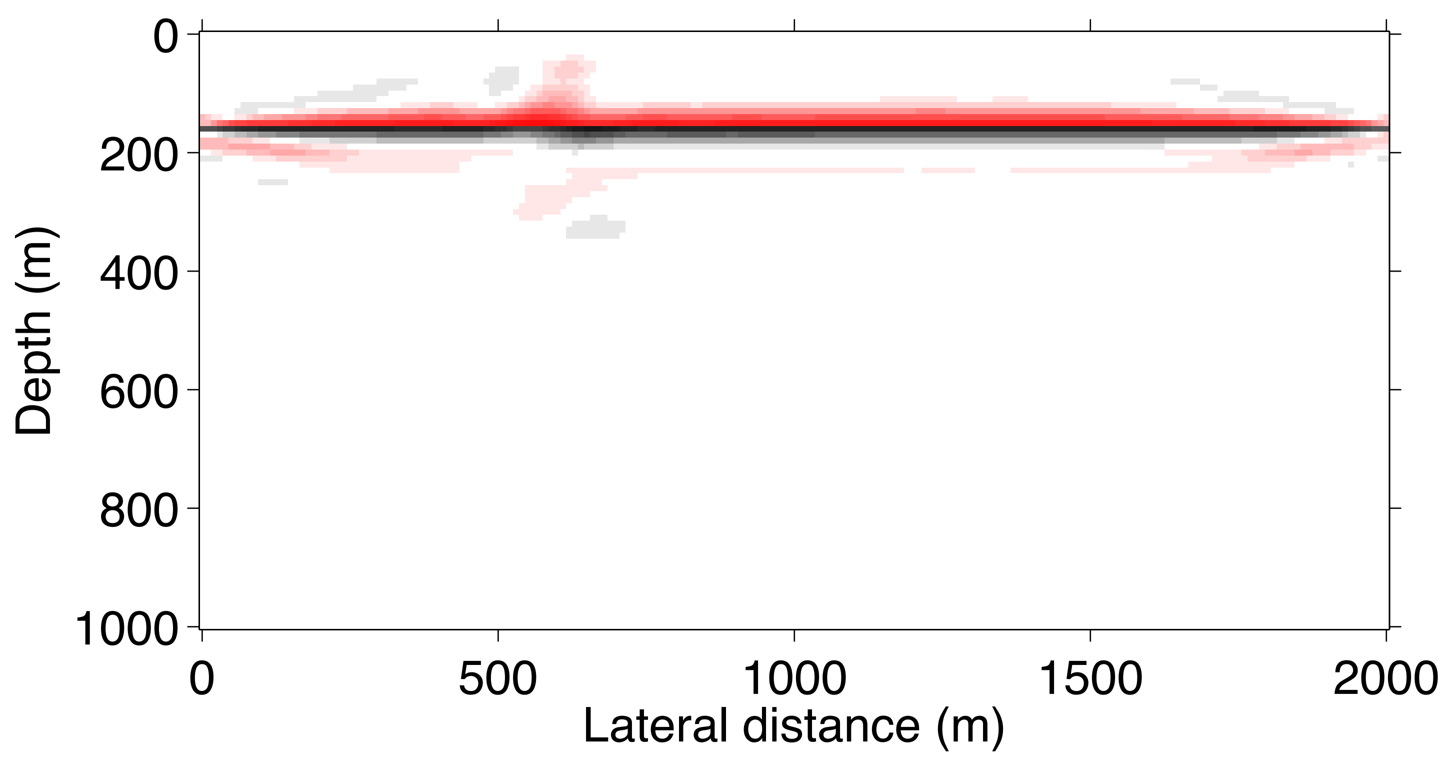


Future work and further extension

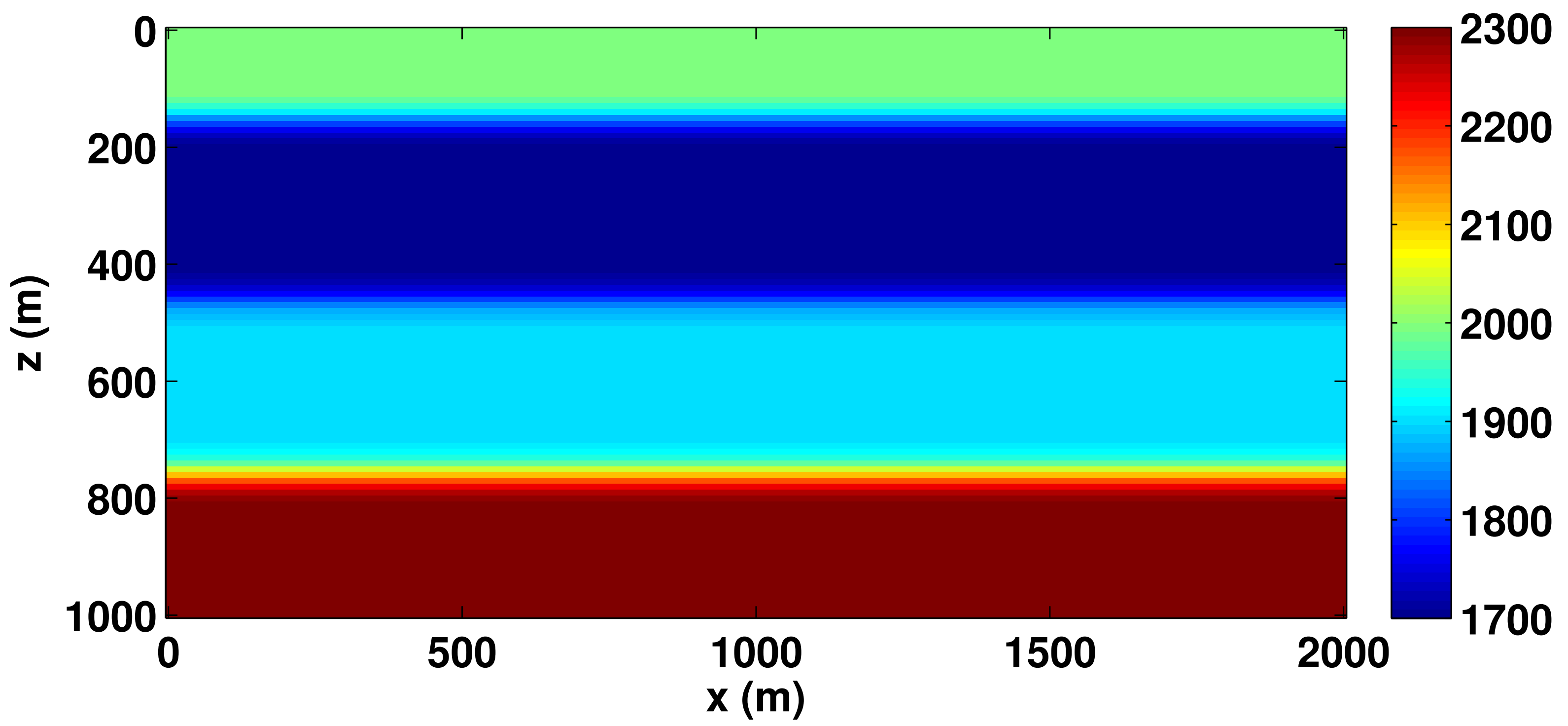
Chapter 8



Imaging



Velocity model building



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Thank you!

Questions?

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