# Efficient Seismic Imaging with Spectral Projector and Joint Sparsity

Lina Miao

SLIM

University of British Columbia

# Outline

Efficient depth stepping migration with the two-way wave equation

- spectral projector

Efficient sparsity promoting seismic imaging

- joint sparsity

Efficient large scale optimization algorithm

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**Efficient depth stepping migration with the two-way wave equation**

- **spectral projector**

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- joint sparsity

Efficient large scale optimization algorithm

# Problem formulation

$$p_{tt} = v(z,x)^2(p_{xx} + p_{zz})$$
$$p(x,0,t) = 0,\ t \leq 0$$
$$\begin{cases} p(x,z,0) = f(x) \\ p_t(x,z,0) = g(x) \end{cases}$$

$$\xrightarrow{\text{Fourier Transform}}$$

$$\hat{p}_{zz} = \left[ -\left( \frac{\omega}{v(x,z)} \right)^2 - D_{xx} \right] \hat{p} \equiv L\hat{p}$$
$$\begin{cases} \hat{p}(x,z,\omega) = q(x,z,\omega) \\ \hat{p}_z(x,z,\omega) = q_z(x,z,\omega) \end{cases}$$

Two way wave-equation as an initial value problem

Unstable with evanescent wave components

# Problem formulation

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$\xrightarrow{\text{Spectral Projector } \mathcal{P}}$

$$\hat{p}_{zz} = \mathcal{P}L\mathcal{P}\hat{p}$$
$$\hat{p}(x,z_n,\omega) = q(x,z_n,\omega)$$
$$\hat{p_z}(x,z_n,\omega) = q_z(x,z_n,\omega)$$

Two way wave-equation as an initial value problem

Unstable with evanescent wave components

Stabilized initial value problem with spectral projector

L. Auslander and A. Tsao. On parallelizable eigensolvers. Citeseer, 1991.

N. J. Higham. Functions of matrices: theory and computation. Siam, 2008.

# Spectral projector

Geophysically → Unstable extrapolation operator with propagating and **evanescent** waves

$$L = V\Lambda V^*$$

Mathematically → Indefinite matrix with negative and **positive** eigenvalues

Get rid of all **evanescent** waves

$$k_z^2 = \left(\frac{\omega}{v(x,z)}\right)^2 - k^2 \geq 0$$

$$\lambda = -\left(\frac{\omega}{v(x,z)}\right)^2 + k^2 \leq 0$$

Set all **positive** eigenvalue to be 0

Stable extrapolation operator with only propagating waves

$$\mathcal{P}L\mathcal{P} = V\tilde{\Lambda}V^*$$

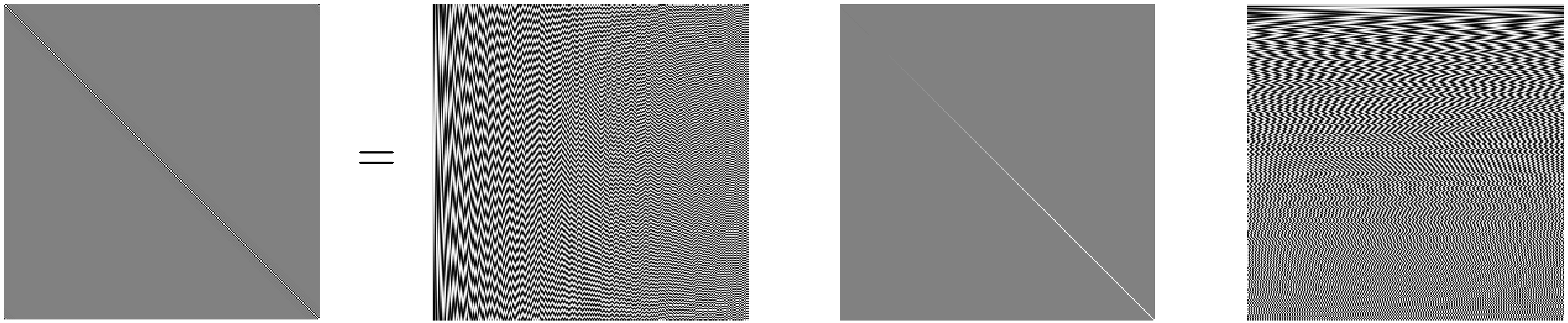
Definite matrix with only negative eigenvalues

$$\mathcal{P} = (I - \mathrm{sign}(L))/2$$

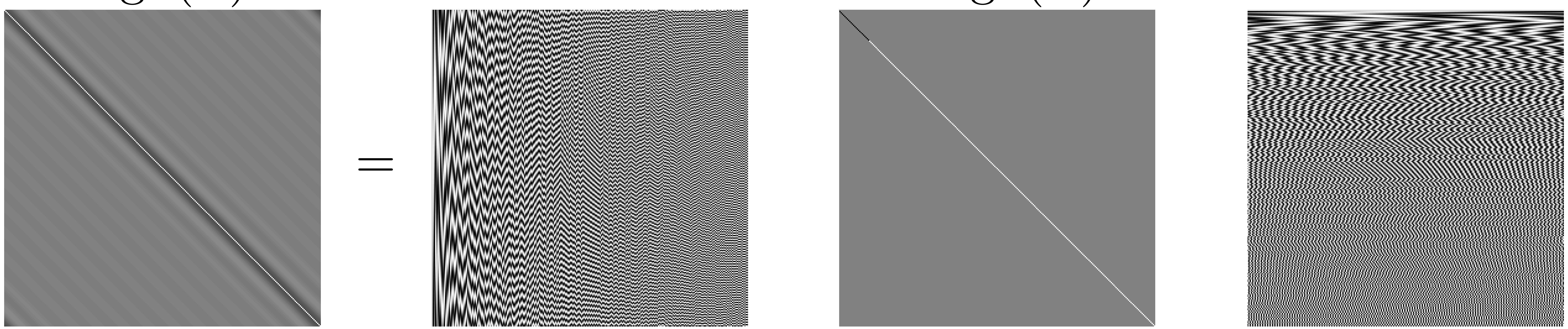
$$\mathrm{sign}(L) = V\mathrm{sign}(D)V^*$$

# Matrix sign function

$$L = V * \Lambda * V^*$$



$$\mathrm{sign}(L) = V * \mathrm{sign}(\Lambda) * V^*$$



C. S. Kenney and A. J. Laub. The matrix sign function. Automatic Control, IEEE Transactions on, 40(8):1330–1348, 1995.

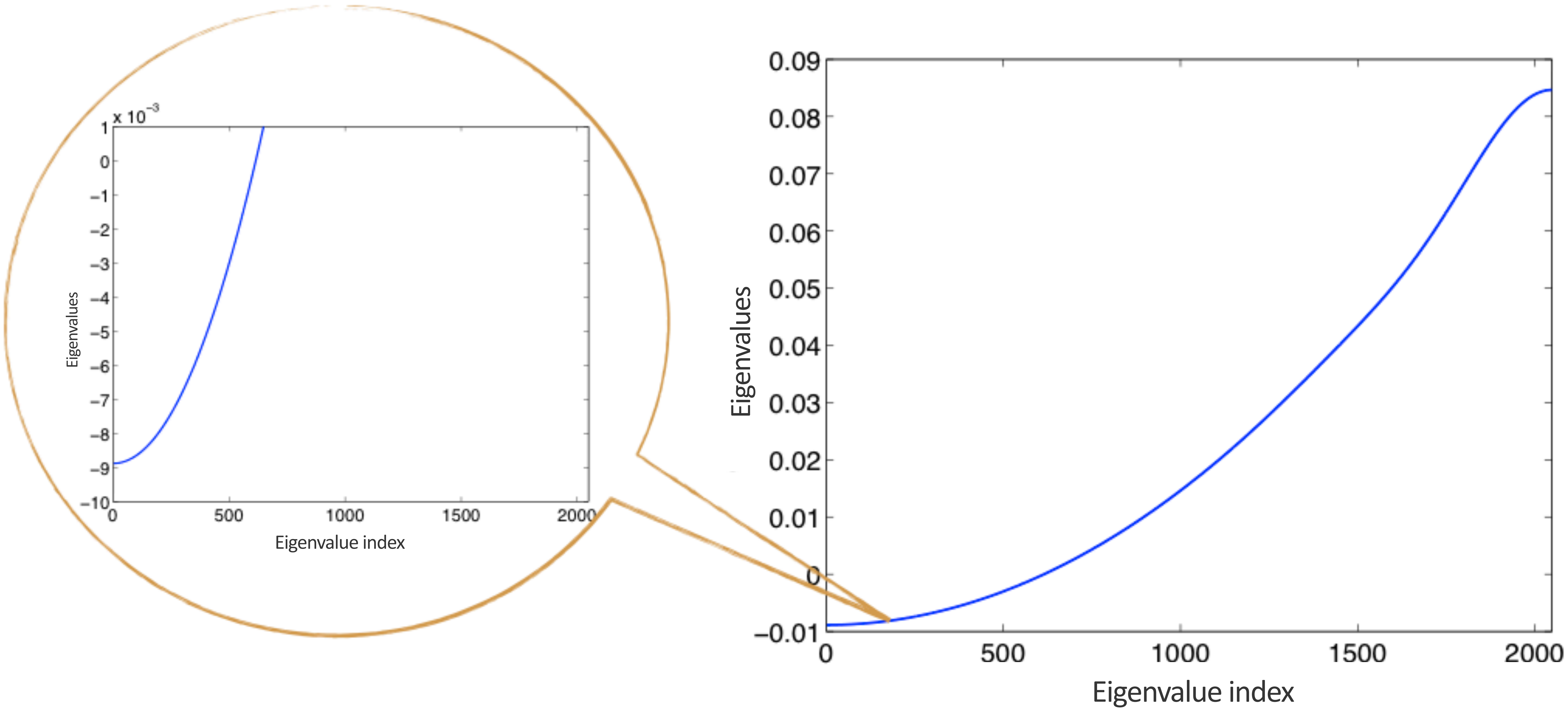
# Newton-Schulz iteration for the matrix sign function

**Algorithm** Newton-Schulz Iteration for the Matrix Sign Function ( Auslander and Tsao (1991))

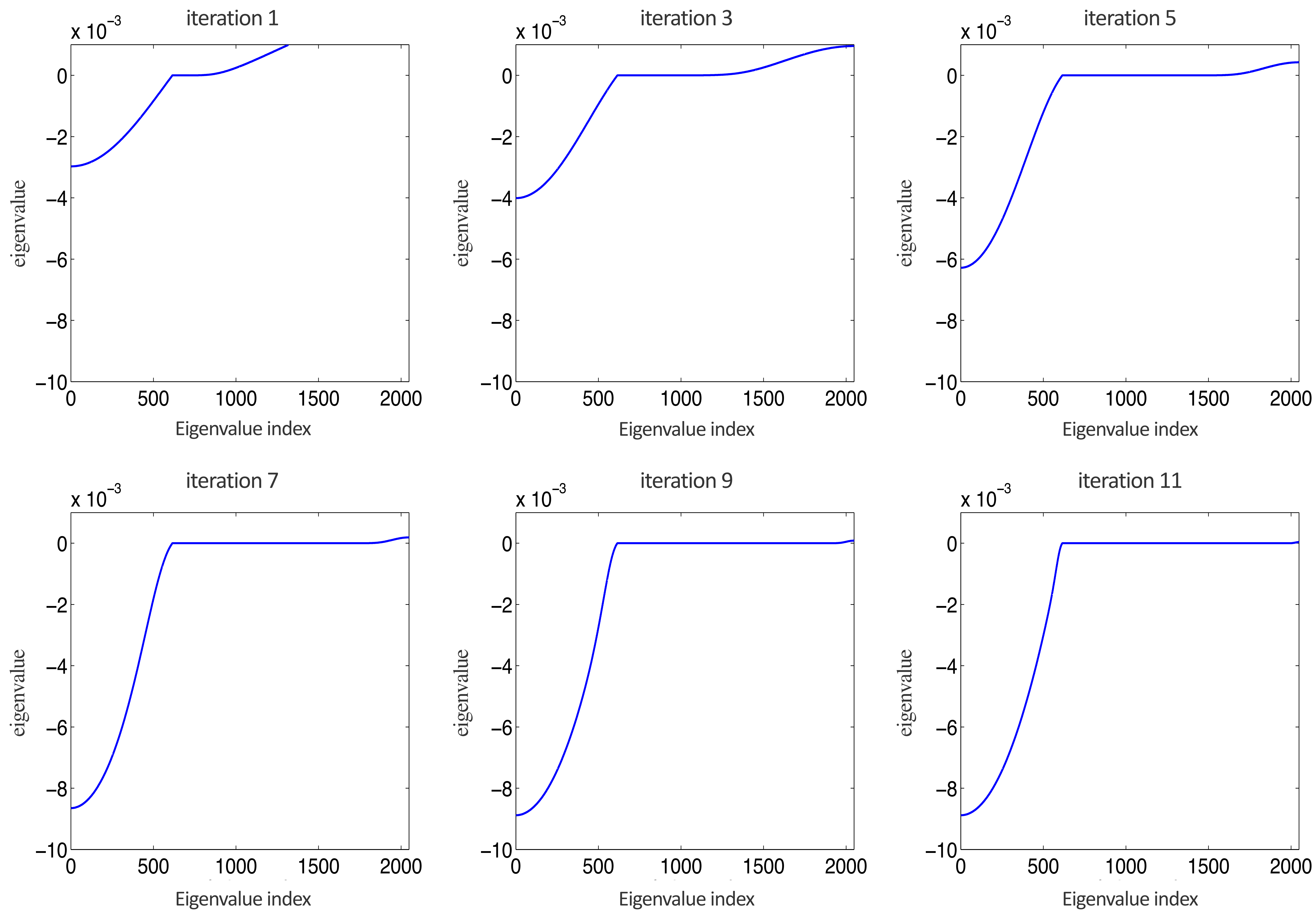
**Input:** Self adjoint matrix $L$
**Output:** $S := \text{sign}(L)$

1. Initialize $S_0 = L/||L||_2$, where $||L||_2$ stands for the 2 norm of matrix L
2. For $k = 1......N$, $S_{k+1} = \frac{3}{2}S_k - \frac{1}{2}S_k^3$

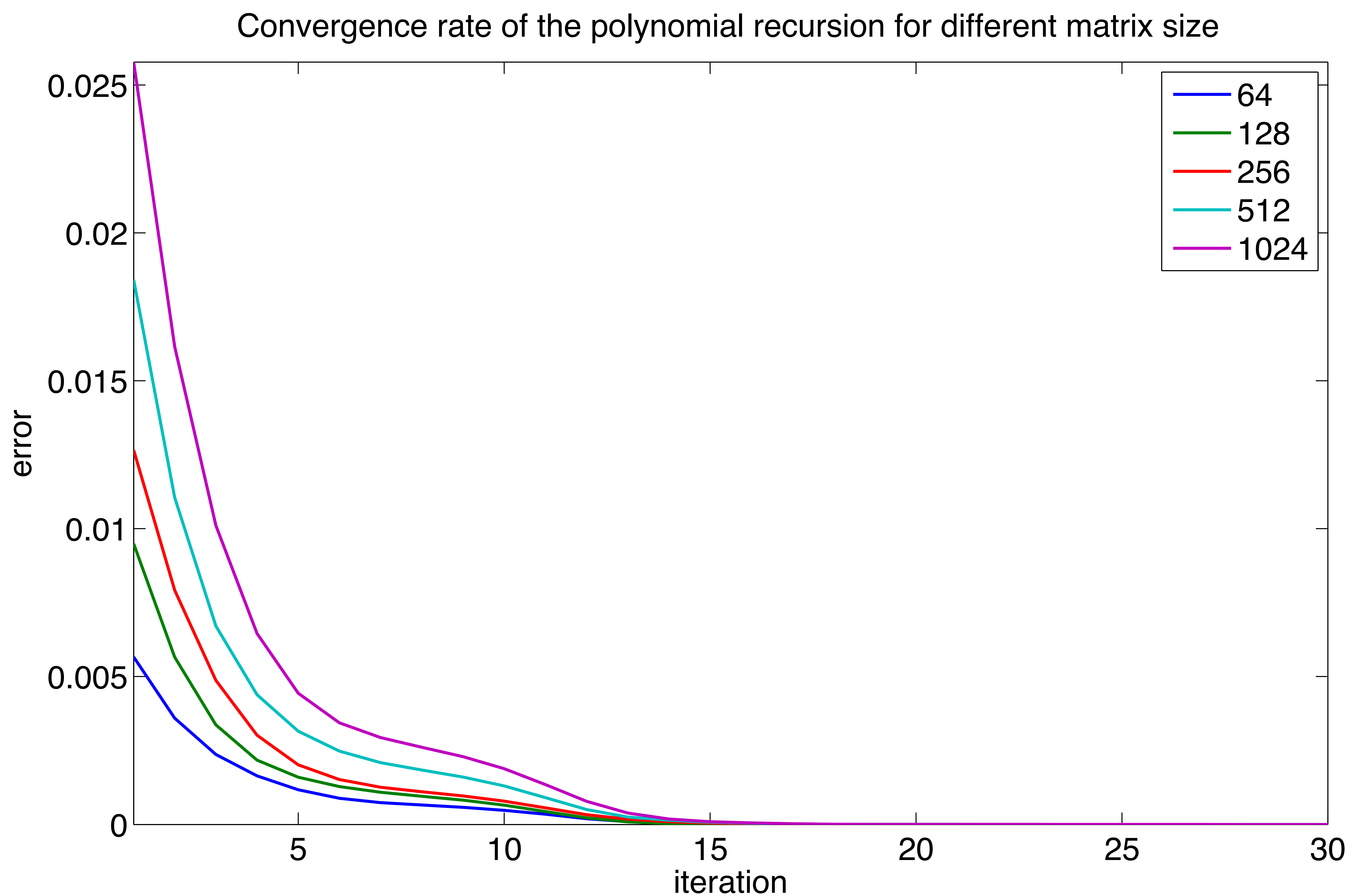
# Spectral projector (matrix sign function) via polynomial recursion



# Spectral projector (matrix sign function) via polynomial recursion



# Sign function computation via polynomial recursion



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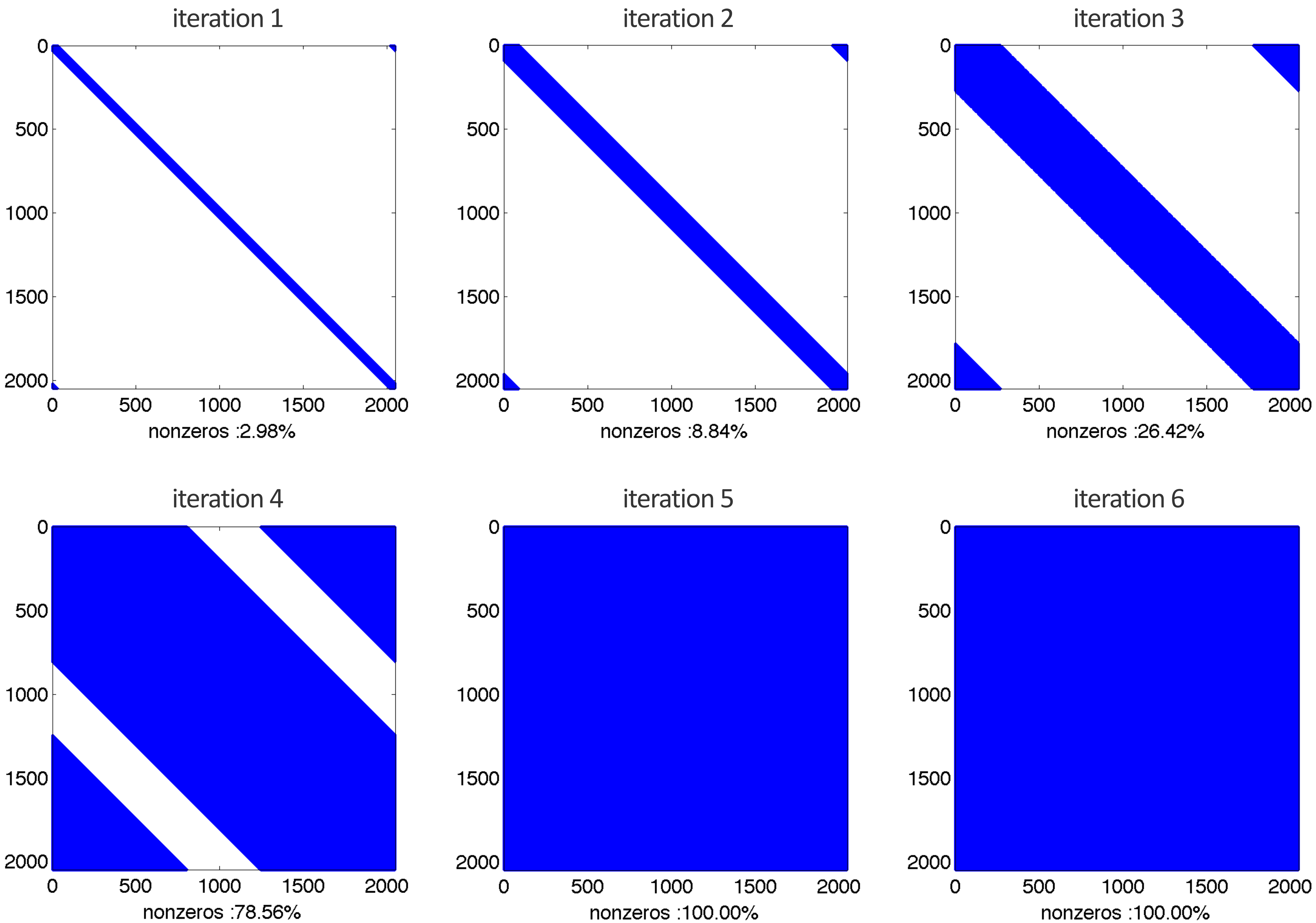
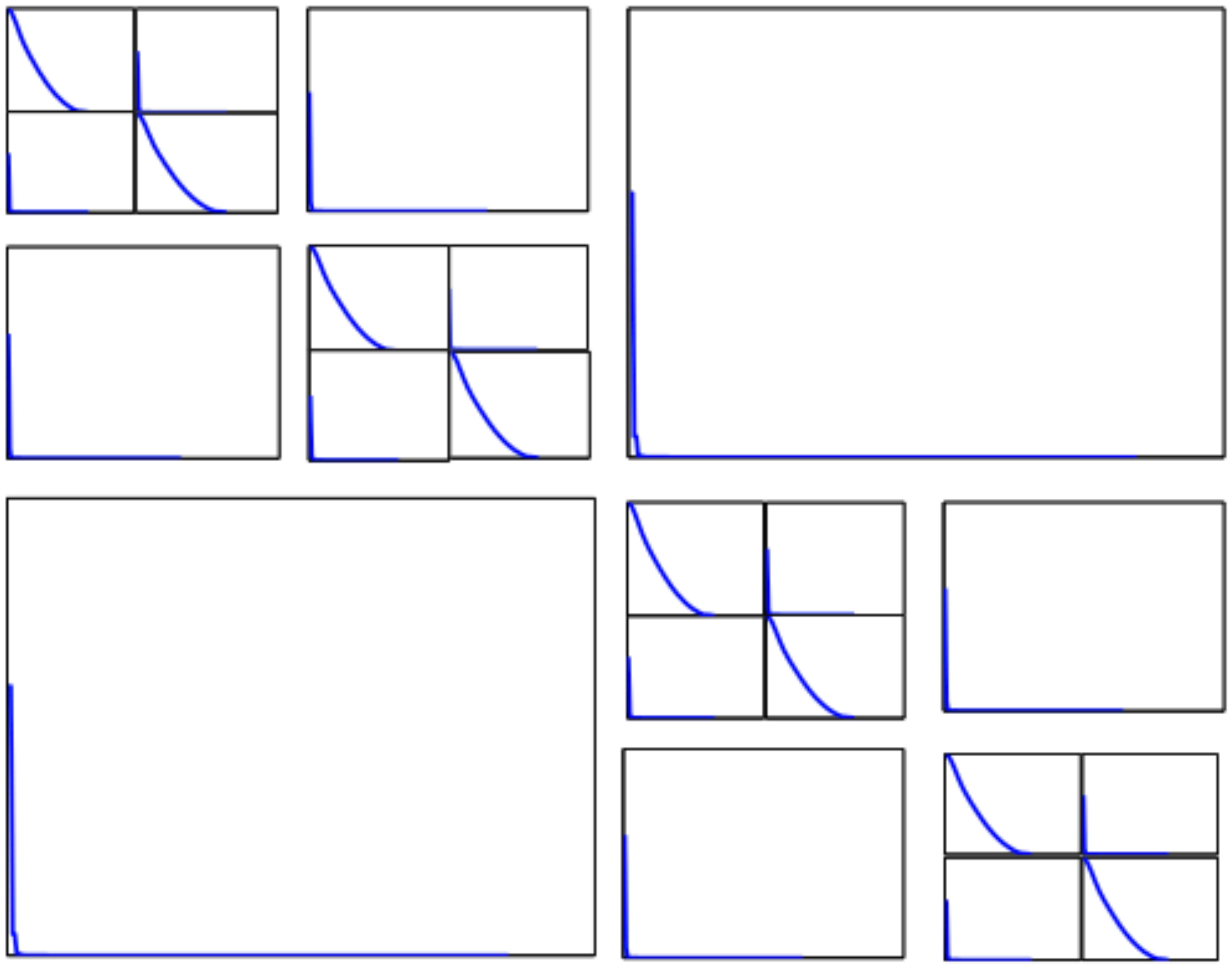
# Matrix sparsity in the polynomial recursion



J. Xia, Y. Xi, and M. Gu. A superfast structured solver for toeplitz linear systems via randomized sampling. SIAM Journal on Matrix Analysis and Applications, 33(3):837–858, 2012.

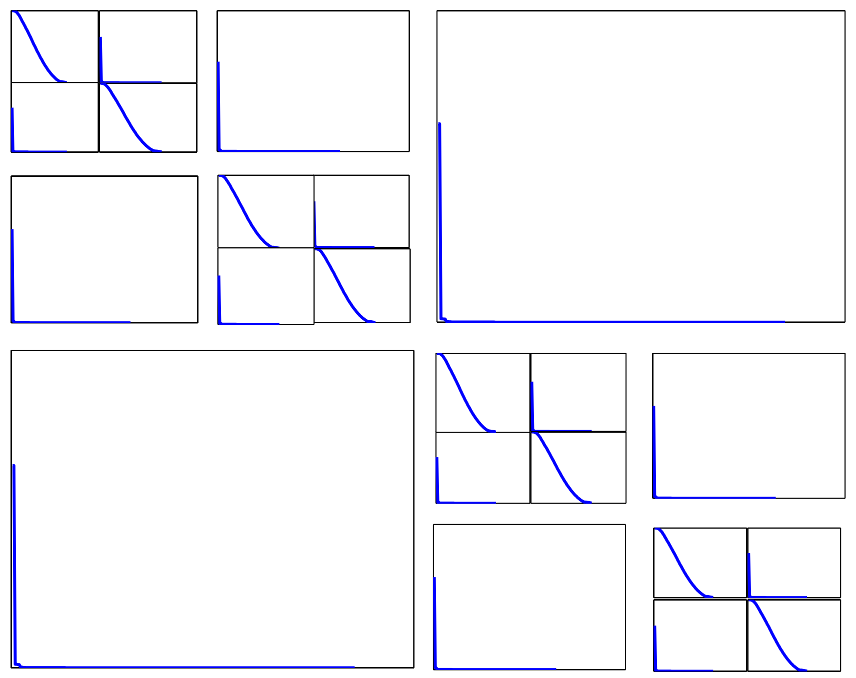
W. Lyons. Fast algorithms with applications to pdes. PhD thesis, UNIVER- SITY of CALIFORNIA, 2005.

# Matrix spectrum with HSS partition

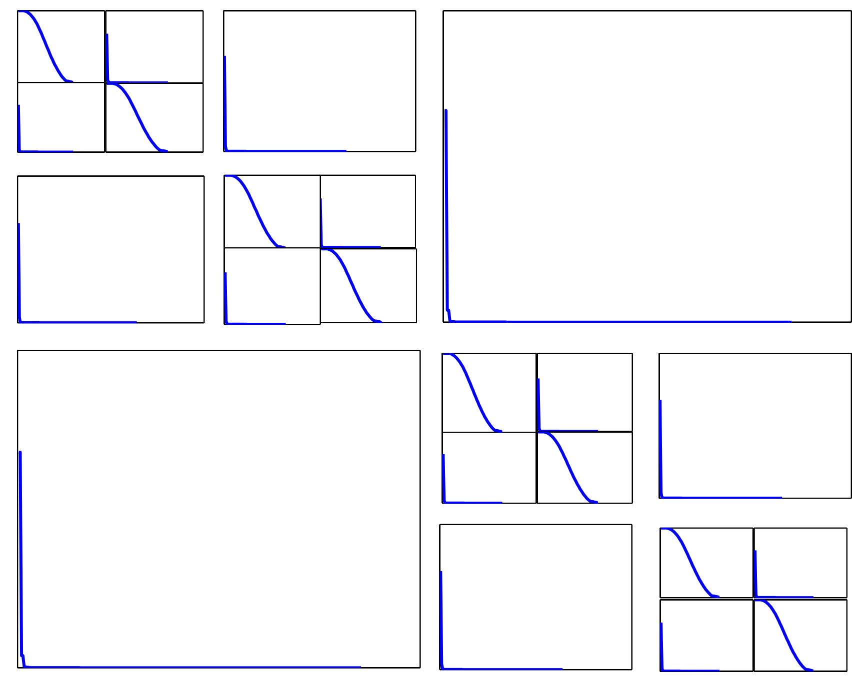


# Matrix spectrum with HSS partition in Newton-Schulz Iteration

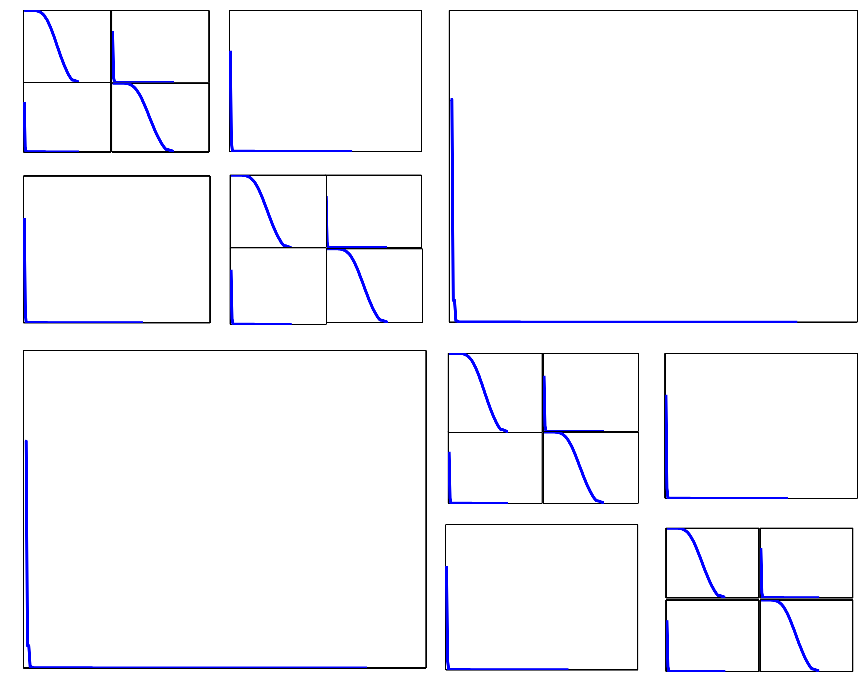
iteration 1



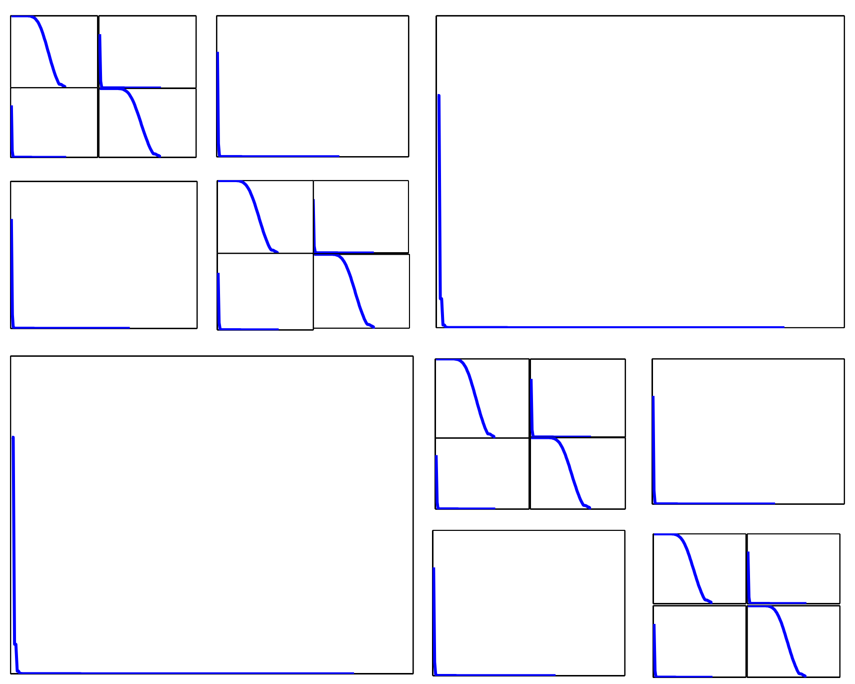
iteration 2



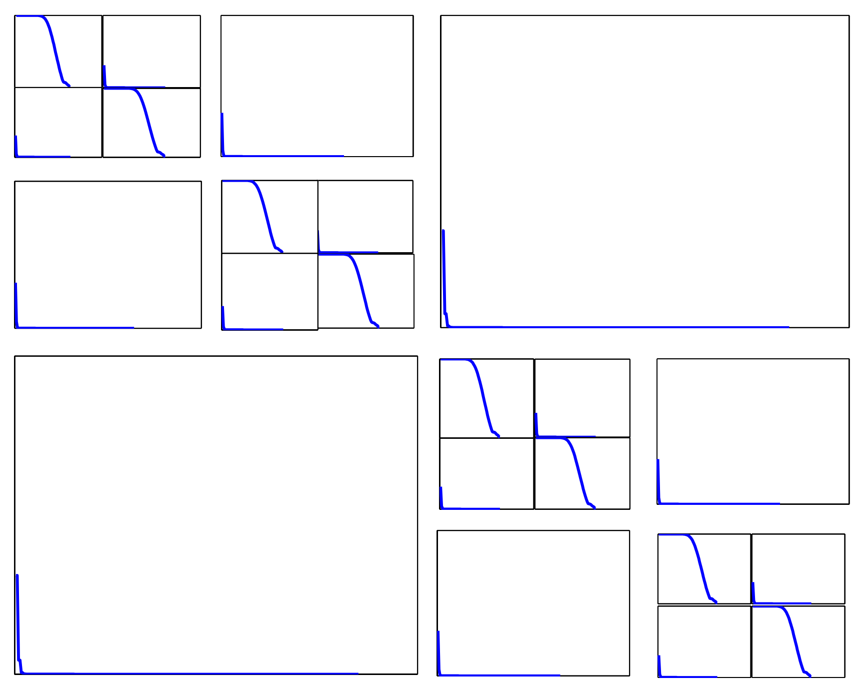
iteration 3



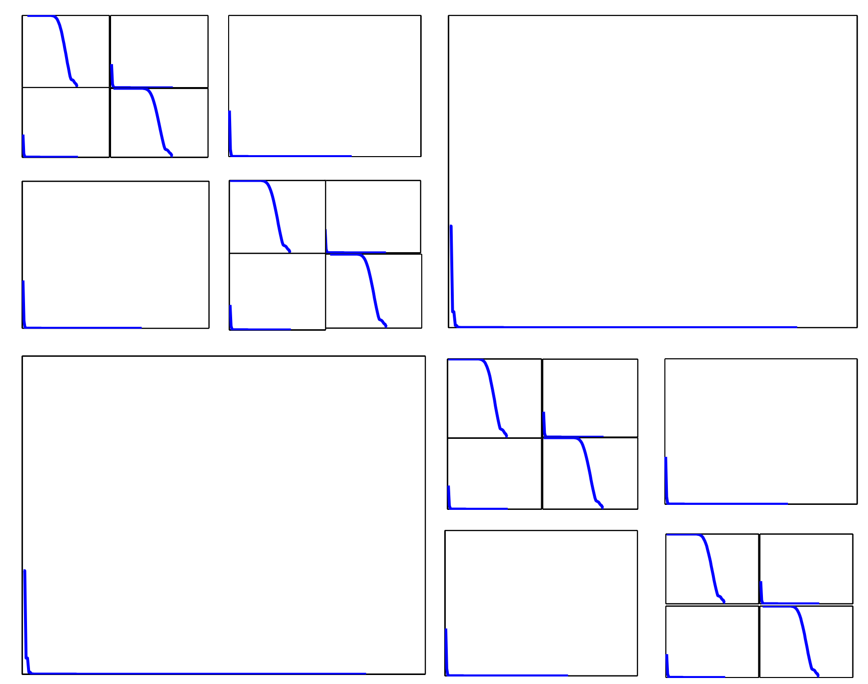
iteration 4



iteration 5



iteration 6



J. Xia. On the complexity of some hierarchical structured matrix algorithms. SIAM Journal on Matrix Analysis and Applications, 33(2):388–410, 2012.

# Hierarchically semi-separable matrix representation

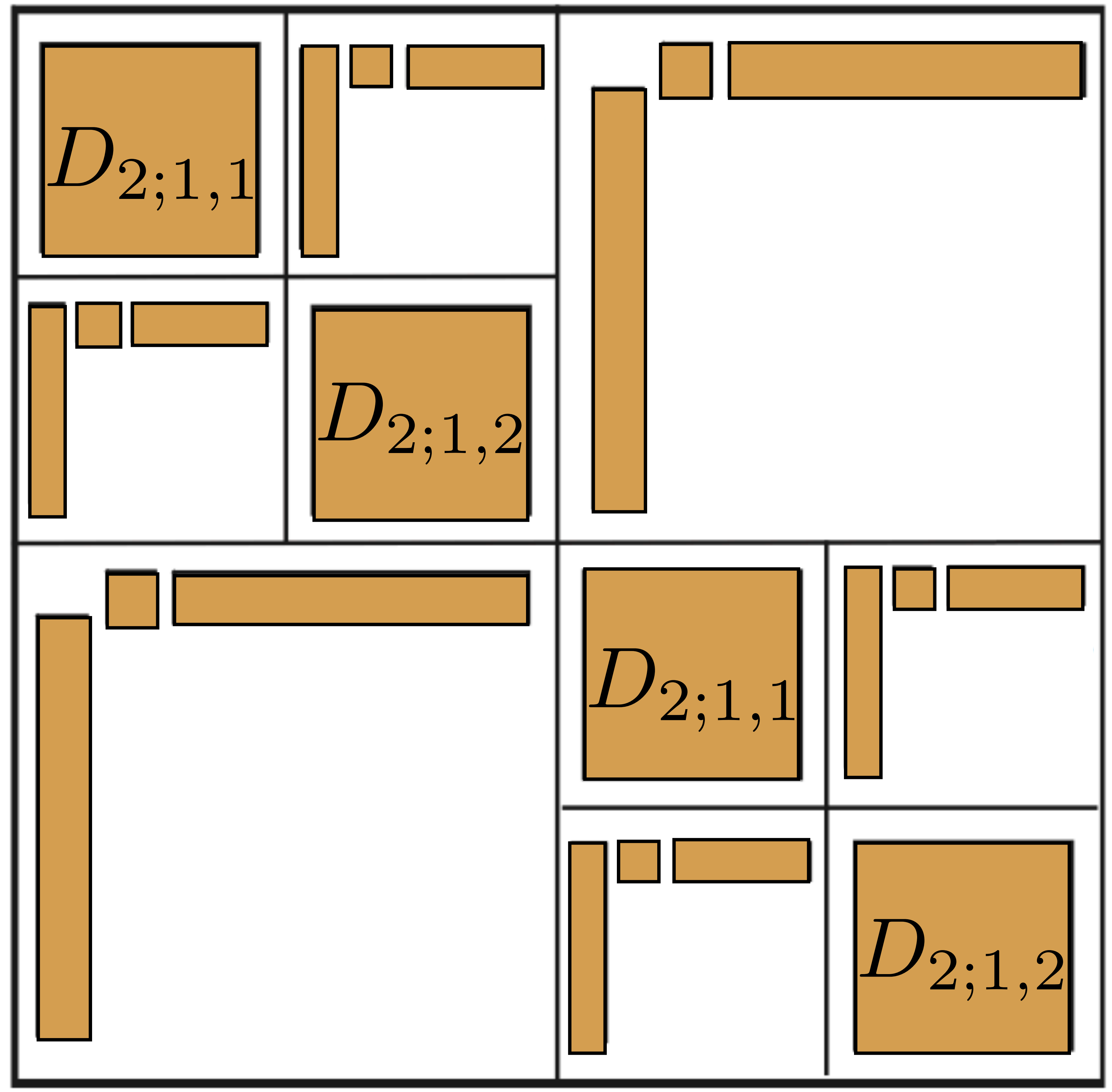
$$A = \begin{pmatrix} A_{1;1,1} & A_{1;1,2} \\ A_{1;2,1} & A_{1;2,2} \end{pmatrix}$$

$$\downarrow$$

$$\mathbf{A} = \begin{pmatrix} D_{1;1,1} & (USV^*)_{1;1,2} \\ (USV^*)_{1;2,1} & D_{1;2,2} \end{pmatrix}$$

$$\downarrow$$

$$\mathbf{A} = \begin{pmatrix} \begin{pmatrix} D_{2;1,1} & (USV^*)_{2;1,2} \\ (USV^*)_{2;2,1} & D_{2;2,2} \end{pmatrix} & (USV^*)_{1;1,2} \\ (USV^*)_{1;2,1} & \begin{pmatrix} D_{2;1,1} & (USV^*)_{2;1,2} \\ (USV^*)_{2;2,1} & D_{2;2,2} \end{pmatrix} \end{pmatrix}$$



Z. Sheng, P. Dewilde, and S. Chandrasekaran. Algorithms to solve hierar- chically semi-separable systems. pages 255–294, 2007.

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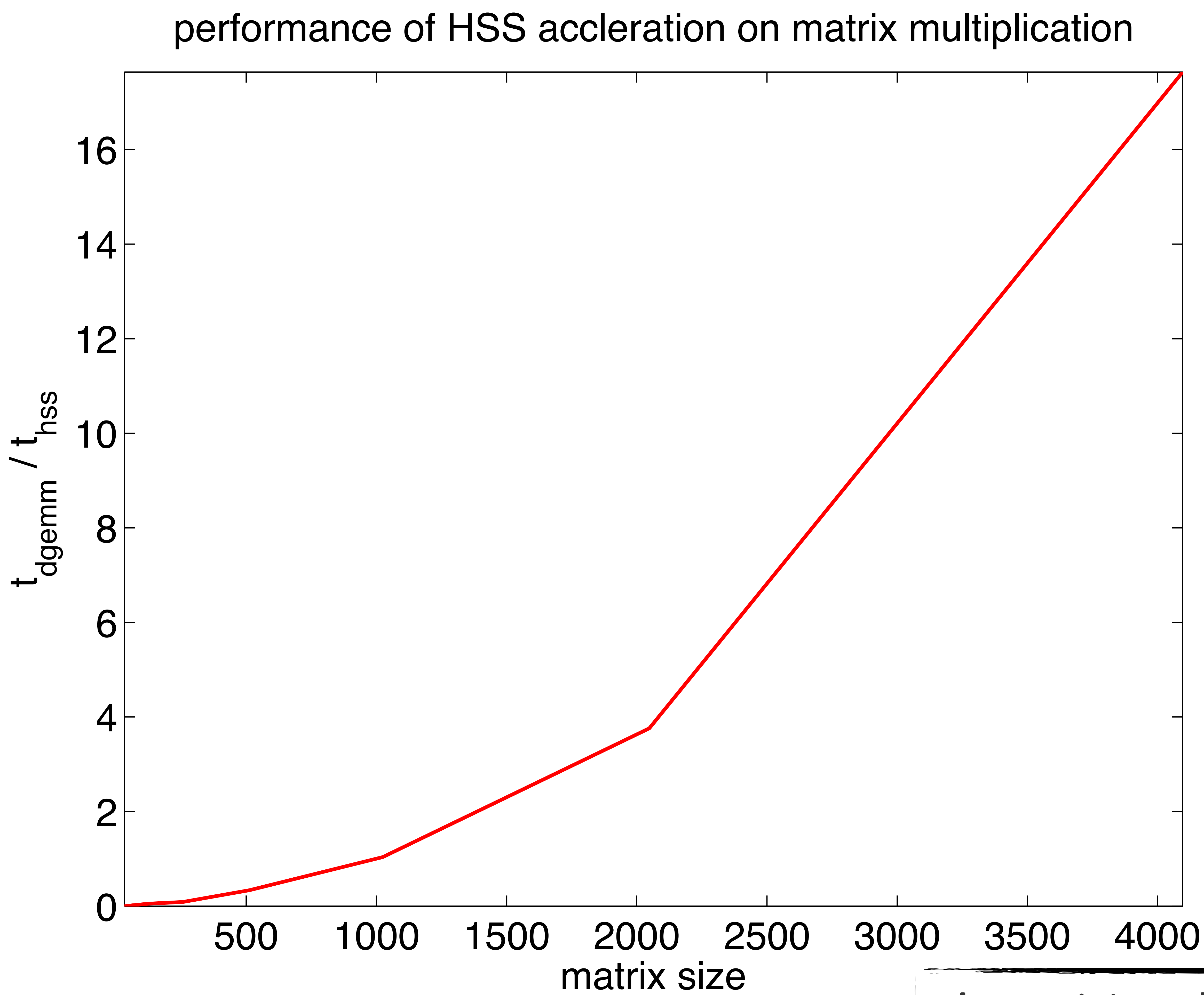
# Complexity analysis of HSS operations

| Operation | Complexity with HSS | Complexity without HSS |
|---|---|---|
| Matrix-Vector Multiplication | $\mathcal{O}(nk^2)$ | $\mathcal{O}(n^2)$ |
| Matrix-Matrix Multiplication | $\mathcal{O}(nk^3)$ | $\mathcal{O}(n^3)$ |
| Matrix addition | $\mathcal{O}(nk^2)$ | $\mathcal{O}(n^2)$ |
| Compression | $\mathcal{O}(nk^3)$ | Not Applicable |
| LU Decomposition | $\mathcal{O}(nk^3)$ | $\mathcal{O}(n^3)$ |
| Inverse | $\mathcal{O}(nk^3)$ | $\mathcal{O}(n^3)$ |
| Transpose | $\mathcal{O}(nk)$ | $\mathcal{O}(n^2)$ |
| Construct HSS (Xia (2012)) | $\mathcal{O}(n^2r)$ | Not Applicable |

k is maximum rank of the off-diagonal matrices

W. Lyons. Fast algorithms with applications to pdes. PhD thesis, UNIVER- SITY of CALIFORNIA, 2005.

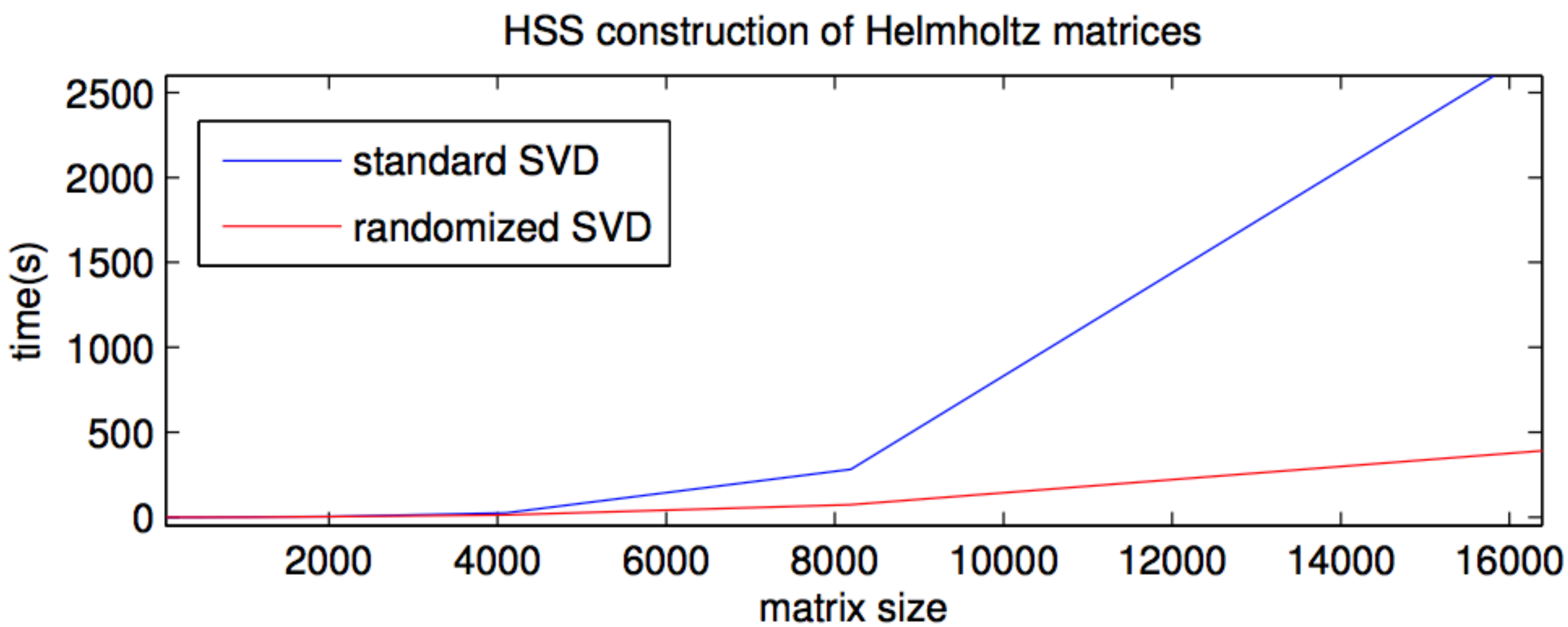
# Computation of matrix-matrix multiplication with HSS



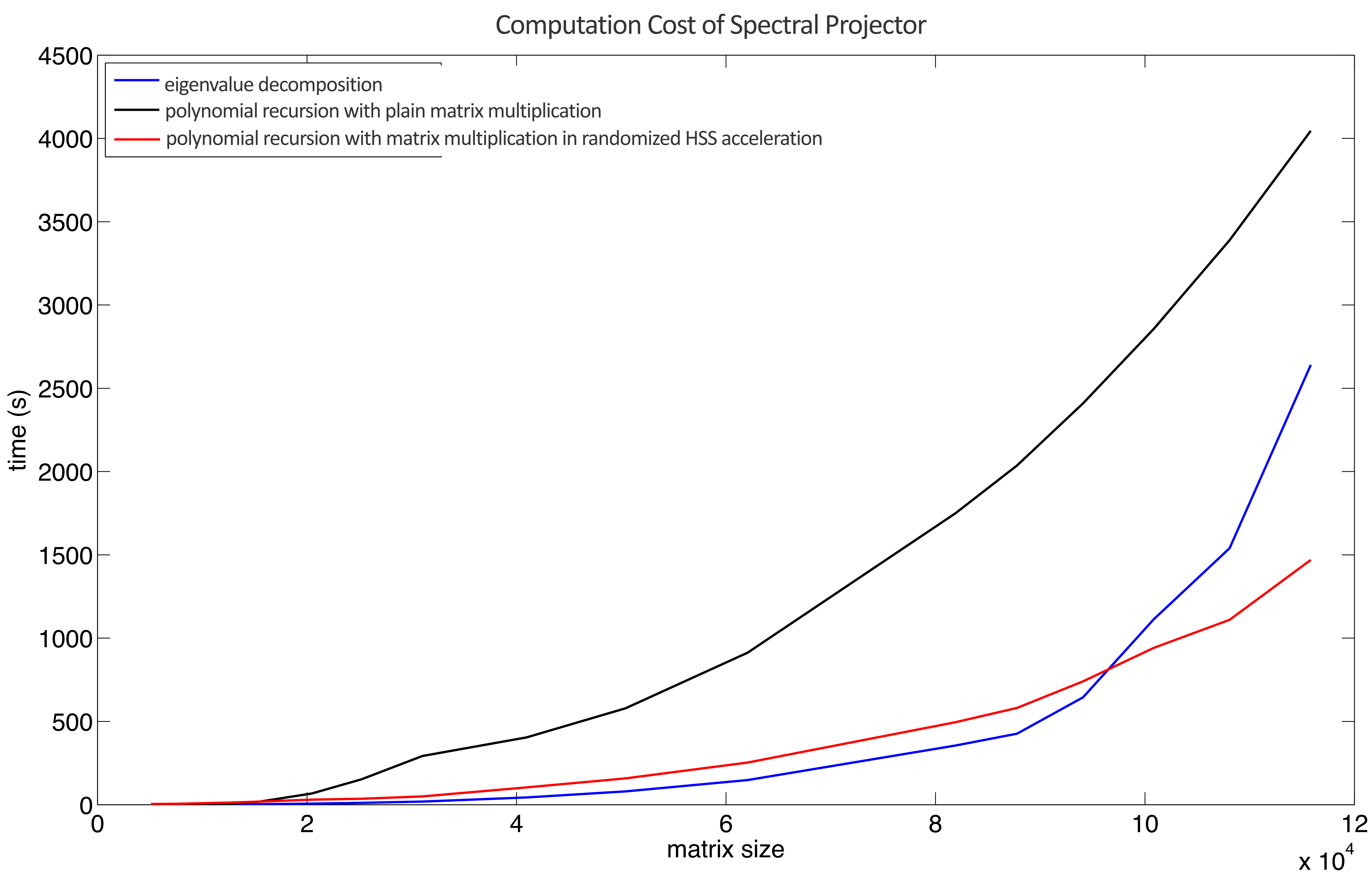

**dgemm** is Lapack routine for matrix-matrix multiplication

M. Tygert and Rokhlin. A randomized algorithm for approximating the svd of a matrix. 2007. URL https://www.ipam.ucla.edu/publications/ setut/setut_7373.pdf.
Halko, Nathan, Per-Gunnar Martinsson, and Joel A. Tropp. "Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions." *SIAM review* 53.2 (2011): 217-288.
B. Jumah and F. J. Herrmann. Dimensionality-reduced estimation of primaries by sparse inversion. 2012. URL https: //www.slim.eos.ubc.ca/Publications/Private/Submitted/ Journal/bander2012dre/bander2012dre.pdf.

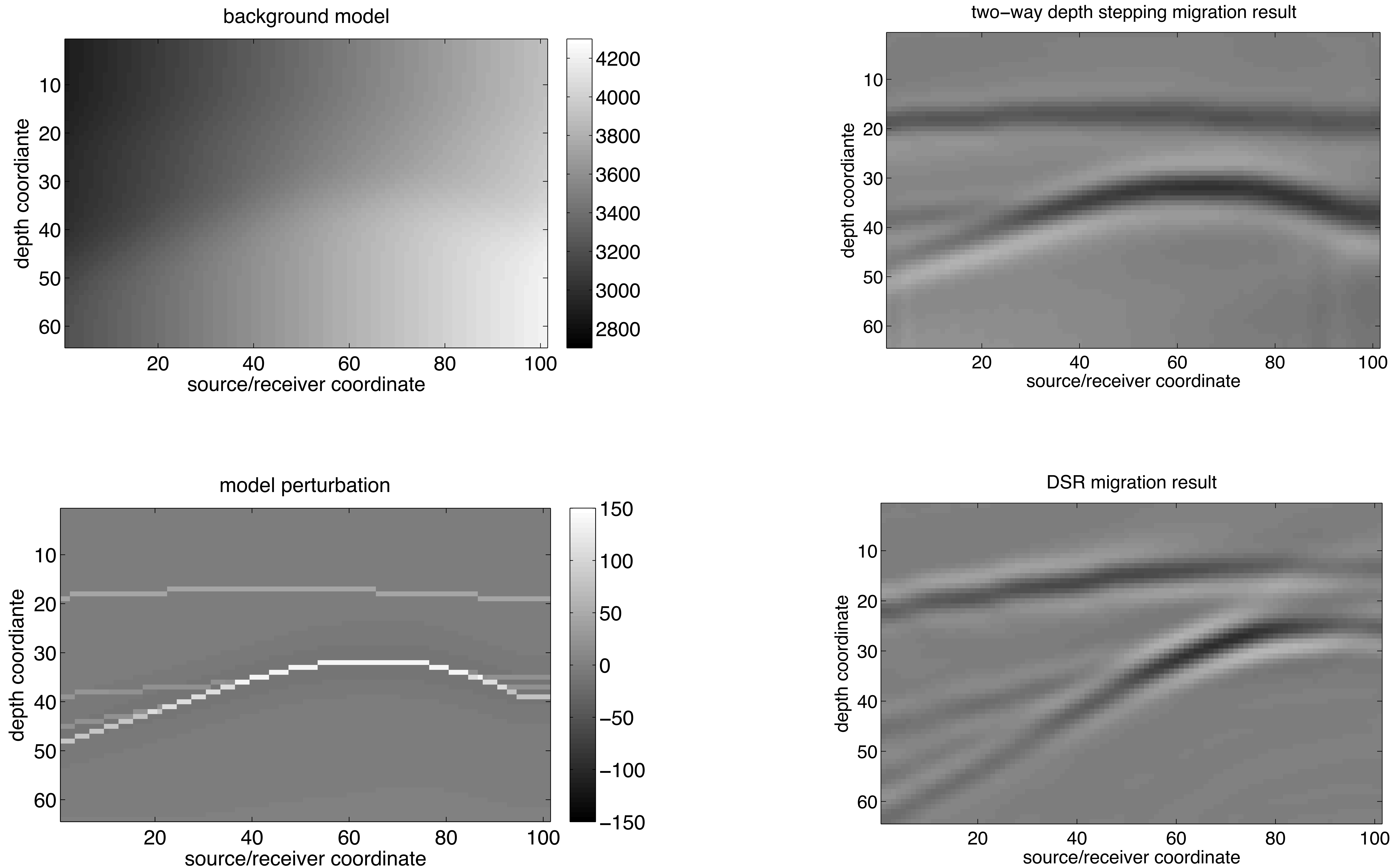
# Randomized HSS



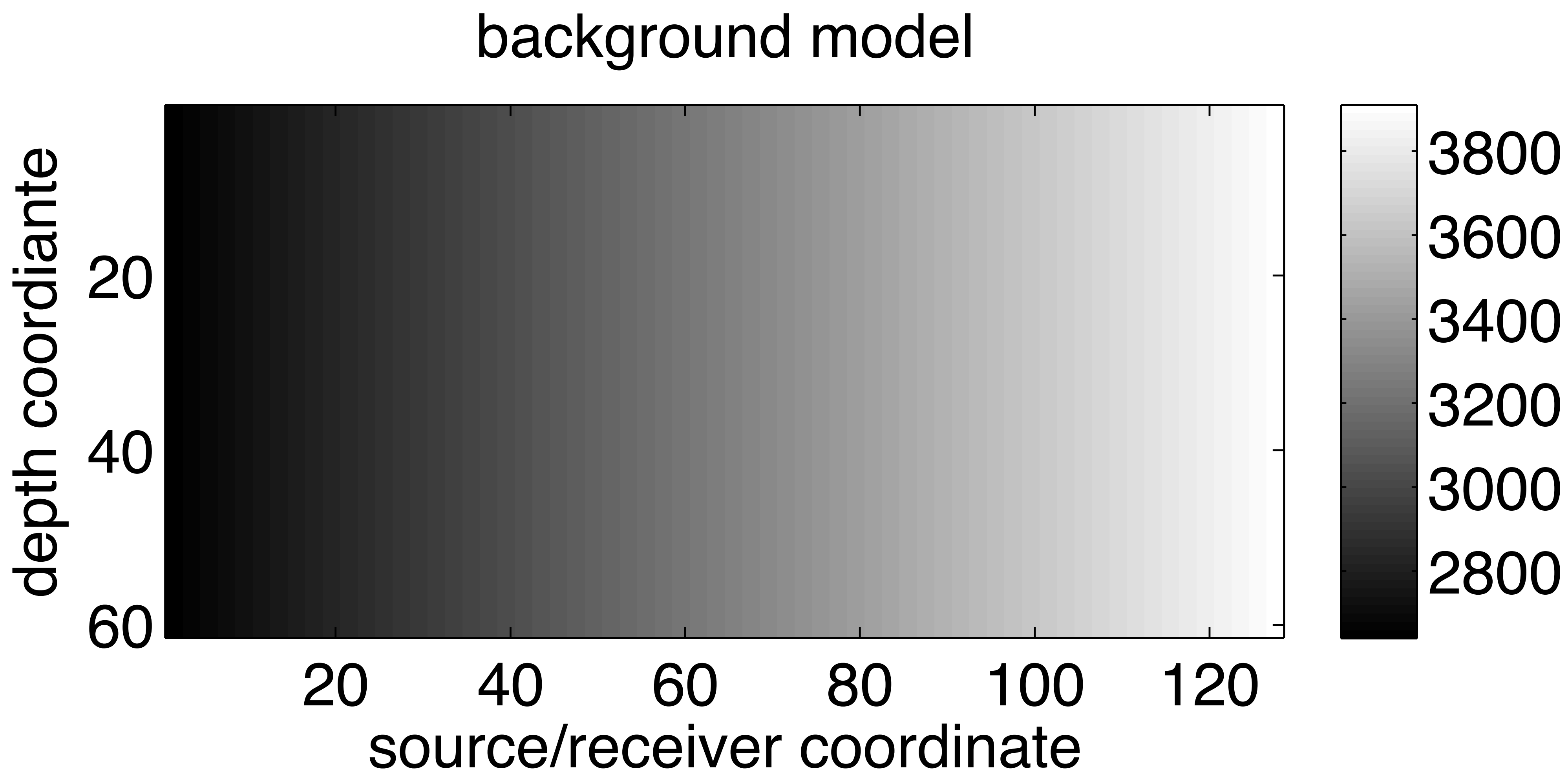
# Computation of spectral projector with HSS

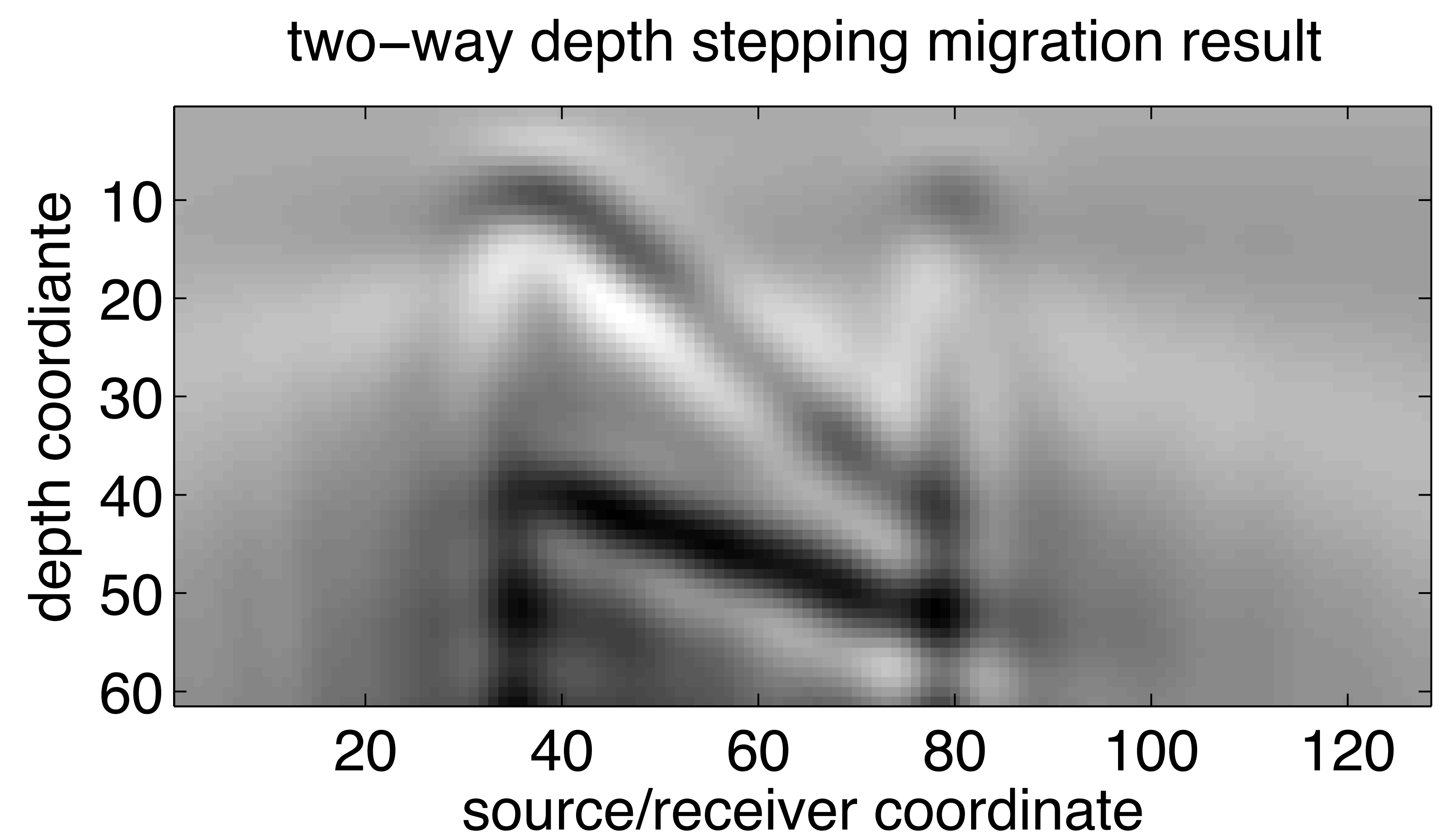


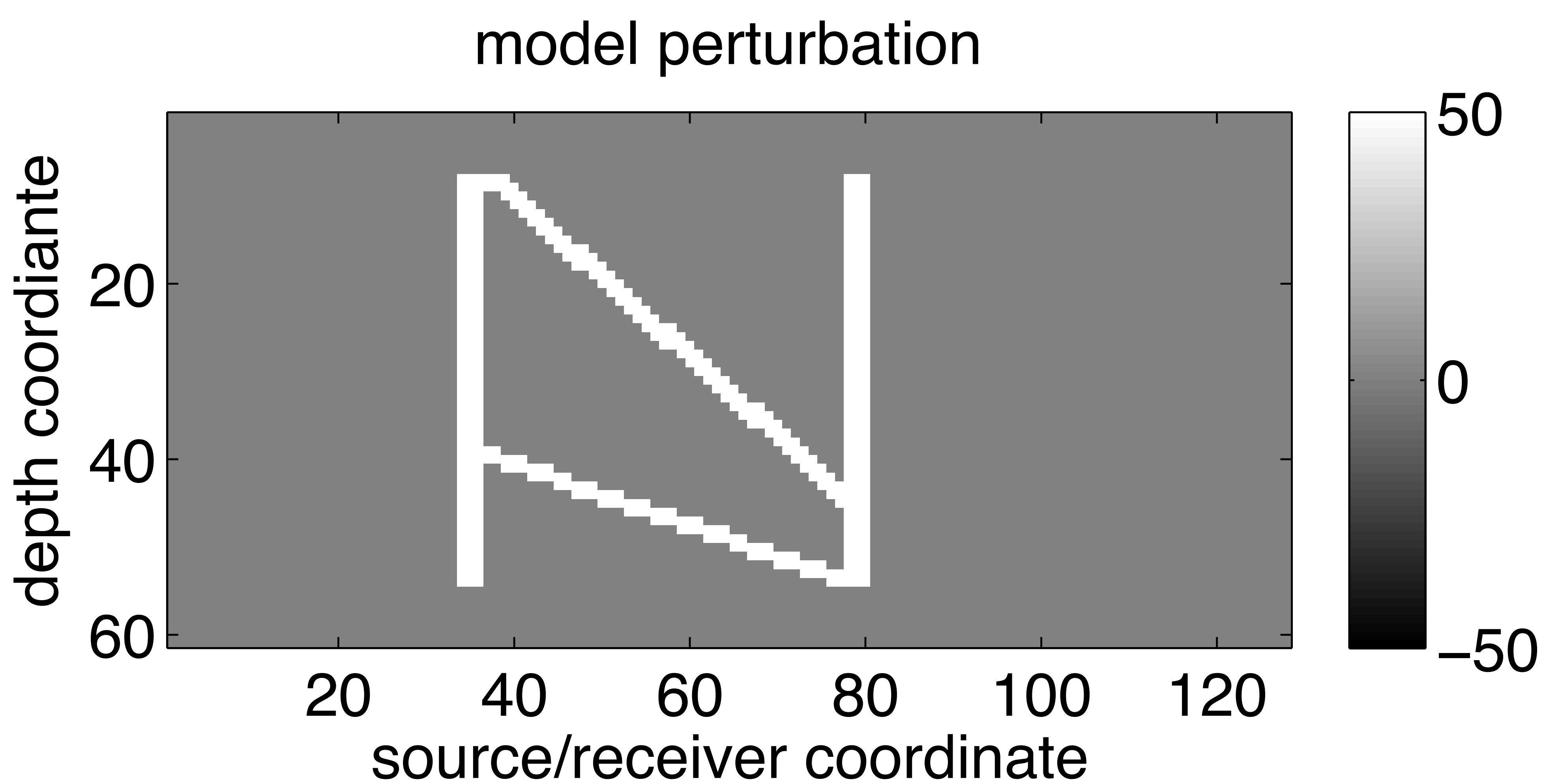
# Migration example

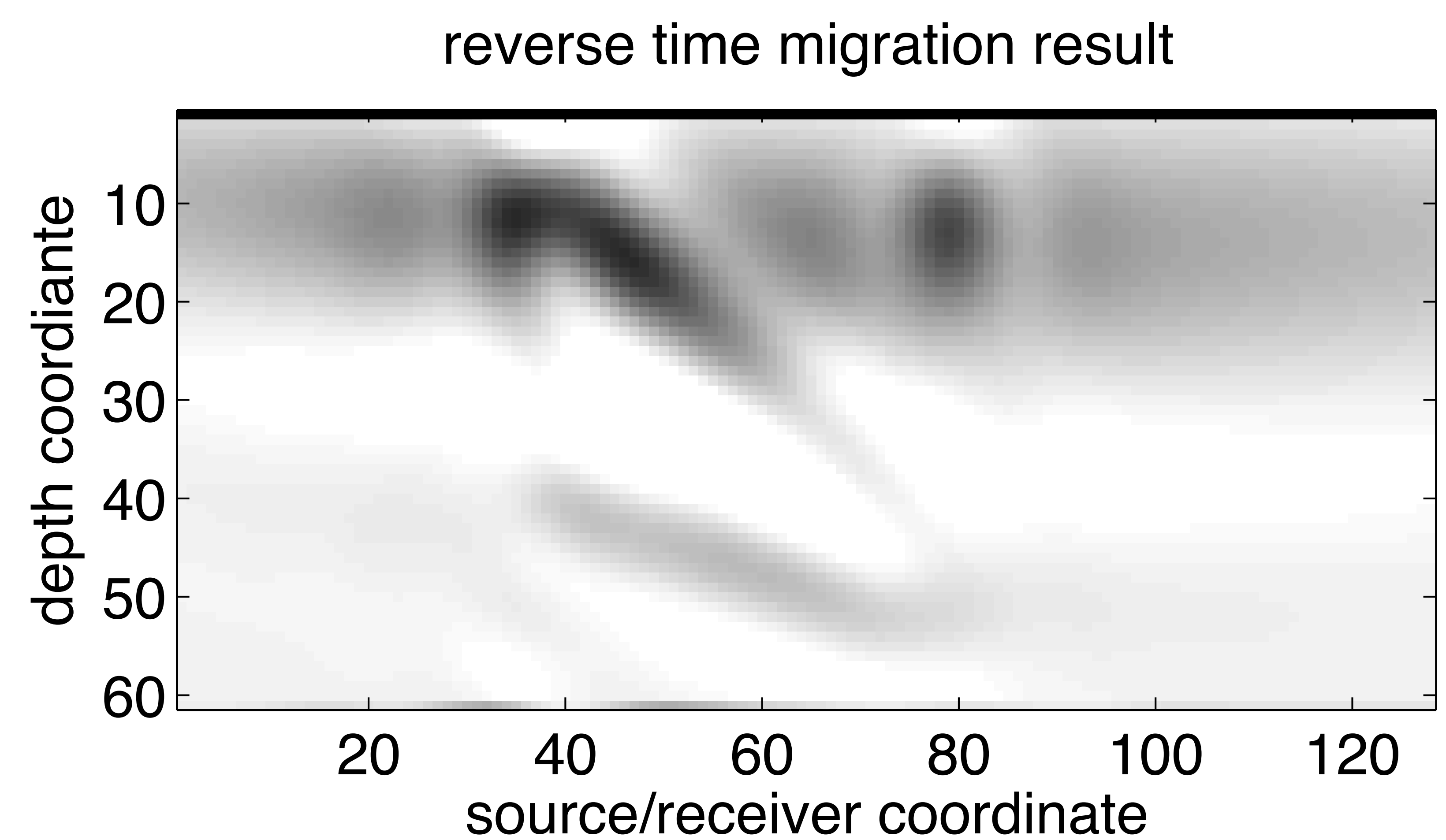


# Migration example



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# Conclusion

- **Spectral projector** computed with **polynomial recursion** using **randomized HSS techniques** provides an **efficient two-way** wave equation based **depth extrapolation** migration for **laterally variant medium**.
- Better image quality with the steep events (90 degrees) compared to reverse time migration (RTM).

## Outline

Efficient depth stepping migration with the two-way wave equation

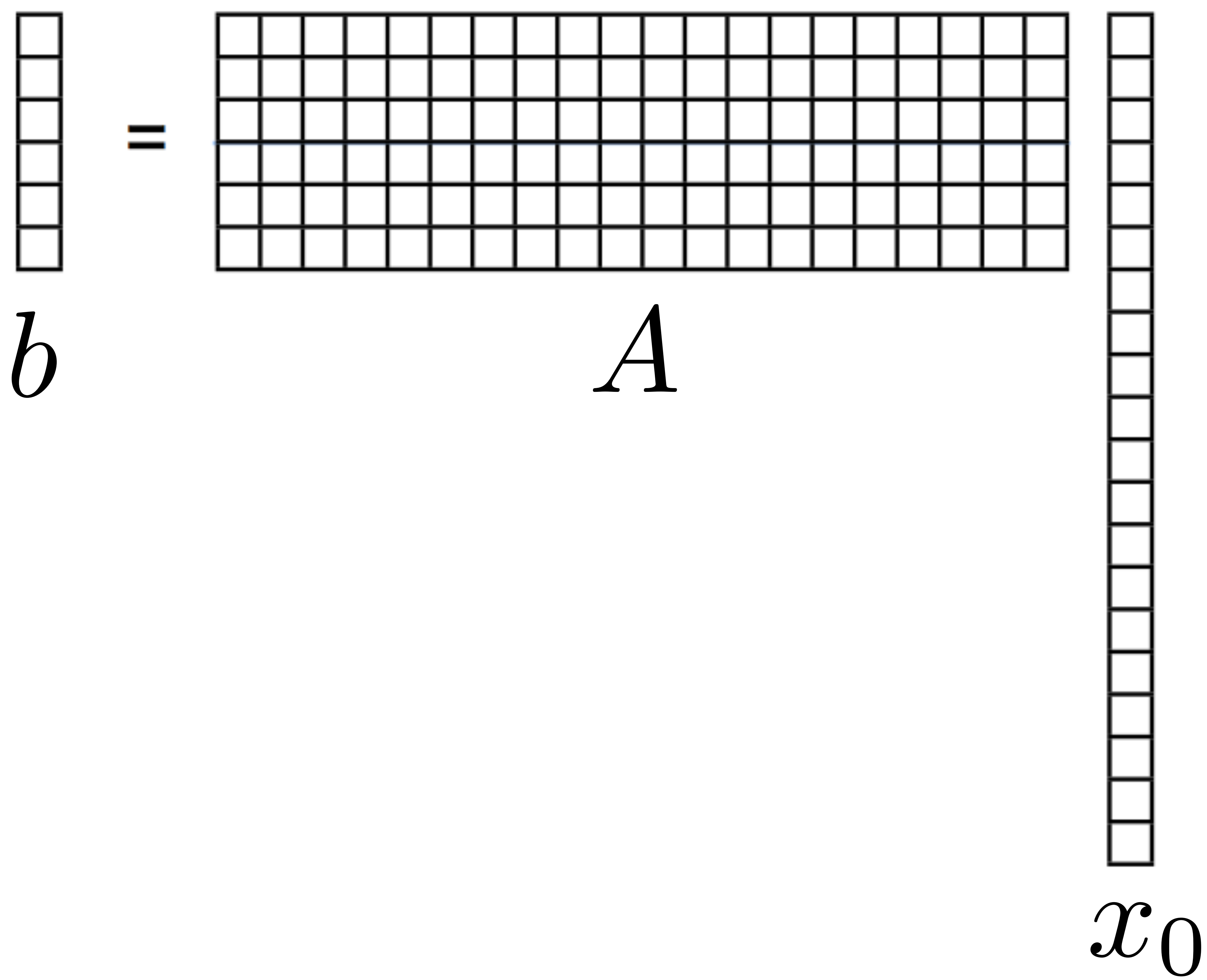
- spectral projector

**Efficient sparsity promoting seismic imaging**

- **joint sparsity**

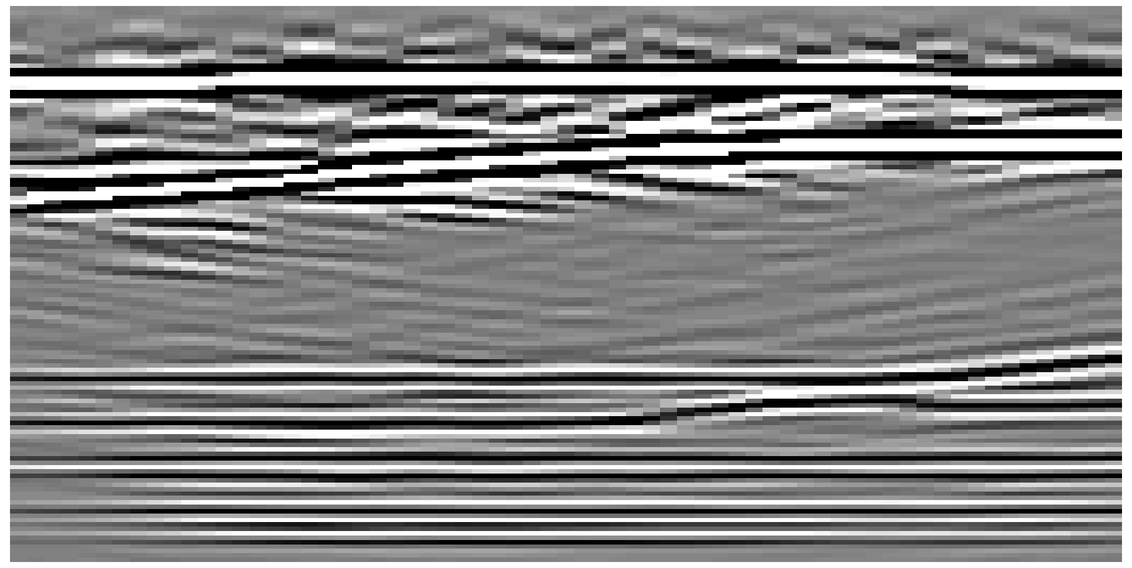
Efficient large scale optimization algorithm

# Sparsity promoting seismic imaging




$$\boxed{\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax - b\|_2 \leq \sigma \end{array}}$$

migration image

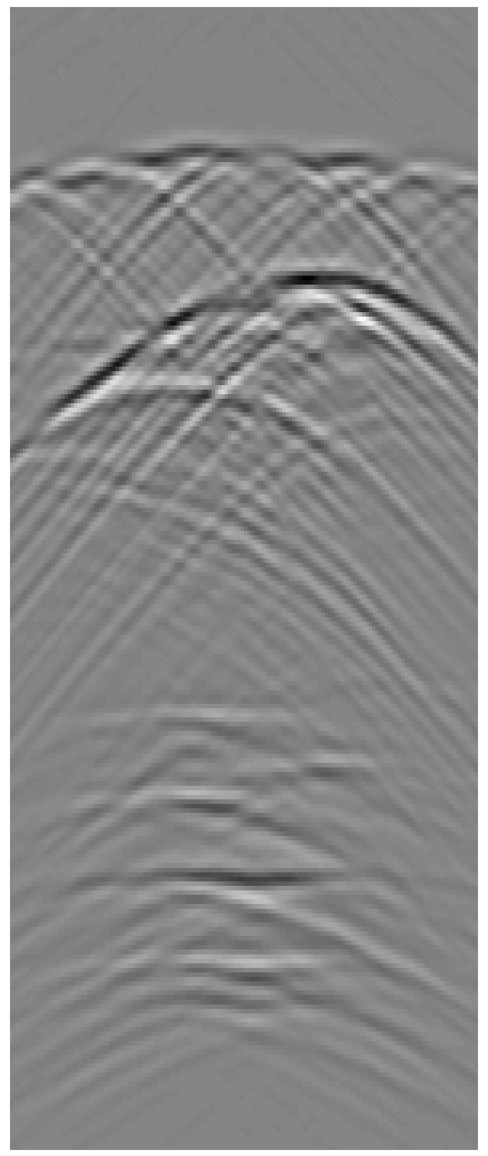


$\ell_1$

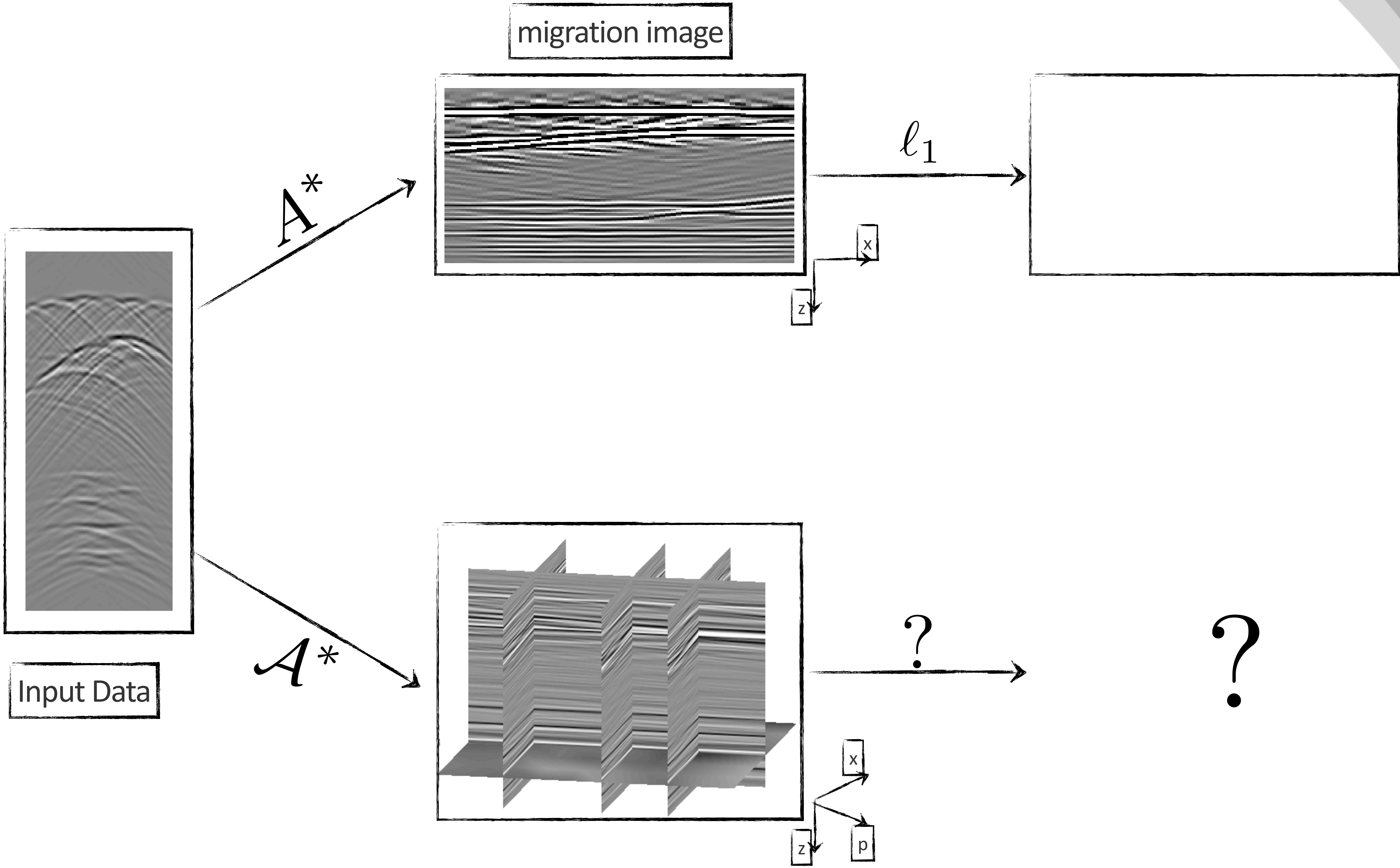
x

z

$A^*$



Input Data


migration image
$A^*$
$\ell_1$
x
z
Input Data
$\mathcal{A}^*$
?
?
x
z
p

# Joint sparsity recovery

| | Sparsity Promoting Compressed Sensing | Joint Sparsity Promoting Compressed Sensing |
|---|---|---|
| Recovery Scheme | $\min \; \|\|x\|\|_1 \;$ subject to $\; Ax = b$ <br> $where \;\; \|\|x\|\|_1 = \sum_{i=1}^{n} \|x_i\|$ | $\min \; \|\|X\|\|_{p,q} \;$ subject to $\; \mathcal{A}(X) = b$ <br> $where \;\; \|\|X\|\|_{p,q} = \left( \sum_{j=1}^{n} \|\|X^{j\rightarrow}\|\|_q^p \right)^{1/p}$ |

$X^{j\rightarrow}$ *is the (column) vector whose entries form the* $j^{\text{th}}$ *row of* $X$.

# Joint sparsity recovery (sum-of-norms formulation)

$$||X||_{p,q} = \left( \sum_{j=1}^{n} ||X^{j\rightarrow}||_q^p \right)^{1/p}$$

$\xrightarrow{p=1,\, q=1}$

$$||X||_{1,1} = \left( \sum_{j=1}^{n} ||X^{j\rightarrow}||_1^1 \right)^{1/1}$$

Equivalent to $\ell_1$ norm of vectorized $X$

$\xrightarrow{p=1,\, q=2}$

$$||X||_{1,2} = \left( \sum_{j=1}^{n} ||X^{j\rightarrow}||_2^1 \right)^{1/1}$$

Equivalent to sum of $\ell_2$ norm of rows of $X$

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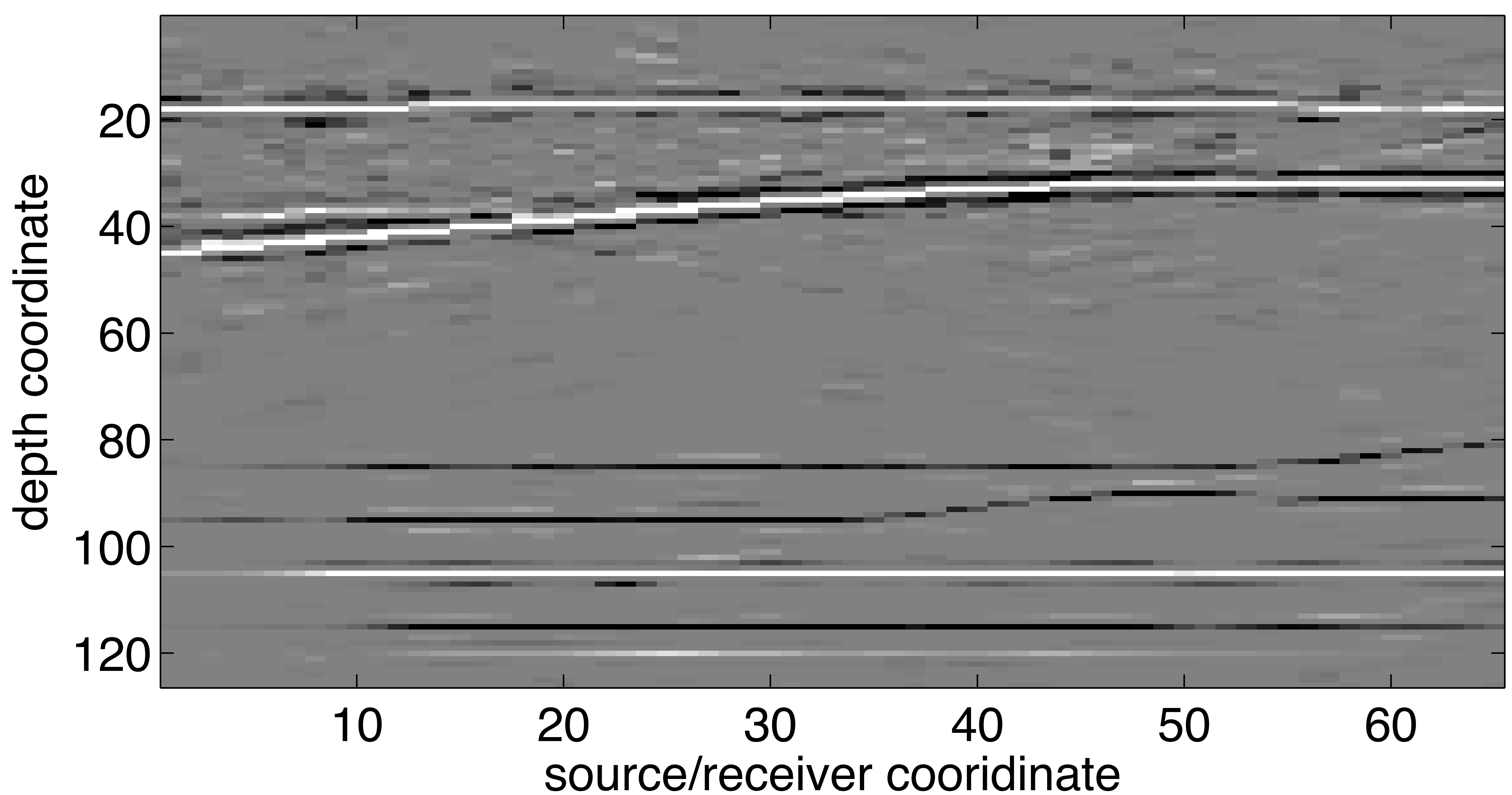
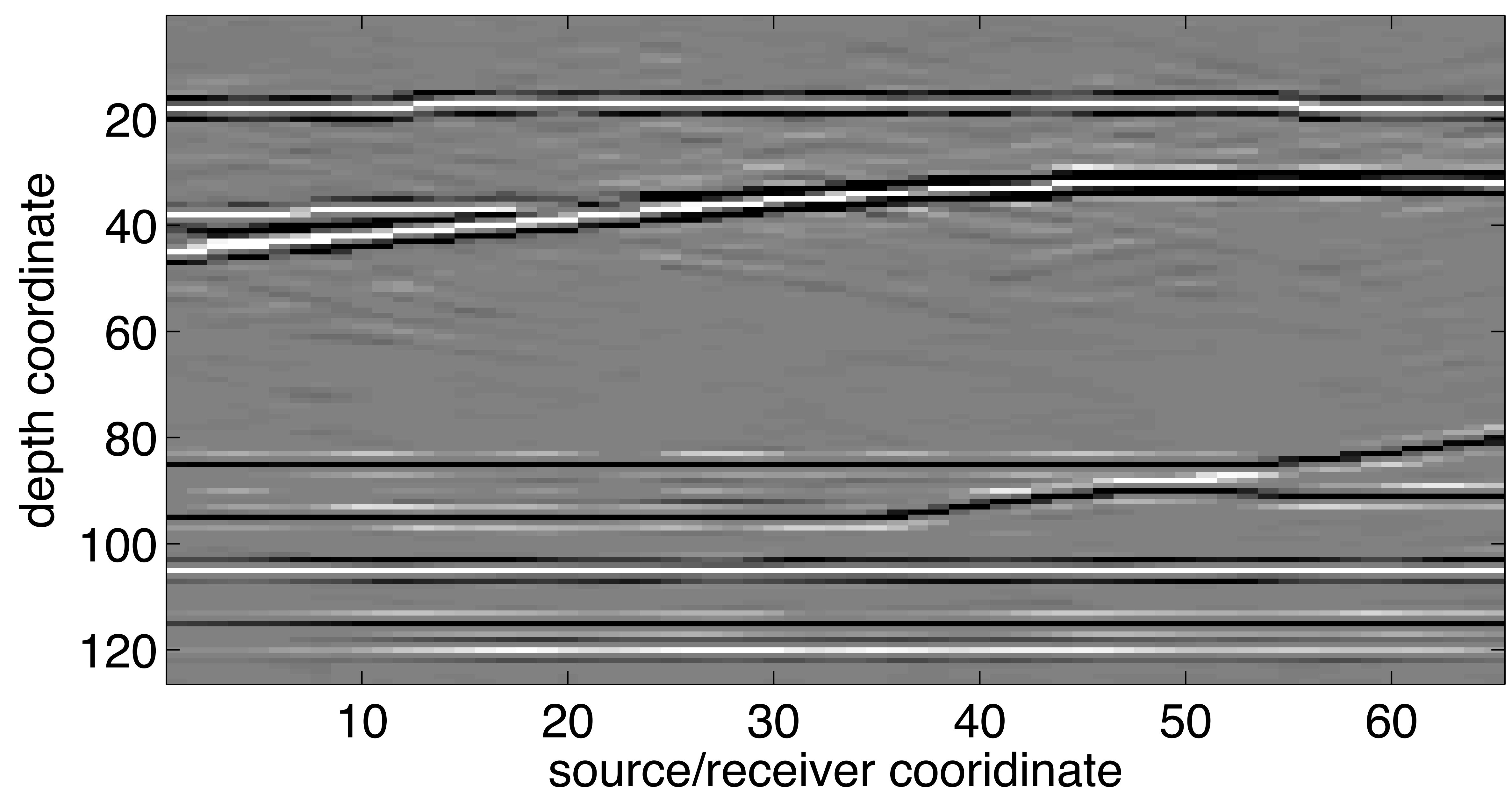
# Evaluation

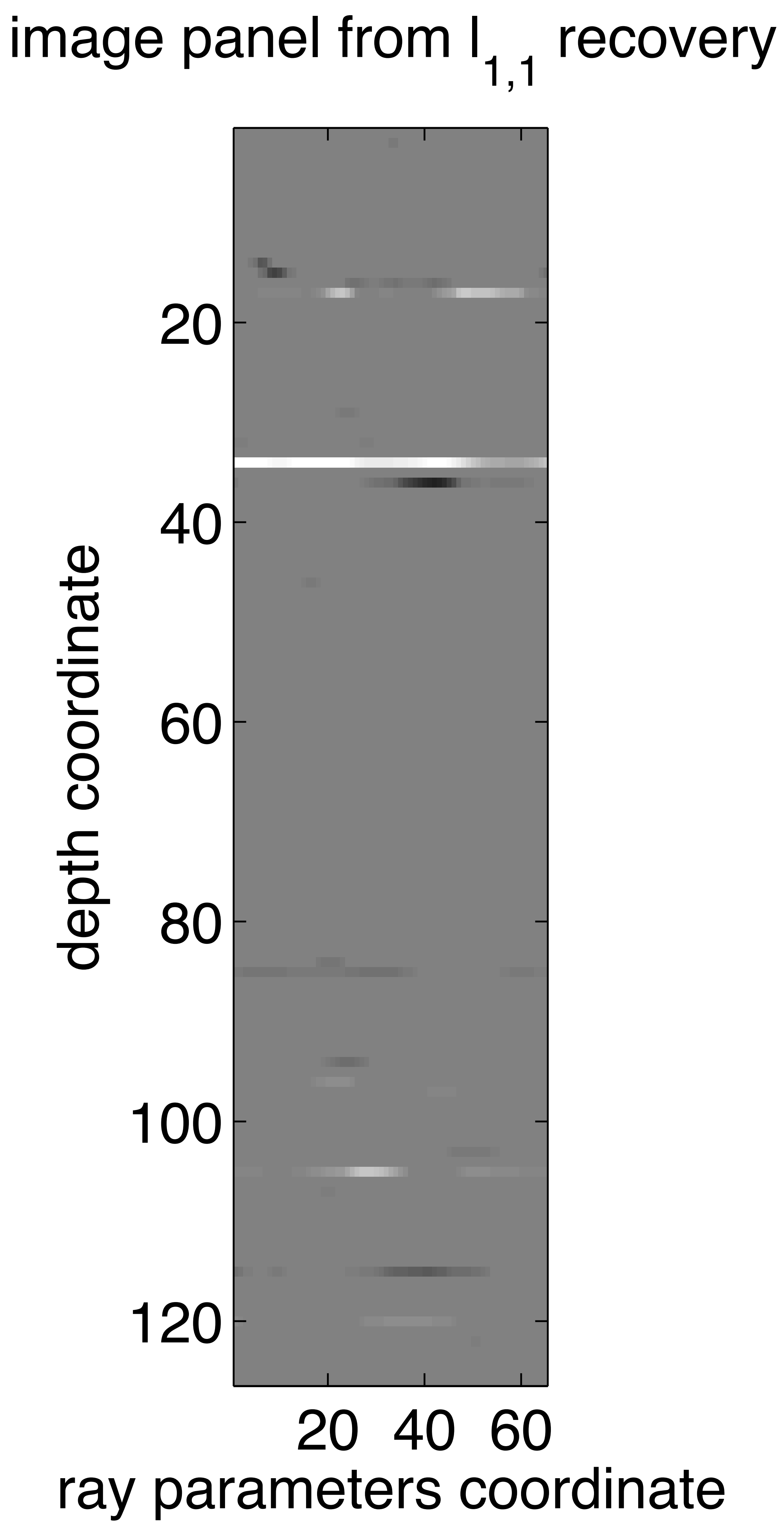
migration image with $l_{1,1}$ sparsity promoting

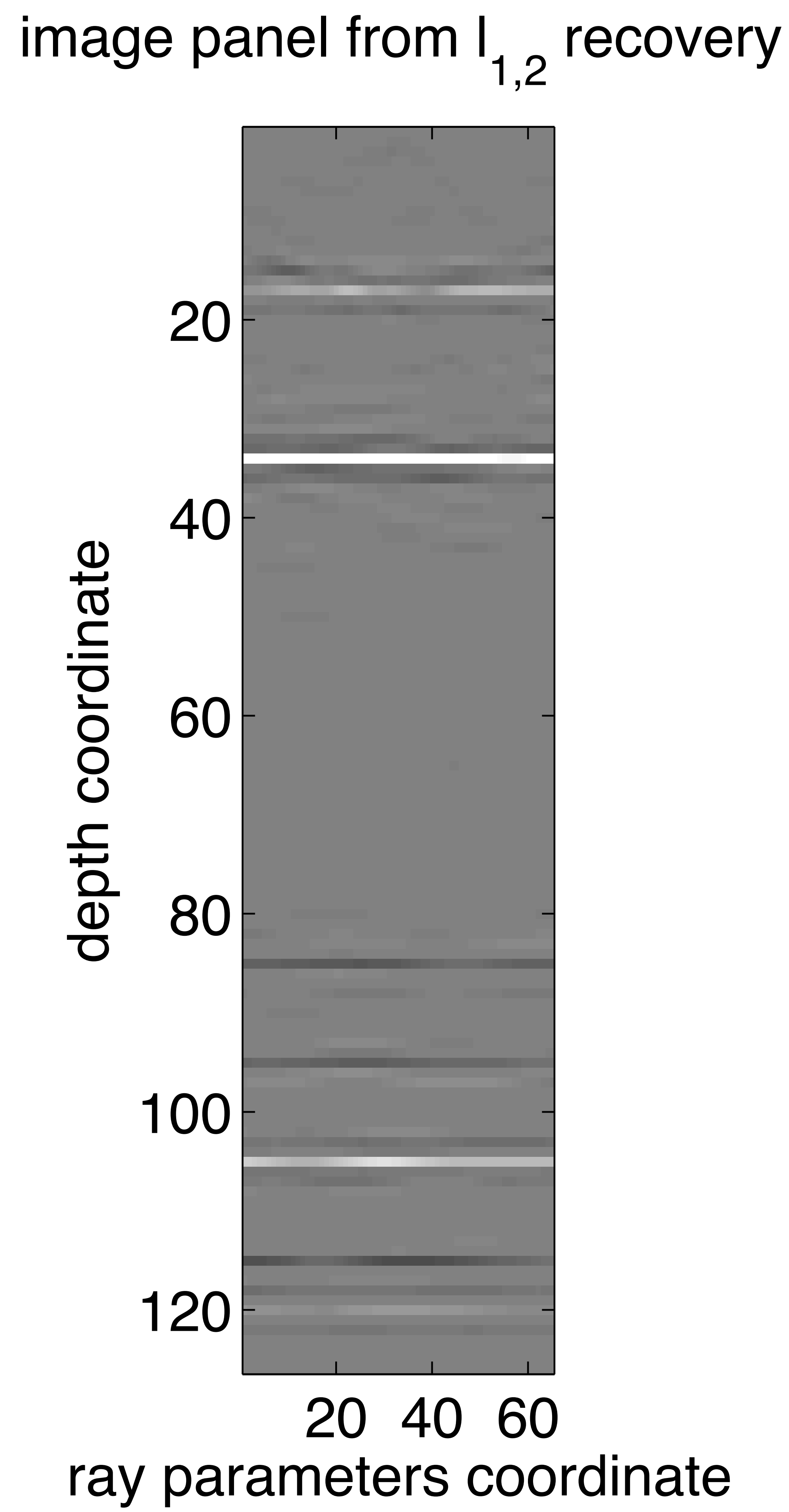



migration image with $l_{1,2}$ sparsity promoting



# Evaluation (Image volume)



# Conclusion

- Joint sparsity($\ell_{1,2}$ norm) promoting recovery generates better result images (migration image and image volume panel) compared to sparsity($\ell_{1,1}$ norm).
- The joint sparsity would be potentially useful in applications such as migration velocity analysis (MVA) or amplitude versus offset (AVO).

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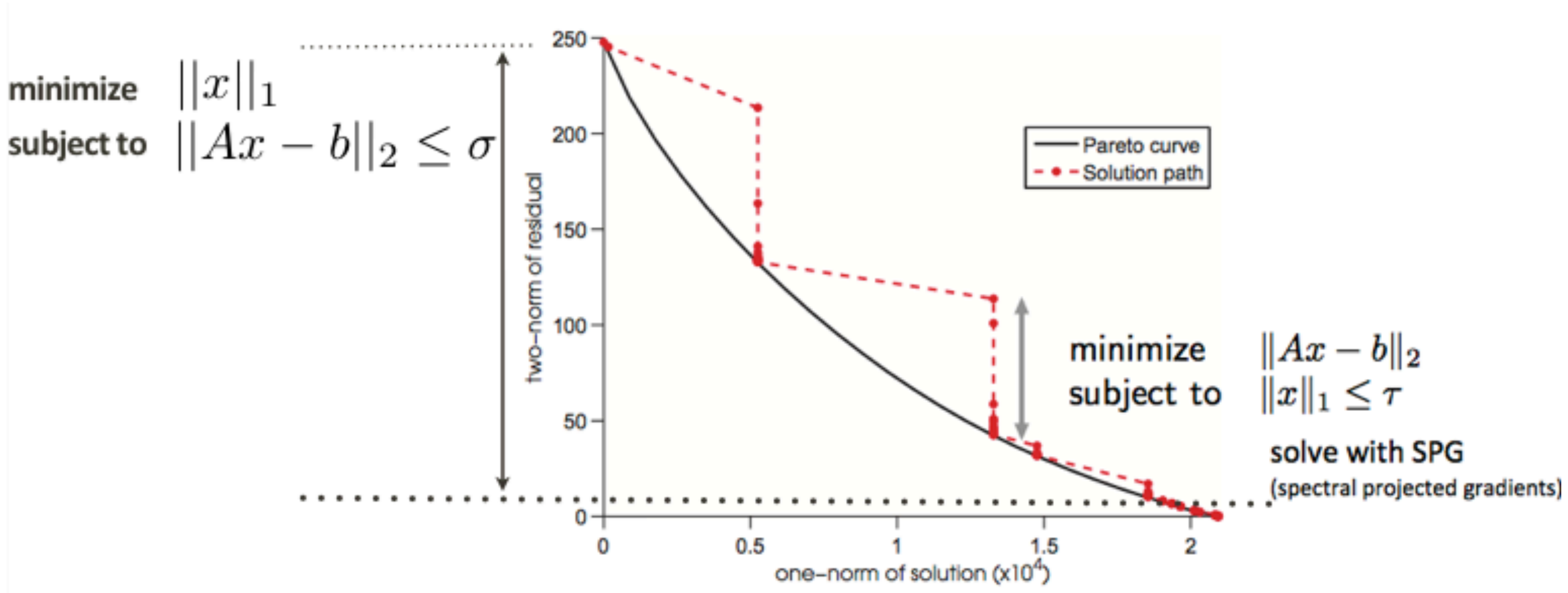
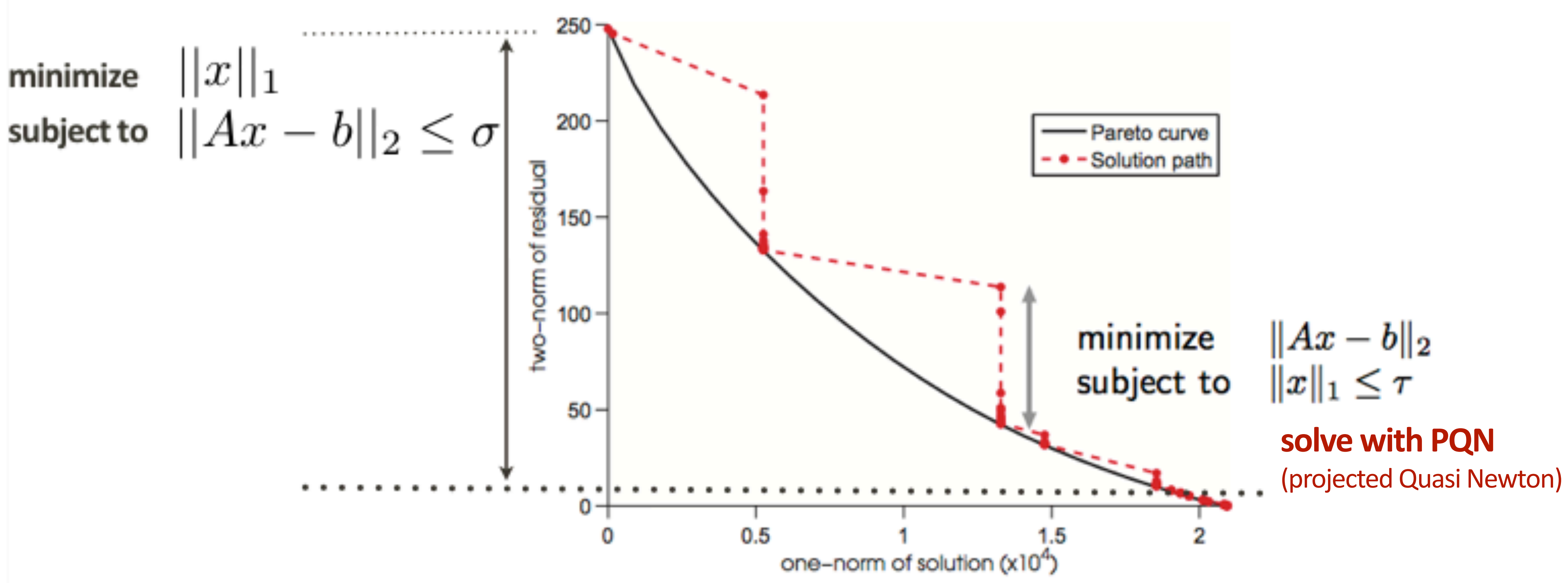
**Efficient large scale optimization algorithm**

# Large scale optimization (SPGl1)




Solution Path of Solver SPGl1

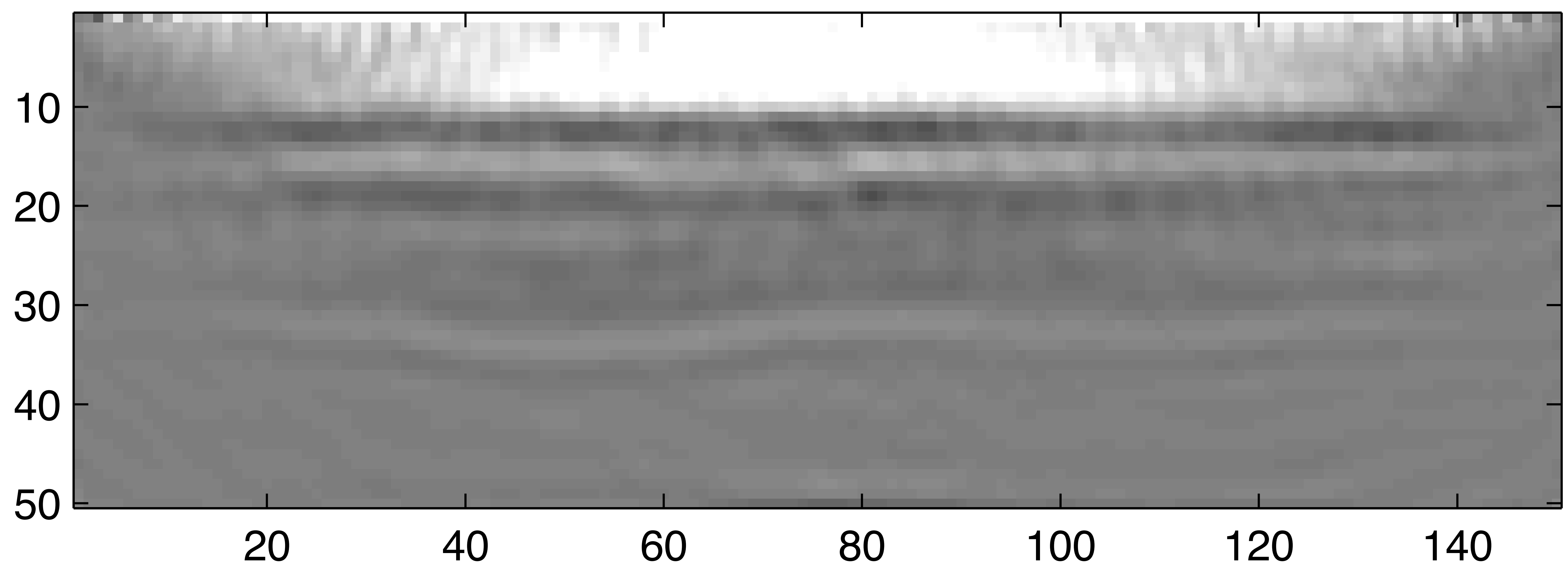
# Large scale optimization (PQNl1)




Solution Path of Solver PQN1

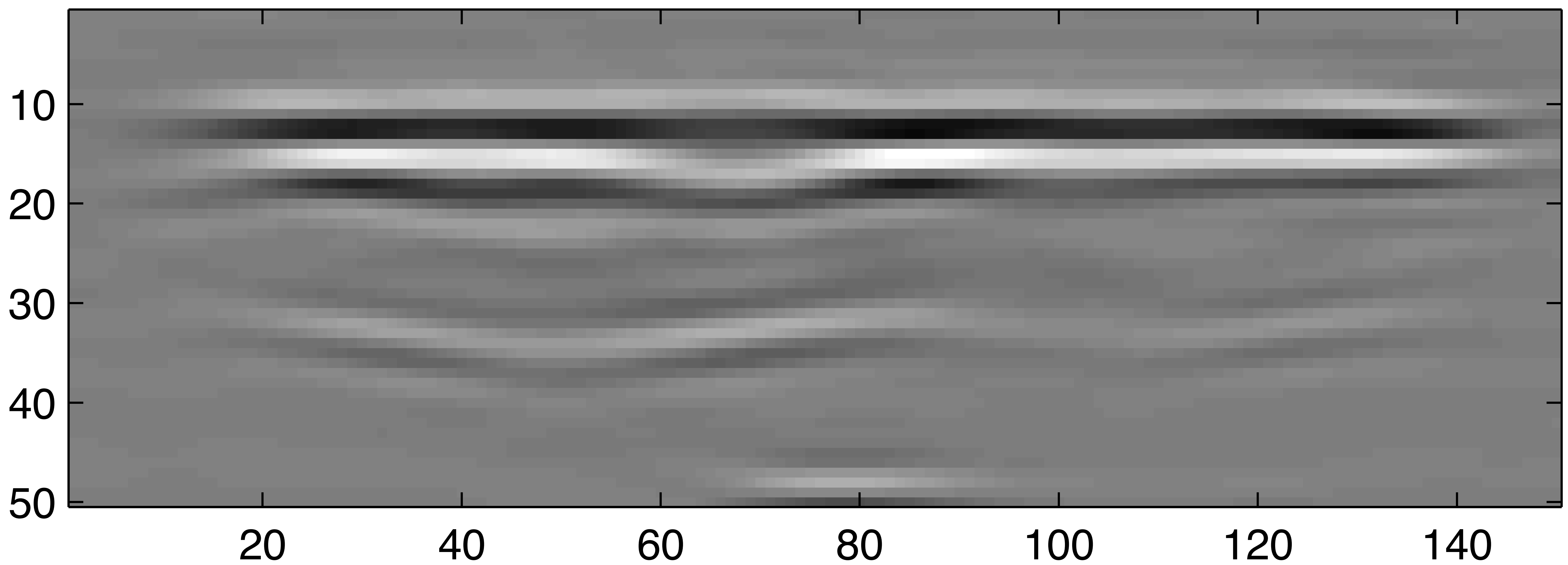
# SPGl$_1$ vs PQNl$_1$
## (seismic imaging example)

Reverse time migration result



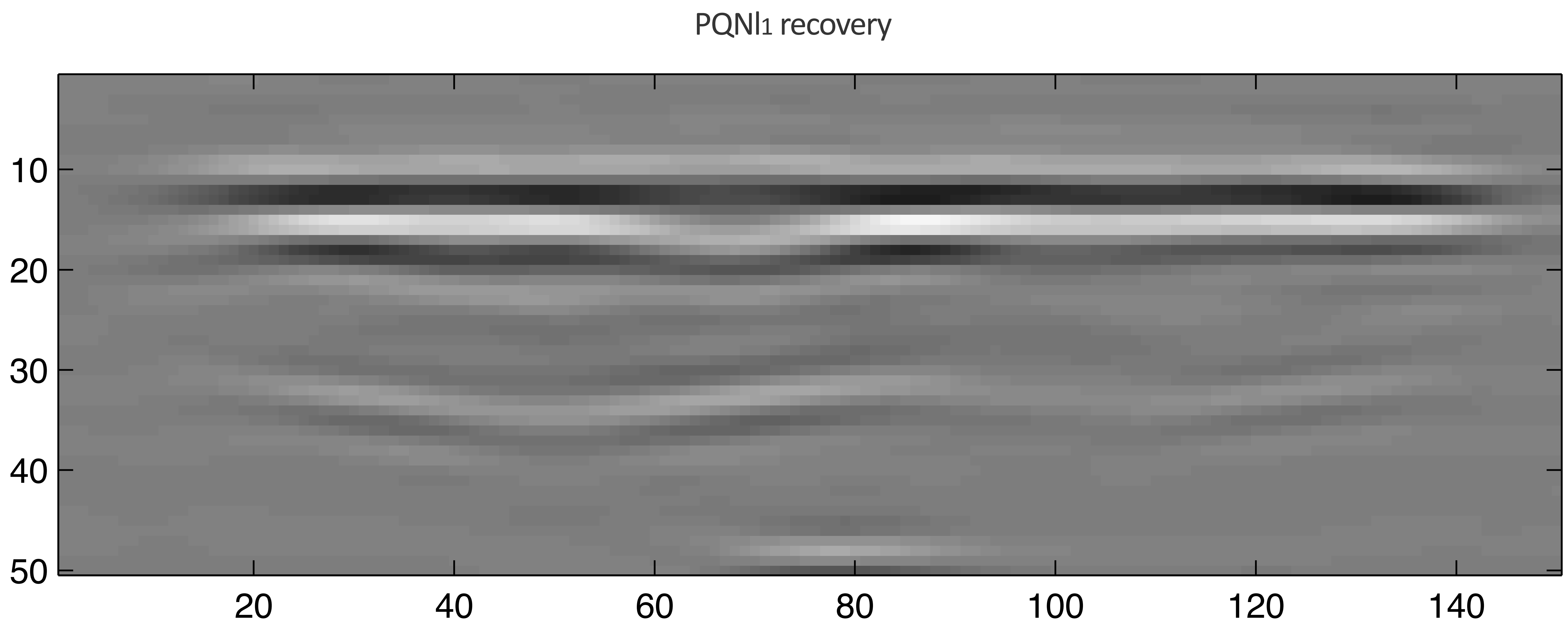
# SPGl1 vs PQNl1
## (seismic imaging example)

SPGl1 recovery



# SPGl$_1$ vs PQNl$_1$

## (seismic imaging example)



# SPGl1 vs PQNl1

## (seismic imaging example)

| | ITERATION | MAT-VEC | TIME(s) |
|---|---|---|---|
| SPGL1 | 100 | 156+101 | 39375 |
| PQNL1(5) | 36 | 45+45 | 9278.7 |
| PQNL1(10) | 35 | 44+44 | 7832.4 |
| PQNL1(20) | 35 | 44+44 | 6515.3 |
| PQNL1(30) | 35 | 44+44 | 6335.9 |

# Acknowledgements

## Thank you for your attention !

[https://www.slim.eos.ubc.ca/](https://www.slim.eos.ubc.ca/)

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