# Sampling and reconstruction of seismic wavefields in the curvelet domain

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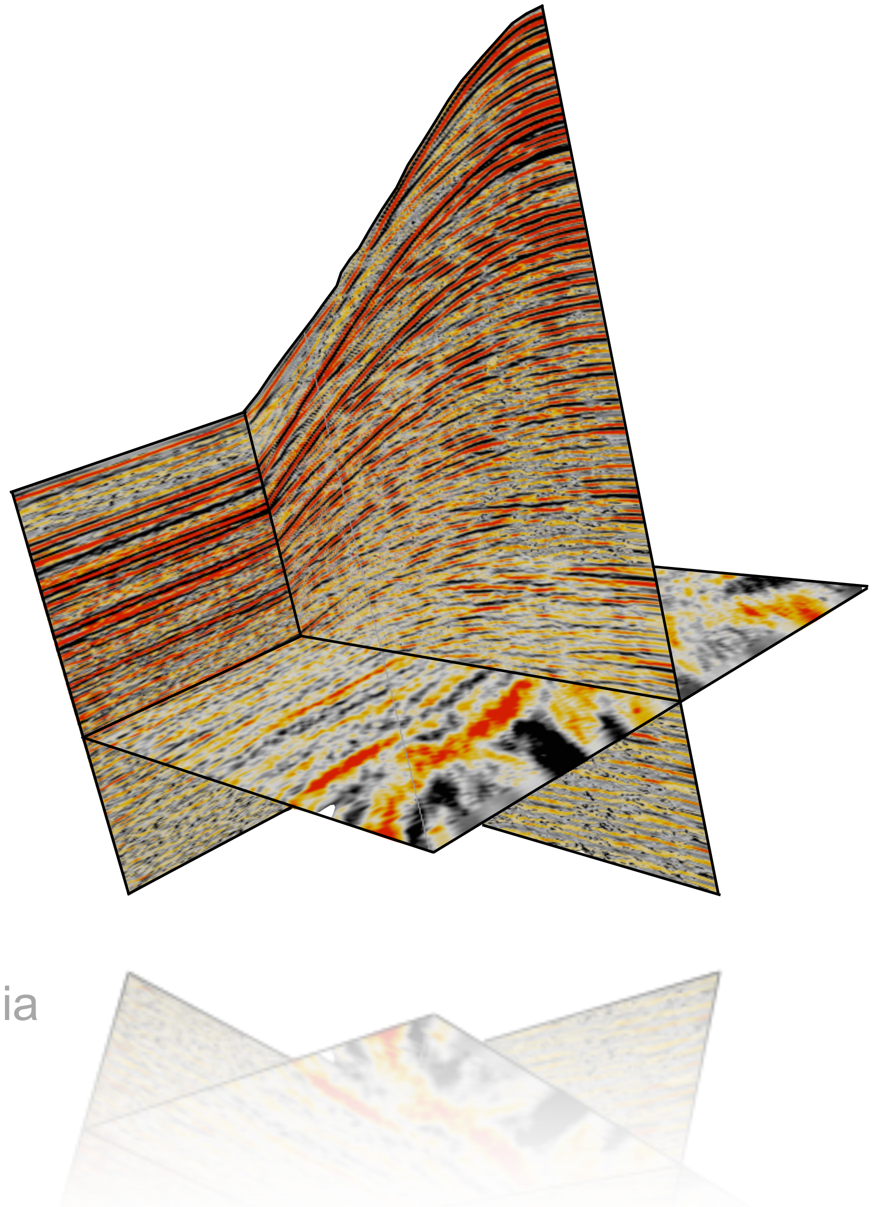


Final Doctoral Oral Examination
Room 203, Graduate Student Centre, The University of British Columbia
Thursday, April 10th, 2008 - 4PM

# Problematic

- acquisition irregularities create
  - **uneven illumination** of the subsurface
    - distorted image amplitudes (acquisition footprint)
  - **aliasing**, at least locally, when the acquisition geometry has large holes
    - image artifacts
    - erroneous predictions of coherent noise, e.g., multiples

- coarse spatial sampling creates
  - **aliasing**
    - image artifacts
    - erroneous predictions of coherent noise, e.g., multiples

# Wavefield reconstruction methods

- filter-based methods [Spitz'91, Fomel'00]
  - convolve the incomplete data with an interpolating filter

- wavefield-operator-based methods [Canning and Gardner'96, Biondi et al.'98, Stolt'02]
  - explicitly include wave propagation
  - require knowledge of velocity model
  - computationally intensive

- transform-based methods [Sacchi et al.'98, Trad et al.'03, Zwartjes and Sacchi'07]
  - fastest approaches
  - no explicit link with wave propagation

Performance of most aforementioned methods deteriorates for data with acquisition irregularities.

# Key contributions

- new wavefield reconstruction method that handles both regular and irregular acquisition geometries
  - curvelet reconstruction with sparsity-promoting inversion (CRSI) [Herrmann and Hennenfent‘08]

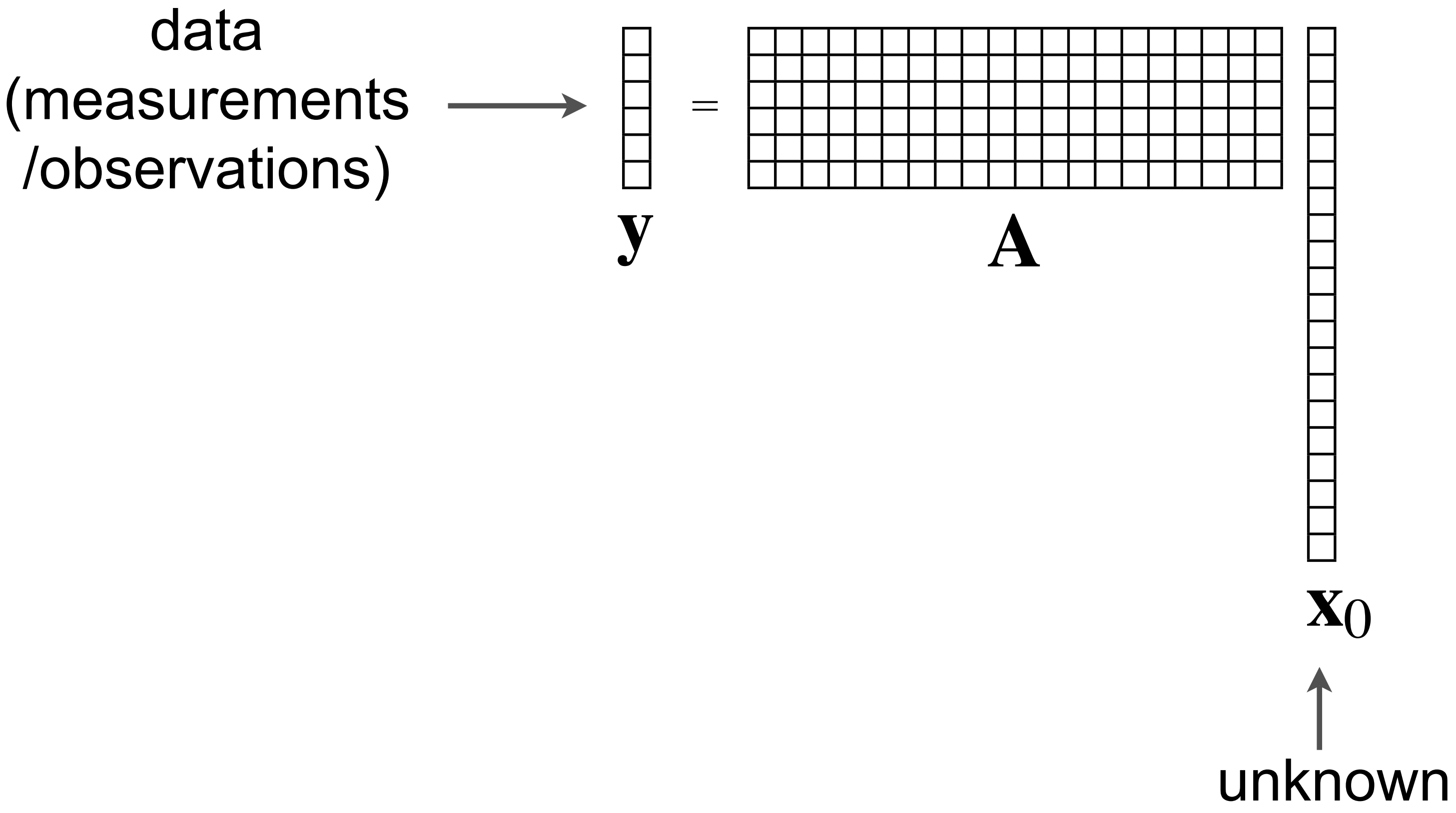
- extension of the fast discrete curvelet transform to handle irregular seismic data
  - nonequispaced fast discrete curvelet transform (NFDCT) [Hennenfent and Herrmann‘06]

- new coarse sampling schemes that maximize performance of CRSI
  - jittered undersampling schemes [Hennenfent and Herrmann‘08]

- new large-scale, one-norm solver
  - iterative soft thresholding with cooling (ISTc) [Herrmann and Hennenfent’08, Hennenfent et al.‘08]

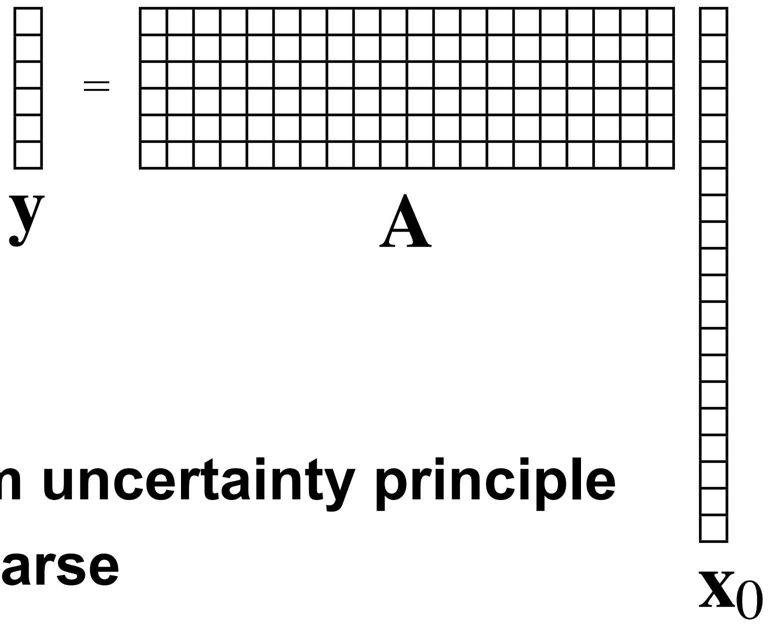
# Problem statement

Consider the following (severely) underdetermined system of linear equations



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

# Perfect recovery



- conditions
  - **A** obeys the **uniform uncertainty principle**
  - $\mathbf{x}_0$ is **sufficiently sparse**
- procedure

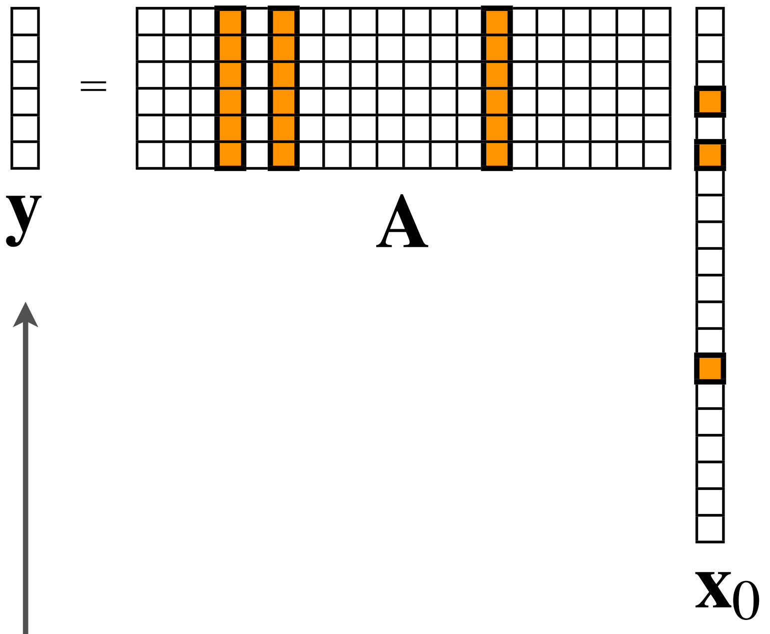
$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

- performance
  - *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

[Candès et al.'06]
[Donoho'06]

# Simple example

restriction
operator



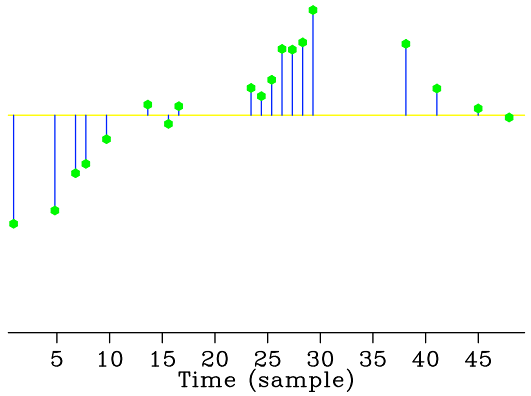
with

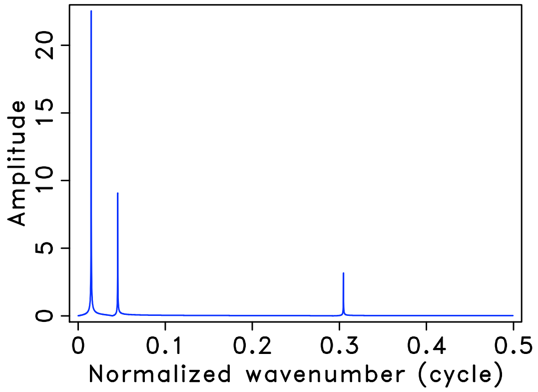
$$\mathbf{A} := \mathbf{R}\mathbf{F}^H$$

Fourier
transform

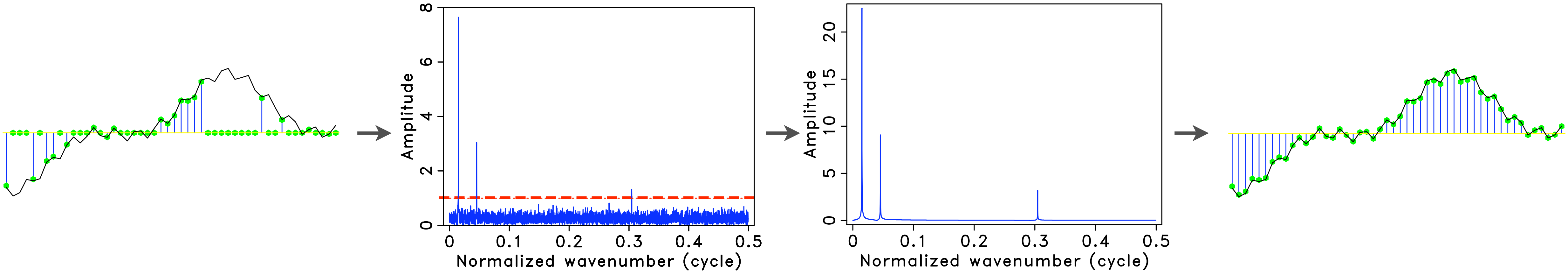
signal

Fourier coefficients
(sparse)





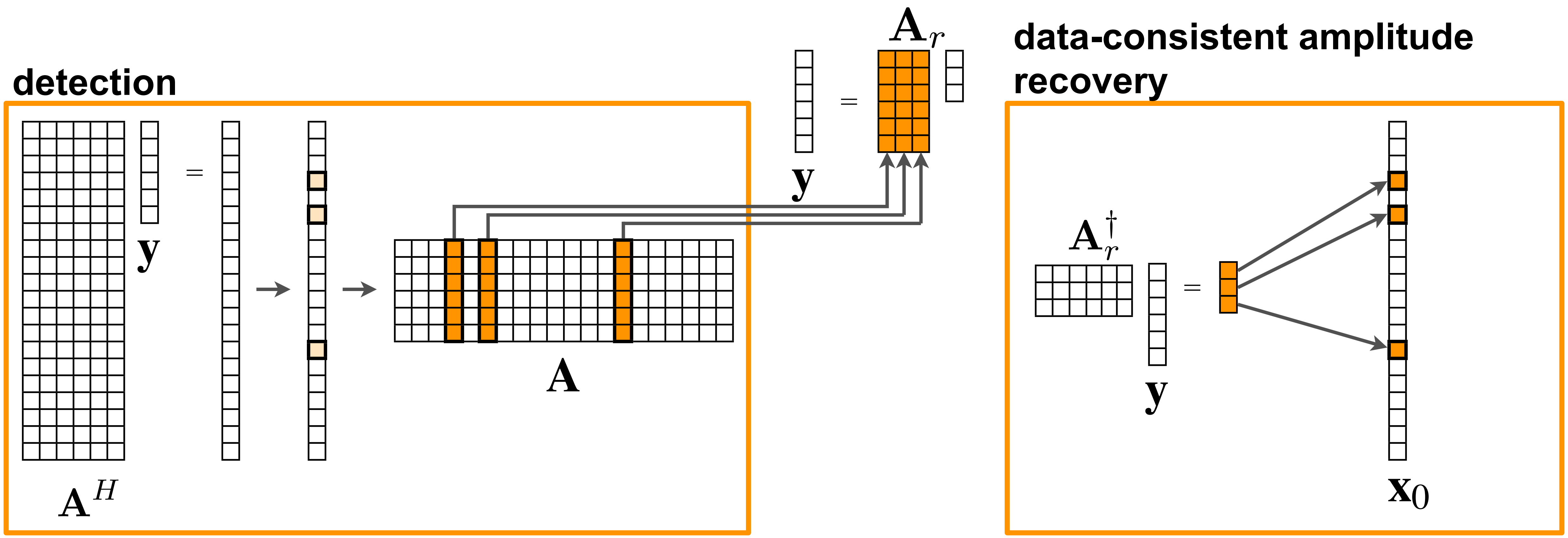
# NAIVE sparsity-promoting recovery



inverse
Fourier
transform

detection +
data-consistent
amplitude recovery

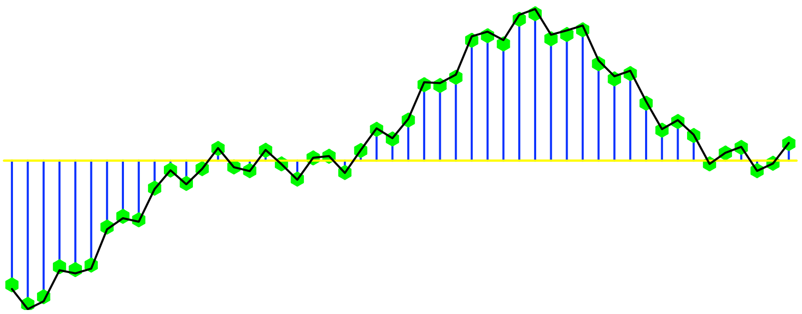
Fourier
transform



**detection**

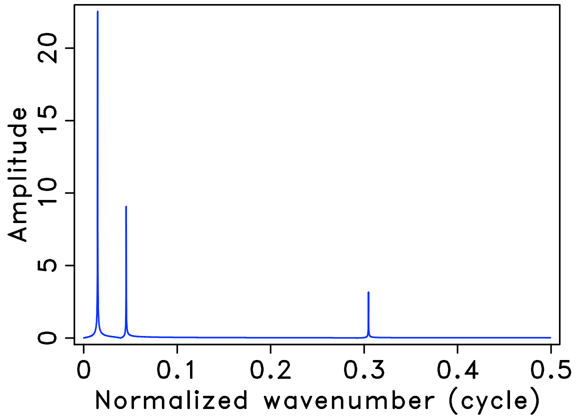
**data-consistent amplitude recovery**

# Coarse sampling schemes



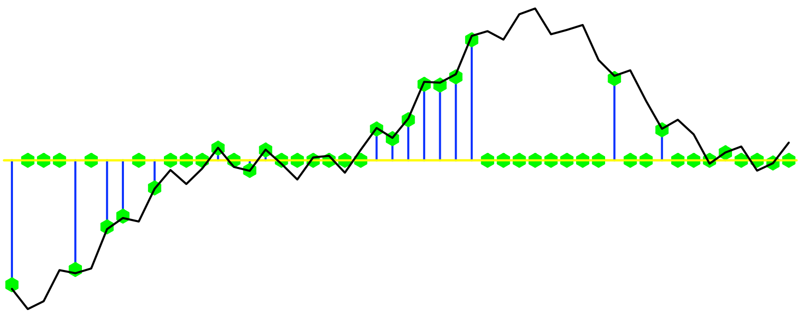
Fourier

transform



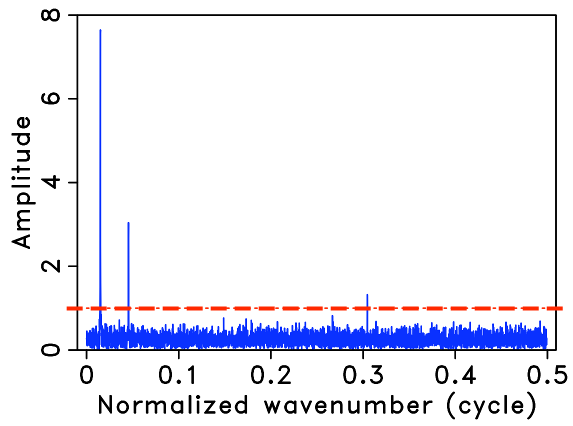
few significant coefficients

**3-fold under-sampling**



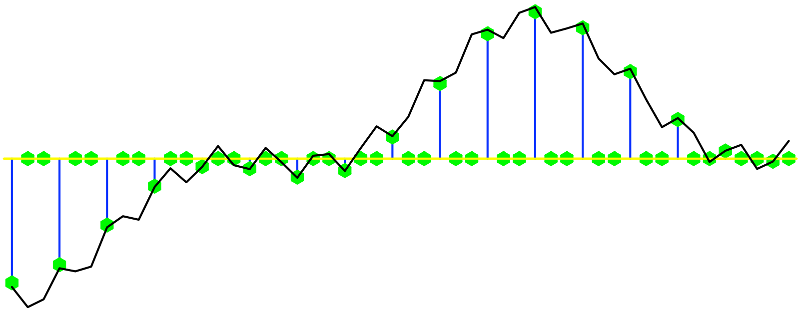
Fourier

transform



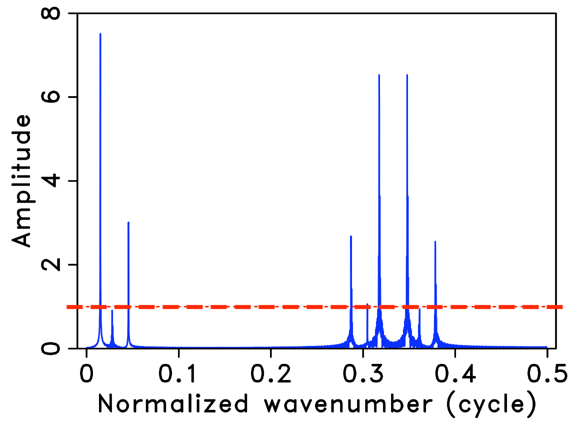
✓

significant coefficients detected



Fourier

transform

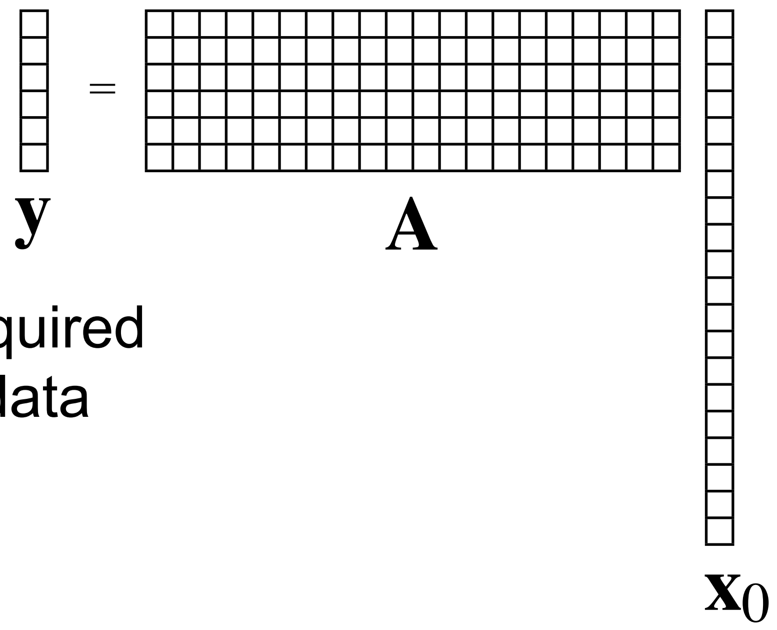


✗

ambiguity

[Hennenfent and Herrmann'08]

# Sparsity-promoting wavefield reconstruction



acquired
data

complete wavefield
(transform domain)

restriction operator

with $\mathbf{A} := \mathbf{R}\mathbf{S}^H$

sparsifying transform
for seismic data

Interpolated data given by $\tilde{\mathbf{f}} = \mathbf{S}^H \tilde{\mathbf{x}}$ with

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$

[Sacchi et al.‘98]
[Xu et al.‘05]
[Zwartjes and Sacchi‘07]
[Herrmann and Hennenfent‘08]

# Key elements

☐ *sparsifying transform*

- typically **localized** in the time-space domain to handle the complexity of seismic data

☐ *advantageous coarse sampling*

- generates incoherent random undersampling “noise” in the sparsifying domain
- does not create large gaps
  - because of the limited spatiotemporal extent of transform elements used for the reconstruction

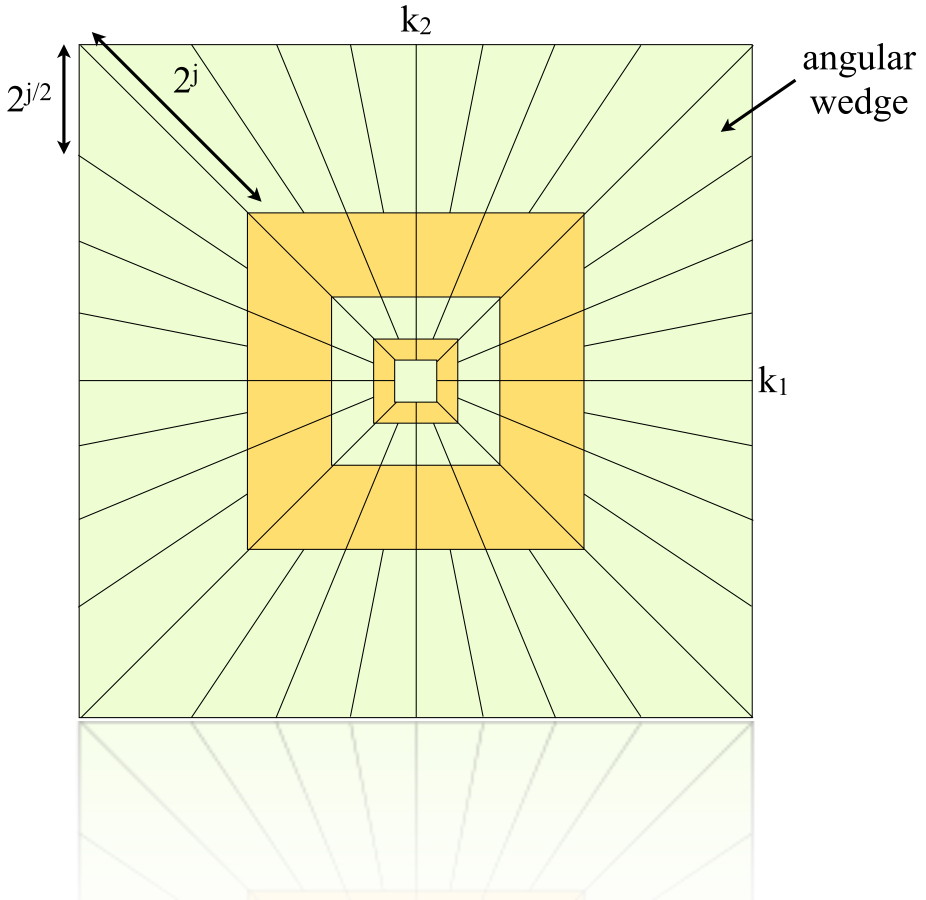
☐ *sparsity-promoting solver*

- requires few matrix-vector multiplications

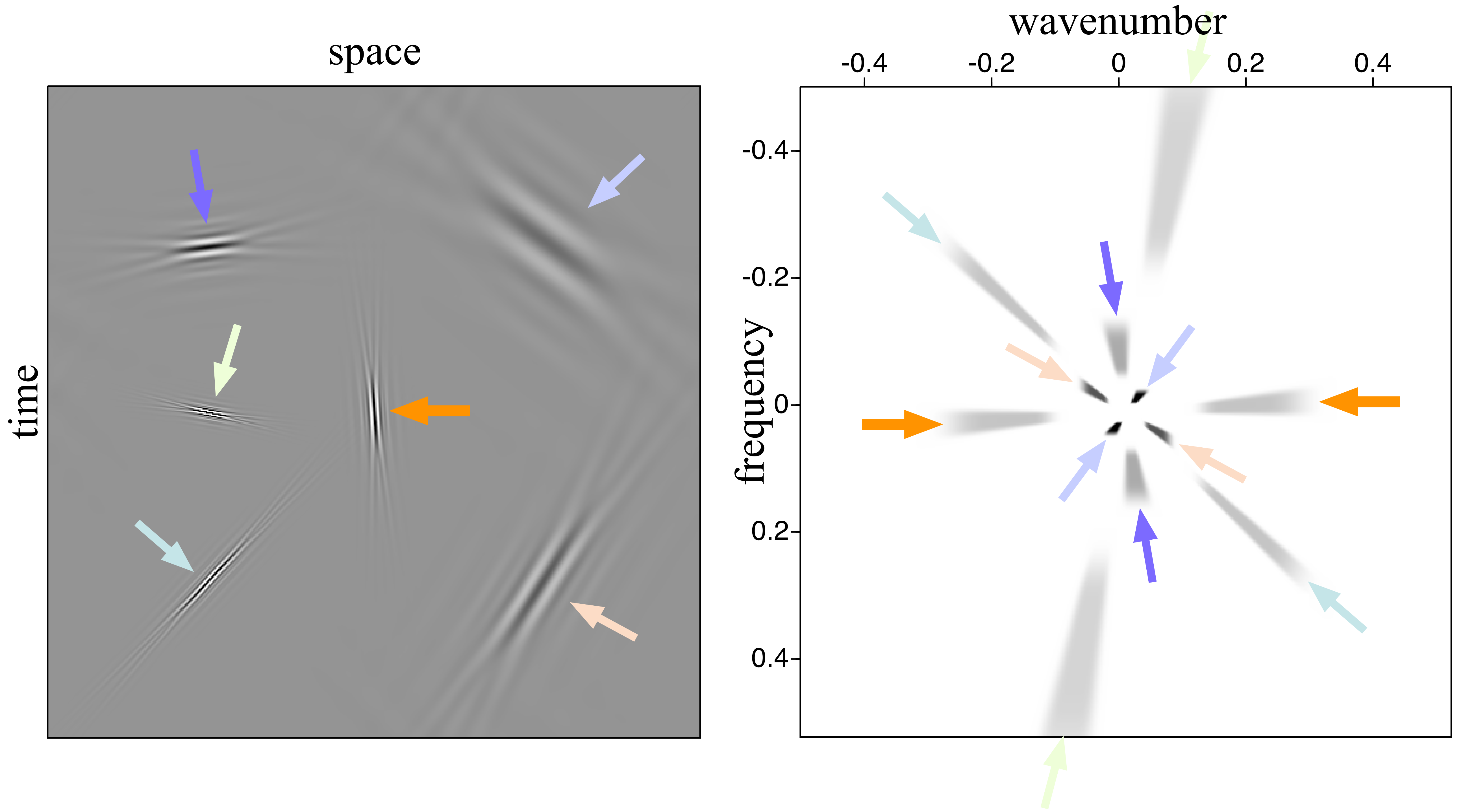
# Representations for seismic data

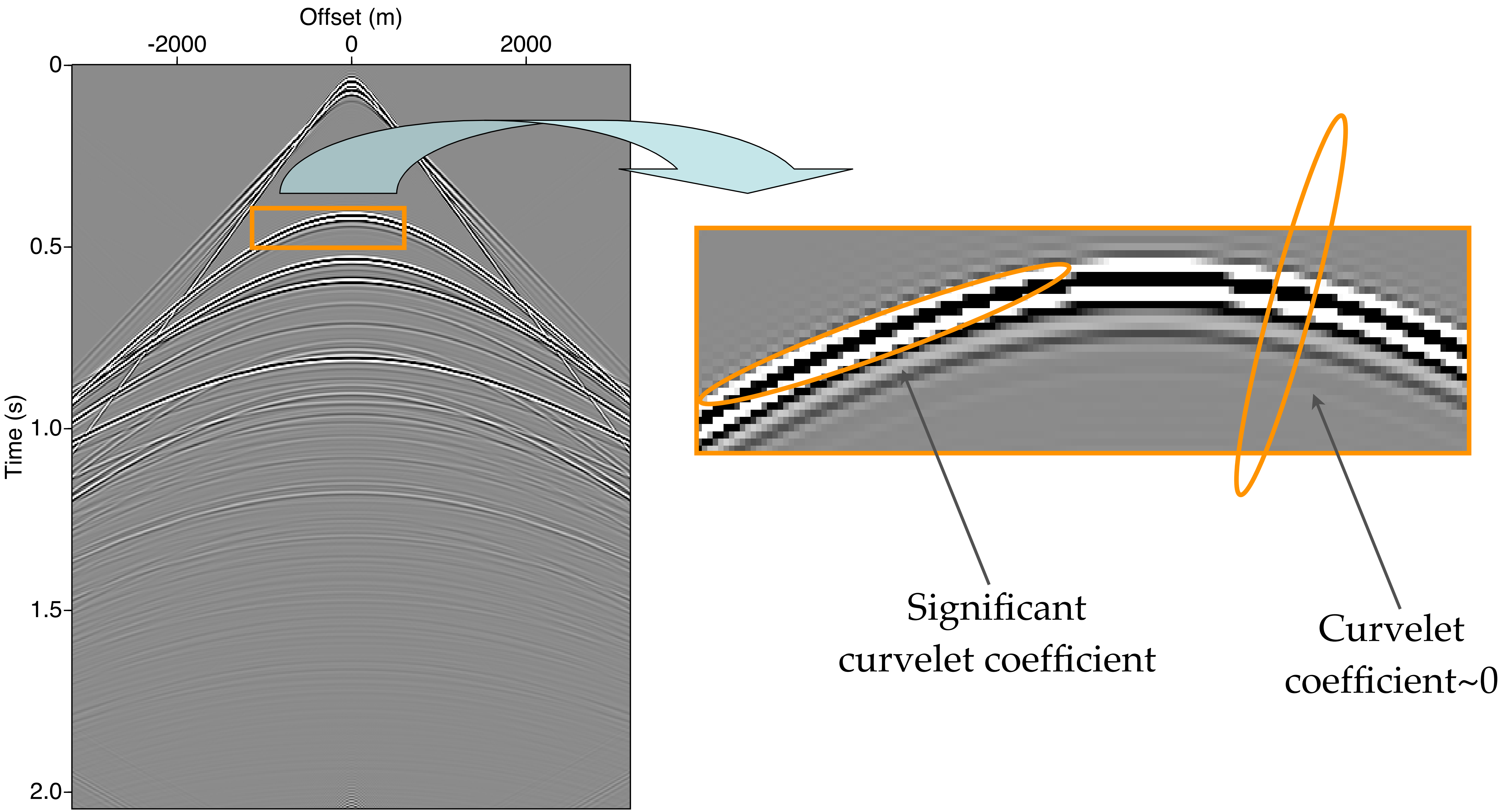
| Transform | Underlying assumption |
|---|---|
| FK | plane waves |
| linear/parabolic Radon transform | linear/parabolic events |
| wavelet transform | point-like events (1D singularities) |
| **curvelet transform** | **curve-like events (2D singularities)** |

- curvelet transform
  - **multiscale**: tiling of the FK domain into dyadic coronae
  - **multidirectional**: coronae sub-partitioned into angular wedges, # of angles doubles every other scale
  - **anisotropic**: parabolic scaling principle
  - **local**



# 2D discrete curvelets

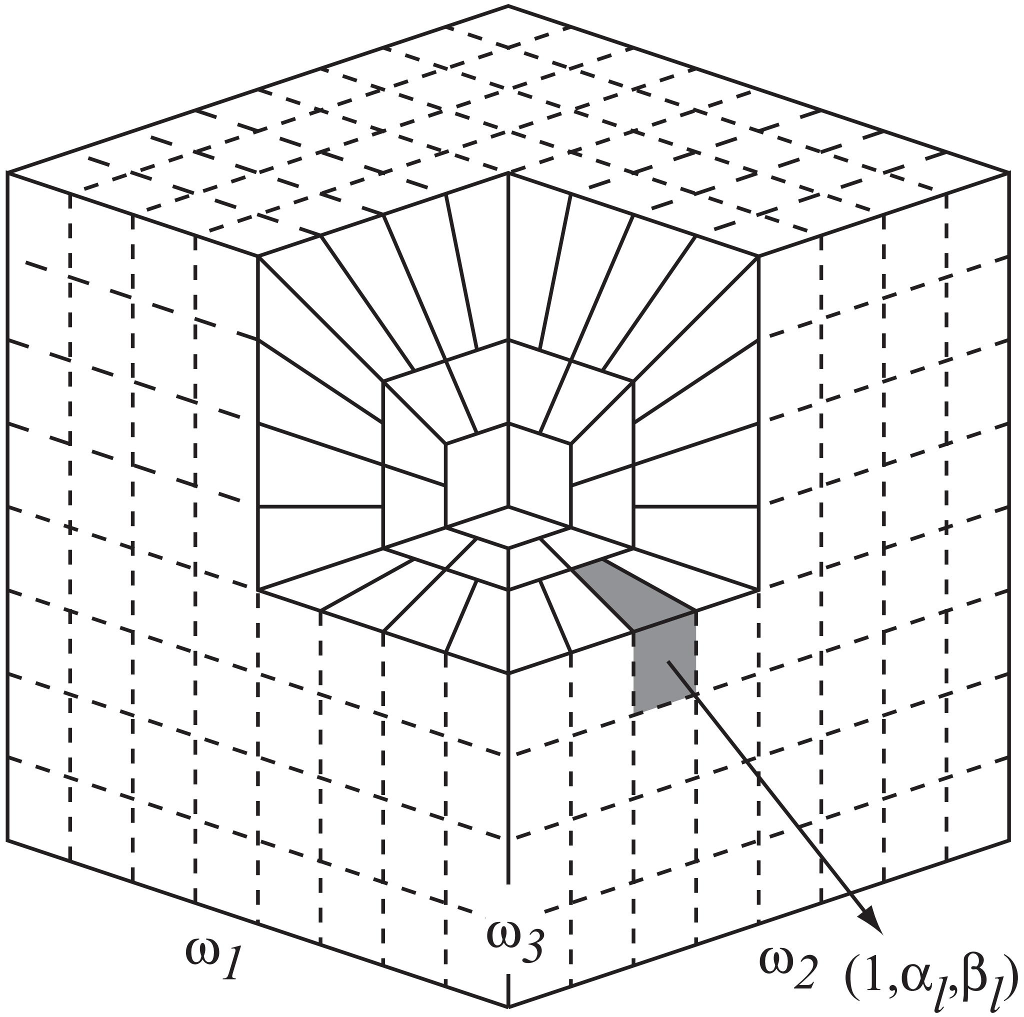


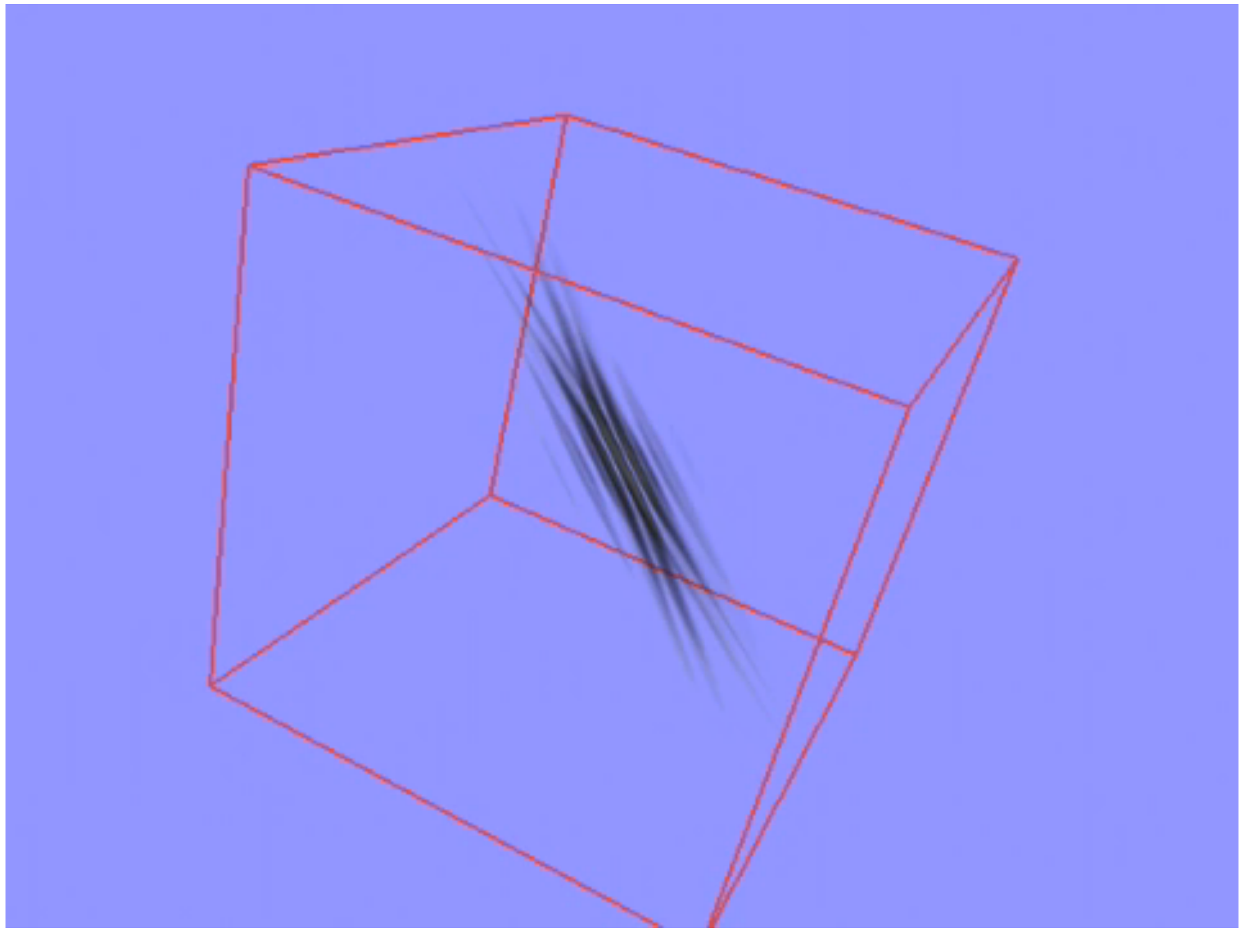


Significant
curvelet coefficient

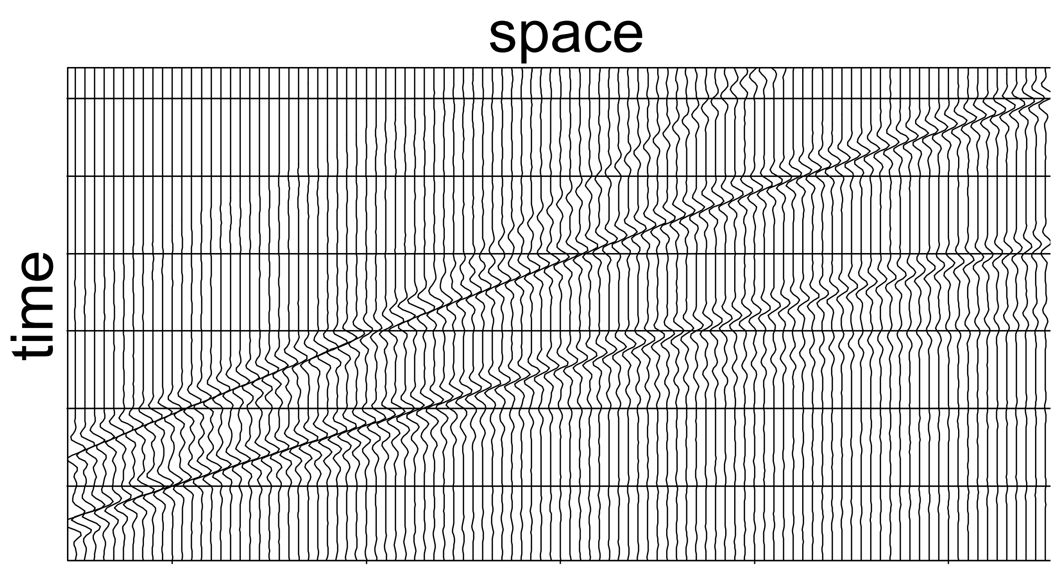
Curvelet
coefficient~0

# 3D discrete curvelets





# 2D nonequispaced fast discrete curvelets

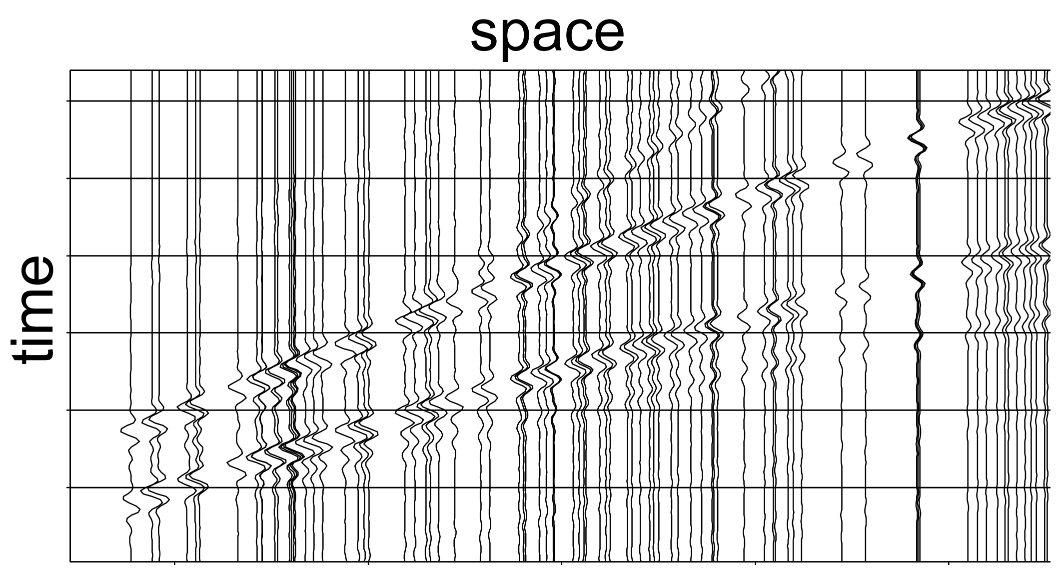


processed

→

using

**fast discrete curvelet transform**

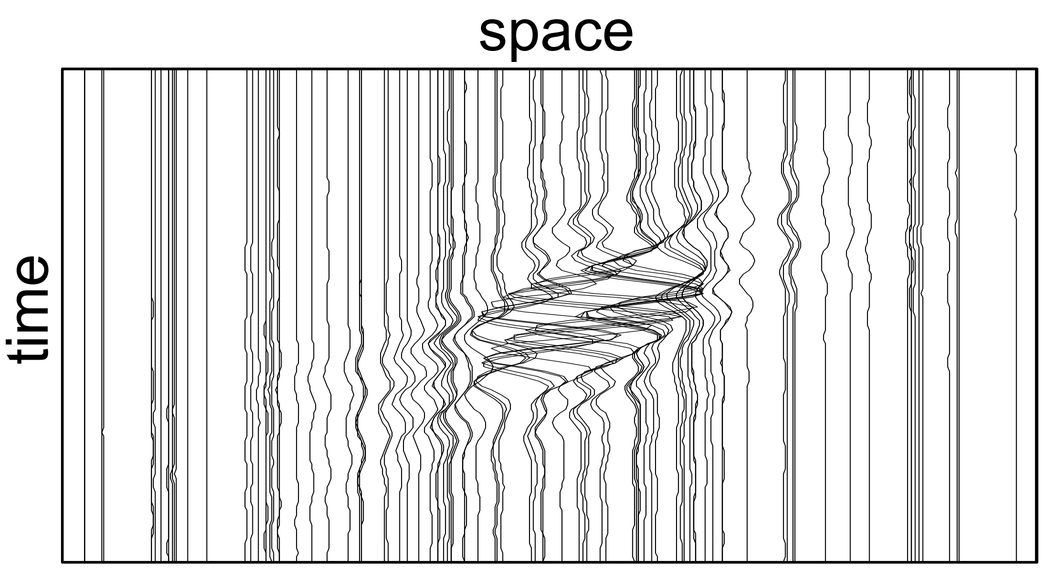


**data with acquisition irregularities**

processed

→

using



**“seismic” curvelet**

[Hennenfent and Herrmann‘06]

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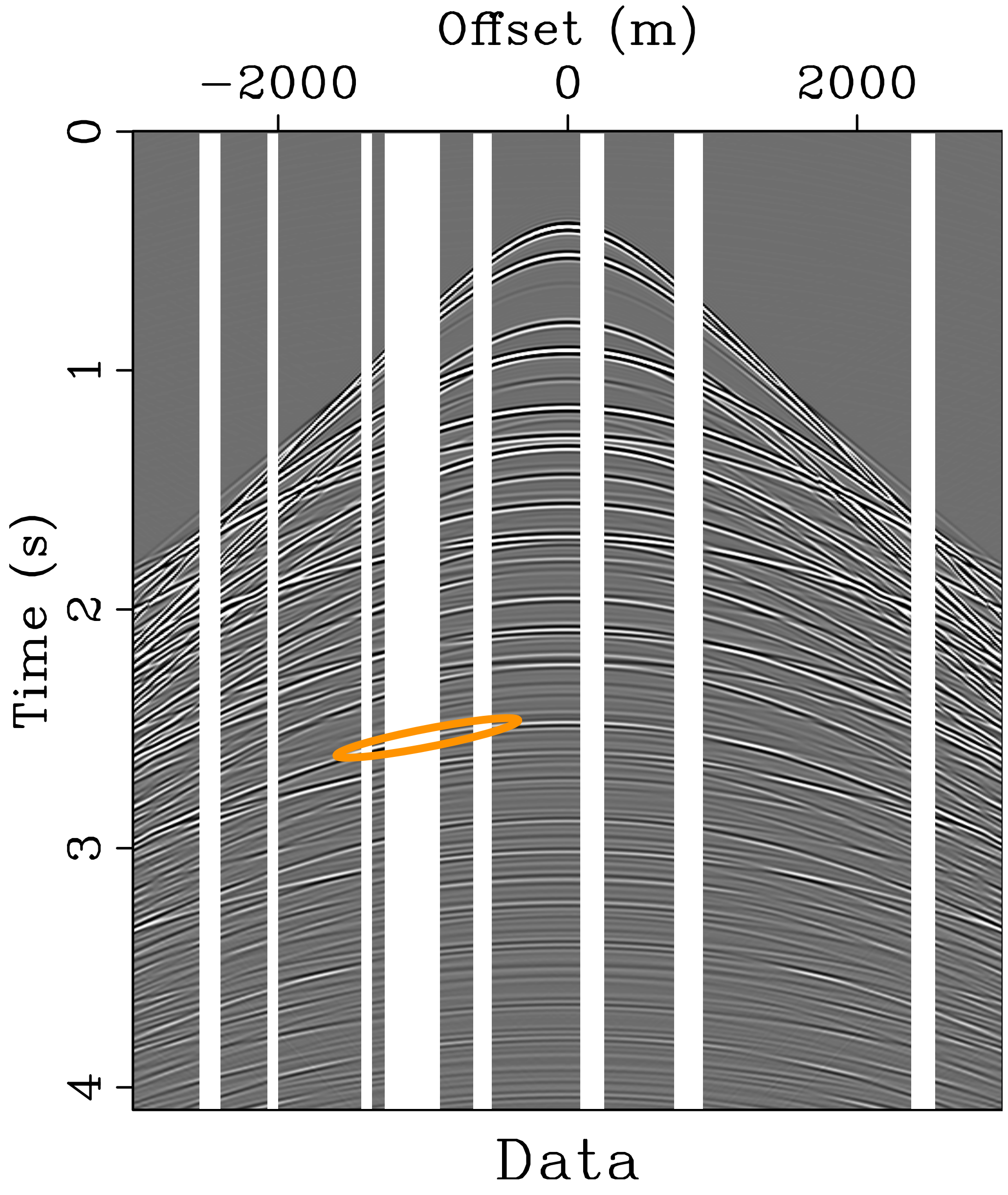
☐ *sparsity-promoting solver*

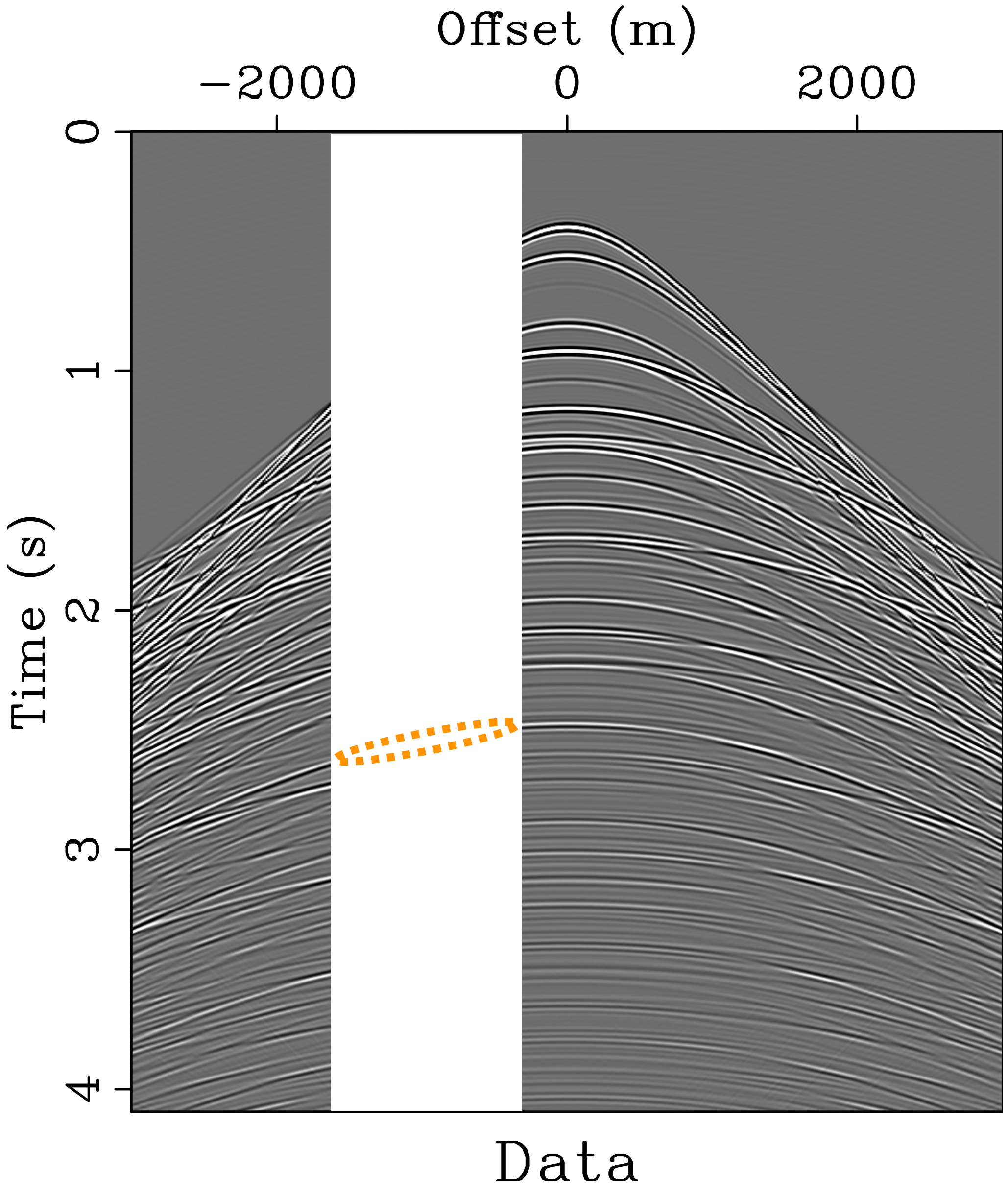
- requires few matrix-vector multiplications

# Localized transform elements & gap size

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x}$$



✓



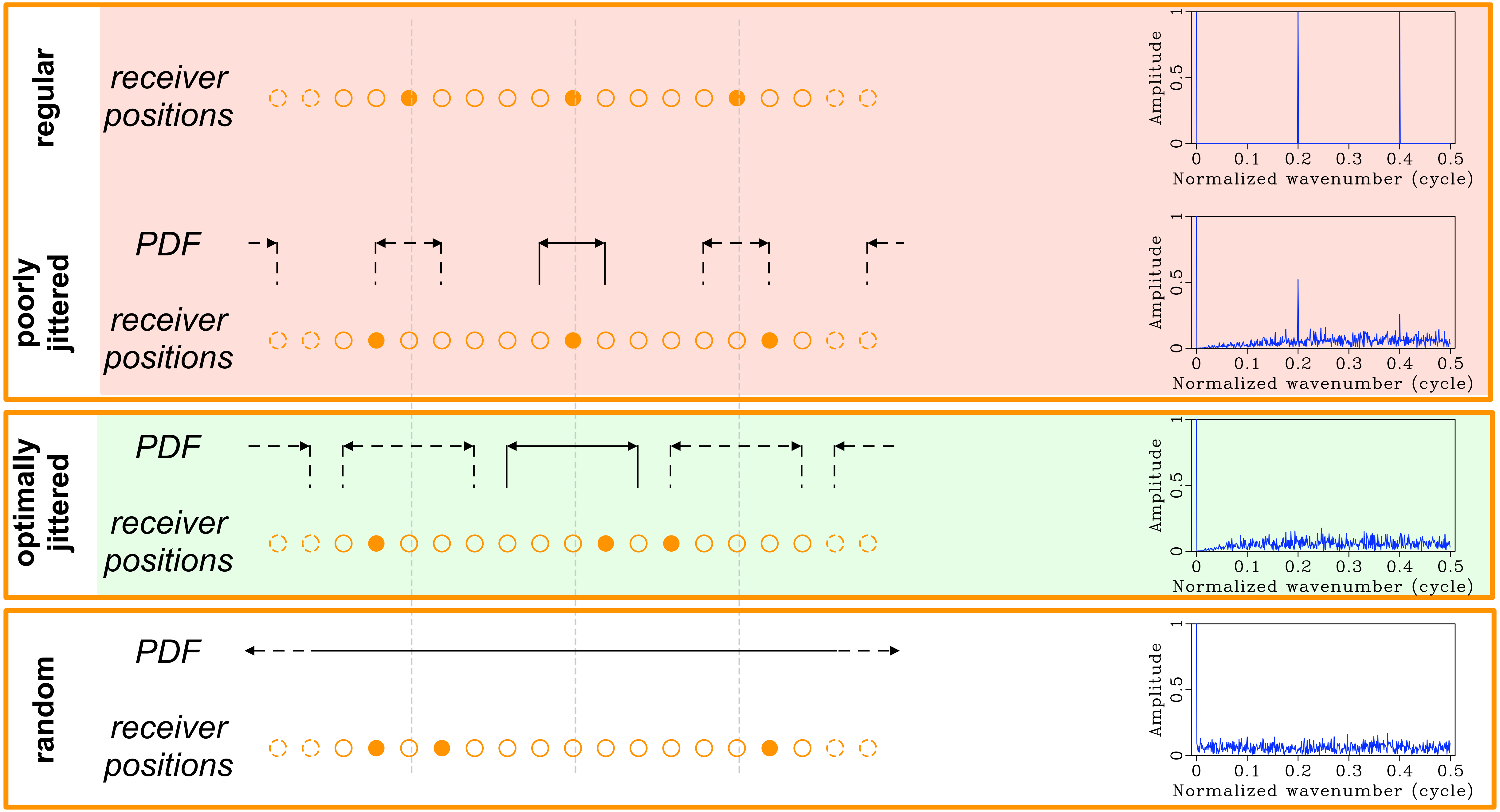
✗

# Discrete random jittered undersampling

Type

Sampling scheme

*Typical* spatial convolution kernel



**regular**

*receiver positions*

**poorly jittered**

*PDF*

*receiver positions*

**optimally jittered**

*PDF*

*receiver positions*

**random**

*PDF*

*receiver positions*

[Hennenfent and Herrmann‘08]

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# Approaches

- quadratic programming [many references!]

$$\mathrm{QP}_\lambda : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1$$

- basis pursuit denoise [Chen et al.'95]

$$\mathrm{BP}_\sigma : \quad \min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma$$

- LASSO [Tibshirani'96]

$$\mathrm{LS}_\tau : \quad \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 \leq \tau$$

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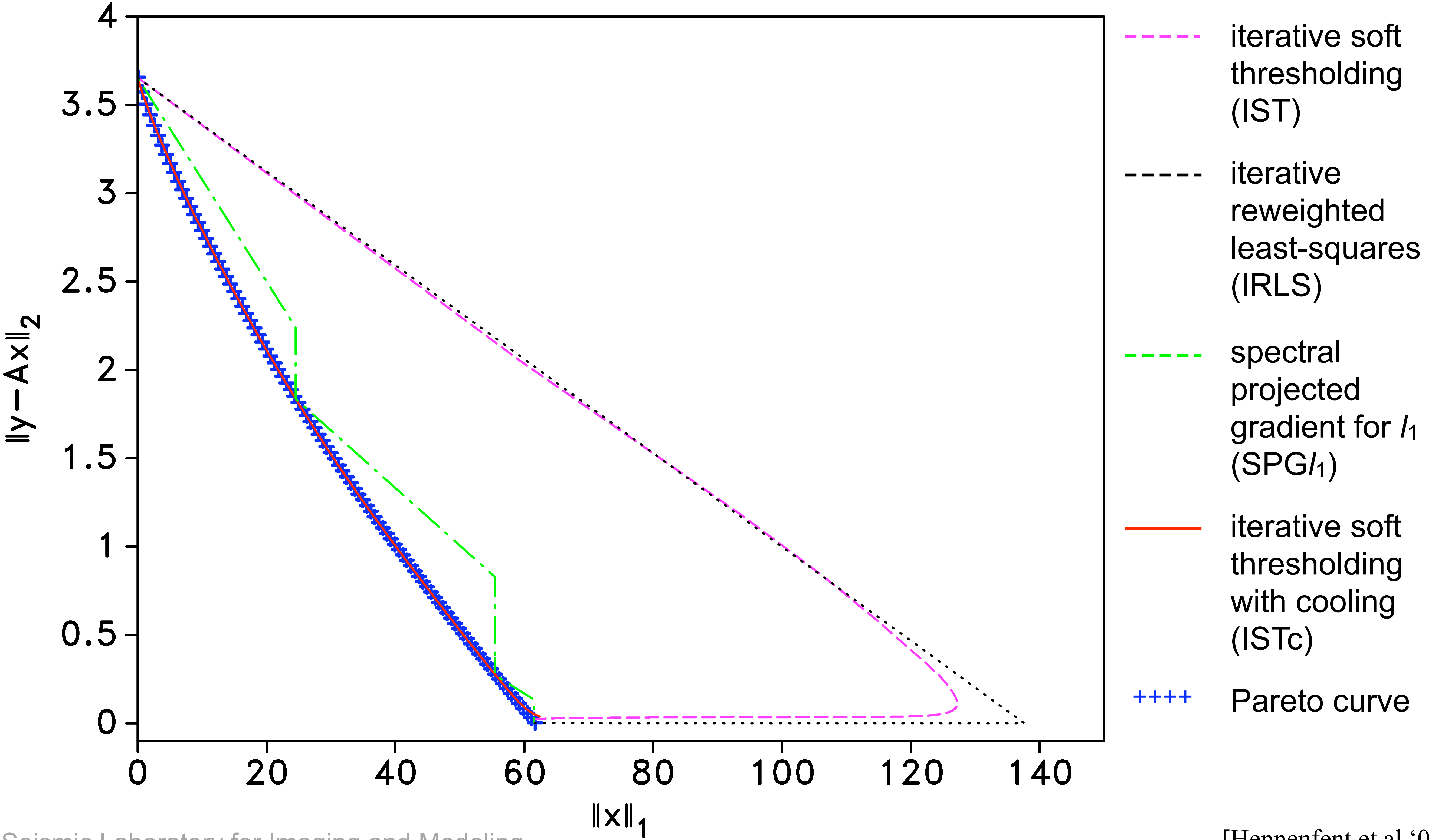
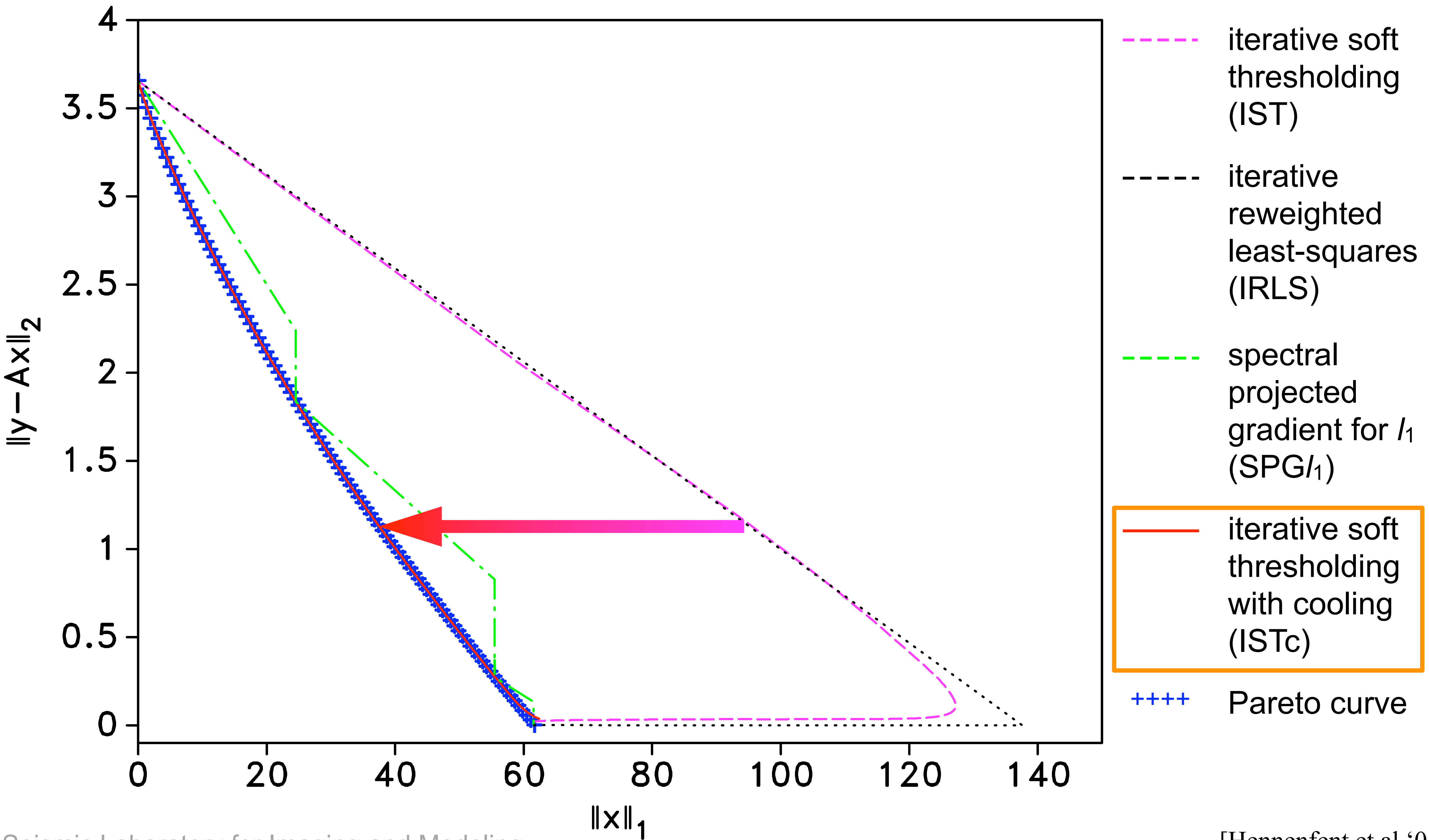
# One-norm solvers



[Hennenfent et al.‘08]

# One-norm solvers



[Hennenfent et al.'08]

# Key elements

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☑ *sparsity-promoting solver*

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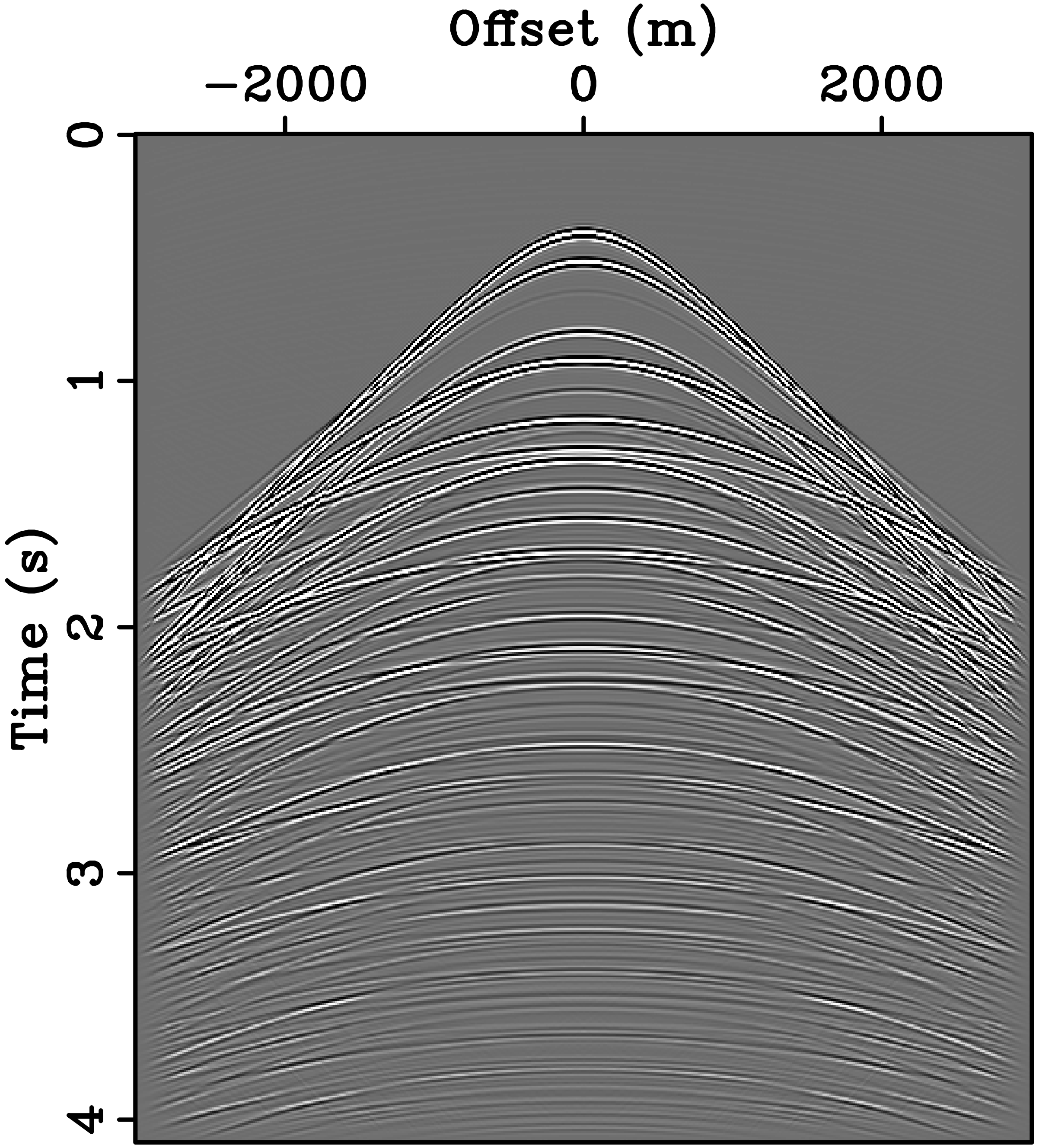
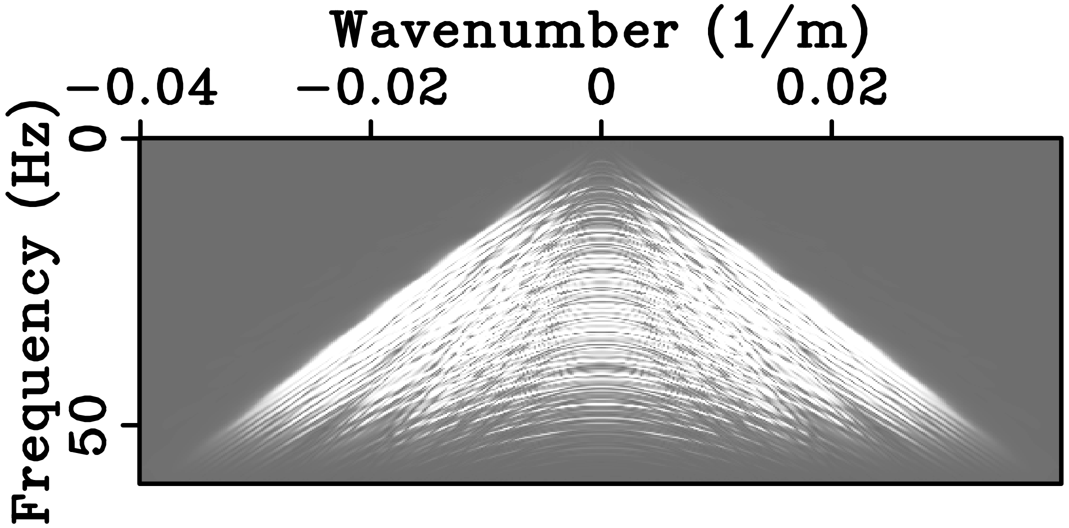
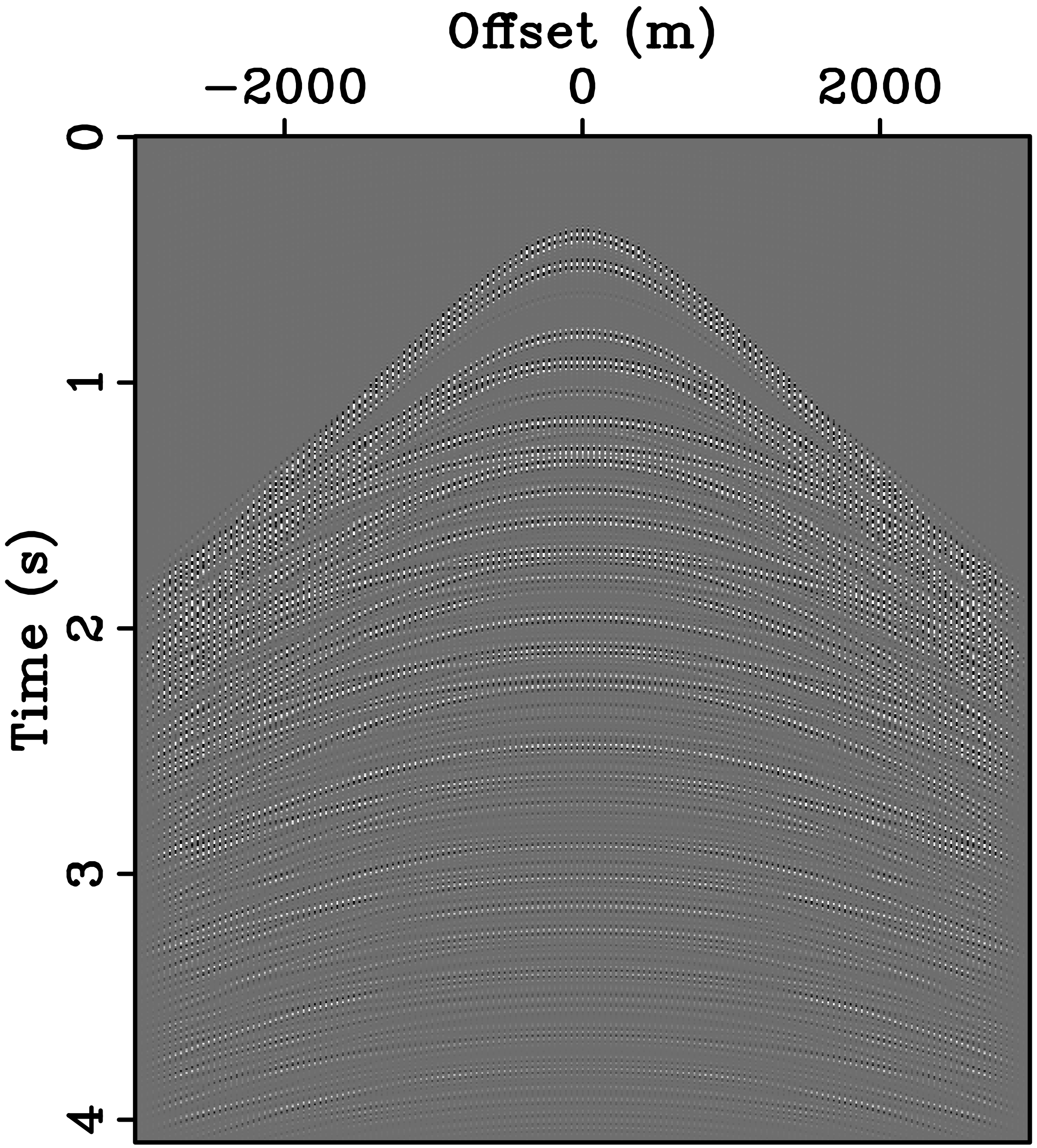
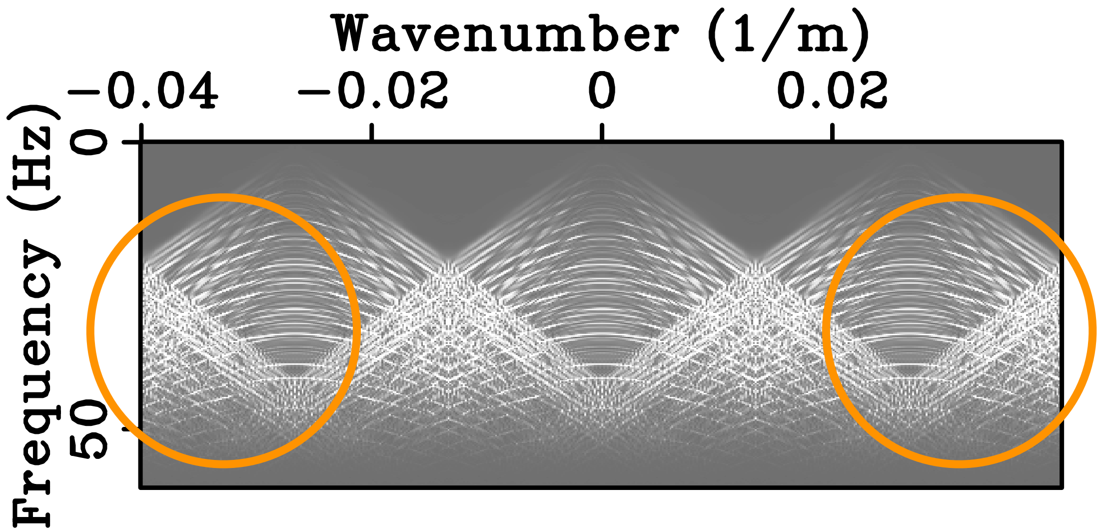
# Model


Offset (m)
−2000
0
2000
Time (s)
0
1
2
3
4

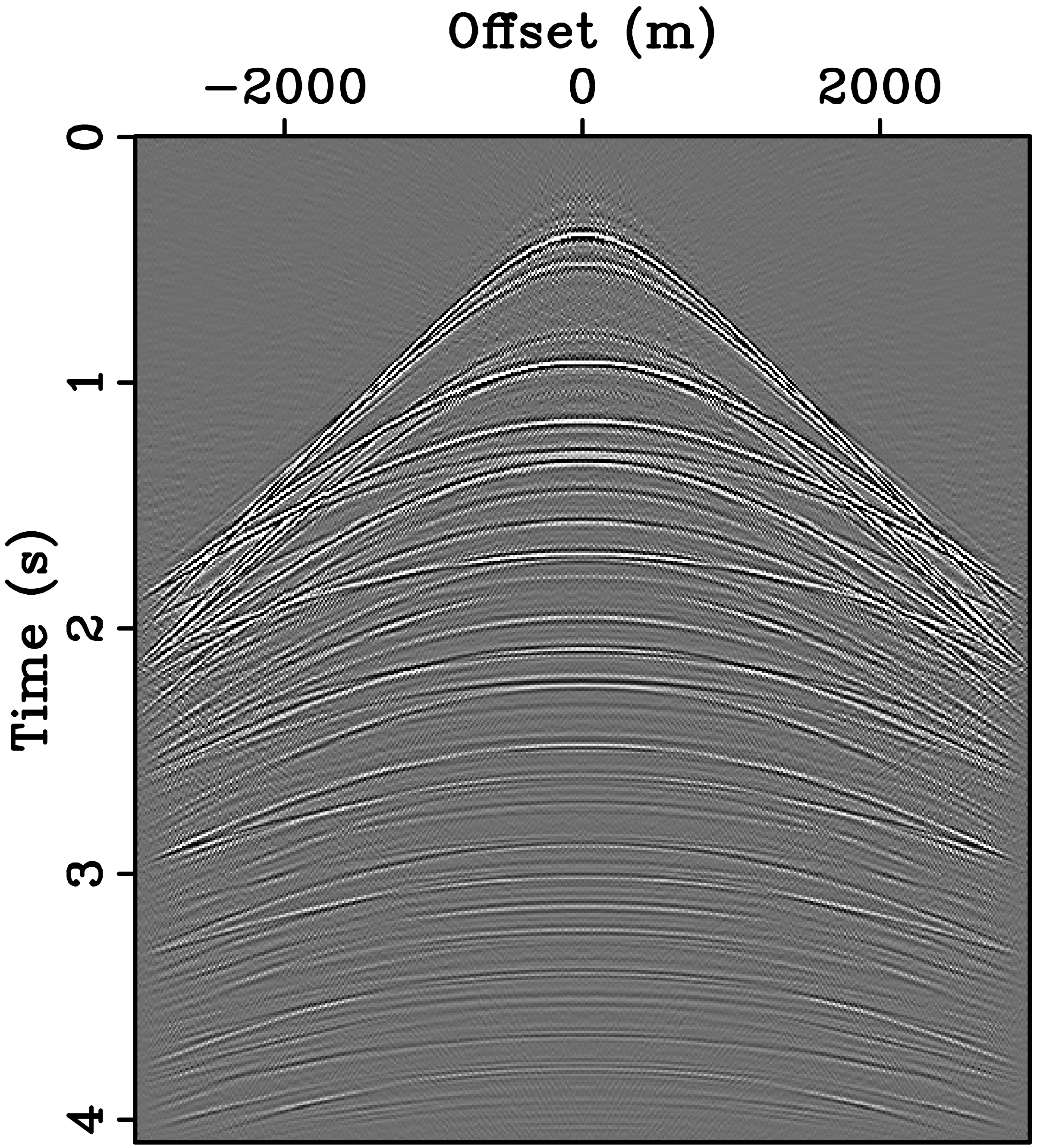

Wavenumber (1/m)
−0.04
−0.02
0
0.02
Frequency (Hz)
0
50

# Regular 3-fold undersampling

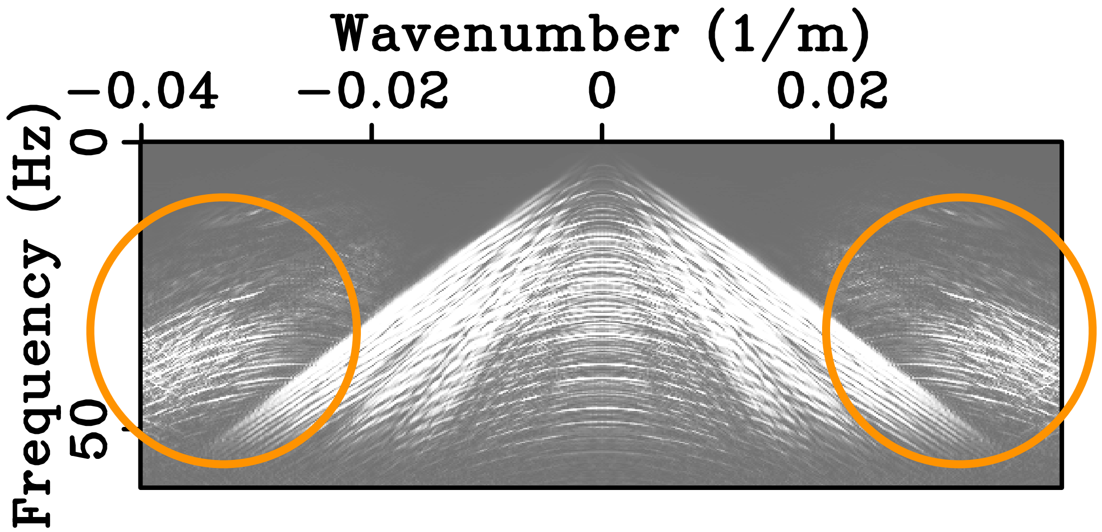




# CRSI from regular 3-fold undersampling

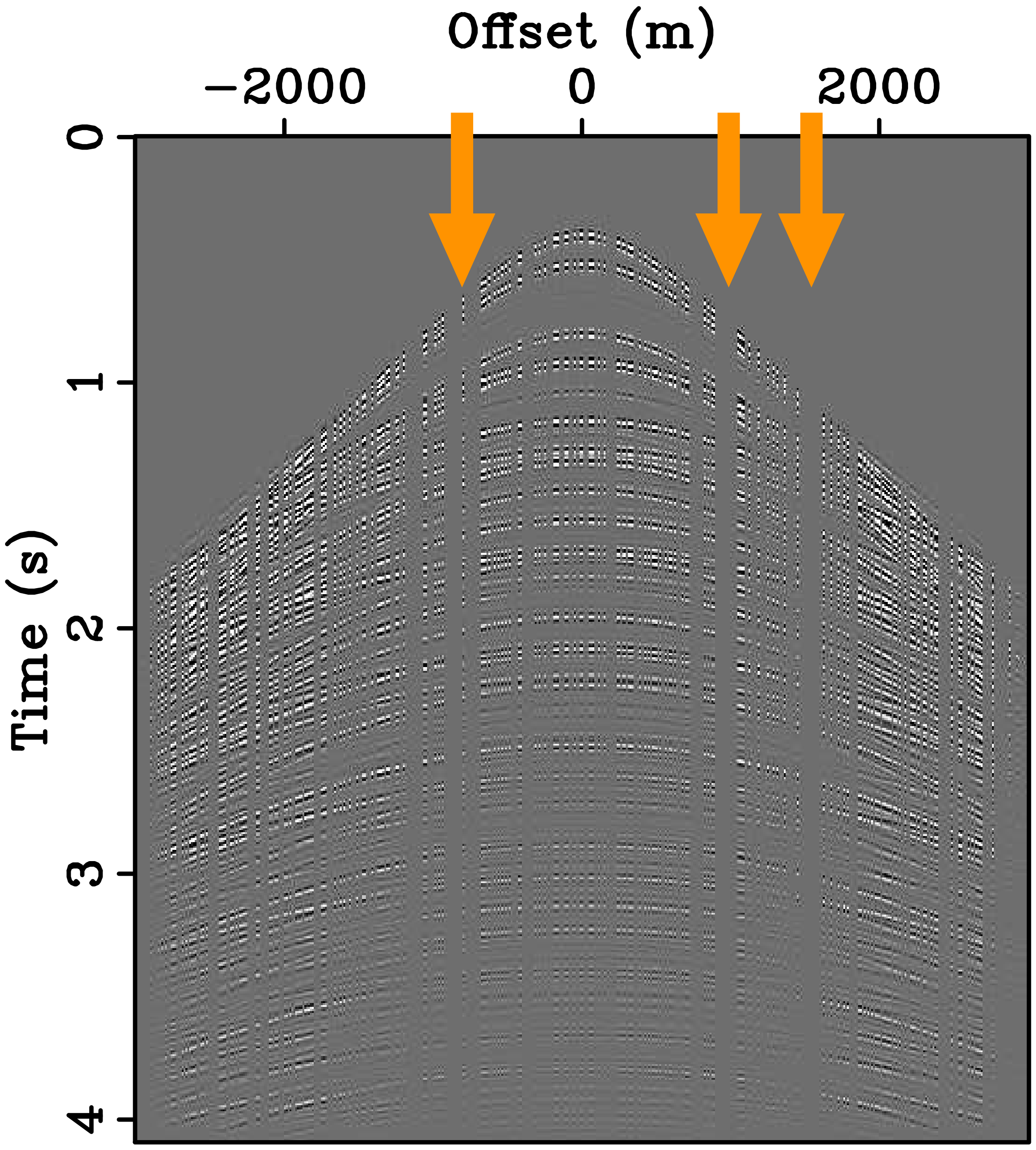


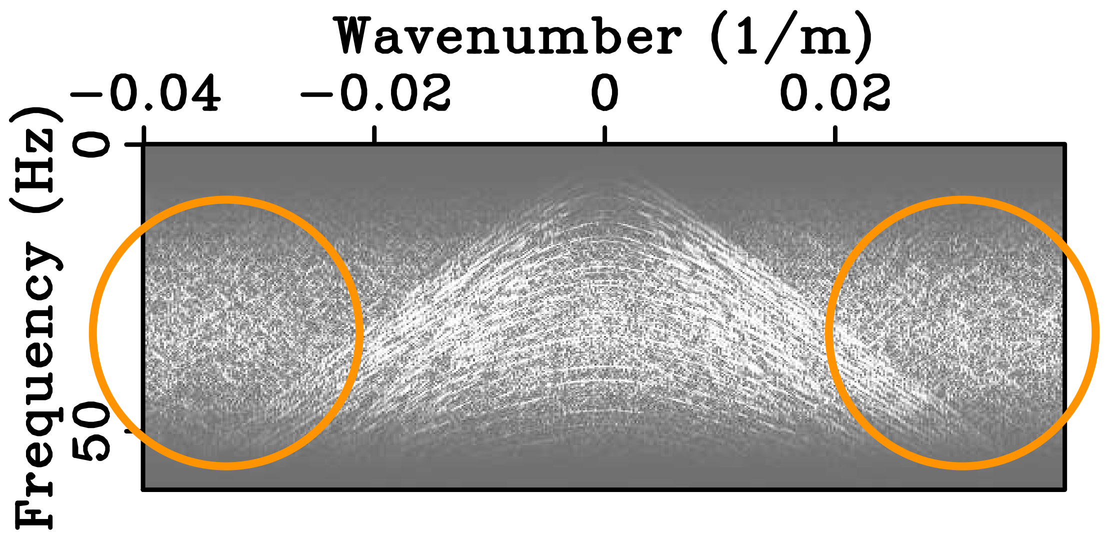
SNR = 6.92 dB



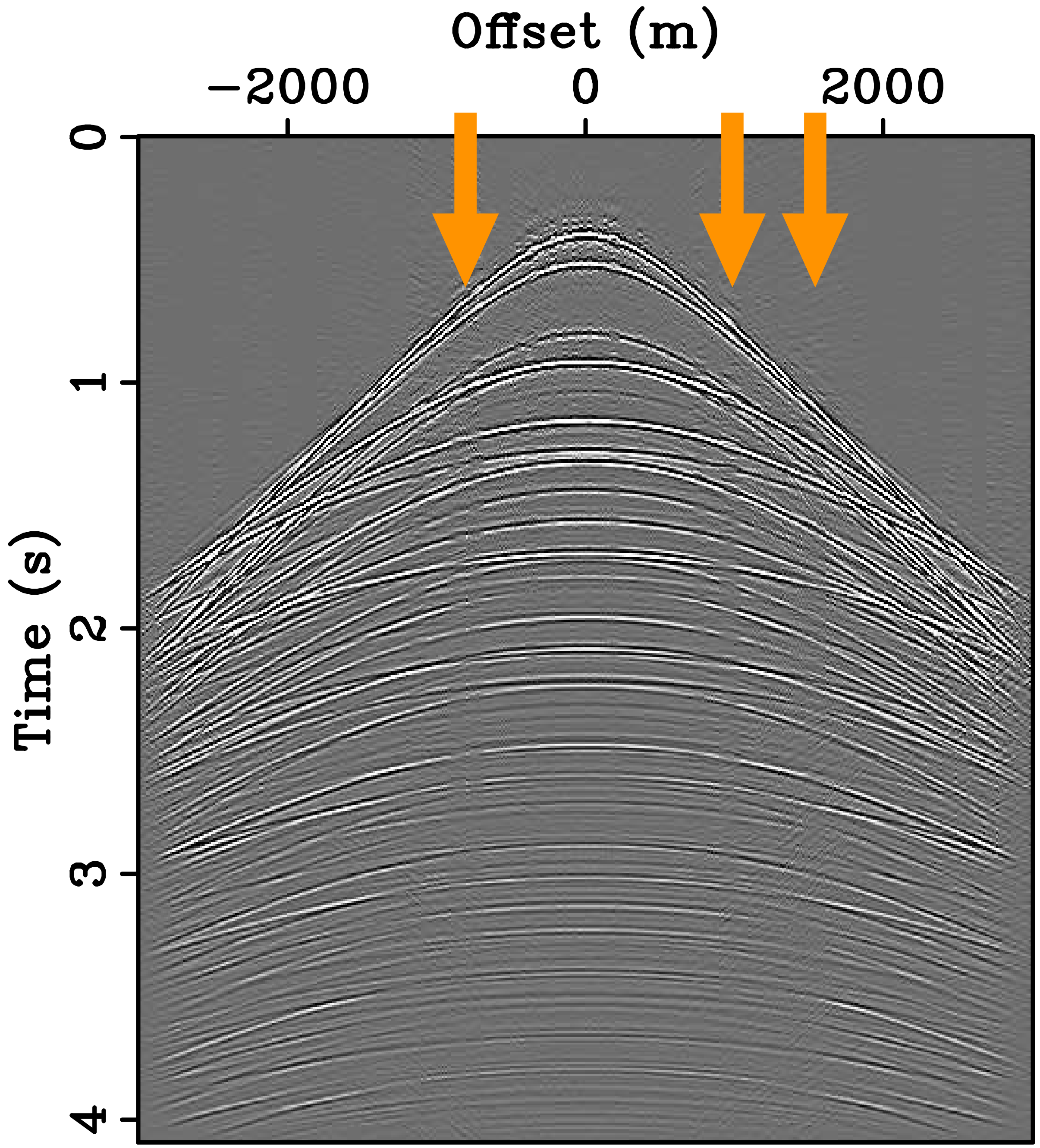
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

# Random 3-fold undersampling

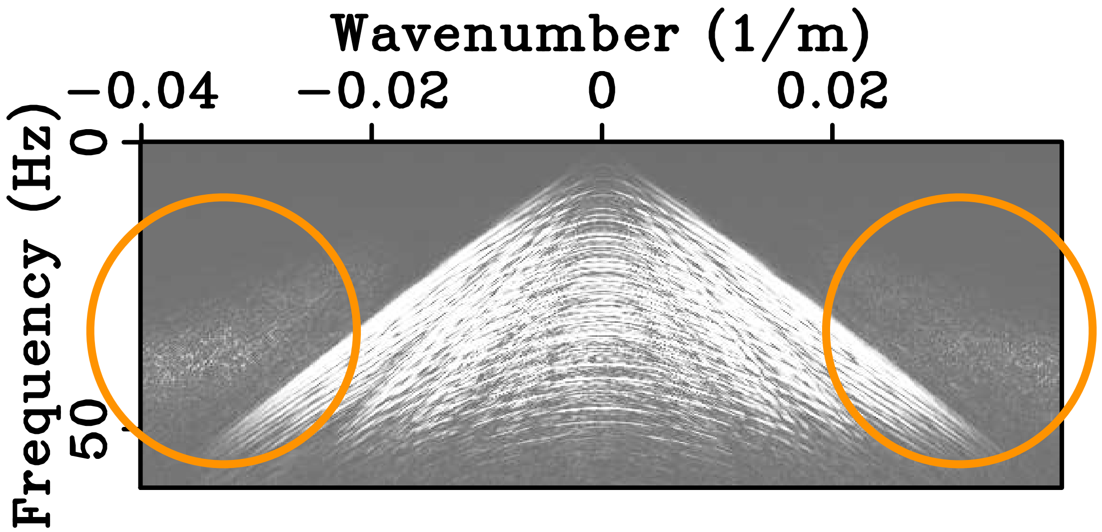




# CRSI from random 3-fold undersampling

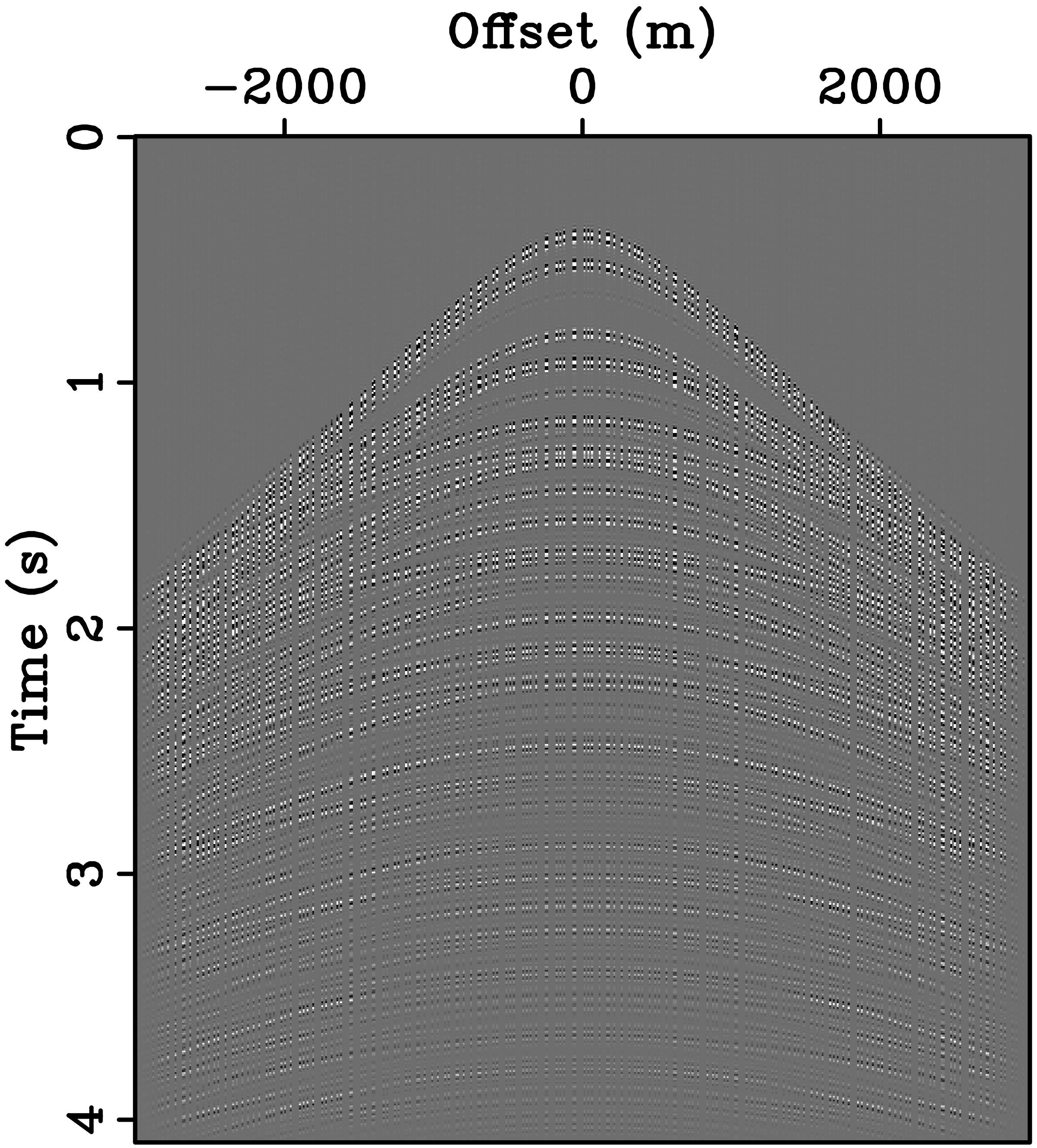


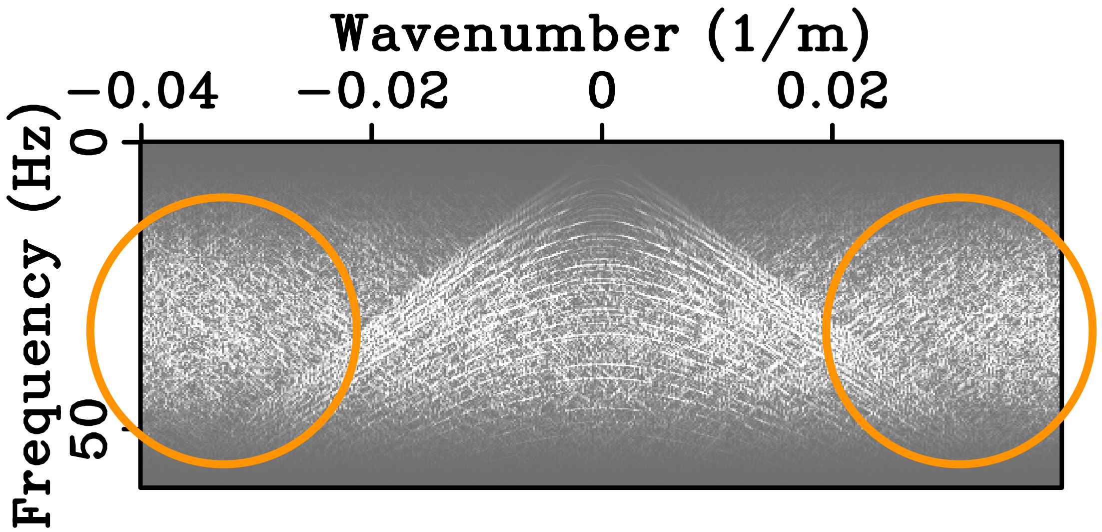
SNR = 9.72 dB



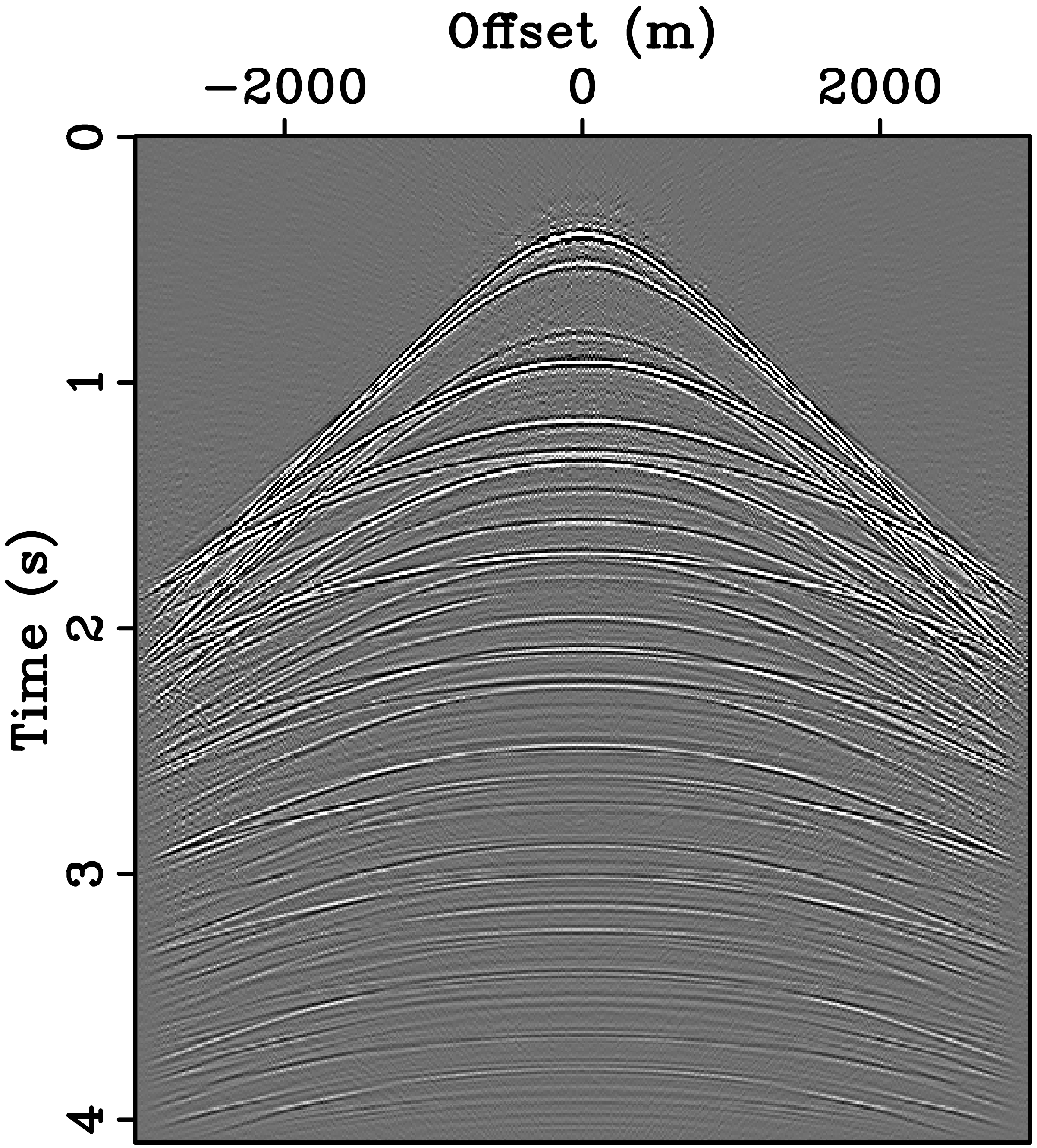
$$\mathrm{SNR} = 20 \times \log_{10}\left(\frac{\|\mathrm{model}\|_2}{\|\mathrm{reconstruction\ error}\|_2}\right)$$

# Optimally-jittered 3-fold undersampling

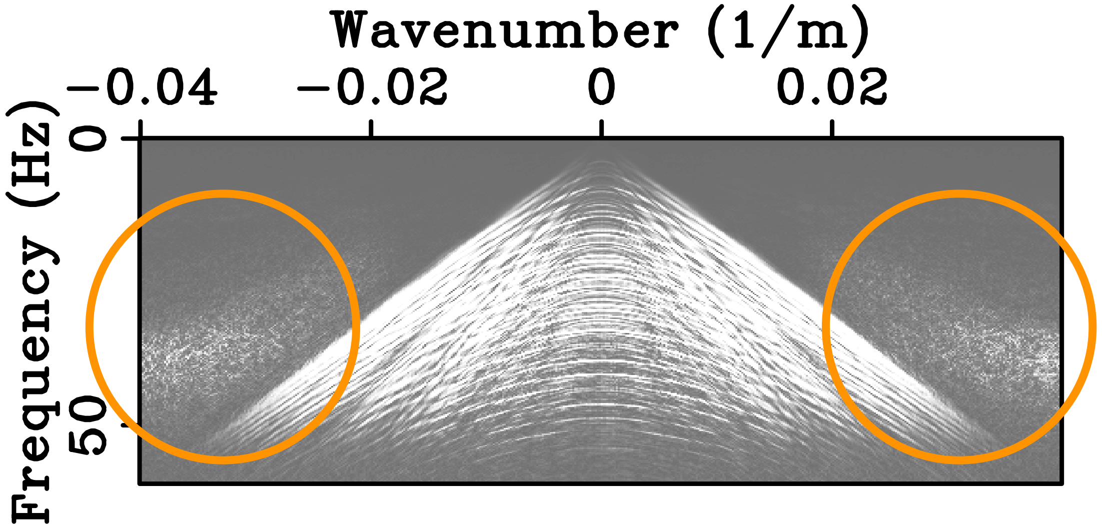




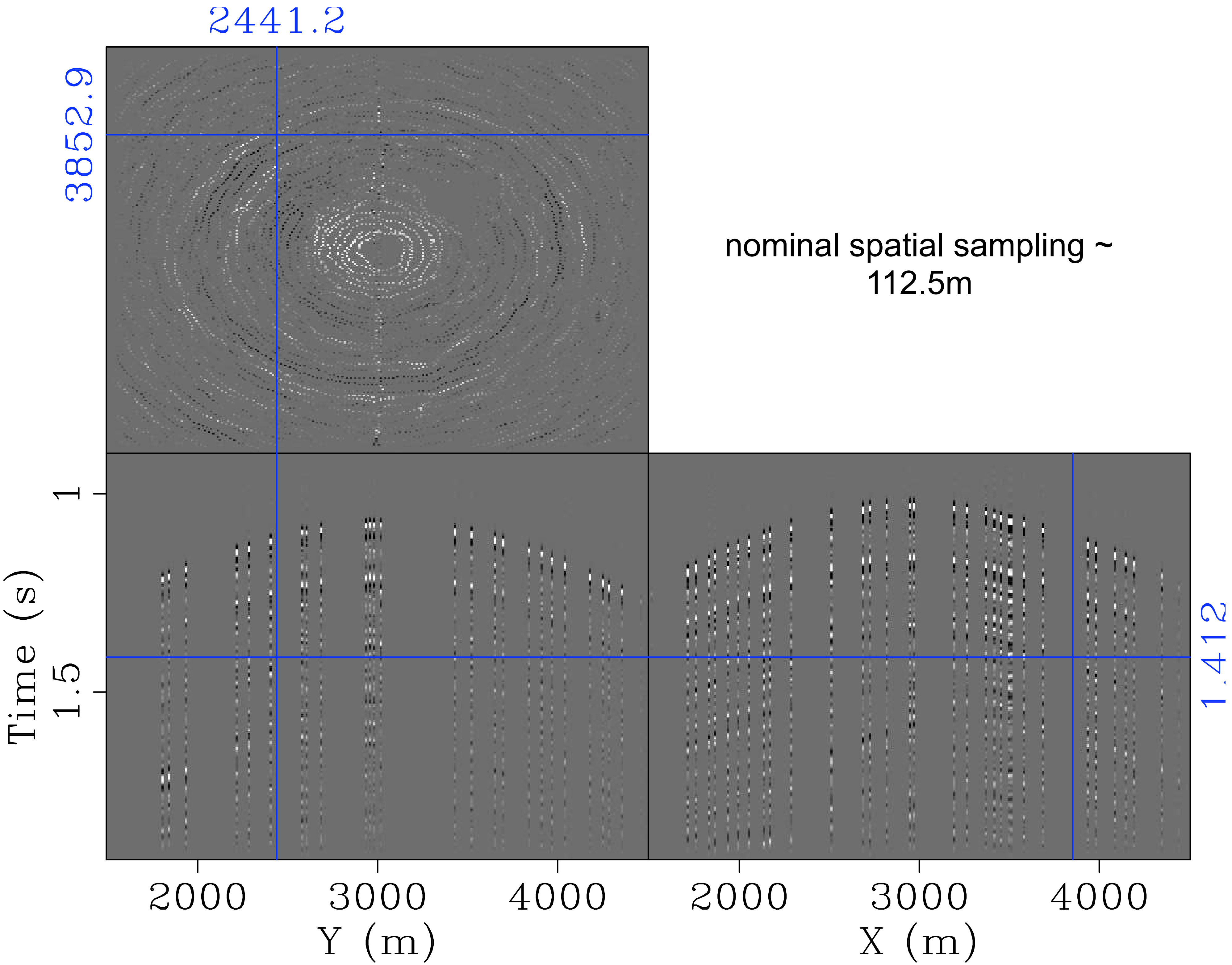
# CRSI from opt.-jittered 3-fold undersampling



SNR = 10.42 dB

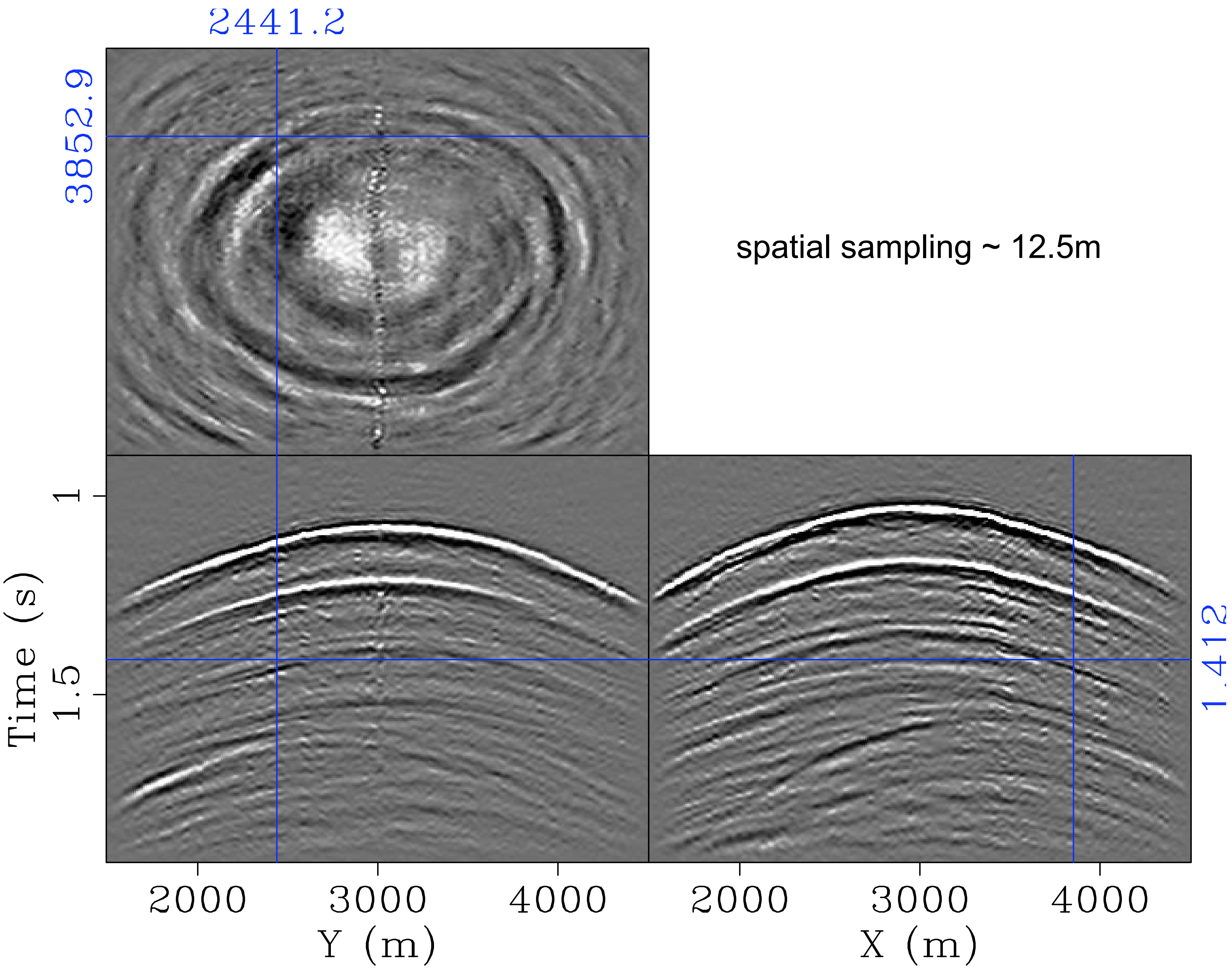


# Data



nominal spatial sampling ~ 112.5m

# CRSI



spatial sampling ~ 12.5m

# Conclusions

- new wavefield reconstruction method that handles both regular and irregular acquisition geometries
  - curvelet reconstruction with sparsity-promoting inversion (CRSI) [Herrmann and Hennenfent‘08]
- extension of the fast discrete curvelet transform to handle irregular seismic data
  - nonequispaced fast discrete curvelet transform (NFDCT) [Hennenfent and Herrmann‘06]
- new coarse sampling schemes that maximize performance of CRSI
  - jittered undersampling schemes [Hennenfent and Herrmann‘08]
- new large-scale, one-norm solver
  - iterative soft thresholding with cooling (ISTc) [Herrmann and Hennenfent’08, Hennenfent et al.‘08]

# Opportunities

- paradigm shift
  - from an assumption of band-limited to **sparse representation for seismic data**
  - from linear to **nonlinear wavefield sampling theory**

- design of advantageous coarse sampling schemes
  - same image quality at a **lower acquisition cost**
  - **better image quality** at a given acquisition cost

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