Optimization strategies for transformed domain sparseness- and continuity- enhancing imaging: Theoretical considerations

SLIM
Seismic Laboratory for Imaging and Modeling

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Context

Least-squares migration & migration deconvolution [Nemeth '99, Chavent '99, de Hoop '00, Hu '01, Kuhl '03]

Sparseness/minimal structure constrained imaging [Wang '03-'05]

Continuity enhancement with anistropic diffusion [Kuhl, Fehmers, Imhof, Schertzer '03]

Curvelet frames [Stein '93, Smit '97, Candes & Donoho, Do '02, Demanet '05, Ying '05]

Decon. & Wavelet-Vaguelette/Q-SVD [Donoho '95, Mallat '97,

Regularizations

Preserve frequency content of seismic images:

***sparseness* of directional frames: e.g. curvelet frames** [Candes & Donoho '02, H '03-'05]

***invariance* of curvelet frames under imaging operators** [Douma '04, Demanet '03-'05, H & M '04]

***continuity* along reflectors through Curvelets & anisotropic TV/diffusion** [H & M '03-05]

Seismic imaging

[Nemeth '99, Chavent '99, de Hoop '00, Hu '01, Kuhl '03]

The forward problem:

$$\overbrace{\mathbf{d}}^{\text{data}} = \underbrace{\mathbf{K}}_{\text{scat. oper.}}\overbrace{\mathbf{m}}^{\text{model/refl.}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

Conventional inverse problem:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \underbrace{\|\mathbf{d}-\mathbf{K}\mathbf{m}\|_2^2}_{\text{data misfit}} + \underbrace{\lambda\|\mathbf{m}\|_p^p}_{\text{regularization}}$$

Non-linear imaging

[Donoho '95, Candes '01, H & M '03-'05]

Wish list: seek a *transformed domain*

- **Where atoms remain *invariant* under physics of wave propagation**
- ***That is sparse*, energy concentration**
- ***Optimal* for *curved* reflectors**
- ***Local* both in space & spatial freq.**

Aim to exploit sparseness of curvelet frames in a non-linear estimation & optimization procedure!

Curvelets

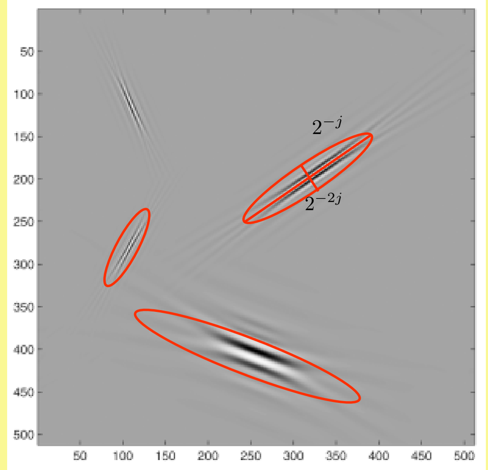
[Candes, Donoho, Demanet, Ying '02-'05]

Tight frames

Partitioning of the 2-D/3-D Fourier domain into angular wedges of second dyadic coronae

Parabolic scaling law

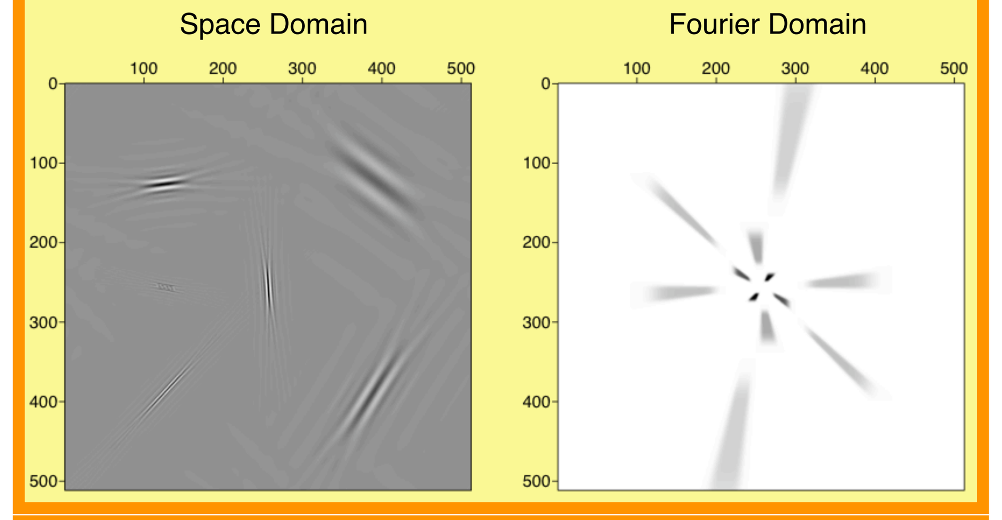
n log n



Numerical construction

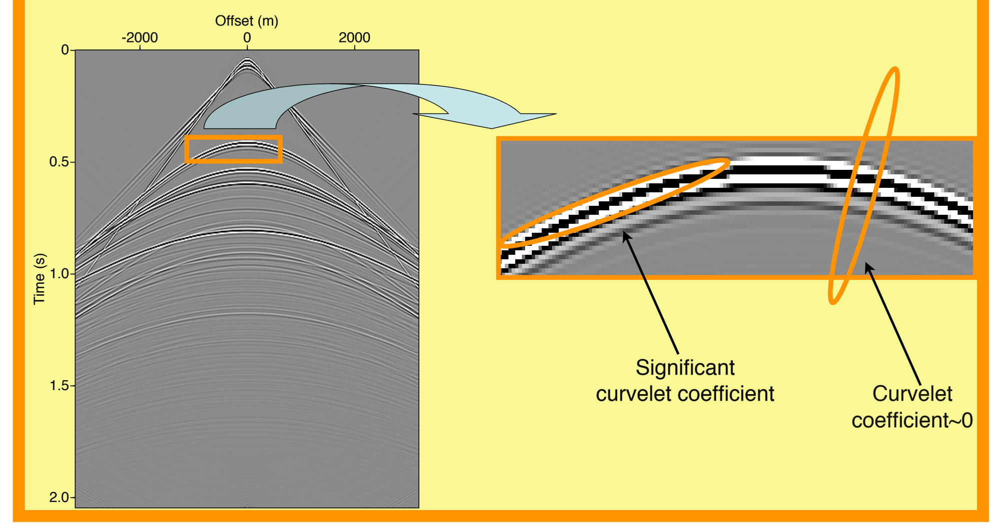
[Candes & Donoho '02-'05, Do '02, Demanet '05, Ying '05]

Localized in both Fourier and space domain

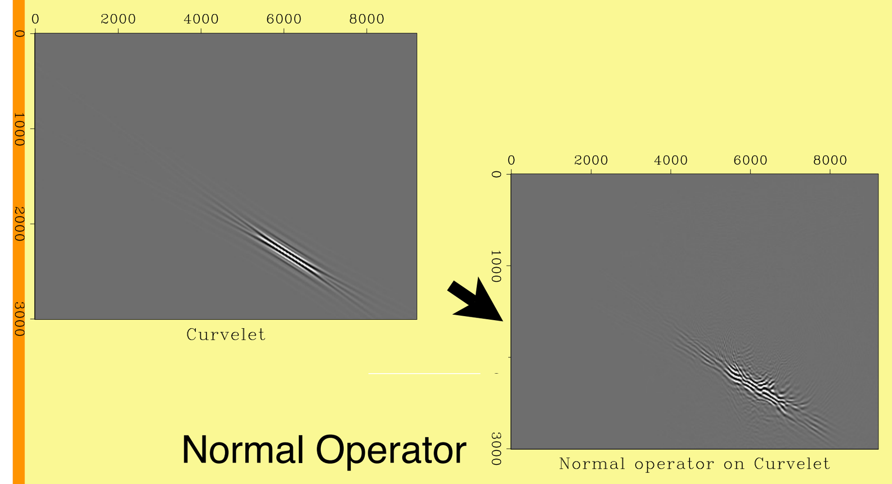


Curvelets & Seismic

[Douma '04, H, H & M '03-'05]



Curvelets & waves



Sparseness & continuity constrained imaging and inversion

Combine curvelet atoms

- ***Sparseness & locality***
- ***Invariance* under scattering-migration**

$$\mathbf{C}\mathbf{K}^T\mathbf{K}\mathbf{C}^T\cdot \approx \mathbf{\Gamma}^2 \longleftarrow \text{Quasi-singular values}$$

with regularization functionals

- **Enhance the sparseness (ℓ^1 -norm)**

Iterative denoising

[Donoho '98, Mallat '98 Candes '02, Starck '04, Daubechies '05]

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{d}-\mathbf{F}\mathbf{x}\|_2^2 + \|\mathbf{x}\|_1^1$$

with $\mathbf{m} = \mathbf{F}\mathbf{x},\ \mathbf{F}\cdot \triangleq \mathbf{C}^T\cdot$

- **sparse superposition of curvelet atoms**
- ***multiple* Landweber iterations:**

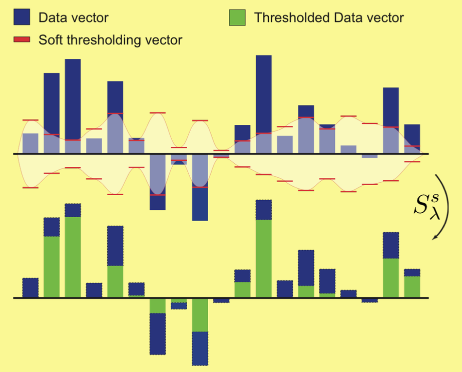
$$\mathbf{x}^m = S^s_{w(\lambda_m)}\left(\mathbf{x}^{m-1} + \mathbf{F}^*\left(\mathbf{d}-\mathbf{F}\mathbf{x}^{m-1}\right)\right)$$

Soft thresholding

[Donoho '98, Mallat '98 Candes '02, Starck '04, Daubechies '05]

$$\mathbf{x}^m = S^s_{w(\lambda_m)}\left(\mathbf{x}^{m-1} + \mathbf{F}^*\left(\mathbf{d}-\mathbf{F}\mathbf{x}^{m-1}\right)\right)$$

$$S_\lambda(x) = \begin{cases} x - \text{sign}(x)\lambda & |x| \geq \lambda \\ 0 & |x| < \lambda. \end{cases}$$



Preconditioning

[Donoho '95, Candes '01, Mallat '97, Neelamani '03, Daubechies '05, H & M '04-'05]

Compose modeling operator and model with

$$\mathbf{K}\cdot \mapsto \mathbf{F}\cdot \triangleq \mathbf{K}\left(-\Delta\right)^{-1/2}\mathbf{C}^T\mathbf{\Gamma}^{-1}\cdot$$

and $\mathbf{m} \mapsto \mathbf{\Gamma}\mathbf{C}\mathbf{m}$ **such that approximately**

$$\langle \mathbf{F}\mathbf{f}, \mathbf{F}\mathbf{g}\rangle \approx \langle \mathbf{f}, \mathbf{g}\rangle \quad \text{or} \quad \mathbf{F}^T\mathbf{F}\cdot \approx \mathbf{Id}\cdot$$

Also use $\mathbf{E}\{\varepsilon\varepsilon^T\} \approx \mathbf{Id}$ with $\varepsilon = \mathbf{F}^T\mathbf{n},\ \mathbf{n} \in N(0,1)$

$$\mathbf{F}^T\mathbf{d} = \mathbf{F}^T\mathbf{F}\mathbf{x} + \mathbf{F}^T\mathbf{n} \rightarrow \mathbf{y} = \overbrace{\mathbf{A}}^{\approx \mathbf{I}}\mathbf{x} + \underbrace{\mathbf{n}}_{\text{'white'}}$$

Sparseness & continuity constrained imaging and inversion

Two different optimization strategies

1) divide-and-conquer first threshold then

$$\hat{\mathbf{m}}: \quad \min_{\mathbf{m}} J_a(\mathbf{m}) \quad \text{s.t.} \quad |\mathbf{x}-\hat{\mathbf{x}}_0|_\mu \leq \mathbf{e}_\mu \quad \forall\mu$$

2) jointly minimize sparsness & continuity

$$\hat{\mathbf{m}}: \quad \arg\min_{\mathbf{m}} J(\mathbf{m}) \quad \text{s.t.} \quad \|d-\mathbf{K}\mathbf{m}\|_2 \leq N\sigma_n$$

Quasi-Newton: fast convergence.

Sparseness & continuity constrained imaging and inversion

Advantages of 1) compared to 2):

- **Existence of simple solver**
- **Compact and simple form**

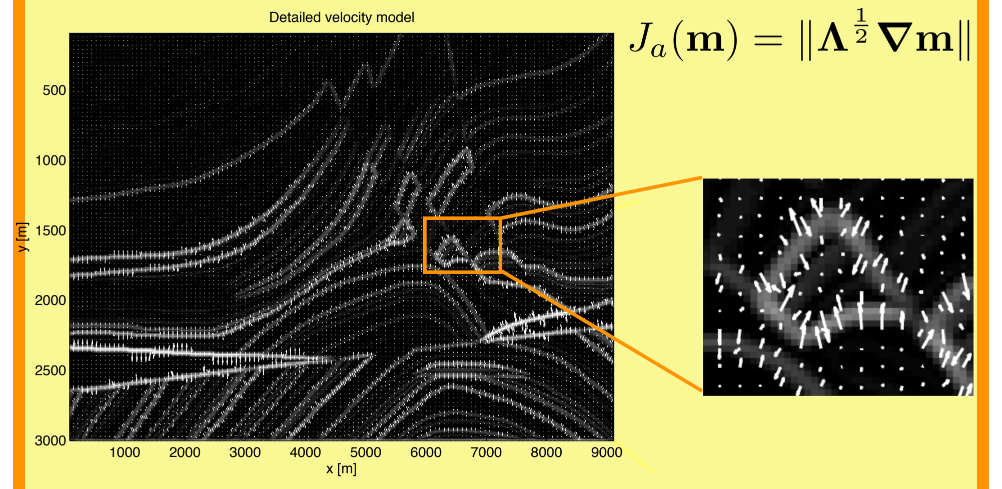
Disadvantages of 1) compared to 2):

- **Satisfies the second statistical moment.**
- **Does not control over the data tolerance.**
- **not flexible**

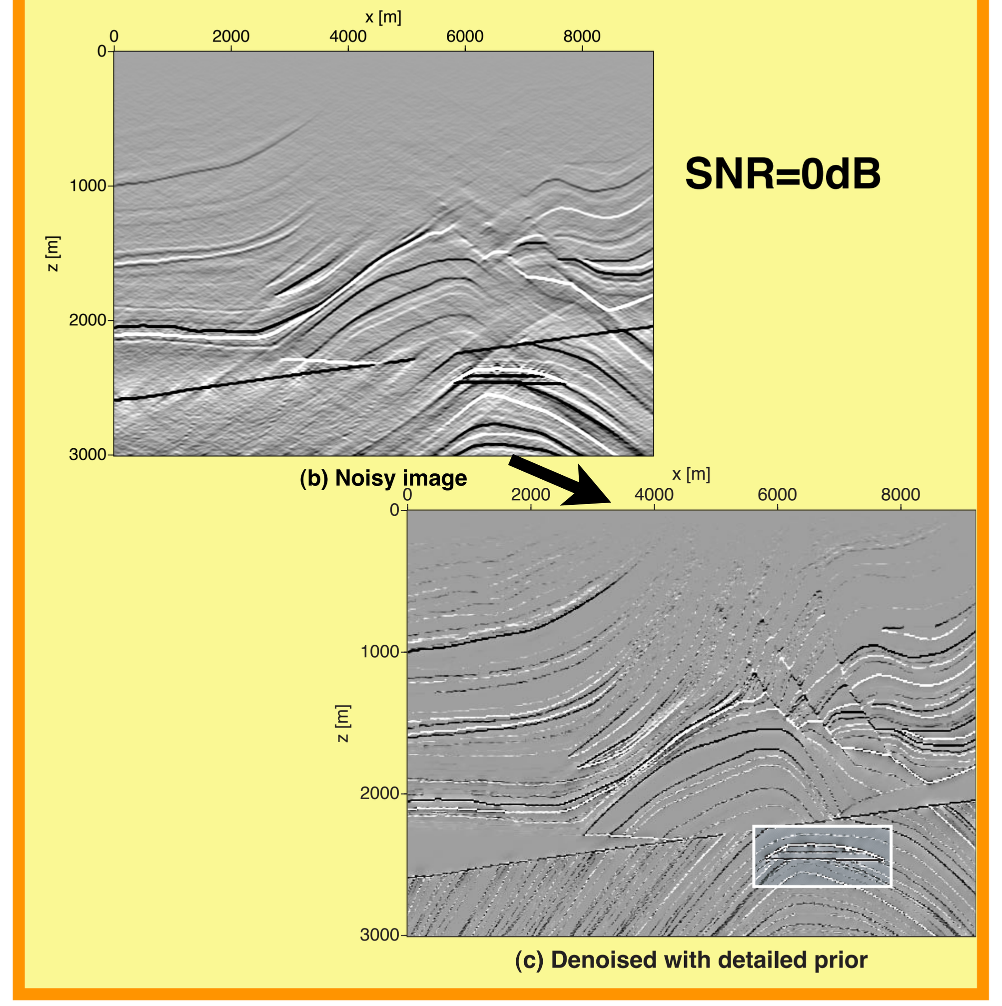
Anisotropic diffusion

[Fehmers, Imhof, Schertzer '03]

$$J_a(\mathbf{m}) = \|\mathbf{\Lambda}^{\frac{1}{2}}\nabla\mathbf{m}\|$$



Non-linear imaging



Sparsity- & continuity-enhanced imaging

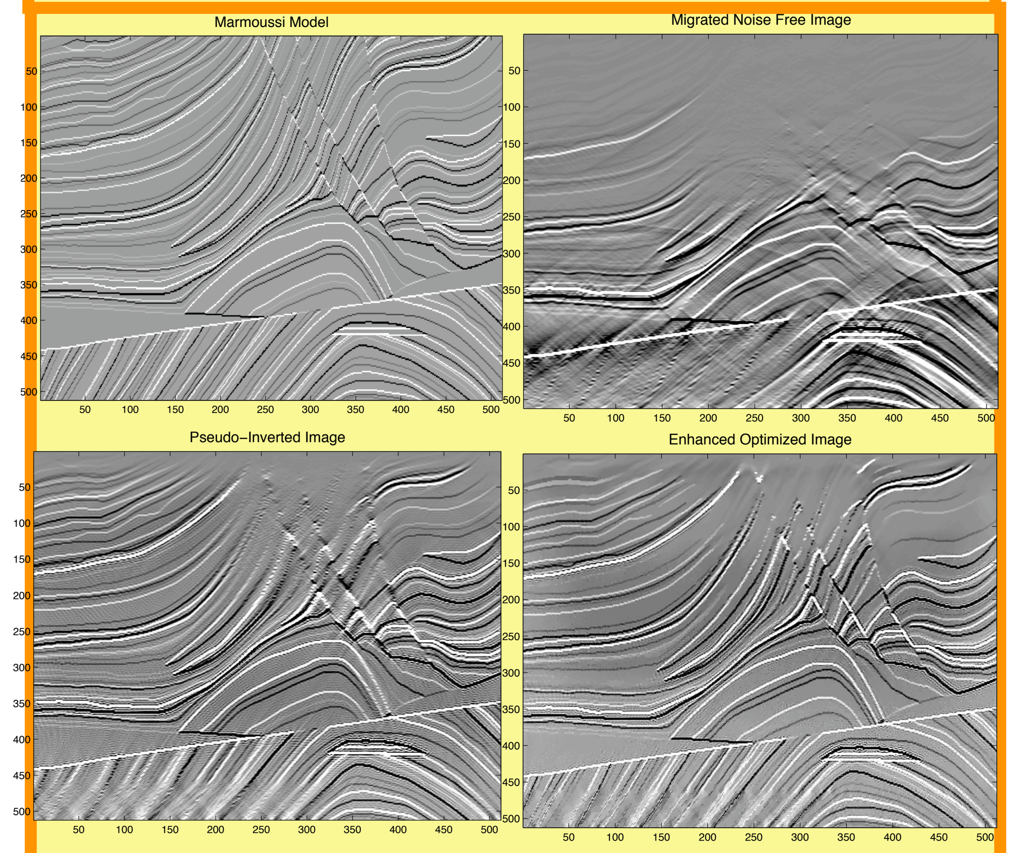
Results were obtained for SNR=0dB

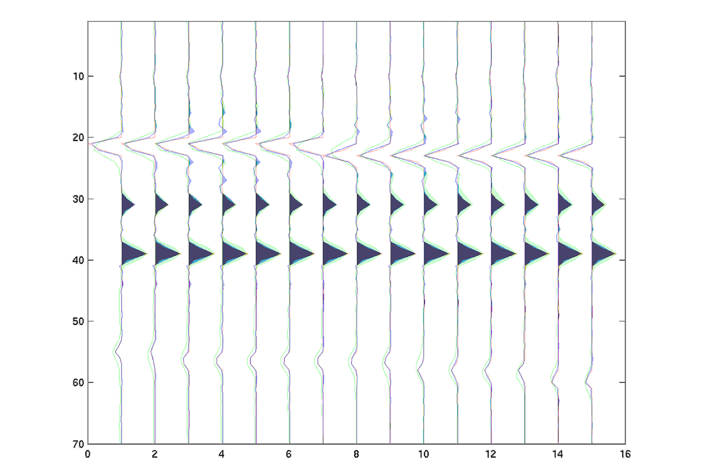
Sparseness & continuity constraints improved the results

Non-trivial even for noise-free case

- **bad illumination**
- **null space modeling operator**

Apply strategy II: *joint* minimization of both constraints.





Original(red), Migrated(green), Enhanced (blue)

Acknowledgments

Authors of CurveLab [Candes, Donoho, Demanet and Ying '05]

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