Seismic Deconvolution Revisited With Curvelet Frames

SLIM Seismic Laboratory for Imaging and Modeling

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Summary

We propose an efficient iterative curvelet-regularized deconvolution algorithm that exploits continuity along reflectors in seismic images. Curvelets are a new multiscale transform that provide sparse representations for images (such as seismic images) that comprise smooth objects separated by piece-wise smooth discontinuities. Our technique combines conjugate gradient-based convolution operator inversion with noise regularization that is performed using non-linear curvelet coefficient shrinkage (thresholding). The shrinkage operation leverages the sparsity of curvelet representations.

Context

Two different types of deconvolution:
- statistical (deconvolution operator estimated)
  - least-squares [Robinson, 57]
  - homomorphic [Buttkus, 75]
  - maximum-likelihood [Chi et al., 84]
- deterministic (deconvolution operator "known")
  - minimum structure/l_1-norm [Oldenburg, 81; Ulrych & Walker, 82, Sacchi et al., 94]
  - ForWaRD [Neelamani, 04]

Our approach is **deterministic** and aims at:
- boosting high frequencies
- being robust under noise
- exploiting
  - continuity along reflectors (2-D/3-D)
  - sparseness in transformed domain

Curvelet Frames [Candes & Donoho, 99]
- What are they?
  - tight frames
  - partitioning of the 2-D/3-D Fourier domain into angular wedges of second dyadic coronae
  - parabolic scaling law (length^2 ≈ width)

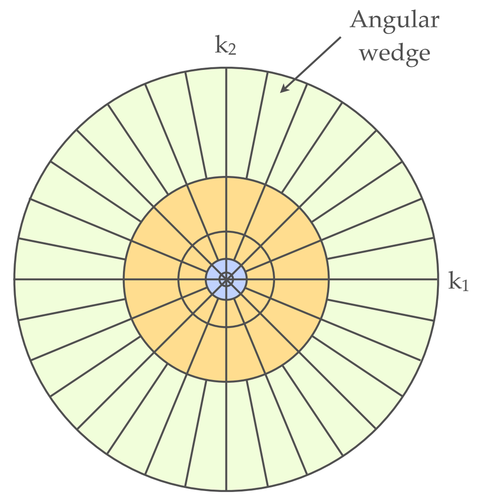

Figure 1: Partitioning of the 2-D Fourier domain.

- Curvelet properties
  - multi-scale
  - multi-directional
  - highly anisotropic
  - localized both in space & frequency

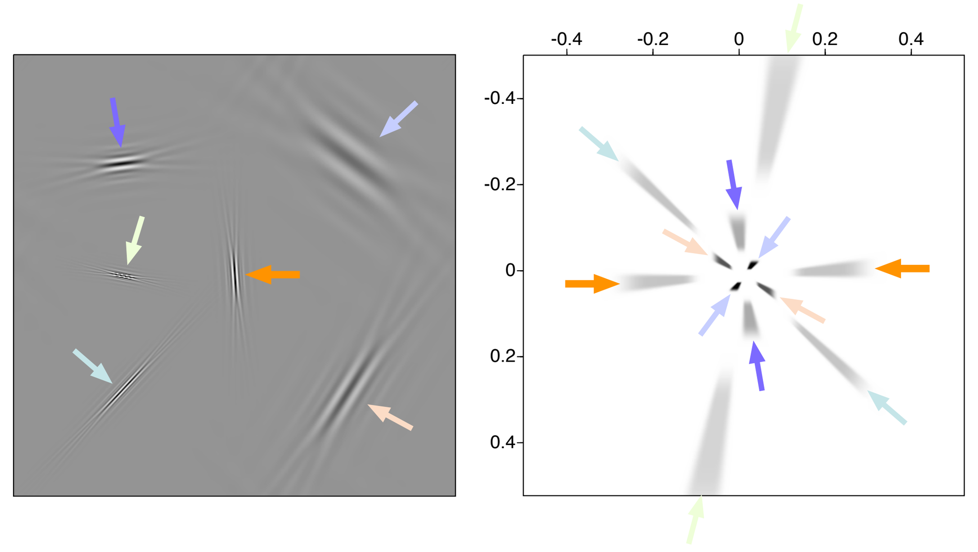

Figure 2: Spatial (left) and frequency (right) viewpoint of six real curvelets at different scales and angles. As opposed to complex curvelets, real curvelets live in two angular wedges symmetric about the zero frequency point.

- Curvelets & seismic data

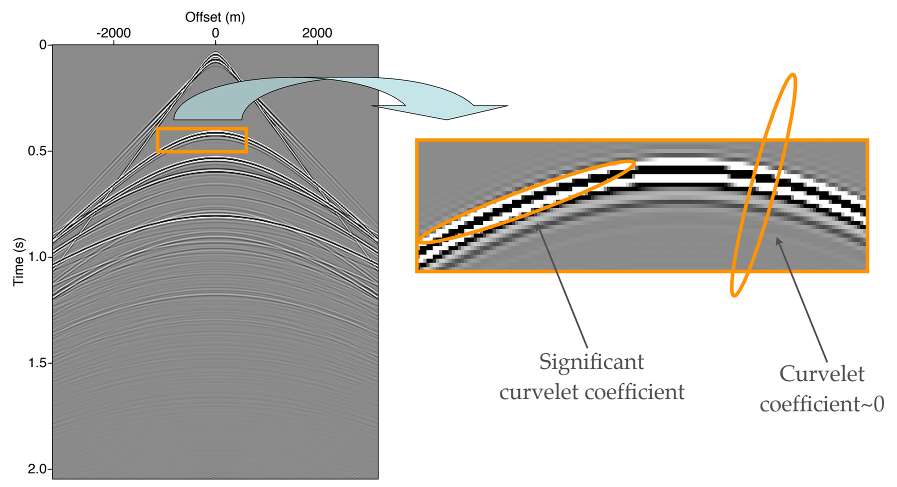

Figure 3: Curvelets whose essential support does not overlap or does overlap but is not tangent with the discontinuities make small frame coefficients. Curvelets whose essential support does overlap with the singularity and is nearly tangent to the singularity make large frame coefficients.

Curvelets are close to optimal to represent objects with arbitrary piece-wise smooth discontinuities (i.e. the energy of the objects is concentrated in a very limited number of frame coefficients). This property makes curvelets suitable for representing wavefronts and reflectors, which may be faulted, within 2-D/3-D seismic datasets.

Curvelets for Seismic Deconvolution

- Our forward problem

$$\mathbf{d} = \mathbf{K}\mathbf{C}^*\mathbf{x} + \mathbf{n} \quad \& \quad \mathbf{m} = \mathbf{C}^*\mathbf{x}$$

(data → $\mathbf{d}$; convolution operator → $\mathbf{K}$; curvelet composition operator → $\mathbf{C}^*$; curvelet coefficients → $\mathbf{x}$; noise → $\mathbf{n}$; reflectivity → $\mathbf{m}$)

- Our inverse problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1 \quad \text{with} \quad \mathbf{F} = \mathbf{K}\mathbf{C}^*$$

In words, we want to deconvolve the data and stabilize the process by imposing a sparseness constraint on the curvelet representation of the model.

- Existing methods for l_1-norm optimization
  - Iterative Re-weighted Least-Squares (IRLS) [Gersztenkorn et al., 86]
  - Landweber iterations and soft thresholding [Daubechies, 05]

$$\mathbf{x}^m = \mathcal{S}^s_{\lambda_m}\left[\mathbf{x}^{m-1} + \mathbf{F}^*(\mathbf{d} - \mathbf{F}\mathbf{x}^{m-1})\right]$$

with

$$S^s_\lambda(\mathbf{x}) = \begin{cases} \mathbf{x} - \text{sign}(\mathbf{x})\lambda & |\mathbf{x}| \geq \lambda \\ 0 & |\mathbf{x}| < \lambda, \end{cases}$$

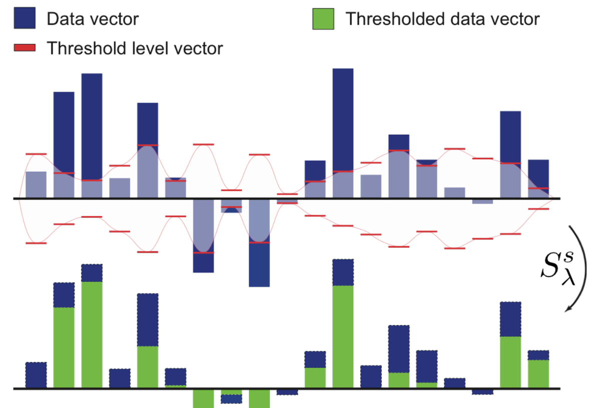

Figure 4: Soft thresholding is an element-by-element operation whose result only depends upon the magnitude of the data and the corresponding threshold.

- Our algorithm
  - Initialize L_{max}, number of conjugate gradient (CG) iterations per layer N, varying threshold level vector **T**=**W**.L_{max} with **W** reference threshold level vector.
  - Update $\hat{\mathbf{x}}$
    - $\alpha_n = \text{CG}(\hat{\mathbf{x}}_{n-1}, N)$,
    - Soft threshold the α_n coefficient with the threshold **T** and obtain $\hat{\mathbf{x}}_n$.
  - Update the threshold by **T** = **T**- **W**.
  - If **T** > **W**, return to Step 2. Else, finish.

2-D Synthetic Examples

- Example 1

The true Marmoussi reflectivity is convolved with a Ricker wavelet. White Gaussian noise is added (*SNR ~ 8dB*).

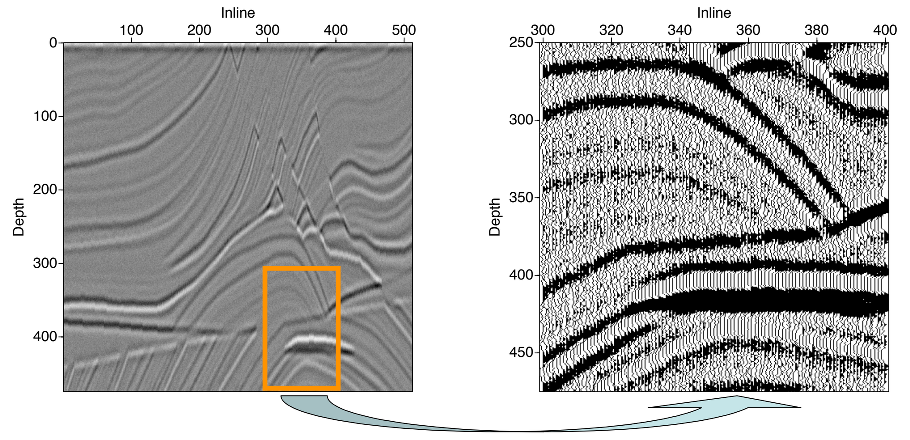

Figure 5: Noisy data.

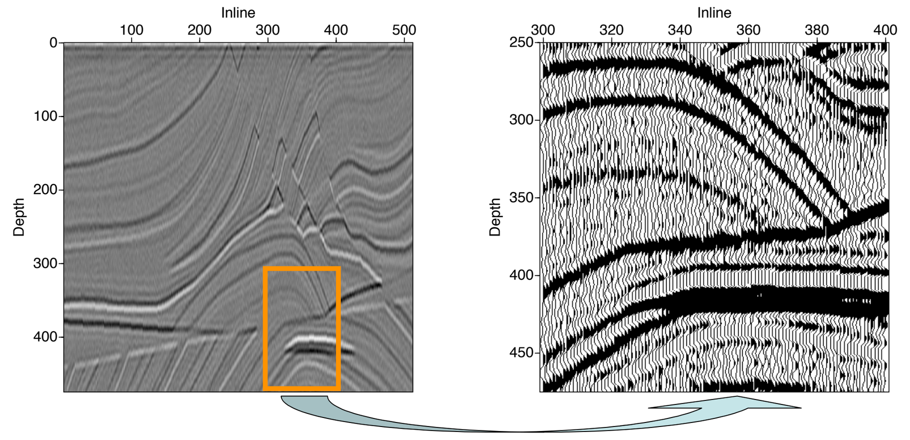

Figure 6: Wiener-based deconvolution improves the frequency content of the section but suffers from noise and gives rise to ringing effects.

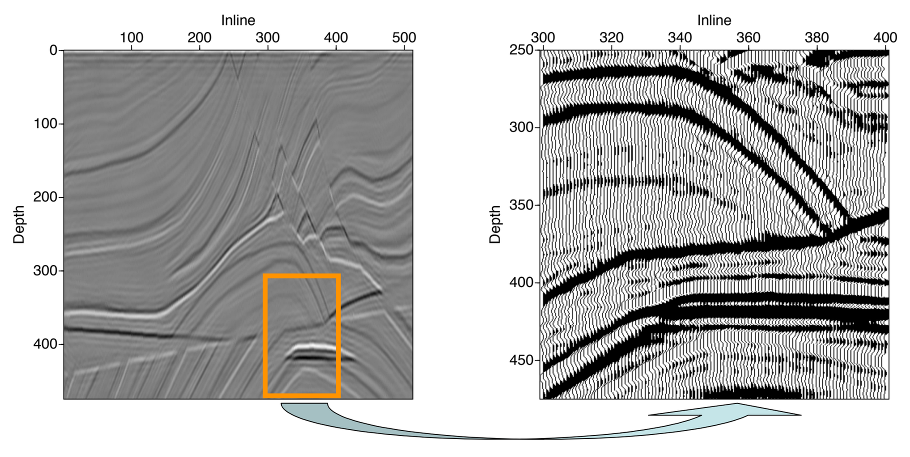

Figure 7: Curvelet-based deconvolution outperforms Wiener-based deconvolution by providing an estimate with a broader spectrum, less ringing due to noise and highly enhanced continuity along reflectors.

- Example 2

The constant-velocity Kirchhoff modeling operator and the true Marmoussi reflectivity are used to make broadband common-receiver noise-free data that are then convolved with a Ricker wavelet (peak frequency 15Hz). White Gaussian noise is added (*SNR ~ 8dB*).

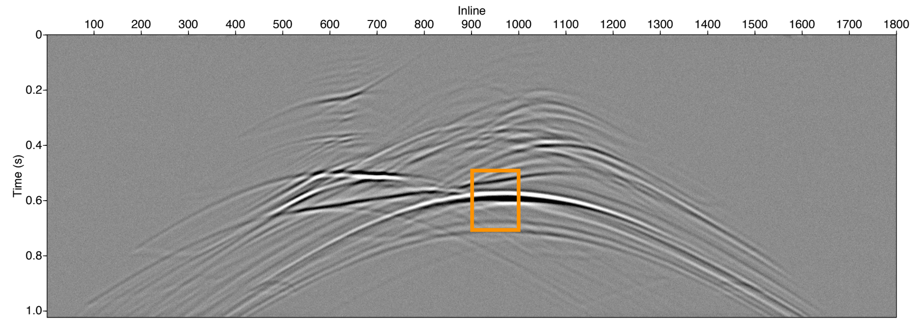

Figure 8: Noisy data

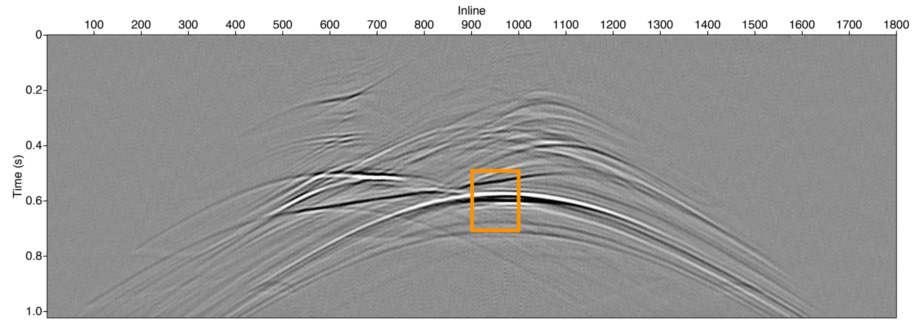

Figure 9: Wiener-based deconvolution improves the frequency content of the section but suffers from noise and gives rise to ringing effects.

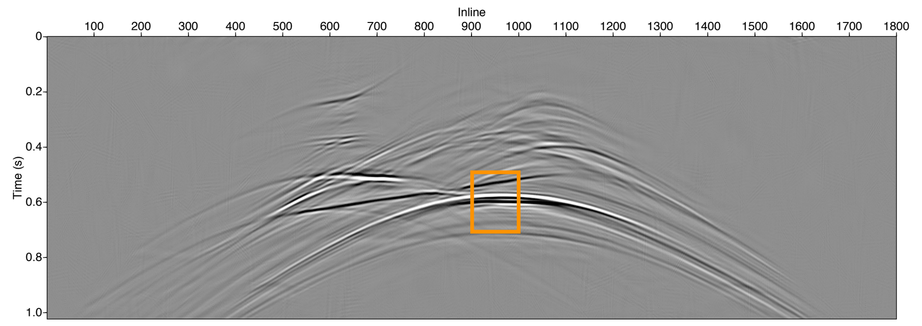

Figure 10: Curvelet-based deconvolution outperforms Wiener-based deconvolution by providing an estimate with a broader spectrum, less ringing due to noise and highly enhanced continuity along wavefronts.

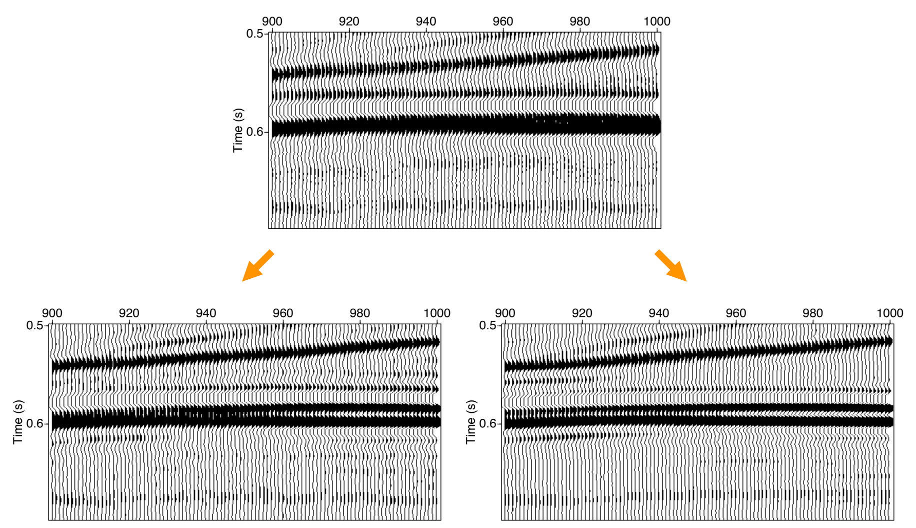

Figure 11: Windowed wiggle plot of the noisy data (top), the Wiener-based deconvolution estimate (bottom left), and the curvelet-based deconvolution estimate (bottom right).

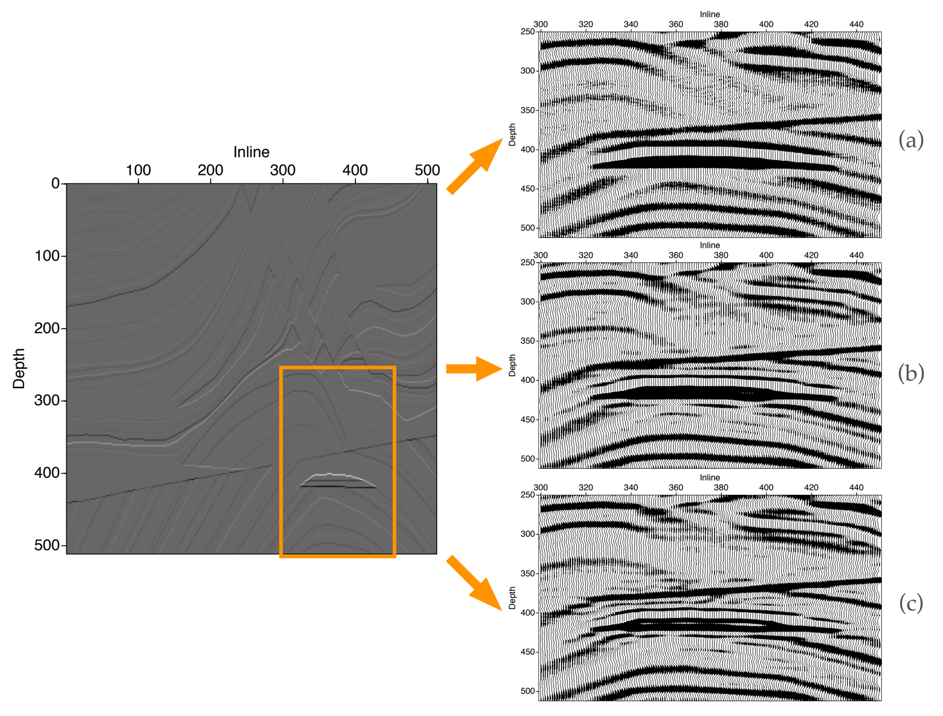

Figure 12: True Marmoussi reflectivity (left) and windowed wiggle plot of the image obtained by simple least-squares common-receiver migration using noisy data (a), Wiener-based deconvolution estimate (b), and curvelet-based deconvolution estimate (c).

Conclusions

We developed and demonstrated a new iterative curvelet-regularized deconvolution algorithm which has the following properties:
- 2-D as opposed to trace-by-trace (3-D extension straightforward)
- fast by combining Conjugate-Gradient iterations and Fast Discrete Curvelet Transform
- robust under noise
- explore smoothness along reflectors. This information from many traces stabilizes the 2-D deconvolution

Our algorithm can be applied to a wide range of applications (e.g. migration, primary-multiple separation) as long as:
1. the model has intermittent regularity, e.g. reflectors on smooth curves with faults and pinch outs
2. the covariance matrix of the noise is near diagonal in curvelet frames

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