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# **Abstract**

Explore and compare 2D sampling methodologies for curvelet-based reconstruction, with bluenoise spectra: Poisson Disk sampling, Farthest Point sampling & 2D extension of jittered sampling. Better recovery and results compared to other more traditional sampling protocols.

# Motivation

# Practical considerations:

- -- Seismic data volumes are usually extremely large and expensive to acquire.
- -- Often incomplete, with traces missing due to complex acquisition constraints.

# <u>Issues of sampling:</u>

- -- Competing requirements between high-resolution recovery and saving measurements.
- -- Limits of Shannon/Nyquist sampling theorem.

# Insights from compressive sensing:

- -- Seismic data volumes can be recovered from sub-Nyquist sampling by exploiting combination of randomized sampling and transform-domain sparsity promotion.
- -- Curvelet-based Recovery by Sparsity-promoting Inversion (CRSI) developed by Hennenfent & Herrmann(2008) [1]. Used with jittered sampling turning coherent aliases to incoherent noise.
- -- Our aim is to extend this work to 2-D sampling for X-line geometries (see Fig.1).

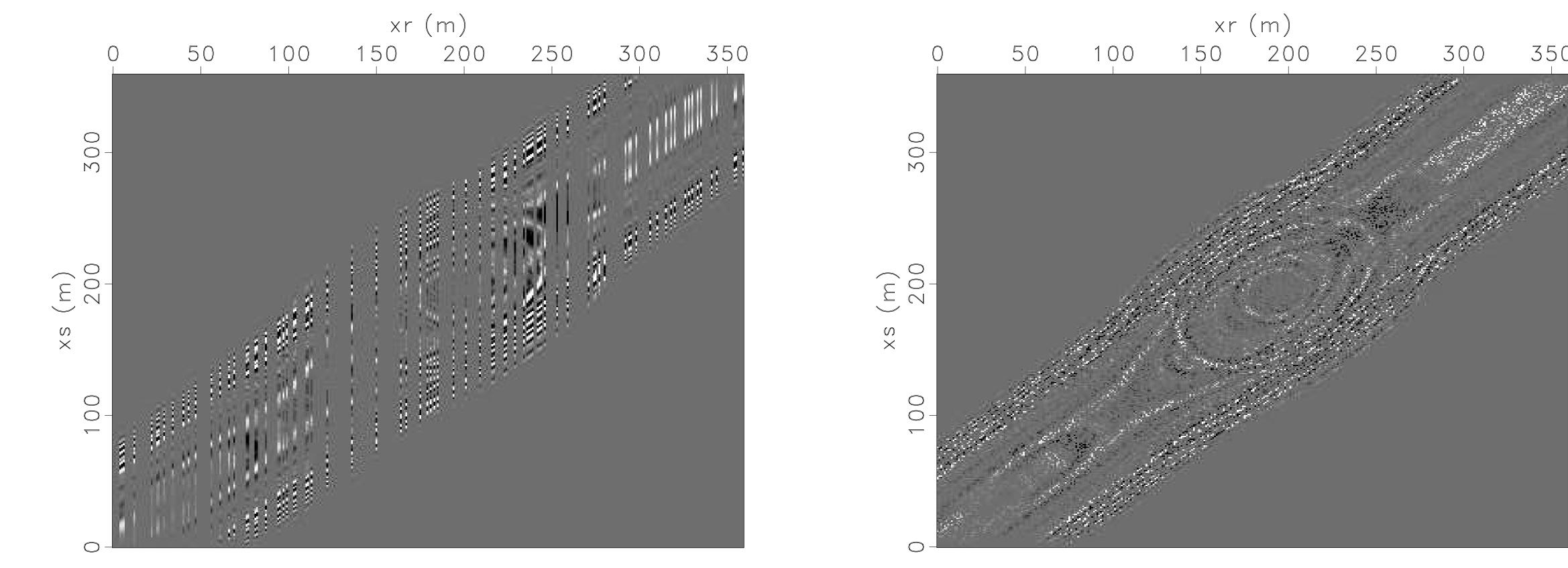


Fig.1: 25% random samples on time slices via: (left) 1D and (right) 2D sampling.

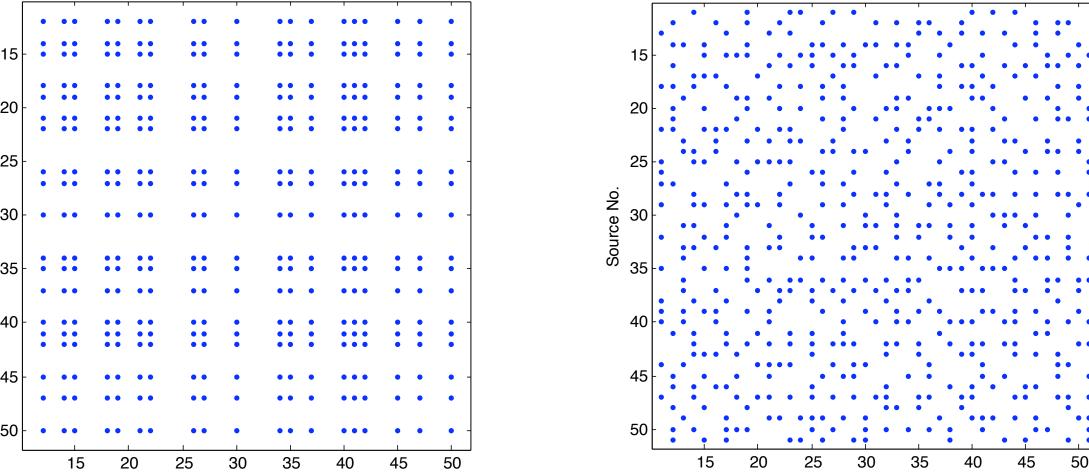
# **CRSI** reconstruction method:

$$\mathbf{y} = \mathbf{Rm}, \mathbf{R} \in \mathbb{R}^{n \times m}, \mathbf{m} \in \mathbb{R}^m, m >> n,$$
  $\mathbf{P}_{\sigma} : \qquad \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \left\{ \begin{array}{l} \mathbf{RC}^H \mathbf{x} = \mathbf{y}, \\ \widetilde{\mathbf{m}} = \mathbf{C}^H \widetilde{\mathbf{x}} \end{array} \right.$  Results:

In the above, **C** is discrete curvelet transform, **y** is incomplete observed data, **m** is unknown complete model to be determined and x is set of curvelet coefficients of m.

The matrix **R** is the restriction matrix, which is determined by the subsampling scheme we use, and which we want to design to maximize reconstruction quality.

R can be separable or non-separable.



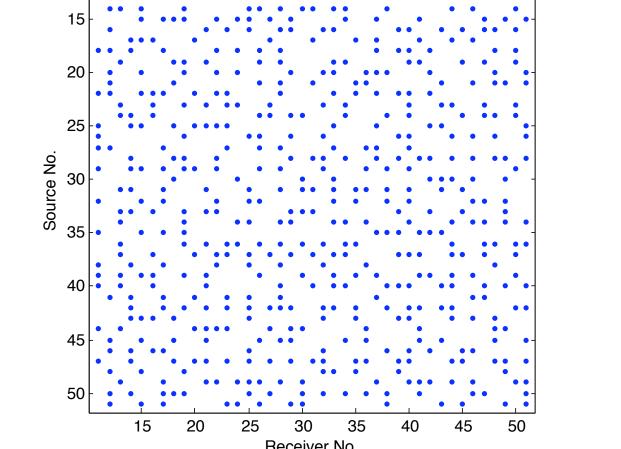


Fig. 2: Separable jittered sampling (left) and non-separable jittered sampling (right)

# Sampling methods:

Periodic sampling leads to coherent and hence difficult to remove aliased energy.

Randomized sampling converts coherent aliases into easy-to-remove noise.

Unfortunately, gap size not controlled leading to amplitude errors in CRSI where large gaps occur. Jittered sampling controls gap sizes & has blue-noise spectrum so artifacts pushed to high freqs.

This control leads to favorable conditions for curvelet-based recovery.

Poisson disk sampling offers alternative to jittered sampling by selecting n points at random iteratively, and only keeping sample if sufficient distance away from all previous samples (see Fig. 3(c)).

Farthest point sampling also leads excellent anti-aliasing properties, with implementation usually based on Voronoi diagrams [4], as shown in Fig. 3(d).

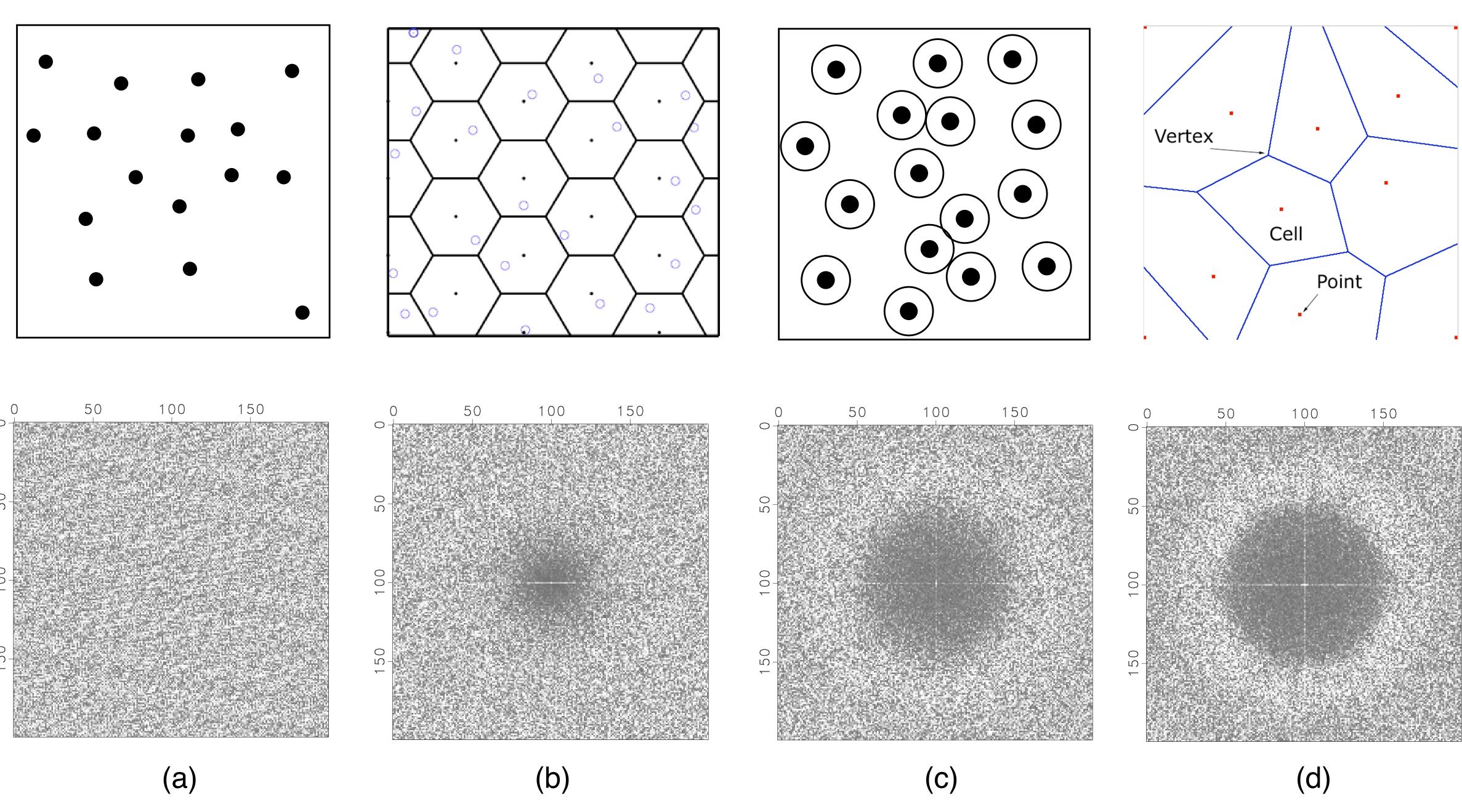
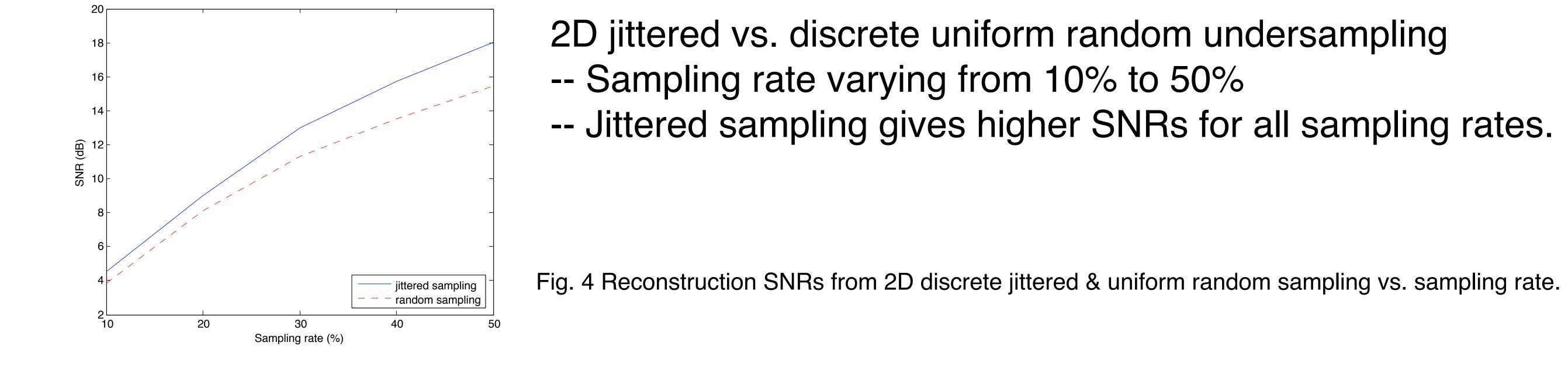


Fig. 3 Sampling schemes (top row) and their corresponding spectra (bottom row) of (a) discrete uniform random sampling, (b) jittered sampling, (c) Poisson disk sampling, (d) Farthest point sampling.



3D seismic X-line geometries (Fig. 5):

- -- 25% samples taken for each of the sampling methods.
- -- Jittered sampling better than discrete uniform random sampling.
- -- Poisson Disk and Farthest Point sampling slightly better than jittered sampling.

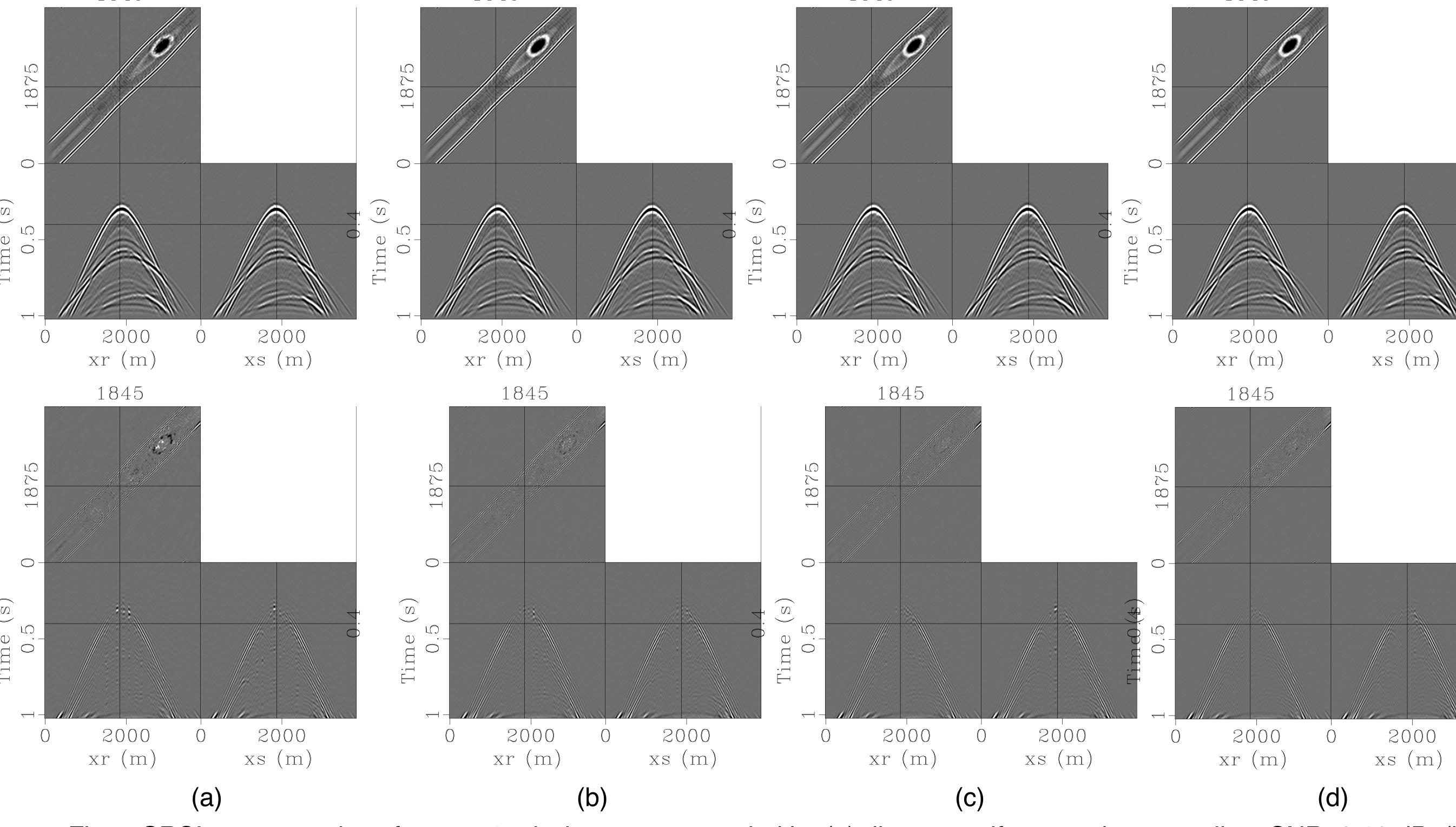


Fig. 5 CRSI reconstructions from 75% missing traces sampled by (a) discrete uniform random sampling, SNR=8.13 dB, (b) jittered sampling, SNR=8.43 dB (c) Poisson Disk sampling, SNR=8.48 dB (d) Farthest Point sampling, SNR=8.50 dB. The top row shows reconstructions, and bottom row shows their residuals.

# Conclusions:

Explored undersampling schemes with blue-noise characteristics for curvelet-based interpolation. Extended jittered sampling to 2D and tested two other blue-noise pattern sampling schemes, Farthest Point & Poisson Disk sampling.

Applied all these sampling techniques to 2D and 3D seismic data interpolation by CRSI, and obtained very good results, compared to methods in the literature.

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