Curvelet-Domain Matched Filtering with Frequency-Domain Regularization and an Application to Primary-Multiple Separation

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Introduction

Shown by our group previously that zero-order pseudodifferential operators, modeling various seismic operators, can be represented by diagonal weighting in curvelet domain.

Smoothness constraint introduced in phase space regularizing solution to make unique.

Here, we use recent results in Demanet and Ying (2008) on discrete symbol calculus to impose further smoothness constraint, this time in frequency domain.

Faster convergence realized with improved quality results.

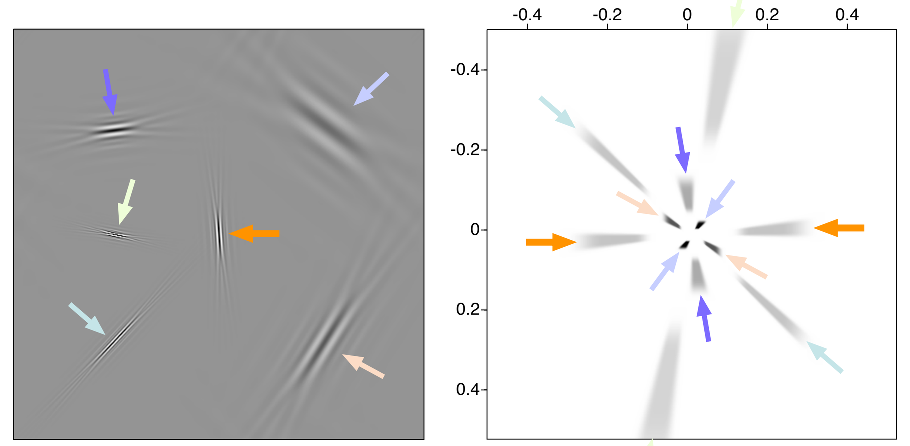
Curvelets in Seismic Imaging

Curvelets model seismic signals well - oscillate in one direction, smooth in the other.

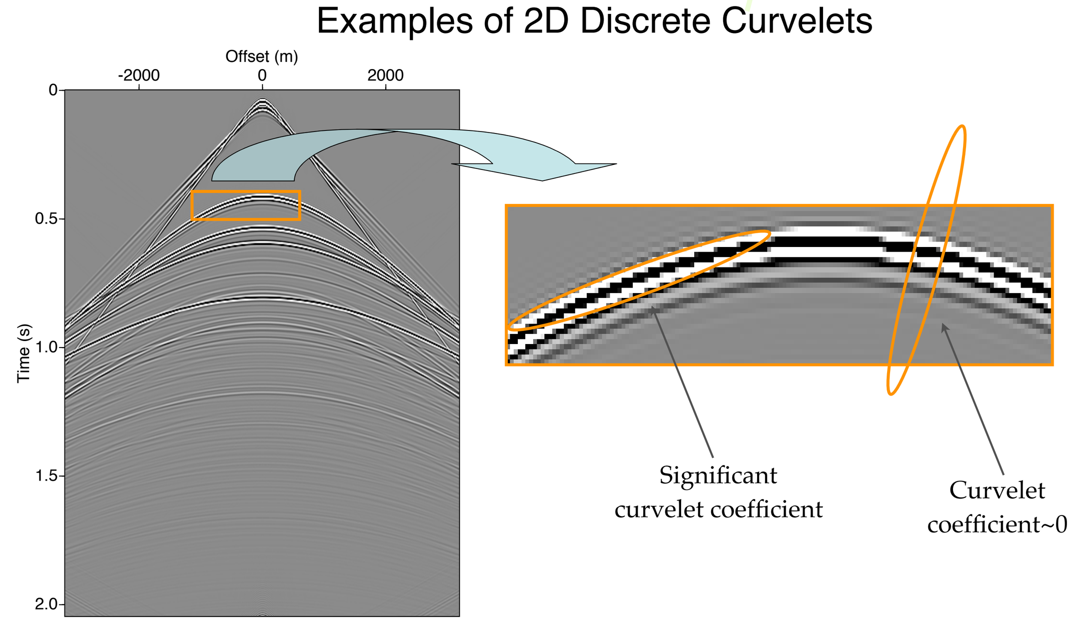
Seismic signals are *sparse* in curvelet domain.

Curvelets mostly localized both in space and frequency.

Parabolic scaling: length ∝ width², multidirectional and multiscale.



Examples of 2D Discrete Curvelets



Pseudodifferential Operators

A pseudodifferential operator (PsDO) is a scale/dip-dependent convolution.

Given seismic signal f, and denoting its Fourier transform with a hat superscript, the PsDO Ψ acts on f by the formula:

$$\Psi(\mathbf{f})(\mathbf{x}) = \int_{\Omega} a(\mathbf{x},\zeta)e^{-i\mathbf{x}\cdot\zeta}\hat{\mathbf{f}}(\zeta)d\zeta$$

a(x,ζ) is called the *symbol* of the PsDO Ψ.

Matched Filtering in Curvelet Domain

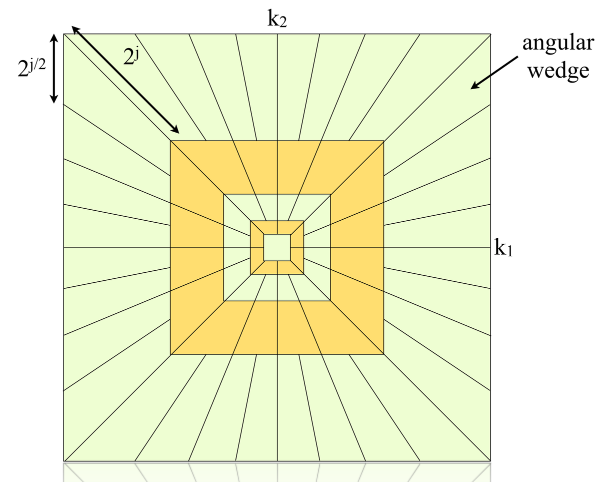
Curvelet-domain matched filtering based on approximation of PsDOs by diagonal weighting in curvelet domain.

Because of redundancy of curvelet transform, this fact may be expressed by:

$$\Psi \approx \mathbf{C}^{\mathbf{T}}\mathbf{D}_{\Psi}\mathbf{C}$$

with **C** = 2D discrete curvelet transform, $\mathbf{C}^{\mathbf{T}}$ its adjoint, and $\mathbf{D}_{\Psi}$ = diagonal matrix.

It may be proven that elements of diagonal of $\mathbf{D}_{\Psi}$ (also called the scaling vector) are given by the symbol a(x,ζ) of operator Ψ evaluated at curvelet centres in phase space.



Division of Phase Space for Discrete Curvelet

Because of nonuniqueness, there are many possible scaling vectors which are solutions.

From theory, we know that symbol a(x,ζ) is smooth in phase space.

Therefore, can *regularize* the solution to make it unique.

We add a smoothness term, so that nonlinear least squares problem to solve becomes:

$$\arg\min_{\mathbf{z}} \frac{1}{2}||\mathbf{C}^{\mathbf{T}}\mathrm{diag}(\mathbf{Cf})e^{\mathbf{z}} - \mathbf{g}||_2^2 + \frac{\lambda^2}{2}||\mathbf{L}e^{\mathbf{z}}||_2^2$$

The parameter λ controls the amount of smoothness in phase space.

L is a sharpening operator: $\mathbf{L} = [\mathbf{D}_1^{\mathbf{T}}\ \mathbf{D}_2^{\mathbf{T}}\ \mathbf{D}_\theta^{\mathbf{T}}\ \mathbf{D}_{scale}^{T}]^T$

Frequency-Domain Regularization

Can do better! According to work on discrete symbol calculus, symbol will also be smooth in frequency.

Does not follow directly from smoothness in phase space.

E.g. consider following symbol: $a(\mathbf{x},\zeta) = \sin\left(x_1 + \frac{|\zeta|}{100}\right)$

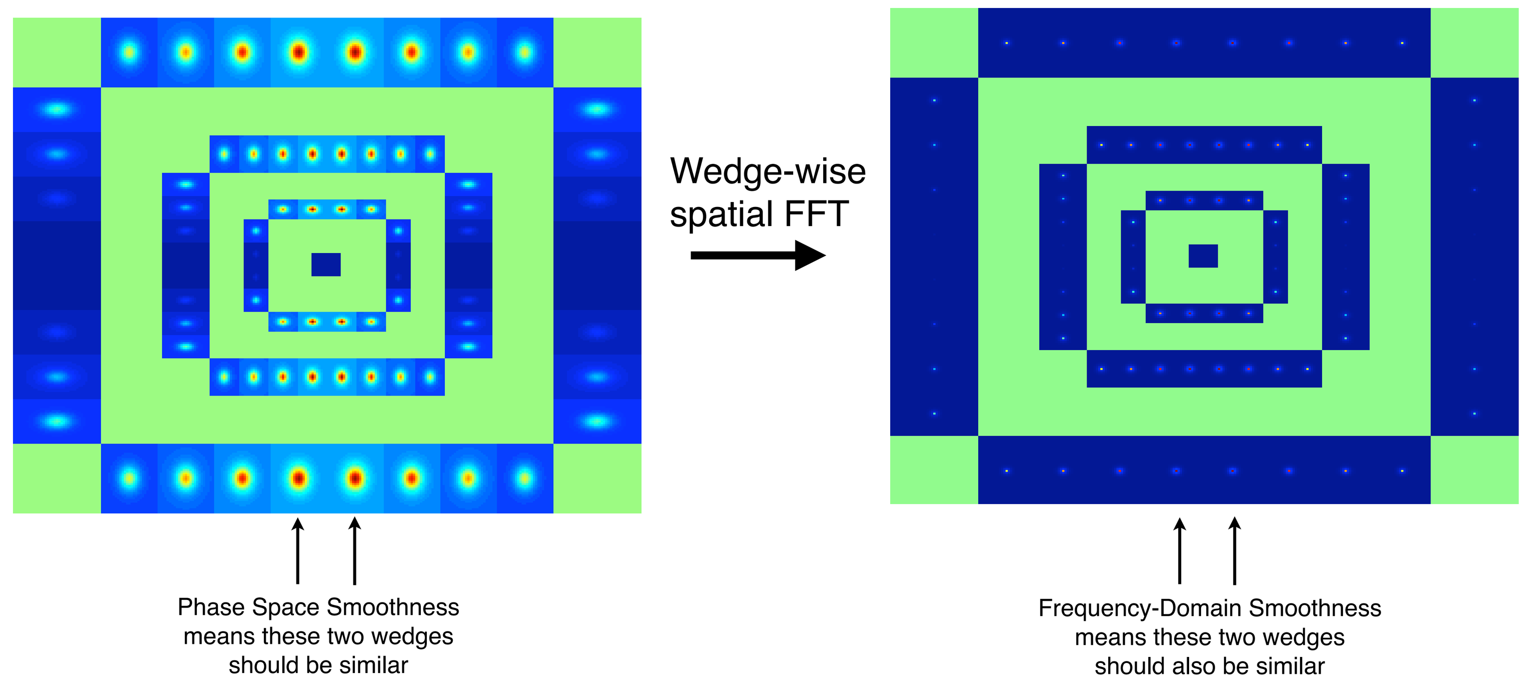
It is clearly smooth, however Fourier transform is: $\hat{a}(\eta,\zeta) = e^{-\frac{i\eta|\zeta|}{100}}\frac{\pi}{i}\left[\delta(\eta_1 - 1) - \delta(\eta_1 + 1)\right]$

This is not a smooth function in frequency because of Dirac impulse spikes.

But for PsDOs we're interested in, frequency smoothness also holds. So can use model:

$$\arg\min_{\mathbf{z}} \frac{1}{2}||\mathbf{C}^{\mathbf{T}}\mathrm{diag}(\mathbf{Cf})e^{\mathbf{z}} - \mathbf{g}||_2^2 + \frac{\lambda^2}{2}||\mathbf{L}e^{\mathbf{z}}||_2^2 + \frac{\mu^2}{2}||\mathbf{M}_\zeta\mathbf{R}\mathbf{F}_{\mathbf{x}}e^{\mathbf{z}}||_2^2$$

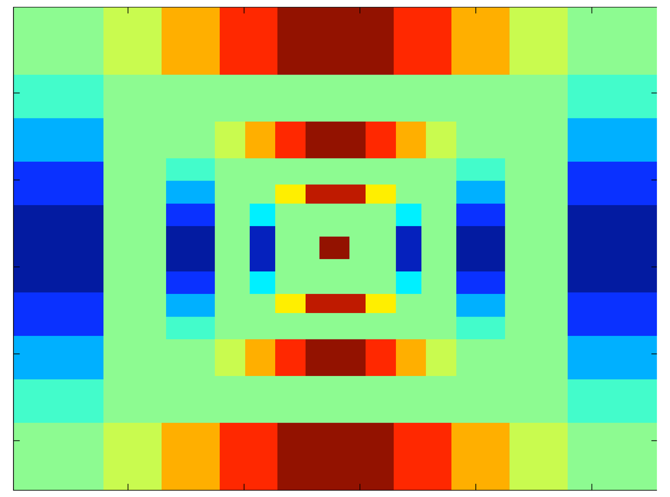
F is a spatial Fourier transform, **R** is a restriction matrix to reduce redundancy (scaling vector is always real), and $\mathbf{M}_\zeta$ is a frequency-domain sharpening operator.



Experiment on Synthetic Operator

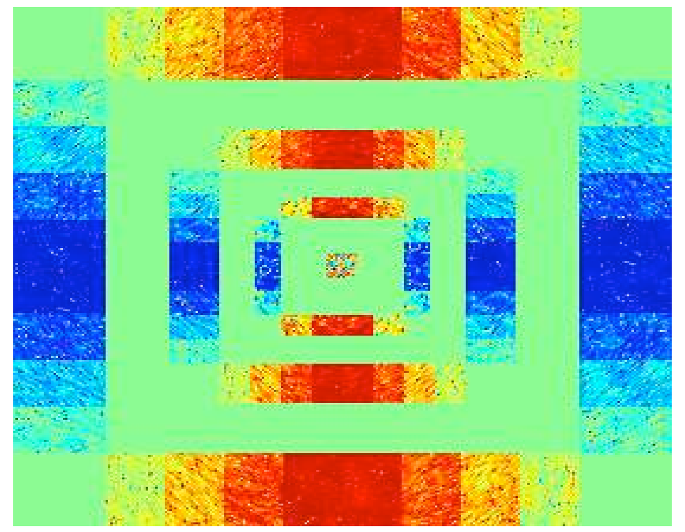
First measure performance on synthetic operator so can compare with ground truth.

The operator we use is $a_1(\mathbf{x},\zeta) = \sin^2(\theta)$ for which wedge plot is shown below

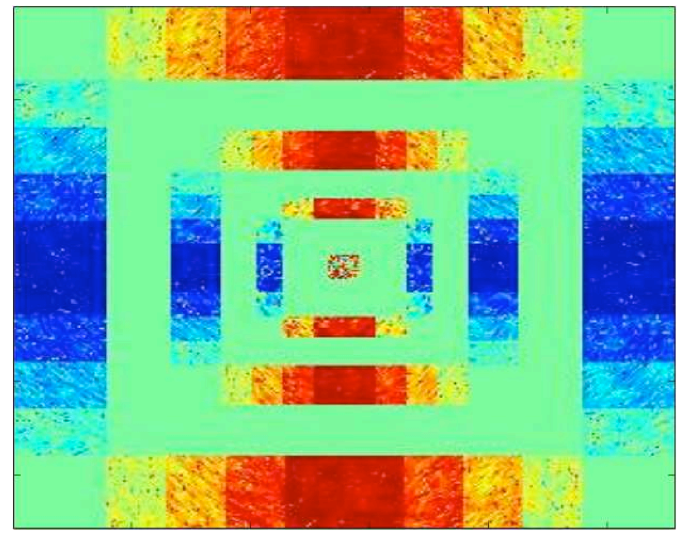


Ground Truth Wedge Plot of Scaling Vector
$a_1(\mathbf{x},\zeta) = \sin^2(\theta)$

We obtain the following results. Result on left is without new frequency-domain regularization while result on right is with new regularization.



Scaling Vector Wedge Plot, no Frequency-Domain Regularization



Scaling Vector Wedge Plot with Frequency-Domain Regularization

The result on right is clearly better (closer to ground truth) - only in 20 iterations!

The result without this new regularization is worse and it took 50 iterations to get it.

Primary-Multiple Separation

Pseudodifferential operator concept can also be applied to primary-multiple separation.

In that case, assume we have multiples predicted from another algorithm (e.g. SRME [Verschuur, 1992]), so then map between predicted multiples $\mathbf{m_P}$ and true multiples $\mathbf{m_T}$ is given by the equation: $\mathbf{B}\mathbf{m}_{\mathrm{P}} = \mathbf{m}_{\mathrm{T}}$

It may be shown that operator **B** can be expressed as zero-order pseudodifferential operator. So we can apply curvelet-domain matched filter as above.

Therefore, $\mathbf{B} \approx \mathbf{C}^{\mathbf{T}}\mathrm{diag}(\mathbf{w})\,\mathbf{C}$

In practice, do not have true multiples, so instead use total data **d** as an approximation.

$$\mathbf{d} \approx \mathbf{C}^{\mathbf{T}}\mathrm{diag}(\mathbf{w})\,\mathbf{C}\mathbf{m}_{\mathrm{P}}$$

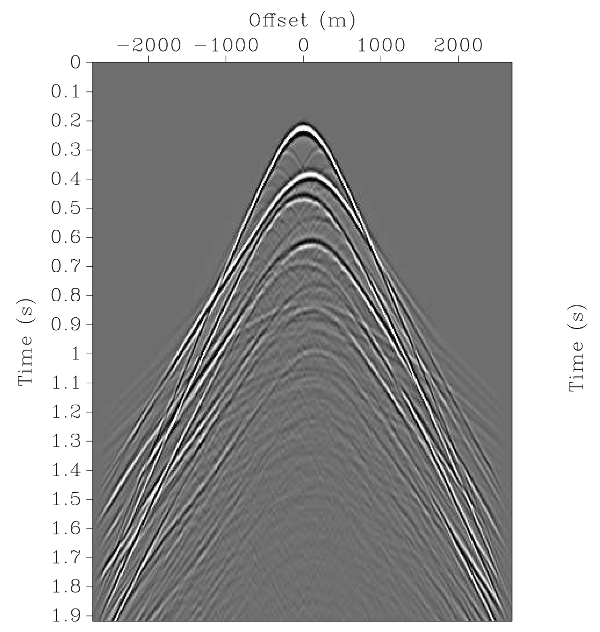
Primary-Multiple Separation and Frequency Regularization

Curvelet-domain matched filter used successfully for primary-multiple separation.

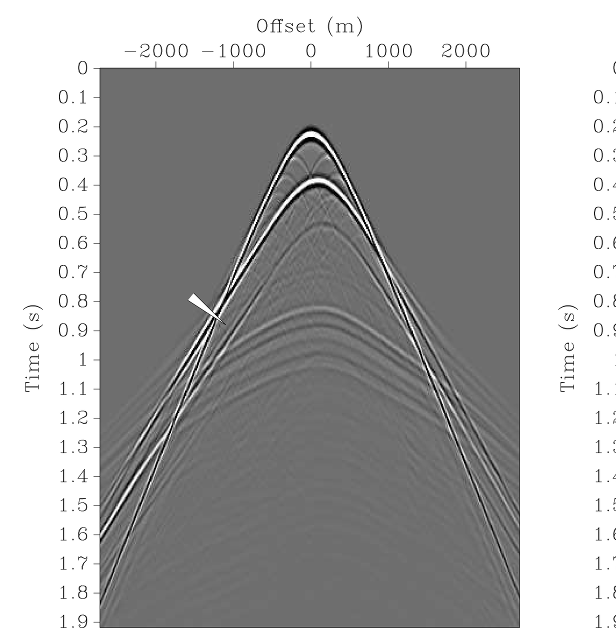
The coefficient of the new regularization term is μ.

The case when μ=0 corresponds to case of no new frequency-domain regularization.

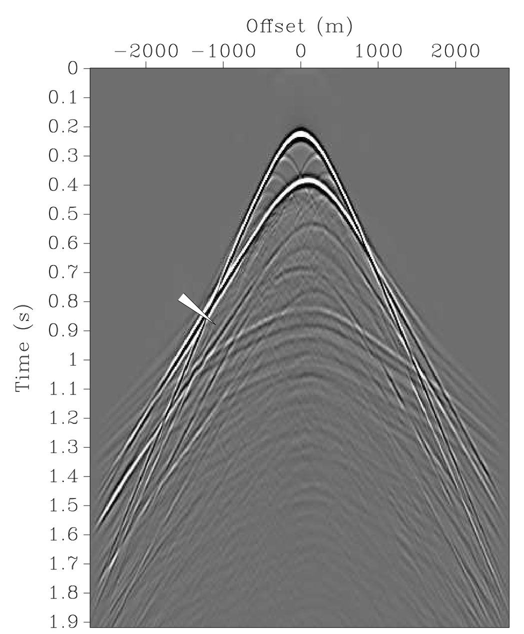
As μ increases, so does the new regularization.



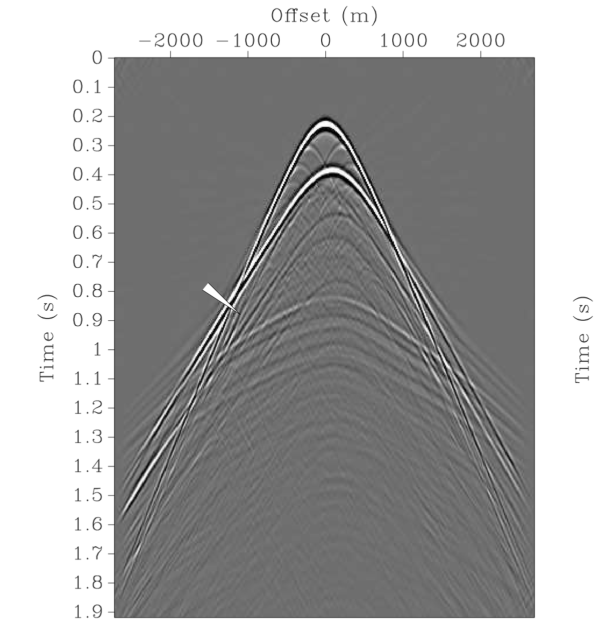
Total data (primaries + multiples)



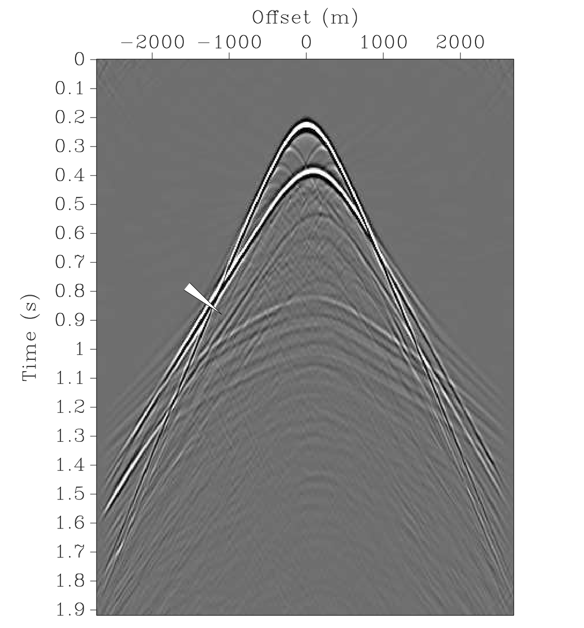
Ground-truth primaries



SRME-predicted primaries



Estimated primaries from matched filter, μ=0, SNR=8.996



Estimated primaries with new regularization, μ=1, SNR=9.148

Above are some results showing effect of new regularization (data already matched for wavelet with Fourier matching - not proposed frequency domain regularization!)

Clearly, new regularization improves accuracy of recovered primaries.

More remarkable is this is only in 35 iterations while 50 for old model.

Can be extended to 3D data and also real seismic data.

References

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