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Irregular grid tensor completion

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Abstract: Low rank tensor completion has recently garnered the attention of researchers owing to the ubiquity of tensors in the sciences and the theoretical and numerical challenges compared to matrix completion. Here we consider a tensor completion scheme where the data arises from sampling a continuous function. When the sampling grid is not periodic, the resulting tensor may not be low rank in the Hierarchical Tucker sense, which can adversely affect reconstruction quality when there are missing samples. In order to compensate for this off-grid sampling, we introduce a resampling operator (here, the non-uniform Fourier transform) that accounts for this non-uniformity moreso than merely treating the sampling grid as periodic. Numerical experiments demonstrate that this approach can improve reconstruction quality when there is relatively little data.

Motivation and approach:

For continuous signals, low rank tensor completion can be posed as completing the values of a function sampled on a regular grid, i.e., for a function $f(x^1, x^2, x^3, x^4)$

$$X_{i_1,i_2,i_3,i_4} = f(x^1_{i_1}, x^2_{i_2}, x^3_{i_3}, x^4_{i_4}) \qquad x^j_{i_j} = x^j_0 + i_j \Delta x^j$$
$$i_j = 1, 2, \ldots, n_j \quad j = 1, \ldots, 4$$

We assume that we have the values of the tensor X at a subset of indices $\Omega \subset [n_1] \times [n_2] \times [n_3] \times [n_4]$, where $[n_j] = \{1, 2, \ldots, n_j\}$. Let G denote the regular sampling grid given by $x^j_{i_j}$.

We can represent the underlying function in the Hierarchical Tucker format, which recursively decomposes subspaces of various matrix unfoldings of the tensor. Specifically, for a specific dimension tree T, a tensor X can be expressed in Hierarchical Tucker format if, for each $t \in T$, there exists matrices U_t and 3-tensors B_t such that

$$\text{vec}(X) = U_{t_r} \otimes U_{t_l} B_{t_{\text{root}}} \qquad t \text{ is the root of } T$$
$$U_t = U_{t_r} \otimes U_{t_l} B_t \qquad t \text{ is not a leaf of } T$$

The set of Hierarchical Tucker tensors parametrizes a smooth manifold (first studied in [1]) and using the techniques of [2] we can interpolate tensors with missing entries in this format when sampling grid is regular.

We consider a perturbation of the sampling grid in each coordinate, so that

$$\widetilde{x_{i_j}}^j = x^j_{i_j} + \delta x^j_{i_j} \qquad |\delta x^j_{i_j}| < |\Delta x^j|.$$

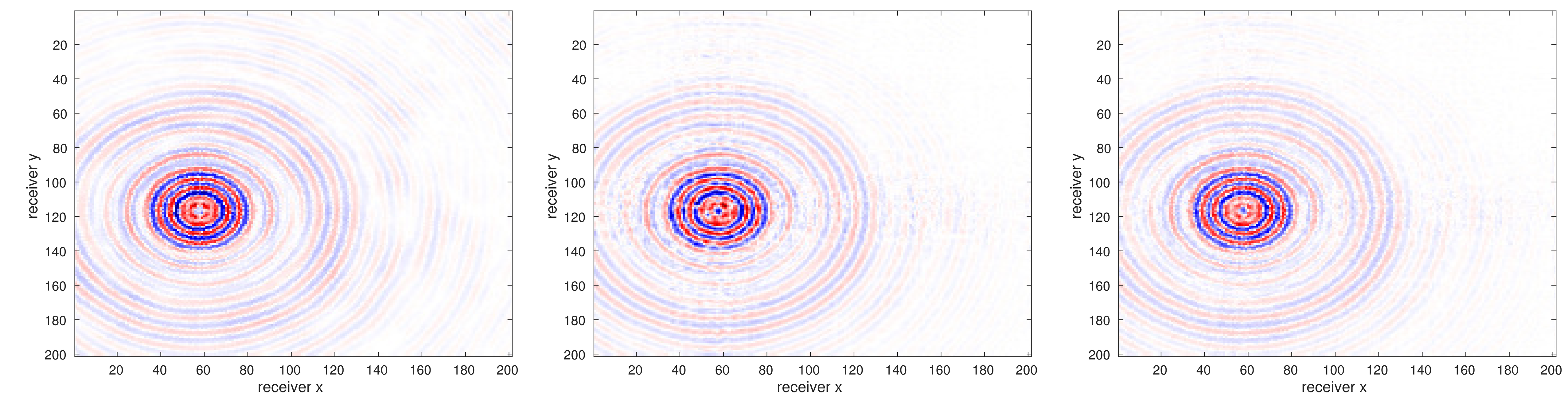
The resulting irregular grid, denoted $\widetilde{G}$, increases the ranks in the various matrix modes of tensor X, rendering it high rank. We follow the approach of [3] for irregular grid matrix completion and introduce a resampling operator $R : G \to \widetilde{G}$ that interpolates function samples from the regular grid to the irregular grid on which the data is acquired. By incorporating this operator in to the optimization program, assuming that it is sufficiently accurate for the class of functions to which belongs, we can properly

References

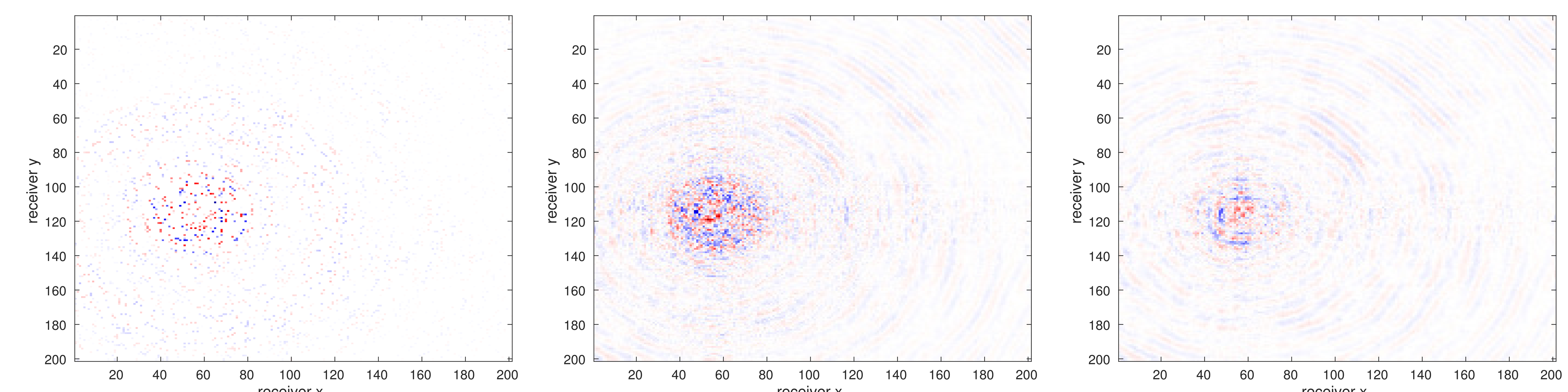
[1] A. Uschmajew and B. Vandereycken. The geometry of algorithms using hierarchical tensors. Linear Algebra and its Applications, 439(1):133--166, July 2013.
[2] C. Da Silva and F. J. Herrmann. Optimization on the hierarchical tucker manifold – applications to tensor completion. Linear Algebra and its Applications, 481(0):131 - 173, 2015. (to appear)
[3] R. Kumar, O. Lopez, E. Esser, and F. J. Herrmann. Matrix completion on unstructured grids : 2-D seismic data regularization and interpolation. EAGE conference, 2015.
[4] Leslie Greengard and June-Yub Lee. Accelerating the nonuniform fast fourier transform. SIAM Review, 46(3):443-454, 2004.

Experiments

Here we consider a (temporal) frequency slice from a seismic data volume having 68 x 68 sources and 401 x 401 receivers. Let G_{401} denote the 401 x 401 receiver grid. We regularly subsample G_{401} to a coarser 201 x 201 grid, denoted G_{201}. On this coarser grid, we randomly perturb the grid points to nearby points on G_{401}, so that approximately 25% of the subsampled grid is unperturbed. This is to avoid generating 'inverse crime' data by resampling the volume through some other means such as spline interpolation. For recovery, we consider using the Non-uniform Fourier Transform (NFT) [4] and merely treating the irregular grid as a regular 201 x 201 grid (effectively nearest neighbour interpolation). We randomly remove 90% of the receivers from the data and recover the tensor using Hierarchical Tucker manifold optimization. Given that seismic data is much better represented via complex sinusoids than piecewise constant functions, using the NFT greatly improves the reconstruction quality of the algorithm.



True data | Nearest Neighbour SNR 6.95 dB | NFT SNR 11.0 dB



Input data 90% missing receivers | Nearest Neighbour Difference | NFT Difference

Conclusion

We have demonstrated that, by accounting for the off-grid nature of the sampling grid rather than merely treating it as periodic, we can improve the reconstruction quality of the resulting tensors when there is missing data. Given the many open theoretical questions surrounding standard tensor completion, it remains to be seen how the theoretical results of [3] will translate to the tensor case.