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BEYOND ℓ_1-NORM MINIMIZATION FOR SPARSE SIGNAL RECOVERY

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Abstract

For over a decade, sparse signal recovery has been dominated by the basis pursuit denoise (BPDN) problem:

$$\min_{u\in\mathbb{R}^N} \|u\|_1 \text{ subject to } \|Au-y\|_2 \le \epsilon \qquad \text{(BPDN)}$$

We propose the WSPGL1 algorithm that finds sparse solutions to underdetermined linear systems of equations. WSPGL1 is a modification of the SPGL1 algorithm in which the sequence of LASSO subproblems are replaced by a sequence of weighted LASSO subproblems with constant weights applied to a support estimate. The support estimate is derived from the data and is updated at every iteration.

The WSPGL1 algorithm recovers less sparse signals than BPDN at no additional computational cost. Moreover, the sparse recovery performance approaches that of iterative re-weighted ℓ_1 (IRWL1) of Candès, Wakin, and Boyd, although it does not match it in general.

Spectral Projected Gradient for ℓ_1 Minimization

- Solve a sequence of LASSO subproblems:

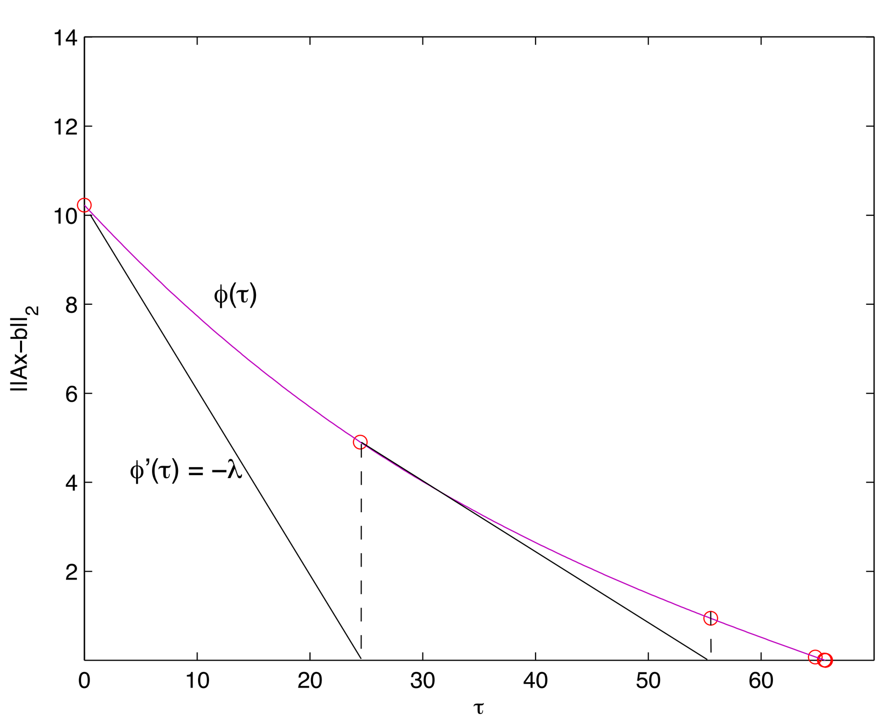
$$\min_{u\in\mathbb{R}^N} \|Au-y\|_2 \text{ subject to } \|u\|_1 \le \tau \qquad (\text{LS}_\tau)$$

- Start with an initial τ_0 and update τ according to the relation

$$\tau_{t+1} = \tau_t + \frac{\|r^t\|_2 - \epsilon}{\|A^H r^t\|_\infty / \|r^t\|_2},$$

where $r^t = y - Ax^t$ and x^t is the solution to LS_τ at iteration t.

- The update of τ traverses a Pareto curve which traces the optimal trade-off between the least-squares fit and the one-norm of the solution.



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WSPGL1 - A Better Sparse Recovery Algorithm

- Solve a sequence of ***weighted*** LASSO subproblems:

$$\min_{u\in\mathbb{R}^N} \|Au-y\|_2 \text{ subject to } \|u\|_{1,\mathrm{w}} \le \tau \qquad (\text{LS}_{\tau,\mathrm{w}})$$

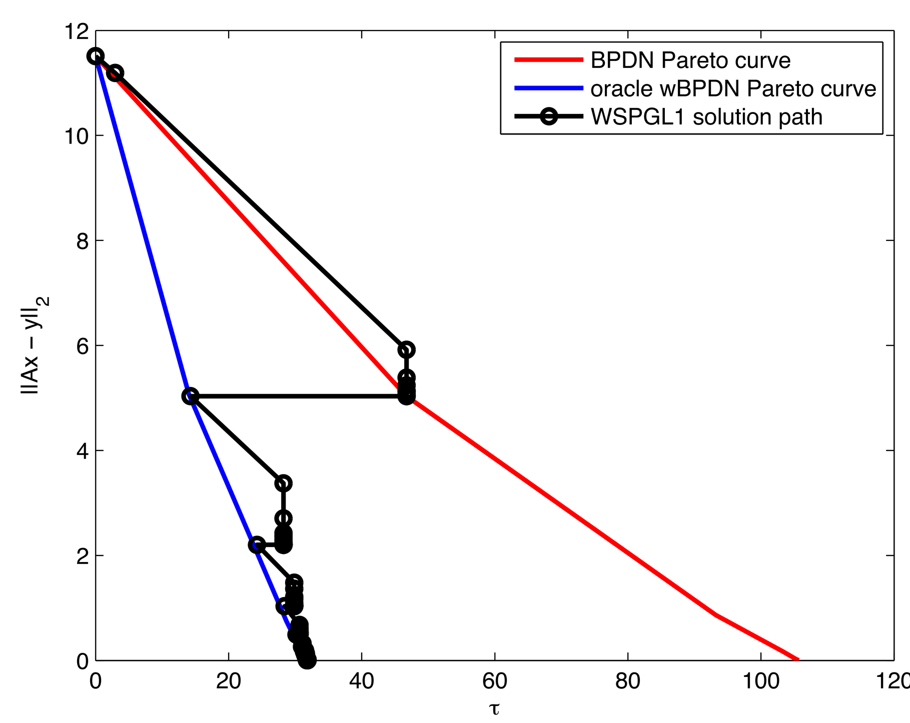
where the weighted one-norm is given by

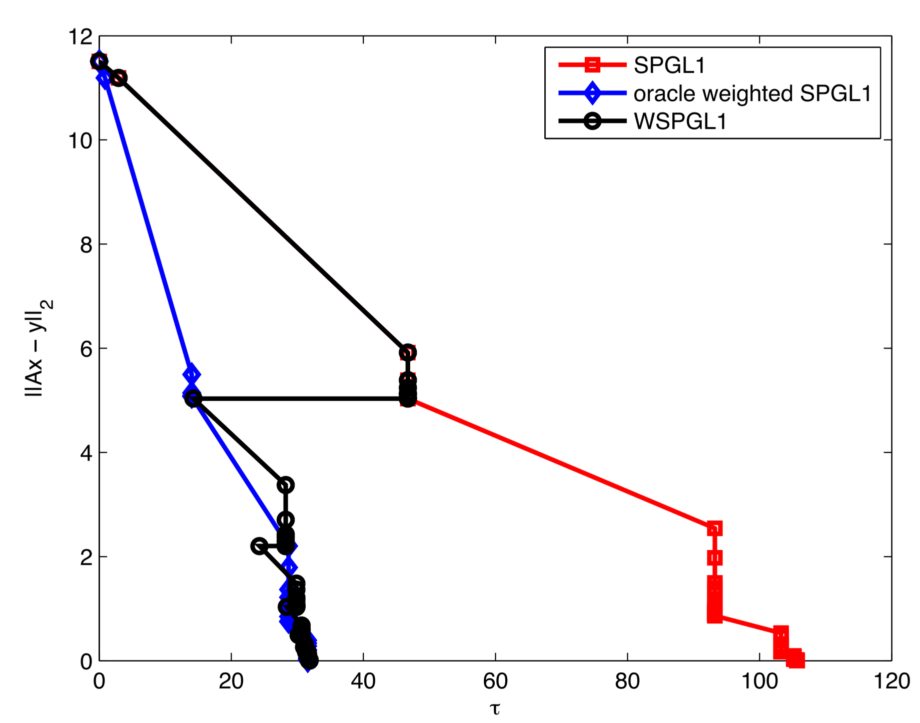
$$\|x\|_{1,\mathrm{w}} = \sum_{i=1}^{N} \mathrm{w}_i |x_i|, \quad \mathrm{w}_i = \begin{cases} \omega, & i\in\Lambda \\ 1, & i\in\Lambda^c \end{cases}$$

and Λ is a support estimate from the previous LASSO subproblem.

Algorithm 1 The WSPGL1 algorithm

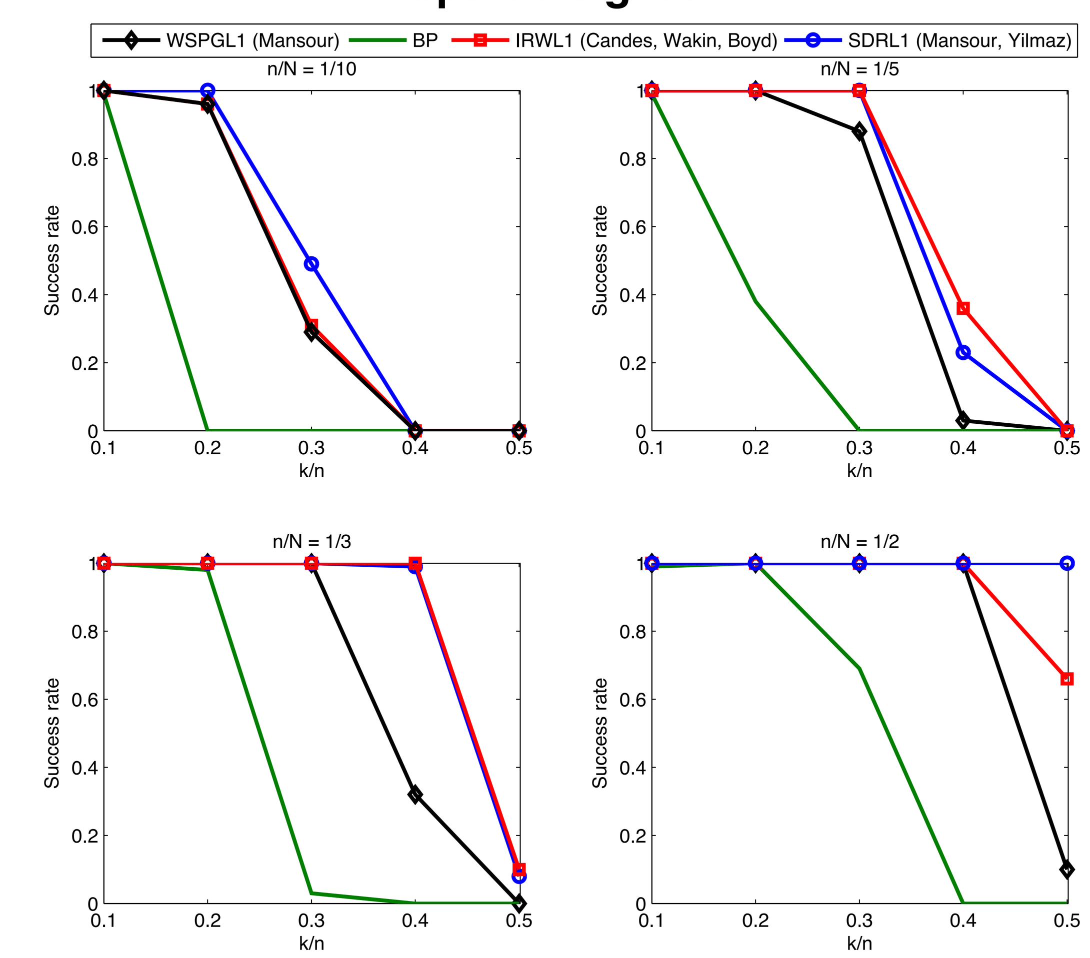
Input $y = Ax + e$, ϵ, $k = n/\left(2\log(N/n)\right)$, $\omega\in[0,1]$
Output $x^{(t)}$
Initialize $\mathrm{w}_i^{(0)} = 1$ for all $i \in \{1\dots N\}$
$t = 0$, $x^{(0)} = 0$, $r^{(0)} = y$, $\tau_0 = 0$
loop
$t = t + 1$
$\Lambda = \mathrm{supp}(x^{(t-1)}|_k)$, $\mathrm{w}_i = \begin{cases} \omega, & i\in\Lambda \\ 1, & i\in\Lambda^c \end{cases}$
$\tau'_{t-1} = \|x^{(t-1)}\|_{1,\mathrm{w}}$
$\tau_t = \tau'_{t-1} + \frac{\|r^{(t-1)}\|_2 - \epsilon}{\|A^H r^{(t-1)}\|_{\infty,\mathrm{w}}/\|r^{(t-1)}\|_2}$
$x^{(t)} = \arg\min_u \|Au-y\|_2$ s.t. $\|u\|_{1,\mathrm{w}} \le \tau_t$
$r^{(t)} = y - Ax^{(t)}$
end loop



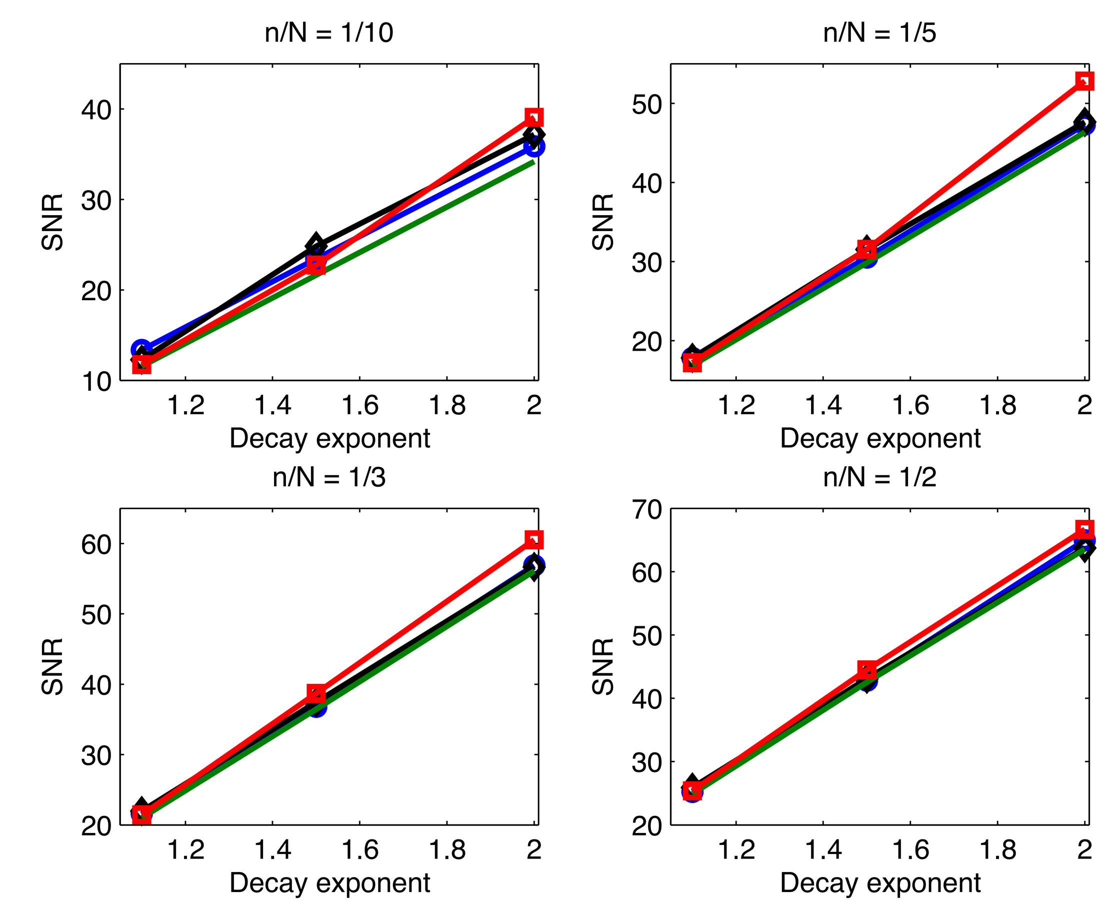


Experimental Results

Sparse signals



Compressible Signals



Runtime Comparison