**Approximate Message Passing Meets Exploration Seismology**

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**Abstract:** Seismic imaging hinges on large-scale sparsity promoting solvers to remove artifacts caused by efforts to reduce (computational) costs. Including "Onsager’s message term" improves convergence by canceling correlations between model iterates and linearized forward models. Unfortunately, the asymptotic arguments of approximate message passing are oft violated in practice. By using Montanari’s heuristic argument of “statistical equivalence”, we break correlations by selecting independent experiments via randomized subsampling.

**Current imaging paradigm**

Linear forward model: Imaging:

\[ \mathbf{A} \mathbf{x}_0 = \mathbf{b} \quad \mathbf{x} = \mathbf{A}^H \mathbf{b} \]

**Dimensionality reduction with CS**

- ability to exploit model-space structure
- reduces size data volumes
- reduces wave simulation costs
- at the price of introducing interference

**Convex optimization with cooling**

Solve

\[ \text{minimize } \| \mathbf{x} \|_1 \text{ subject to } \mathbf{A} \mathbf{x} = \mathbf{b} \]

via a series of LASSO subproblems

\[ \text{minimize } \frac{1}{2} \| \mathbf{b} - \mathbf{A} \mathbf{x} \|_2 \text{ subject to } \| \mathbf{x} \|_1 \leq \tau \]

for appropriately chosen increasing \( \tau \)'s.

- uses Pareto trade-off curve
- suffers from harmful correlation buildup
- solvers need too many matrix-vector products

**Approximate Message passing**

Add “Onsager”-term to iterative thresholding

\[ \mathbf{x}^{t+1} = \eta_t \left( \mathbf{A}^* \mathbf{r}^t + \mathbf{x}^t \right) \]

\[ \mathbf{r}^t = \mathbf{b} - \mathbf{A} \mathbf{x}^t - \frac{\| \mathbf{x}^{t+1} \|_0}{n} \mathbf{r}^{t-1} \]

for \( \mathbf{A}^{n \times N} \in n^{-1/2} N(0,1) \) and \( N \to \infty \)

Statistically equivalent to [Montanari, '12]

\[ \mathbf{x}^{t+1} = \eta_t \left( \mathbf{A}^* \mathbf{r}^t + \mathbf{x}^t \right) \]

\[ \mathbf{r}^t = \mathbf{b}_t - \mathbf{A}_t \mathbf{x}^t \]

by drawing new independent \( \{ \mathbf{b}_t, \mathbf{A}_t \} \) for each iteration.

- Onsager term cancels harmful correlations
- renewals yield similar decorrelation
- improves convergence data-rich problems

**Possibility to better & speedup imaging...**

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