Massive 3D seismic data compression & interpolation w/ on-the-fly data extraction

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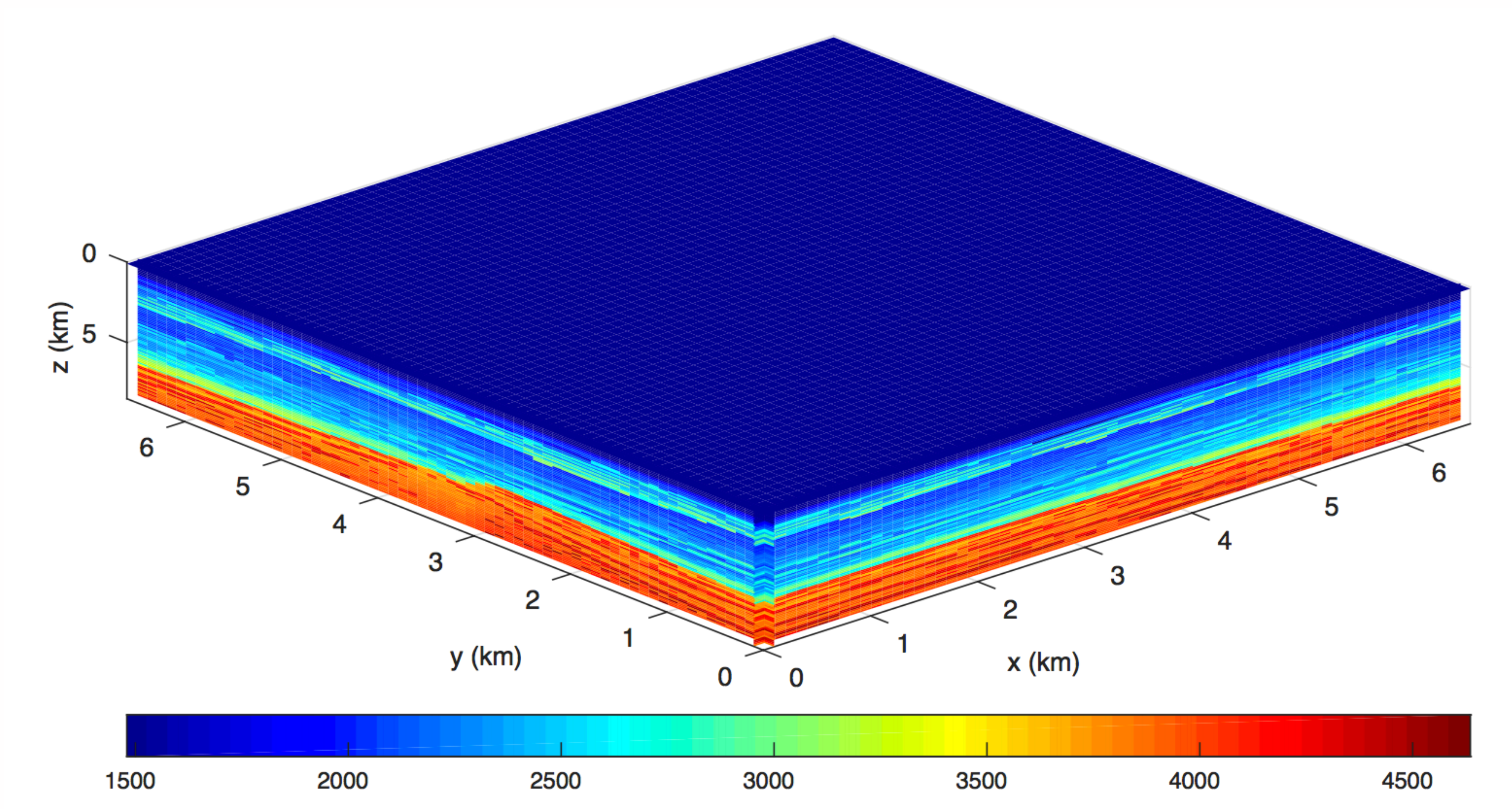
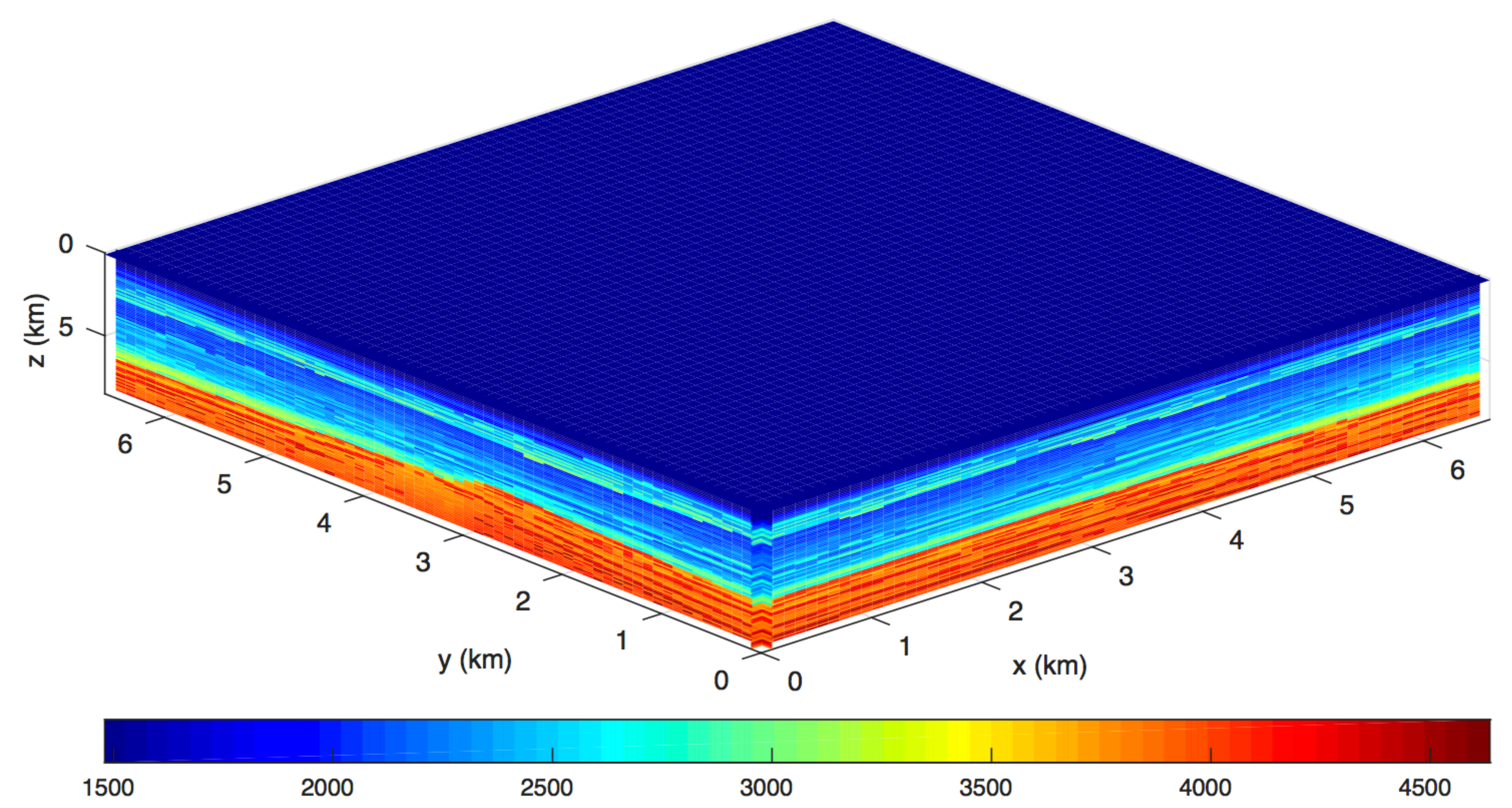
Motivation

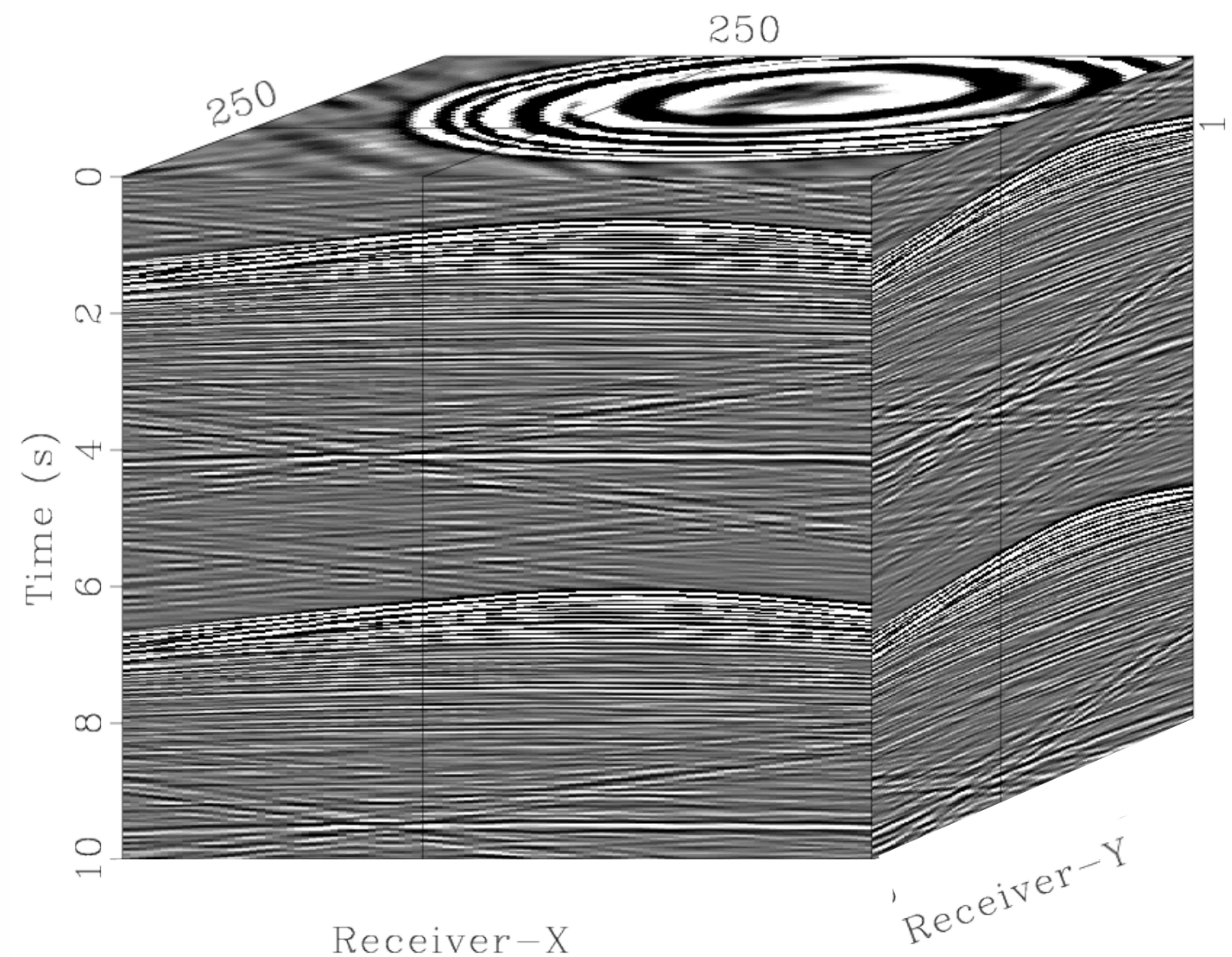

0
5
z (km)
6
5
4
3
2
1
0
y (km)
0
1
2
3
4
5
6
x (km)
1500
2000
2500
3000
3500
4000
4500

Motivation

- **Enormous** volumes of seismic data

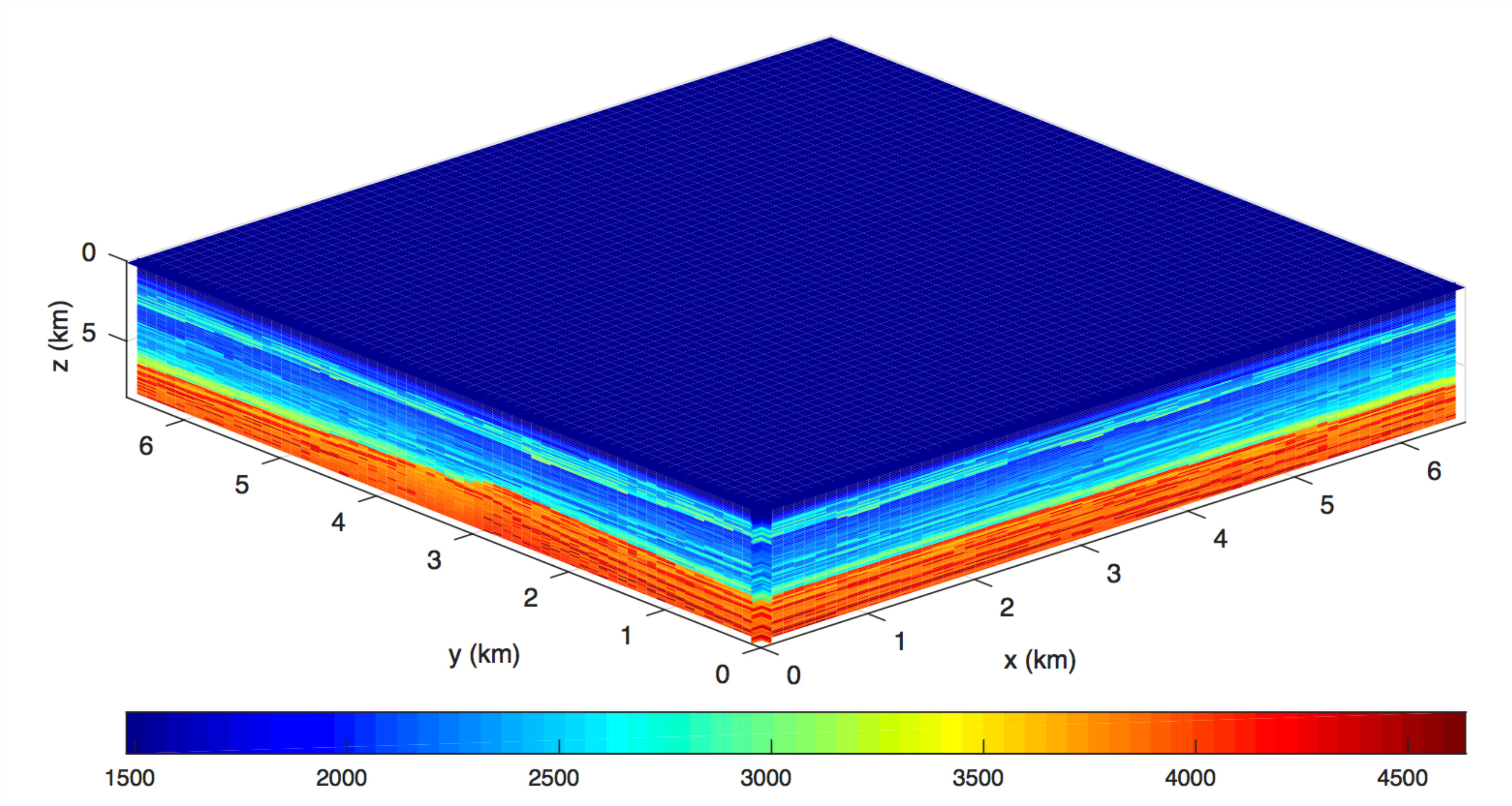
~1TB



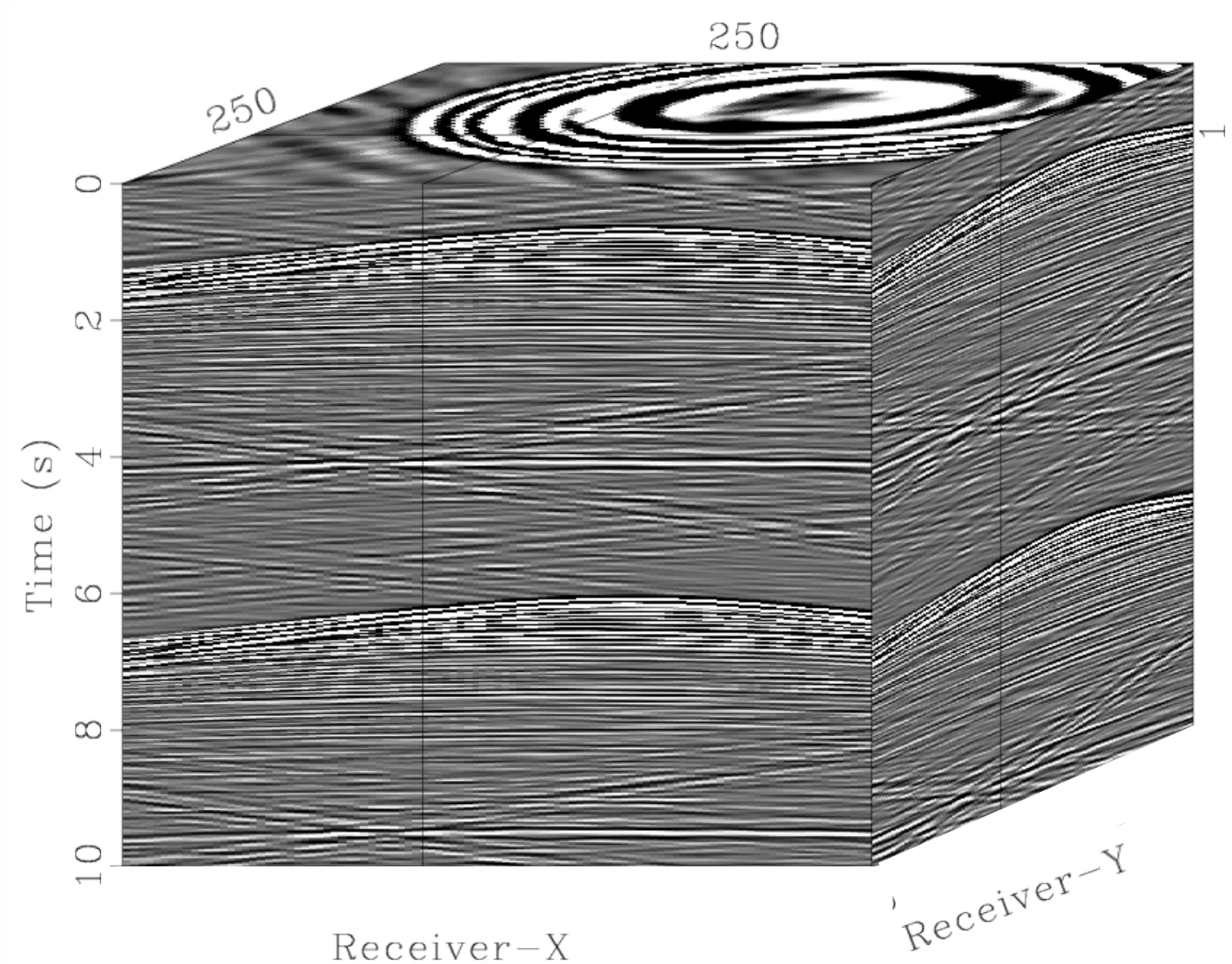


Motivation

- **Challenging** in inversion



~1TB



Motivation

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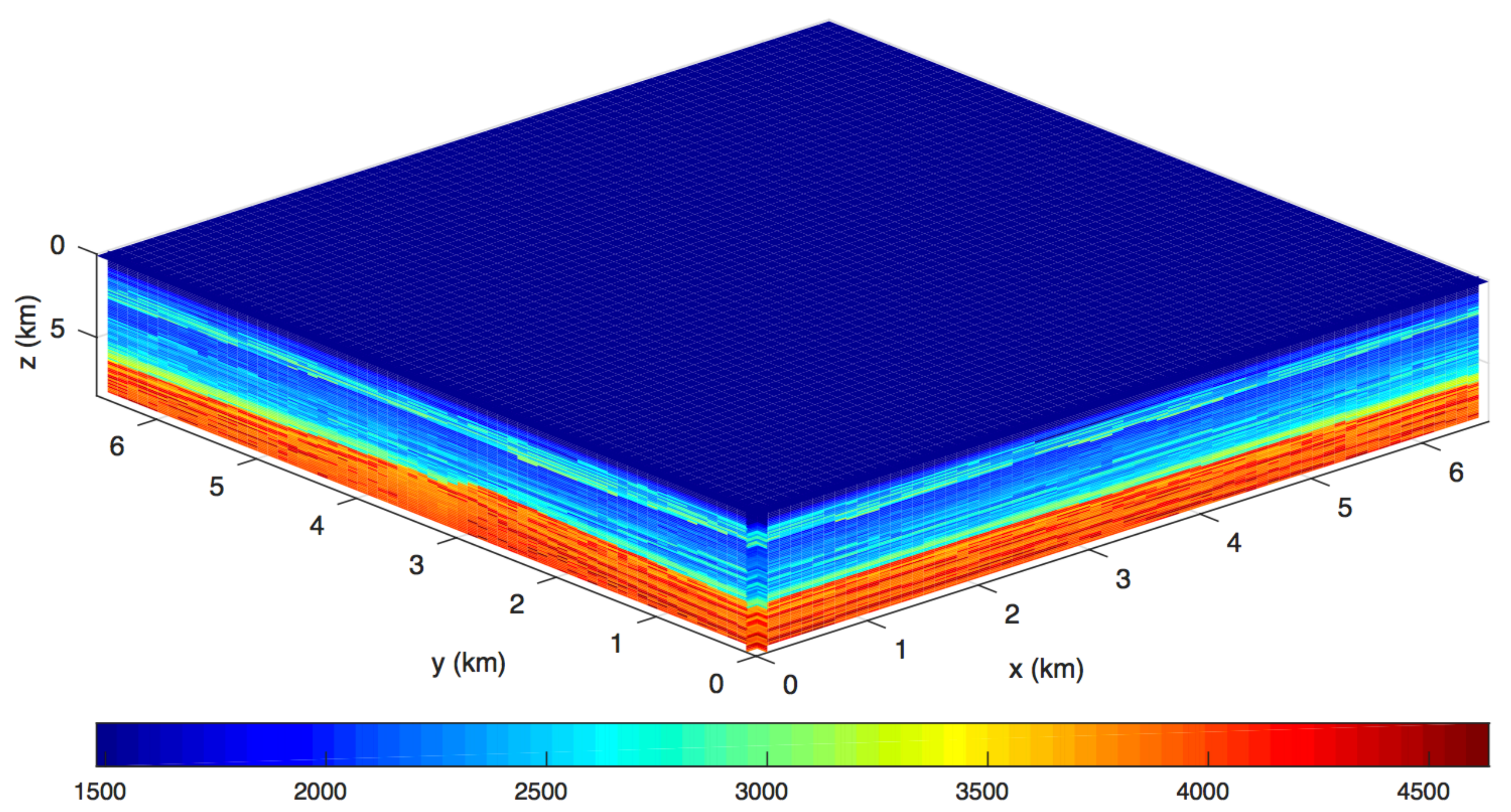
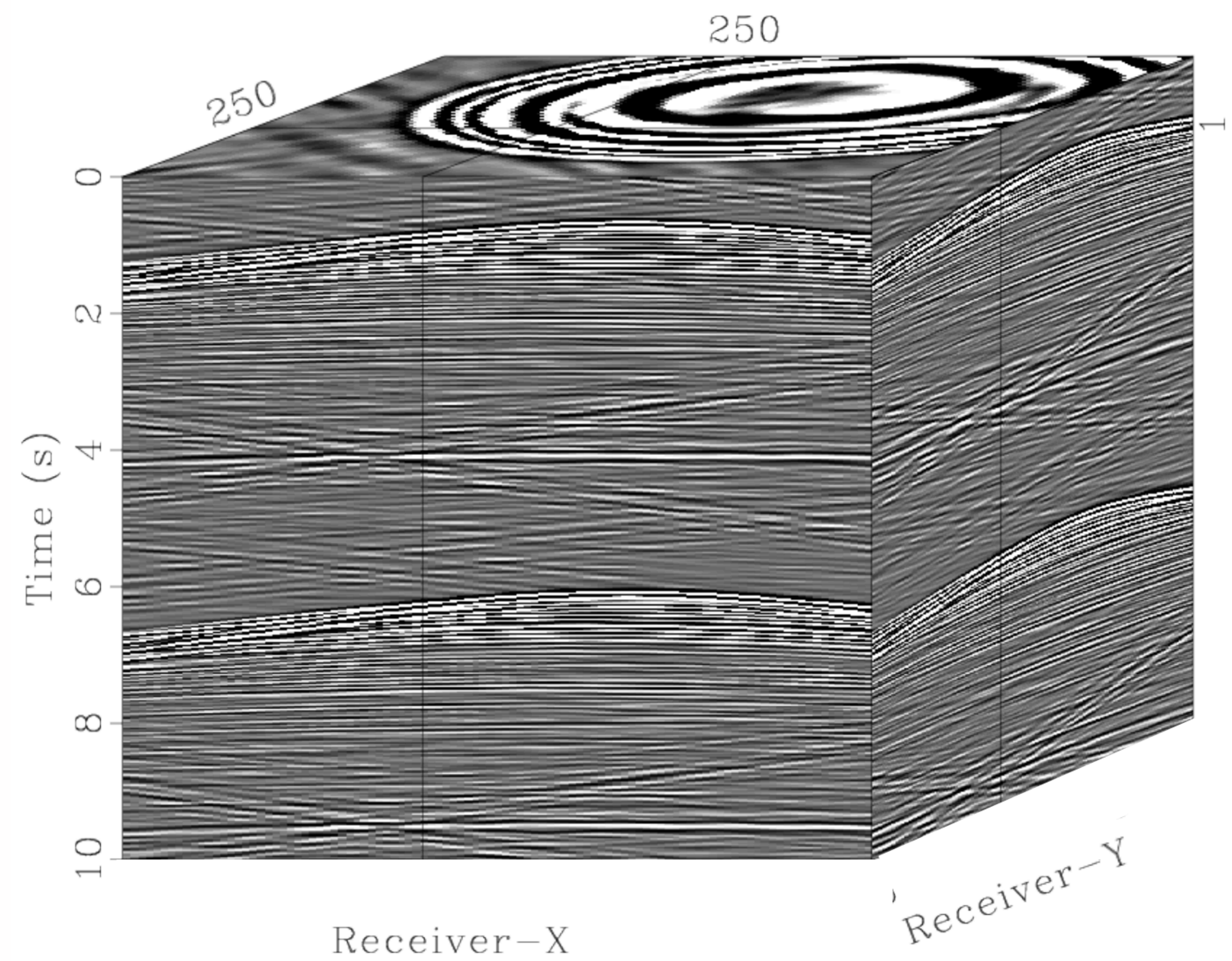


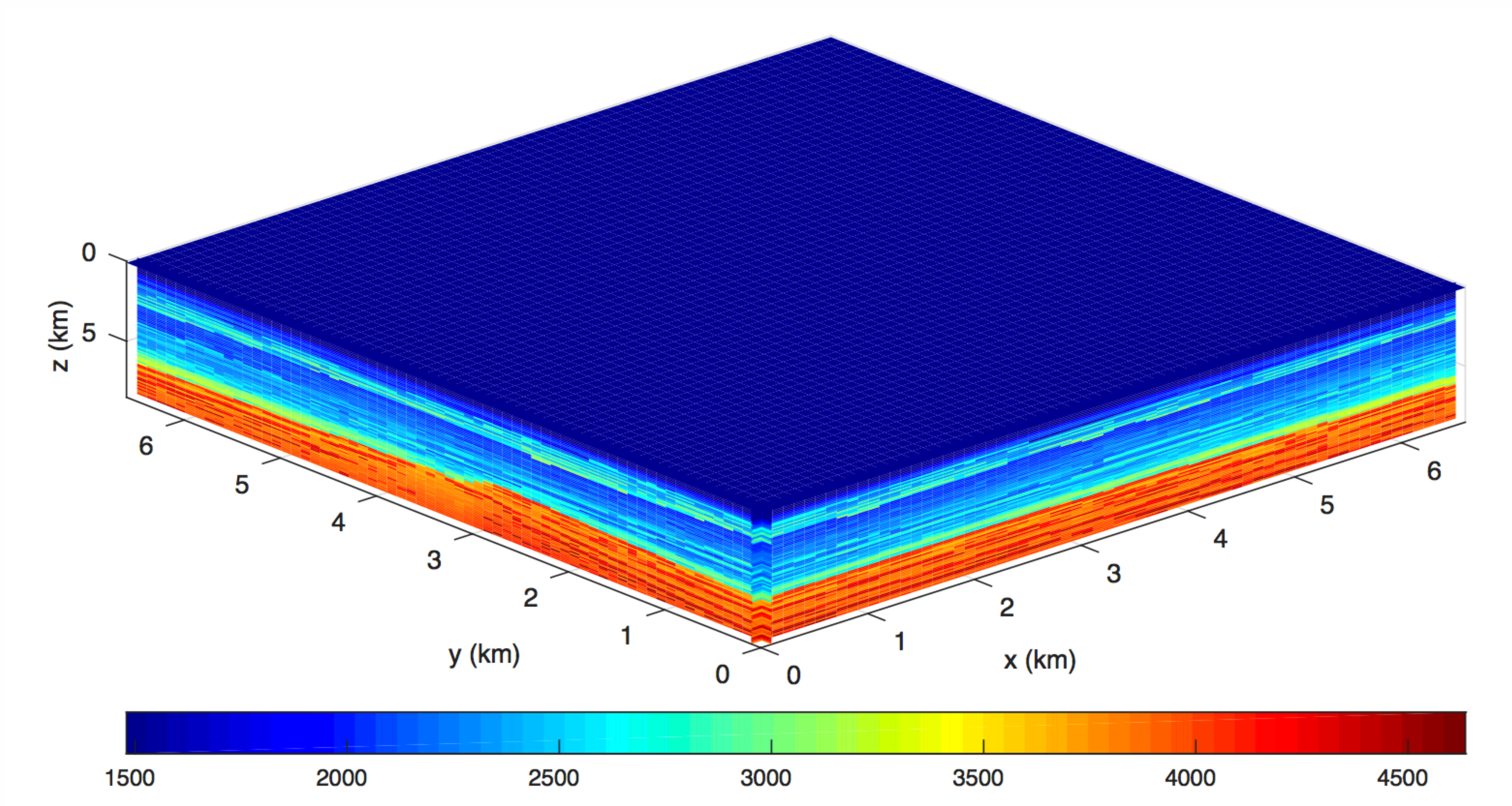
?

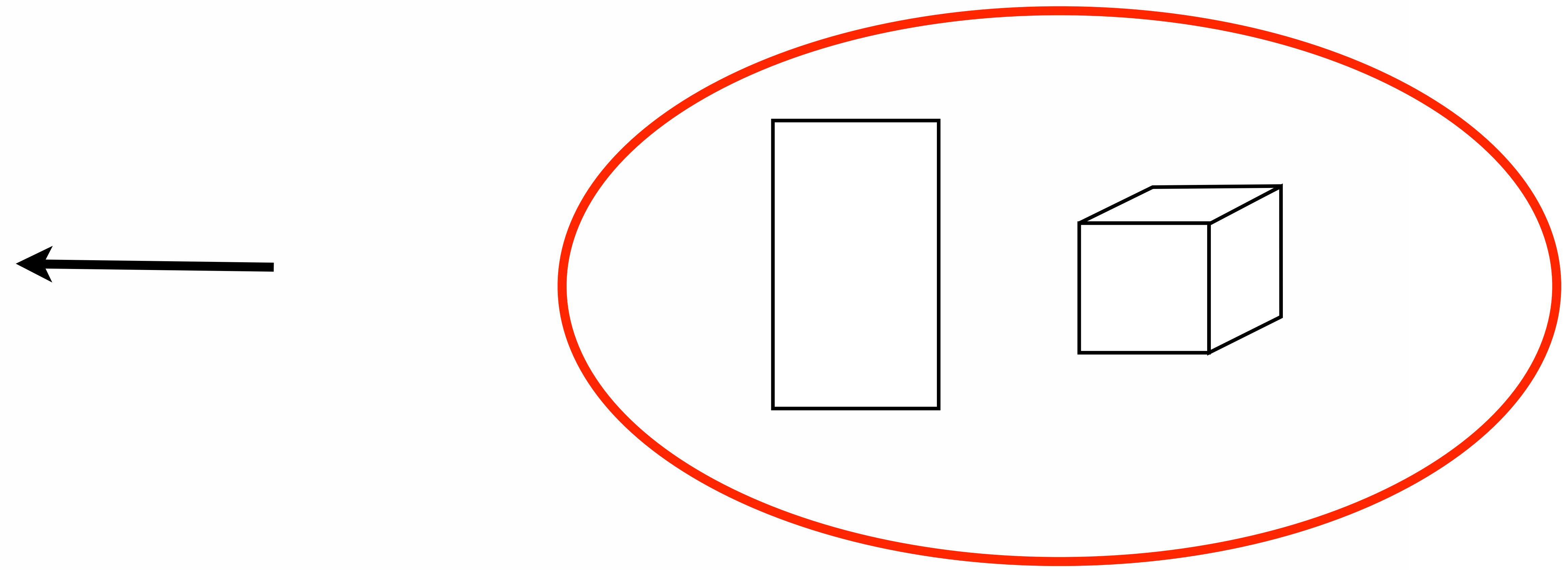


Motivation

- **How about in this way**

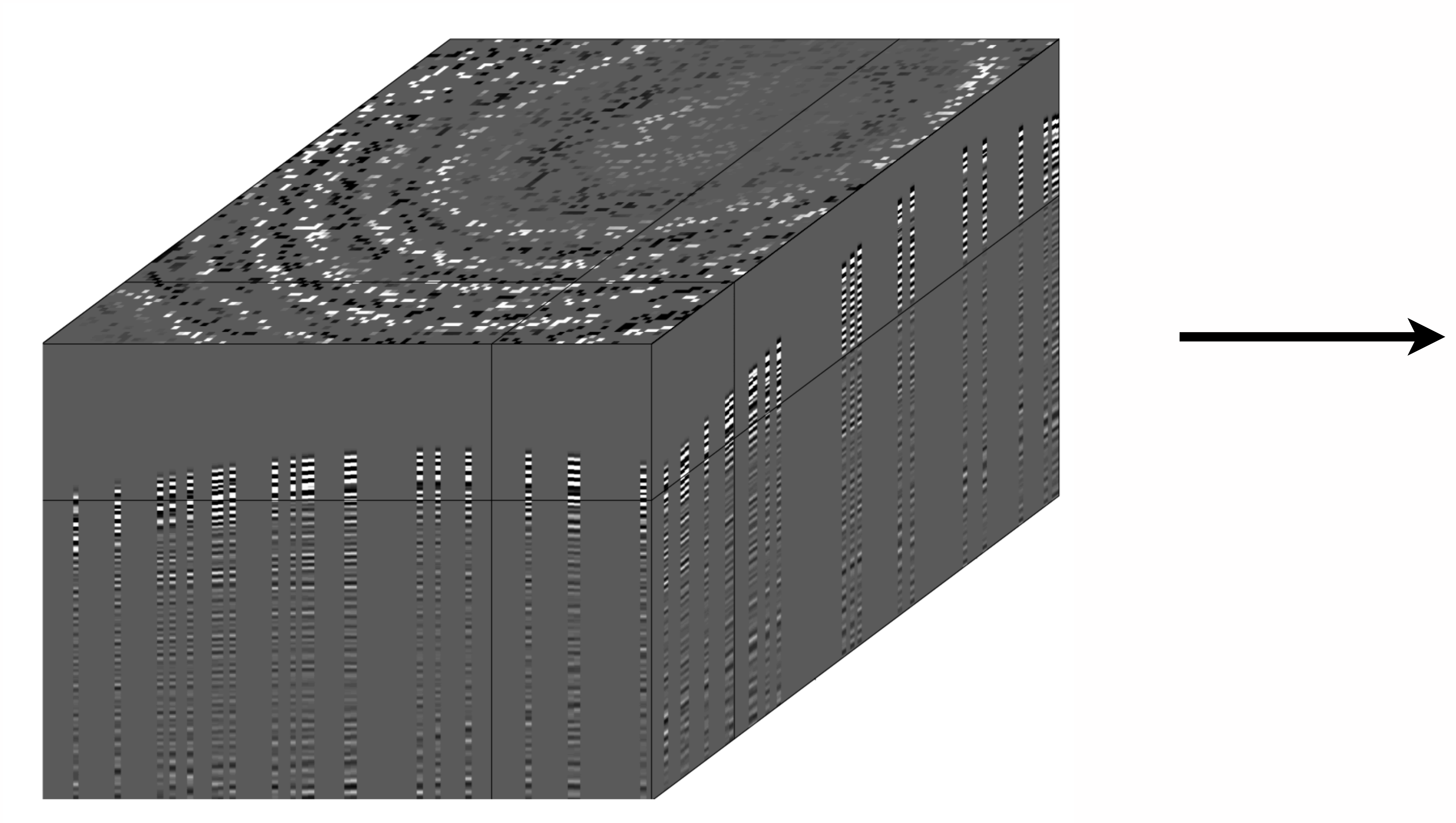
~1GB





Motivation

- **Missing data scenarios**



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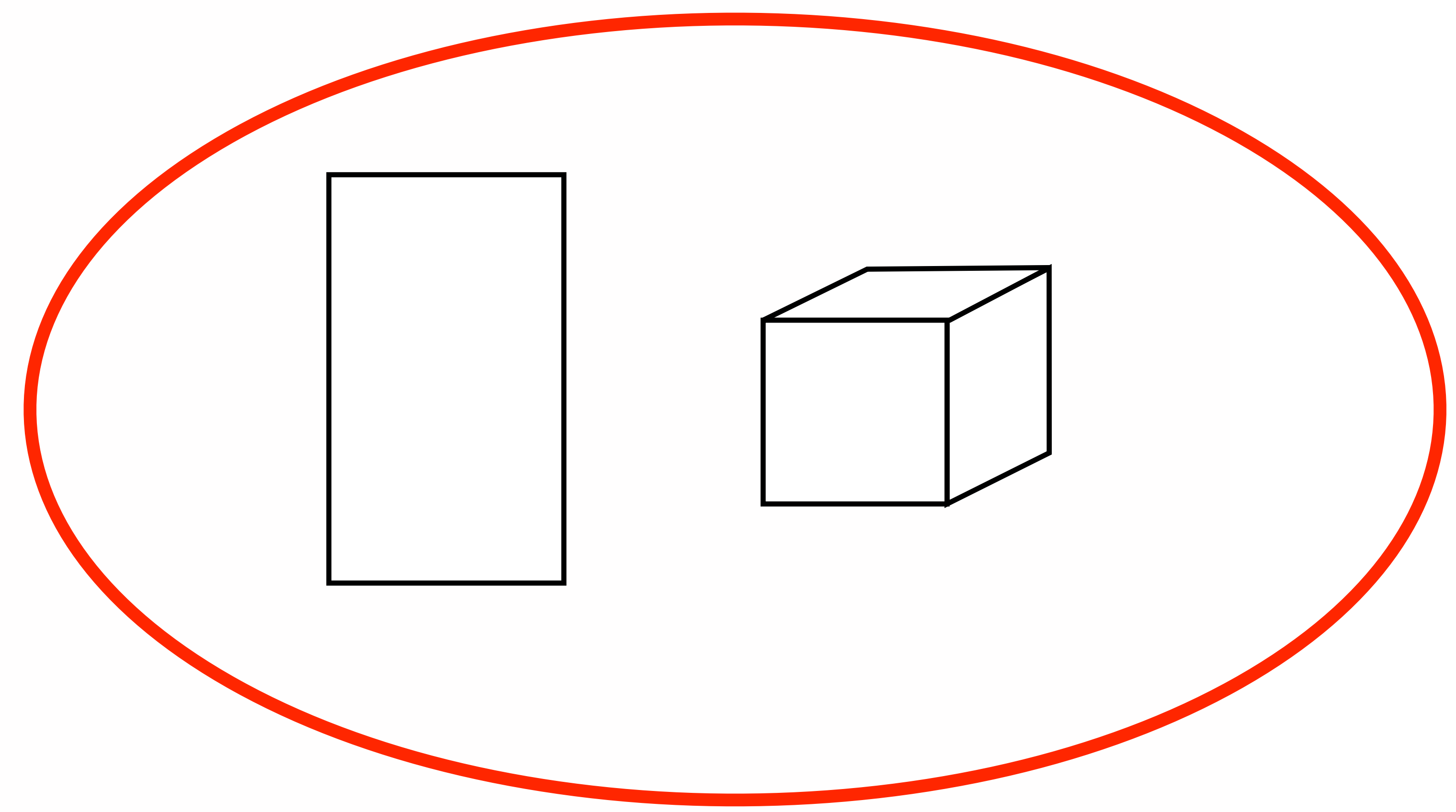
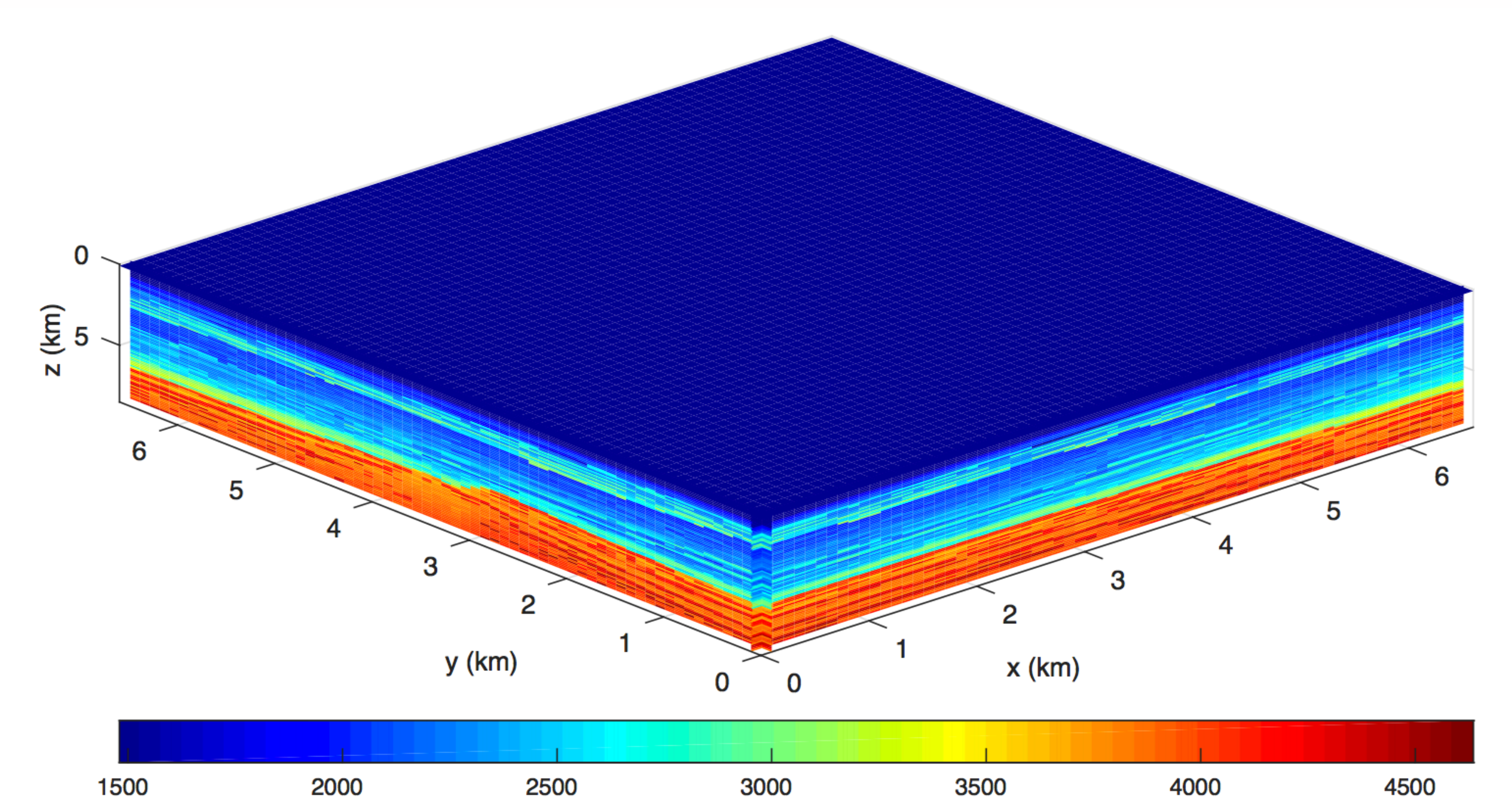
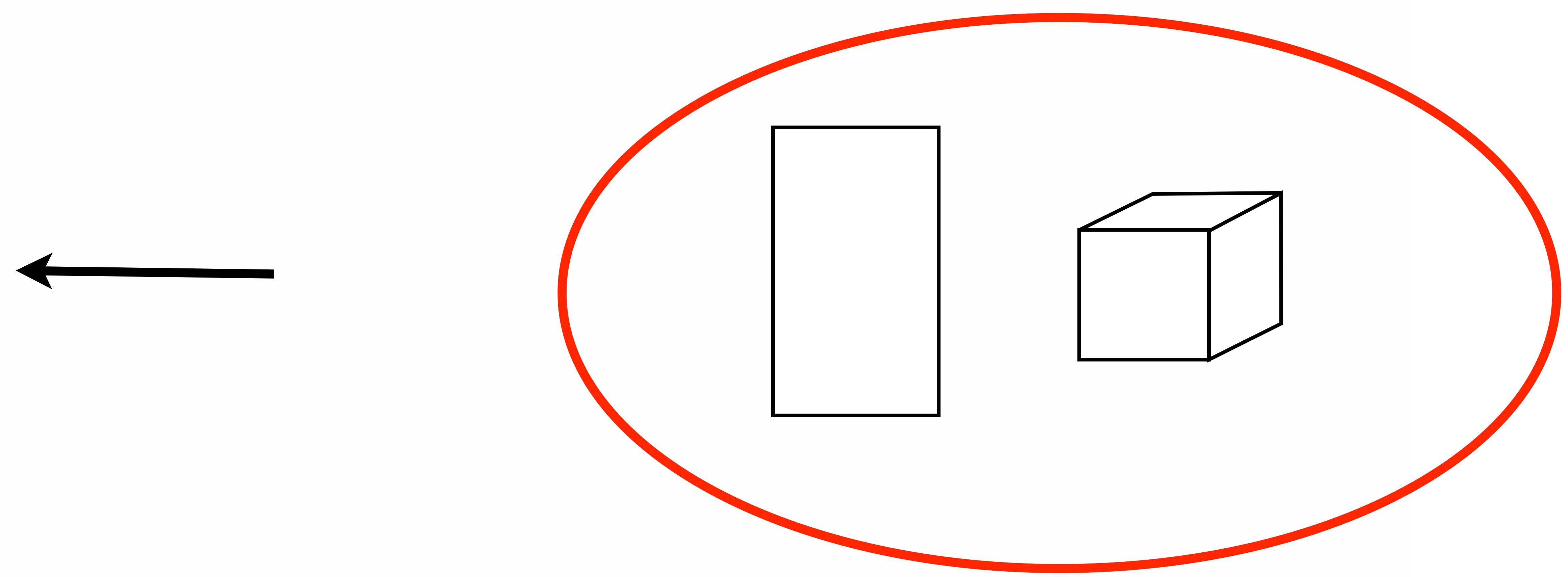


Motivation

- **can still in this way?**

~1GB





Motivation

How to fight "curse of dimensionality" ?

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Situation:

- seismic data is redundant
- low-rank format can be exploited at low frequencies

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How to fight “curse of dimensionality” ?

Situation:

- seismic data is redundant
- low-rank format can be exploited at low frequencies

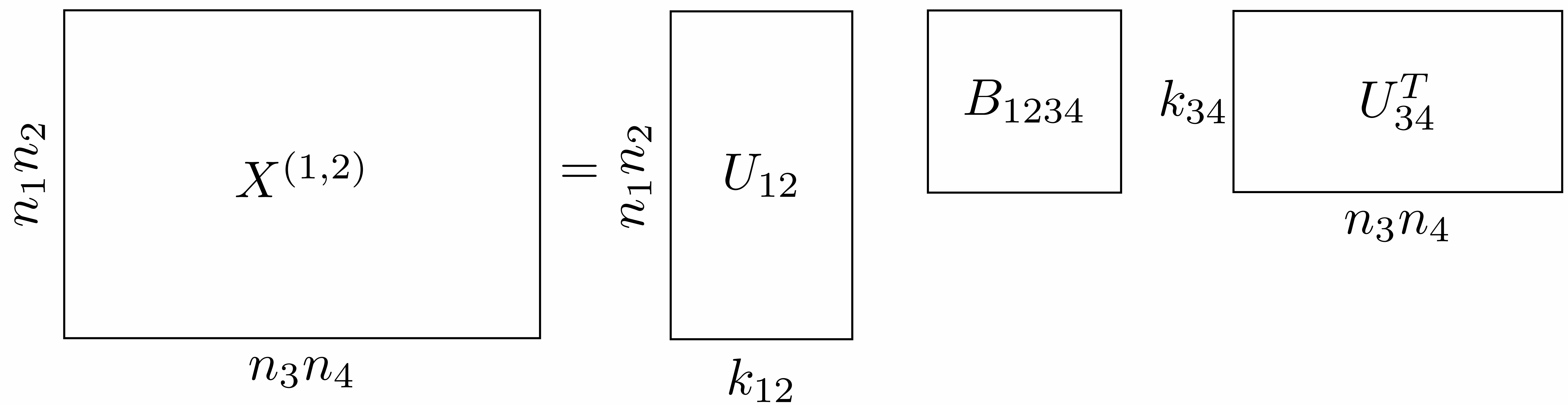
Solution:

- represent in hierarchical Tucker (HT) format
- interpolate HT format when missing data
- work w/ full data volume w/o forming them for later downstream processes, e.g. FWI

Da Silva, C., and F. J. Herrmann, 2015, Optimization on the hierarchical tucker manifold–applications to tensor completion: Linear Algebra and its Applications, 481, 131–173.

Hierarchical Tucker representation

$$X = n_1 \times n_2 \times n_3 \times n_4 \text{ tensor}$$



"SVD-like" factorization

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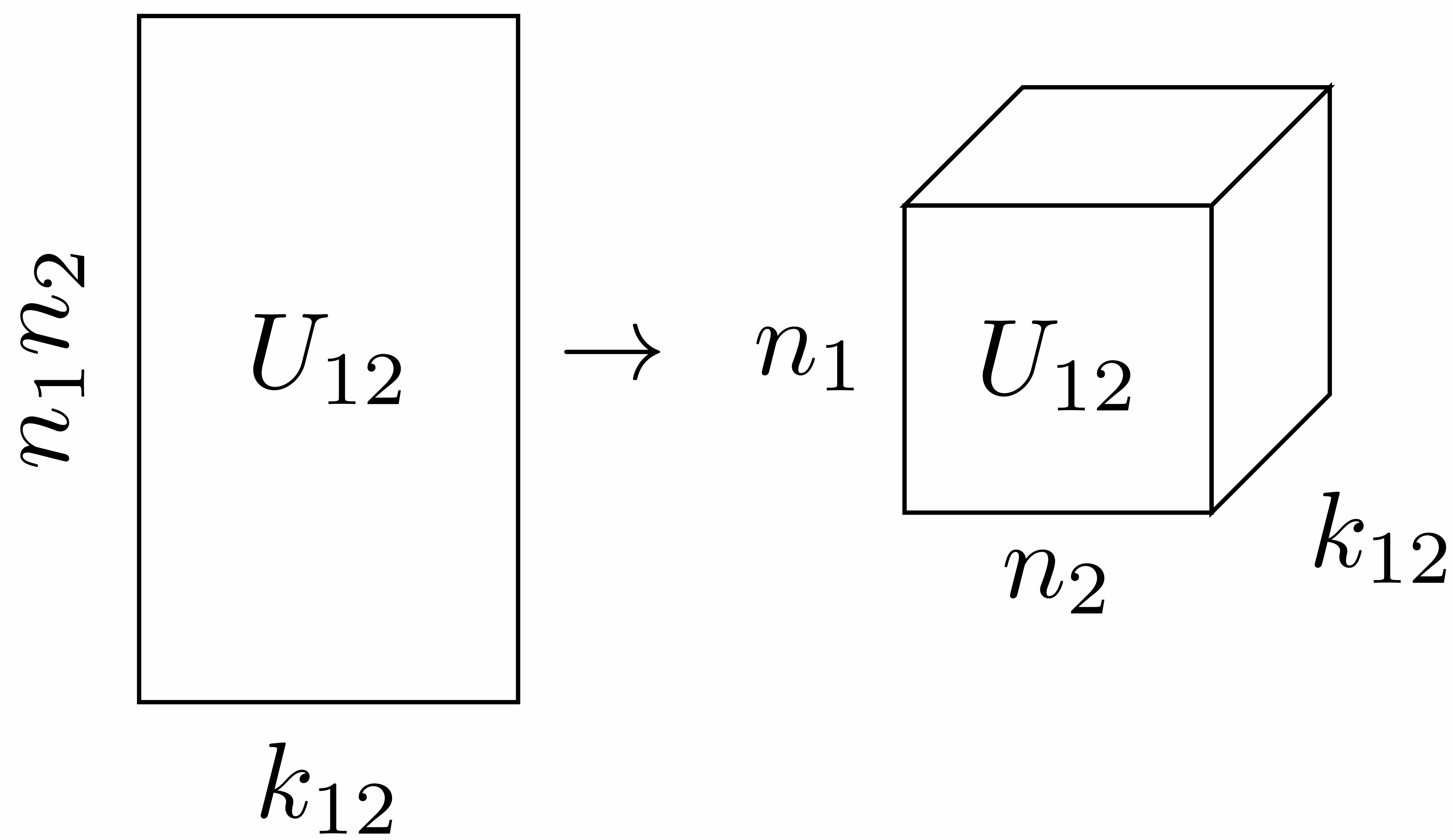

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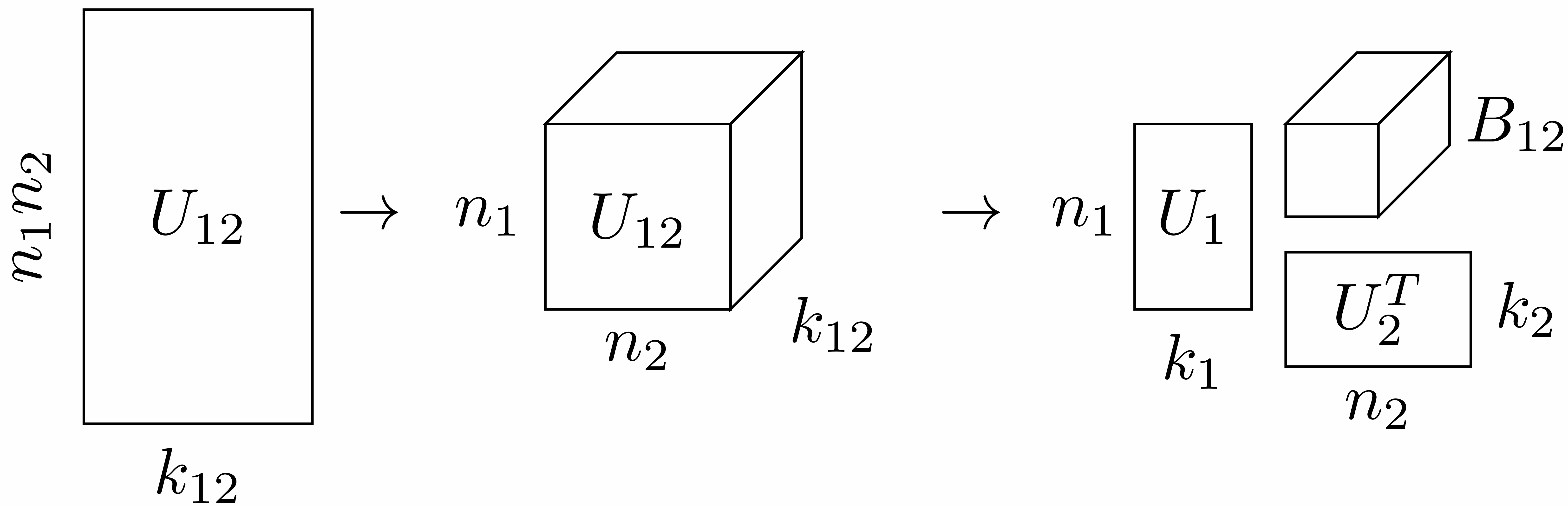


Hierarchical Tucker representation

This format is extremely storage-efficient

- not necessarily store **intermediate** matrices U_{12} and U_{34}
- storage $\leq dNk + (d-2)k^3 + k^2$
- compare to N^d parameters needed to store for the full data
- **computationally tractable** for high-dimensional problem

  $(k << N)$

Hierarchical Tucker representation

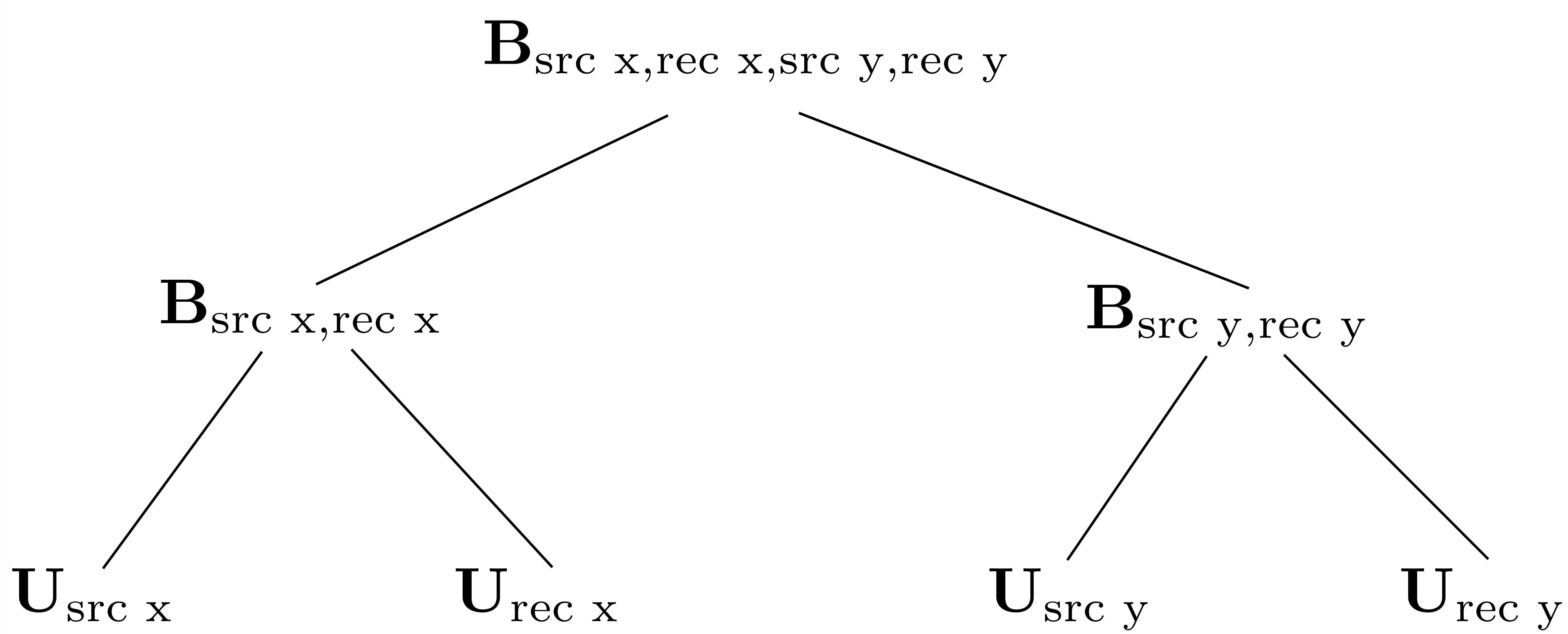
- a $100 \times 100 \times 100 \times 100$ tensor, max HT rank 20
- full storage: $100^4 = 10^8$ parameters
- HT storage: 24400 values
- compression ratio: 99.97%

Kumar, R., Da Silva, C., Akalin, O., Aravkin, A. Y., Mansour, H., Recht, B., & Herrmann, F. J. (2015). Efficient matrix completion for seismic data reconstruction. Geophysics, 80(5), V97-V114.

Da Silva, C., and F. J. Herrmann, 2015, Optimization on the hierarchical tucker manifold–applications to tensor completion: Linear Algebra and its Applications, 481, 131–173.

Seismic hierarchical Tucker

Given a frequency slice with coordinate (src x, src y, rec x, rec y), we introduce the **non-canonical dimension tree** for seismic data.



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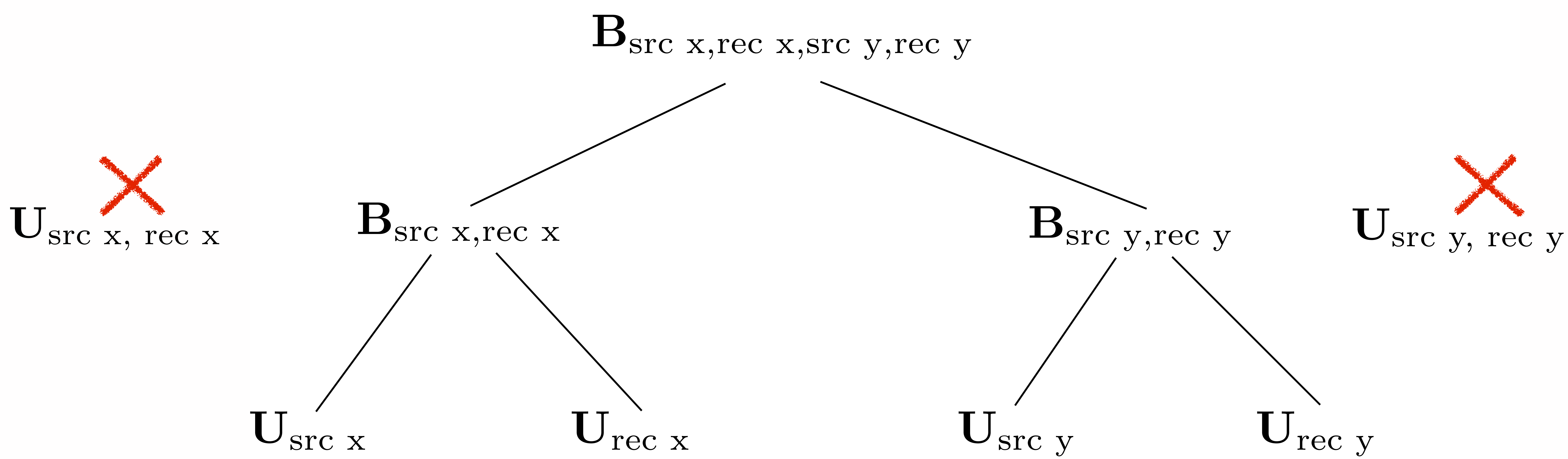

Non-canonical vs. canonical – 396 x 396 x 50 x 50 volume (~5.8 GB)

	Frequency (Hz)	Parameter Size	SNR	Compression Ratio
Non-canonical	3	71MB	42.8	98.8%
canonical	3	501MB	42.9	91.6%
Non-canonical	6	421MB	43.0	92.9%
canonical	6	1194MB	43.1	79.9%

Seismic hierarchical Tucker

We can compress low-frequency seismic data in HT in either case listed below

- **full data**

Seismic hierarchical Tucker

We can compress low-frequency seismic data in HT in either case listed below

- **full data** $\longrightarrow$ HT **truncation** algorithm detail in *Tobler, 2012*

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Seismic hierarchical Tucker

We can compress low-frequency seismic data in HT in either case listed below

- **full data** $\longrightarrow$ HT **truncation** algorithm detail in *Tobler, 2012*
- **missing data**

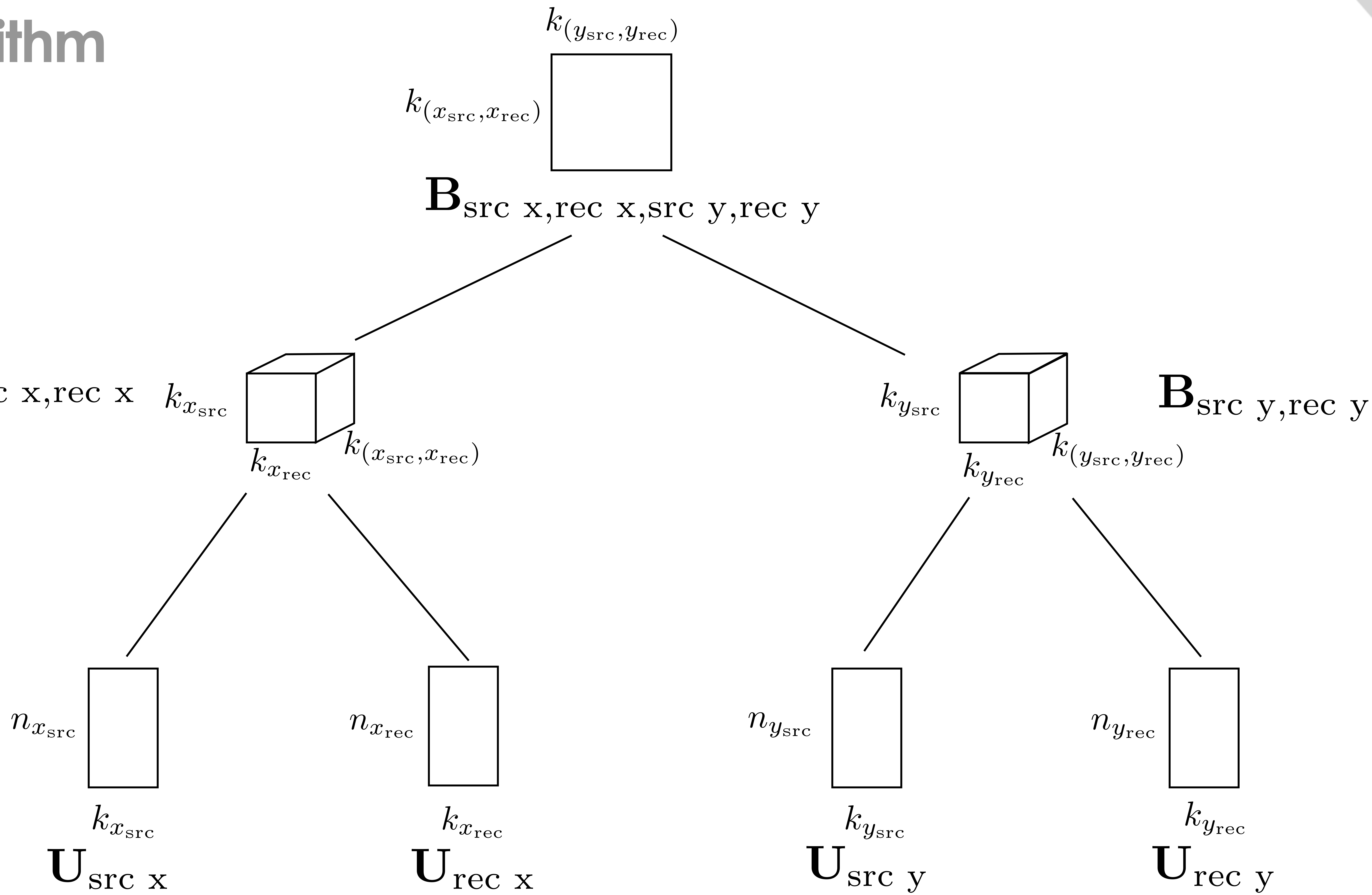
Seismic hierarchical Tucker

We can compress low-frequency seismic data in HT in either case listed below

- **full data** ⟶ HT **truncation** algorithm detail in *Tobler, 2012*
- **missing data** ⟶ **interpolate** HT format described in *Da Silva & Herrmann, 2015*

On-the-fly extraction of shots/receivers

Algorithm



$k_{(y_{\mathrm{src}},y_{\mathrm{rec}})}$
$k_{(x_{\mathrm{src}},x_{\mathrm{rec}})}$
$\mathbf{B}_{\text{src x,rec x,src y,rec y}}$
$\mathbf{B}_{\text{src x,rec x}}$
$k_{x_{\mathrm{src}}}$
$k_{(x_{\mathrm{src}},x_{\mathrm{rec}})}$
$k_{x_{\mathrm{rec}}}$
$k_{y_{\mathrm{src}}}$
$\mathbf{B}_{\text{src y,rec y}}$
$k_{(y_{\mathrm{src}},y_{\mathrm{rec}})}$
$k_{y_{\mathrm{rec}}}$
$n_{x_{\mathrm{src}}}$
$k_{x_{\mathrm{src}}}$
$\mathbf{U}_{\text{src x}}$
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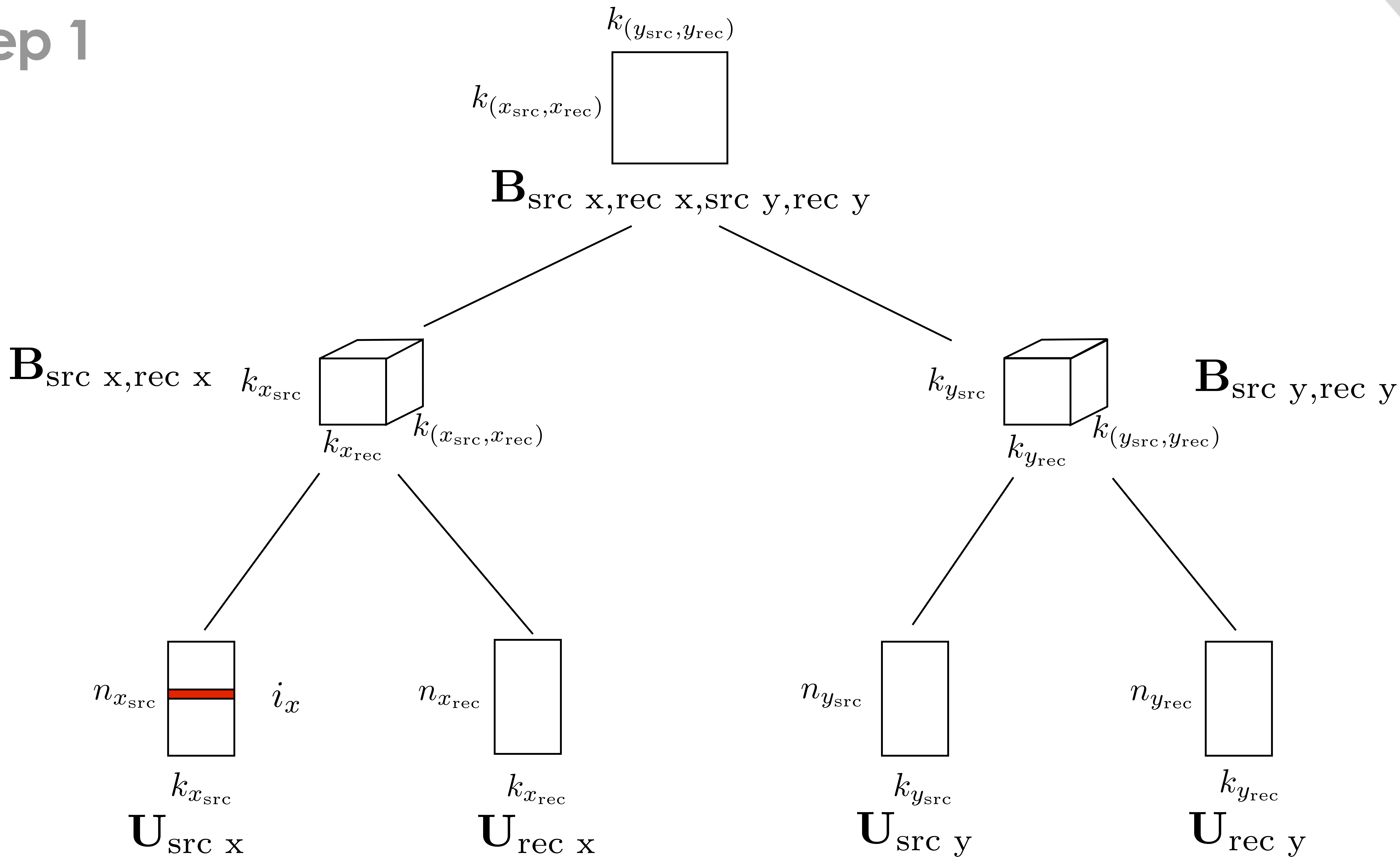
$k_{x_{\mathrm{rec}}}$
$\mathbf{U}_{\text{rec x}}$
$n_{y_{\mathrm{src}}}$
$k_{y_{\mathrm{src}}}$
$\mathbf{U}_{\text{src y}}$
$n_{y_{\mathrm{rec}}}$
$k_{y_{\mathrm{rec}}}$
$\mathbf{U}_{\text{rec y}}$

Step 1



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$k_{(x_{\text{src}}, x_{\text{rec}})}$
$\mathbf{B}_{\text{src x,rec x,src y,rec y}}$
$\mathbf{B}_{\text{src x,rec x}}$
$k_{x_{\text{src}}}$
$k_{x_{\text{rec}}}$
$k_{(x_{\text{src}}, x_{\text{rec}})}$
$k_{y_{\text{src}}}$
$\mathbf{B}_{\text{src y,rec y}}$
$k_{y_{\text{rec}}}$
$k_{(y_{\text{src}}, y_{\text{rec}})}$
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$k_{x_{\text{rec}}}$
$\mathbf{U}_{\text{rec x}}$
$k_{y_{\text{src}}}$
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$\mathbf{U}_{\text{rec y}}$

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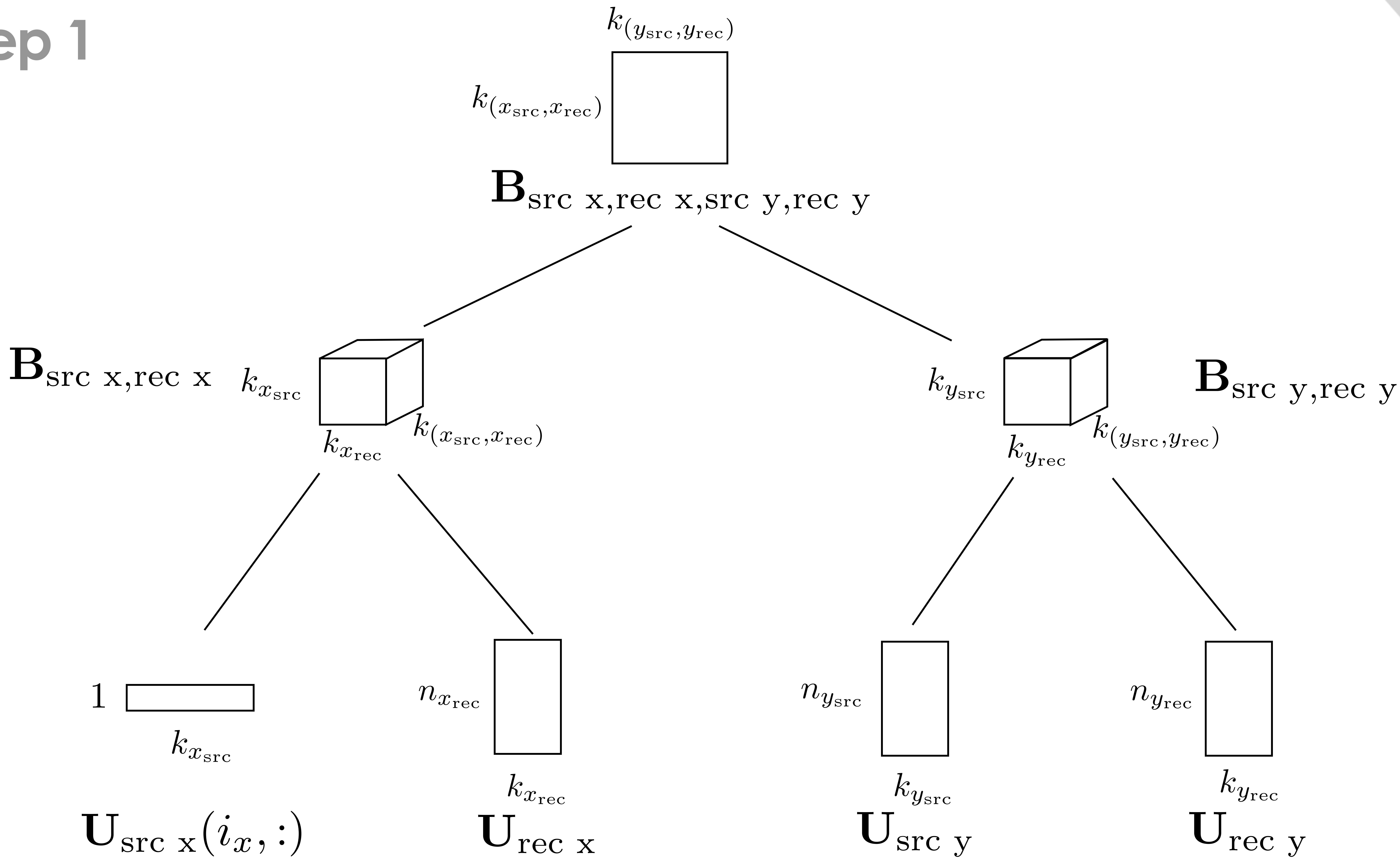
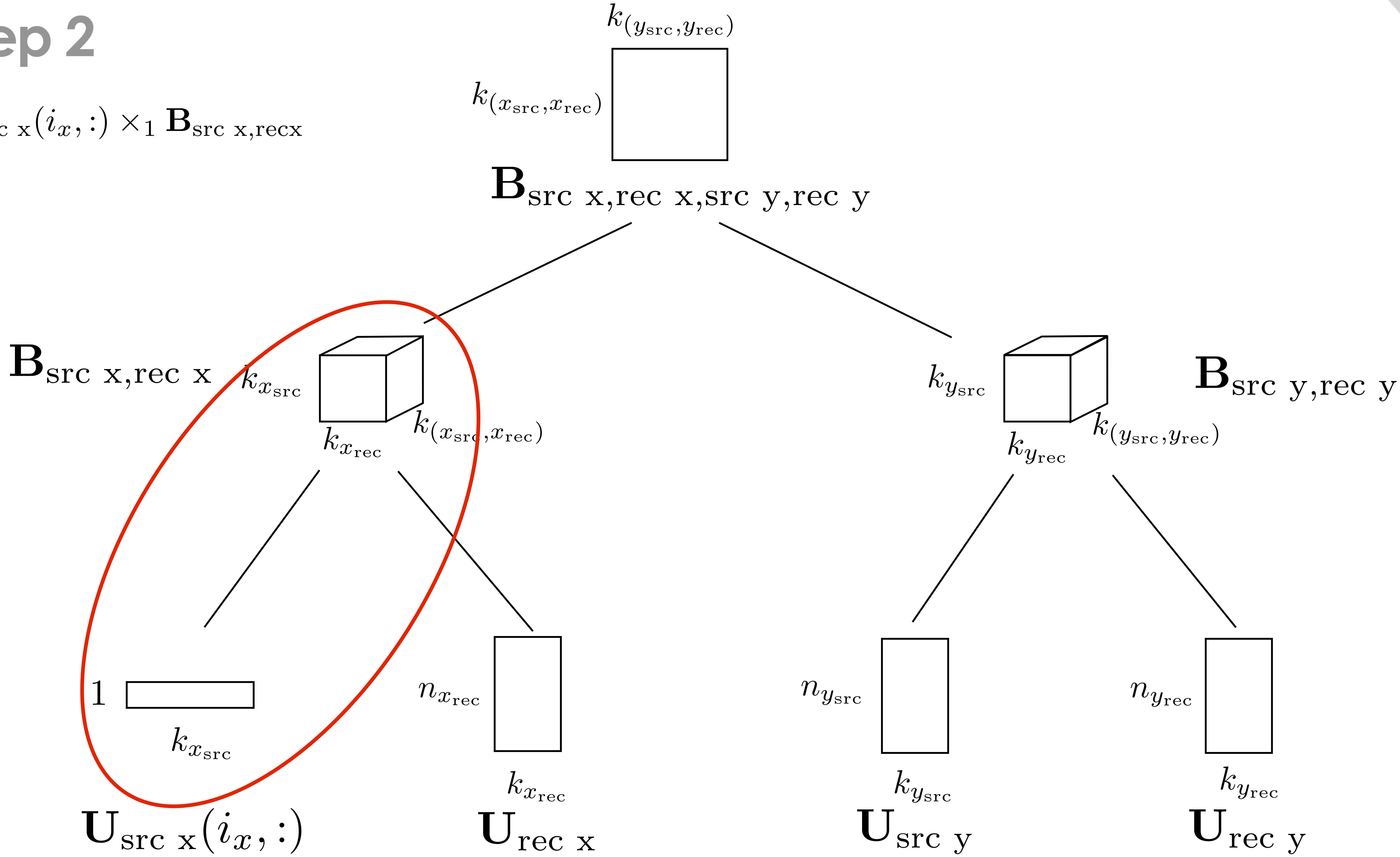


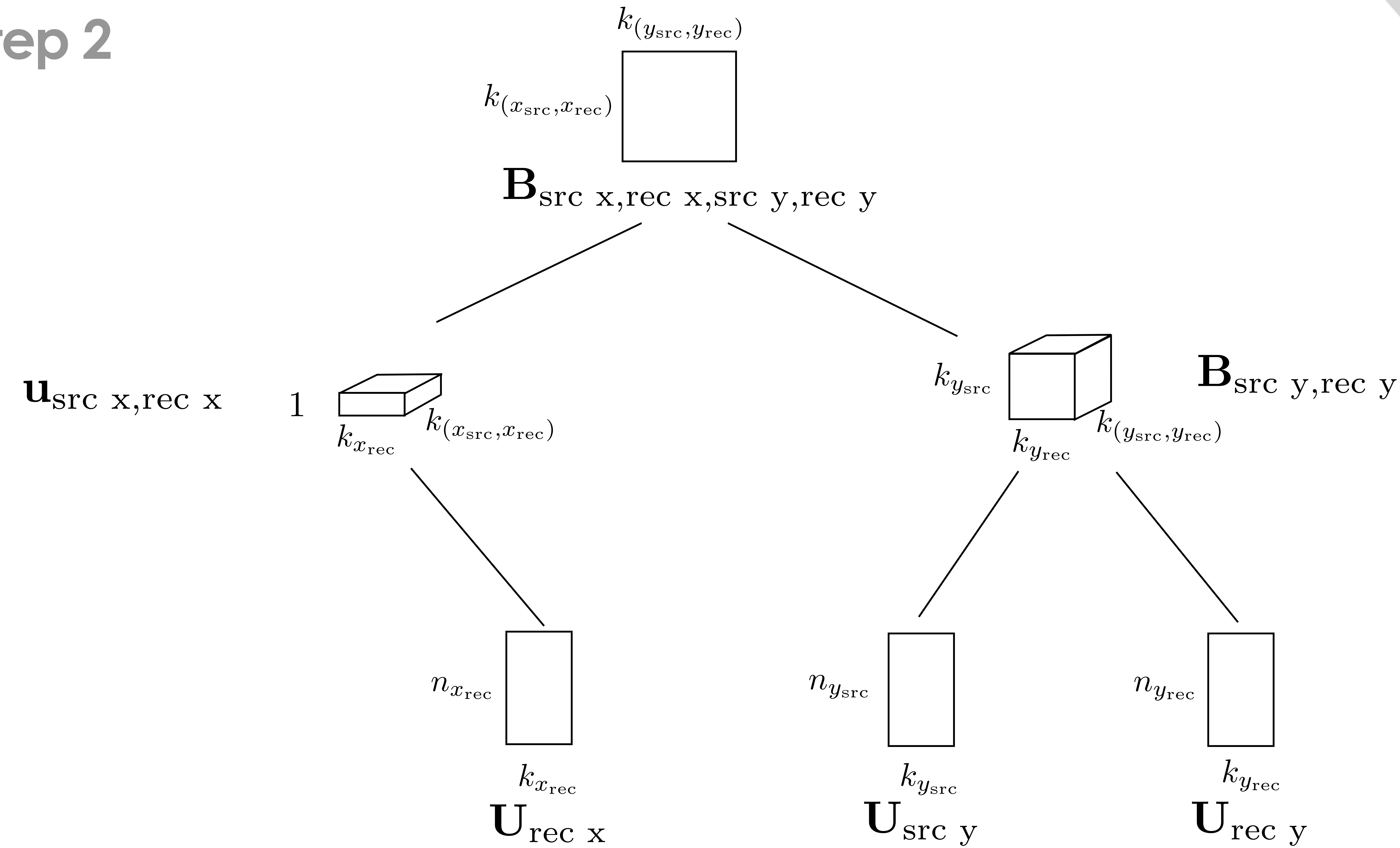
$k_{(y_{\mathrm{src}}, y_{\mathrm{rec}})}$
$k_{(x_{\mathrm{src}}, x_{\mathrm{rec}})}$
$\mathbf{B}_{\text{src x,rec x,src y,rec y}}$
$\mathbf{B}_{\text{src x,rec x}}$
$k_{x_{\mathrm{src}}}$
$k_{x_{\mathrm{rec}}}$
$k_{(x_{\mathrm{src}}, x_{\mathrm{rec}})}$
$k_{y_{\mathrm{src}}}$
$\mathbf{B}_{\text{src y,rec y}}$
$k_{y_{\mathrm{rec}}}$
$k_{(y_{\mathrm{src}}, y_{\mathrm{rec}})}$
1
$k_{x_{\mathrm{src}}}$
$n_{x_{\mathrm{rec}}}$
$k_{x_{\mathrm{rec}}}$
$n_{y_{\mathrm{src}}}$
$k_{y_{\mathrm{src}}}$
$n_{y_{\mathrm{rec}}}$
$k_{y_{\mathrm{rec}}}$
$\mathbf{U}_{\text{src x}}(i_x, :)$
$\mathbf{U}_{\text{rec x}}$
$\mathbf{U}_{\text{src y}}$
$\mathbf{U}_{\text{rec y}}$

Step 2

$\mathbf{U}_{\text{src x}}(i_x,:) \times_1 \mathbf{B}_{\text{src x,recx}}$



Step 2



$k_{(y_{\mathrm{src}},y_{\mathrm{rec}})}$
$k_{(x_{\mathrm{src}},x_{\mathrm{rec}})}$
$\mathbf{B}_{\text{src x,rec x,src y,rec y}}$
$\mathbf{u}_{\text{src x,rec x}}$
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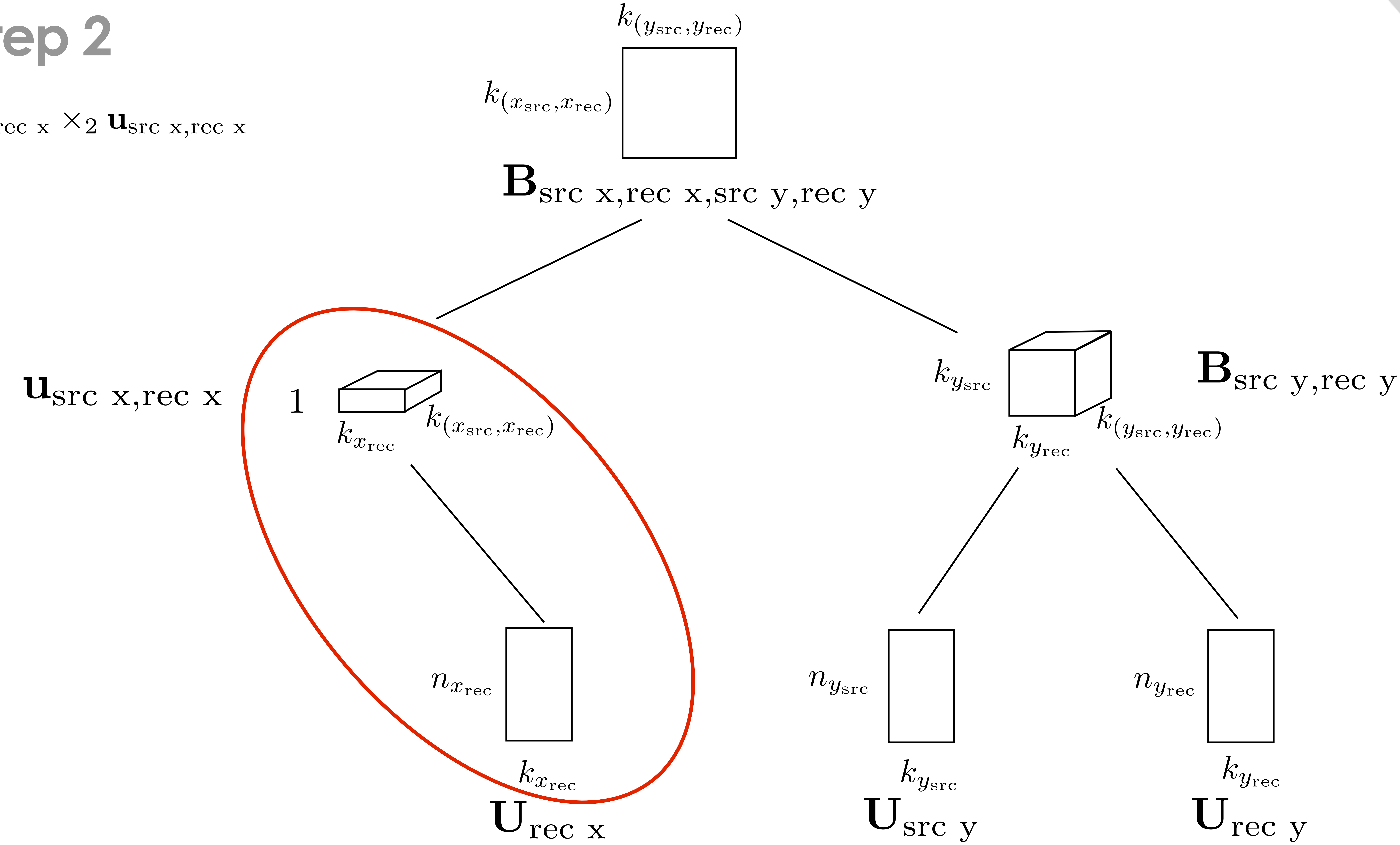
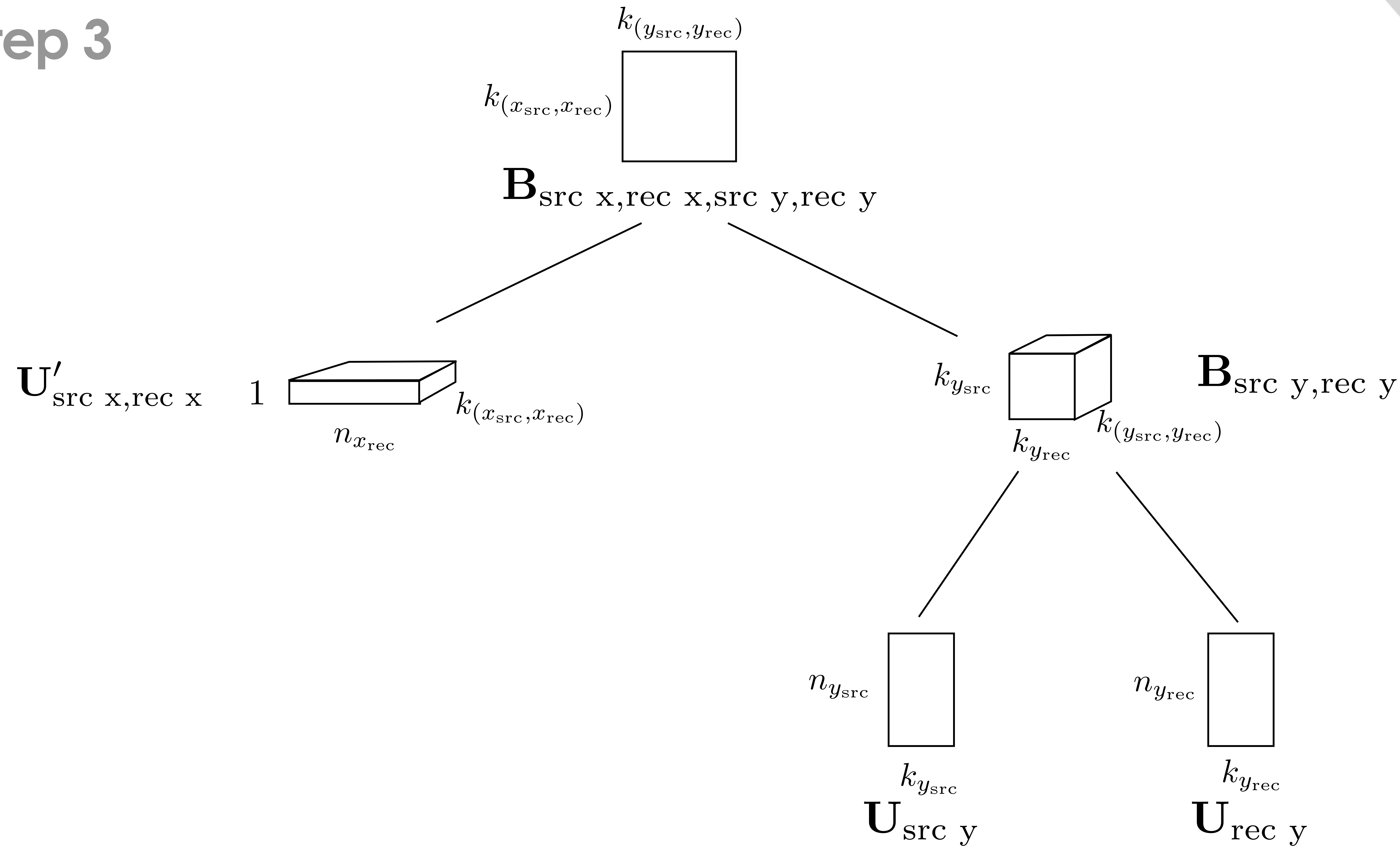
$k_{x_{\mathrm{rec}}}$
$k_{(x_{\mathrm{src}},x_{\mathrm{rec}})}$
$k_{y_{\mathrm{src}}}$
$k_{y_{\mathrm{rec}}}$
$k_{(y_{\mathrm{src}},y_{\mathrm{rec}})}$
$\mathbf{B}_{\text{src y,rec y}}$
$n_{x_{\mathrm{rec}}}$
$k_{x_{\mathrm{rec}}}$
$\mathbf{U}_{\text{rec x}}$
$n_{y_{\mathrm{src}}}$
$k_{y_{\mathrm{src}}}$
$\mathbf{U}_{\text{src y}}$
$n_{y_{\mathrm{rec}}}$
$k_{y_{\mathrm{rec}}}$
$\mathbf{U}_{\text{rec y}}$

Step 2

$\mathbf{U}_{\text{rec x}} \times_2 \mathbf{u}_{\text{src x,rec x}}$



Step 3



$k_{(y_{\text{src}},y_{\text{rec}})}$
$k_{(x_{\text{src}},x_{\text{rec}})}$
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$\mathbf{U}'_{\text{src x,rec x}}$
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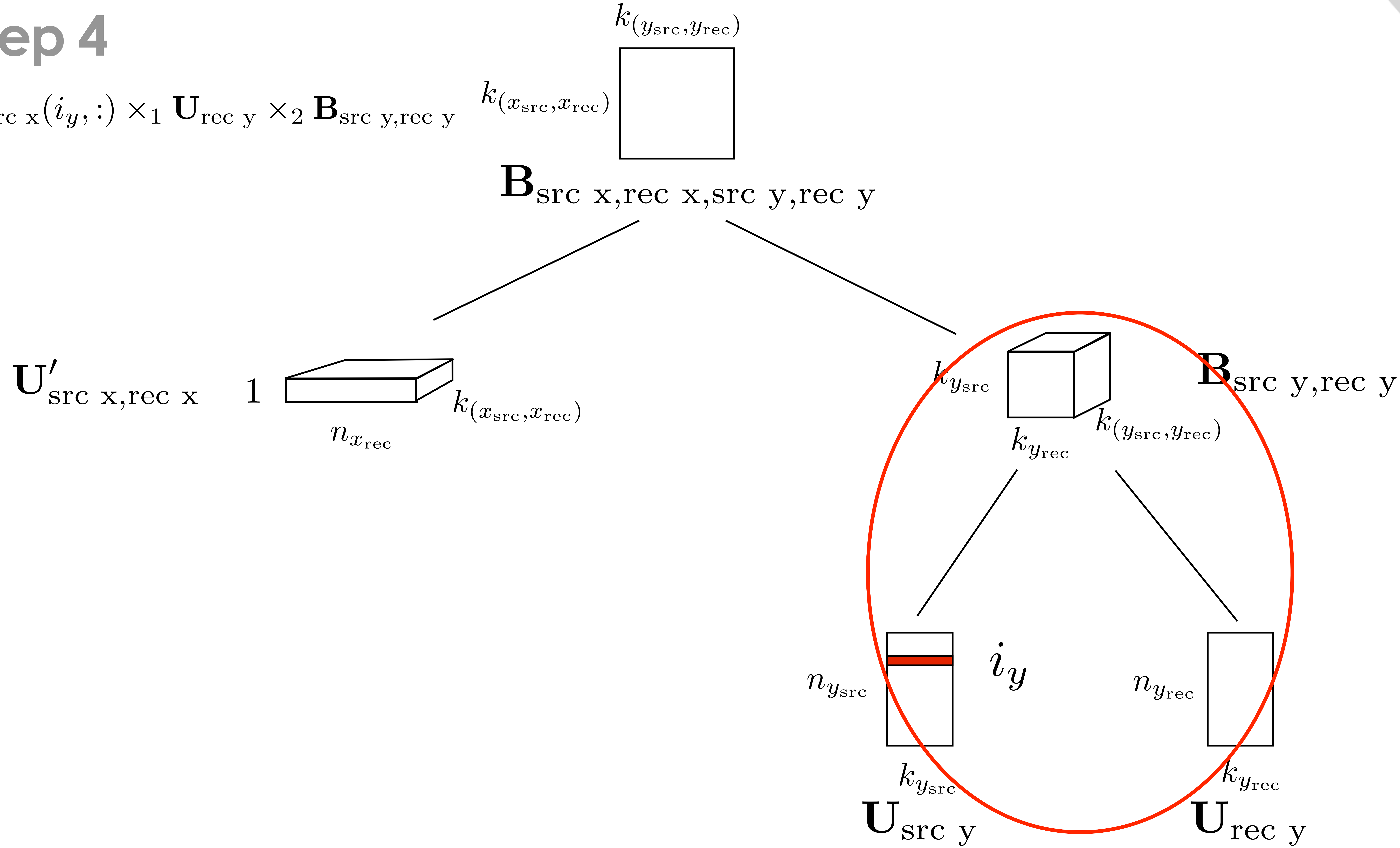
$n_{x_{\text{rec}}}$
$k_{y_{\text{src}}}$
$\mathbf{B}_{\text{src y,rec y}}$
$k_{(y_{\text{src}},y_{\text{rec}})}$
$k_{y_{\text{rec}}}$
$n_{y_{\text{src}}}$
$n_{y_{\text{rec}}}$
$k_{y_{\text{src}}}$
$k_{y_{\text{rec}}}$
$\mathbf{U}_{\text{src y}}$
$\mathbf{U}_{\text{rec y}}$

Step 4

$\mathbf{U}_{\text{src x}}(i_y,:) \times_1 \mathbf{U}_{\text{rec y}} \times_2 \mathbf{B}_{\text{src y,rec y}}$



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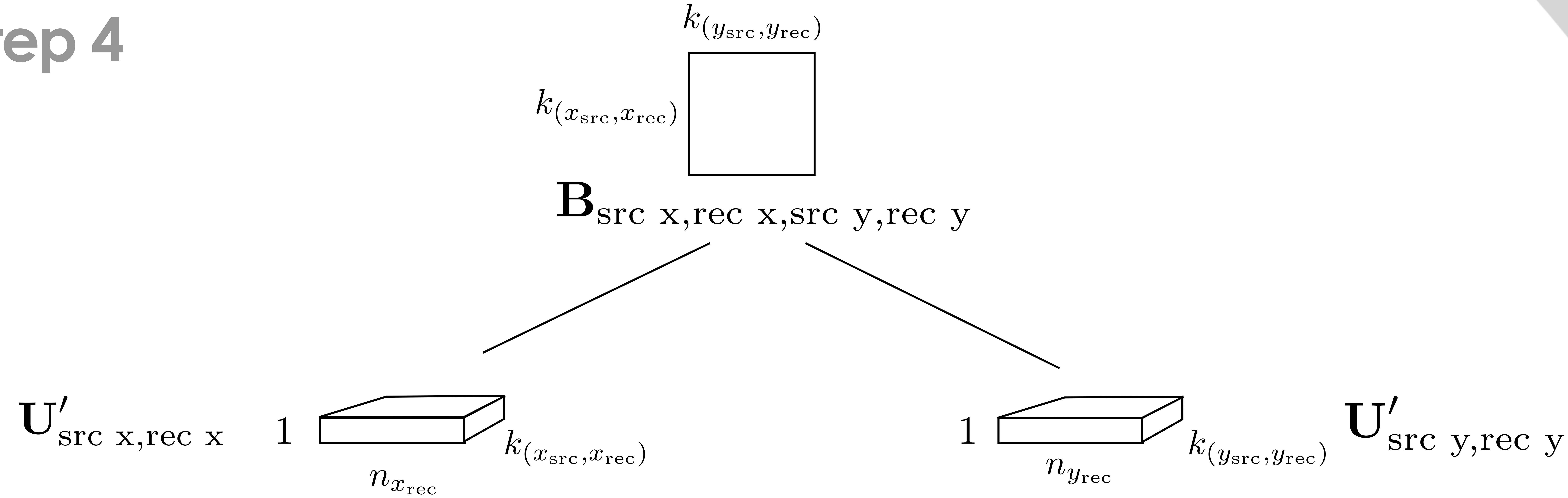
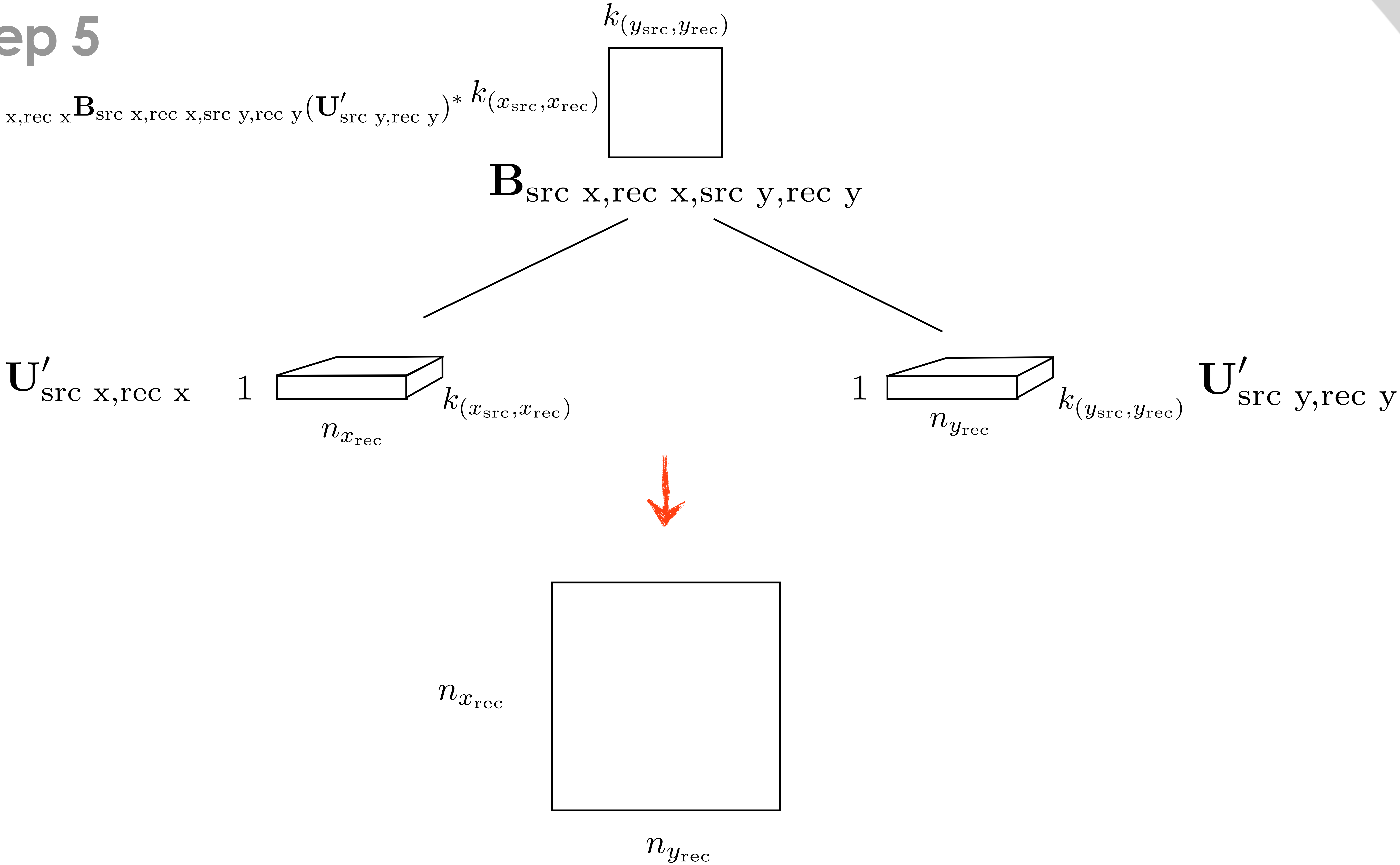


$k_{(y_{\text{src}},y_{\text{rec}})}$
$k_{(x_{\text{src}},x_{\text{rec}})}$
$\mathbf{B}_{\text{src x,rec x,src y,rec y}}$
$\mathbf{U}'_{\text{src x,rec x}}$
1
$k_{(x_{\text{src}},x_{\text{rec}})}$
$n_{x_{\text{rec}}}$
1
$n_{y_{\text{rec}}}$
$k_{(y_{\text{src}},y_{\text{rec}})}$
$\mathbf{U}'_{\text{src y,rec y}}$

Step 5

$\mathbf{U}'_{\text{src x,rec x}}\mathbf{B}_{\text{src x,rec x,src y,rec y}}(\mathbf{U}'_{\text{src y,rec y}})^*$



On-the-fly extraction of shots/receivers

Once our data in HT representation

- Kronecker product or matrix-matrix multiplication
- intermediate qualities **much smaller** than ambient dimensionality
- extract common **receiver gather** in an analogous way
- compute **simultaneous** shots/receivers gathers

Case study 1: 3D FWI

FWI examples

3D FWI with stochastic optimization algorithm

- a **subset** of full shots per iteration
- partially minimize least-square objective function
- **LBFGS** w/ bound constrains, i.e. **minimum & maximum** velocities allowed
- **single freq.** inverted at a time

FWI examples

For HT compressed data

- over **90%** reduced data volume in size
- cheaply store **compressed** form of the full data on every node
- automatically determine **indices** for shots per iteration
- **query-based** access to the data volume on-the-fly w/ our proposed algorithm

FWI examples

Computational environment:

- SENAI Yemoja cluster
- 50 nodes, 256GB RAM each, 20 CPU cores
- 8 Parallel Matlab workers per node
- modeling code, WAVEFORM (Da Silva and Herrmann, 2016)

FWI examples on Overthrust model

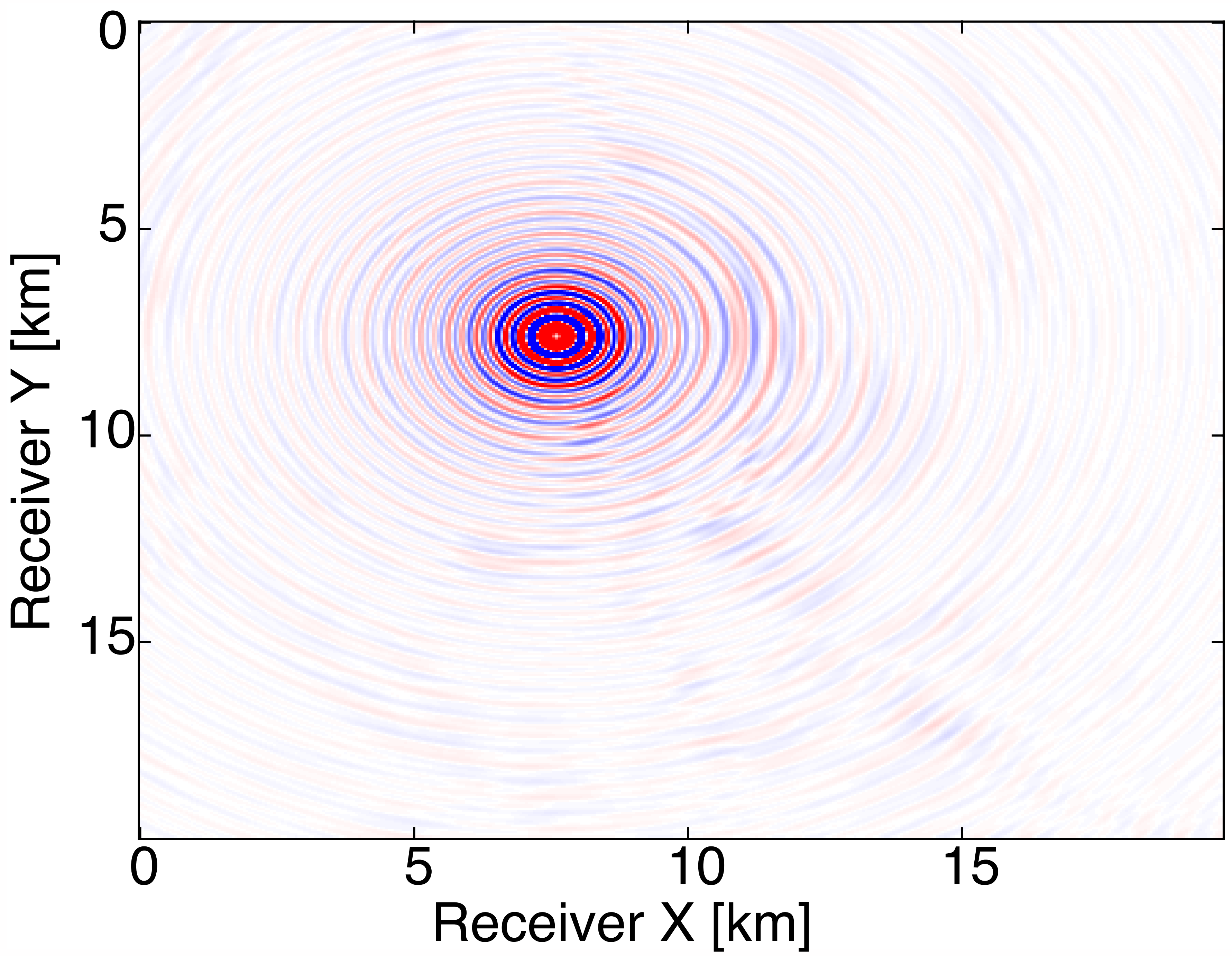
Model 1:

- 3D **Overthrust** model
- 20km x 20km x 4.6 km, adding 500m water
- 50m x 50m x 50 m spacing

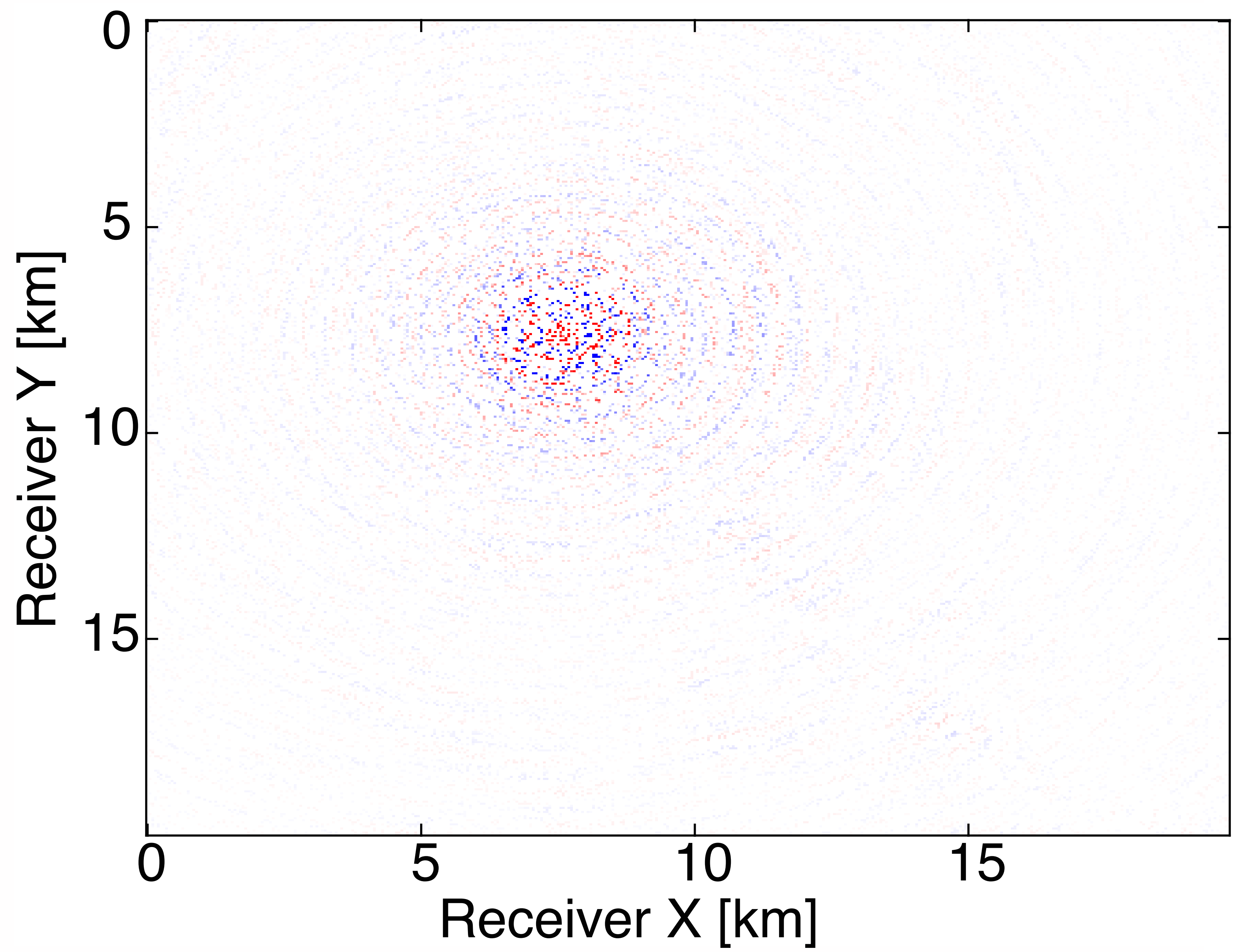
Data:

- 50 x 50 sources, 200m interval
- 396 x 396 receivers, 50m interval
- Ricker wavelet, 10Hz peak frequency
- 3Hz - 6Hz ranging, 1Hz interval
- **remove 80%** of random receivers

FWI examples on Overthrust model

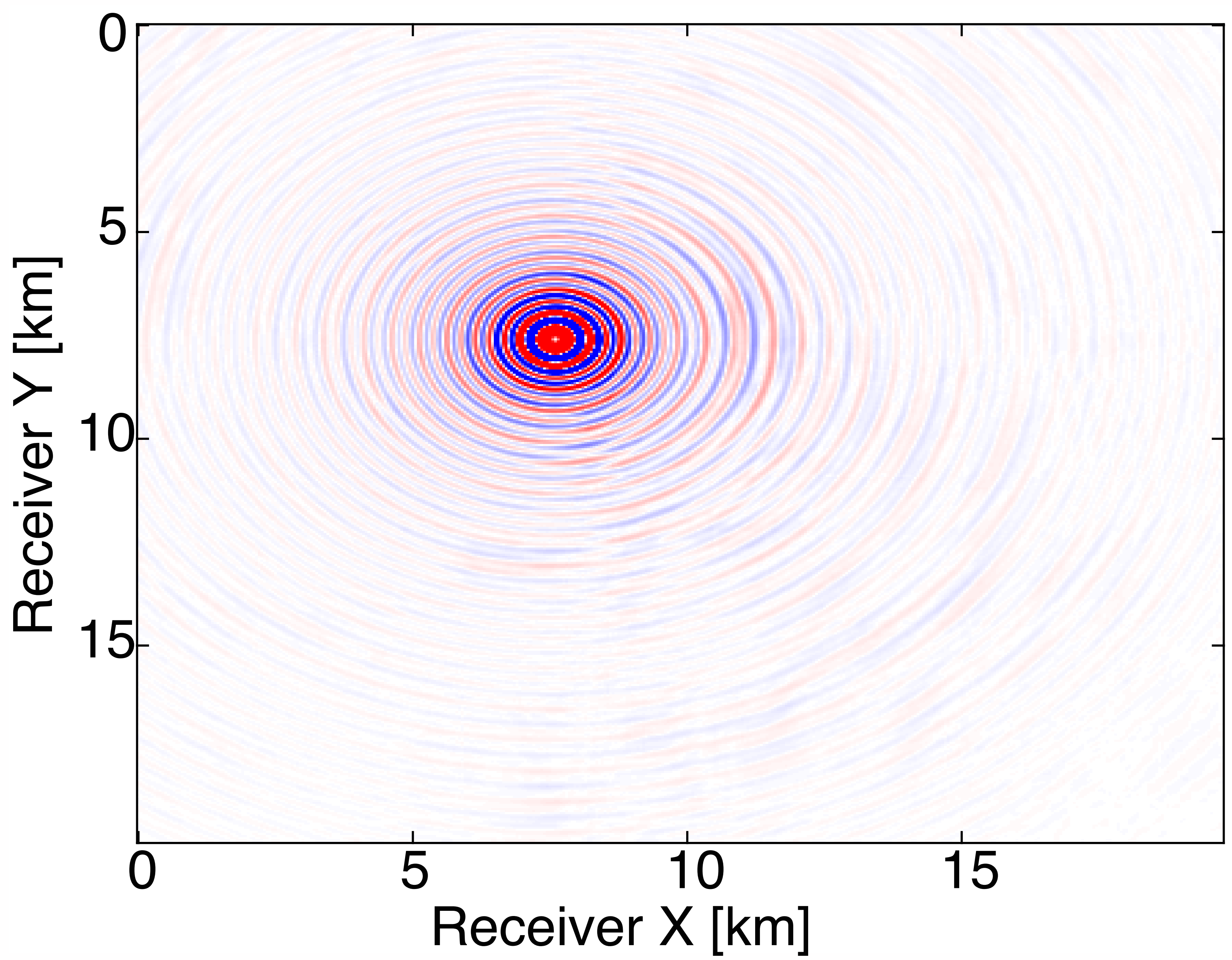


True data

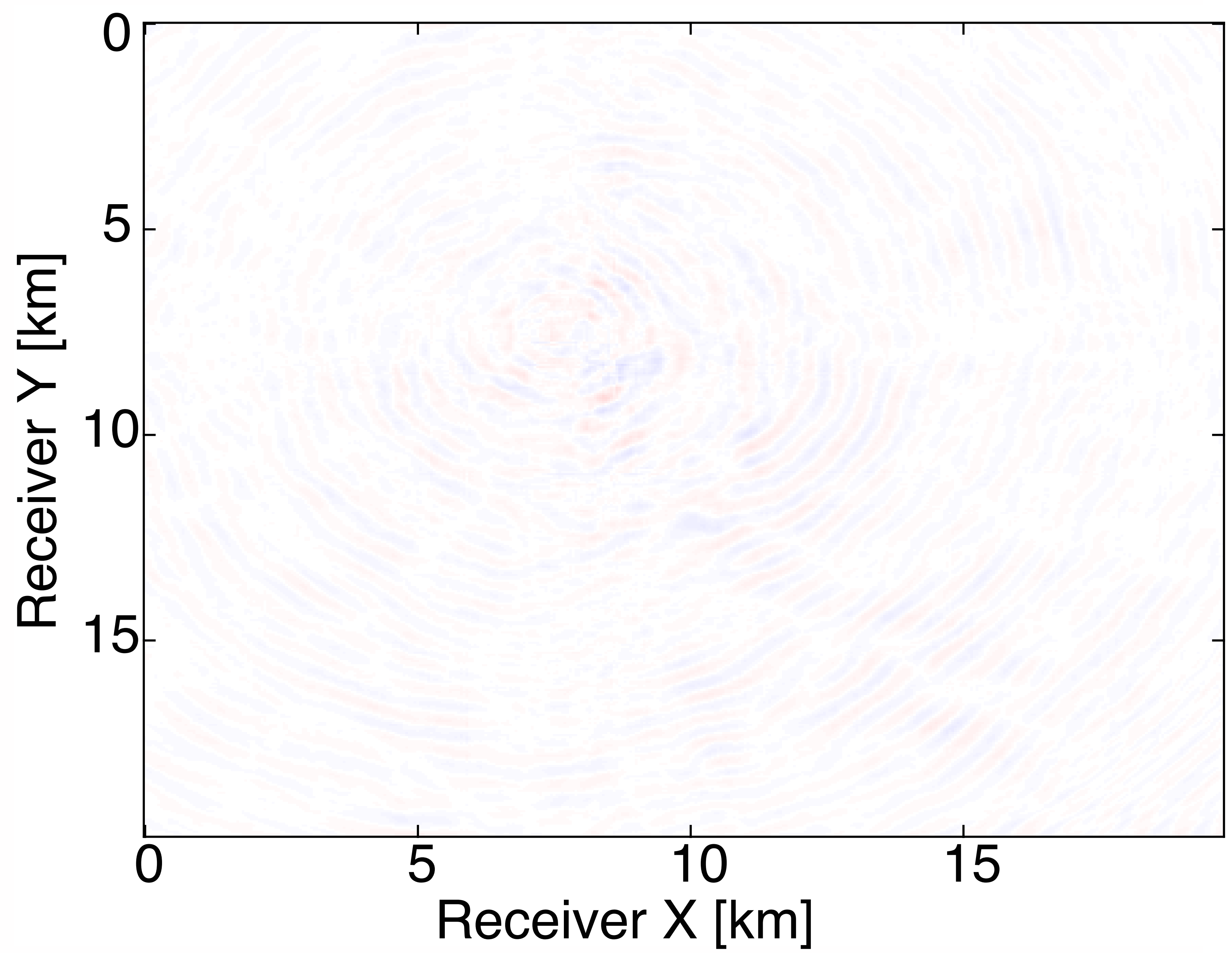


Missing 80% data

FWI examples on Overthrust model



Extract from compressed data



Residual

FWI examples on Overthrust model

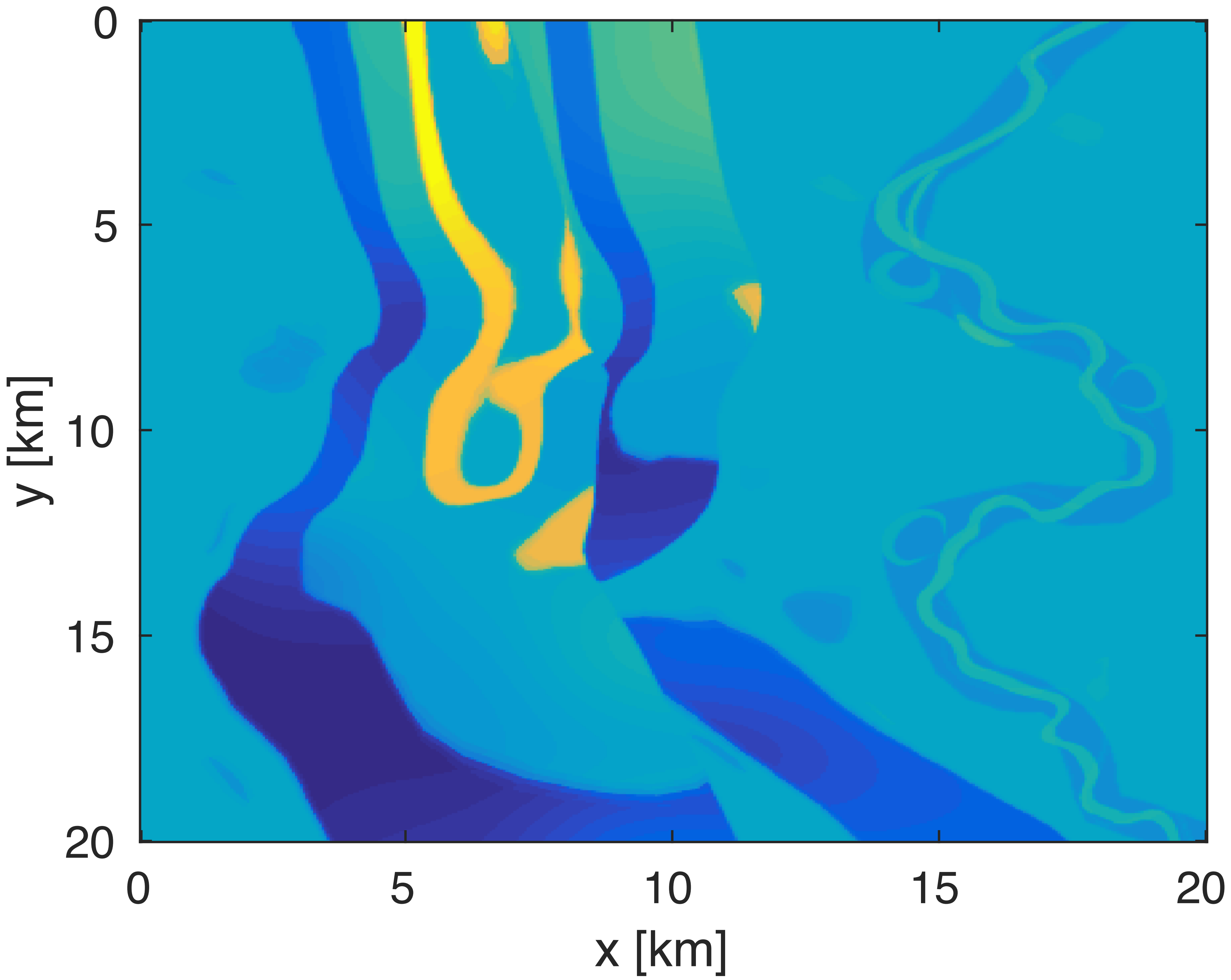
Stochastic FWI results inverted w/

- **full** data
- compressed **HT parameters** recovered from interpolation

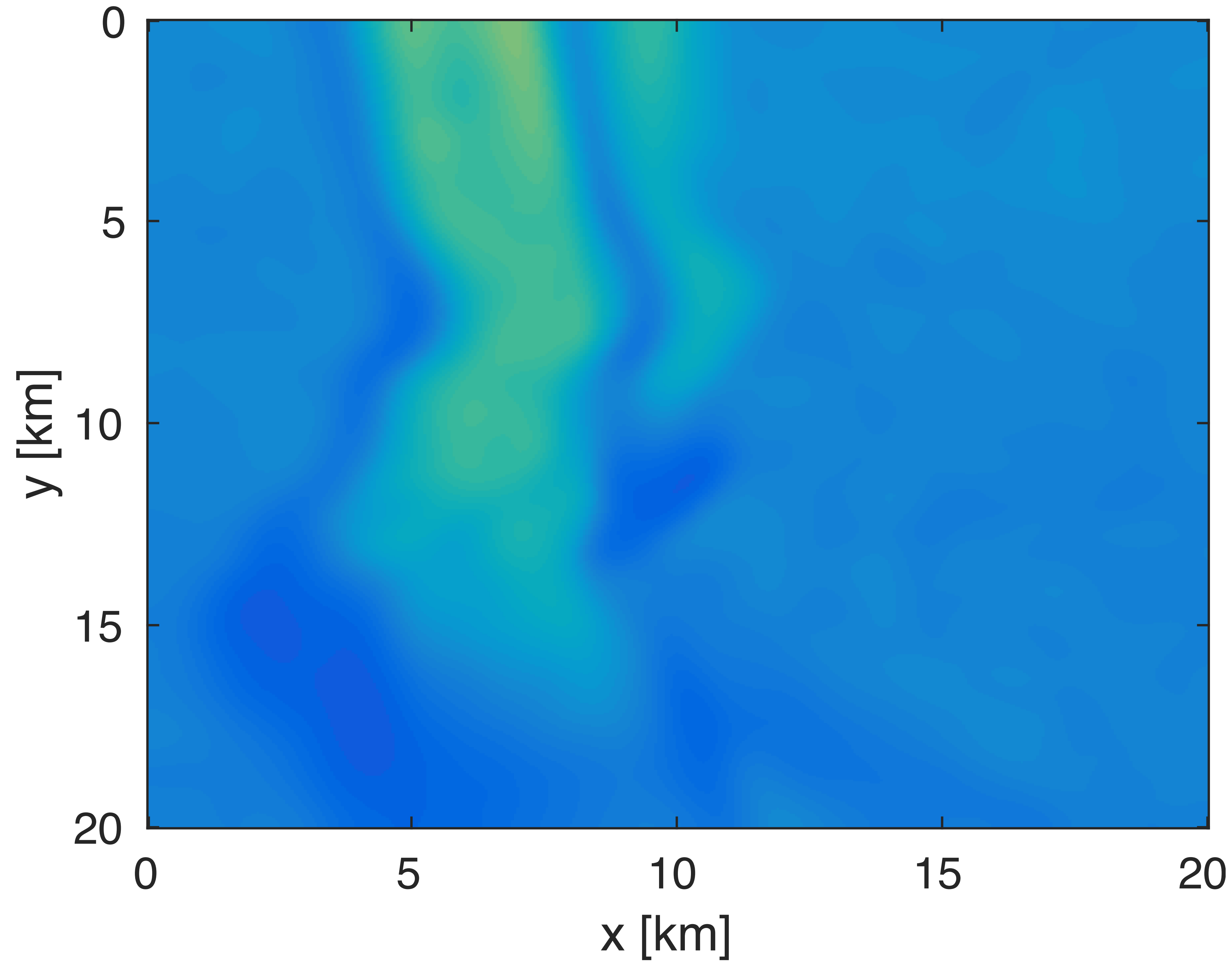
Same source indices for two examples

- **same** number of **PDE** solves
- **three** passes through the data

Z = 1000m depth slice

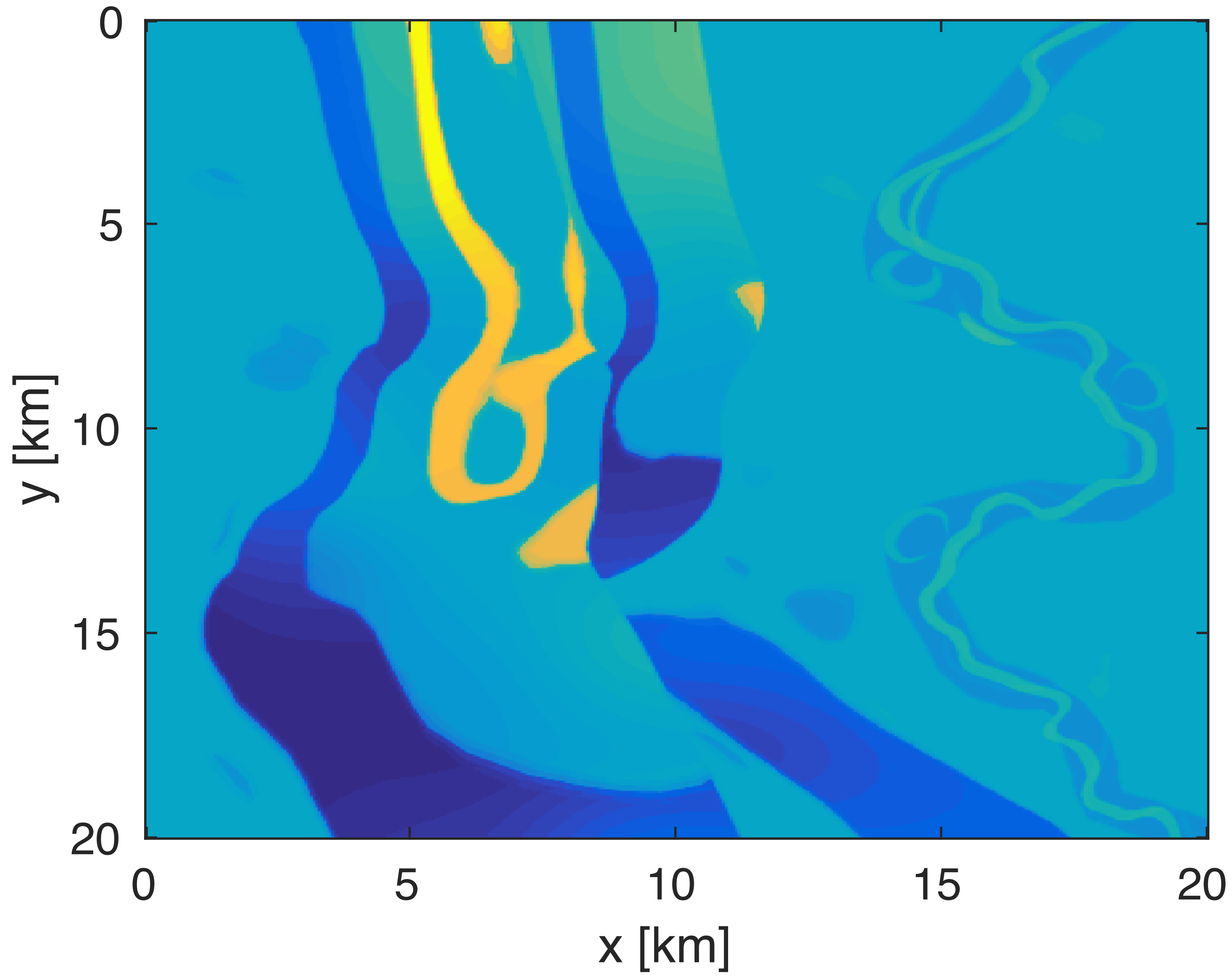


True model

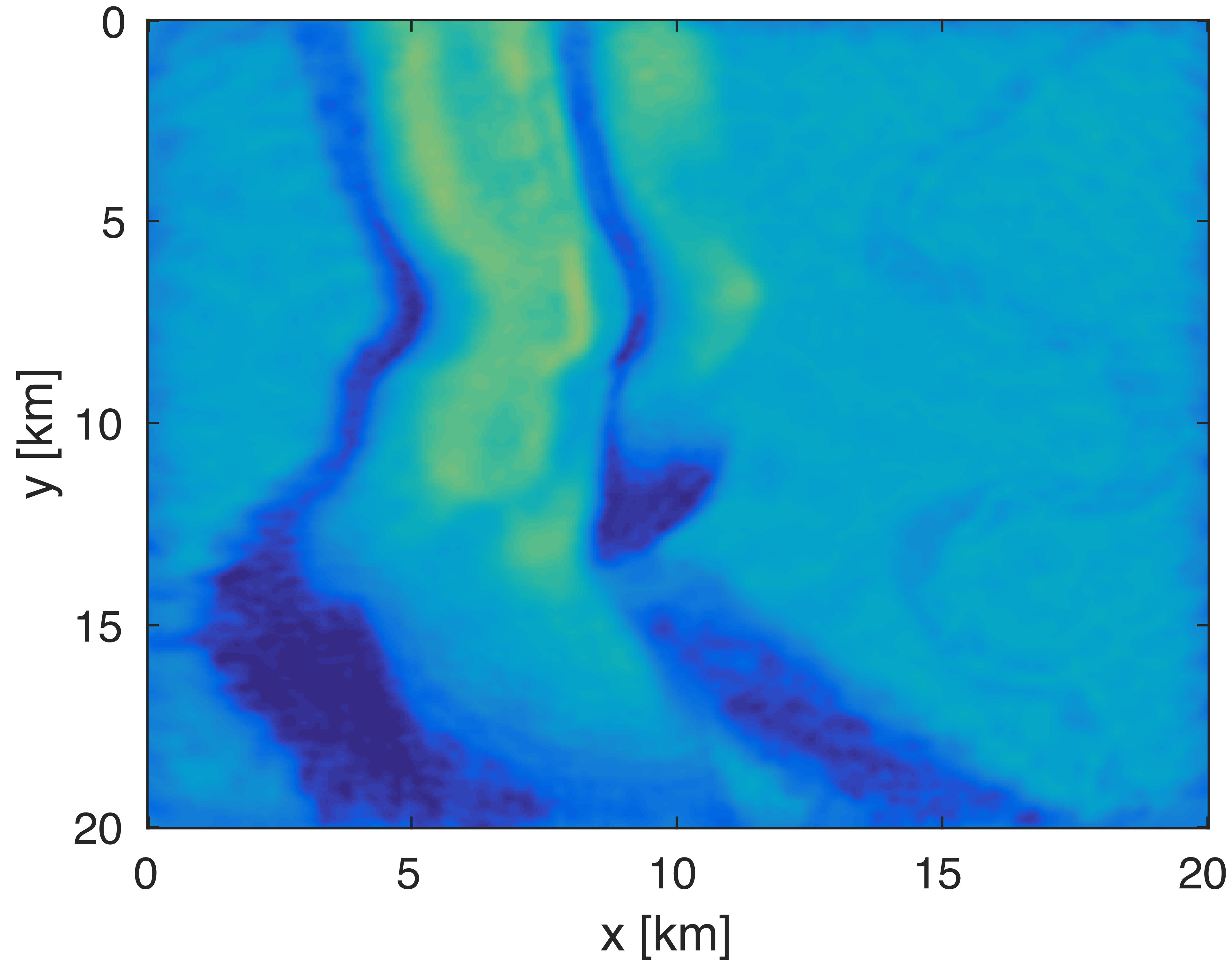


Initial model

Z = 1000m depth slice

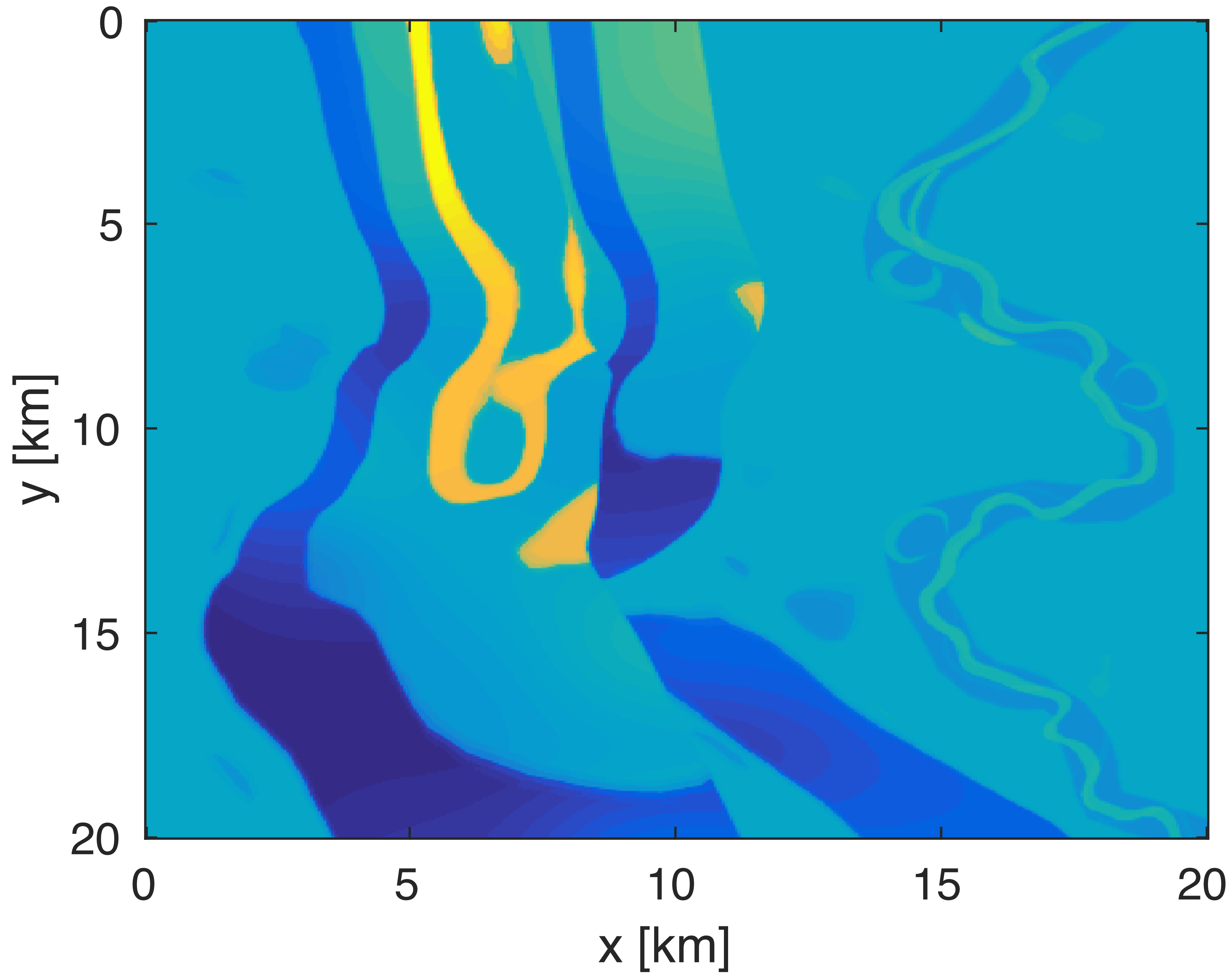


True model

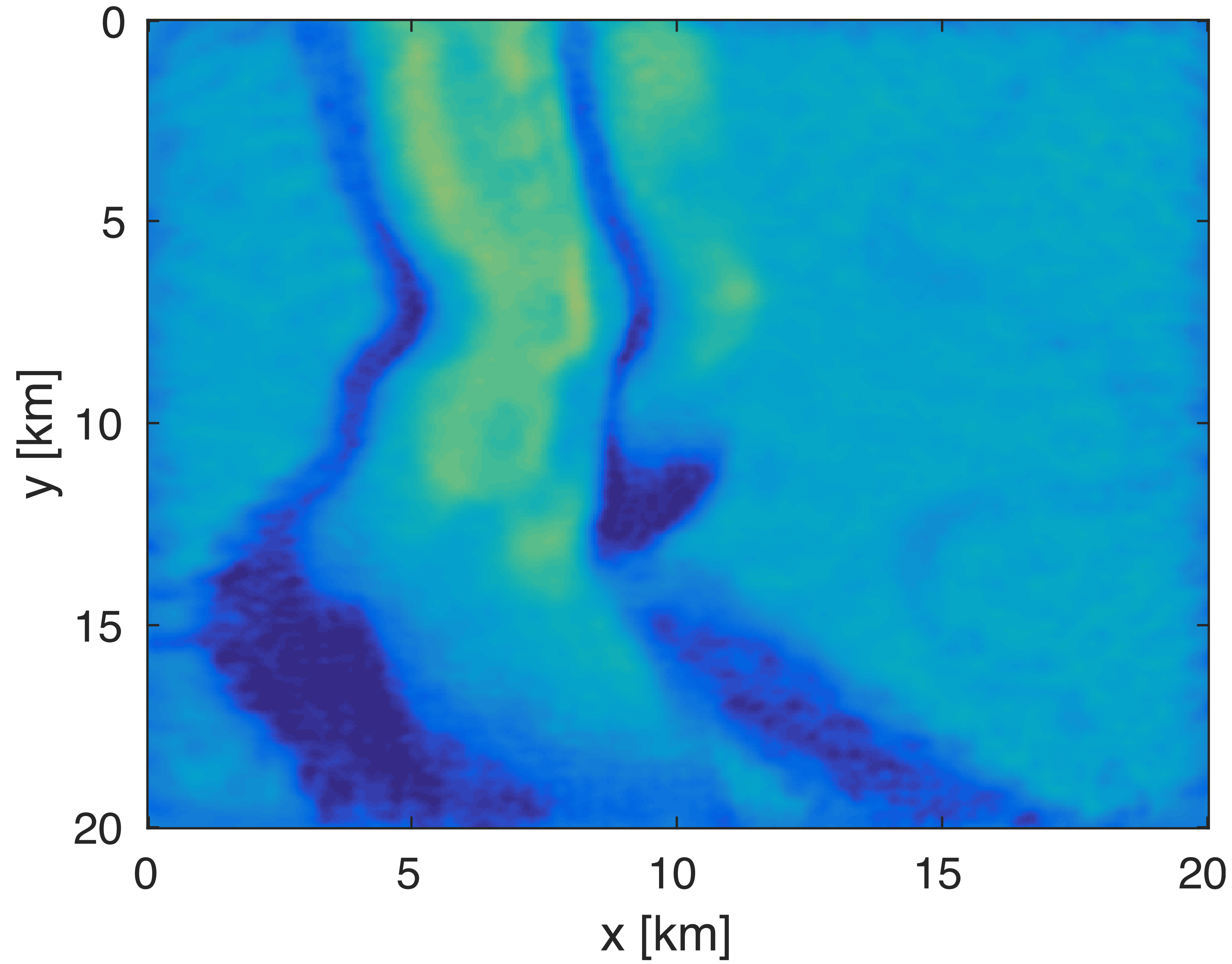


Full data

Z = 1000m depth slice

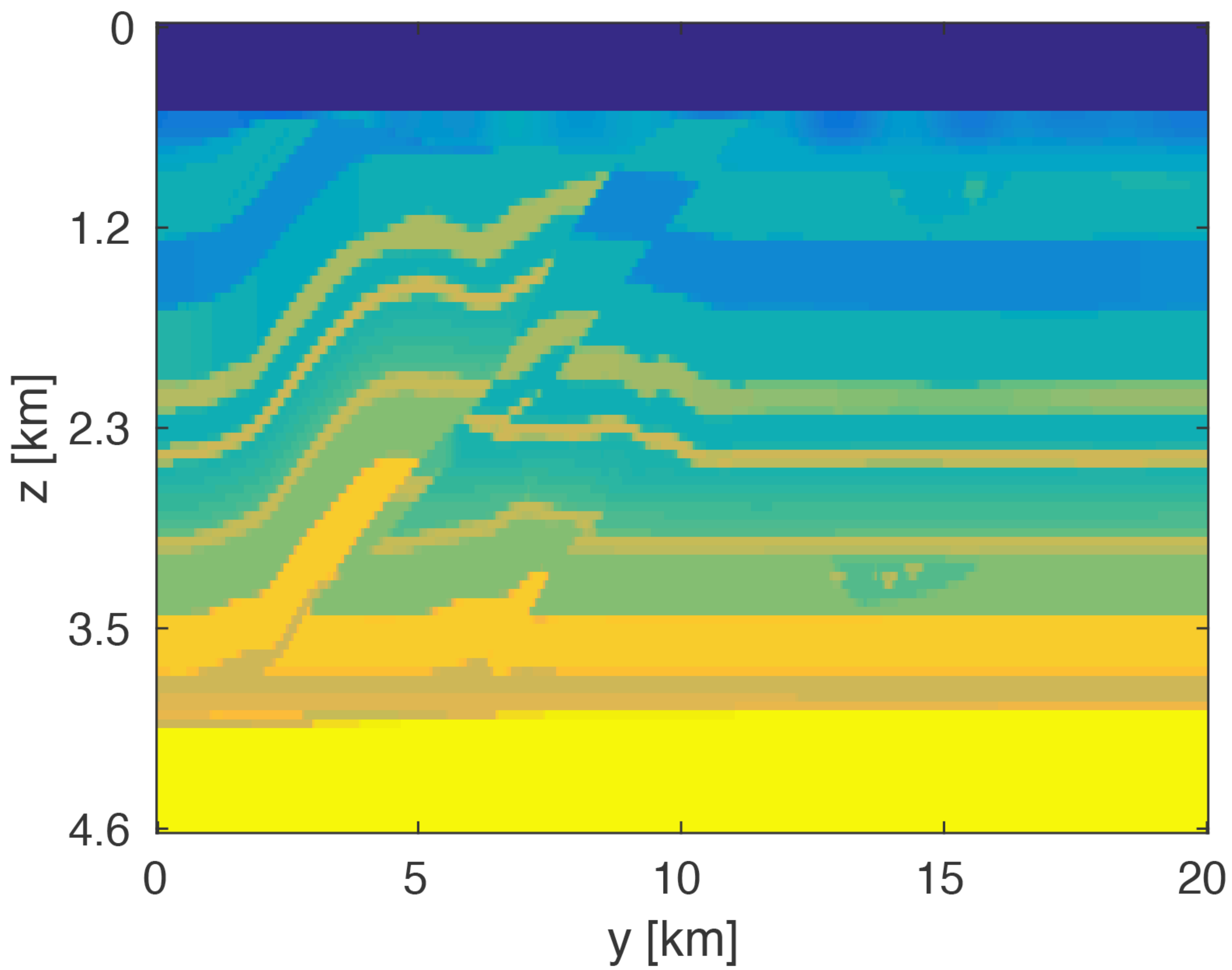


True model

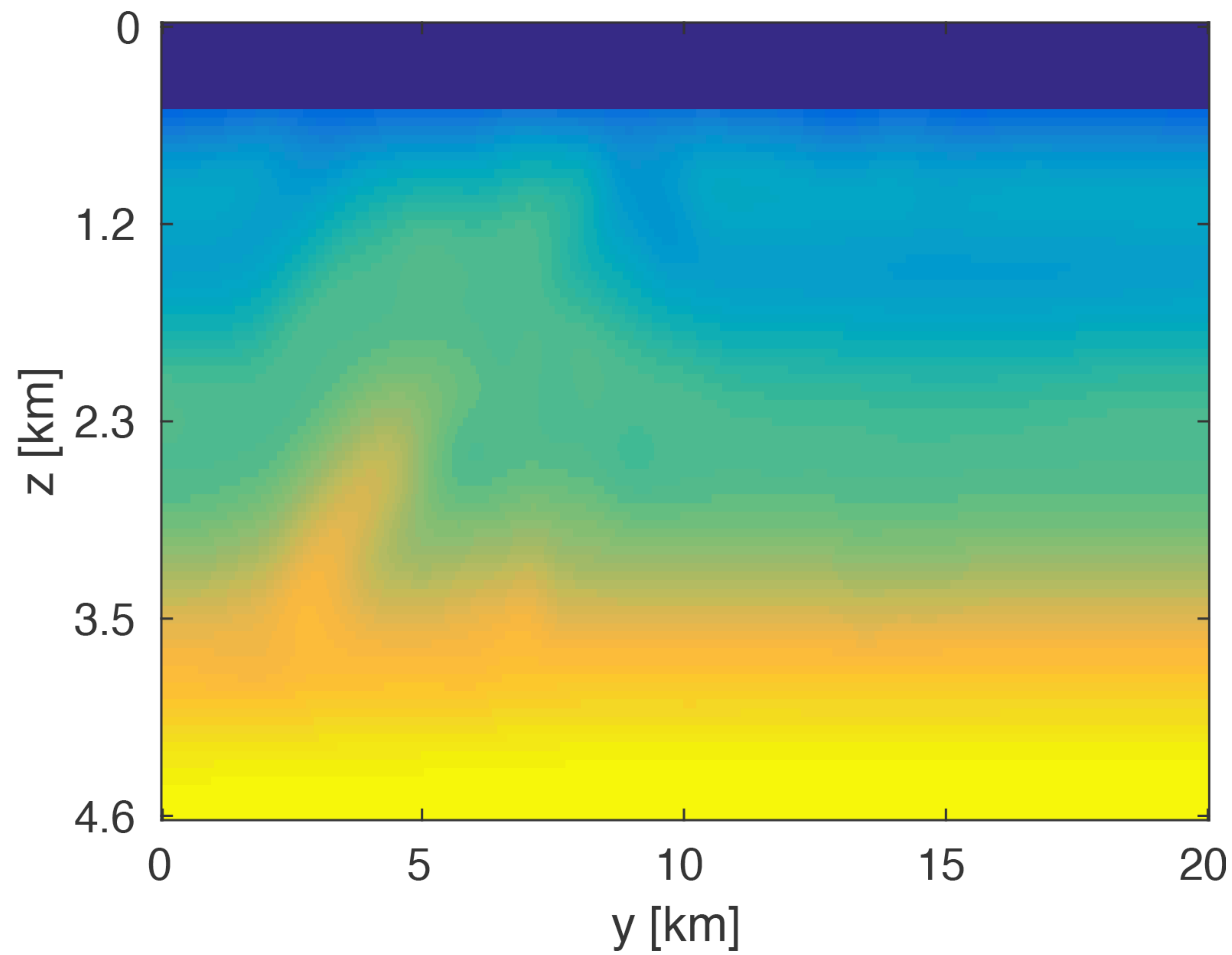


Compressed data

x = 12.5km lateral slice



True model



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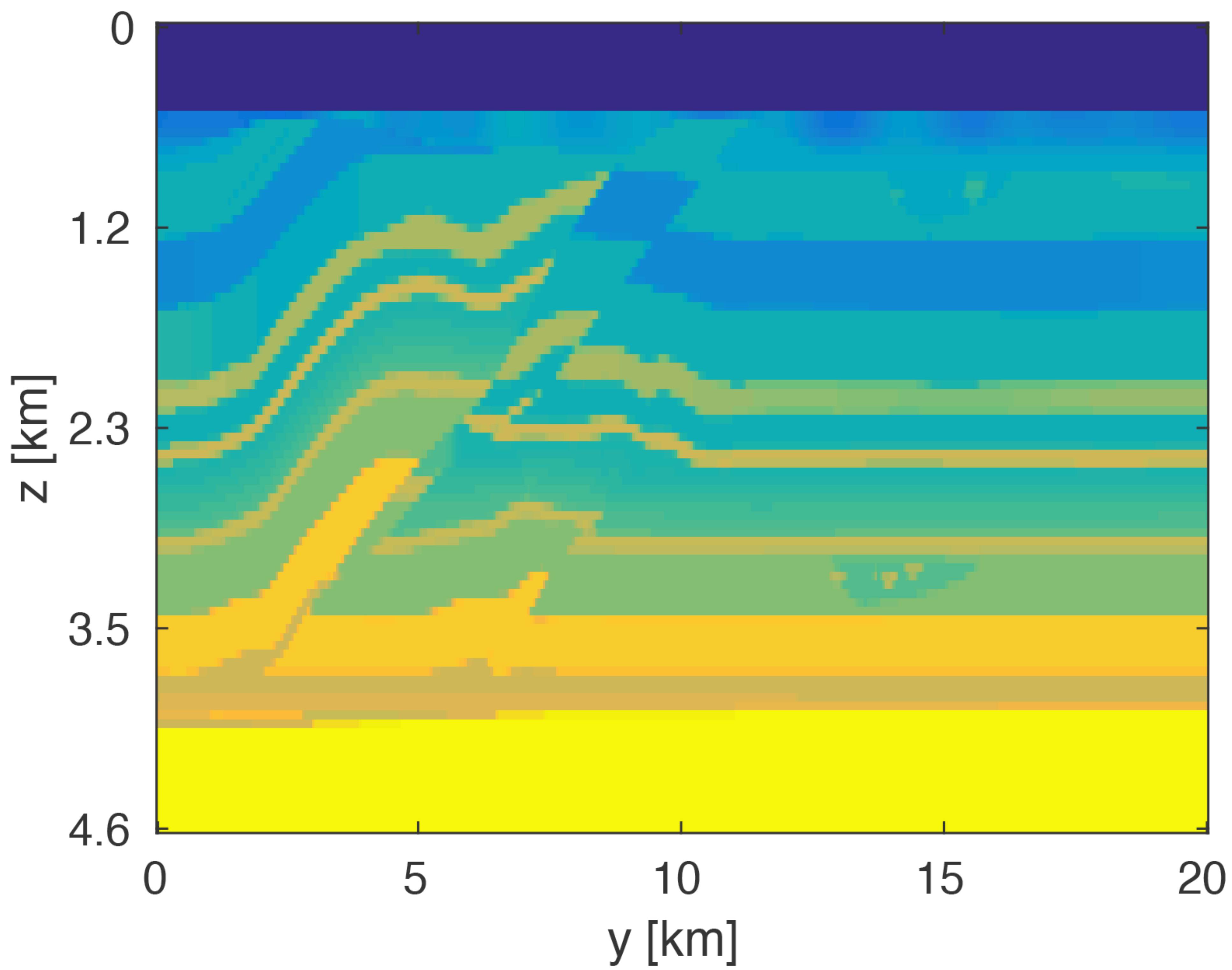
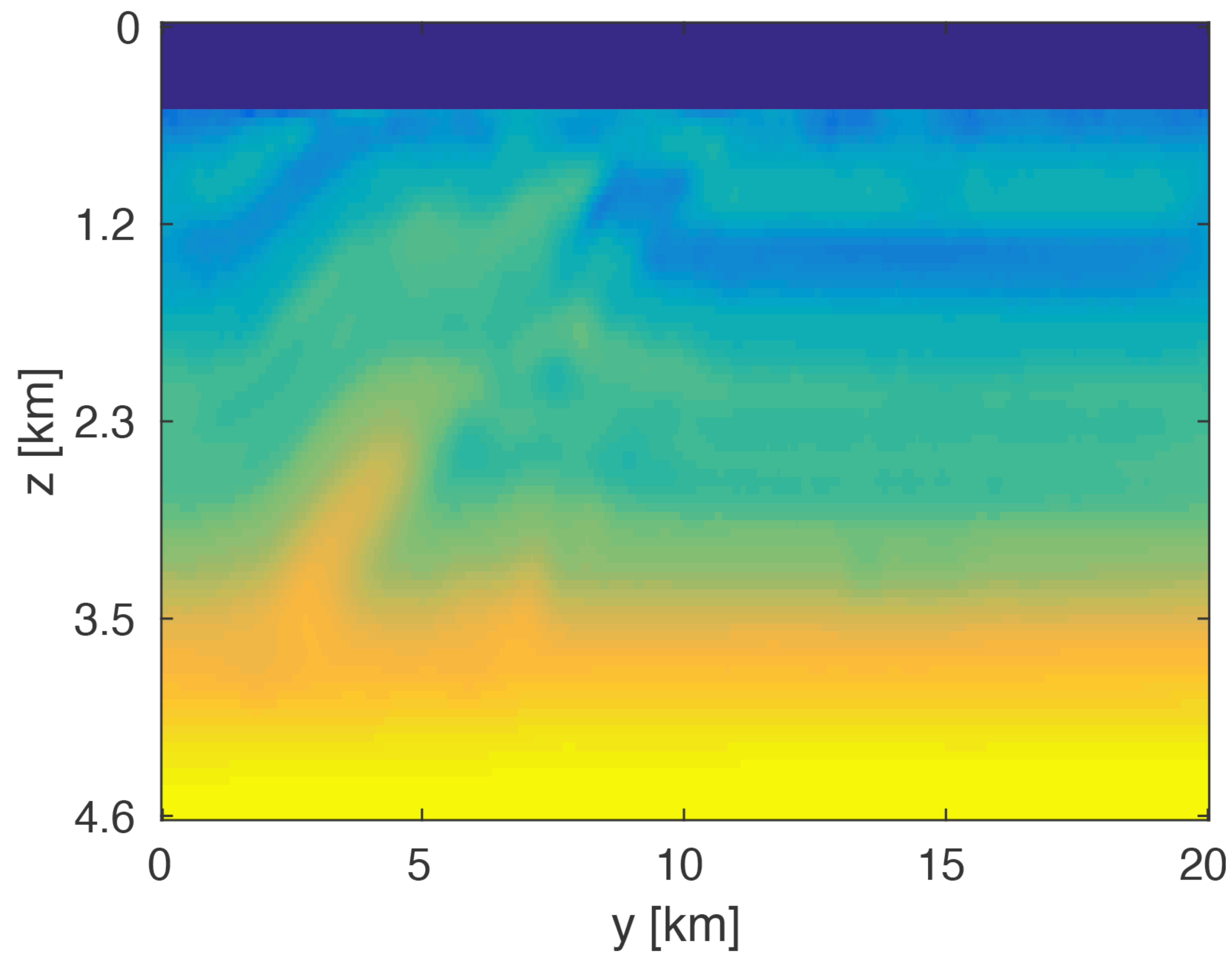
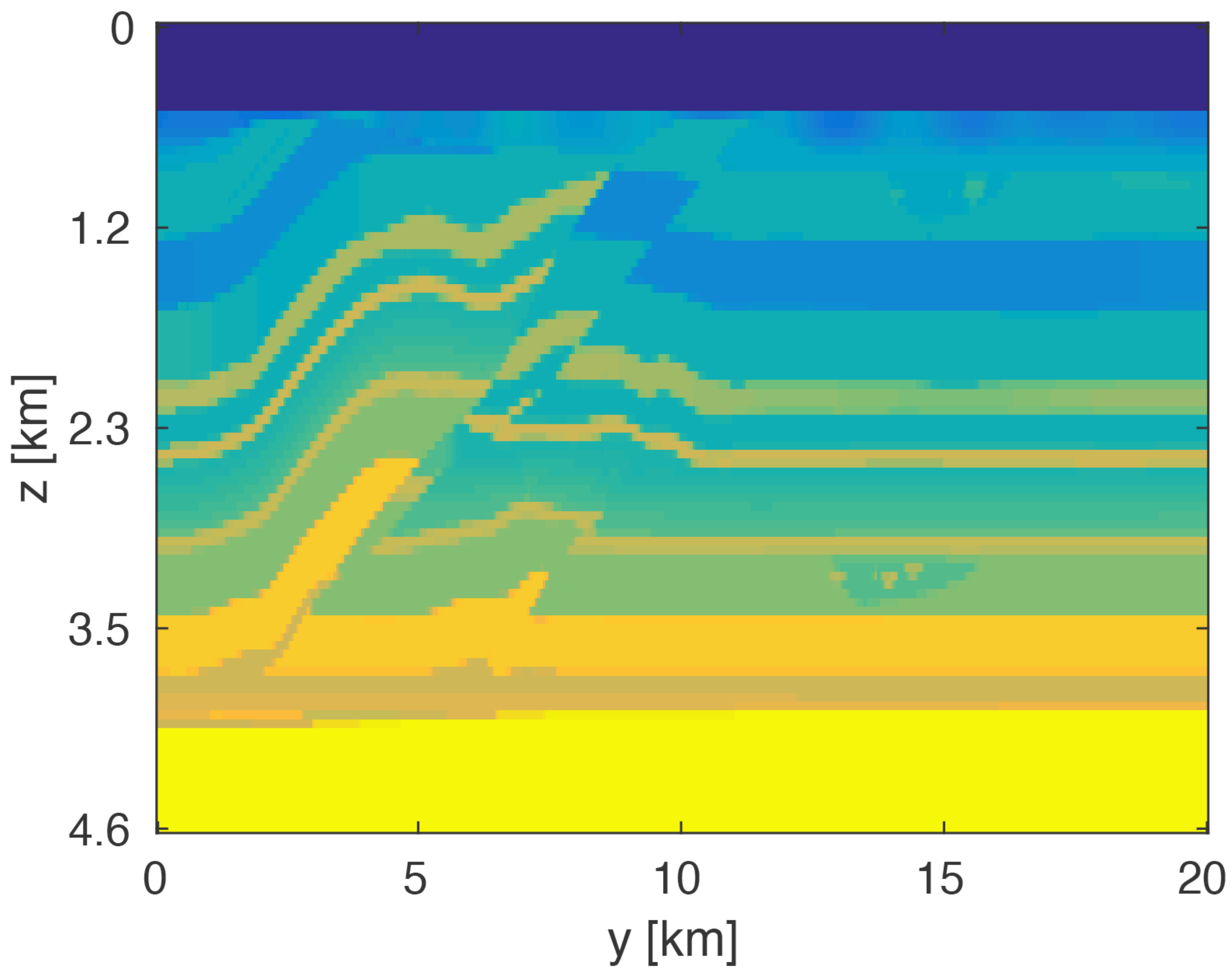


True model

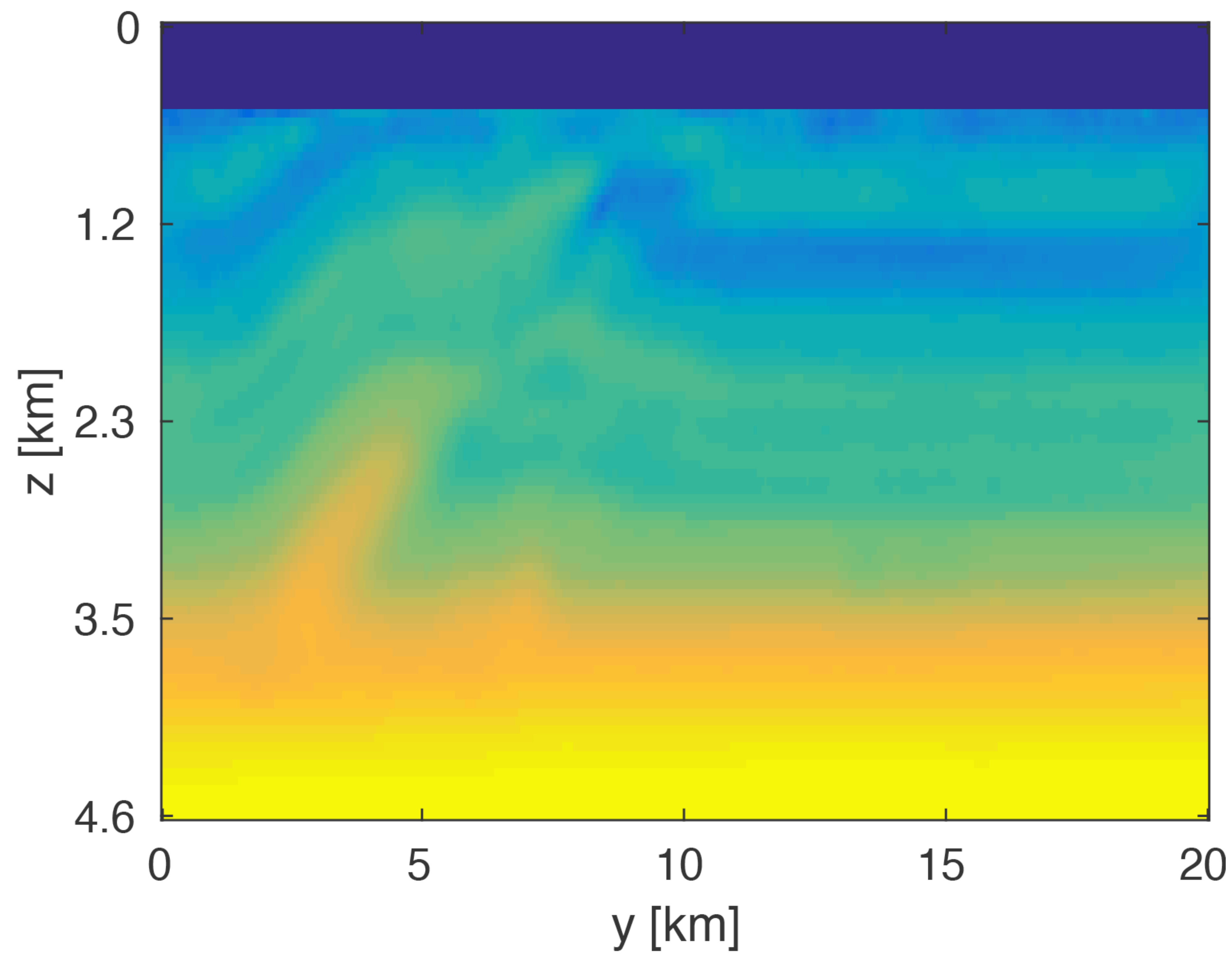


Full data

x = 12.5km lateral slice



True model



Compressed data

FWI examples on BG model

Model 2:

- 3D **BG** model
- 10km x 10km x 1.8 km
- 50m x 50m x 12 m spacing

Data:

- 49 x 49 sources, 200m interval
- 196 x 196 receivers, 50m interval
- Ricker wavelet, 10Hz peak frequency
- 3Hz - 6Hz ranging, 0.25Hz interval
- **remove 75%** of random receivers

FWI examples on BG model

Stochastic FWI results inverted w/

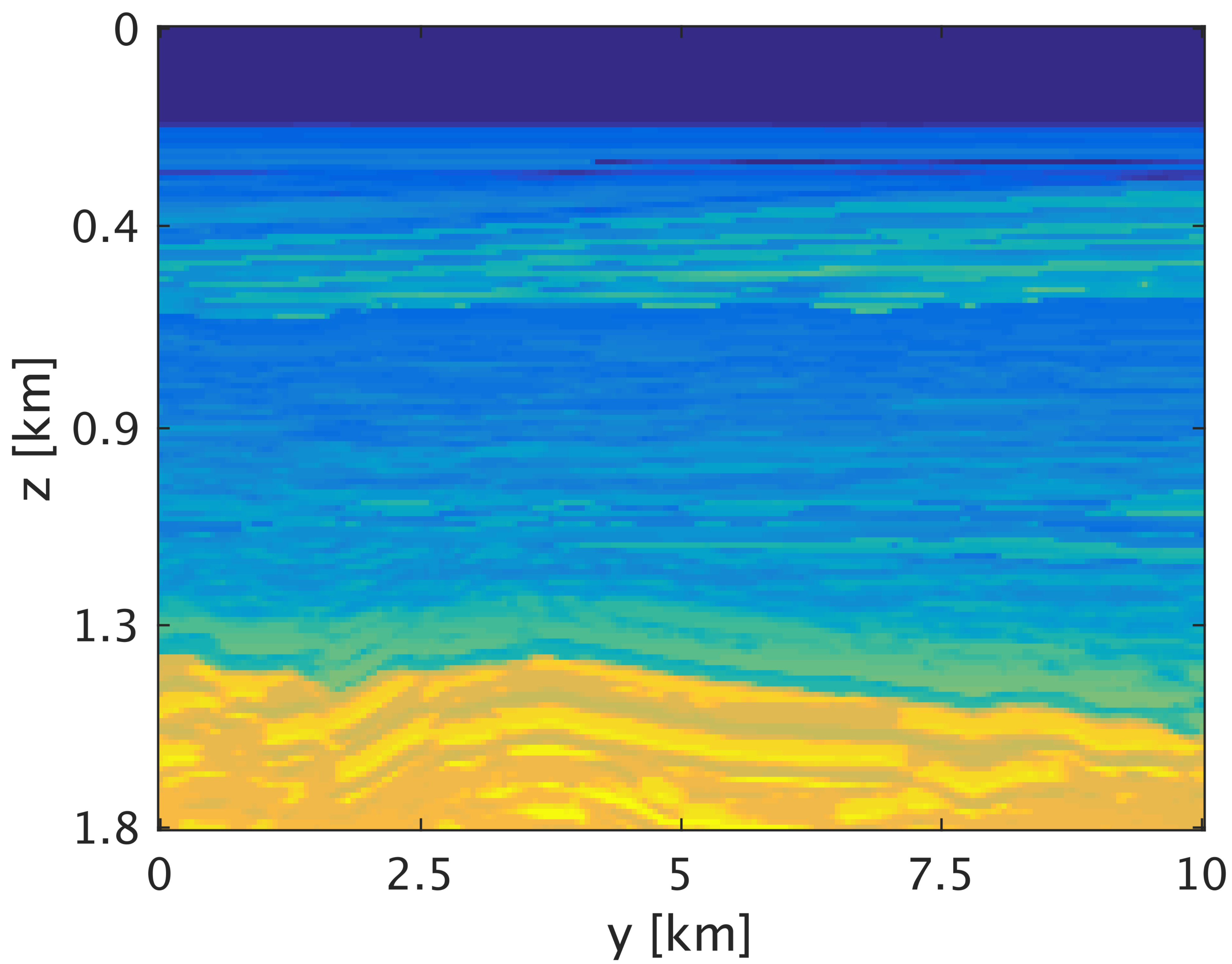
- **full** data
- compressed **HT** via **truncation**
- **subsampled** data
- compressed **HT** recovered from **interpolation**

Same source indices for four examples

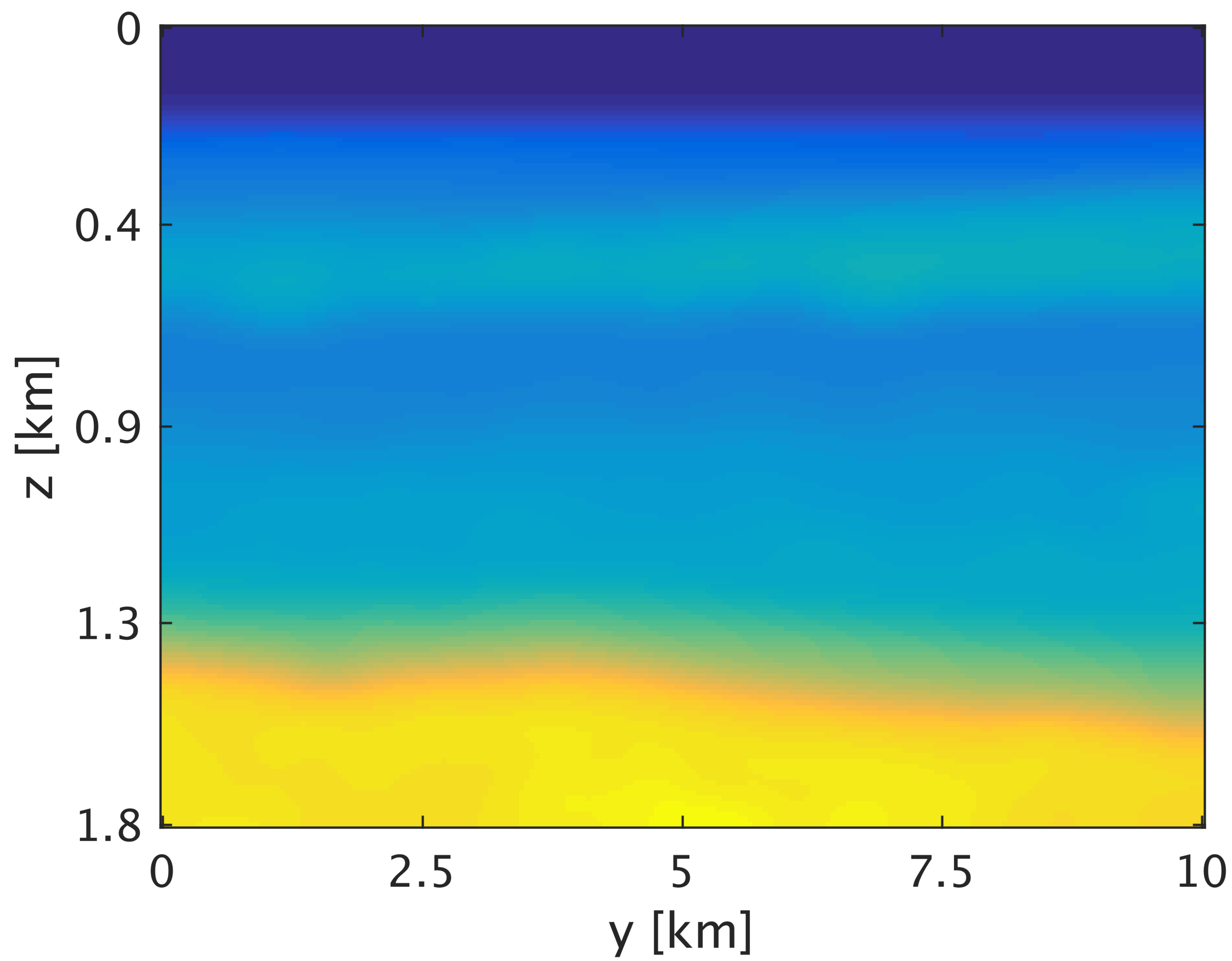
- **same** number of **PDE** solves
- **three** passes through the data

Full data & Compressed data

x = 4900m lateral slice

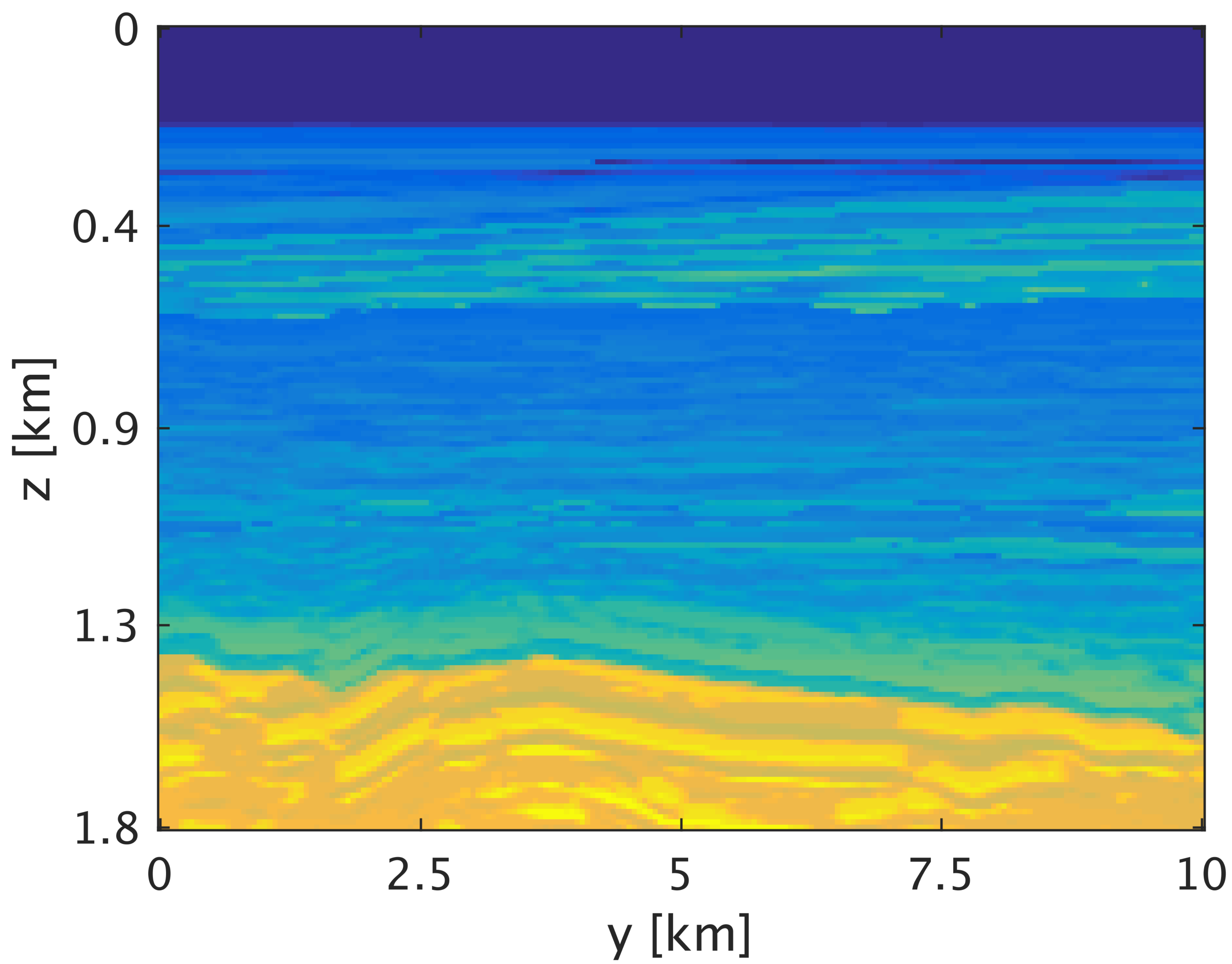


True model

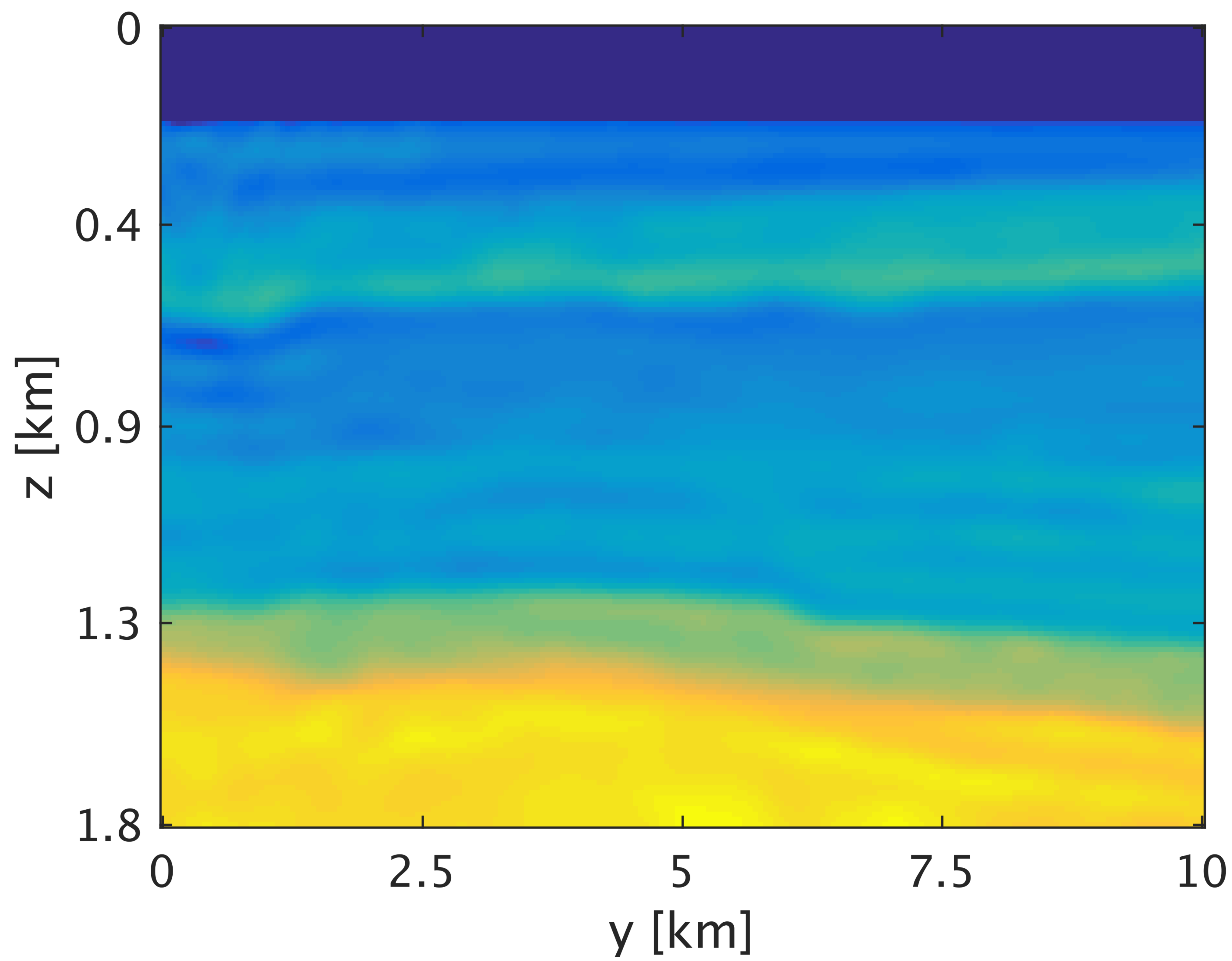


Initial model

x = 4900m lateral slice

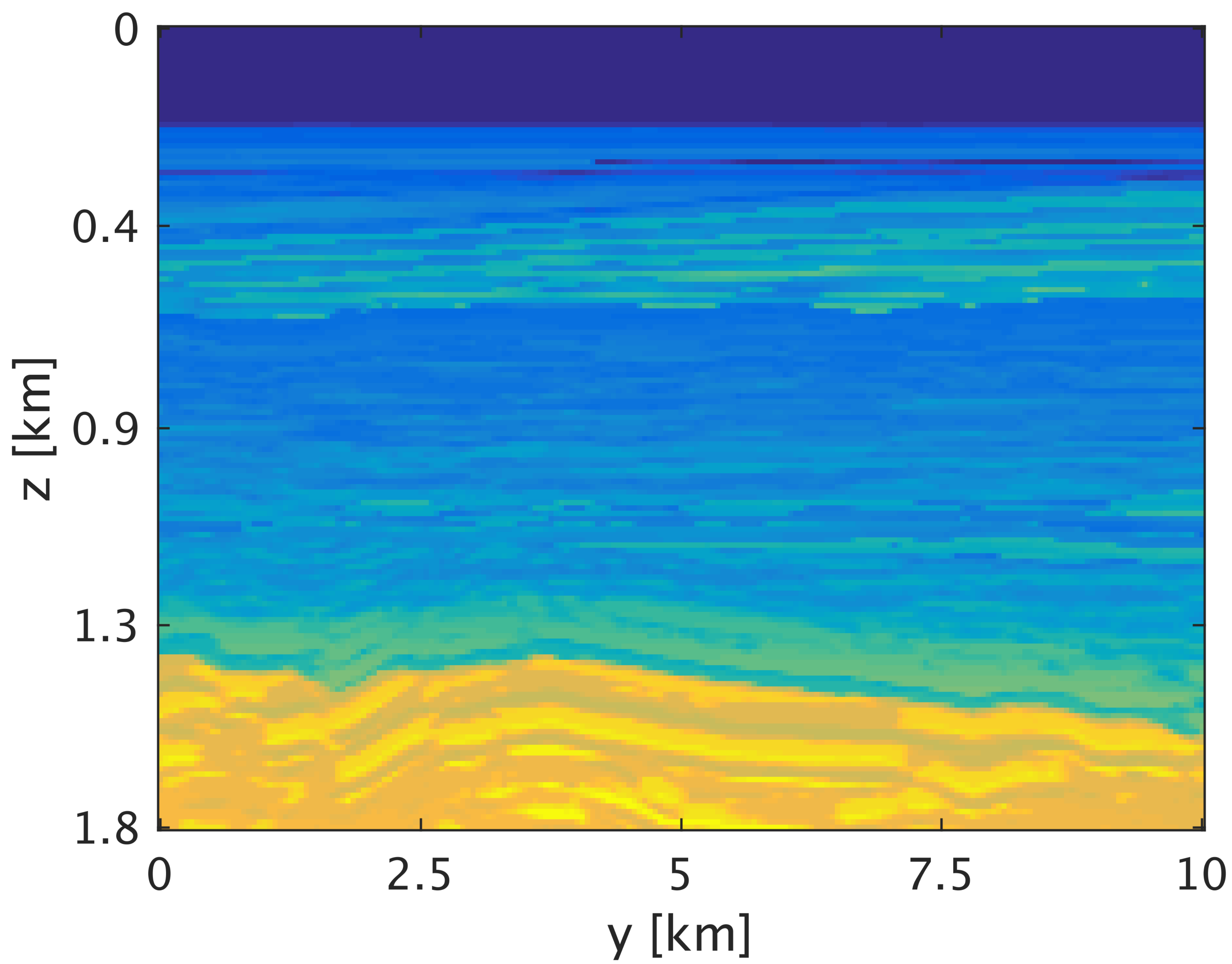


True model

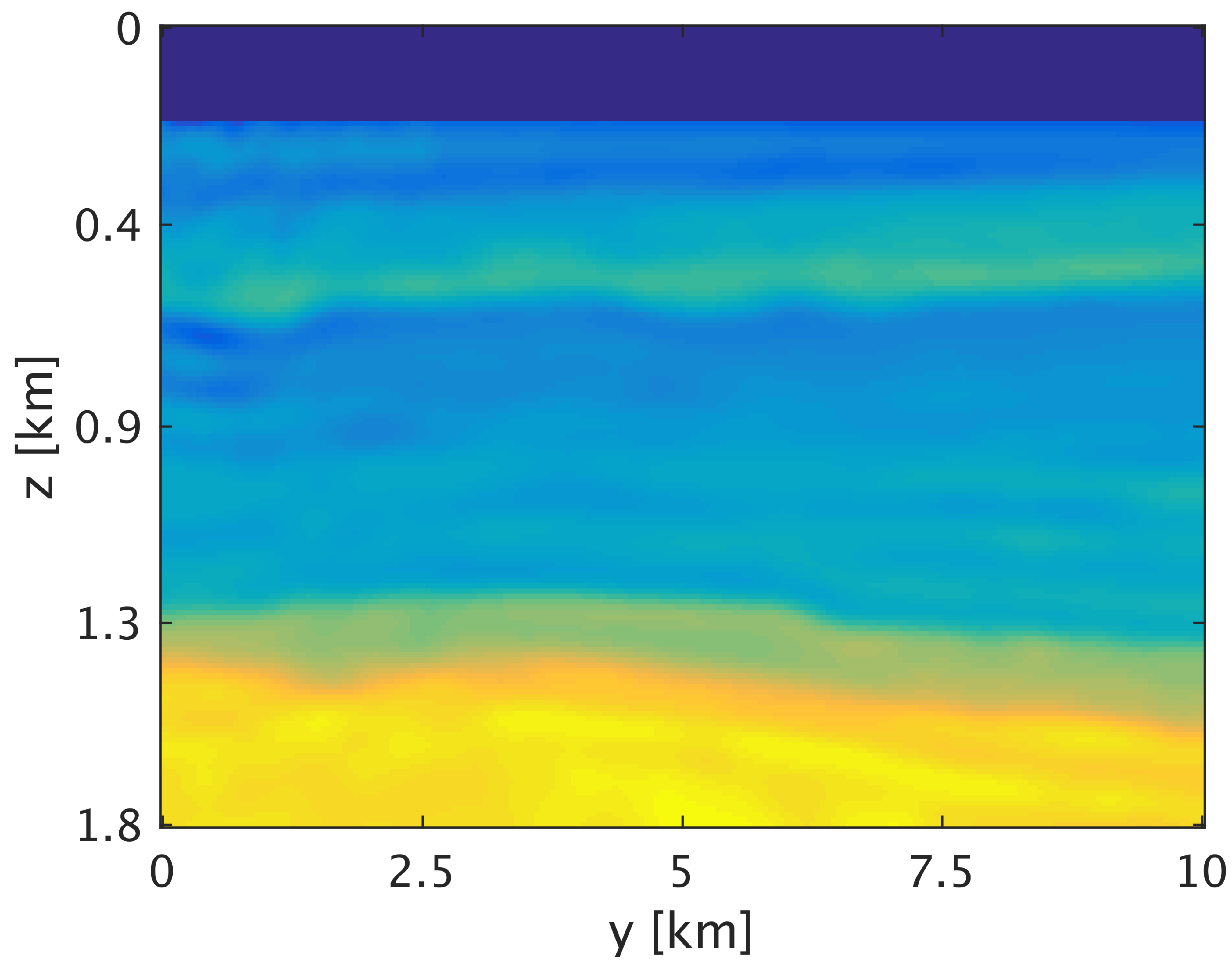


Full data

x = 4900m lateral slice

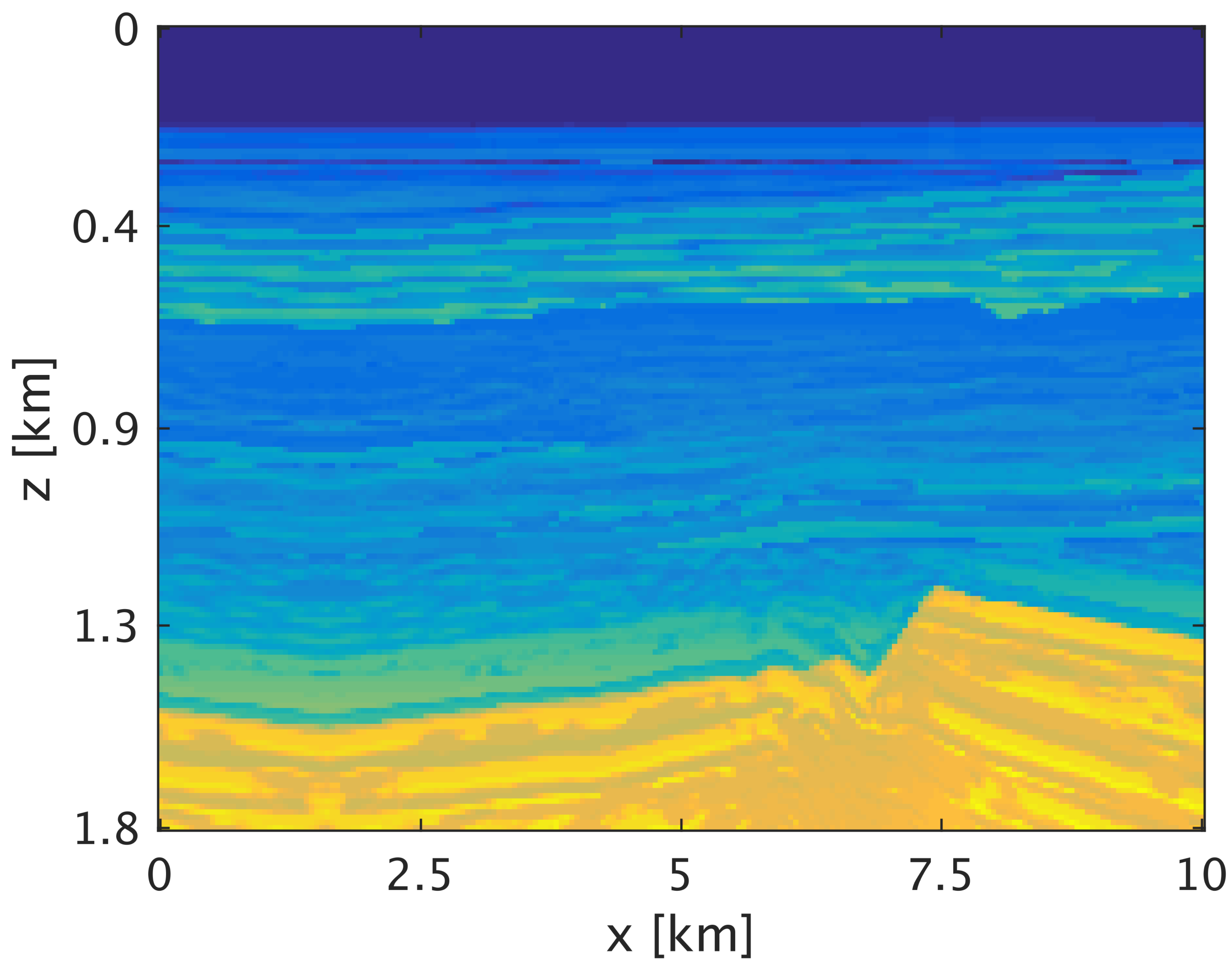


Full data

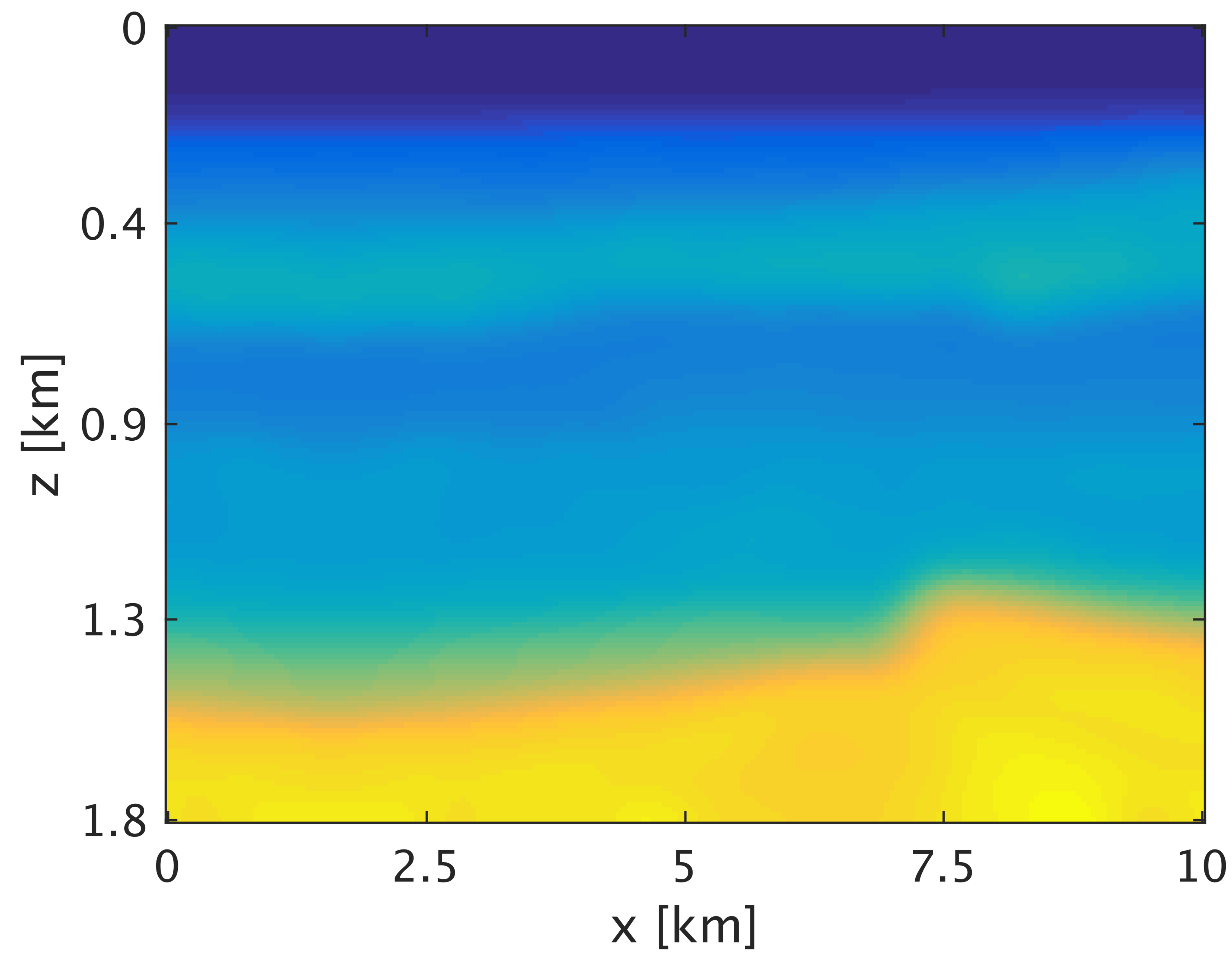


Compressed data

y = 5650m lateral slice

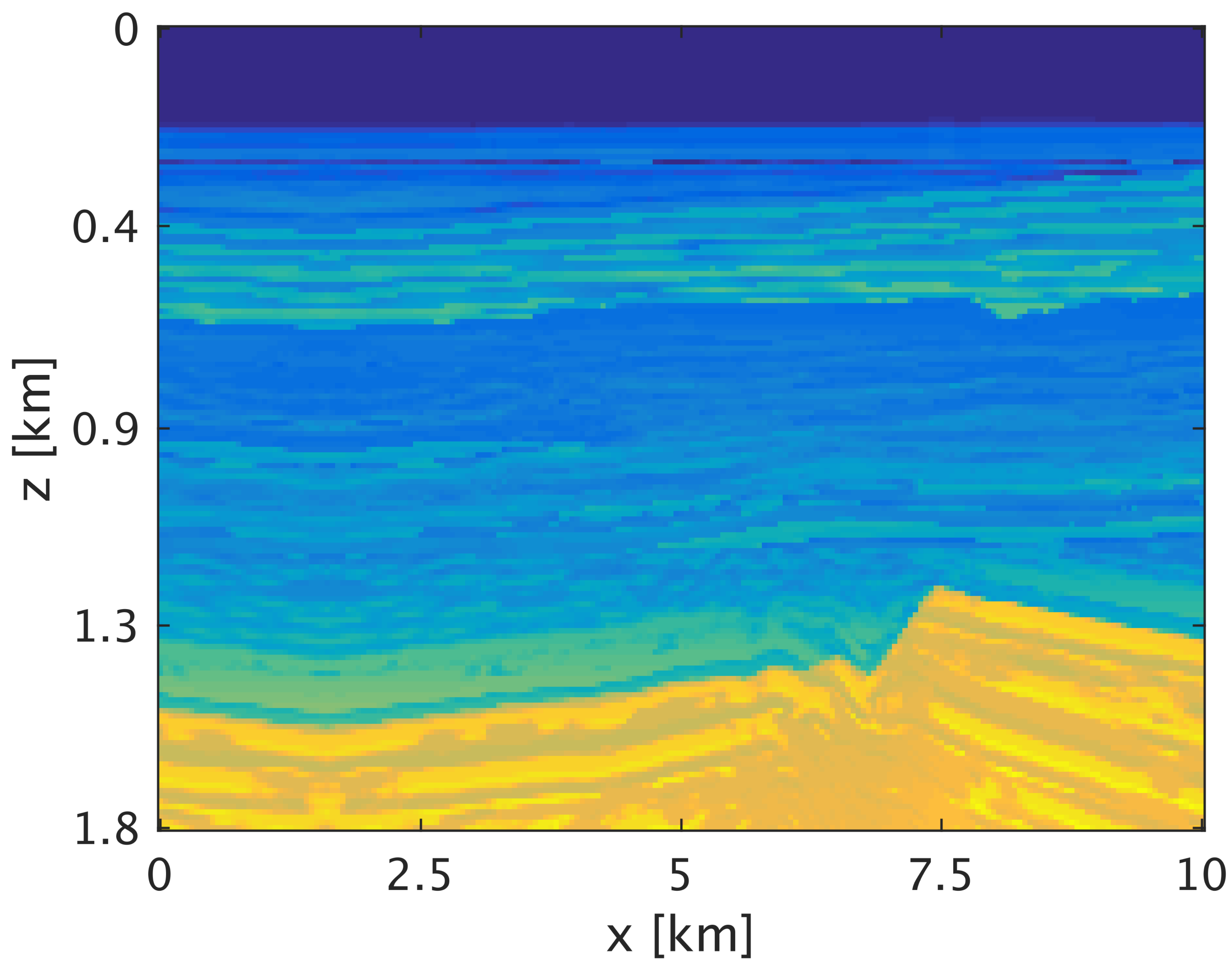


True model

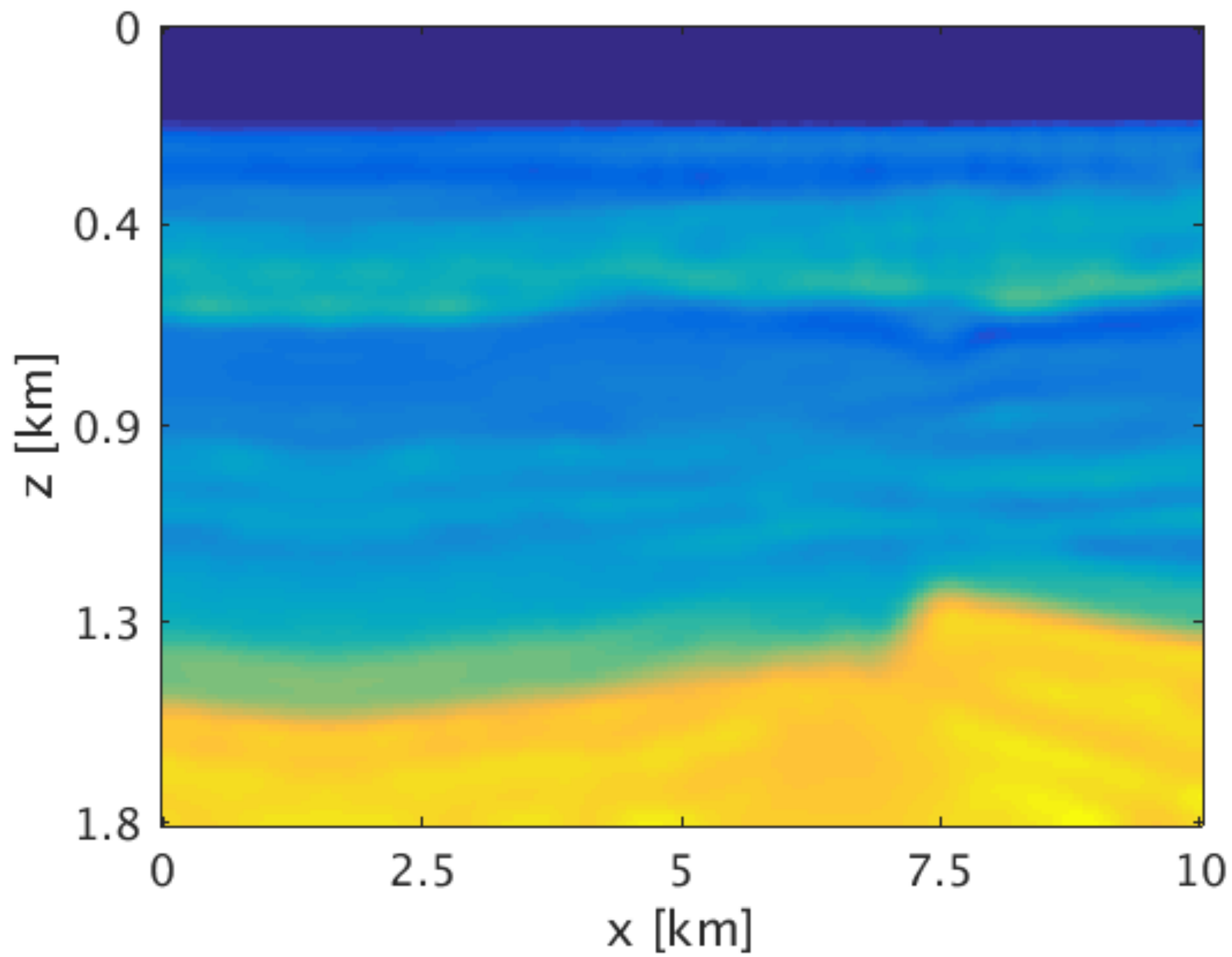


Initial model

y = 5650m lateral slice

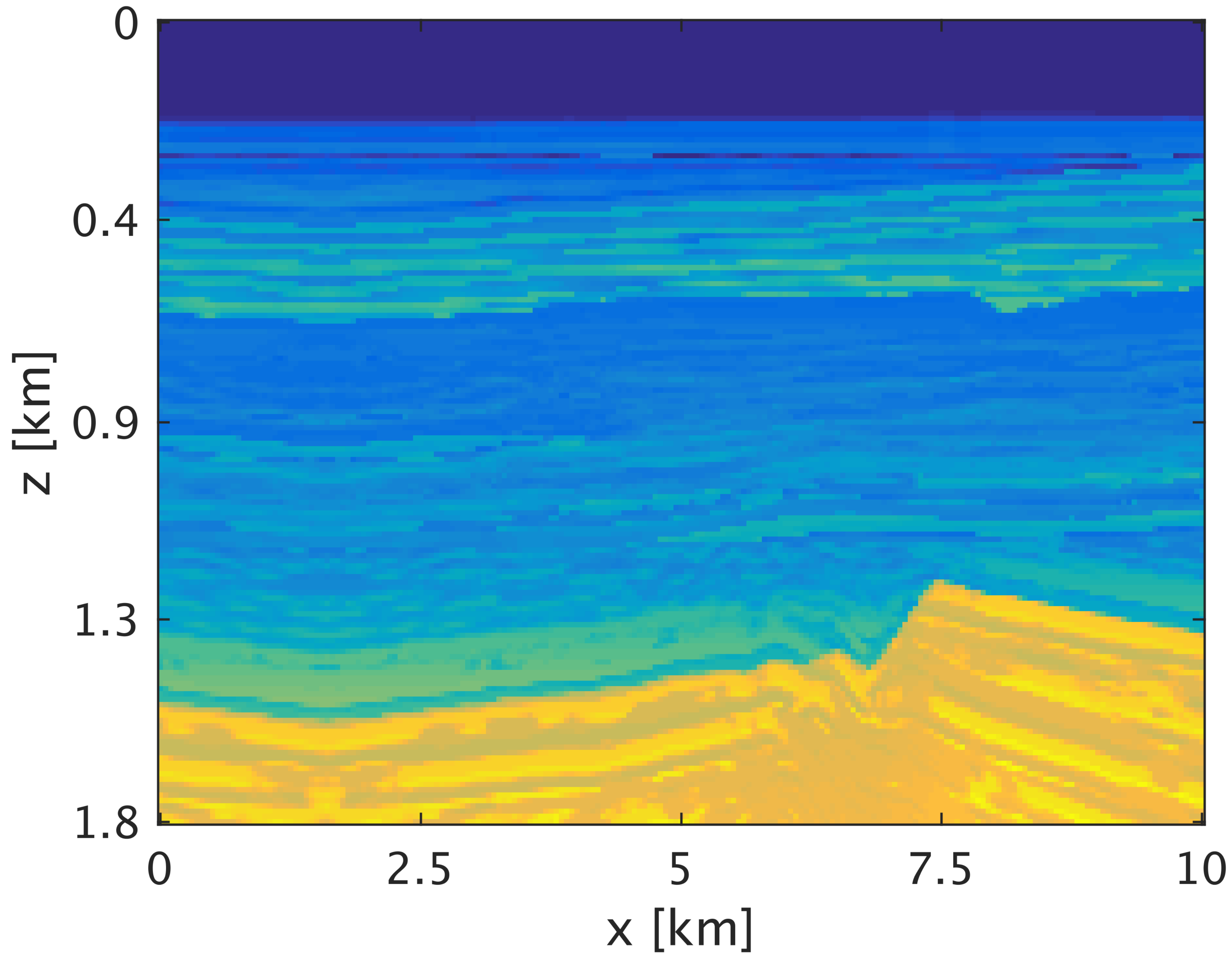


True model

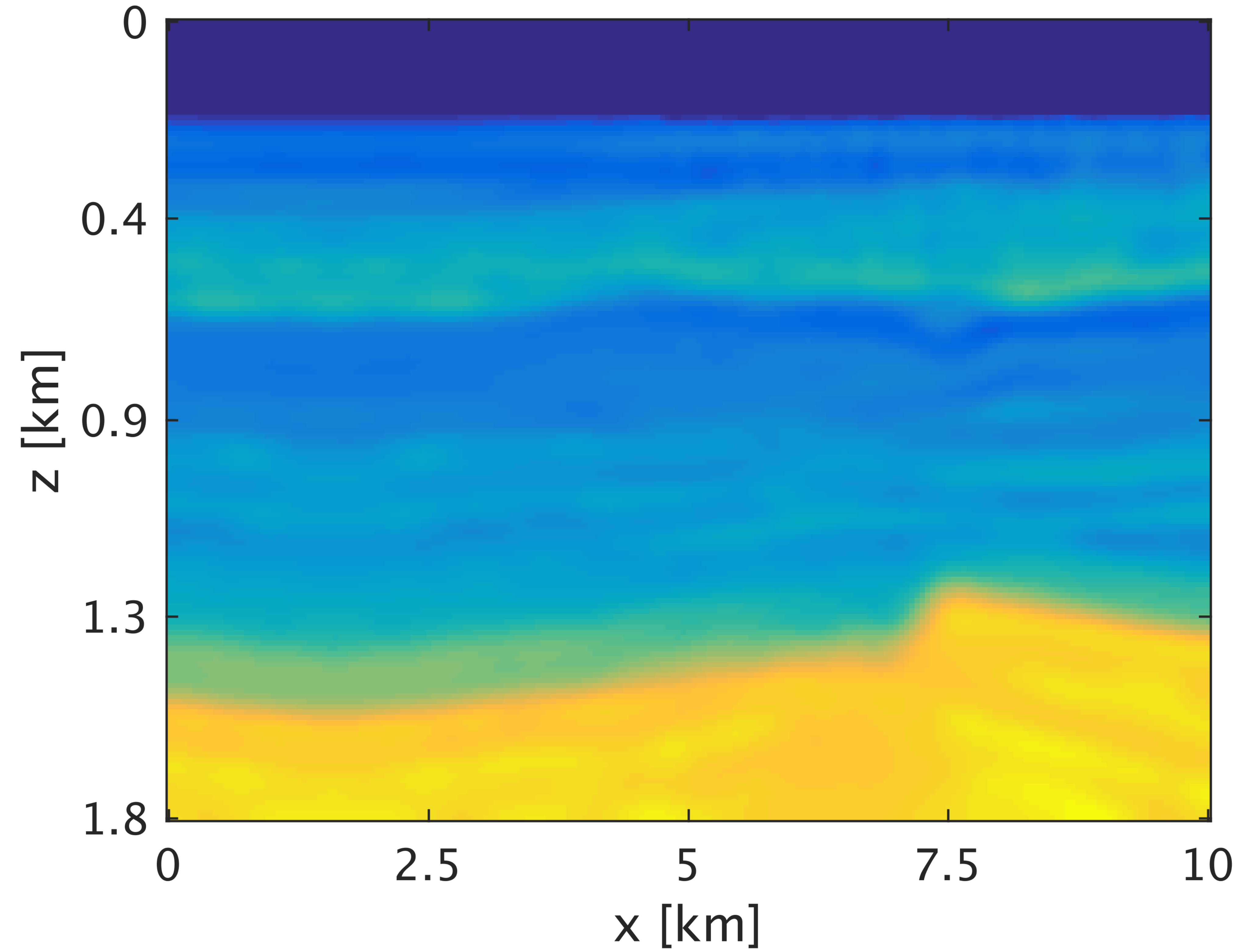


Full data

y = 5650m lateral slice

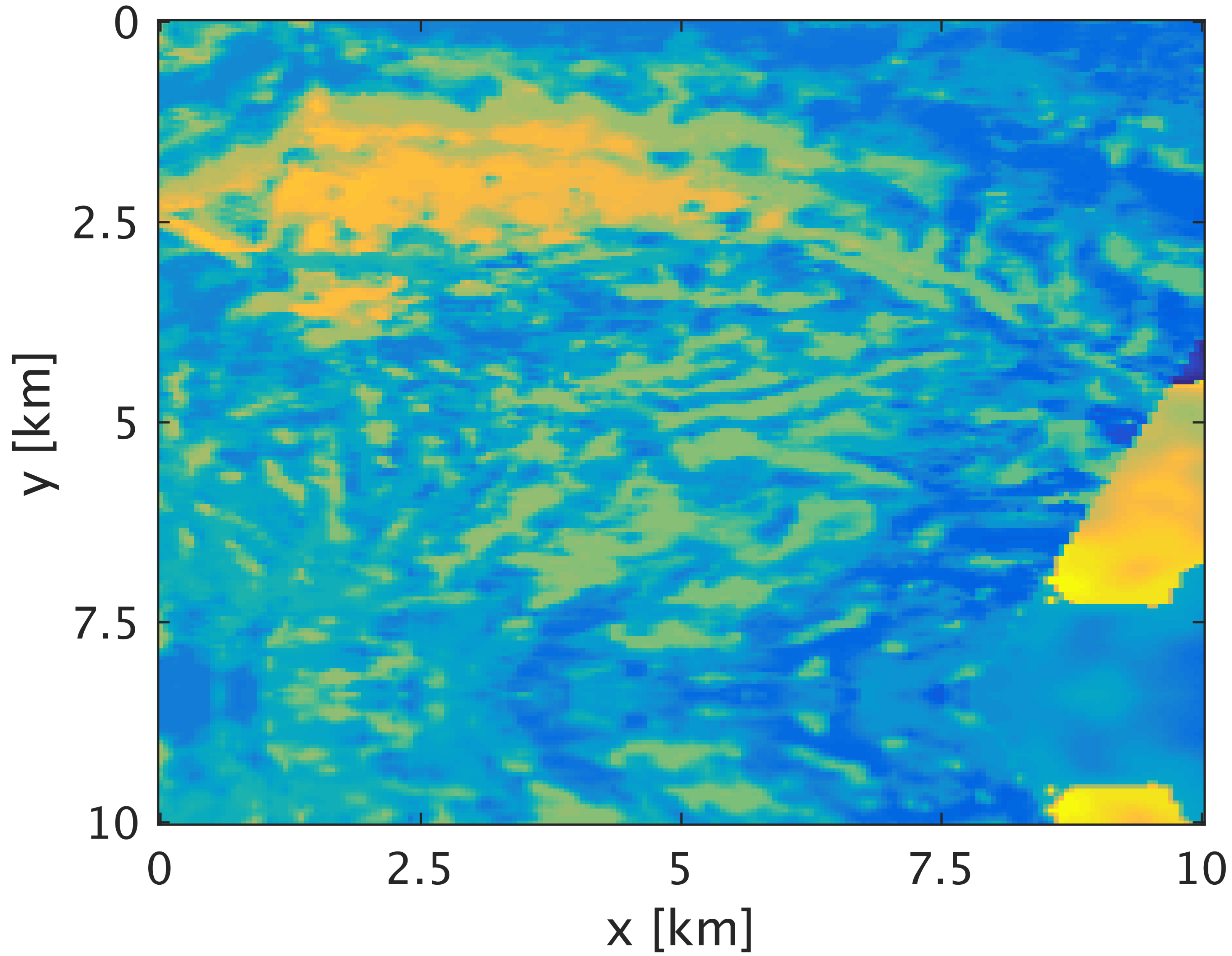


True model

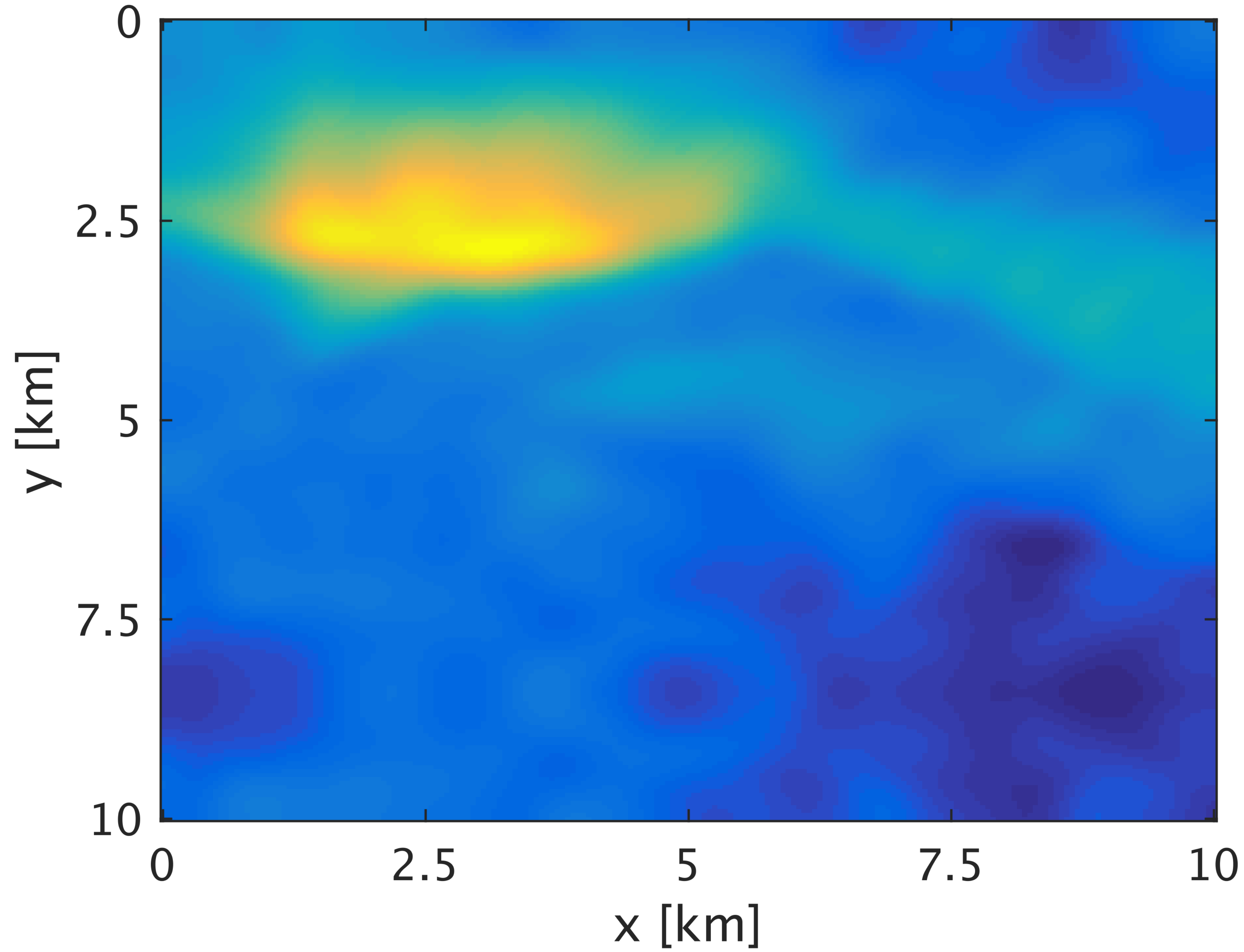


Compressed data

z = 1200m depth slice

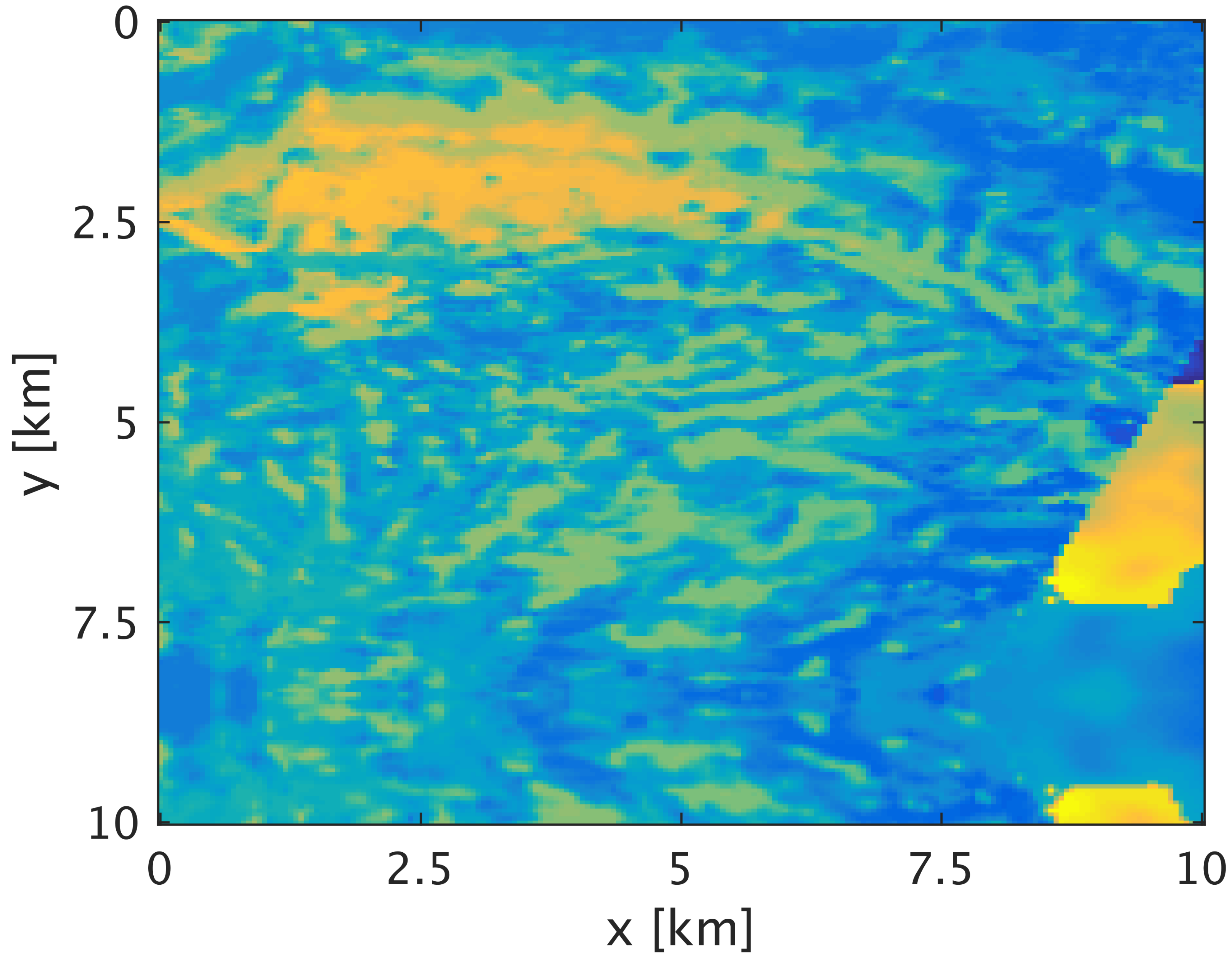


True model

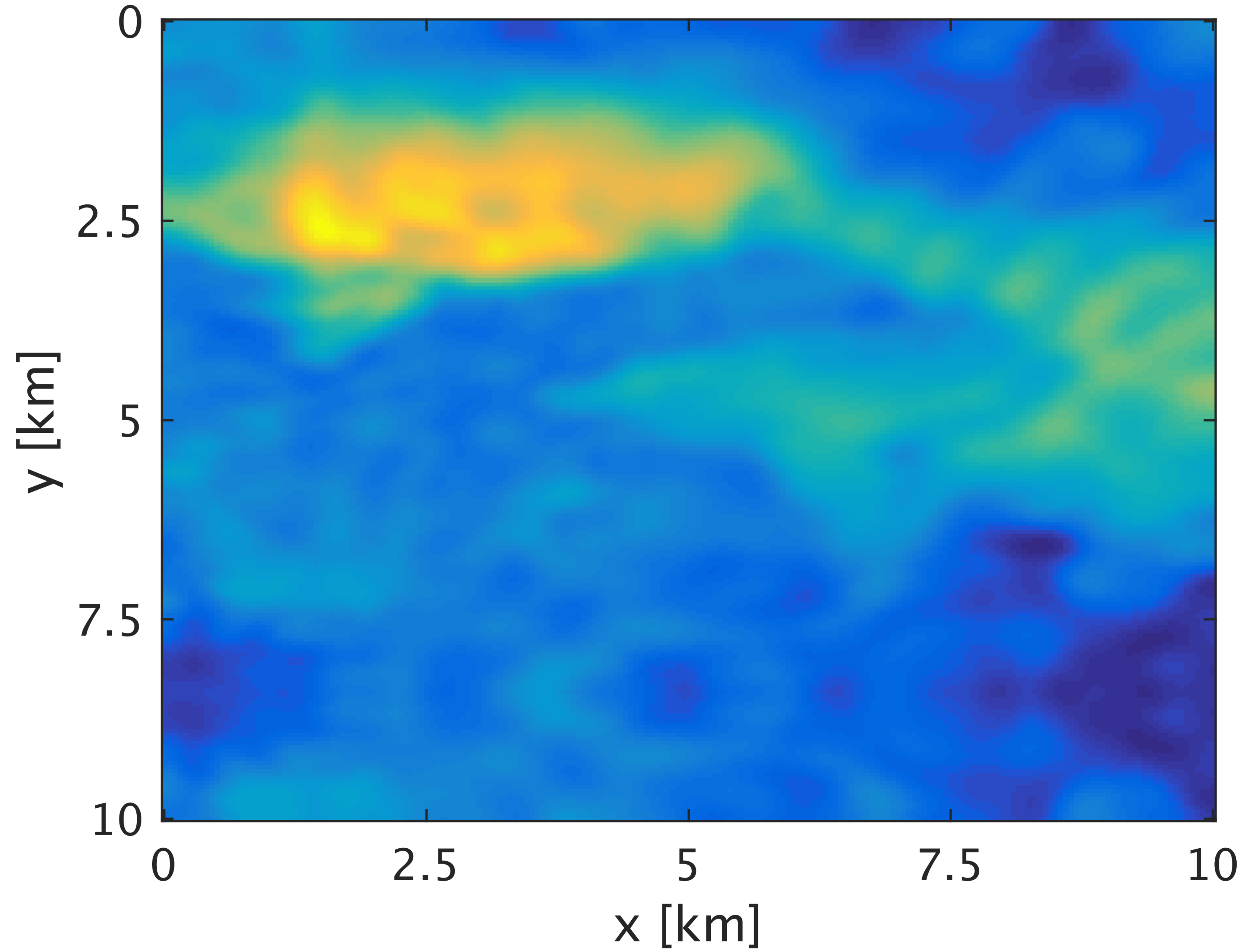


Initial model

z = 1200m depth slice

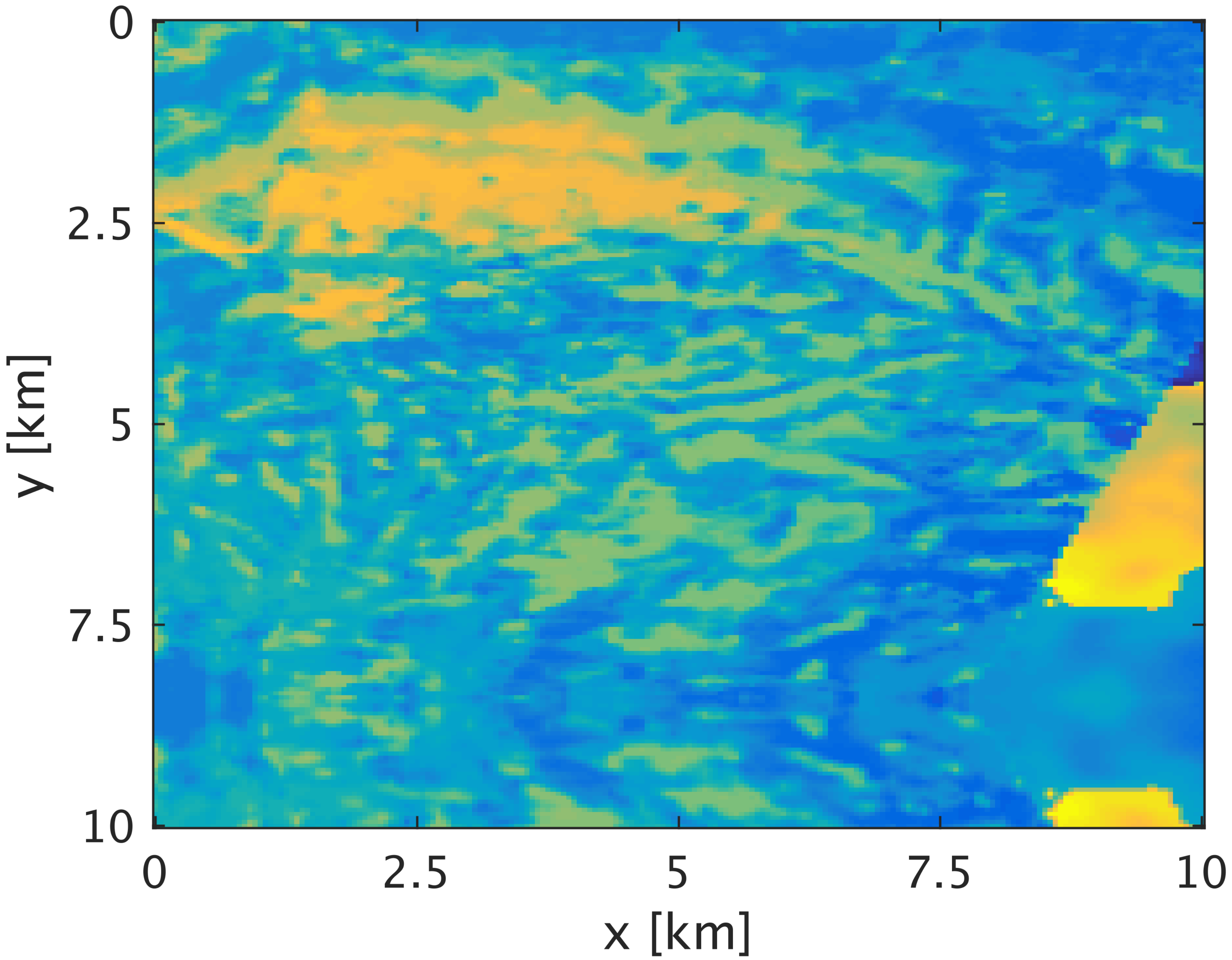


True model

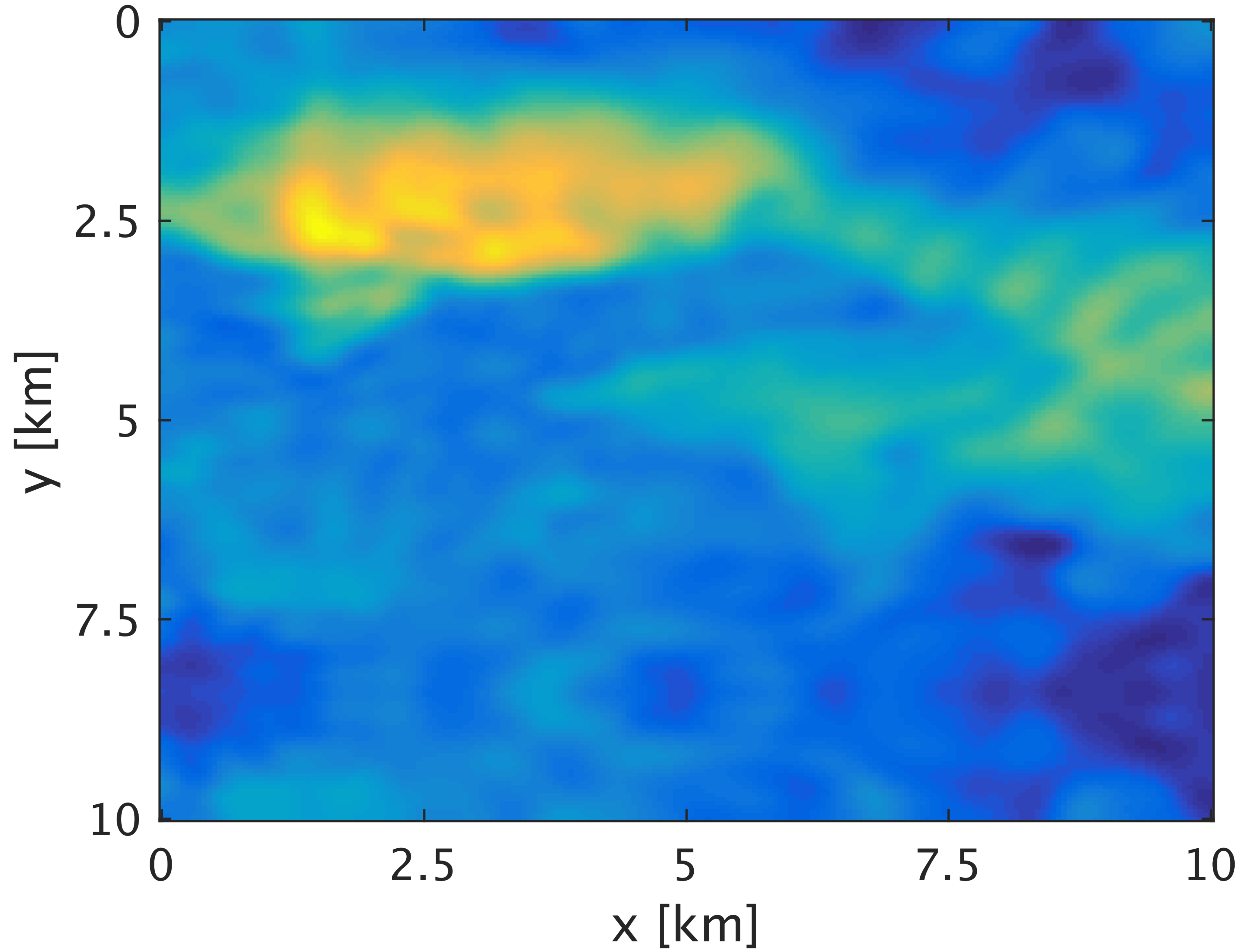


Full data

z = 1200m depth slice



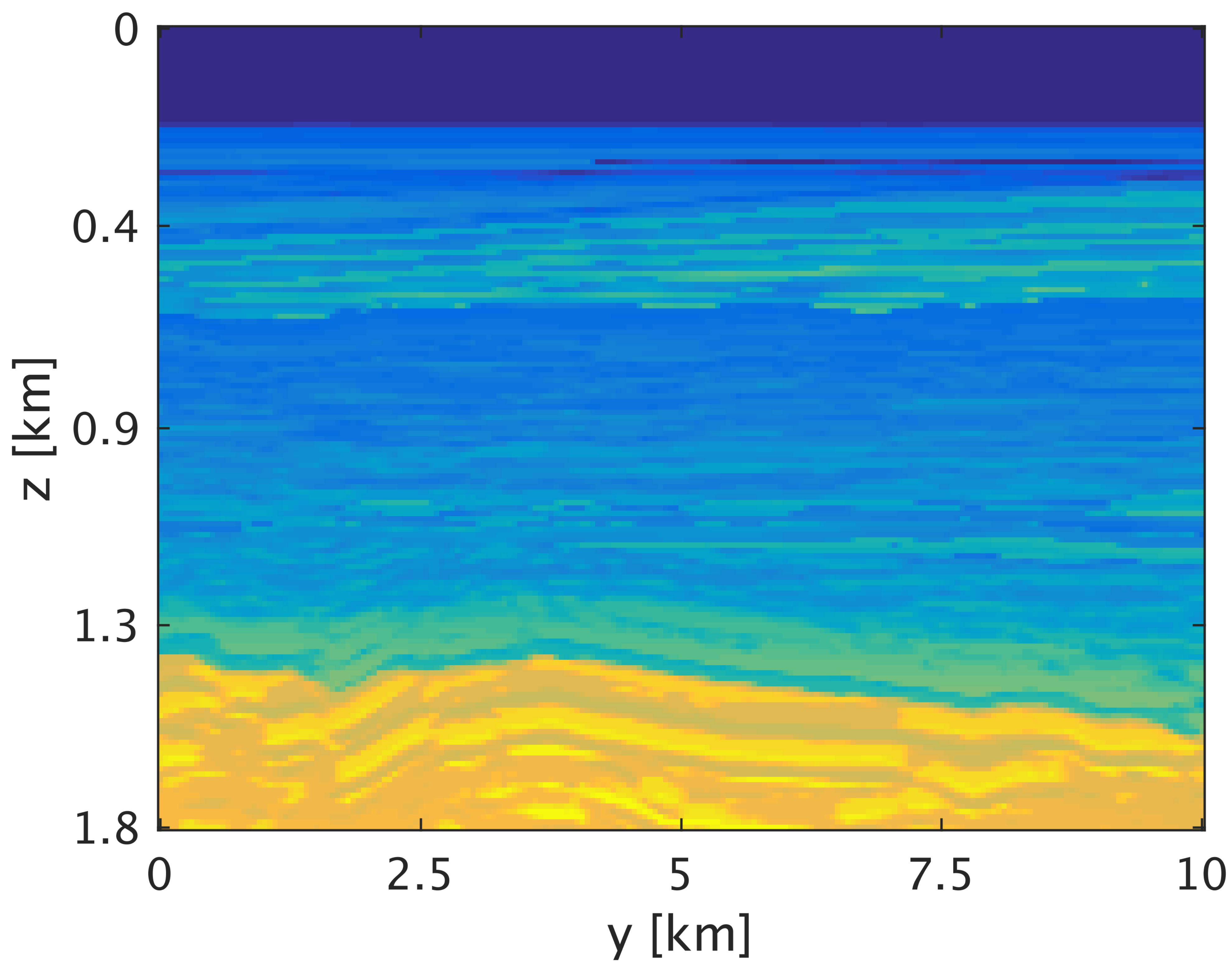
True model



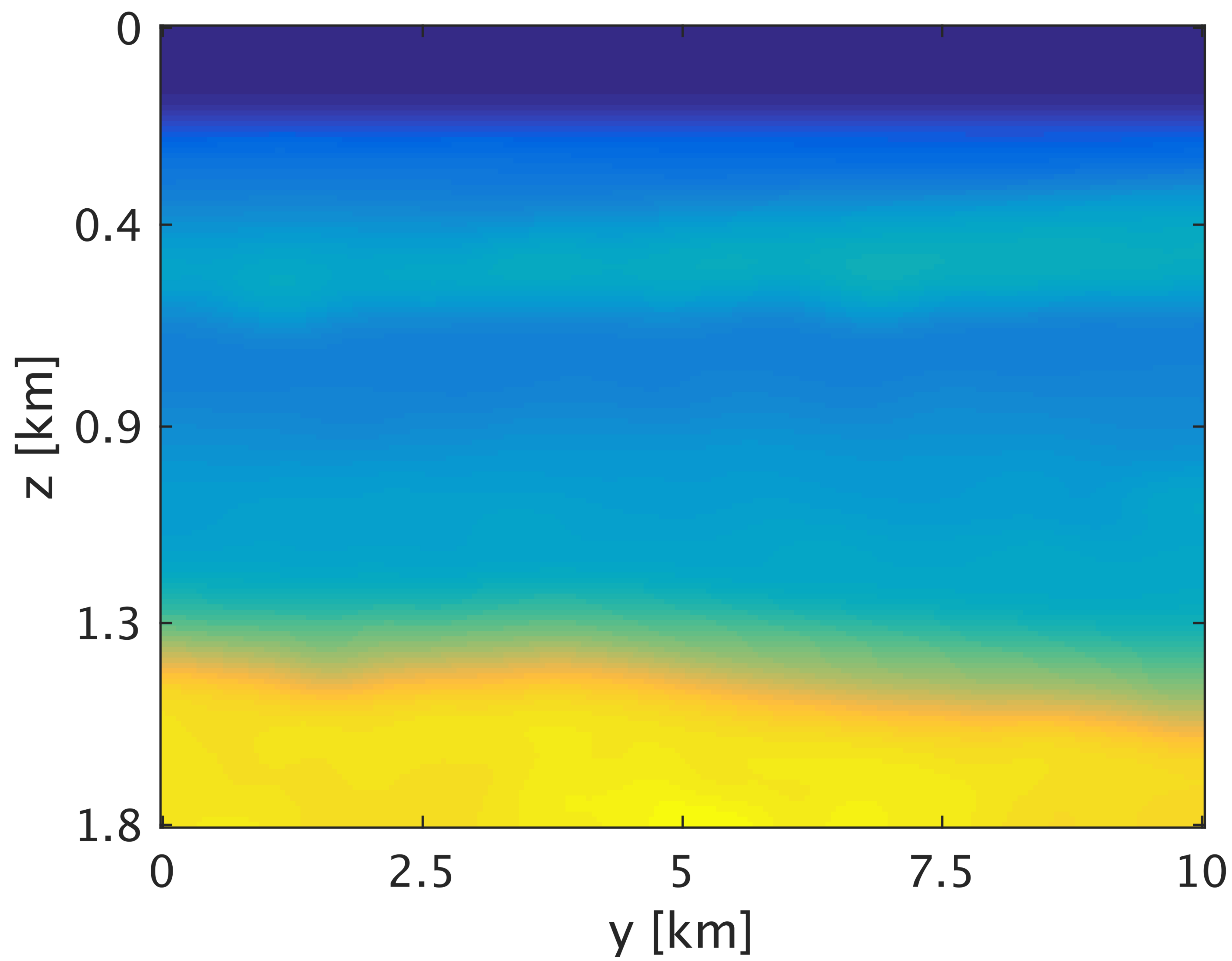
Compressed data

Subsampled data & Interpolated data

x = 4900m lateral slice

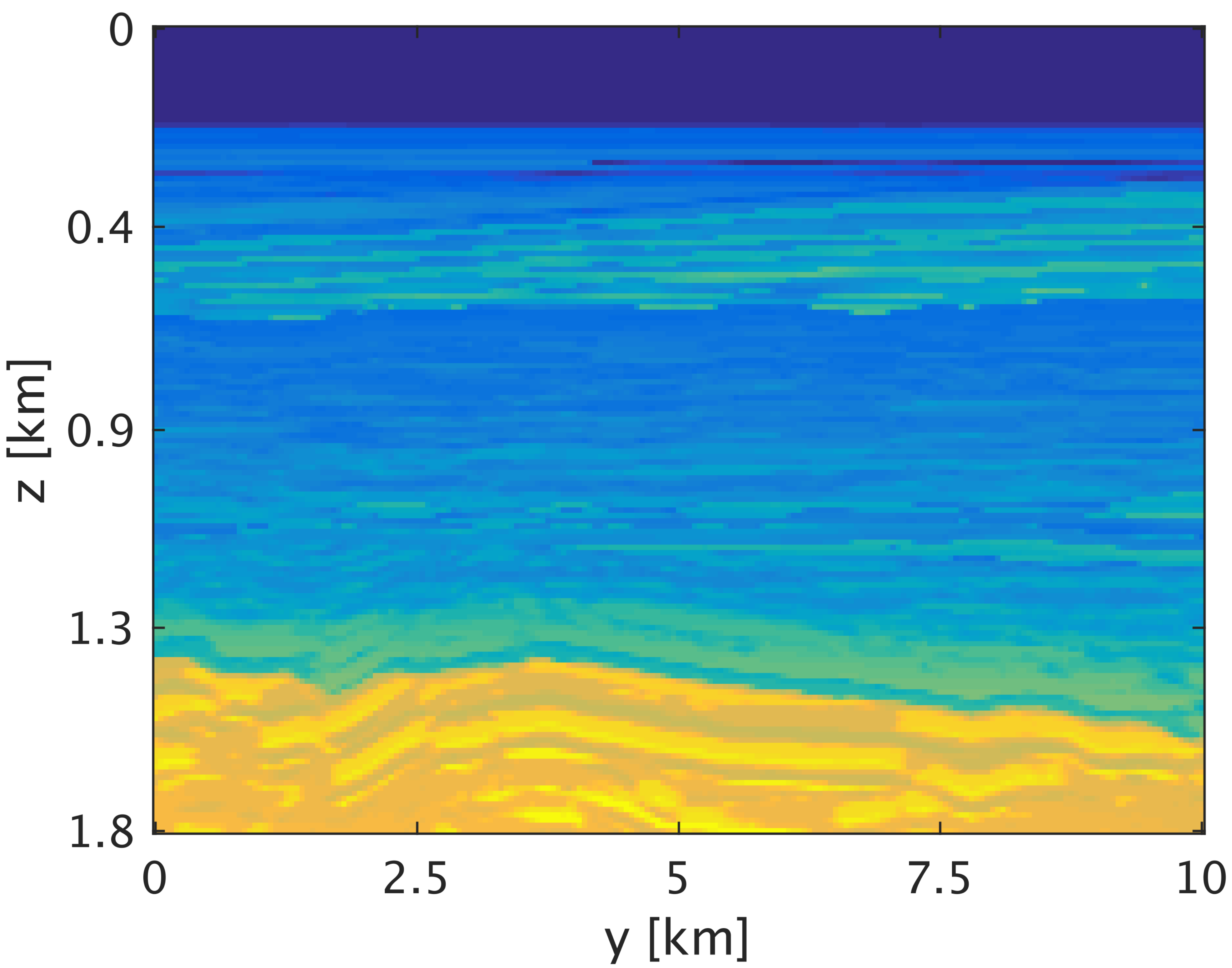


True model

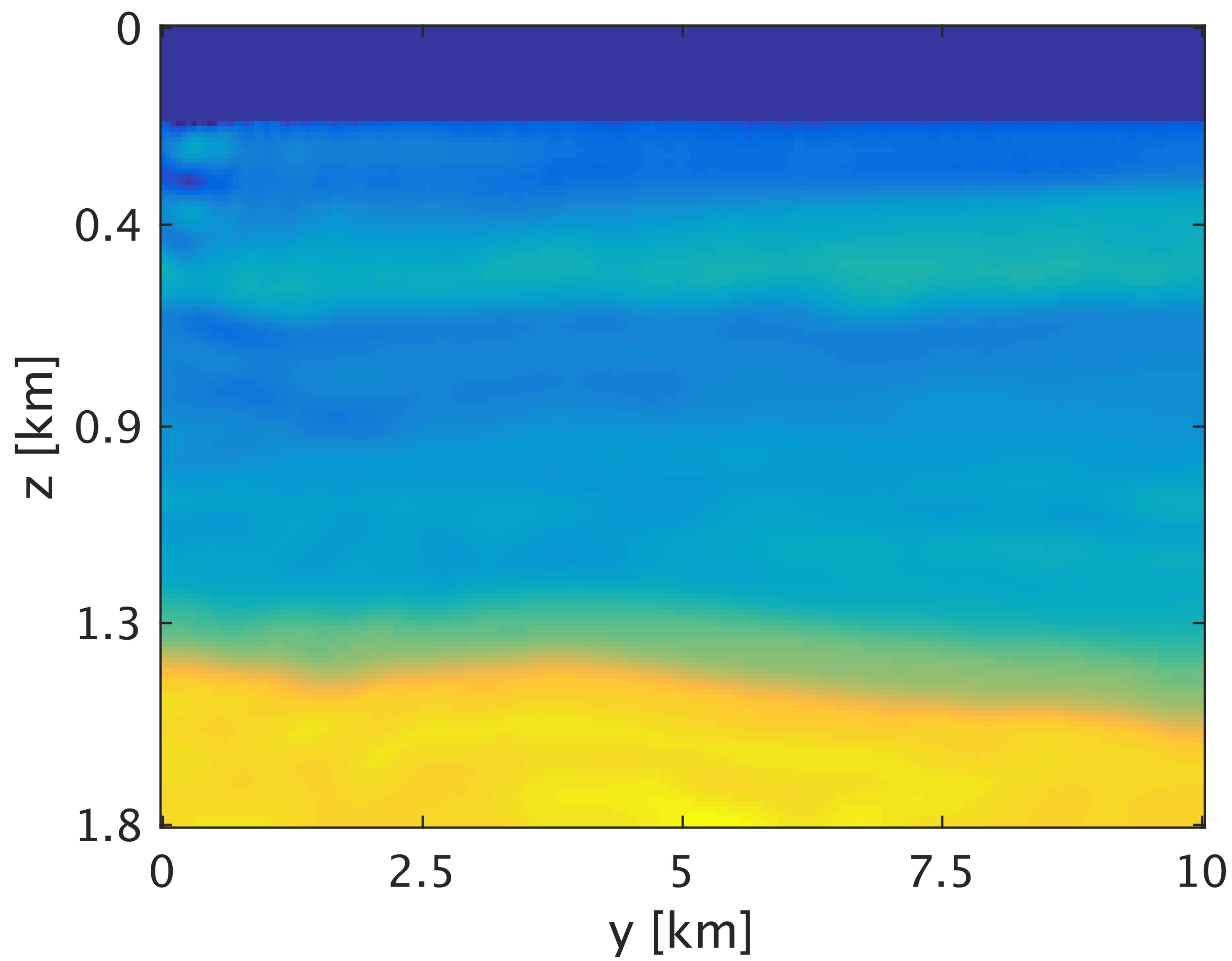


Initial model

x = 4900m lateral slice

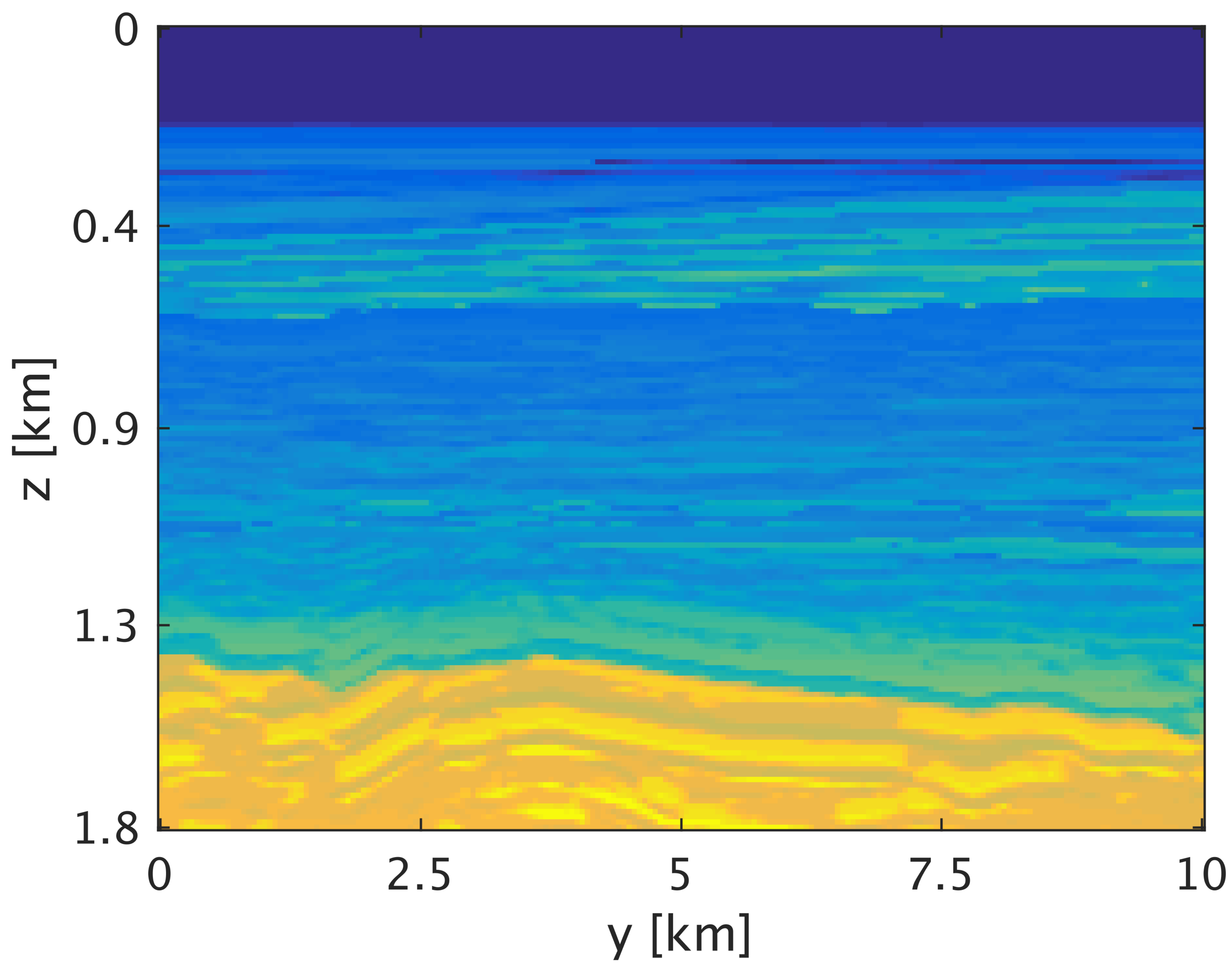


True model

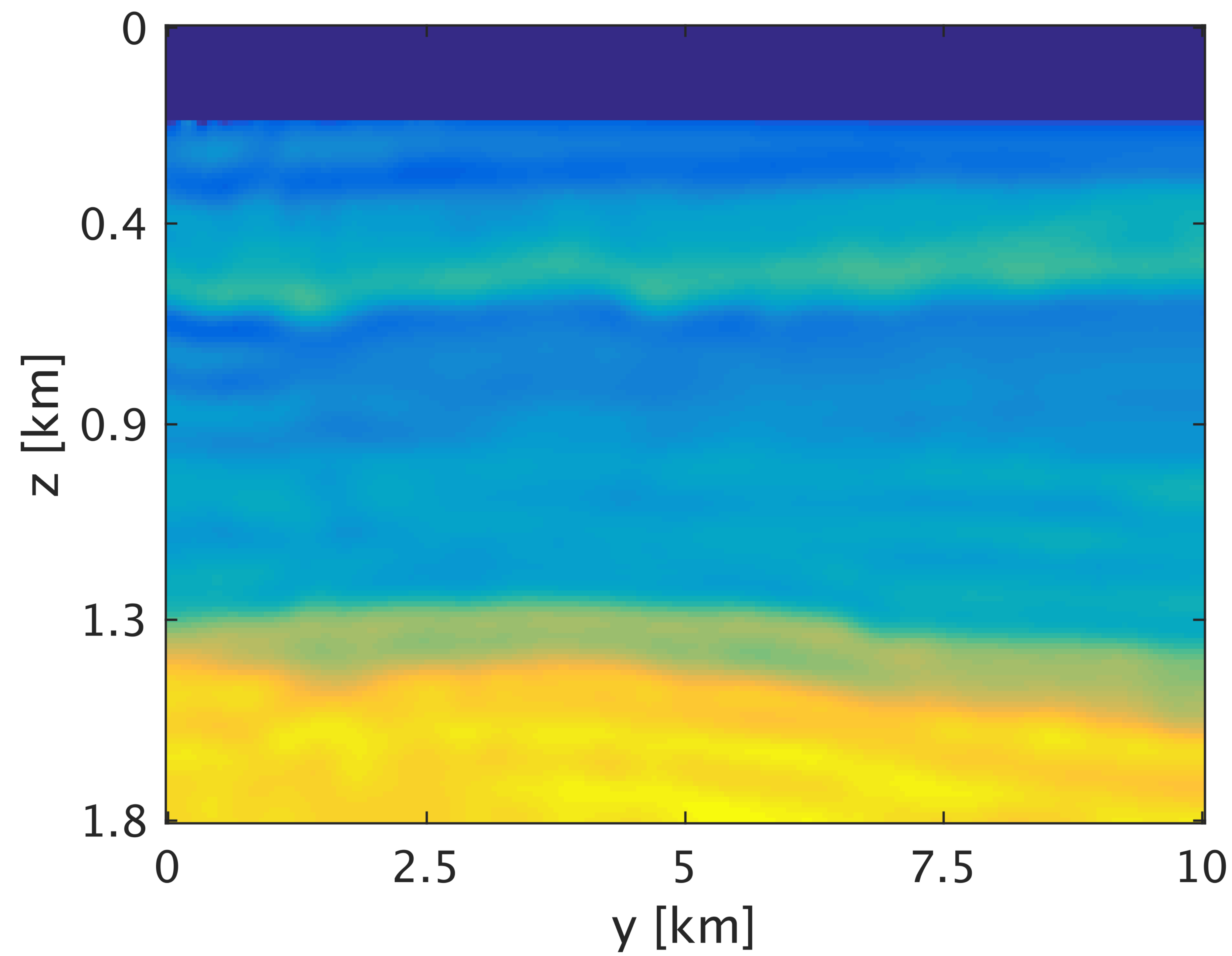


Subsampled data

x = 4900m lateral slice

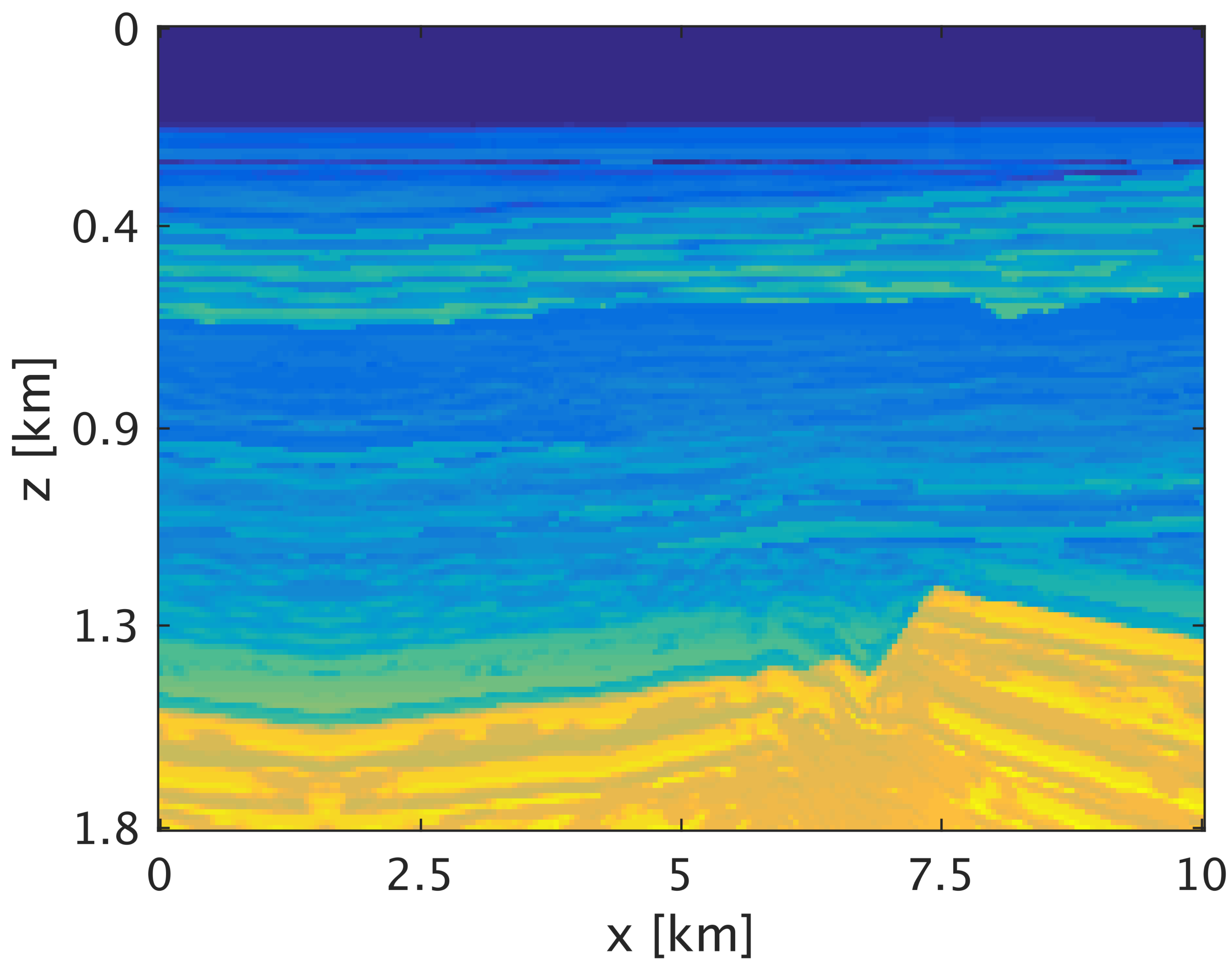


Full data

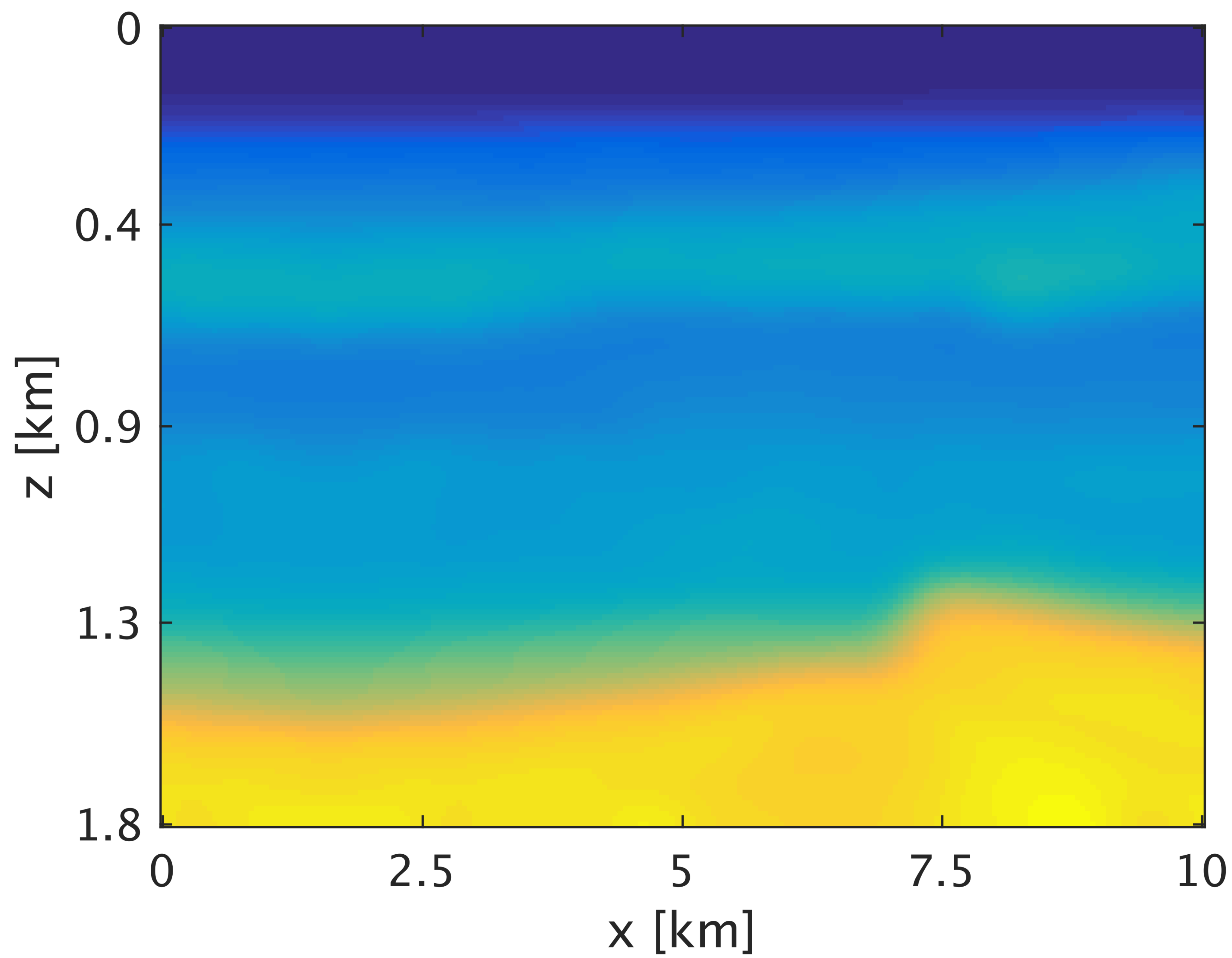


Interpolated data

y = 5650m lateral slice

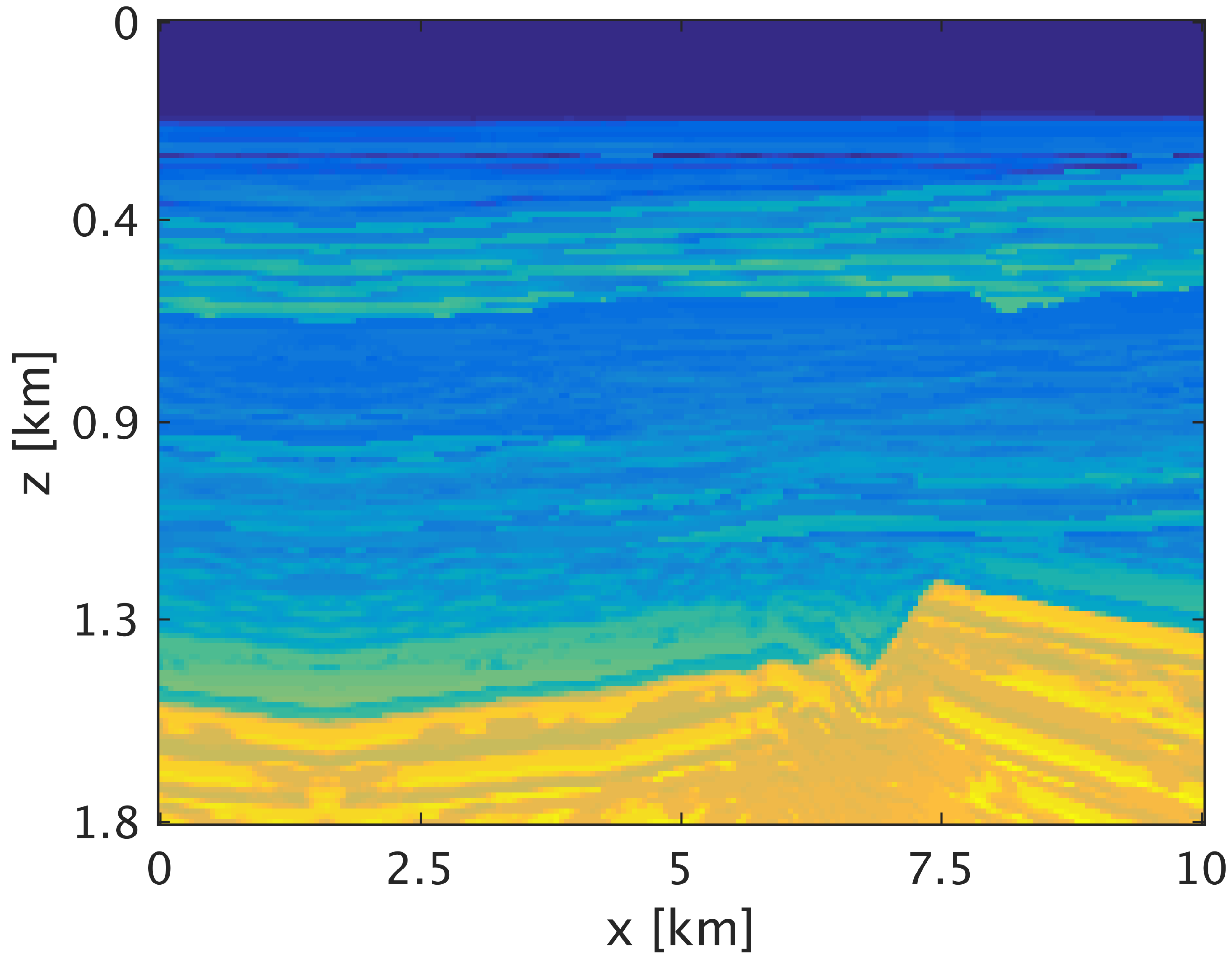


True model

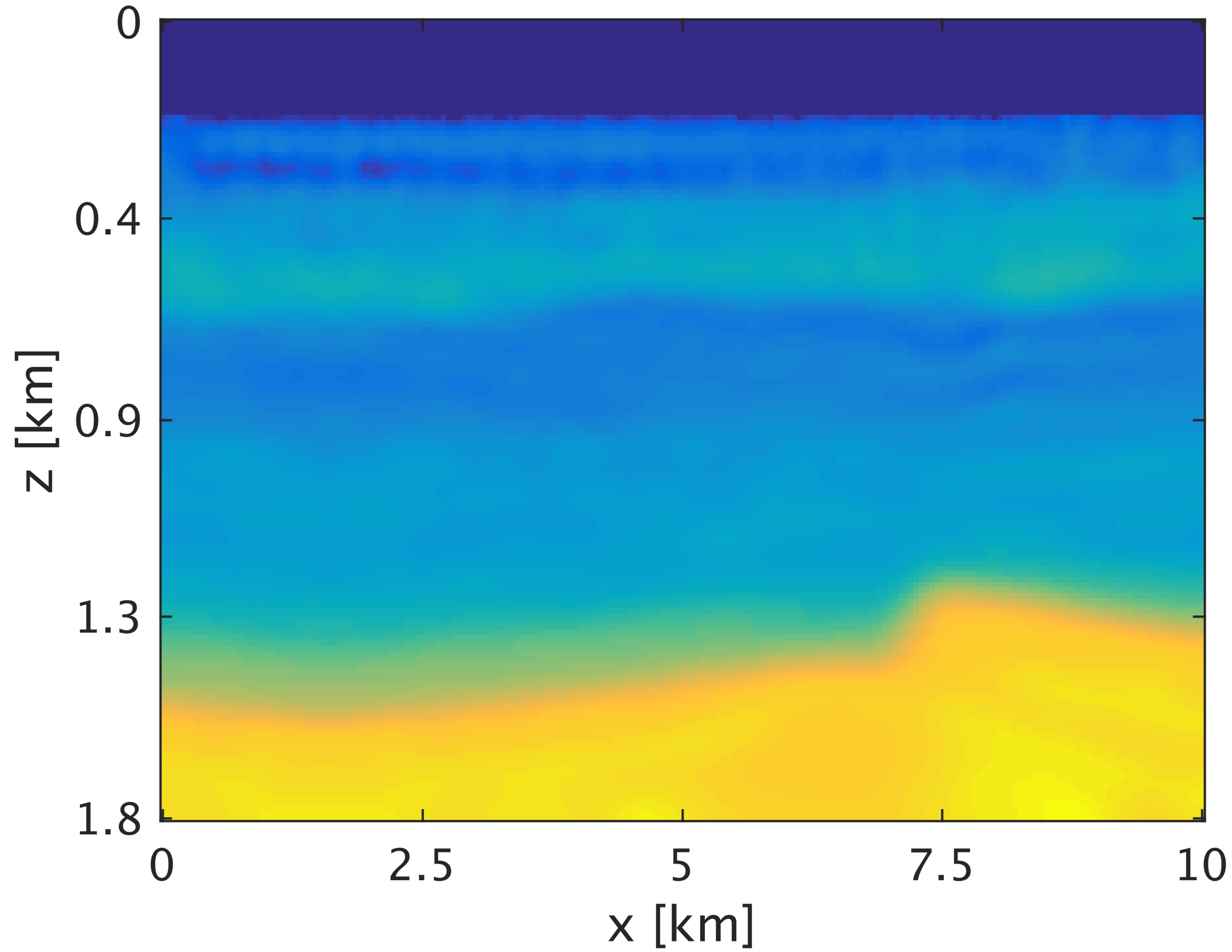


Initial model

y = 5650m lateral slice

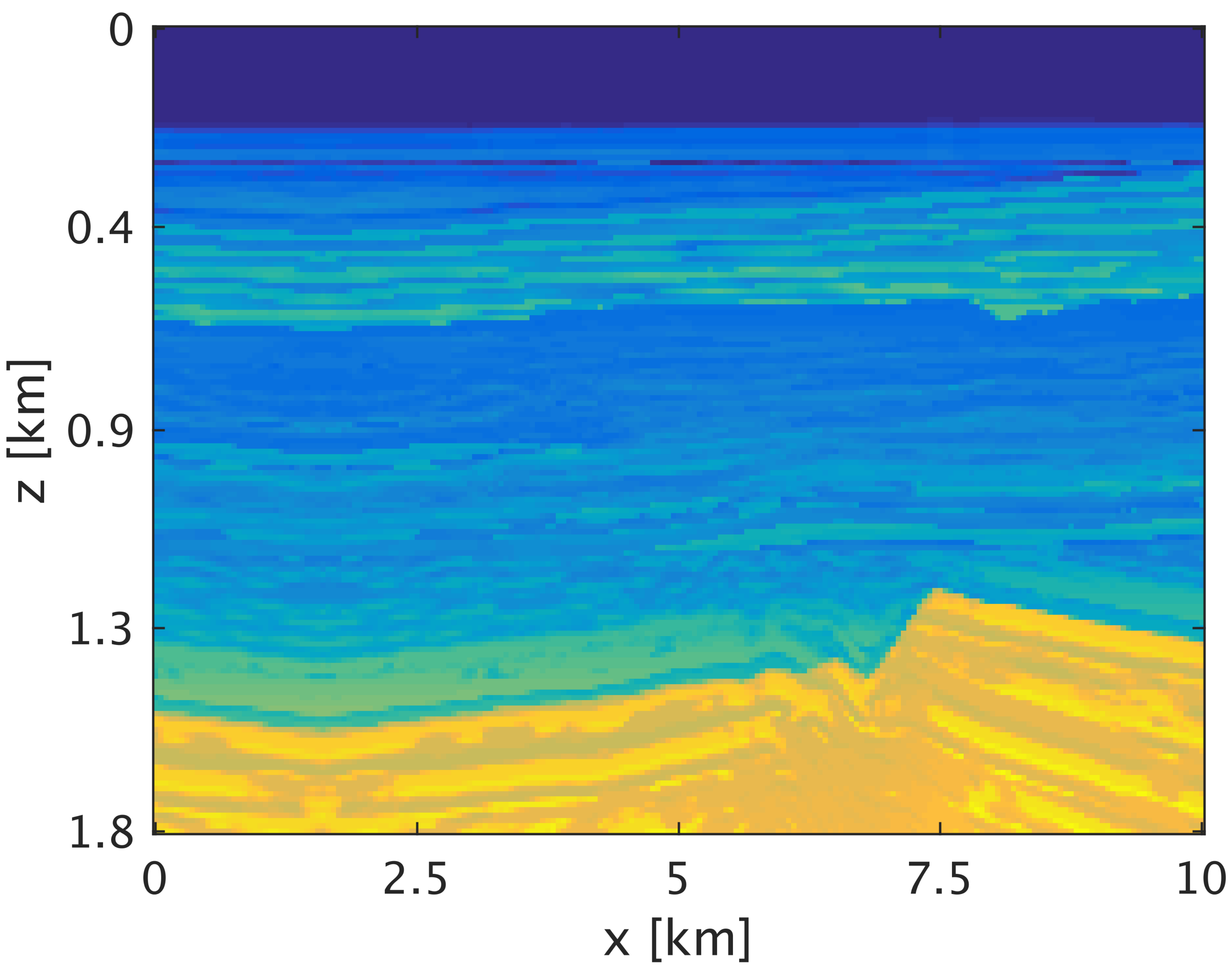


True model

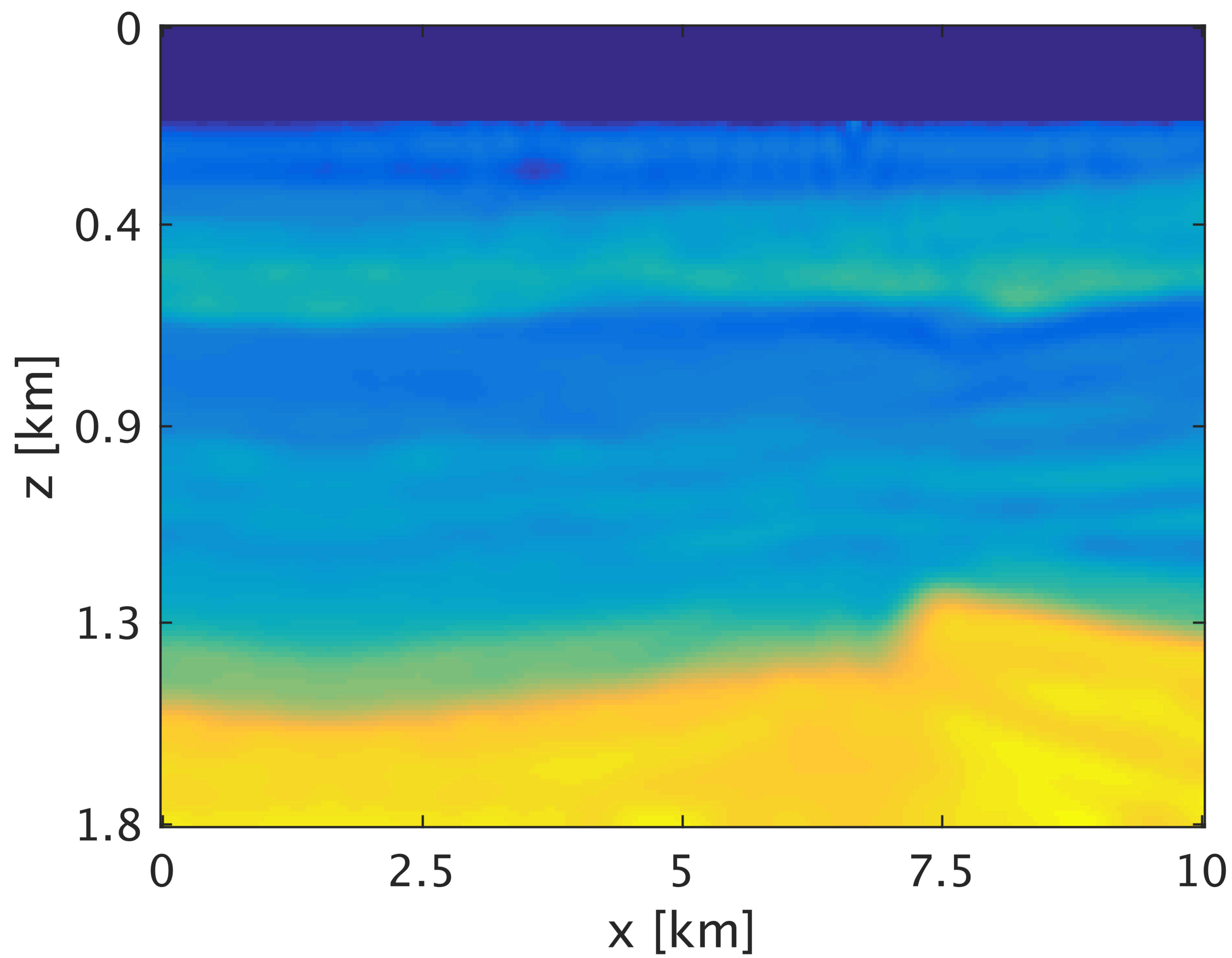


Subsampled data

y = 5650m lateral slice

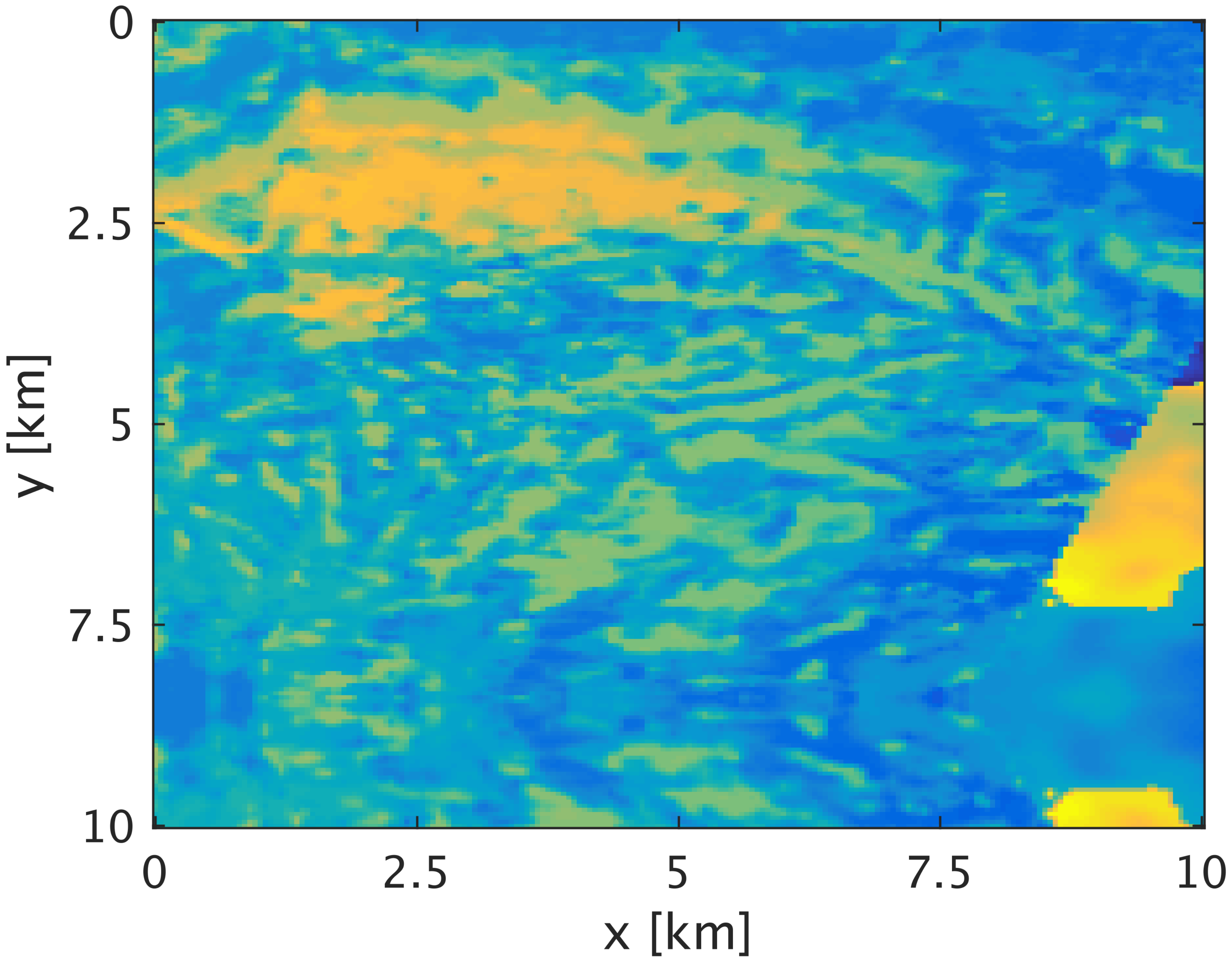


True model

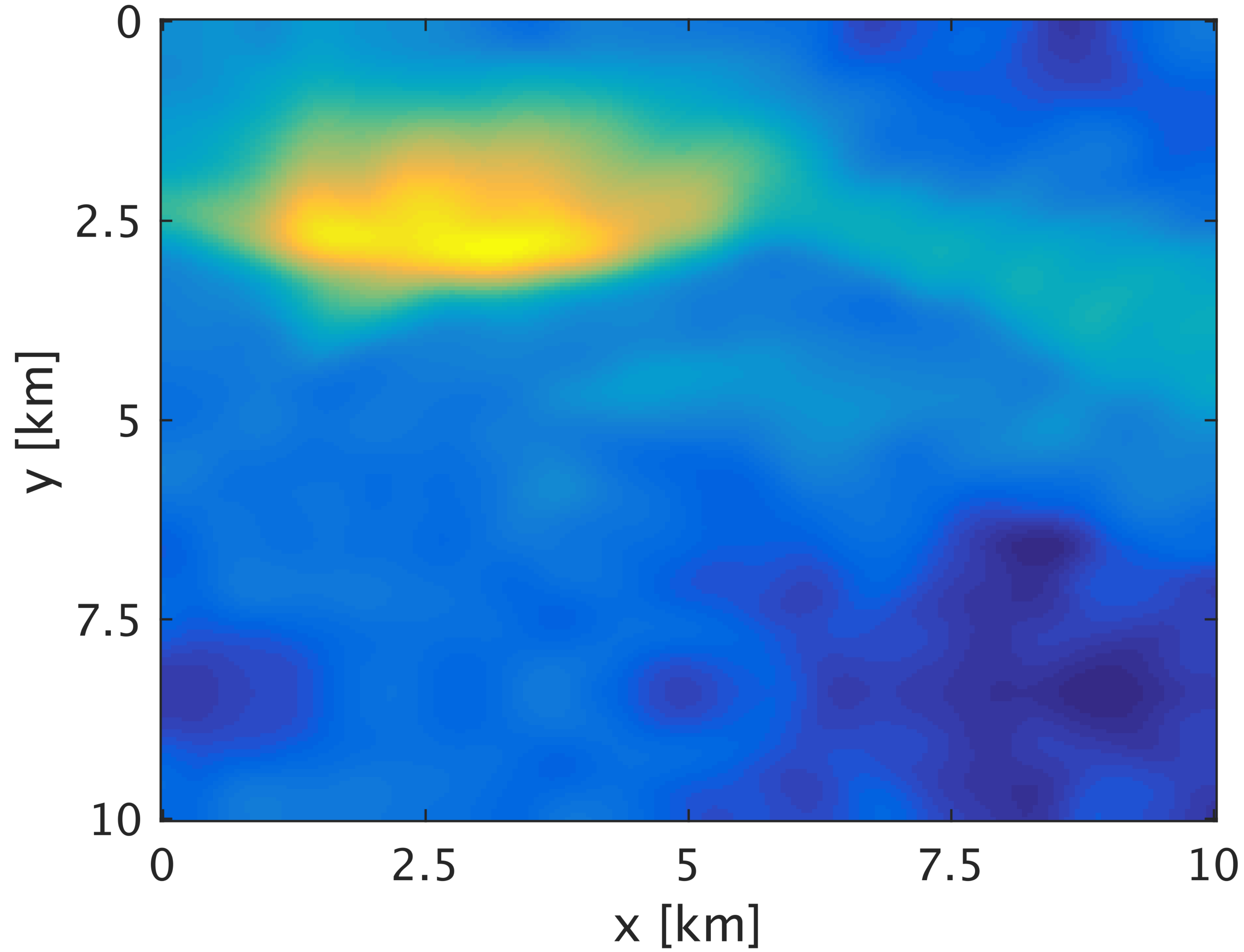


Interpolated data

z = 1200m depth slice

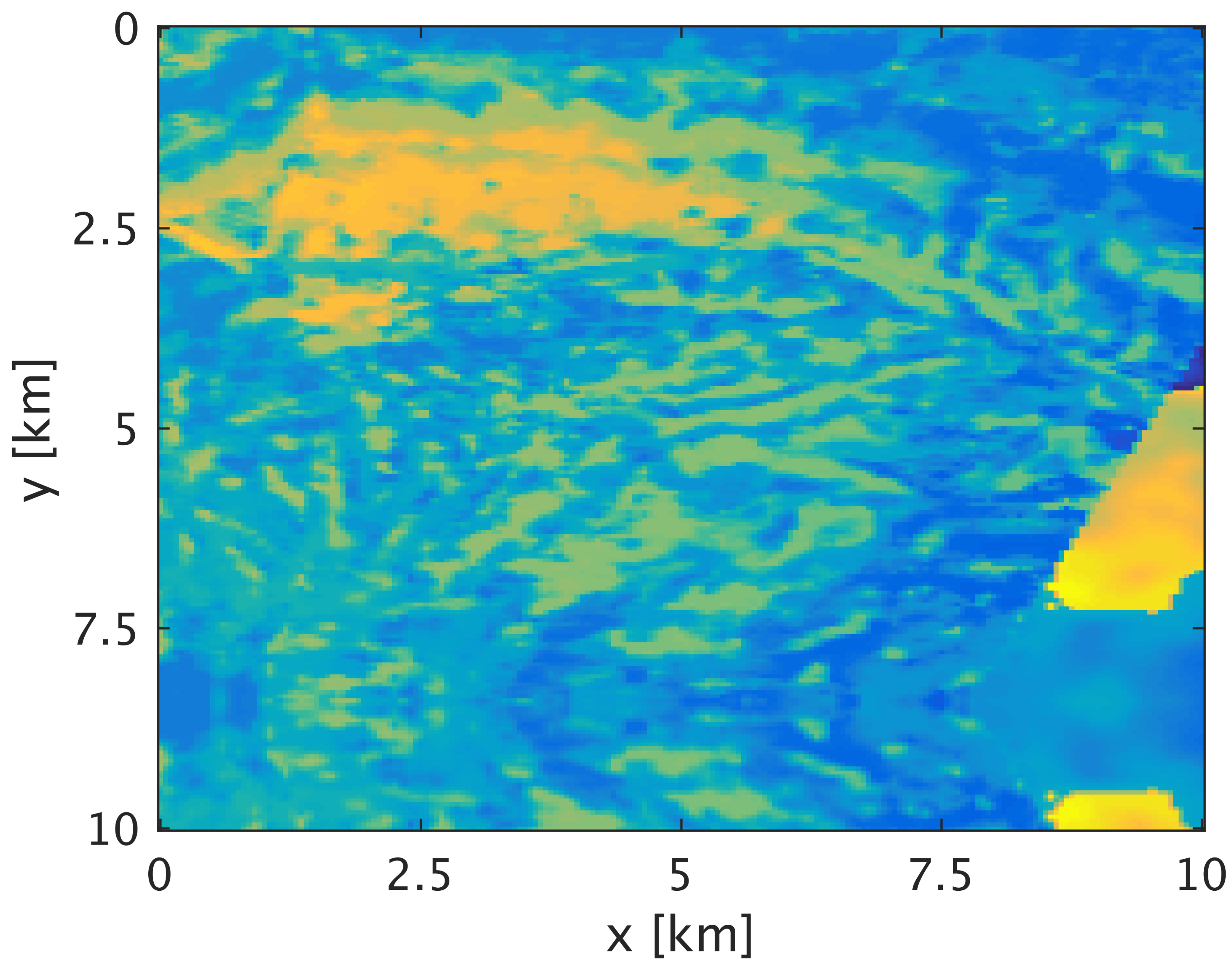


True model

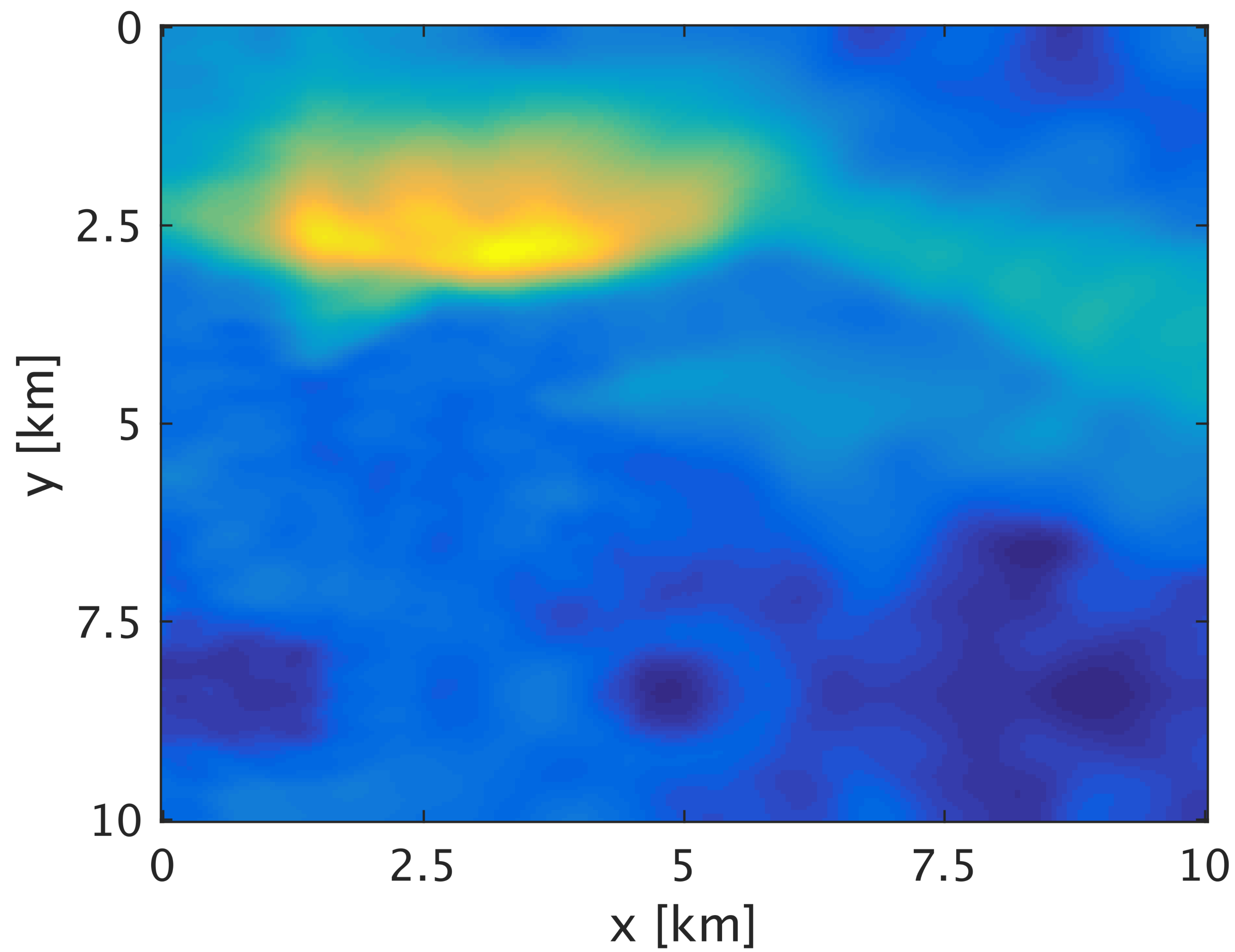


Initial model

z = 1200m depth slice

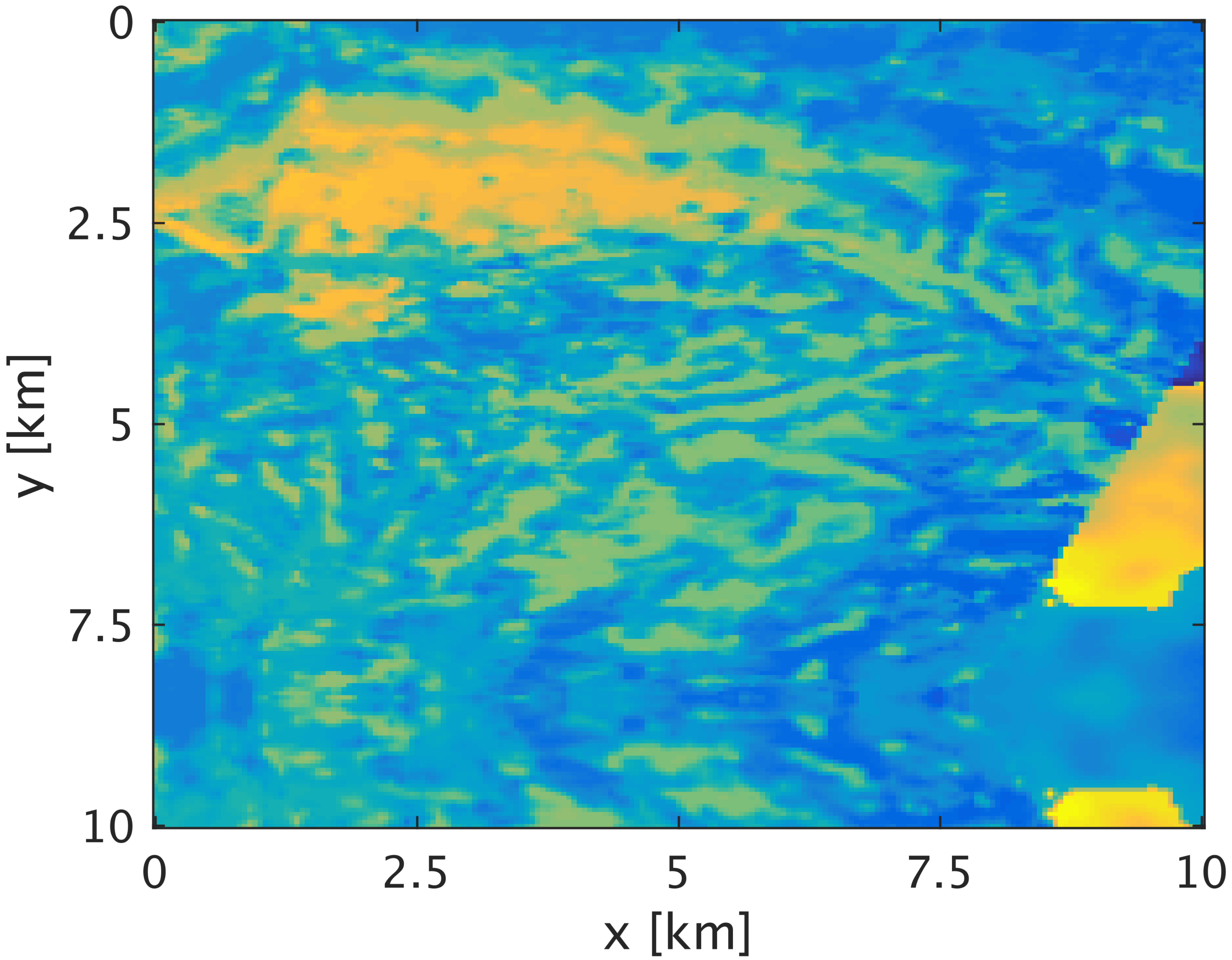


True model

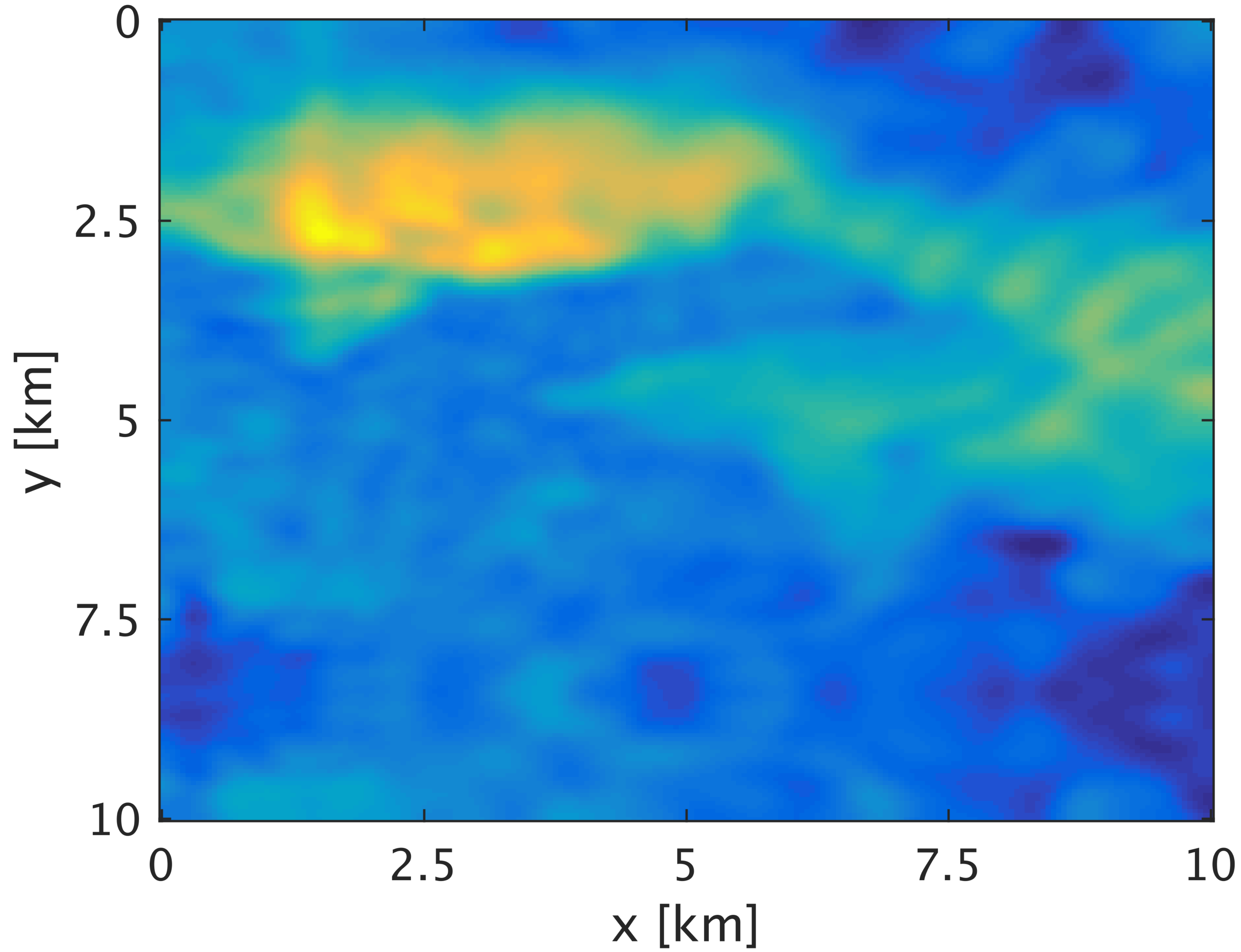


Subsampled data

z = 1200m depth slice



True model



Interpolated data

Case study 2: Extended Images

Extended images

Given two way wave equations, we define the **source** wavefield U and **receiver** wavefield V as

$$H(\mathbf{m})U = P_s^T Q$$
$$H(\mathbf{m})^* V = P_r^T D$$

where

$\mathbf{m}$: slowness

$H(\mathbf{m})$: discretization of the Helmholtz operator

$Q\ D$: source function and data matrix

$P_s^T\ P_r^T$: samples the wavefield at the source and receiver positions

Extended images

Organize wavefields in monochromatic data **matrices** where each **column** represents a **common** shot gather

Express image volume **tensor** for **single** frequency as a **matrix**

$$E = VU^*$$

Extended images

Image volume too l**arge** to form and too **expensive**

Instead, **probe** volume with **tall** matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_\ell]$

$$\widetilde{E} = EW = H^{-*} P_r^\top D Q^* P_s H^{-*} W$$

where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents **single** scattering points

Proposed method

$$\tilde{E} = H^{-*} P_r^T D Q^* P_s H^{-*} w$$

SLIM

Proposed method

$$\tilde{E} = H^{-*} P_r^T D Q^* P_s H^{-*} w$$

Proposed method

$$\tilde{E} = H^{-*} P_r^T D Q^* P_s H^{-*} w$$

$$\downarrow$$

$$v \in \mathbb{C}^{n_s \times 1}$$

Proposed method

$$\tilde{E} = H^{-*} P_r^T D Q^* P_s H^{-*} w$$

$$\downarrow$$

$$\tilde{E} = H^{-*} P_r^T D v$$

Algorithm 1

$$\tilde{E} = H^{-*} P_r^T D v$$

Single common shot gather extraction technique

Algorithm 1

$$\tilde{E} = H^{-*} P_r^T Dv$$

Single common shot gather extraction technique

Input: $\mathbf{v} = Q^* P_s \mathbf{H}^{-*} \mathbf{w}$
Output: $\mathbf{z} = \mathbf{D}\mathbf{v}$
For each source index $\mathbf{i} = (i_{\mathrm{src}_x}, i_{\mathrm{src}_y})$

1. Extract the common shot gather from the data using our proposed, resulting in $\mathbf{D_i}$;
2. Scale $\mathbf{D_i}$ by a scalar $\mathbf{v}_{\mathrm{i}}$ to produce $\mathbf{z}$;
3. Update $\mathbf{z}$ with addition of previous $\mathbf{z}$.

Algorithm 2

$$\tilde{E} = H^{-*} P_r^T Dv$$

Simultaneous common shot gathers extraction technique

Algorithm 2

$$\tilde{E} = H^{-*} P_r^T Dv$$

Simultaneous common shot gathers extraction technique

Input: $\mathbf{v} = Q^* P_s \mathbf{H}^{-*} \mathbf{w}$
Output: $\mathbf{z} = \mathbf{D}\mathbf{v}$
For source indices $\mathbf{i} = (i_{\mathrm{src}_x}, \mathrm{src}_y)$

1. Extract simultaneous common shot gathers from the data using our proposed, resulting in $\mathbf{D_i}$
2. Multiply $\mathbf{D_i}$ with $\mathbf{v}(i,:)$ to produce $\mathbf{z}$
3. Update $\mathbf{z}$ with addition of previous $\mathbf{z}$

Common Image-point gather

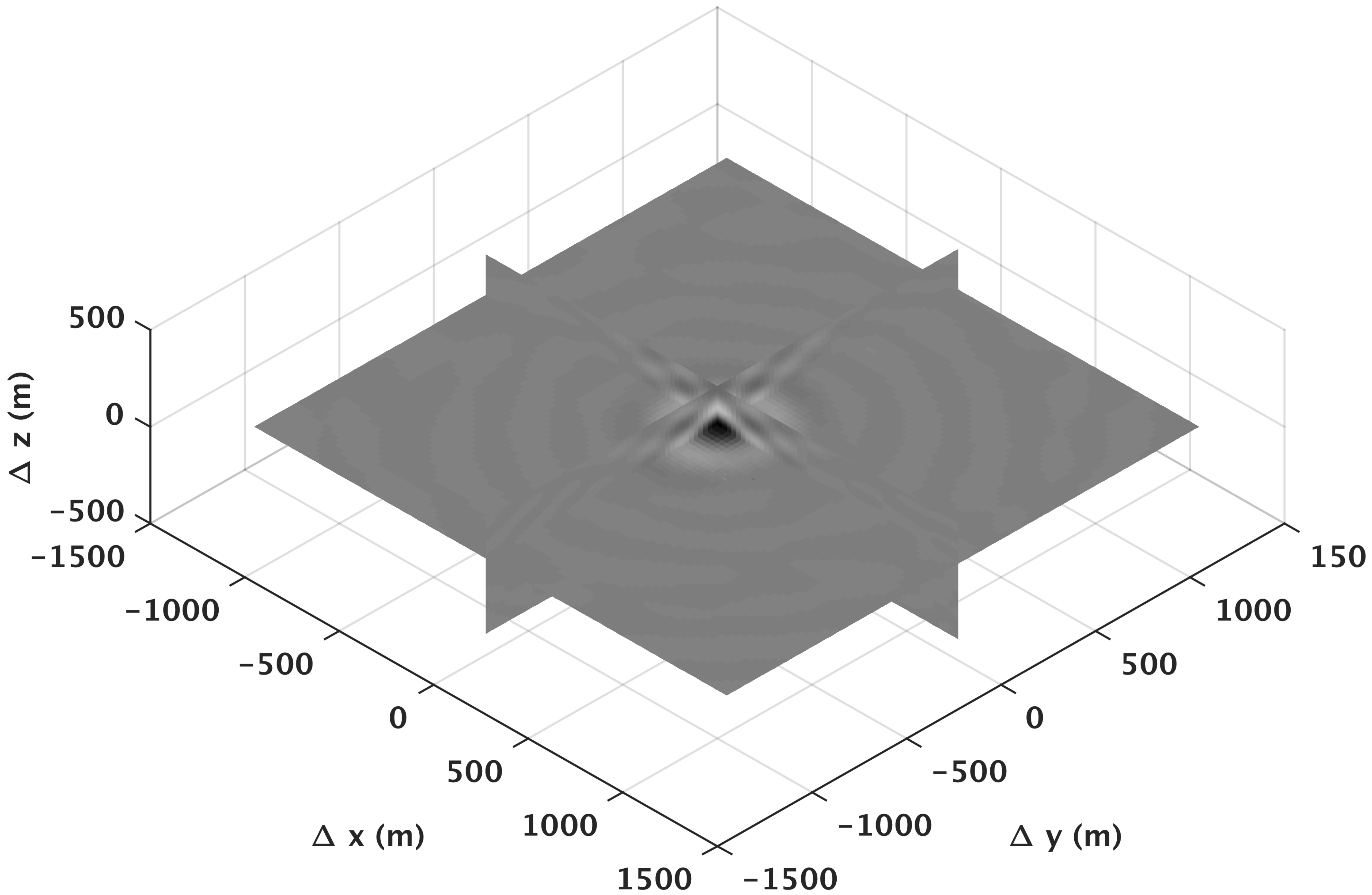
Model:

- 3D **BG** model
- 1.25km x 1.25km x 0.39 km
- 25m x 25m x 6 m spacing

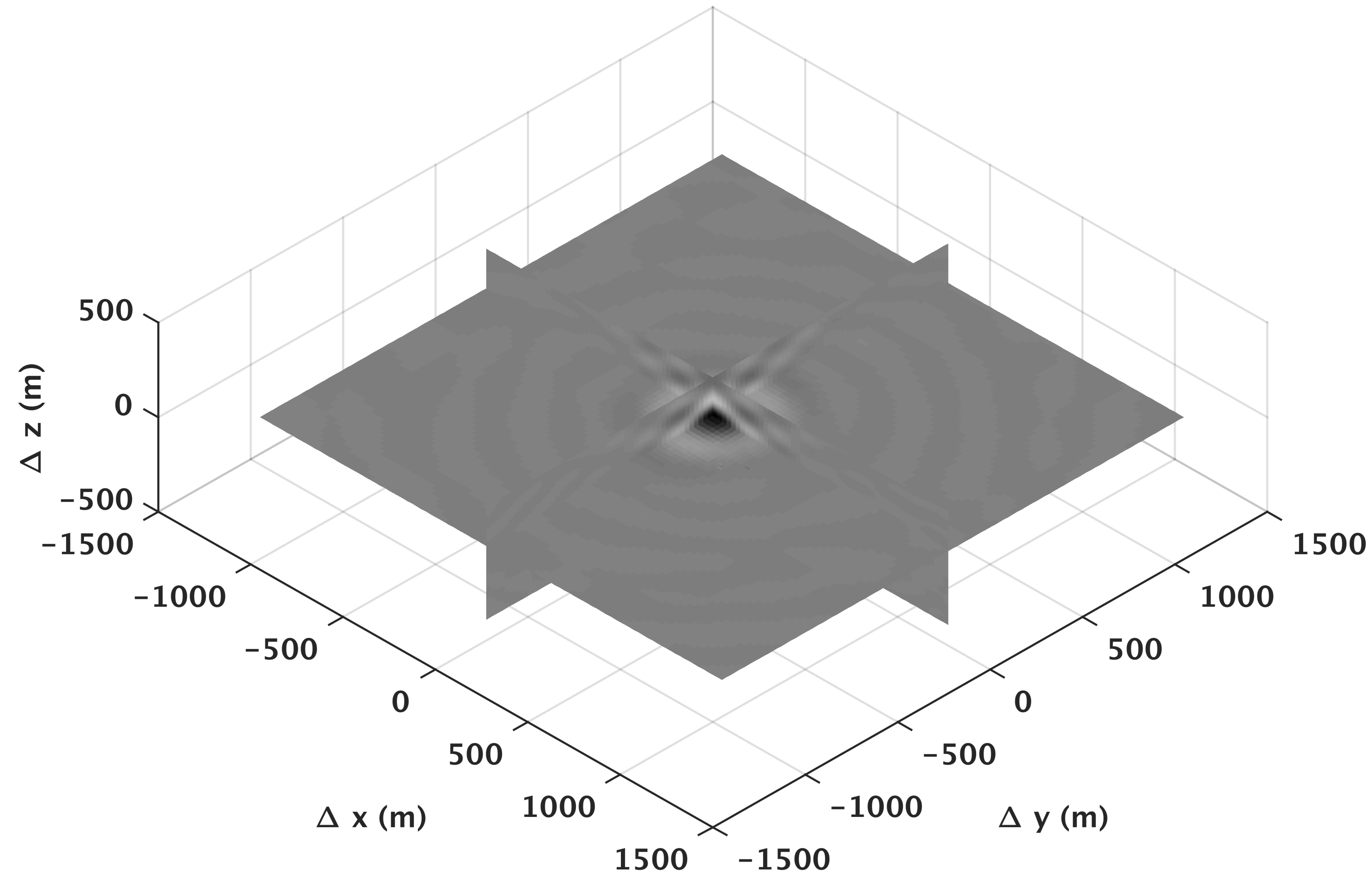
Experiment details:

- OBN acquisition
- 1156 sources (75m spacing), 2601 receivers (50m spacing)
- Ricker wavelet, 15 peak frequency
- 5-12 Hz, 0.5Hz interval

Common-point gather at (1250m, 1250m, 390m)

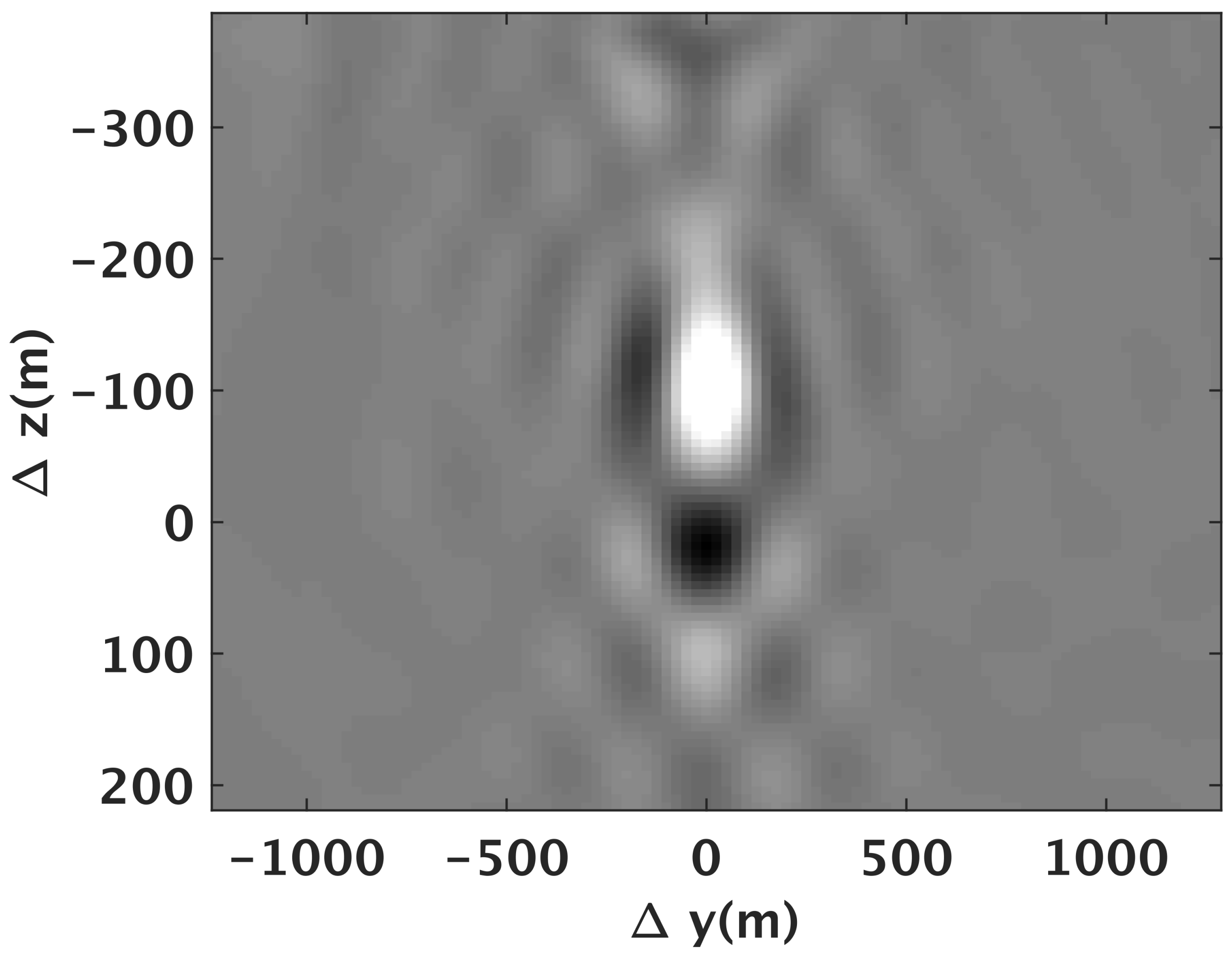


Full data

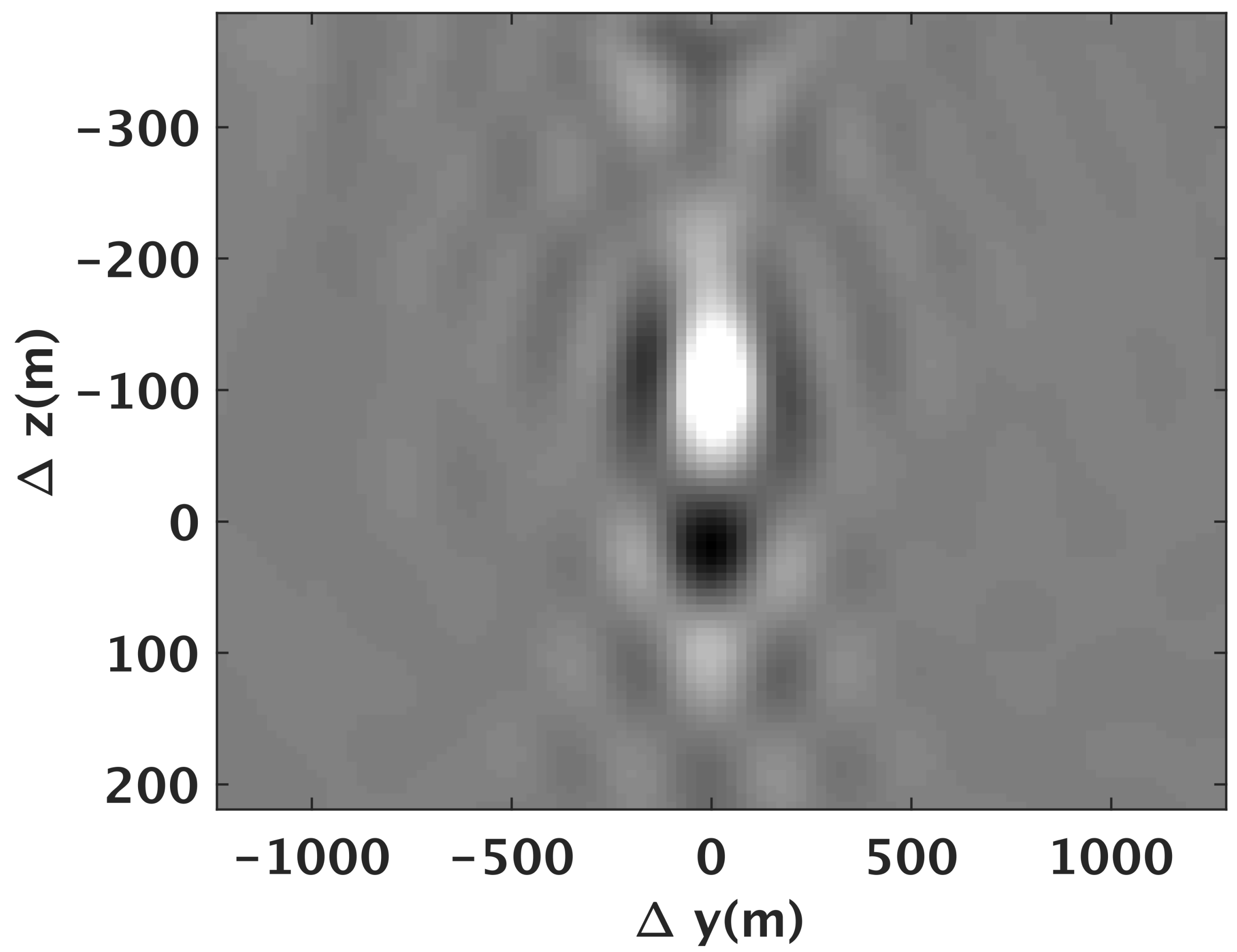


Compressed data

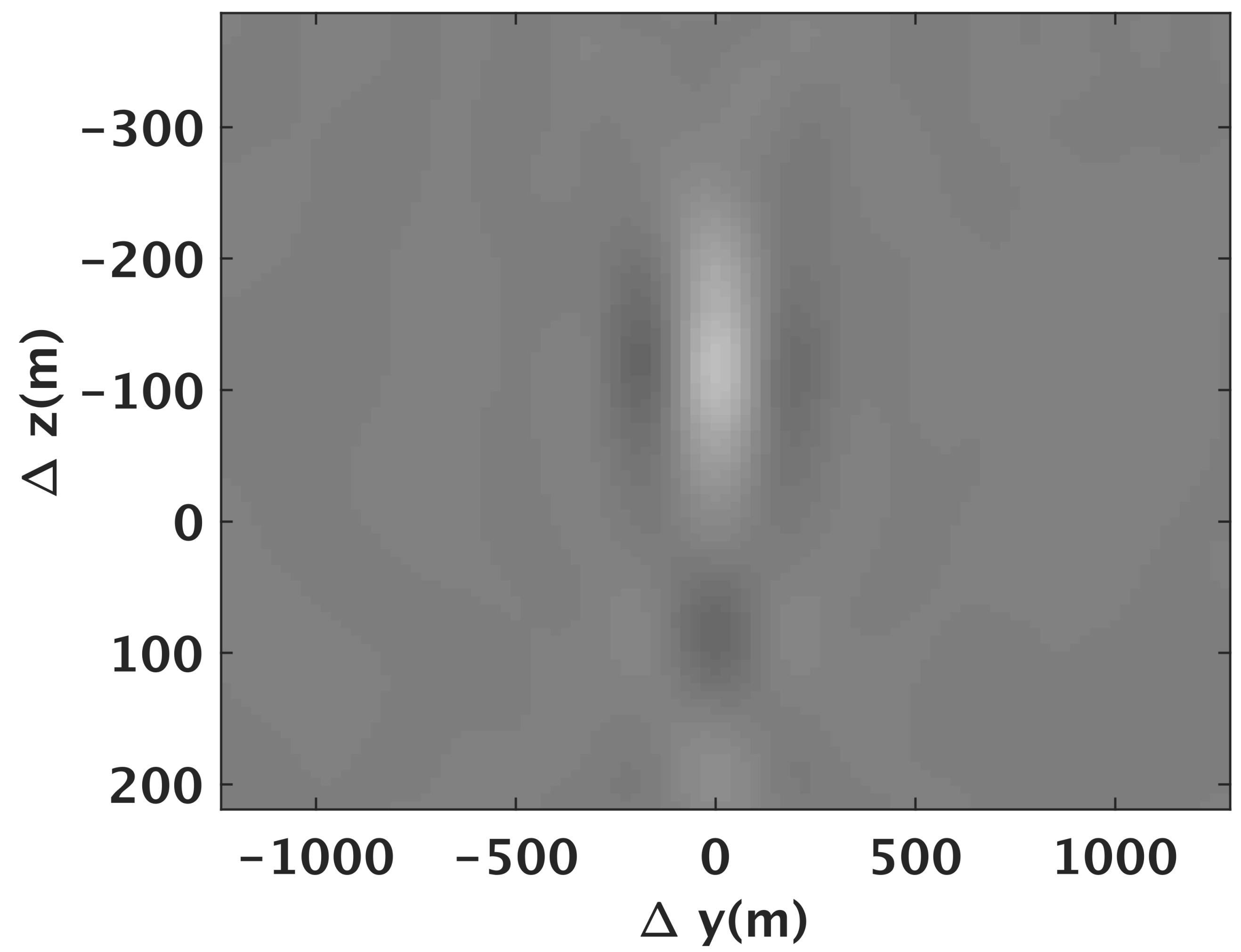
Along lateral offset direction



Full data

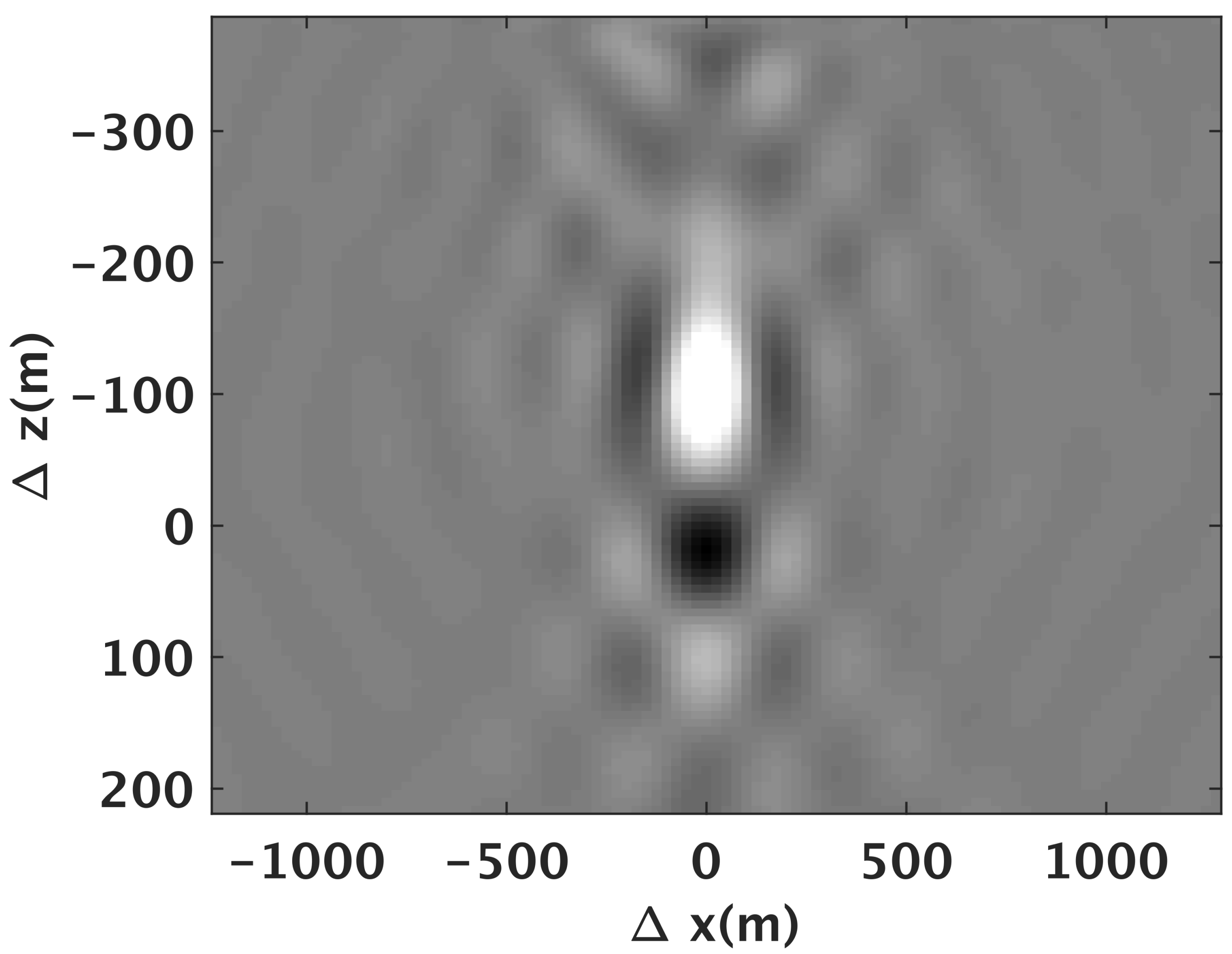


Compressed data

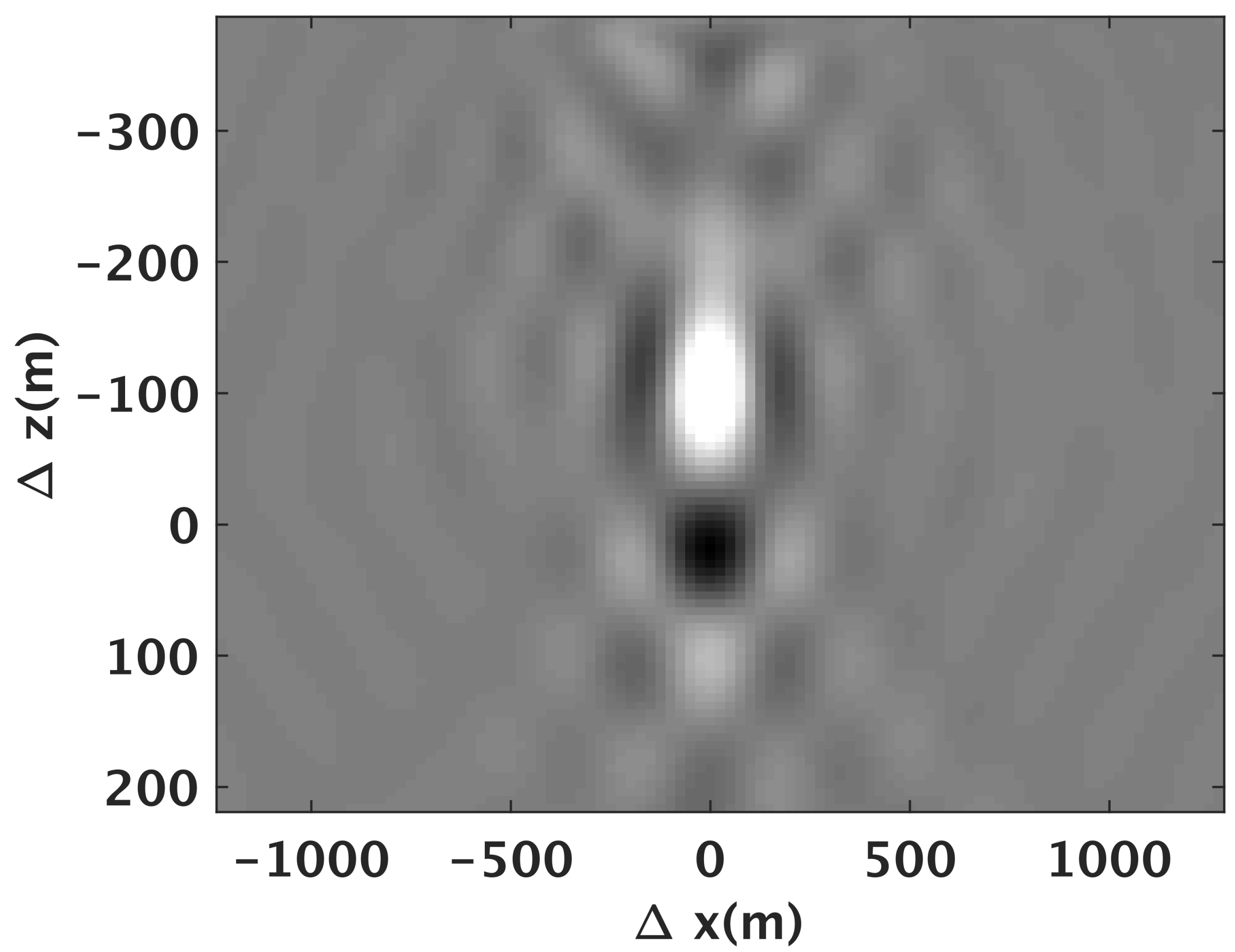


Difference x 100

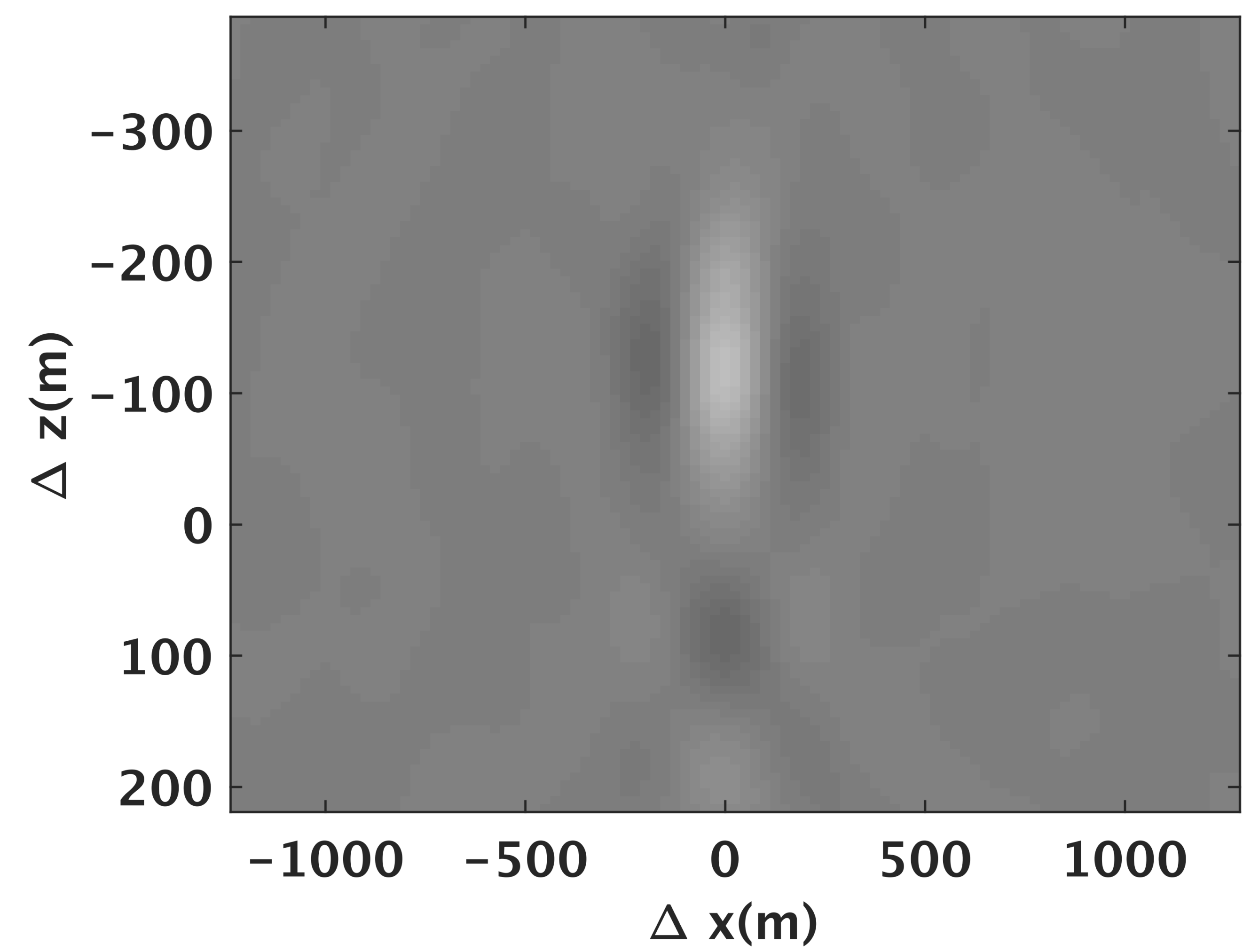
Along lateral offset direction



Full data

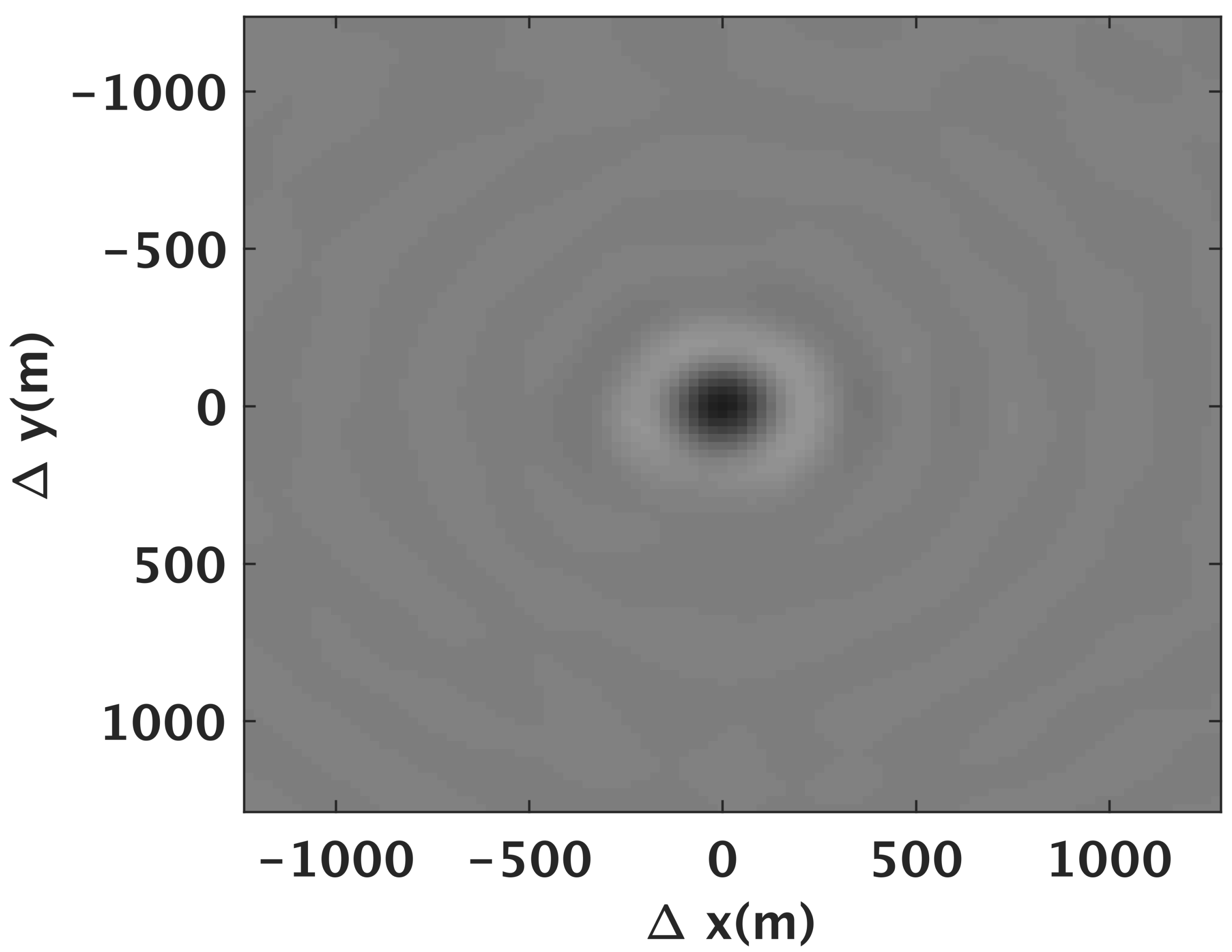


Compressed data

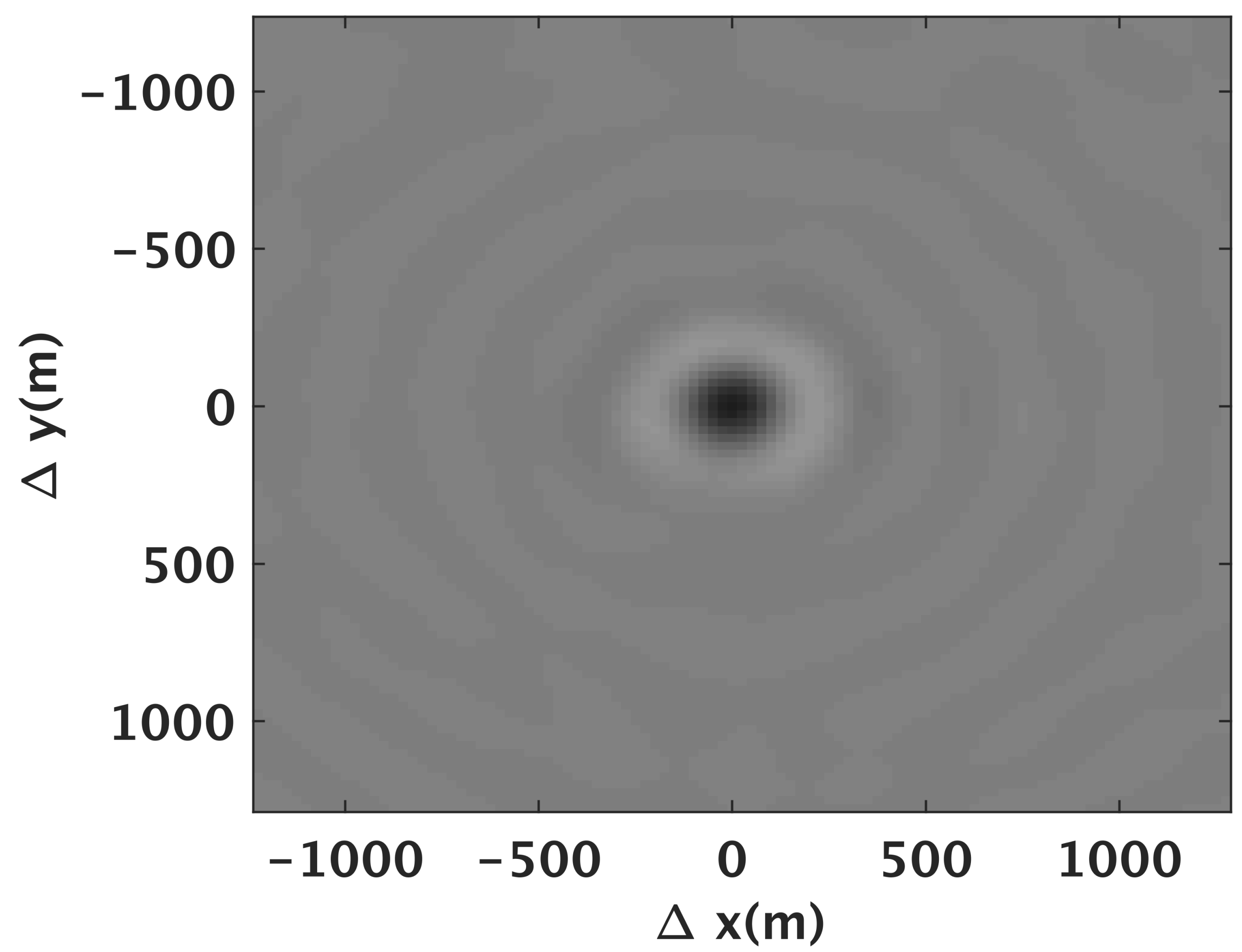


Difference x 100

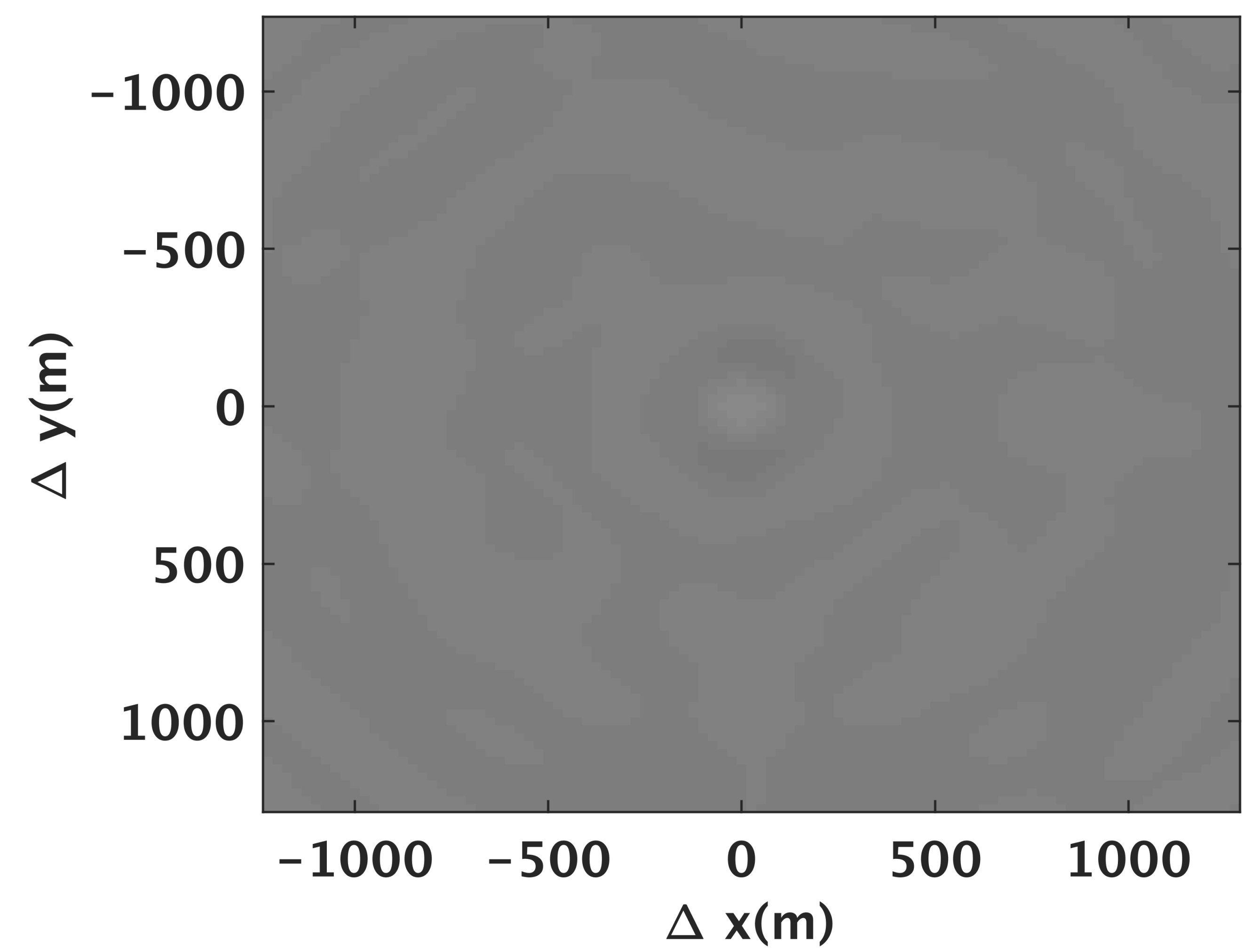
Along vertical offset direction



Full data



Compressed data



Difference x 100

Conclusions

- **high** compression ratio is achievable
- reduces **memory & computational** costs when combined w/ stochastic optimization/ probing technique
- codes **easily embedded** into other processing frameworks
- leads to major reduction in IO for **low-frequency** full waveform inversion & extended images
- suitable for both **fully sampled** data and **missing random** receivers/shots

Acknowledgements

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SLIM

Acknowledgements

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Thank you for your attention