Imaging with multiples in shallow water

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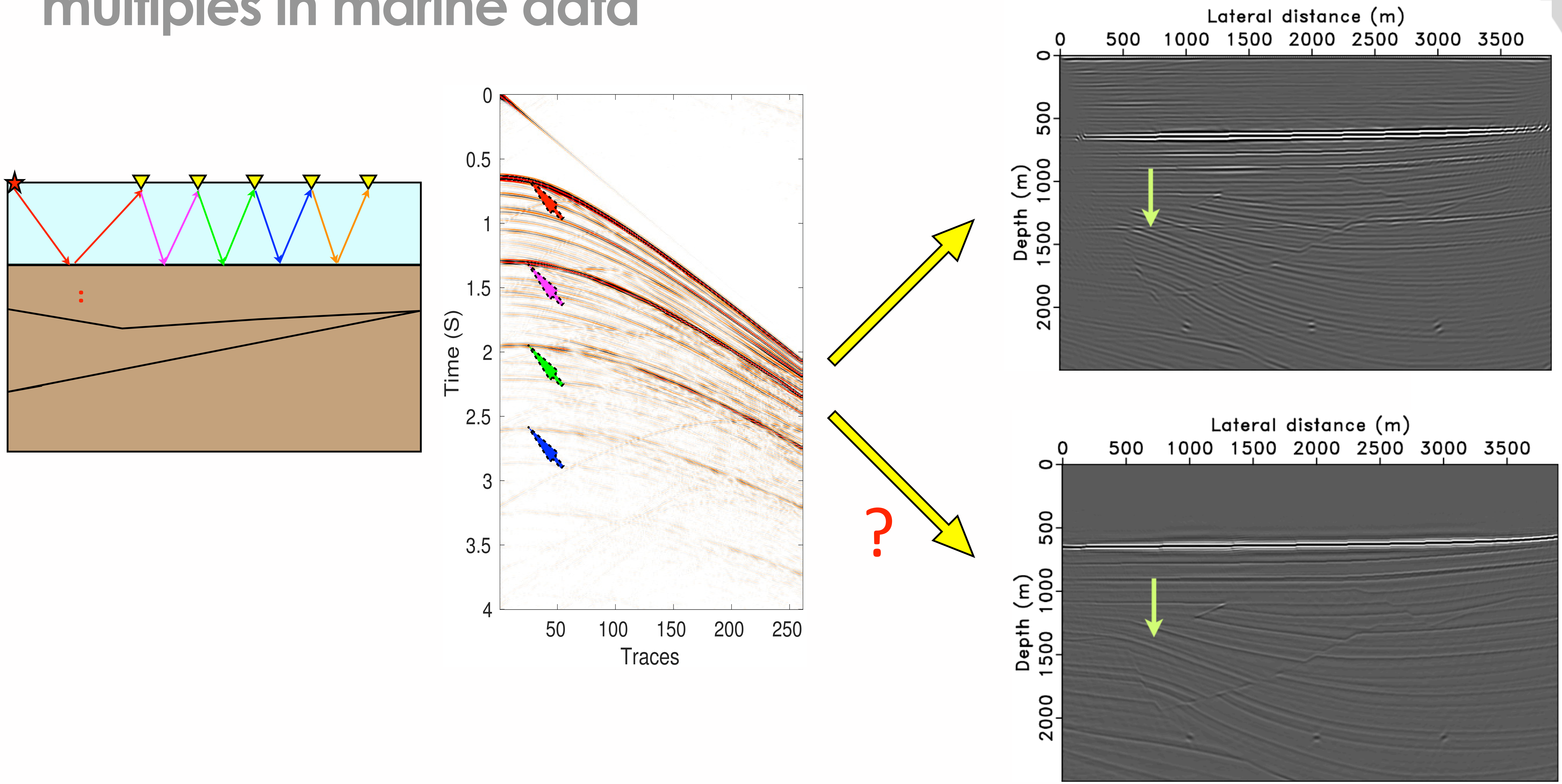
Situation
multiples in marine data



From Ning Tu, EAGE, 2013

Verschuur et al., 1992

How to get reliable images?

SRME relation $\mathcal{P}_j = \mathcal{G}_j(\mathcal{Q}_j + \mathcal{R}_j\mathcal{P}_j)$

minimize energy misfit $\mathcal{P}_{0j} = \mathcal{P}_j - \mathcal{R}_j\mathcal{Q}^{-1}\mathcal{P}_{0j}\mathcal{P}_j$

$\mathcal{P}$: total up-going wavefield

$\mathcal{G}$: surface-free dipole Green's function

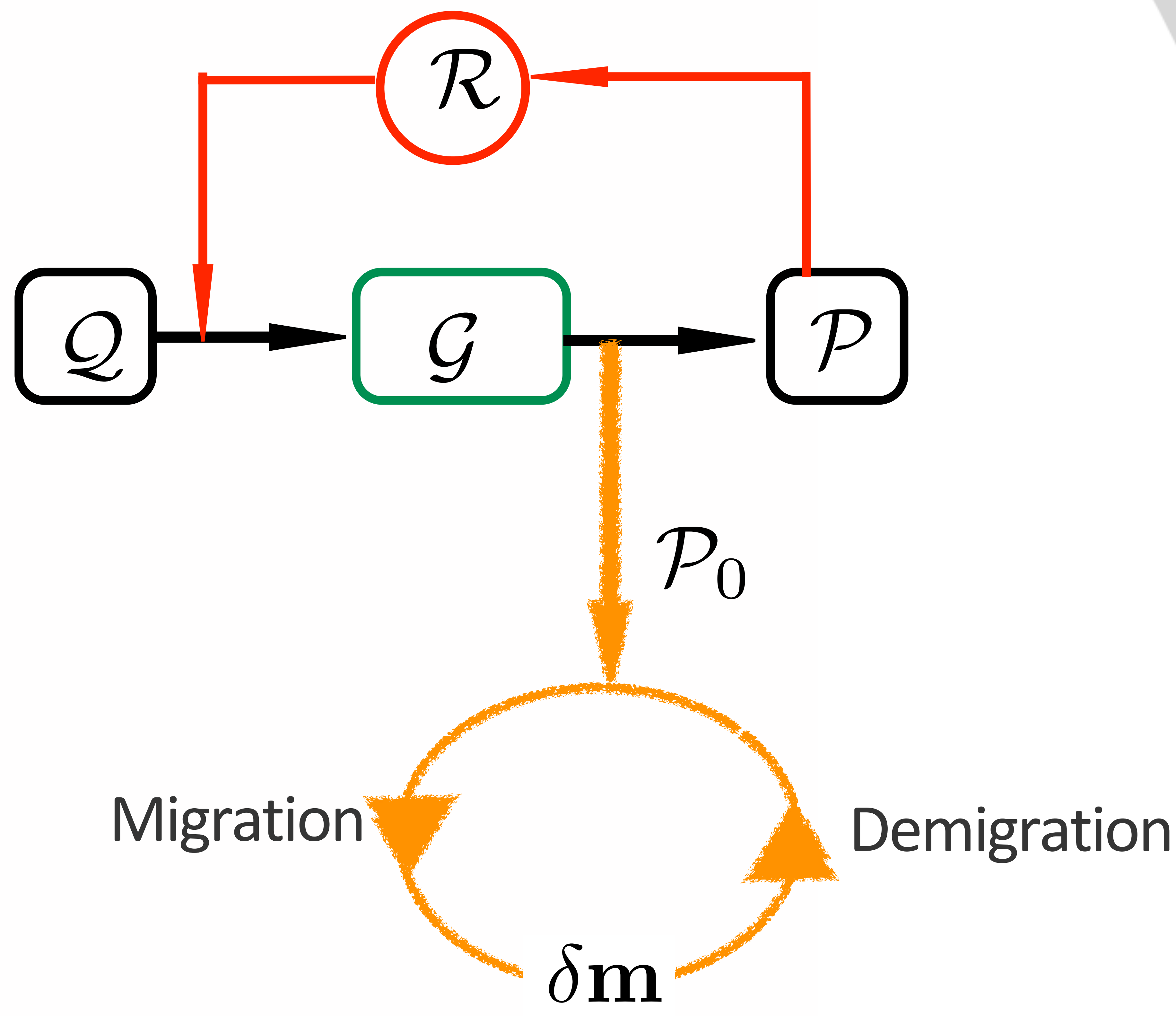
$\mathcal{Q}$: point-source wavefield $= \omega_j I$

$\mathcal{R}$: surface reflectivity

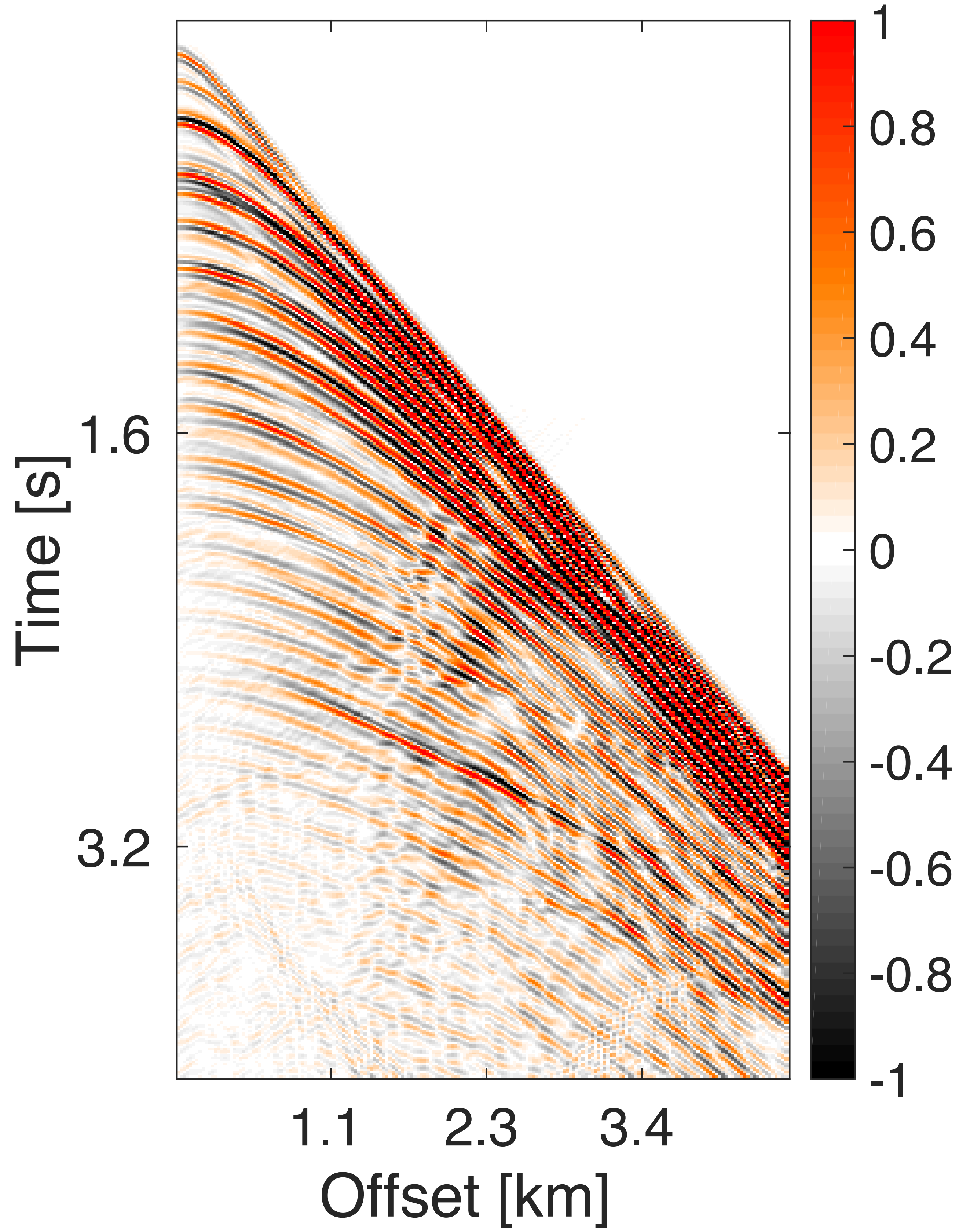
j : frequency index

$\mathcal{P}_0$: up-going primaries

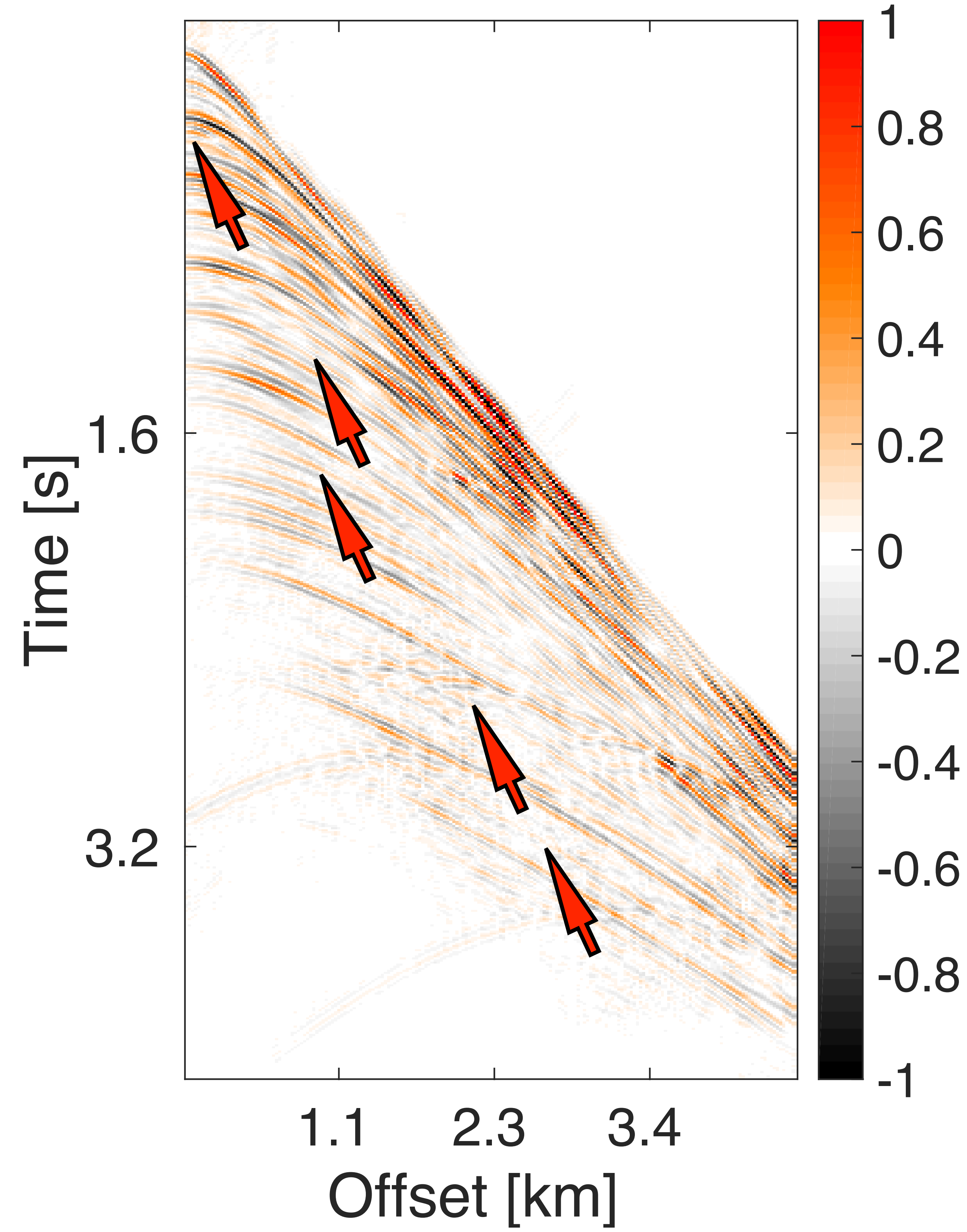
$\delta\mathbf{m}$: model perturbation



Problem in shallow water
Shot gathers

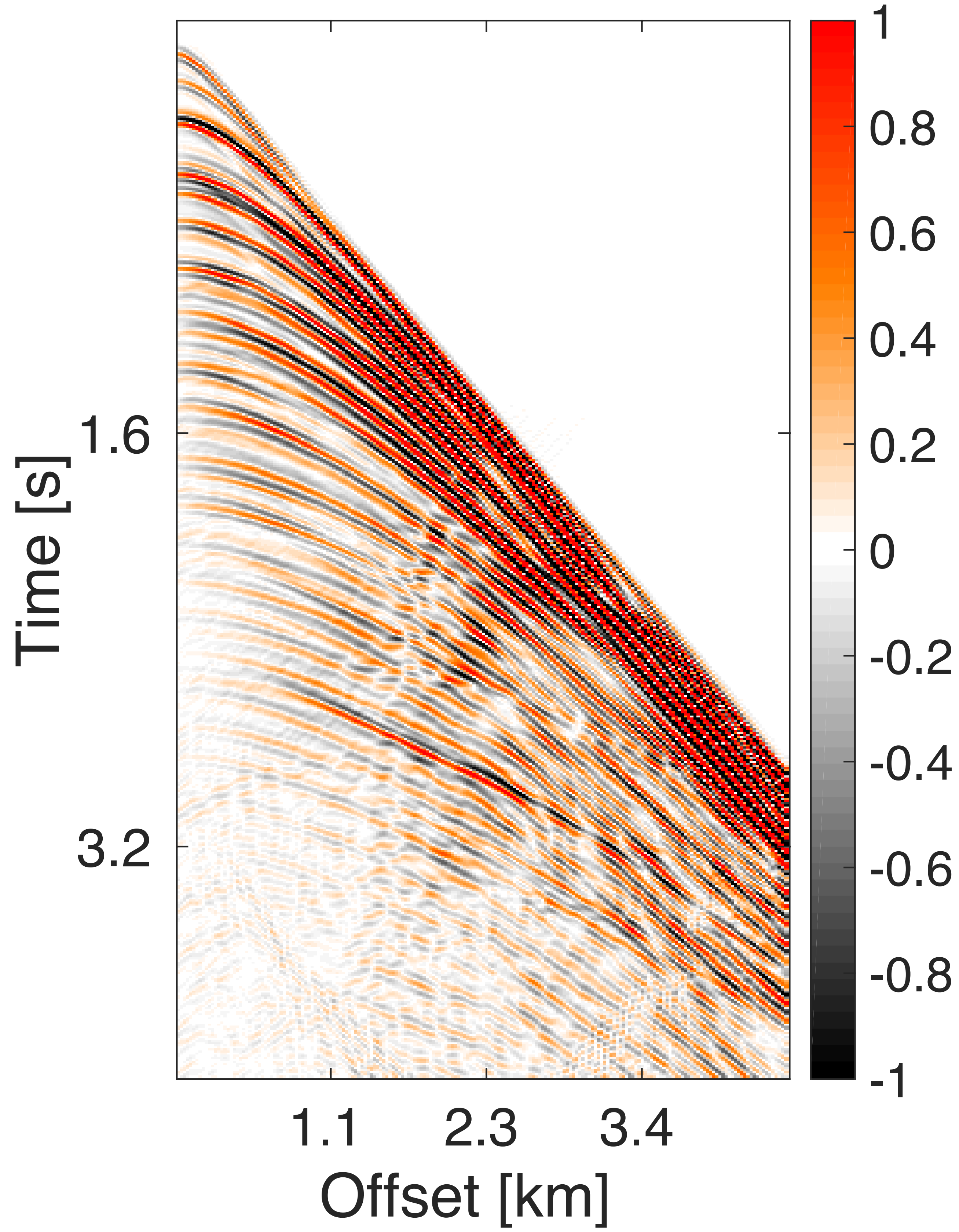


Total data



Primaries by SRME

Problem in shallow water
Shot gathers



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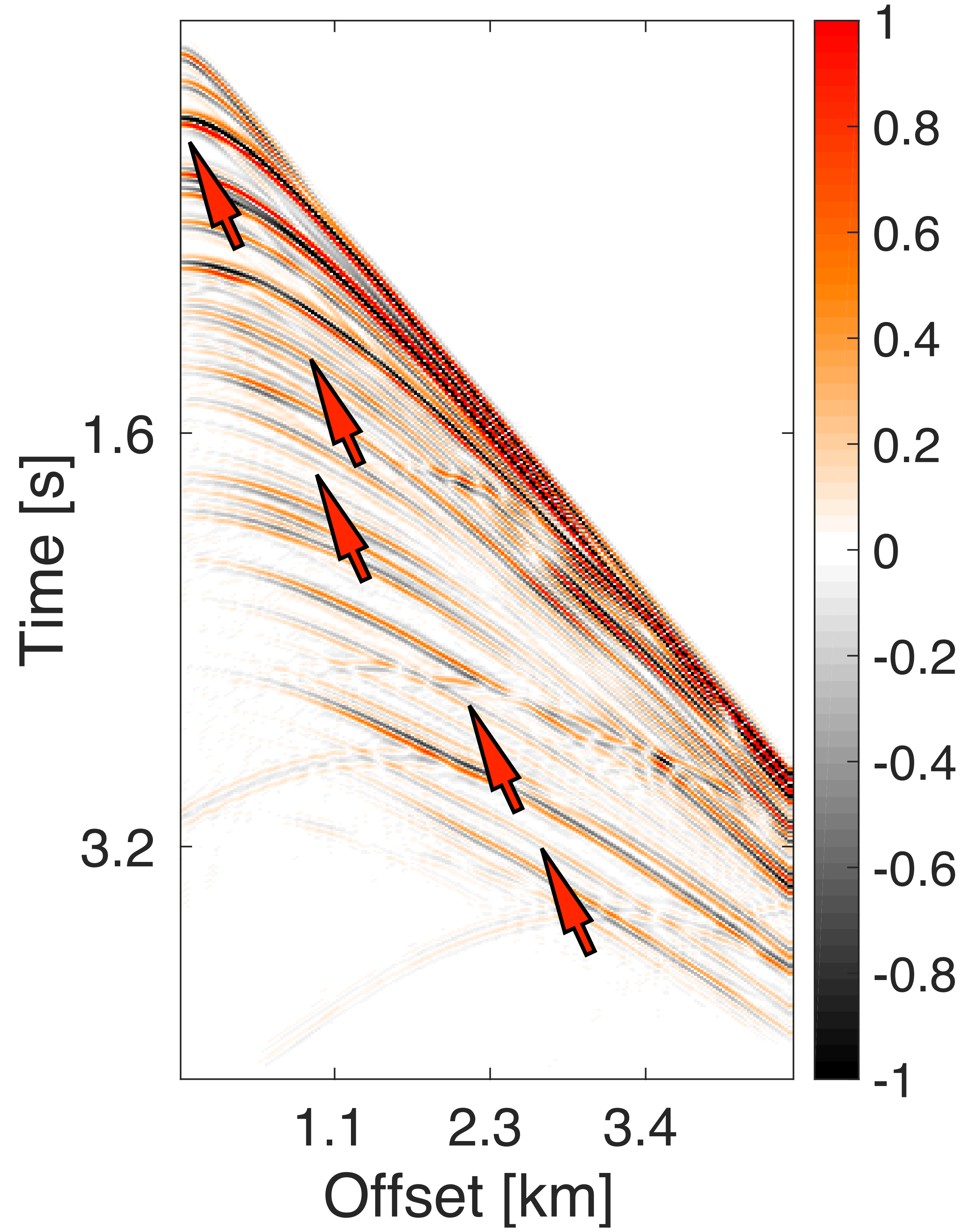


True primaries

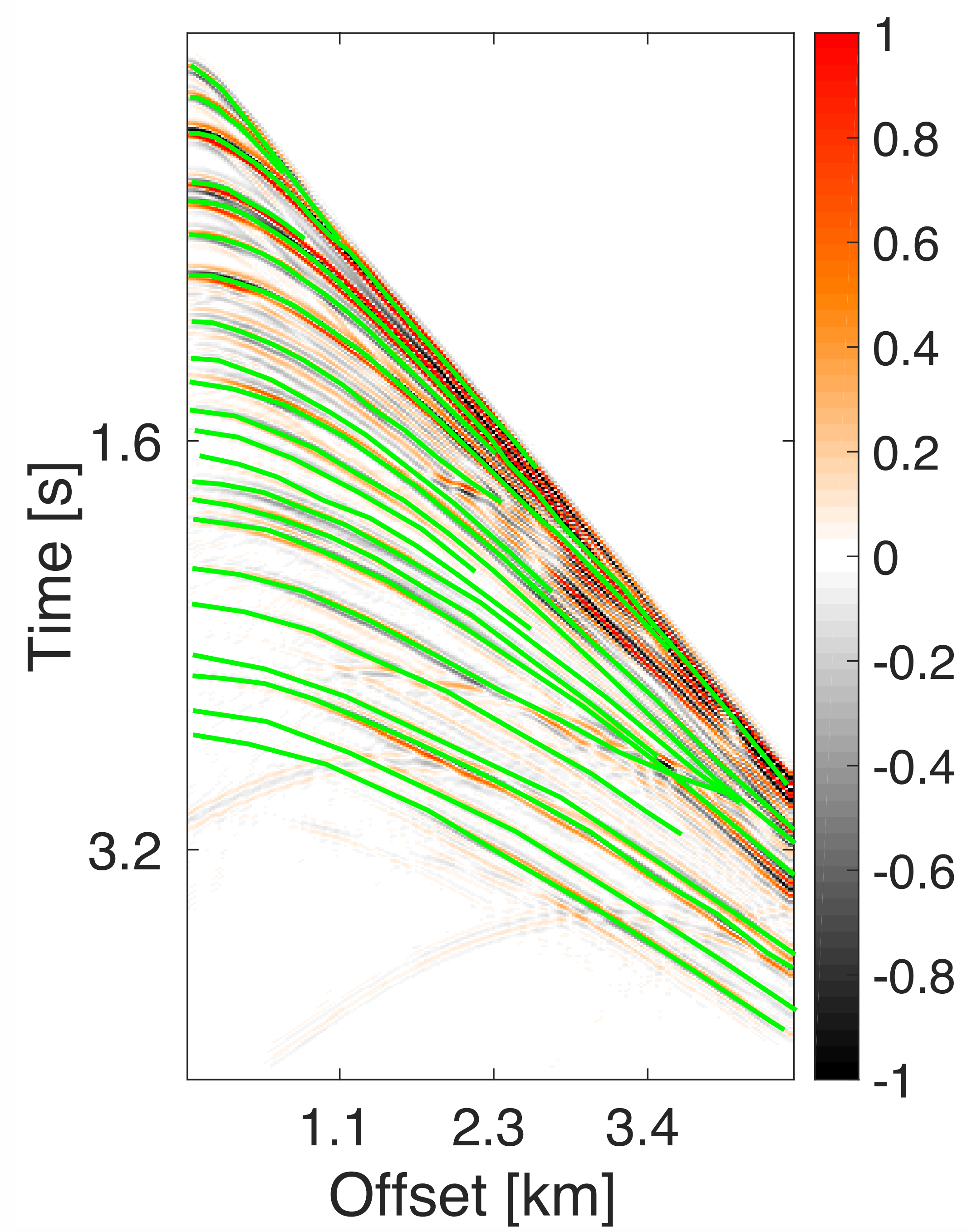
Verschuur et al., 1992

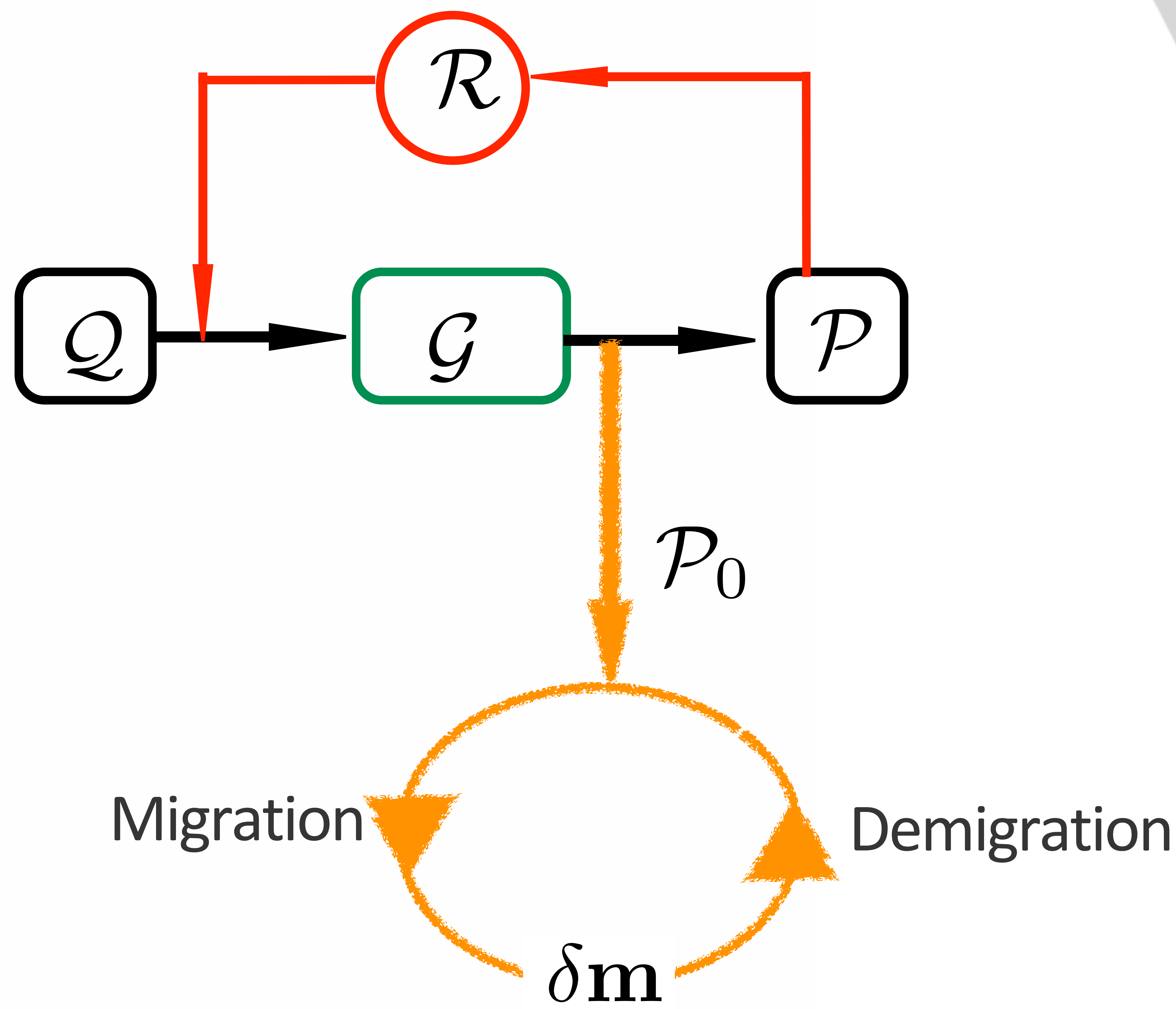
How to get reliable images?

SRME relation $\mathcal{P}_j = \mathcal{G}_j(\mathcal{Q}_j + \mathcal{R}_j\mathcal{P}_j)$

minimize energy misfit $\mathcal{P}_{0j} = \mathcal{P}_j - \mathcal{R}_j\mathcal{Q}^{-1}\mathcal{P}_{0j}\mathcal{P}_j$

invert sparse Green function $\mathcal{G}_j$





Tu N, Herrmann F J. Fast imaging with surface-related multiples by sparse inversion[J]. Geophysical Journal International, 2015, 201(1): 304-317.

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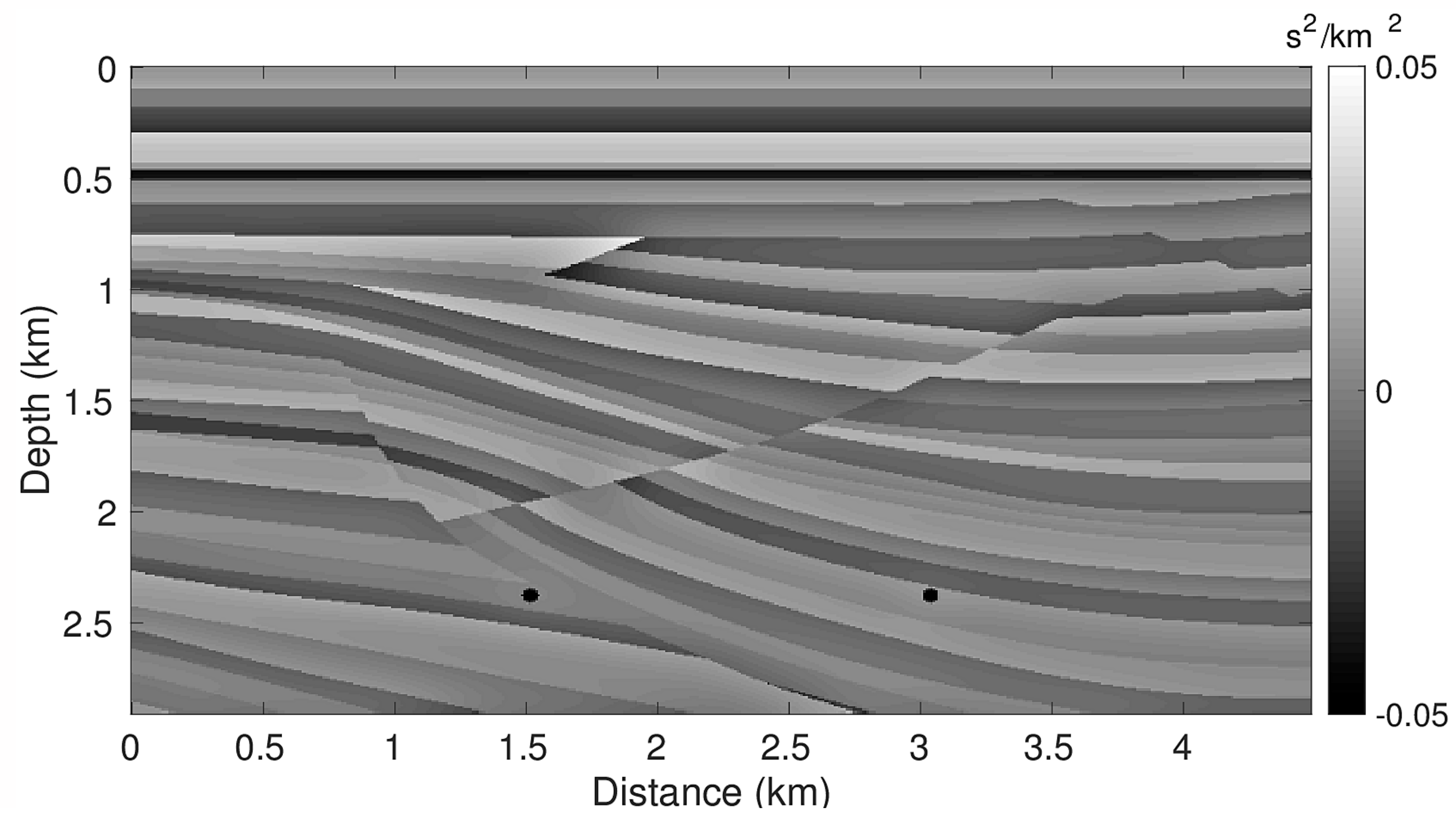
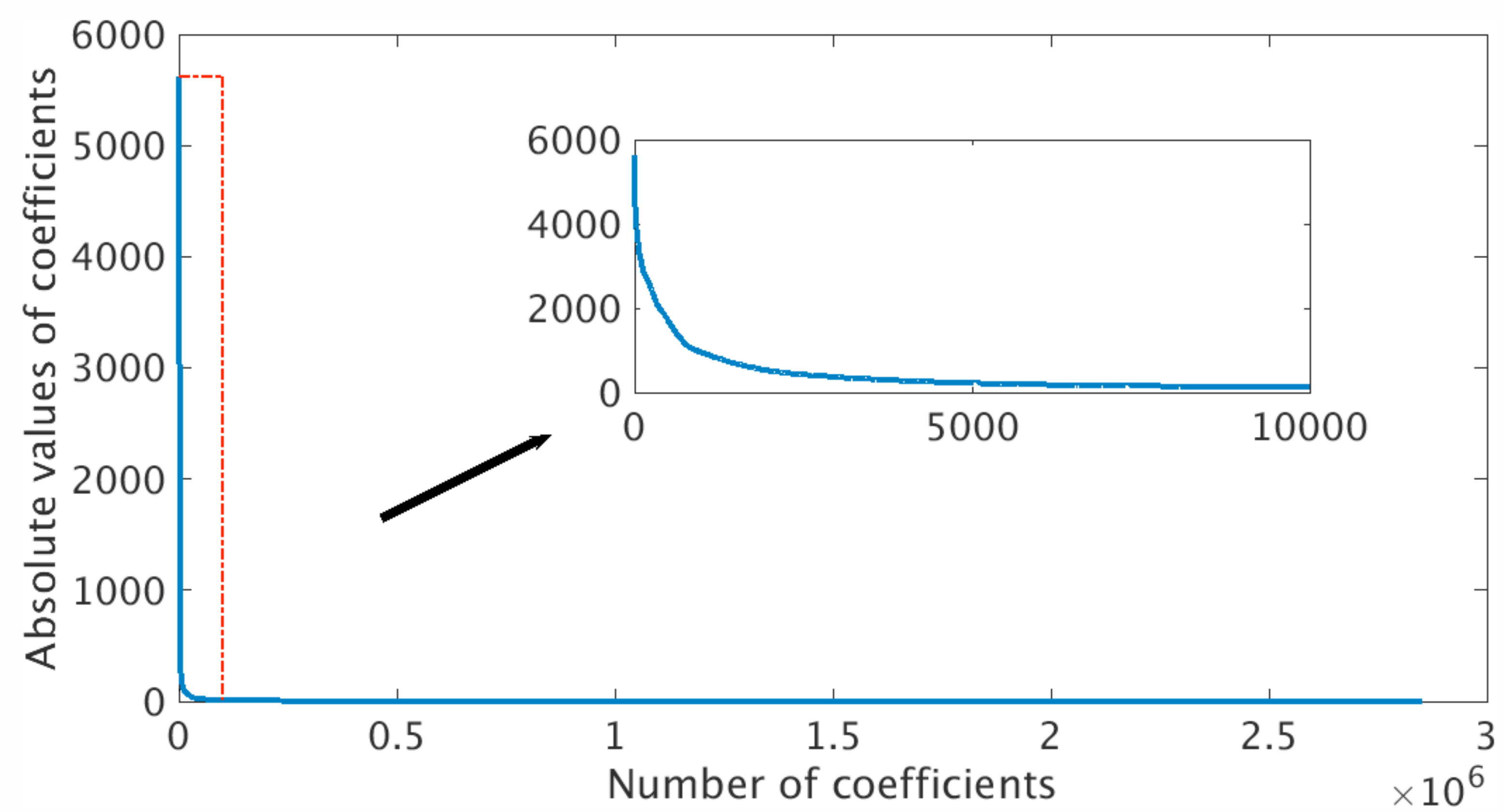
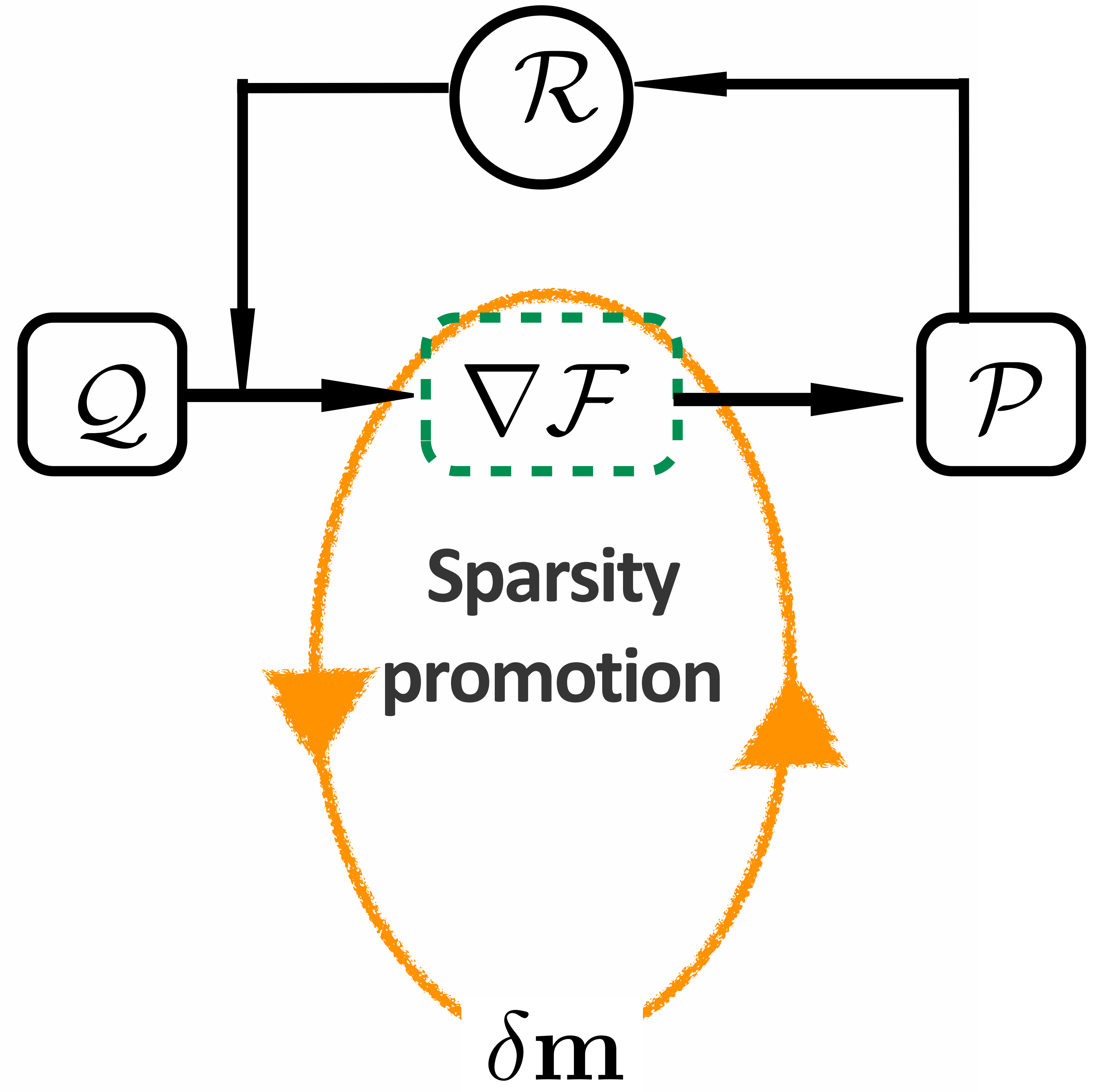
How to get reliable images?



Model perturbation



Sorted Curvelet coefficients



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Motivation

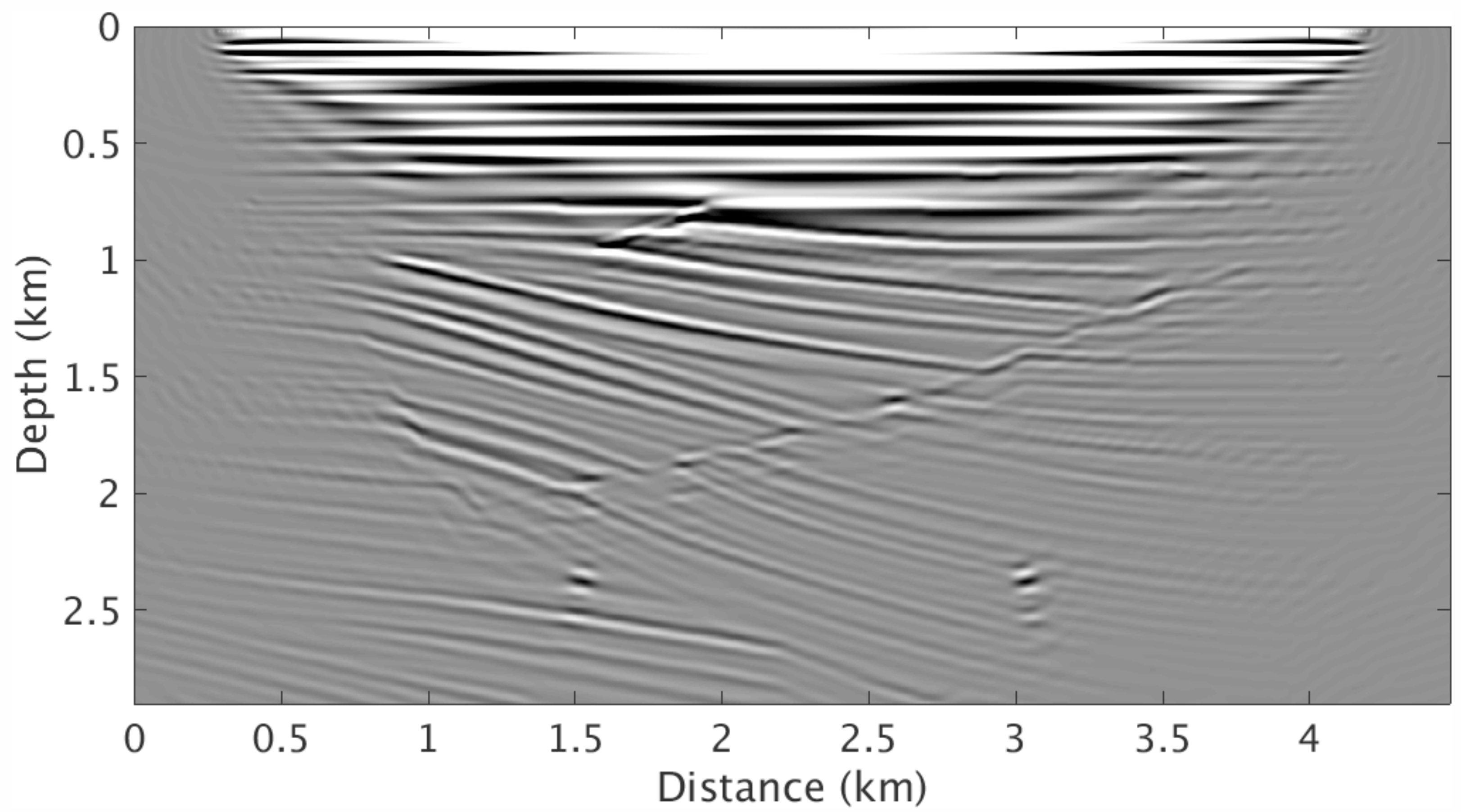
Challenges for primary prediction from shallow water multiples:

- SRME often fails to predict primaries because of “leakage”
- EPSI is too expensive

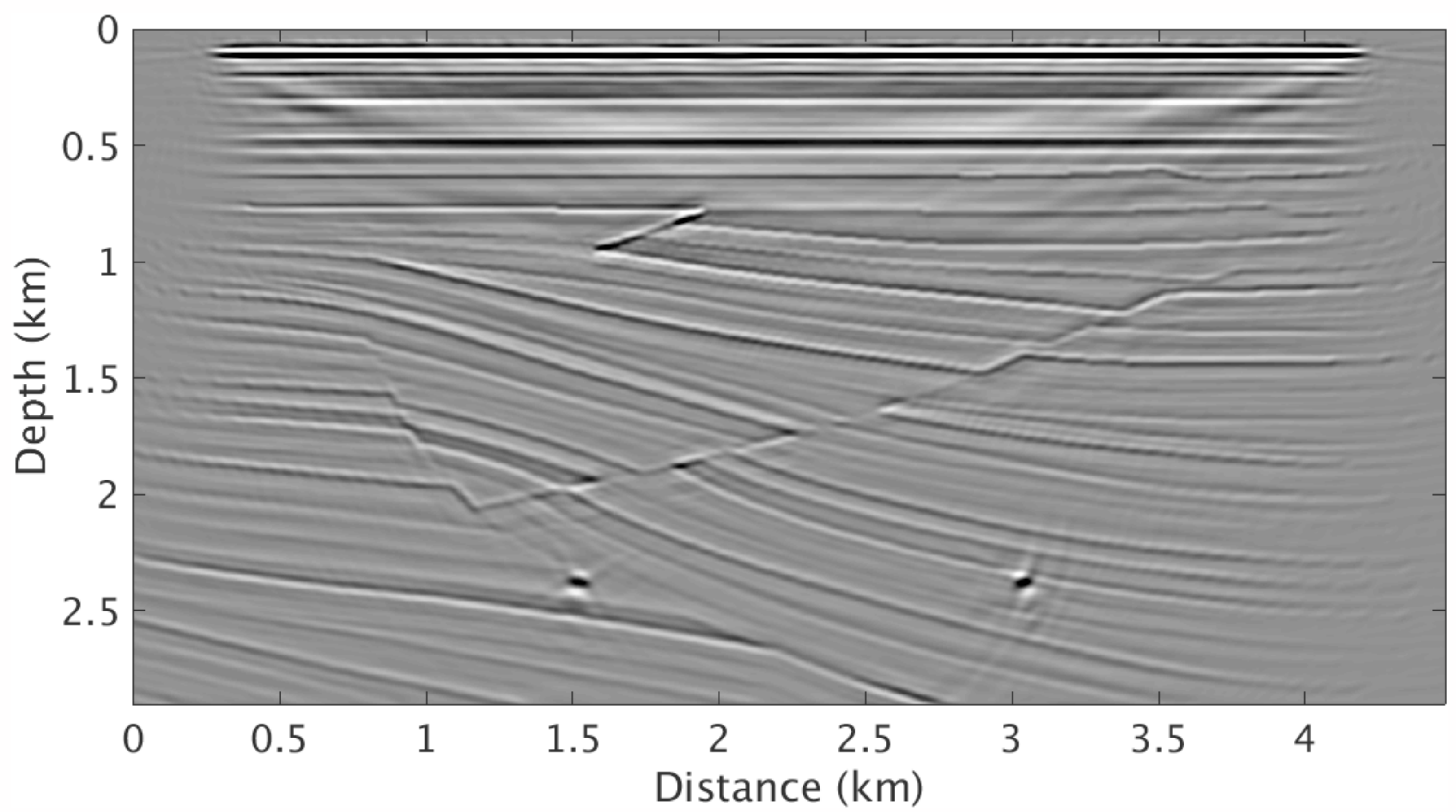
Other challenges:

- image artifacts from surface-related multiples
- computational costs
- time-domain implementation

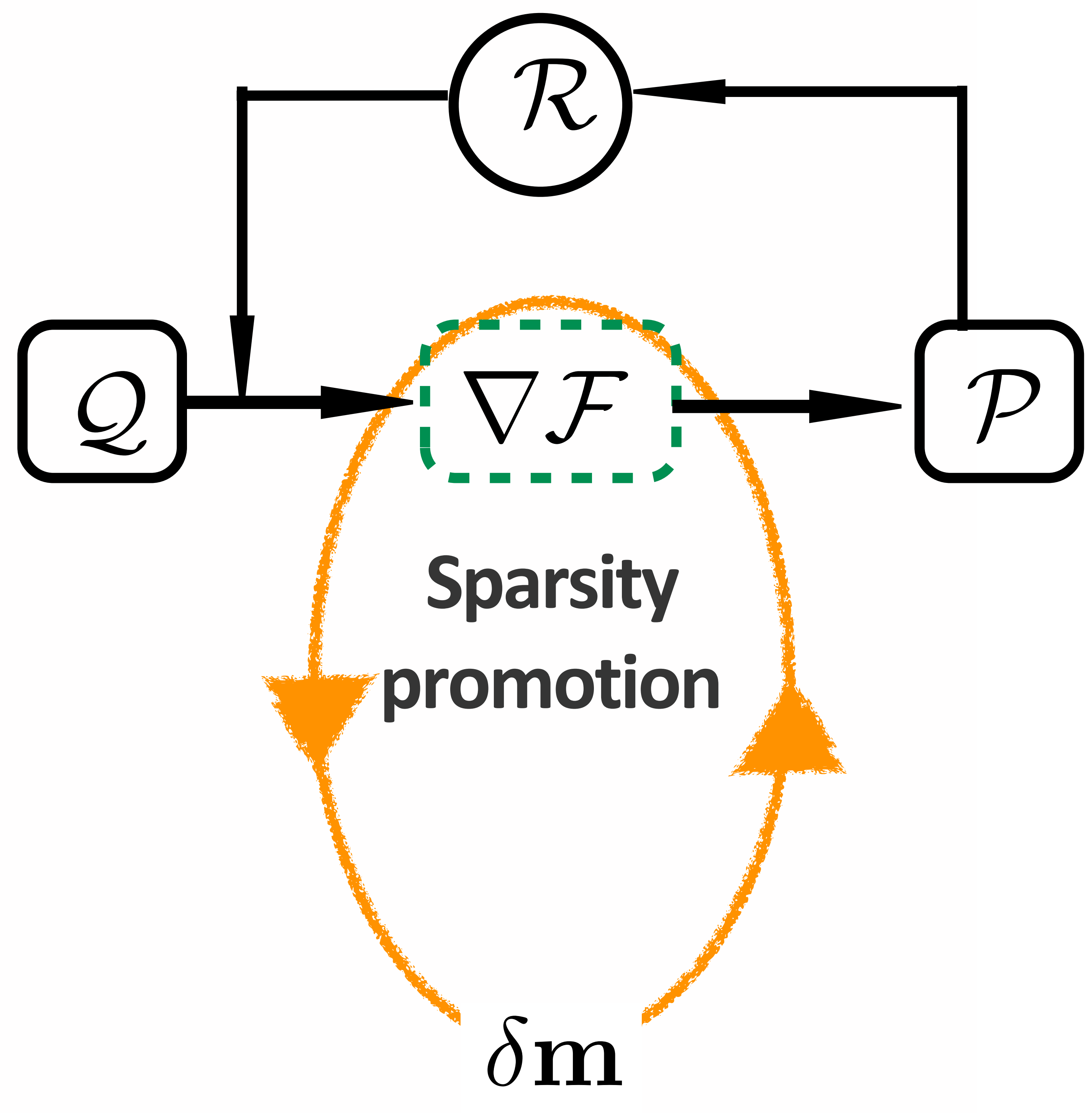
How to get reliable images?



Migration of total data



Joint inversion w/ areal source



Solution

Incorporate surface-related multiples directly into imaging

- w/ SRME relation
- WE solver does multi-D convolutions implicitly
- simple implementation via linearized Bregman projections (LBP)

Tu, N. & Herrmann, F. J. “Least-squares migration of full wavefield with source encoding”,
74th EAGE Conference & Exhibition incorporating SPE EUROPEC 2012

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Eliminating dense matrix-matrix products
[SRME relation & wave-equation solver]

Combine linearized modelling w/ free-surface physics:

$$\begin{aligned}\mathcal{P}_j &\approx \nabla\mathcal{F}_j\left[\mathbf{m}_0,\delta\mathbf{m};\mathcal{I}\right]\left(\mathcal{Q}_j-\mathcal{P}_j\right)\\ &= \nabla\mathcal{F}_j[\mathbf{m}_0,\delta\mathbf{m};\mathcal{Q}_j-\mathcal{P}_j]\\ &= \nabla\mathcal{F}_j[\mathbf{m}_0;\mathcal{Q}_j-\mathcal{P}_j]\delta\mathbf{m}.\end{aligned}$$

→ Dense matrix-matrix products

→ Wave-equation solves with total downgoing data injected as “areal” source

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Eliminating dense matrix-matrix products
[SRME relation & wave-equation solver]

Combine linearized time-domain modelling w/ free-surface physics:

$$\mathbf{P} \approx \left(\nabla\mathbf{F}_{\mathbf{m}}[\mathbf{m}_0, \boldsymbol{\rho}_0; \mathbf{Q} - \mathbf{P}] \quad \nabla\mathbf{F}_{\boldsymbol{\rho}}[\mathbf{m}_0, \boldsymbol{\rho}_0; \mathbf{Q} - \mathbf{P}]\right) \begin{pmatrix} \delta\mathbf{m} \\ \delta\boldsymbol{\rho} \end{pmatrix}$$

$$\approx \nabla\mathbf{F}_{\mathbf{m}}[\mathbf{m}_0, \boldsymbol{\rho}_0; \mathbf{Q} - \mathbf{P}]\delta\mathbf{m}'$$

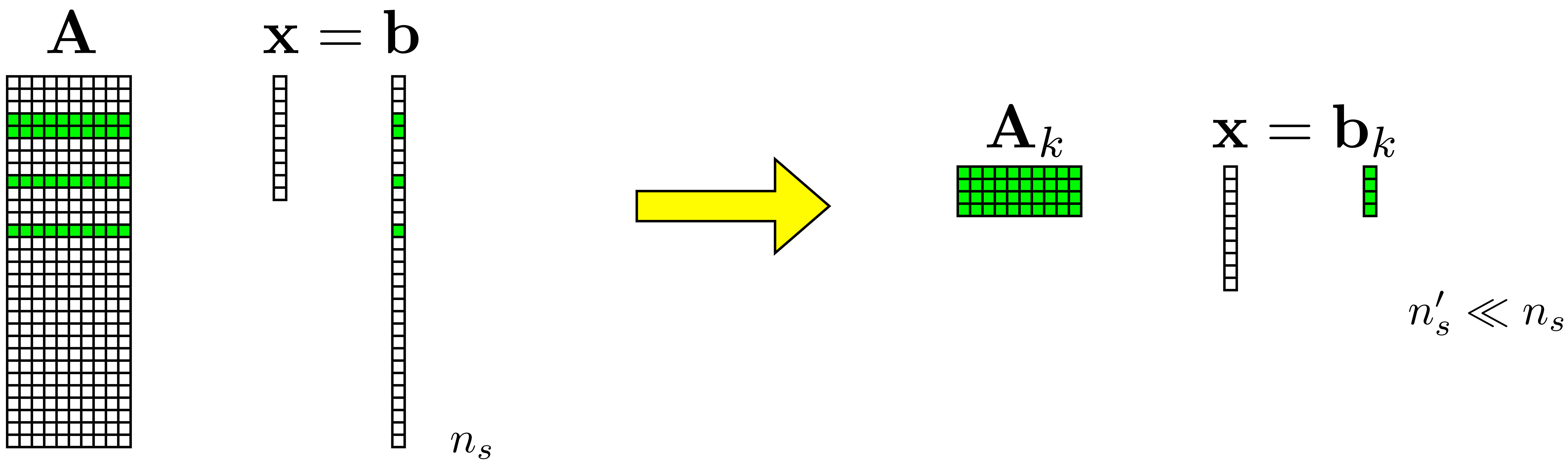
Felix J. Herrmann,Ning Tu and Ernie Esser, “Fast ‘online’ migration with Compressive Sensing”, EAGE Annual Conference Proceeding, 2015, vol. 60, p. 696-712, 2012
Lorenz, Dirk A. Wenger, Stephan, “A sparse Kaczmarz solver and a linearized Bregman method for online compressed sensing”, arXiv:1403.7543

Yang M, Witte P, Fang Z and Felix J. Herrmann, “Time-domain sparsity-promoting least-squares migration with source estimation”, SEG Technical Program Expanded Abstracts, 2016. 4225-4229.

LBP via randomized subsampling

Randomized subsets of $\mathbf{A}$, $\mathbf{b}$ for linearized Bregman method:

1. $\quad$ **for** $k = 0, 1, \cdots$
2. $\qquad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}^*_{r(k)}(\mathbf{A}_{r(k)}\mathbf{x}_k - \mathbf{b}_{r(k)})$
3. $\qquad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
4. $\quad$ **end for**



Joint SP-LSRTM w/ primaries & multiples

$$\min_{\mathbf{x}} \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|_2^2$$

$$\text{subject to } \sum_i \|\nabla\mathbf{F}_i(\mathbf{m}_0, \rho_0, \mathbf{Q}_i - \mathbf{P}_i)\mathbf{C}^T\mathbf{x} - \mathbf{P}_i\|_2 \leq \sigma,$$

Areal source

1. Initialize $\mathbf{x}_0 = \mathbf{0}$, $\mathbf{z}_0 = \mathbf{0}$, $\mathbf{Q}$, λ, batchsize $n_s' \ll n_s$
2. for $k = 0, 1, \cdots$
3. Randomly choose shot subsets $\mathcal{I} \in [1 \cdots n_s]$, $|\mathcal{I}| = n_s'$
4. $\mathbf{A}_k = \{\nabla\mathbf{F}_i(\mathbf{m}_0, \rho_0, \mathbf{Q}_i - \mathbf{P}_i)\mathbf{C}^T\}_{i\in\mathcal{I}}$
5. $\mathbf{b}_k = \{\mathbf{P}_i\}_{i\in\mathcal{I}}$
6. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k\mathbf{A}_k^T\mathbb{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$
7. $\mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
8. end

note: $S_\lambda(\mathbf{z}_{k+1}) = \text{sign}(\mathbf{z}_{k+1})\max\{0, |\mathbf{z}_{k+1}| - \lambda\}$

$\mathbb{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k) = \max\{0, 1 - \frac{\sigma}{\|\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k\|_2}\} \cdot (\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$

$t_k = \|\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k\|^2 / \|\mathbf{A}_k^T(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)\|^2$

Experiment

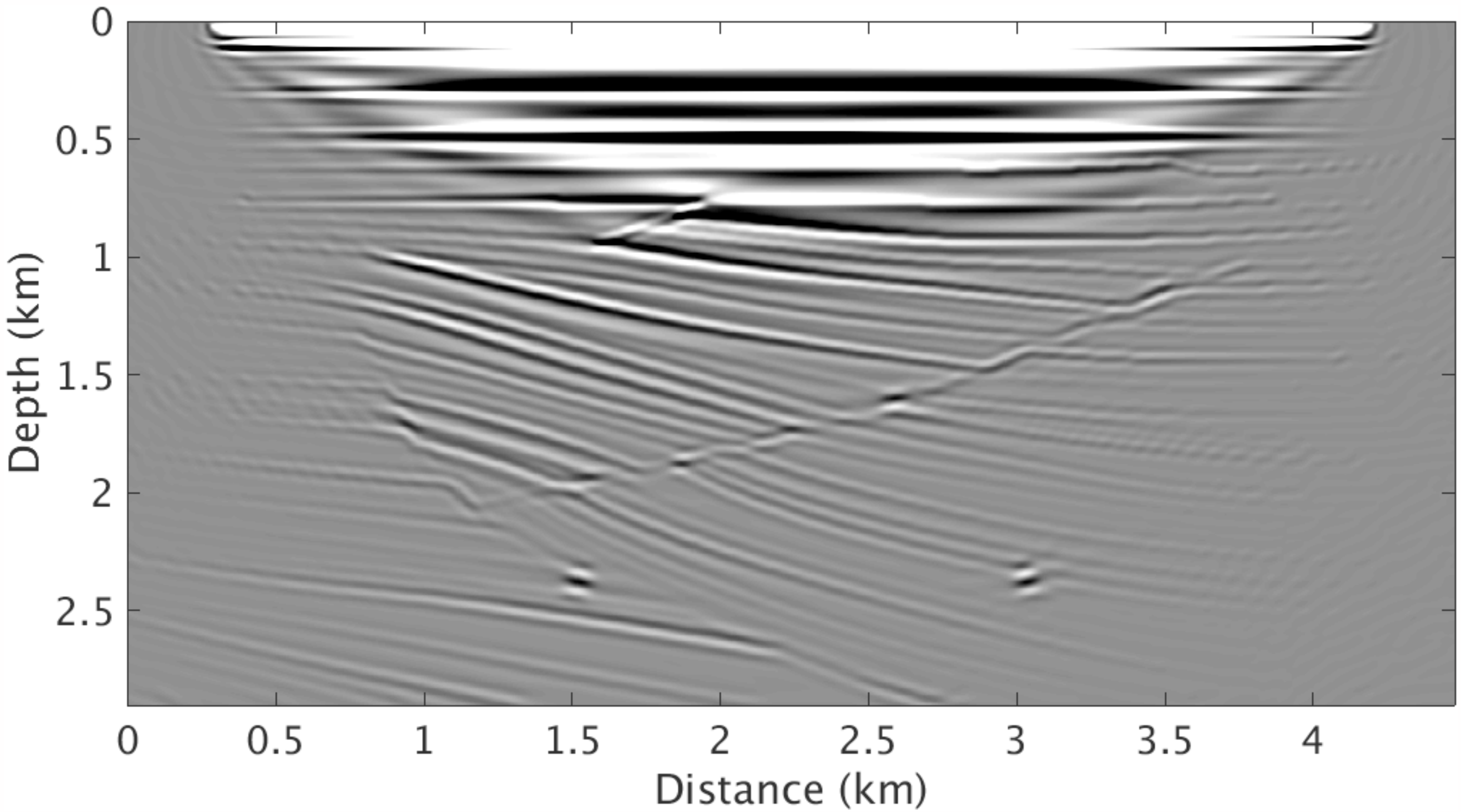
Data:

- 261 sources and receivers
- Ricker wavelet centered at 15 Hz
- generated with free surface, $\mathbf{F}[\mathbf{m}, \rho] - \mathbf{F}[\mathbf{m}_0, \rho_0]$
- source-side deghost

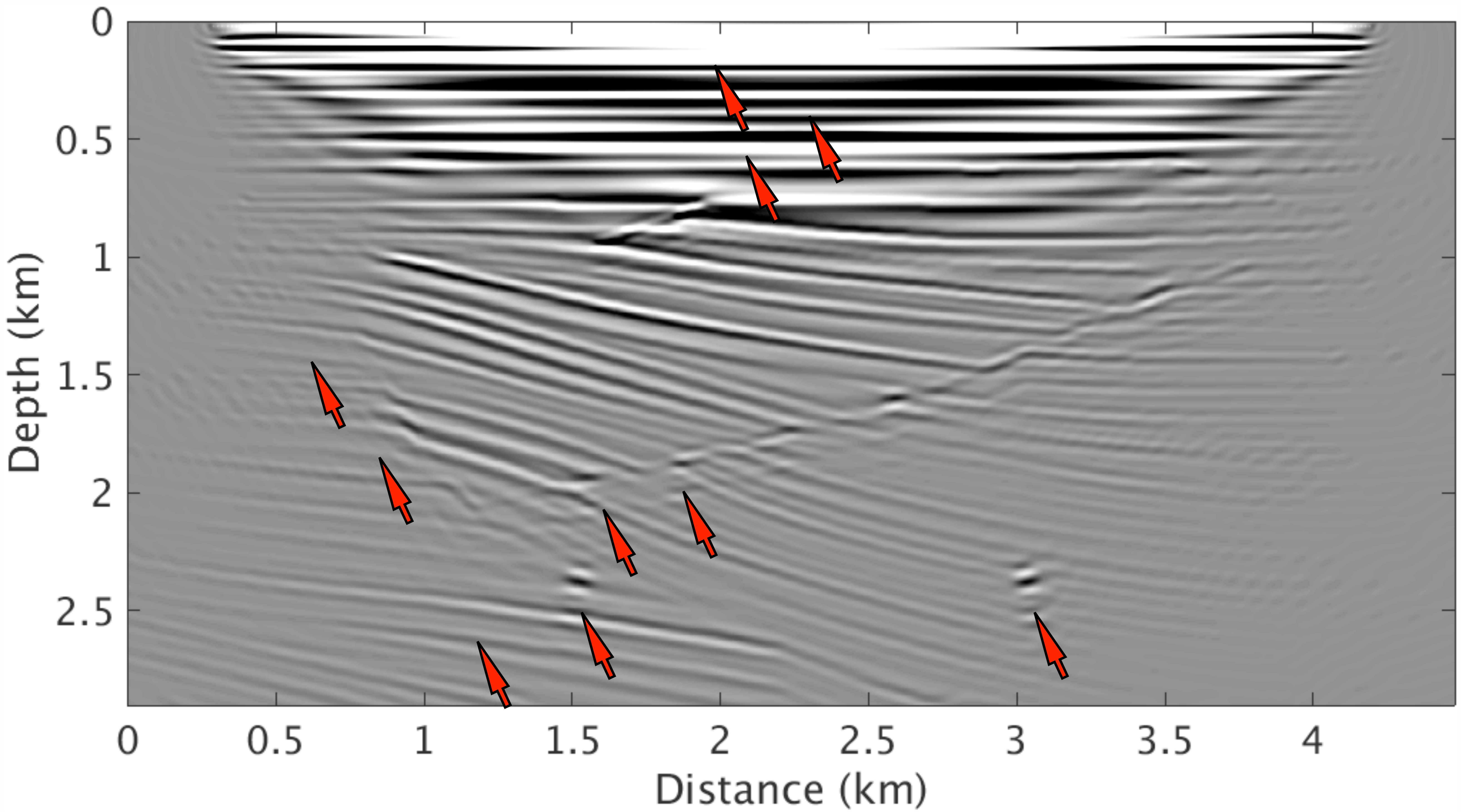
Experiments:

- dipole source setting
- one pass through the data with batch sizes 2.5% data
- randomized subset of shots
- true source wavelet

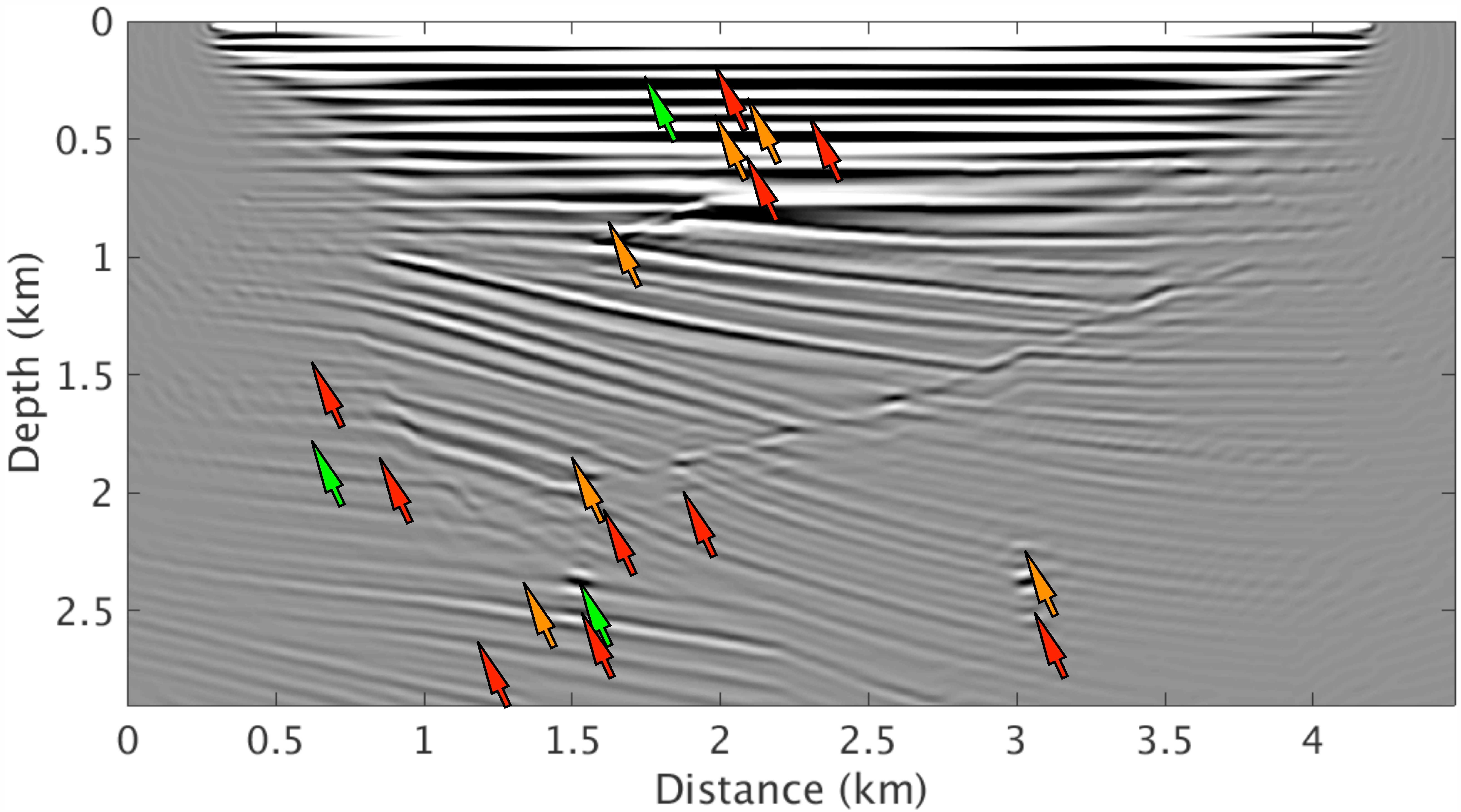
RTM of primaries



RTM of total data



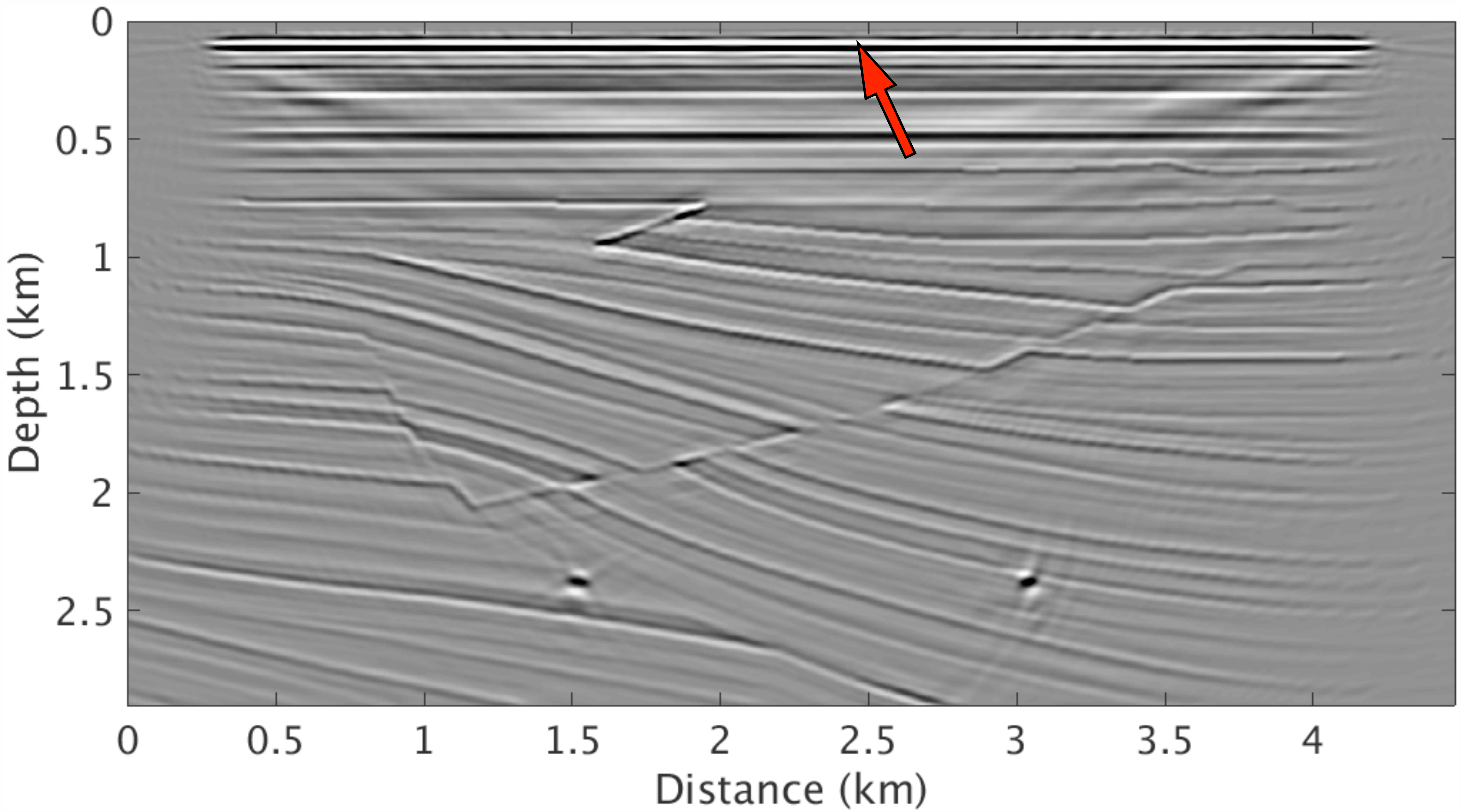
RTM of total data w/ areal source



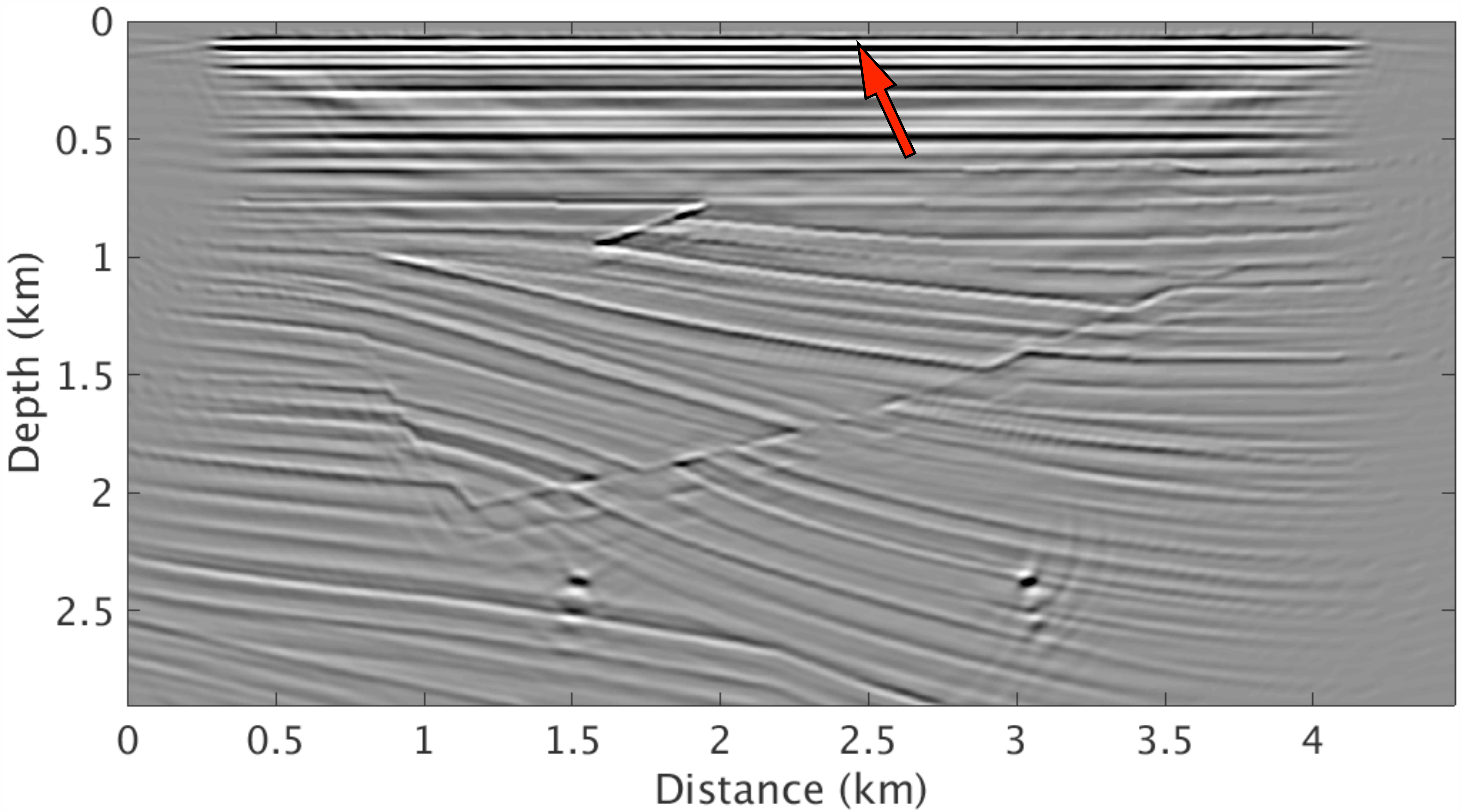
Stronger at correct position

More artifacts

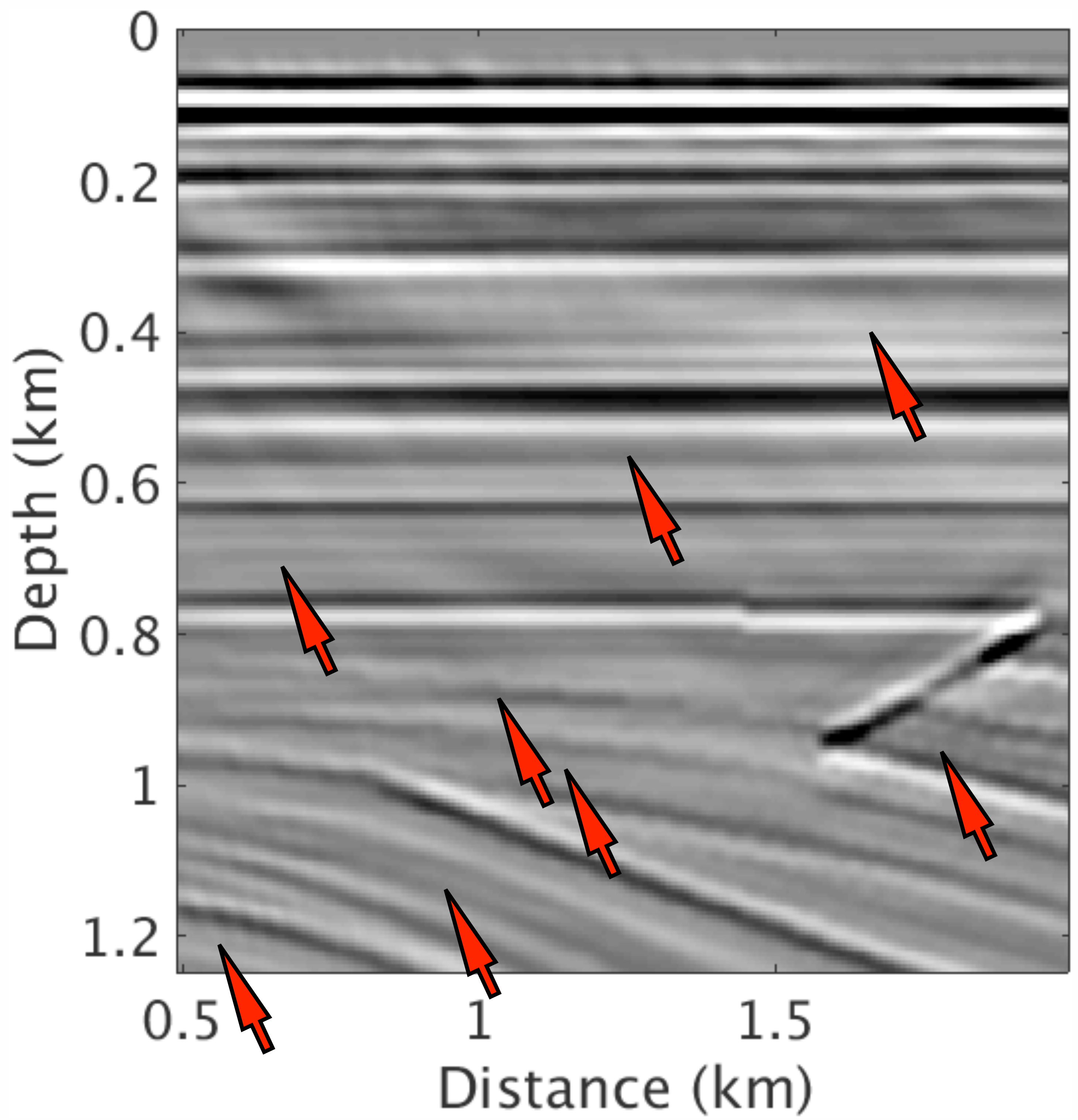
Joint SP-LSRTM w/ primaries & multiples
w/ areal source



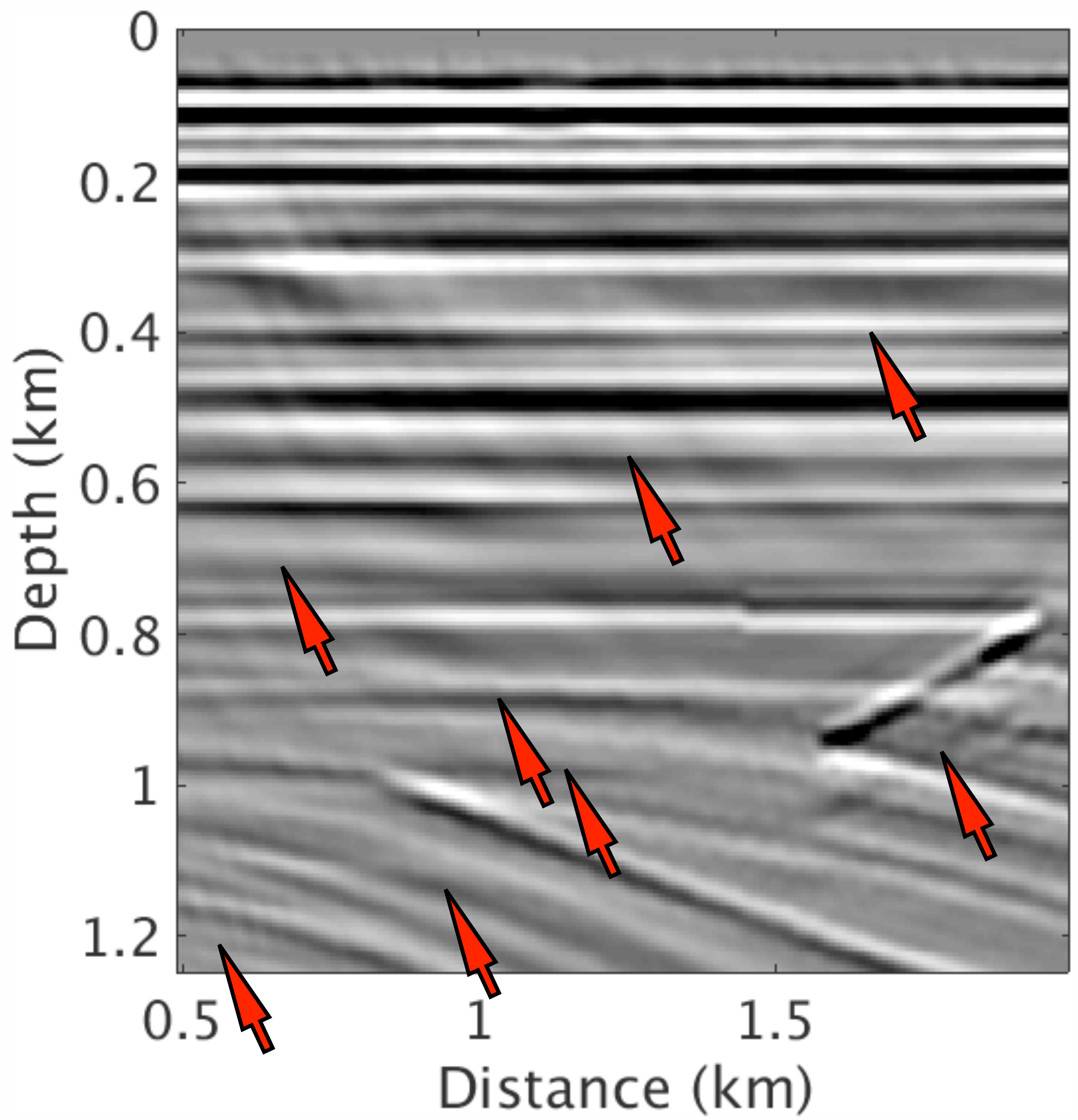
Joint SP-LSRTM w/ primaries & multiples
w/o areal source



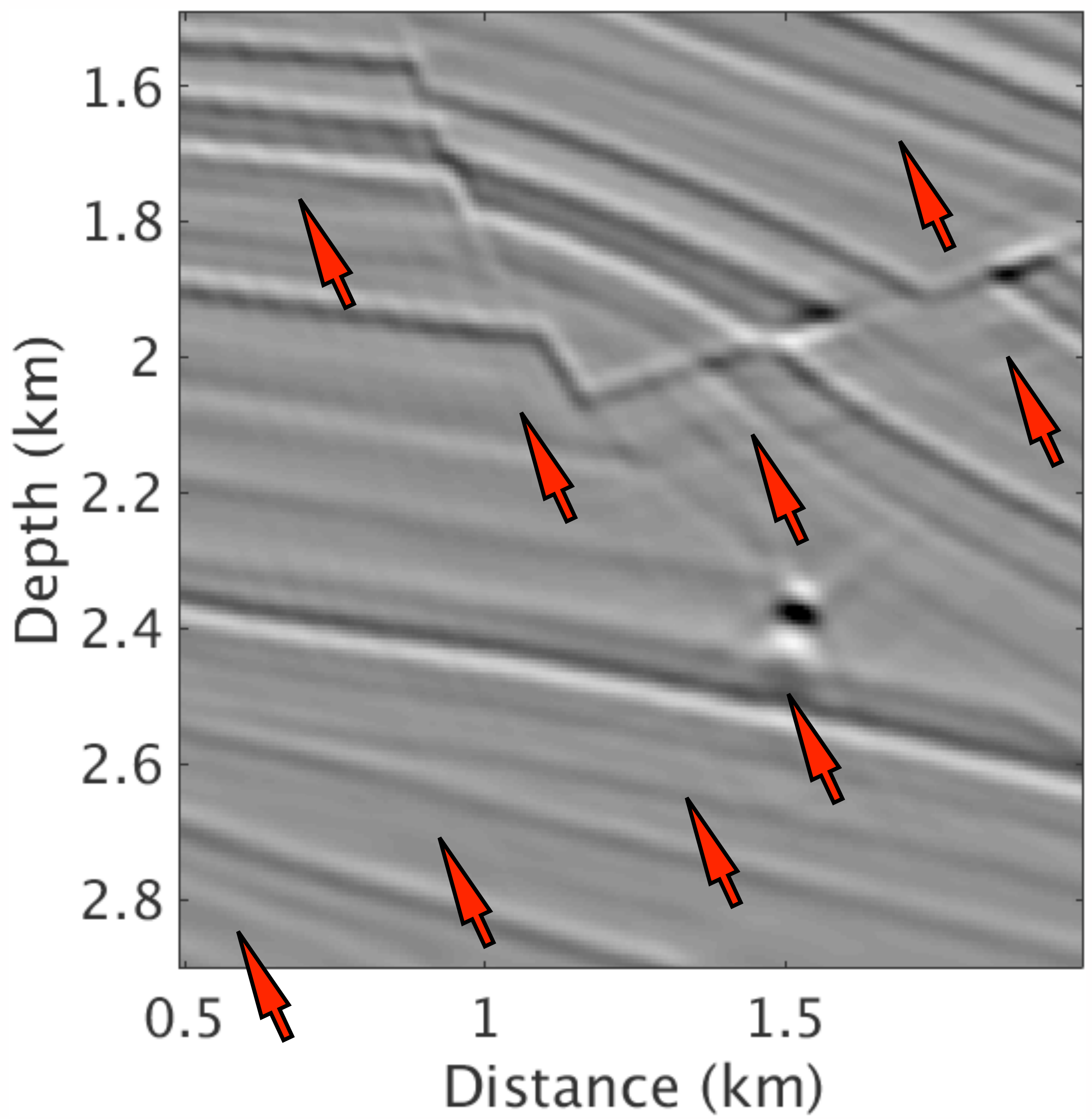
Joint SP-LSRTM w/ primaries & multiples
w/ areal source, zoomed



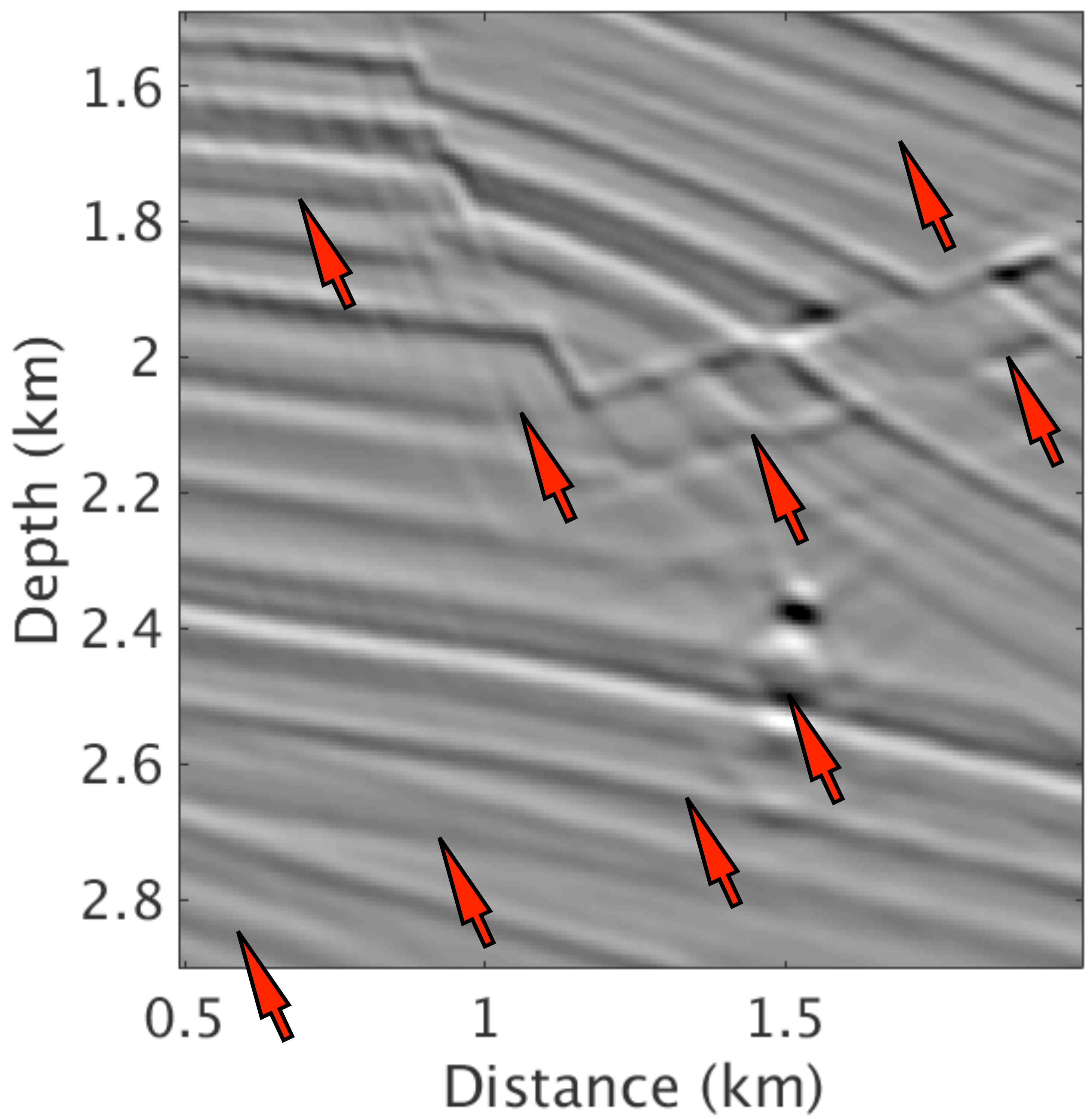
Joint SP-LSRTM w/ primaries & multiples
w/o areal source, zoomed



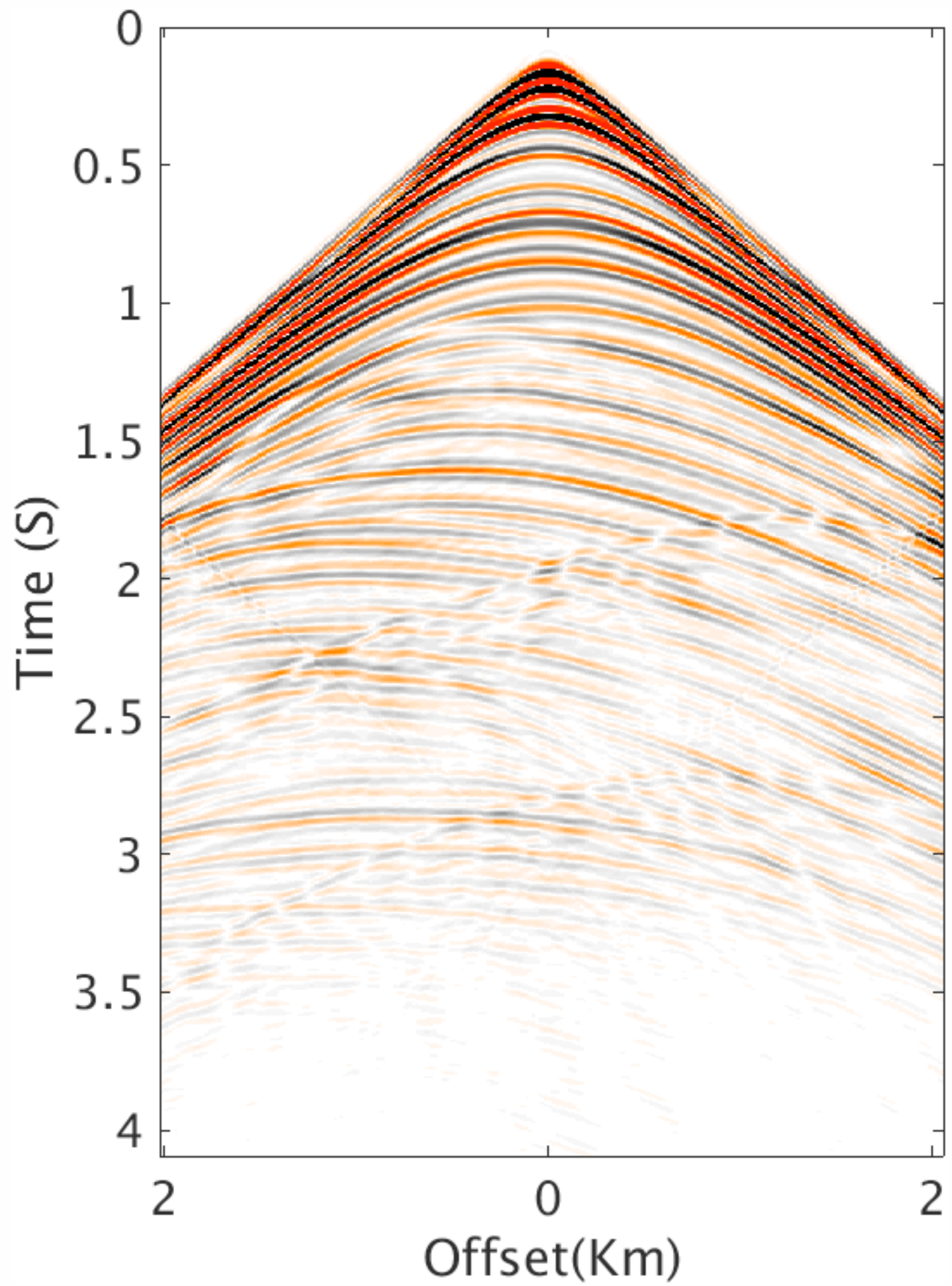
Joint SP-LSRTM w/ primaries & multiples
w/ areal source, zoomed



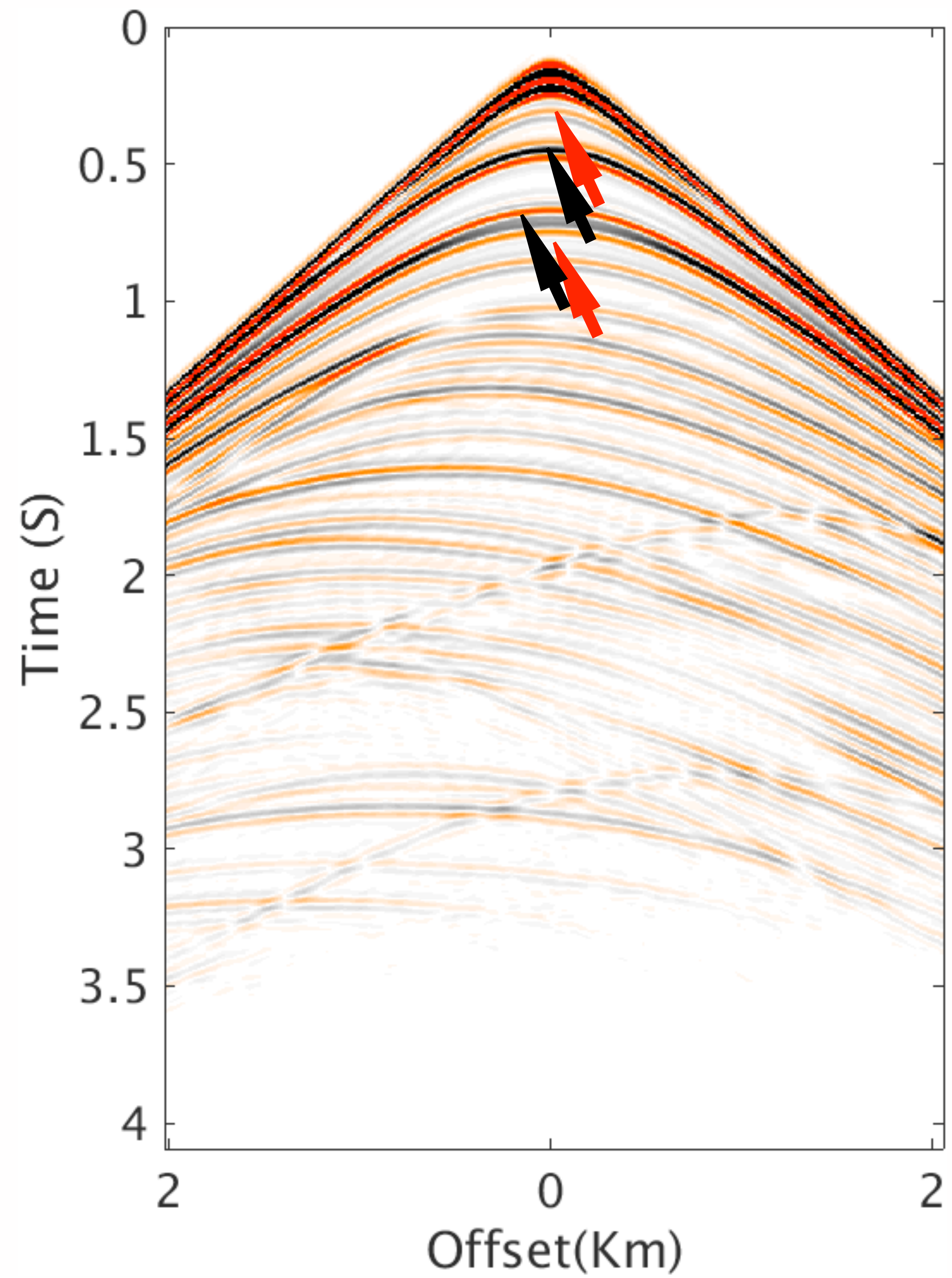
Joint SP-LSRTM w/ primaries & multiples
w/o areal source, zoomed



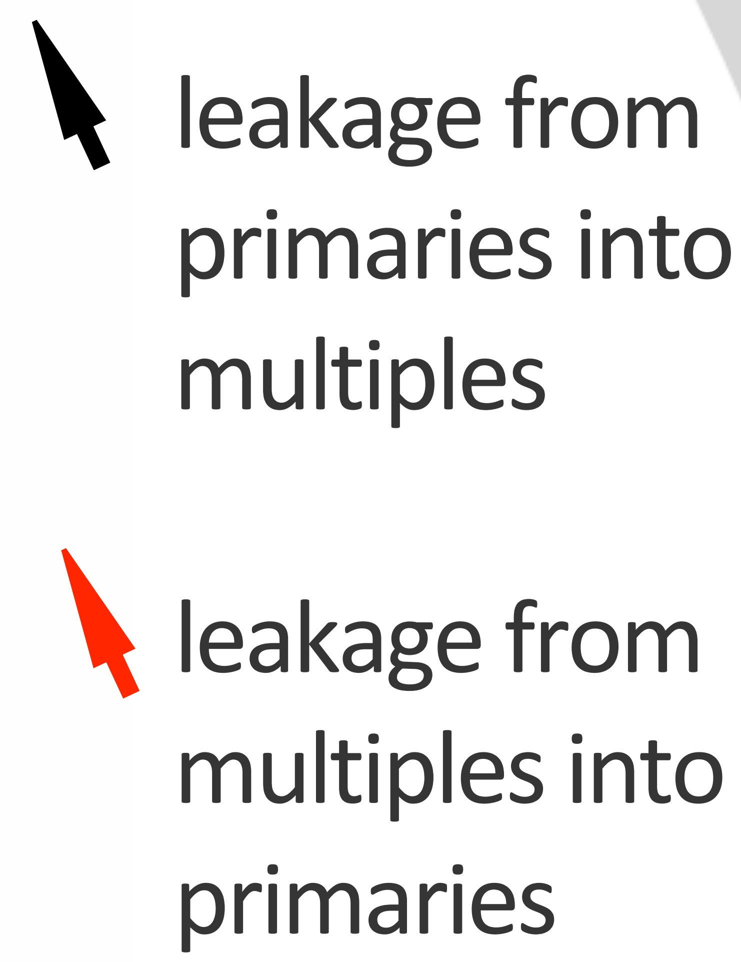
Shot gathers



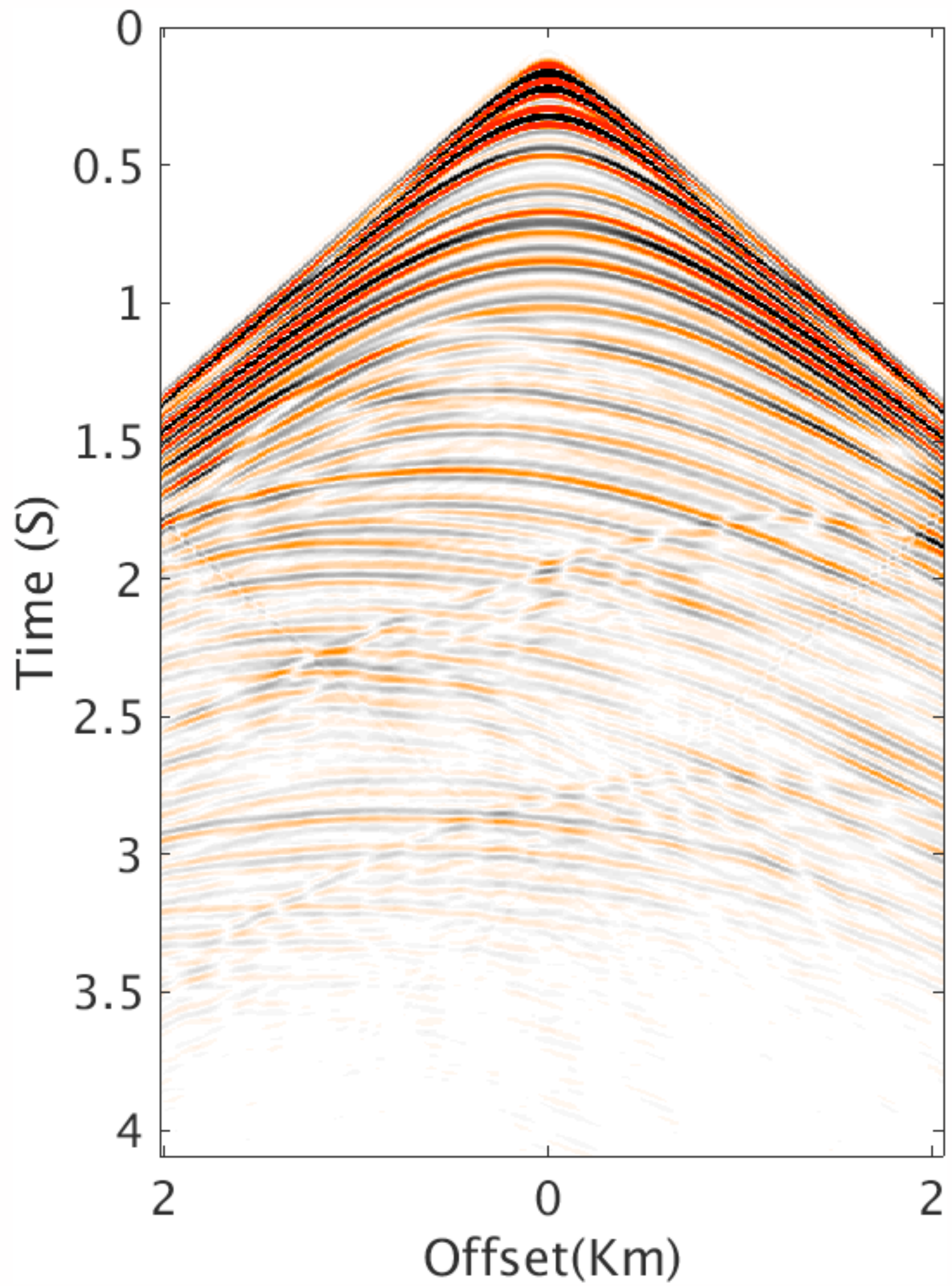
Total data



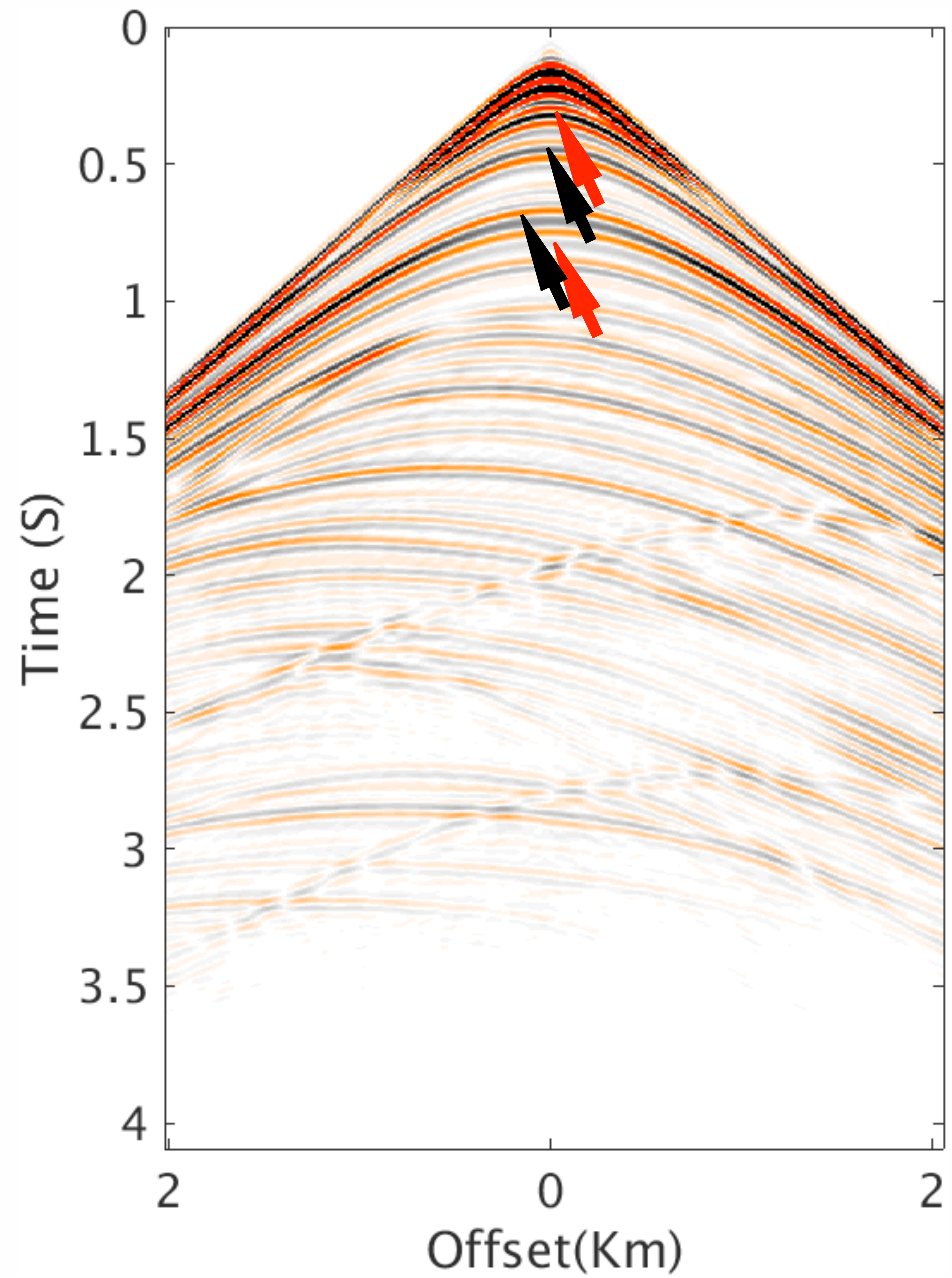
Synthetic primaries



Shot gathers

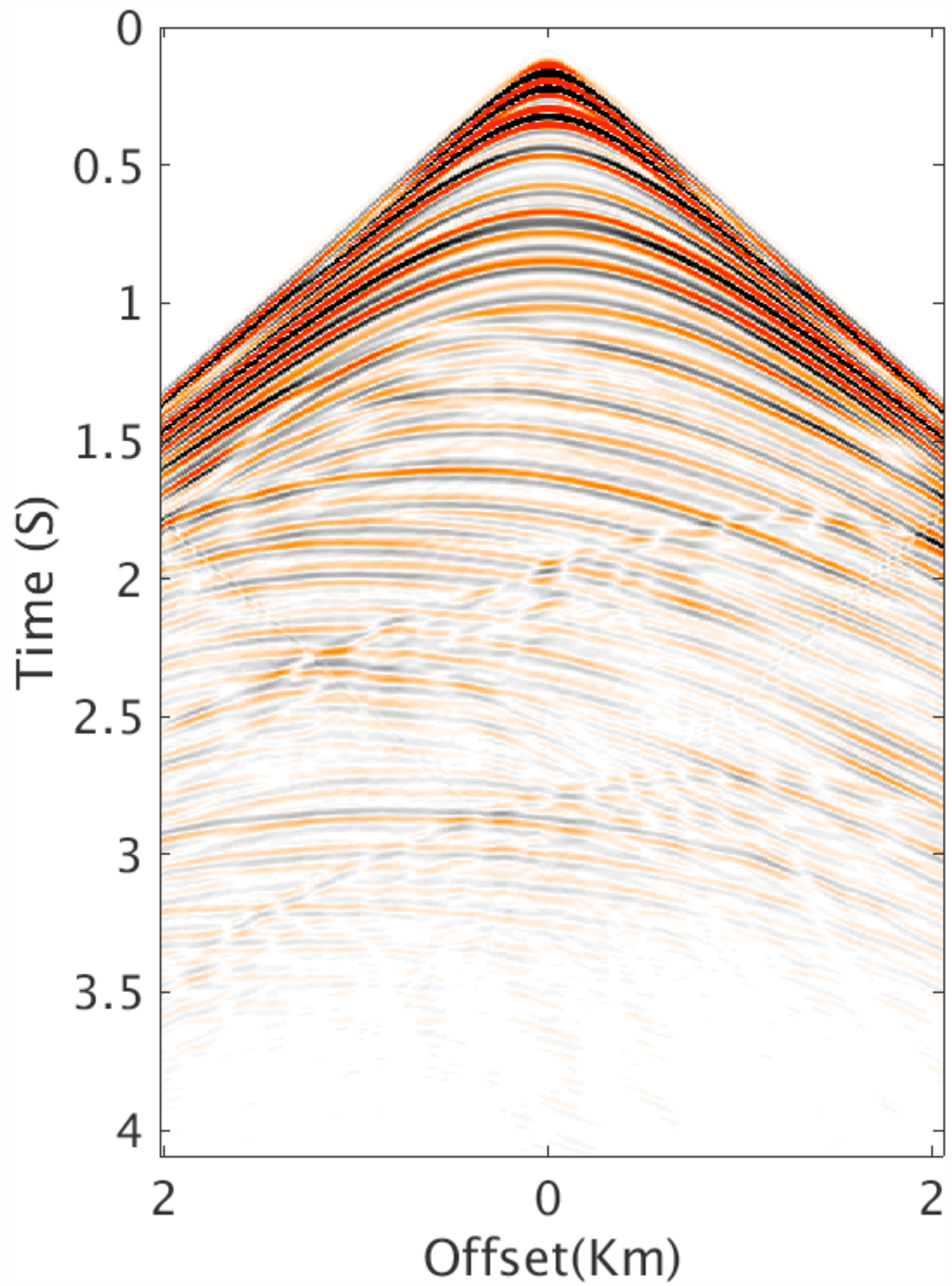


Total data

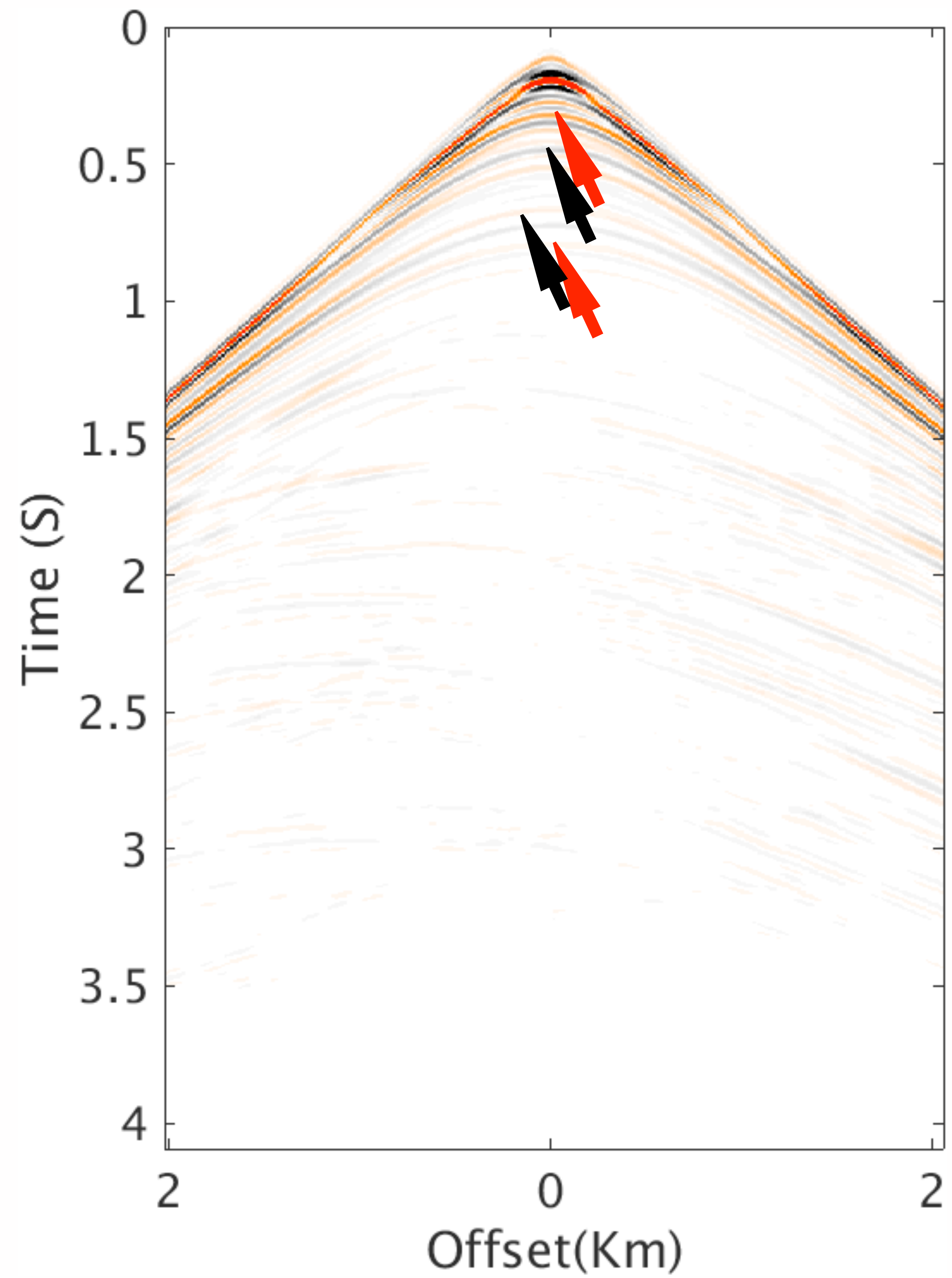


Recovered primaries by inversion w/ areal source

Shot gathers

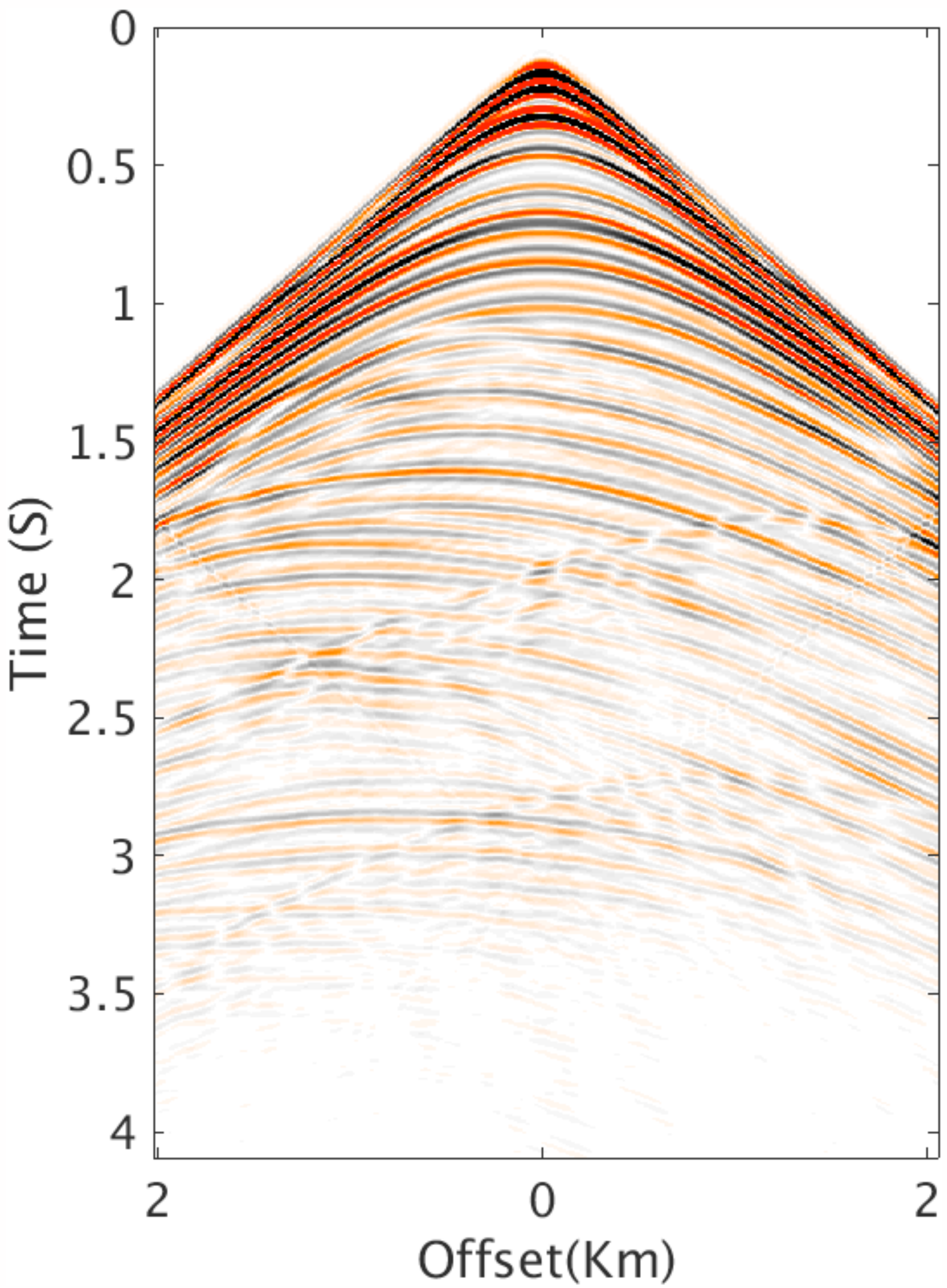


Total data

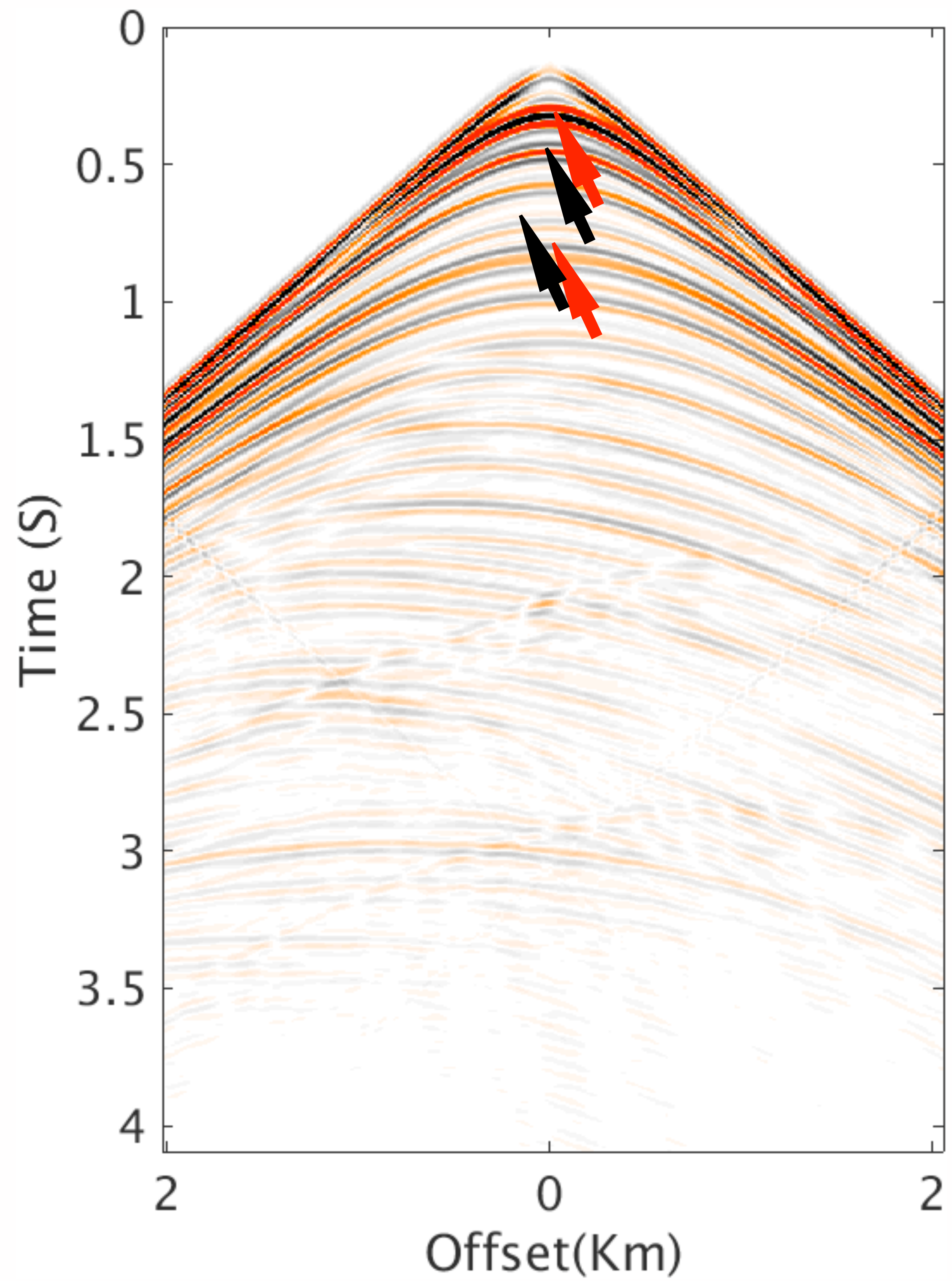


Errors in primaries

Shot gathers

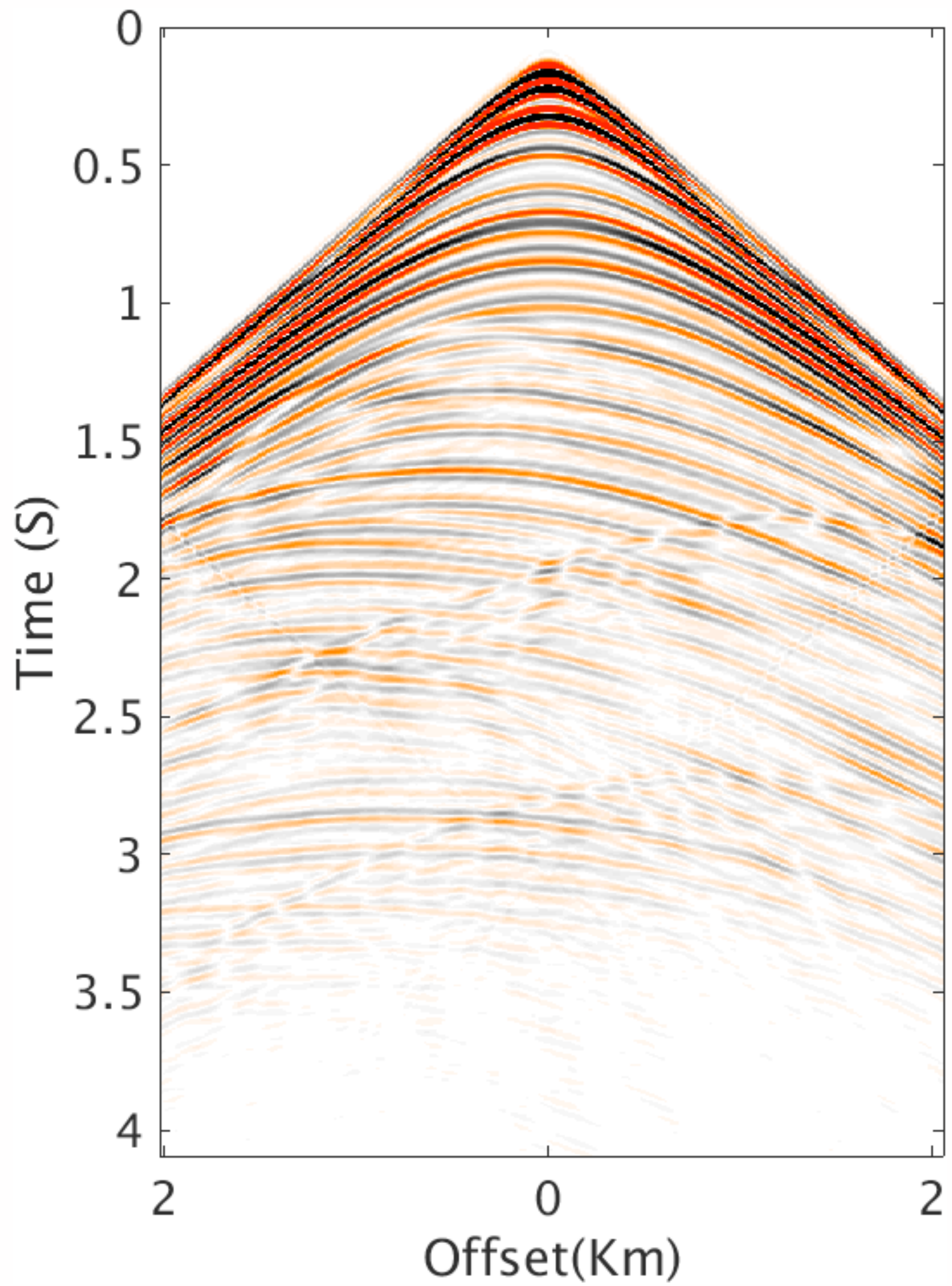


Total data



Multiples

Shot gathers



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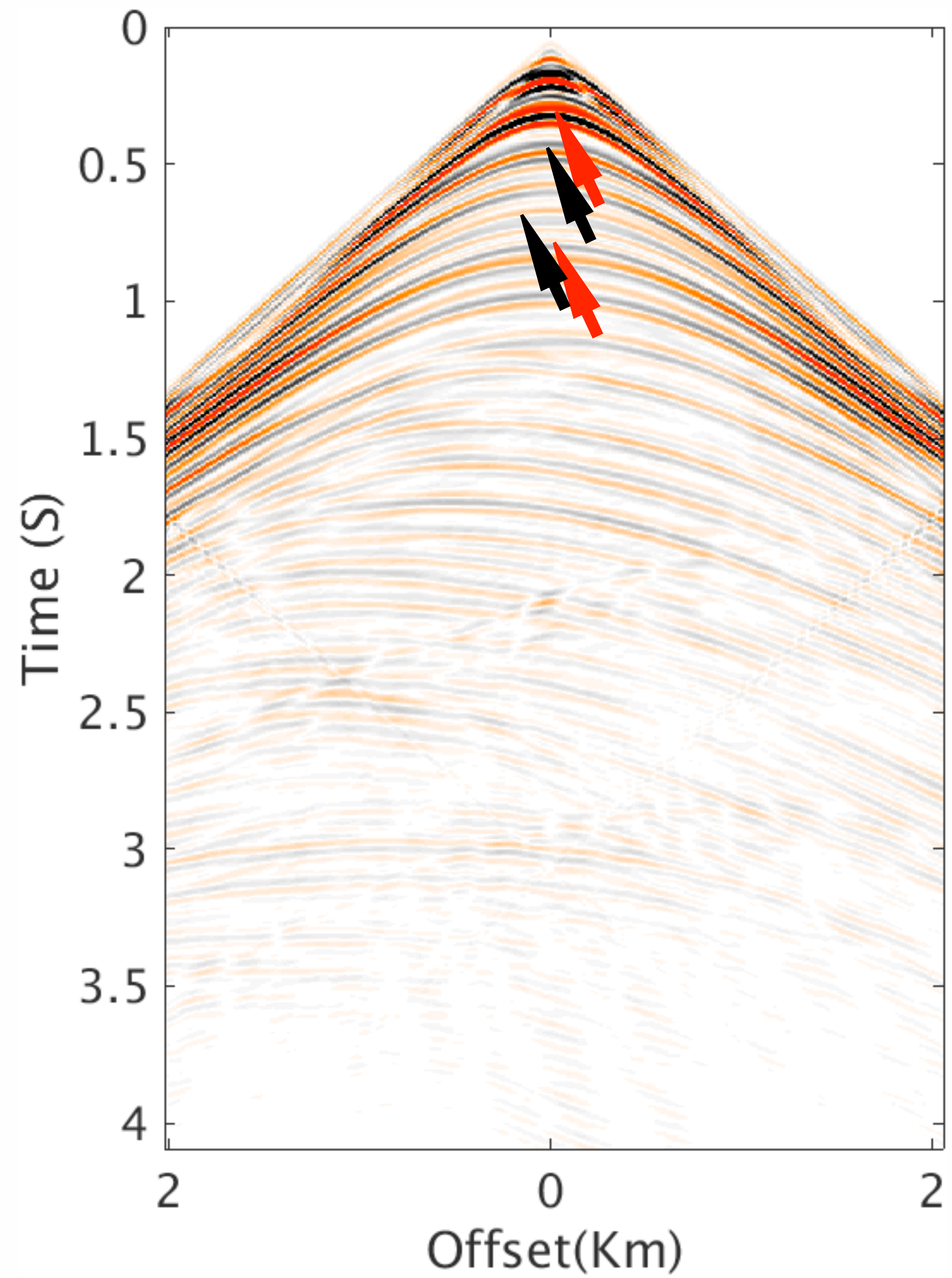


Recovered multiples

Conclusions

- Joint inversion w/ primaries & multiples via areal source gives reasonable images
  - w/ artifacts suppressed & no pre-processing
  - need only 1 data path thanks to rerandomization

- In shallow water
  - SRME always fails
  - our joint inversion succeeds

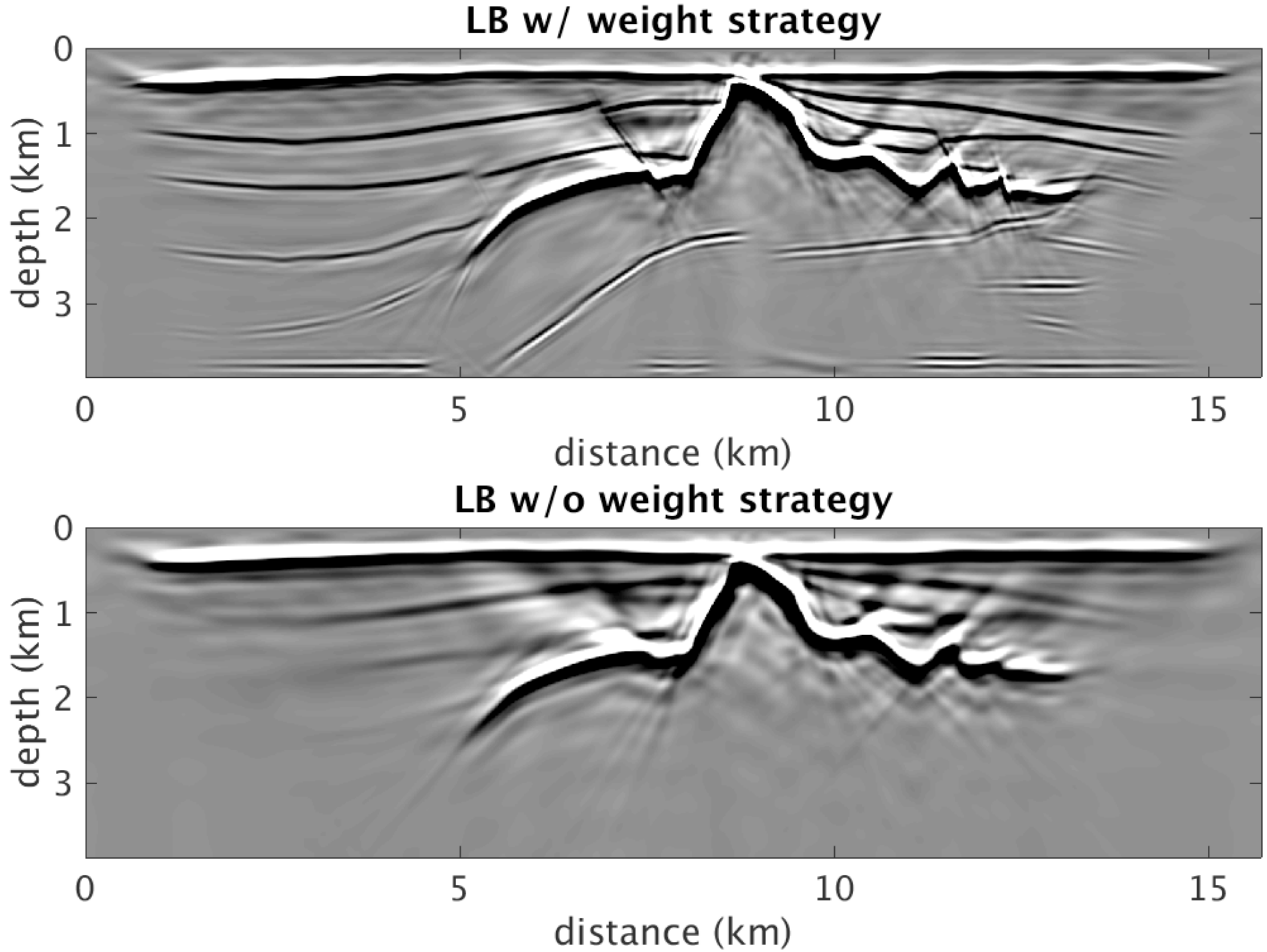
Future work

- Fix phase error in the image of joint SP-LSRTM w/ multiples & primaries
- Implement in 3D
- Accelerate convergence of SP-LSRTM

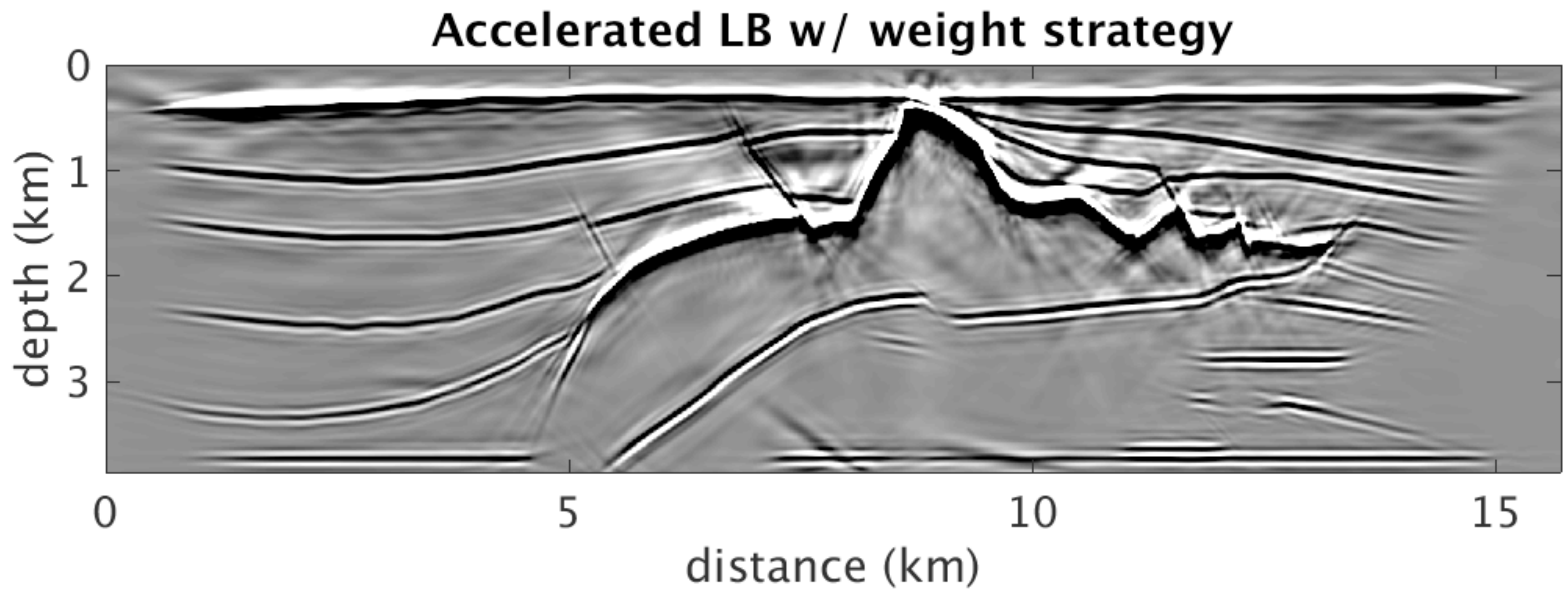
Accelerate and weight strategy on LB

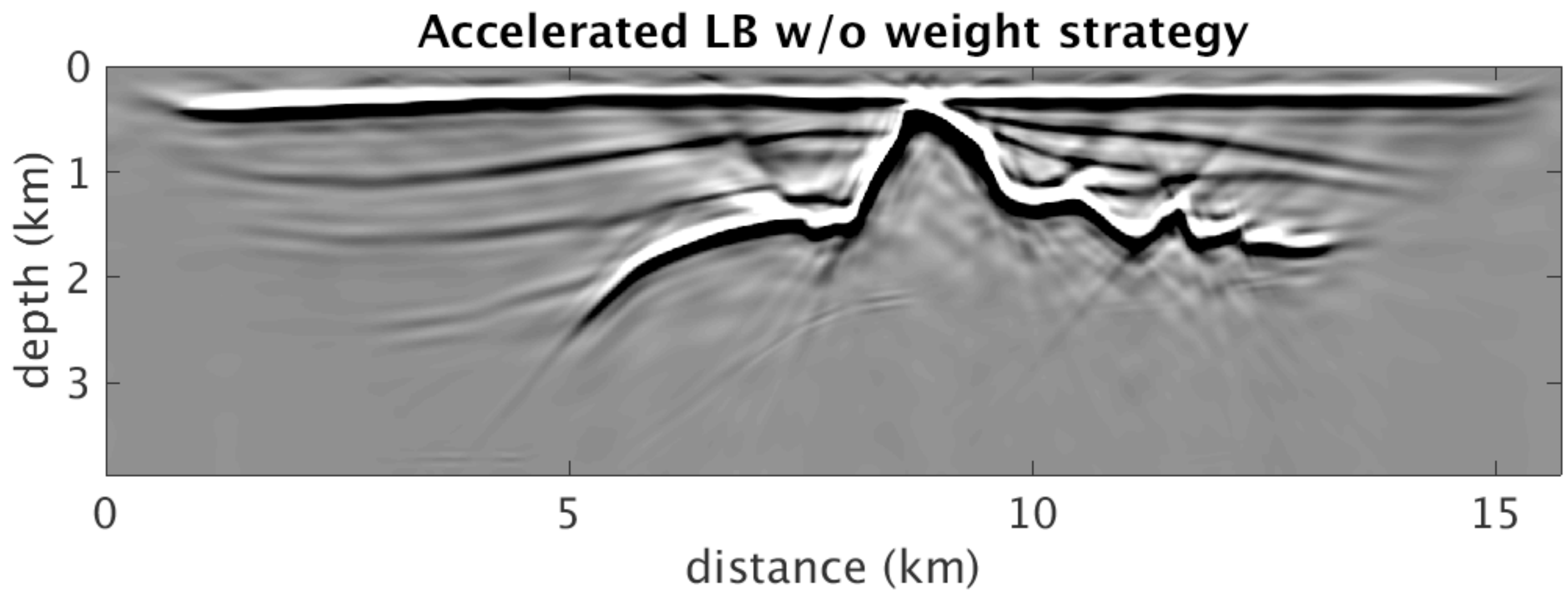
Experiment set on SEG salt model

SEG salt model, linear data, 1 data pass

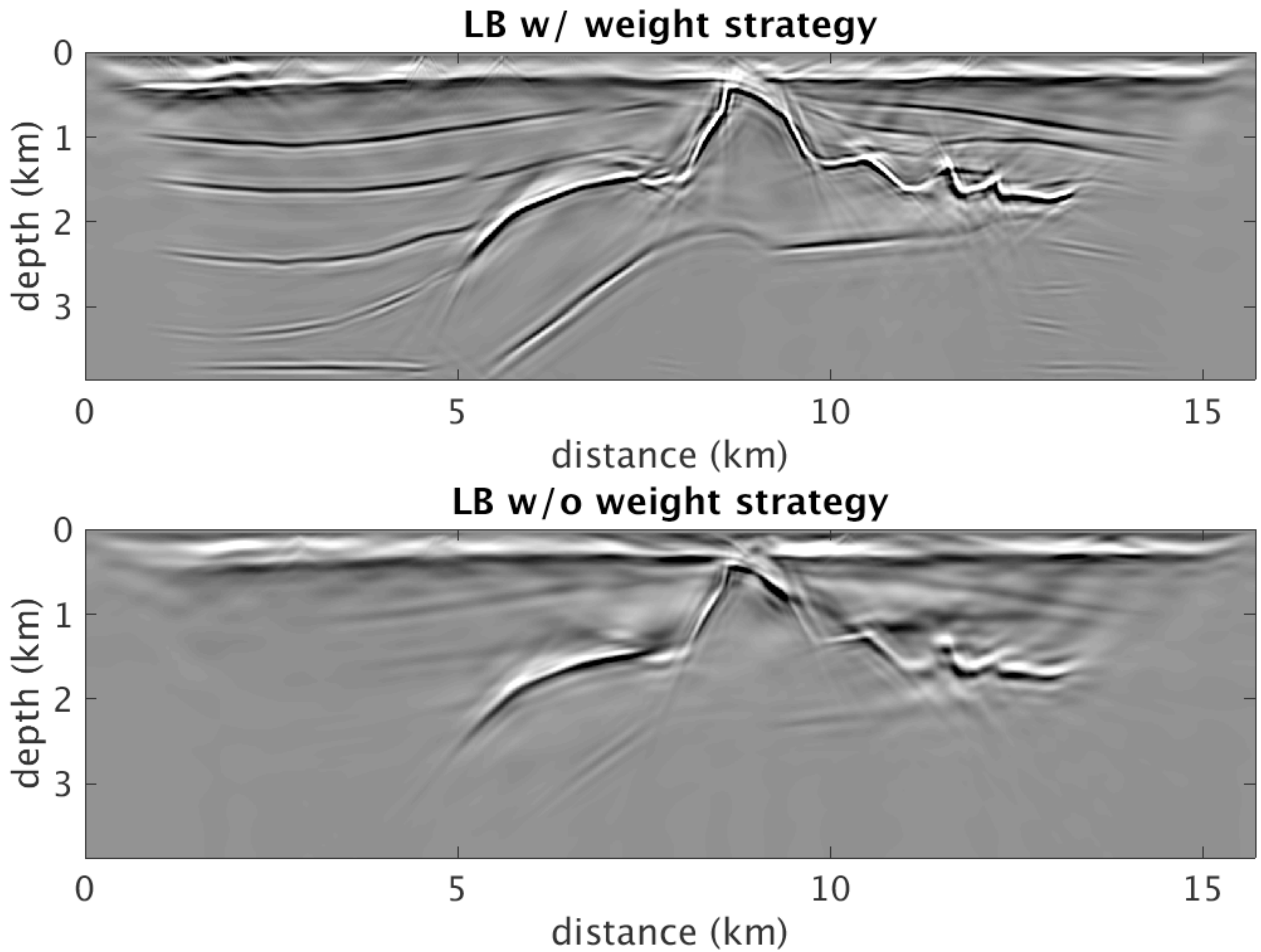


SEG salt model, linear data, 1 data pass

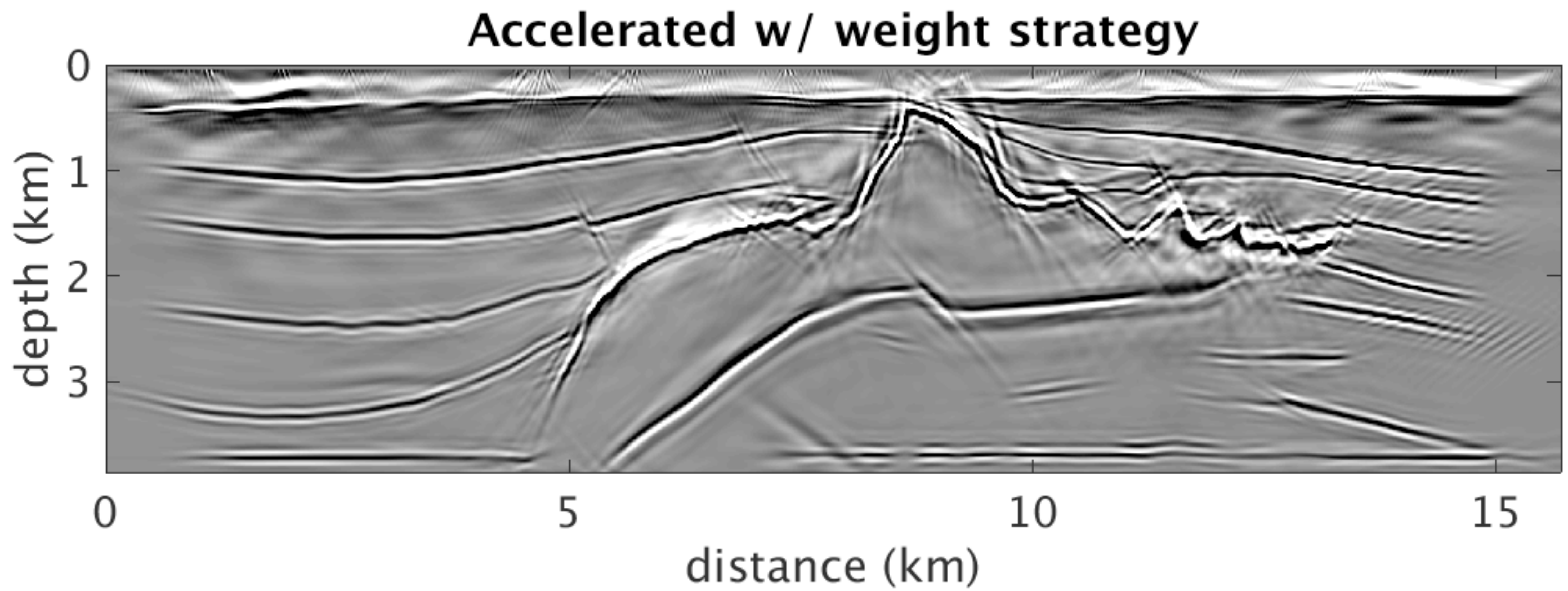


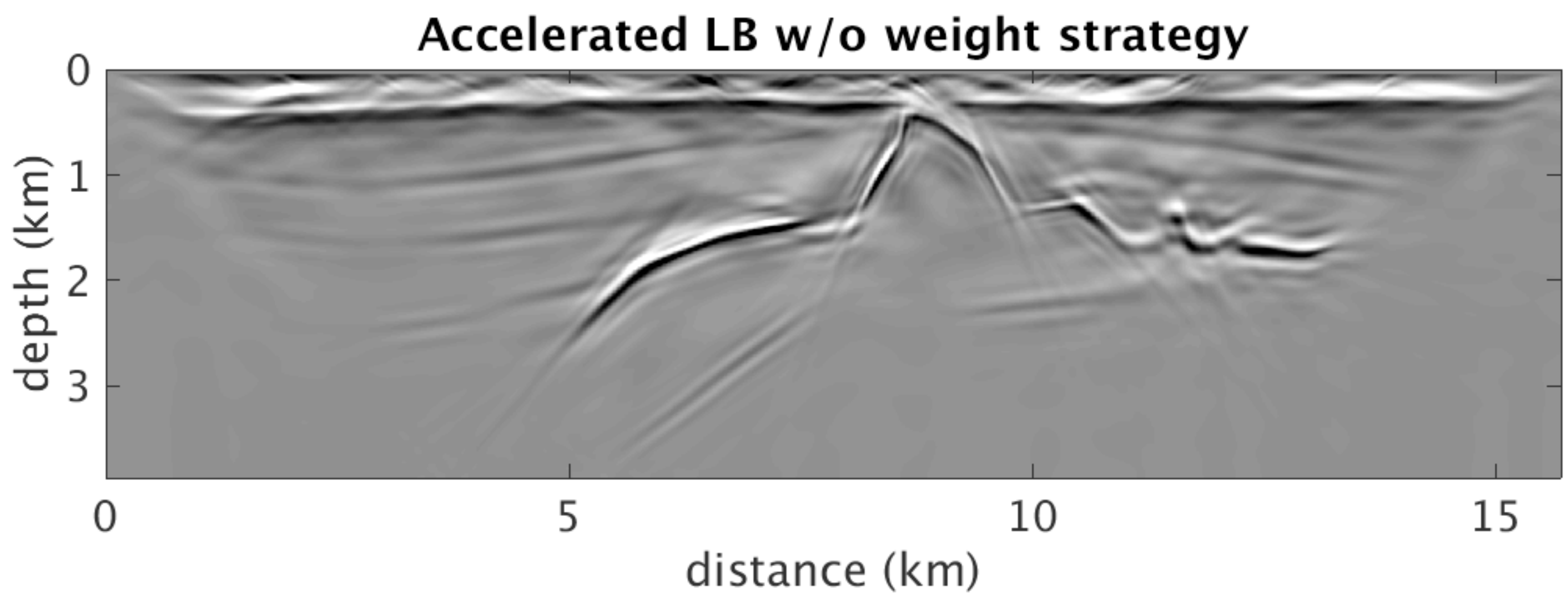


SEG salt model, nonlinear data, 1 data pass



SEG salt model, nonlinear data, 1 data pass





Acknowledgements

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Thank you for your attention