Imaging with multiples in shallow water

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Situation multiples in marine data

From Ning Tu, EAGE, 2013
How to get reliable images?

SRME relation \[ P_j = G_j (Q_j + \mathcal{R}_j P_j) \]

minimize energy misfit \[ P_{0j} = P_j - \mathcal{R}_j Q^{-1} P_{0j} P_j \]

- \( P \): total up-going wavefield
- \( G \): surface-free dipole Green’s function
- \( Q \): point-source wavefield = \( \omega_j I \)
- \( \mathcal{R} \): surface reflectivity
- \( j \): frequency index
- \( P_0 \): up-going primaries
- \( \delta m \): model perturbation

Migration

Demigration

\( G \)

\( P \)

\( Q \)
Problem in shallow water

Shot gathers

Total data

Primaries by SRME
Problem in shallow water
Shot gathers

Total data
True primaries
How to get reliable images?

SRME relation
\[ p_j = g_j (q_j + r_j p_j) \]

minimize energy misfit
\[ p_{0j} = p_j - r_j q_j^{-1} p_{0j} p_j \]

invert sparse Green function \( g_j \)
How to get reliable images?


Sorted Curvelet coefficients
Motivation

Challenges for primary prediction from shallow water multiples:
  ▶ SRME often fails to predict primaries because of “leakage”
  ▶ EPSI is too expensive

Other challenges:
  ▶ image artifacts from surface-related multiples
  ▶ computational costs
  ▶ time-domain implementation
How to get reliable images?

Joint inversion w/ areal source

Migration of total data

Sparsity promotion

\( \delta m \)
Solution

Incorporate surface-related multiples directly into imaging
  ▶ w/ SRME relation
  ▶ WE solver does multi-D convolutions implicitly
  ▶ simple implementation via linearized Bregman projections (LBP)
Eliminating dense matrix-matrix products
[SRME relation & wave-equation solver]

Combine linearized modelling w/ free-surface physics:

\[
\mathcal{P}_j \approx \nabla \mathcal{F}_j [m_0, \delta m; \mathcal{I}] (Q_j - \mathcal{P}_j) \\
= \nabla \mathcal{F}_j [m_0, \delta m; Q_j - \mathcal{P}_j] \\
= \nabla \mathcal{F}_j [m_0; Q_j - \mathcal{P}_j] \delta m.
\]

Dense matrix-matrix products
Wave-equation solves
with total downgoing data
injected as “areal” source
Eliminating dense matrix-matrix products
[SRME relation & wave-equation solver]

Combine linearized time-domain modelling w/ free-surface physics:

\[ P \approx (\nabla F_m[m_0, \rho_0; Q - P] \quad \nabla F_\rho[m_0, \rho_0; Q - P]) \begin{pmatrix} \delta m \\ \delta \rho \end{pmatrix} \]

\[ \approx \nabla F_m[m_0, \rho_0; Q - P] \delta m' \]
LBP via randomized subsampling

Randomized subsets of $A$, $b$ for linearized Bregman method:

1. \textbf{for} $k = 0, 1, \ldots$
2. $z_{k+1} = z_k - t_k A_r^*(A_r x_k - b_r)$
3. $x_{k+1} = S_\lambda(z_{k+1})$
4. \textbf{end for}

Joint SP-LSRTM w/ primaries & multiples

$$\min_{x} \lambda \|x\|_1 + \frac{1}{2} \|x\|_2^2$$

subject to $${\sum}_i \|\nabla F_i(m_0, \rho_0, Q_i - P_i)C^T x - P_i\|_2 \leq \sigma,$$

Areal source

1. Initialize $x_0 = 0, z_0 = 0, Q, \lambda$, batchsize $n'_s \ll n_s$
2. for $k = 0, 1, \ldots$
3. Randomly choose shot subsets $I \in [1 \cdots n_s], |I| = n'_s$
4. $A_k = \{\nabla F_i(m_0, \rho_0, Q_i - P_i)C^T\}_{i \in I}$
5. $b_k = \{P_i\}_{i \in I}$
6. $z_{k+1} = z_k - t_k A_k^T P_{\sigma}(A_k x_k - b_k)$
7. $x_{k+1} = S_\lambda(z_{k+1})$
8. end

note: $S_\lambda(z_{k+1}) = \text{sign}(z_{k+1}) \max\{0, |z_{k+1}| - \lambda\}$

$$P_{\sigma}(A_k x_k - b_k) = \max\{0, 1 - \frac{\sigma}{\|A_k x_k - b_k\|_2}\} \cdot (A_k x_k - b_k)$$

$$t_k = \|A_k x_k - b_k\|^2 / \|A_k^T (A_k x_k - b_k)\|^2$$
Experiment

Data:
- 261 sources and receivers
- Ricker wavelet centered at 15 Hz
- generated with free surface, $F[m, \rho] - F[m_0, \rho_0]$
- source-side deghost

Experiments:
- dipole source setting
- one pass through the data with batch sizes 2.5% data
- randomized subset of shots
- true source wavelet
RTM of primaries
RTM of total data
RTM of total data w/ areal source

Stronger at correct position

More artifacts
Joint SP-LSRTM w/ primaries & multiples
w/ areal source
Joint SP-LSRTM w/ primaries & multiples
w/o areal source
Joint SP-LSRTM w/ primaries & multiples w/ areal source, zoomed
Joint SP-LSRTM w/ primaries & multiples
w/o areal source, zoomed
Joint SP-LSRTM w/ primaries & multiples w/ areal source, zoomed
Joint SP-LSRTM w/ primaries & multiples
w/o areal source, zoomed
Shot gathers

- Total data
- Synthetic primaries

- Leakage from primaries into multiples
- Leakage from multiples into primaries
Shot gathers

Total data

Recovered primaries by inversion w/ areal source
Shot gathers

- Total data
- Errors in primaries
Shot gathers

Total data

Multiples
Conclusions

‣ Joint inversion w/ primaries & multiples via areal source gives reasonable images
  - w/ artifacts suppressed & no pre-processing
  - need only 1 data path thanks to rerandomization

‣ In shallow water
  - SRME always fails
  - our joint inversion succeeds
Future work

- Fix phase error in the image of joint SP-LSRTM w/ multiples & primaries
- Implement in 3D
- Accelerate convergence of SP-LSRTM
Accelerate and weight strategy on LB

Experiment set on SEG salt model
SEG salt model, linear data, 1 data pass

**LB w/ weight strategy**

**LB w/o weight strategy**
SEG salt model, linear data, 1 data pass

Accelerated LB w/ weight strategy

depth (km)

0 1 2 3

distance (km)

0 5 10 15

Accelerated LB w/o weight strategy

depth (km)

0 1 2 3

distance (km)

0 5 10 15
SEG salt model, nonlinear data, 1 data pass

LB w/ weight strategy

LB w/o weight strategy
SEG salt model, nonlinear data, 1 data pass

Accelerated w/ weight strategy

Accelerated LB w/o weight strategy
Acknowledgements

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.
Thank you for your attention