Multi-domain target-oriented imaging using extreme-scale matrix factorization

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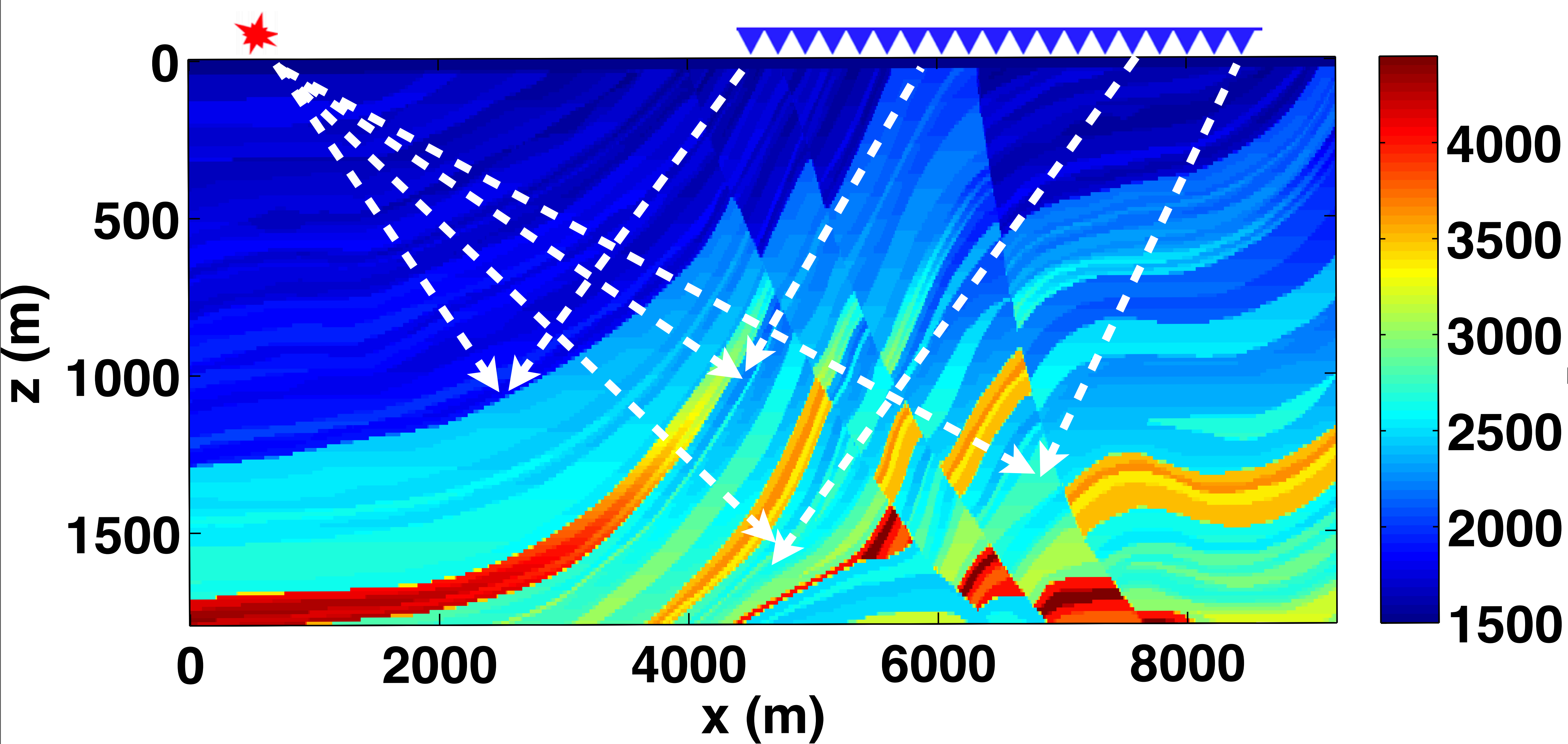
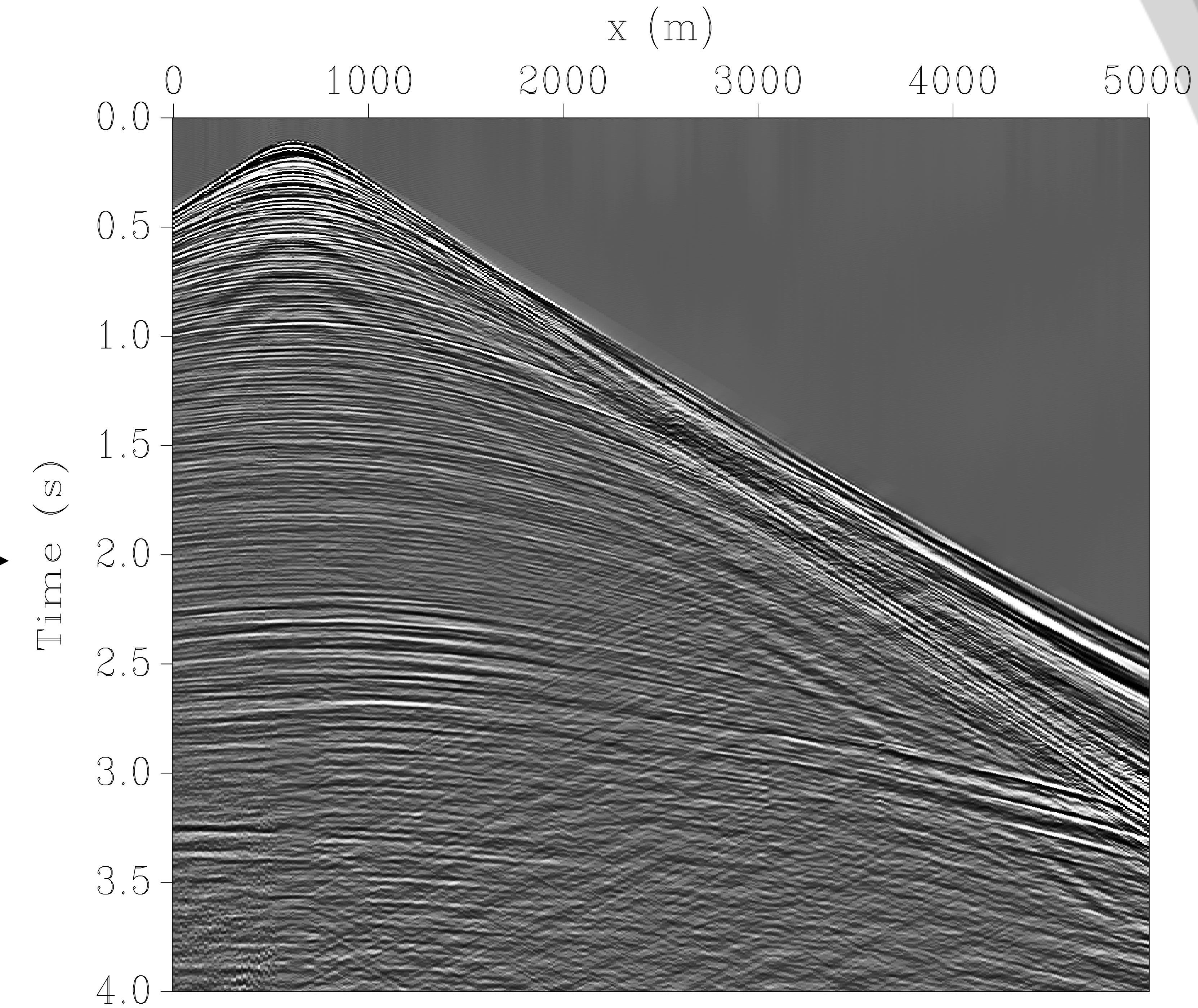
Exploration Seismology



Forward simulation



Objective

Mapping of reflection/diffraction seismic data to their true subsurface positions

Claerbout, 1970, Symes and Carazzone, 1991, Biondi and Symes, 2004, Sava and Vasconcelos, 2011

Seismic Imaging

- Forward propagate source wavefields
- Back propagate receiver wavefields
- Cross-correlate wavefields at the subsurface locations

Zero offset (migration)

All offsets (Extended image volume)

Claerbout, 1970, Symes and Carazzone, 1991, Biondi and Symes, 2004, Sava and Vasconcelos, 2011

Seismic Imaging w/ extensions

- Conventional imaging extracts zero-offset section only
- Extension/lifting corresponds to new experiment w/ sources/receivers anywhere in subsurface
- Near isometry

Zero offset (migration)

All offsets (Extended image volume)

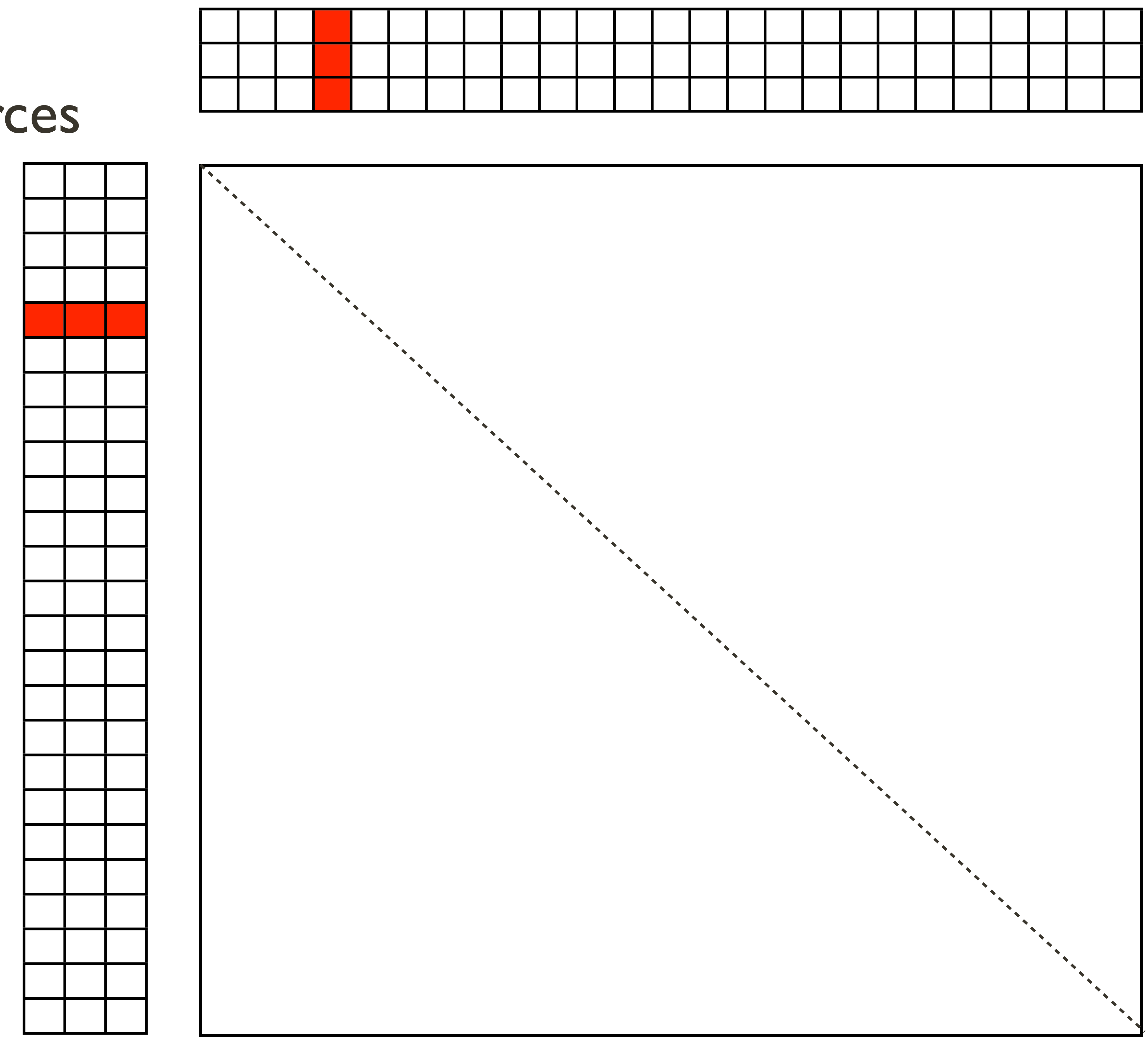
Seismic Imaging w/ extensions

- Parametrized by subsurface horizontal offset or angles
- Computed & stored for small subsets of offsets/angles
- Do not explore underlying low-rank structure

Zero offset (migration)

All offsets (Extended image volume)

Imaging—Dimensionality in 2D

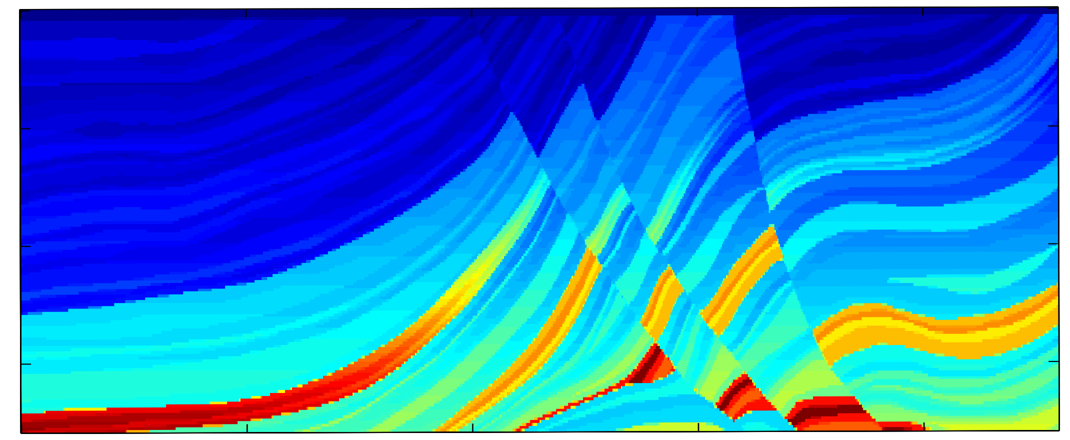


Migration : (n_x x n_z)

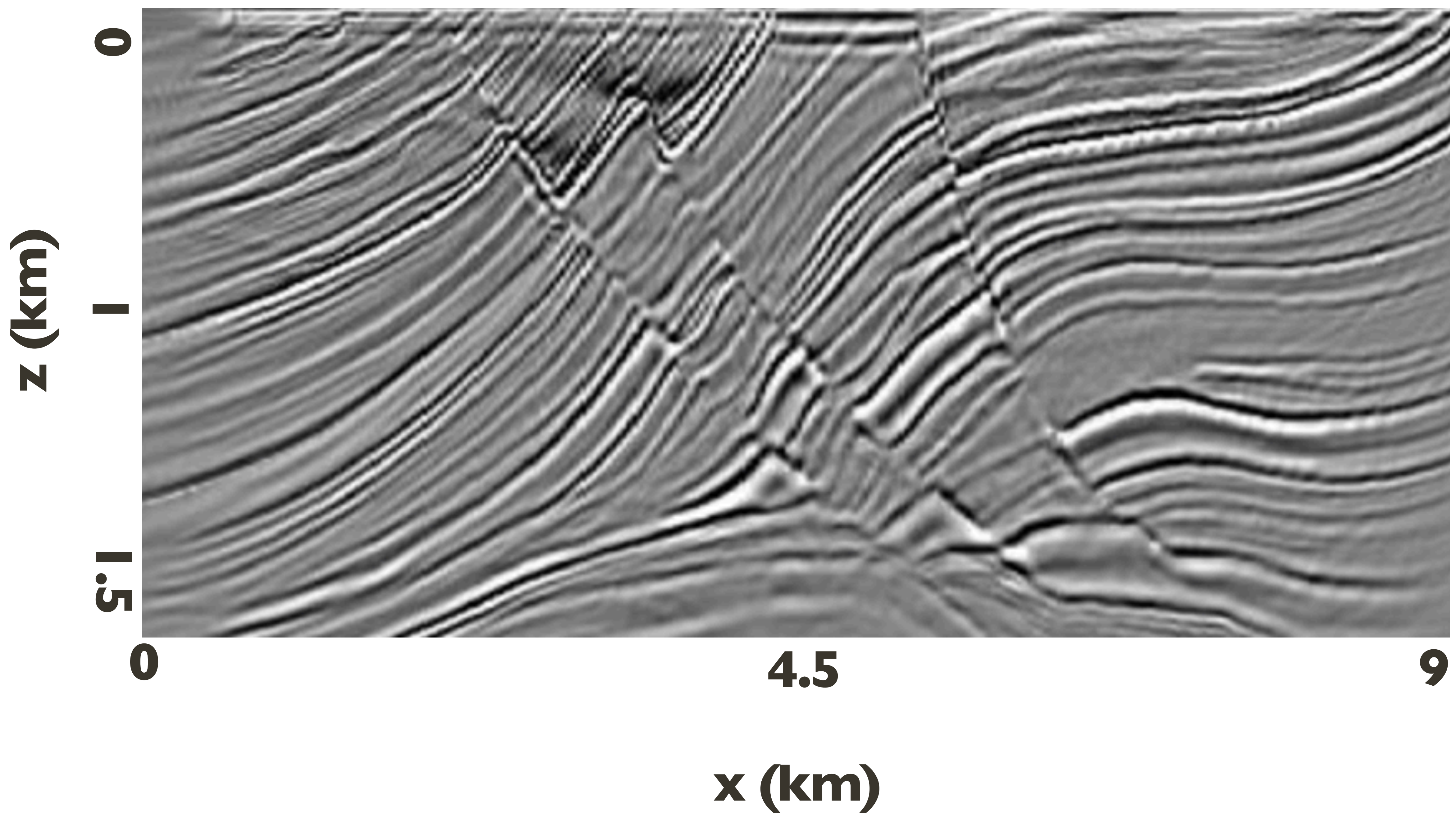
Extended image volume : (n_x x n_z) x (n_x x n_z) x ω

n_x : number of gridpoints in x-direction
n_z : number of gridpoints in z-direction
ω : number of frequencies

Imaging

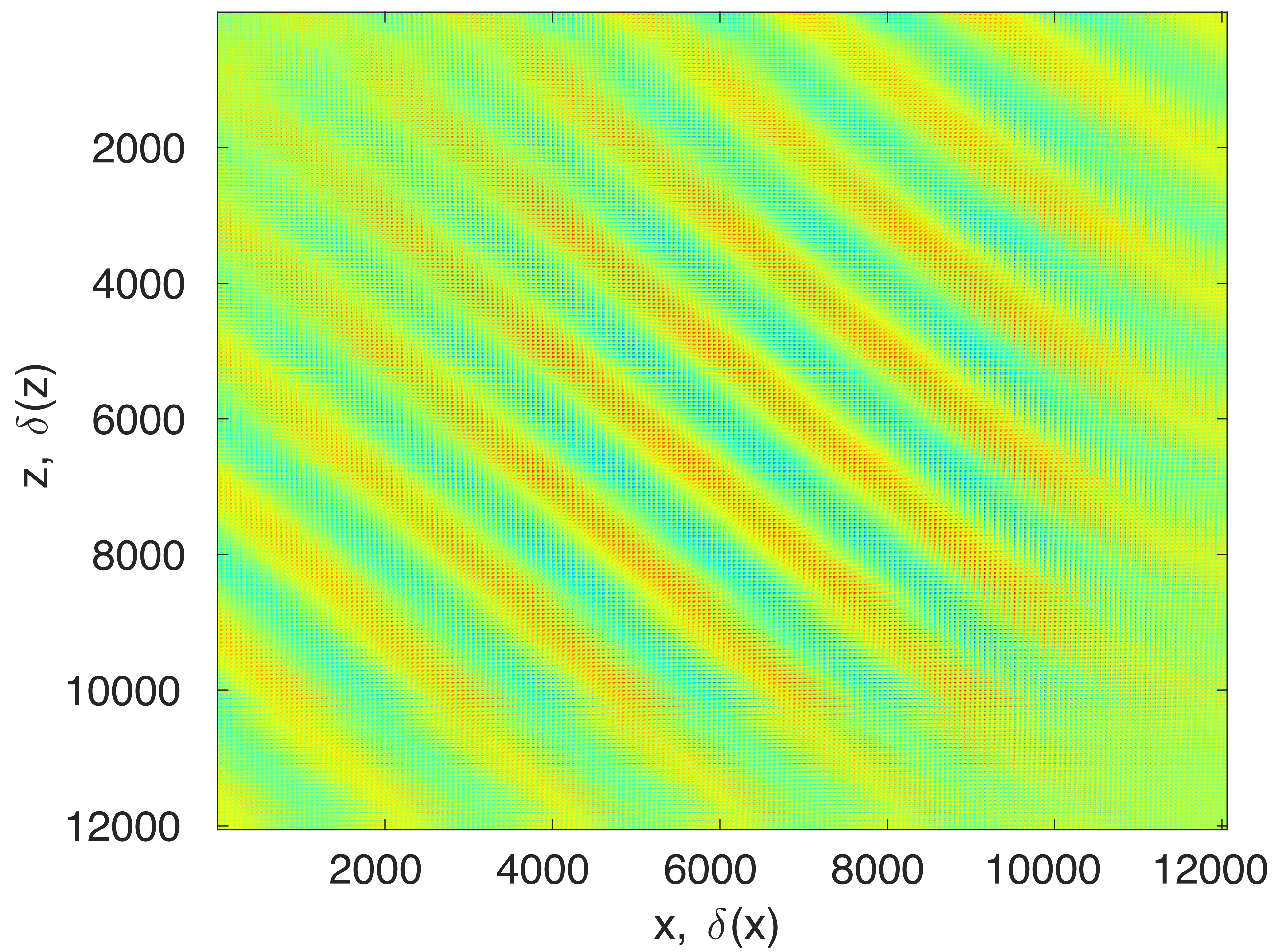


2D Matrix: (n_x x n_z)



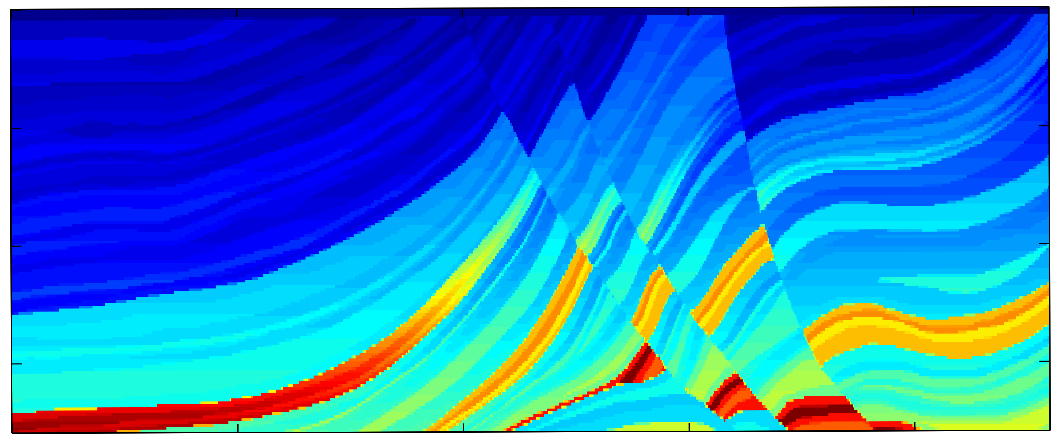
Migration

4D Tensor: (n_x x n_z) x (n_x x n_z)

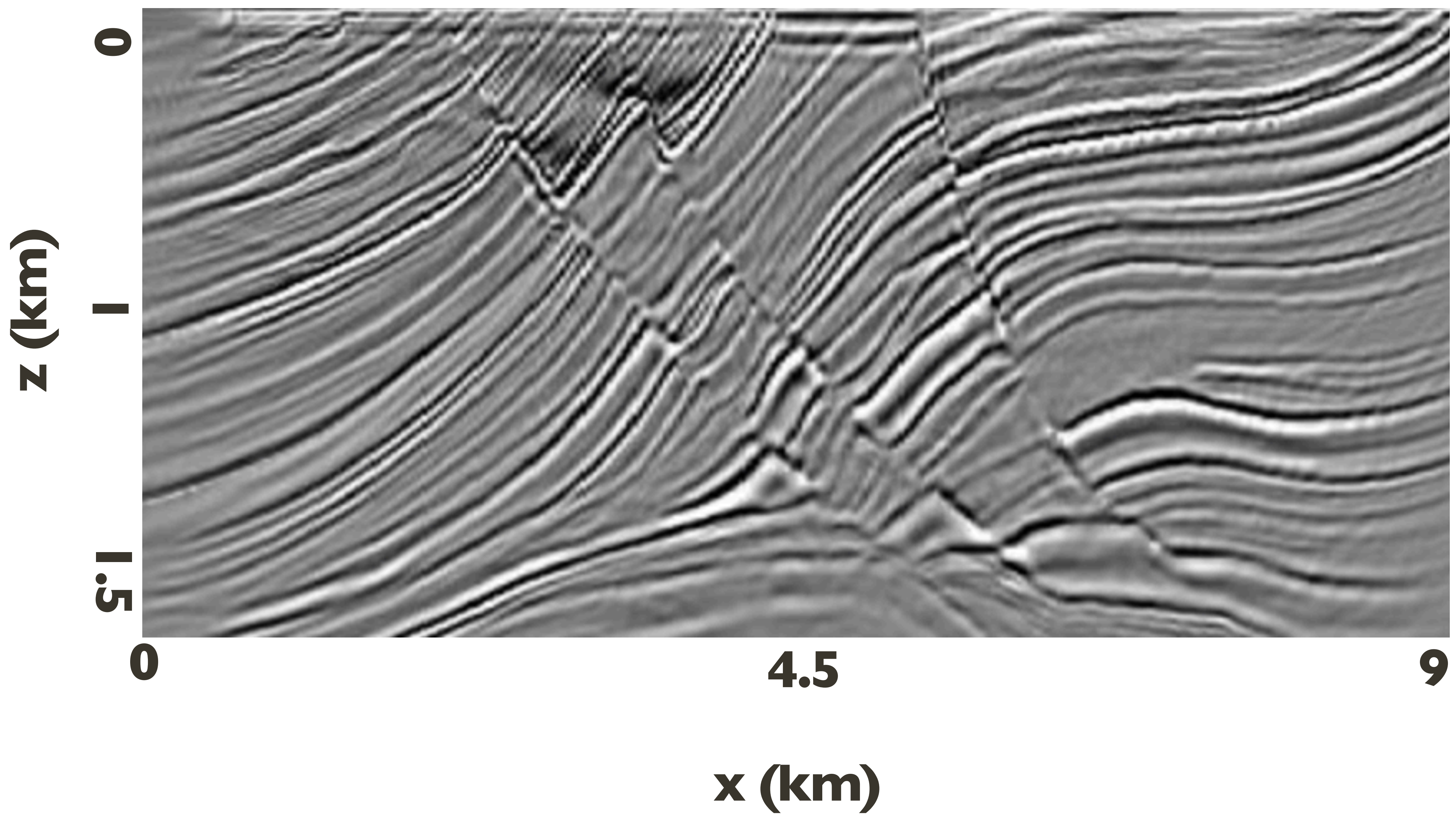


full-subsurface image volume

Imaging

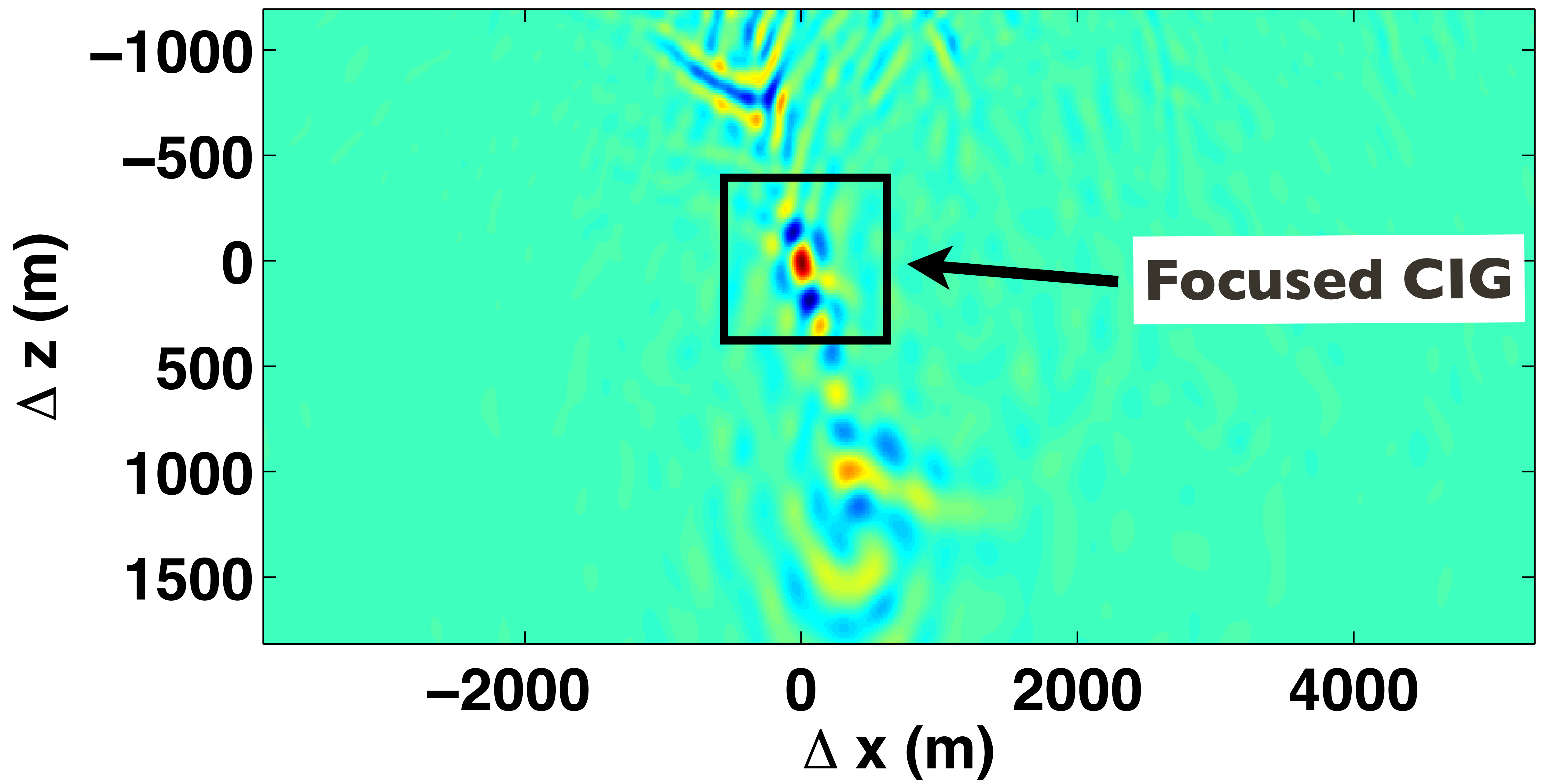


2D Matrix: (n_x x n_z)



Migration

2D Matrix: (n_x x n_z)



A column from
full-subsurface image volume

Applications of Image Volume

- Wave-equation migration-velocity analysis via principle of focusing
- Targeted imaging
- Access to directivity pattern (amplitude versus *full* offset)
- Image gathers as QC tool for validating the velocity inversion

Extended images

Given two-way wave equations, source and receiver wavefields are defined as

$$H(\mathbf{m})U = P_s^T Q$$
$$H(\mathbf{m})^* V = P_r^T D$$

where

$H(\mathbf{m})$: discretization of the Helmoltz operator

Q : source

D : data matrix

P_s, P_r : samples the wavefield at the source and receiver positions

$\mathbf{m}$: slowness

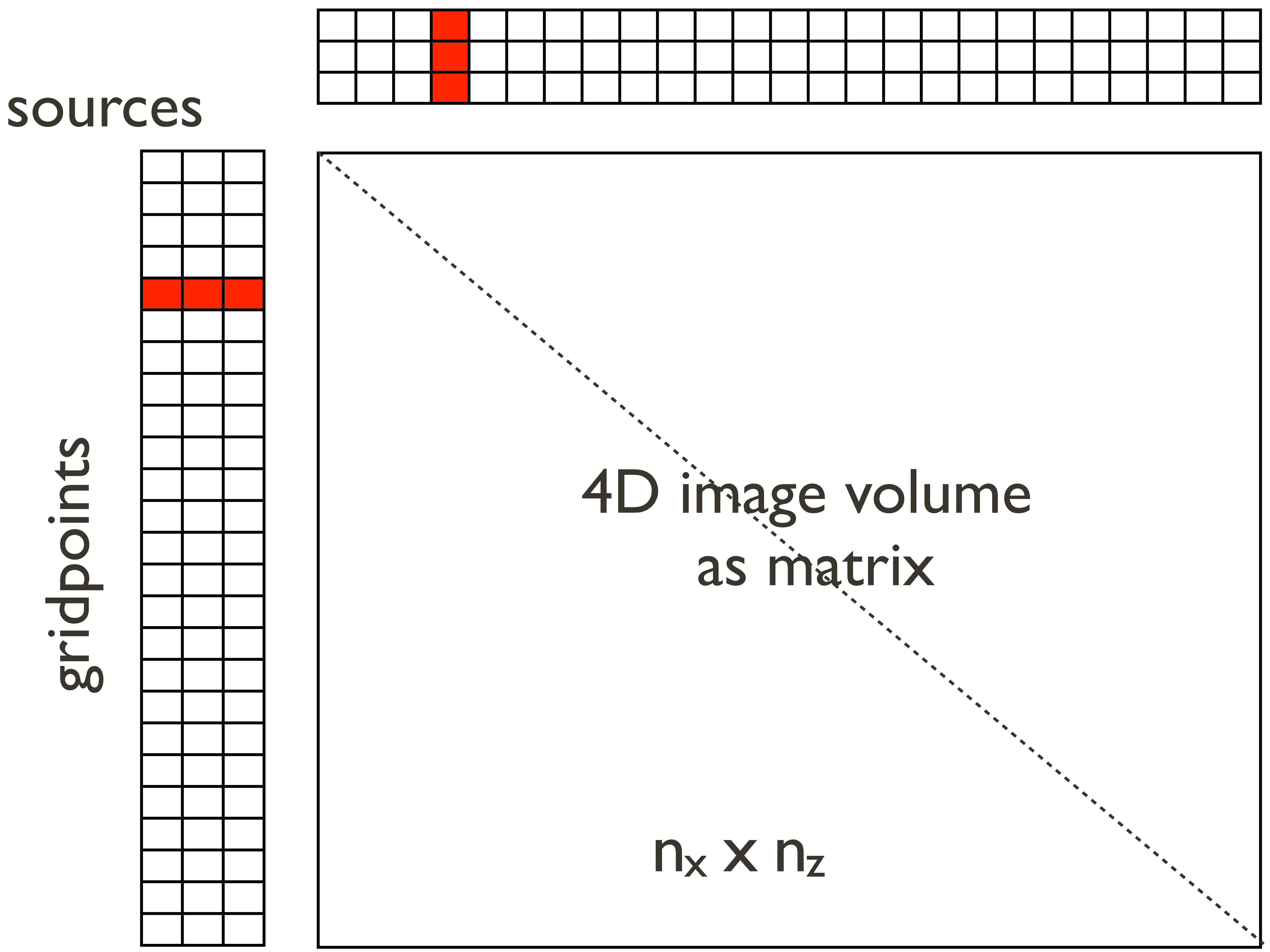
Extended images

Organize wavefields in monochromatic data *matrices* where each *column* represents a *common* shot gather

Express image volume *tensor* for *single* frequency as a *matrix*

$$E = UV^*$$

Extended images



In 3D, E is 6D tensor for each monochromatic slice

Computational bottleneck

Computation of full-subsurface offset volumes is prohibitively expensive in 3D (storage & computation time)

Can not form full *E* but *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

Extended images

Too expensive to compute *(storage and computational time)*

Instead, *probe* volume with *tall* matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_l]$

$$\widetilde{E} = EW = H^{-1} P_s^T Q D^* P_r H^{-1} W$$

van Leeuwen et al., 2013, 2016

where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents *single* scattering points

Computational costs

Full subsurface offset *extended* images:

	# of PDE solves	"flops for correlations"
conventional	$2N_s$	$N_s \times N_h$
mat-vecs	$2N_x$	$N_s \times N_r$

N_s - # of sources
N_r - # of receivers
N_h - # of subsurface offsets
N_x - # of sample points

3D BG Compass model

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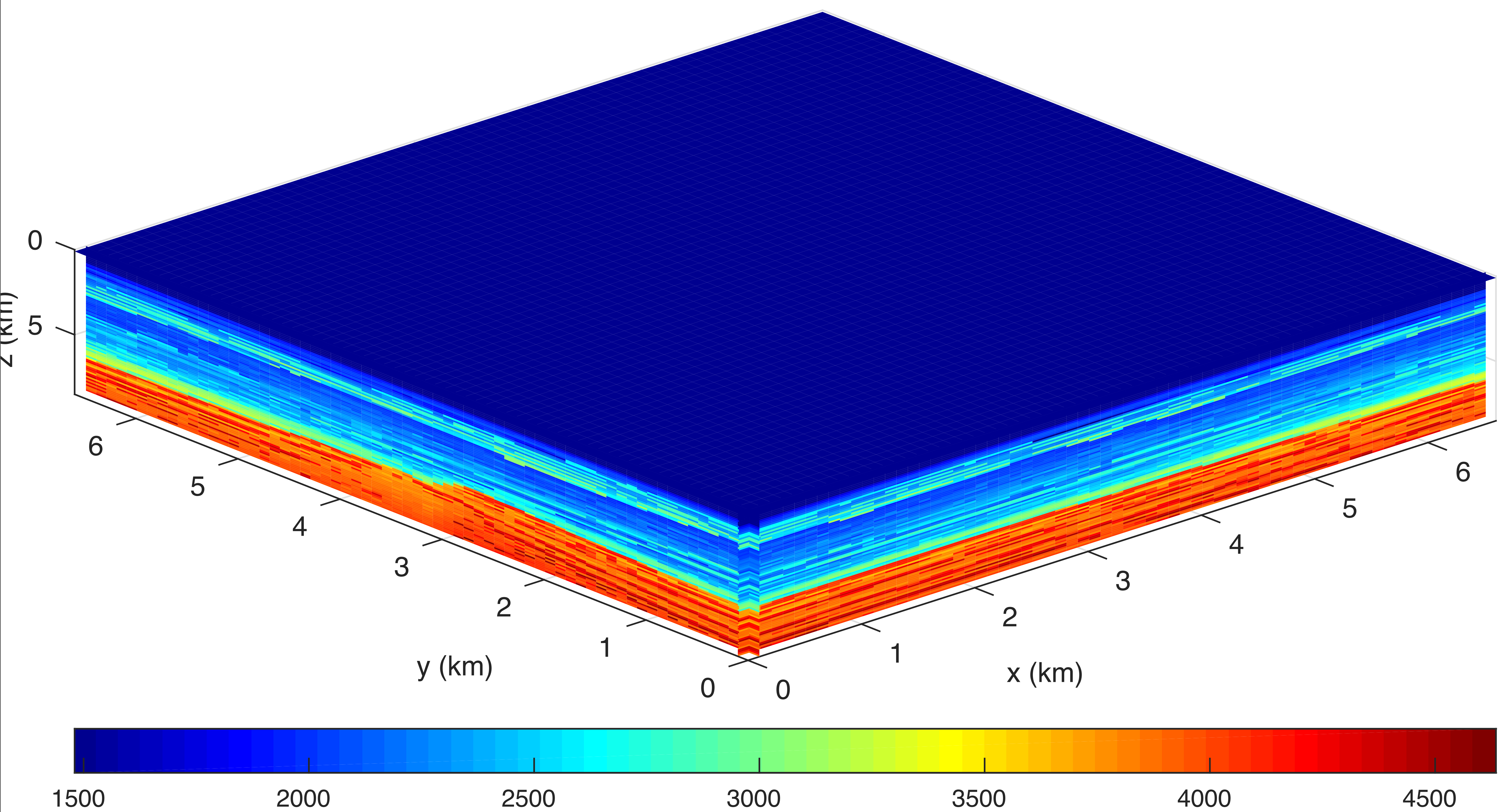
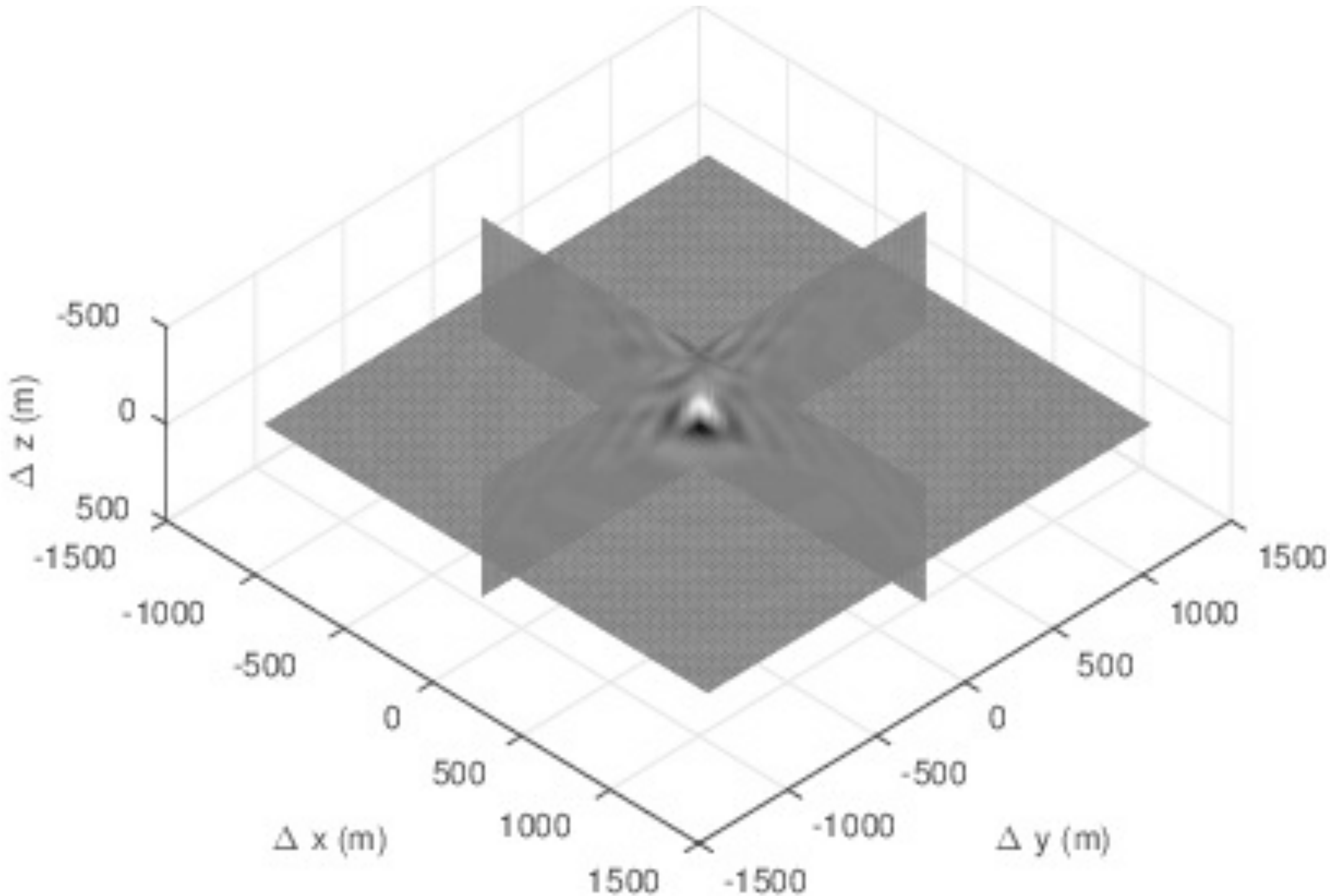


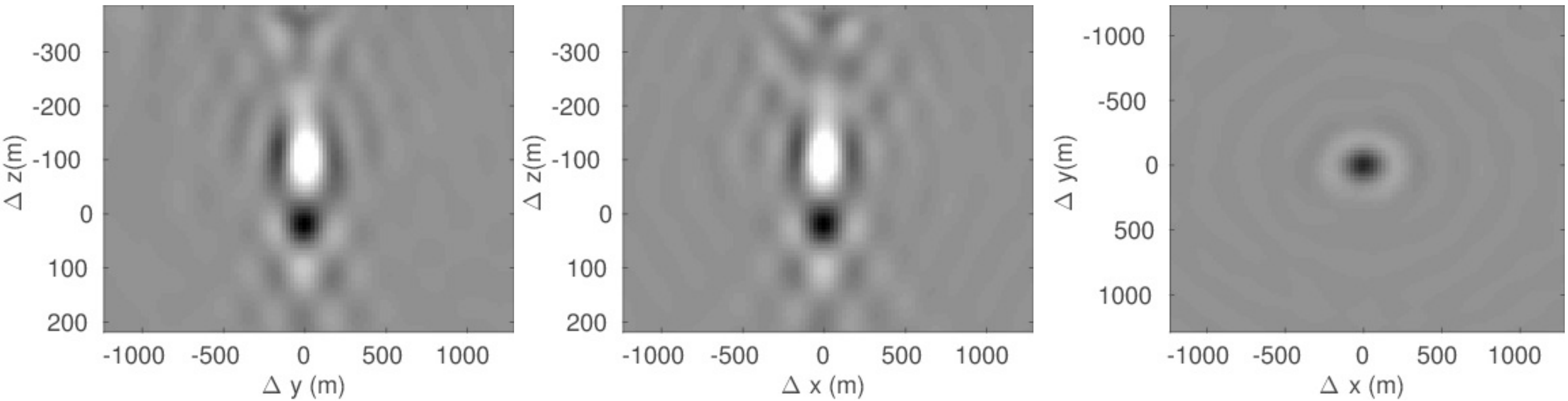
Experimental details

- 1200 source (75 m spacing)
- 2500 receivers (50 m spacing)
- 5-12 Hz
- OBN acquisition
- peak frequency 15 Hz
- One probing vector
- **1500 times faster then conventional method**

Common-image point gather



Cross section across common-image point gather



Target-oriented imaging

Why target imaging?

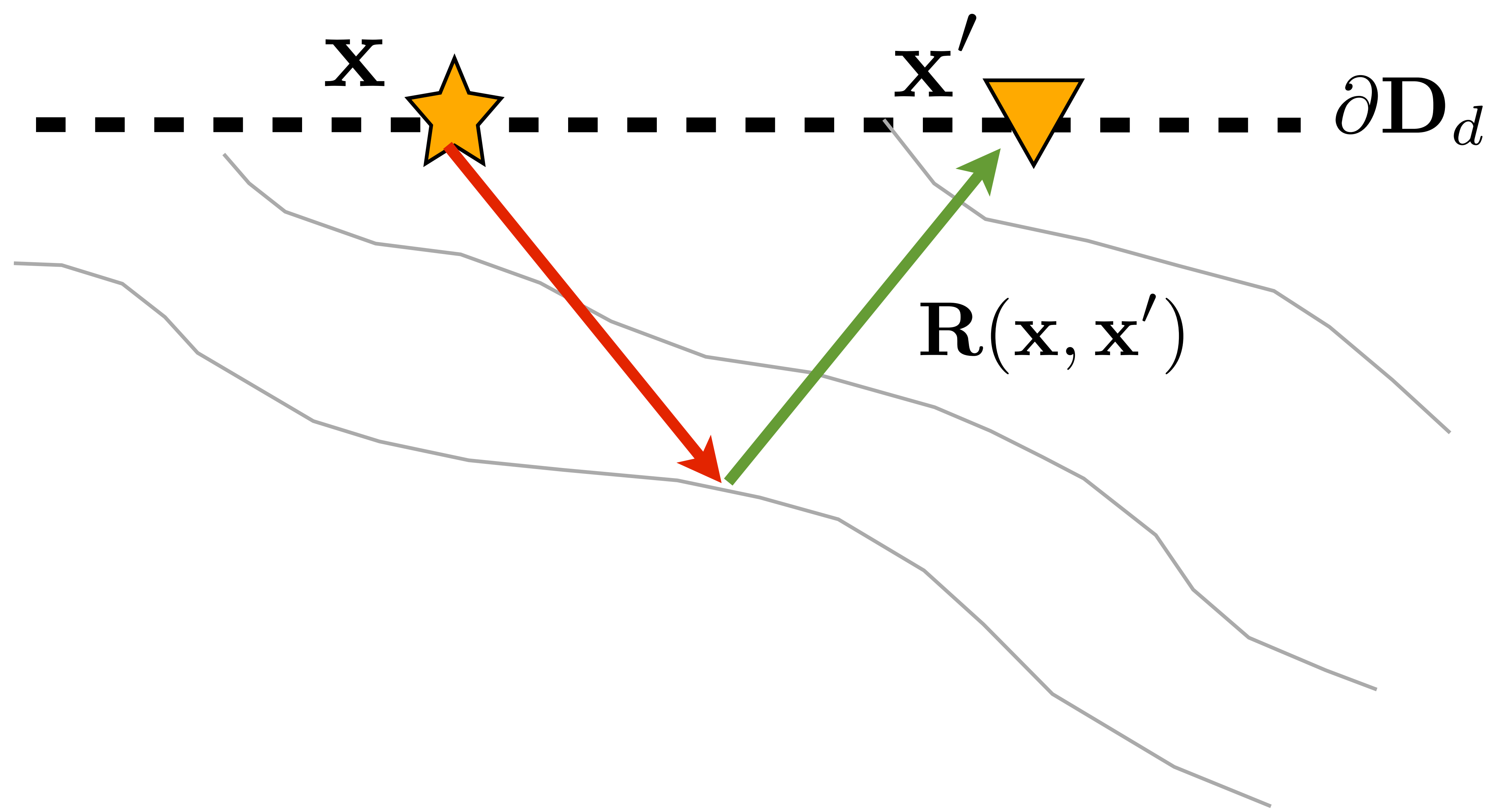
Remove complex overburden

Sub-salt imaging

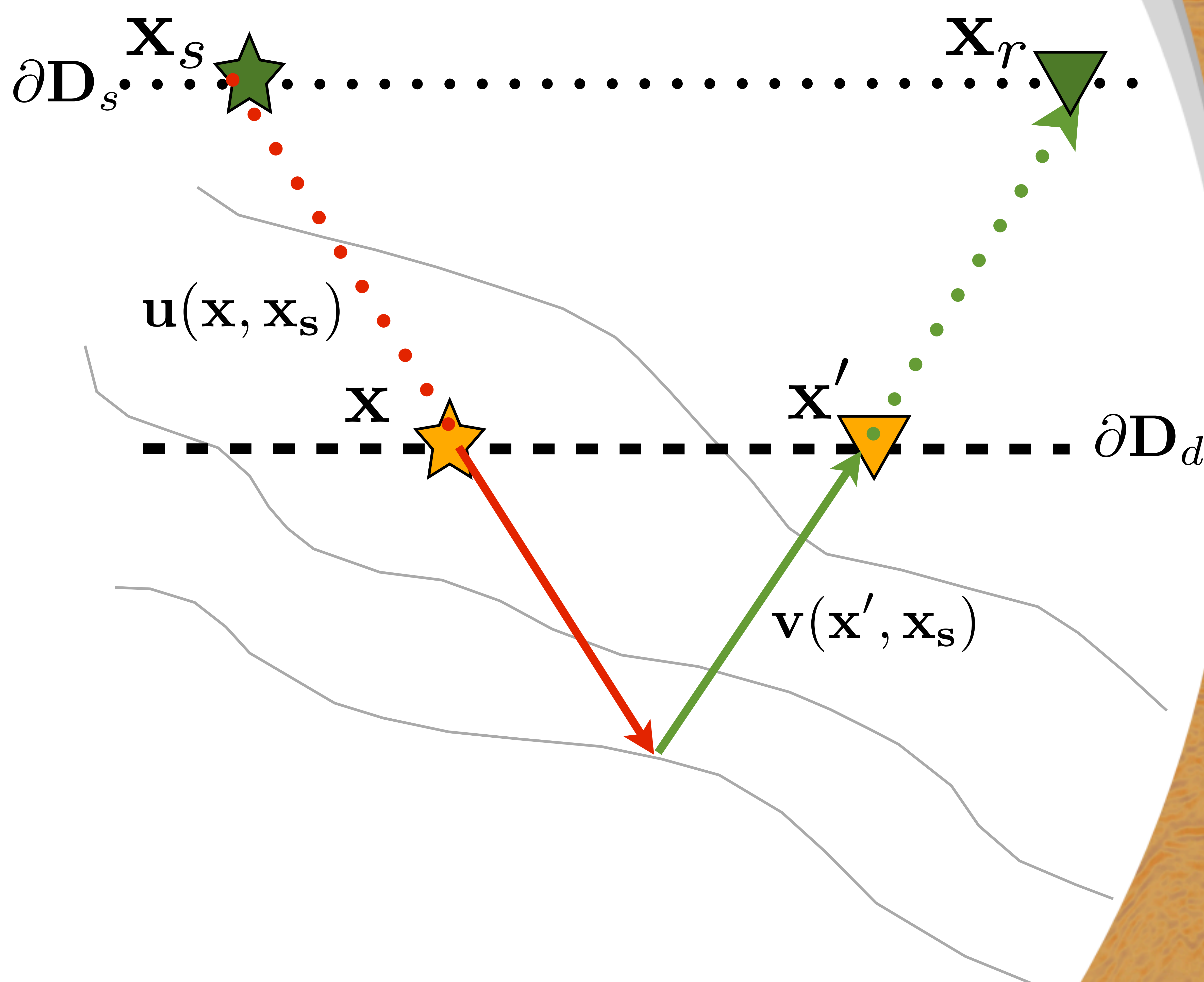
Time-lapse (imaging changes at reservoir level)

Seismic data acquisition

Ideal scenario (subsurface acquisition)



Reality (surface acquisition)



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Interferometric redatuming

Perform wavefield extrapolation followed by up-down decomposition

at the subsurface datum

Apply multidimensional deconvolution of upgoing wavefield with downgoing wavefield to perform target-oriented imaging

Methodology

Given the time-harmonic up- and down-going wavefields, i.e., $v(\mathbf{x}, \mathbf{x}_s)$ and $u(\mathbf{x}', \mathbf{x}_s)$ the reflectivity response at the subsurface datum is defined as:

$$v(\mathbf{x}, \mathbf{x}_s) = \int_{\mathbf{x}' \in \partial D_d} R(\mathbf{x}, \mathbf{x}') u(\mathbf{x}', \mathbf{x}_s) \mathrm{d}^2 \mathbf{x}'$$

In a discrete setting,

$$\mathbf{V} = \mathbf{R}\mathbf{U},$$

where, $\mathbf{R} \in \mathbf{C}^{N_x \times N_x}$, $\mathbf{V} \in \mathbf{C}^{N_x \times N_s}$, $\mathbf{U} \in \mathbf{C}^{N_x \times N_s}$

Methodology

To compute the least-squares estimate of $\mathbf{R}$, we can solve the normal-equation, which results in

$$\underbrace{\mathbf{V}\mathbf{U}^*}_{\mathbf{E}} = \mathbf{R}\underbrace{(\mathbf{U}\mathbf{U}^*)}_{\boldsymbol{\Gamma}},$$

where,

$\mathbf{E}$: extended image volume at the subsurface datum

$\boldsymbol{\Gamma}$: wavefield point-spread function

Conventional framework

Solve the two-way wave-equation to calculate $\mathbf{U}$ and $\mathbf{V}$

Restrict the solution of the wave-equation to the subsurface datum

Perform correlation, i.e., $\mathbf{V}\mathbf{U}^*$ and $\mathbf{U}\mathbf{U}^*$

Perform the Multidimensional deconvolution (MDD) to get $\mathbf{R}$, i.e., $\mathbf{R} = \mathbf{E}\mathbf{\Gamma}^\ddagger$

Too expensive to compute *(storage and computational time)* for all surface sources

Probing techniques

Instead, *probe* $\mathbf{E}$ and $\mathbf{\Gamma}$ with *tall* matrix $\mathbf{W}$, i.e.,

$$\widetilde{\mathbf{E}} = \mathbf{E}\mathbf{W} = \mathbf{H}^{-*}\mathbf{P}_r^T\mathbf{D}\mathbf{Q}^*\mathbf{P}_s\mathbf{H}^{-*}\mathbf{W}$$

$$\widetilde{\mathbf{\Gamma}} = \mathbf{\Gamma}\mathbf{W} = \mathbf{H}^{-1}\mathbf{P}_s^T\mathbf{Q}\mathbf{Q}^*\mathbf{P}_s\mathbf{H}^{-*}\mathbf{W}$$

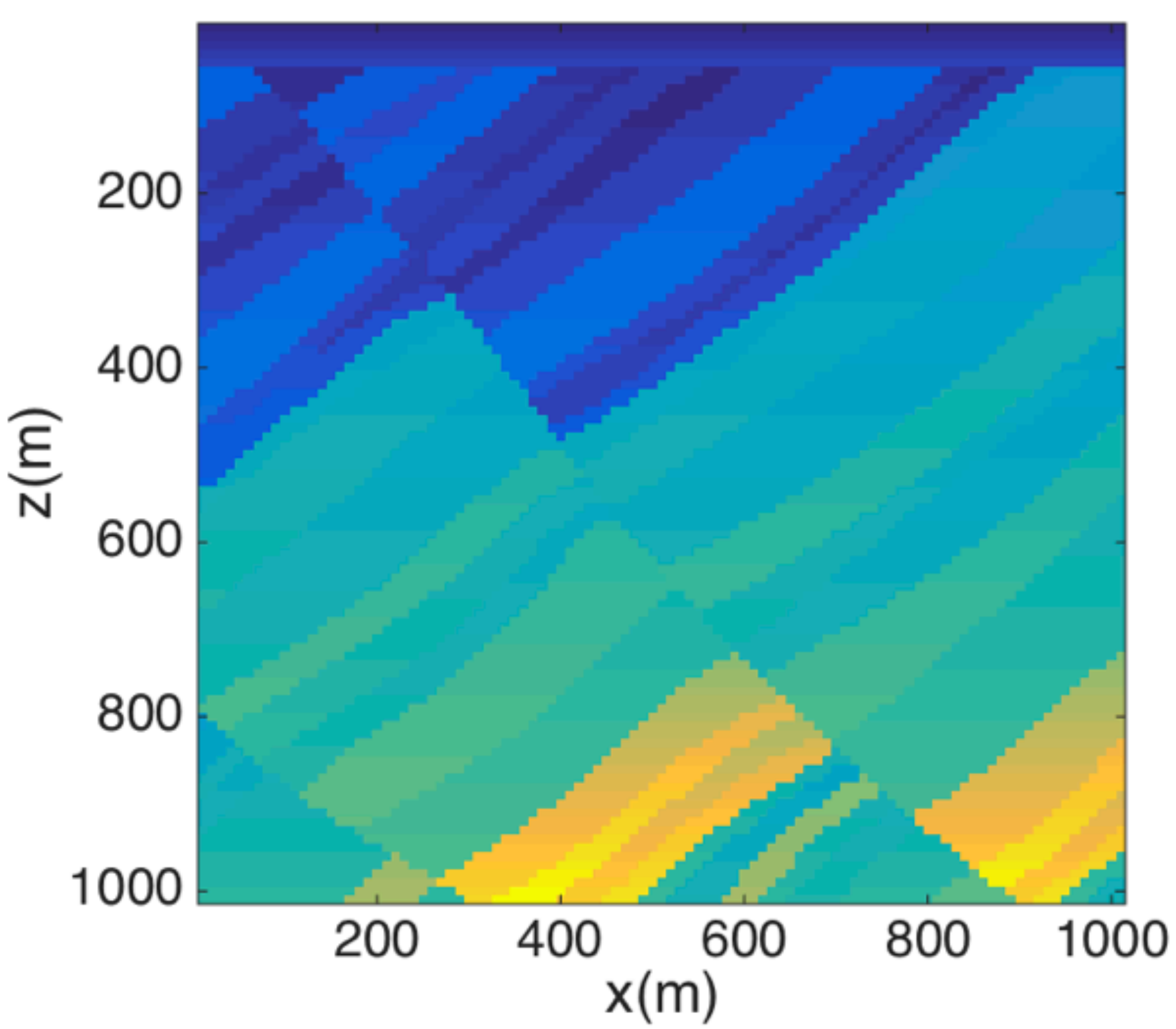
perform MDD to get $\mathbf{R}$, i.e.,

$$\mathbf{R} = \widetilde{\mathbf{E}}\widetilde{\mathbf{\Gamma}}^{\ddagger}$$

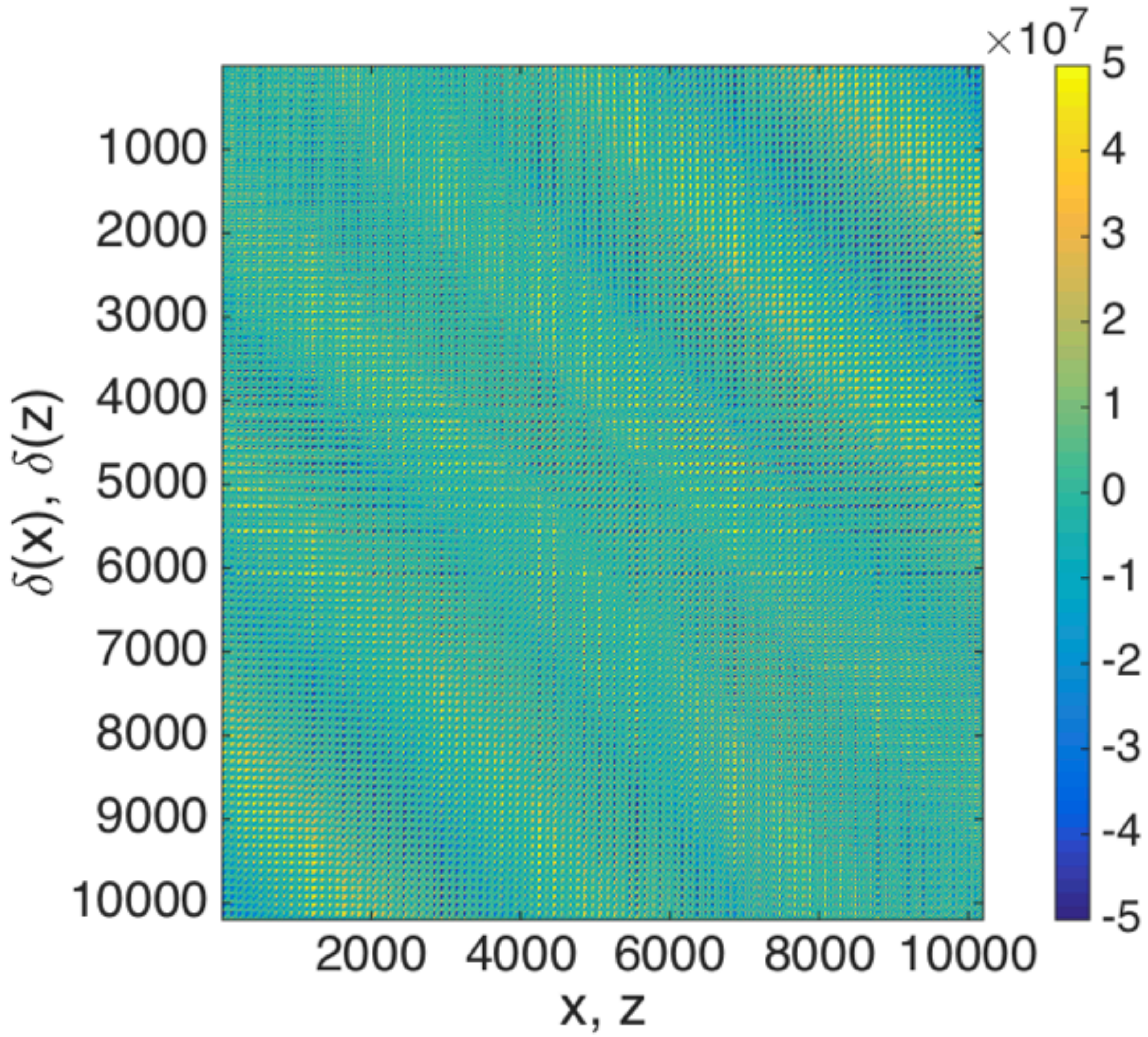
Computationally feasible but limited access to the columns of full-subsurface image volume

Low-rank properties of image volume

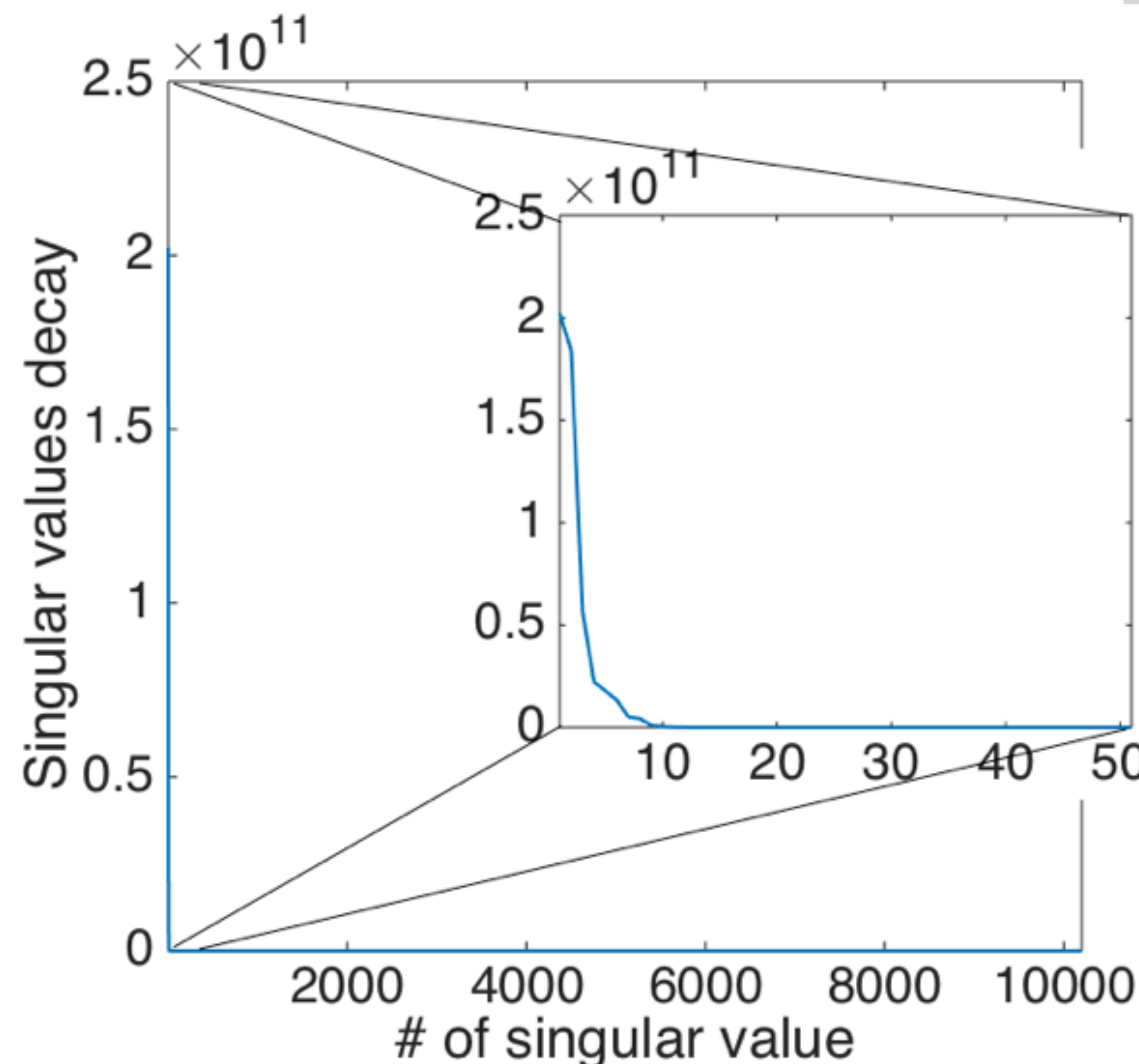
Model (101x101)



Full Image Volume



Singular Values decay



Extreme-scale matrix-factorization

Algorithm from [1]:

1. $Y = EW$ — probe full extended image volume with virtual sources
2. $[Q, R] = \mathrm{qr}(Y)$ — QR factorization
3. $Z = Q^* E$ — probe again with new virtual sources
4. $[U, S, V] = \mathrm{svd}(Z)$ — SVD factorization (first few singular values)
5. $U \leftarrow QU$ — update left singular vectors

We also perform the randomized SVD for Γ

Low-rank across frequencies

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- 3km x 3km subset of Marmousi model
- 301 source, 301 receivers

frequency (Hz)	real rank	full time (s)	estimated rank	new time (s)	SNR (dB)	memory ratio
5	301	4832	25	20	61	1216
10	301	4078	39	27	52	780
15	301	5447	52	34	49	585
20	301	4864	65	42	47	468
25	301	5351	79	50	47	441

Proposed framework

Perform randomized SVD based factorization for $\mathbf{E}$ and $\mathbf{\Gamma}$ and probe the factorized formulation for any subsurface datum using $\mathbf{W}$

$$\widetilde{\mathbf{E}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V'}_1\mathbf{W}$$

$$\widetilde{\mathbf{\Gamma}} = \mathbf{U}_2\mathbf{S}_2\mathbf{V'}_2\mathbf{W}$$

Perform MDD to get $\mathbf{R}$, i.e.,

$$\mathbf{R} = \widetilde{\mathbf{E}}\widetilde{\mathbf{\Gamma}}^{\ddagger}$$

Enabling multi-domain target imaging

Experimental details

375 source (20 m spacing) , 751 receivers (10 m spacing)

5-40 Hz

split-spread acquisition

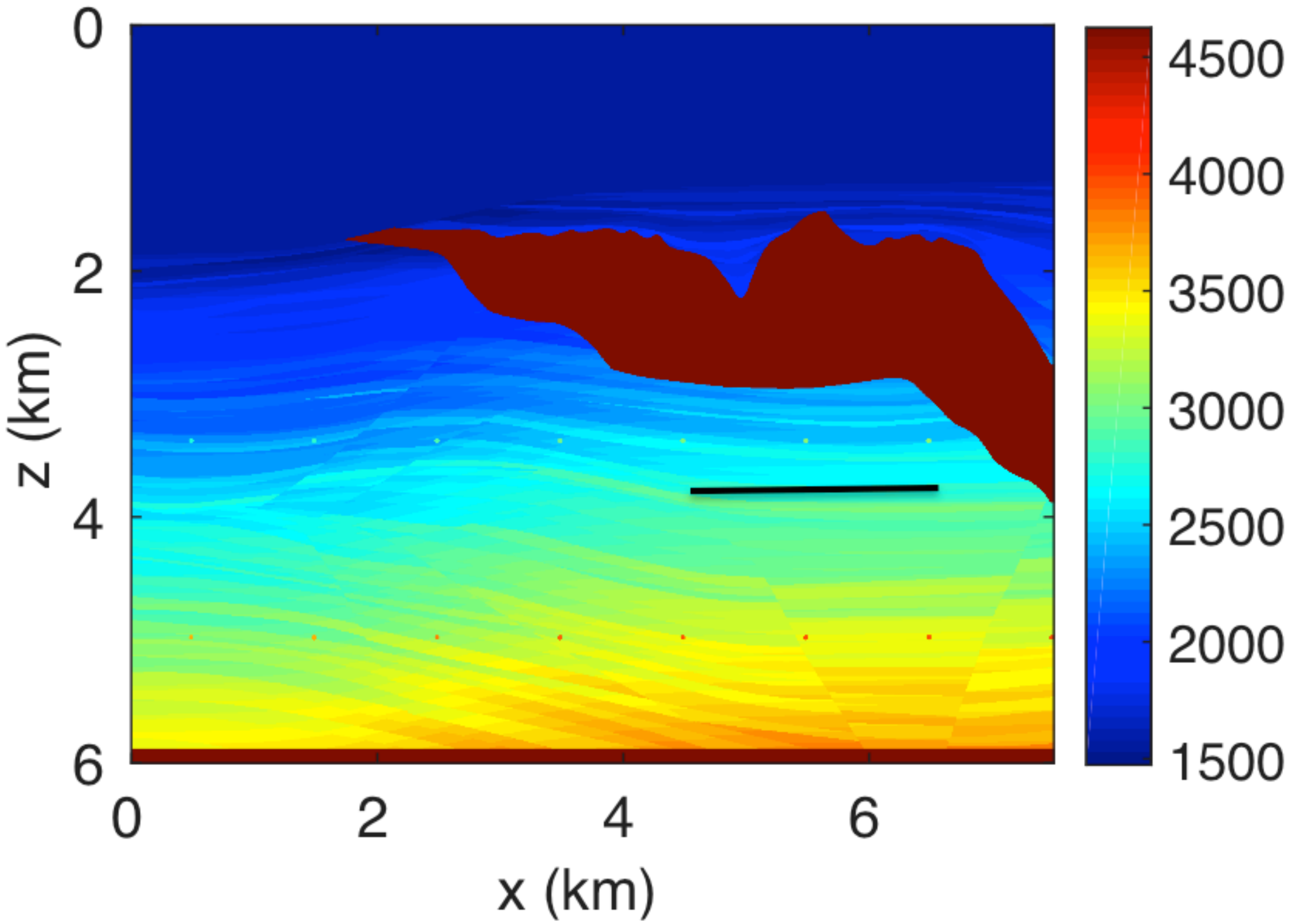
recording length 6s, sampling interval 4ms

peak frequency 25 Hz

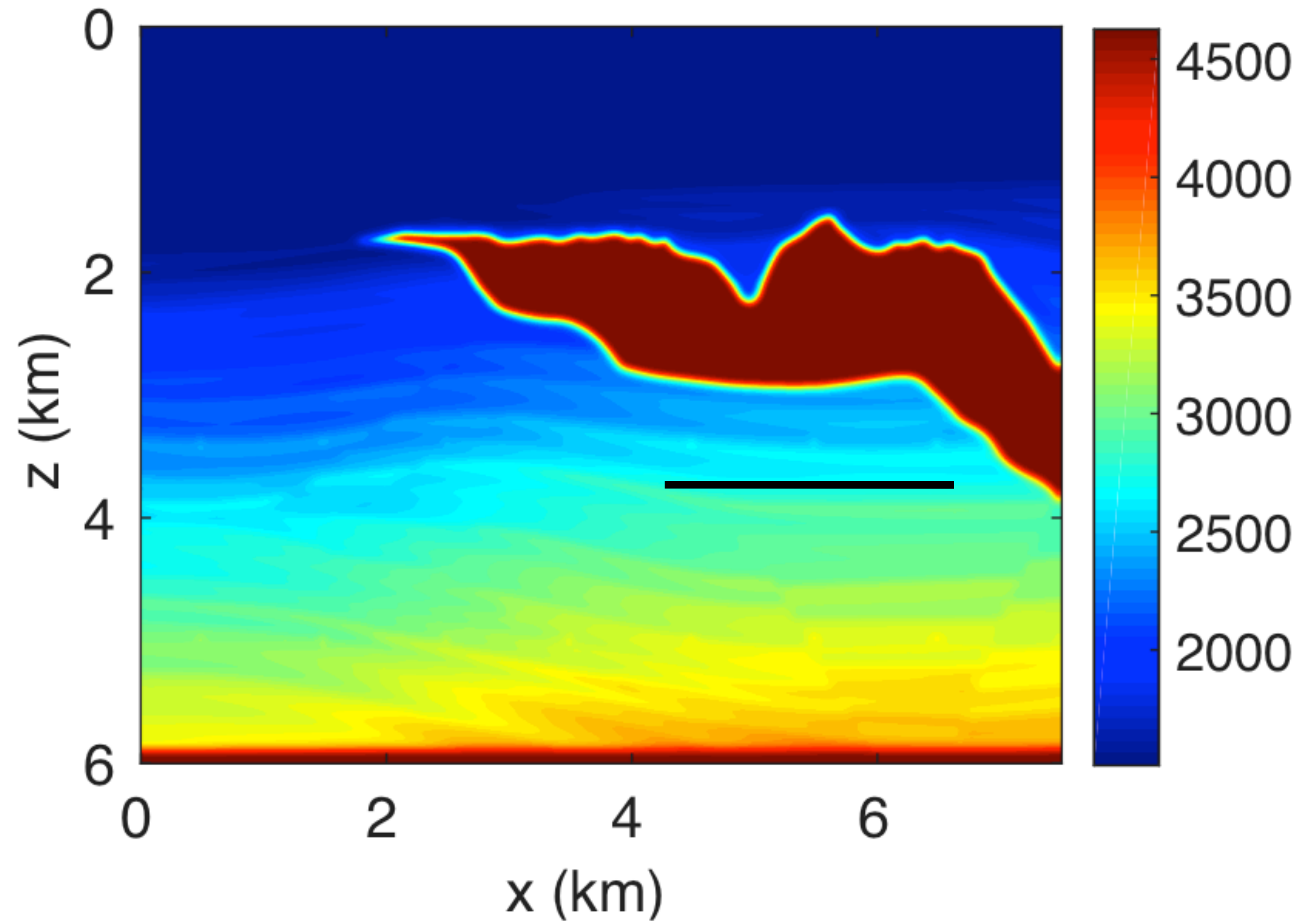
rank varies from 20-100

Sigsbee model

target-imaging datum

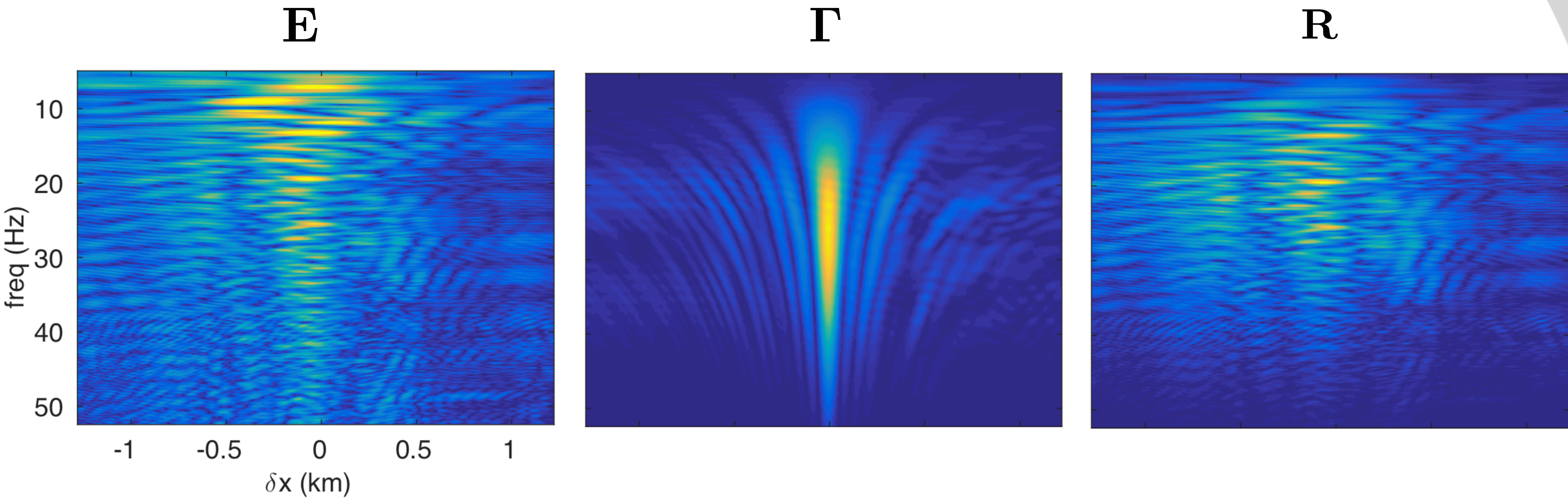


True model



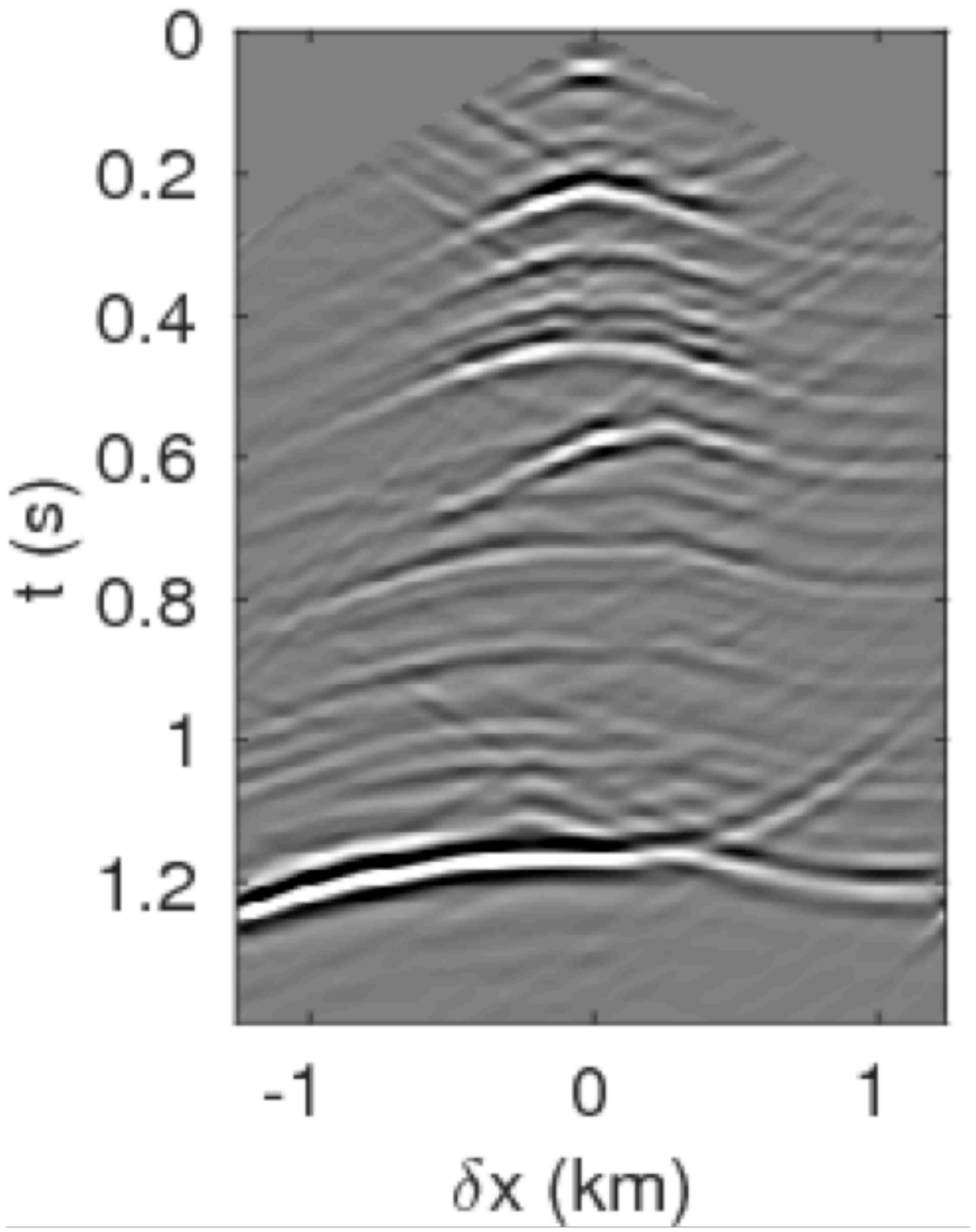
Smooth model

Redatum data (f-x domain)

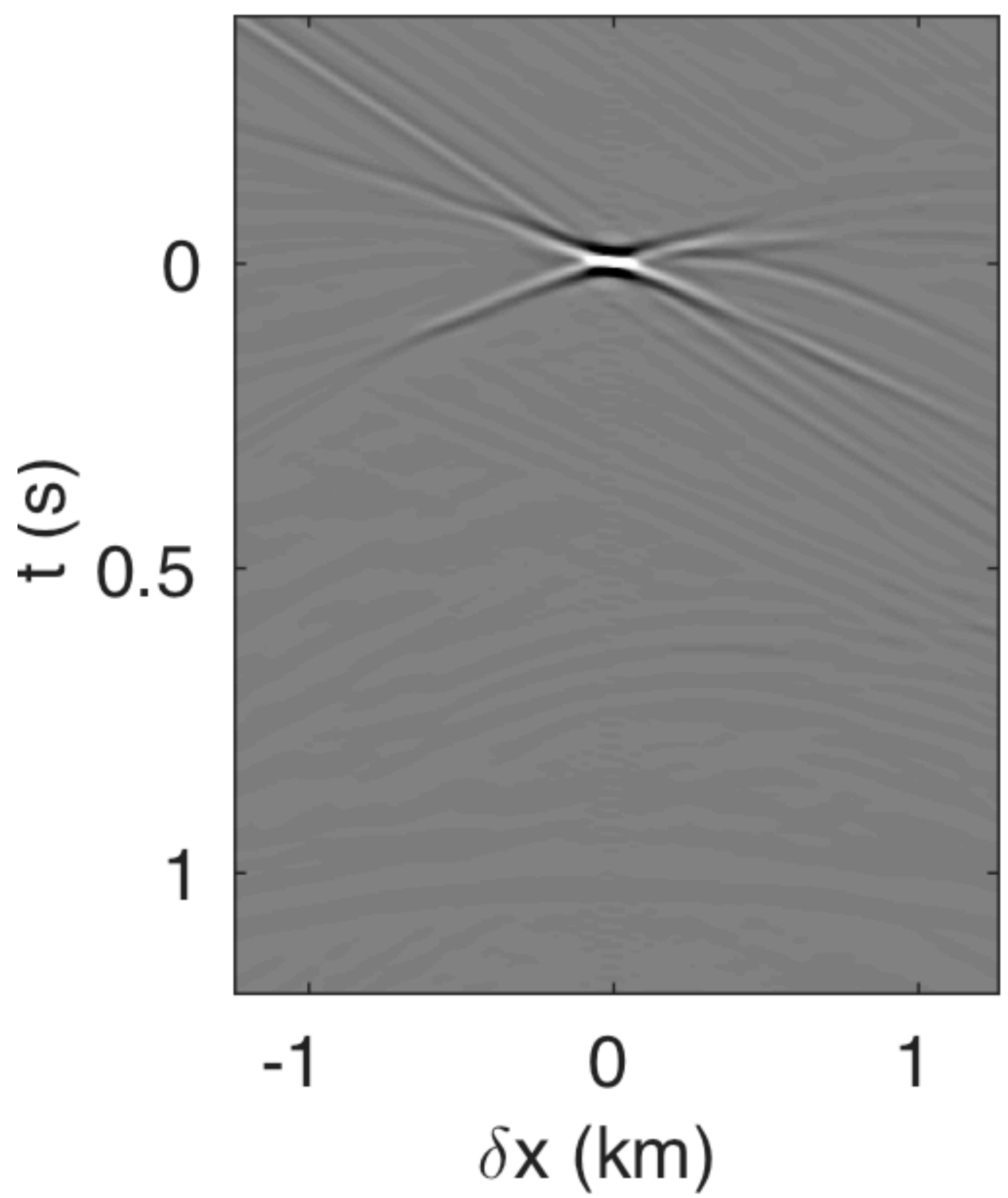


Redatum data (t-x domain)

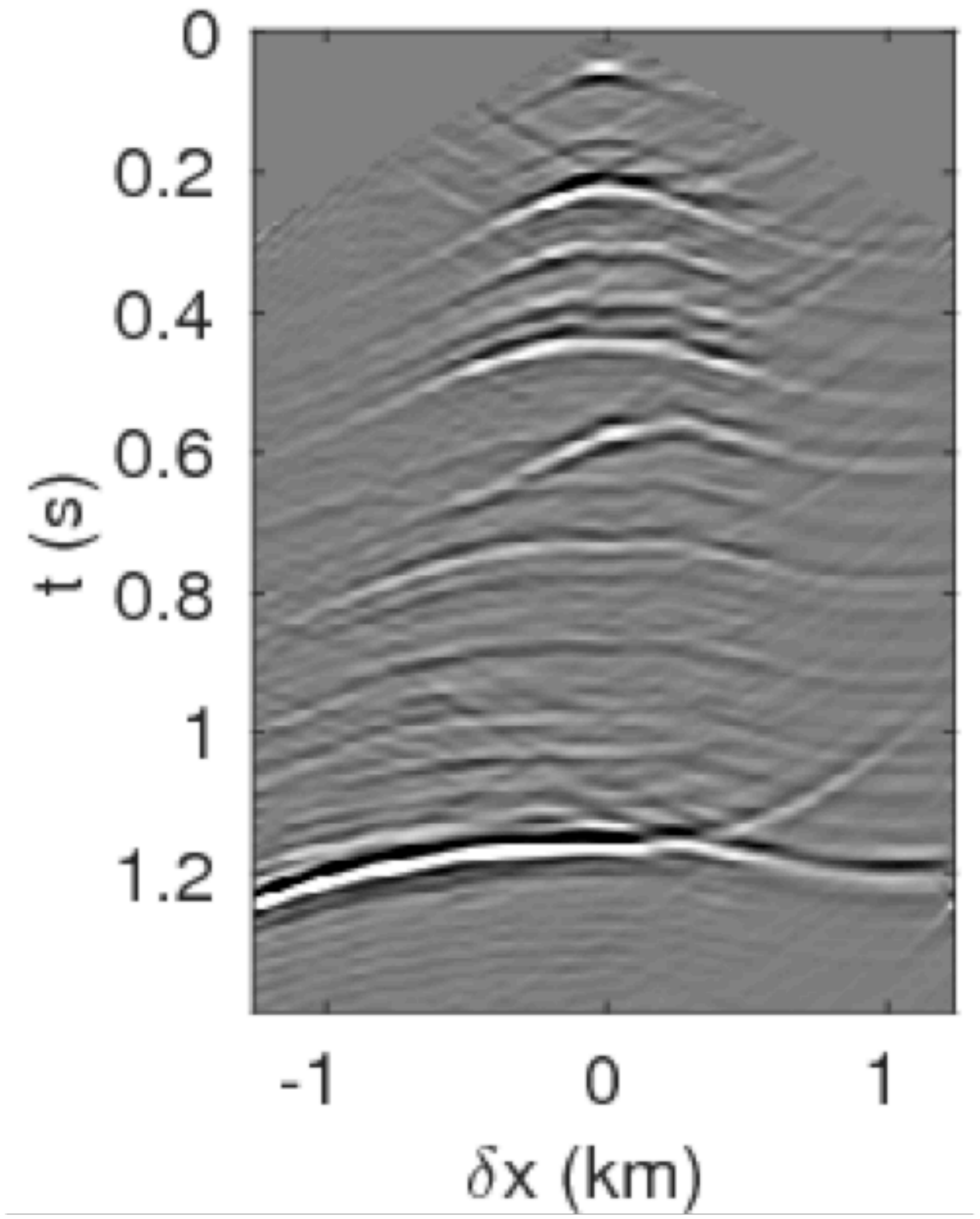
$\mathbf{E}$



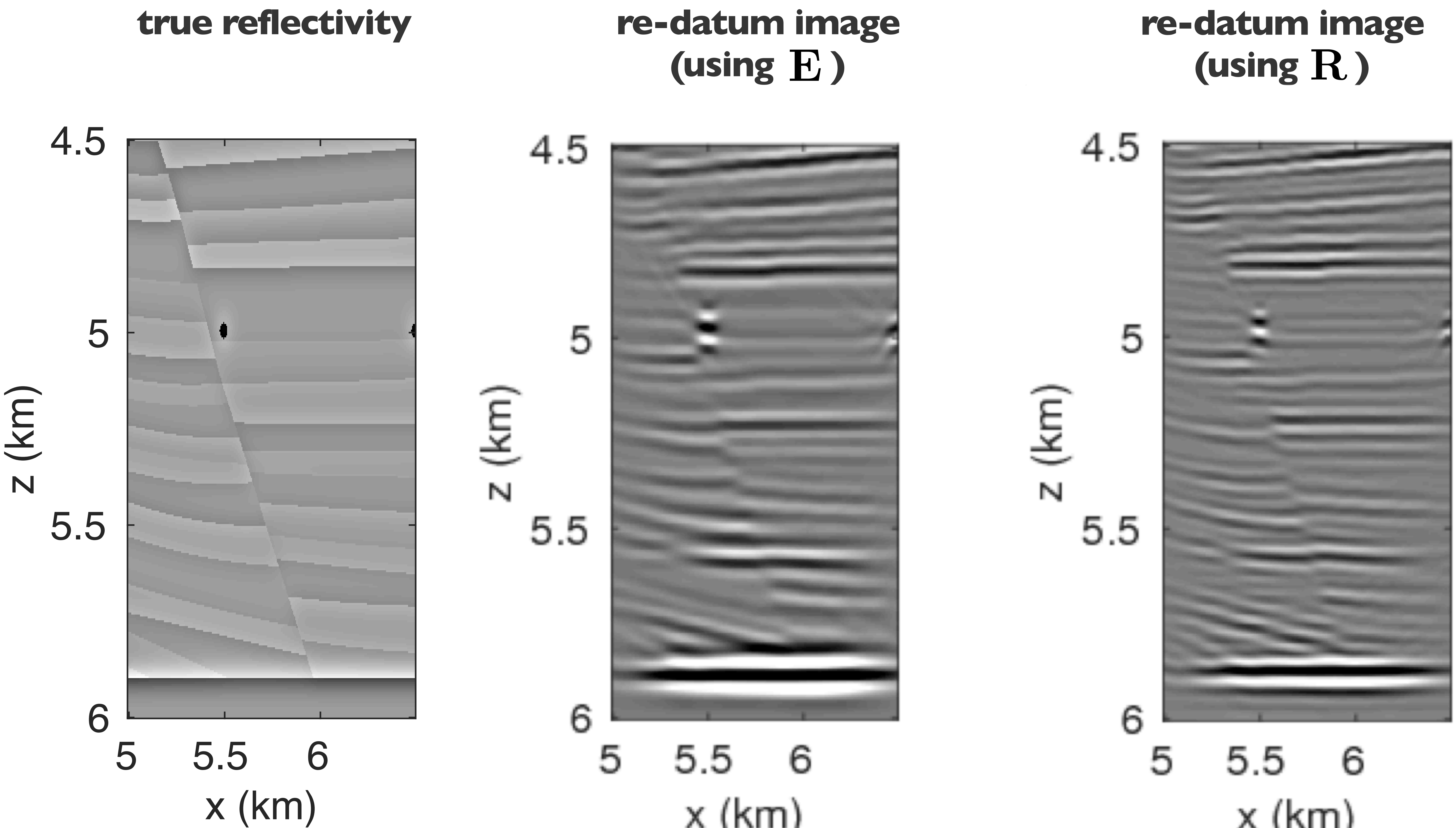
$\mathbf{\Gamma}$



$\mathbf{R}$

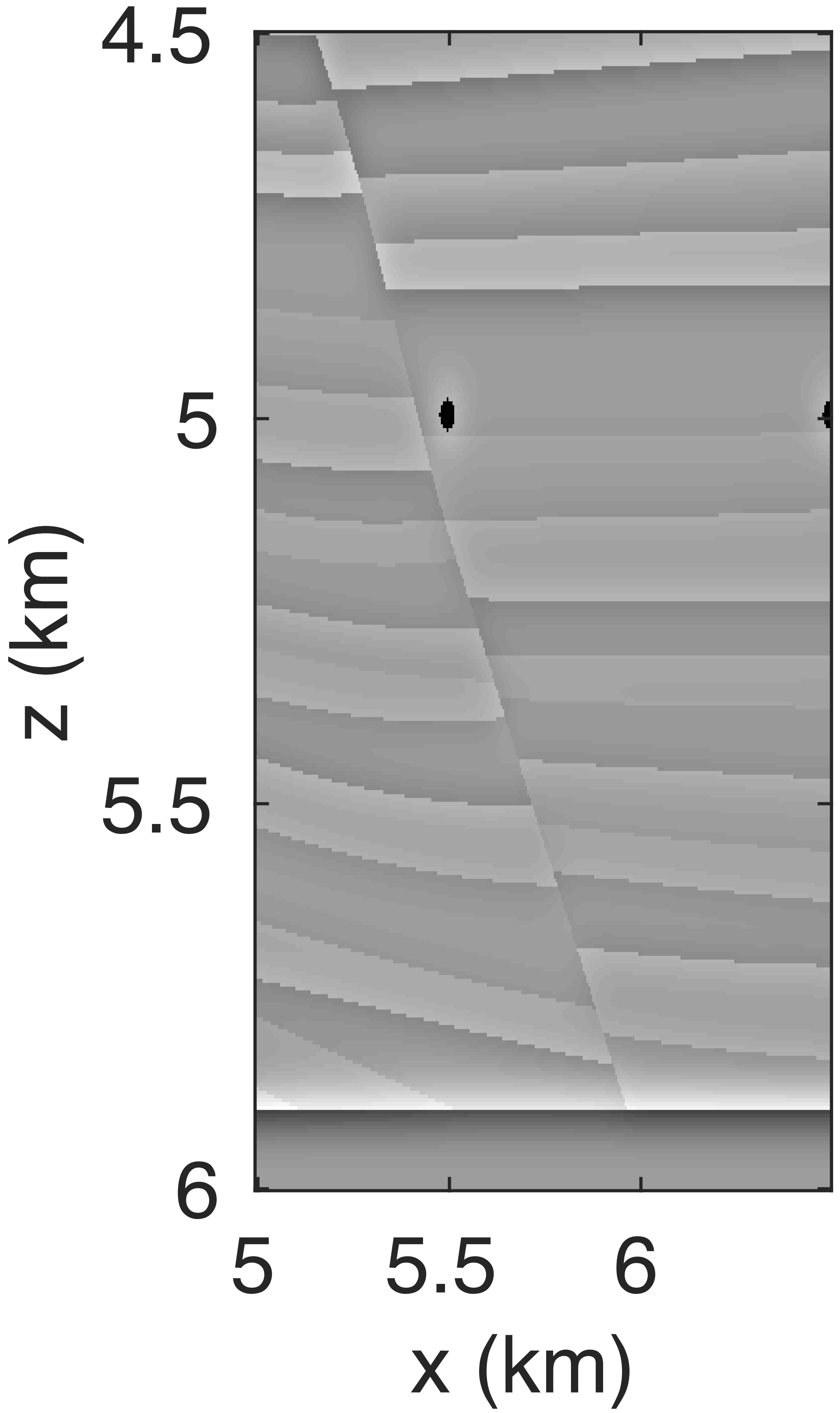


Redatum image

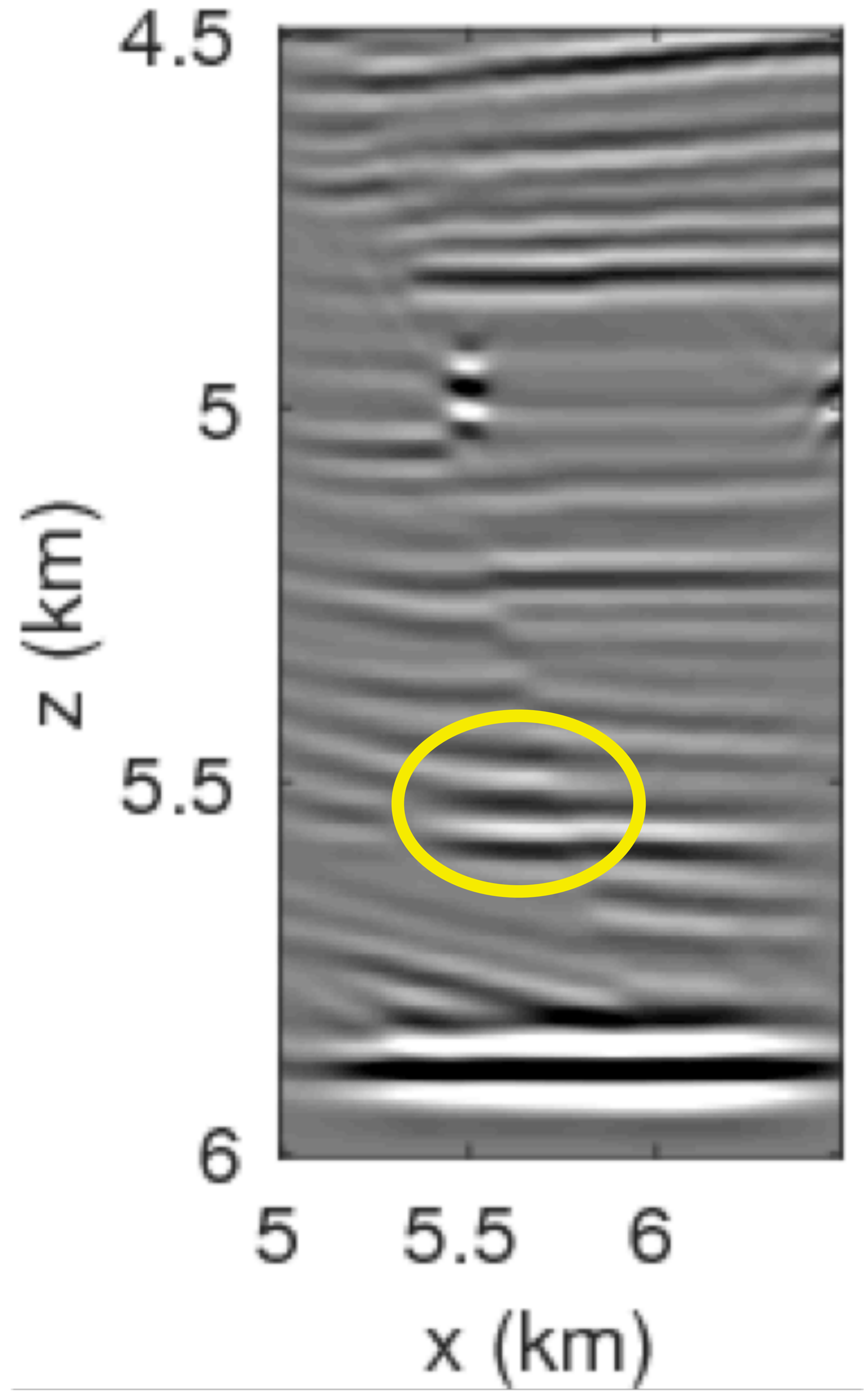


Redatum image

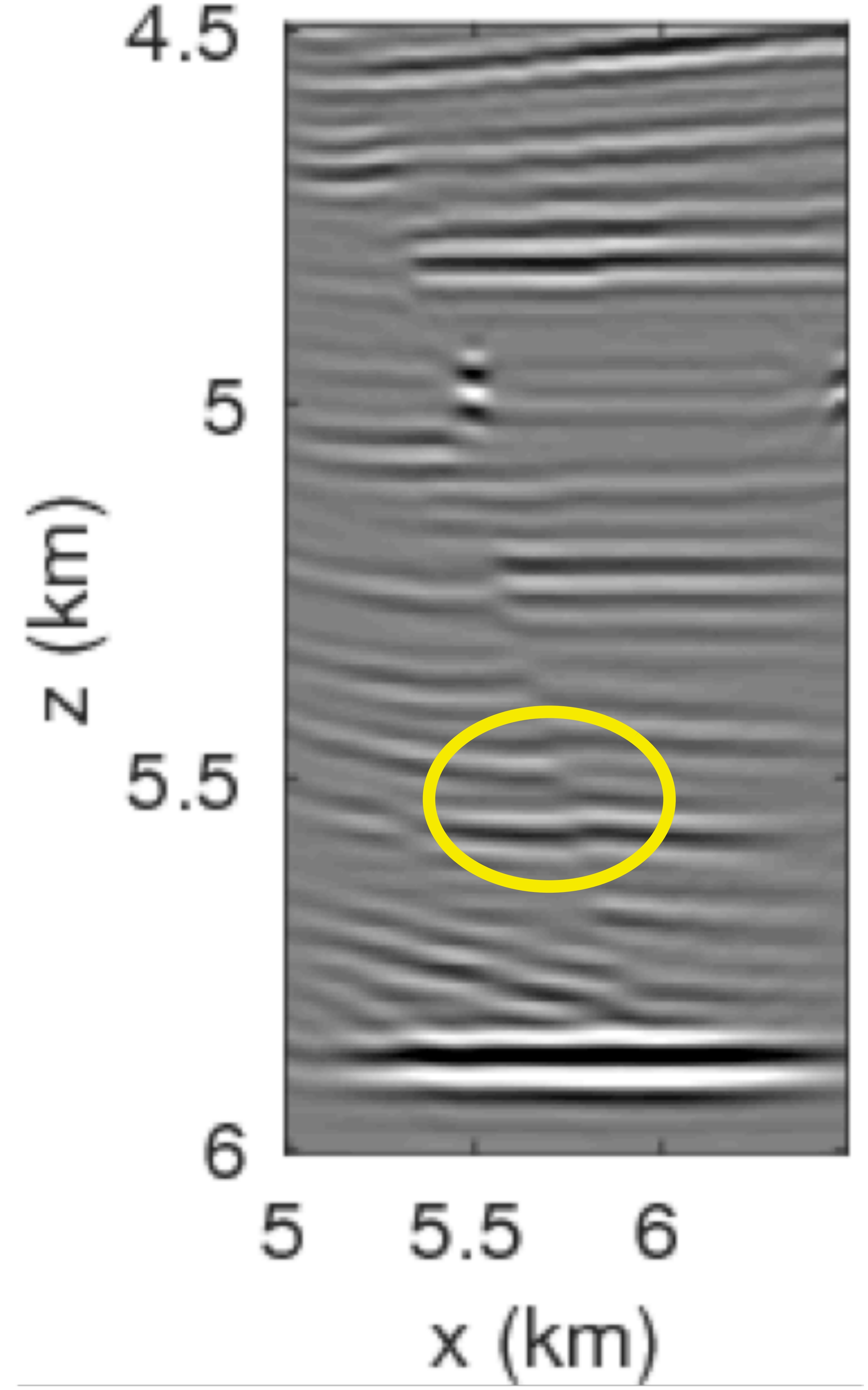
true reflectivity



re-datum image (using $\mathbf{E}$)



re-datum image (using $\mathbf{R}$)



Conclusions

Randomized SVD allows factorization of extreme-scale image volumes

Access to full subsurface image volume for all offsets & all directions

True enabler for target imaging at reservoir levels in 3D

Factorized image volumes can be used for QC or WEMVA

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