Low-rank representation of omnidirectional subsurface extended image volumes

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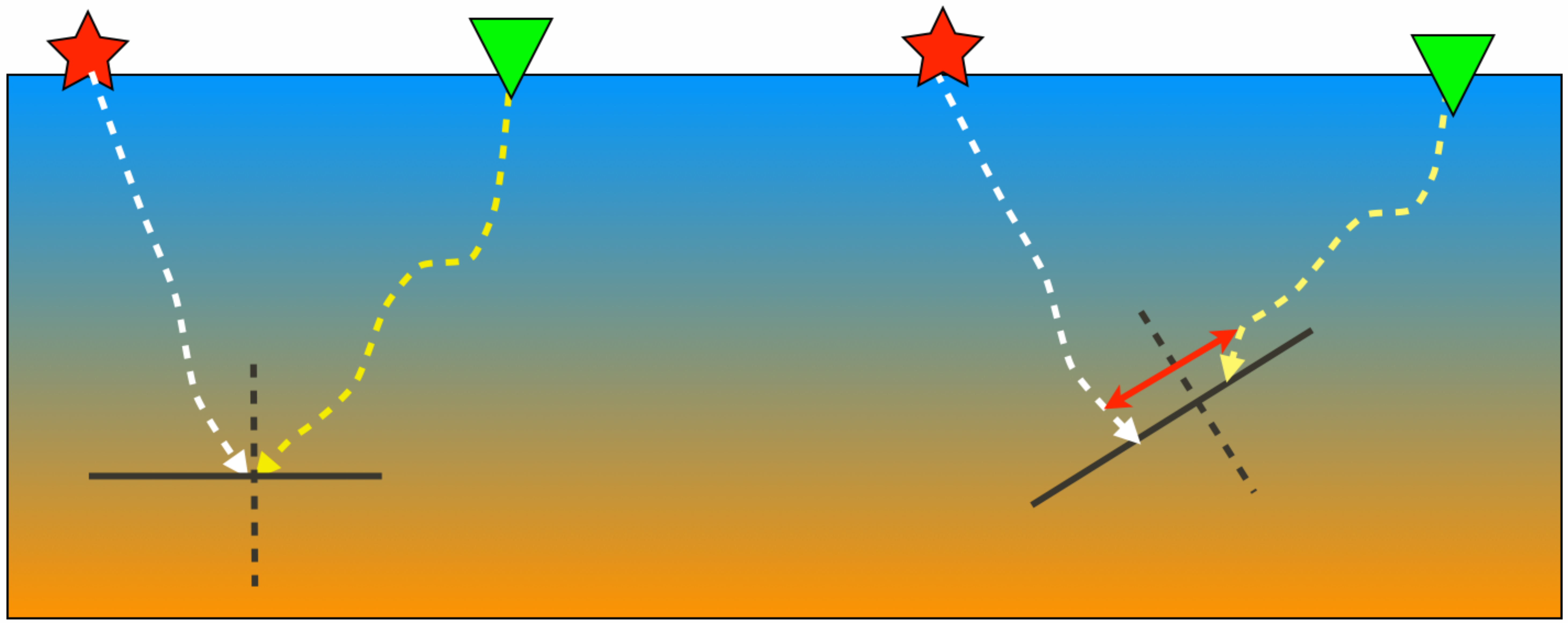
Seismic imaging

- Forward propagate source wavefields
- Back propagate receiver wavefields
- Cross-correlate wavefields at subsurface locations

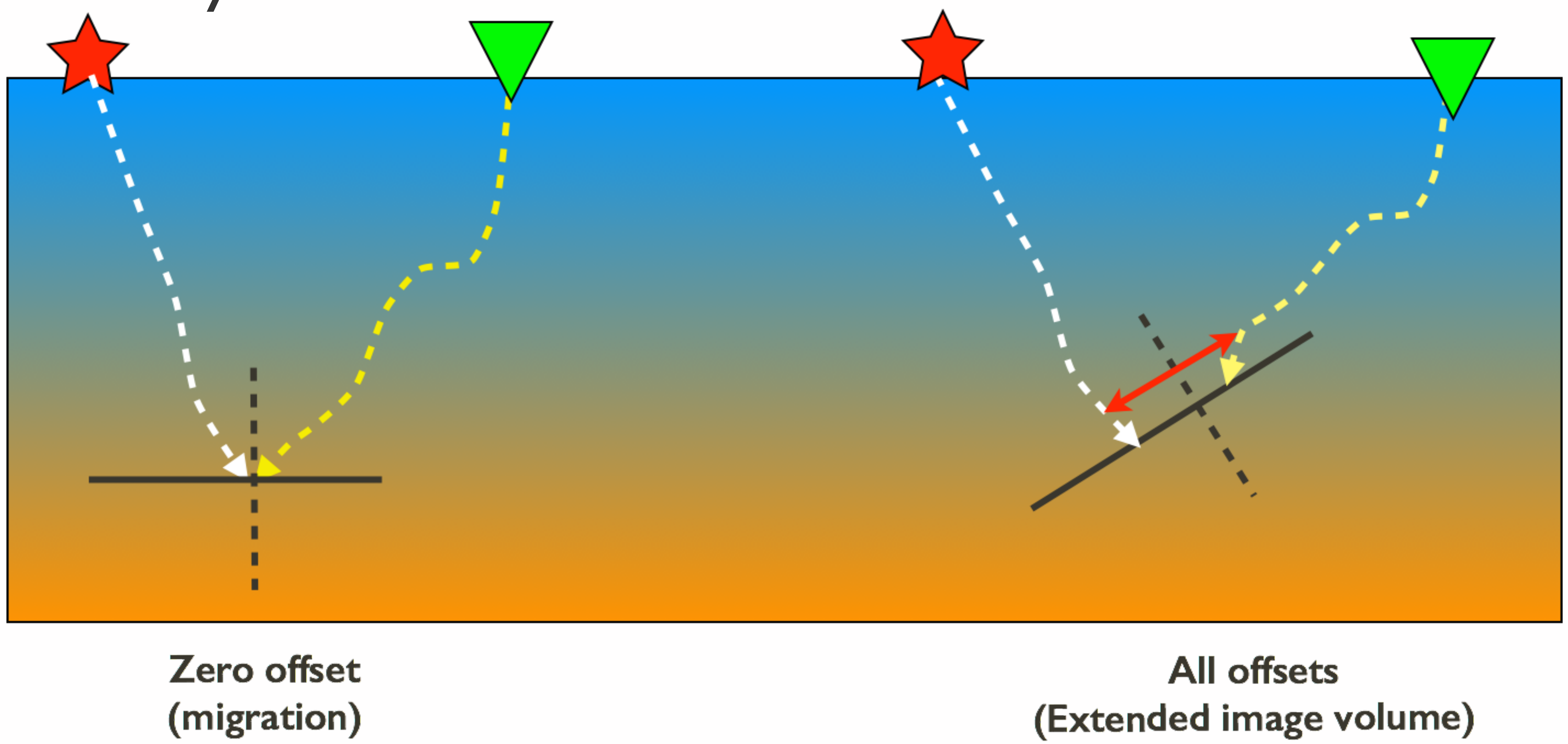


Zero offset
(migration)

All offsets
(Extended image volume)

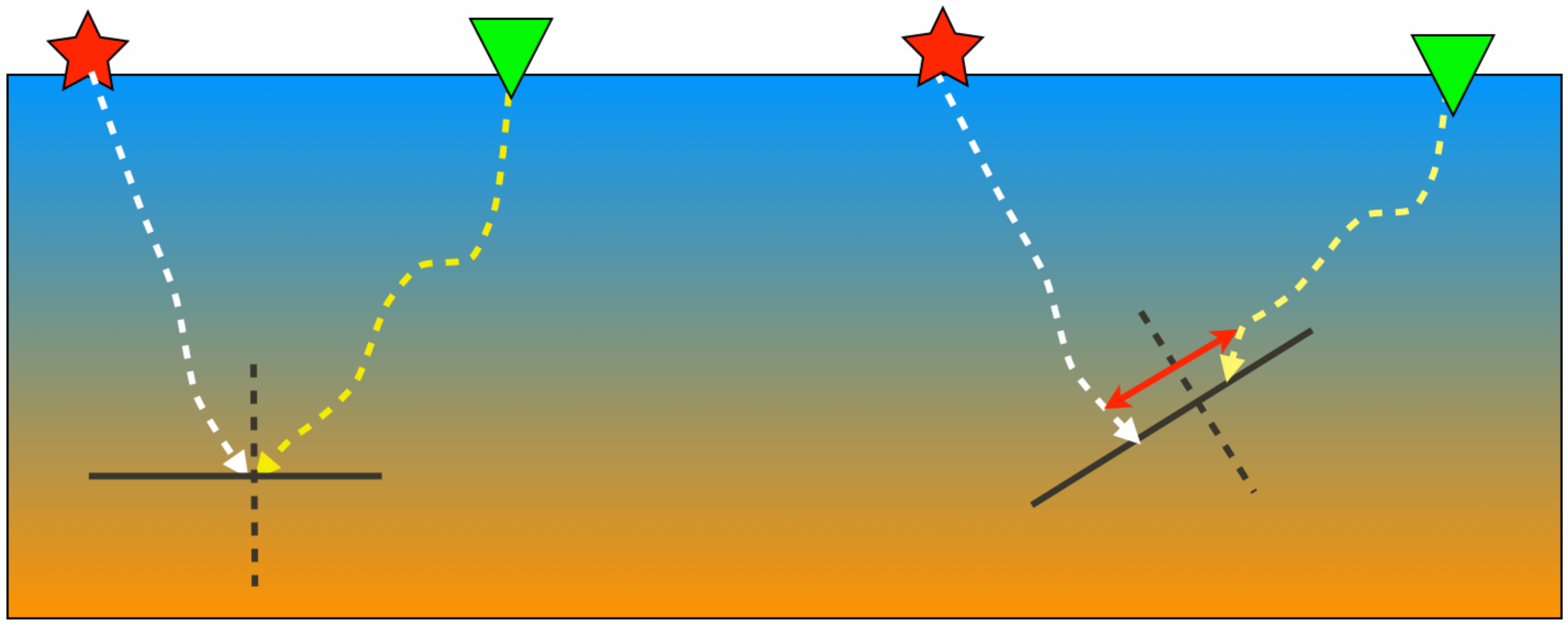
Seismic imaging w/ extensions

- Conventional imaging extracts zero-offset section only
- Extension/lifting corresponds to new experiment w/ sources/receivers anywhere in subsurface
- Near isometry



Seismic imaging w/ extensions

- Parametrized by subsurface horizontal offset or angles
- Computed & stored for small subsets of offsets/angles
- Do not explore underlying low-rank structure



Zero offset
(migration)

All offsets
(Extended image volume)

Motivation and applications

Form subsurface offset image volumes

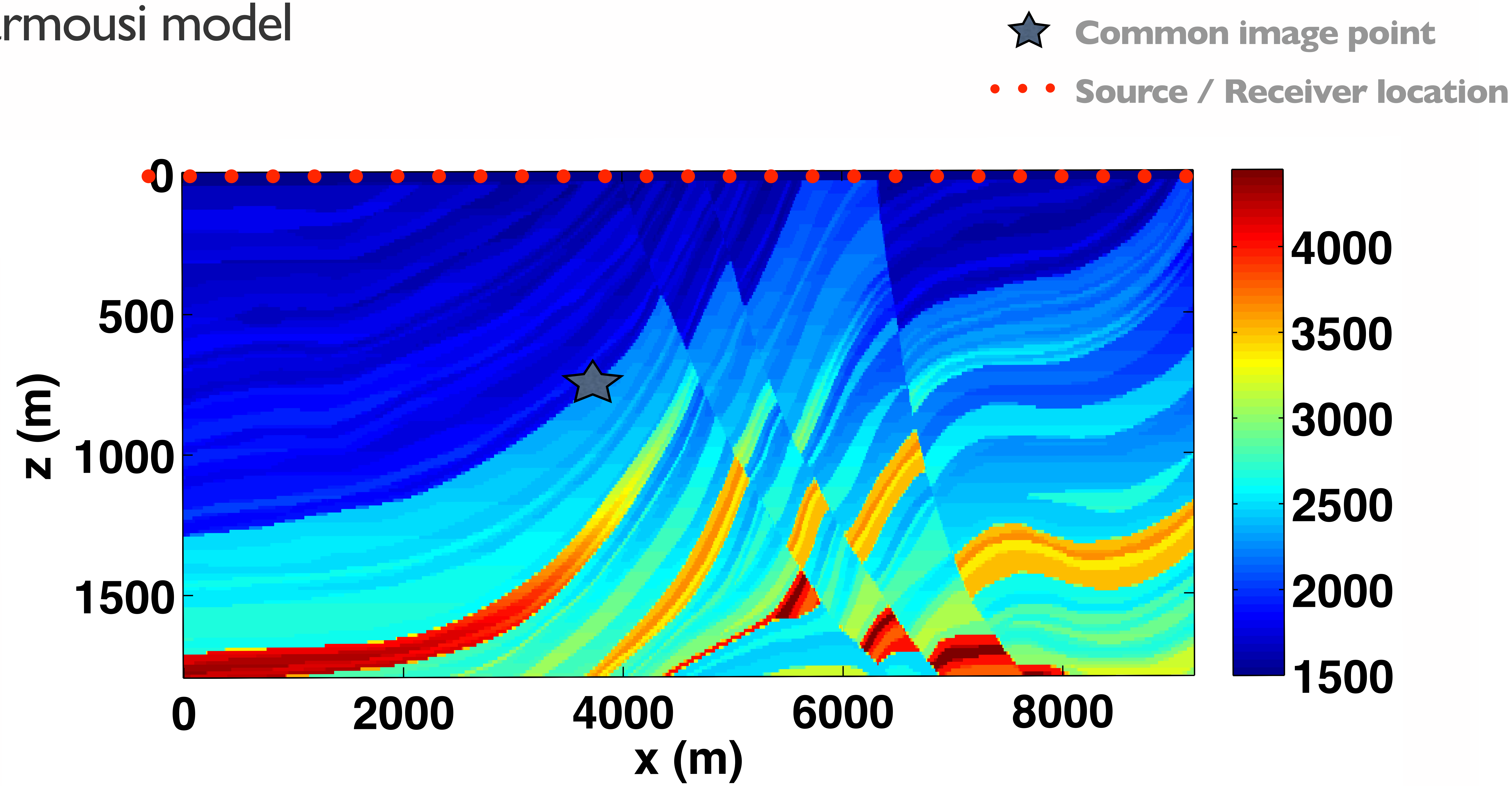
Wave-equation migration velocity analysis & continuation

Targeted imaging

Image gather for QC

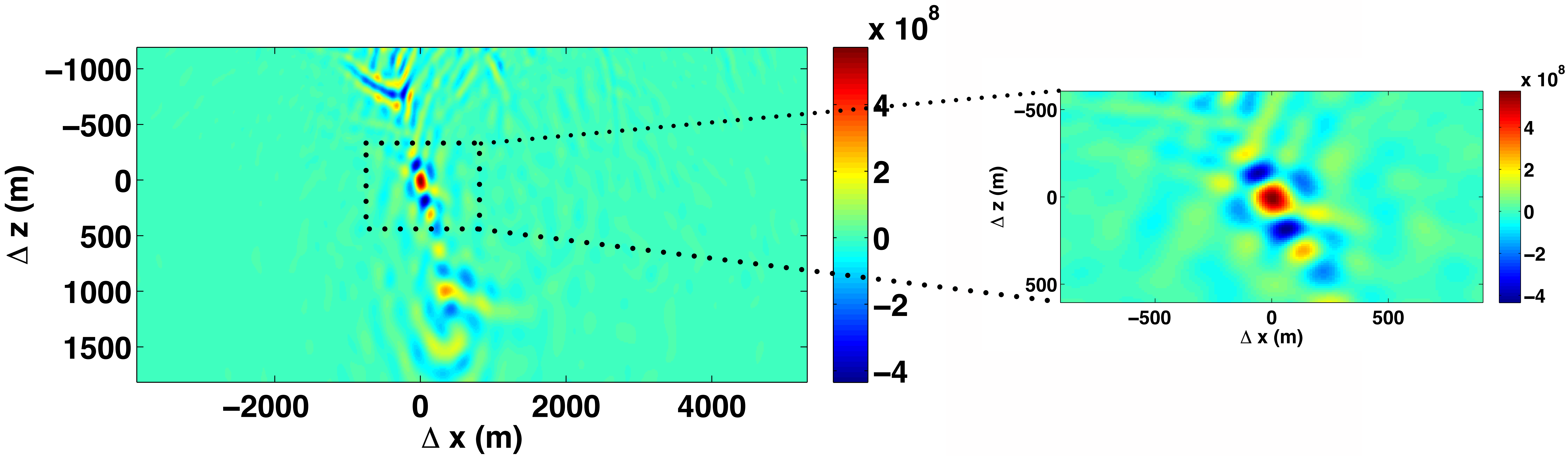
Extended images in 2D

Marmousi model



Extended images in 2D

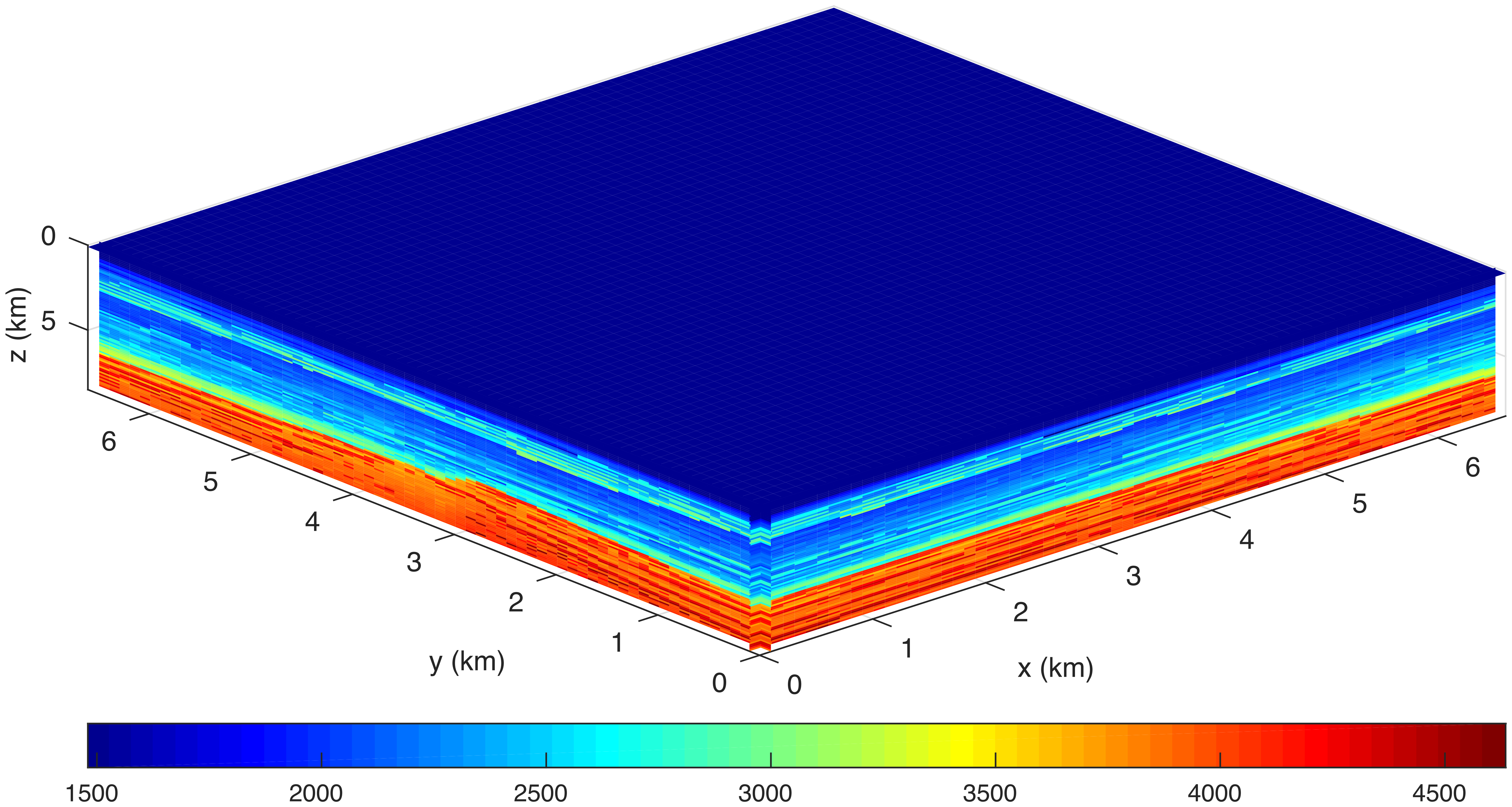
Common image point gather, 3-30 Hz



Δ x : Horizontal offset

Δ z : Vertical offset

Extended images in 3D



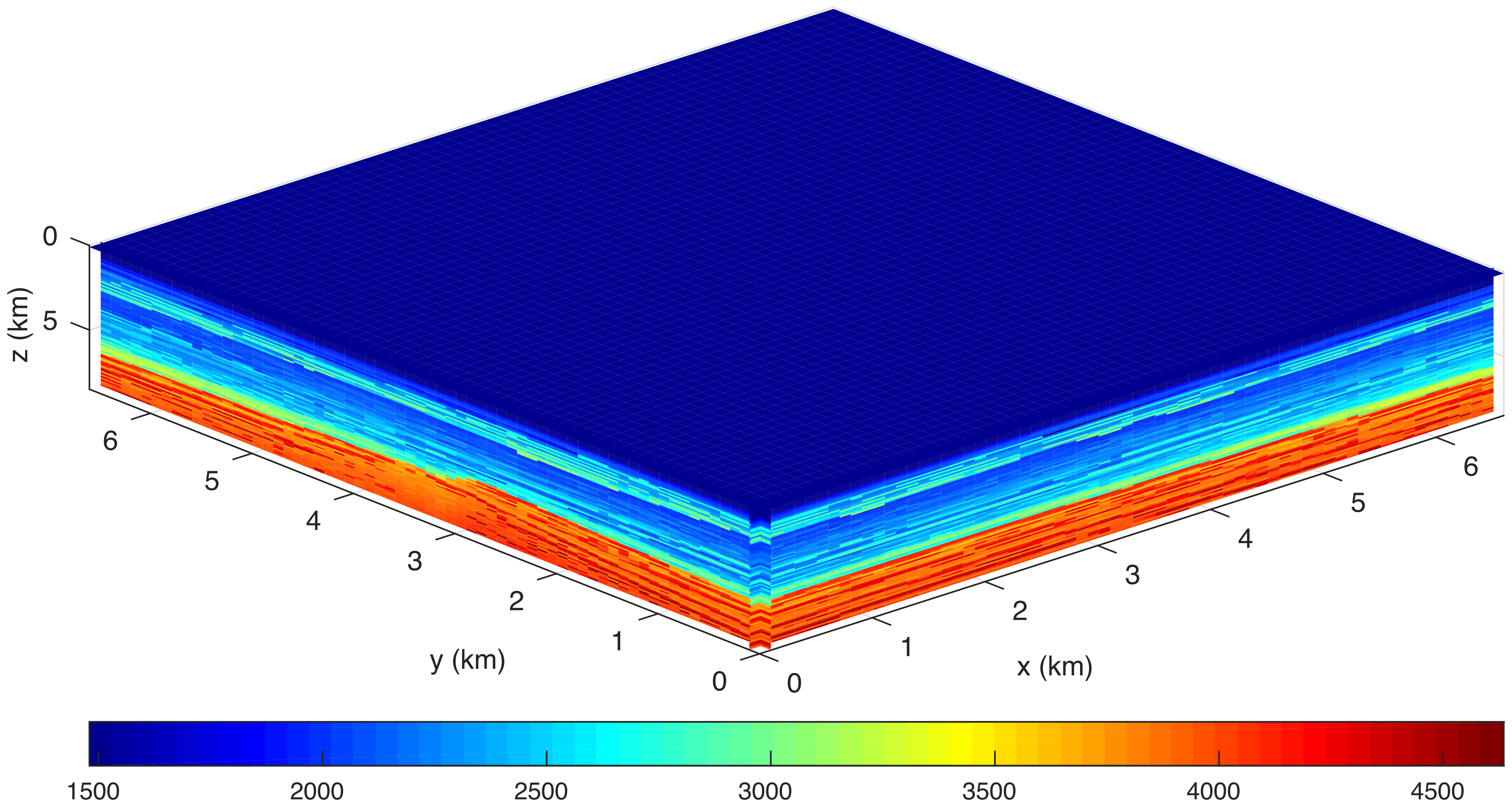
3D BG Compass model

Experimental details

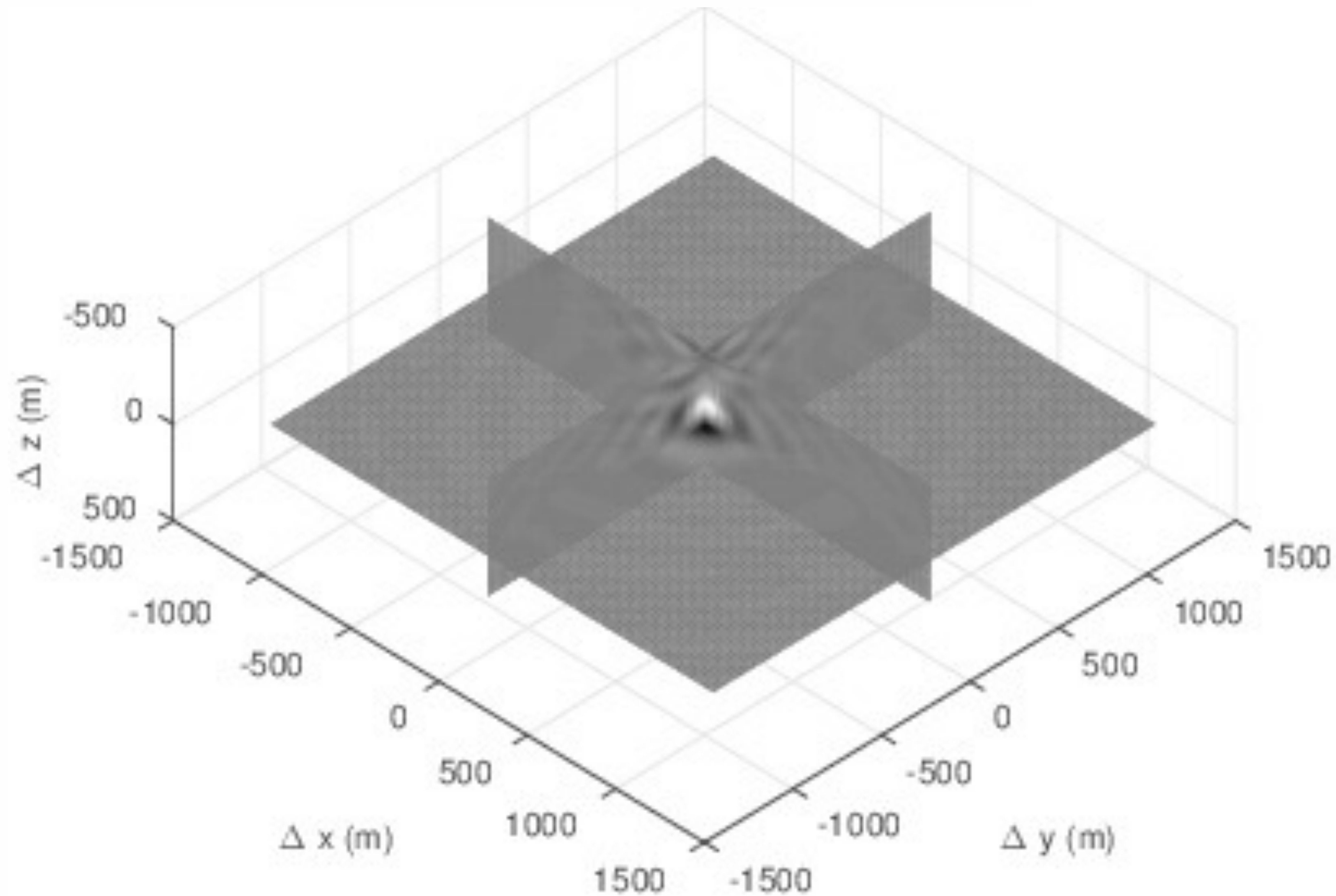
- 1200 source (75 m spacing)
- 2500 receivers (50 m spacing)
- 5-12 Hz
- OBN acquisition
- peak frequency 15 Hz
- One probing vector
- **1500 times faster than conventional method**

Extended images in 3D

3D BG Compass model

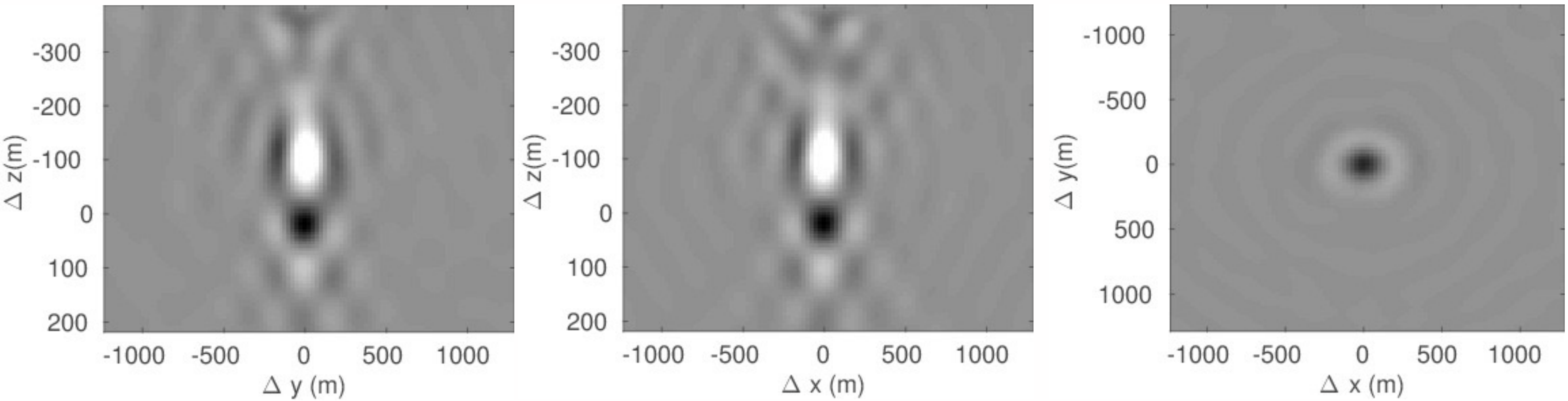


Common-image point gather



Extended images in 3D

Cross section across common-image point gather

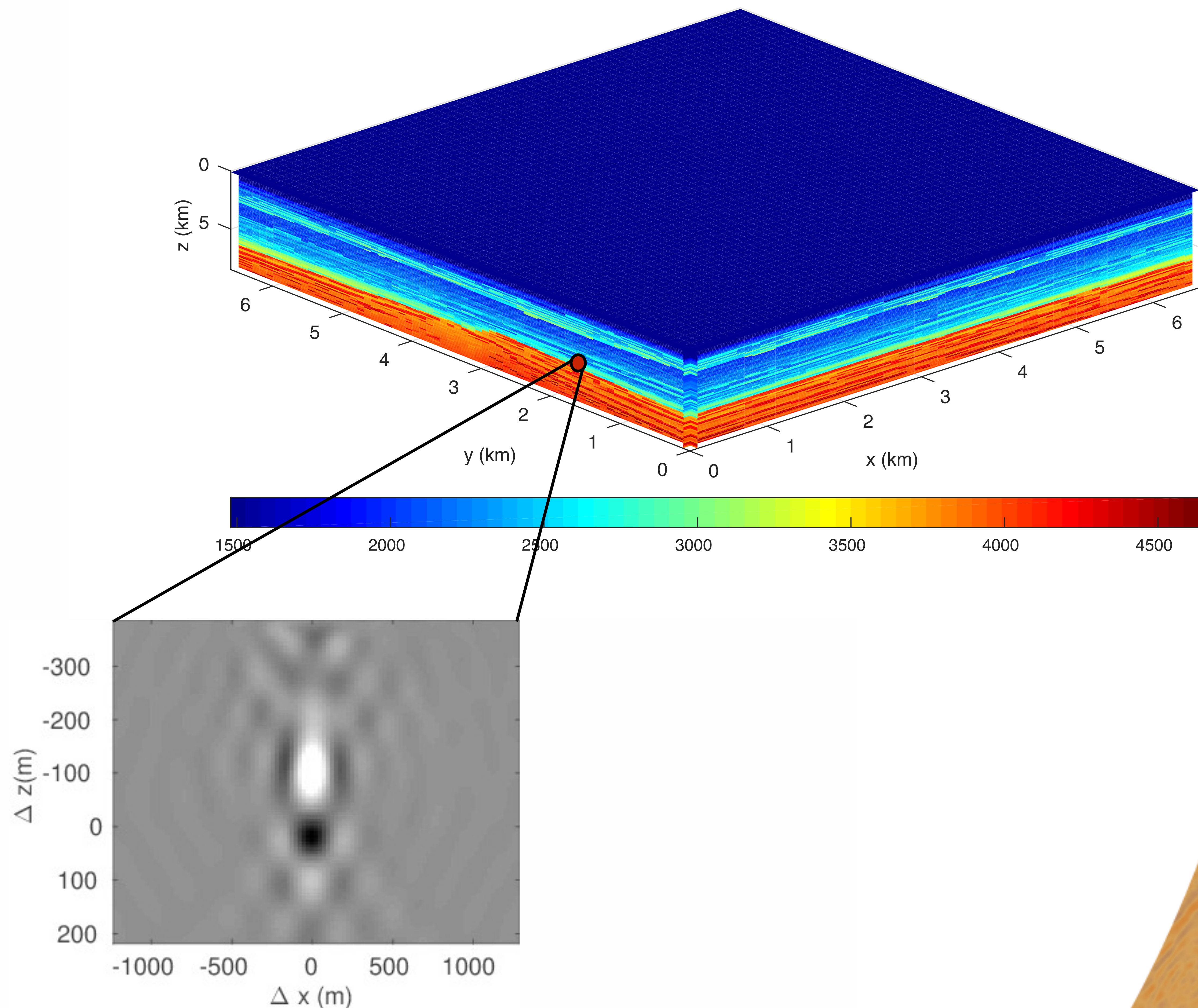


Extended images: difficulties

- use *all* subsurface offsets (6D volume for 3D model)
- 2-way wave-equation

but…. we can never hope to compute or store such an image volume!

Can we work with the volume *implicitly*?



When the dream comes true

Computation of full-subsurface offset volumes is prohibitively expensive in 3D (storage & computation time)

Past

Can **not** form full *E* **but** *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

When the dream comes true

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Past

Can **not** form full *E* **but** *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

Present

Can ~~**not**~~ form full *E* **using** *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

Efficient ways to extract information from highly compressed image volumes

Extended images via probing

Extended images

Given two-way wave equations, source and receiver wavefields are defined as

$$H(\mathbf{m})U = P_s^T Q$$
$$H(\mathbf{m})^* V = P_r^T D$$

where

$H(\mathbf{m})$: discretization of the Helmholtz operator

Q : source

D : data matrix

P_s, P_r : samples the wavefield at the source and receiver positions

$\mathbf{m}$: slowness

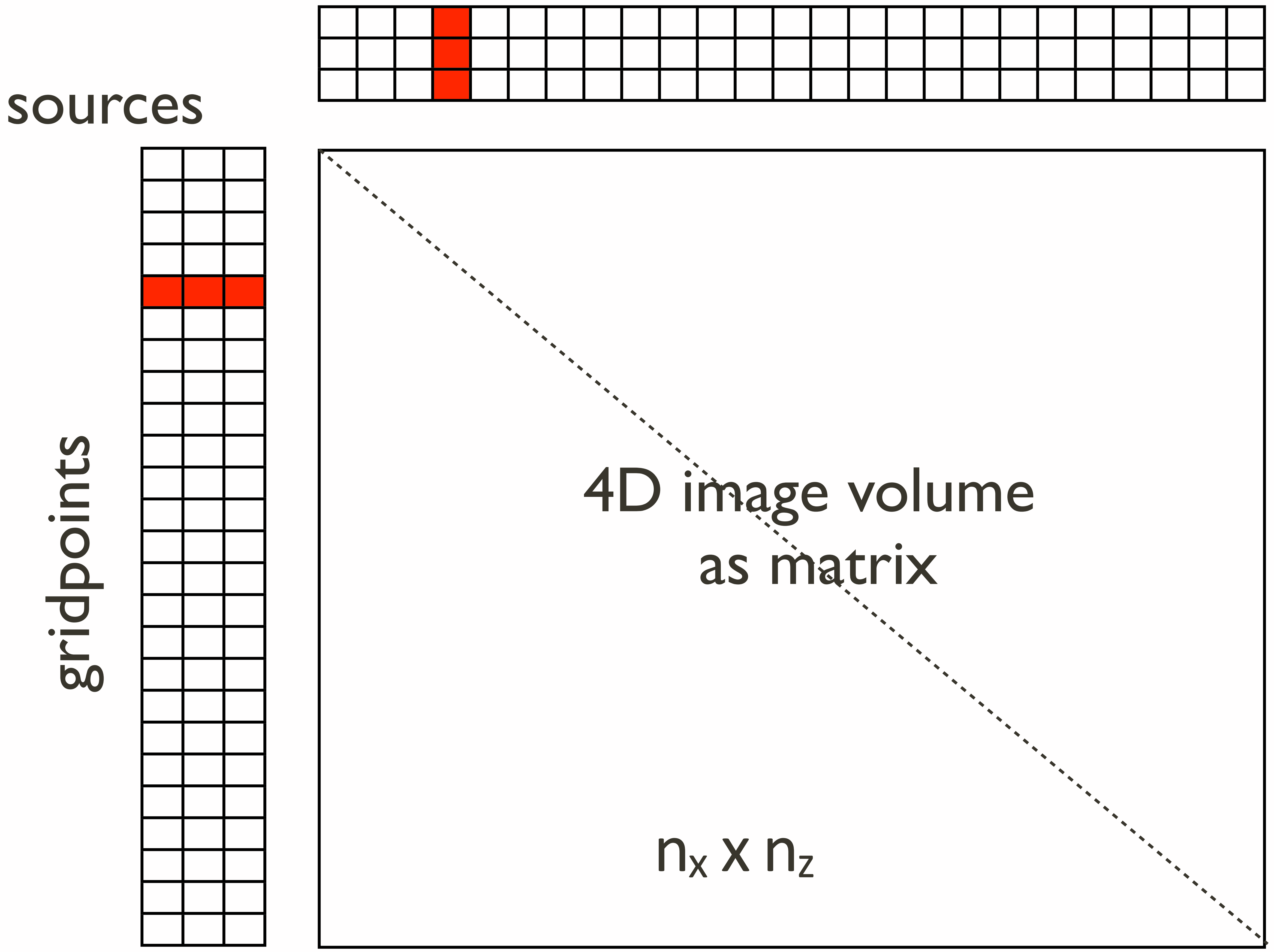
Extended images

Organize wavefields in monochromatic data *matrices* where each *column* represents a *common* shot gather

Express image volume *tensor* for *single* frequency as a *matrix*

$$E = VU^*$$

Extended images



In 3D, E is 6D tensor for each monochromatic slice

Tristan van Leeuwen, Rajiv Kumar, and Felix J. Herrmann, "**Enabling affordable omnidirectional subsurface extended image volumes via probing**", Geophysical Prospecting, 2016

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Extended images (Past)

Too expensive to compute *(storage and computational time)*

Instead, *probe* volume with *tall* matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_\ell]$

$$\widetilde{E} = EW = H^{-*} P_r^\top D Q^* P_s H^{-*} W$$

where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents *single* scattering points

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Extended images (Present)

Too expensive to compute *(storage and computational time)*

Instead, *probe* volume with *tall* matrix $W = [\mathbf{w}_1, \ldots, \mathbf{w}_\ell]$

$$\widetilde{E} = EW = H^{-*} P_r^\top D Q^* P_s H^{-*} W$$

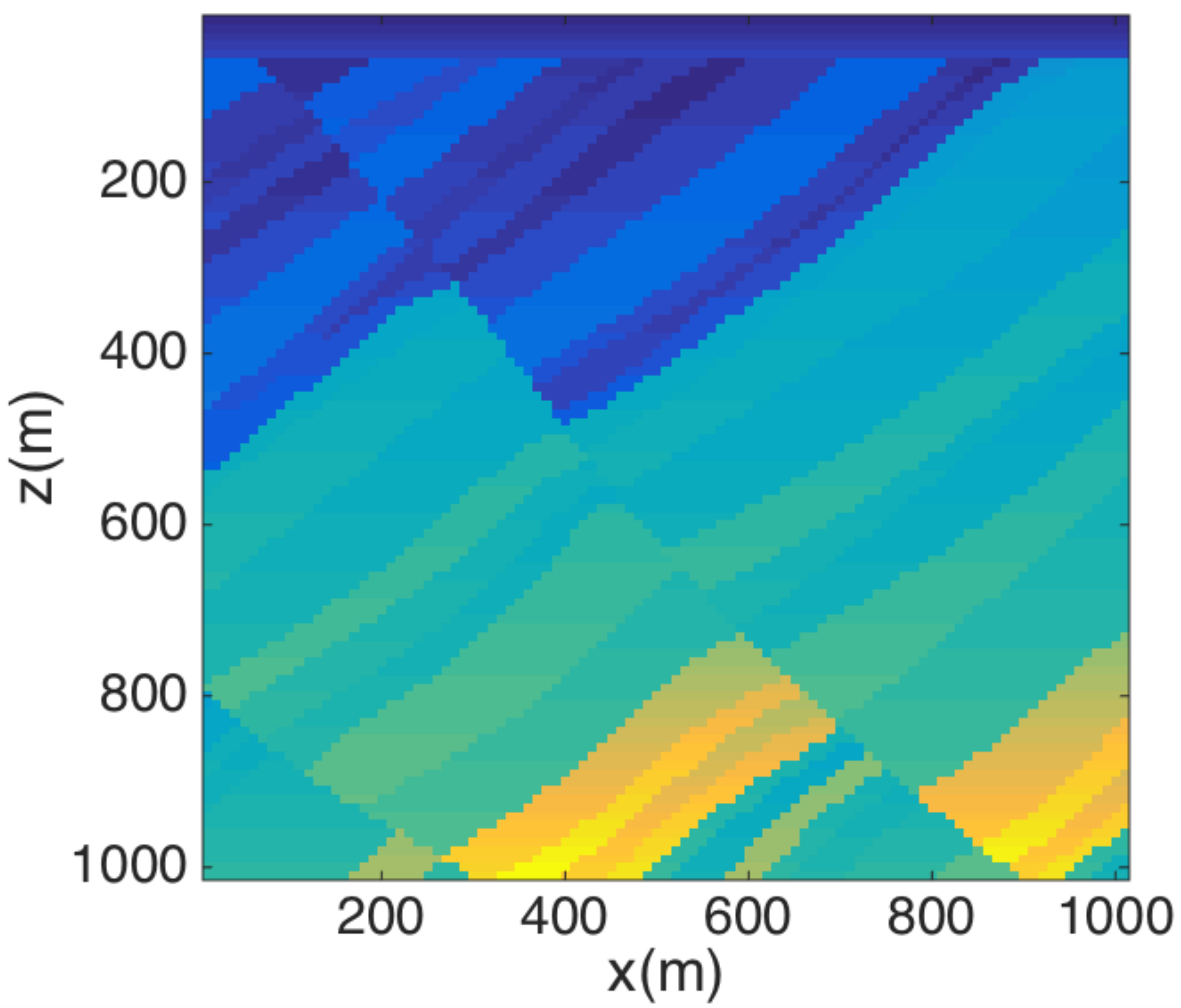
where $\mathbf{w}_i = [0, \ldots, 0, 1, 0, \ldots, 0]$ represents *single* scattering points

Other choice for W ? And how many vectors are needed ?

- random (Gaussian or Rademacher) vectors
- singular vectors from (randomized) SVD

Low-rank representation (5 Hz)

SVD on the monochromatic extended image volume



Model (101x101)

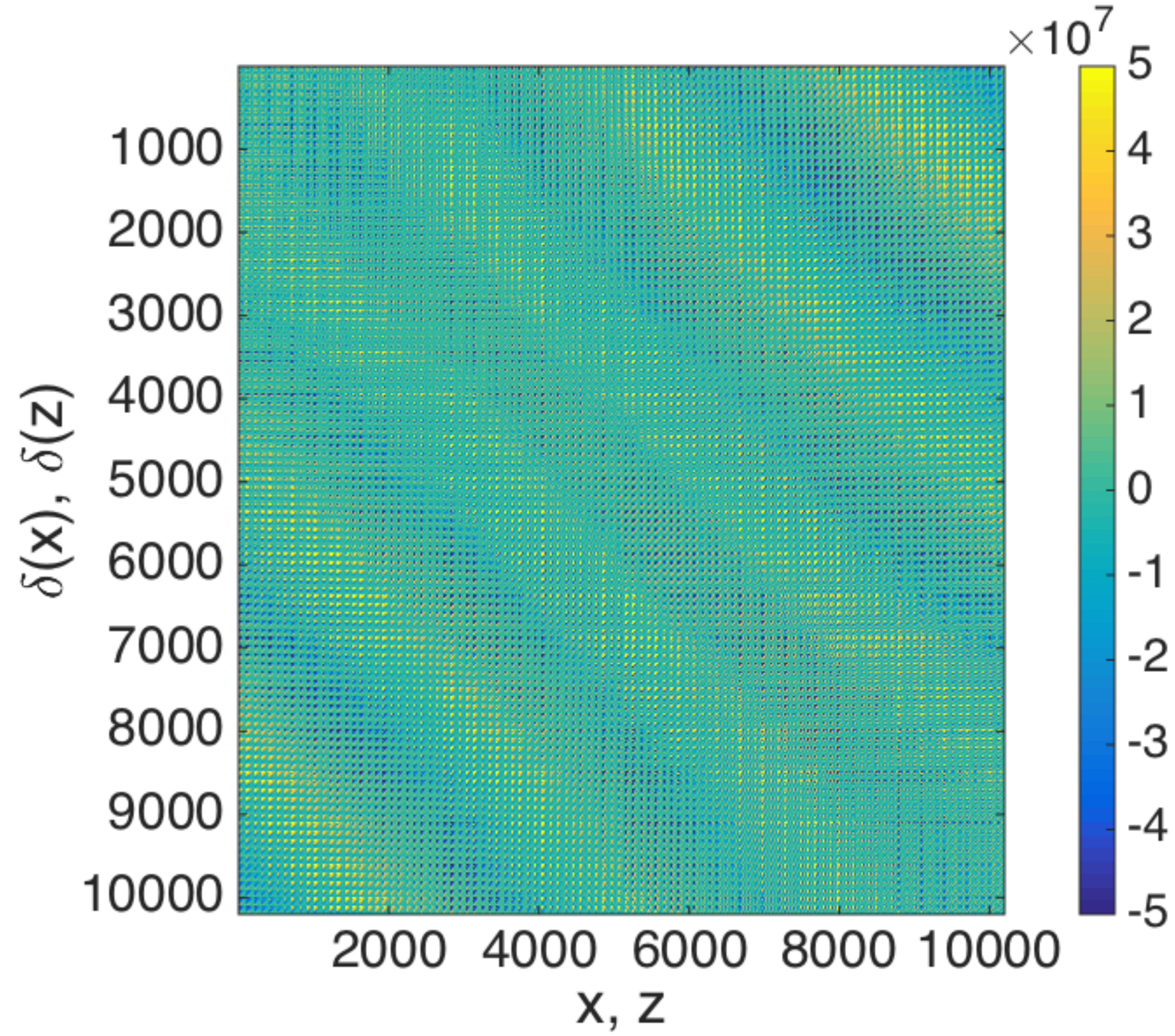
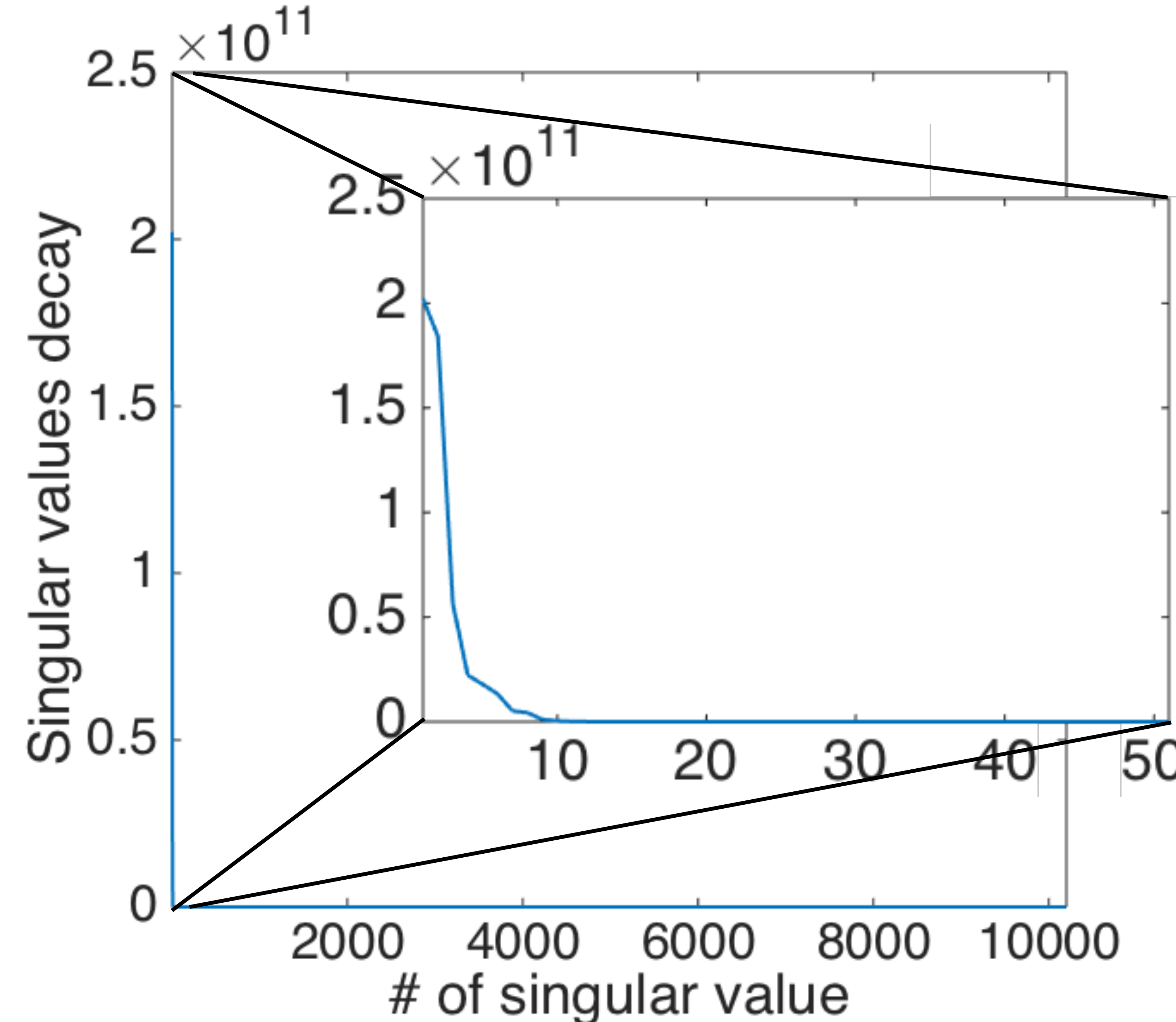


Image Volume (IV)



Singular Values of IV

Rank of the extended image volume

From the formula

$$\widetilde{E} = EW = H^{-*} P_r^{\top} D Q^* P_s H^{-*} W$$

the rank of E is given by the rank of the data matrix D

So, we take r probing vector $W = [w_1, \ldots, w_r]$

— random +1/-1 with probability 0.5

— Gaussian random with 0 mean and variance 1

— our contribution: orthogonal basis of the range of E

[1] Halko et. al, Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions, 2010
[2] Bekas et. al, An Estimator for the Diagonal of a Matrix, 2007

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Orthogonal basis of the range of E

Algorithm:

1. Let $W = [w_1, \ldots, w_r]$ be r Gaussian random vectors
2. Compute $Z = E^*W$ (Z is a $N \times r$ matrix)
3. Compute $[Q, R] = \mathrm{qr}(Z)$ (take only the r first columns of Q)
4. E is fully described by Q (orthogonal probing vectors)
   and EQ (action of E on Q)

Extraction of information of E

— randomized SVD algorithm [1]

— randomized diagonal extraction [2]

Notation: $[Q, EQ]$

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Randomized SVD algorithm

Algorithm from [1]:

1. $Y = EW$ probe full extended image volume with virtual sources
2. $[Q, R] = \mathrm{qr}(Y)$ QR factorization
3. $Z = Q^* E$ probe again with new virtual sources
4. $[U, S, V] = \mathrm{svd}(Z)$ SVD factorization (first few singular values)
5. $U \leftarrow QU$ update left singular vectors

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Randomized SVD algorithm

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Steps 1 to 3 are given by $[Q, EQ]$ (probing only from the right)

if doing so, step 5 becomes an update of right singular vectors: $V \leftarrow QV$

Finally

$$\widetilde{E} = EW = USV^*$$

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Randomized diagonal extraction

Formula from [2]:

$$\mathrm{diag}(E) \approx \left(\sum_{i=1}^{\ell} w_i \odot (E w_i) \right) \oslash \left(\sum_{i=1}^{\ell} w_i \odot w_i \right)$$

for $W = [\mathbf{w}_1, \ldots, \mathbf{w}_\ell]$, +1/-1 with probability 0.5 random vectors

and $\ell \gg N$ (too expensive)

[2] Bekas et. al, An Estimator for the Diagonal of a Matrix, 2007

Randomized diagonal extraction

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for $W = [\mathbf{w}_1, \ldots, \mathbf{w}_\ell]$, +1/-1 with probability 0.5 random vectors
and $\ell \gg N$ (too expensive)

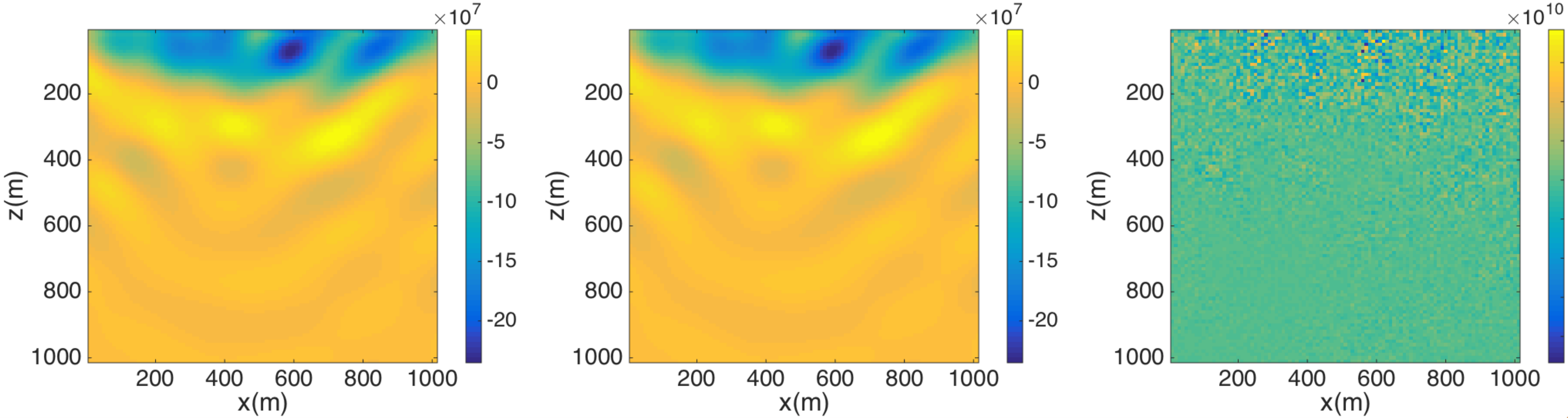
With an orthogonal basis Q:

$$\text{diag}(E) = \sum_{i=1}^{r} q_i \odot (E q_i)$$

Our contribution: take only r vectors spanning an orthogonal basis of the range of E (exact if r is the rank of E)

Orthogonal basis vs random basis

Diagonal extraction of the EIV for different representation (5 Hz, r = 15)



full EIV

E

with orthogonal basis

[Q,EQ]

with Gaussian basis

[W,EW]

Invariance formula for EIVs

Tristan van Leeuwen and Felix J. Herrmann, "Wave-equation extended images: computation and velocity continuation", in *EAGE Annual Conference Proceedings*, 2012.

Invariance formulation for EIVs...

For monochromatic data and sources

$$E = H[m]^{-*} \underbrace{P_r^\top D Q^* P_s}_{invariant} H[m]^{-*}$$

then for two models m_1 and m_2

$$H[m_1]^* E_1 H[m_1]^* = H[m_2]^* E_2 H[m_2]^*$$

Invariance formulation for EIVs…

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$$E = H[m]^{-*} \underbrace{P_r^\top D Q^* P_s}_{invariant} H[m]^{-*}$$

then for two models m_1 and m_2

$$H[m_1]^* E_1 H[m_1]^* = H[m_2]^* E_2 H[m_2]^*$$

we deduce E_2 from E_1

$$E_2 = H[m_2]^{-*} H[m_1]^* E_1 H[m_1]^* H[m_2]^{-*}$$

Only $2r$ PDEs solves!

...from Low-Rank representation

From $[Q_1, E_1 Q_1]$, we get a low-rank formulation for E_1

$$E_1 = L_1 R_1^*$$

with L_1 and R_1 two $N \times r$ matrices given by

$$L_1 = U_1 \sqrt{S_1}$$

$$R_1 = V_1 \sqrt{S_1}$$

$[U_1, S_1, V_1]$ from randomized SVD

New extended image

Now we deduce

$$L_2 = H[m_2]^{-*} H[m_1]^* L_1$$

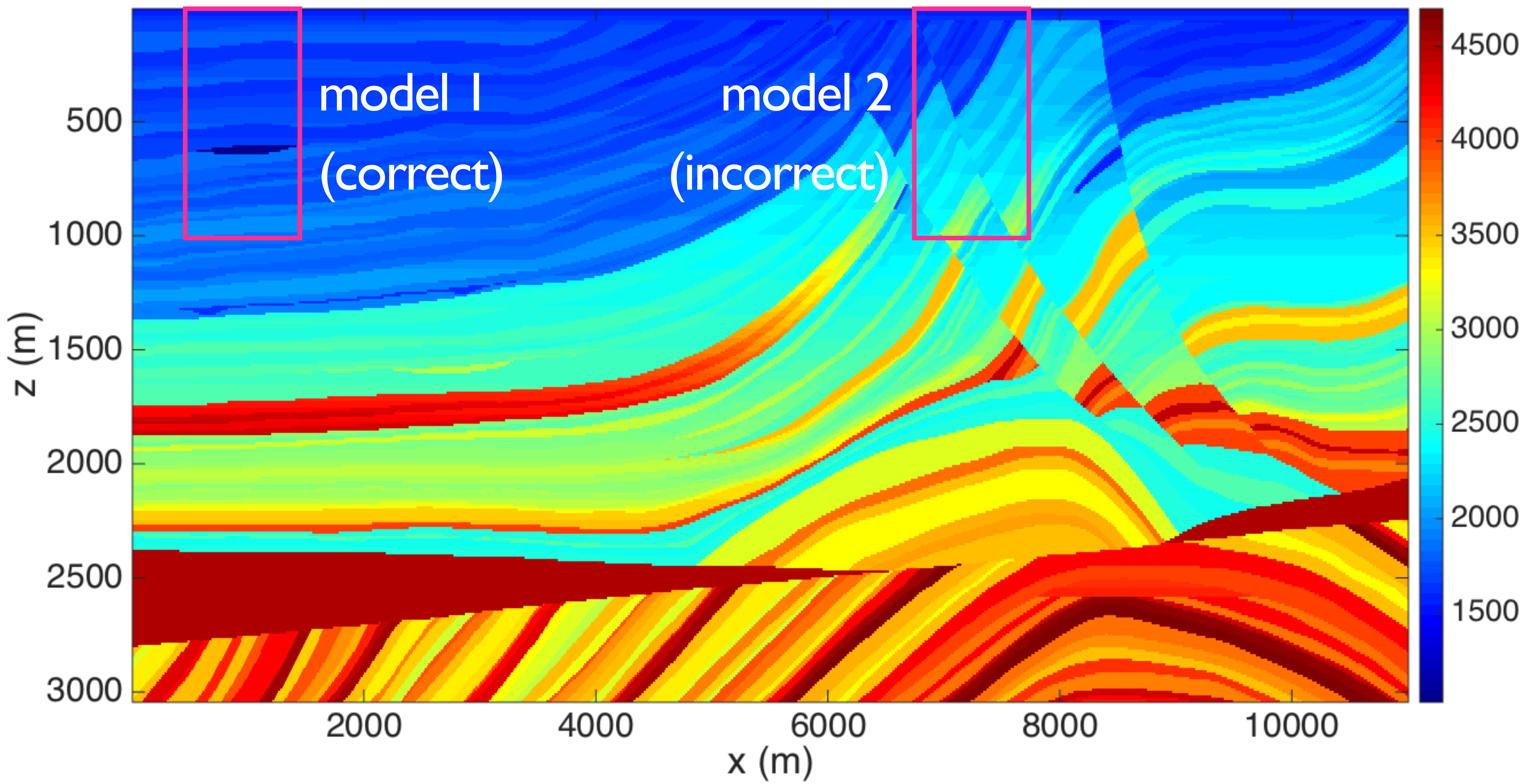
$$R_2 = H[m_2]^{-1} H[m_1] \ R_1$$

to compute

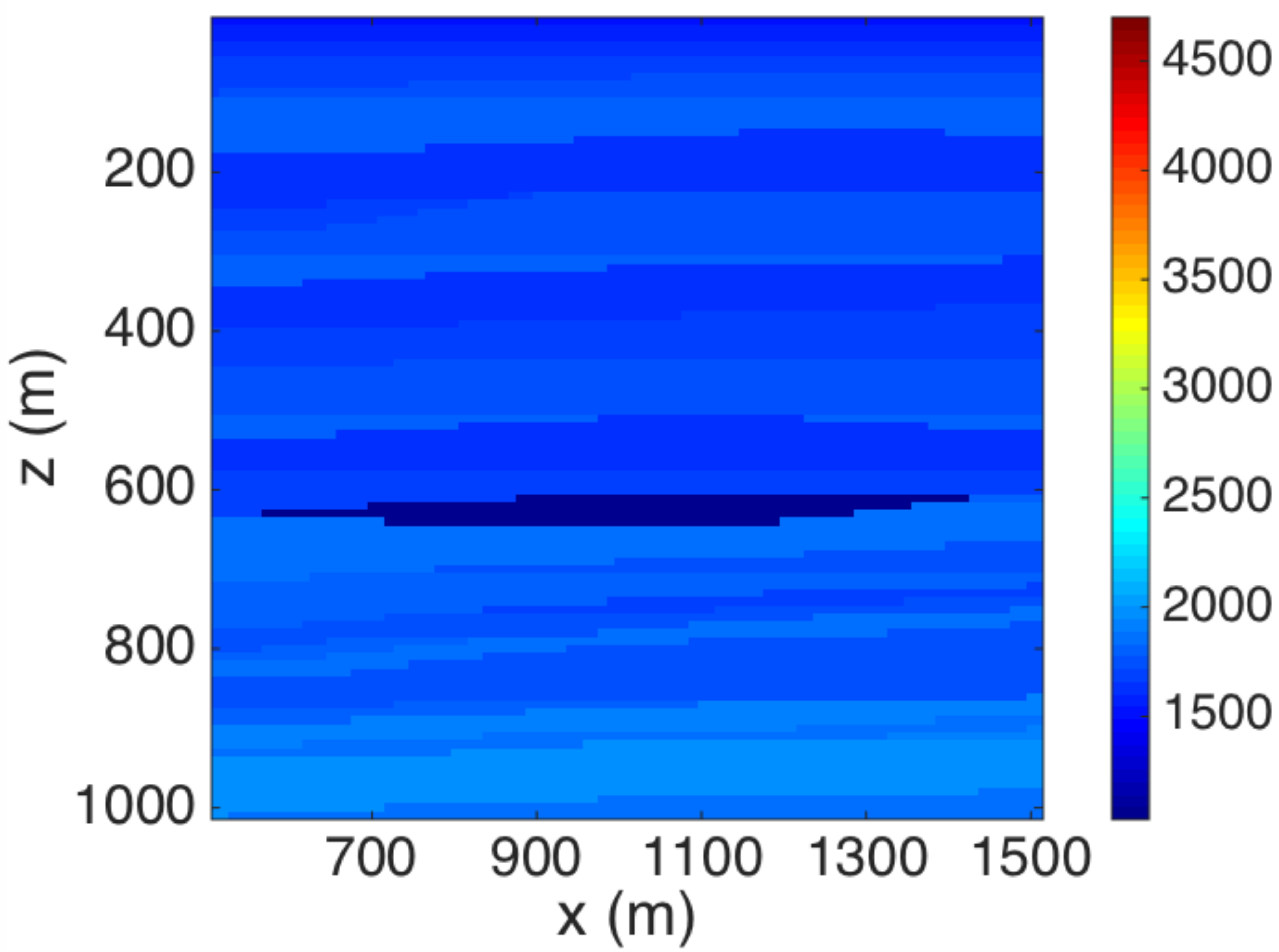
$$E_2 = L_2 R_2^*$$

with *only* $2r$ *extra PDEs solves!*

Invariance formula for EIVs (example 1)

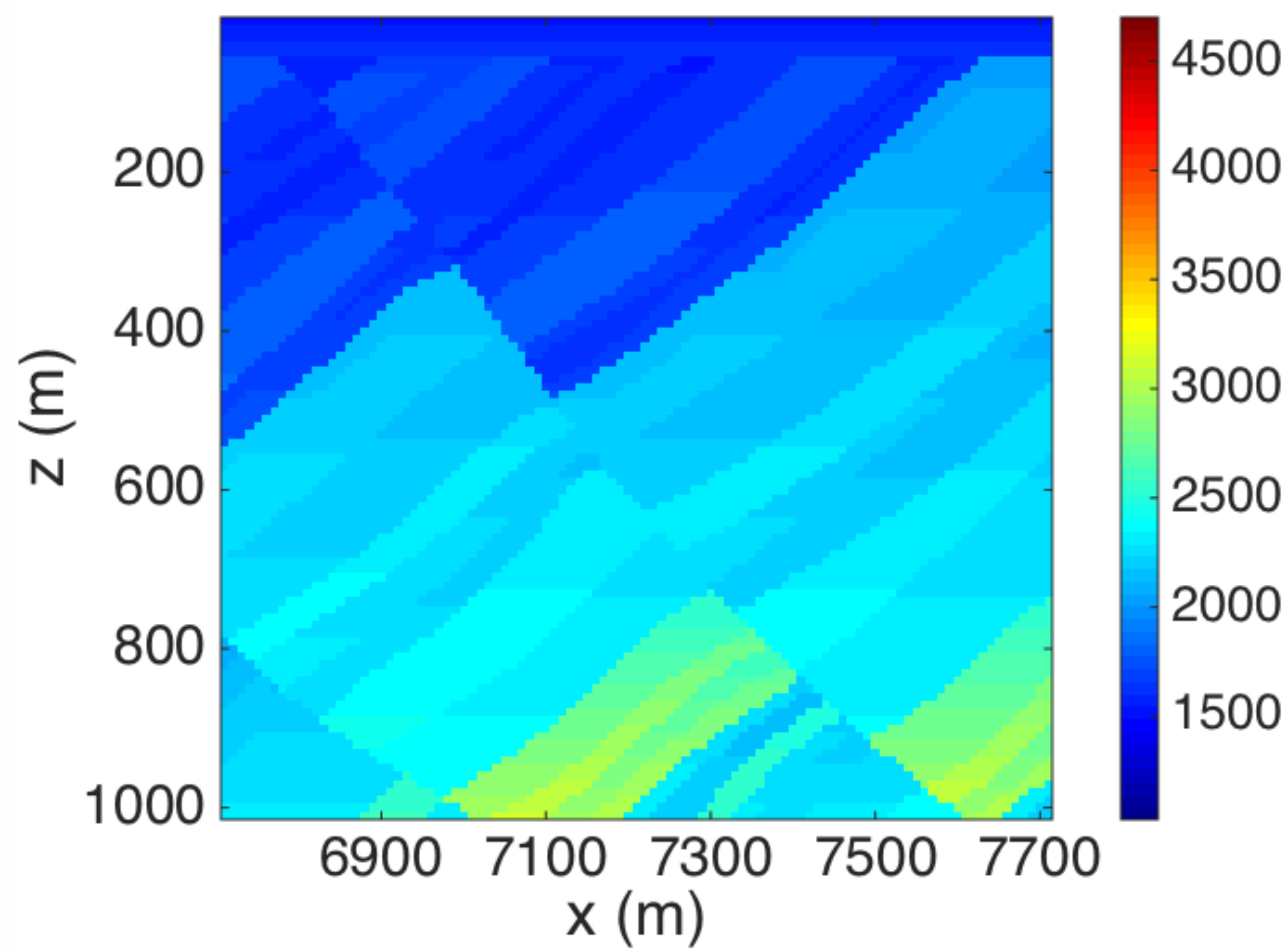


Invariance formula for EIVs (example 1)



background model 1

(correct)

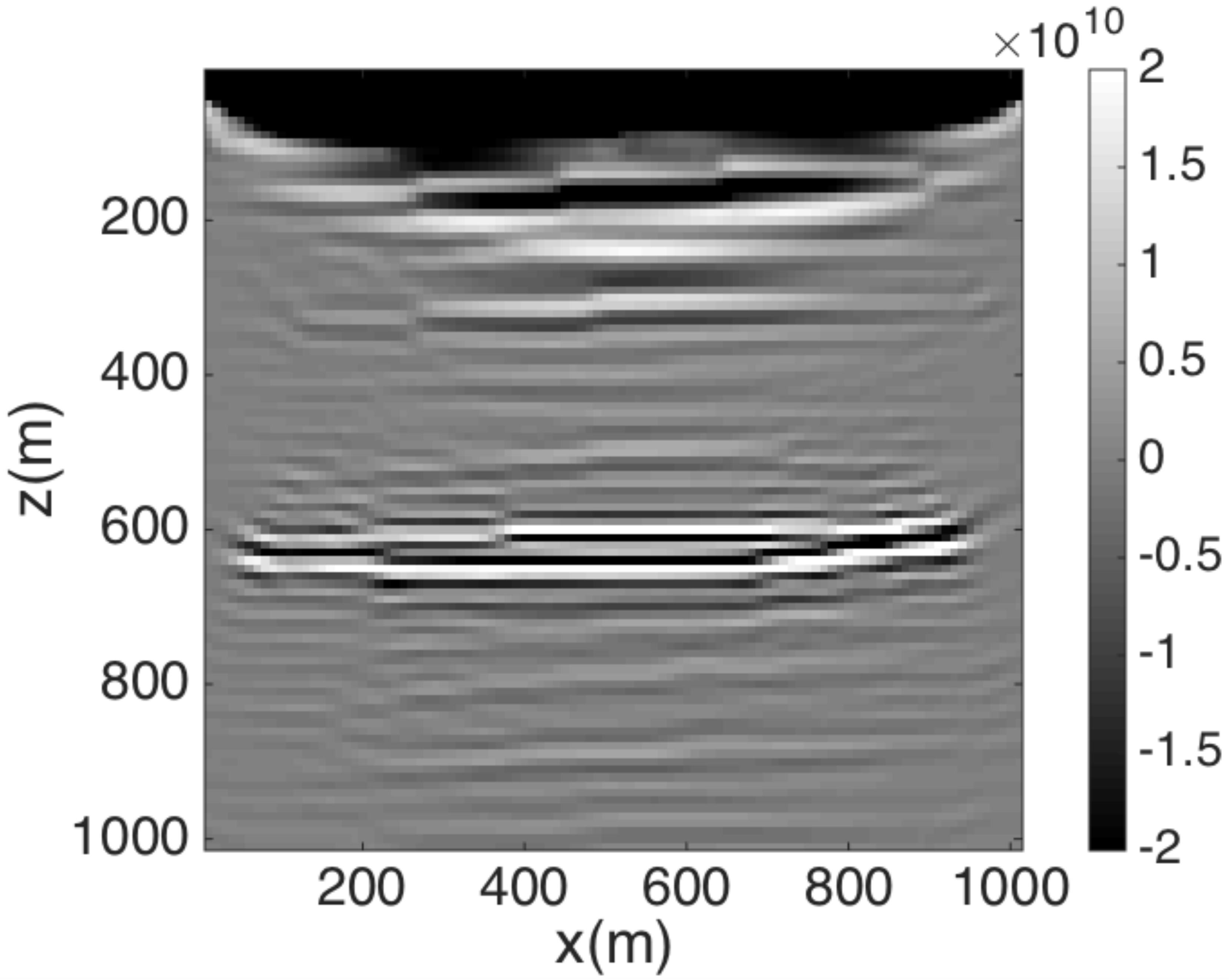


background model 2

(incorrect)

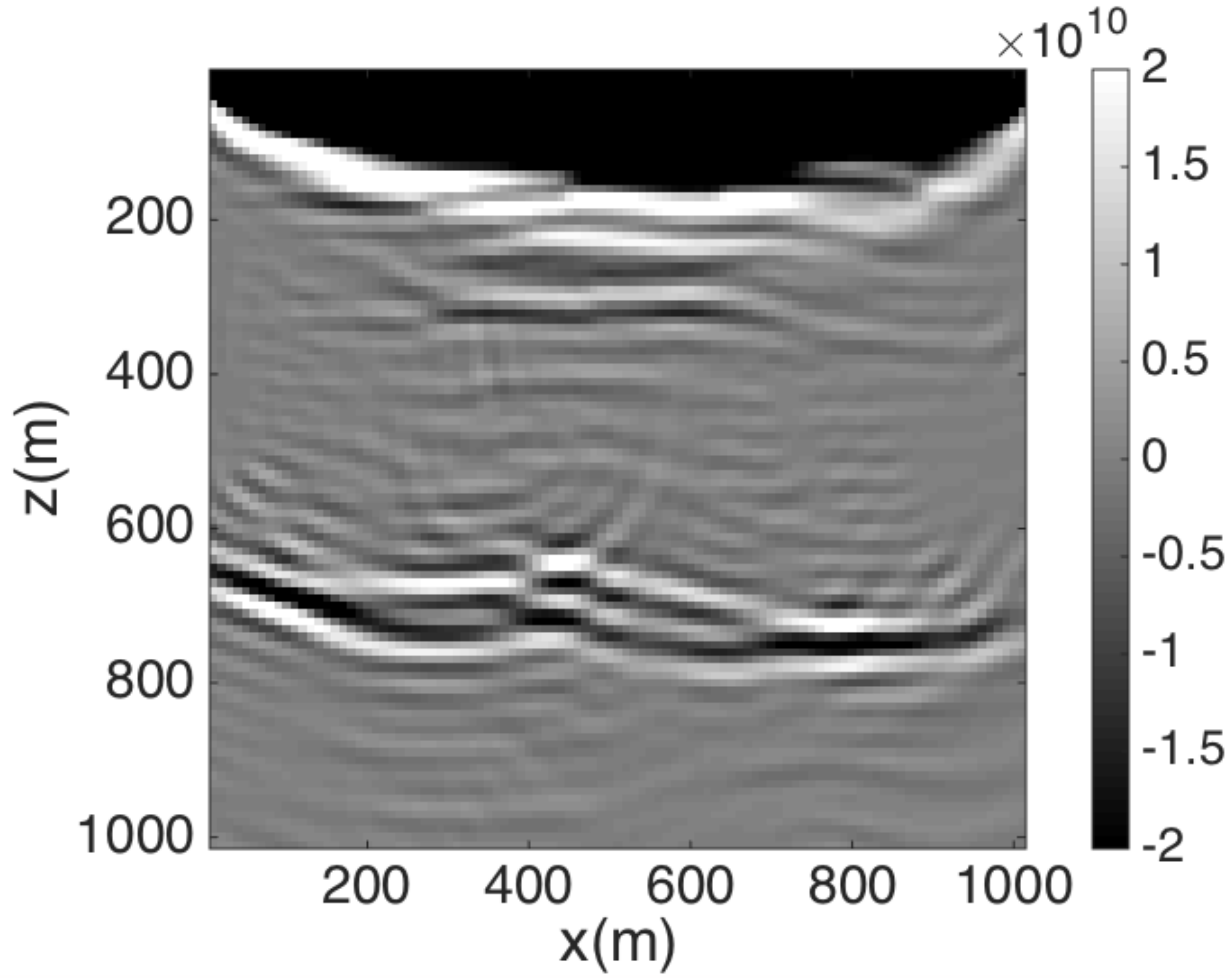
Invariance formula for EIVs (example 1)

Diagonal extraction of the low-rank EIV (5-30 Hz, step 0.5Hz, r = 15-45)



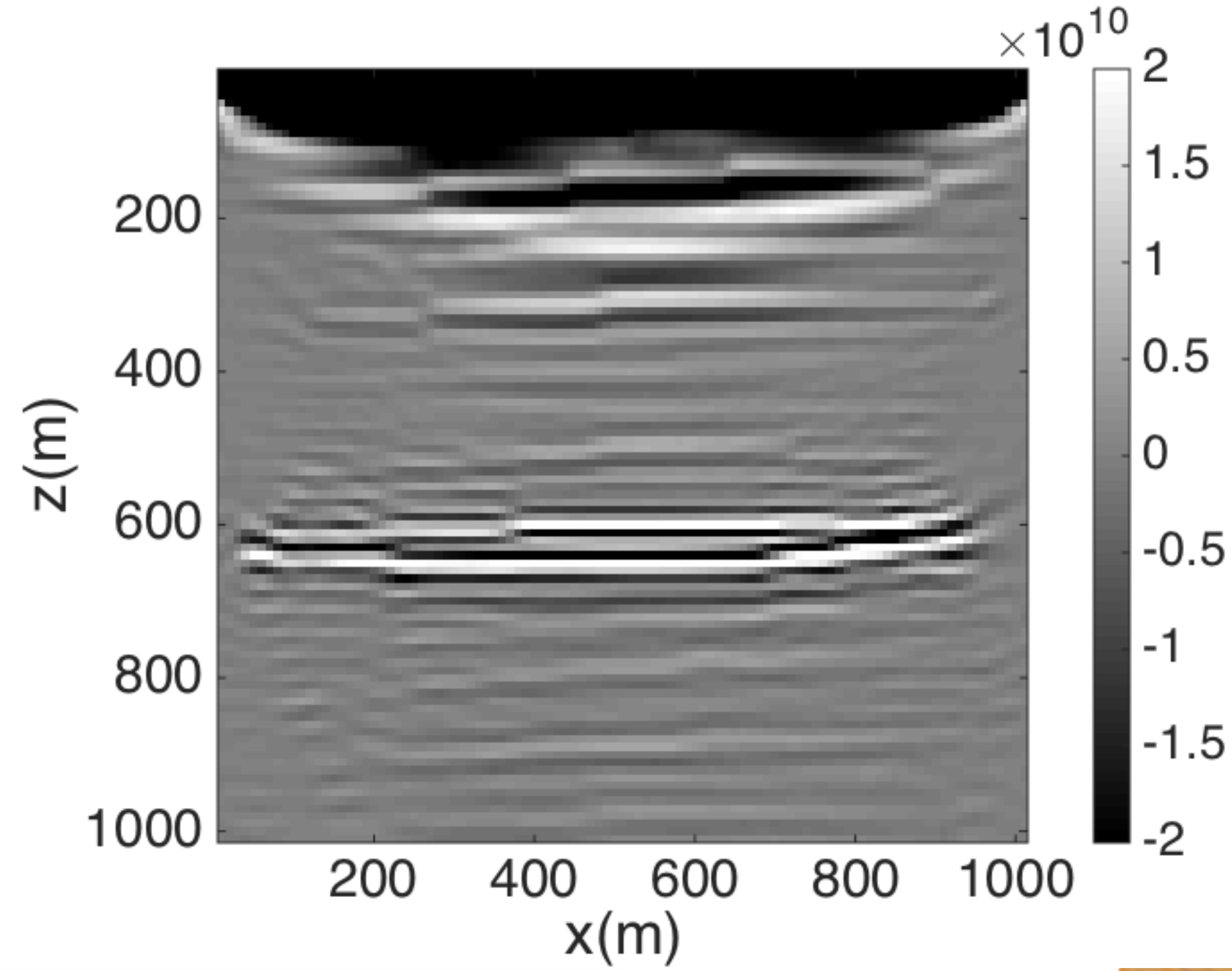
direct reconstruction

model 1



direct reconstruction

model 2

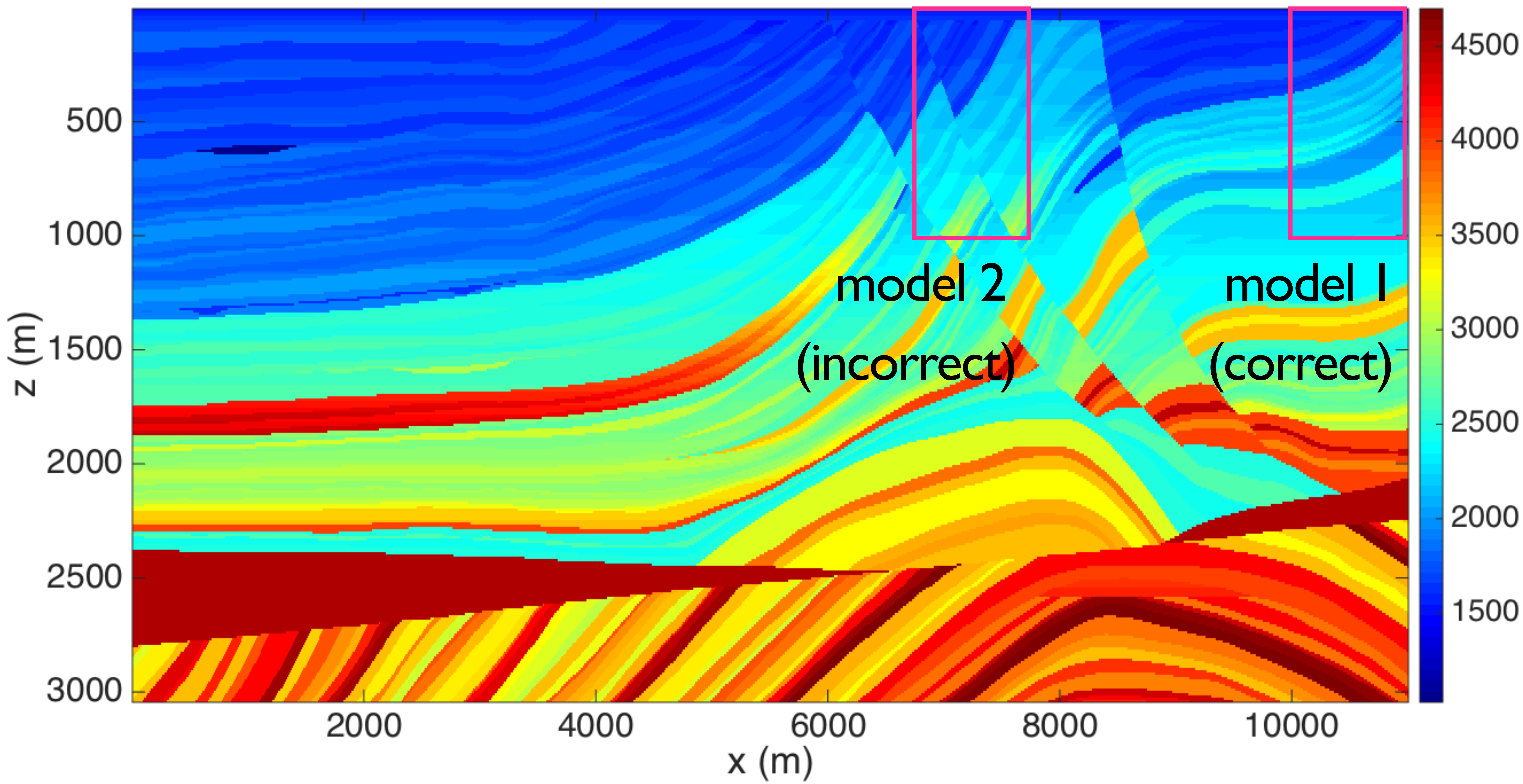


using invariance formula

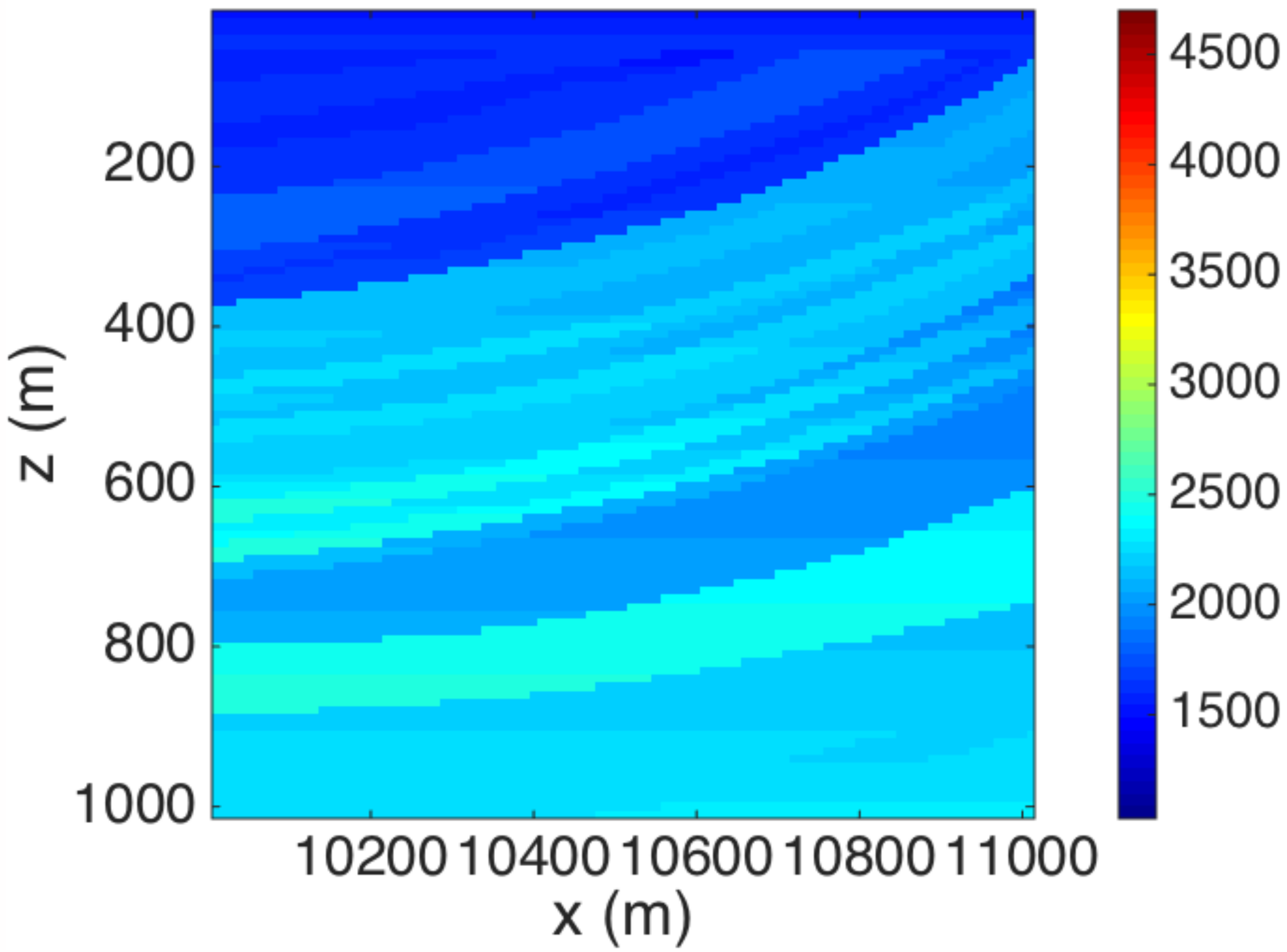
from model 2 to get model 1

from wrong to correct!!!

Invariance formula for EIVs (example 2)

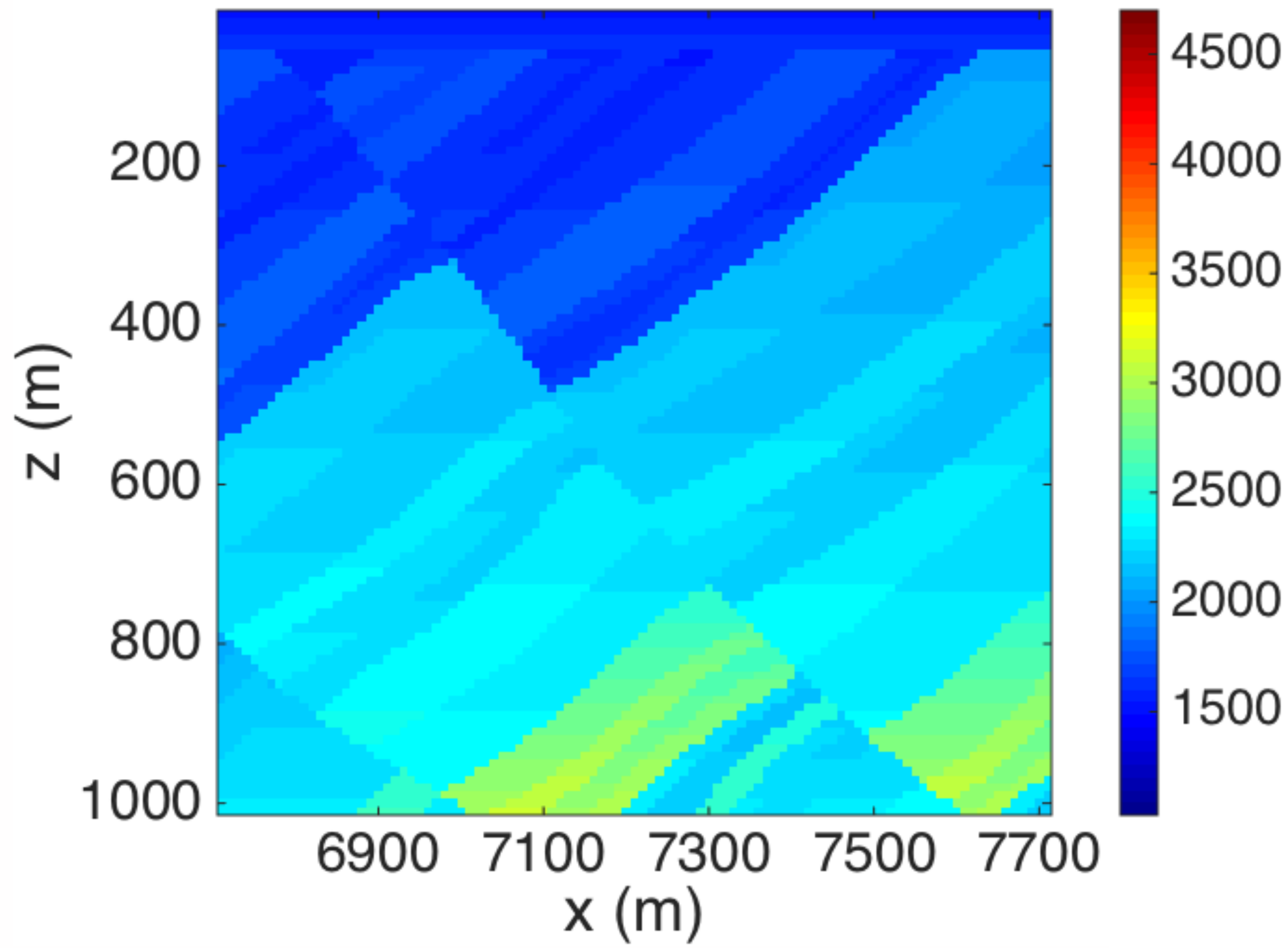


Invariance formula for EIVs (example 2)



background model 1

(correct)

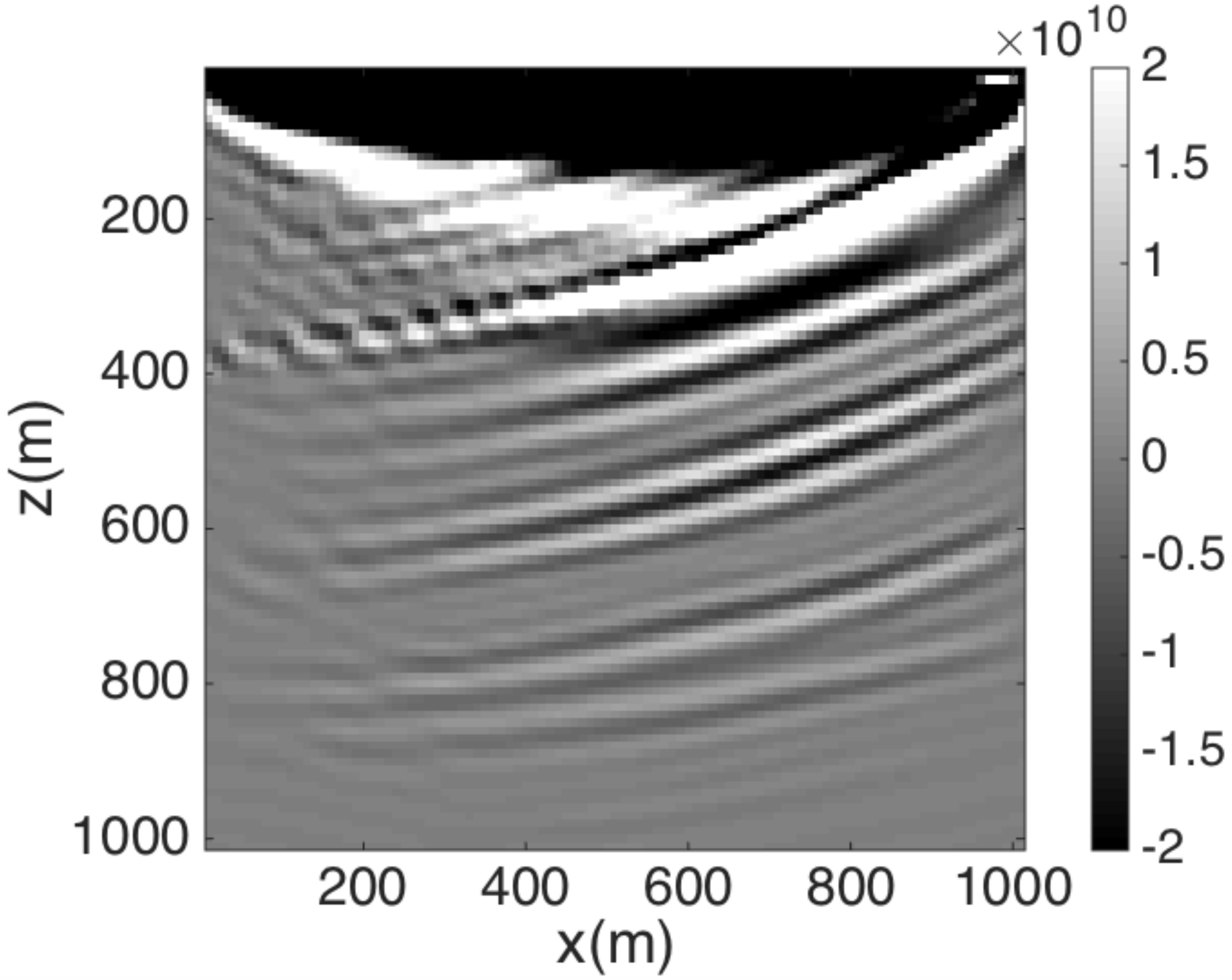


background model 2

(incorrect)

Invariance formula for EIVs (example 2)

Diagonal extraction of the low-rank EIV (5-30 Hz, step 0.5Hz, r = 15-45)



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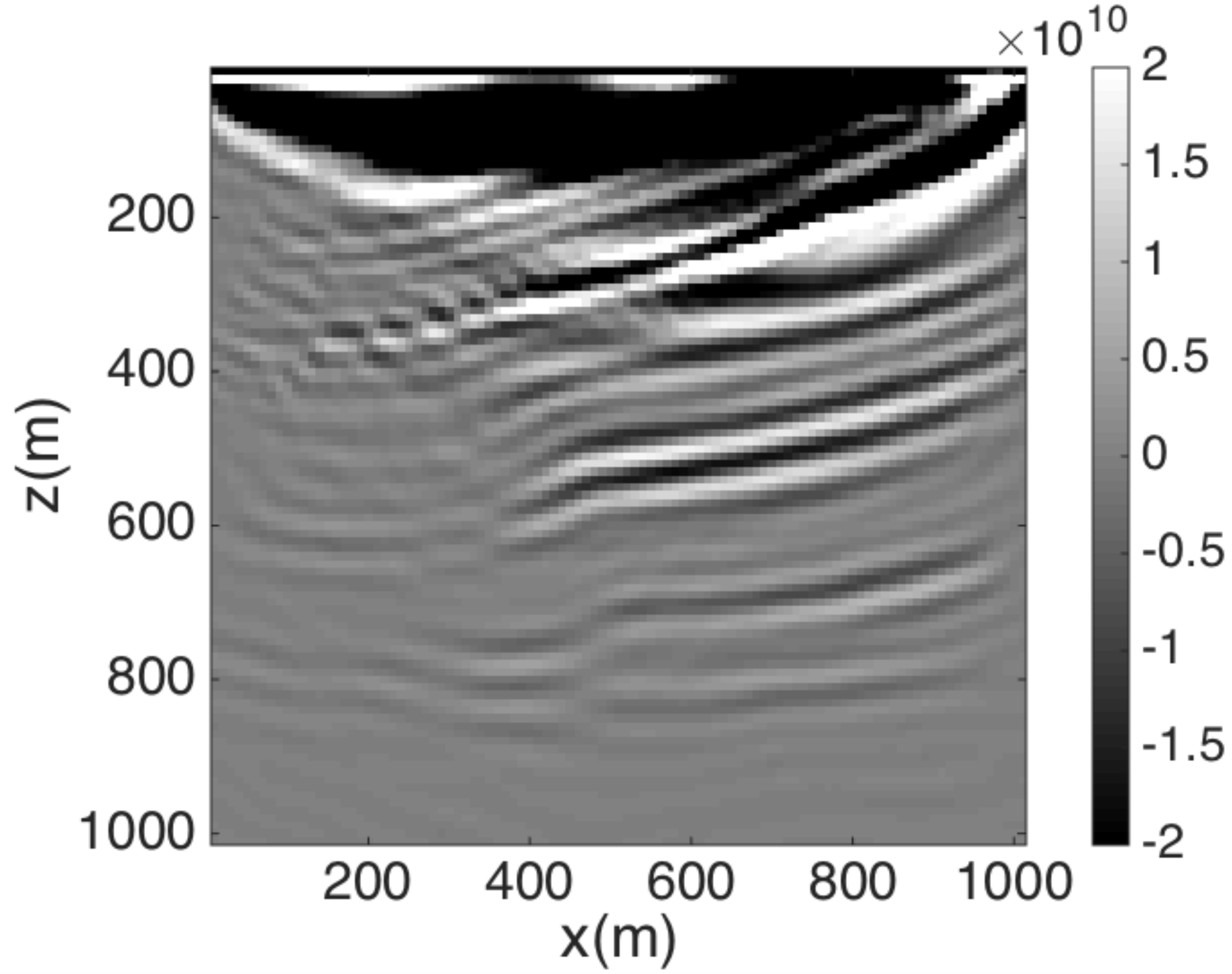
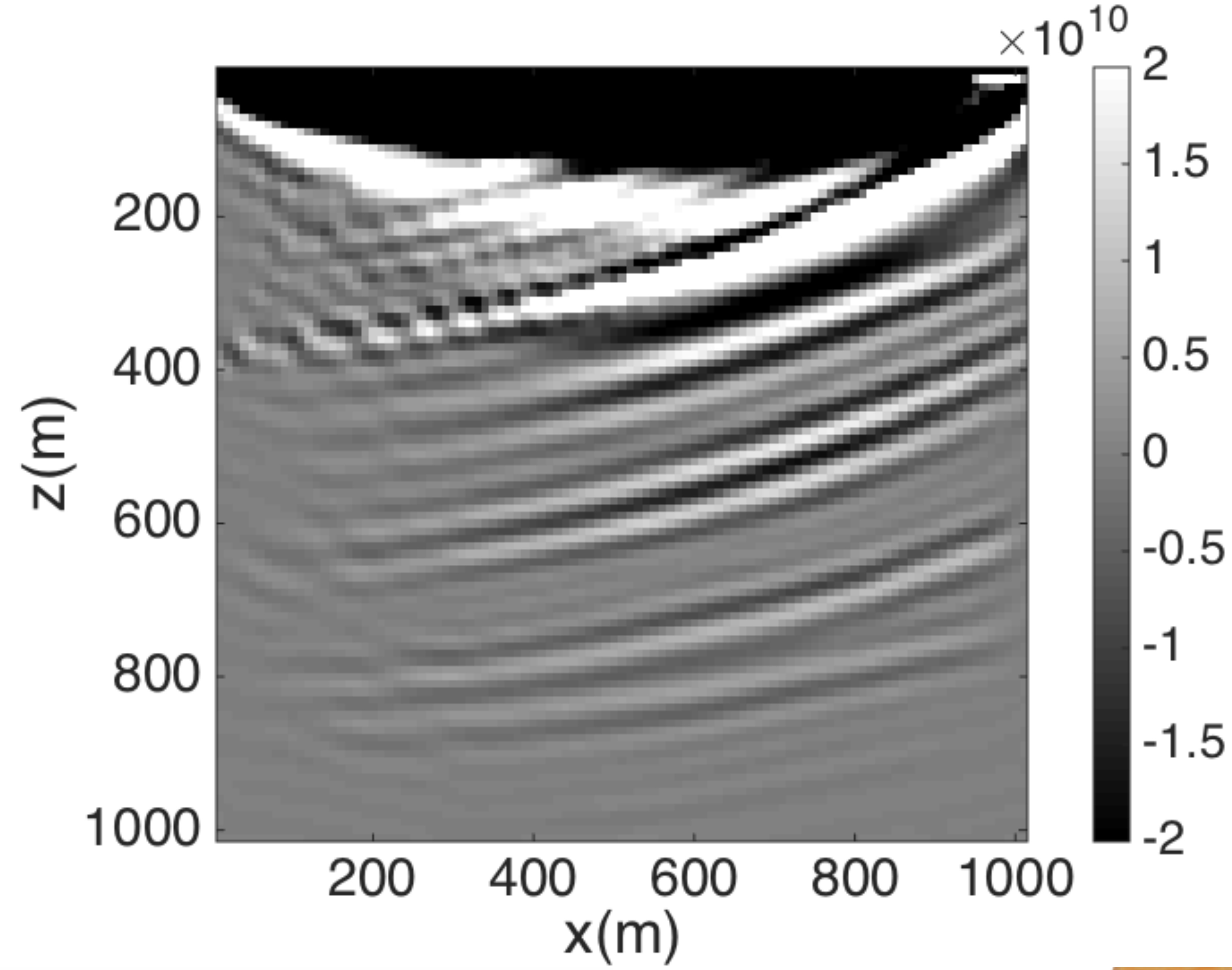


direct reconstruction

model 2



using invariance formula

from model 2 to get model 1

from wrong to correct!!!

Low-rank formulation for least-squares EIVs

Least-squares extended image volume

Aim: build an EIV that fits the data

$$\min_{E} \ \frac{1}{2}\|D - \mathcal{F}(E)\|_F^2$$

with

$$\mathcal{F}(E) = P_r H^{-1} E H^{-1} P_s^\top Q$$

Difficulty: image volume E is too large (storage & computational time)

Our solution: low-rank factorization of $E = LR^*$

with L and R two $N \times r$ matrices

Low-rank least-squares image volume

Least-squares problem for the LR factorization

$$\min_{L,R} \; \frac{1}{2}\Phi(L,R) = \frac{1}{2}\|D - \mathcal{F}(LR^*)\|_F^2$$

for $E = LR^*$

Gradients for least-squares formulation

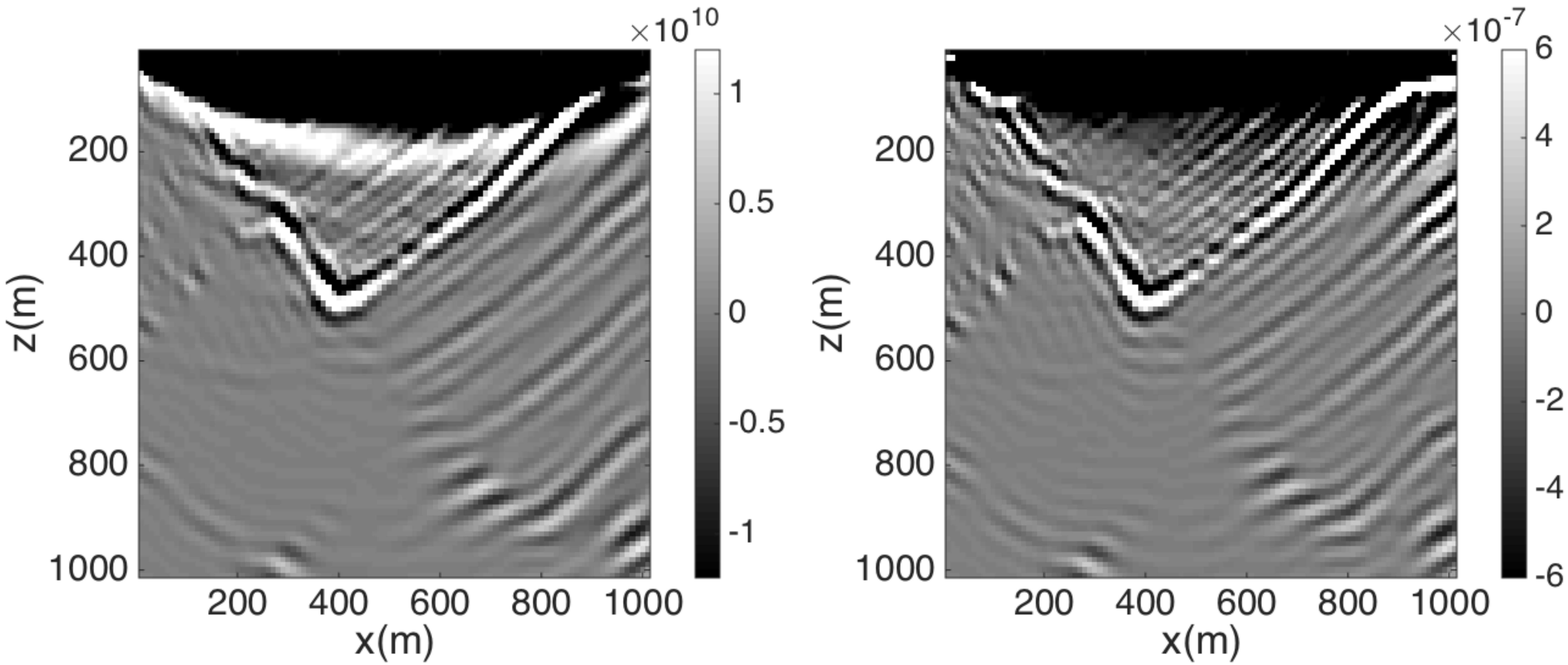
$$\frac{\partial \Phi}{\partial L}(L,R) = H^{-*}P_r^\top (D - \mathcal{F}(LR^*))Q^* P_s H^{-*} R$$

$$\frac{\partial \Phi}{\partial R}(L,R) = H^{-1}P_s^\top Q(D - \mathcal{F}(LR^*))^* P_r H^{-1} L$$

Solution by alternating least-squares on L and on R

Full EIV vs low-rank LS image volume

Diagonal extraction of the EIV for frequencies 5-30 Hz, with steps of 0.5 Hz

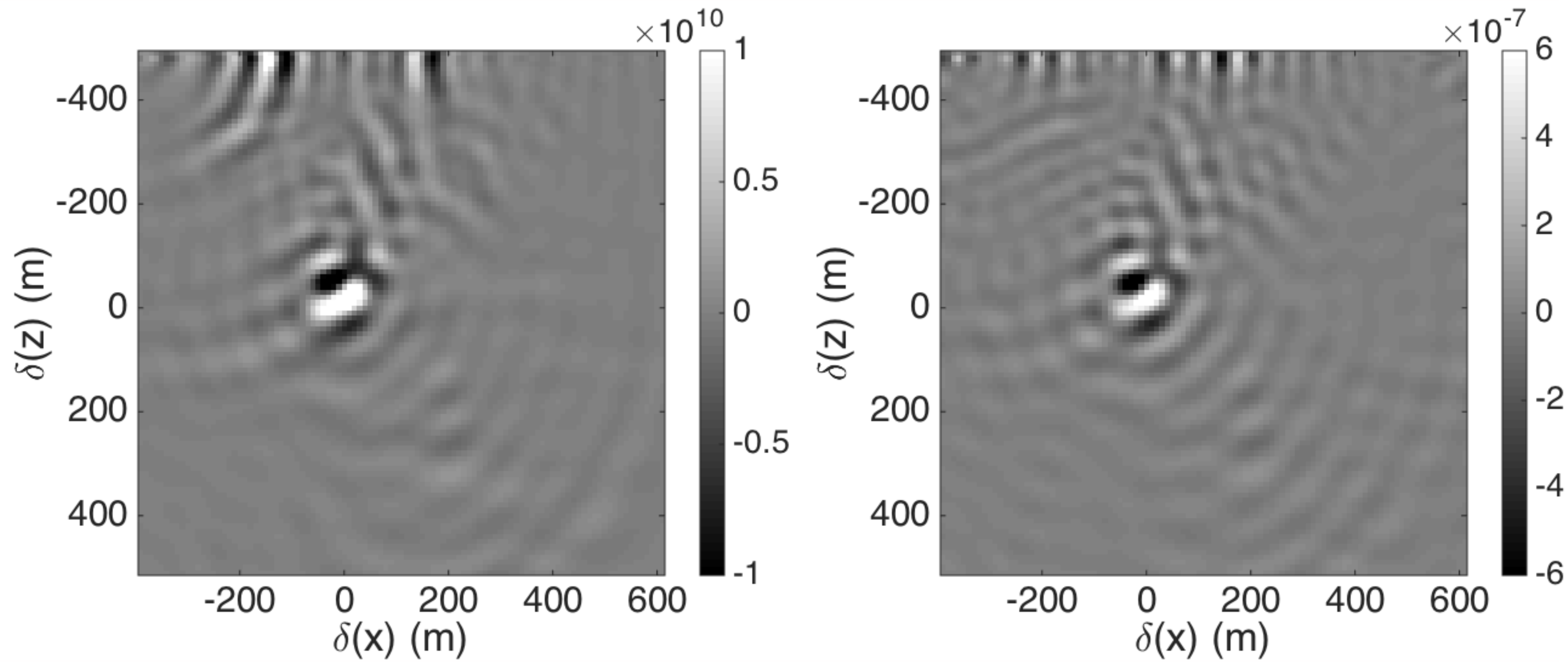


full EIV

Low-rank least-squares EIV

Full EIV vs low-rank LS image volume

Common image point gather of the EIV for 5-30 Hz, with steps of 0.5 Hz



full EIV

Low-rank least-squares EIV

Complexity analysis

Full subsurface offset extended images:

	# of PDE solves	size of EIV
conventional E	2Ns	N x N
mat-vec $\tilde{E} = EW$	2Nx	N x Nx
low-rank L, R	4r	2N x r

Ns = # sources   Nx = # probing points

N = # grid points   r = # estimated rank

Complexity analysis

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we win when Nx << Ns
but usually Nx ~ N
(Dirac probing vectors)

Complexity analysis

Full subsurface offset extended images:

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low-rank L, R	4r	2N x r

Ns = # sources
N = # grid points

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r = # estimated rank

we win when r << Ns
okay from low-rank approx. of data matrix!

Observations & Conclusions

Full-offset image volumes can be formed via probing

Form orthonormal basis that spans its range

— low-rank approximation via randomized SVD

— extract (off)diagonals from image volumes

Form least-squares extended images

— via alternating least-squares on low-rank factors

Natural "parametrization" from linear algebra

Acknowledgements

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

SINBAD

Acknowledgements

The authors wish to acknowledge the SENAI CIMATEC Supercomputing Center for Industrial Innovation, with support from BG Brasil, Shell, and the Brazilian Authority for Oil, Gas and Biofuels (ANP), for the provision and operation of computational facilities and the commitment to invest in Research & Development.

Acknowledgements

The speaker wishes to acknowledge the Swiss National Science Foundation, which partly funded this work.

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

Thank you for your attention