Reconstruction of S-waves from low-cost randomized and simultaneous acquisition

Ali M. Alfaraj, Rajiv Kumar, and Felix J. Herrmann

SINBAD Meeting, Houston

October 3, 2017
Outline

- Advantages of S-waves
- Why S-wave is not commonly used in practice?
- Elastic decomposition
- Jittered subsampling
- Single component reconstruction with:
  - rank minimization
  - sparsity promotion
- Various joint reconstruction formulations with sparsity promotion
- Conclusions
Advantages of S-waves

- Imaging through gas chimneys
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
- Elastic rock properties
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
- Elastic rock properties
- Improve accuracy & confidence
Why S-wave is not commonly used in practice?
Why S-wave is not commonly used in practice?
Why S-wave is not commonly used in practice?

Low S-wave velocity
Why $S$-wave is not commonly used in practice?

Low $S$-wave velocity

Nyquist criterion
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Nyquist criterion
- Denser sampling
- Higher acquisition costs
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Nyquist criterion
- Denser sampling
- Higher acquisition costs

Solution
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Denser sampling
- Higher acquisition costs

Solution:

- Nyquist criterion
- Compressive sensing
Why S-wave is not commonly used in practice?

- Low $S$-wave velocity
- Nyquist criterion
- Denser sampling
- Higher acquisition costs
- Compressive sensing
- Randomized undersampling
- Lower acquisition costs

Solution
Ocean bottom acquisition
Ocean bottom acquisition

\[ z_0 \]

\[ z_1 \]

\[ z_2 \]
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition

\[ \phi^+ - \phi^- + \Psi^+ + \phi^- - \Psi^+ - \phi^- \]

\[ V_z \]

\[ V_x \]

\[ V_y \]

\( z_0 \)

\( z_1 \)

\( z_2 \)
Wavefield decomposition
Elastic wavefield decomposition

\[ d = Nq \]
Elastic wavefield decomposition

\[ \mathbf{d} = \mathbf{Nq} \]

\[
\begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix} =
\begin{pmatrix}
N_1^+ & N_2^+ \\
N_1^- & N_2^-
\end{pmatrix}
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
\]
Elastic wavefield decomposition

\[ d = Nq \]

\[
\begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix}
= \begin{pmatrix}
N_1^+ & N_2^+ \\
N_1^- & N_2^-
\end{pmatrix}
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
\]

At the ocean bottom:

\[ \tau_{xz} = 0 \quad \tau_{zz} = -p \]
Elastic wavefield composition

\[ q = Ld \]

\[
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
= \begin{pmatrix}
L_1^+ & L_1^- \\
L_2^+ & L_2^-
\end{pmatrix}
\begin{pmatrix}
\phi^+ \\
\psi_y^+ \\
\phi^- \\
\psi_y^-
\end{pmatrix}
\]
Multicomponent data
Elastic decomposition

\( \Psi^+ \)

\( \Psi^- \)
Can’t afford dense acquisition
40 m source interval receiver gathers
f-k spectrum, 40 m source interval
Decomposed S-waves
f-k spectrum, 40 m source interval
Can’t afford dense acquisition
Jittered under-sampled acquisition
Jittered under-sampled acquisition

Jittered under-sampled acquisition

Gaps

---

Single component reconstruction \( w\backslash \) (i) rank minimization
Reconstruction w/ rank minimization

\[
\begin{align*}
\min_X \|X\|_* \quad \text{subject to} \quad \|A(X) - b\|_2 & \leq \sigma \\
A &= MS^H \\
\|X\|_* &= \|\lambda\|_1
\end{align*}
\]
Reconstruction w\ rank minimization

\[
\begin{align*}
\min_X \|X\|_* & \quad \text{subject to} \quad \|A(X) - b\|_2 \leq \sigma \\
A = MS^H & \quad \|X\|_* = \|\lambda\|_1 \\
\min_X \|A(X) - b\|_2 & \quad \text{subject to} \quad \|X\|_* \leq \tau \\
X = LR^H & \quad \|X\|_* \leq \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) \\
X & \in \mathbb{C}^{n \times m}, \quad L \in \mathbb{C}^{n \times k}, \quad R \in \mathbb{C}^{m \times k}, \quad k \ll m, n
\end{align*}
\]
Randomly subsampled frequency slices, 25 Hz

\[ P \]

\[ V_x \]

\[ V_z \]
Singular values decay

source-receiver domain

normalized magnitude vs. singular values

- fully sampled data
- randomly subsampled data
Midpoint-offset domain
Singular values decay

**source-receiver domain**

- fully sampled data
- randomly subsampled data

**midpoint-offset domain**

- fully sampled data
- randomly subsampled data
Reconstructed frequency slices, 25 Hz
Densely sampled frequency slices, 25 Hz
Residual

P

V_x

V_z
75% jittered subsampling
Reconstructed receiver gathers

- $P$
- $V_x$
- $V_z$
Densely sampled receiver gathers, 10 m
Residual

- **P**
- **V_x**
- **V_z**
f-k spectrum, 75% jittered subsampling
f-k spectrum, reconstruction
f-k spectrum, densely sampled

- P
- $V_x$
- $V_z$
Reconstructed S-waves

\[ \Psi^+ \]

\[ \Psi^- \]
True S-waves

\[ \Psi^+ \]

\[ \Psi^- \]
Residual
Single component reconstruction w\(\backslash\) (ii) sparsity promotion
Single component reconstruction w/ sparsity promotion

\[
\min_x \| x \|_1 \quad \text{subject to} \quad \| A x - b \|_2 \leq \sigma \\
(\text{BPDN}_\sigma)
\]

\( x \): curvelet coefficients

\( A = MS^H \)
75% jittered subsampling
Reconstructed S-waves

$\Psi^+$

$\Psi^-$
Densely sampled S-waves
Residual

\[ \Psi^+ \]

\[ \Psi^- \]

offset [m]

time [s]
Marmousi II data

\(\rho\)
Densely sampled data

\[ V_x \]

\[ V_z \]

\[ T_{zz} \]
75% jittered subsampling

\[ V_x \]

\[ V_z \]

\[ T_{zz} \]
Reconstructed data
Densely sampled data
Residual

\[
\begin{align*}
\Phi^- & \quad \Phi^+ \\
\Psi^- & \quad \Psi^+
\end{align*}
\]
Joint interpolation decomposition
Joint interpolation decomposition

\[ P \xrightarrow{\text{Interpolation}} V_x \rightarrow V_z \]

(1) Interpolation

\[ P \xrightarrow{\text{Decomposition}} \Psi^+ \rightarrow \Psi^- \]

(2) Decomposition
Joint interpolation decomposition

(1) Interpolation

(2) Decomposition

\[ P \xrightarrow{\text{Interpolation}} V_x \xrightarrow{\text{Interpolation \& Decomposition}} \psi^+ \]

\[ V_z \xrightarrow{\text{Interpolation \& Decomposition}} \psi^- \]
Joint interpolation decomposition with curvelets

\[
\min_x \|x\|_1 \quad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma \quad \text{(BPDM_\sigma)}
\]

\(x\): coefficients of the decomposed data

Sparsifying transform:

\[A_c = MF^H LFS^H\]
75% jittered subsampling

Graphs showing the relationship between time [s] and offset [m] for P, V_x, and V_z.
Joint interpolation decomposition in the curvelet domain
Densely sampled S-waves

\[ \Psi^+ \]

\[ \Psi^- \]
Residual

\[ \Psi^+ \]

\[ \Psi^- \]
75% jittered subsampling

$V_x$

$V_z$

$T_{zz}$
Reconstructed data
Densely data

\begin{align*}
\Phi^- & \\
\Psi^- & \\
\Phi^+ & \\
\Psi^+ & 
\end{align*}
Residual
Joint interpolation decomposition in the f-k domain
Joint interpolation decomposition, f-k

\[
\min_{x} \|x\|_1 \quad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma) \]

\(x\): coefficients of the decomposed data

Sparsifying transform:

\[A_c = MF^H LFS^H\]

\[A_{fk} = MF^H L\]
Reconstructed data, f-k
Residual

Φ⁻

Φ⁺

ψ⁻

ψ⁺
Why curvelets are better?

- Better at capturing curve-like events.
- Sparser representation.
Joint source separation decomposition
Jittered continuous recording, 1 boat, 2 air guns

\[ V_x \]

\[ V_z \]

Zoomed \[ T_{zz} \]
Joint source separation decomposition

\[
\begin{aligned}
\min_{\mathbf{x}} & \quad \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma \\
\mathbf{x} : & \quad \text{curvelet coefficients of the decomposed data} \\
\mathbf{A}_c = & \quad \mathbf{MF}^H \mathbf{LFS}^H \\
\mathbf{M} : & \quad \text{blending matrix}
\end{aligned}
\]
Reconstructed data
Densely data

\( \Phi^- \)

\( \Phi^+ \)

\( \Psi^- \)

\( \Psi^+ \)
Residual

\( \Phi^- \)

\( \Phi^+ \)

\( \Psi^- \)

\( \Psi^+ \)
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
- Minimize parameters selection.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
- Minimize parameters selection.
- Ensure preservation of amplitude ratios.
Conclusions

• Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
Conclusions

- Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
- Coarse regular sampling results in aliasing of the S-waves.
Conclusions

- Acquisition of S-waves is prohibitively expensive without conventional dense acquisition designs.
- Coarse regular sampling results in aliasing of the S-waves.
- Using low-cost jittered under-sampling & simultaneous acquisition with (i) SVD-free rank minimization interpolation & (ii) joint interpolation source separation decomposition, S-waves become feasible to acquire & utilize in practice.
Conclusions

- Acquisition of S-waves is prohibitively **expensive** w/ conventional dense acquisition designs.
- **Coarse regular** sampling results in **aliasing** of the S-waves.
- Using low-cost **jittered under-sampling & simultaneous acquisition** w/ (i) SVD-free rank minimization interpolation & (ii) joint interpolation source separation decomposition, S-waves become feasible to acquire & utilize in practice.
- Utilize the multicomponent data to its available **full extent** at a lower cost compared w/ conventional acquisition.
Future work

- Examining the noise effect is another reason why S-waves are not used, yet another motivation for joint formulations.
- Joint formulations with rank minimization.
- P-S imaging
References

Acknowledgements

I extend my gratitude to Saudi Aramco for sponsoring my Ph.D. studies at the University of British Columbia.

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

Thank you for your attention!