Time-domain least-squares RTM with sparsity promotion on field data

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Motivation

Our first steps with sparsity promoting LSRTM in the time-domain:

- develop robust workflow with little user interaction
- experiments with increasing difficulty

Synthetic data set: linearized data, (non-) inversion crime

↓

Synthetic data set: non-linearized data

↓

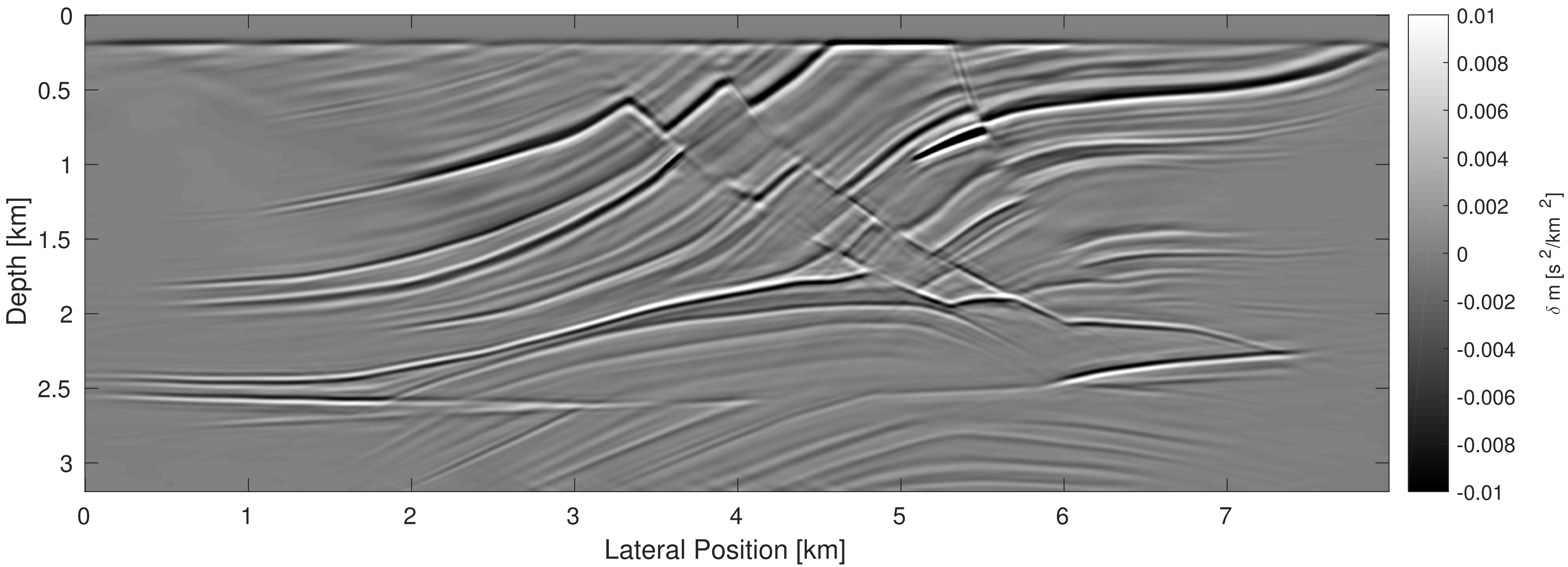
Field data set: Machar 2D

Motivation

Solve sparsity promoting LSRTM w/ linearized Bregman

$$\text{minimize} \quad \lambda||\mathbf{C}\delta\mathbf{m}||_1 + \frac{1}{2}||\mathbf{C}\delta\mathbf{m}||_2^2$$

$$\text{subject to } ||\, \mathbf{J}\delta\mathbf{m} - \delta\mathbf{d}||_2 \leq \sigma$$

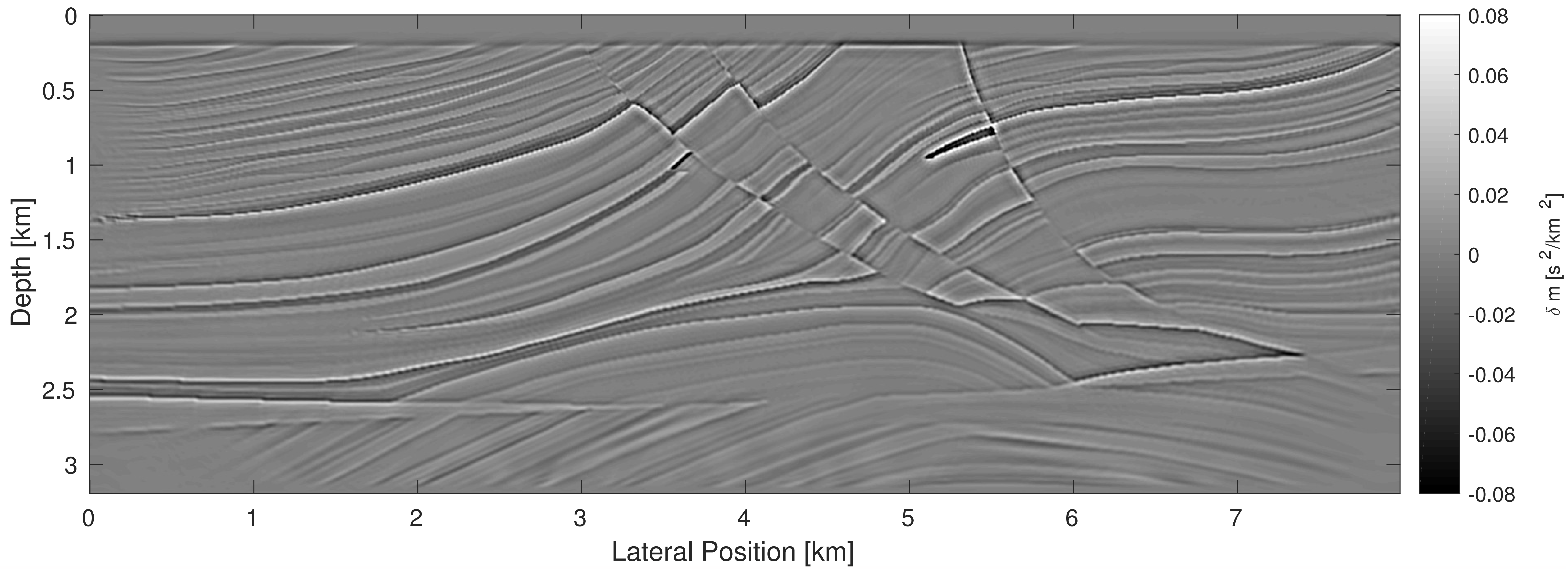


Basic implementation of SP-LSRTM

Motivation

Solve sparsity promoting LSRTM w/ linearized Bregman

$$\begin{aligned}&\text{minimize} \quad \lambda||\mathbf{C}\delta\mathbf{m}||_1 + \frac{1}{2}||\mathbf{C}\delta\mathbf{m}||_2^2 \\ &\text{subject to } ||\,\mathbf{J}\delta\mathbf{m} - \delta\mathbf{d}||_2 \leq \sigma\end{aligned}$$



Pre-conditioned SP-LSRTM w/ λ auto-tuning

Sparsity promoting LSRTM

Problem formulation:

$$\begin{aligned}&\text{minimize} \quad \lambda||\mathbf{C}\delta\mathbf{m}||_1 + \frac{1}{2}||\mathbf{C}\delta\mathbf{m}||_2^2 \\ &\text{subject to } ||\,\mathbf{J}\delta\mathbf{m} - \delta\mathbf{d}||_2 \leq \sigma\end{aligned}$$

$\delta\mathbf{m}$: model perturbation/image

$\delta\mathbf{d}$: linearized data (single scattered data)

$\mathbf{J}$: linearized forward modeling operator (Jacobian)

$\mathbf{C}$: curvelet transform

Sparsity promoting LSRTM

Problem formulation:

$$\text{minimize} \quad \lambda||\mathbf{C}\delta\mathbf{m}||_1 + \frac{1}{2}||\mathbf{C}\delta\mathbf{m}||_2^2$$
$$\text{subject to } ||\ \mathbf{J}\delta\mathbf{m} - \delta\mathbf{d}||_2 \leq \sigma$$

Left- and right-hand preconditioning:

$$\delta\mathbf{m} = \mathbf{M}_R^{-1}\mathbf{x}$$
$$\mathbf{M}_L^{-1}\mathbf{J}\mathbf{M}_R^{-1}\mathbf{x} = \mathbf{M}_L^{-1}\delta\mathbf{d}$$

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Preconditioning

Left-hand preconditioning (data space)

$$\mathbf{M}_L^{-1} = \mathbf{T}_d \mathbf{F}$$

$\mathbf{T}_d$: Topmute

$\mathbf{F}$: Fractional integration $\partial_{|t|}^{-1/2}$

Right-hand preconditioning (model space)

$$\mathbf{M}_R^{-1} = \mathbf{T}_m \mathbf{A}$$

$\mathbf{T}_m$: Topmute

$\mathbf{A}$: Depth scaling

Preconditioned SP-LSRTM

$$\begin{aligned} &\text{minimize} \quad \lambda||\mathbf{Cx}||_1 + \frac{1}{2}||\mathbf{Cx}||_2^2 \\ &\text{subject to } || \underbrace{\mathbf{M}_L^{-1}\mathbf{J}\mathbf{M}_R^{-1}}_{\hat{\mathbf{J}}}\mathbf{x} - \underbrace{\mathbf{M}_L^{-1}\delta\mathbf{d}}_{\mathbf{b}}||_2 \leq \sigma \end{aligned}$$

Algorithm:

$$\begin{array}{ll} 1. & \textbf{for } k = 0, 1, \cdots \\ 2. & \quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \hat{\mathbf{J}}^*_{r(k)} (\hat{\mathbf{J}}_{r(k)}\mathbf{x}_k - \mathbf{b}_{r(k)}) \cdot \max(0, 1 - \frac{\sigma}{||\hat{\mathbf{J}}_{r(k)}\mathbf{x}_k - \mathbf{b}_{r(k)}||_2}) \\ 3. & \quad \mathbf{x}_{k+1} = \mathbf{C}^* S_\lambda(\mathbf{C}\mathbf{z}_{k+1}) \\ 4. & \textbf{end for} \end{array}$$

SP-LSRTM examples w/ linearized data

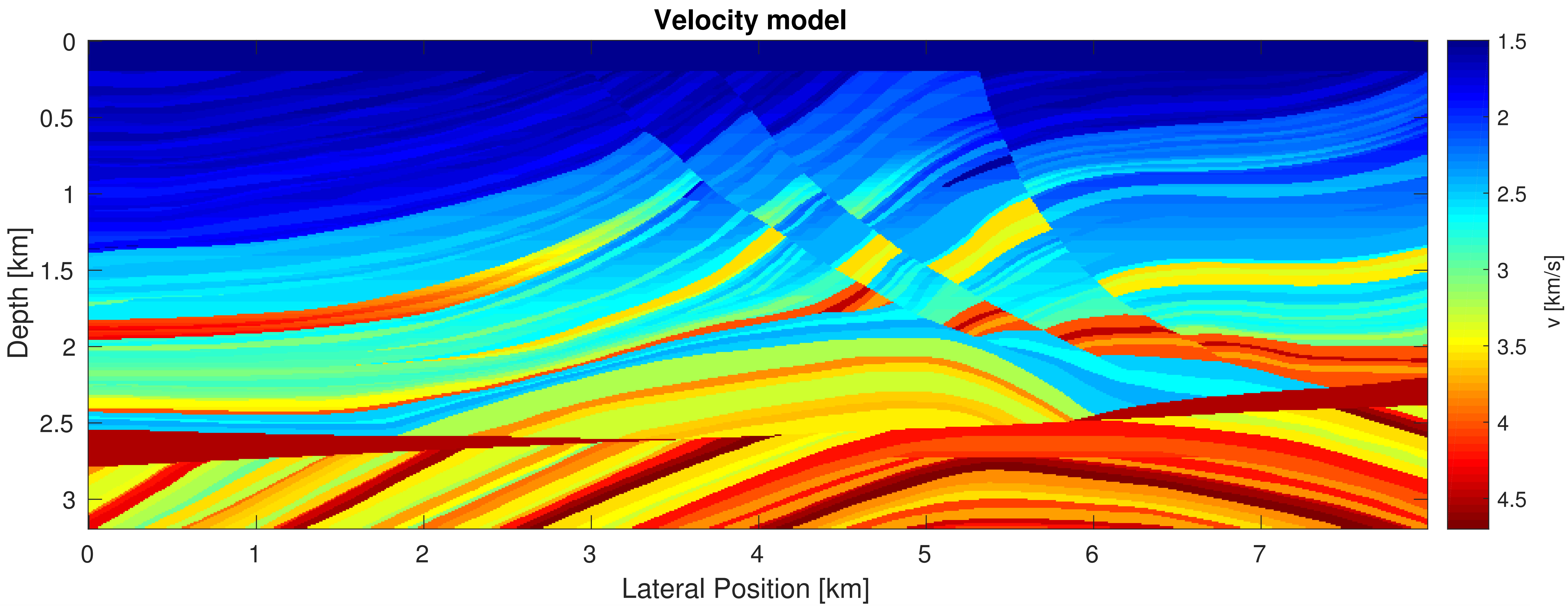
Marmousi model

- 320 shots, 4 seconds recording time
- 30 Hz Ricker wavelet
- 25 m source spacing
- OBNs with 10 m receiver spacing

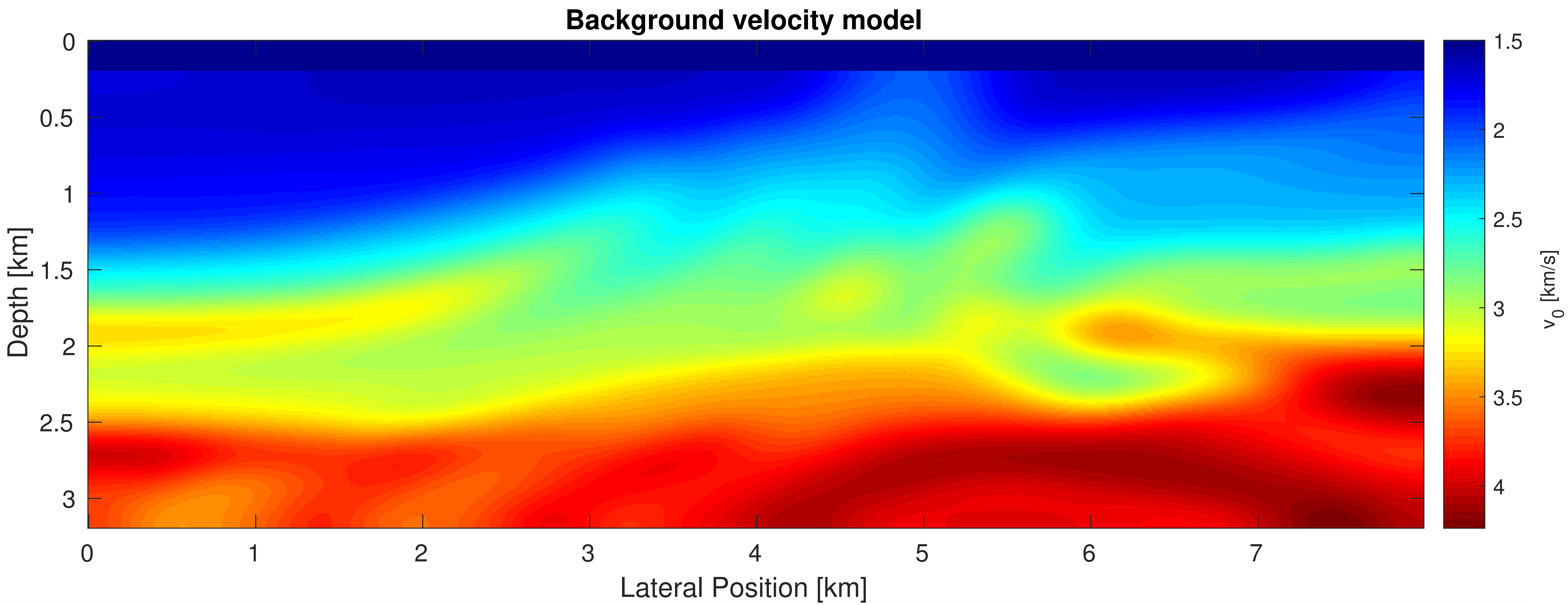
Inversion parameters

- 40 iterations
- 8 shots per iteration (1 data pass)
- linearized observed data (inversion crime)
- no data preprocessing

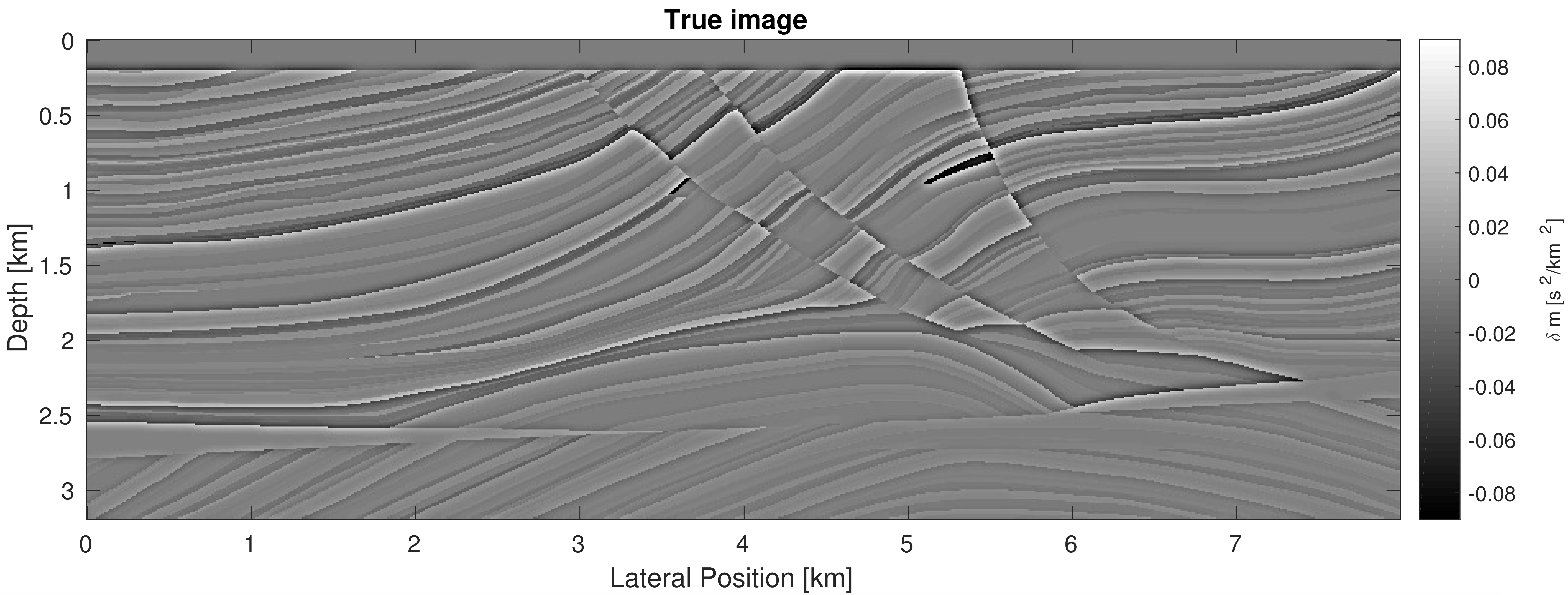
SP-LSRTM: Marmousi



SP-LSRTM: Marmousi



SP-LSRTM: Marmousi



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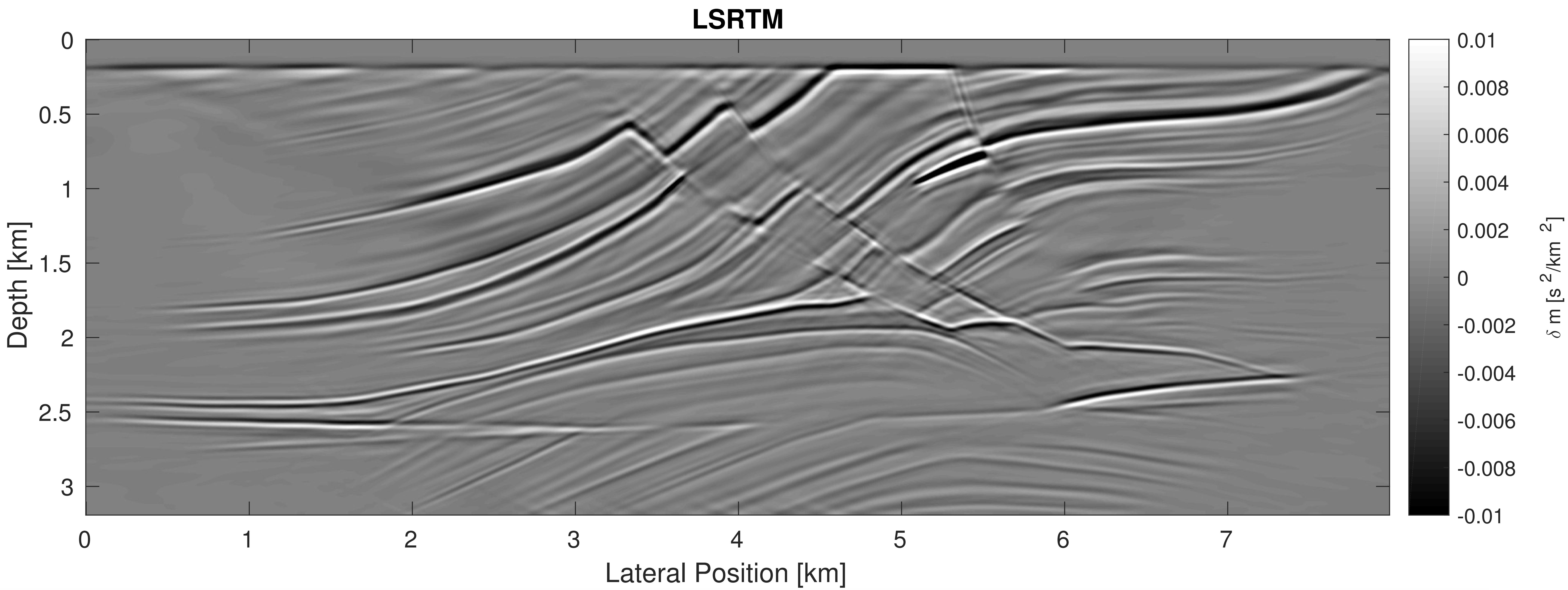
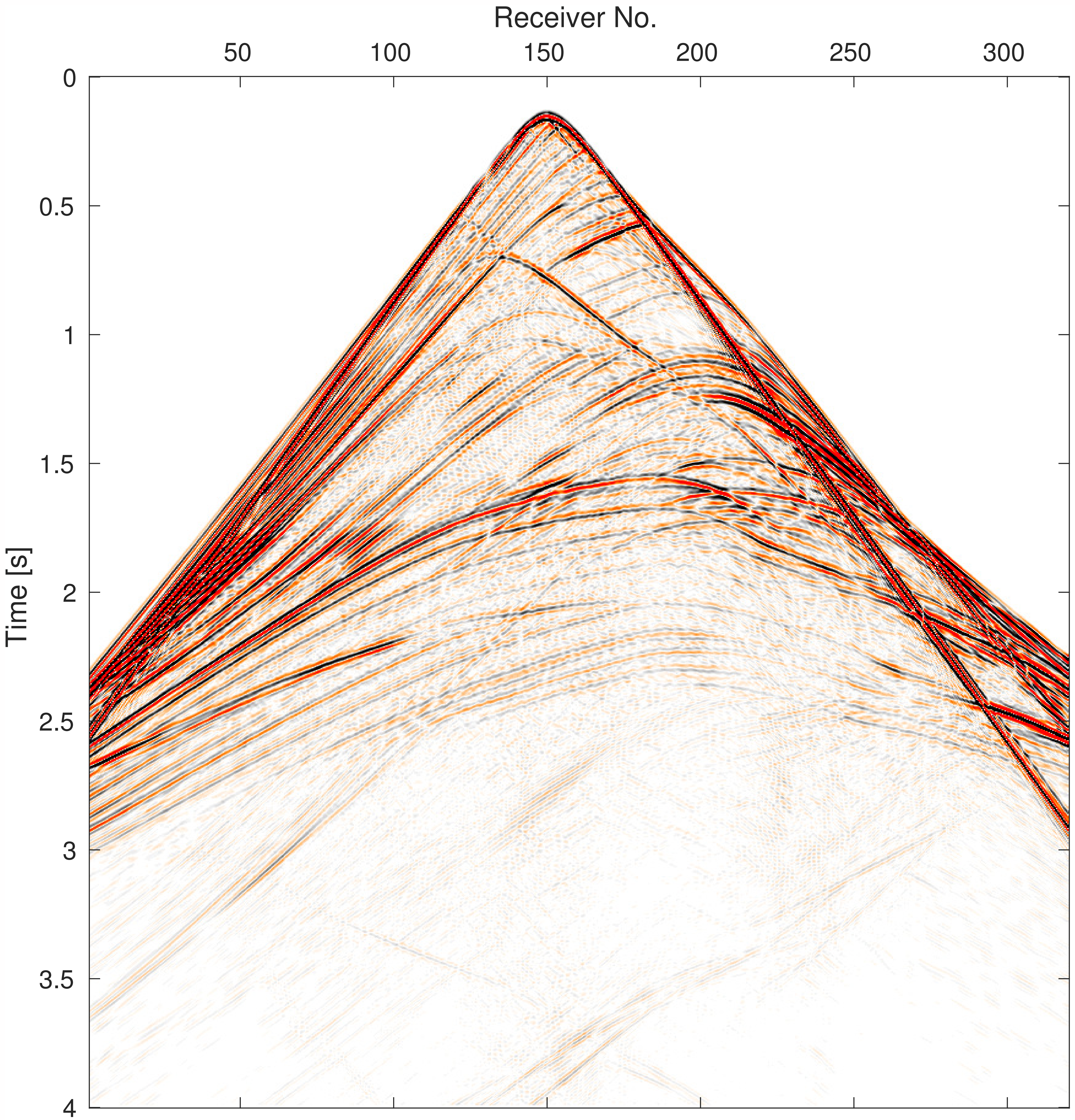


SP-LSRTM: Marmousi



Linearized data

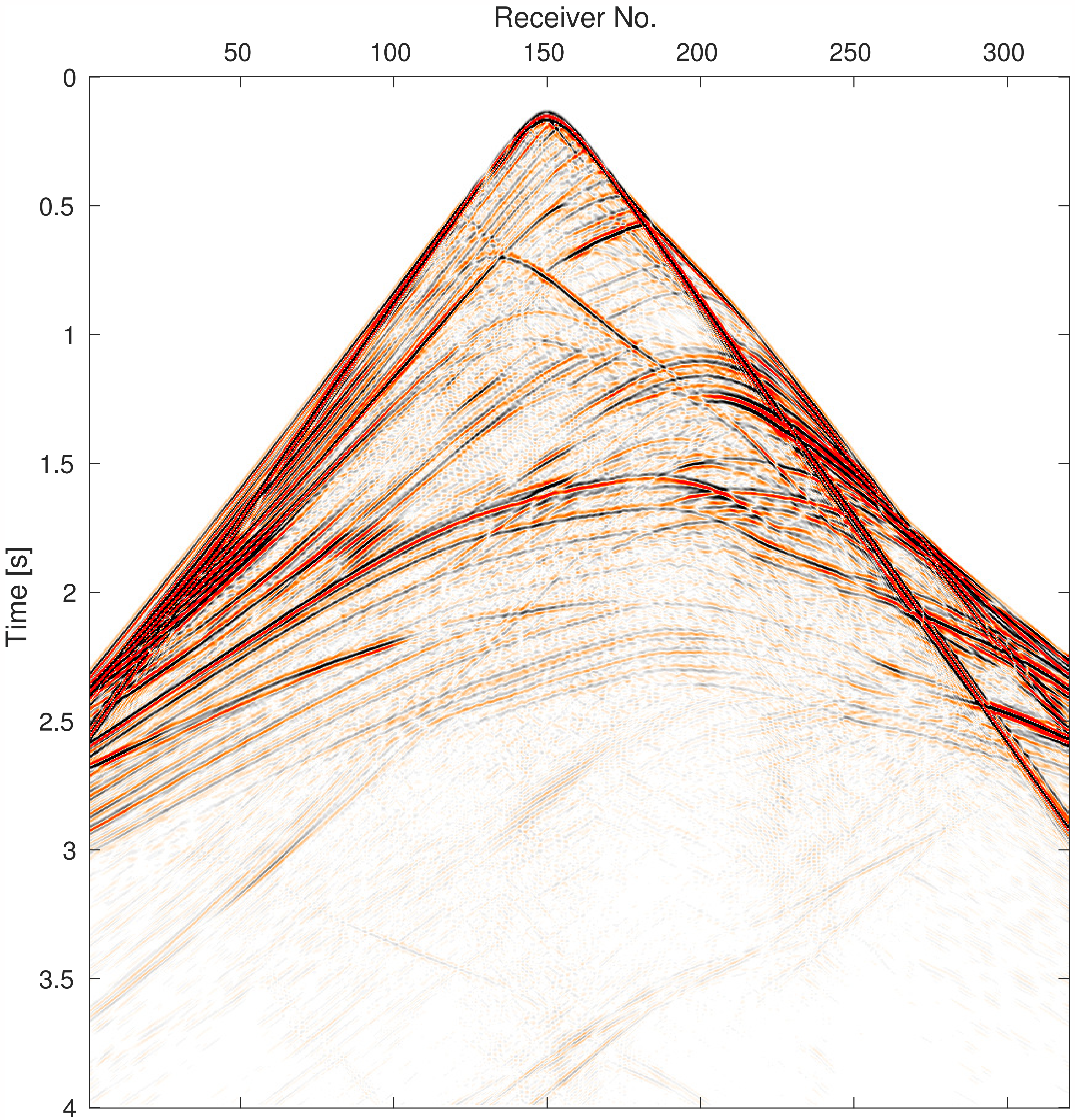
Problem:

- ocean bottom reflection
- backscattered energy in background model ⟶ low frequency updates

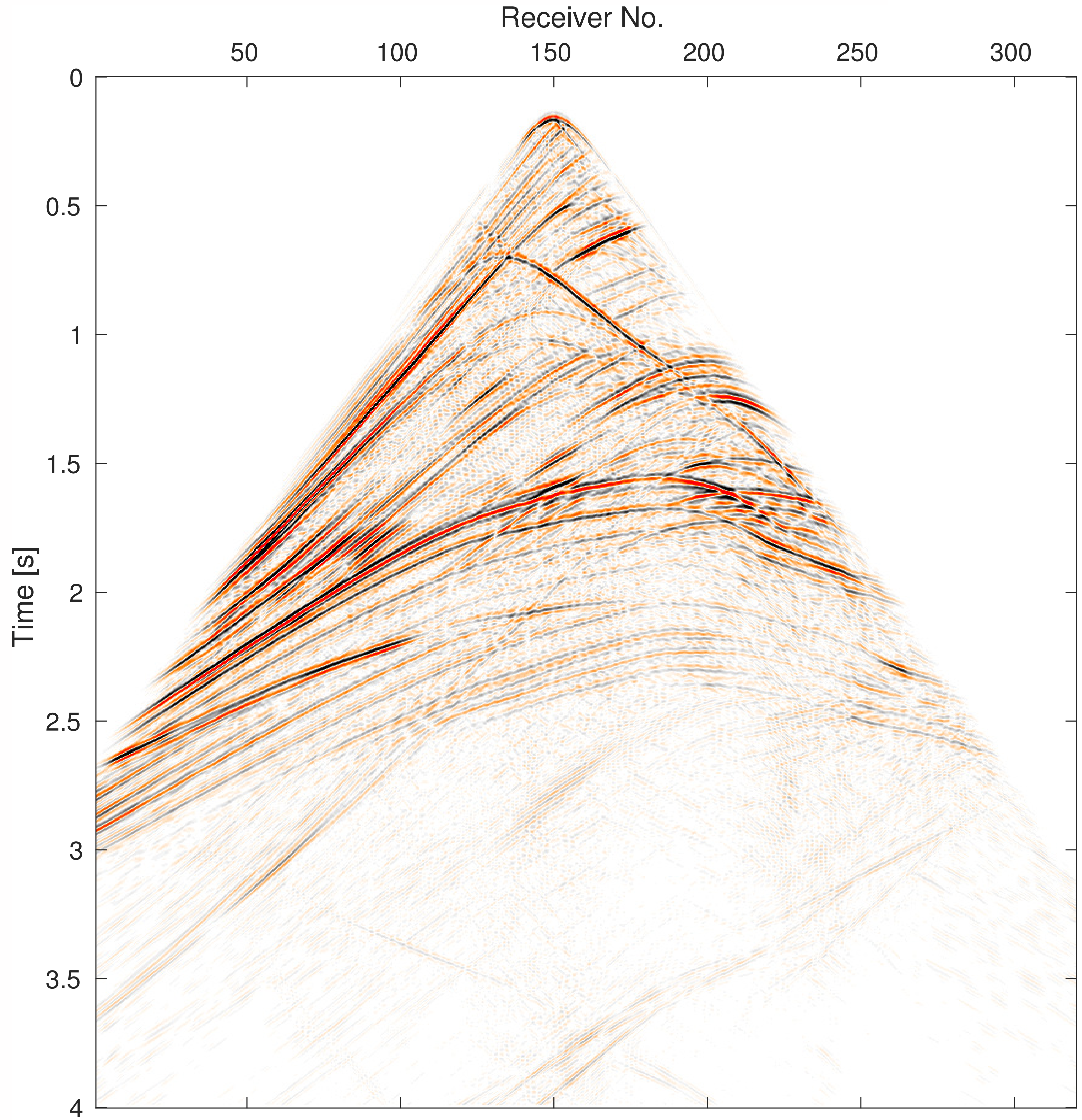
Data-topmute

- mute data outside window of OB reflection
- apply mute at each iteration to observed + synthetic data

SP-LSRTM: Marmousi

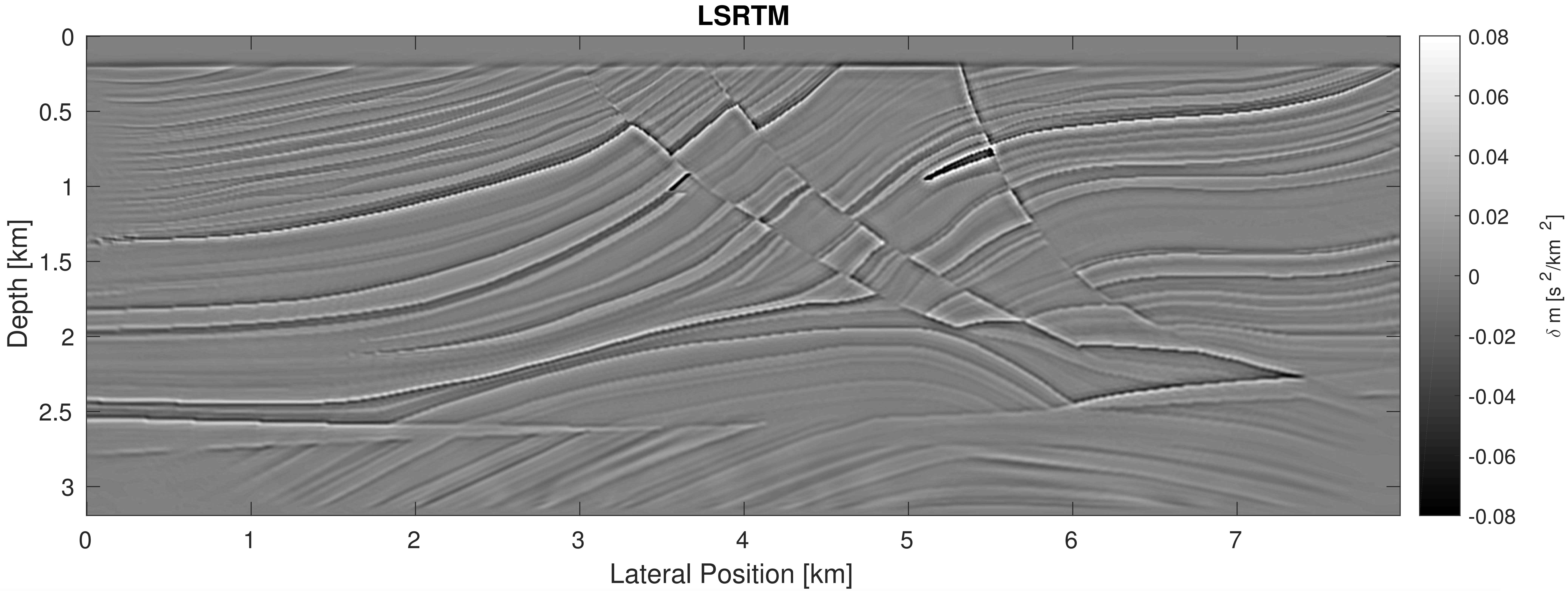


Linearized data



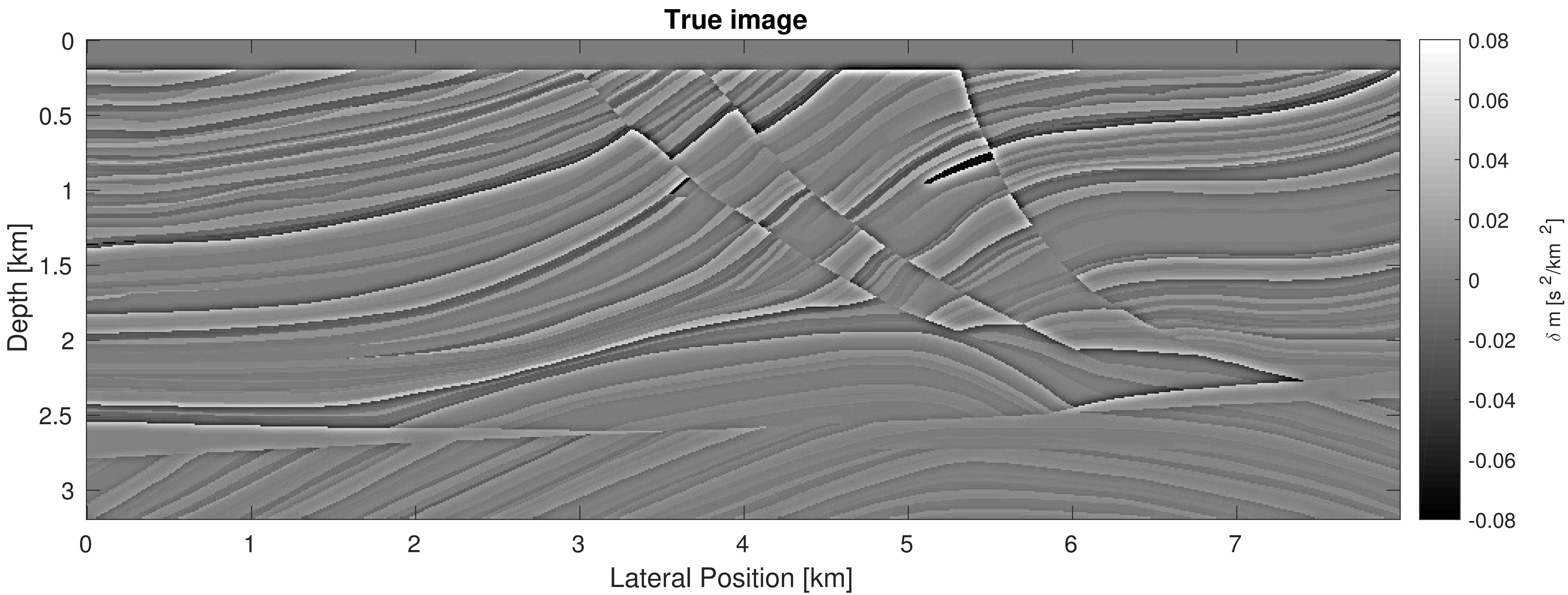
Muted linearized data

SP-LSRTM: Marmousi



Result after 40 iterations with data topmute

SP-LSRTM: Marmousi



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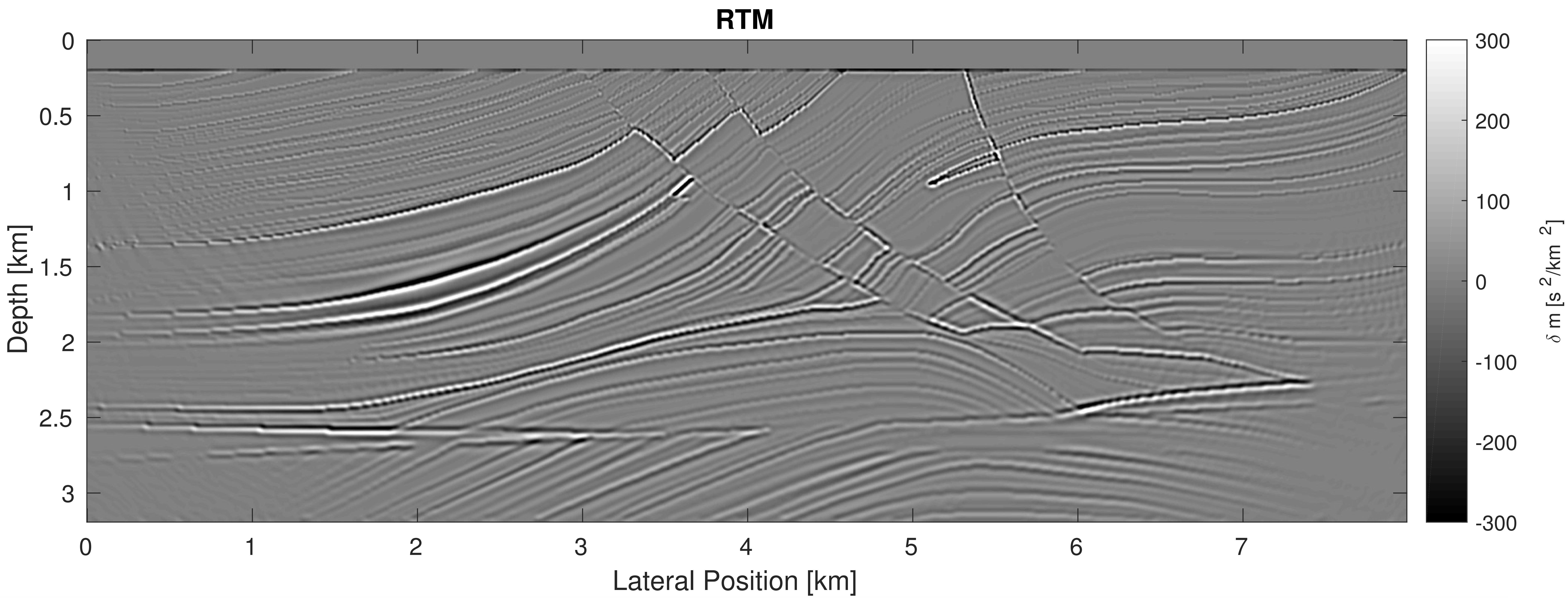

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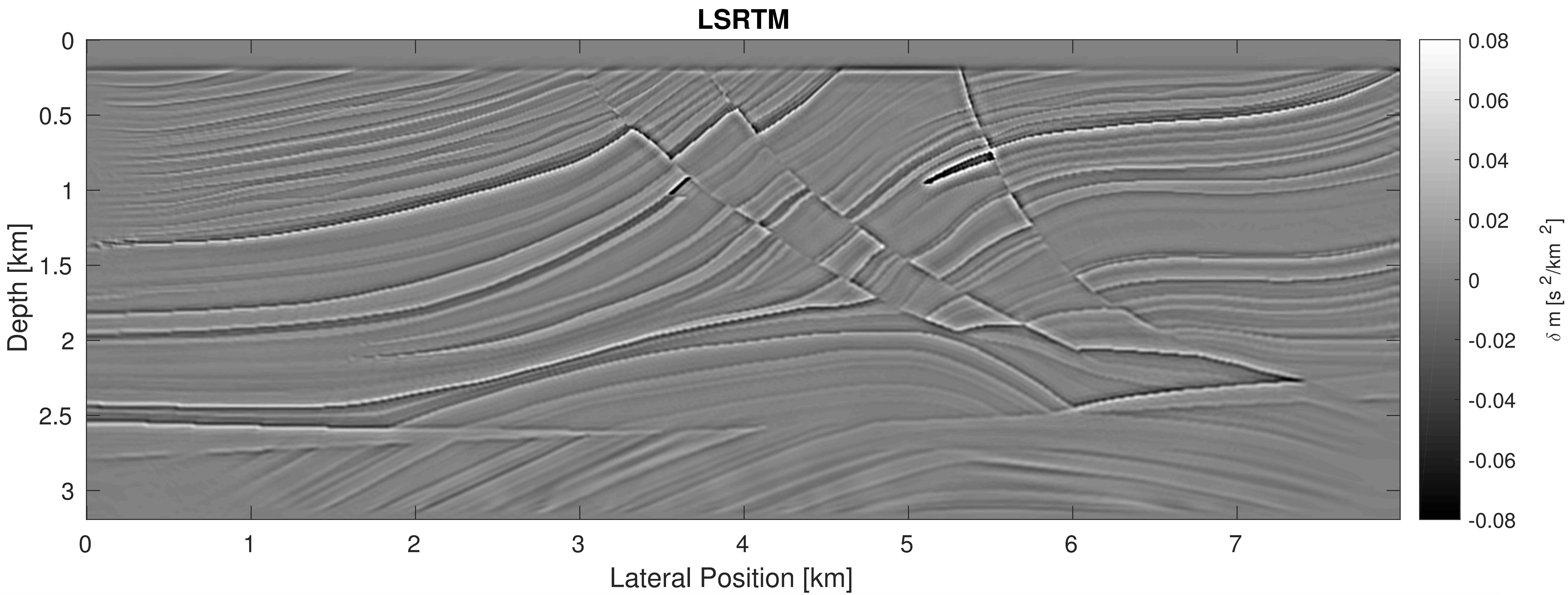
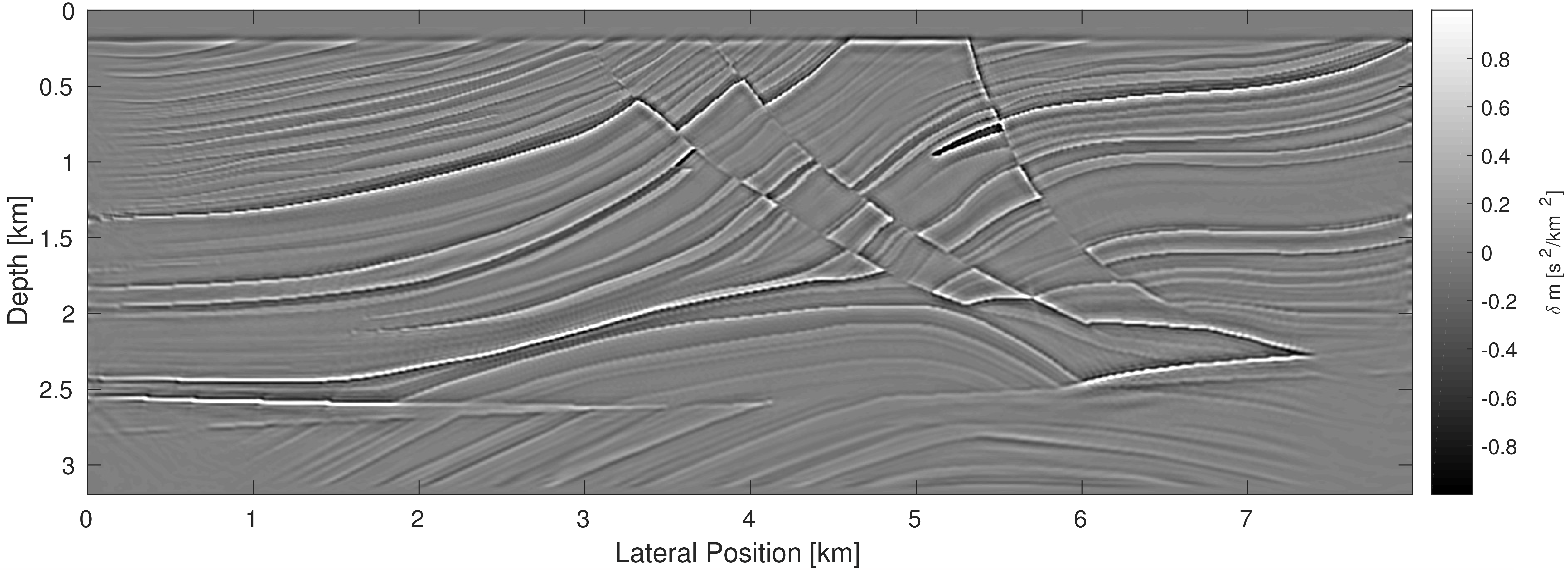


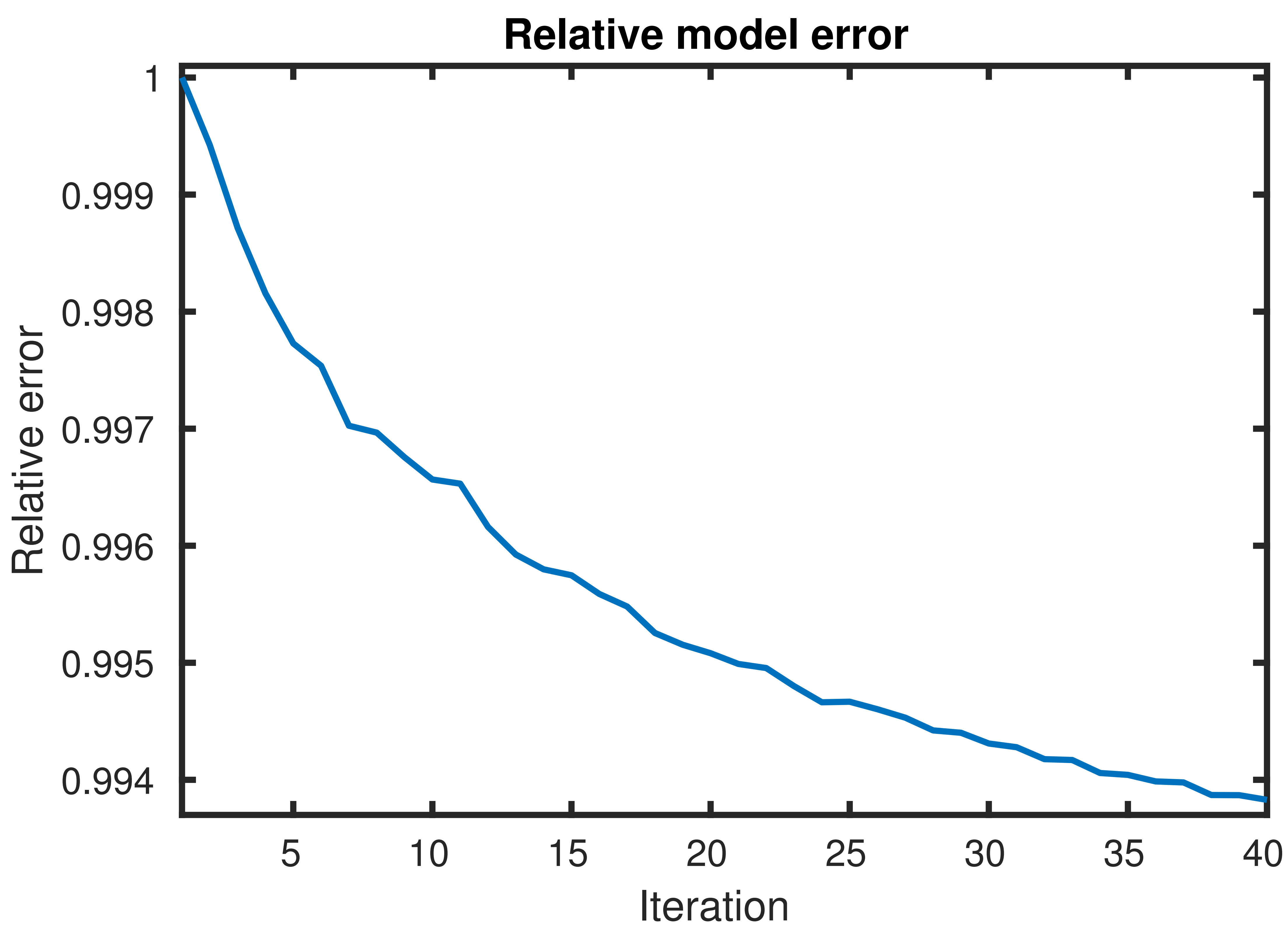
Inverse crime

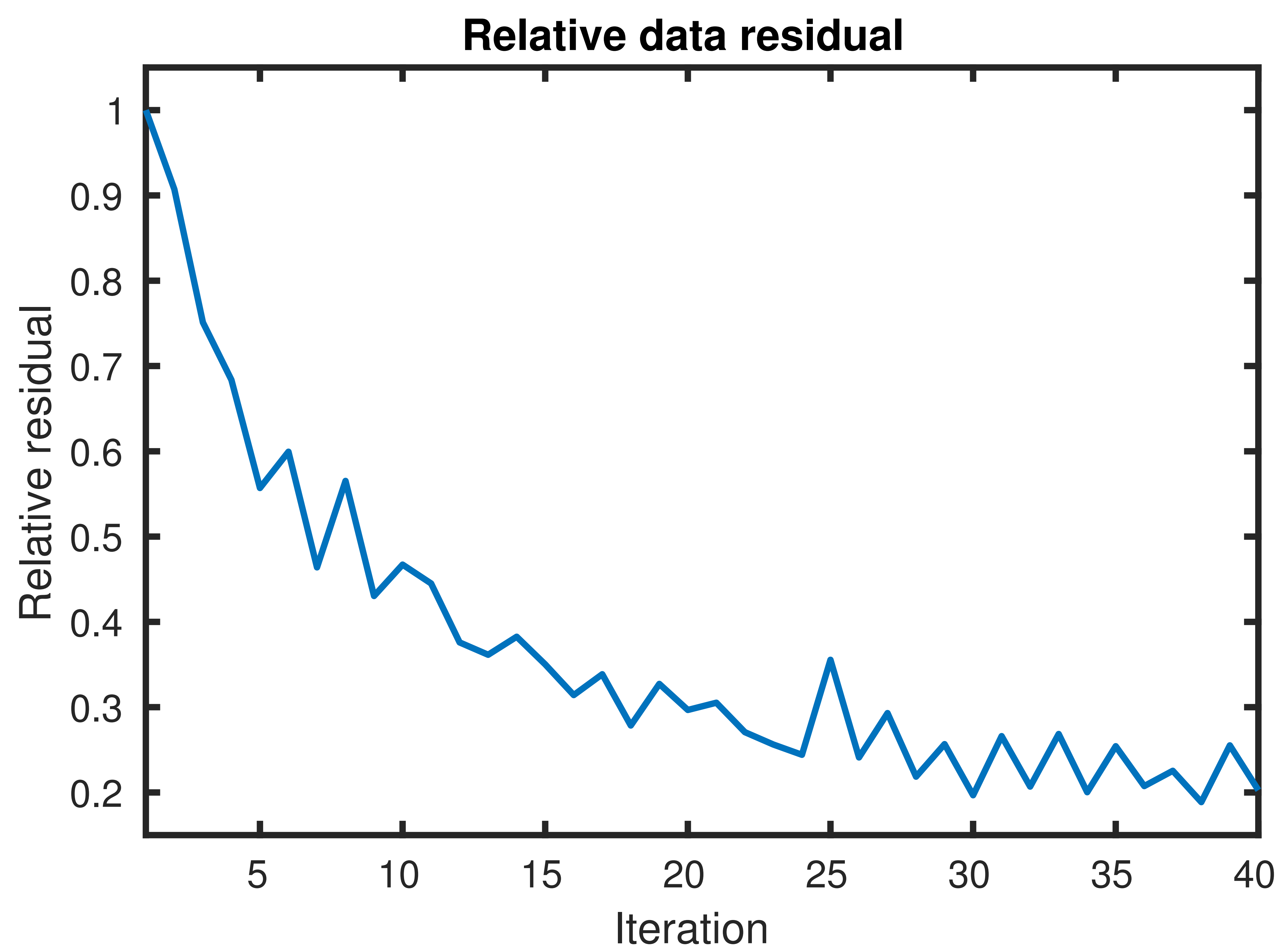
SP-LSRTM: Marmousi



Non-inverse crime (observed data modeled with i-wave)

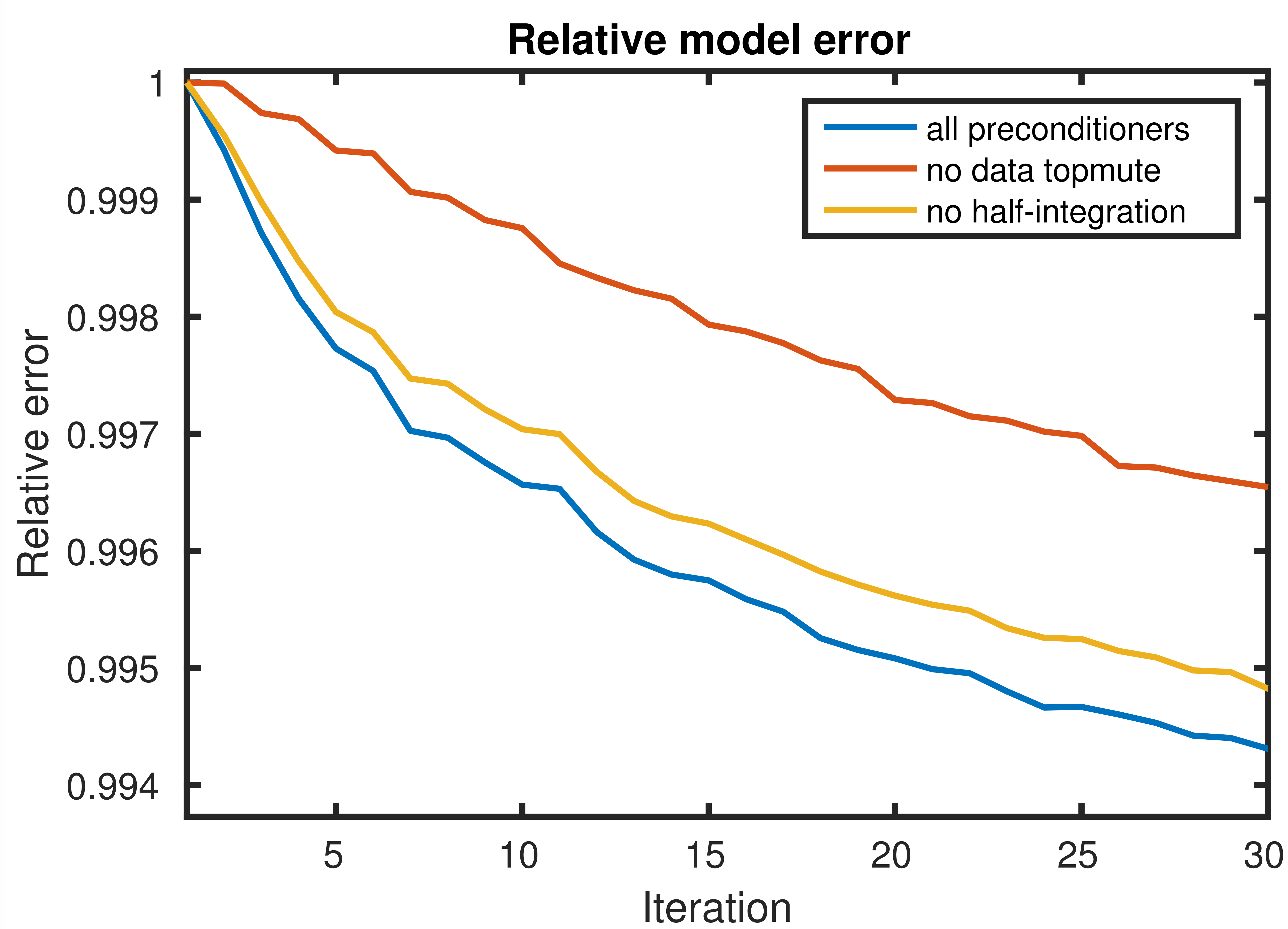
SP-LSRTM: Marmousi





SP-LSRTM: Marmousi

Influence of pre-conditioners on model error:



SP-LSRTM examples w/ linearized data

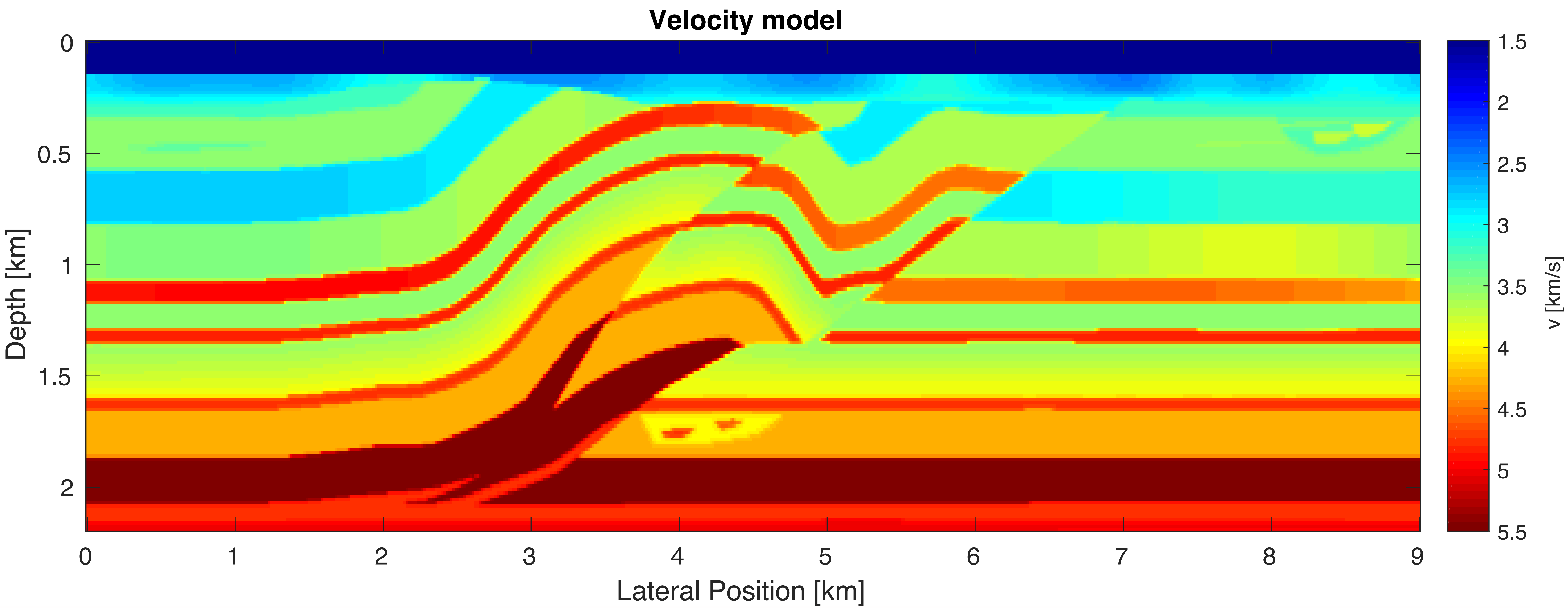
Overthrust model

- 360 shots, 4 seconds recording time
- 30 Hz Ricker wavelet
- 25 m source spacing
- OBNs with 10 m receiver spacing

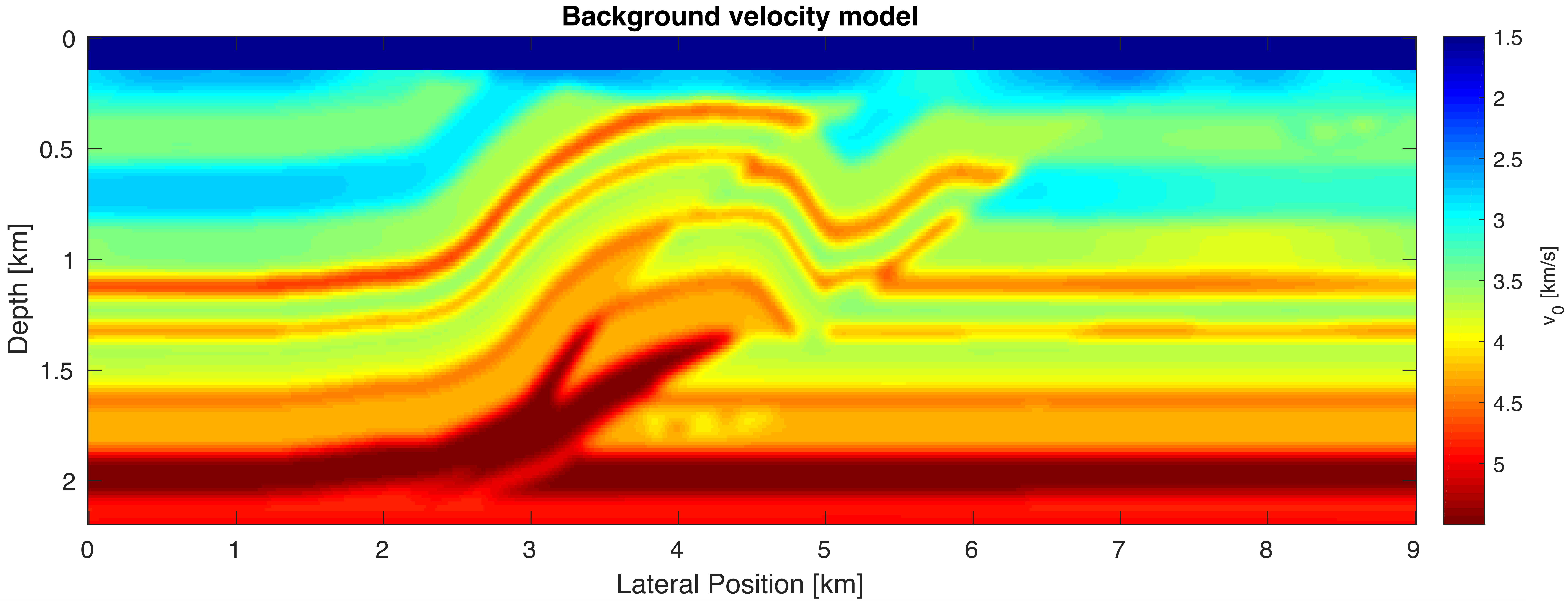
Inversion parameters

- 30 iterations
- 16 shots per iteration (1.3 data passes)
- linearized observed data (inversion crime)

SP-LSRTM: Overthrust



SP-LSRTM: Overthrust



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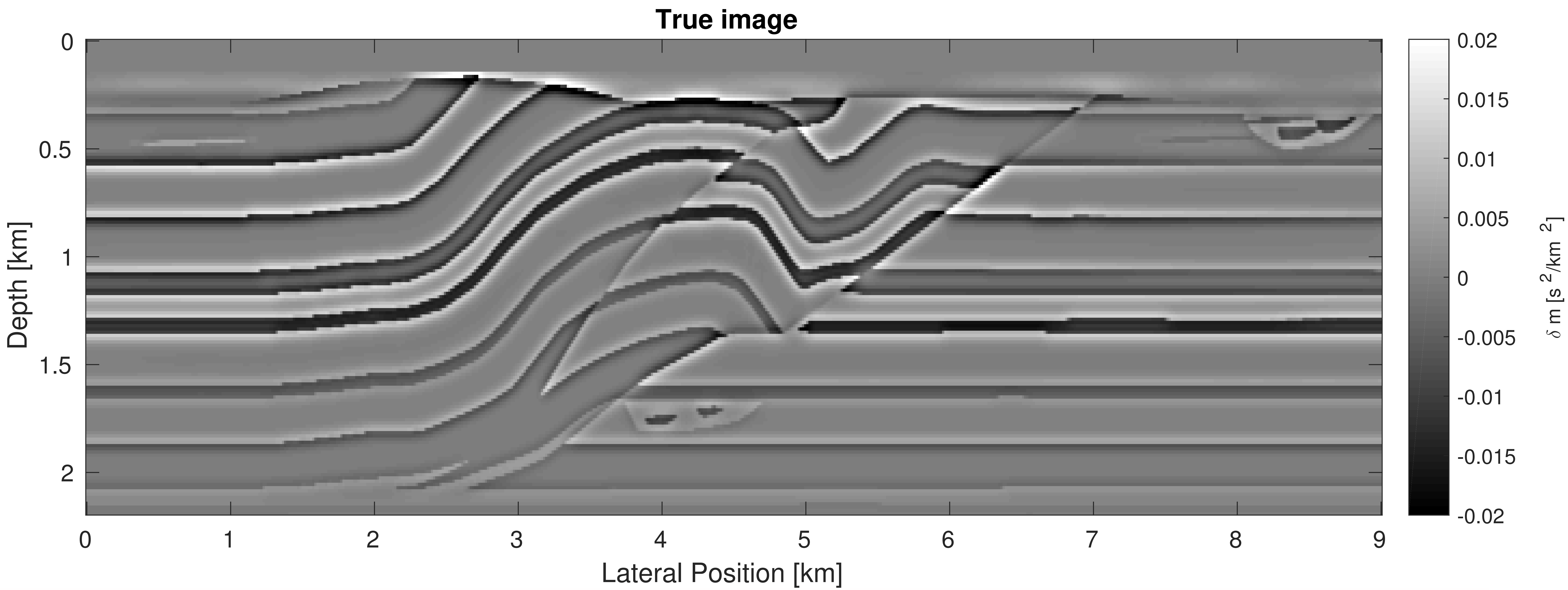
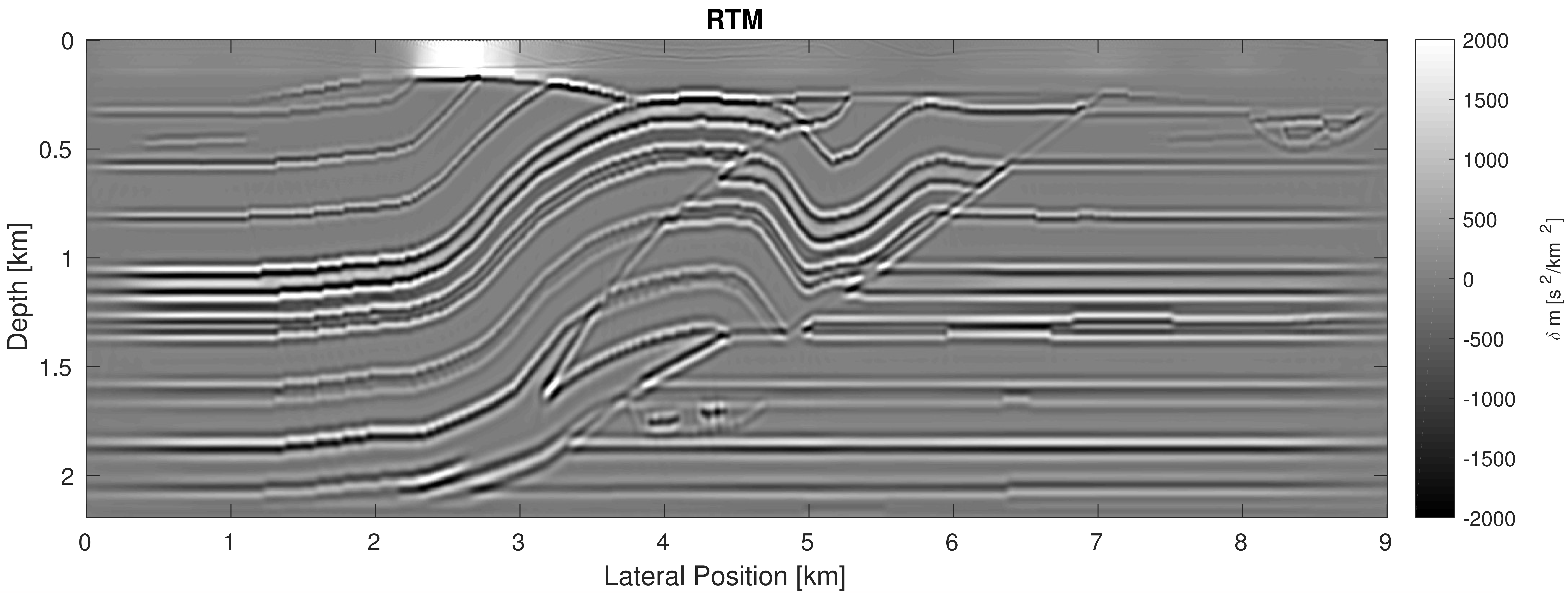


SP-LSRTM: Overthrust



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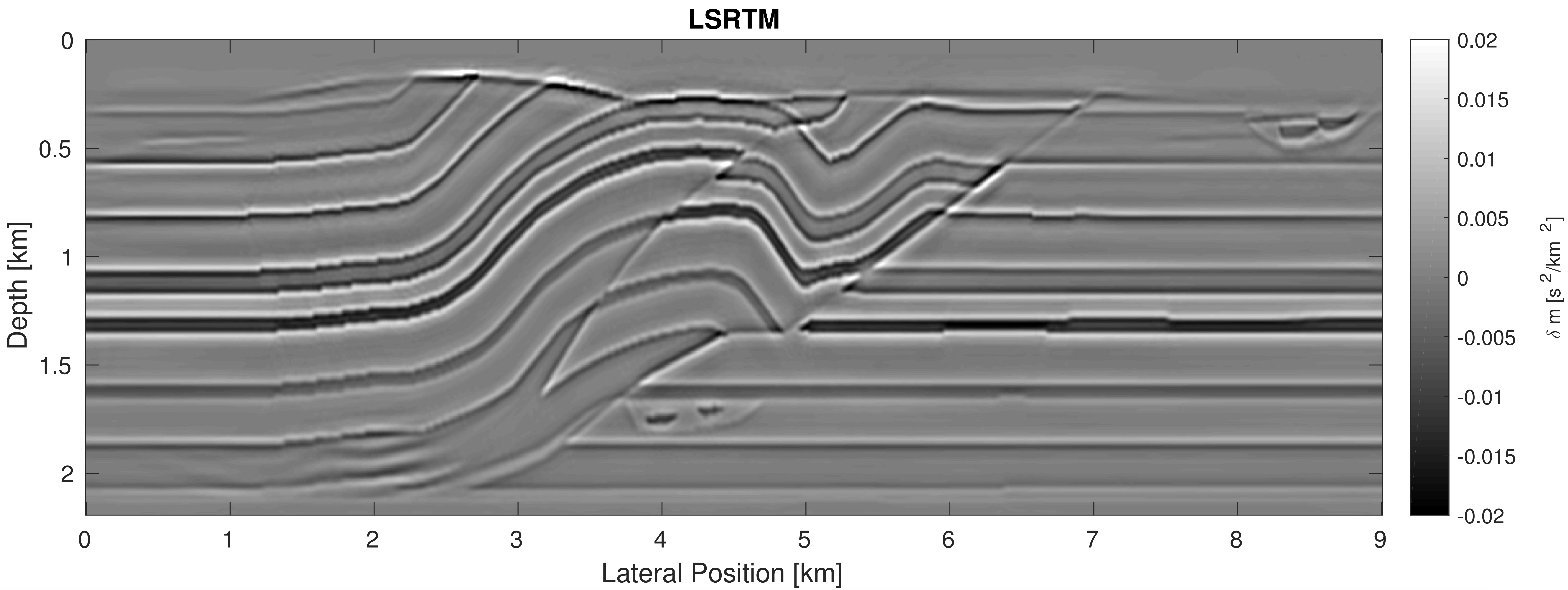

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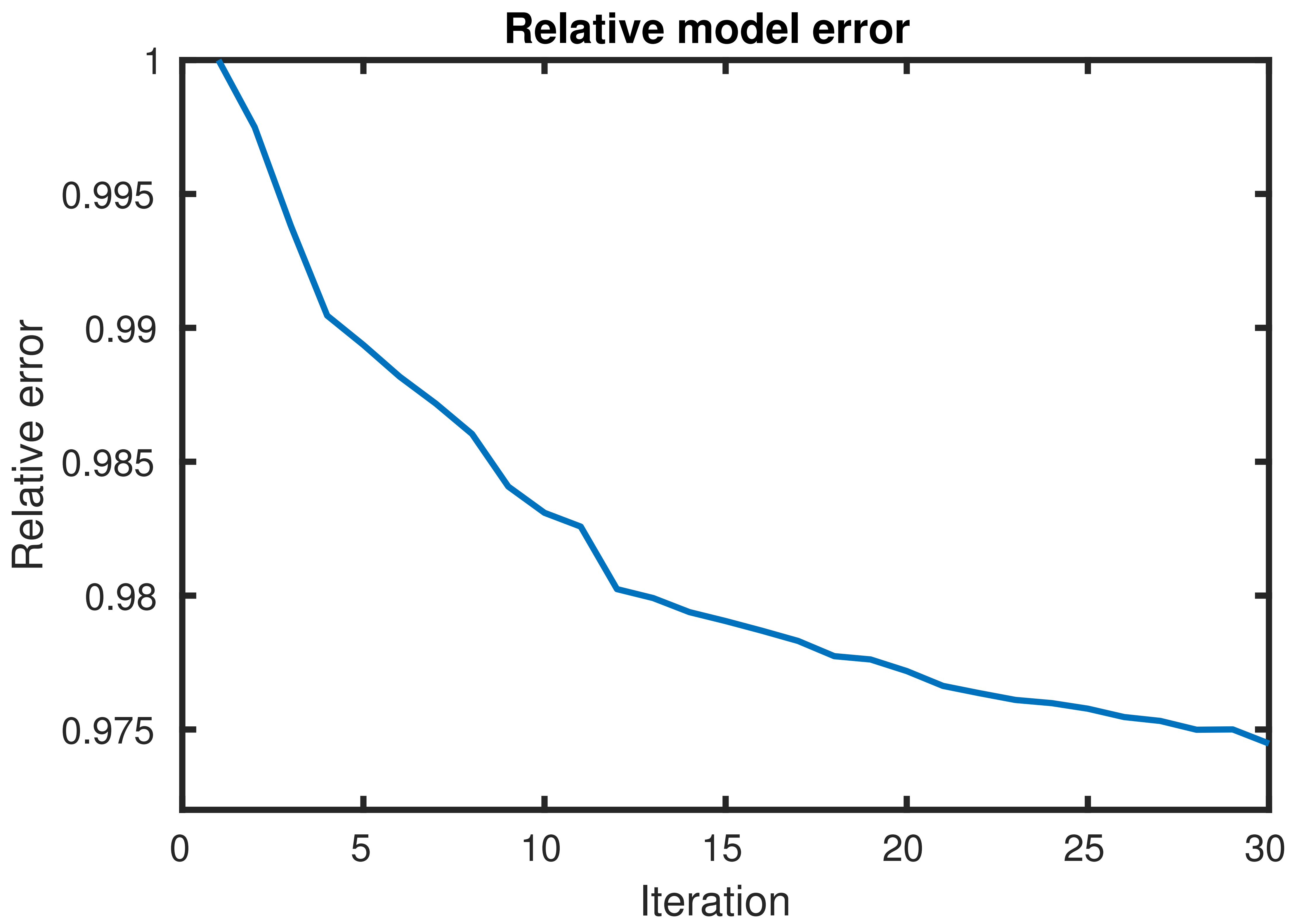

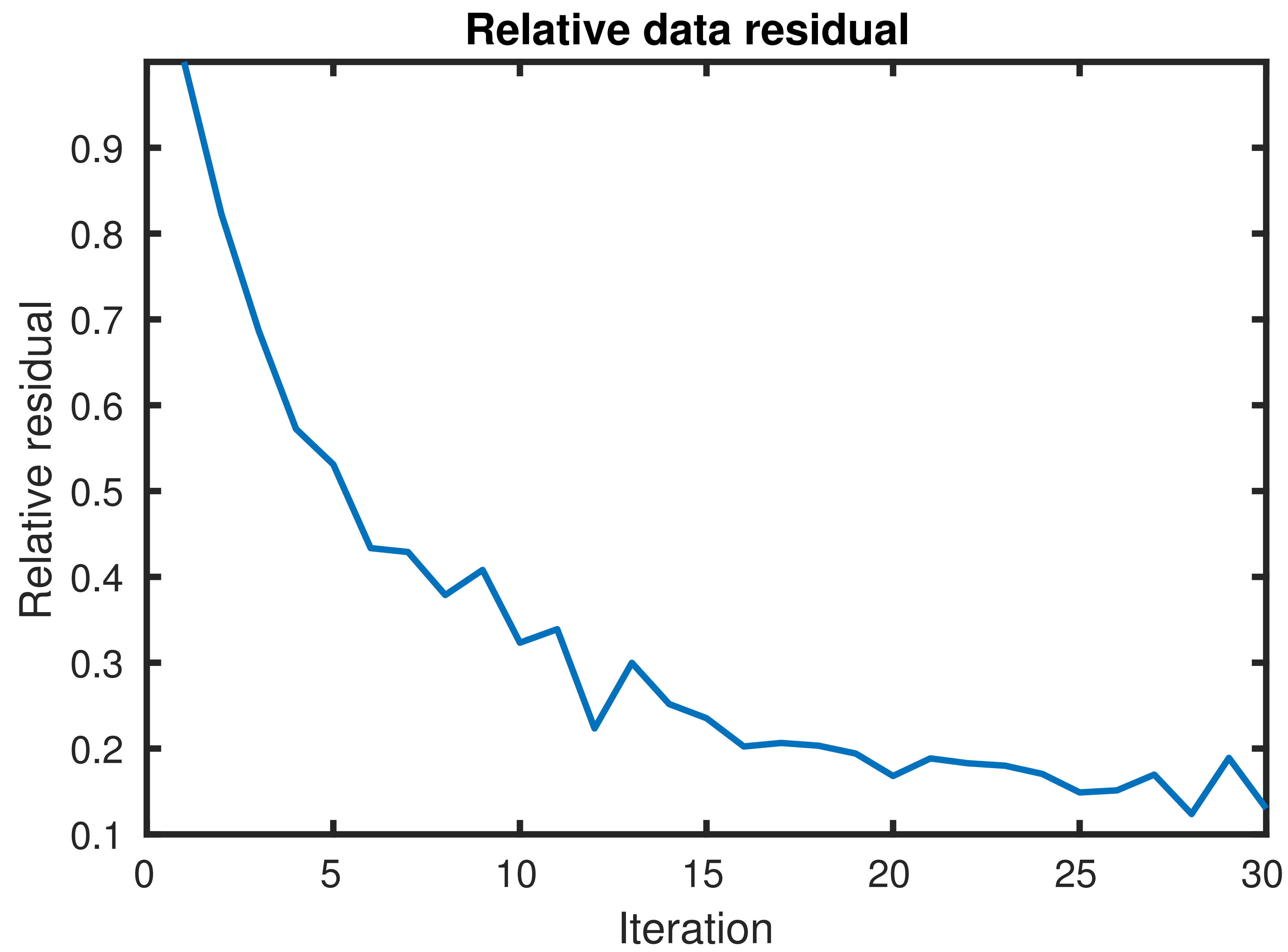

SP-LSRTM with non-linearized data

Sparsity promoting LSRTM with linearized data:

- inverted image close to true image
- noticeable improvement compared to RTM

How does our algorithm behave for non-linearized data?

- Amplitudes of observed data and modeled linearized data can never match

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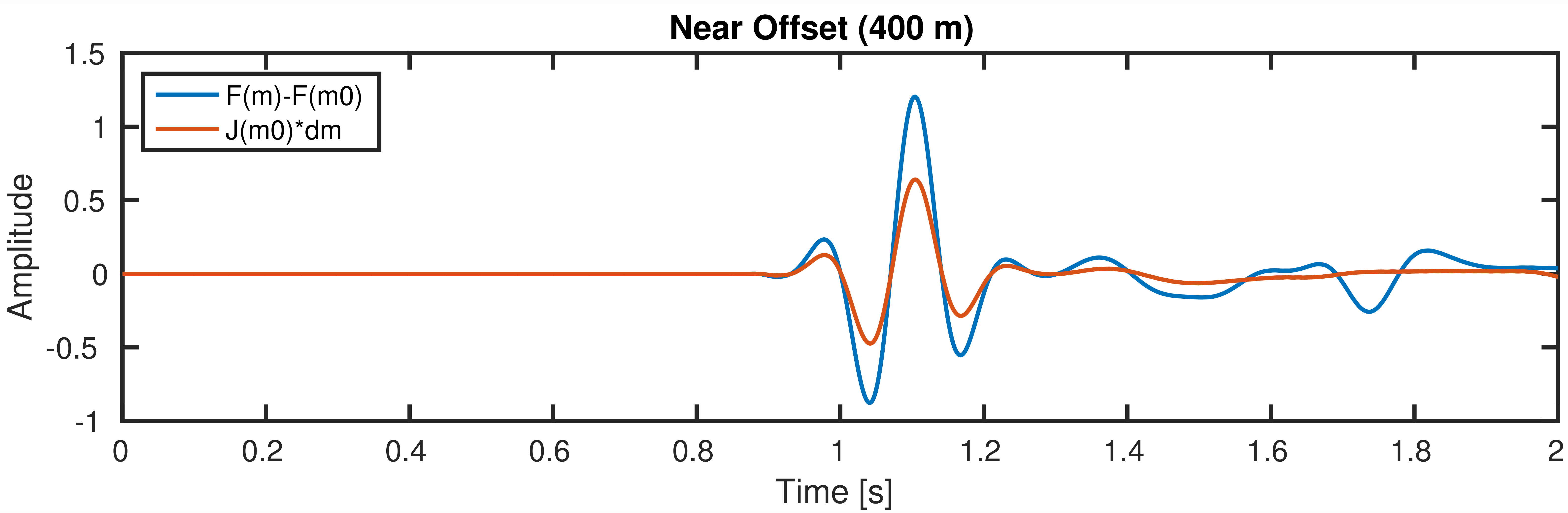

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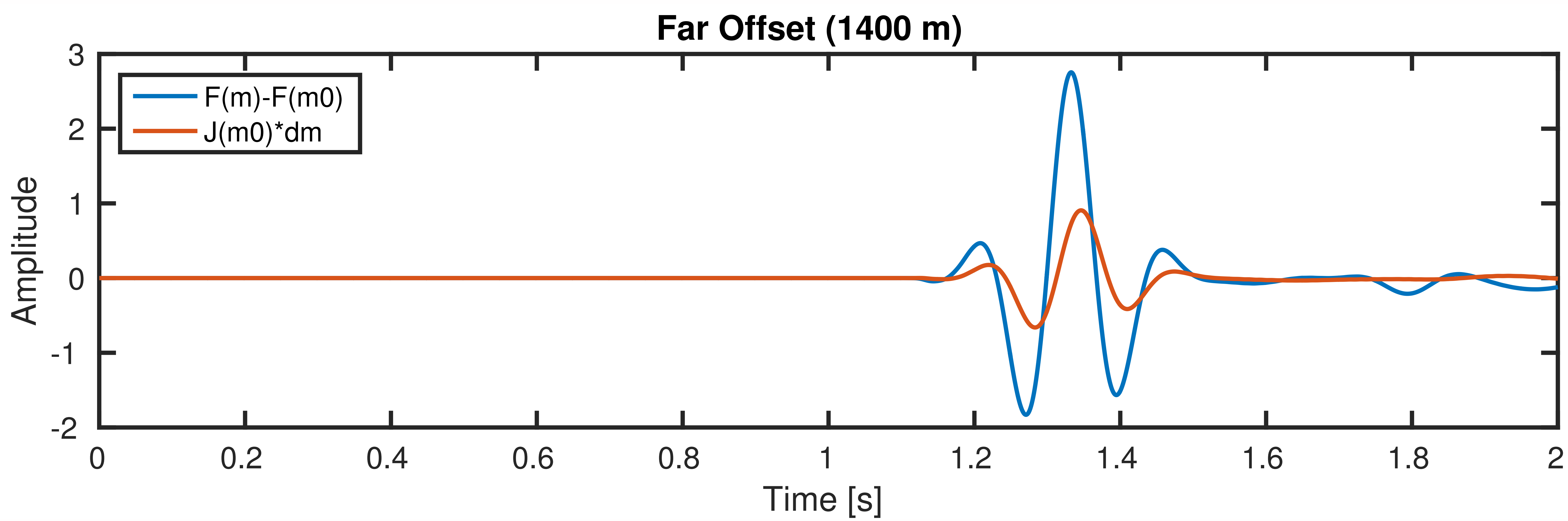

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How does our algorithm behave for non-linearized data?

- Amplitudes of observed data and modeled linearized data can never match

What is the influence of having a correct Jacobian pair?

```
for j=1:maxiter

    # predicted data
    d_pred = J*dm;

    # gradient
    g = J'*(d_pred - d_obs)

    # update image
    dm = dm - t*g

end
```

versus

```
for j=1:maxiter

    # predicted data
    d_pred = model(m0 + dm);

    # gradient
    g = rtm(d_pred - d_obs)

    # update image
    dm = dm - t*g

end
```

SP-LSRTM algorithm w/o Jacobians:

1. **for** $k = 0, 1, \cdots$
2. Demigration:
   $$\delta \mathbf{d}_k = \mathbf{M}_L^{-1} \mathbf{F}_{r(k)} (\mathbf{m}_0 + \mathbf{M}_R^{-1} \mathbf{x}_k) - \mathbf{M}_L^{-1} \mathbf{b}_{r(k)}$$
3. Migration of data residual:
   $$\delta \mathbf{m}_k = (\mathbf{M}_R^{-1})^T \mathbf{F}_{r(k)}^T \Big((\mathbf{M}_L^{-1})^T \delta \mathbf{d}_k \Big)$$
4. Vertical derivative:
   $$\delta \mathbf{m}_k = \mathbf{D}_z \delta \mathbf{m}_k$$
5. Update z:
   $$\mathbf{z}_{k+1} = \mathbf{z}_k - t_k \cdot \delta \mathbf{m}_k$$
6. Update x:
   $$\mathbf{x}_{k+1} = \mathbf{C}^* S_\lambda (\mathbf{C} \mathbf{z}_{k+1})$$
7. Bound projections:
8. $(\mathbf{z}_{k+1}, \mathbf{x}_{k+1}) = \mathcal{P}_{Breg} (\mathbf{z}_{k+1}, \mathbf{x}_{k+1})$
9. **end for**

Projection operator

Projection operator for bound constraints w/ linearized Bregman:

$$(\tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = \mathcal{P}_{Breg}(\mathbf{z}, \mathbf{x})$$

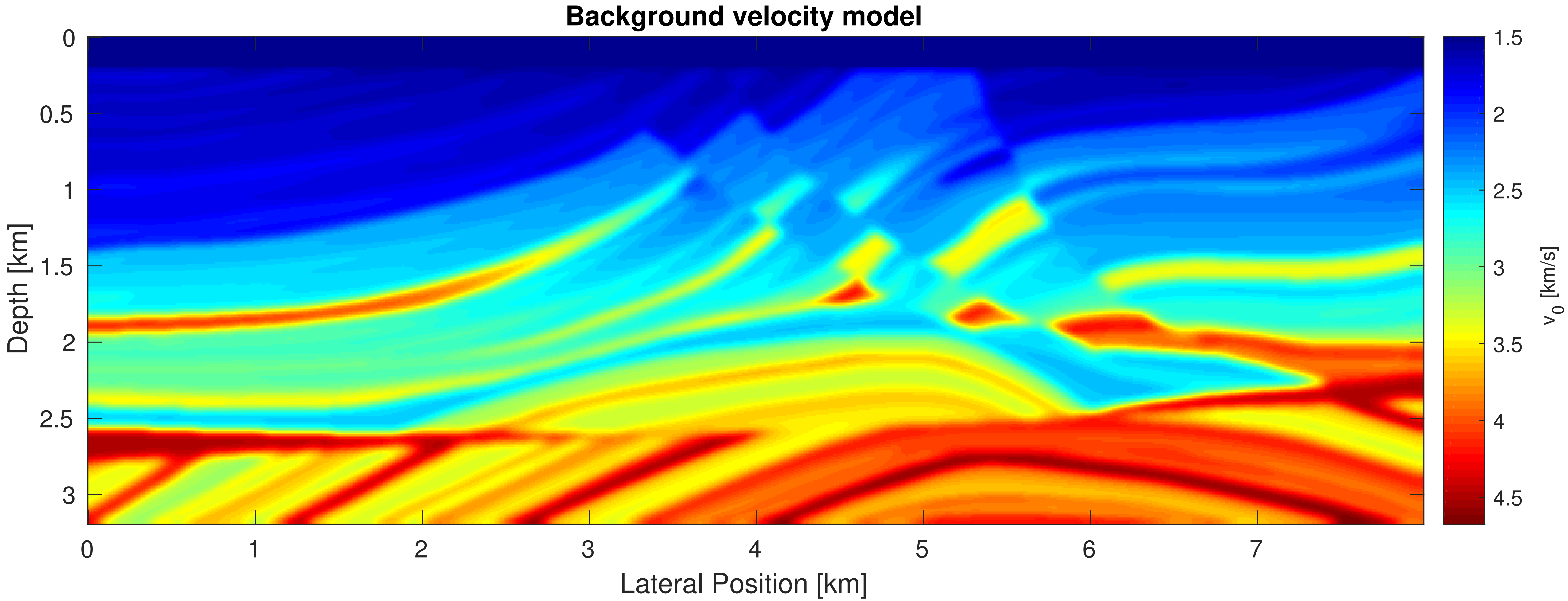
$$\tilde{\mathbf{x}} = \mathcal{P}_B\Big(S_\lambda(\mathbf{x})\Big) = \text{median}\Big(\mathbf{a}, S_\lambda(\mathbf{x}), \mathbf{b}\Big)$$

$\mathbf{a}$: lower bound

$\mathbf{b}$: upper bound

$$\tilde{\mathbf{z}}_j = \begin{cases} \mathbf{z}_j & \mathbf{a}_j \leq S_\lambda(\tilde{\mathbf{x}})_j \leq \mathbf{b}_j \\ \mathbf{b}_j + \lambda & S_\lambda(\tilde{\mathbf{x}})_j > \mathbf{b}_j \\ \mathbf{a}_j - \lambda & S_\lambda(\tilde{\mathbf{x}})_j < \mathbf{a}_j \end{cases}$$

Influence of correct adjoints



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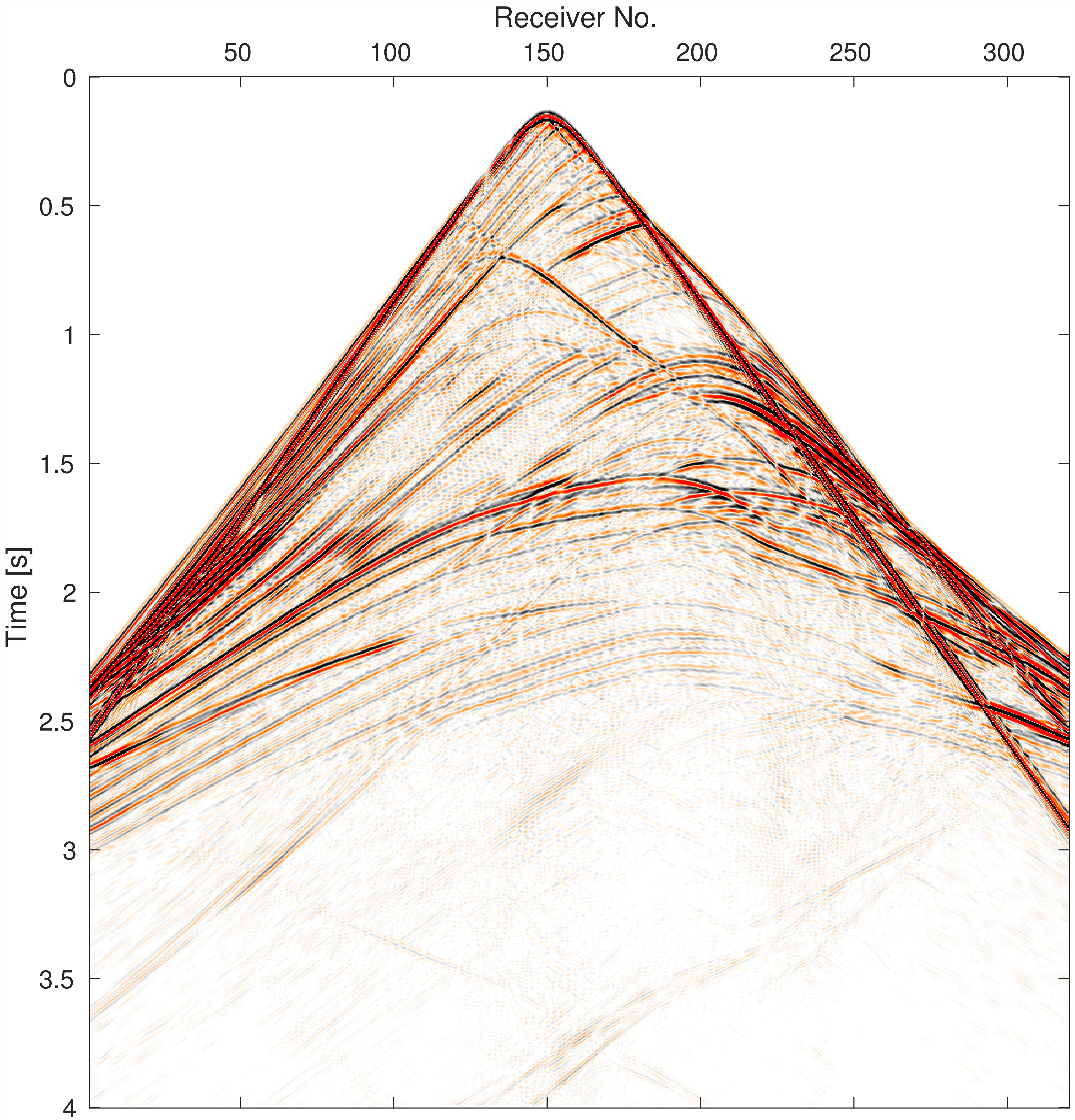
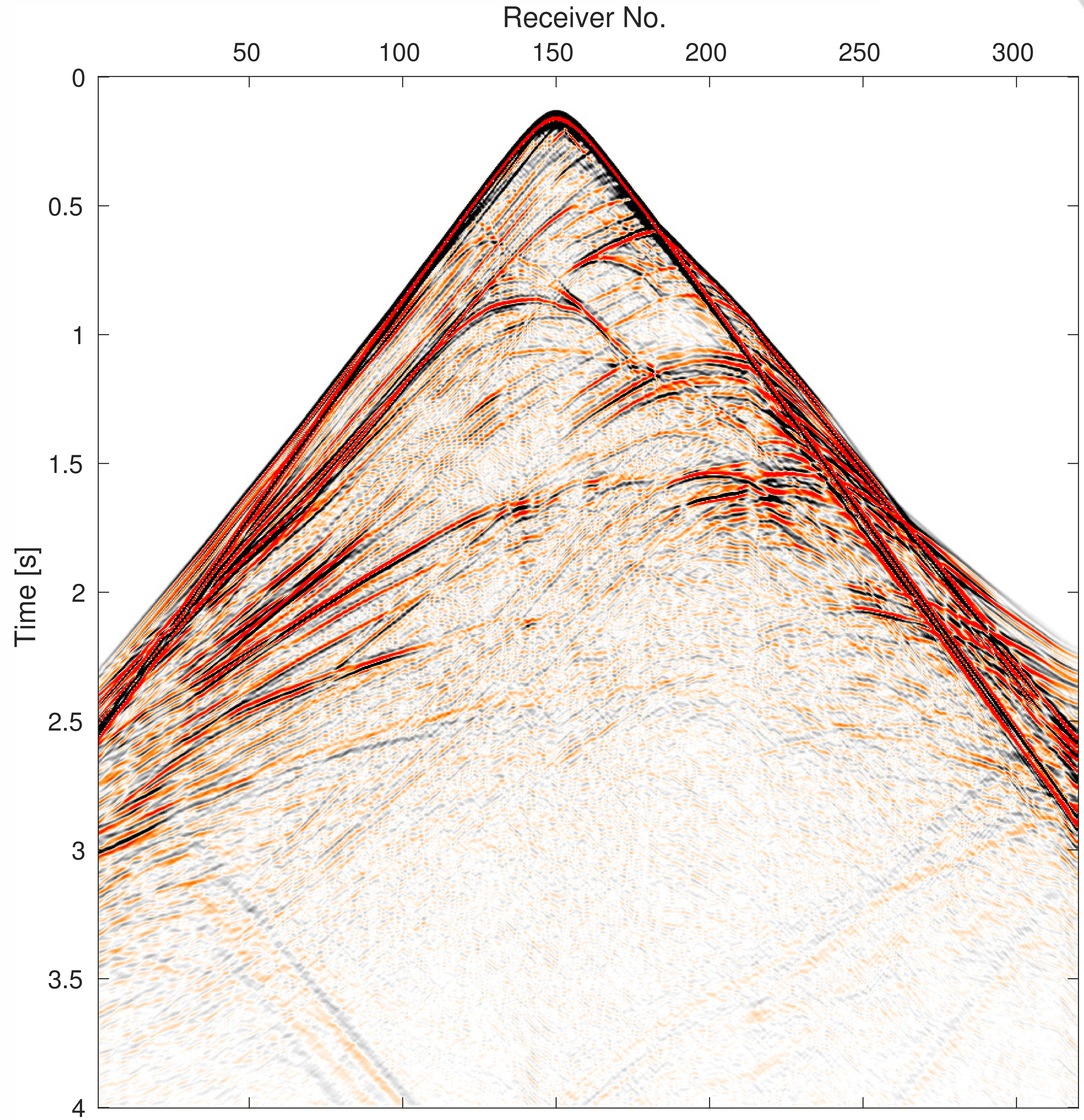
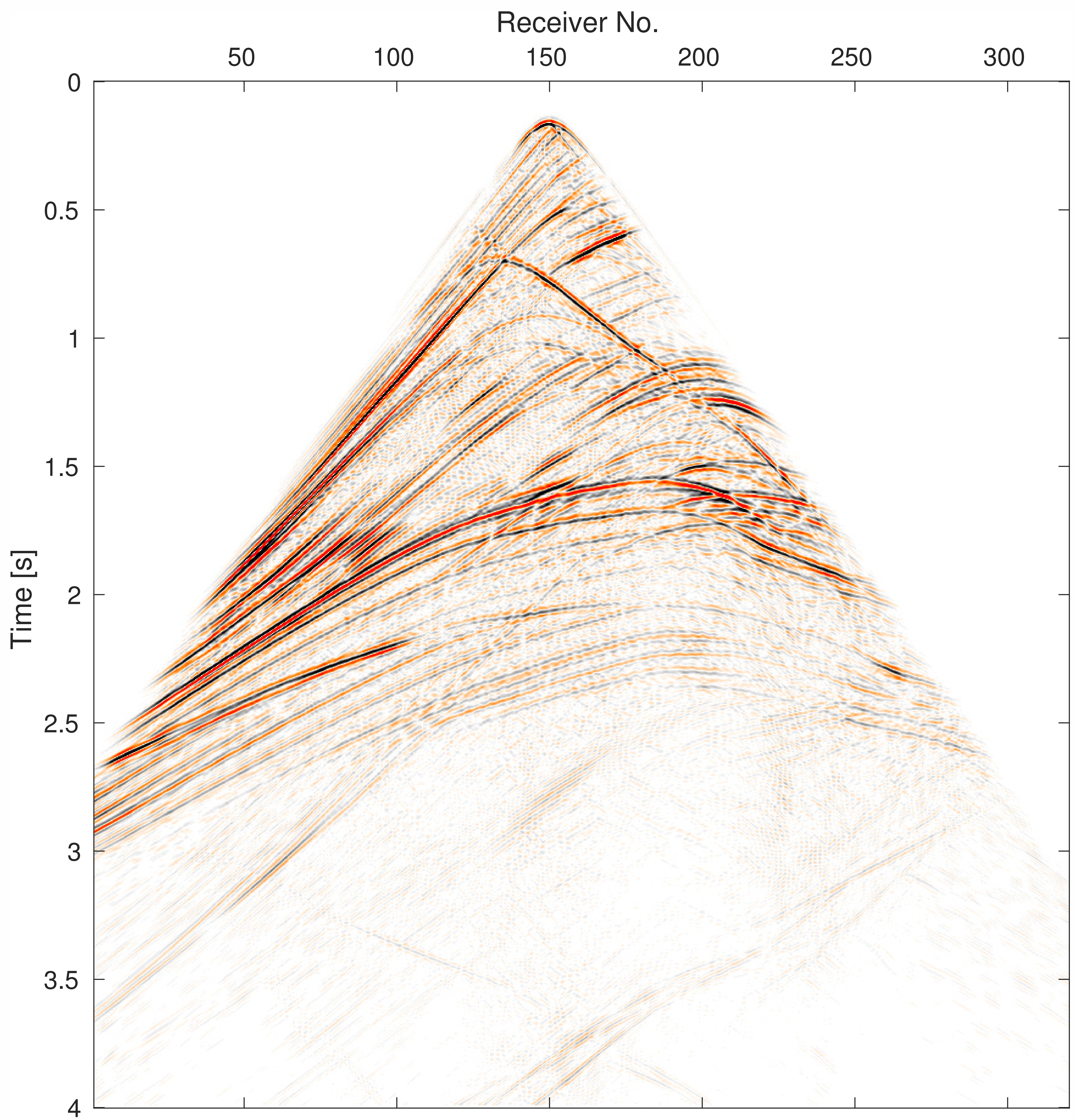


Linearized data

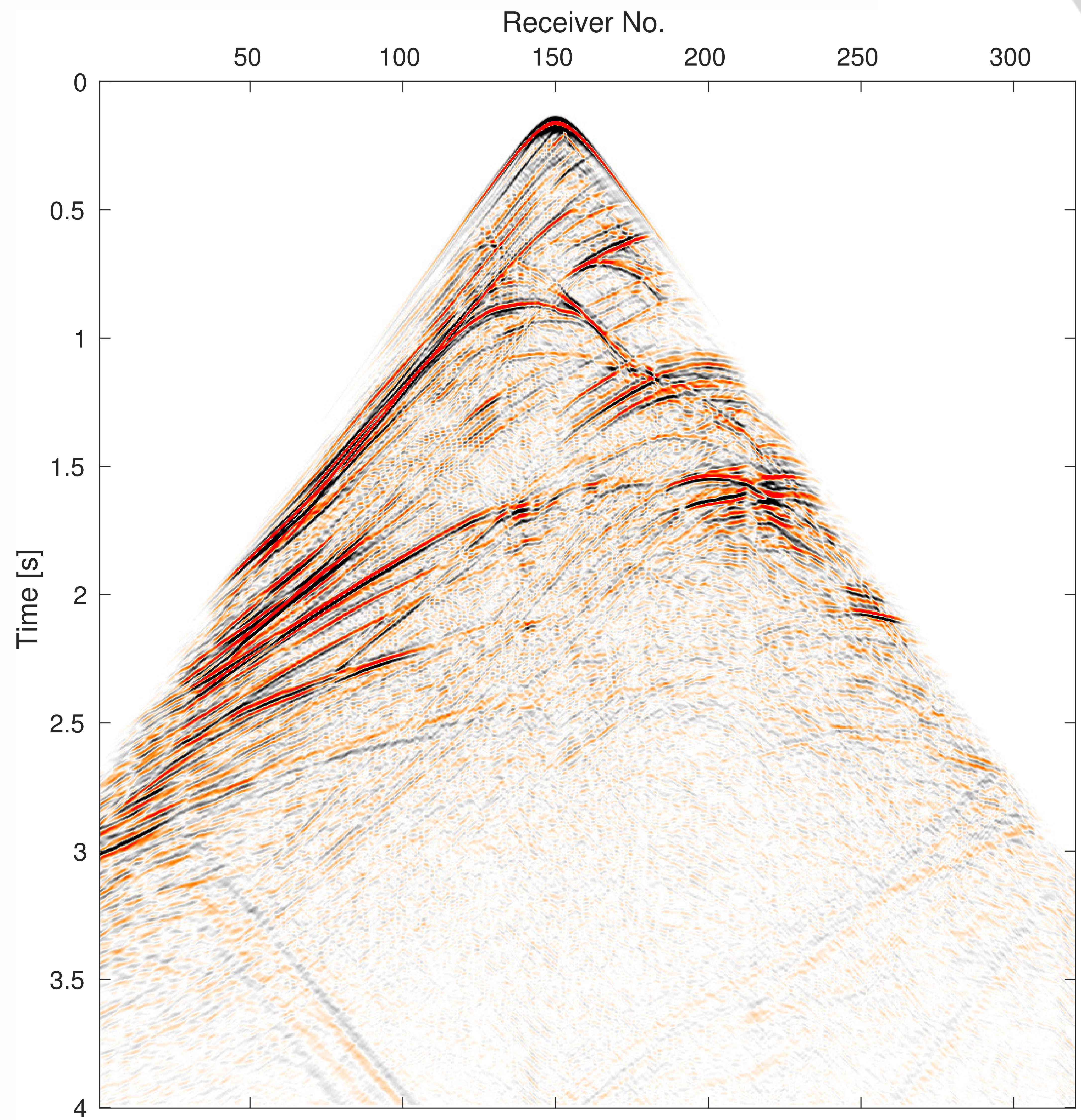


Non-linearized data

Influence of correct adjoints

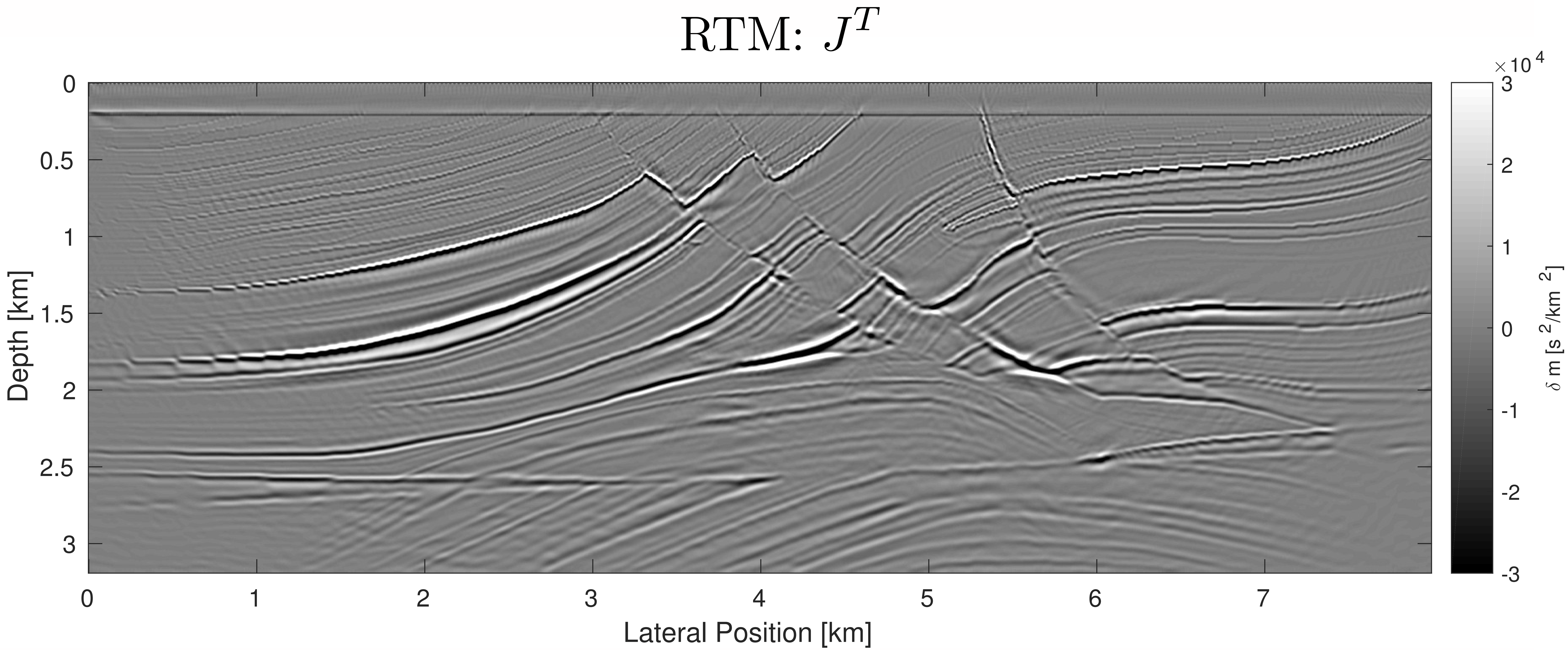


Muted linearized data

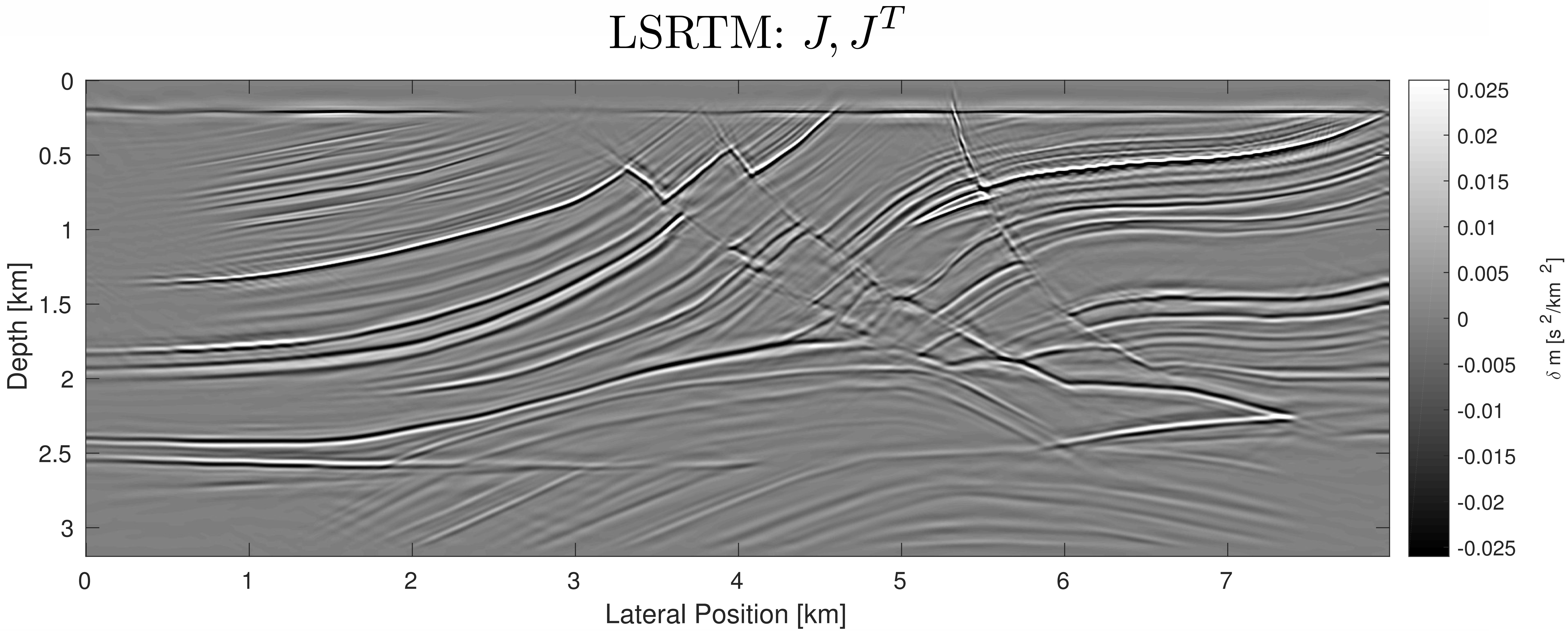


Muted non-linearized data

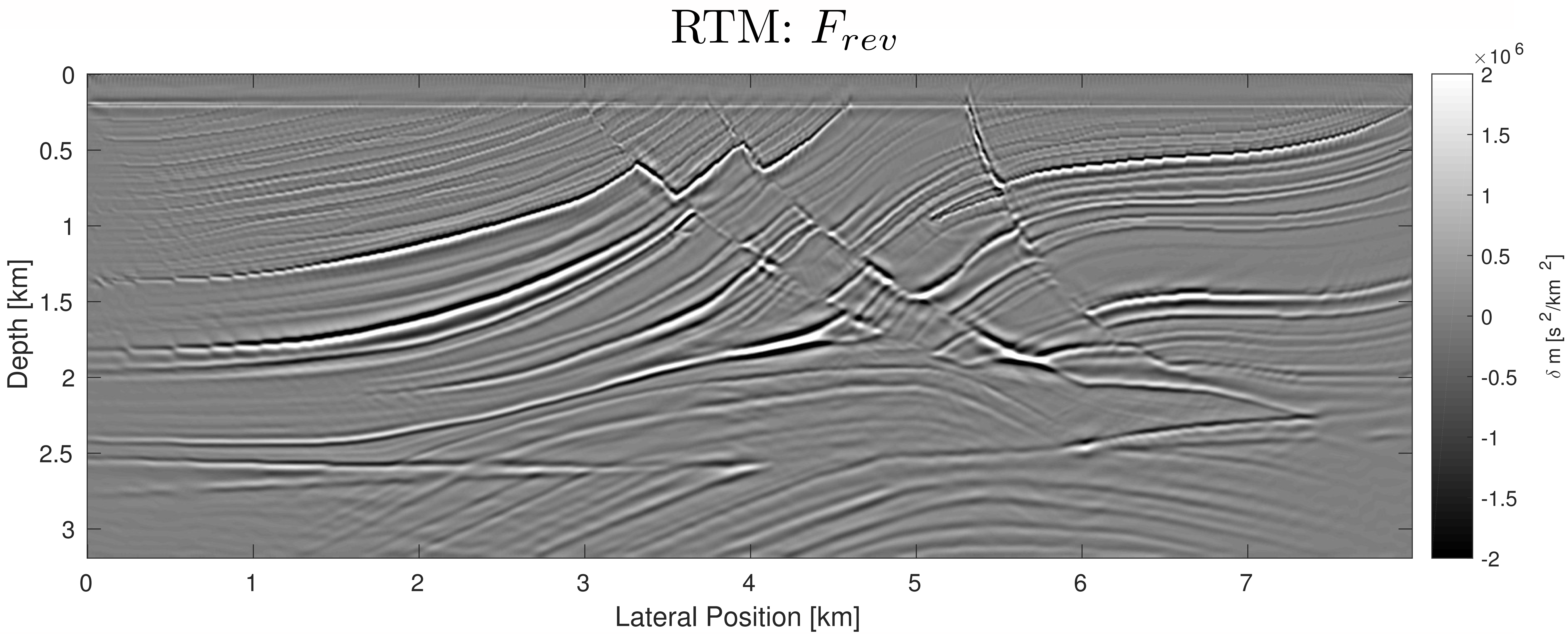
Influence of correct adjoints



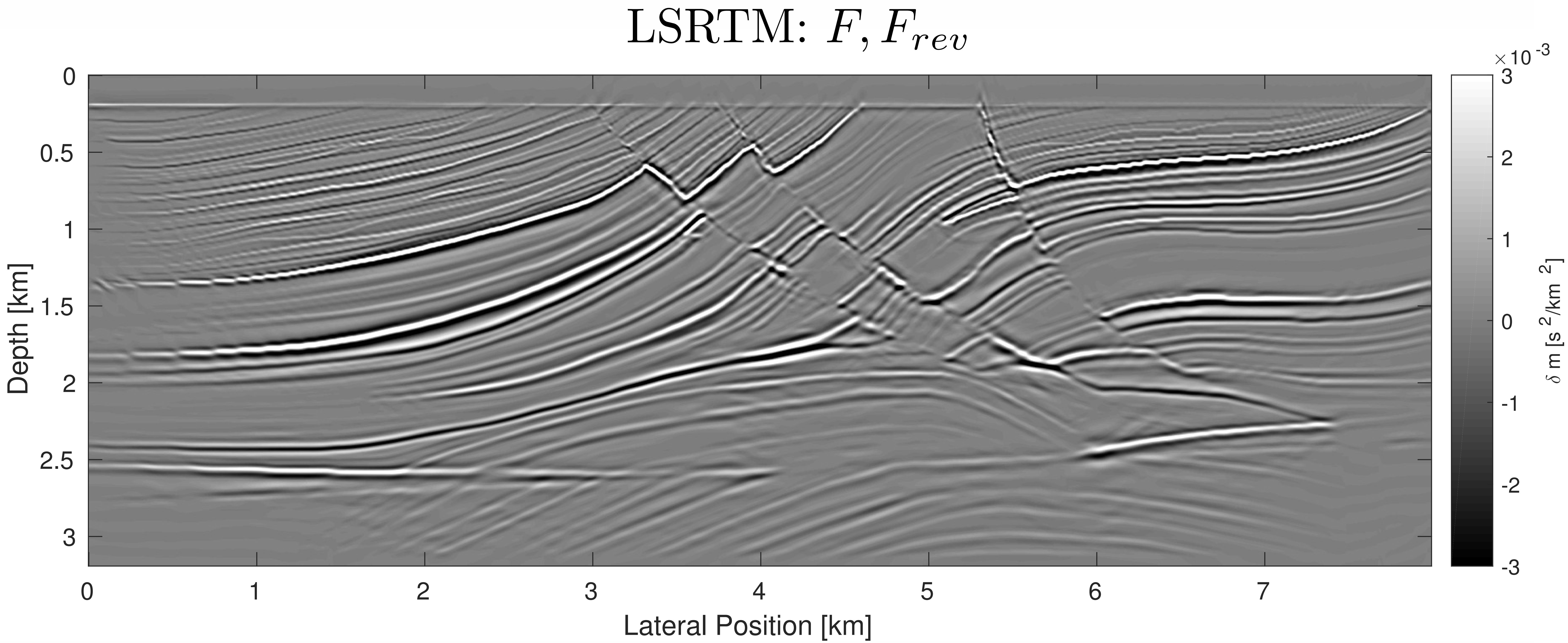
Influence of correct adjoints



Influence of correct adjoints



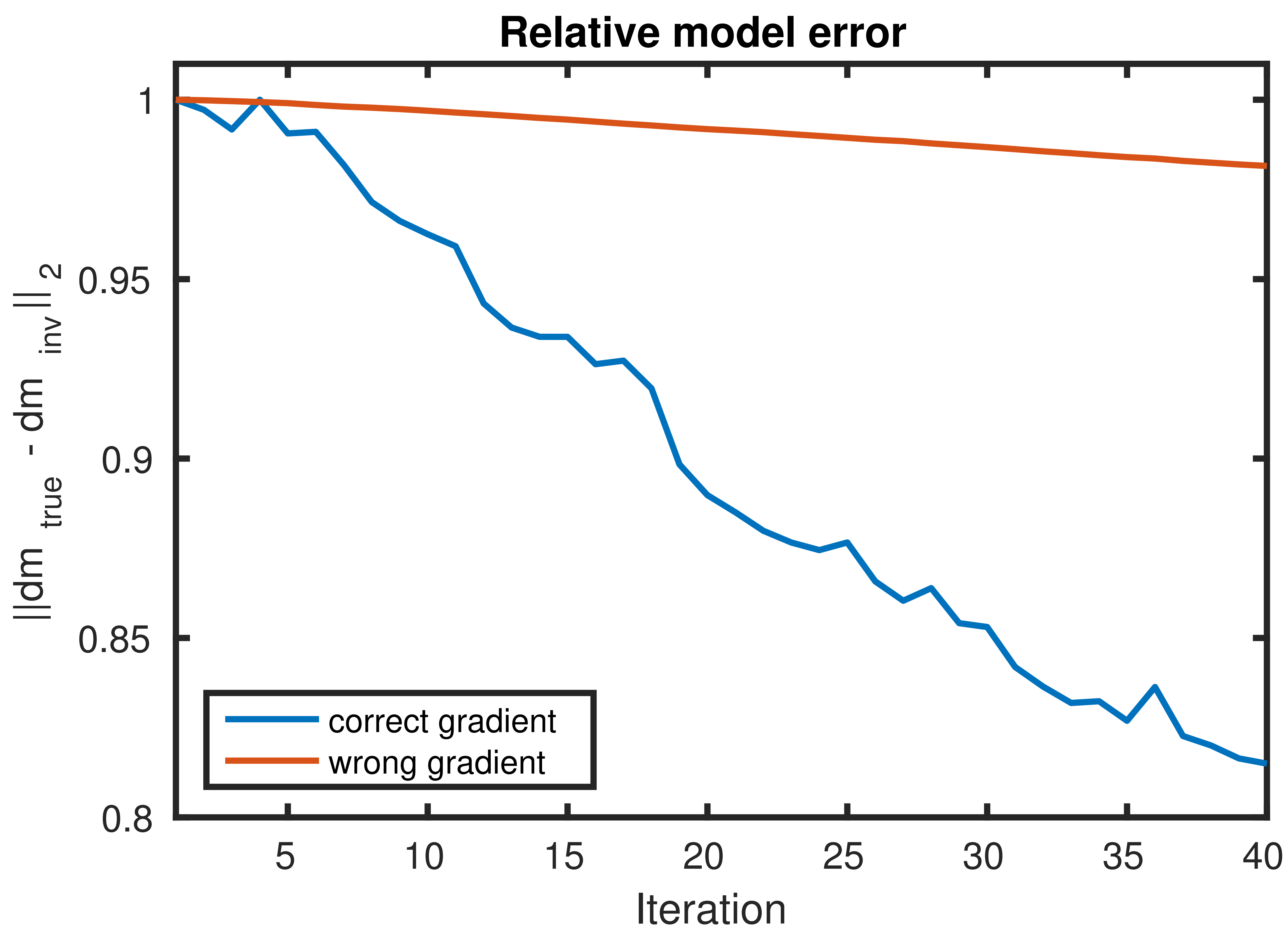
Influence of correct adjoints



Influence of correct adjoints

Sparsity promoting LSRTM w/ correct and incorrect gradient:

- model error decays w/ incorrect gradient
- but: decay w/ correct gradient is much faster



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Field data example

Sparsity promoting least squares RTM on BP Machar field data set:

- Machar oil field in North sea
- 330 shots w/ 8 seconds recording time
- maximum no. of 505 receivers (OBN)

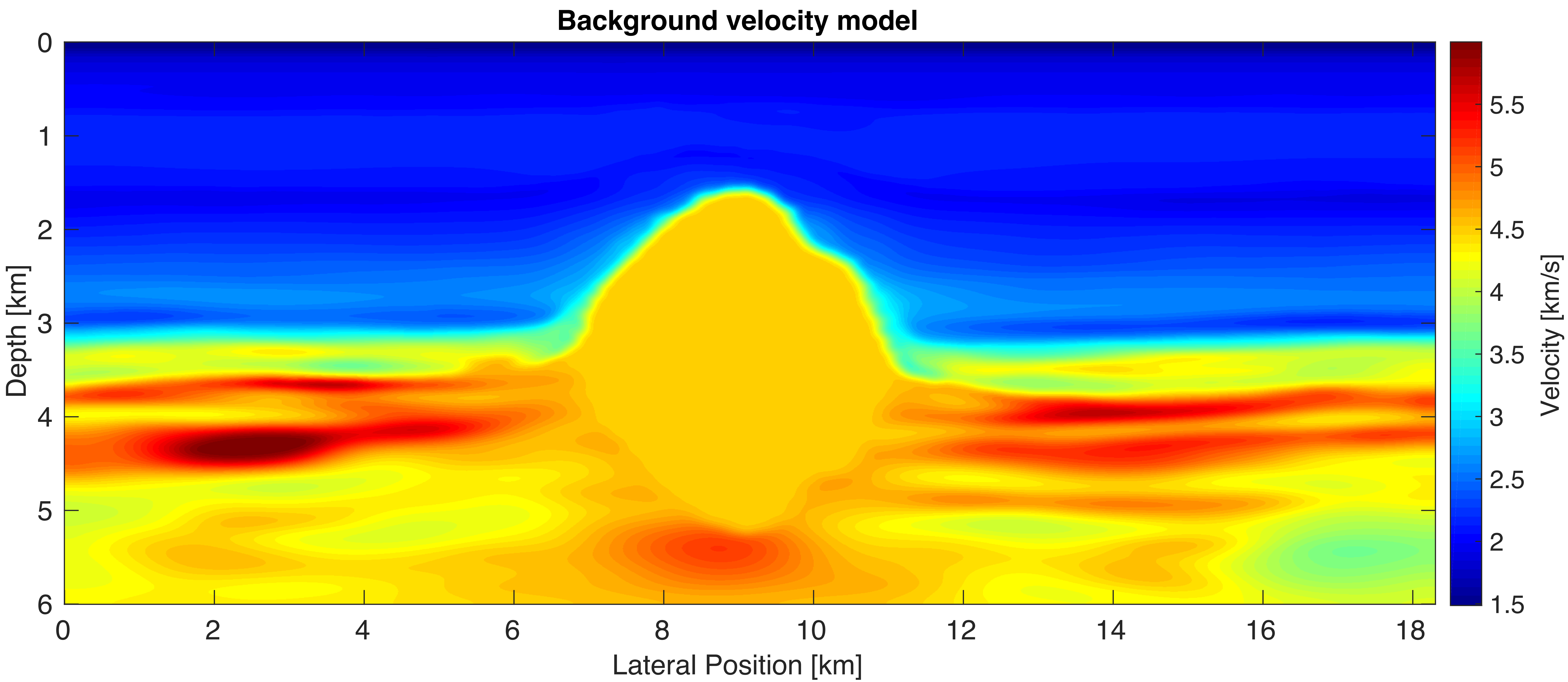
Preprocessing by BP:

- source designature
- mute of direct wave
- multiple removal

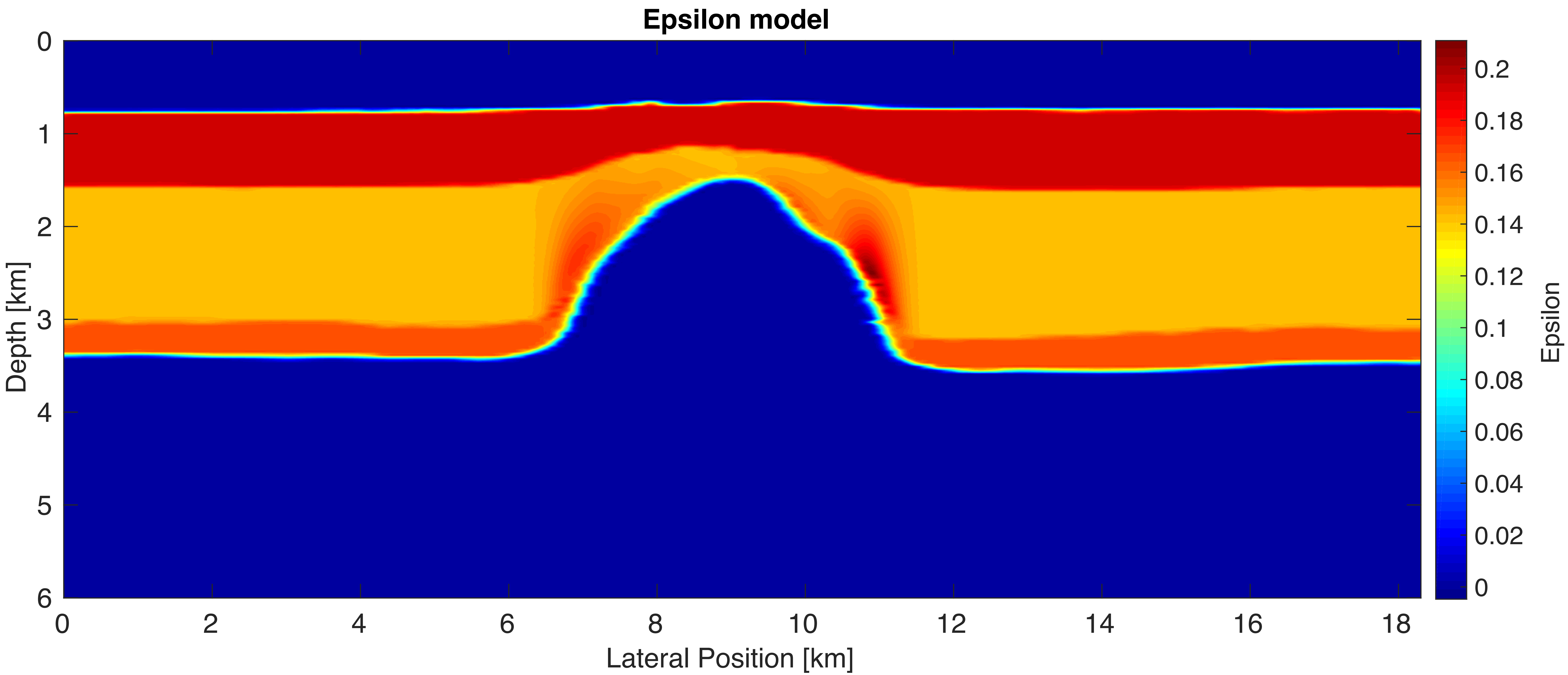
Software:

- Matlab 2D code w/ pseudo-acoustic wave equation
- 10 iterations of linearized Bregman w/ 100 shots per iteration (3 passes through data)
- on the fly source estimation (starting wavelet: 50 Hz Ricker)

Machar model



Machar model



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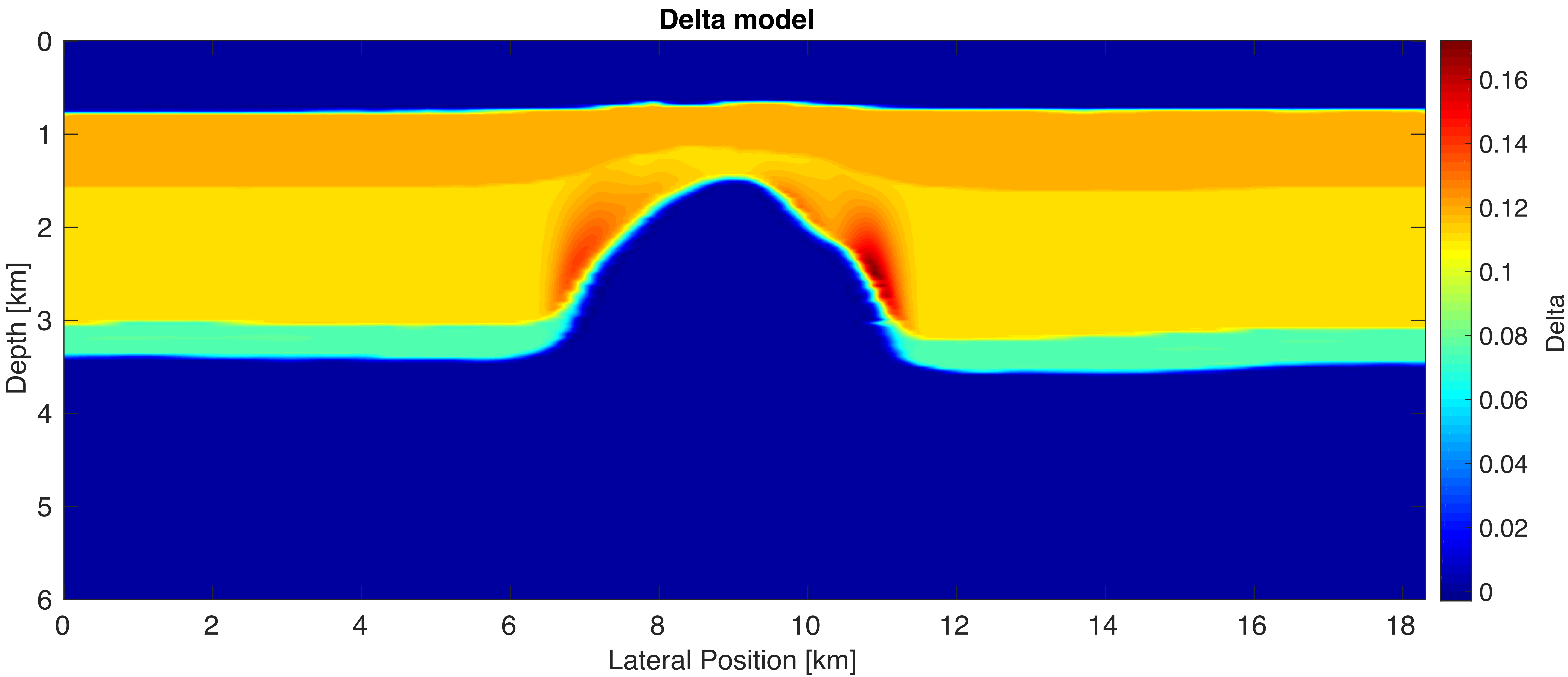


Machar model

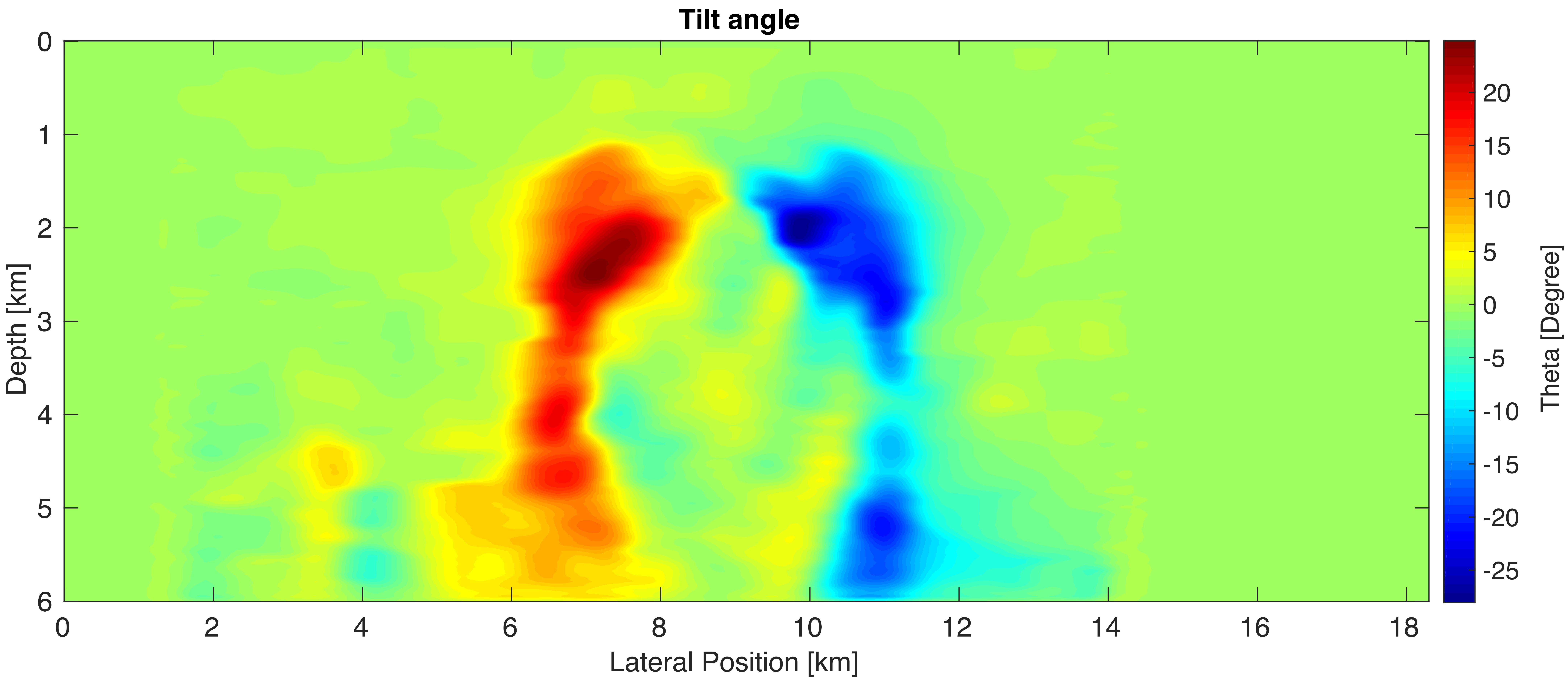
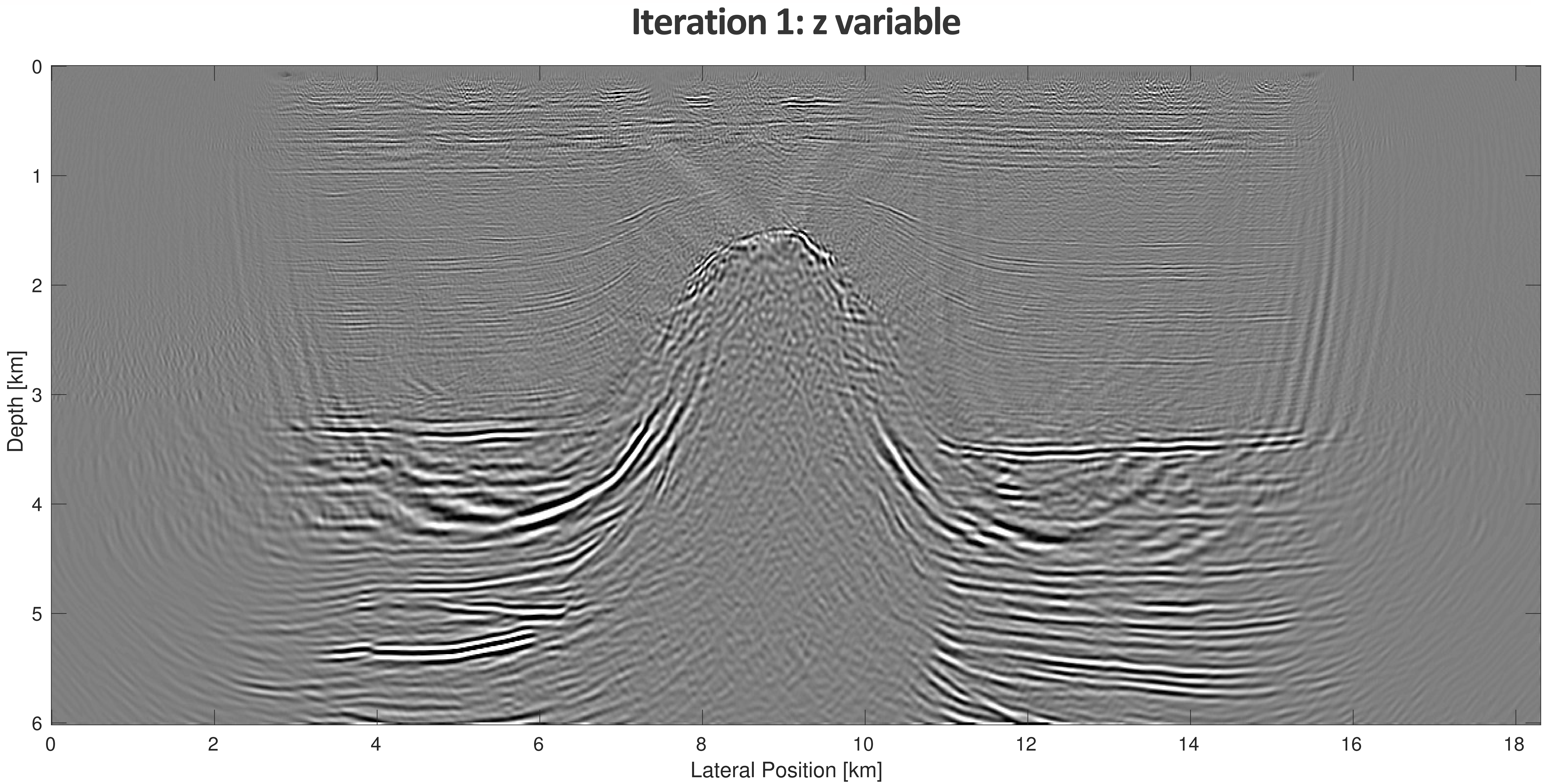


Image updates



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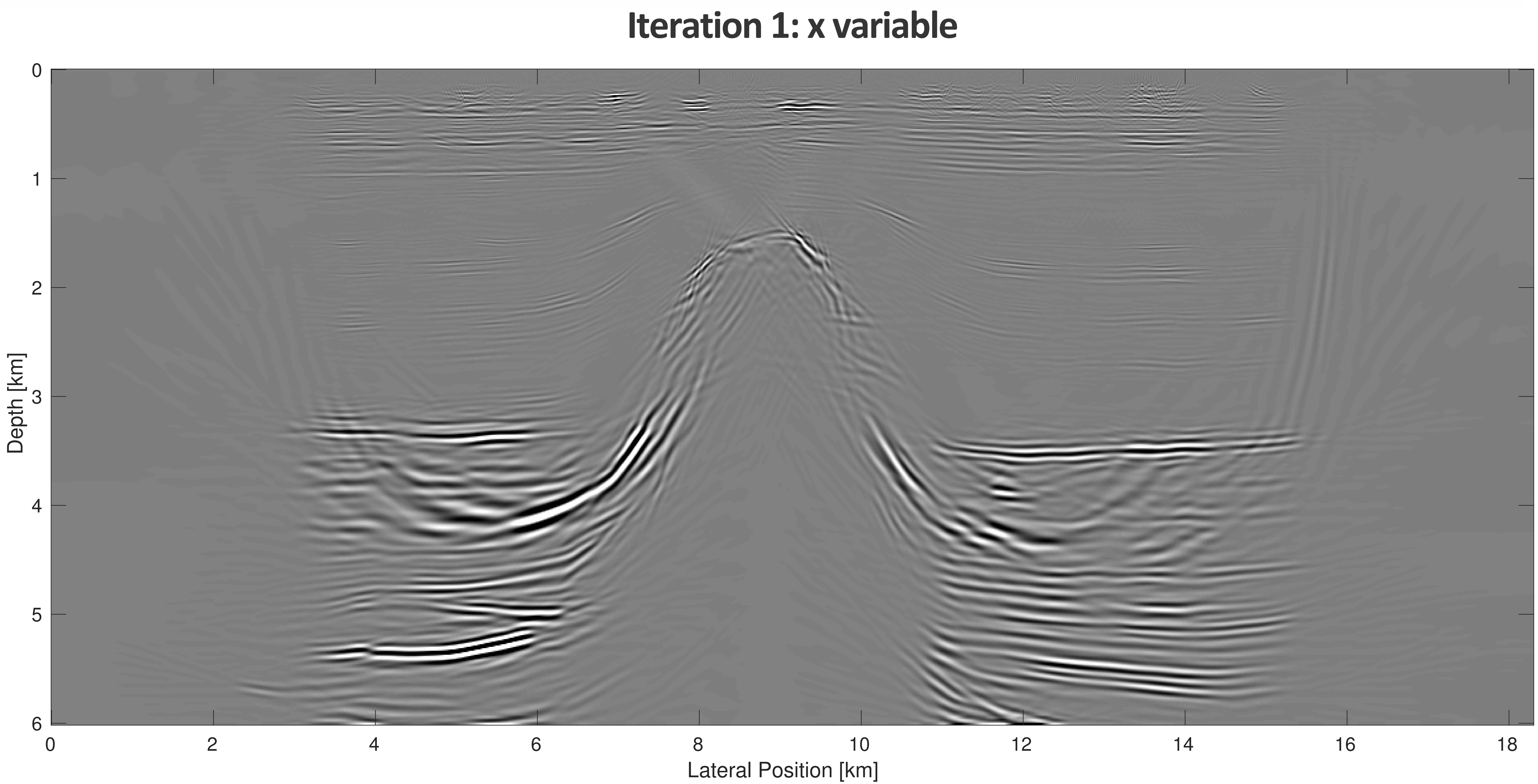
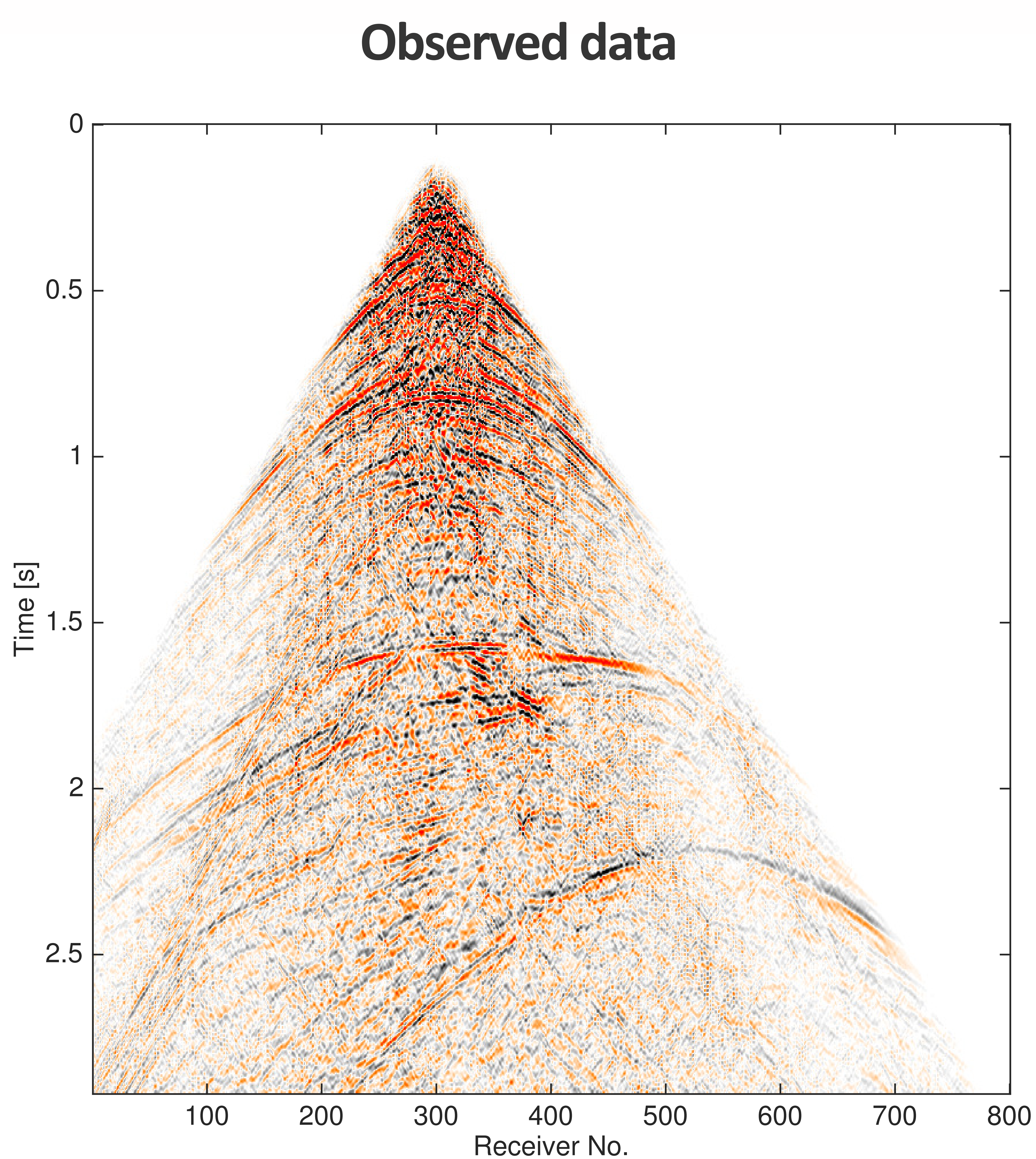
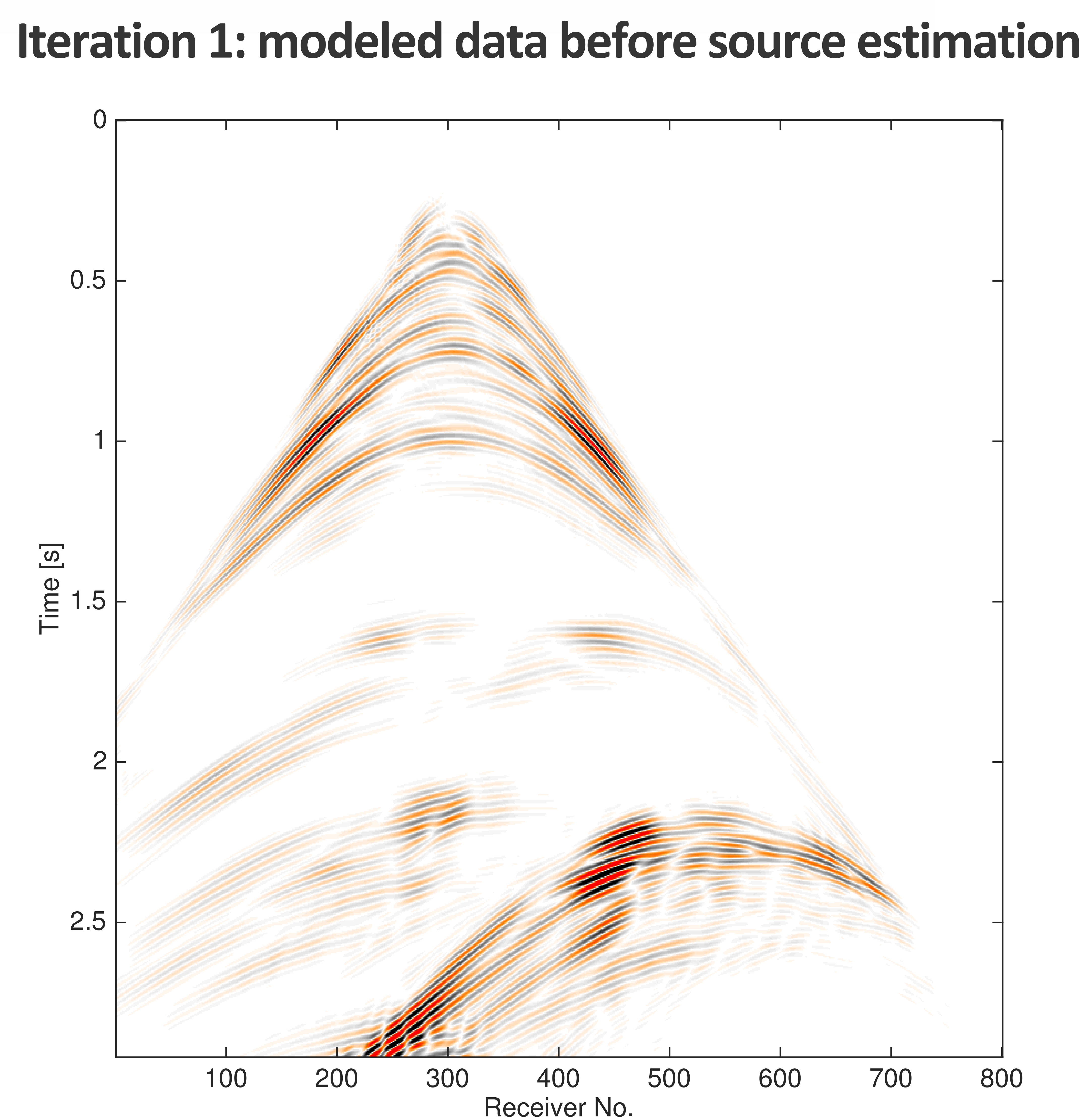


Observed and modeled shot records





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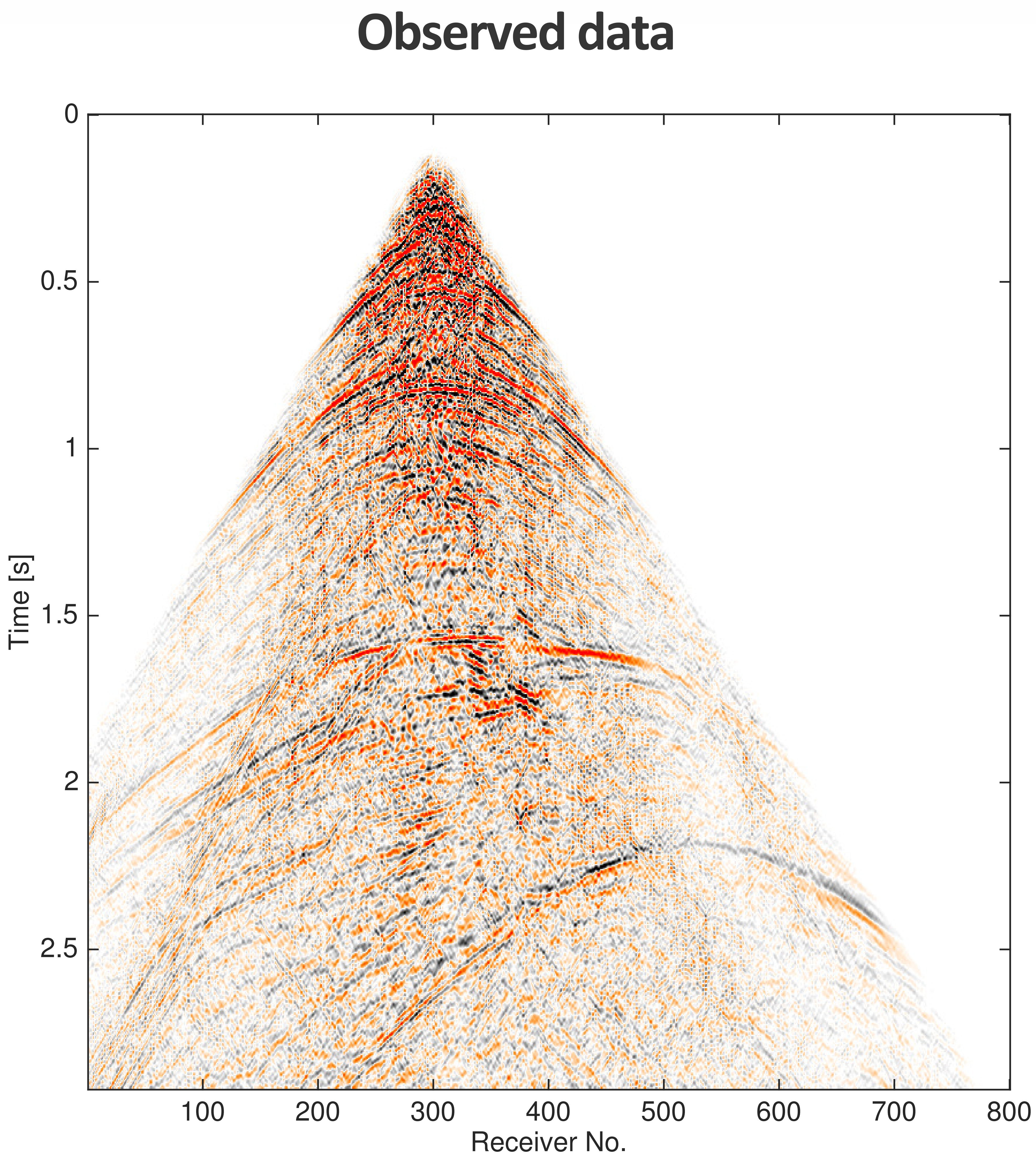
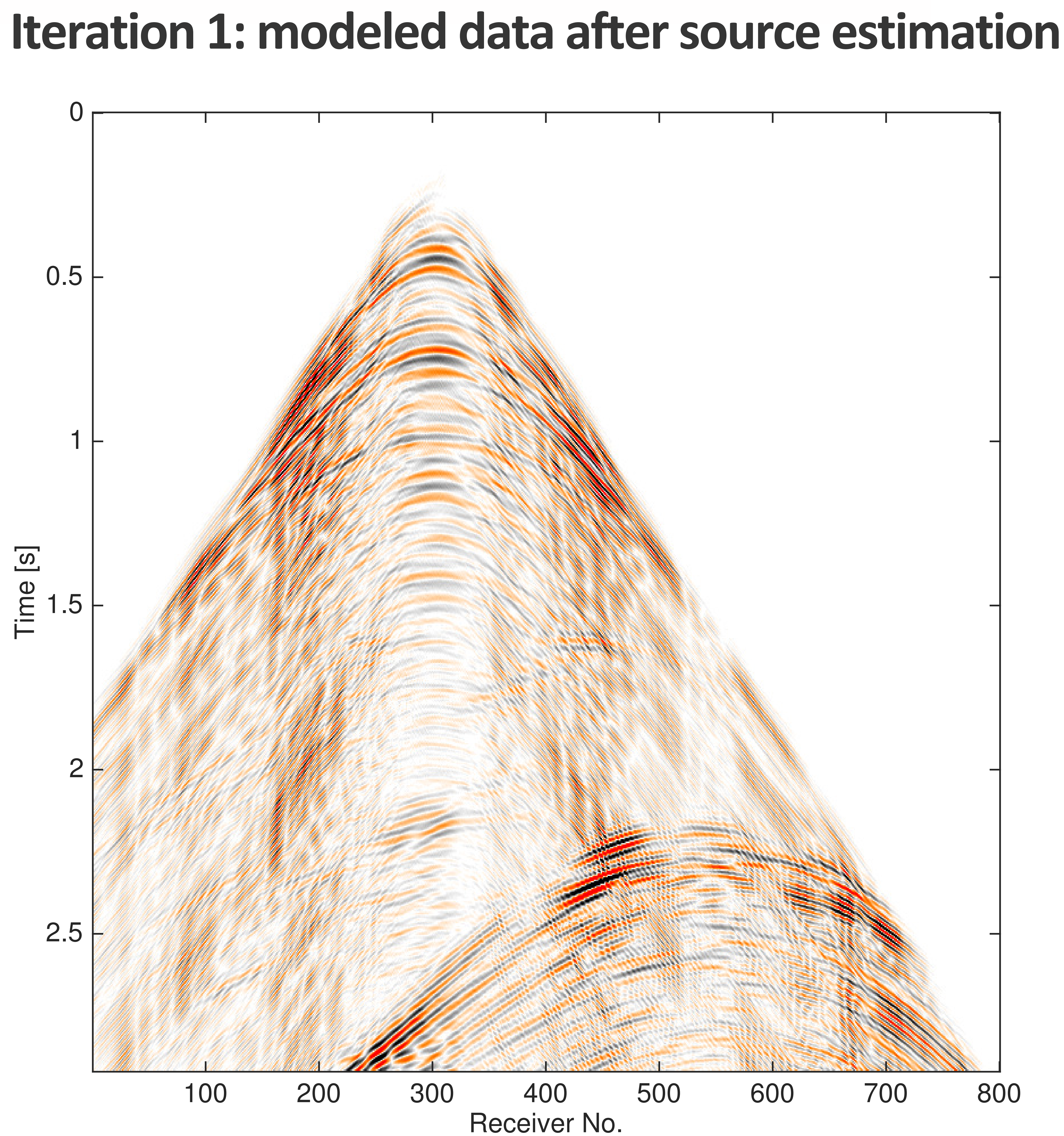
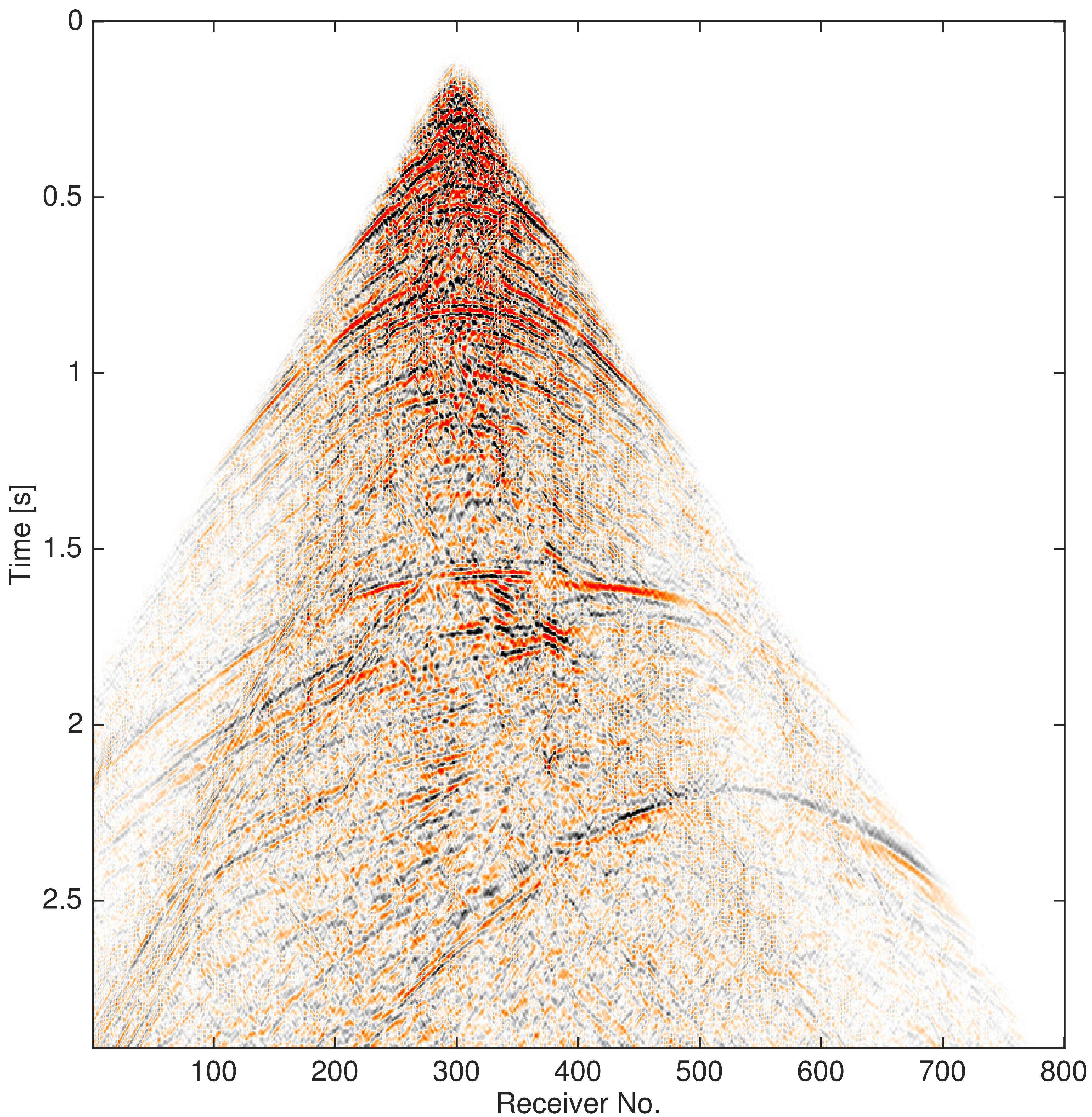


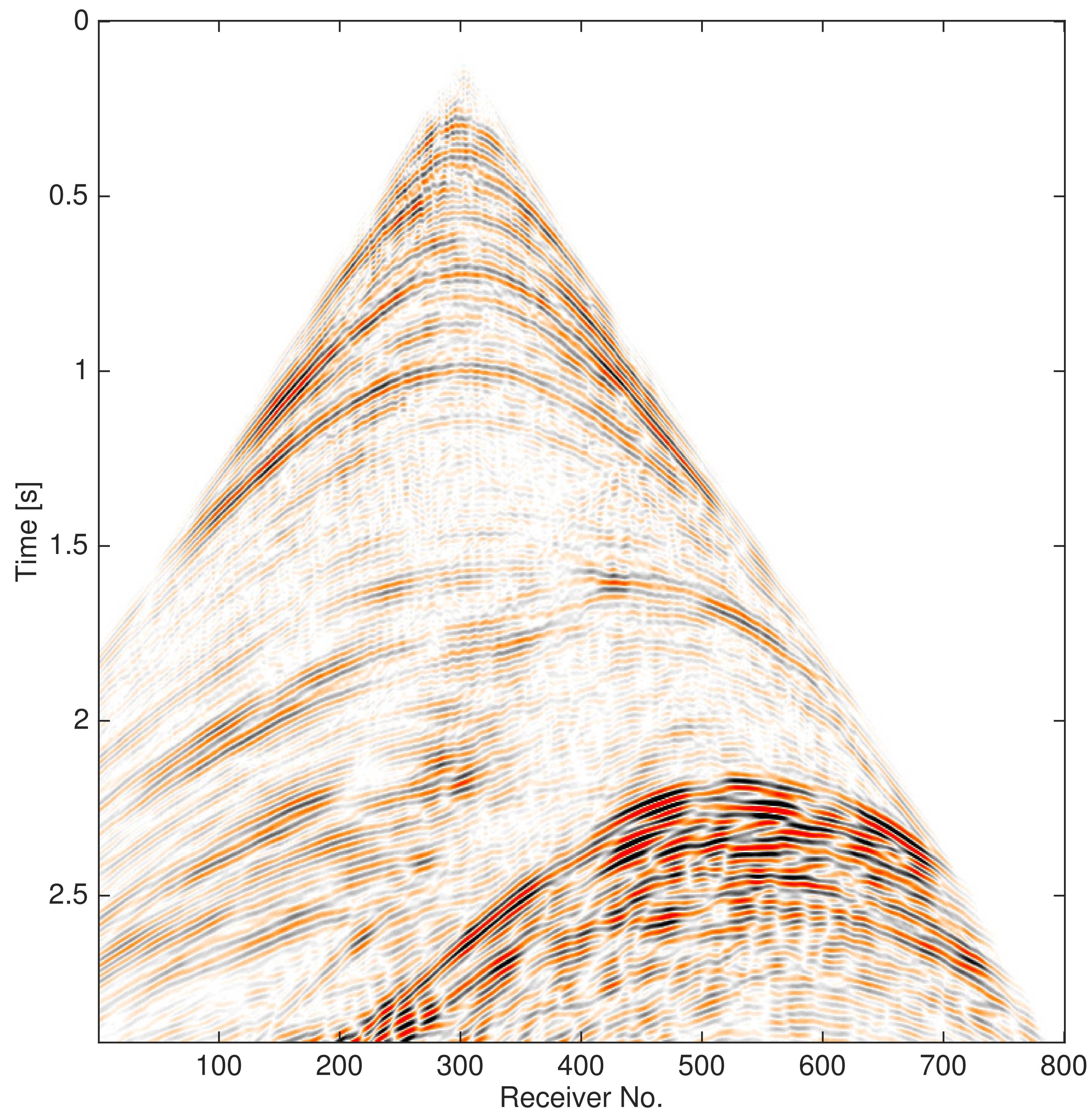


Observed and modeled shot records

Observed data

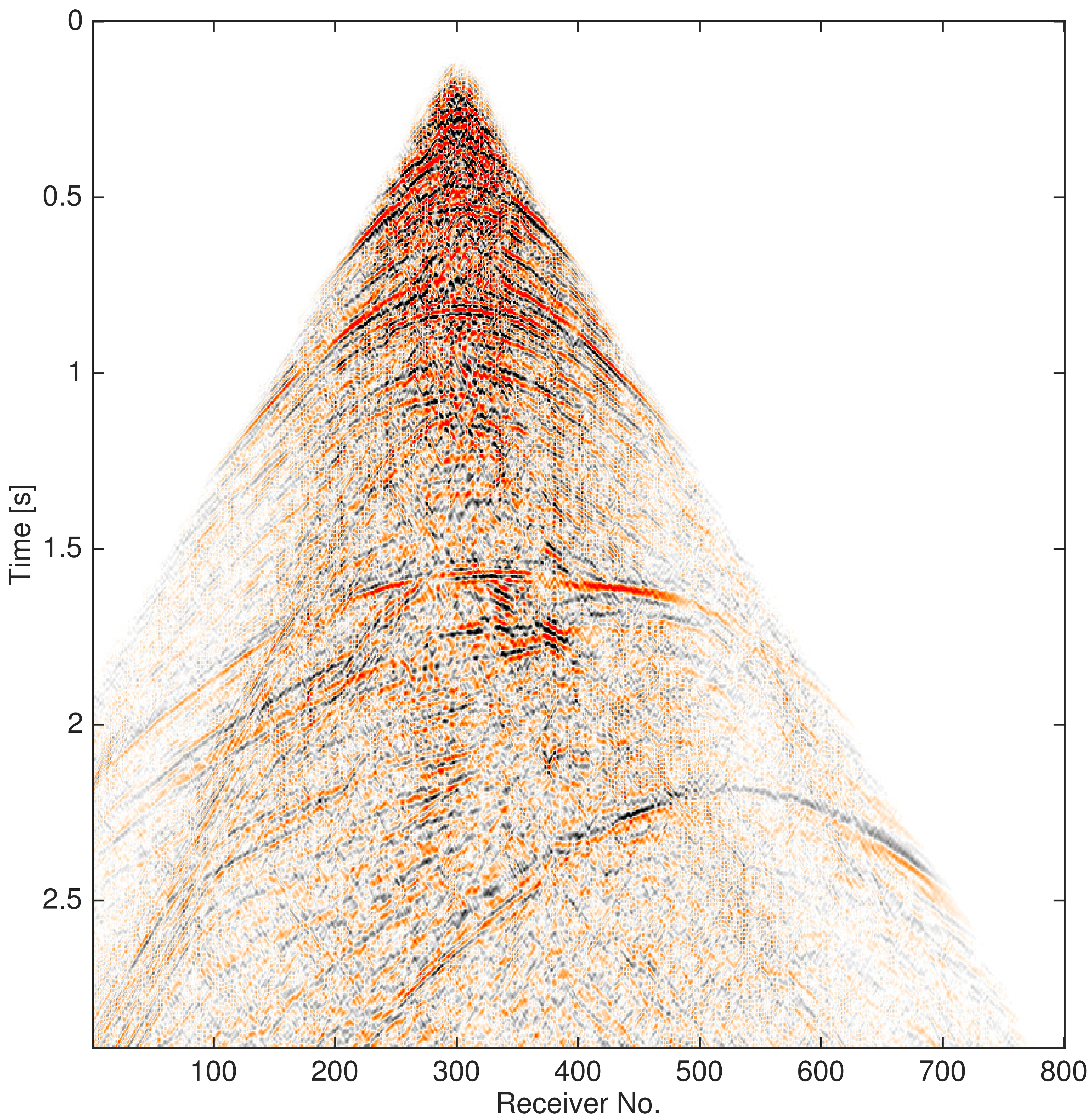


Iteration 10: modeled data before source estimation

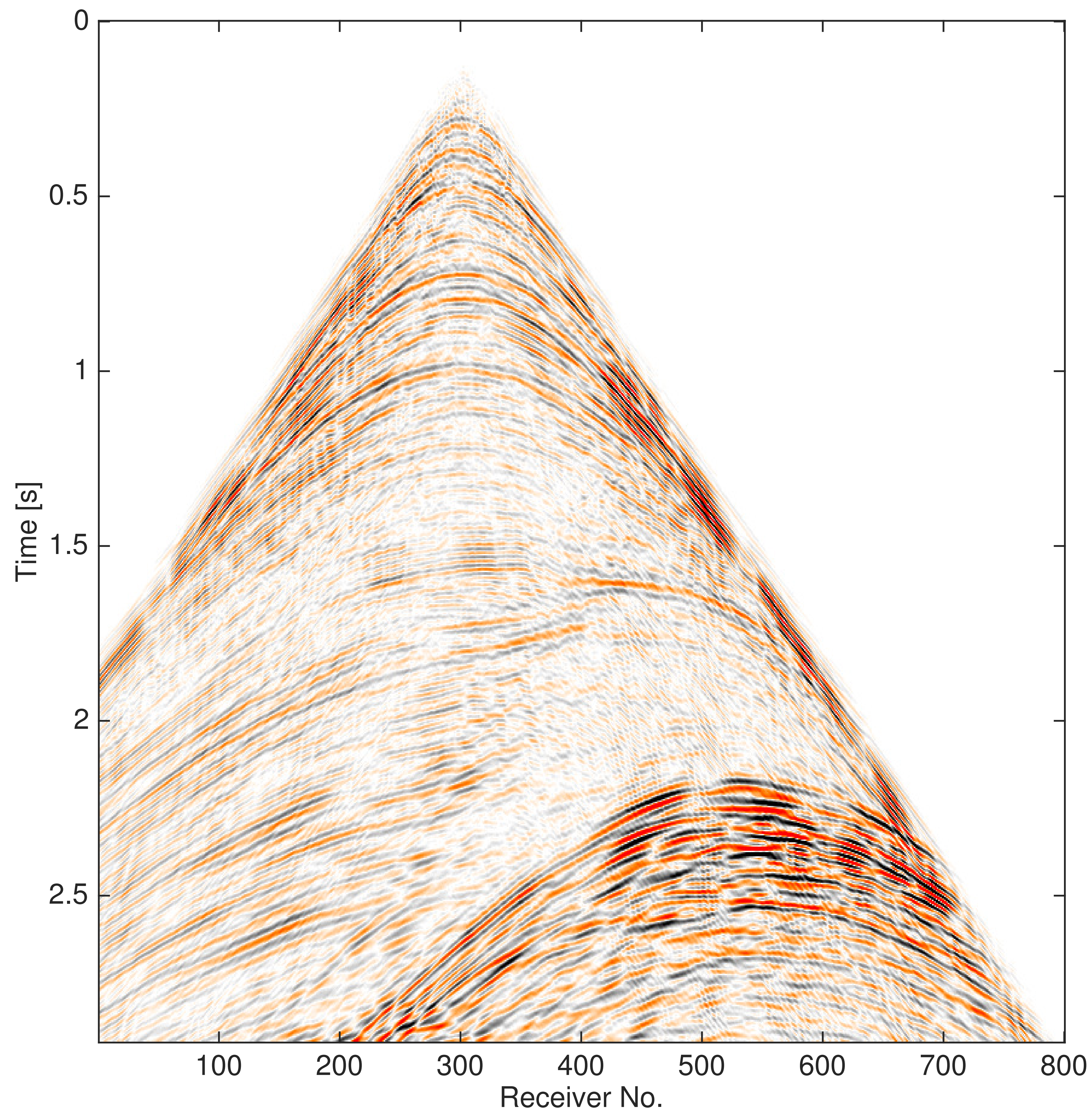


Observed and modeled shot records

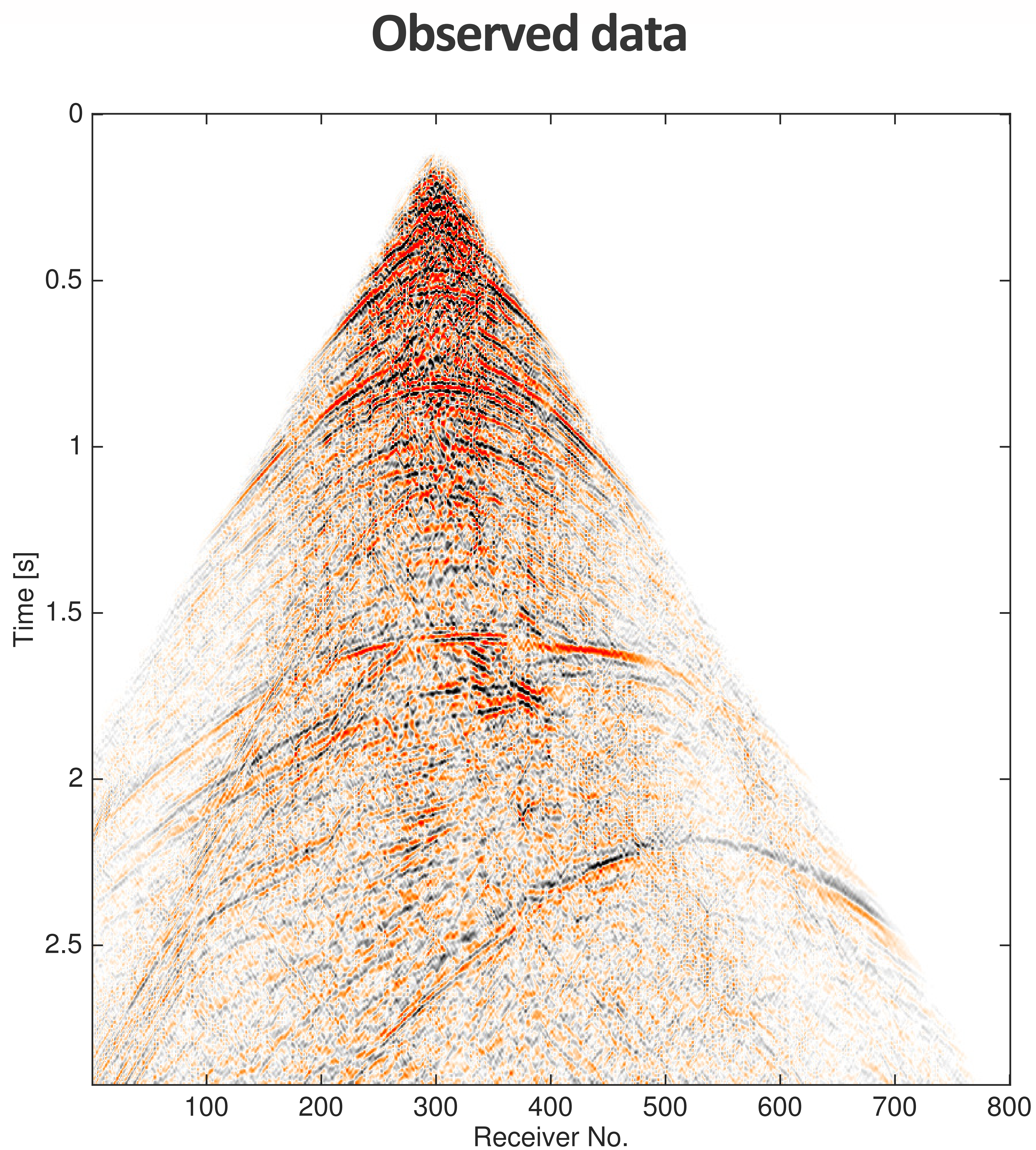
Observed data



Iteration 10: modeled data after source estimation

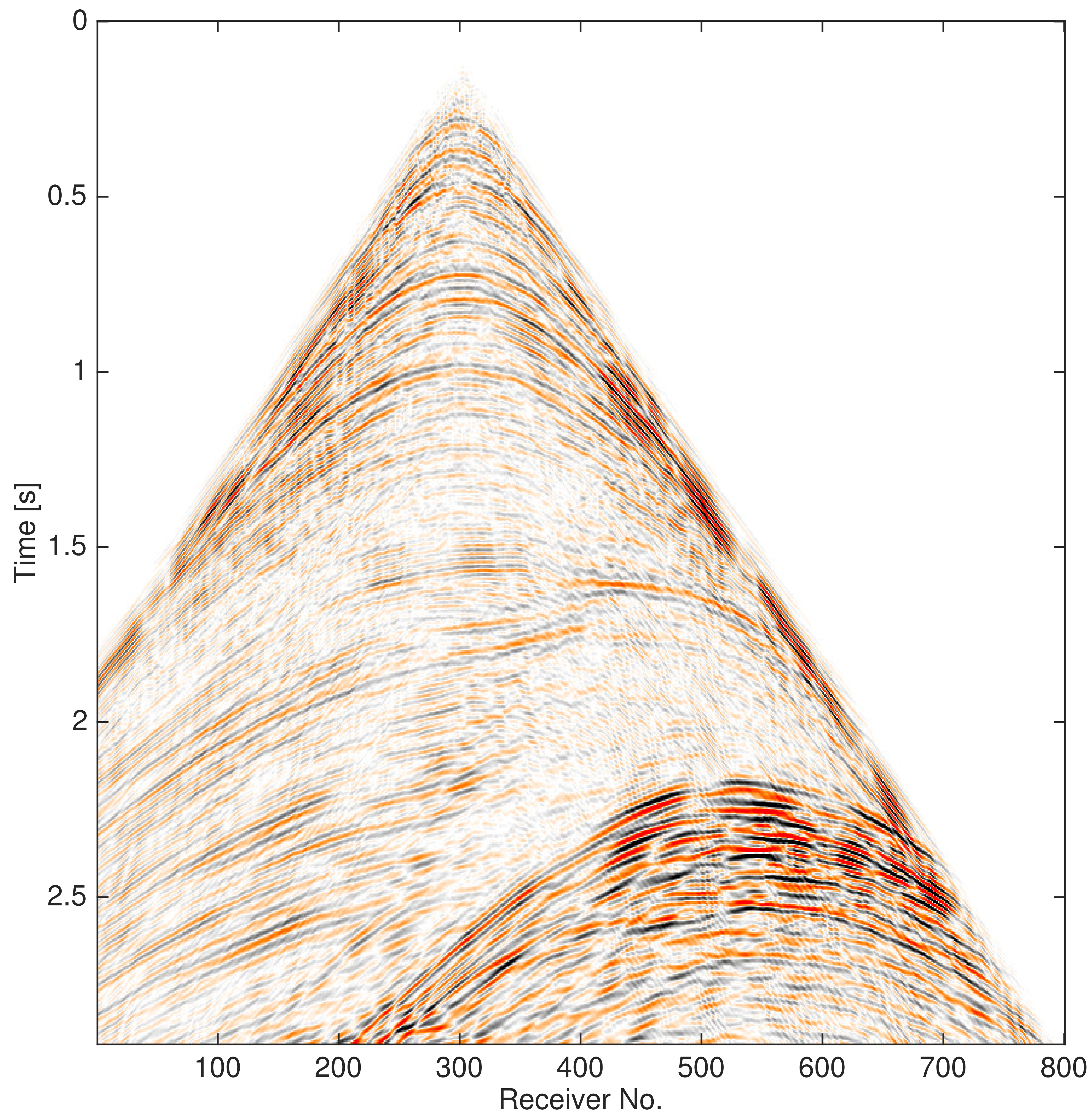


Observed and modeled shot records

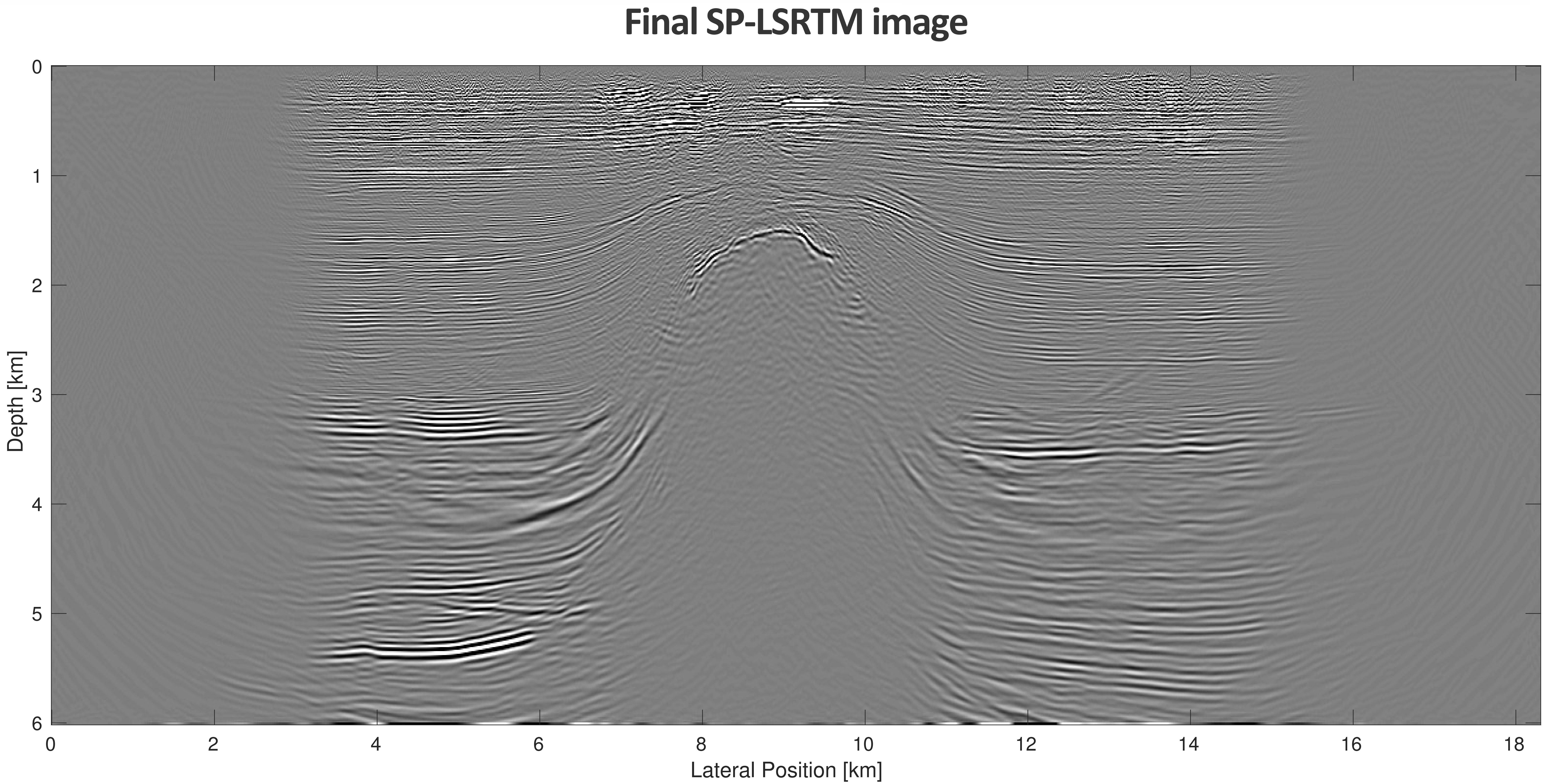


Observed and modeled shot records

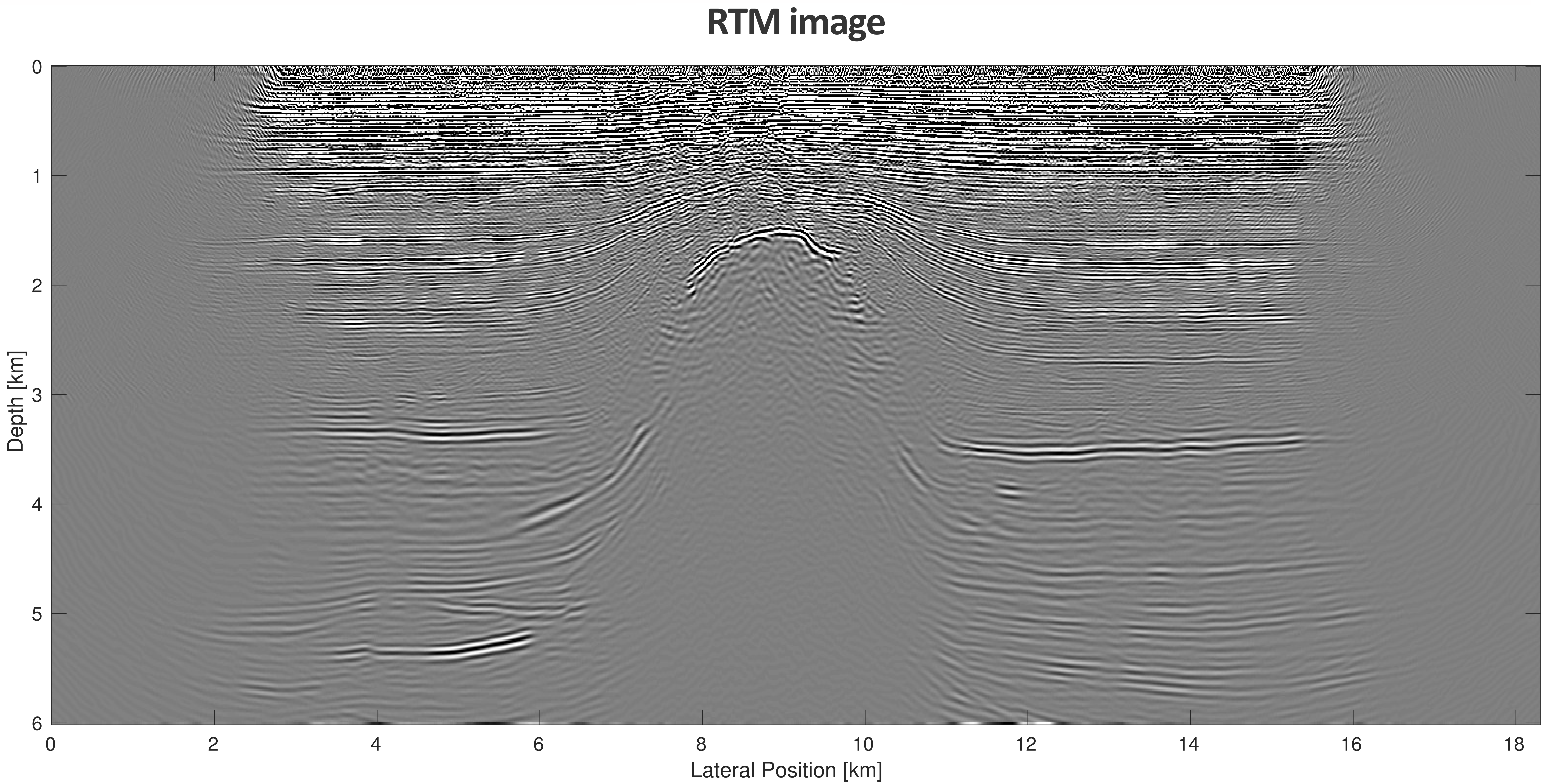
Iteration 10: modeled data after source estimation



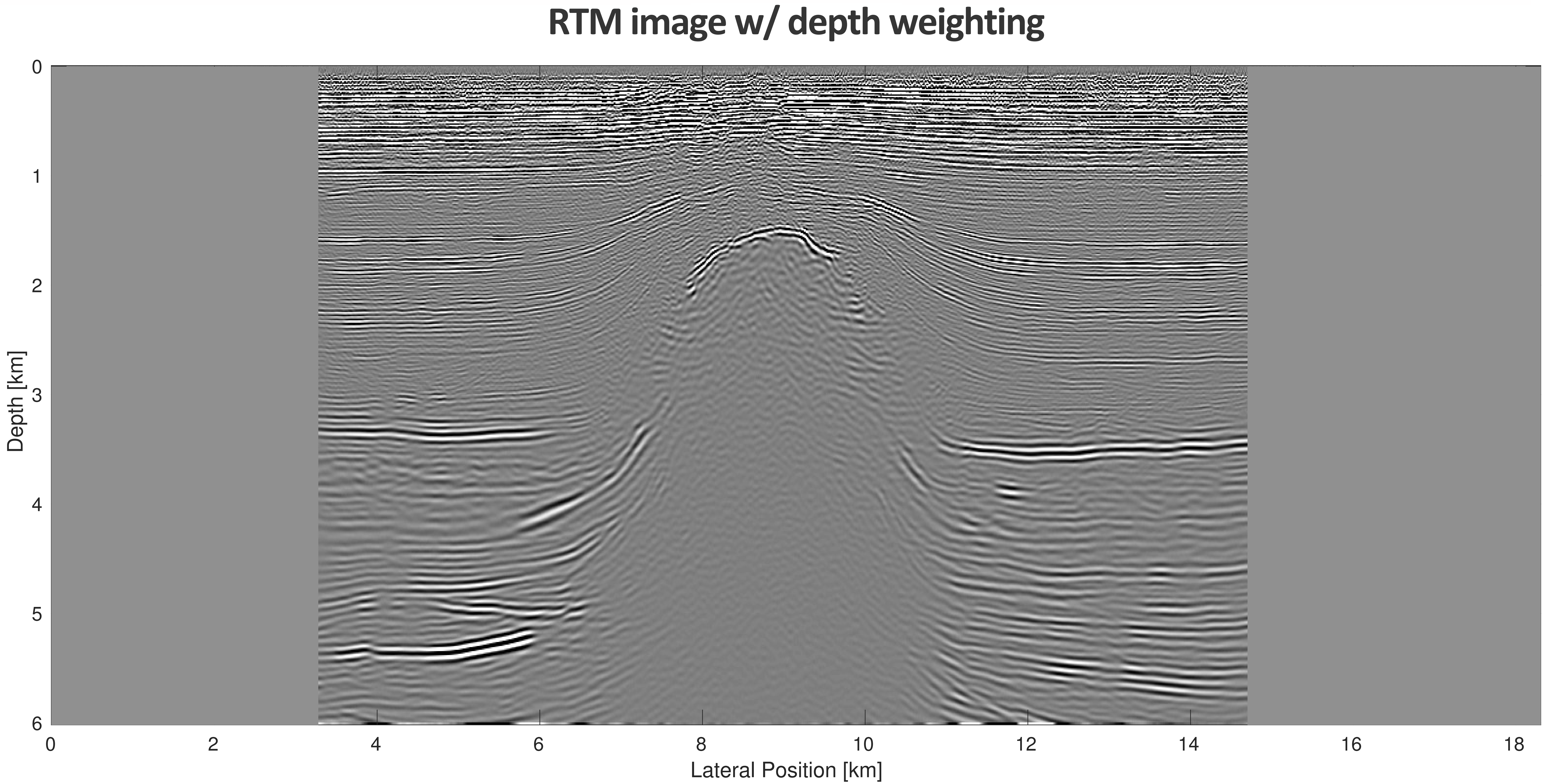
Machar results



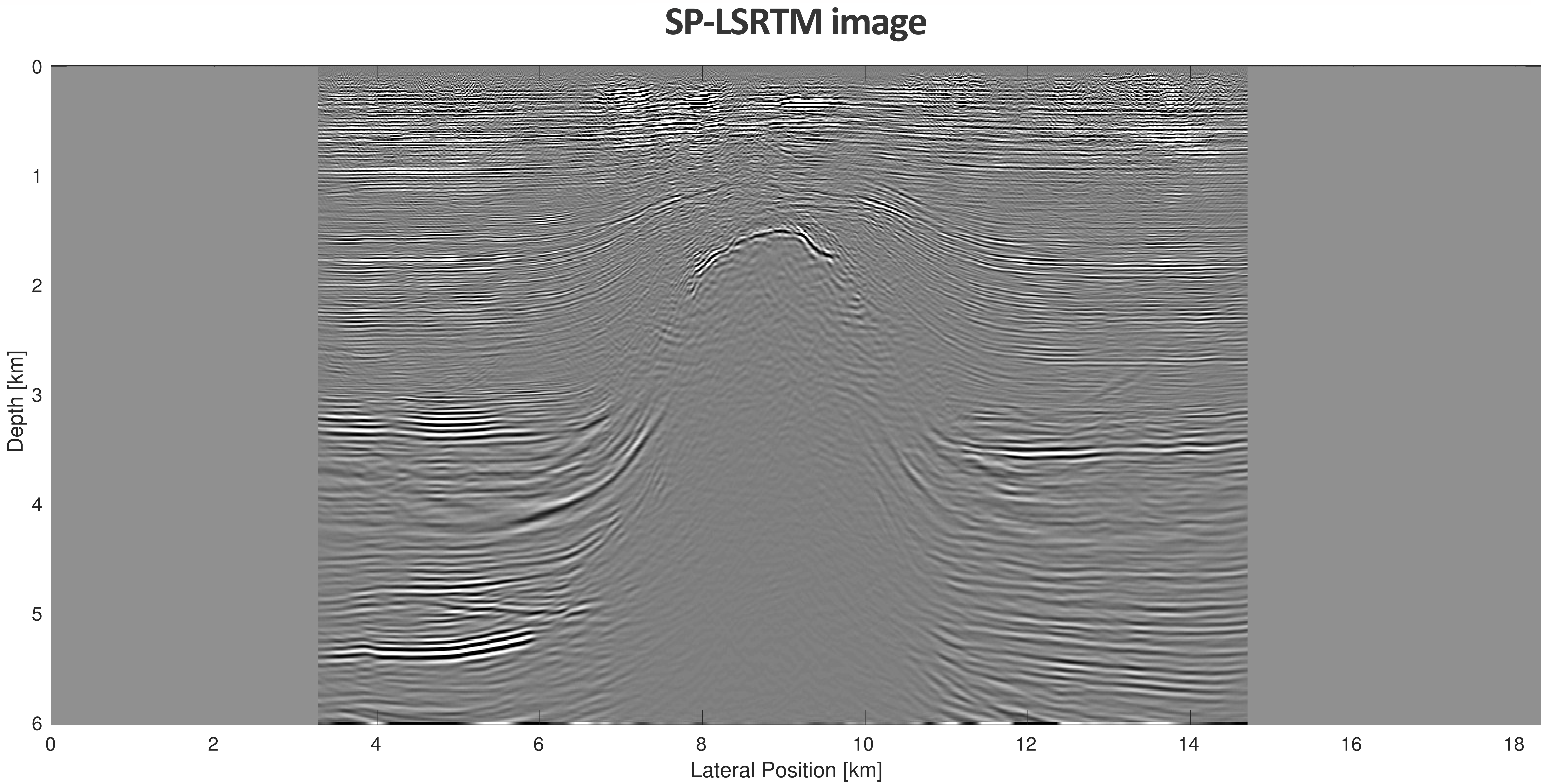
Machar results



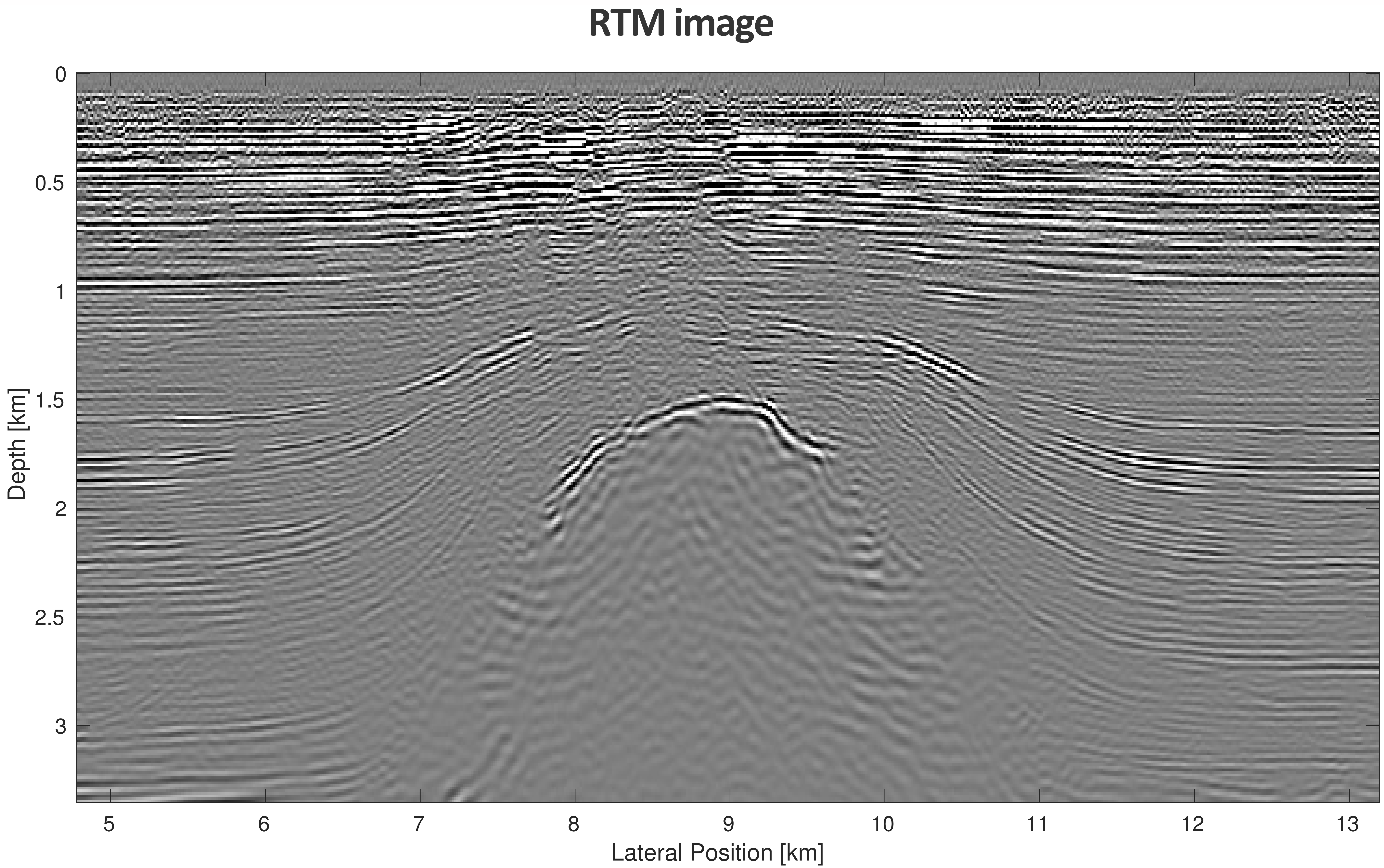
Machar results



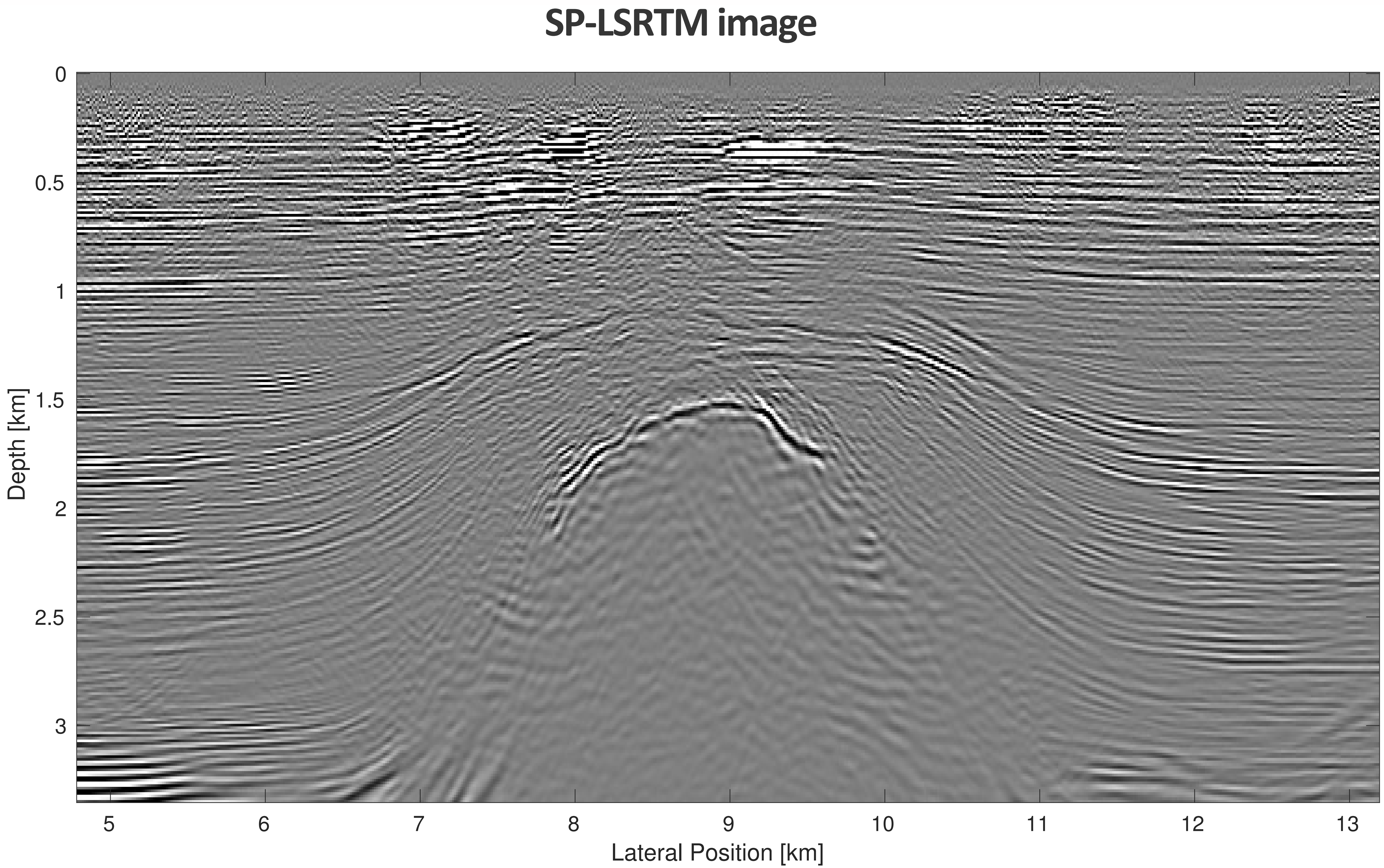
Machar results



Machar results



Machar results



Summary

Introduction of our time-domain LSRTM workflow w/ sparsity promotion:

- Matlab implementation with (pseudo-)acoustic wave equation
- linearized Bregman method as solver
- various pre-conditioners to improve convergence
- source estimation

Influence of correct gradients for LSRTM:

- incorrect gradients + non-linearized data worsen convergence
- still significantly better convergence w/ correct gradient

Outlook

Future projects:

- Application to a sparse data set w/ aliasing
- 3D imaging with our new Julia/Devito workflow

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