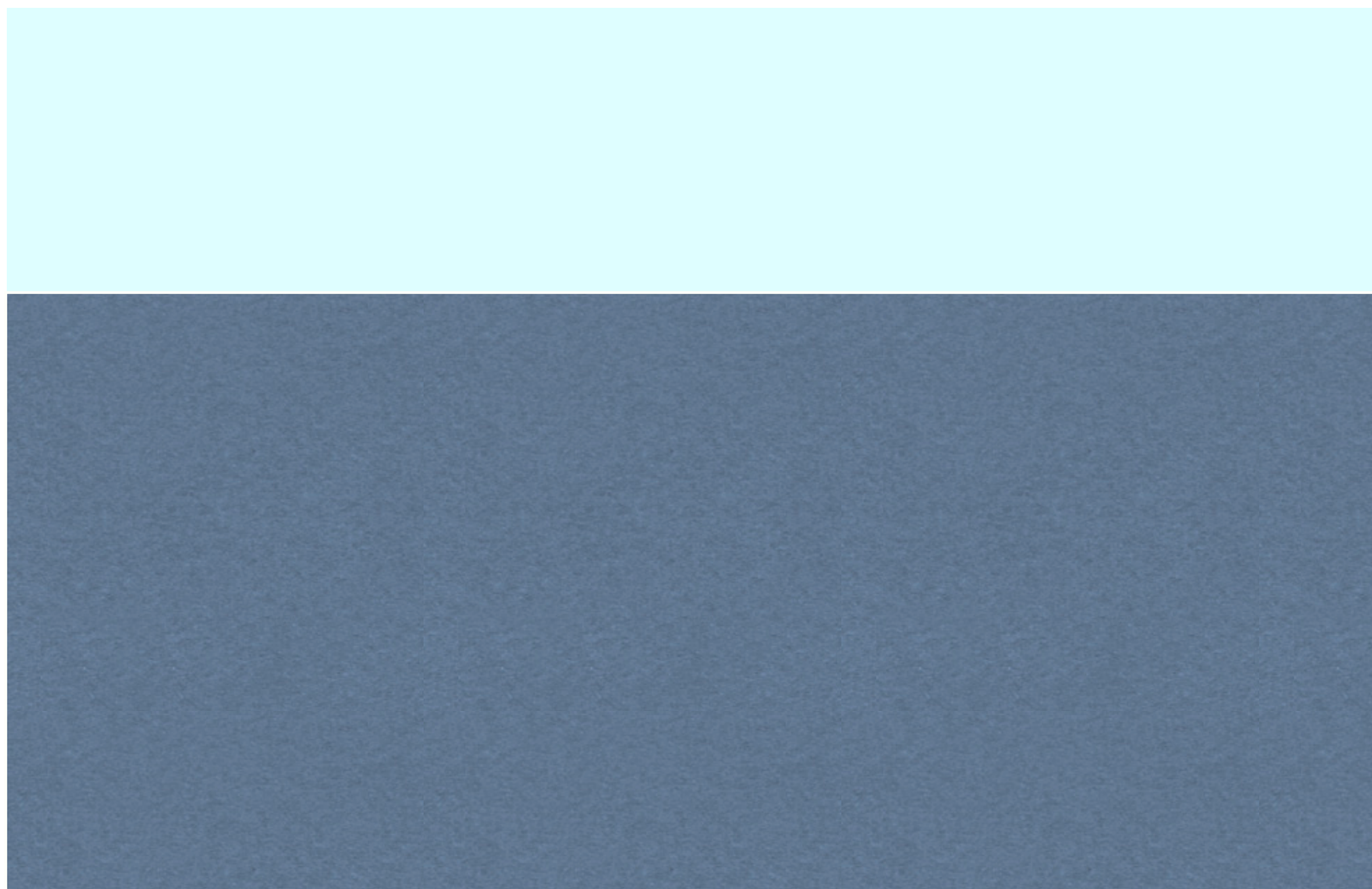


High resolution microseismic source collocation

Shashin Sharan, Rongrong Wang, and Felix J. Herrmann

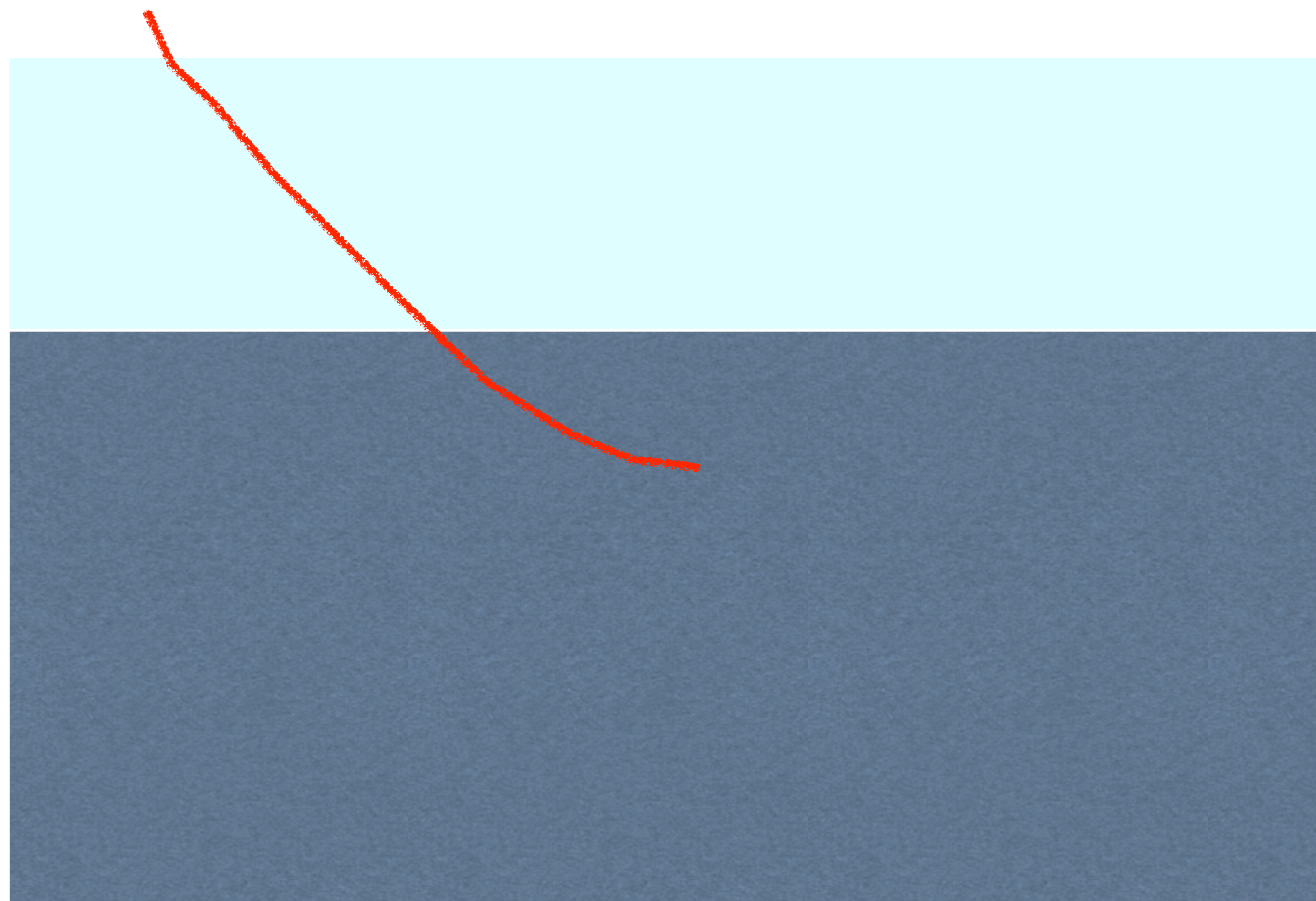


Motivation



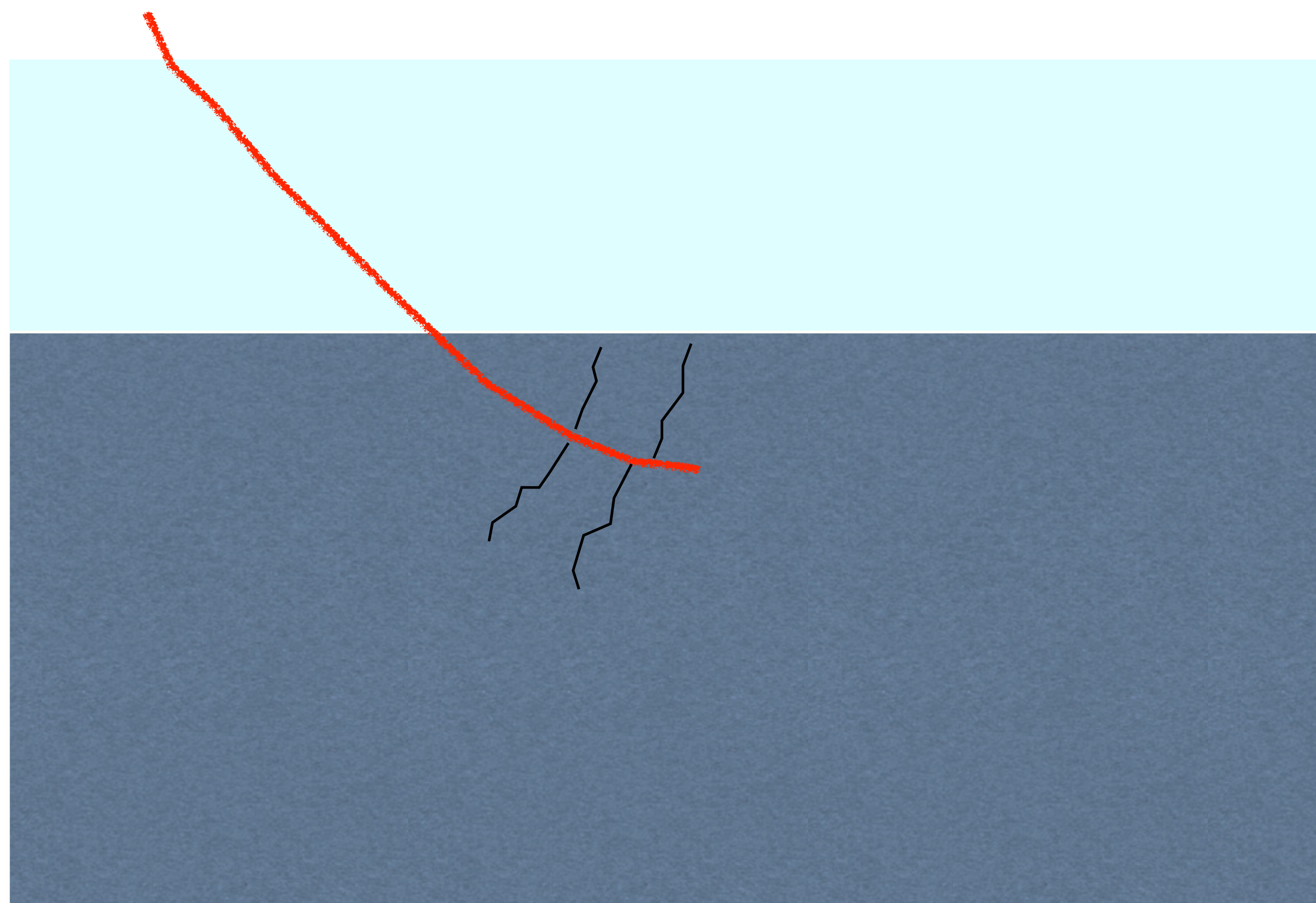
**Unconventional
Reservoir Schematic**

Motivation



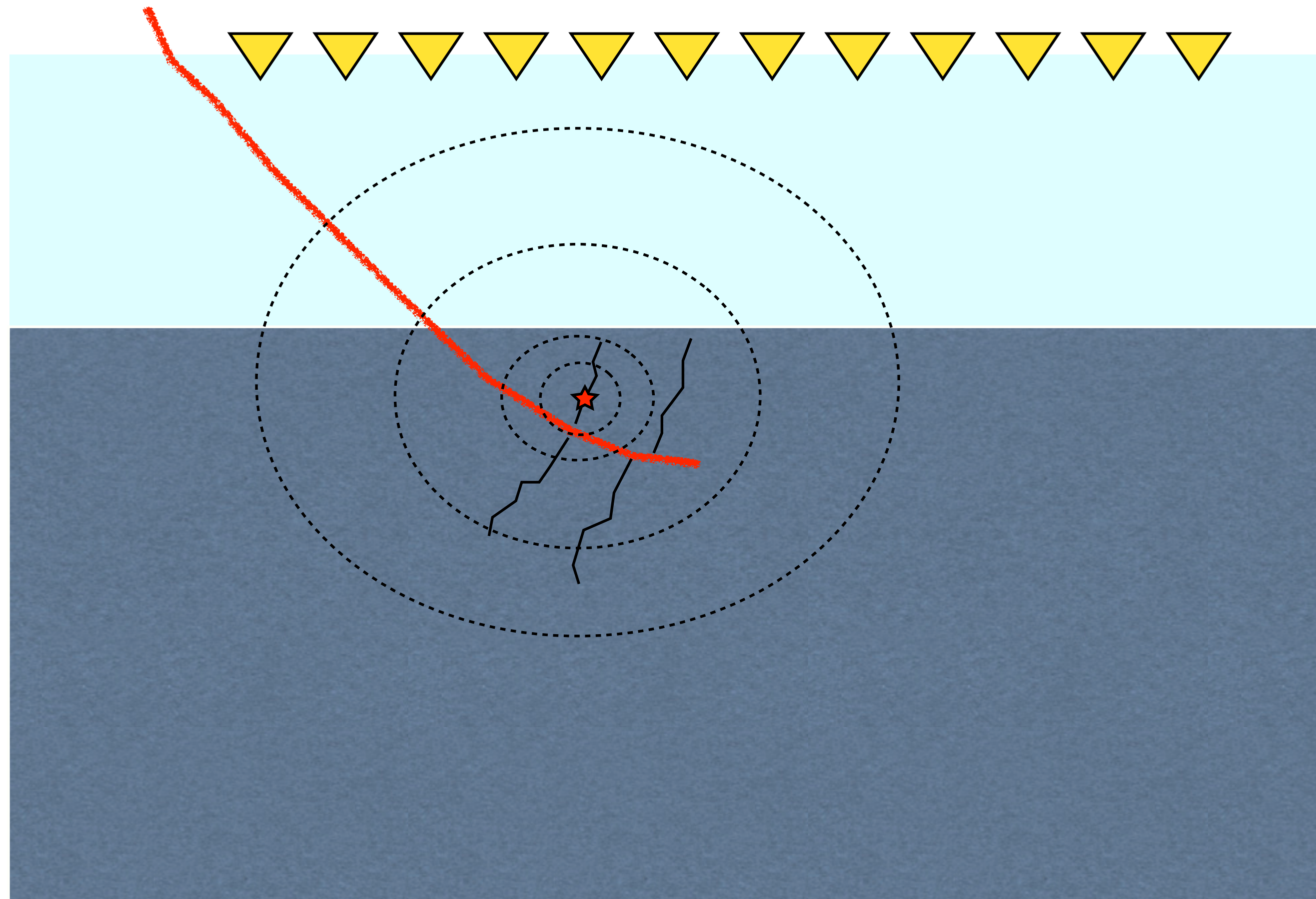
**Unconventional
Reservoir Schematic**

Motivation



**Unconventional
Reservoir Schematic**

Motivation



Unconventional Reservoir Schematic

Motivation behind microseismic imaging

Microseismic benefits

- ▶ Locating fracture at far distance from treatment well
- ▶ Tracer based and sonic log based method fails at far distances

Hazard prevention

- ▶ Activation of pre-existing faults
- ▶ Interference of fractures with wells

Motivation behind microseismic imaging

Reservoir evaluation

Source attribute estimation

- ▶ moment tensor orientation
- ▶ origin time
- ▶ spectral properties of source mechanism

Objectives

Super-resolution via sparsity promotion and “lifting”

Simultaneous estimation of the location of microseismic events & their source time functions

Pre-existing methods

Based on travel-time picking:

- ▶ estimate the origin time
- ▶ estimate the location
- ▶ time consuming
- ▶ no source time function

Pre-existing methods

Imaging based

- ▶ estimates origin time
- ▶ estimates the location

Geometric-mean RTM

- ▶ based on cross-correlation imaging condition
- ▶ wave equation solve for each receiver

Pre-existing methods

Hybrid imaging condition

- ▶ Computationally less intensive
- ▶ Requires grouping of neighboring receivers
- ▶ Lower resolution
- ▶ Receiver group length determination not trivial

FWI based method

$$\underset{\mathbf{f} \in \mathbb{R}^{n_x}, \mathbf{w} \in \mathbb{R}^{n_t}}{\text{minimize}} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{f}\mathbf{w}^T) - \mathbf{d}\|$$

*where $\mathcal{F}[\mathbf{m}] = \mathcal{P}\mathcal{A}[\mathbf{m}]^{-1}$ is the forward modelling operator

\mathbf{f} and \mathbf{w} are the spatial and temporal component of source

Merits

- ▶ alternate estimation approach
- ▶ good estimation when one of the spatial or temporal components known

FWI based method

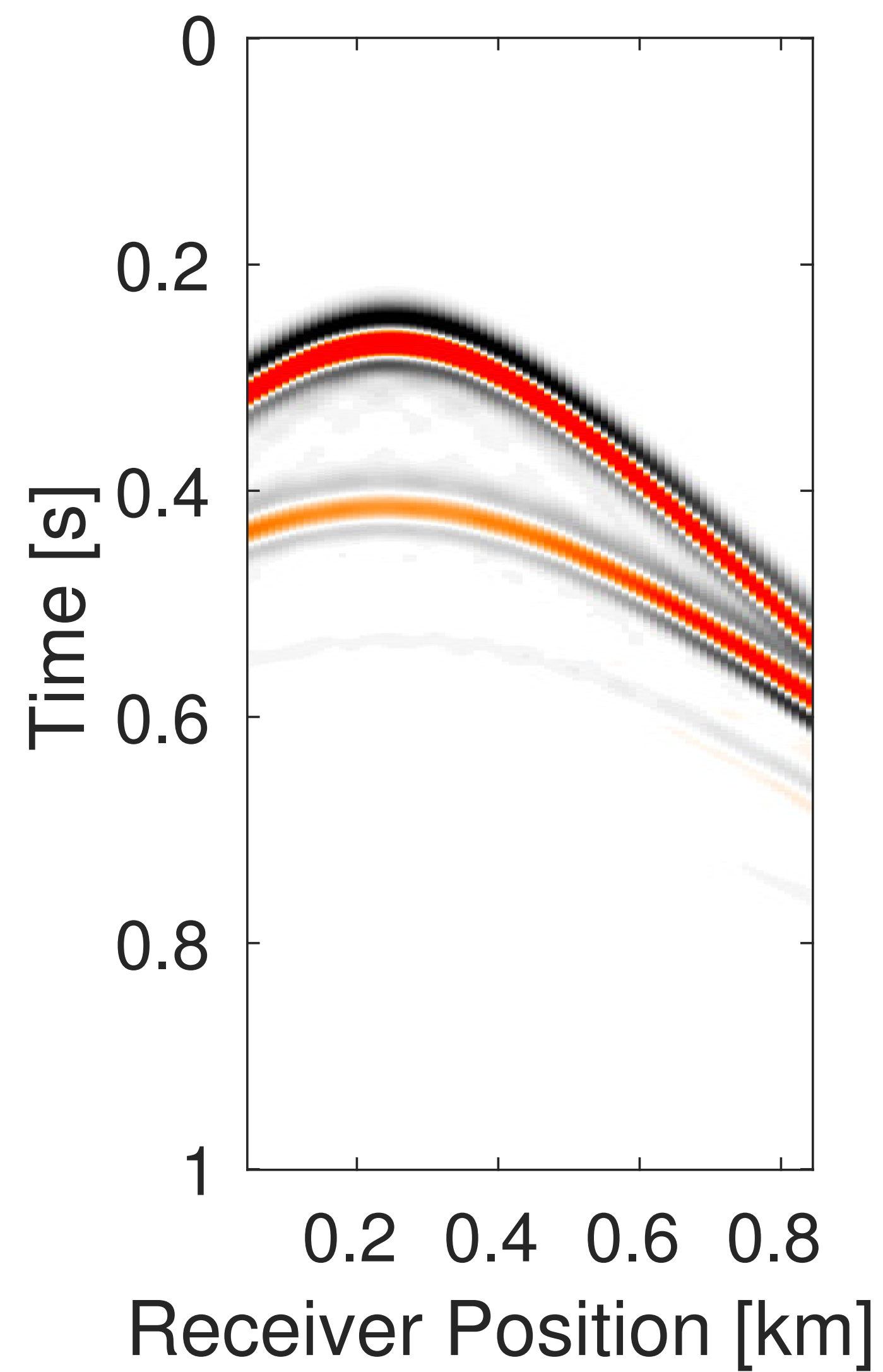
$$\underset{\mathbf{f} \in \mathbb{R}^{n_x}, \mathbf{w} \in \mathbb{R}^{n_t}}{\text{minimize}} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{f}\mathbf{w}^T) - \mathbf{d}\|$$

*where $\mathcal{F}[\mathbf{m}] = \mathcal{P}\mathcal{A}[\mathbf{m}]^{-1}$ is the forward modelling operator

\mathbf{f} and \mathbf{w} are the spatial and temporal component of source

Limitations:

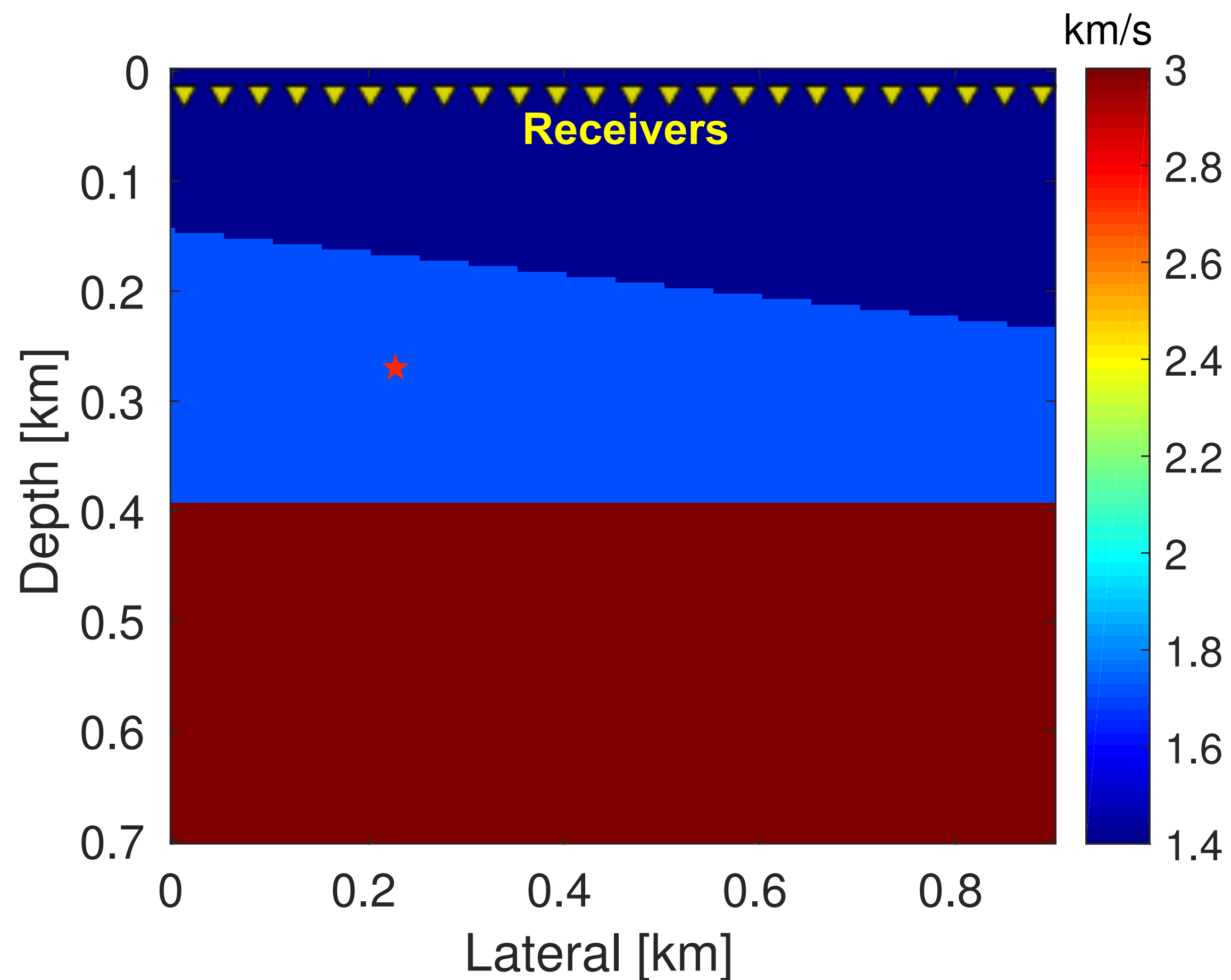
- ▶ poor estimation when both source location & source time function unknown
- ▶ assumes prior on number of sources



Synthetic microseismic data

**Data is simulated using
finite difference time
stepping code**

Experimental setup



Modeling information:

Model size: 0.9 km x 0.7 km

Grid spacing: 10m

Receiver spacing: 10m

Source depth: 0.27 km

Source lateral position: 0.25 km

Wavelet: Ricker wavelet

Receiver depth: 20m

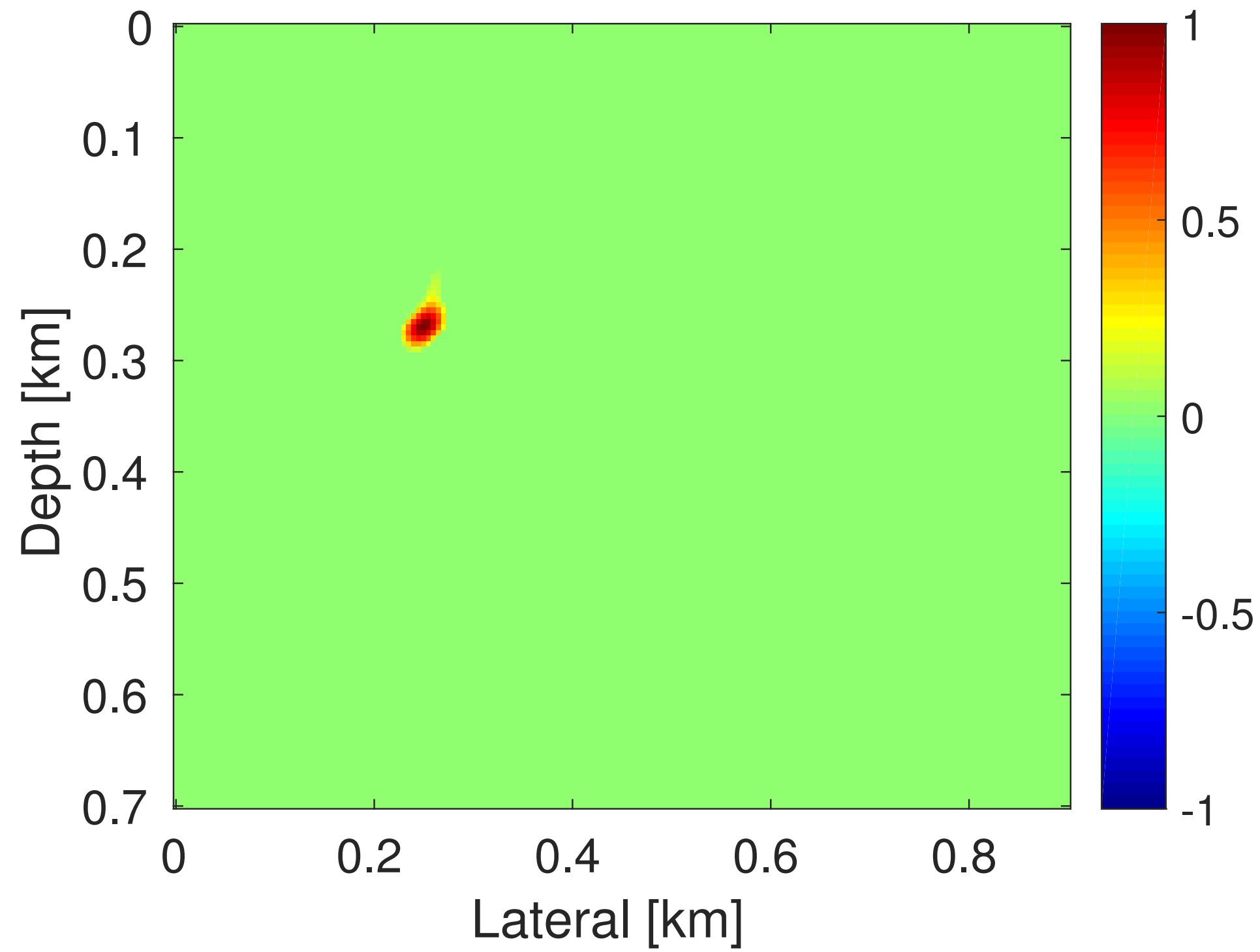
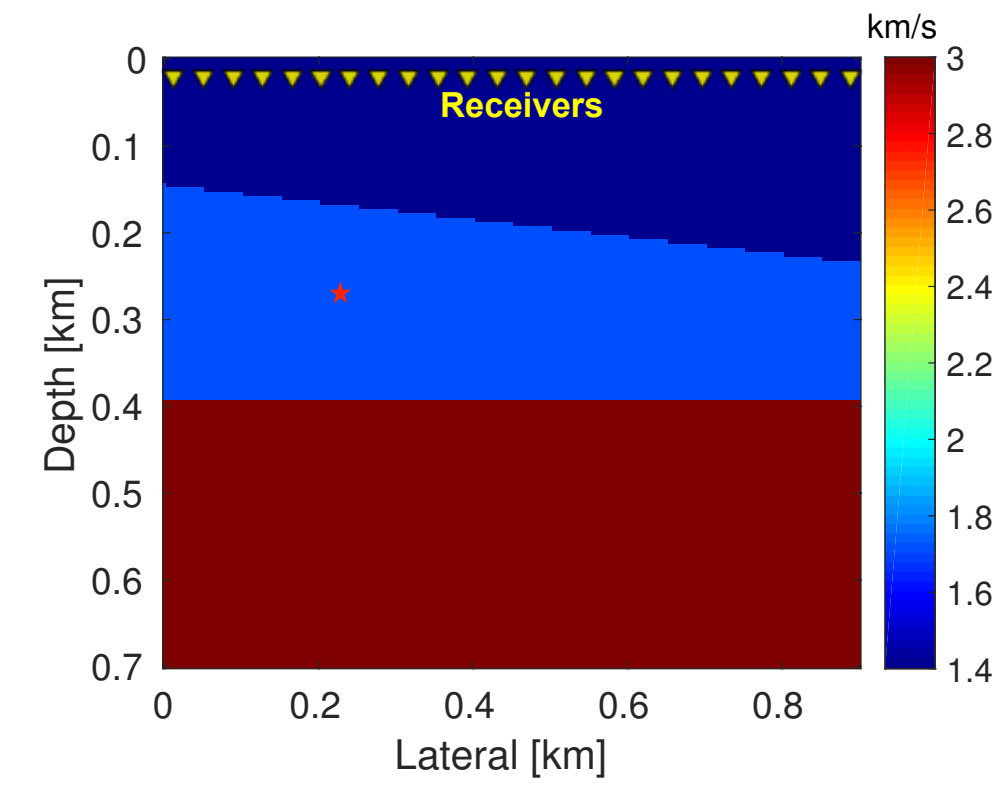
Fixed spread: 0.88km

Sampling interval: 1 ms

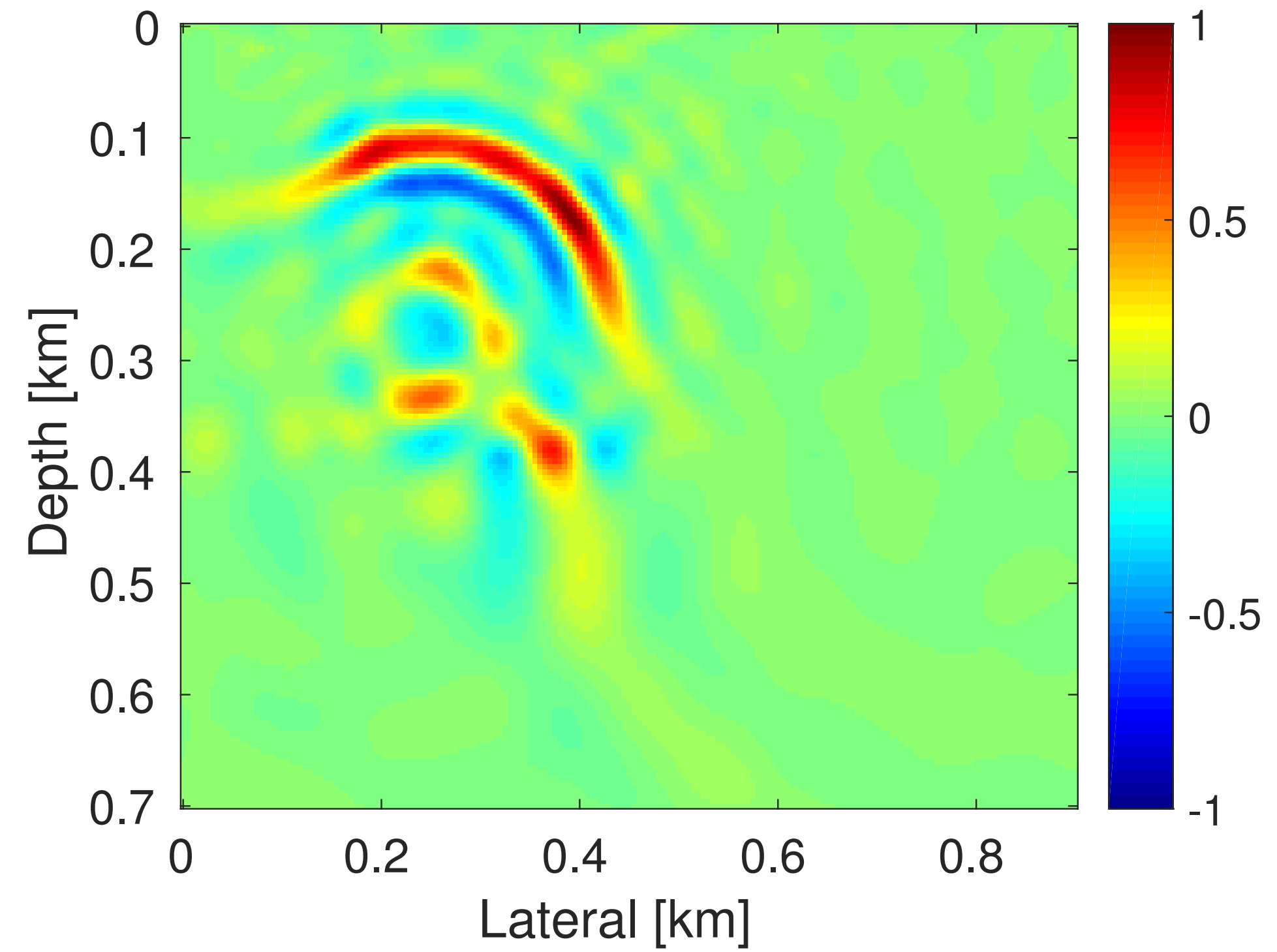
Recording length: 1s

Peak frequency : 20 Hz

Estimated source location

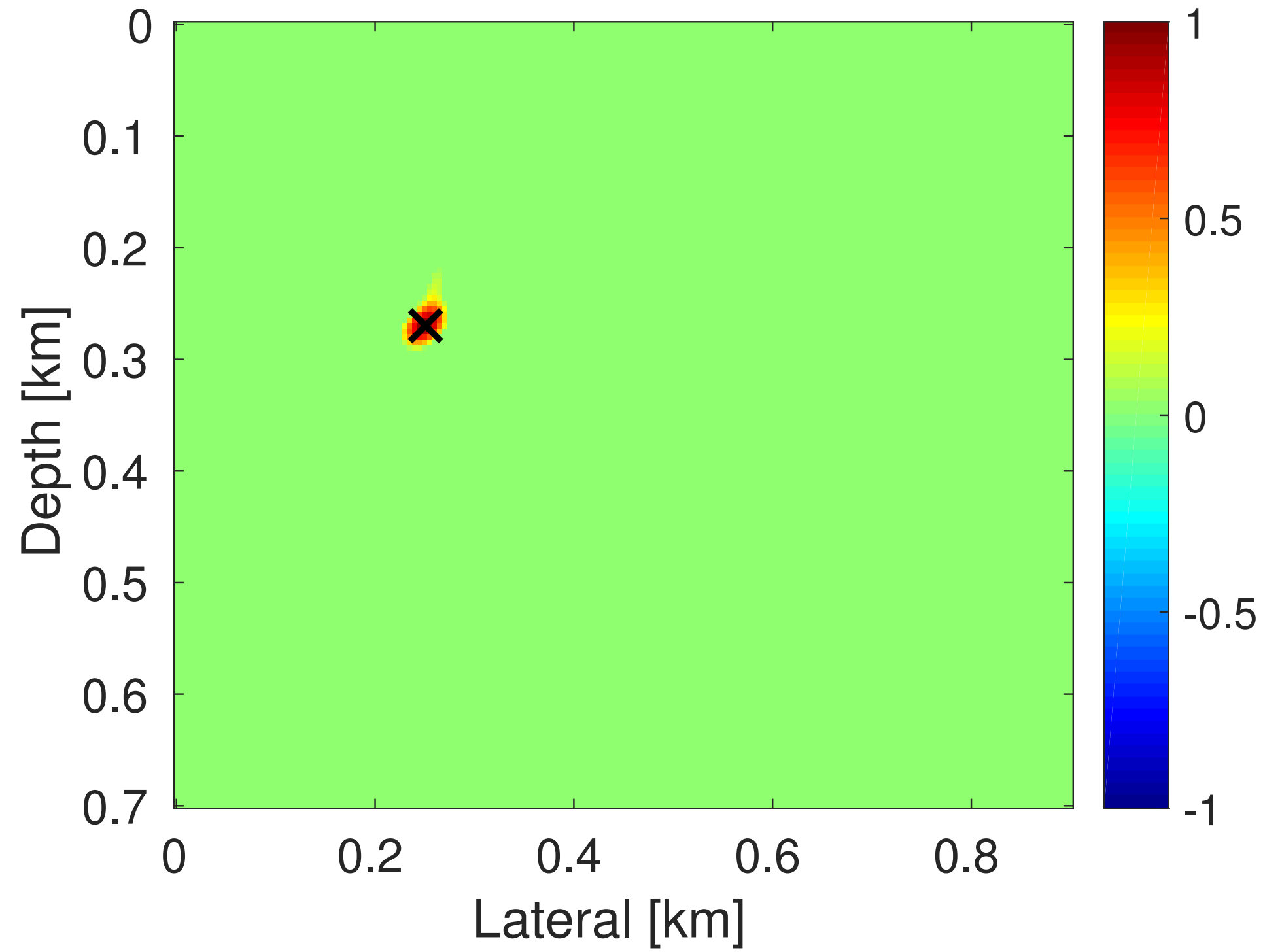
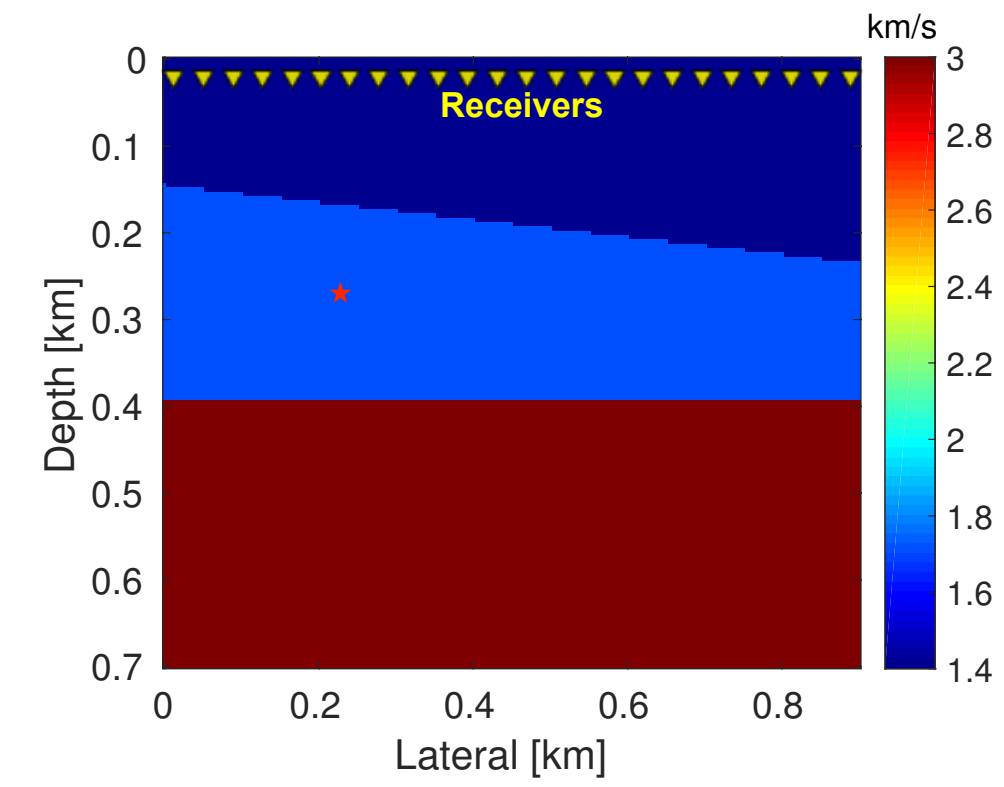


Sparsity-promoting method

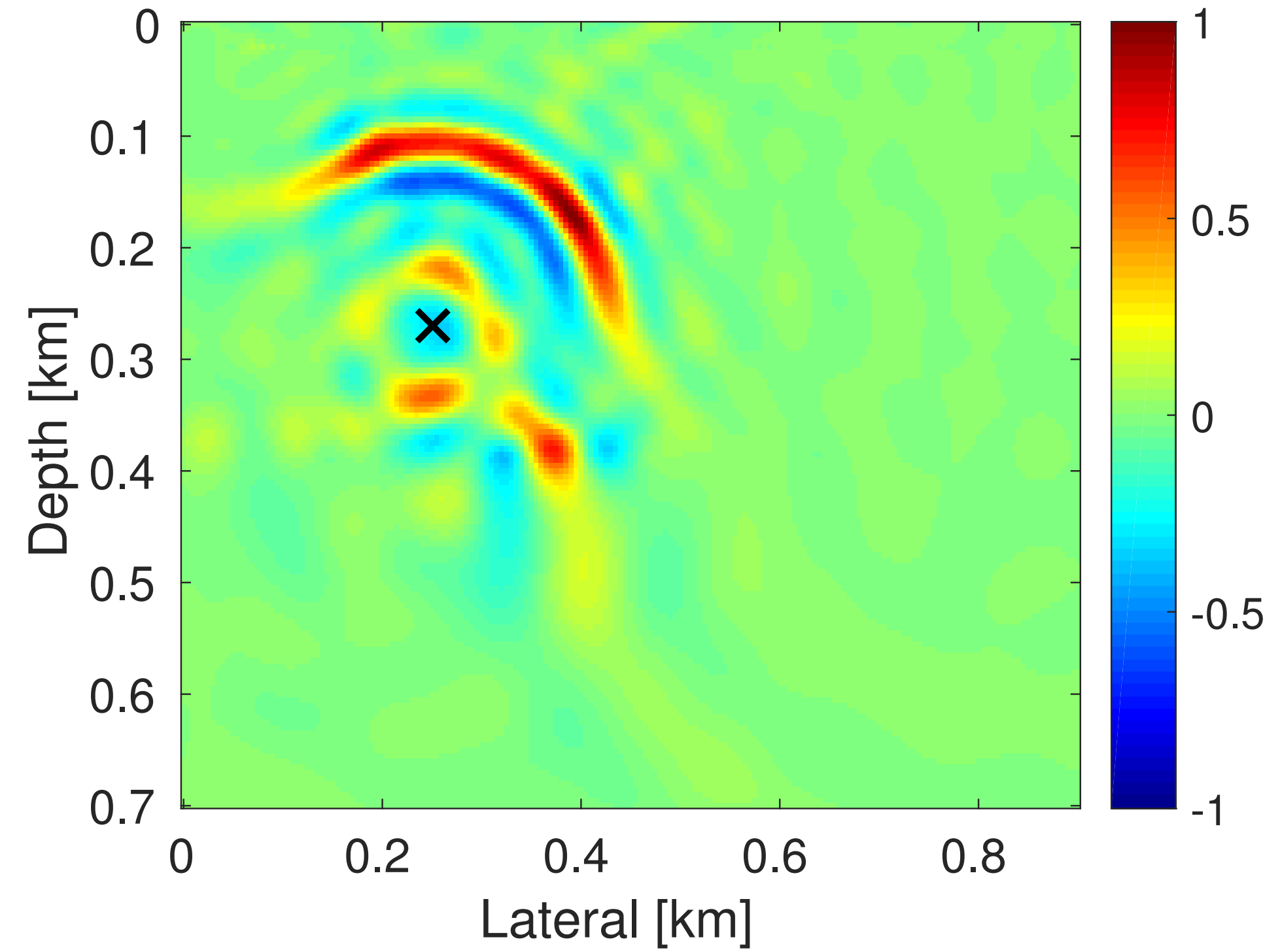


FWI

Estimated source location



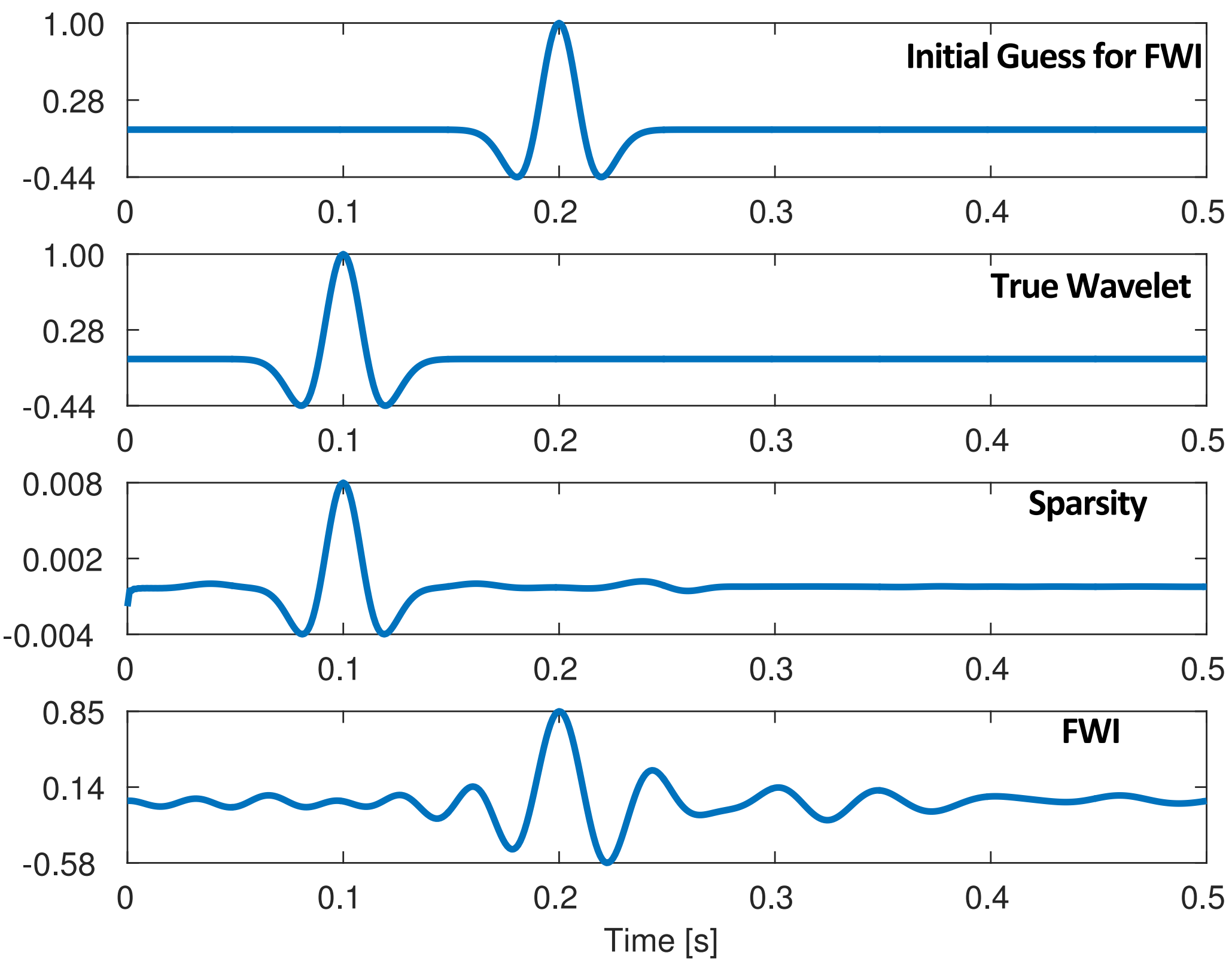
Sparsity-promoting method



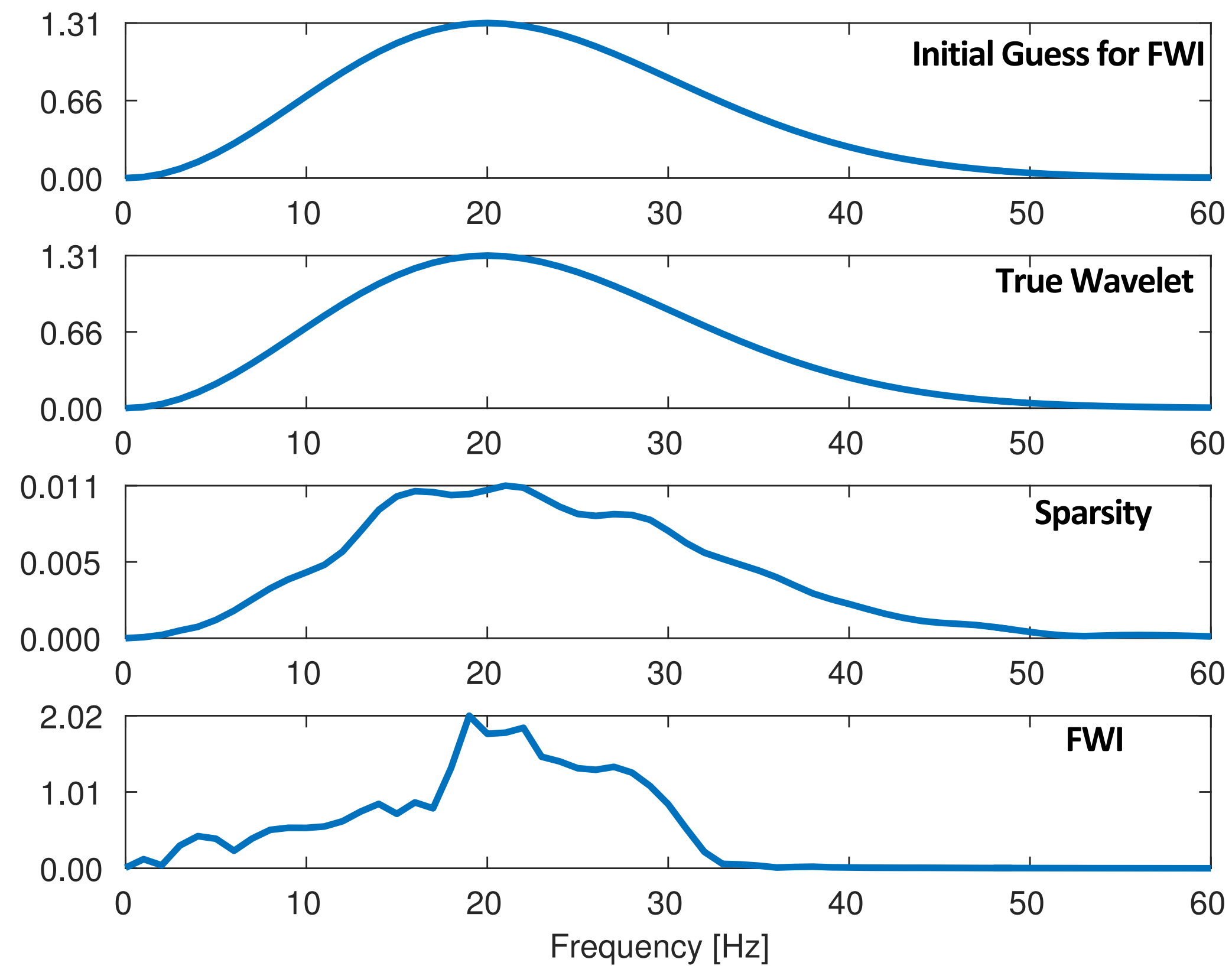
FWI

Wavelet comparison

Wavelet



Spectrum



Our method w/ sparsity promotion

Estimates complete source wavefield in

- ▶ space
- ▶ time

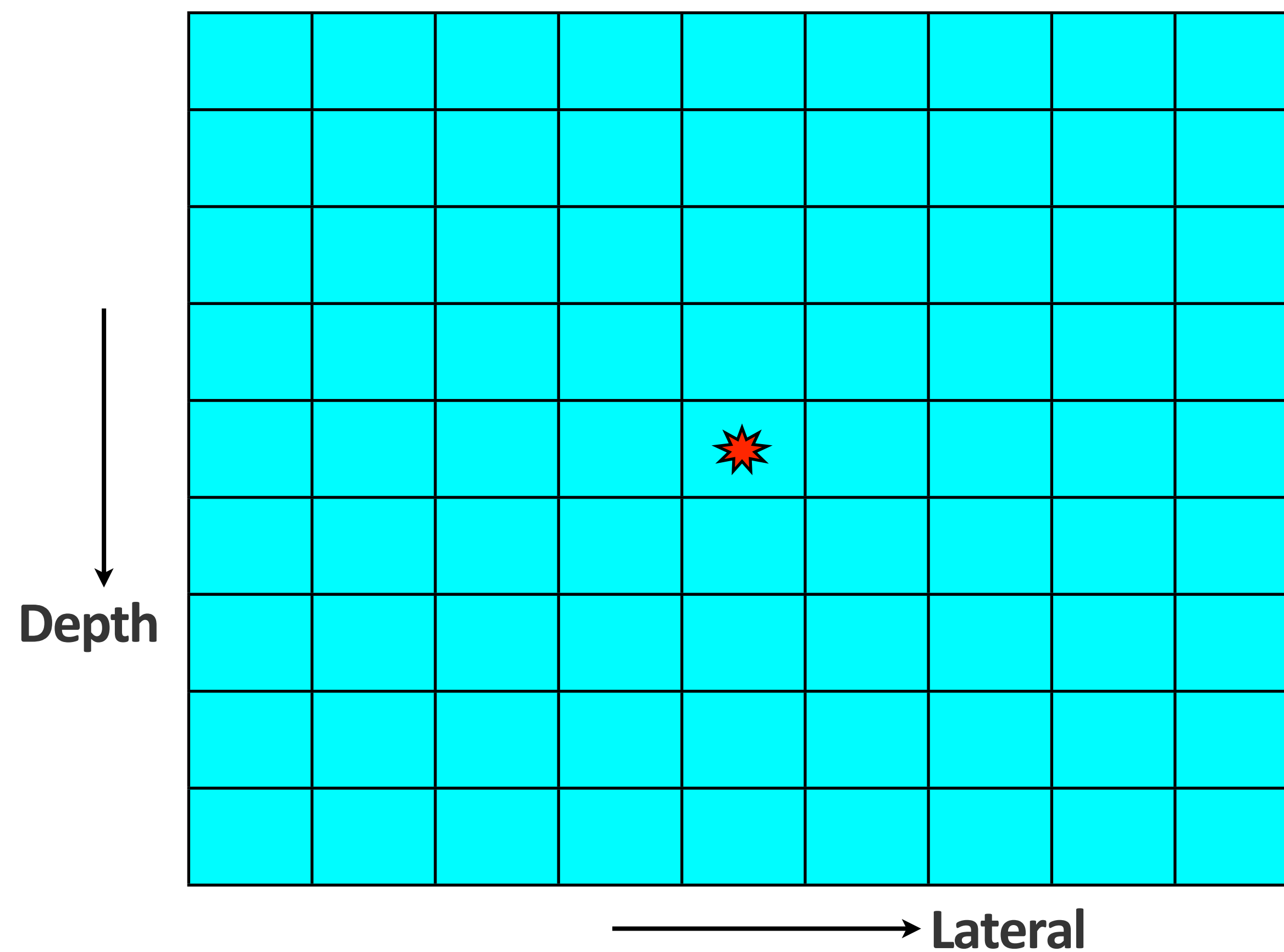
Simultaneous estimation

- ▶ microseismic event location
- ▶ source time function
- ▶ source origin time

Our method w/ sparsity promotion

Assumptions:

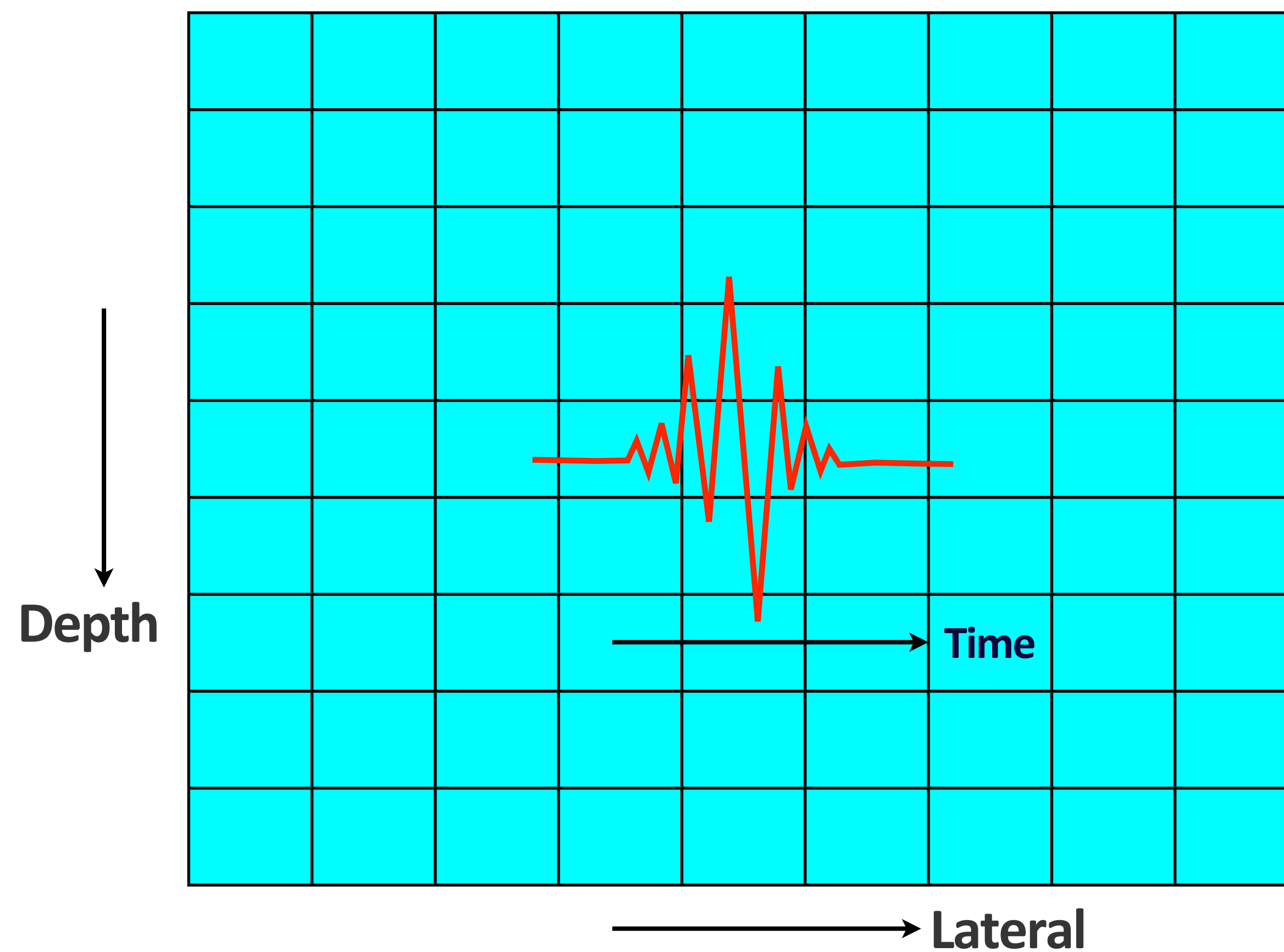
- ▶ localized in space



Our method w/ sparsity promotion

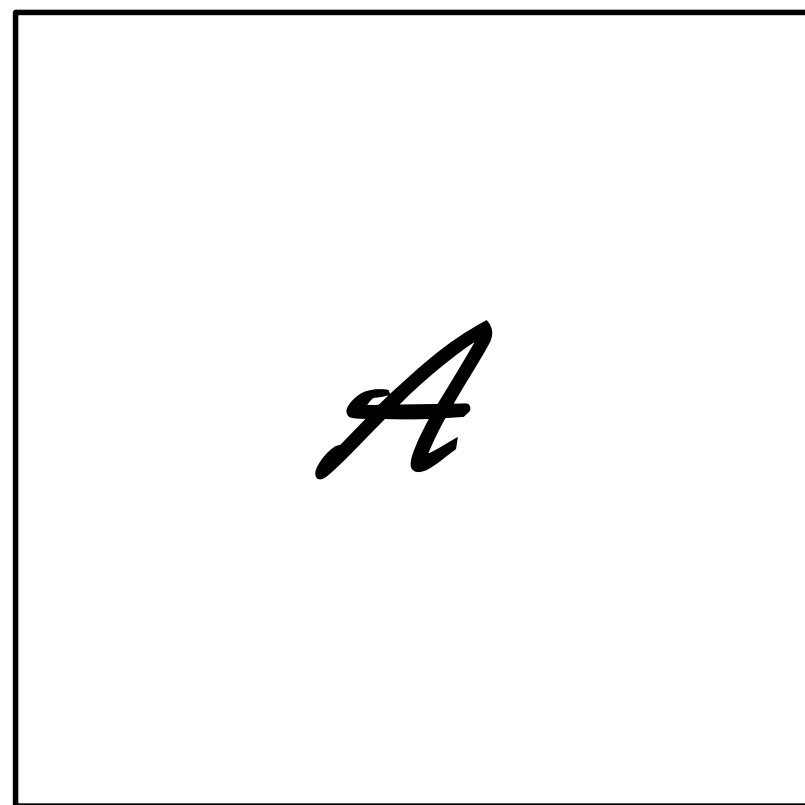
Assumptions:

- ▶ localized in space
- ▶ finite energy along time

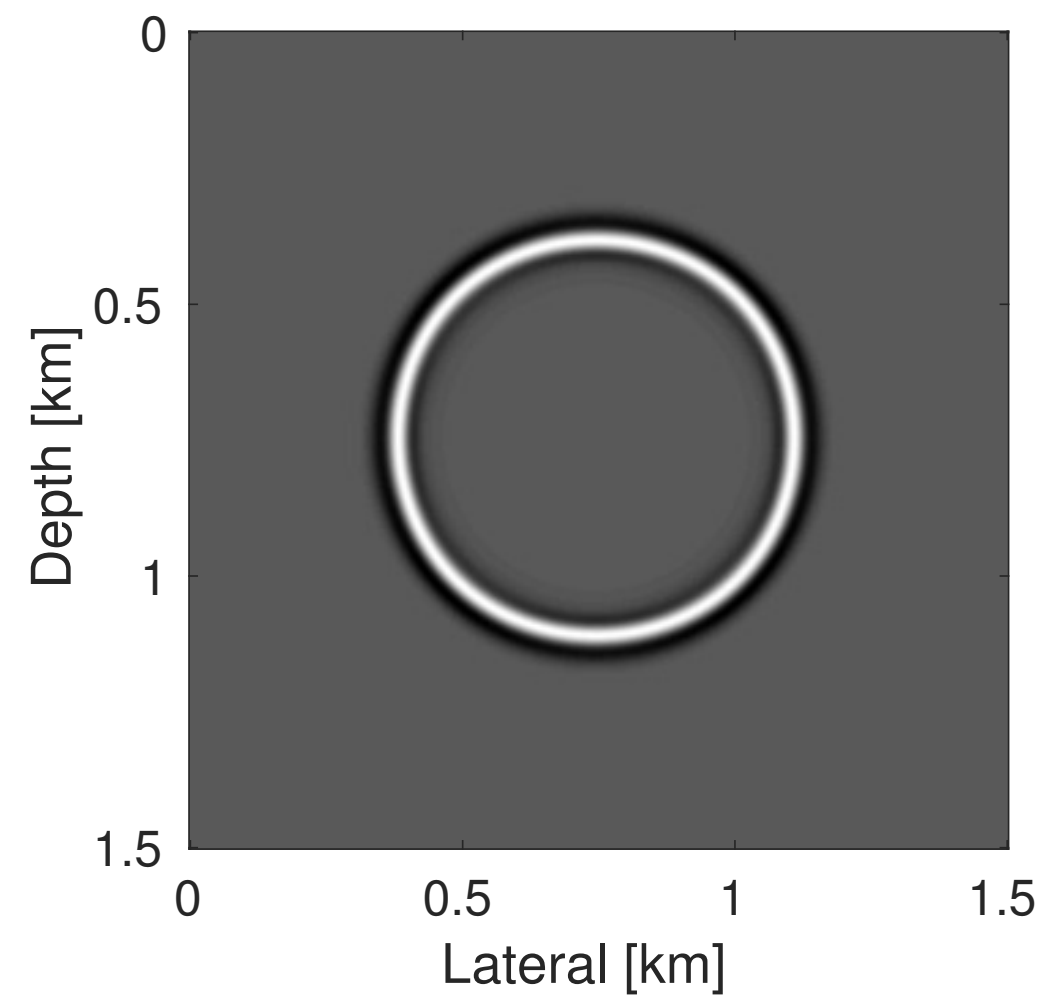


Co-sparsity property of wave equation

Time-stepping operator

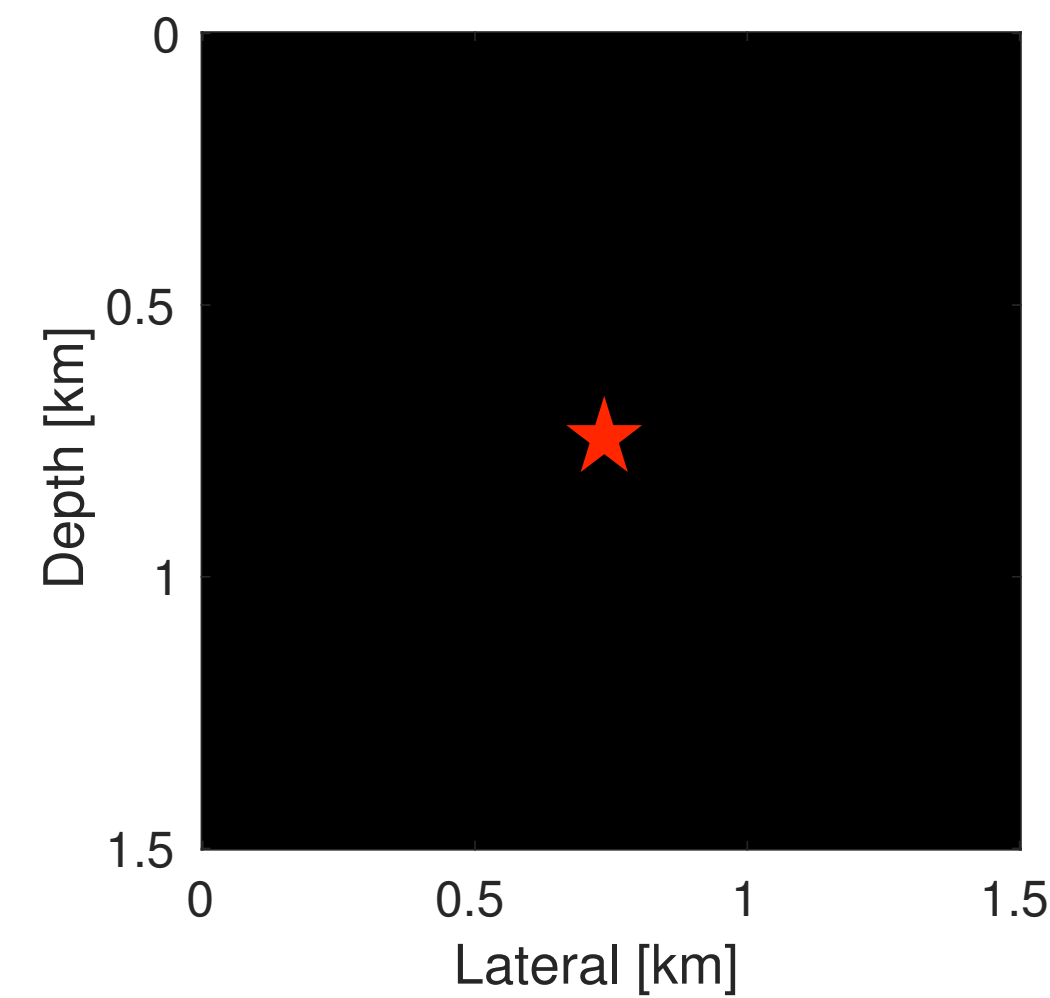


Wavefield

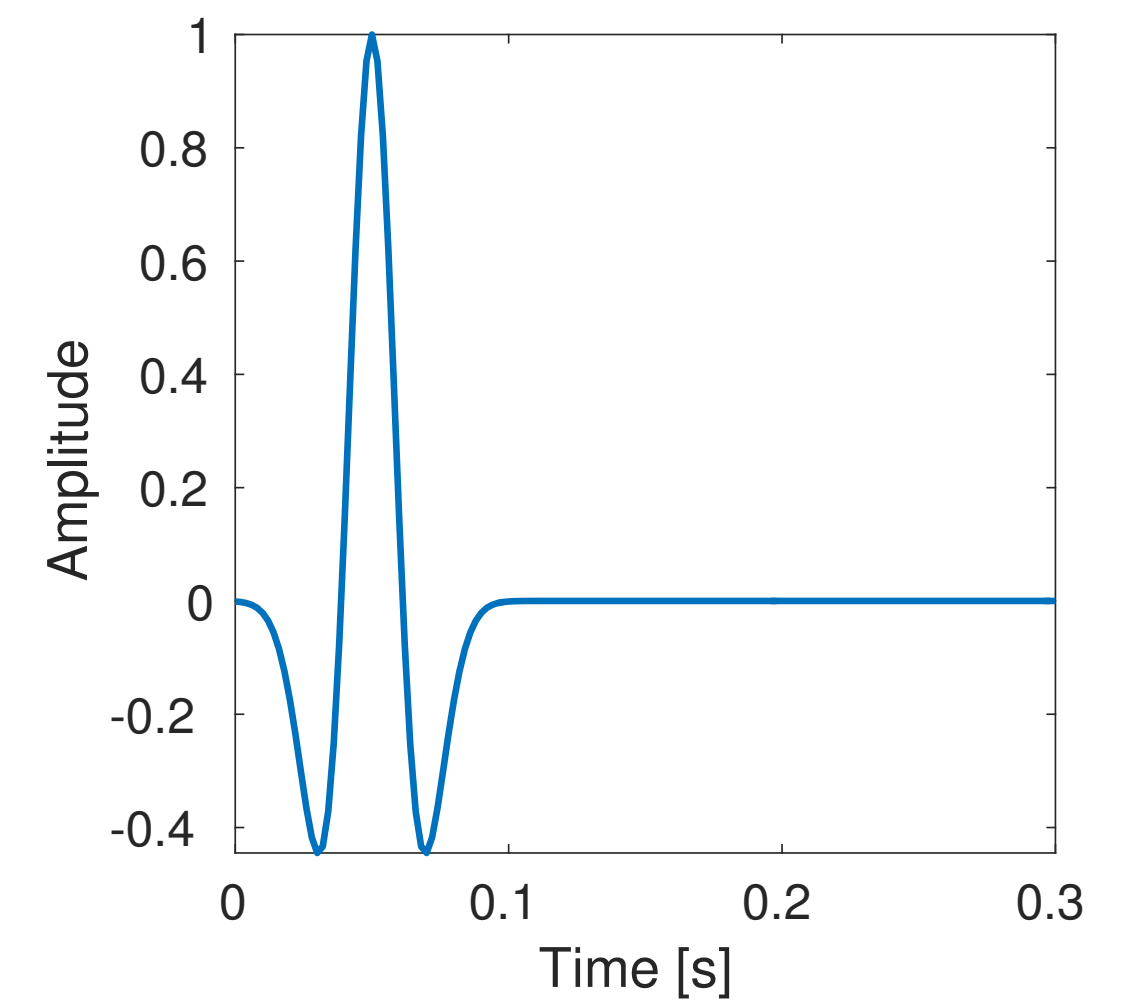


=

Source



Source time function

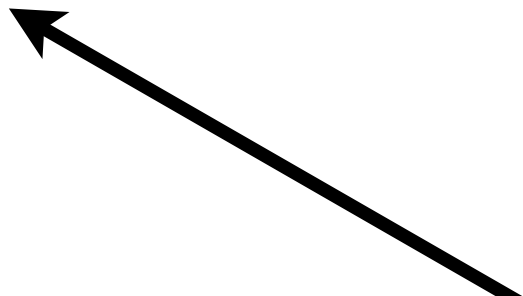


$$\mathcal{A}[\mathbf{m}](\mathbf{u}) = \mathbf{q}$$

Problem statement

$$\begin{aligned} & \underset{\mathbf{u}}{\text{minimize}} \quad \|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1} \\ & \text{subject to} \quad \|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

Problem statement

$$\begin{aligned} & \underset{\mathbf{u}}{\text{minimize}} \quad \|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1} \\ & \text{subject to} \quad \|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$


Problem statement

**Time stepping
operator**

$$\underset{\mathbf{u}}{\text{minimize}} \quad \|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1}$$

$$\text{subject to} \quad \|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon$$

Problem statement

**Time stepping
operator**

$$\underset{\mathbf{u}}{\text{minimize}} \quad \|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1}$$

$$\text{subject to } \|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon$$

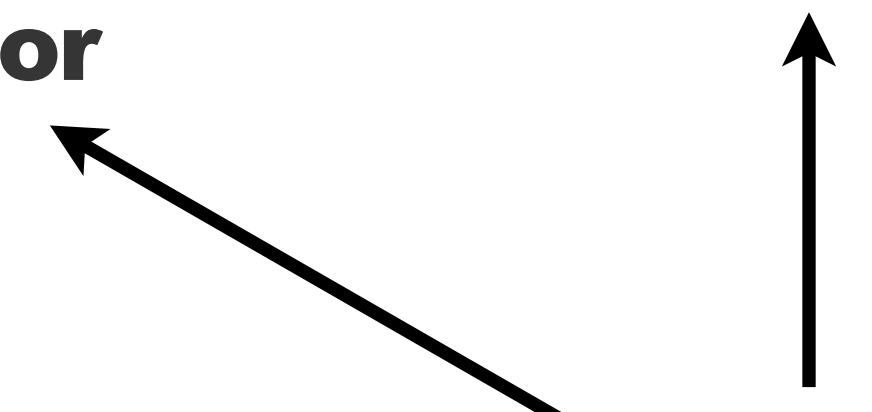
Problem statement

Time stepping operator

Slowness square

minimize $\|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1}$

subject to $\|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon$



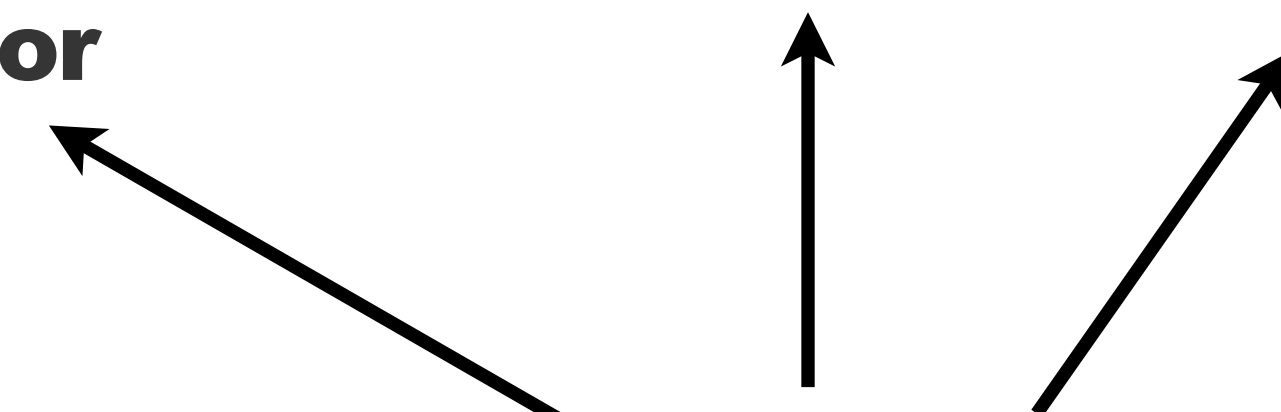
Problem statement

Time stepping operator

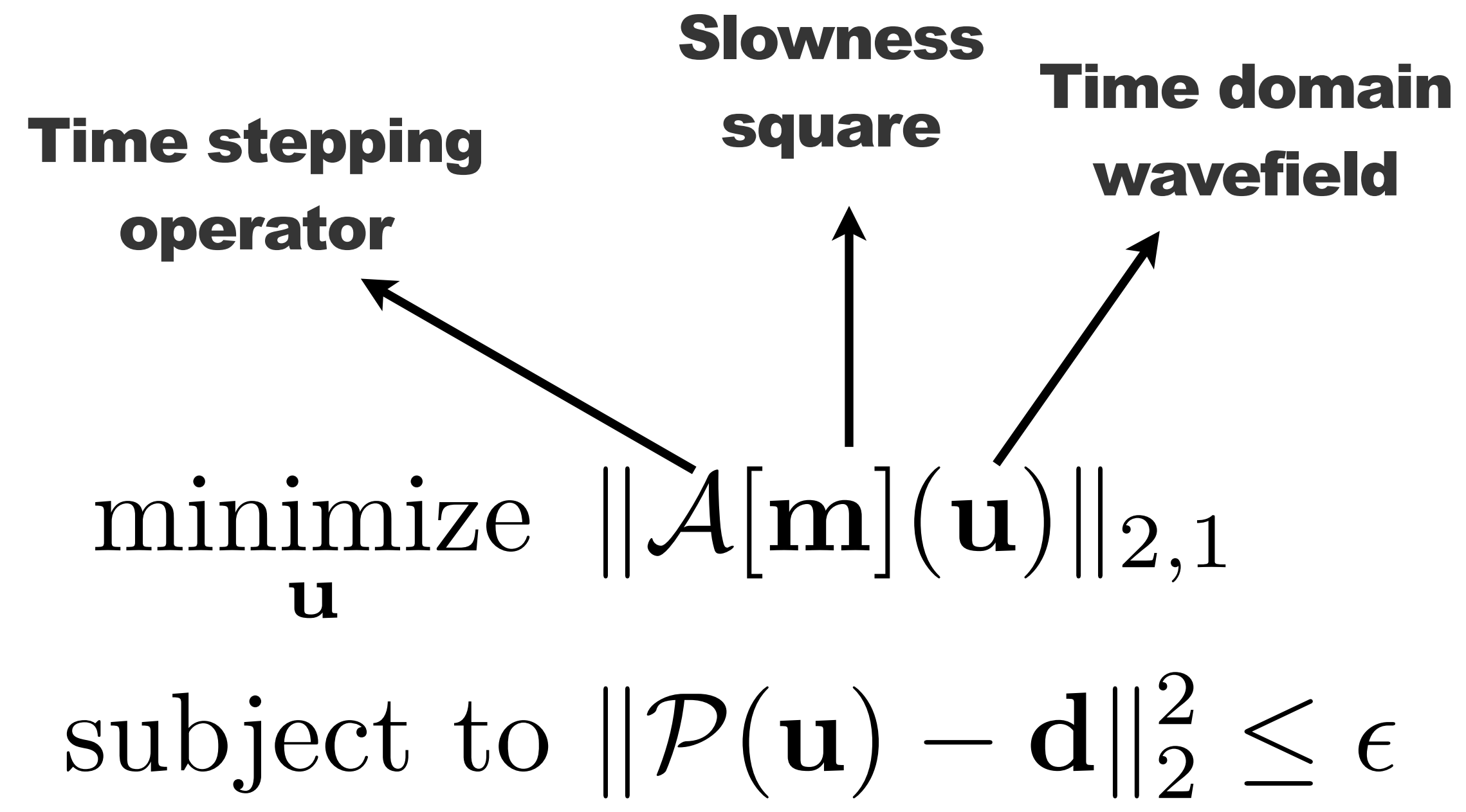
Slowness square

minimize $\|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1}$

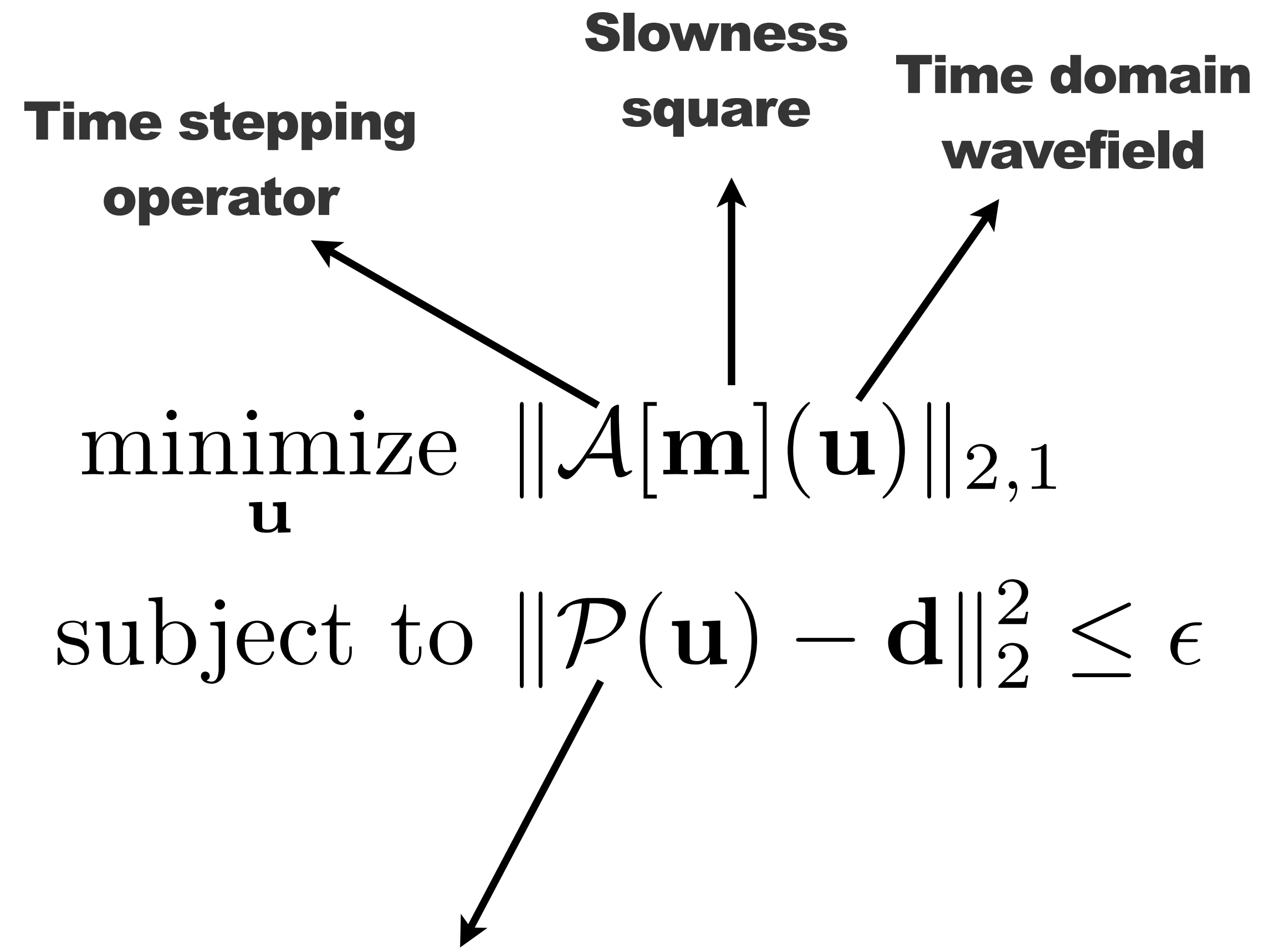
subject to $\|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon$



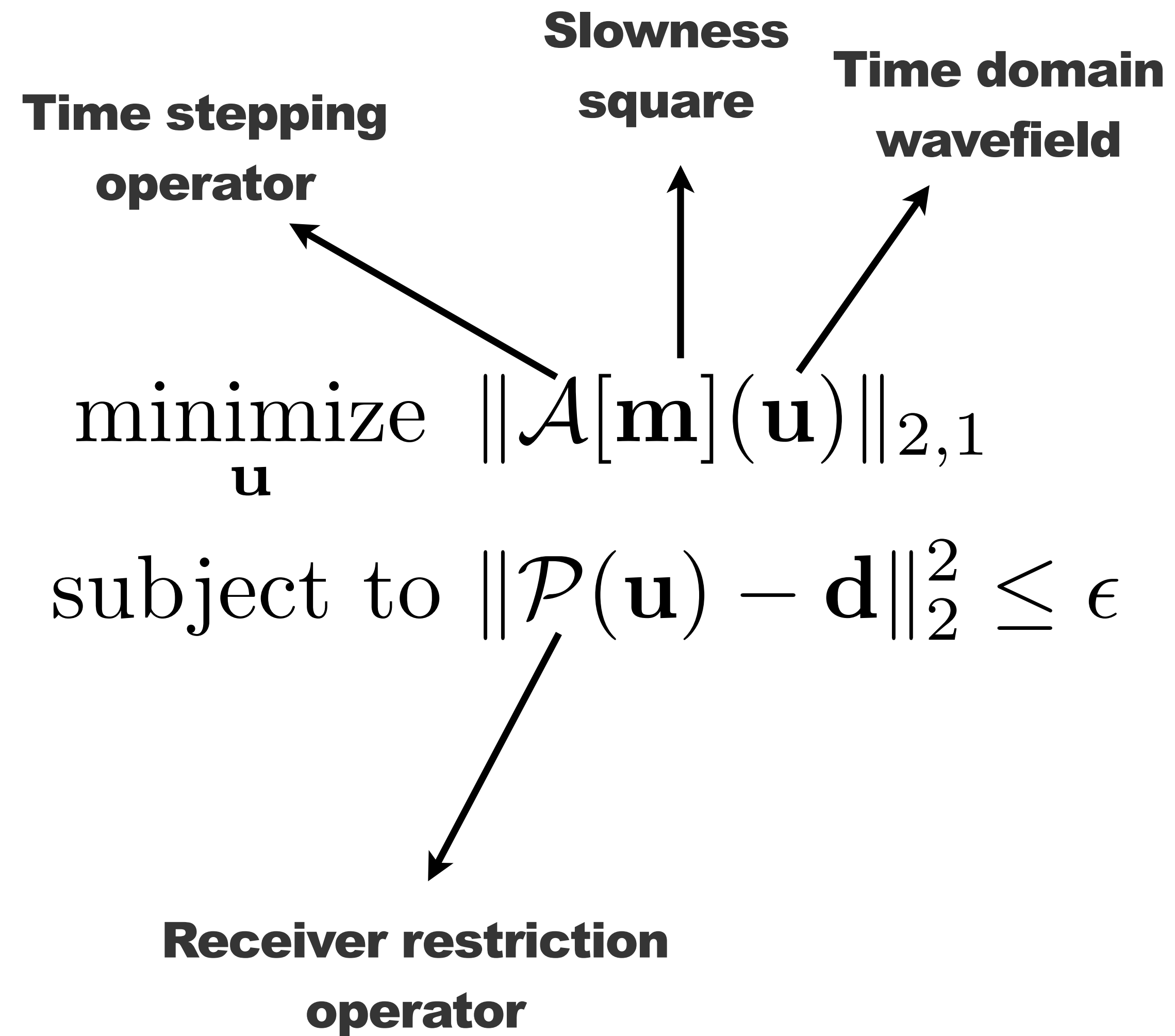
Problem statement



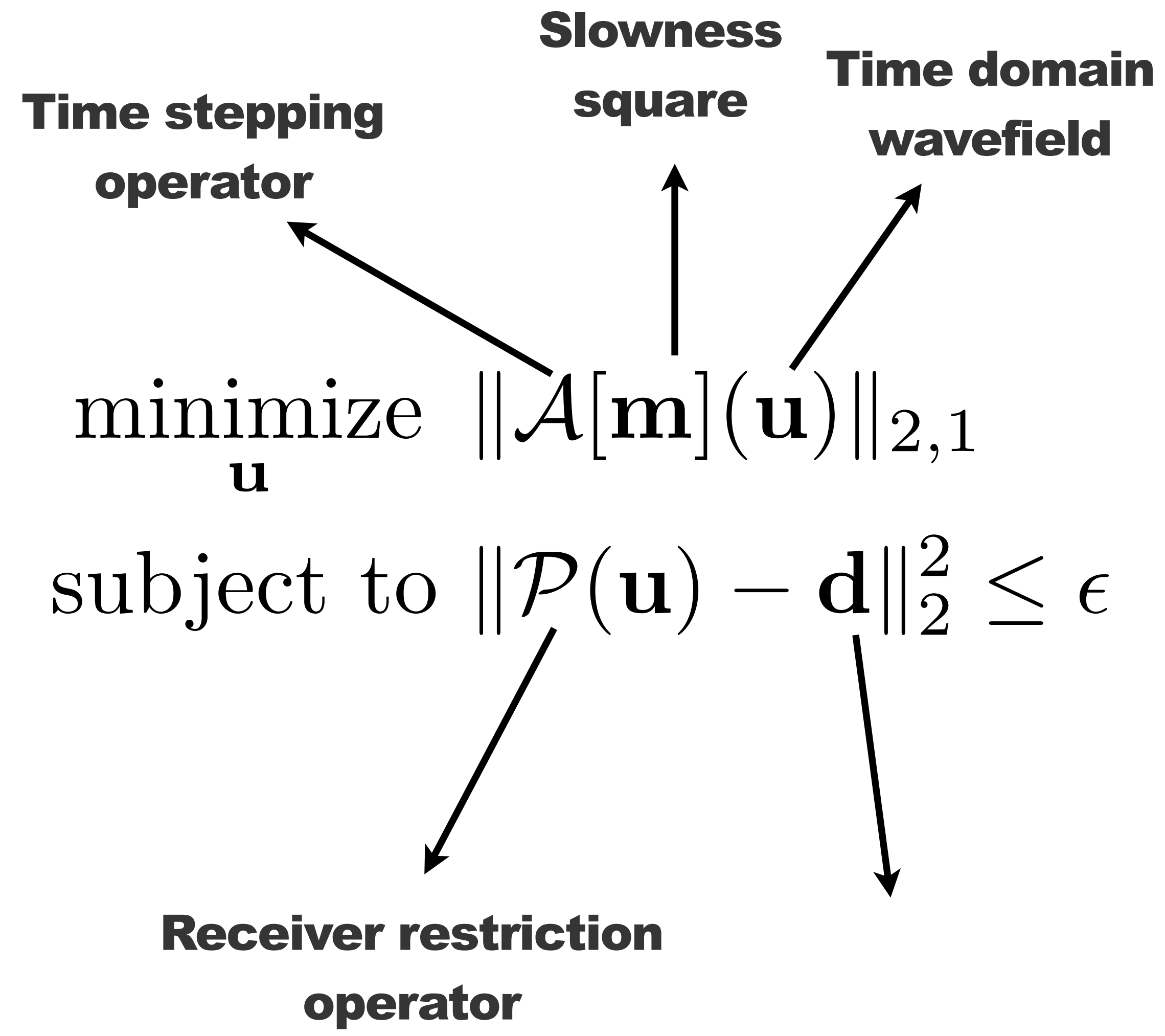
Problem statement



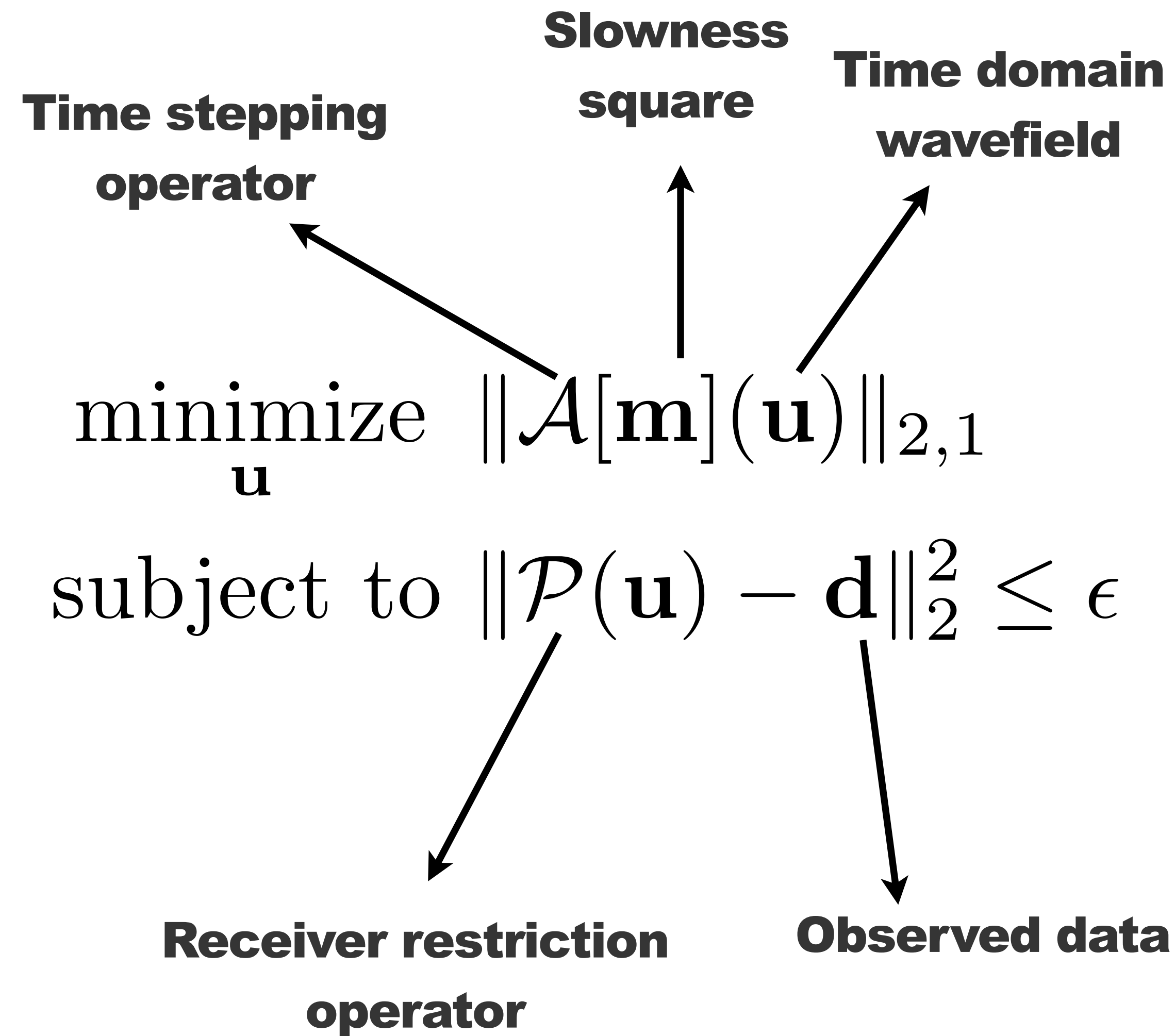
Problem statement



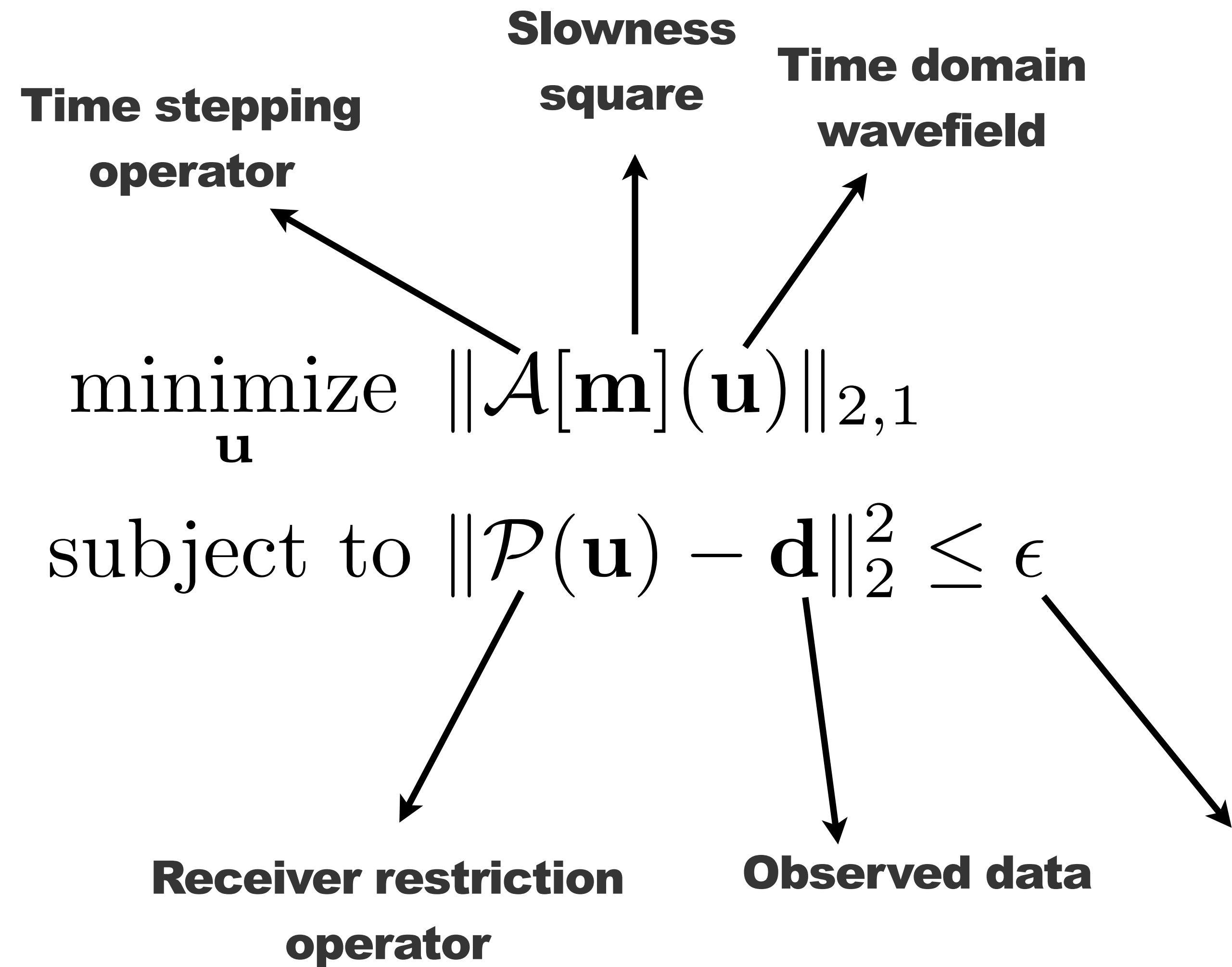
Problem statement



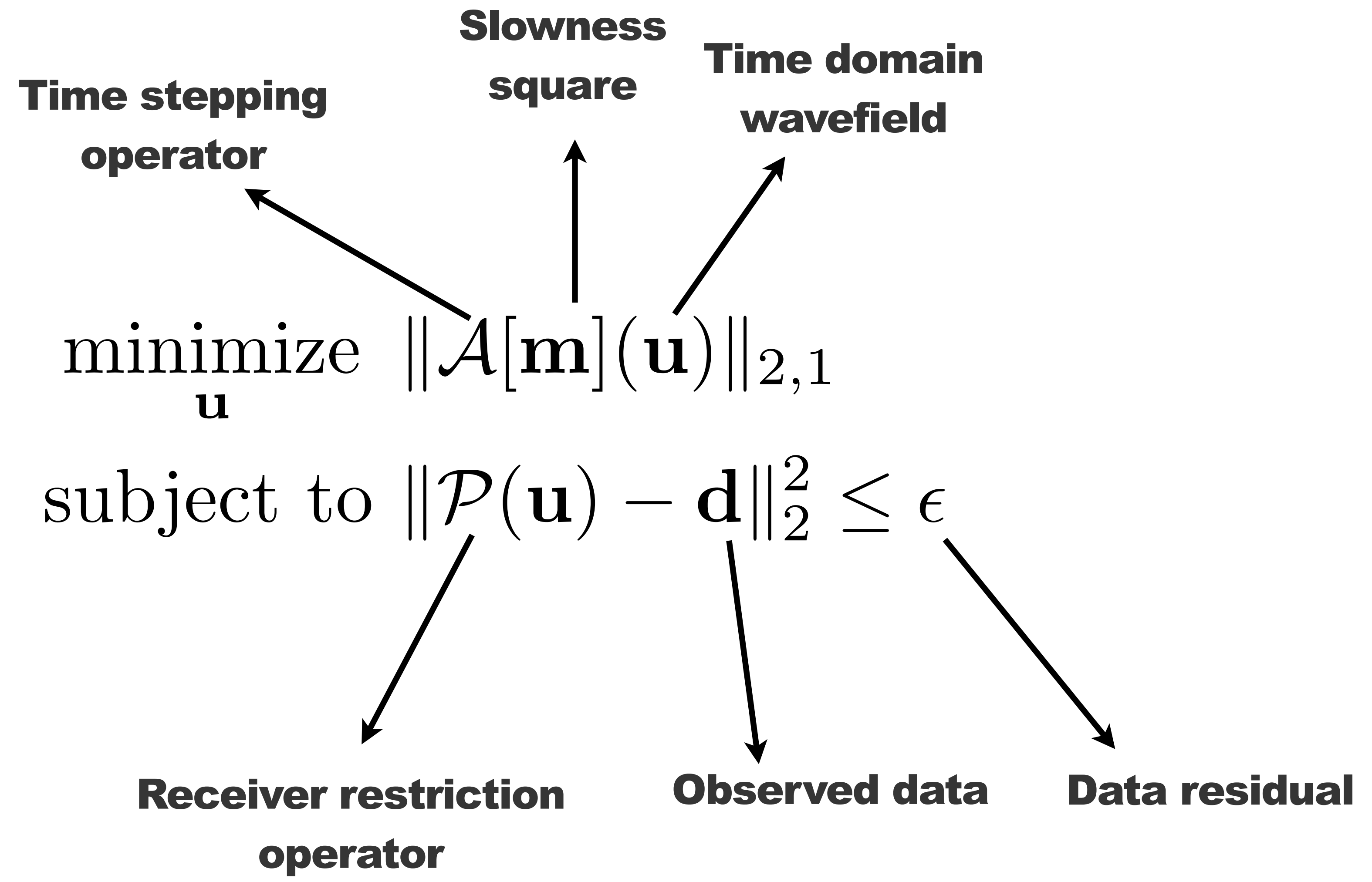
Problem statement



Problem statement



Problem statement



Our method w/ sparsity promotion

$$\begin{aligned} & \underset{\mathbf{u}}{\text{minimize}} \quad \|\mathcal{A}[\mathbf{m}](\mathbf{u})\|_{2,1} \\ & \text{subject to} \quad \|\mathcal{P}(\mathbf{u}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

- ✓ Does not require separable structure of source term into spatial & temporal components
- ✓ Does not require prior information on number of sources
- ✓ Simultaneously estimates location/directivity pattern & source origin/source time function

Method

The above optimization problem is made more tractable by change of variable $\mathcal{A}[\mathbf{m}](\mathbf{u}) = \mathbf{Q}$

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

Method

The above optimization problem is made more tractable by change of variable $\mathcal{A}[\mathbf{m}](\mathbf{u}) = \mathbf{Q}$

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

Similar to classic Basis Pursuit Denoising (BPDN) Problem

Modified Linearized Bregman

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Q}\|_F^2 \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

*where $\|\cdot\|_F$ is the Frobenius norm

- ▶ Recent successful application
- ▶ Three-step algorithm simple to implement
- ▶ Solves slightly relaxed version of original Basis Pursuit Denoising problem

Modified Linearized Bregman

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Q}\|_F^2 \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

*where $\|\cdot\|_F$ is the Frobenius norm

- ▶ Choice of μ controls the trade off between sparsity and the Frobenius norm
- ▶ $\mu \uparrow \infty$ corresponds to solving original BPDN problem

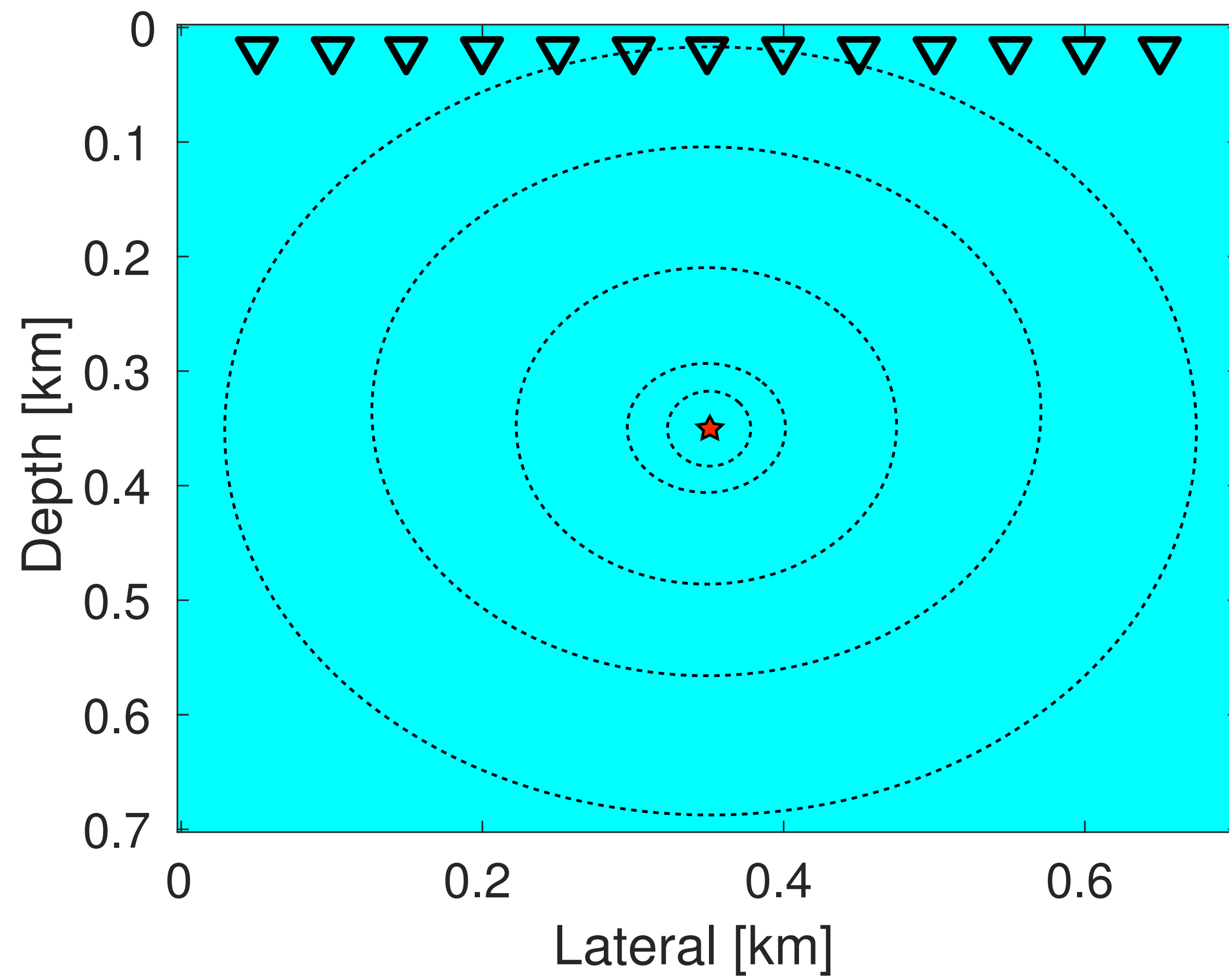
Algorithm

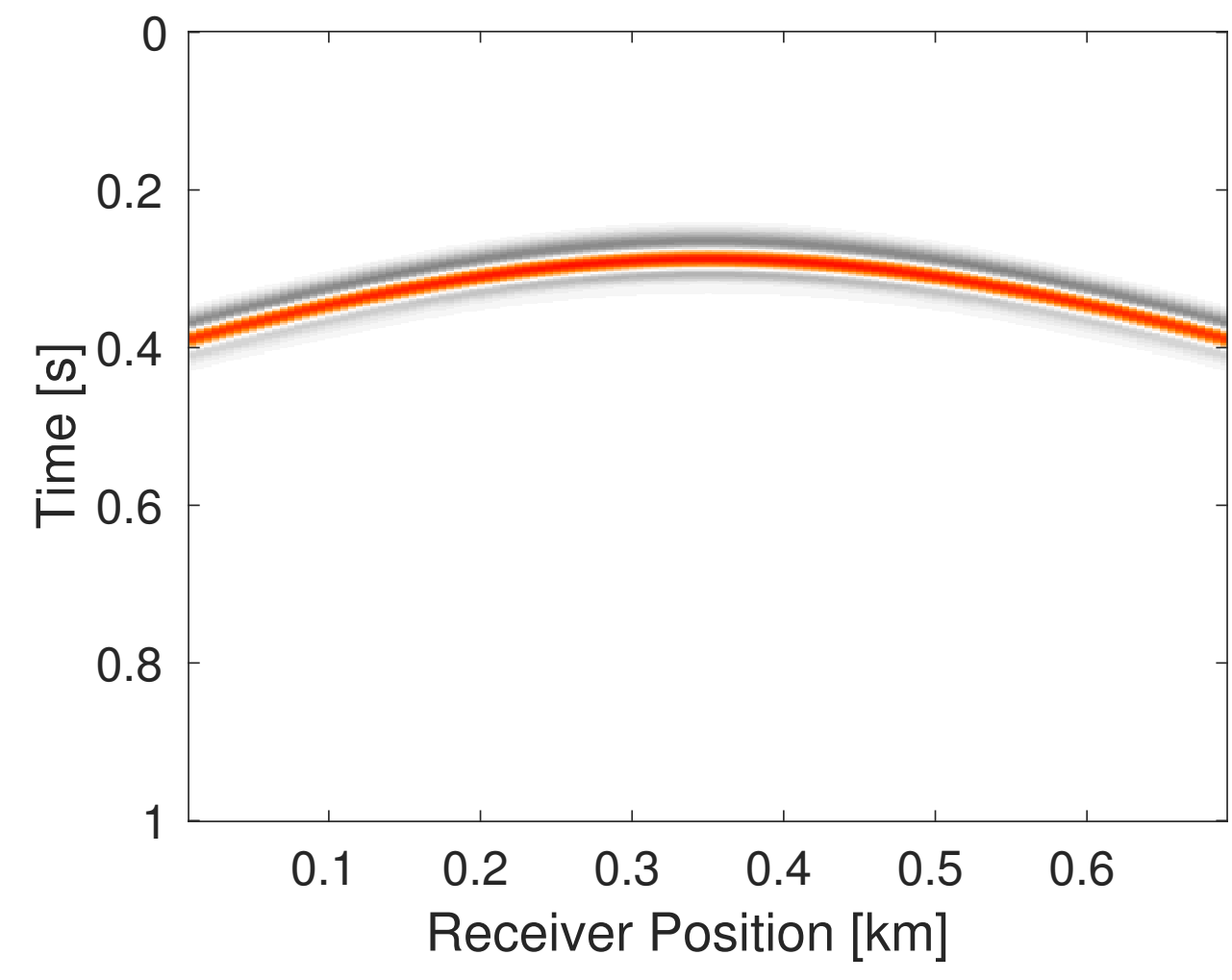
1. **for** $k = 0, 1, \dots$
2. $\mathbf{V}_k = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
3. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
4. $\mathbf{Q}_{k+1} = \text{Prox}_{\mu\|\cdot\|_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
5. **end**

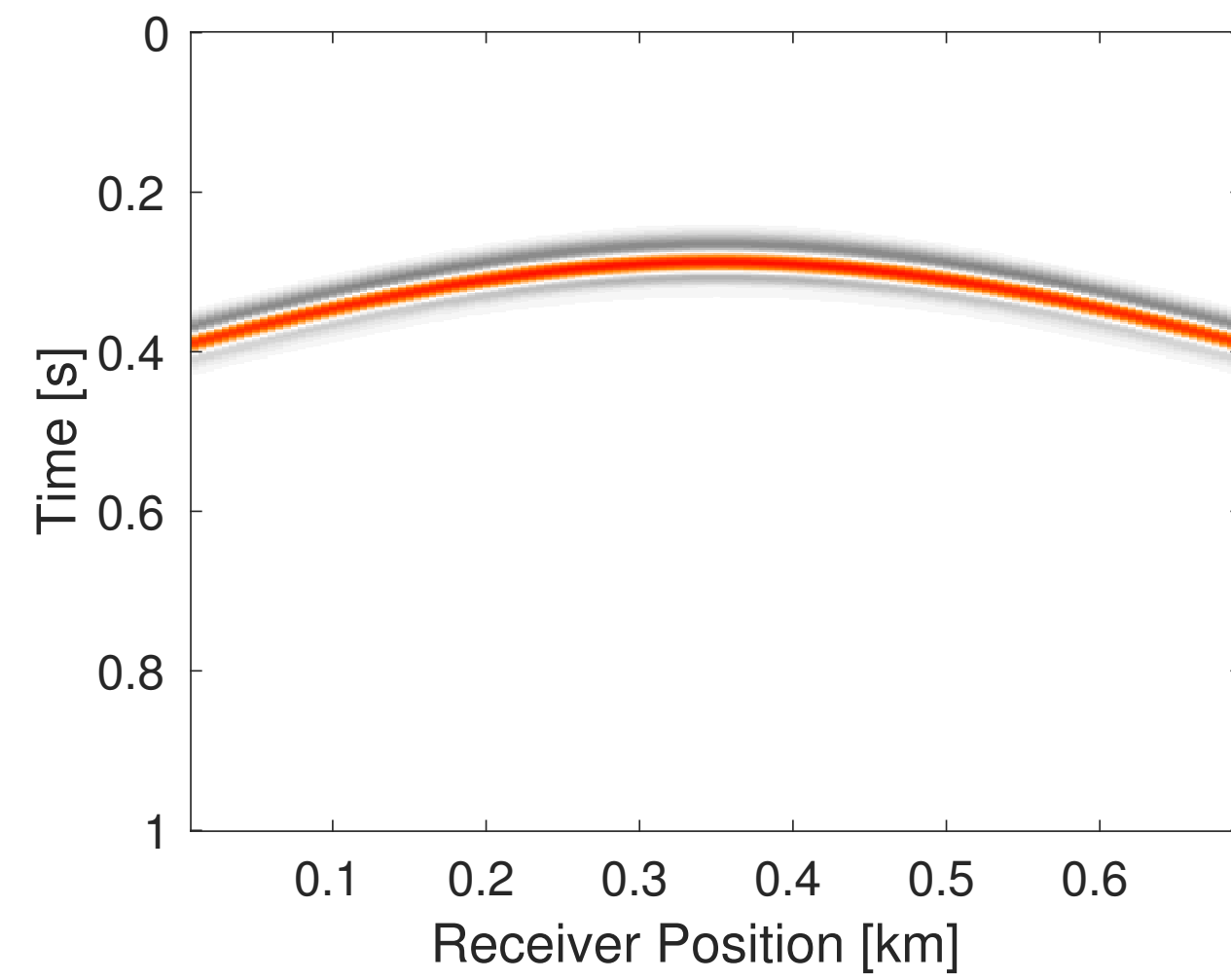
*where $t_k = \frac{\|\mathcal{F}(\mathbf{m})\mathbf{Q}_k - \mathbf{d}\|^2}{\|\mathcal{F}(\mathbf{m})^T(\mathcal{F}(\mathbf{m})\mathbf{Q}_k - \mathbf{d})\|^2}$ is the dynamic step length

* $\text{Prox}_{\mu\|\cdot\|_{2,1}}(c) := \arg \min_b \mu\|b\|_{2,1} + \frac{1}{2}\|c - b\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

* $\Pi_\epsilon(\mathbf{x}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{x}\|}\} \cdot (\mathbf{x})$ the projection on to ℓ_2 norm ball



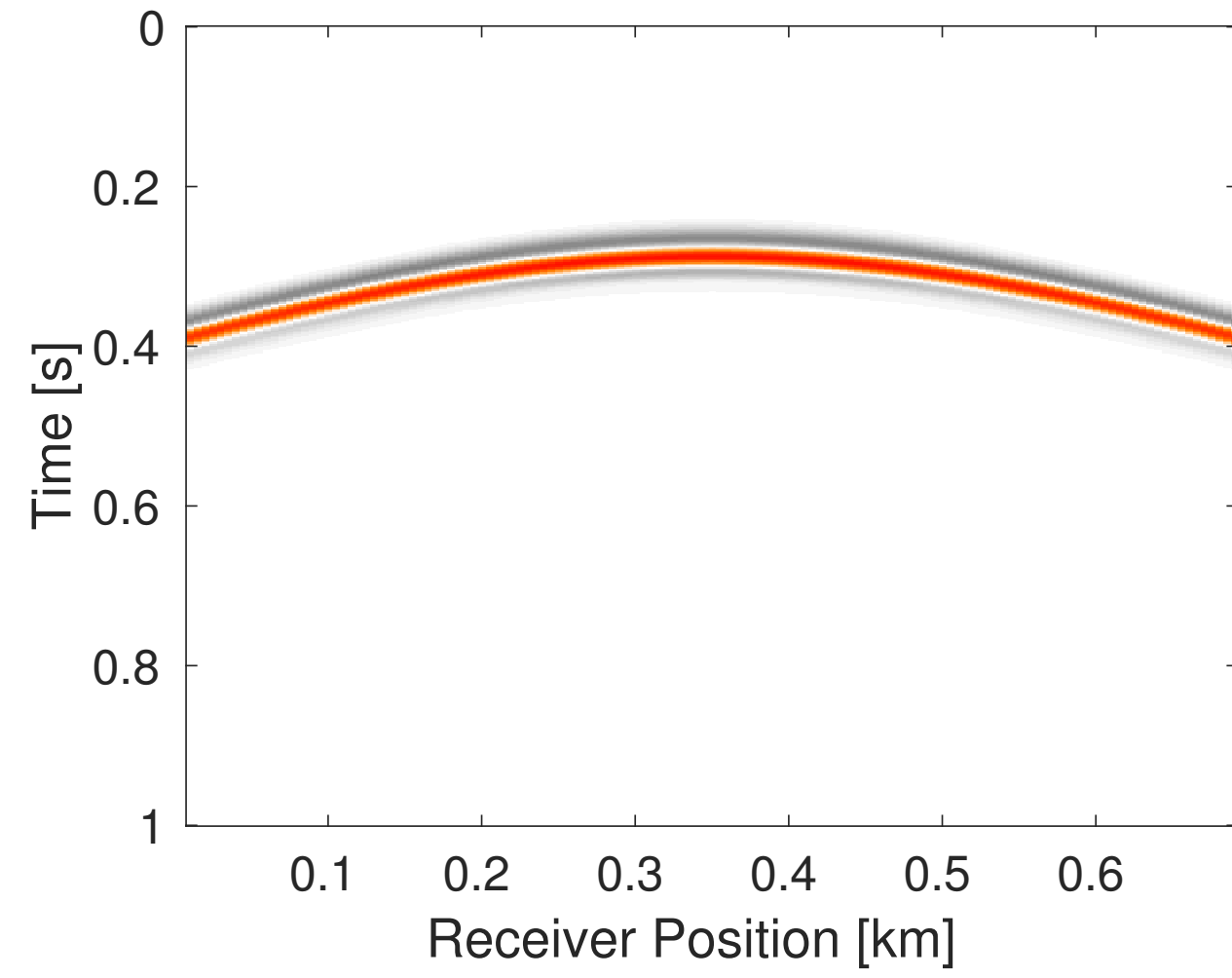




$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$



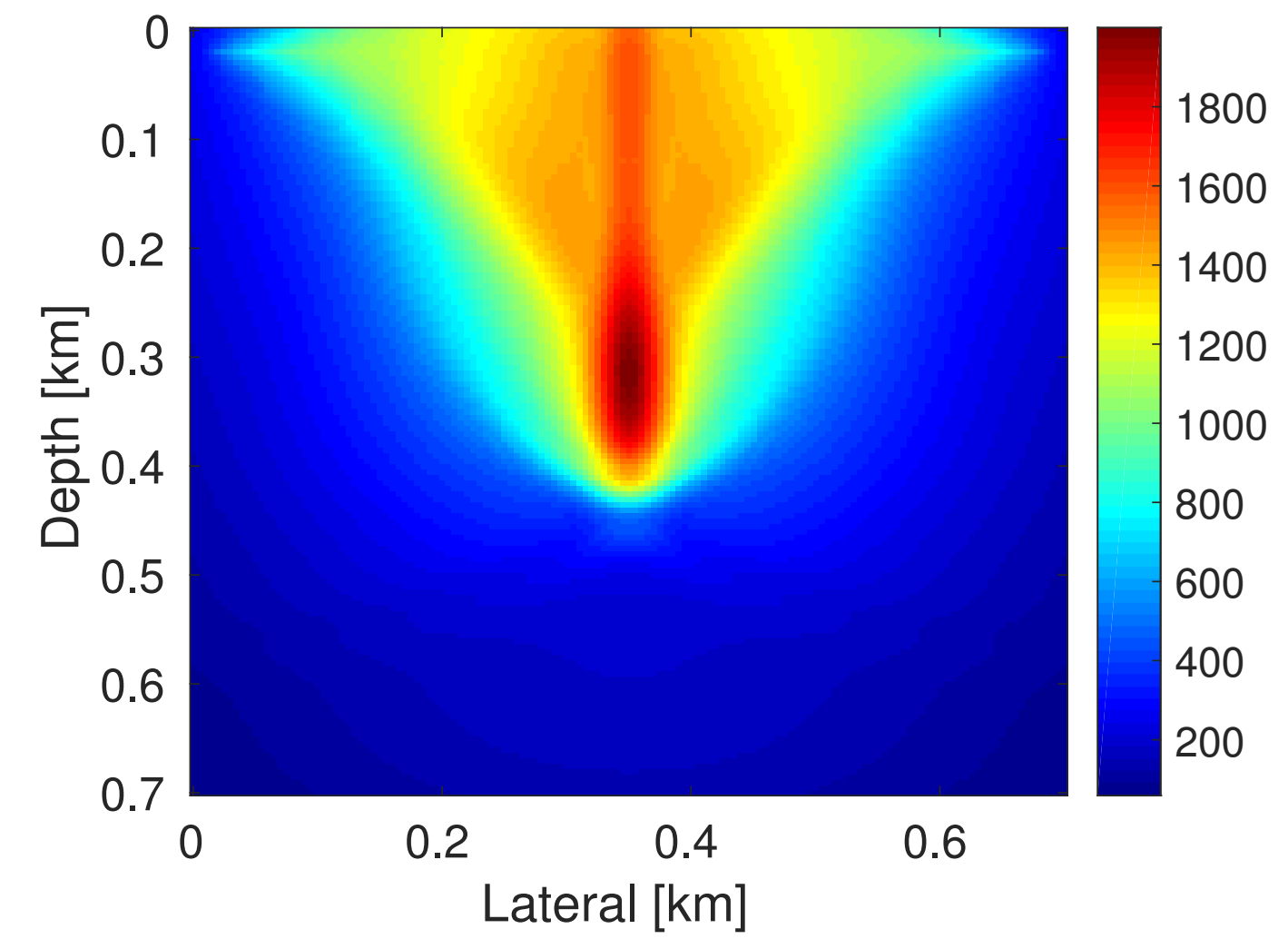
Adjoint solve

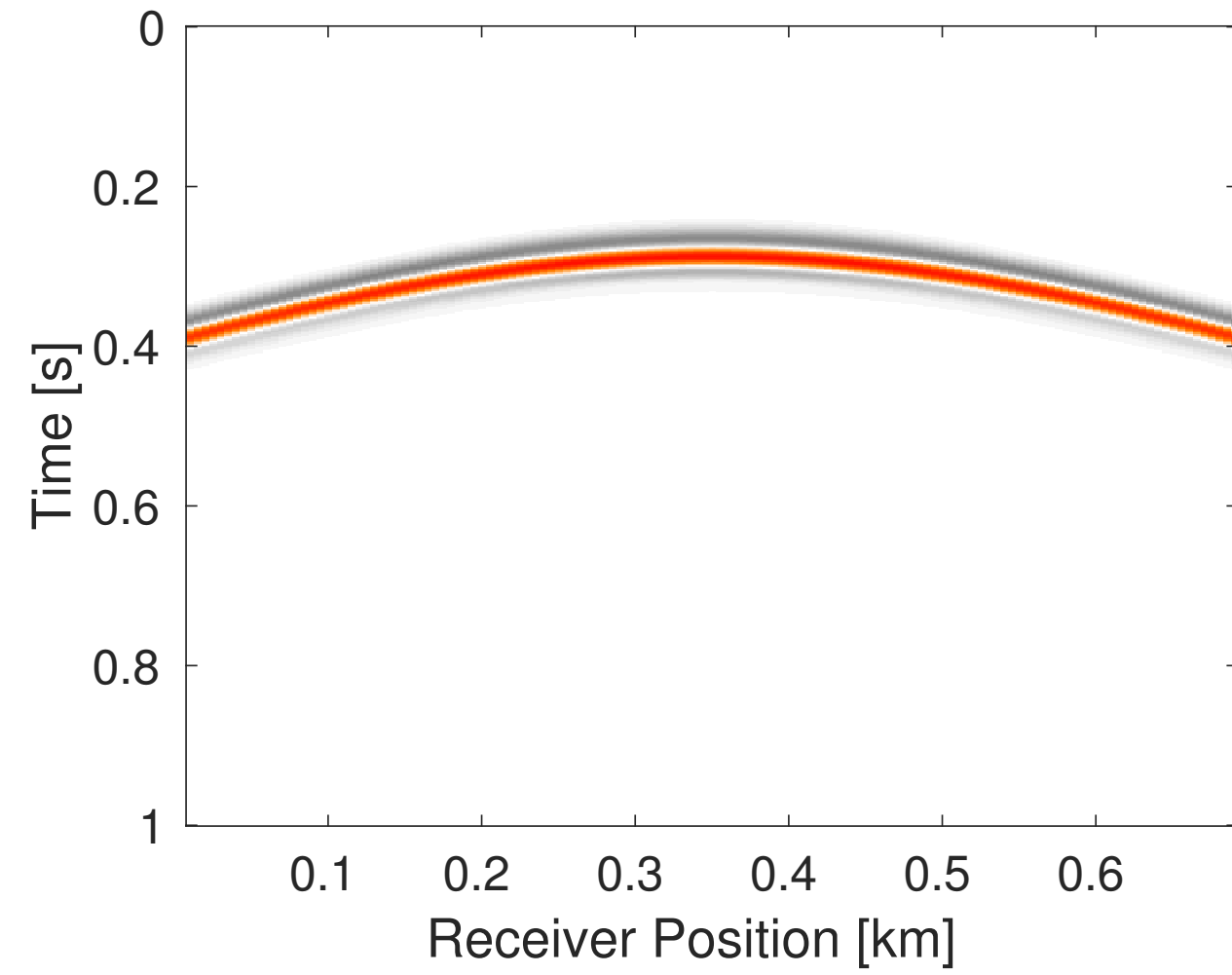


$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$



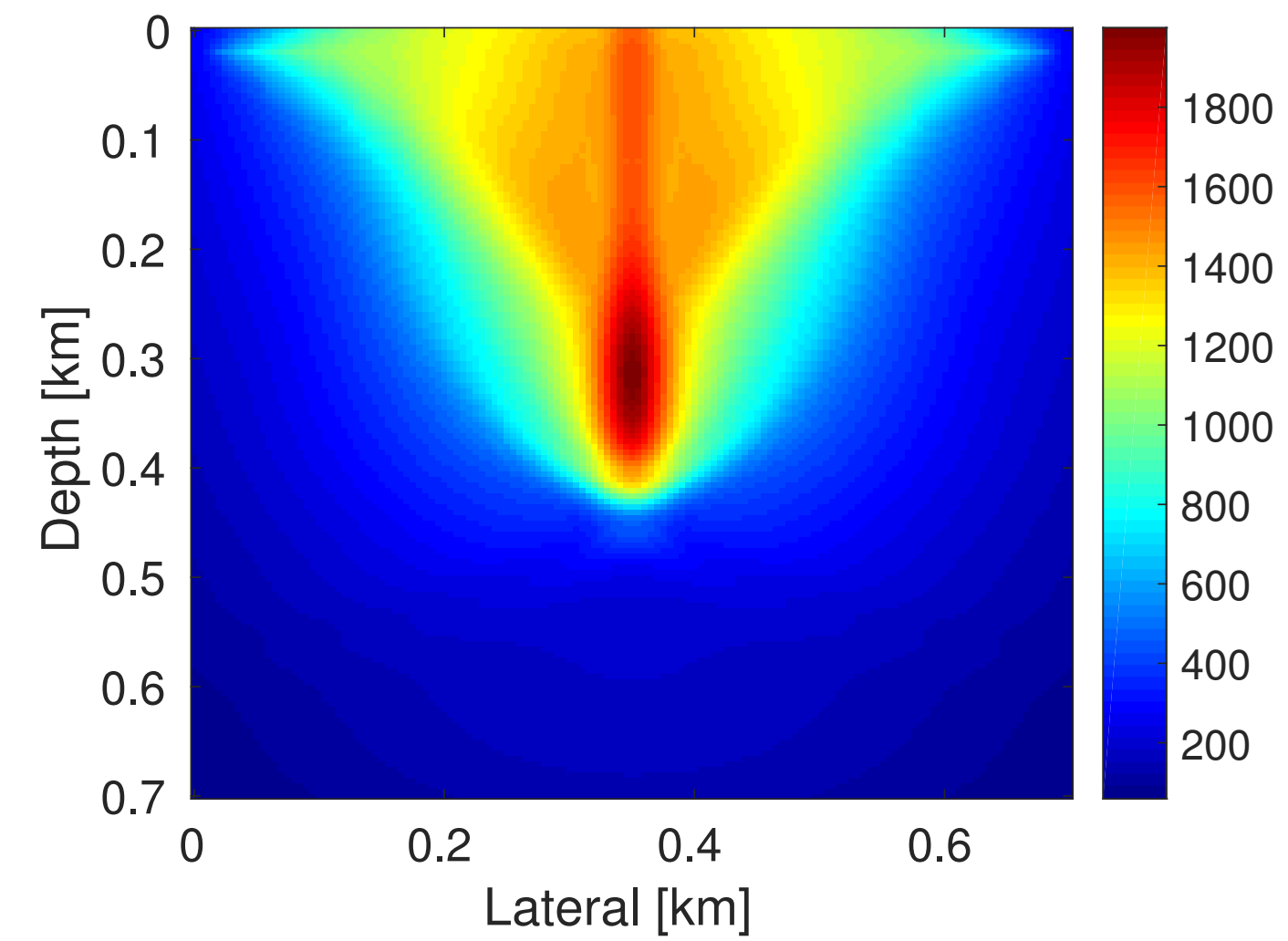
Adjoint solve





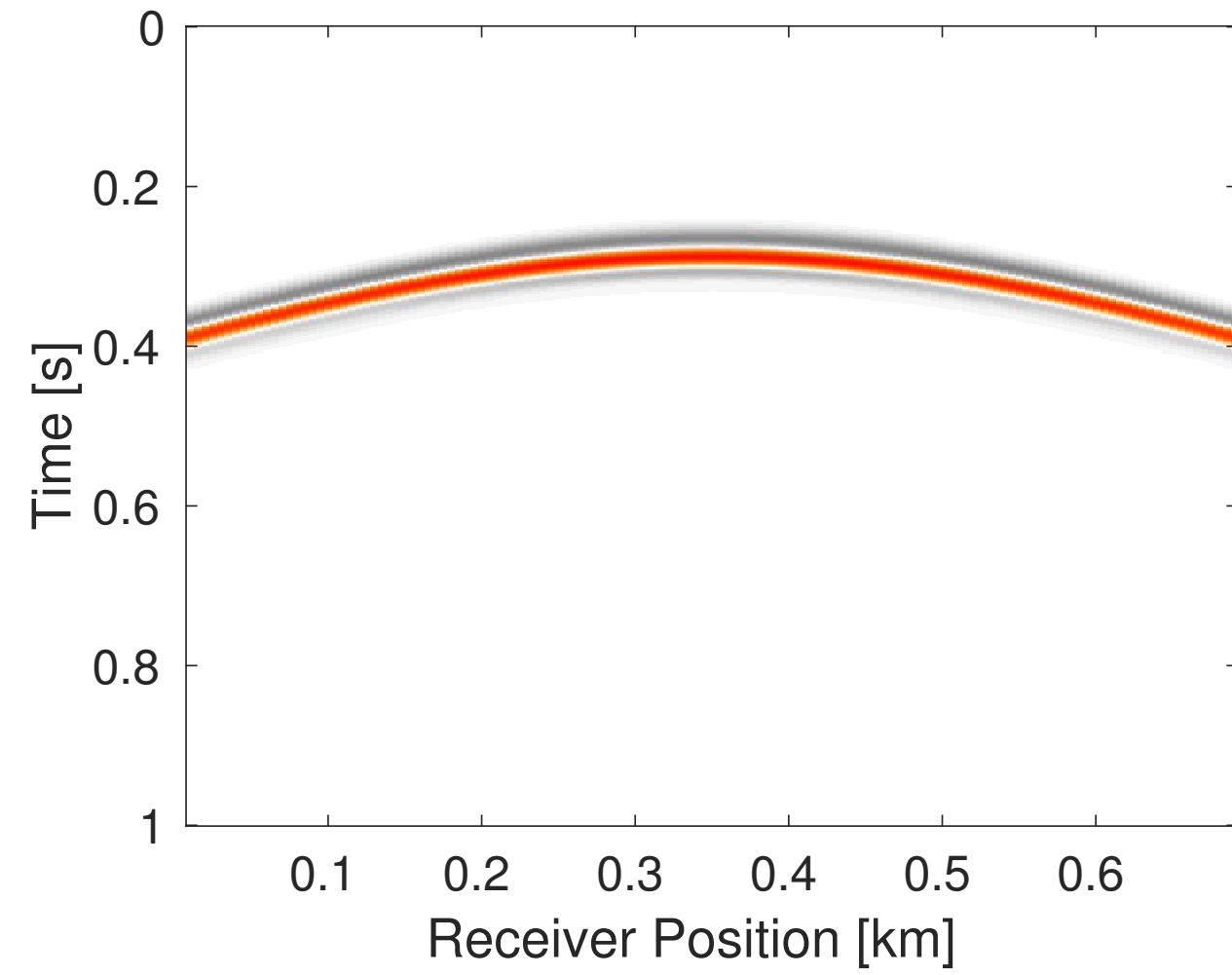
$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$

Adjoint solve



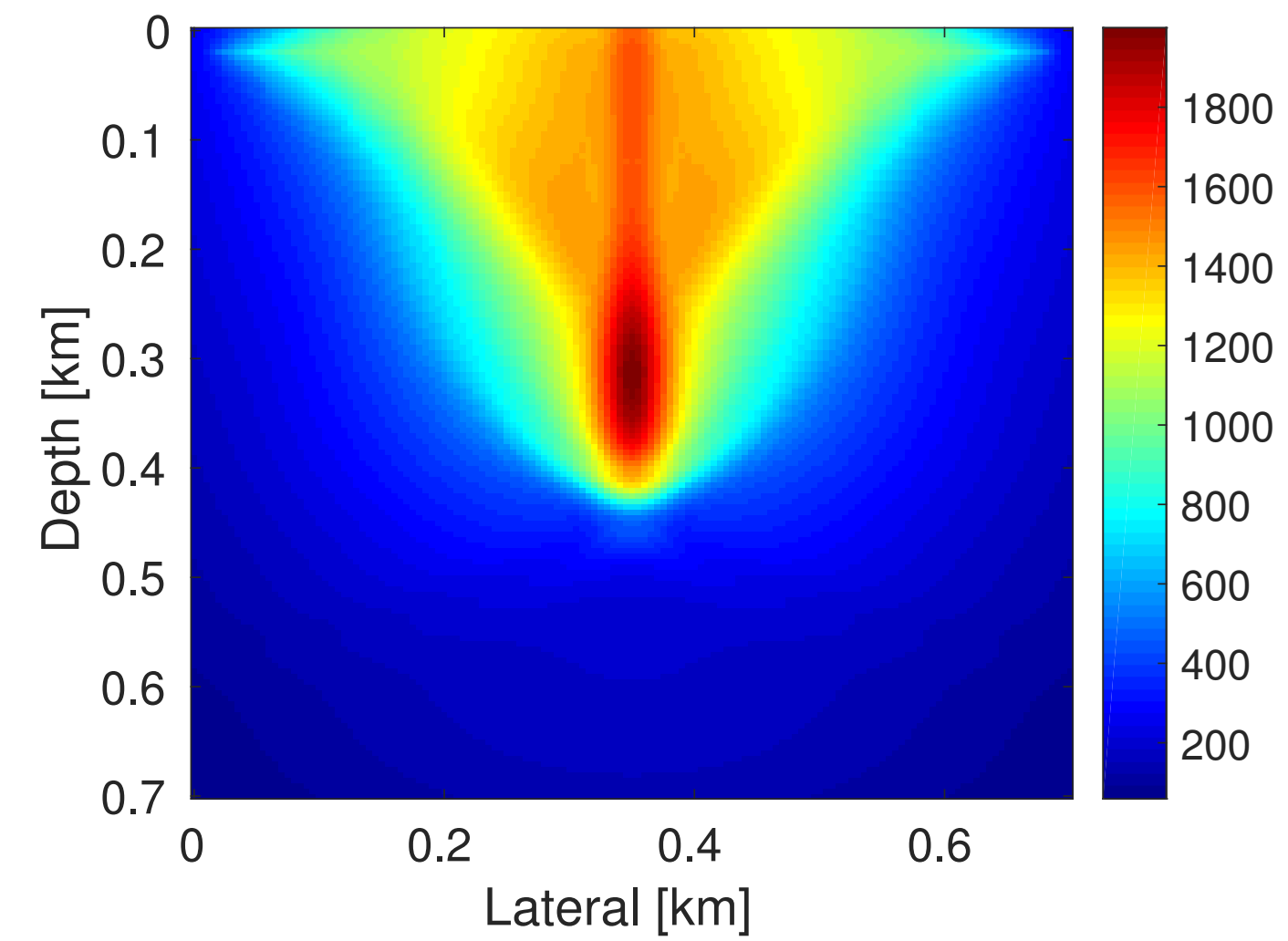
**Auxiliary variable
update**

$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$



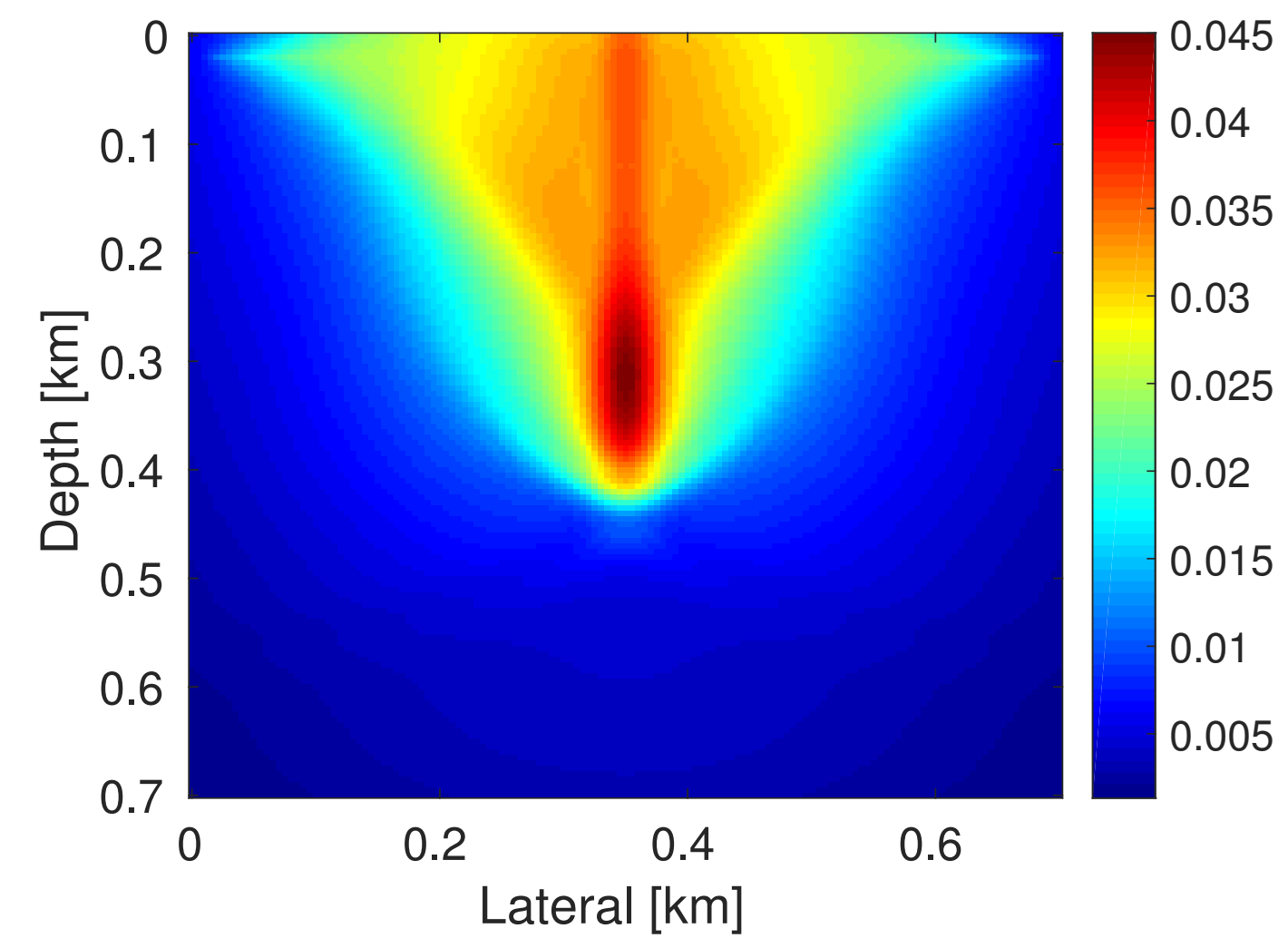
$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$

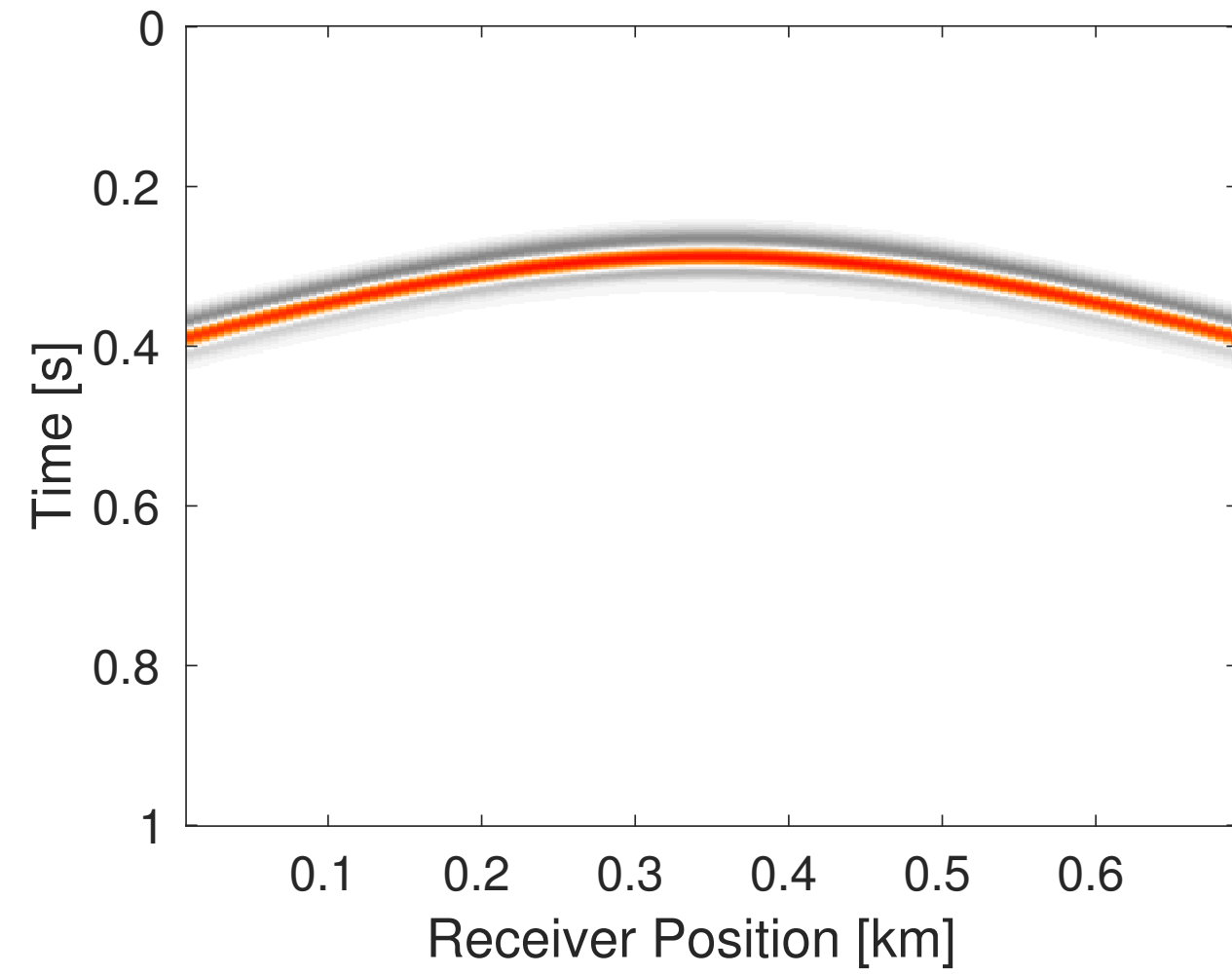
Adjoint solve



**Auxiliary variable
update**

$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$

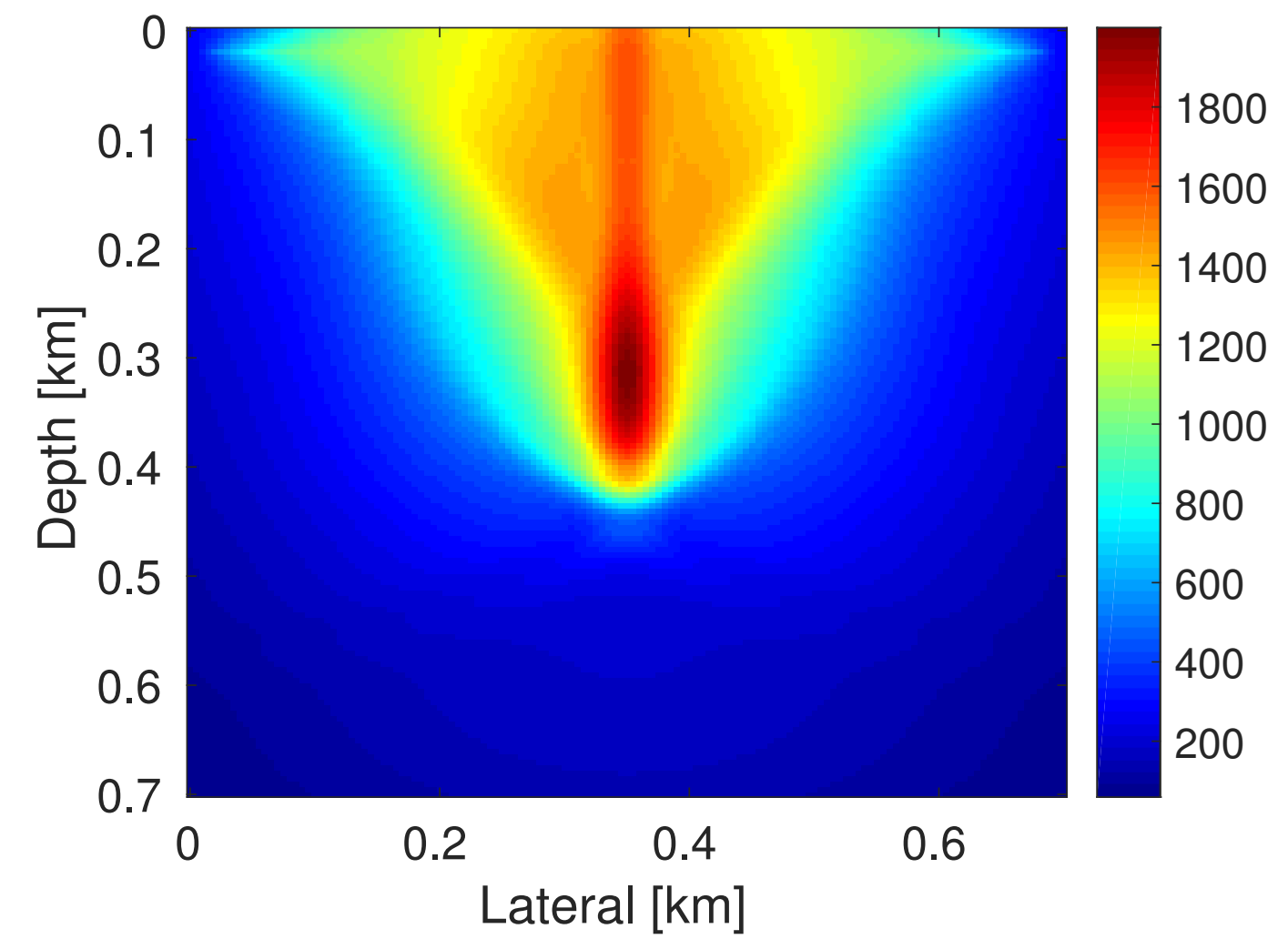




$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$

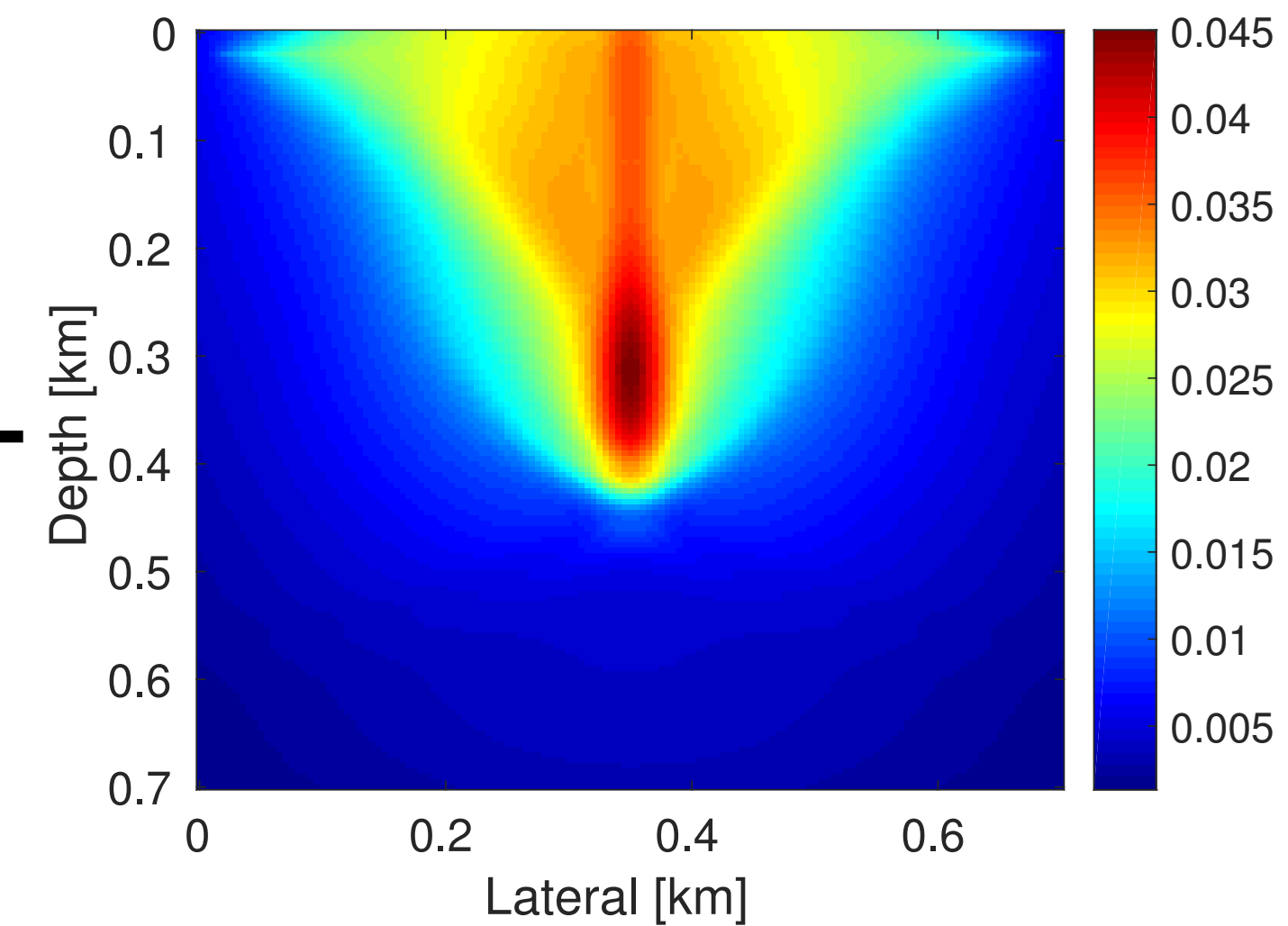


Adjoint solve



**Auxiliary variable
update**

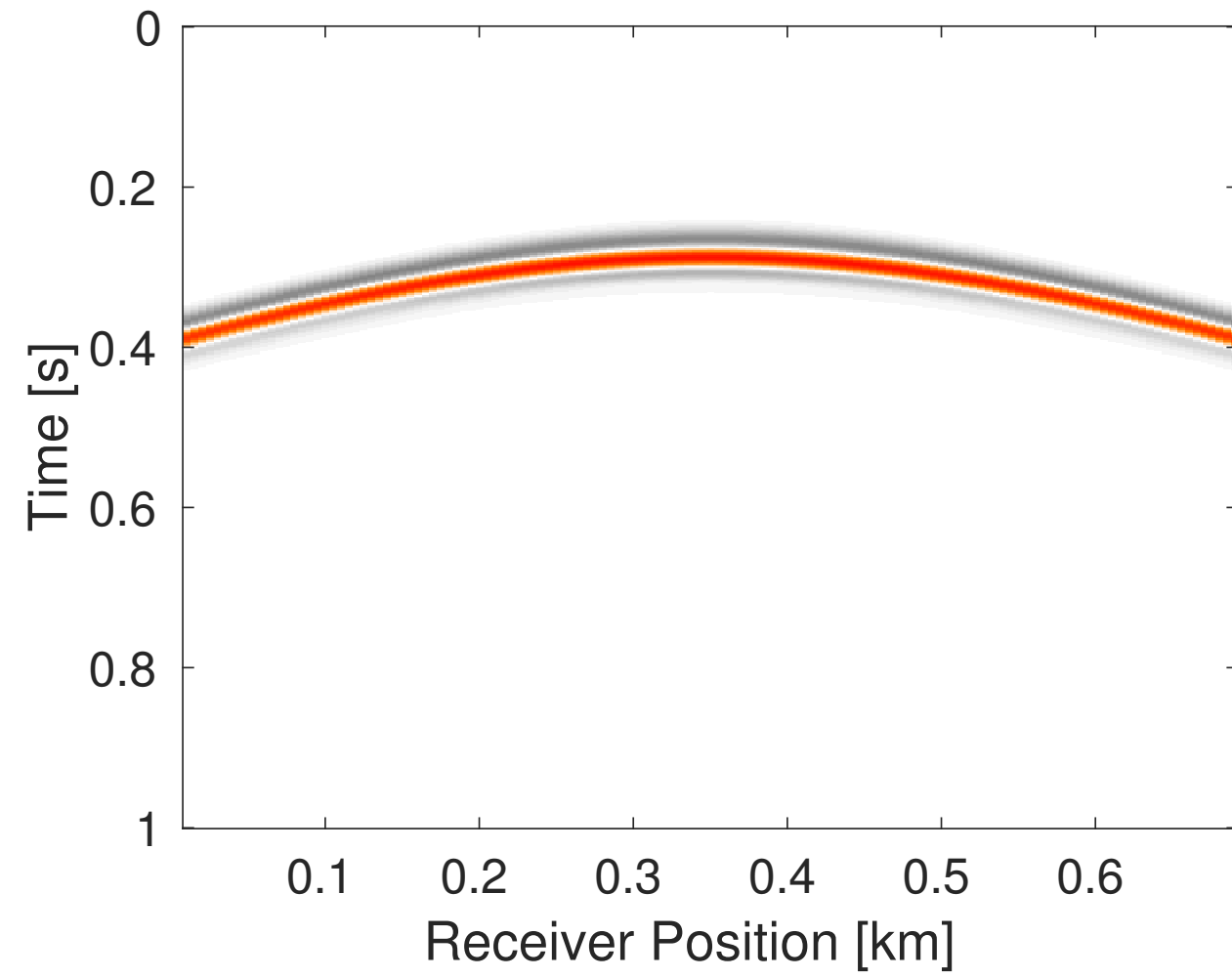
$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$



$$\mathbf{Q}_1 = \text{Prox}_{\mu \|\cdot\|_{2,1}}(\mathbf{Z}_1)$$



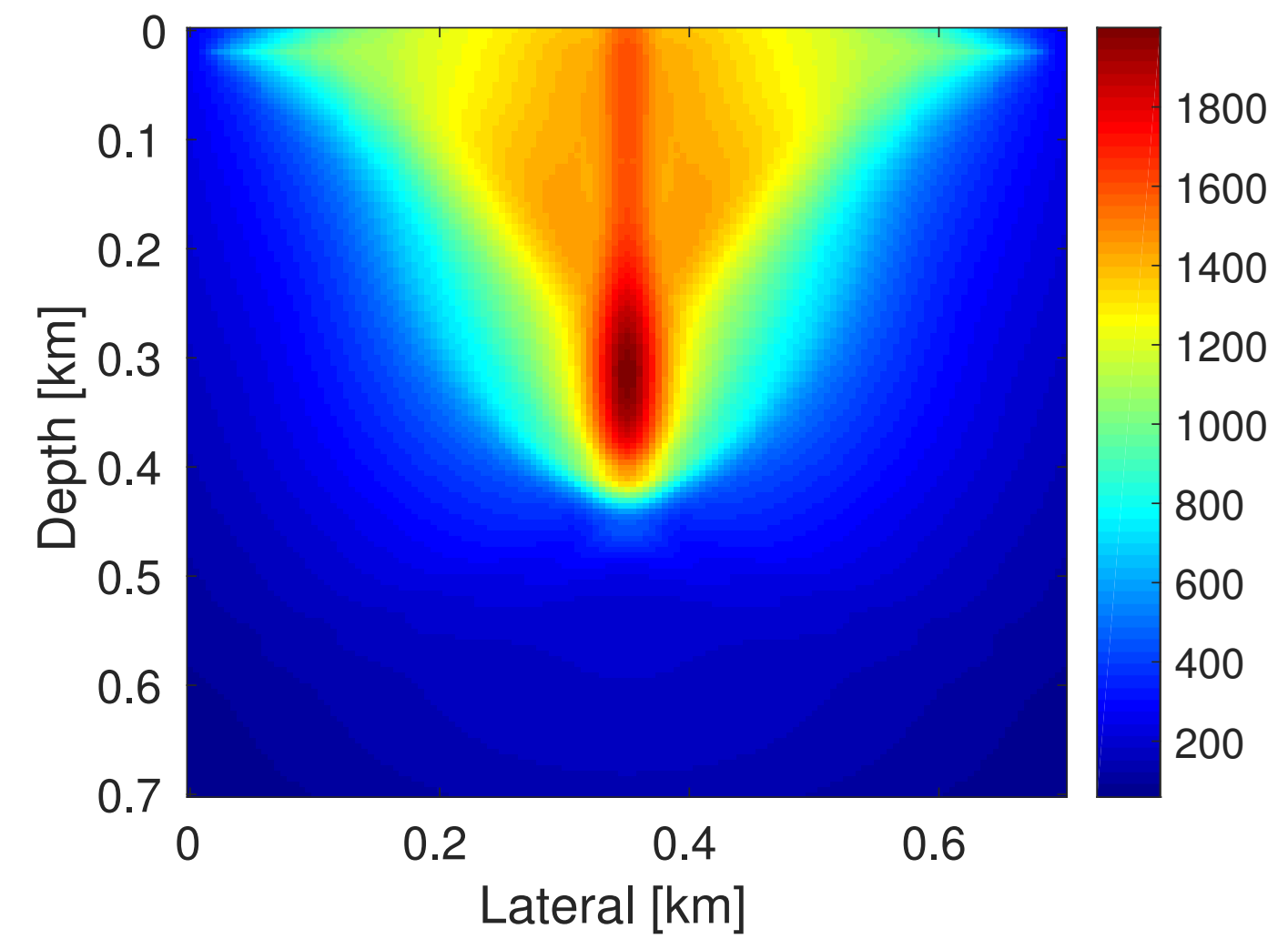
Sparsity promotion



$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$

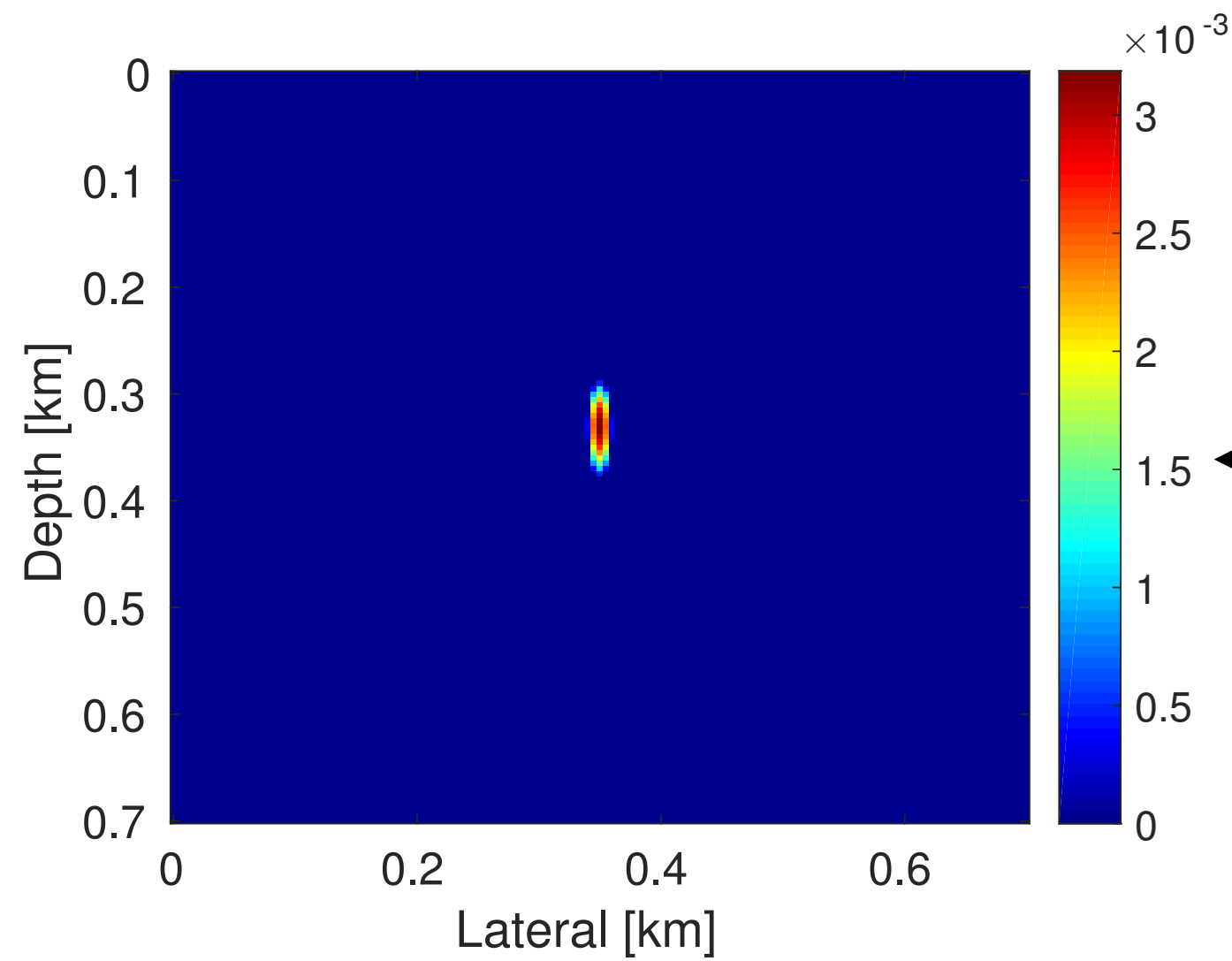


Adjoint solve



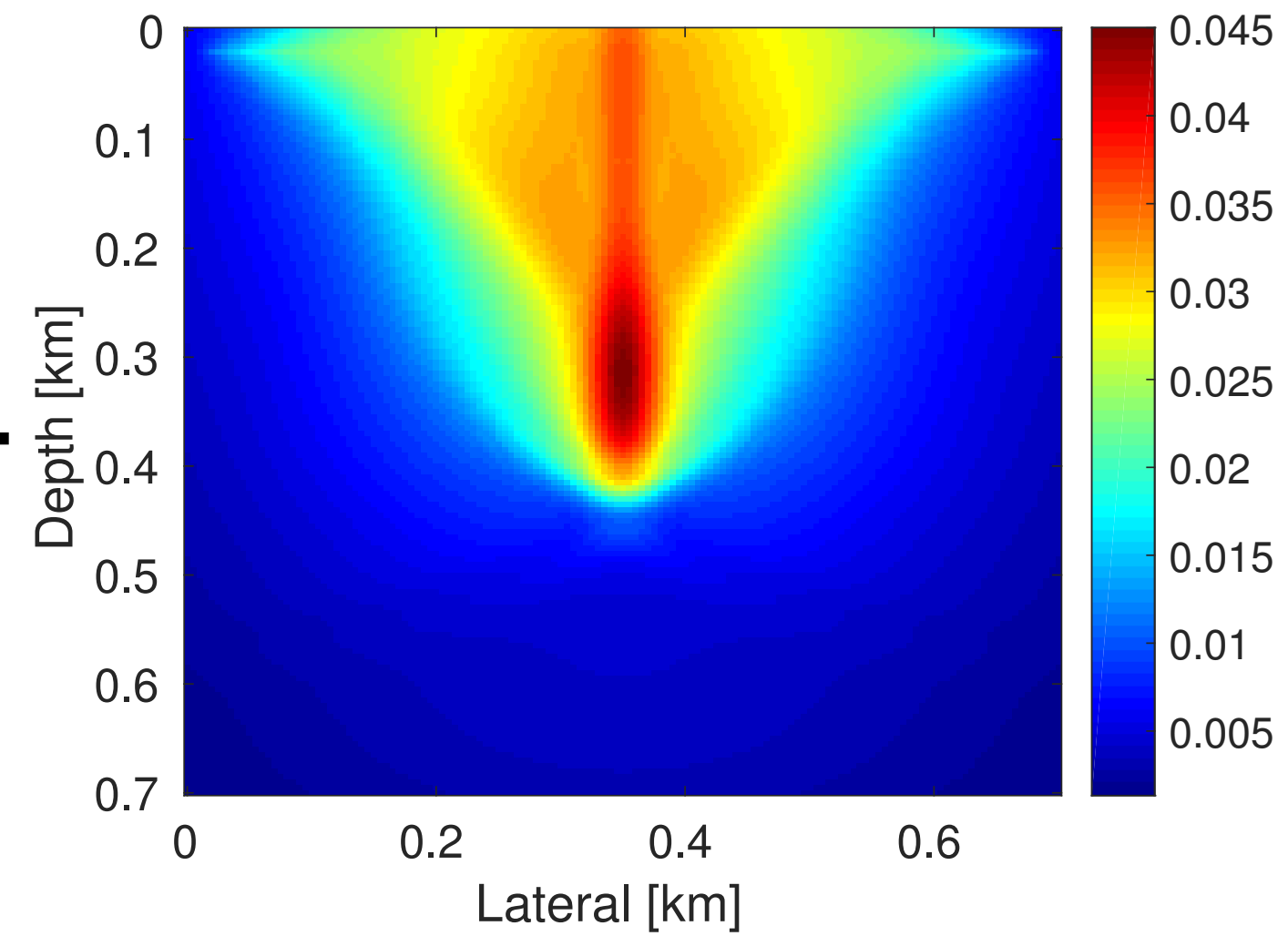
**Auxiliary variable
update**

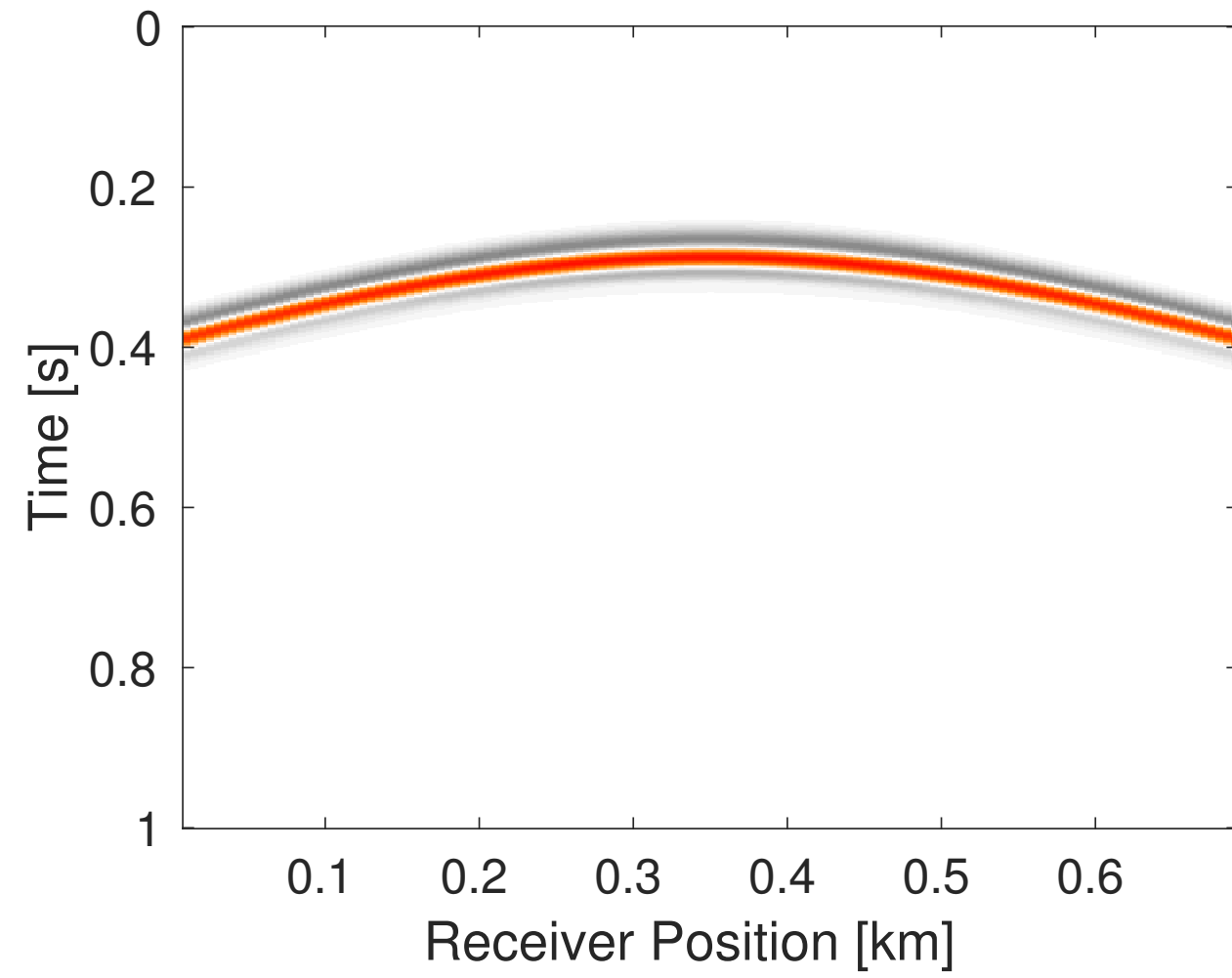
$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$



$$\mathbf{Q}_1 = \text{Prox}_{\mu \|\cdot\|_{2,1}}(\mathbf{Z}_1)$$

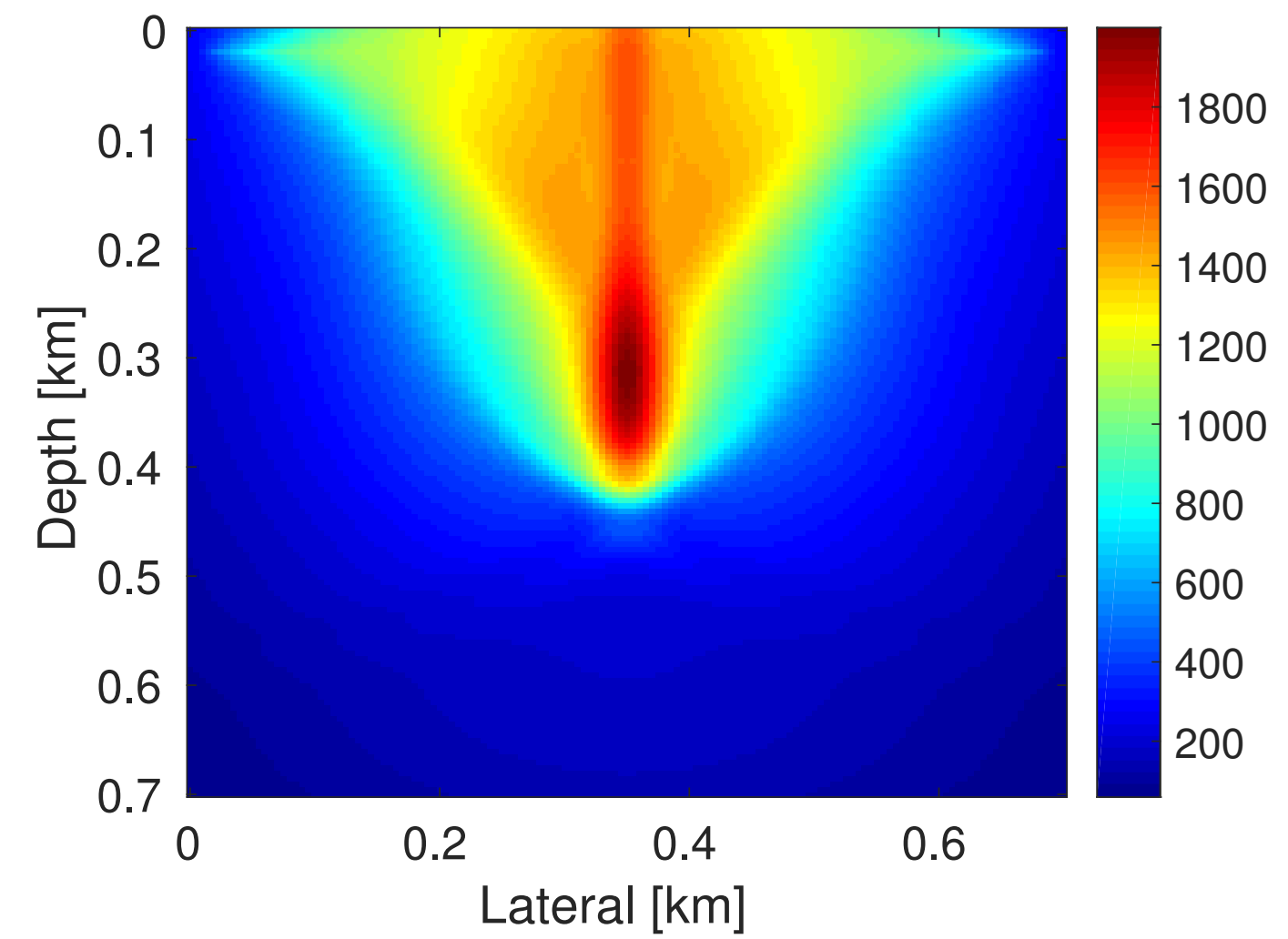
Sparsity promotion





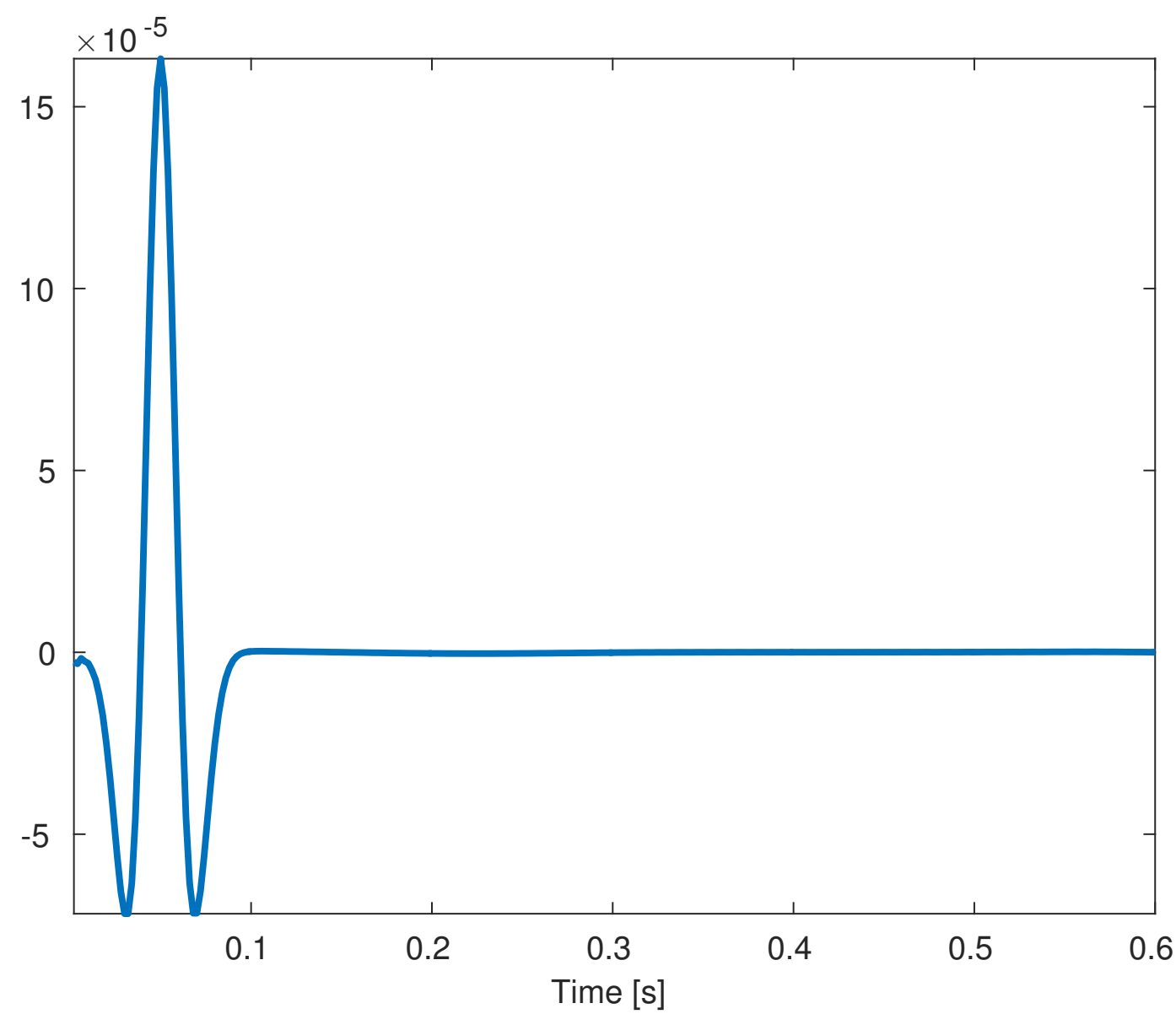
$$\mathbf{V}_1 = \mathcal{F}^T[\mathbf{m}](\Pi_\epsilon(\mathcal{F}(\mathbf{Q}_0) - \mathbf{d}))$$

Adjoint solve



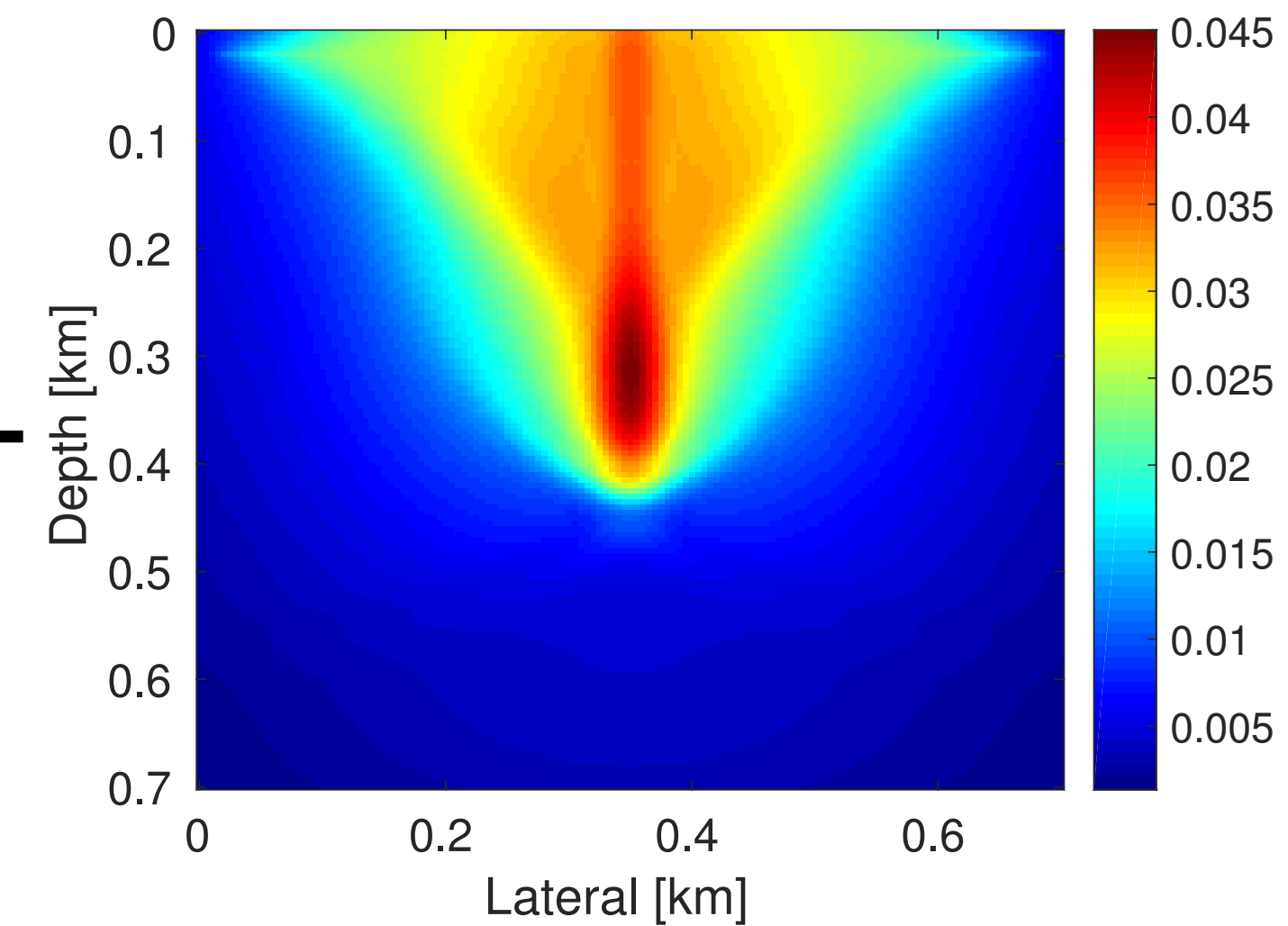
Auxiliary variable update

$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$



$$\mathbf{Q}_1 = \text{Prox}_{\mu \|\cdot\|_{2,1}}(\mathbf{Z}_1)$$

Sparsity promotion



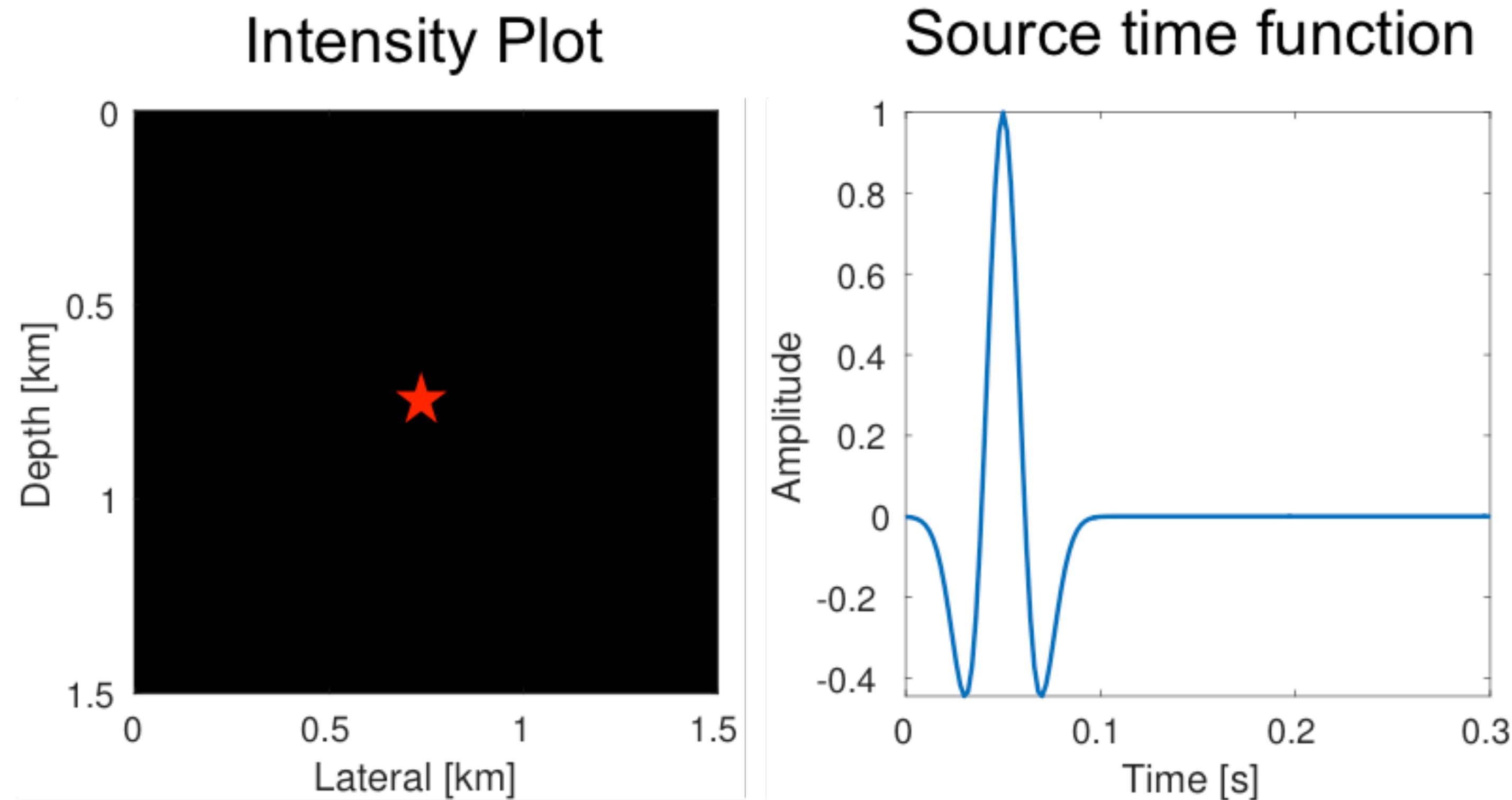
Location and source time function estimation

Source location: estimated as outlier in intensity plot

$$I(\mathbf{x}) = \sum_t | \mathbf{Q}(\mathbf{x}, t) |$$

Source time function: temporal variation of wavefield at estimated source location

Intensity Plot & Source time function



Schematic showing source location as outlier and corresponding source time function

BG compass model example

Objective

- ▶ to show the ability of our method in realistic geological setting

BG compass model example

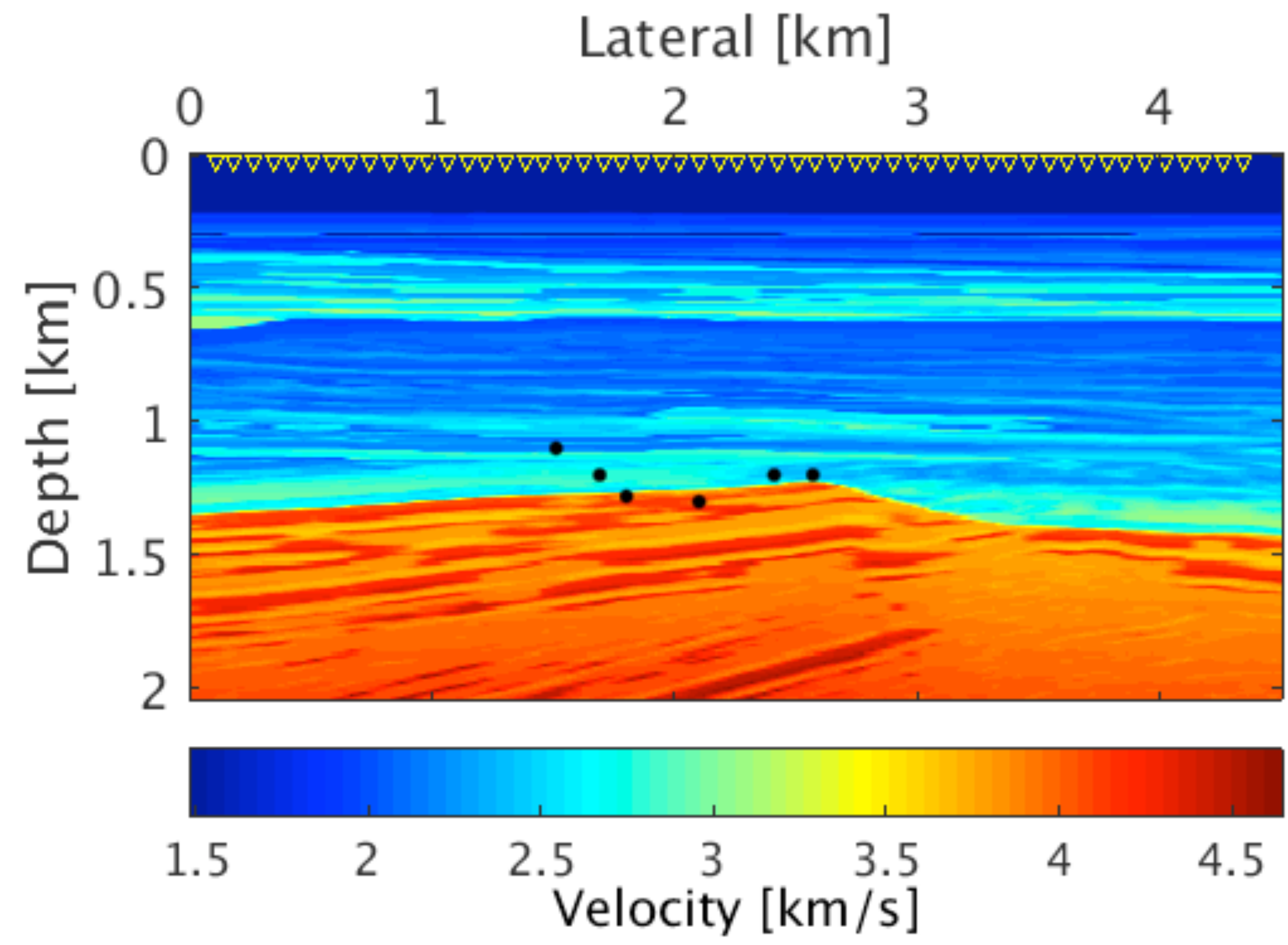
Objective

- ▶ to show the ability of our method in realistic geological setting

Assumptions

- ▶ access to smooth background velocity model
- ▶ noisy data (bandwidth limited random noise up to 45 Hz)

Experimental setup

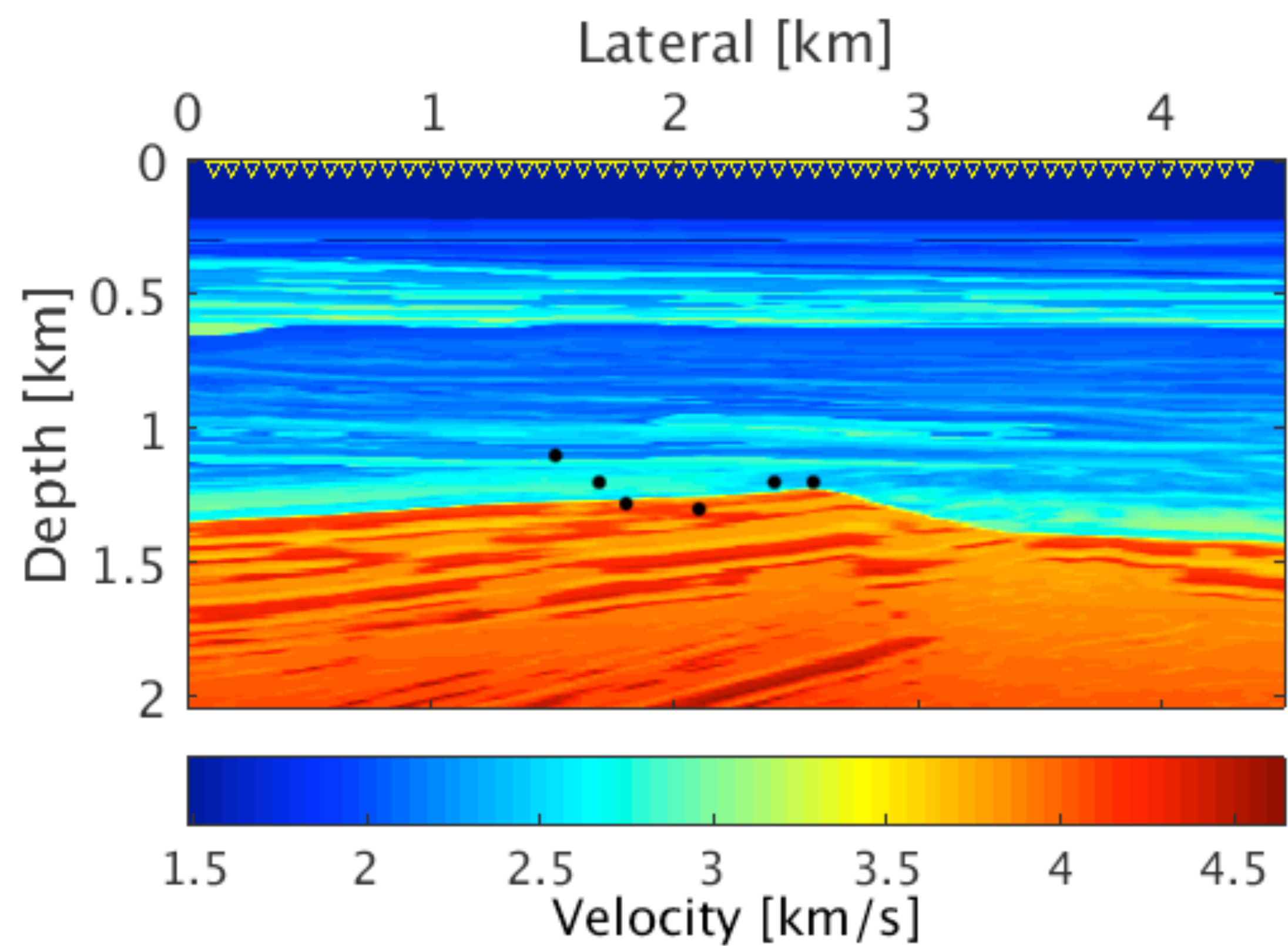


BG Compass velocity model

Modeling information:

- Model size:** 2.04 km x 4.50 km
- Grid spacing:** 10m
- Total number of sources:** 6
- Receiver spacing:** 20m
- Receiver depth:** 20m
- Fixed spread:** 4.30 km
- Sampling interval:** 1 ms
- Recording length:** 2.5 s
- Peak frequency :** 15 Hz & 10 Hz

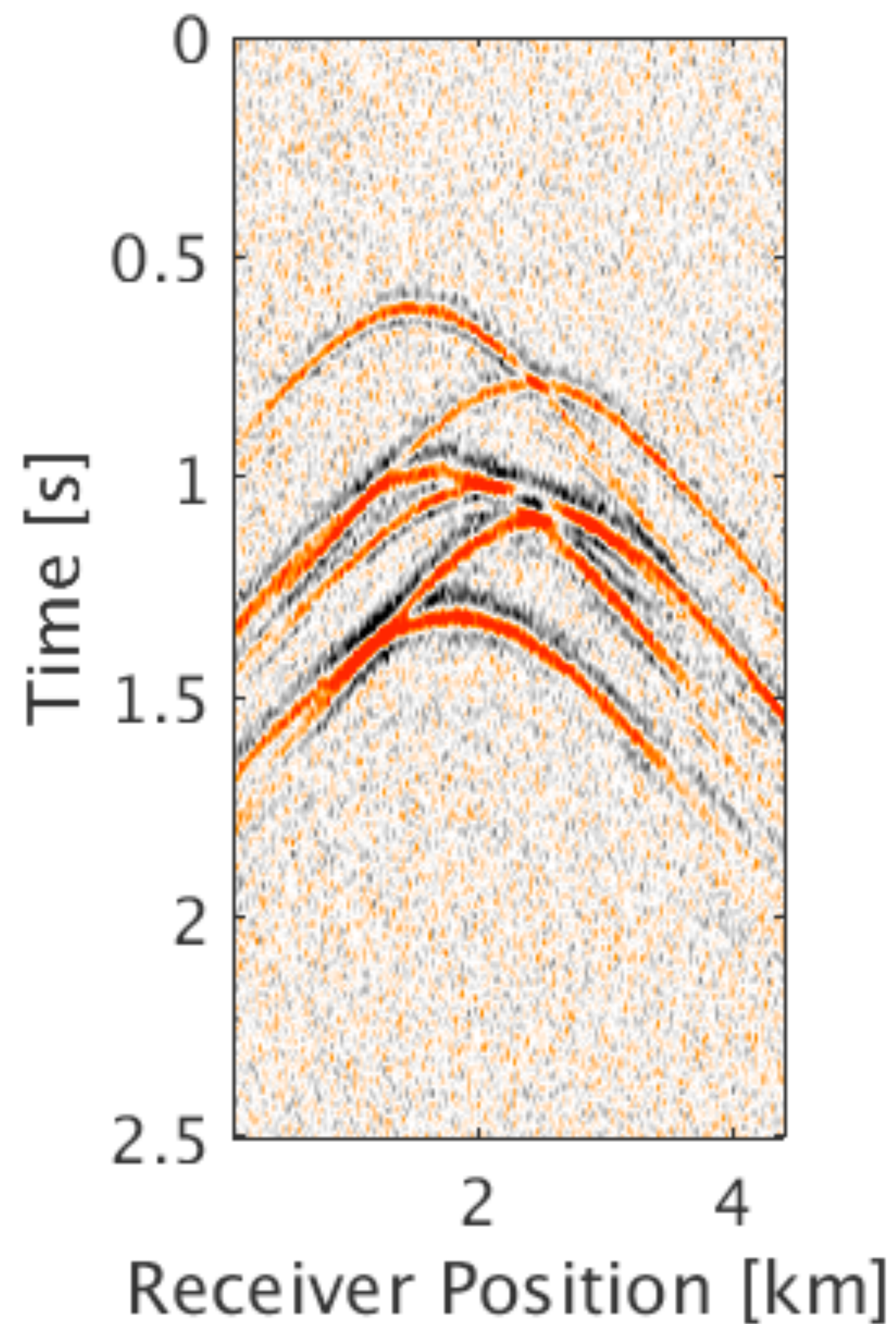
Experimental setup



BG Compass velocity model

Dominant wavelength: 420 m

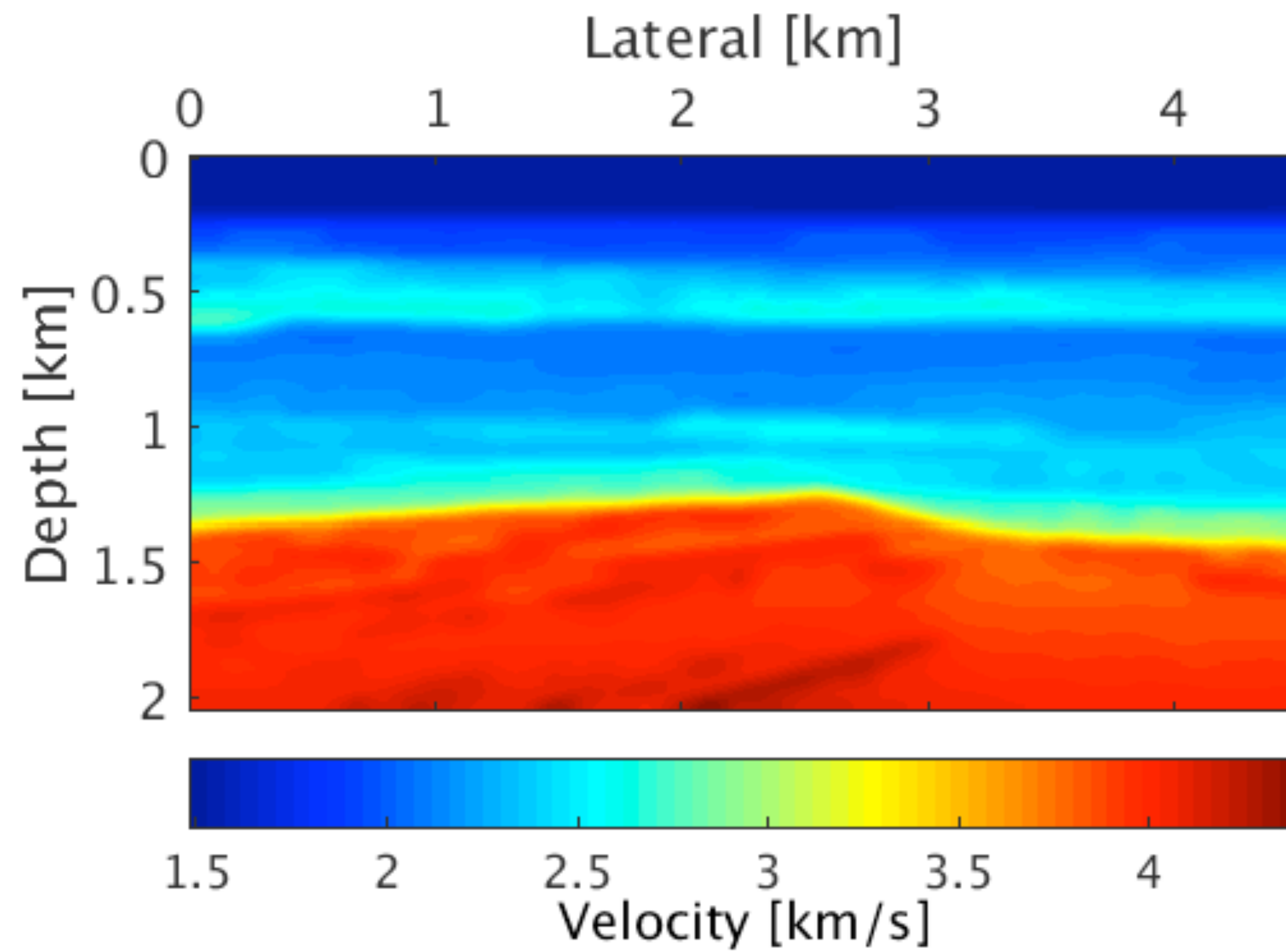
Observed microseismic data



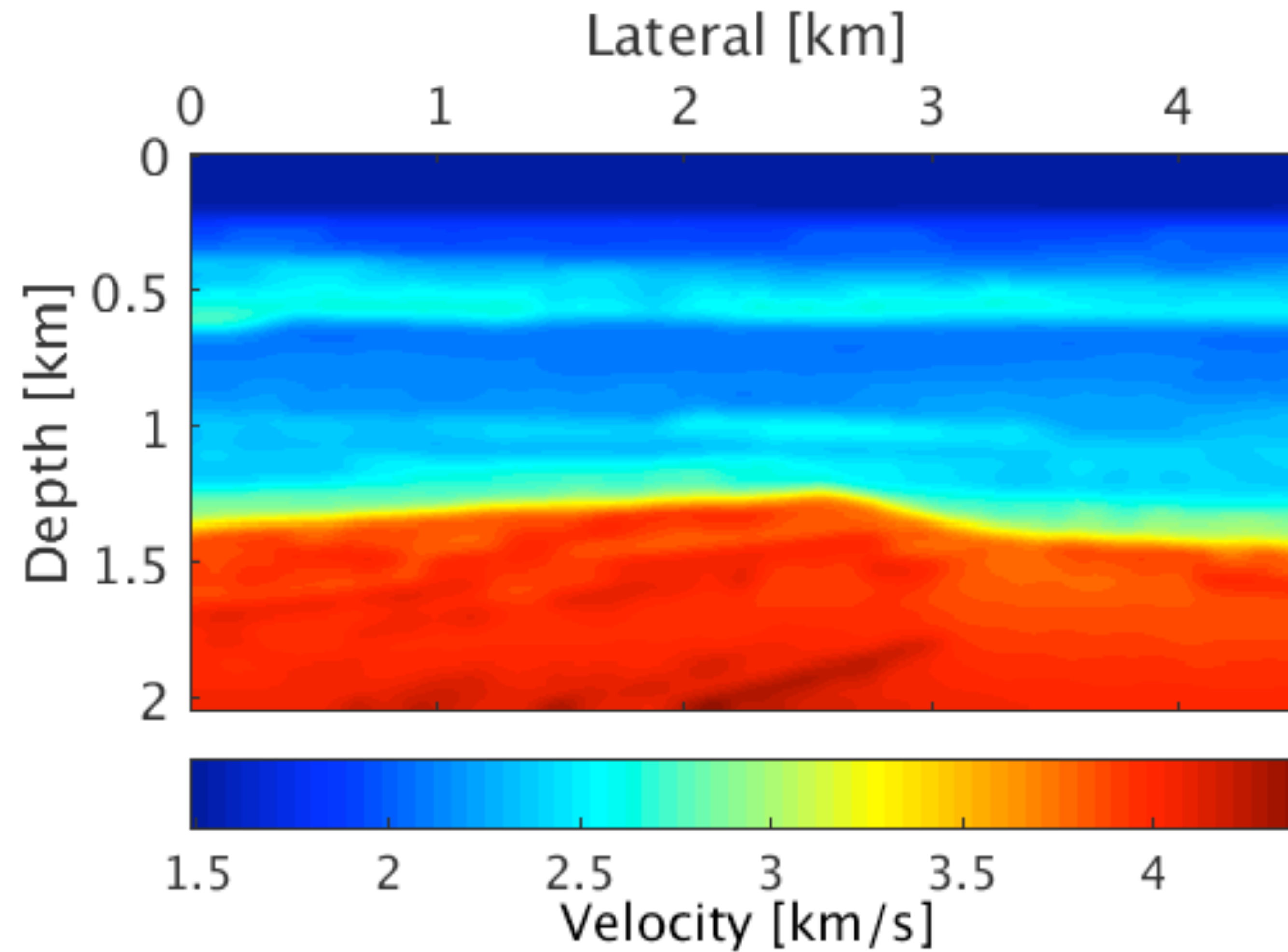
Noisy microseismic data, SNR = 2.83

Data is contaminated with low frequency random noise (up to 45 Hz)

Smooth velocity model

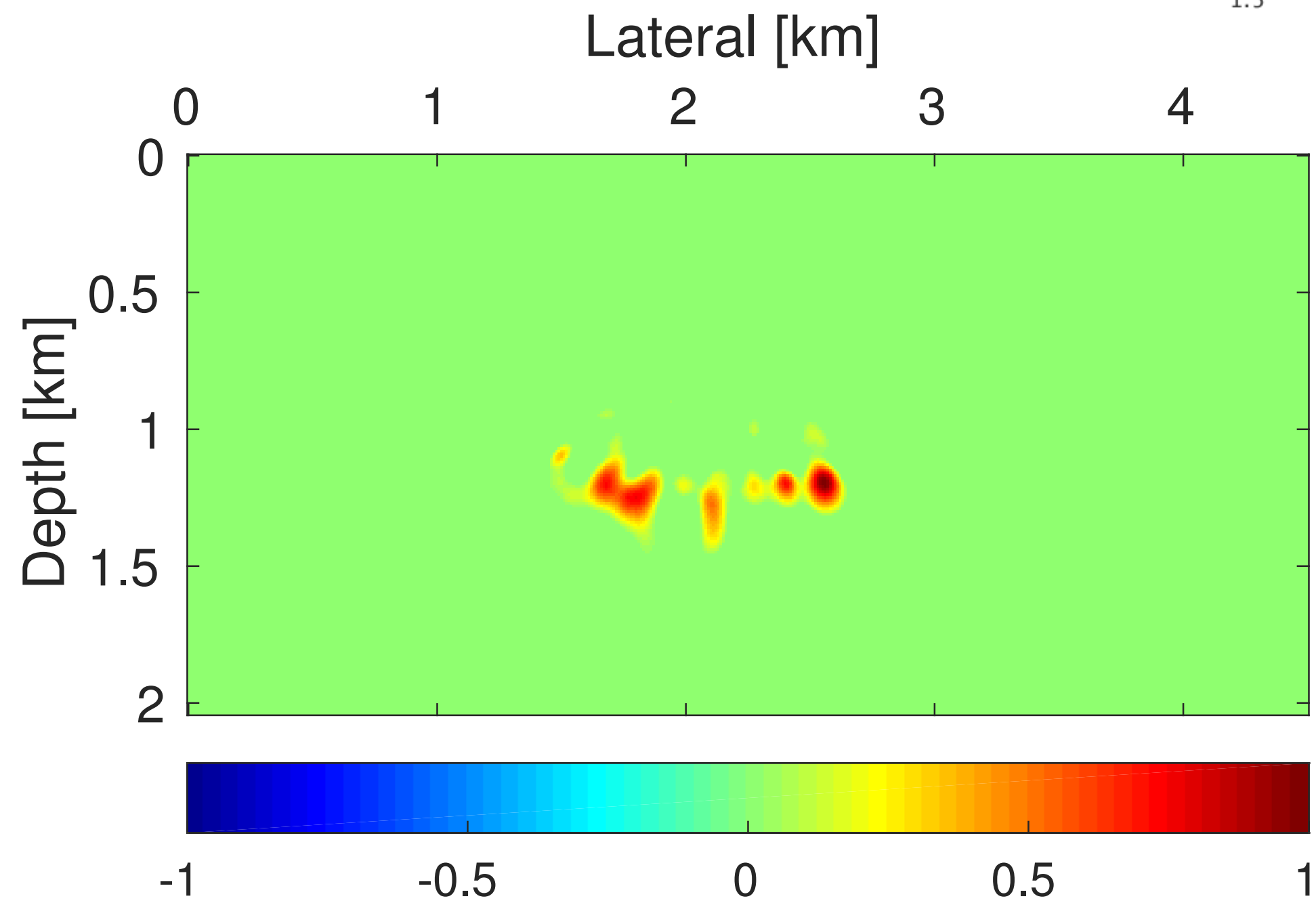
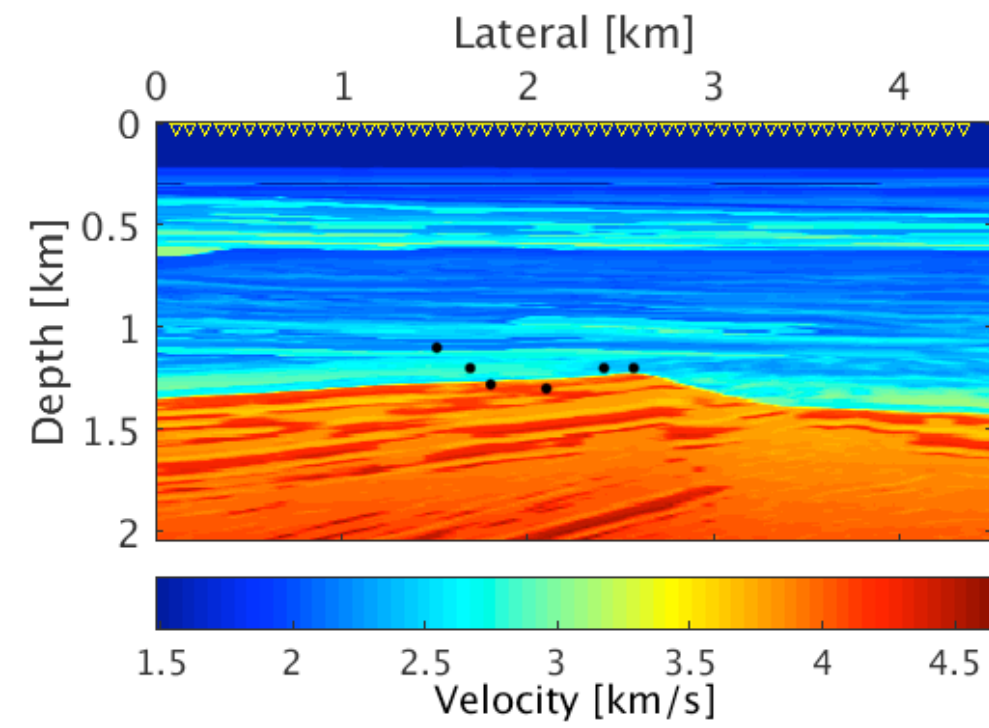


Smooth velocity model

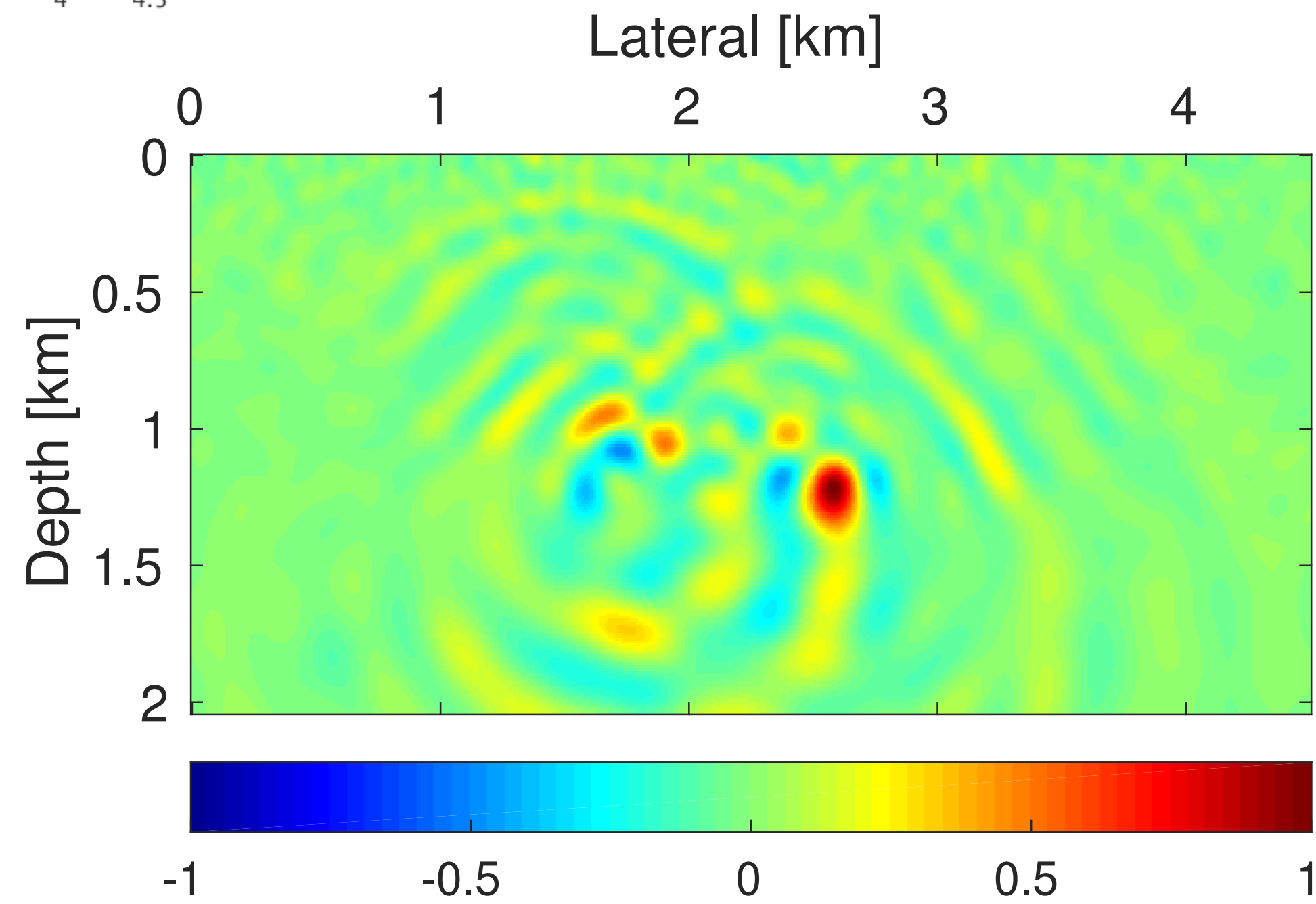


Used for joint microseismic source location and source time function estimation

Estimated Source location (From noisy data)

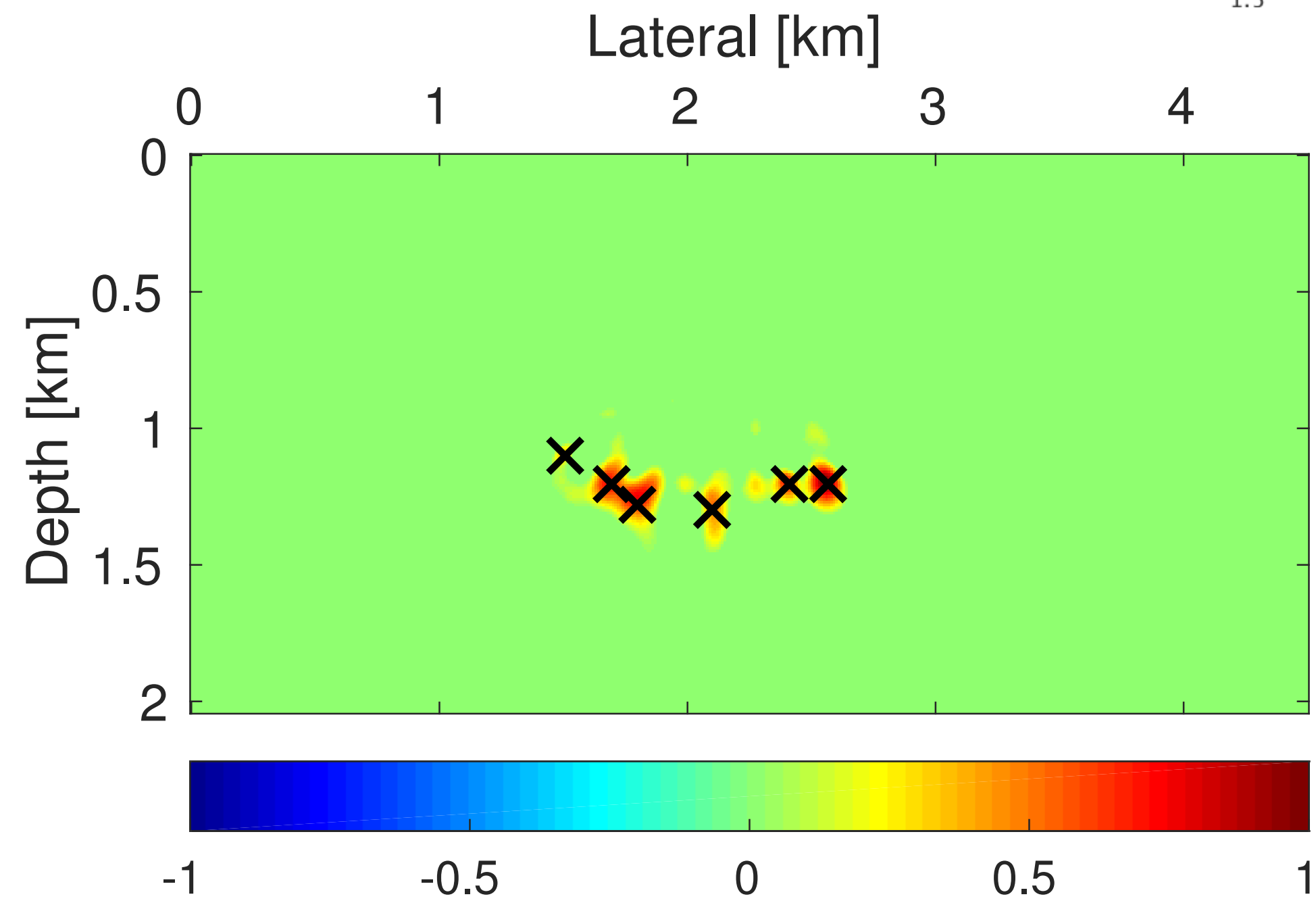
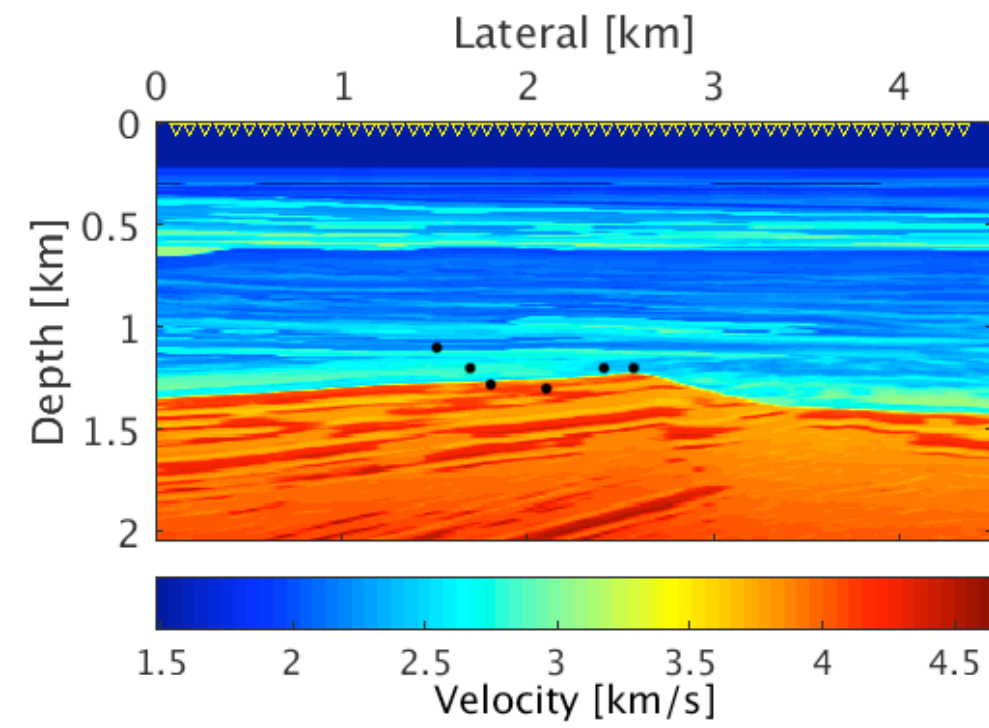


Sparsity-promoting method

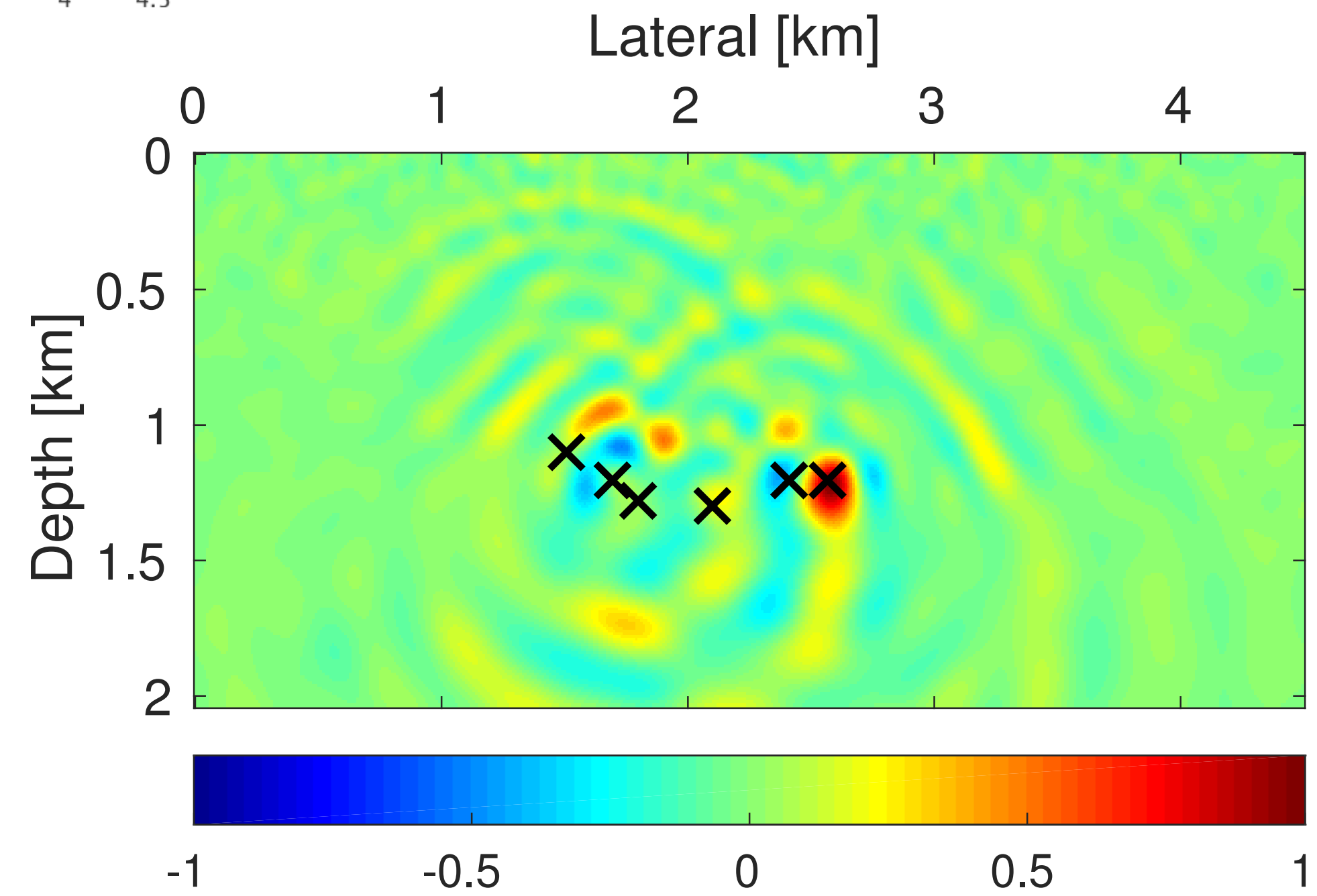


FWI

Estimated Source location (From noisy data)

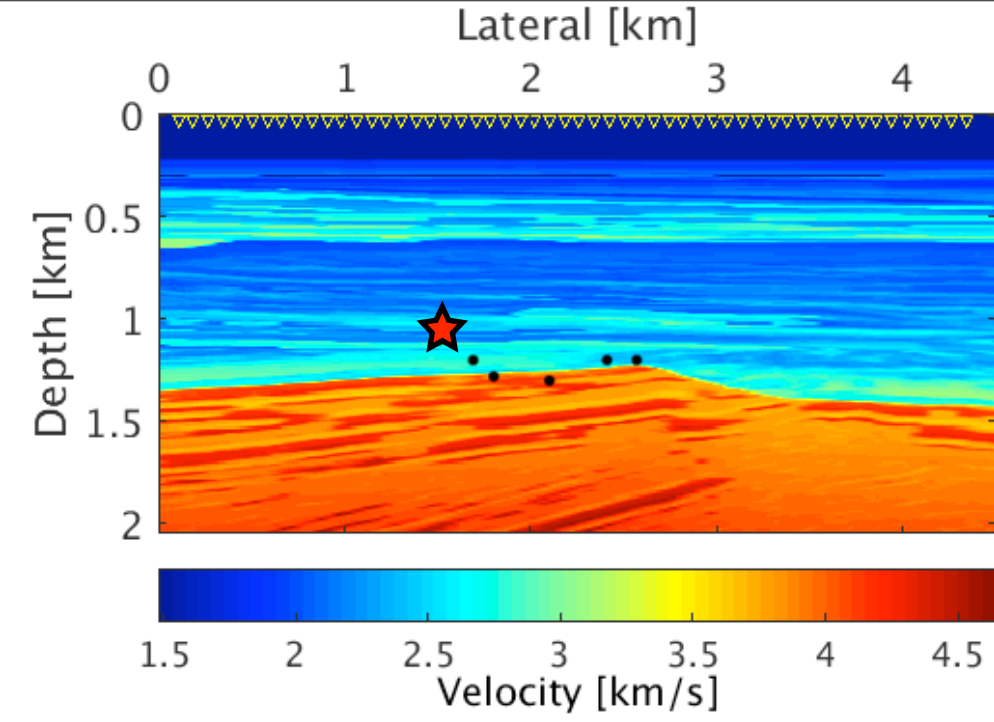


Sparsity-promoting method



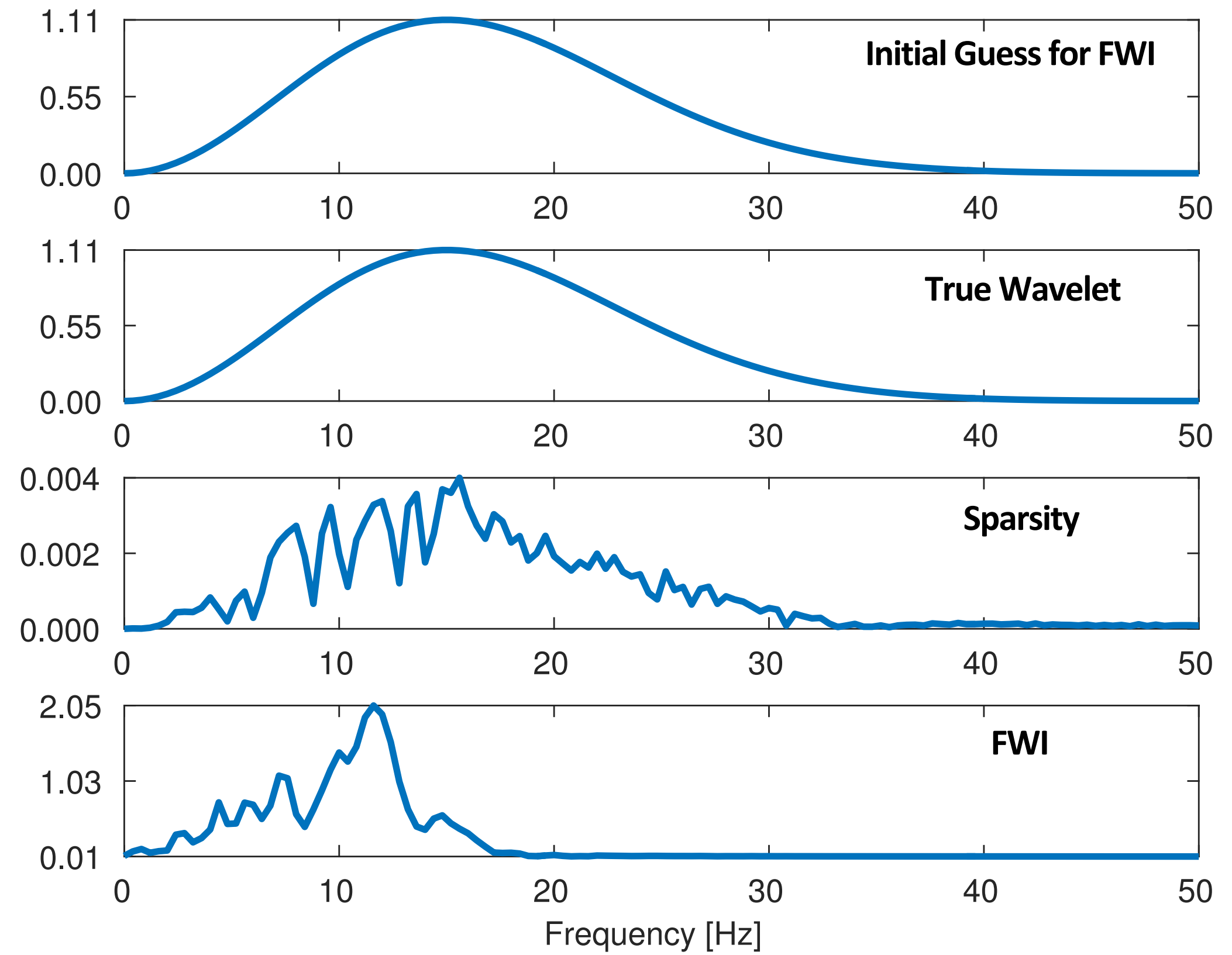
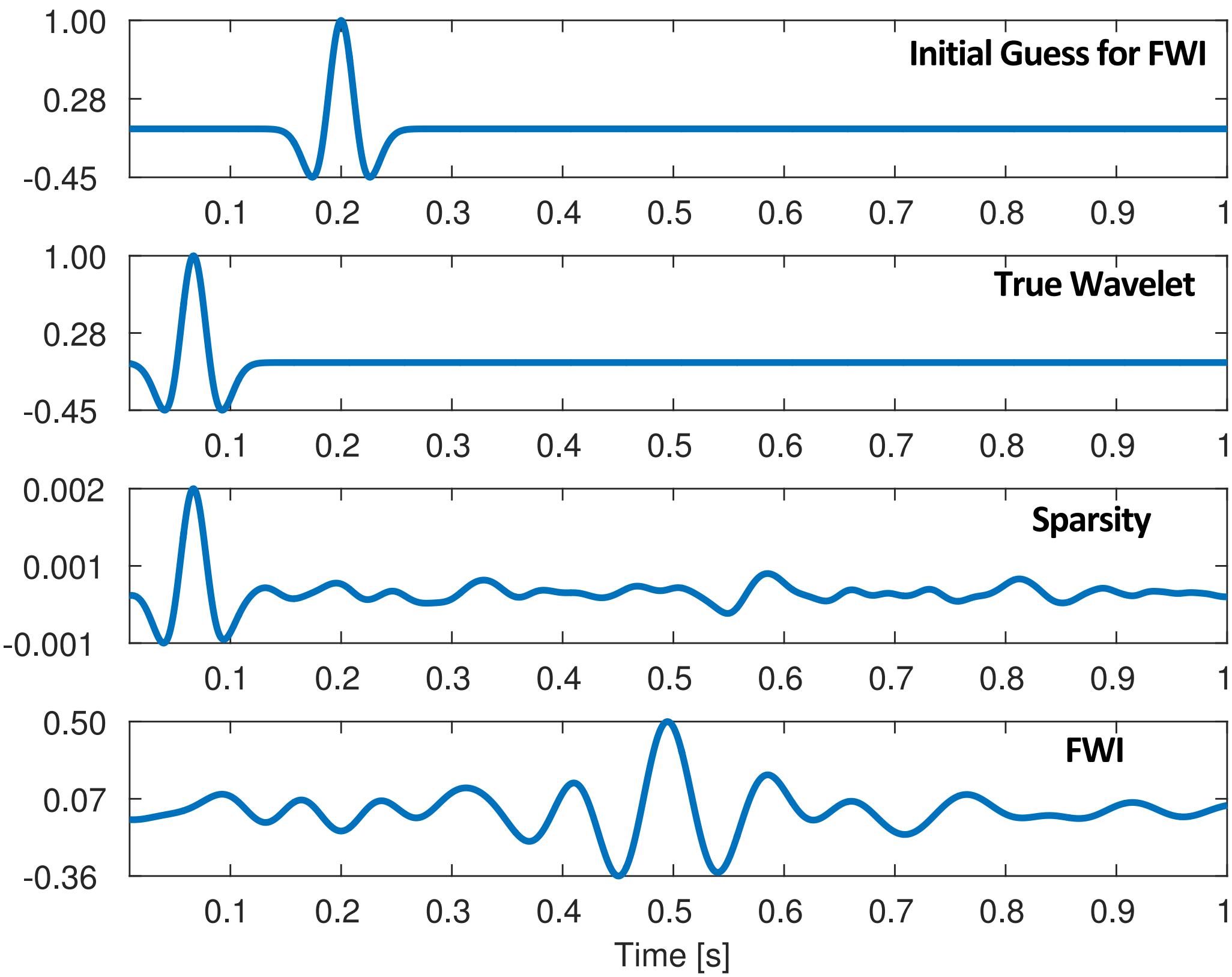
FWI

Location 1

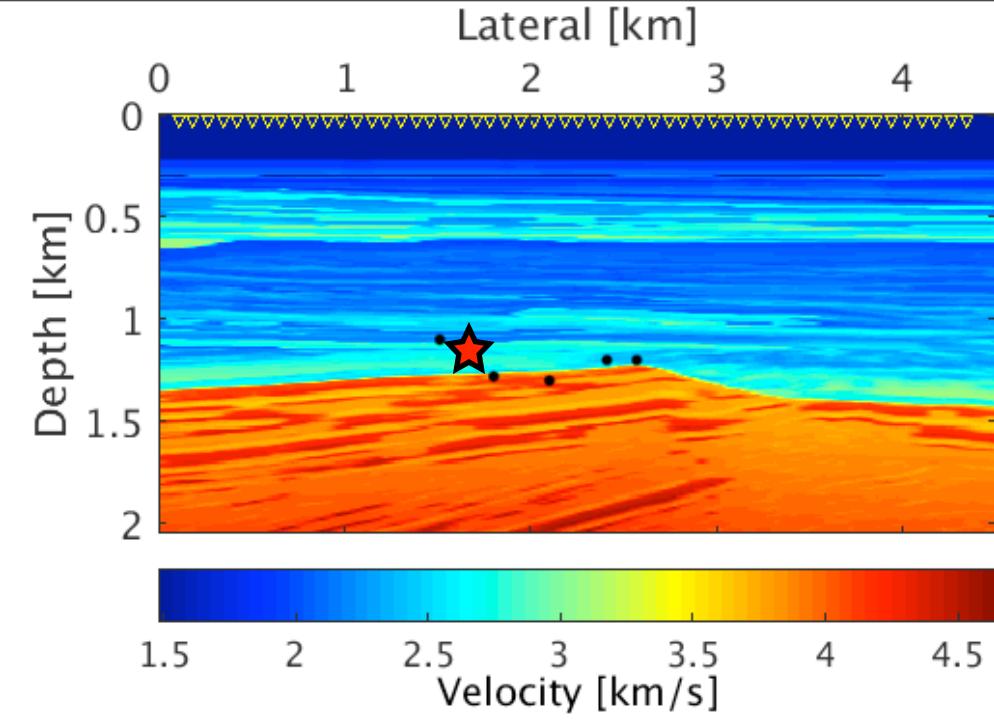


Wavelet

Spectrum

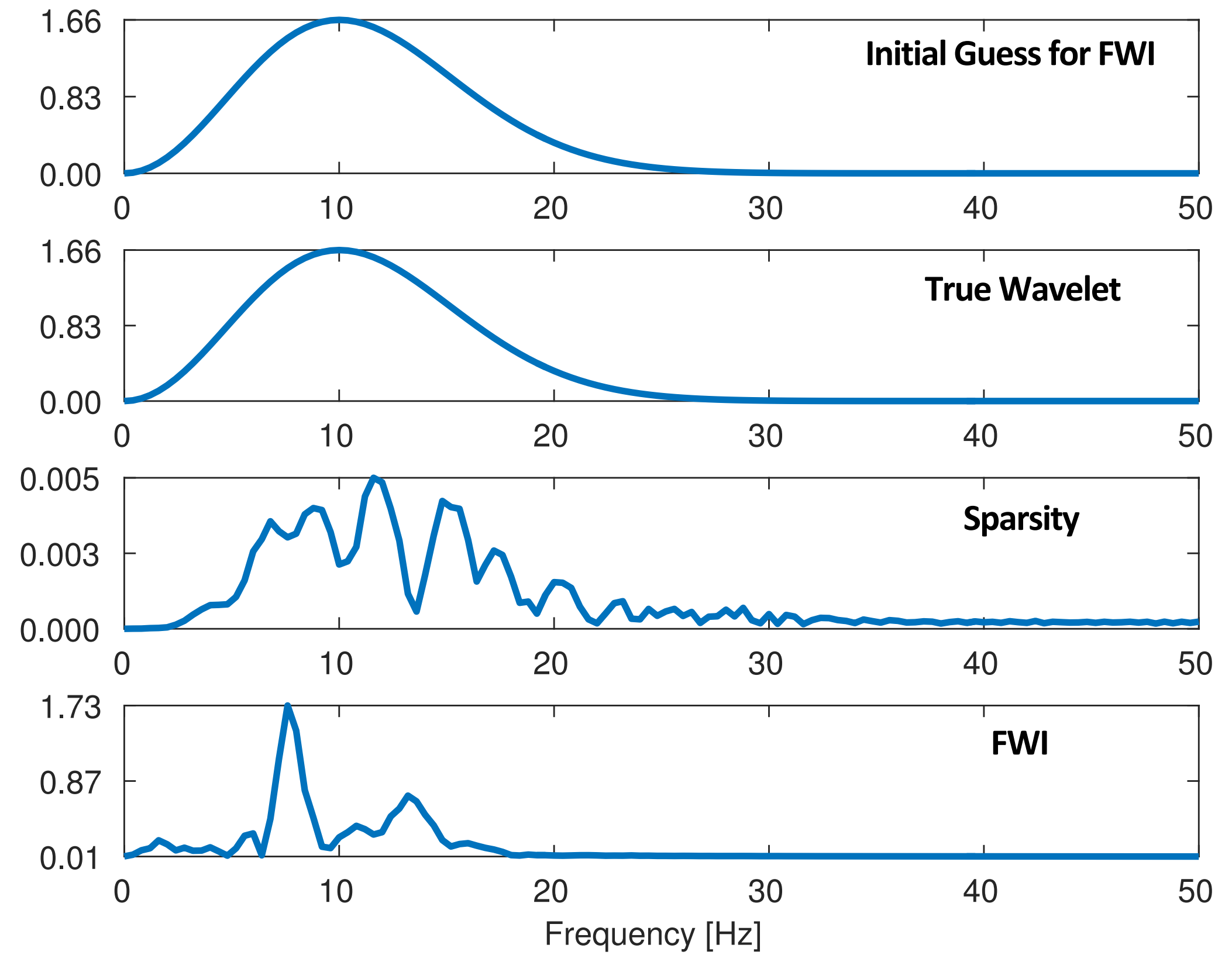
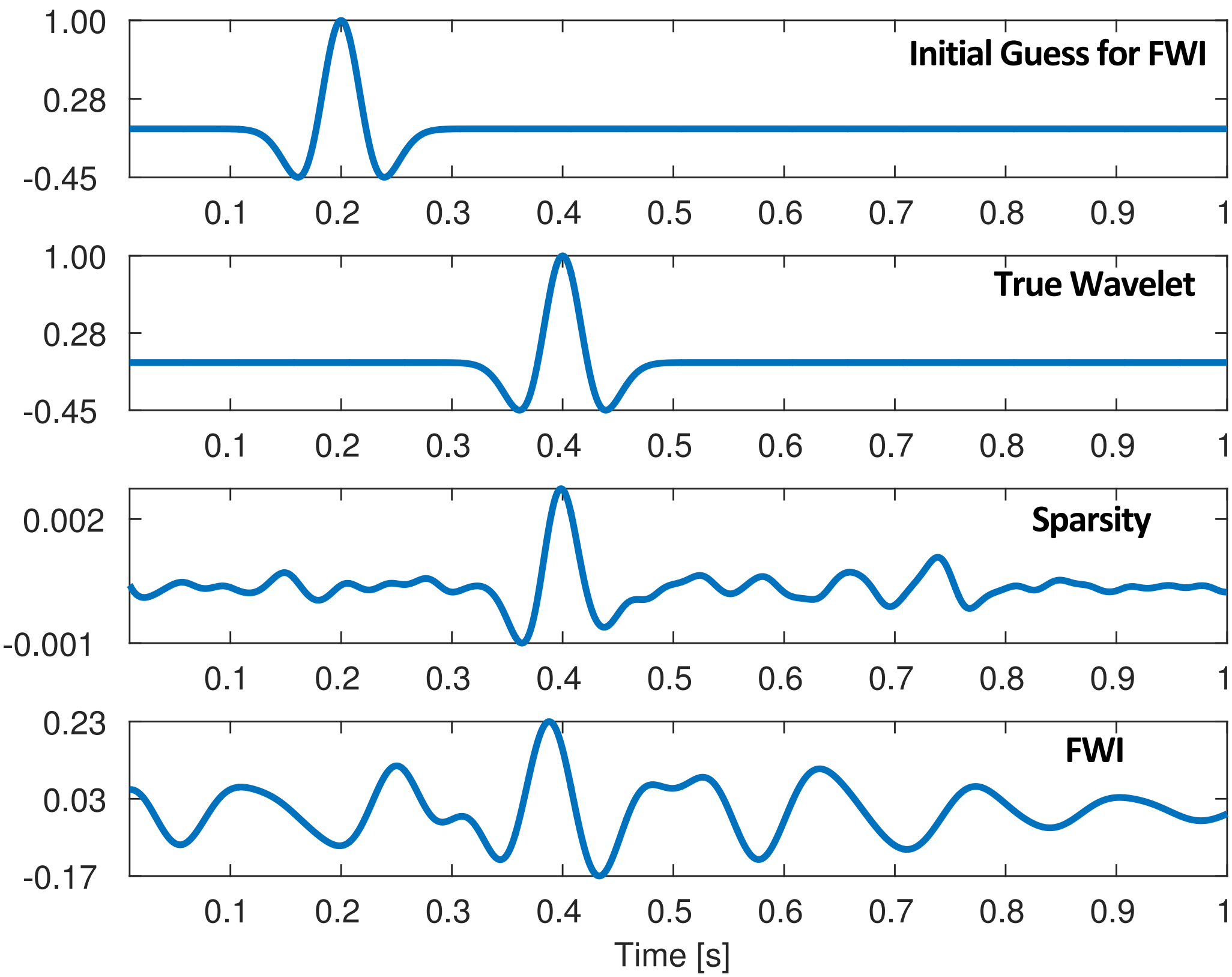


Location 2

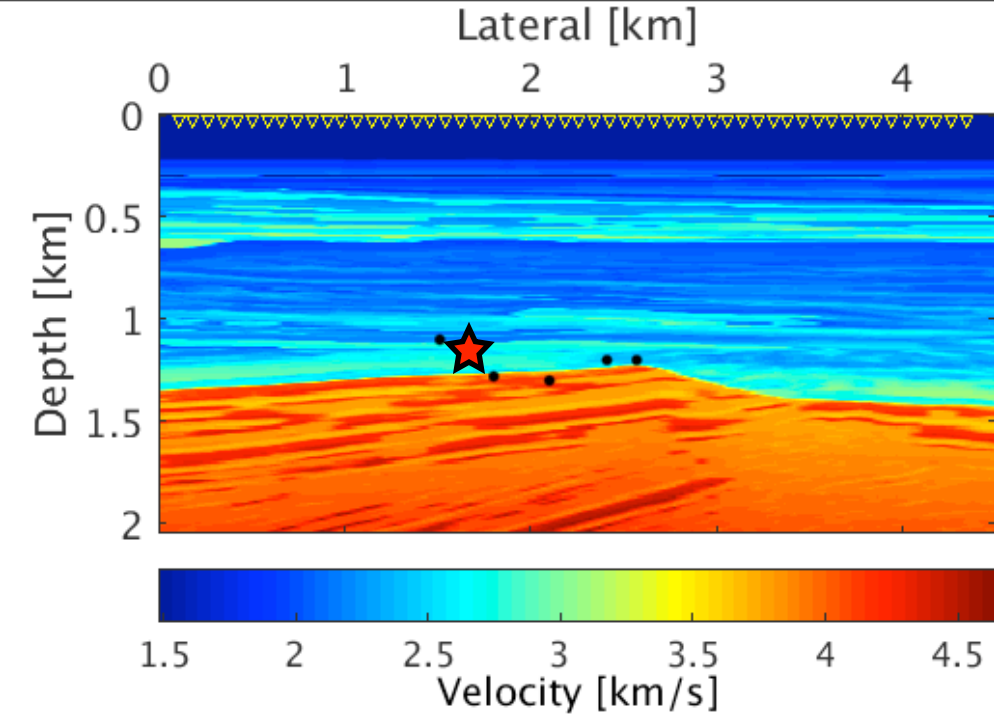


Wavelet

Spectrum

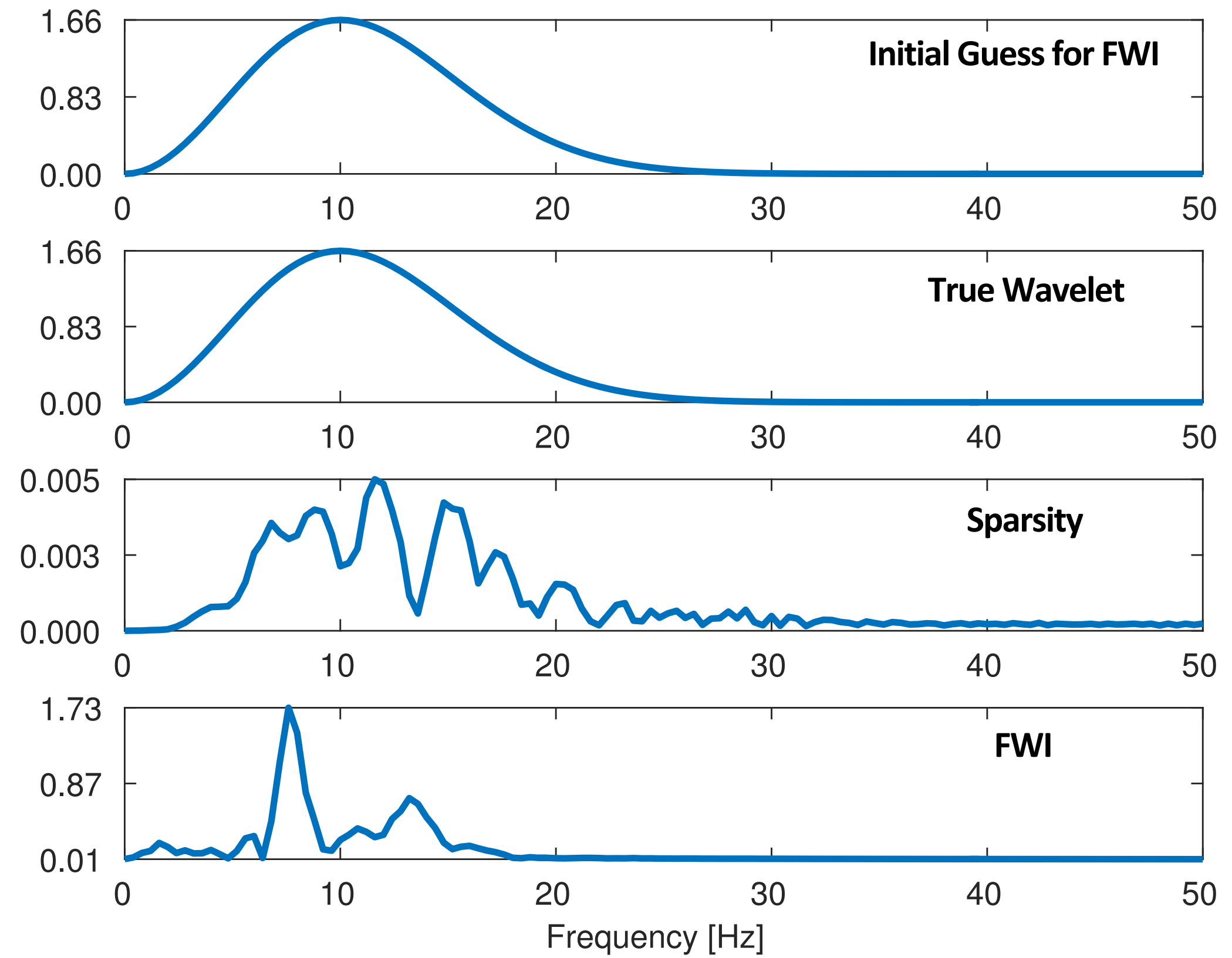
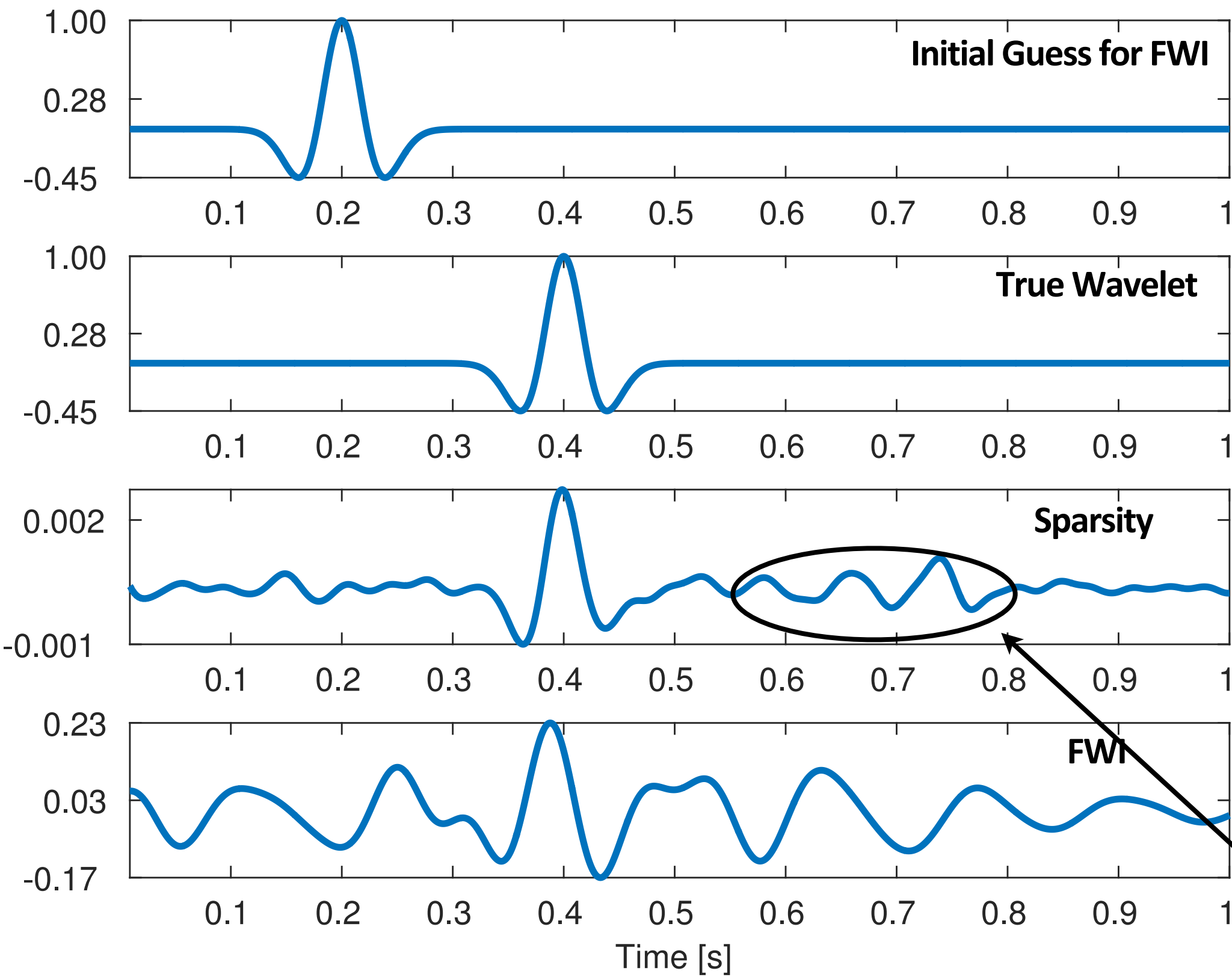


Location 2



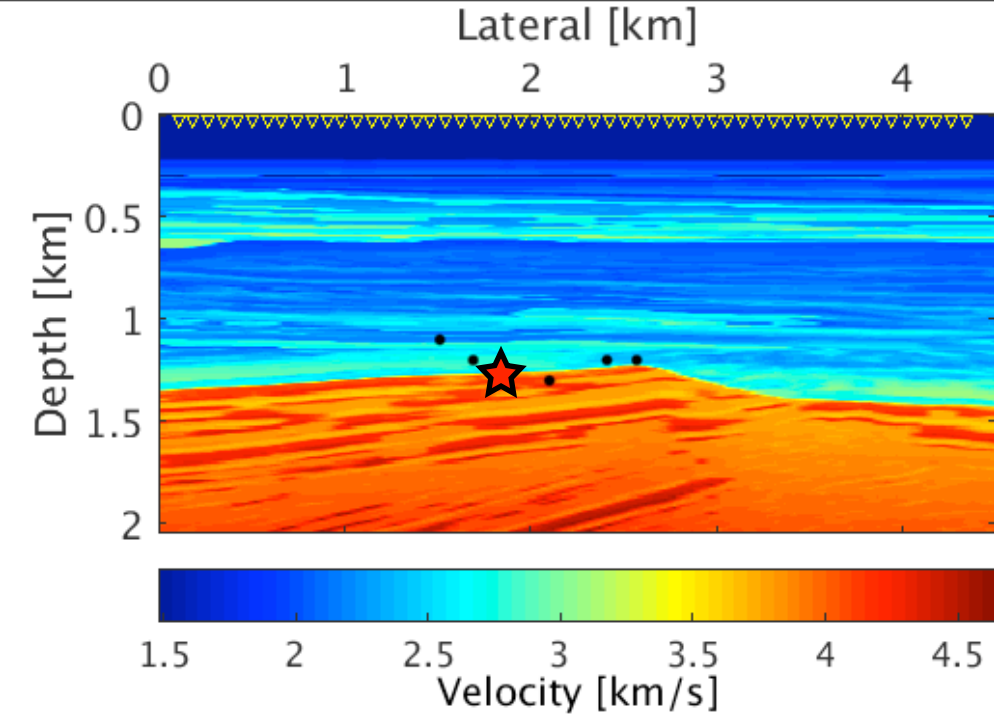
Wavelet

Spectrum



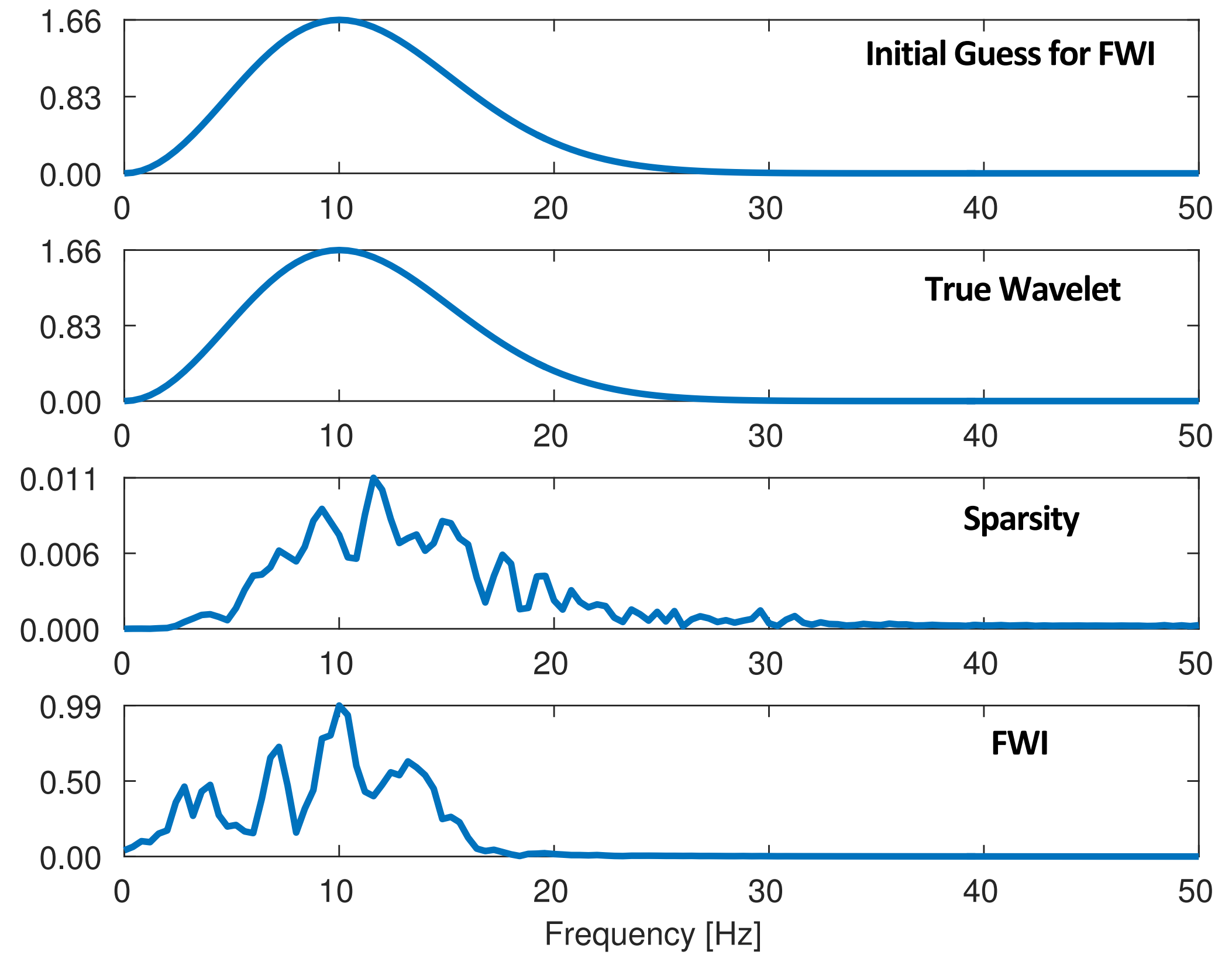
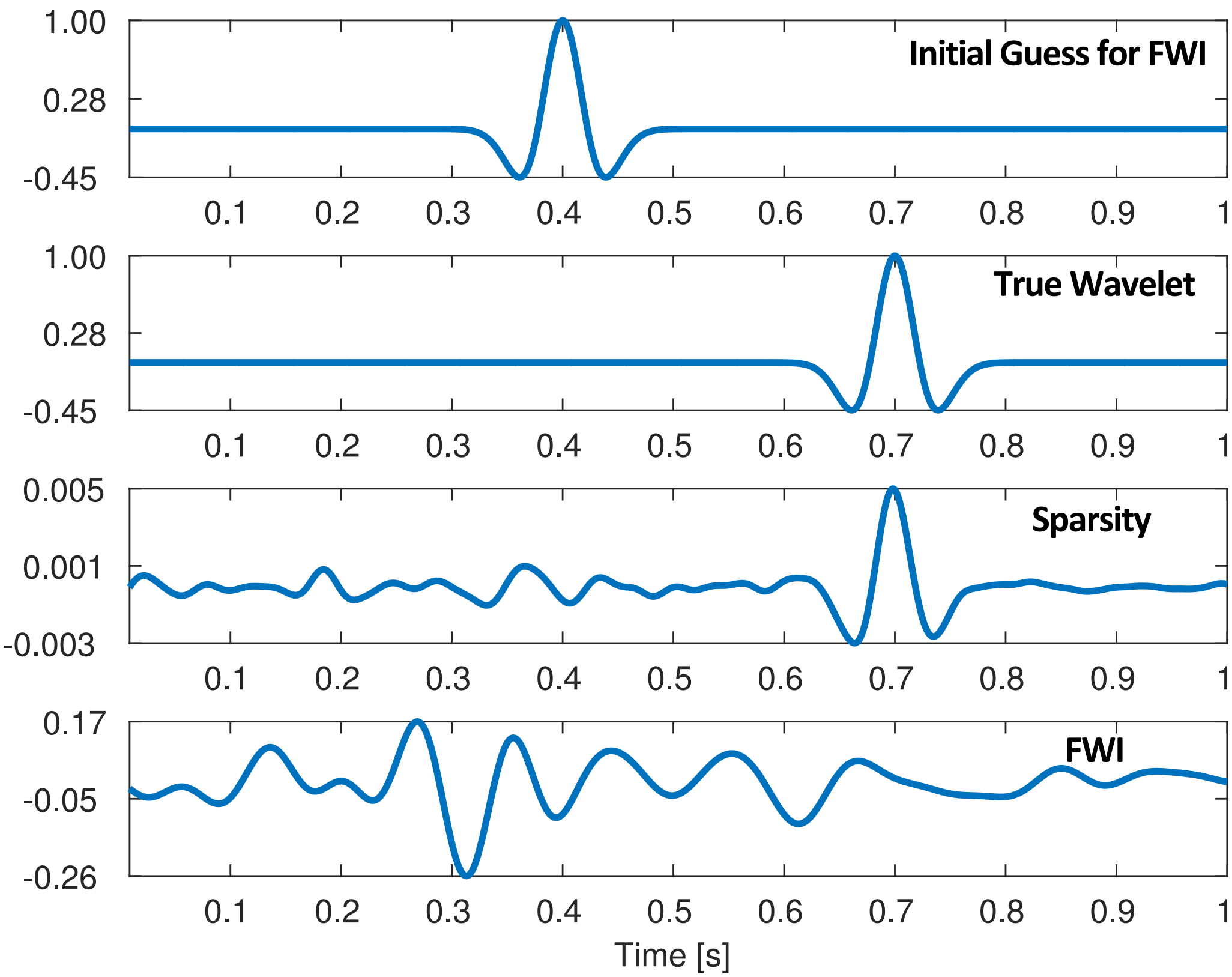
Sources within a wavelength

Location 3

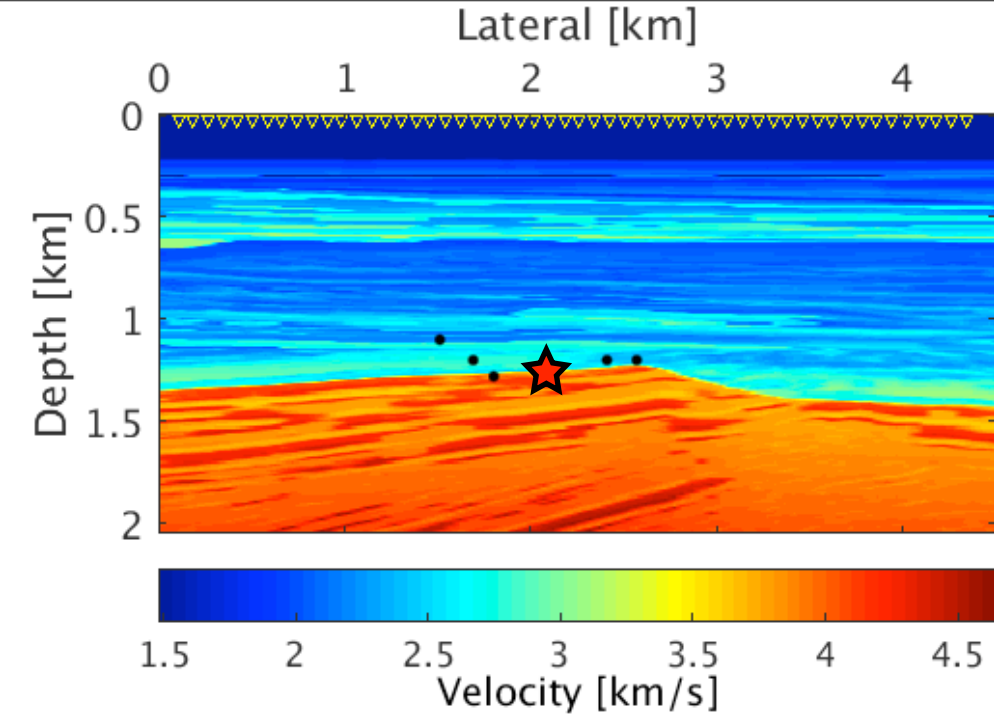


Wavelet

Spectrum

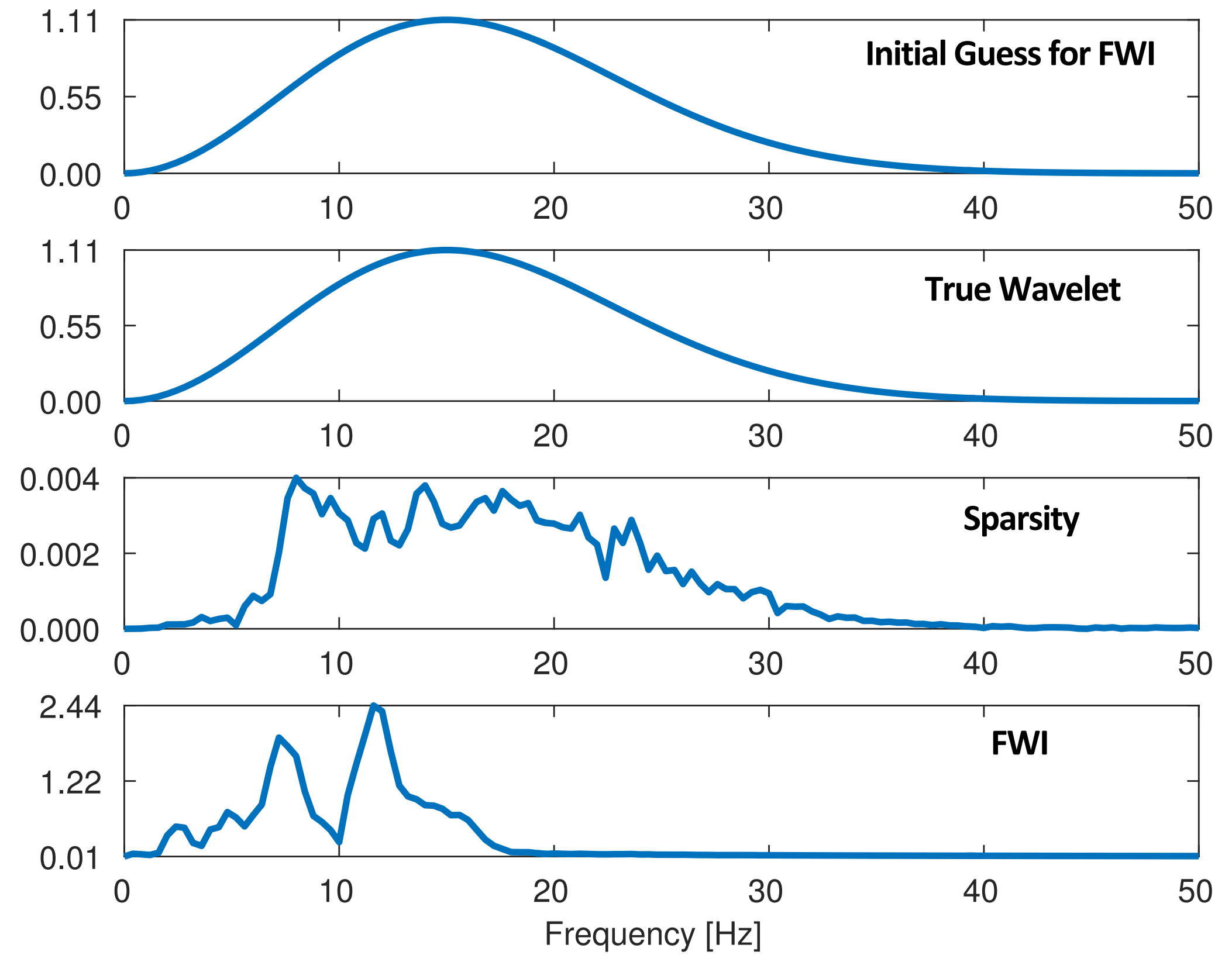
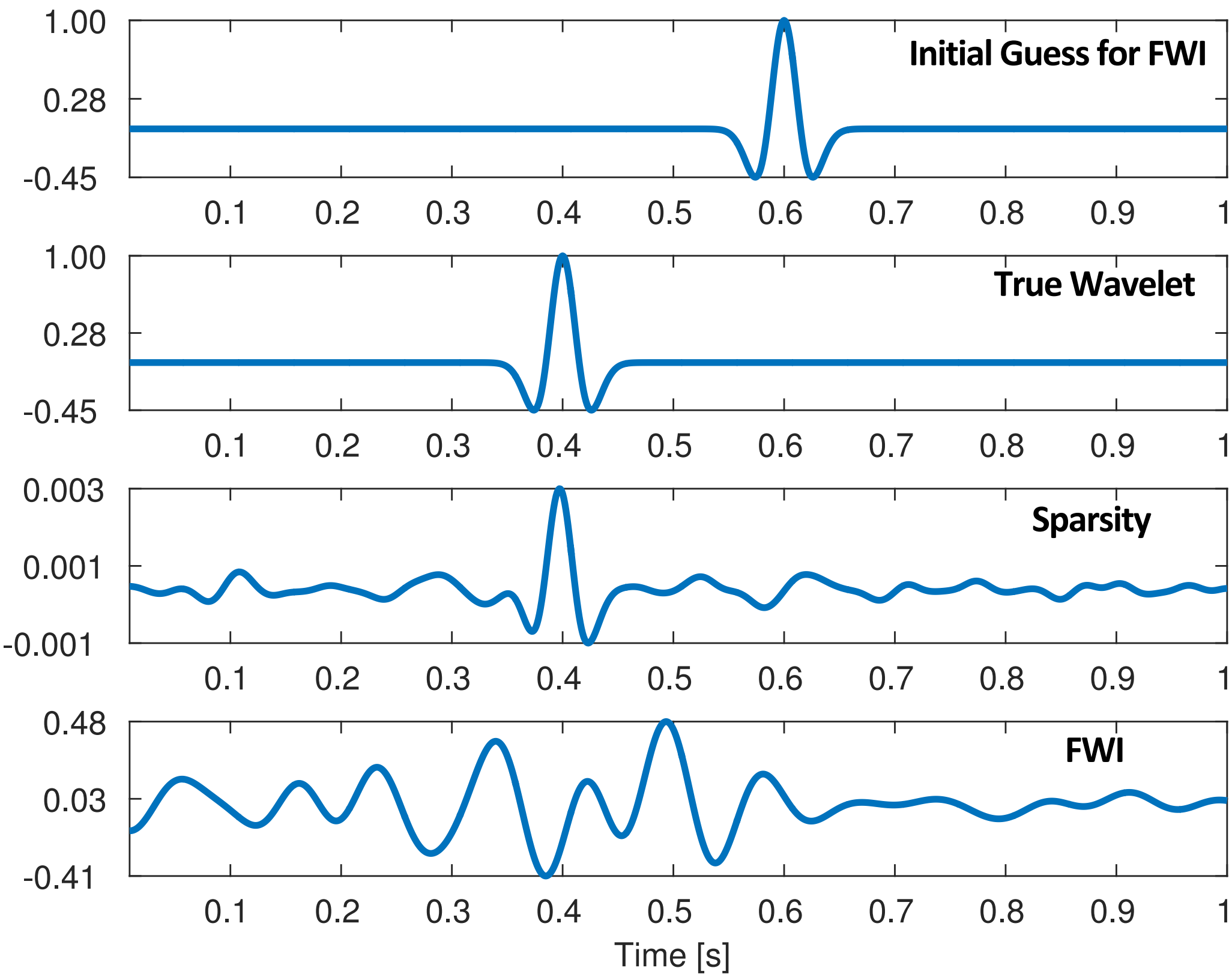


Location 4

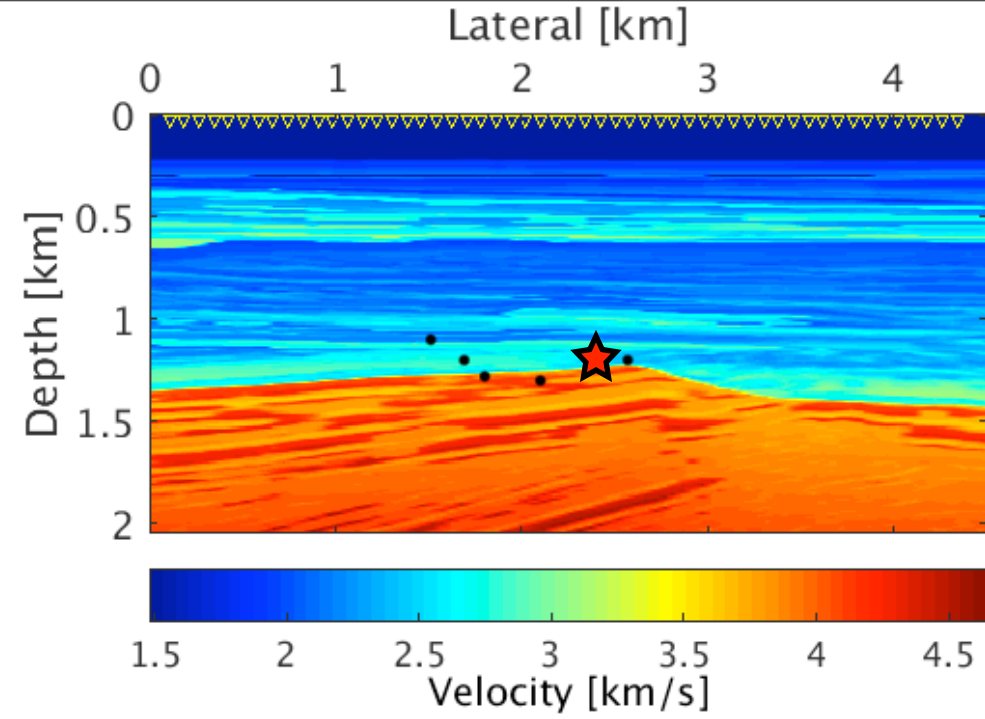


Wavelet

Spectrum

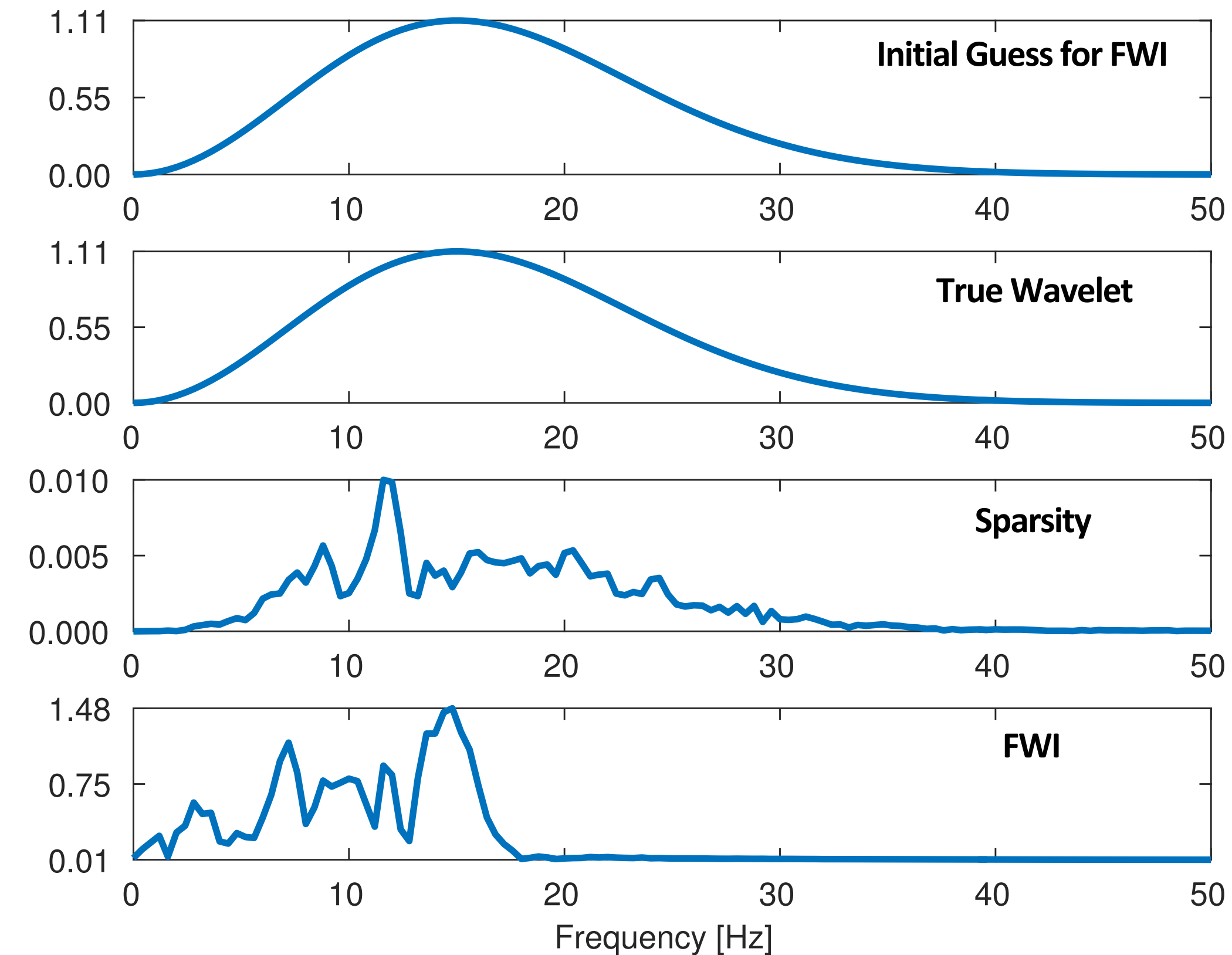
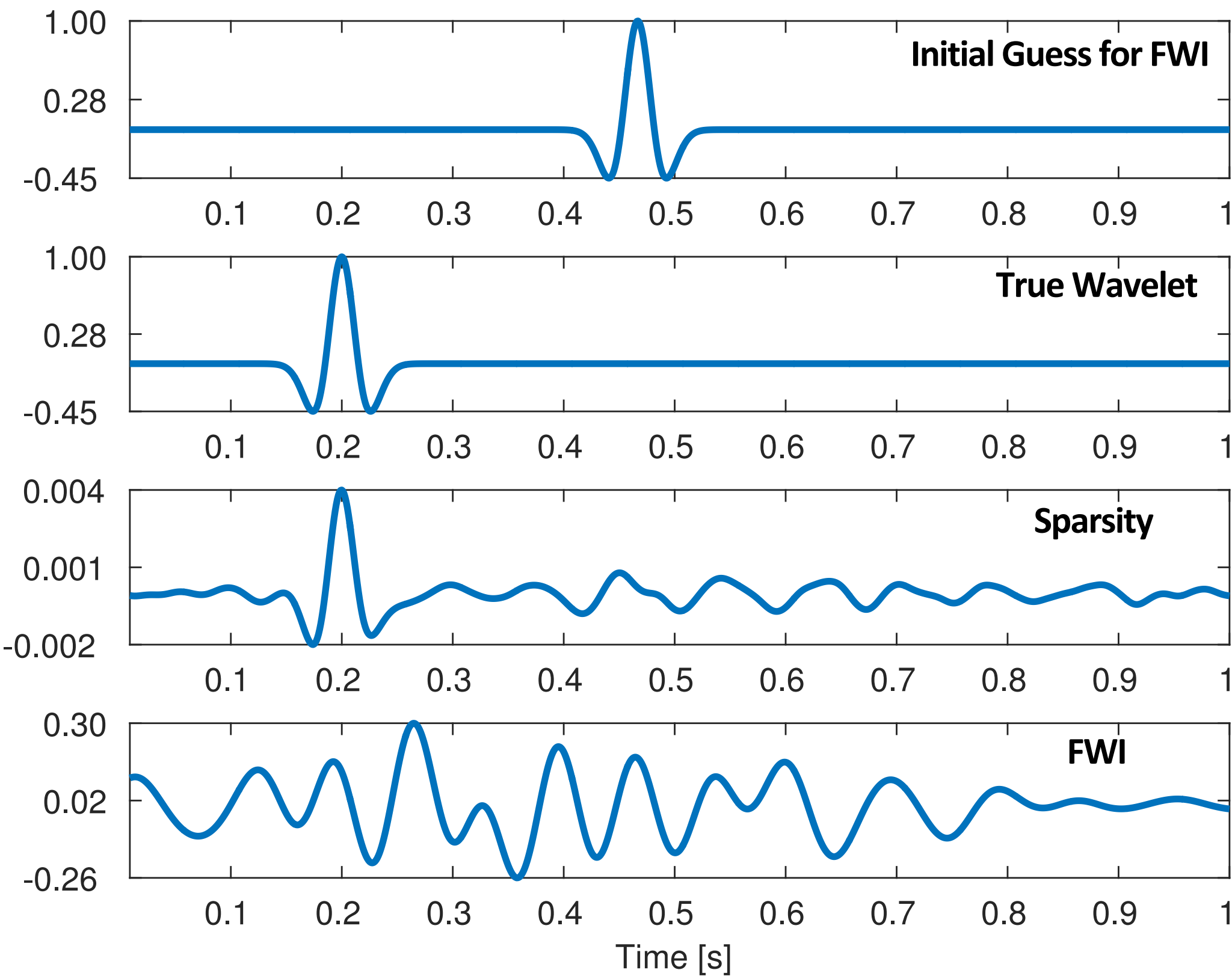


Location 5

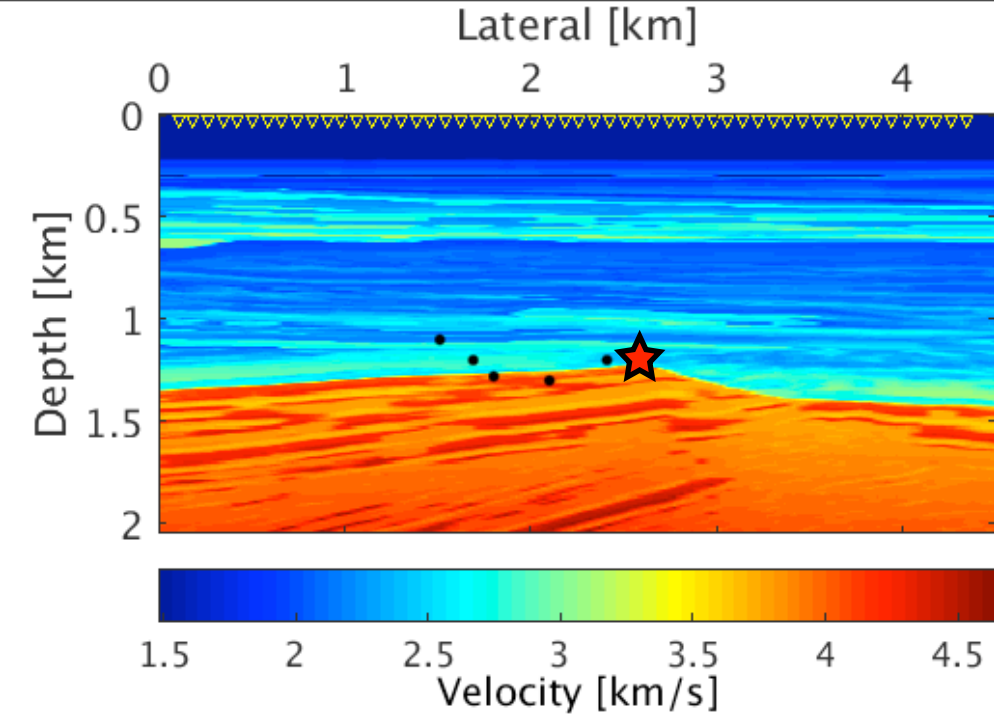


Wavelet

Spectrum

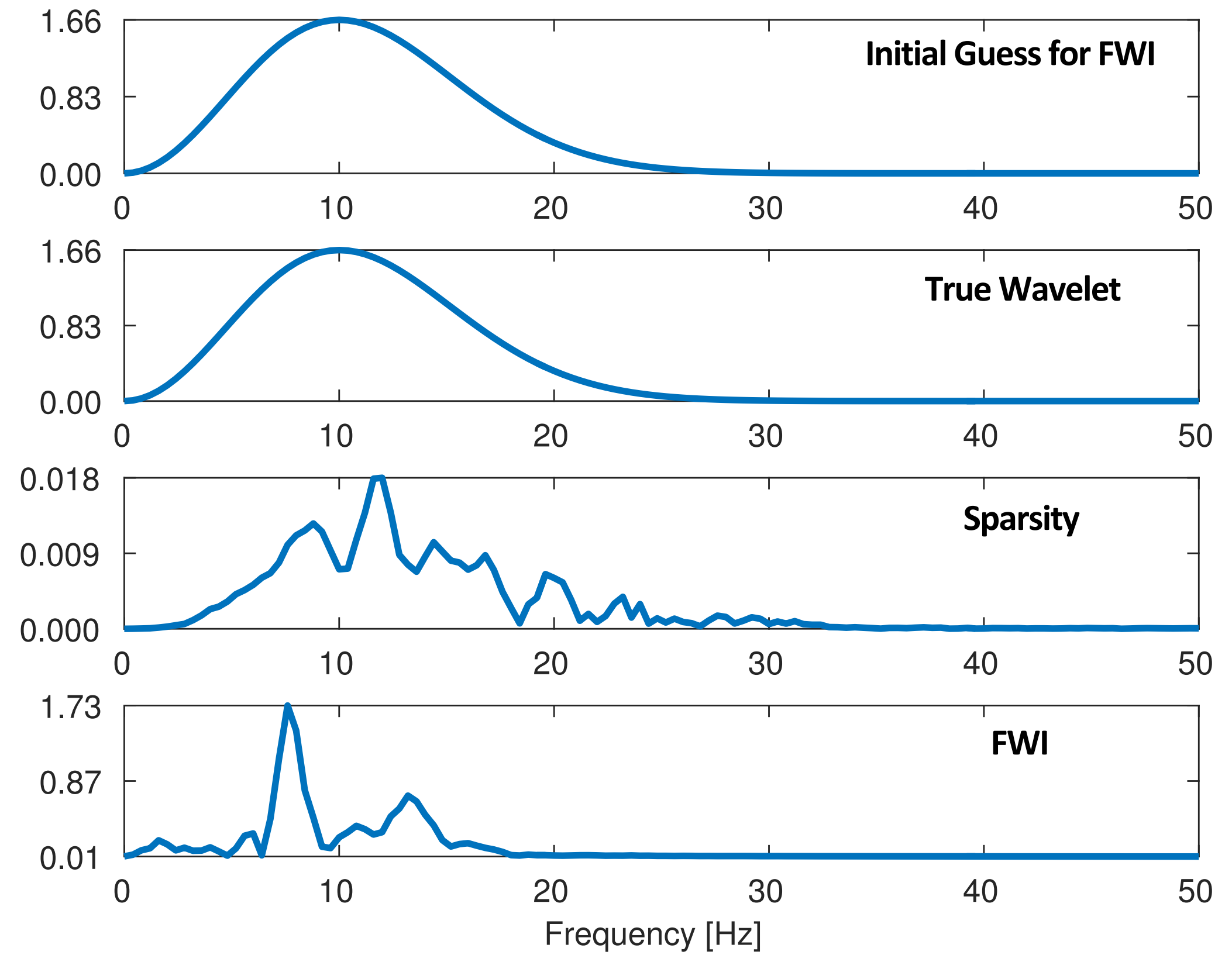
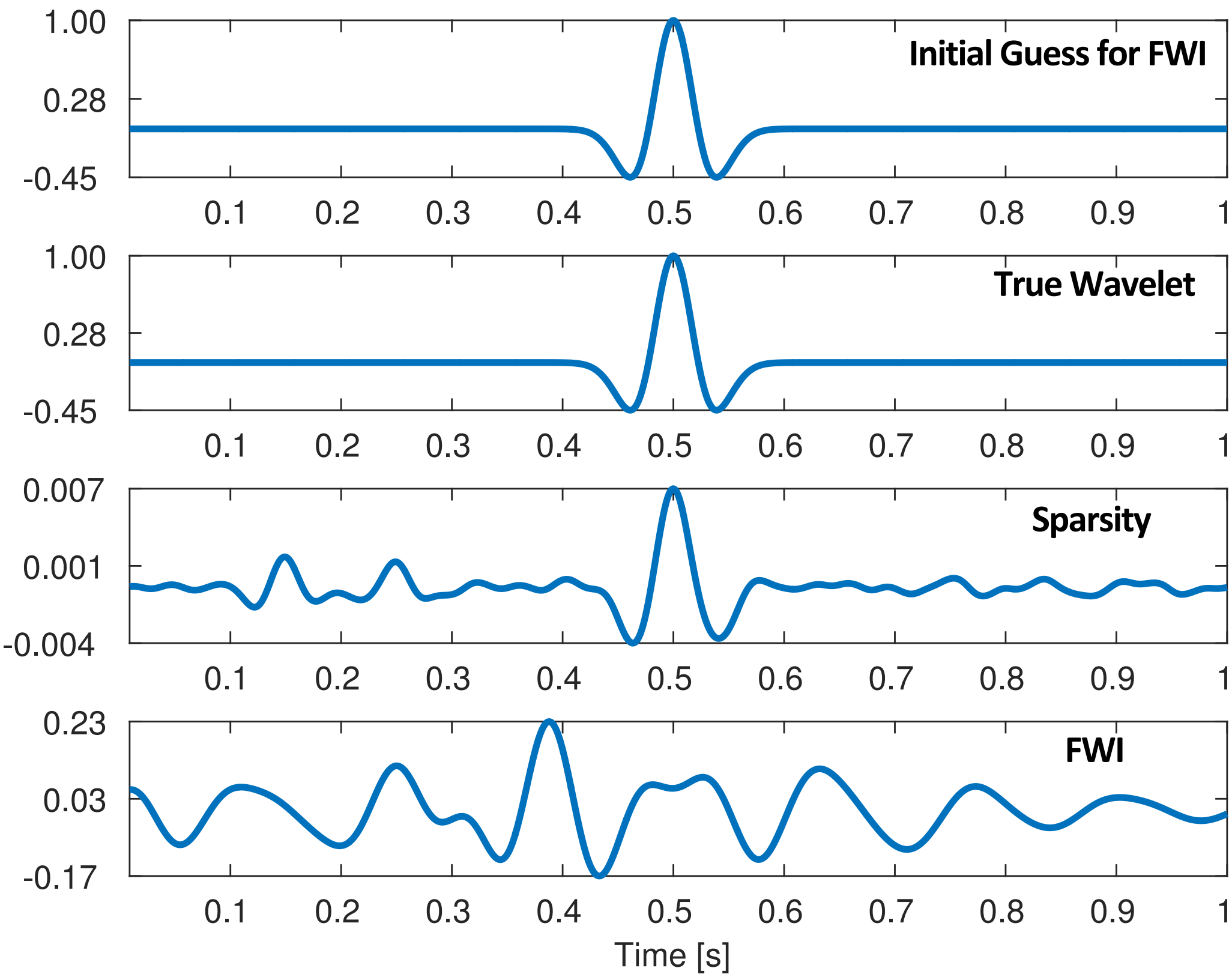


Location 6



Wavelet

Spectrum



Linearized Bregman via LBFGS acceleration

We solve the dual

$$\underset{\mathbf{y}}{\text{minimize}} \quad \mathbf{G}_\mu(\mathbf{y})$$

of the problem

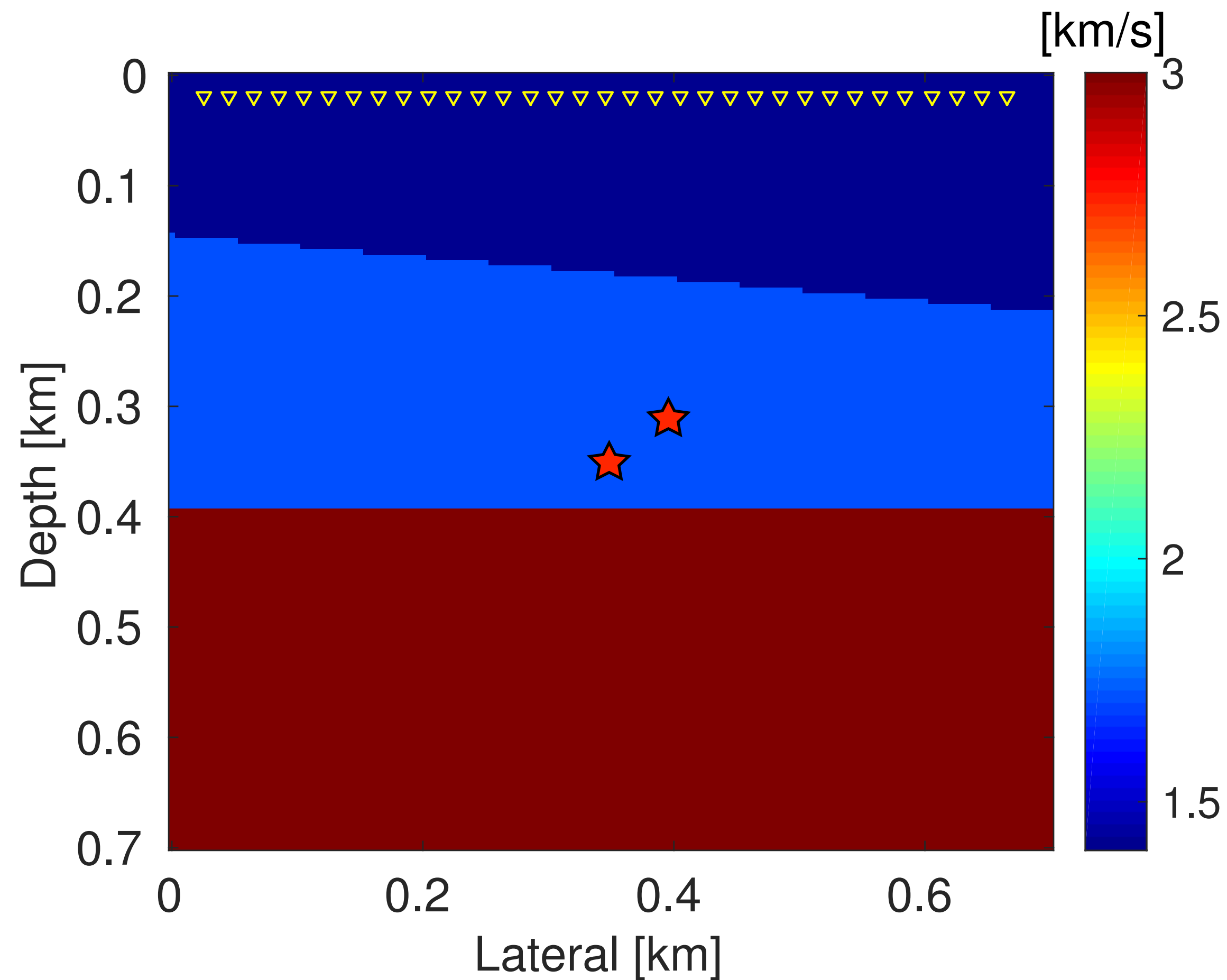
$$\begin{aligned} &\underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Q}\|_F^2 \\ &\text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

via LBFGS acceleration

Case Study

- ▶ Two closely spaced sources
 - Within a wavelength
- ▶ 2.5 D modeling
- ▶ Smooth velocity model
- ▶ Comparison with Hybrid imaging result

Experimental setup



Modeling information:

Model size: 0.7 km x 0.7 km

Grid spacing: 5 m

Receiver spacing: 5 m

Wavelet: Ricker wavelet

Receiver depth: 20 m

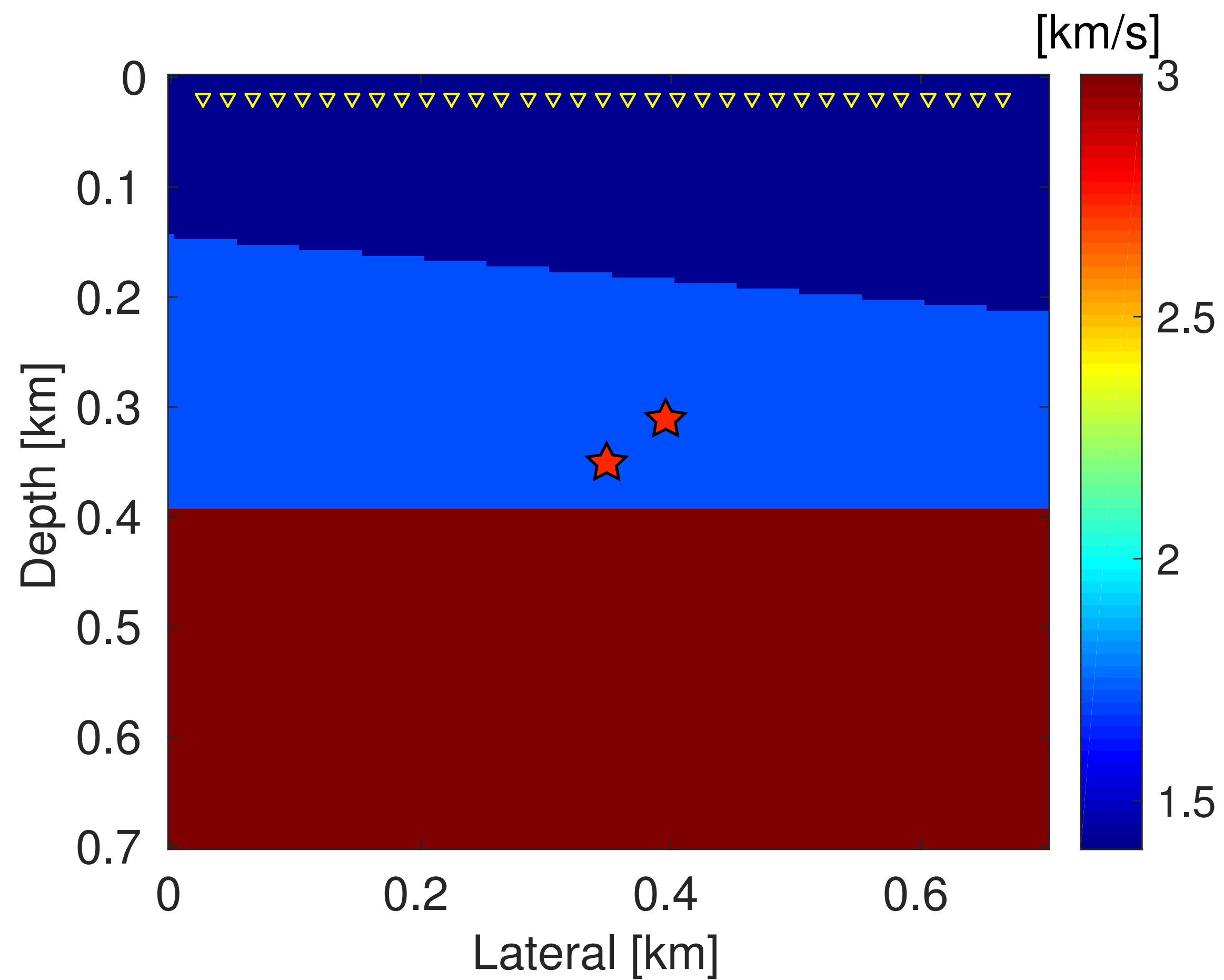
Fixed spread: 0.66 km

Sampling interval: 0.5 ms

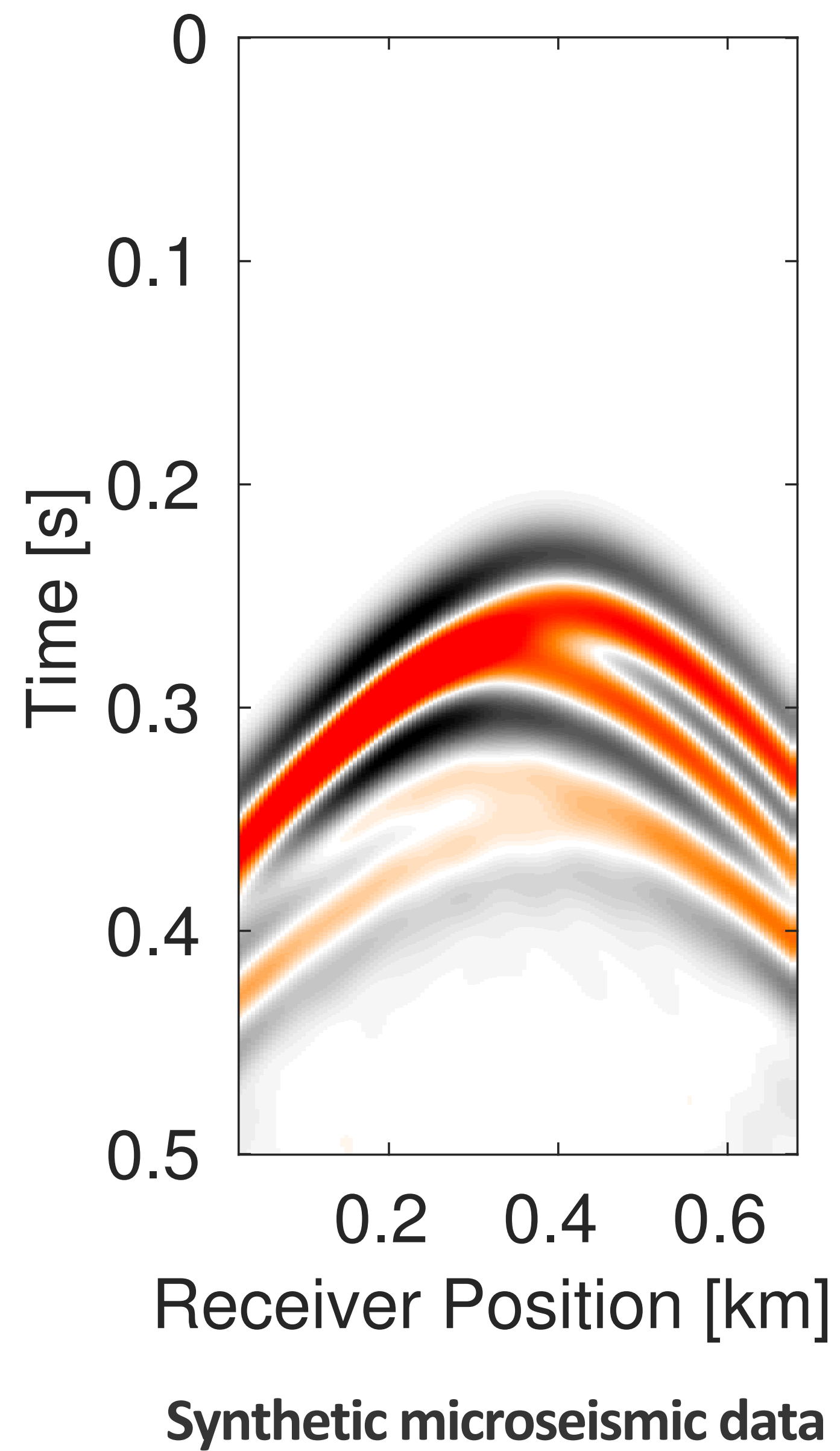
Recording length: 0.5 s

Peak frequency : 15 Hz

Experimental setup

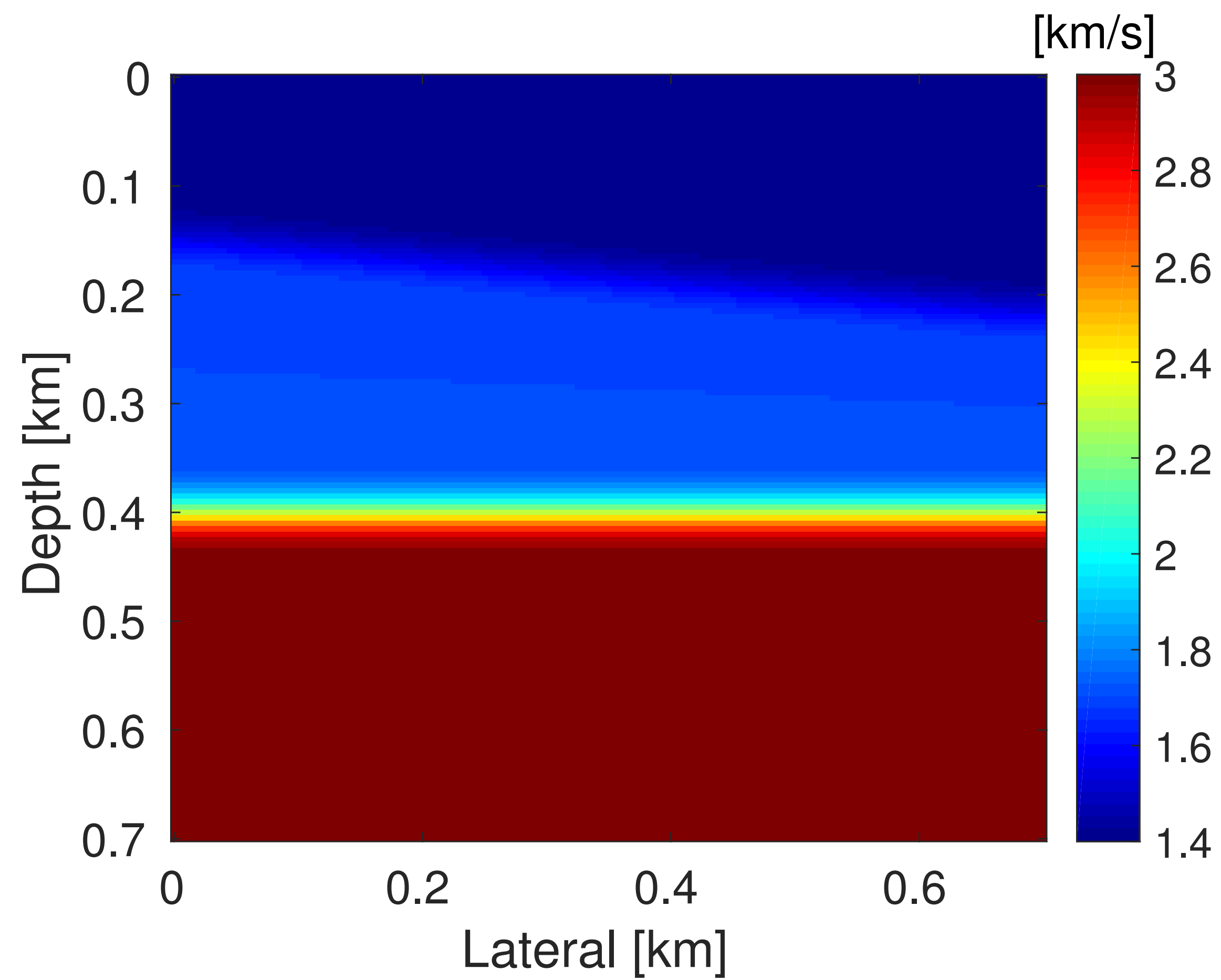


Dominant wavelength: 113 m
Source separation: 62 m

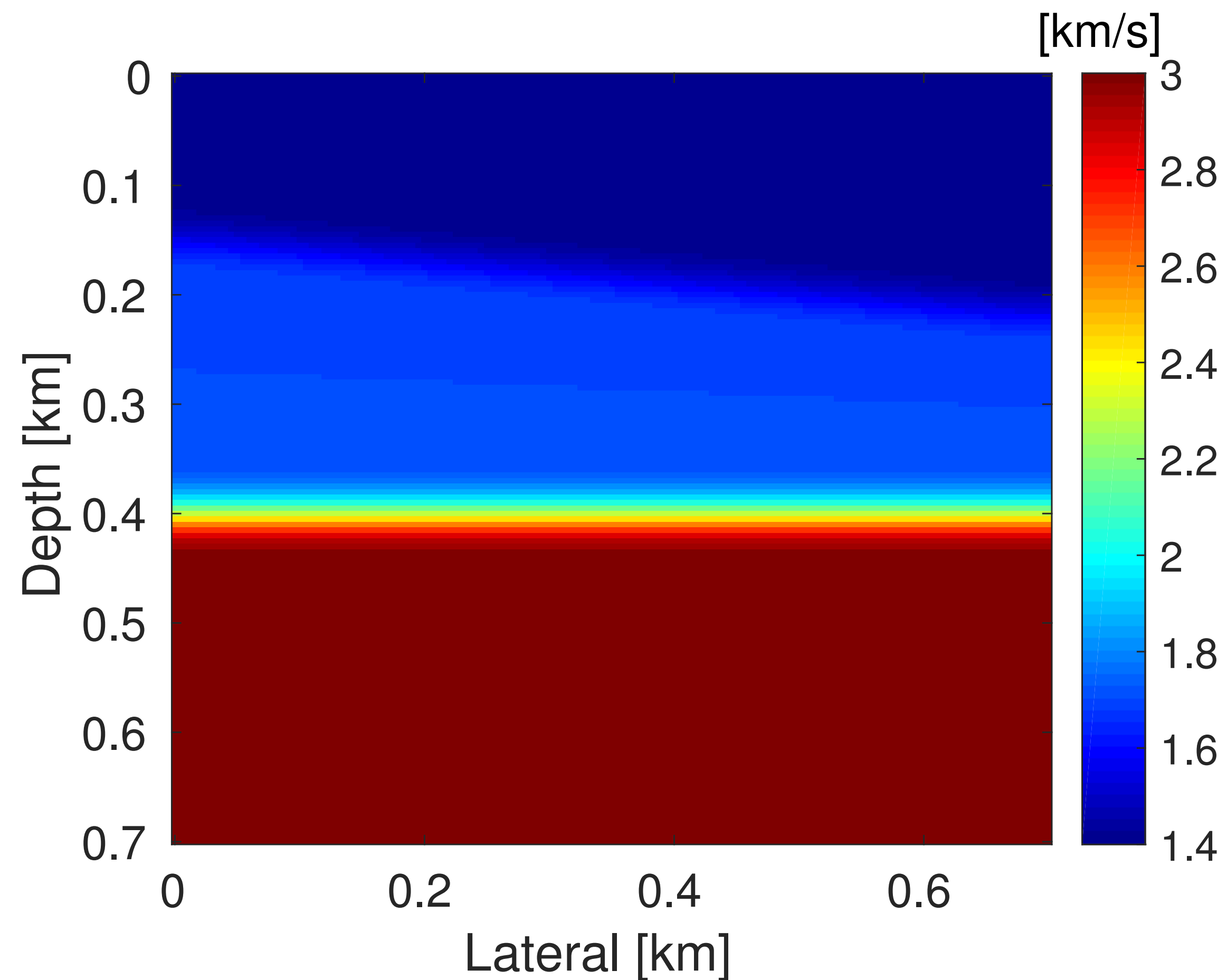


**Data is simulated using
2.5 D finite difference
time stepping code**

Smooth velocity model

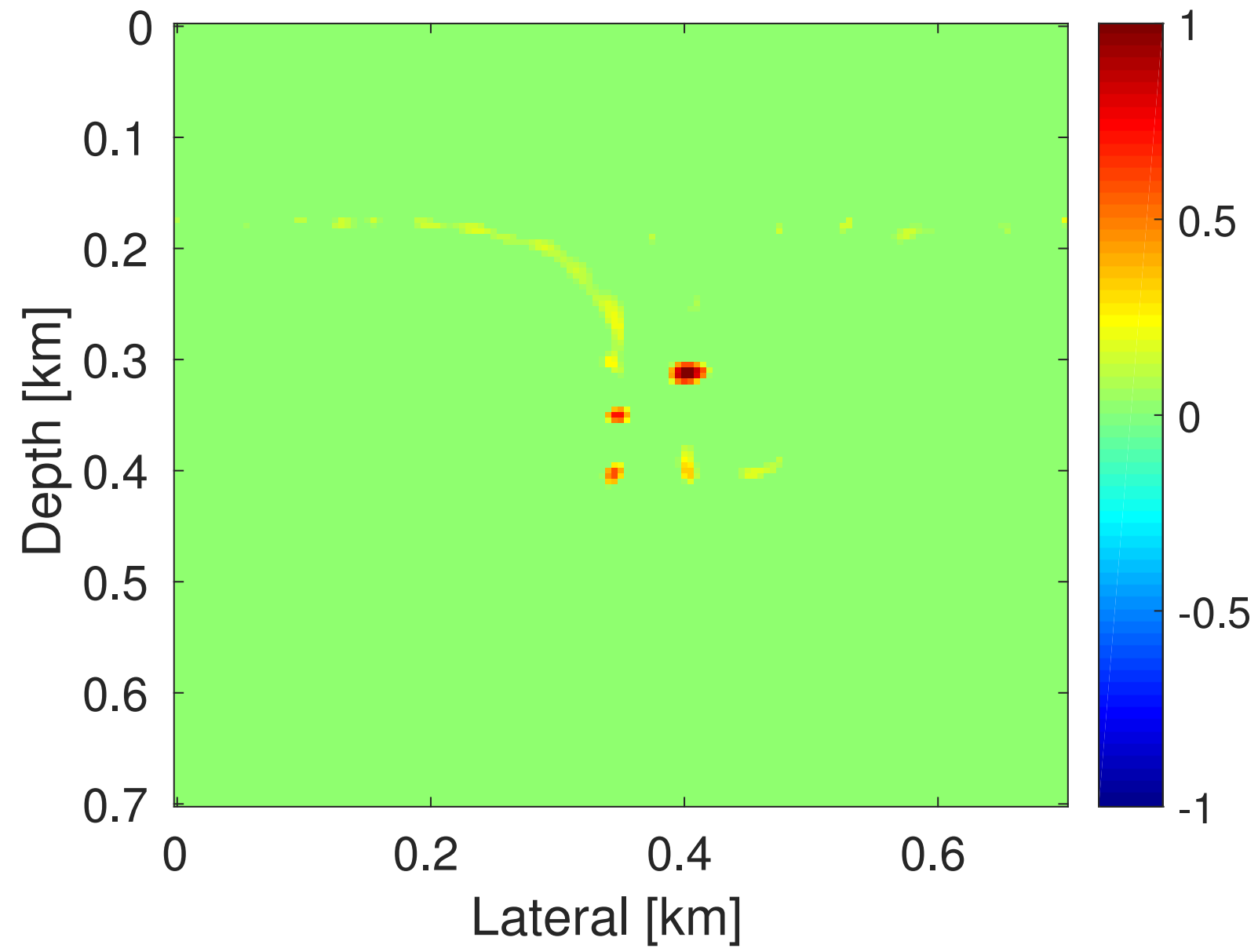


Smooth velocity model

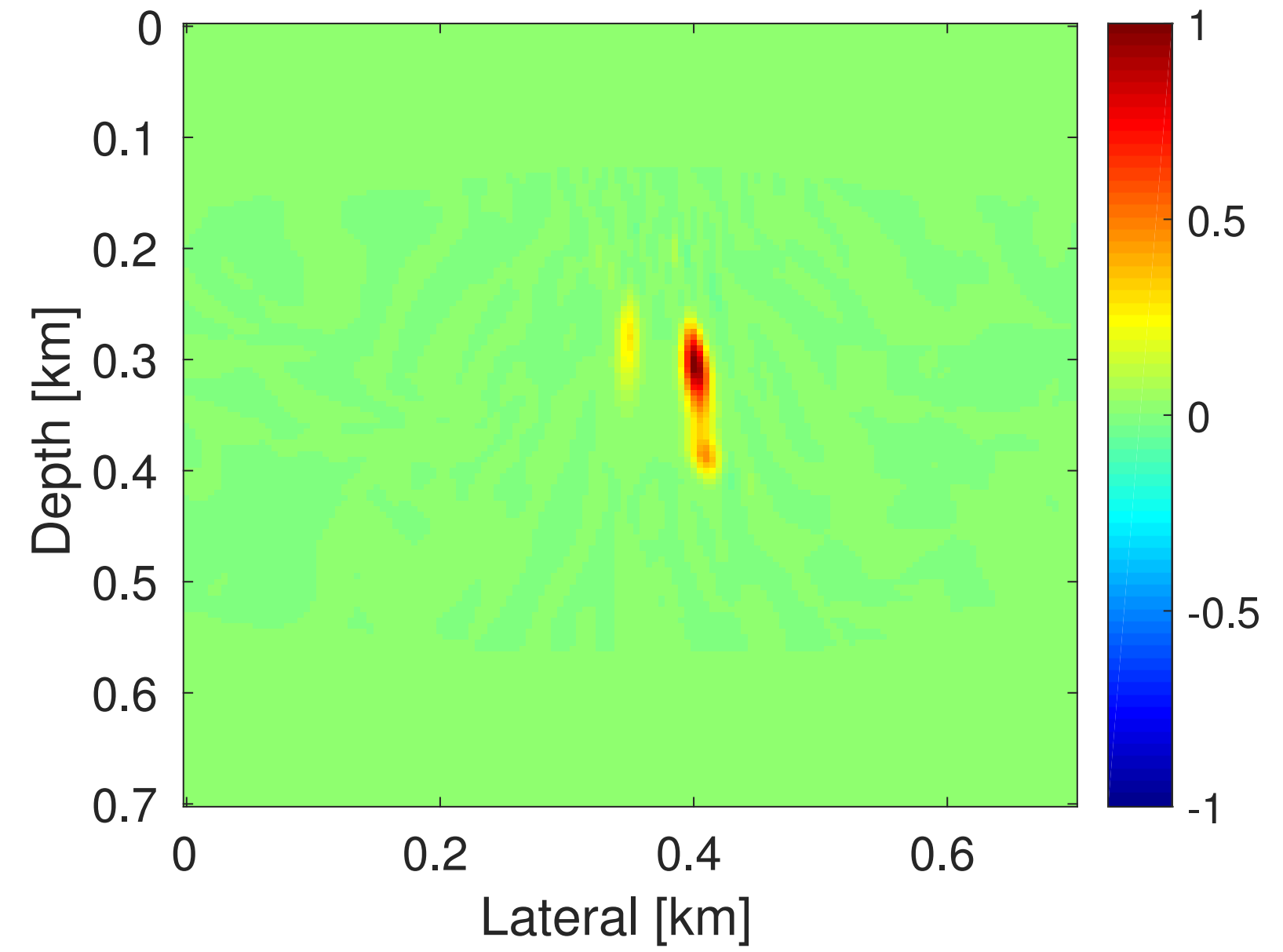


Used for joint microseismic source location & source time function estimation

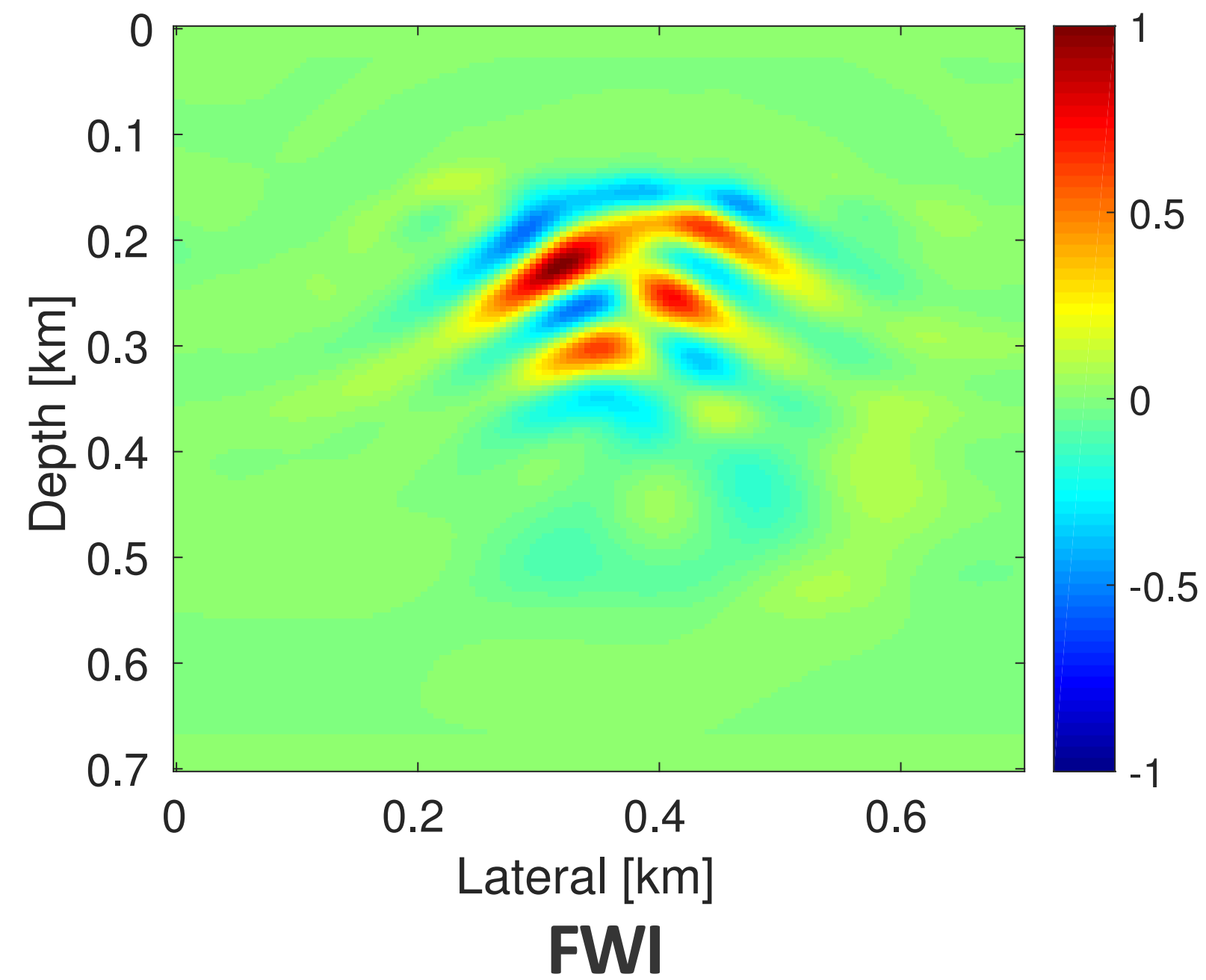
Estimated Source location



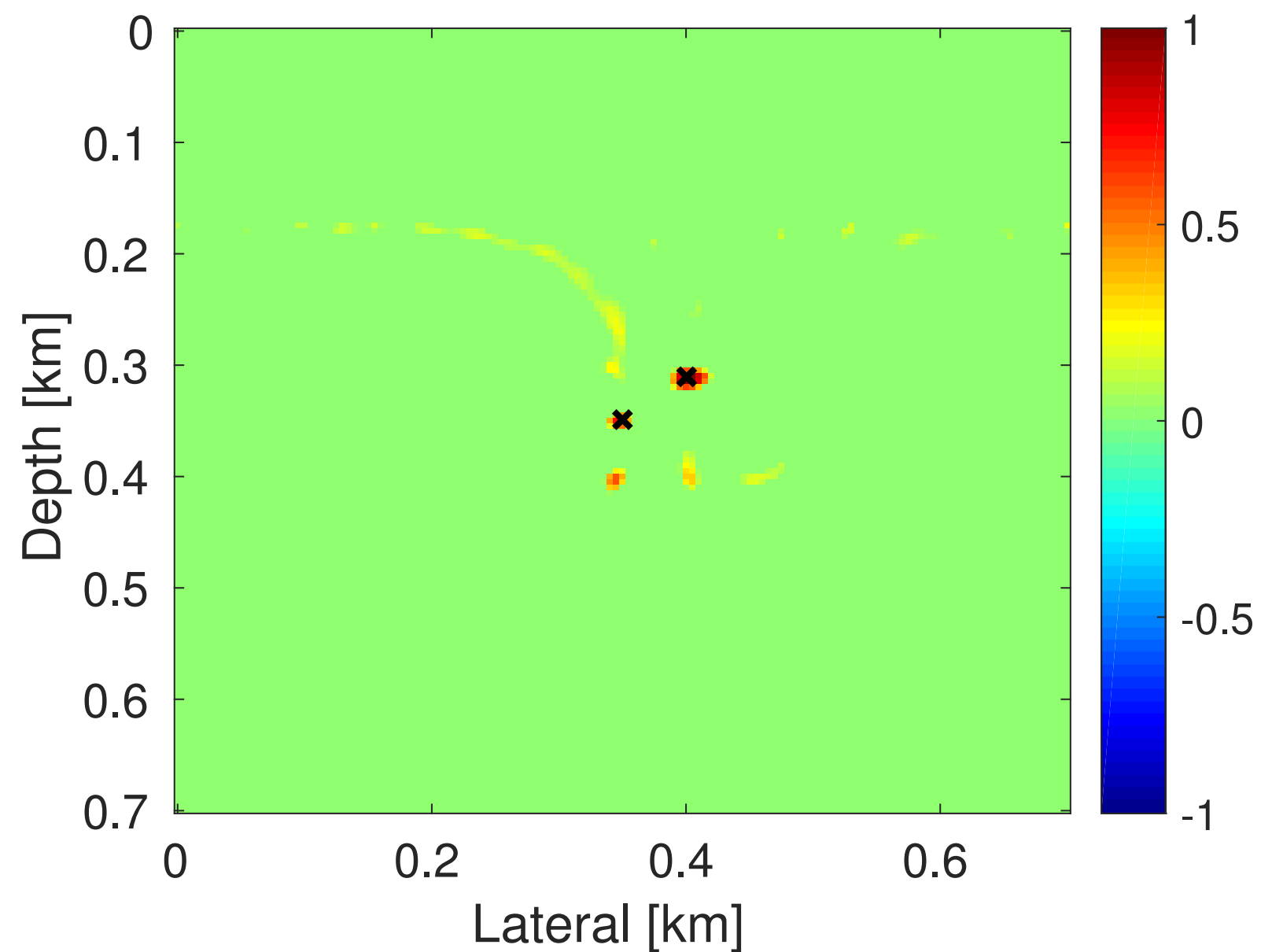
Sparsity-promoting



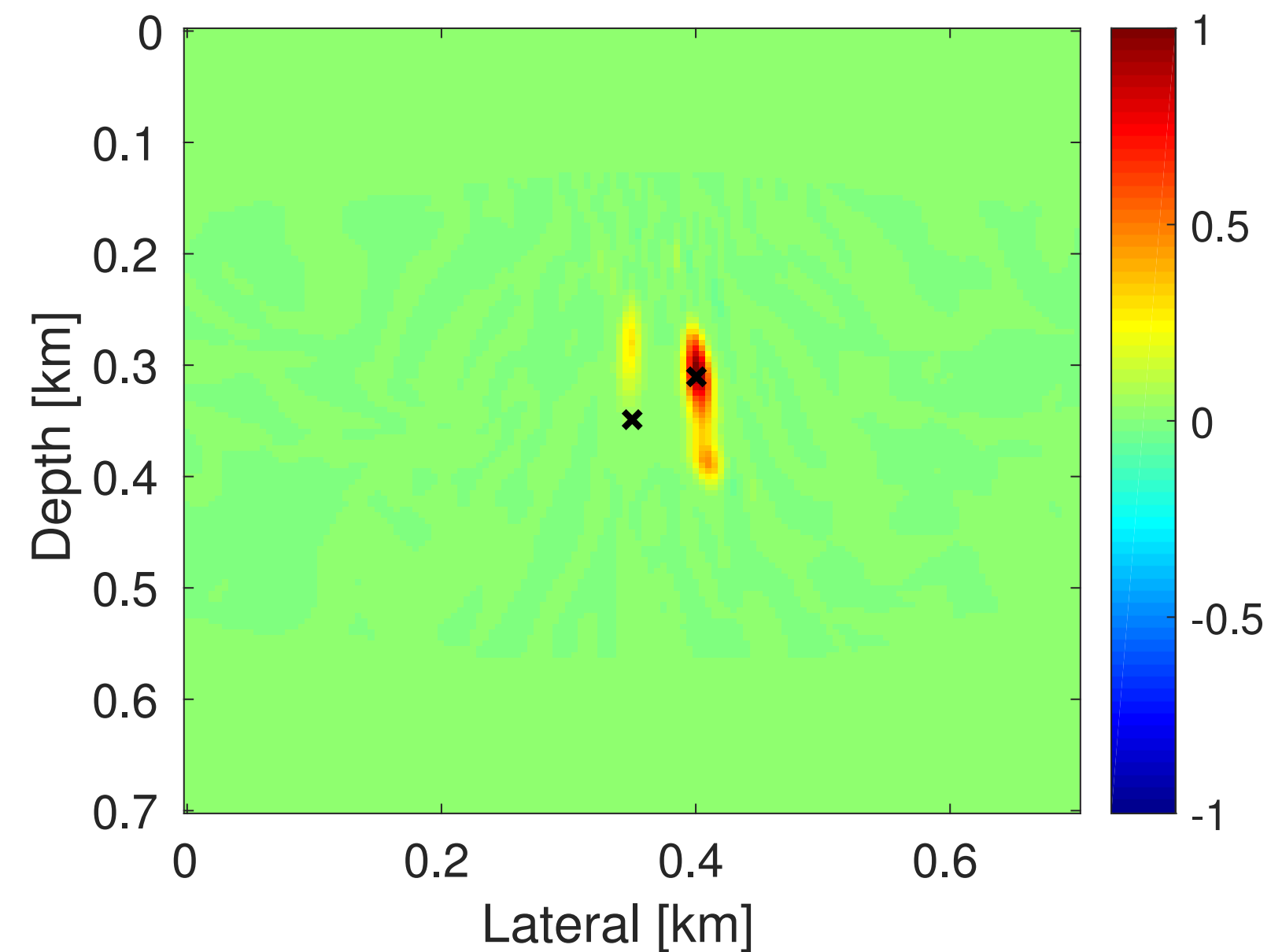
Hybrid Imaging



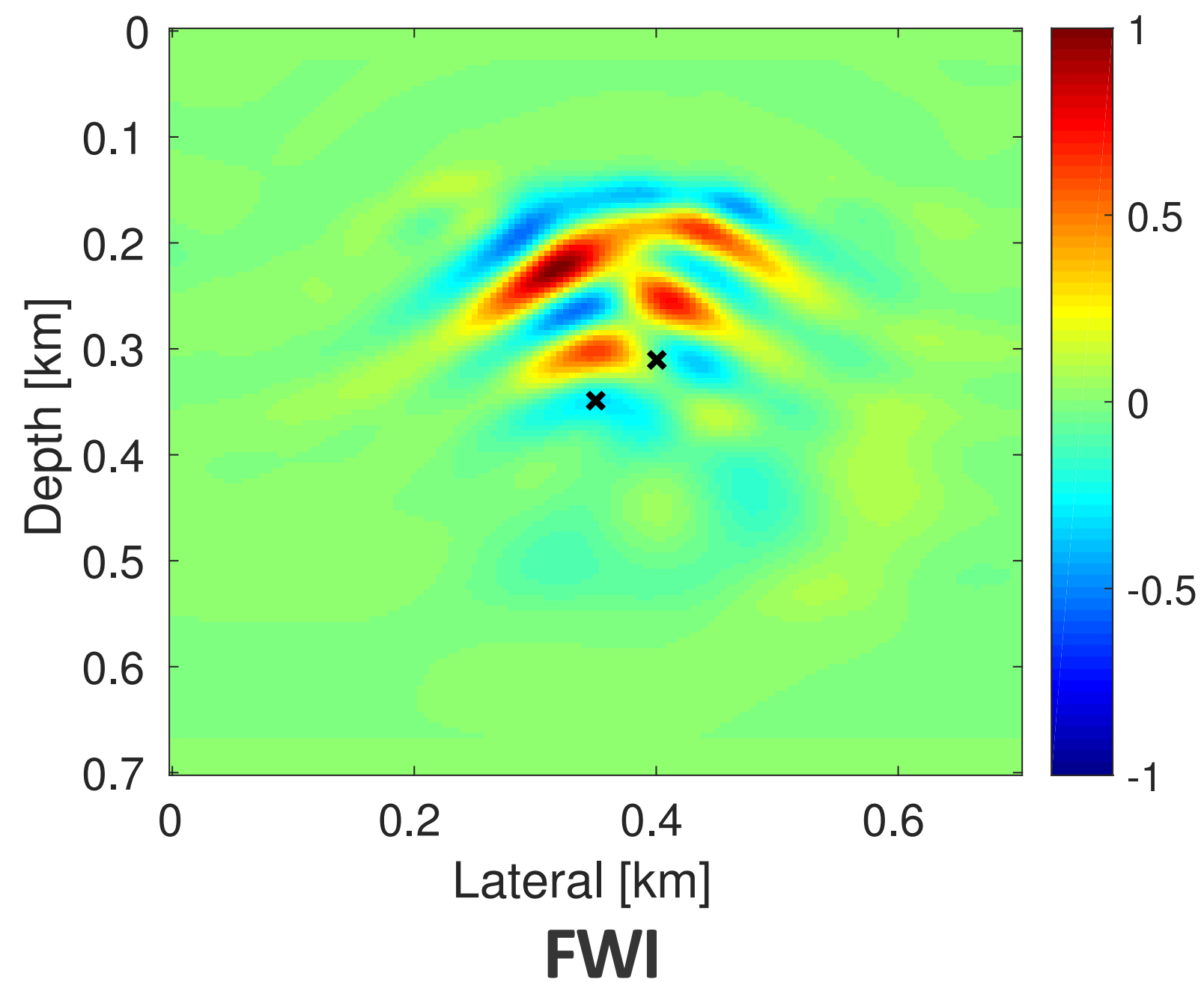
Estimated Source location



Sparsity-promoting

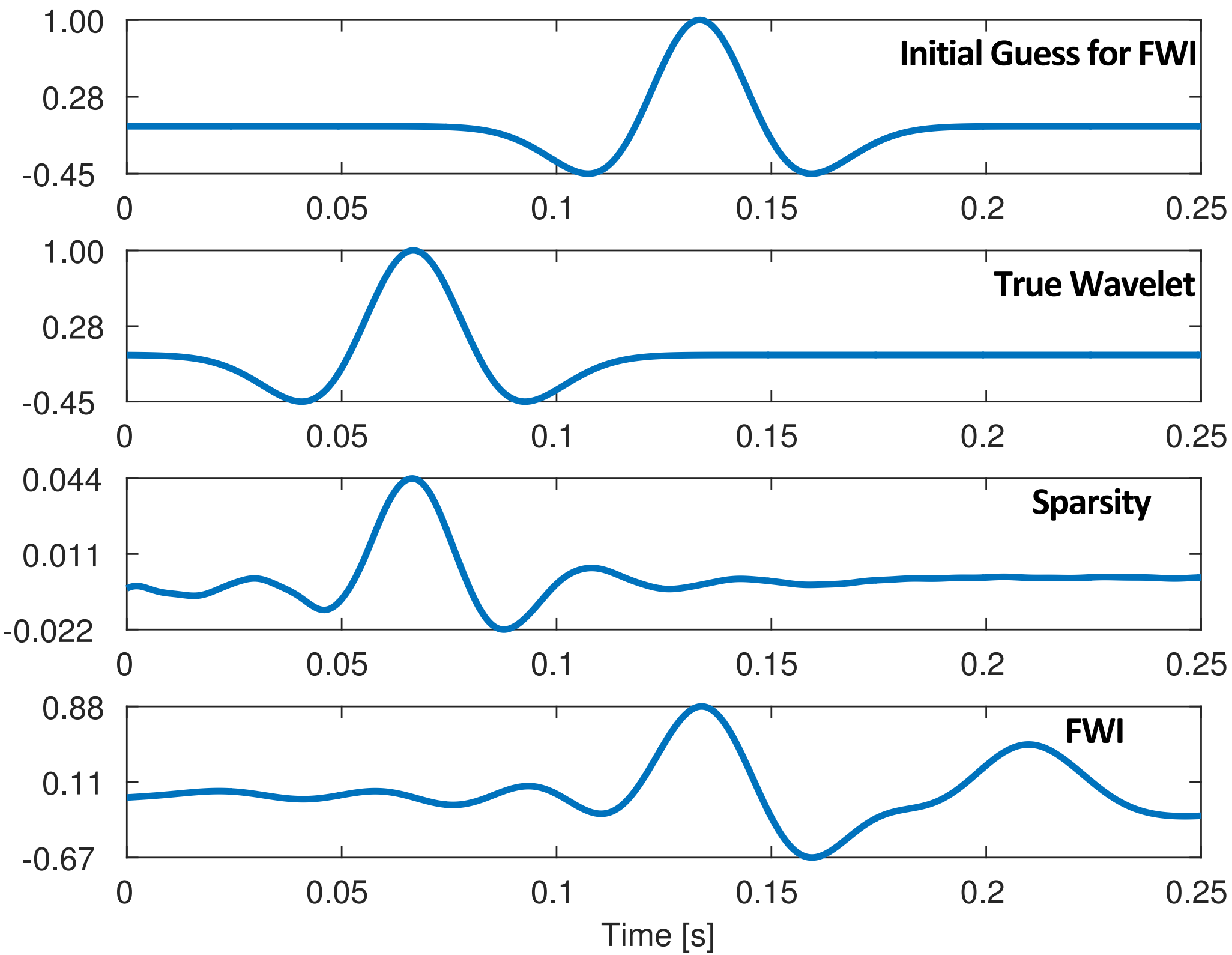


Hybrid Imaging

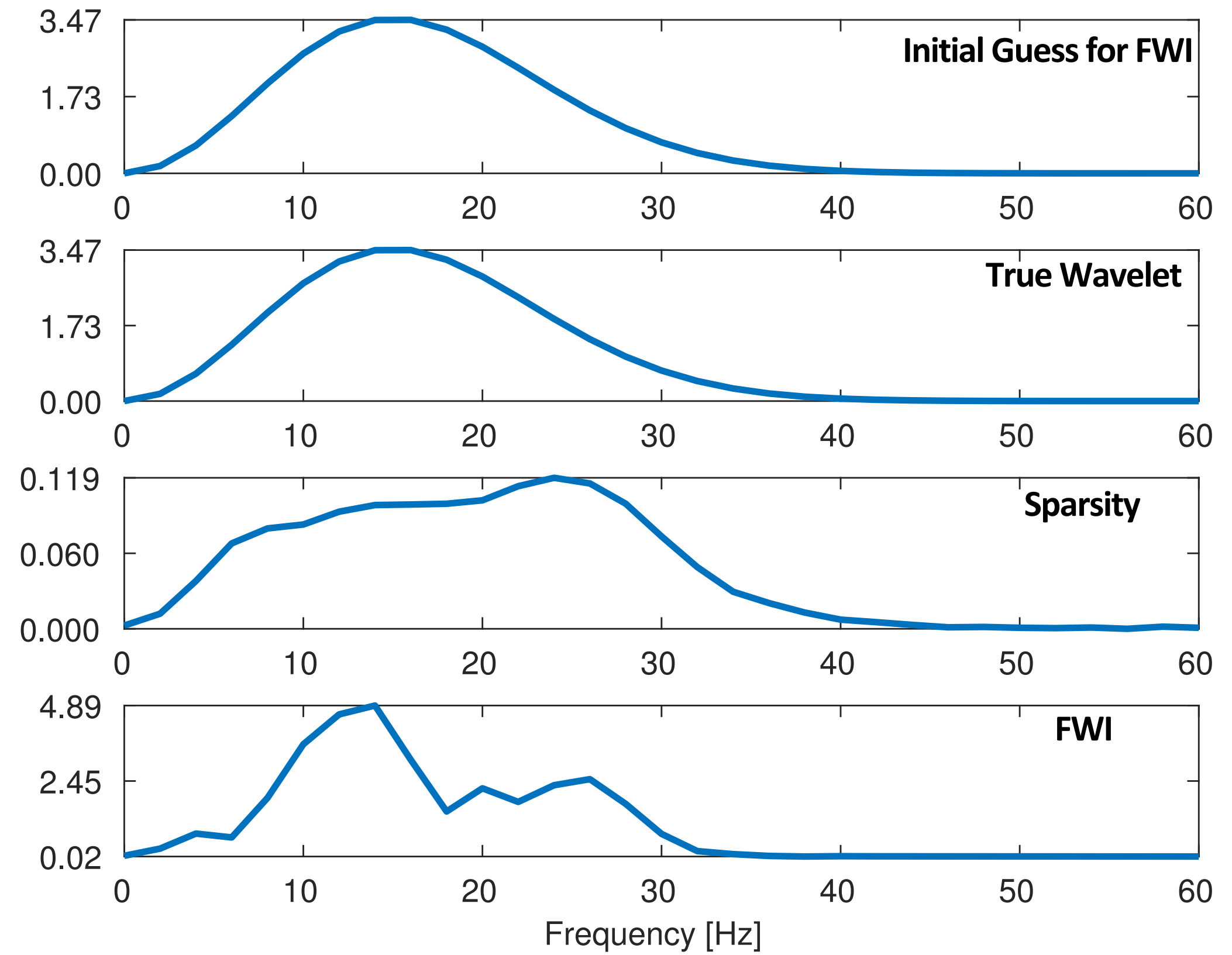


Location 1

Wavelet

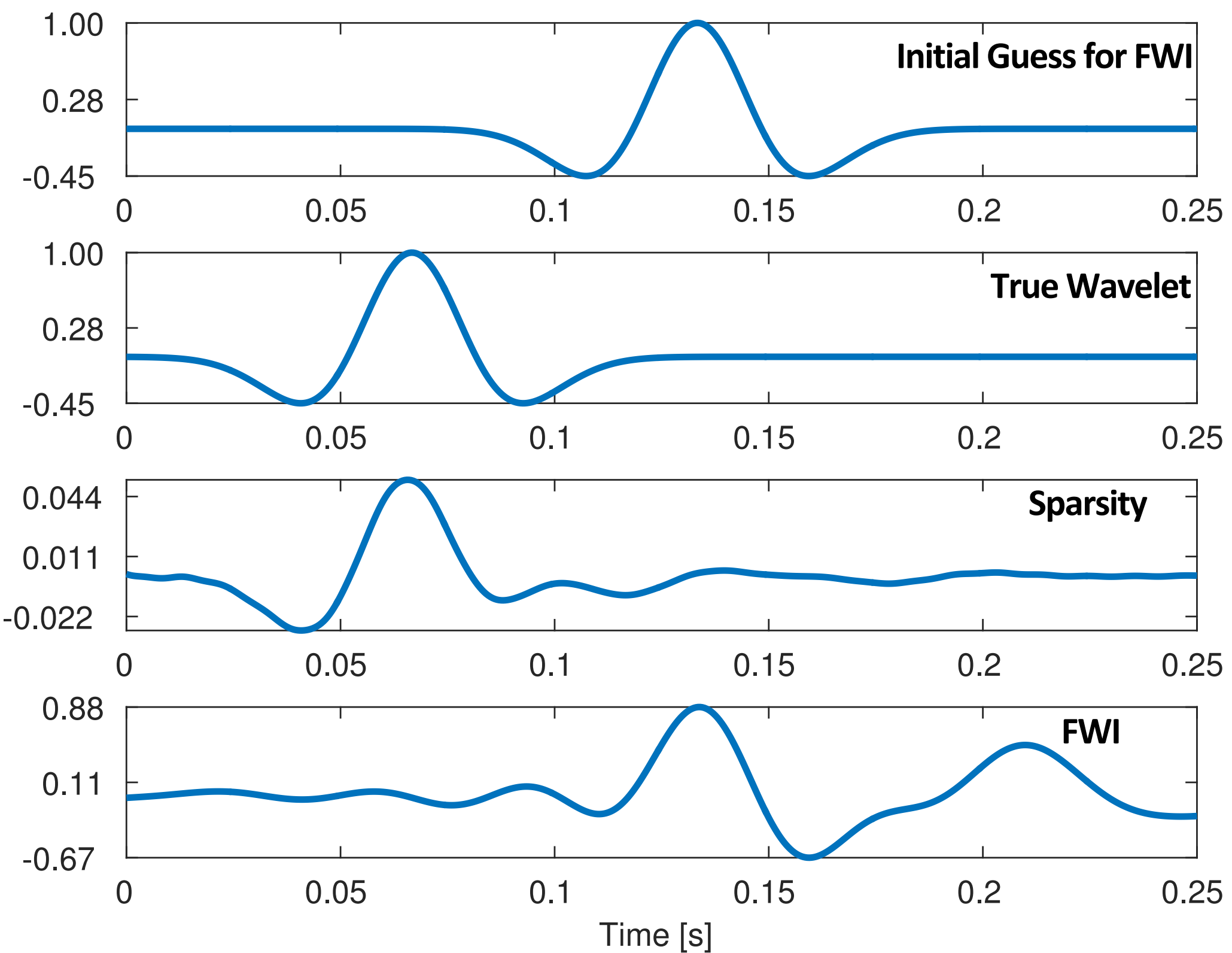


Spectrum

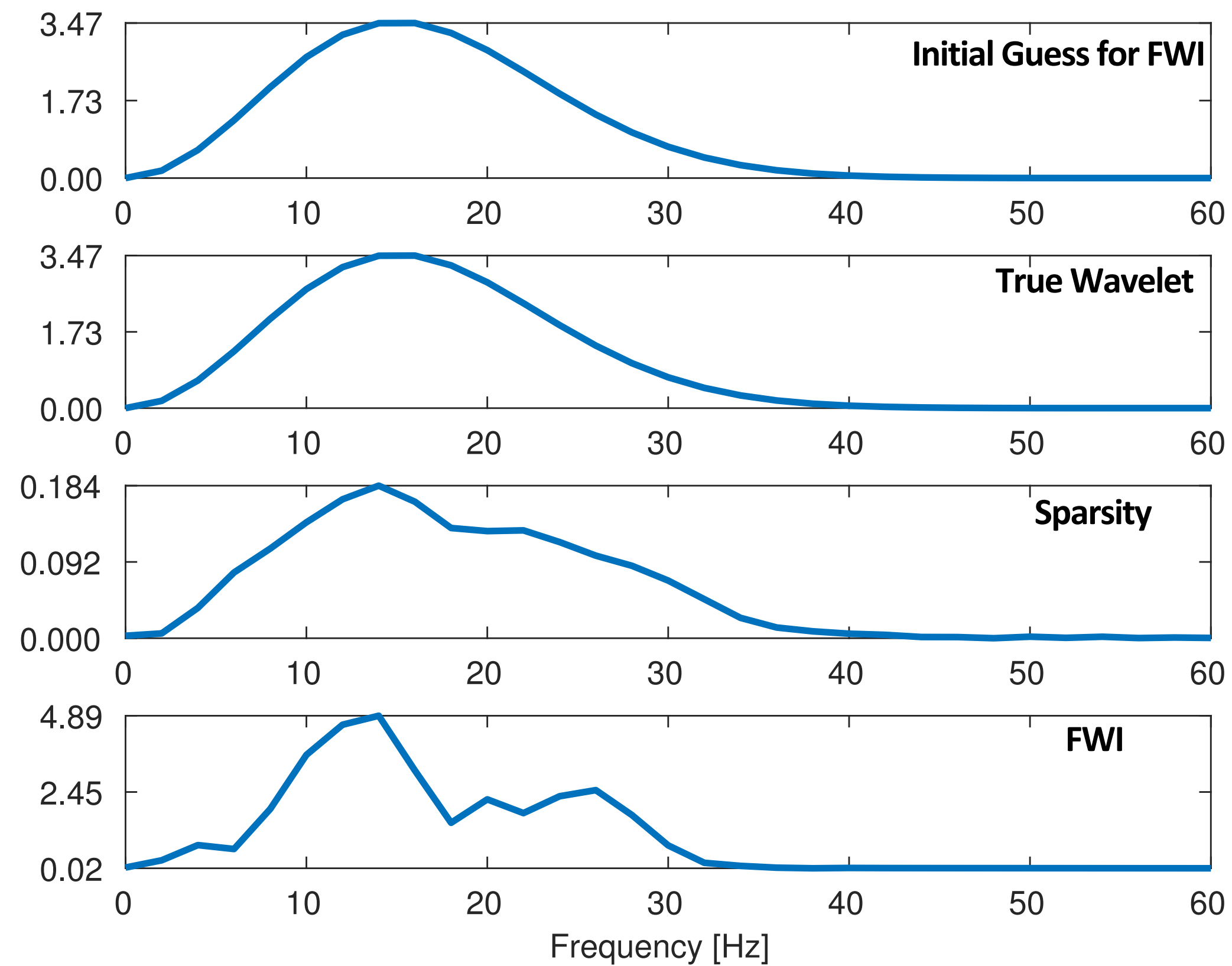


Location 2

Wavelet



Spectrum



Application to dipole sources

Motivation

Earthquake/microseismic source

- ▶ Moment tensor sources
- ▶ Double dipole

Objective

Dipole sources are

- ▶ Directional
- ▶ Can be decomposed horizontal and vertical components

Aim is to

- ▶ locate
- ▶ estimate the directivity by estimating each component
- ▶ estimate the source time function

Method

The original optimization problem

$$\begin{aligned} & \underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Q}\|_F^2 \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

Method

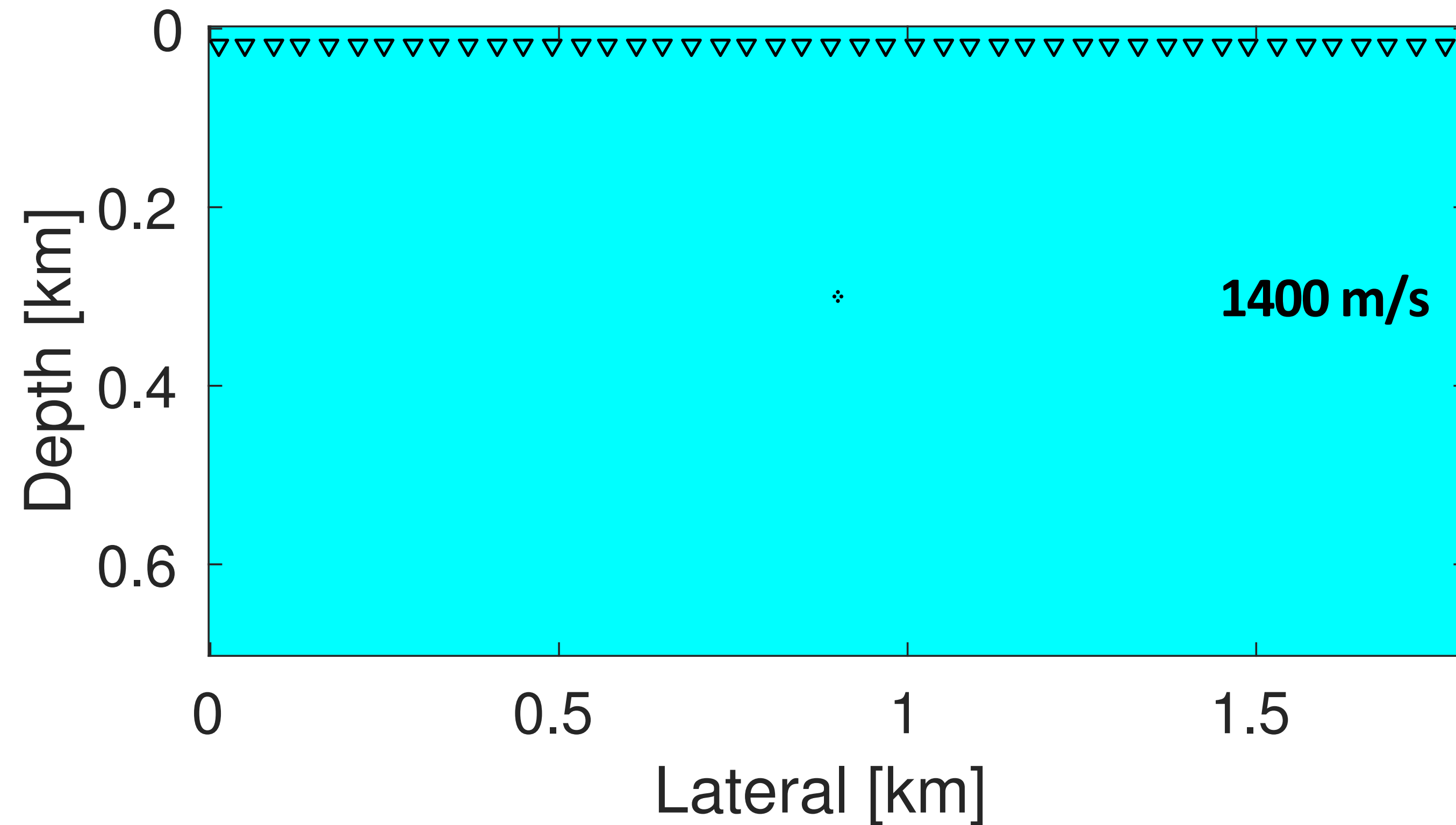
is modified to

$$\begin{aligned} & \underset{\mathbf{S}}{\text{minimize}} \quad \|\mathbf{S}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{S}\|_F^2 \\ & \text{subject to} \quad \|\mathcal{F}[\mathbf{m}](\mathcal{D}(\mathbf{S})) - \mathbf{d}\|_2^2 \leq \epsilon \end{aligned}$$

*where \mathbf{S} is the synthesis matrix containing weights of each dipole component

\mathcal{D} is the dictionary containing all possible horizontal and vertical dipoles for a given dipole source separation

Experimental setup- Double dipole



Modeling information:

Model size: 0.7 km x 1.8 km

Grid spacing: 5 m

Receiver spacing: 5 m

Receiver depth: 20 m

Fixed spread: 1.78 km

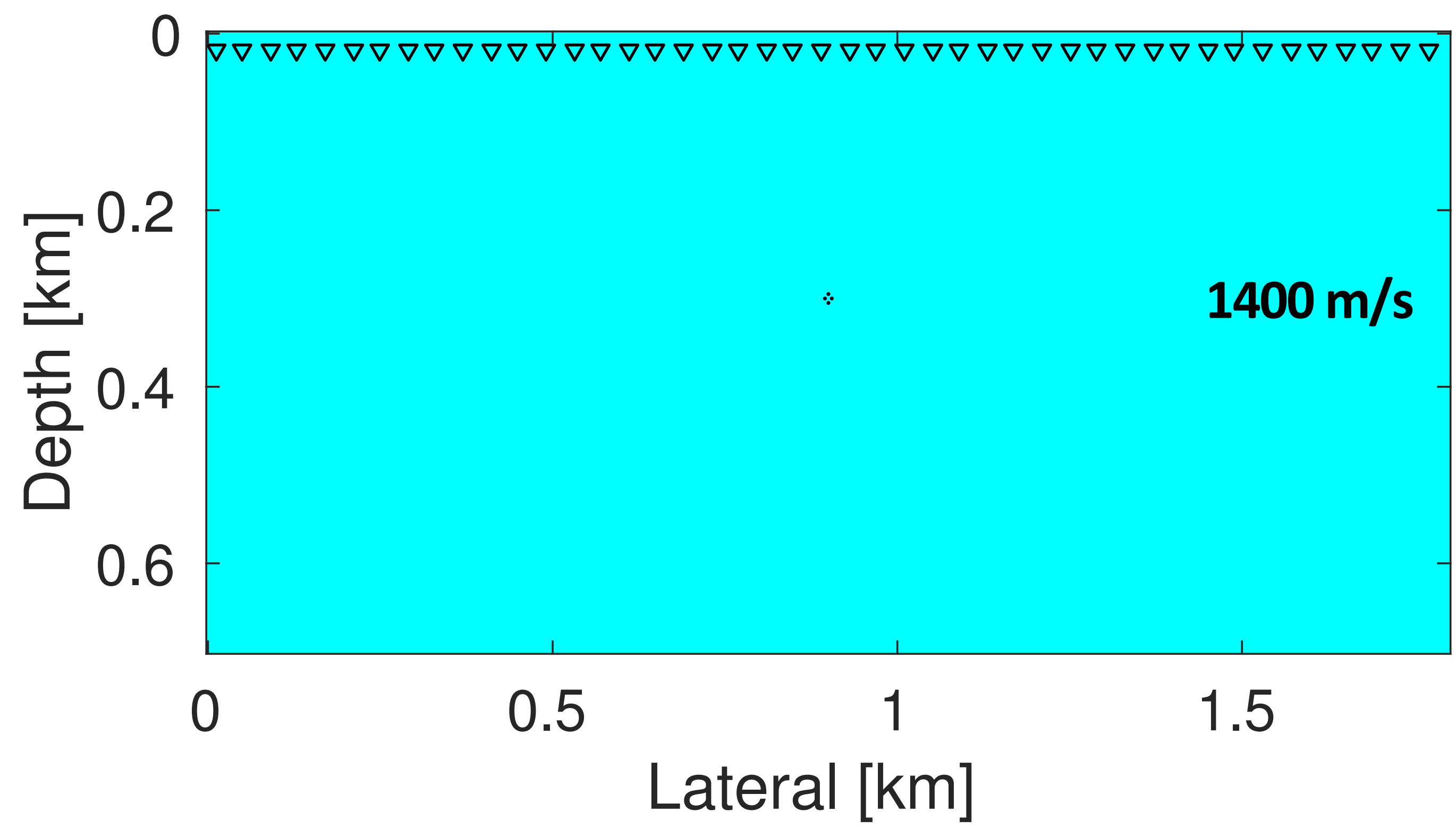
Sampling interval: 1 ms

Recording length: 0.75 s

Peak frequency: 15 Hz

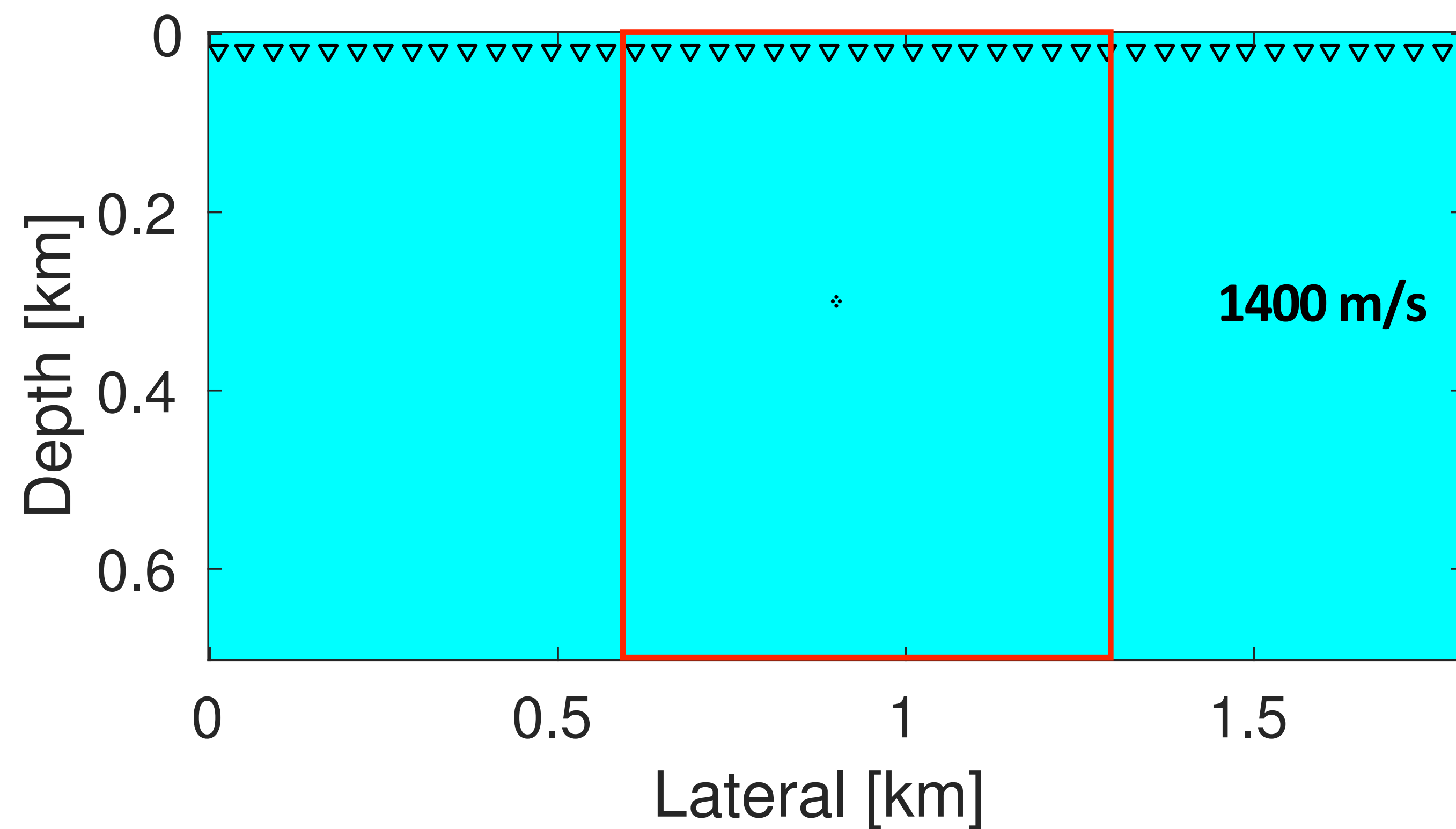
Dipole source separation: 10 m

Experimental setup- Double dipole



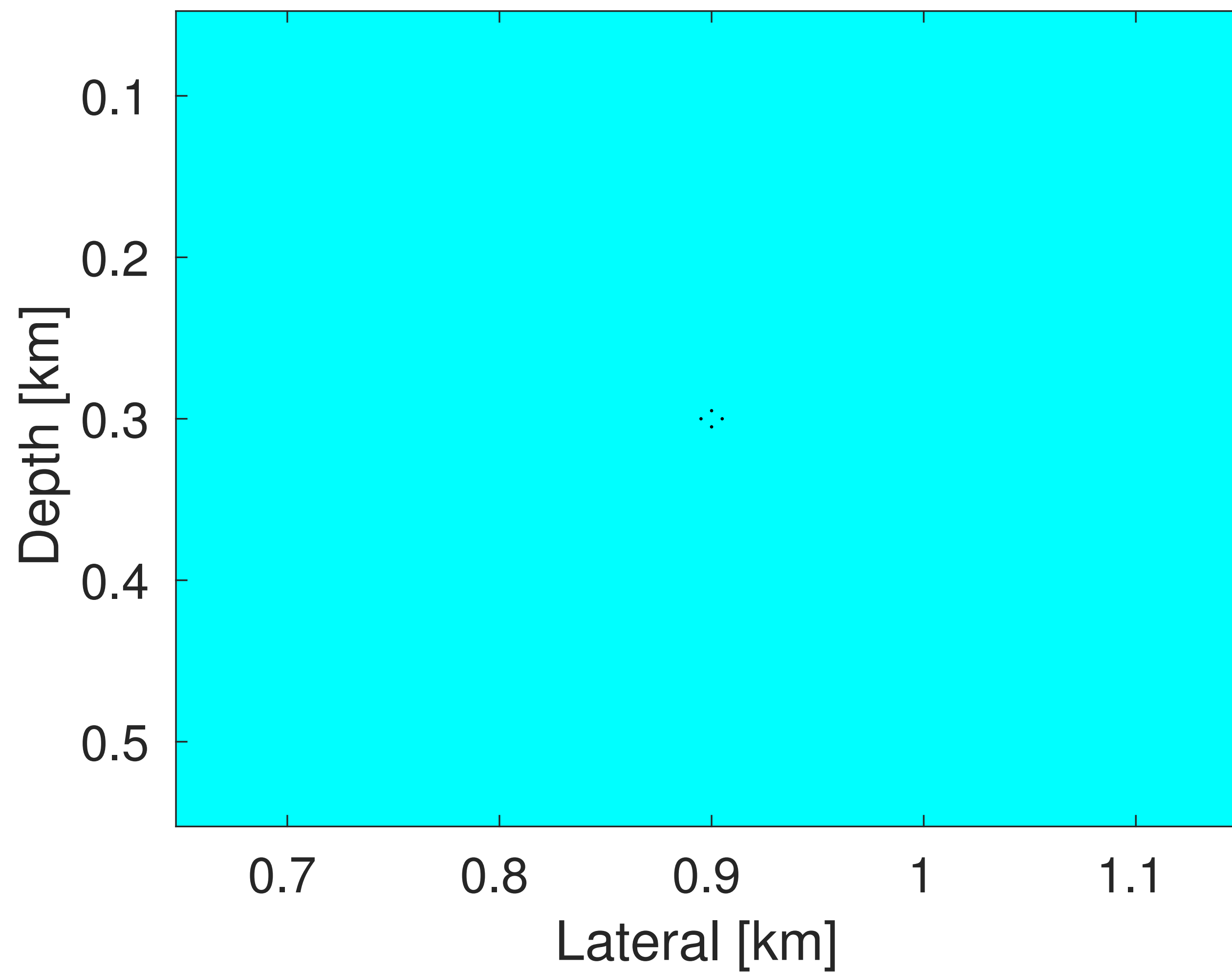
Maximum aperture: 71 degrees

Experimental setup- Double dipole

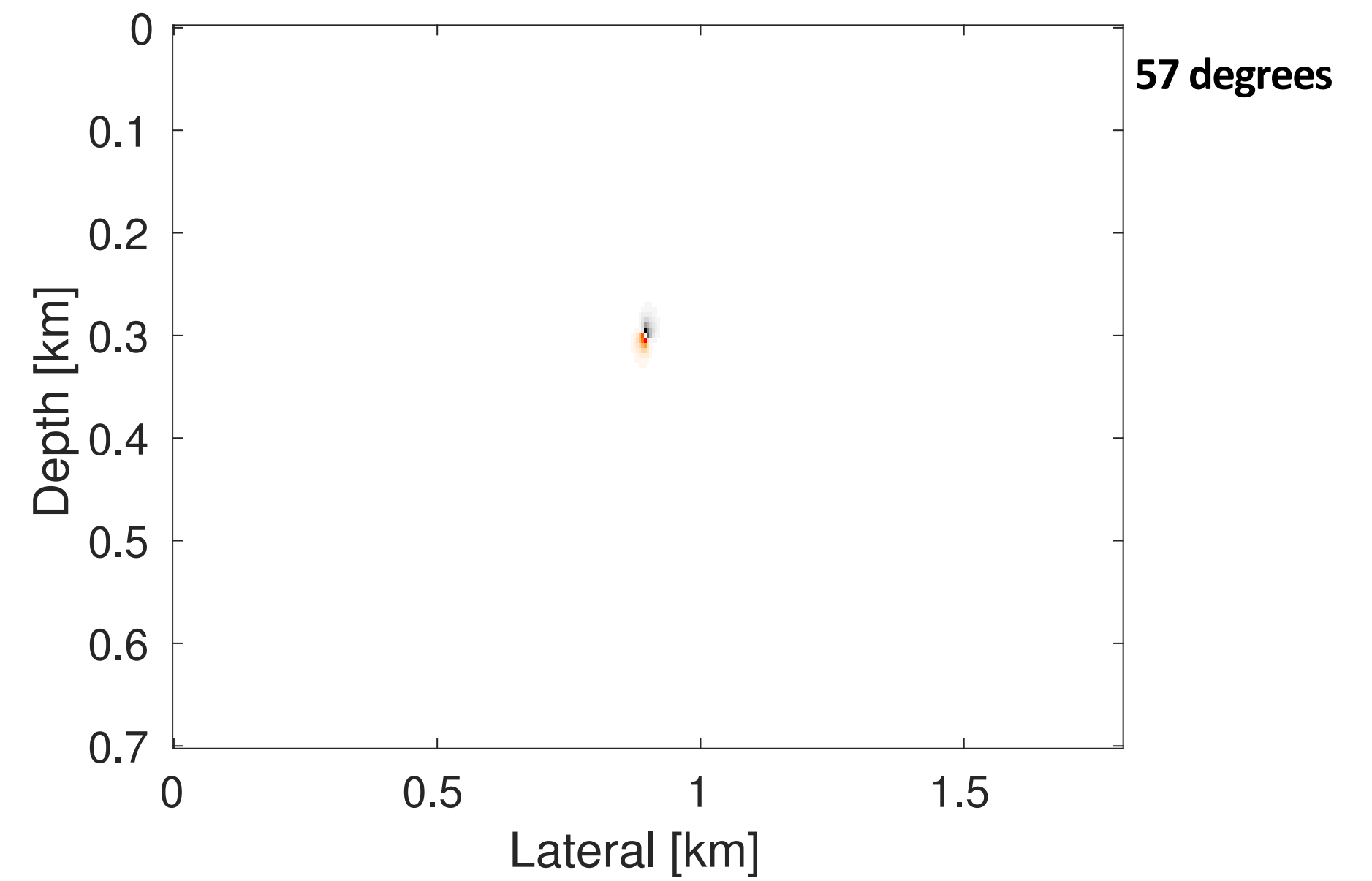
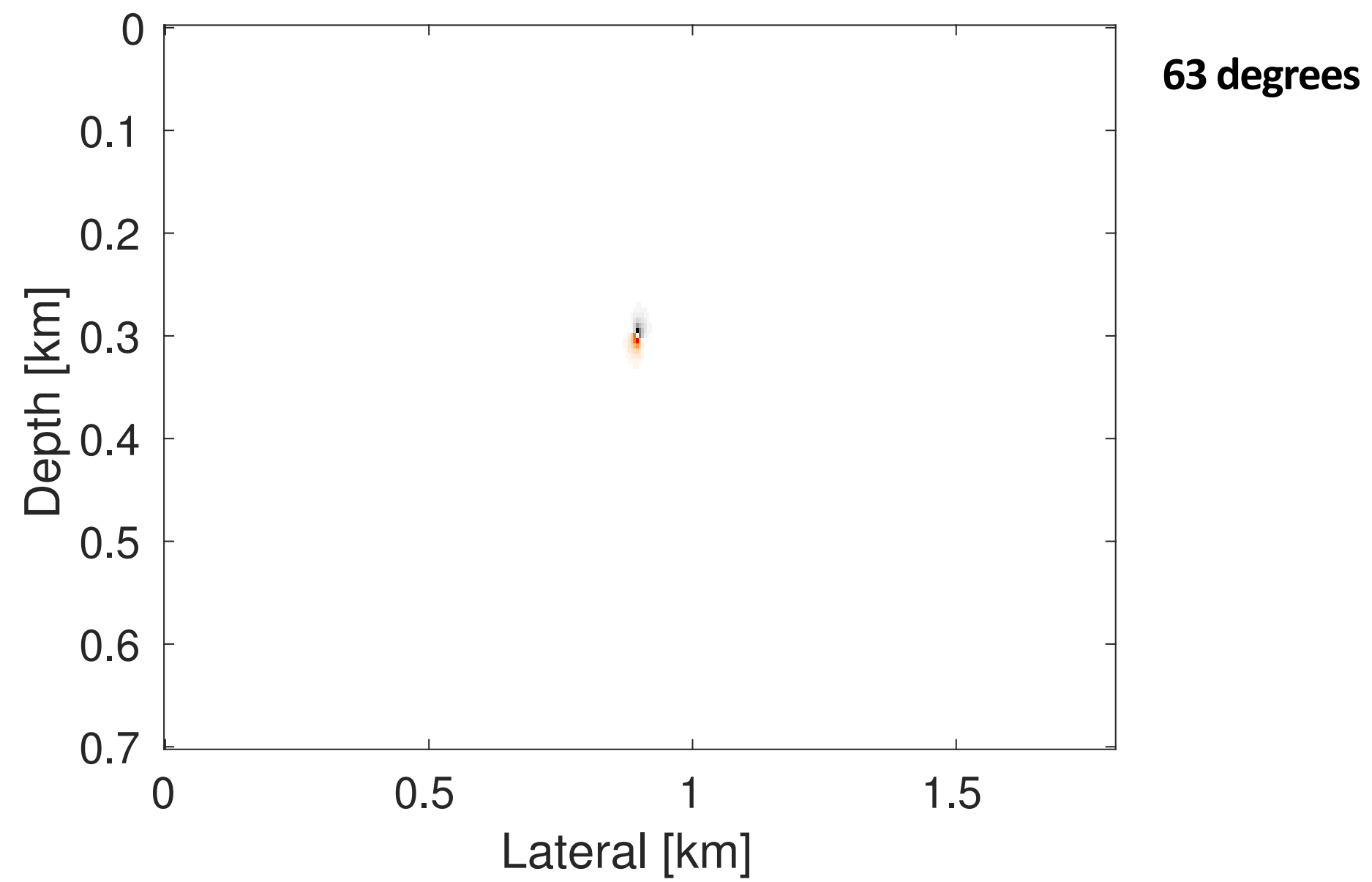
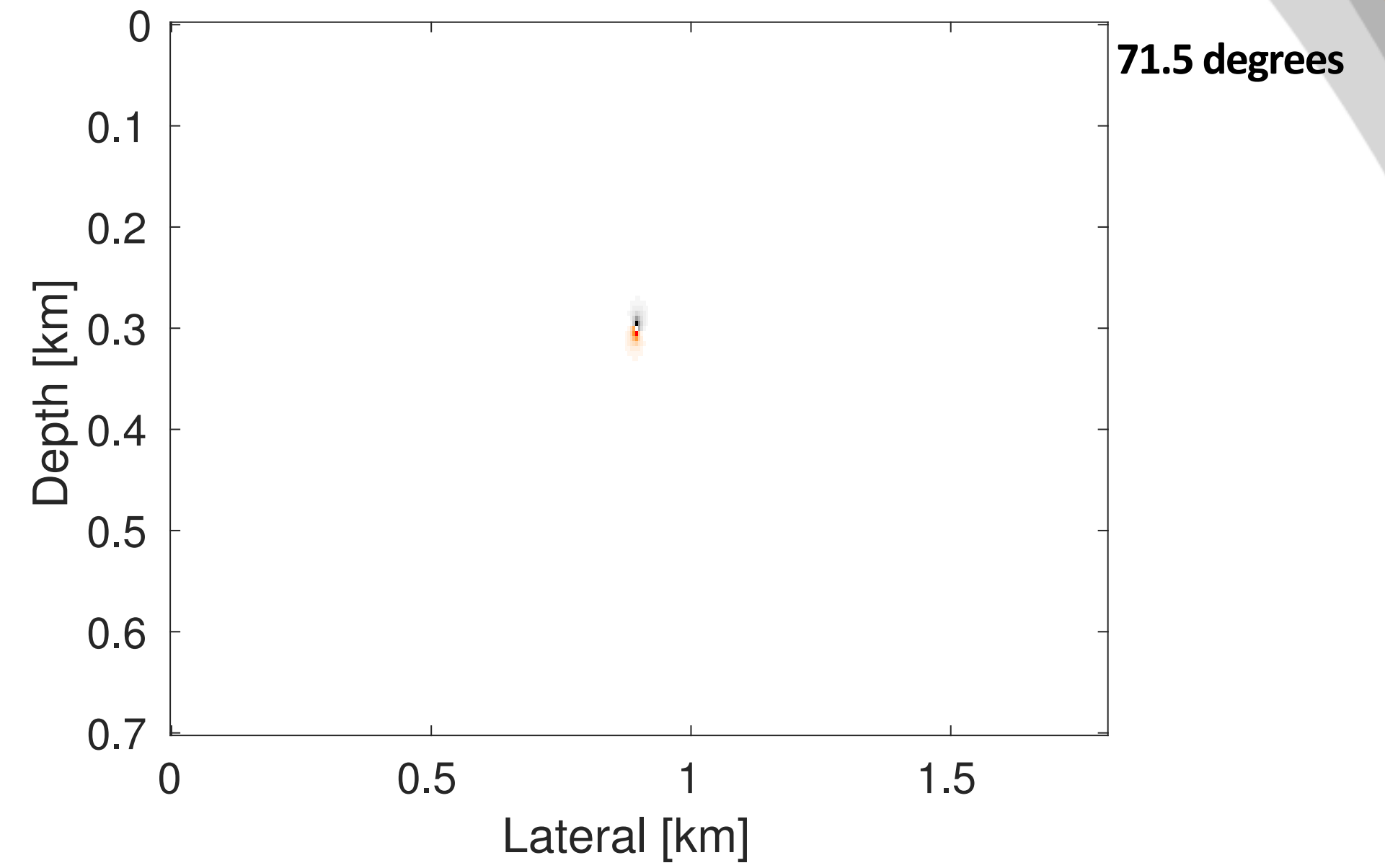
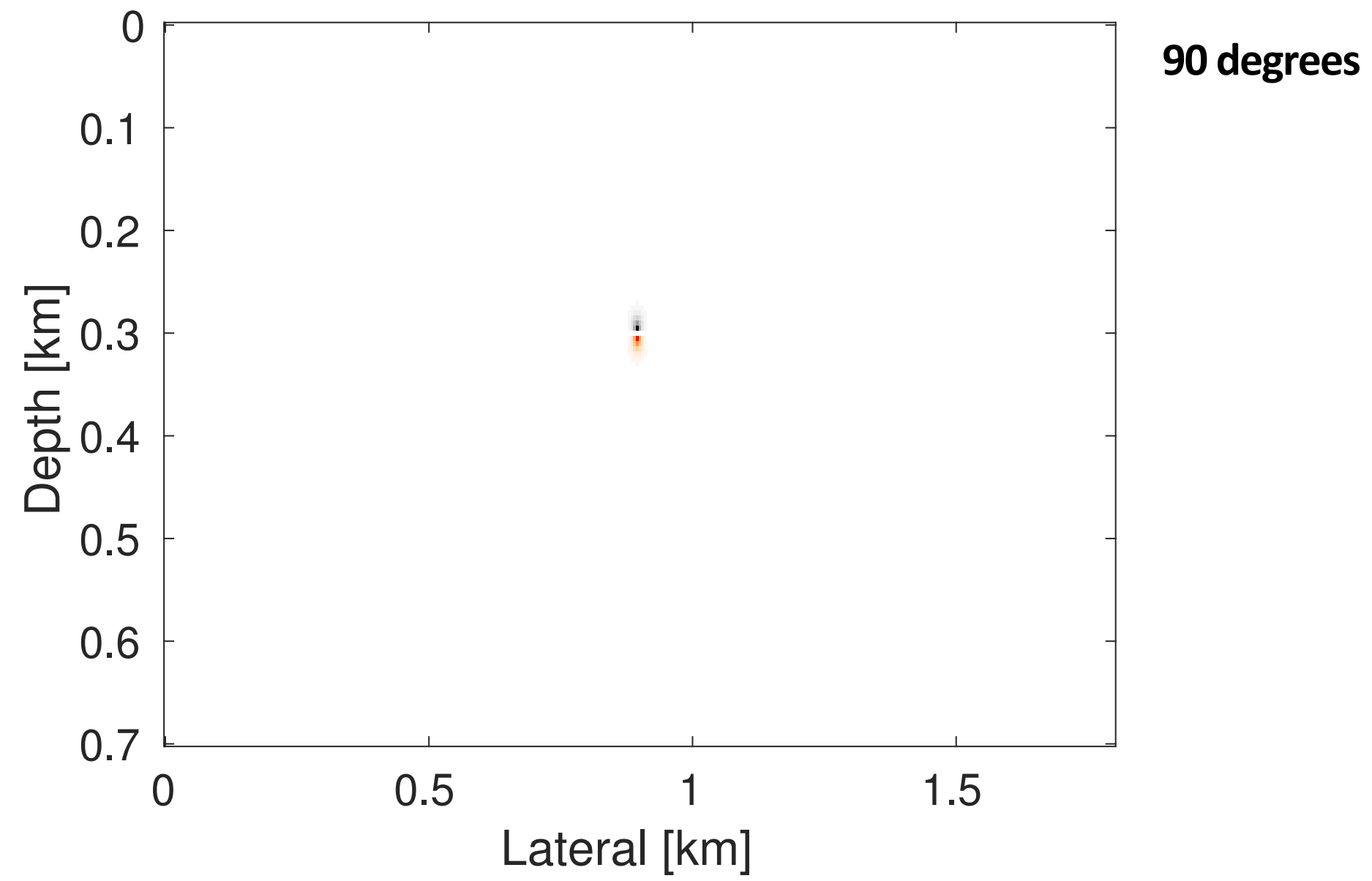


Maximum aperture: 71 degrees

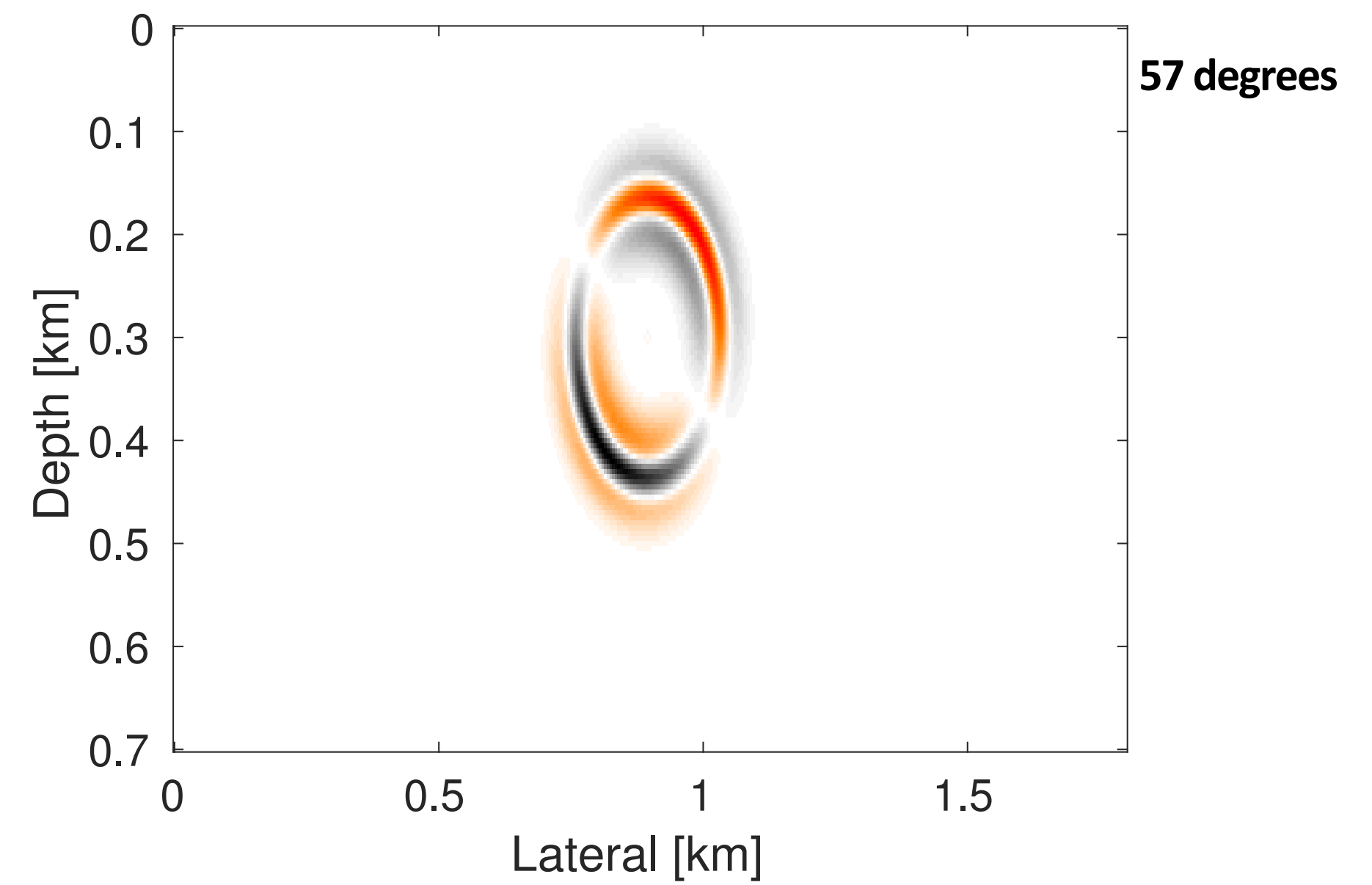
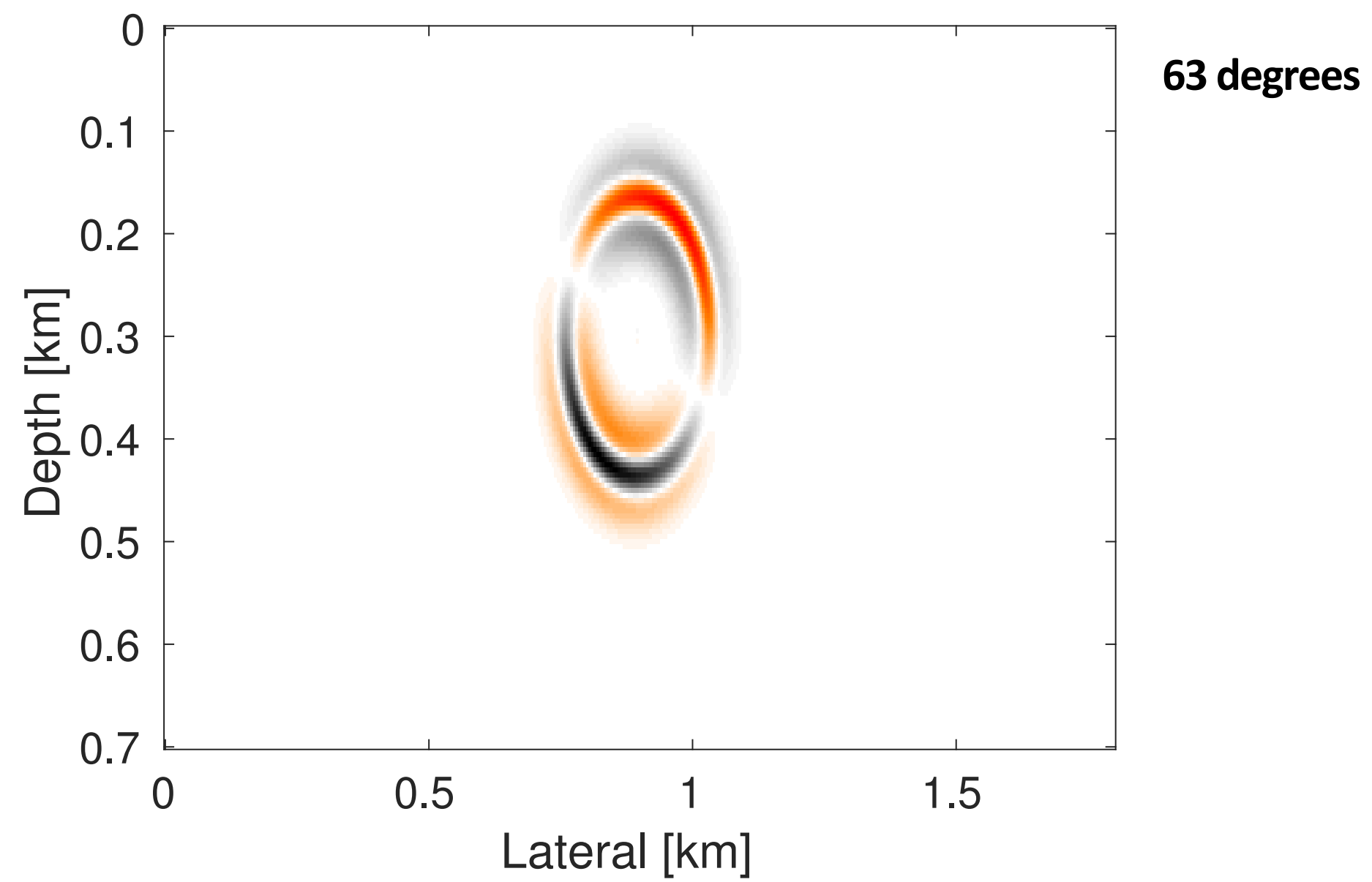
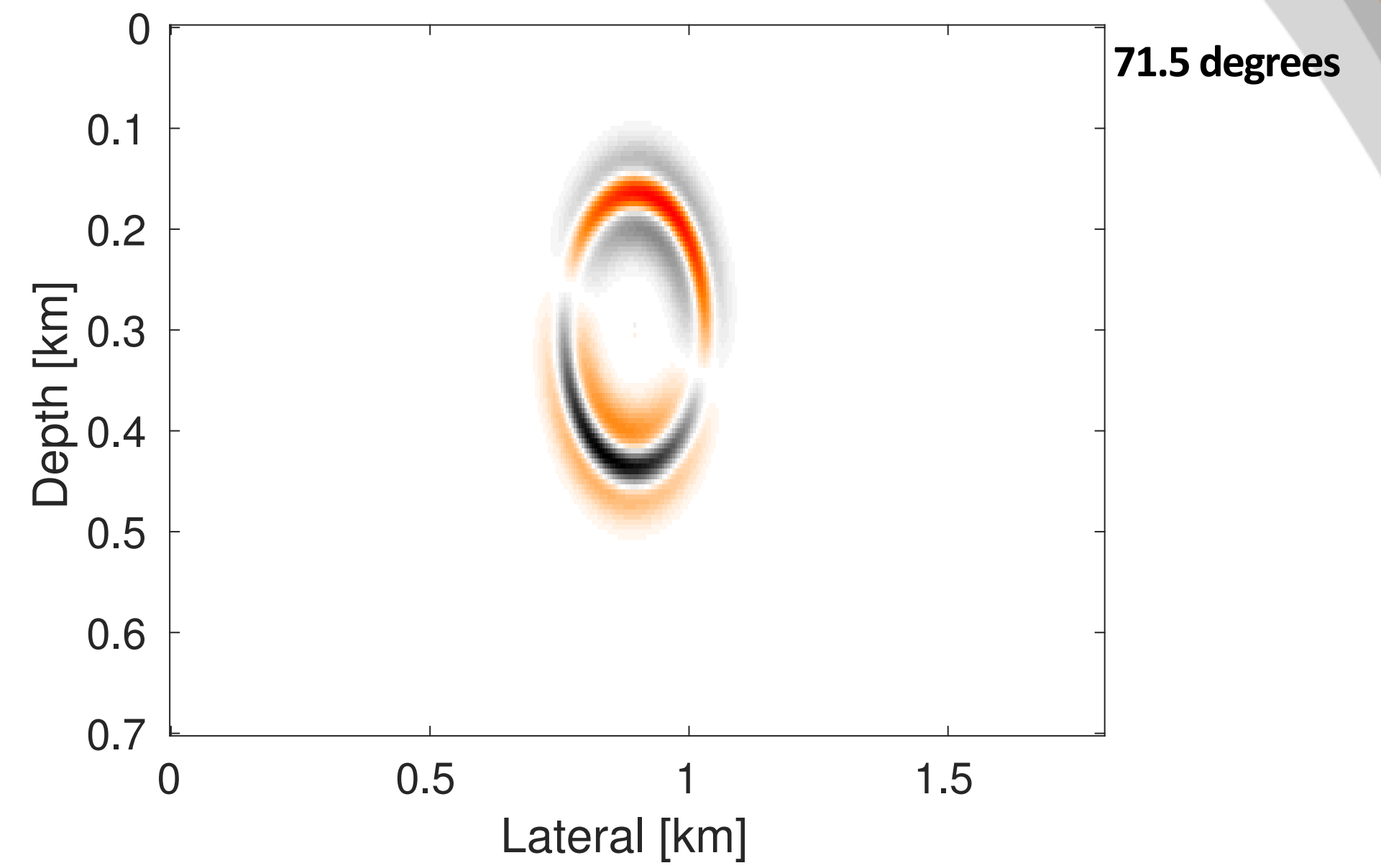
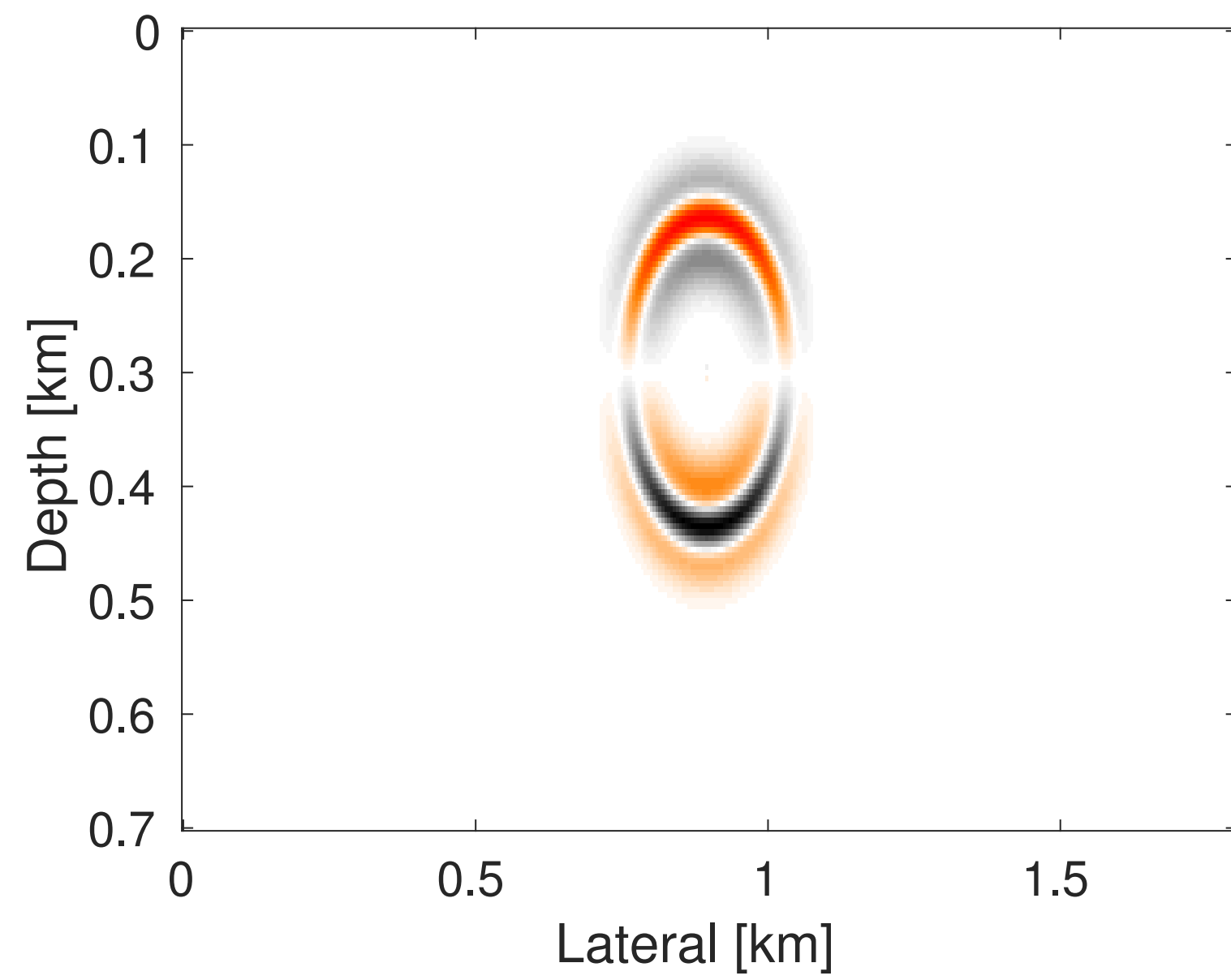
Zoomed



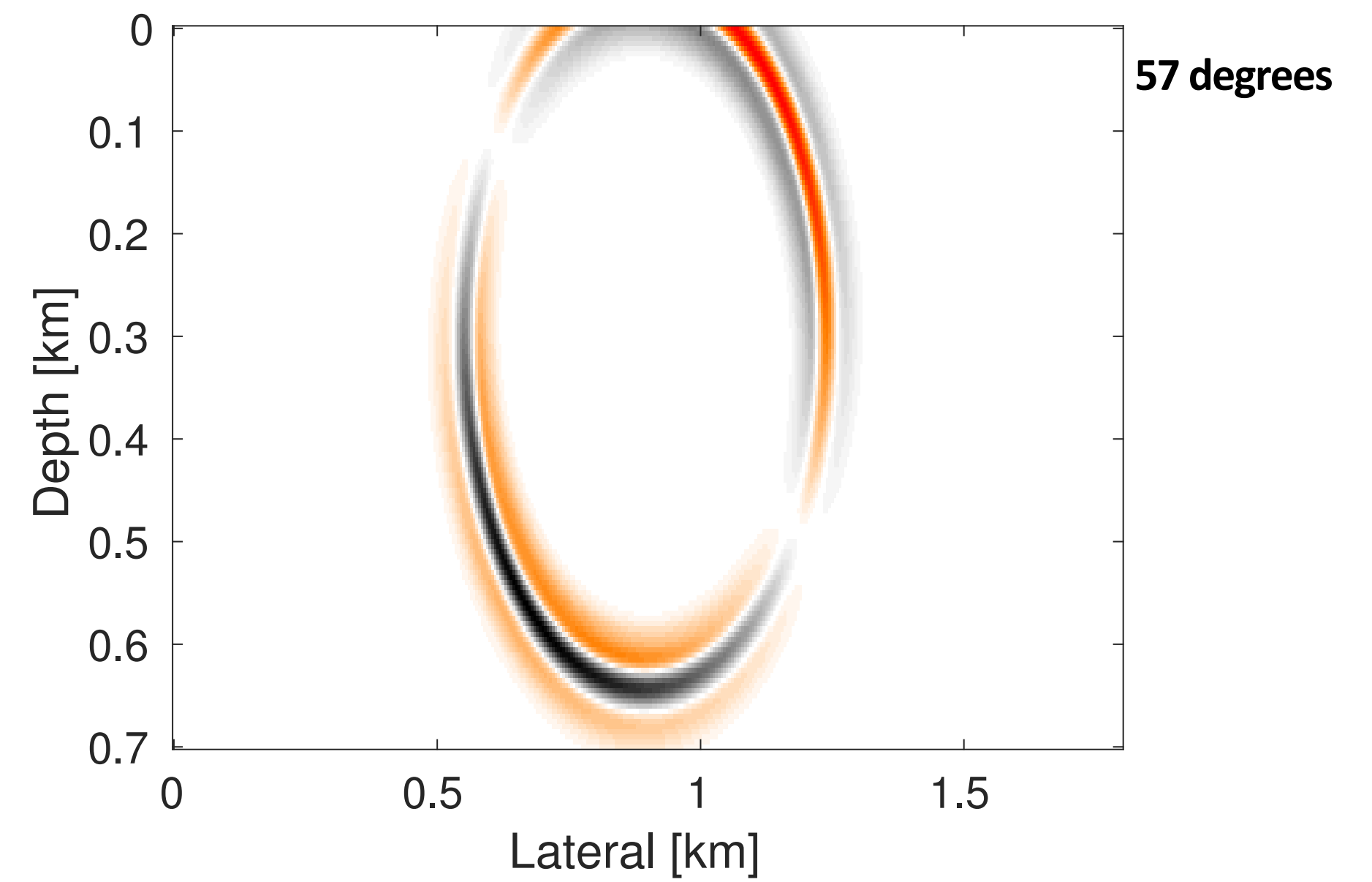
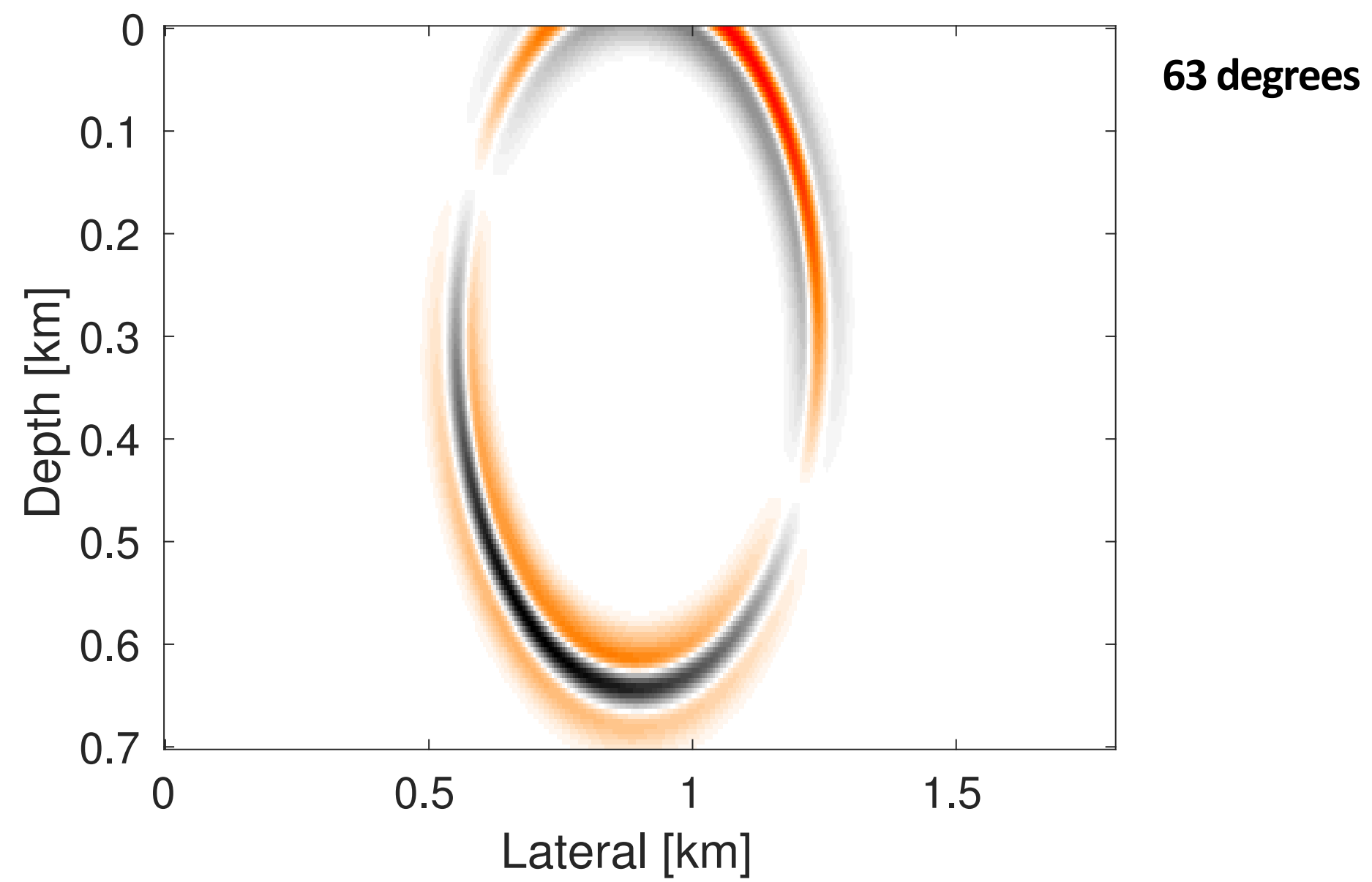
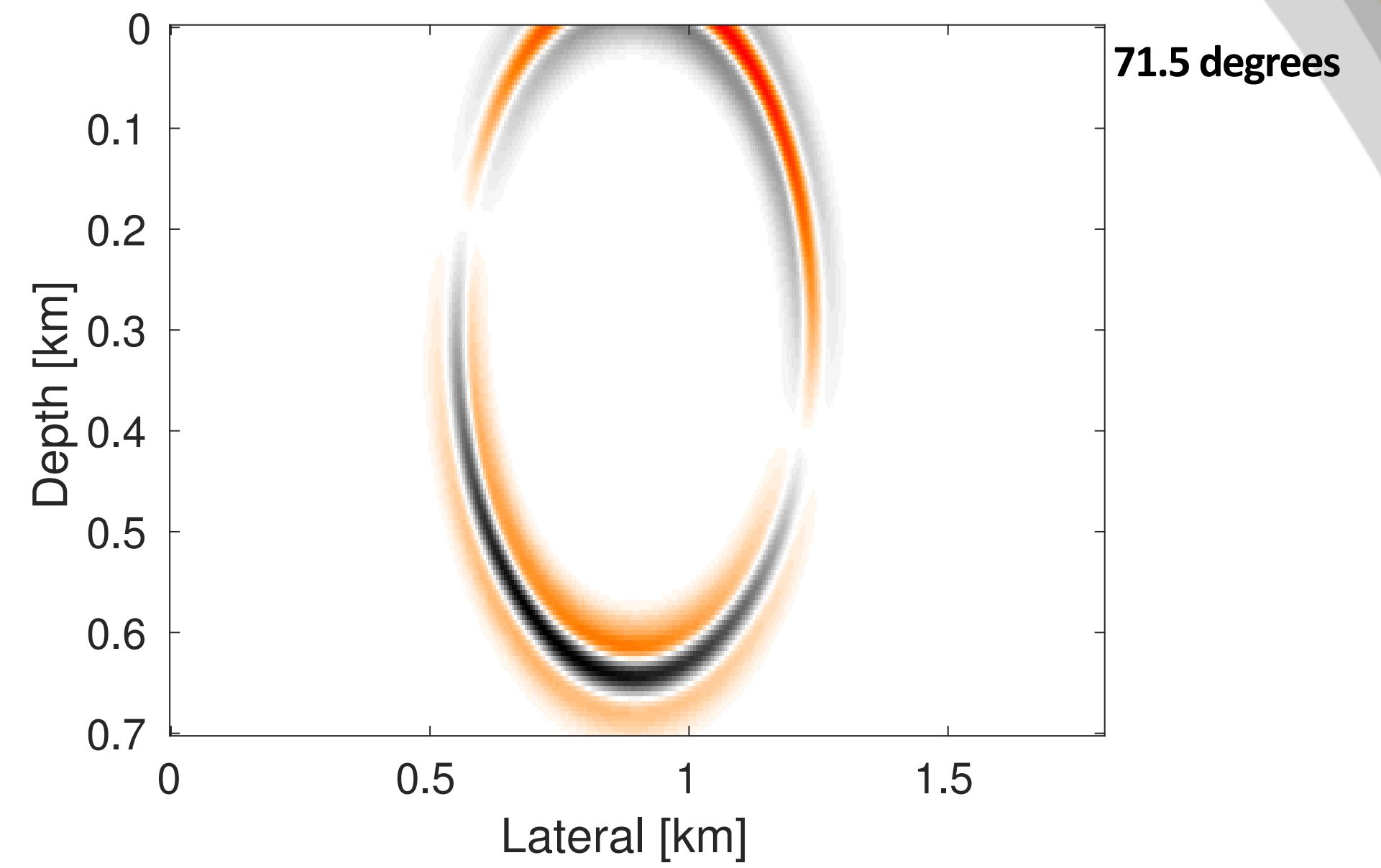
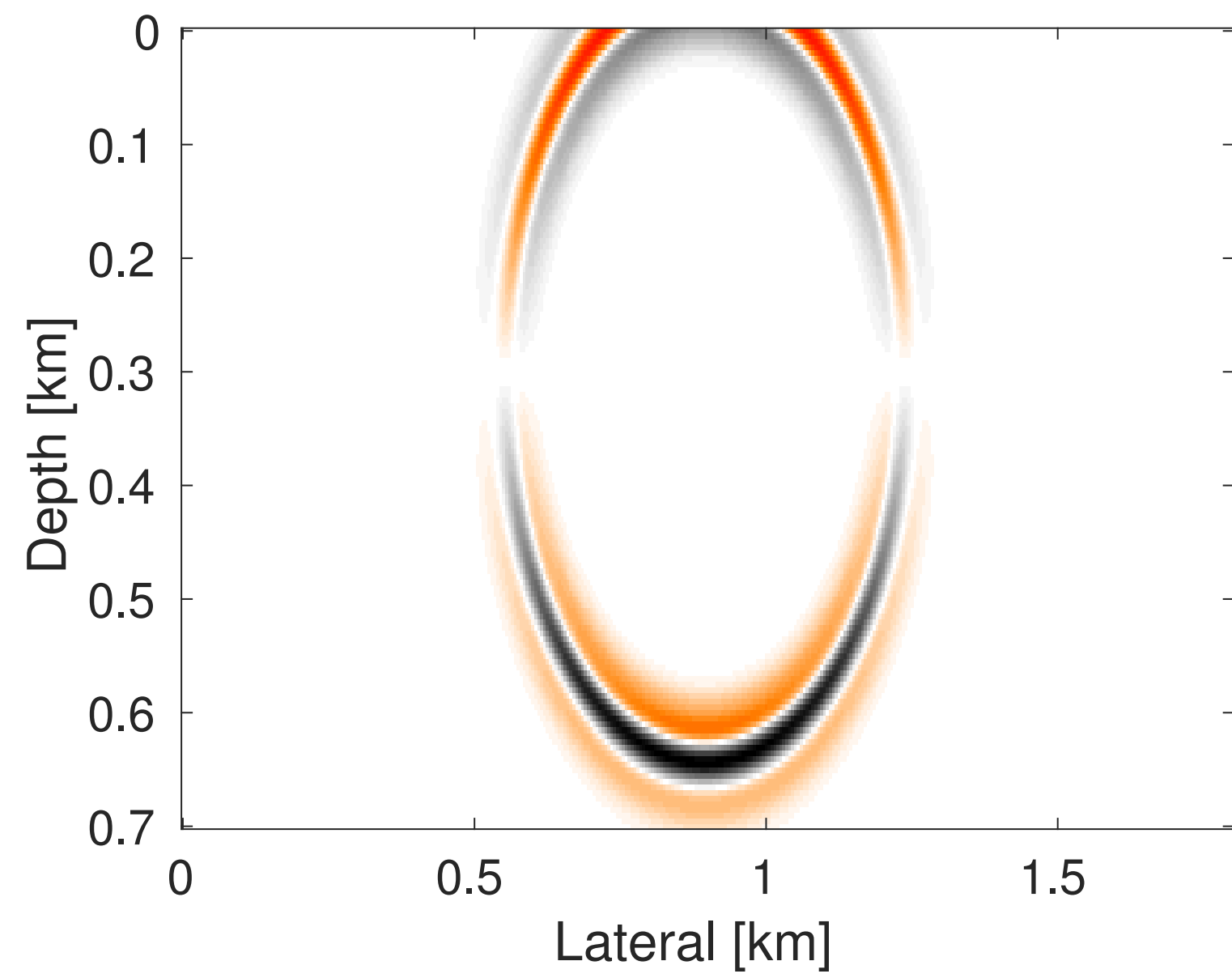
Wavefield at 74 ms



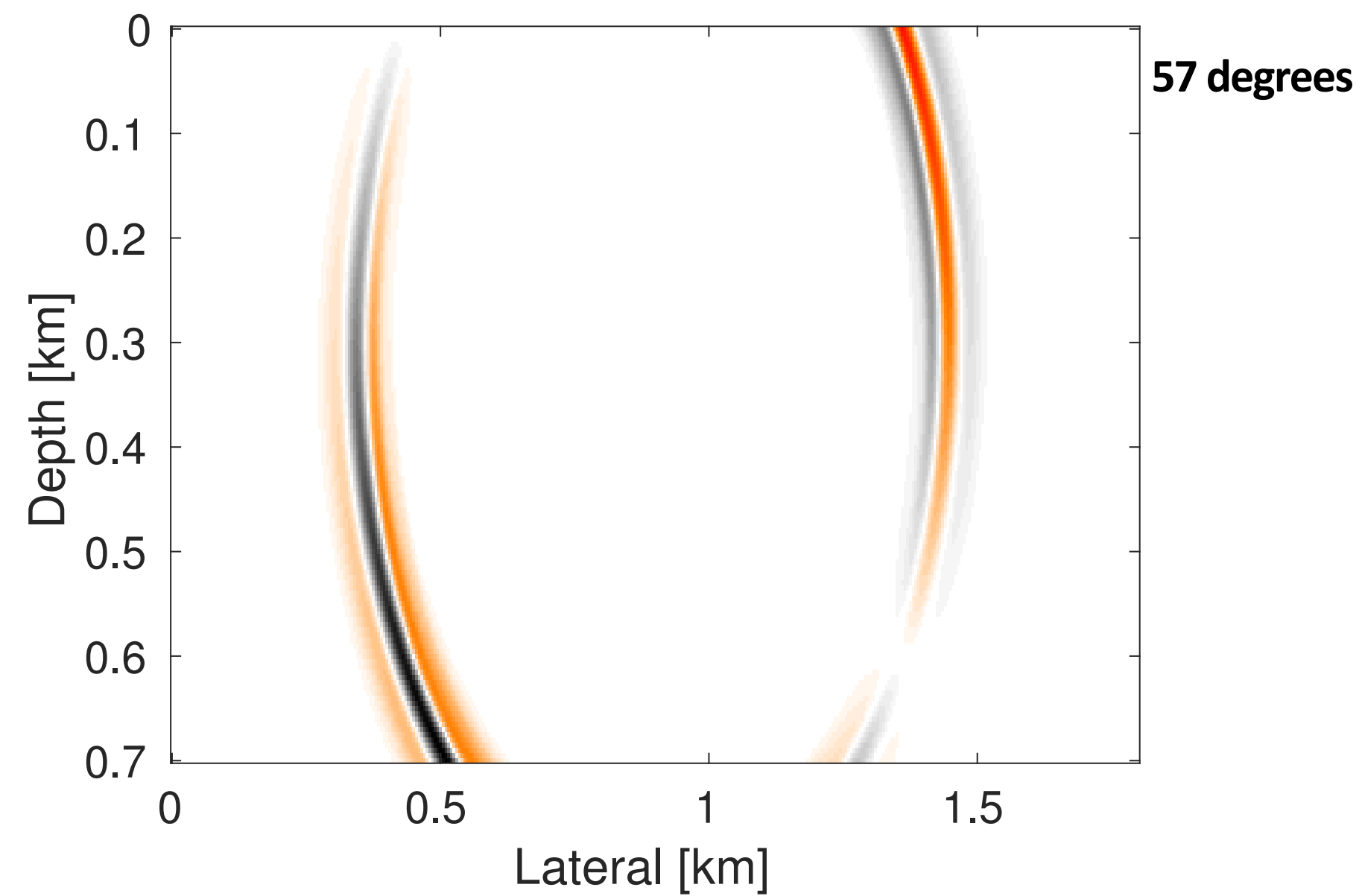
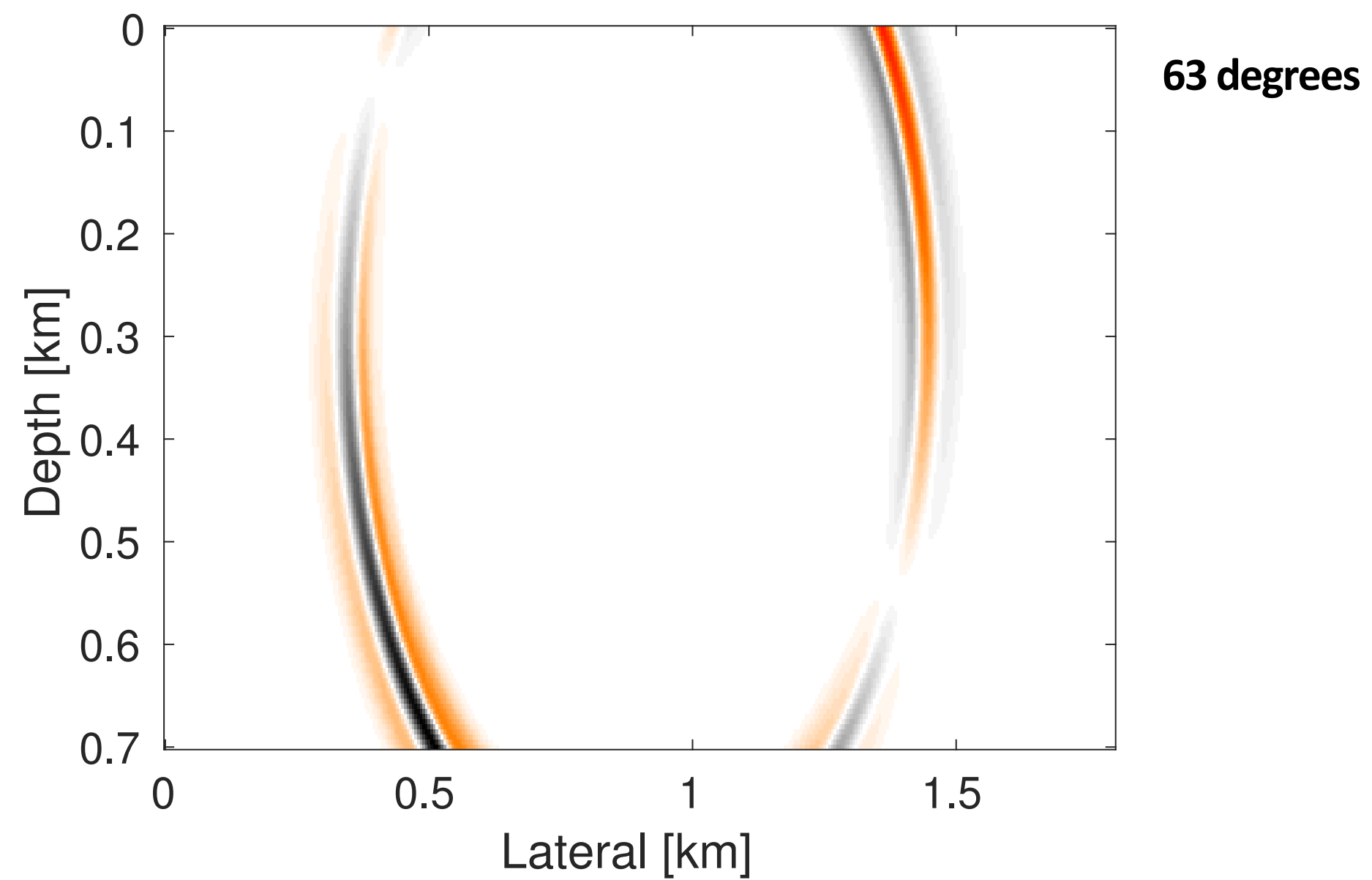
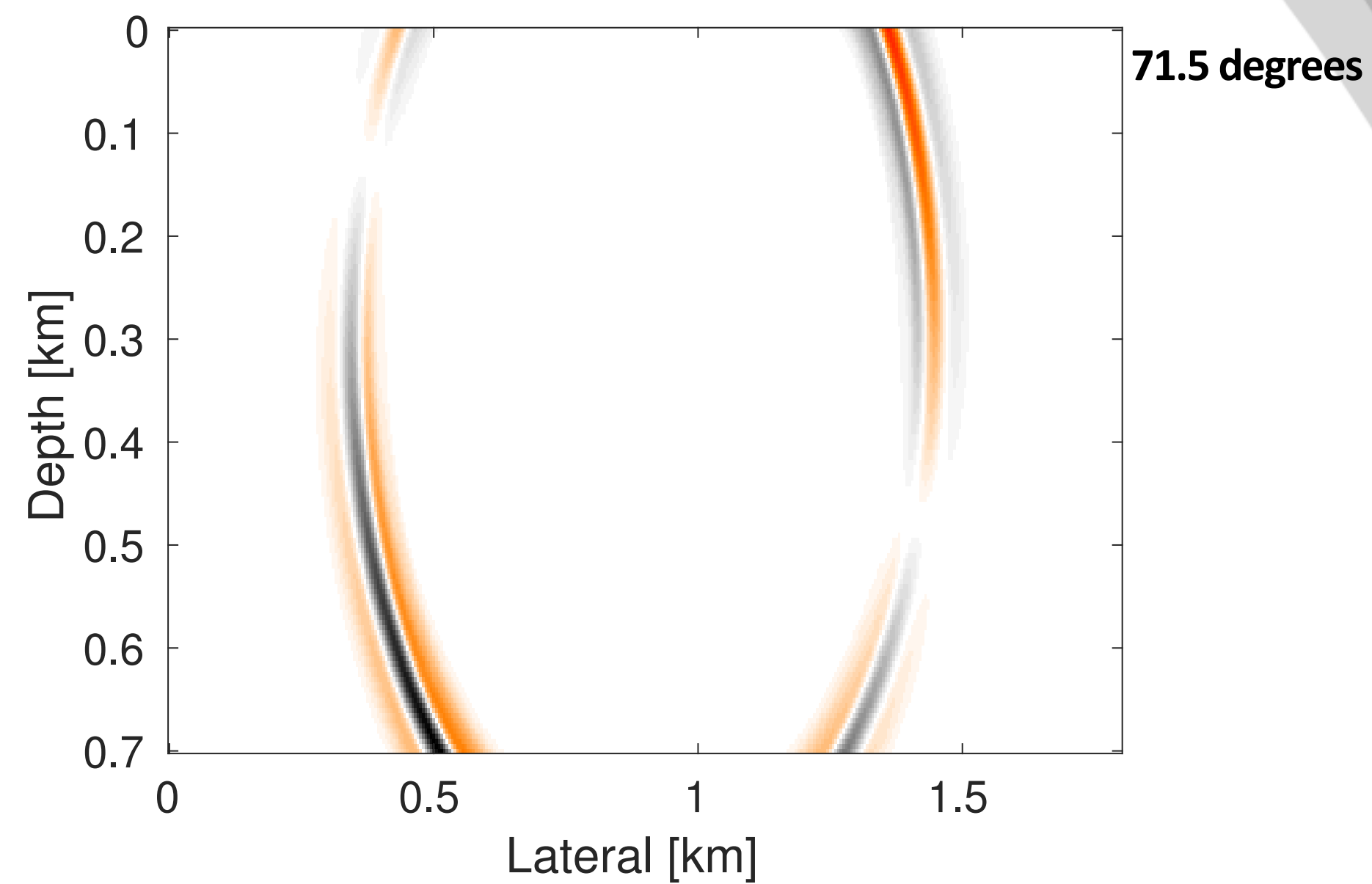
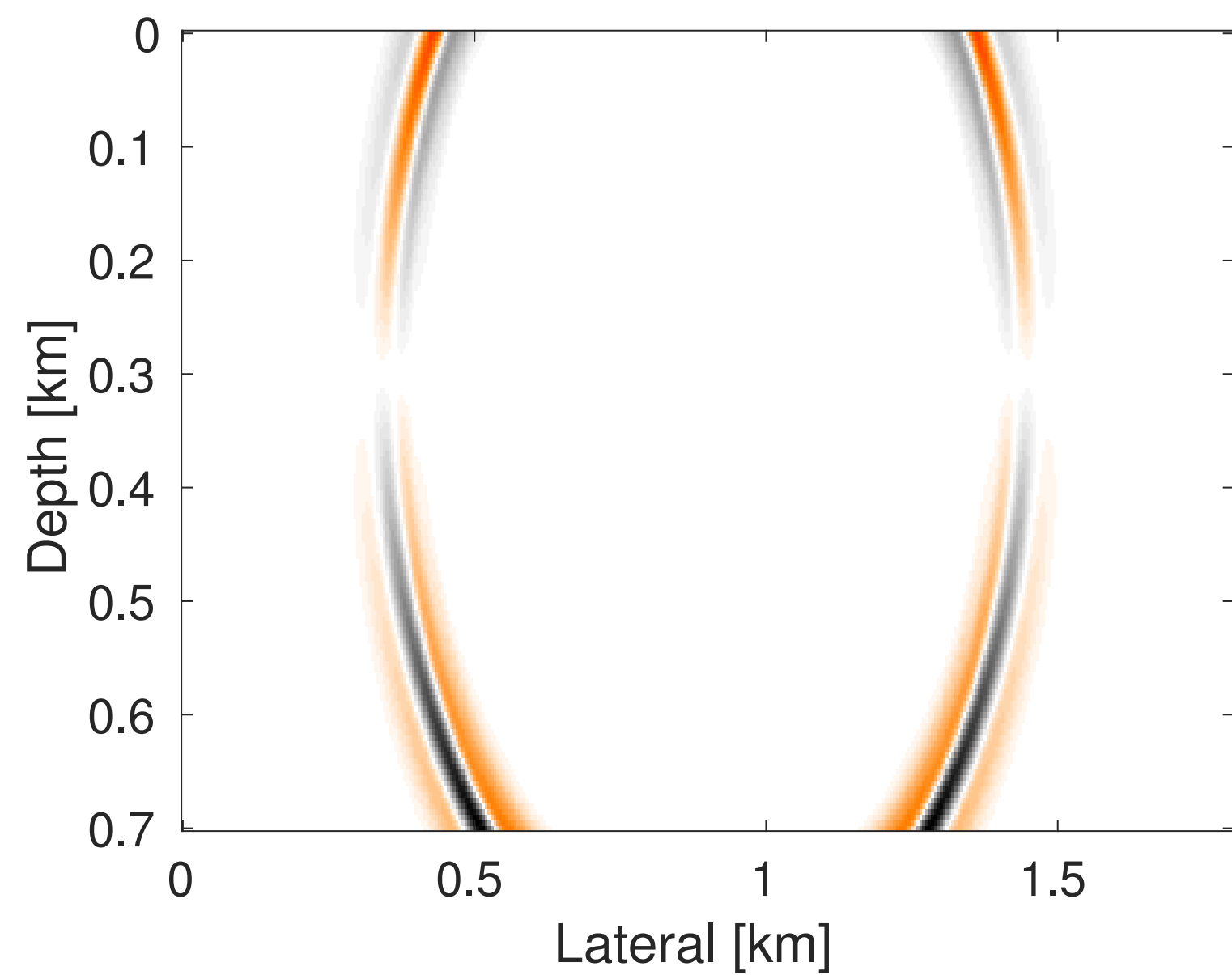
Wavefield at 224 ms



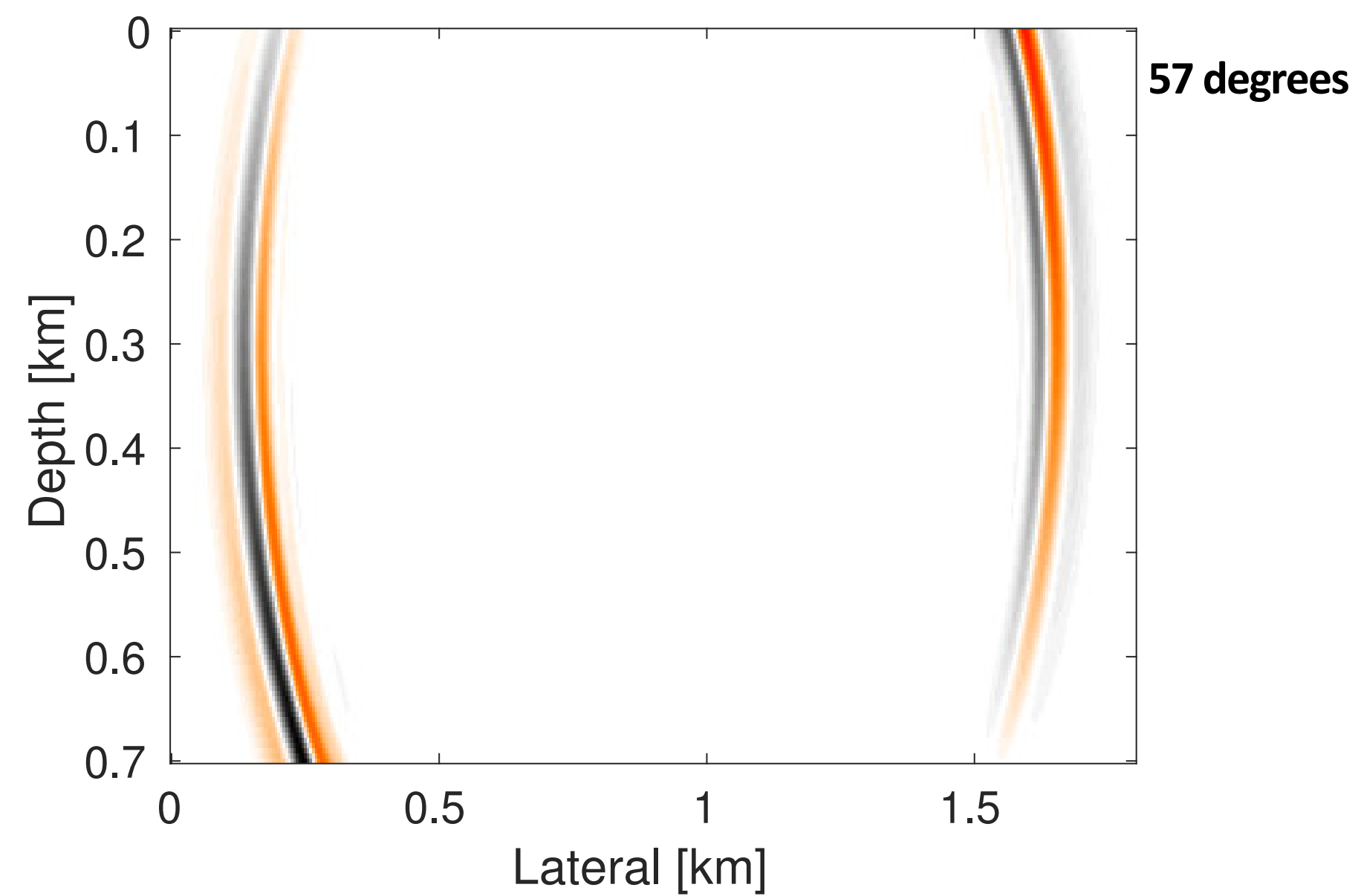
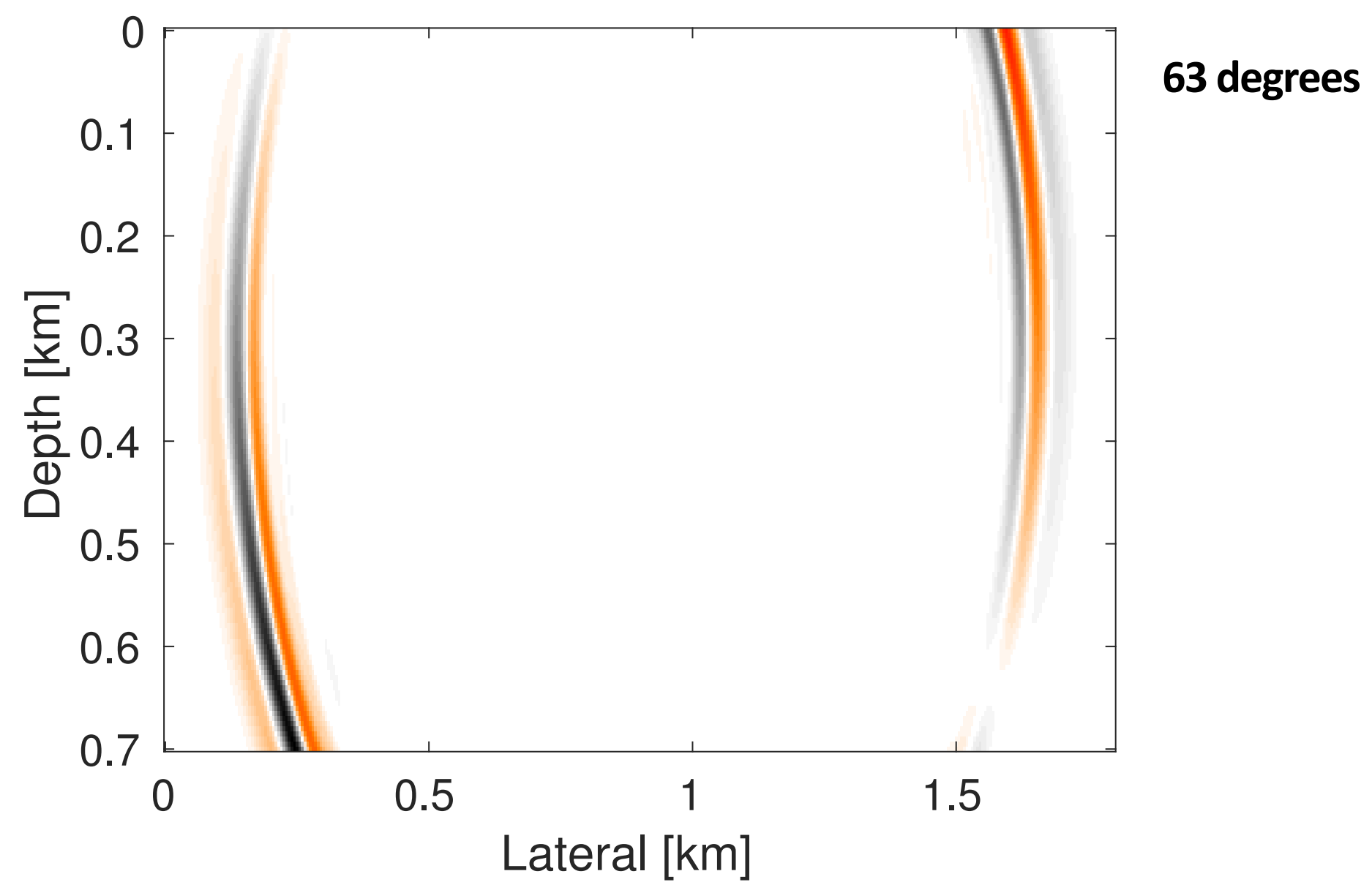
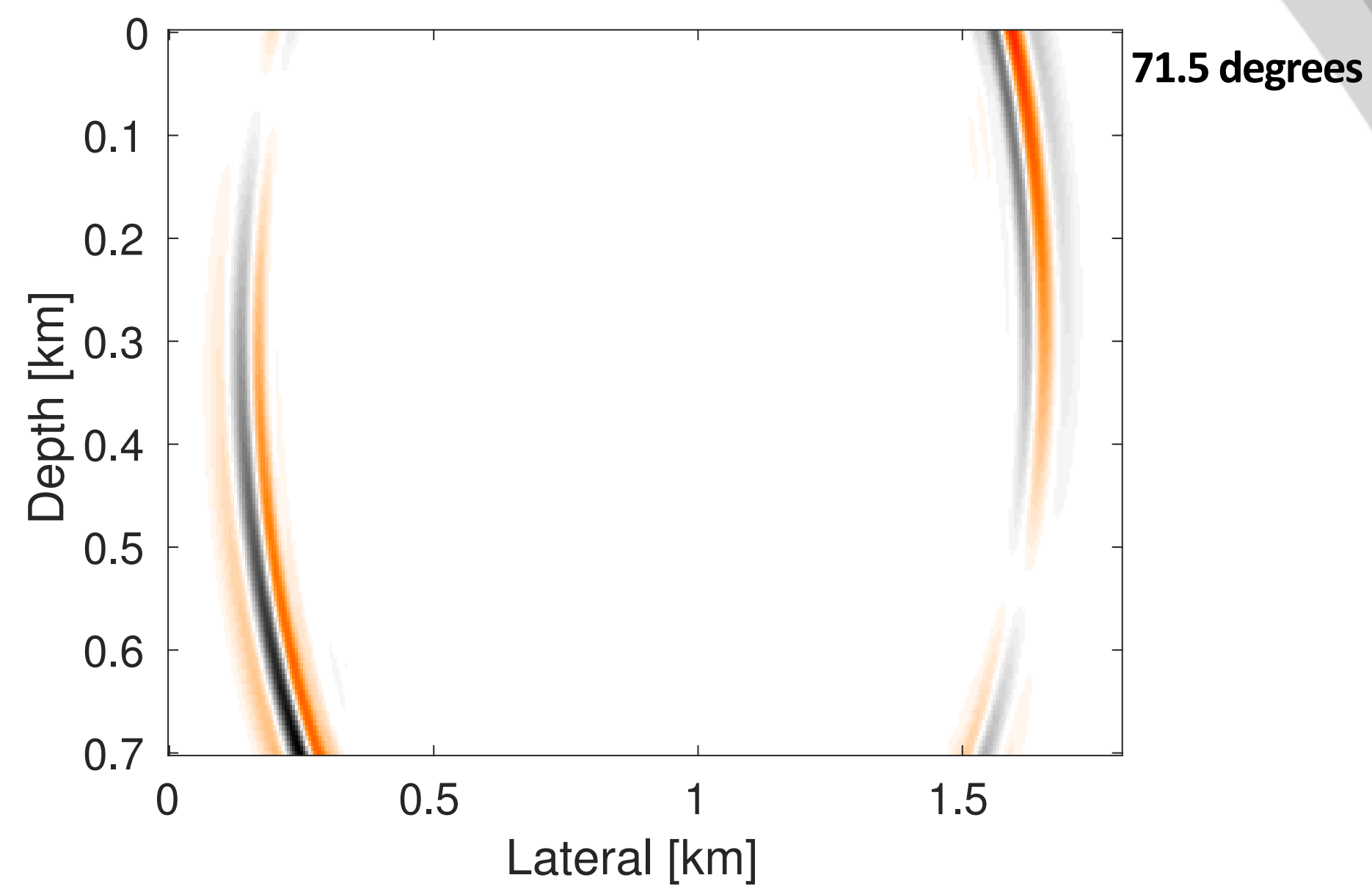
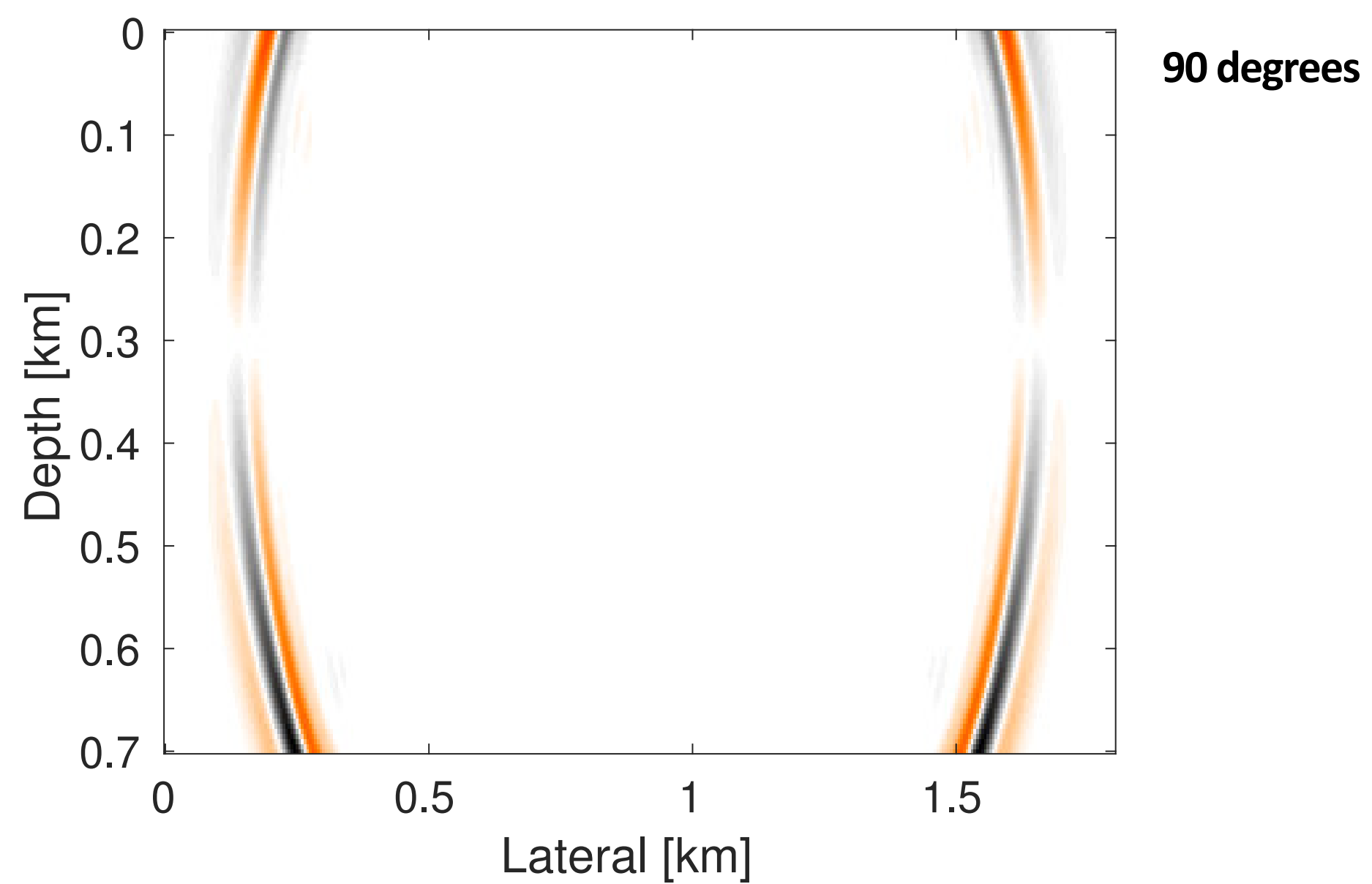
Wavefield at 374 ms



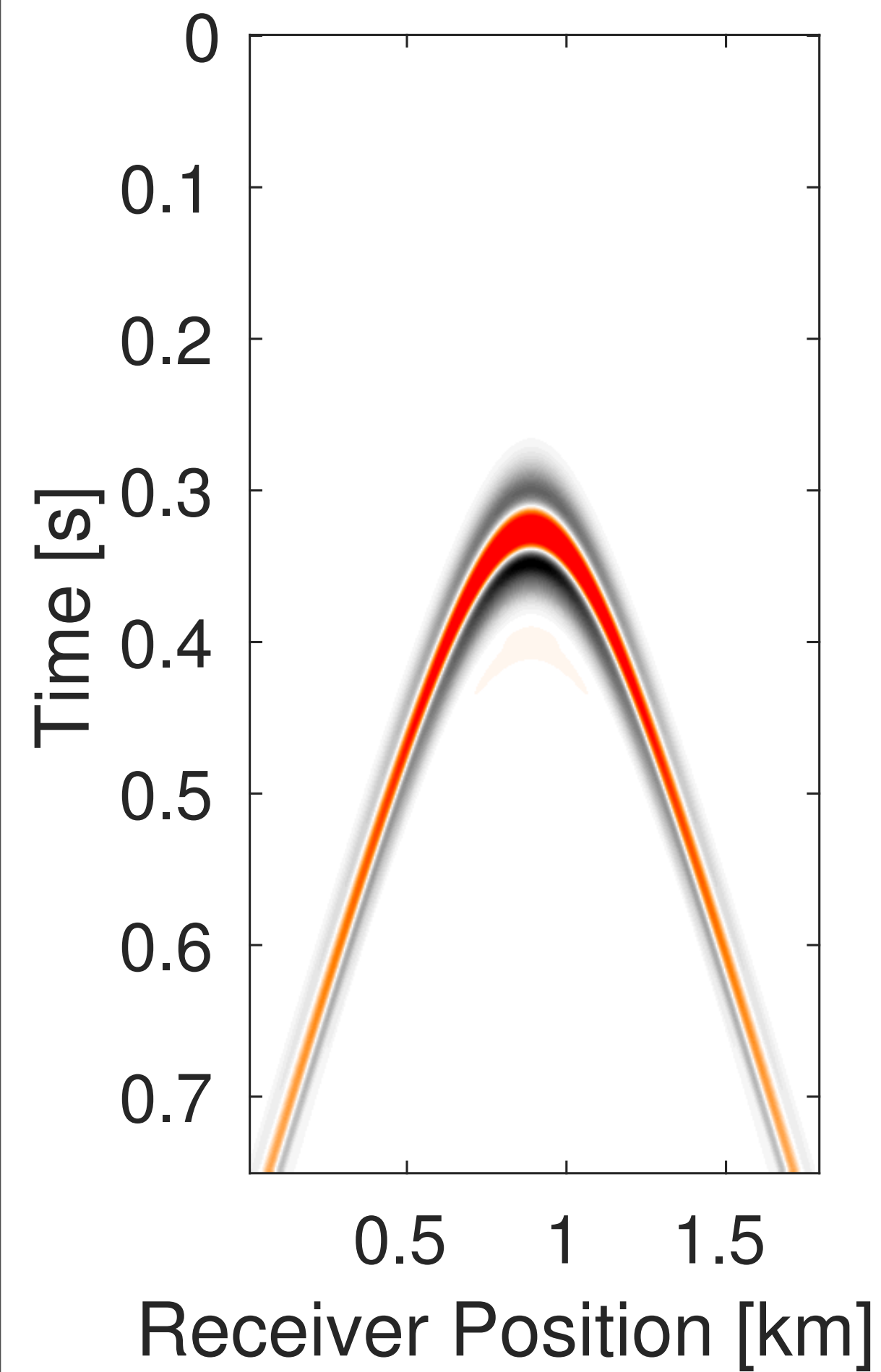
Wavefield at 524 ms



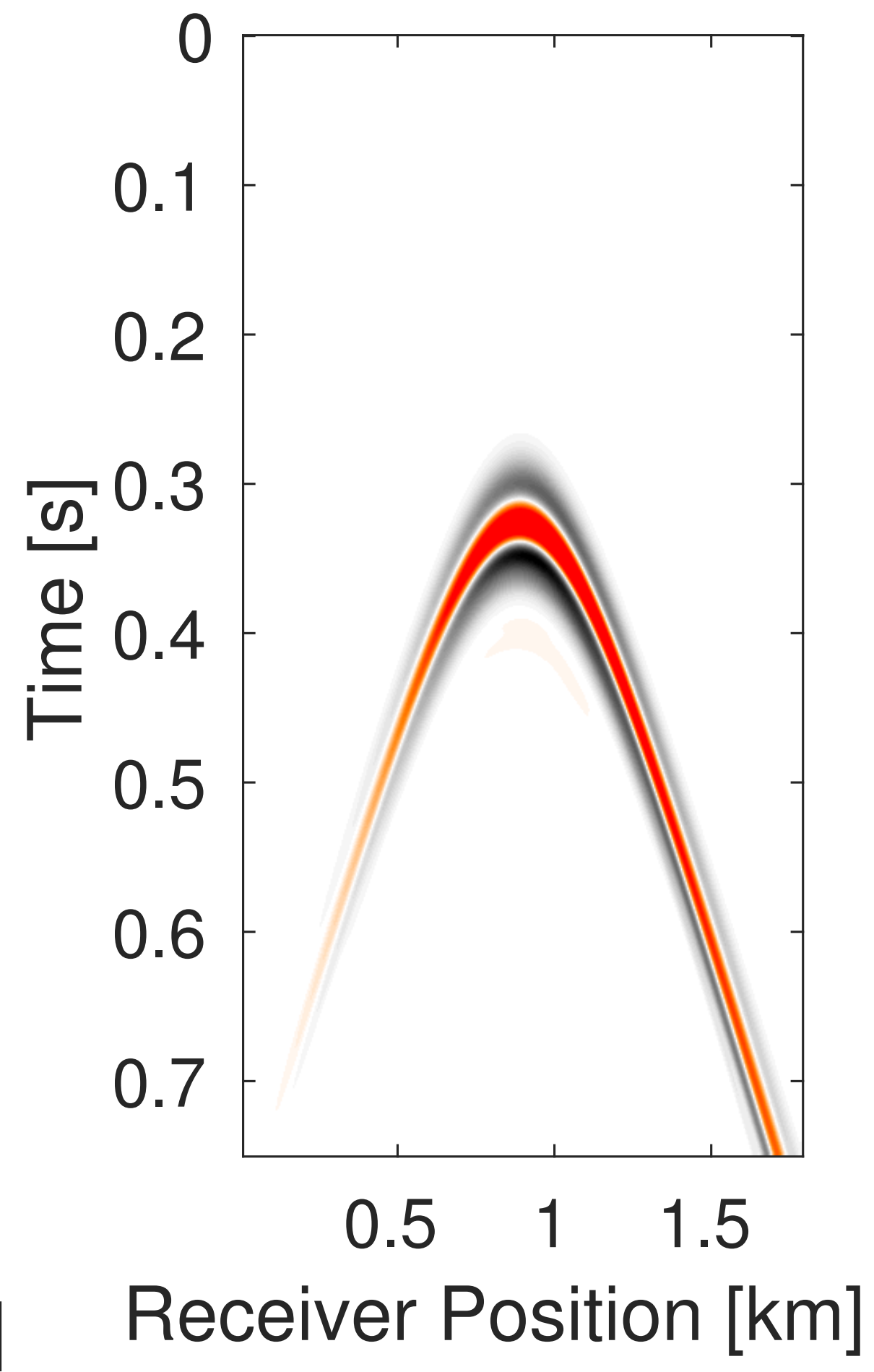
Wavefield at 674 ms



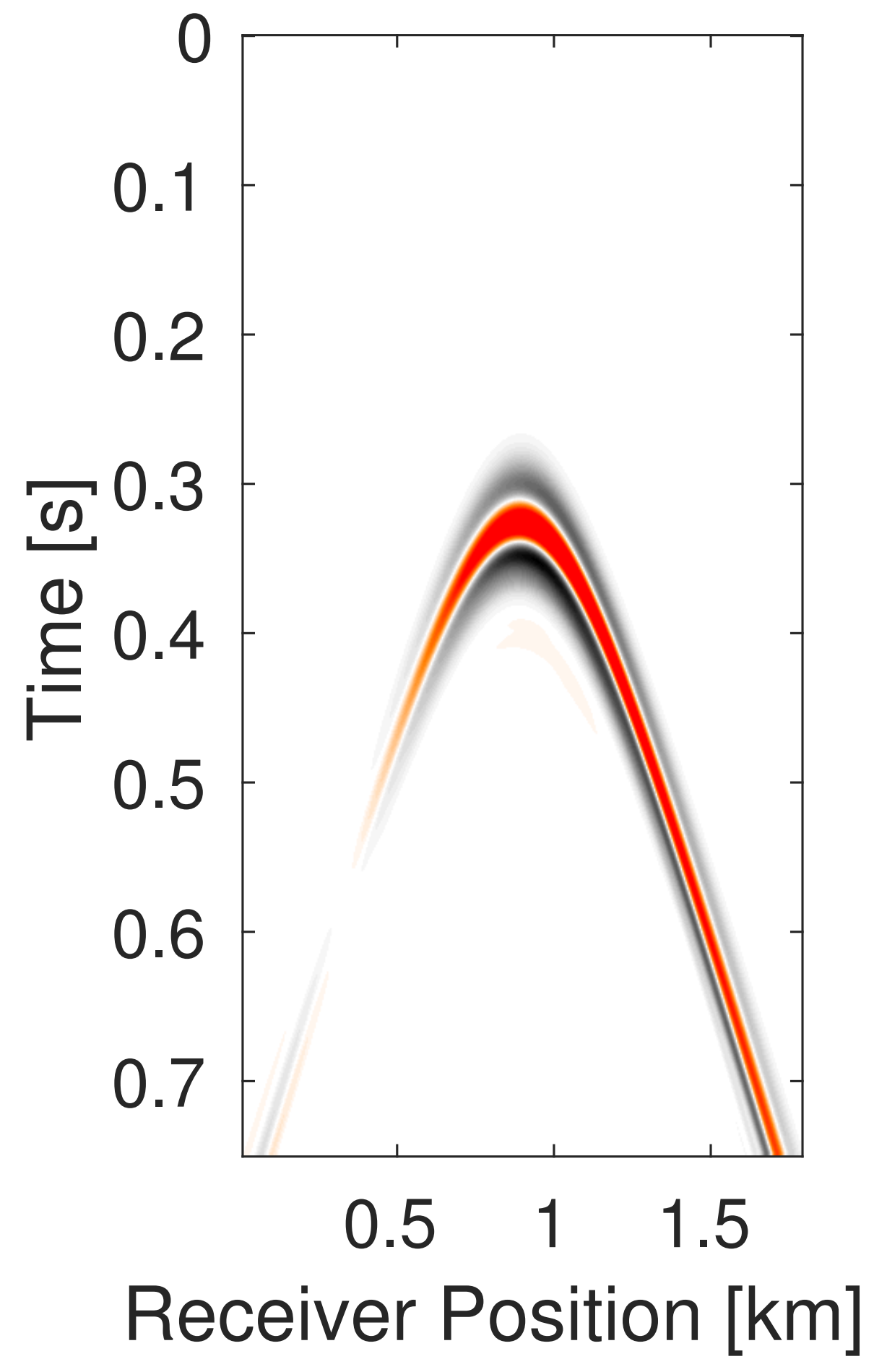
Shot gather with directivity



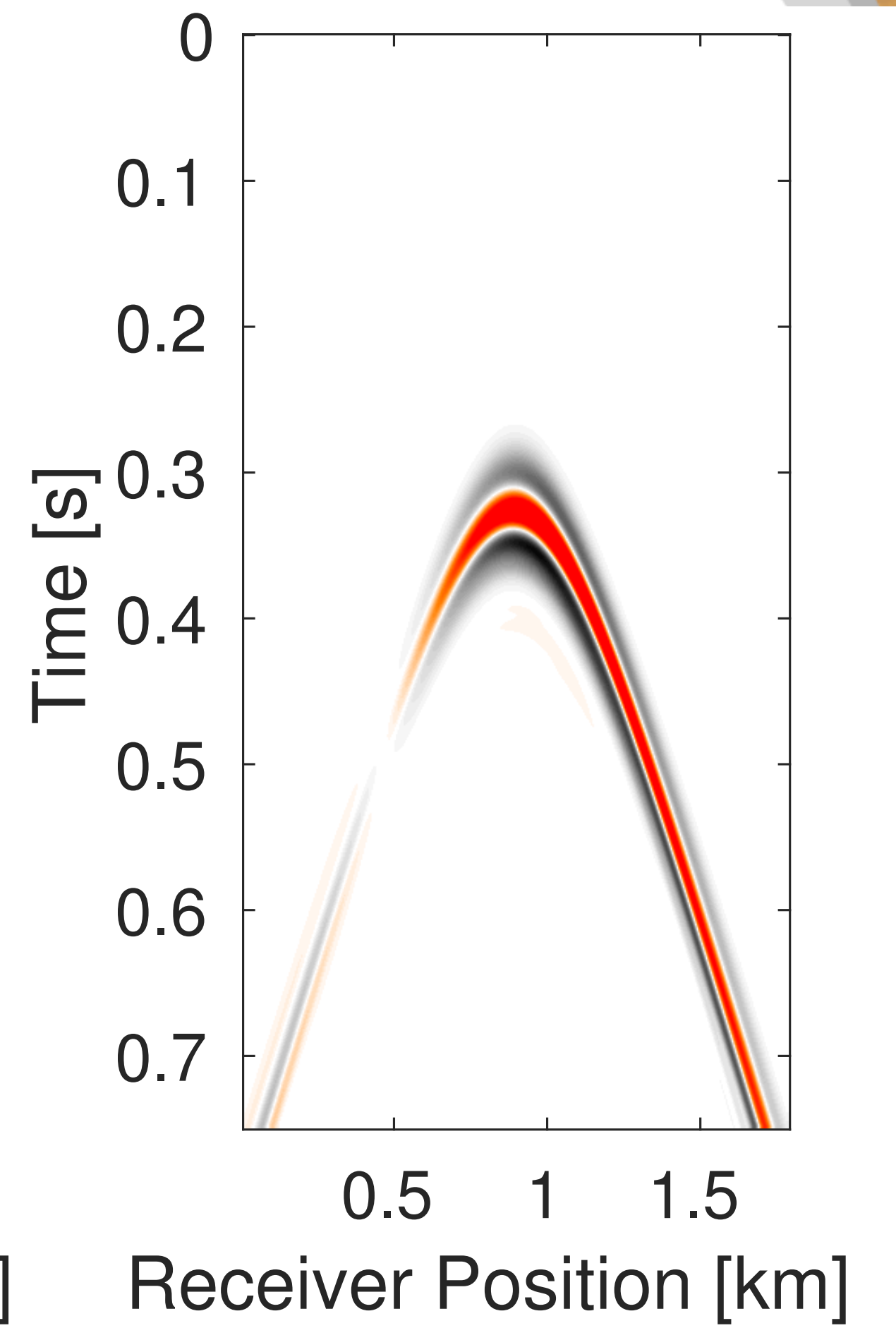
90 degrees



71.5 degrees

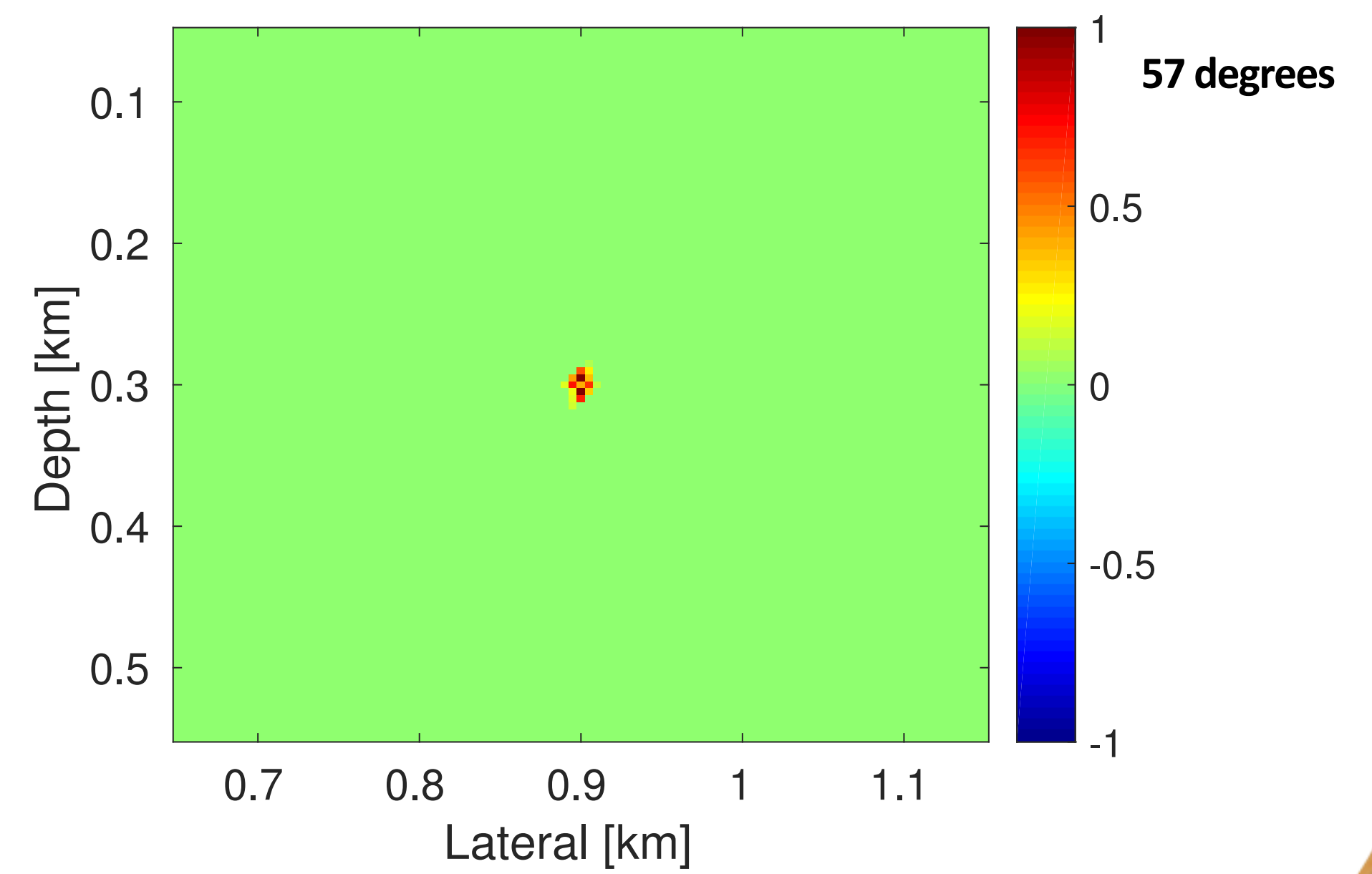
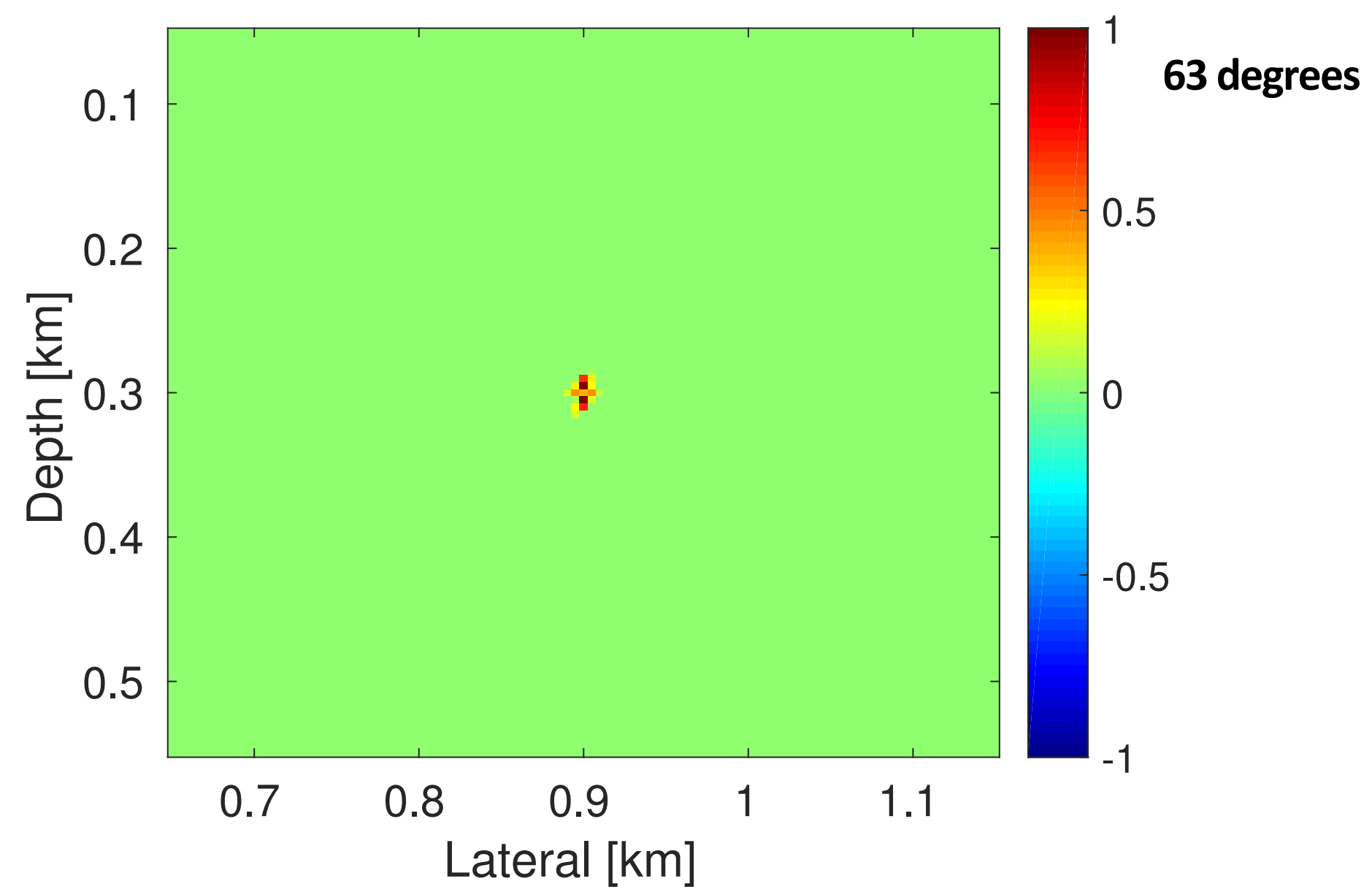
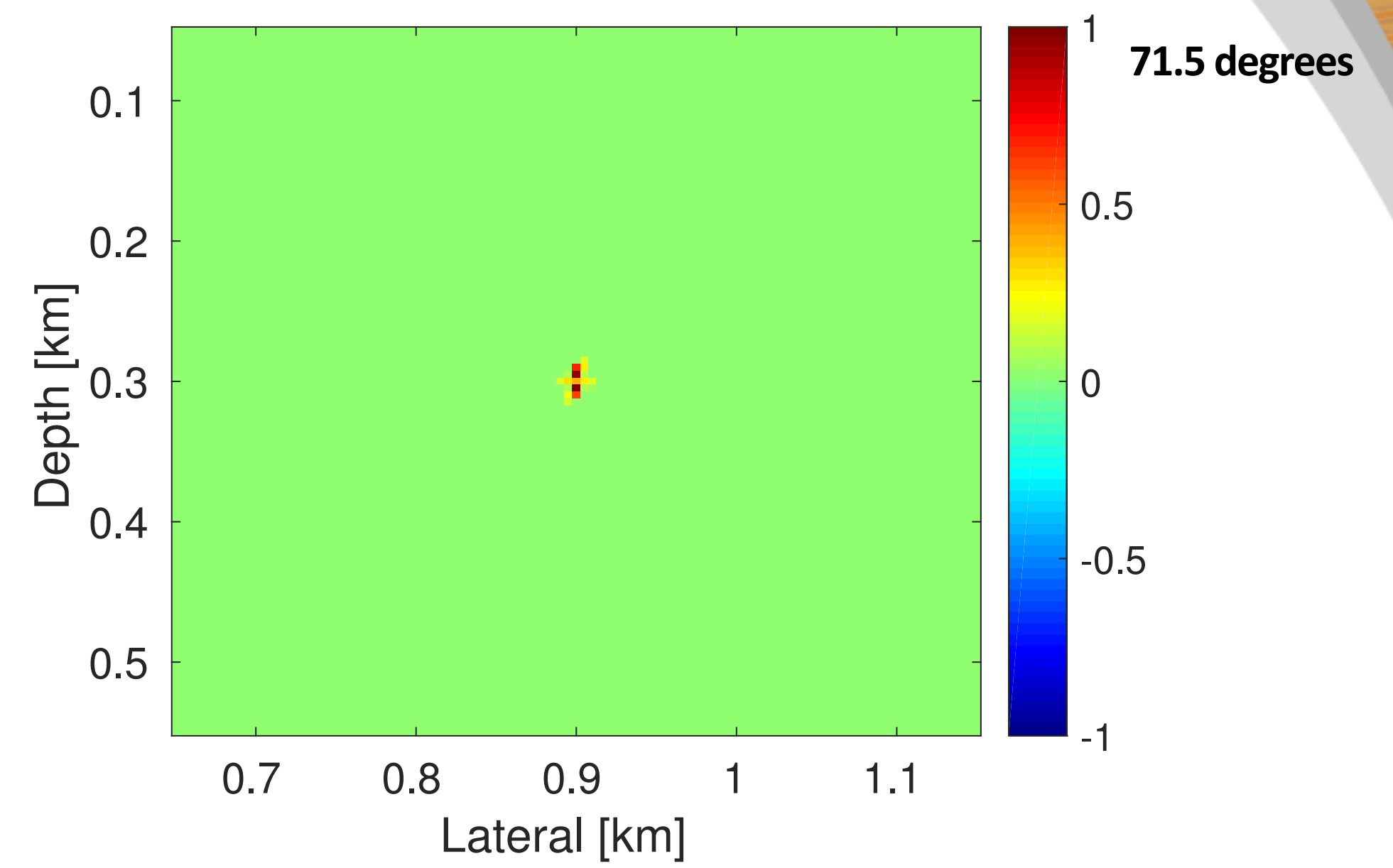
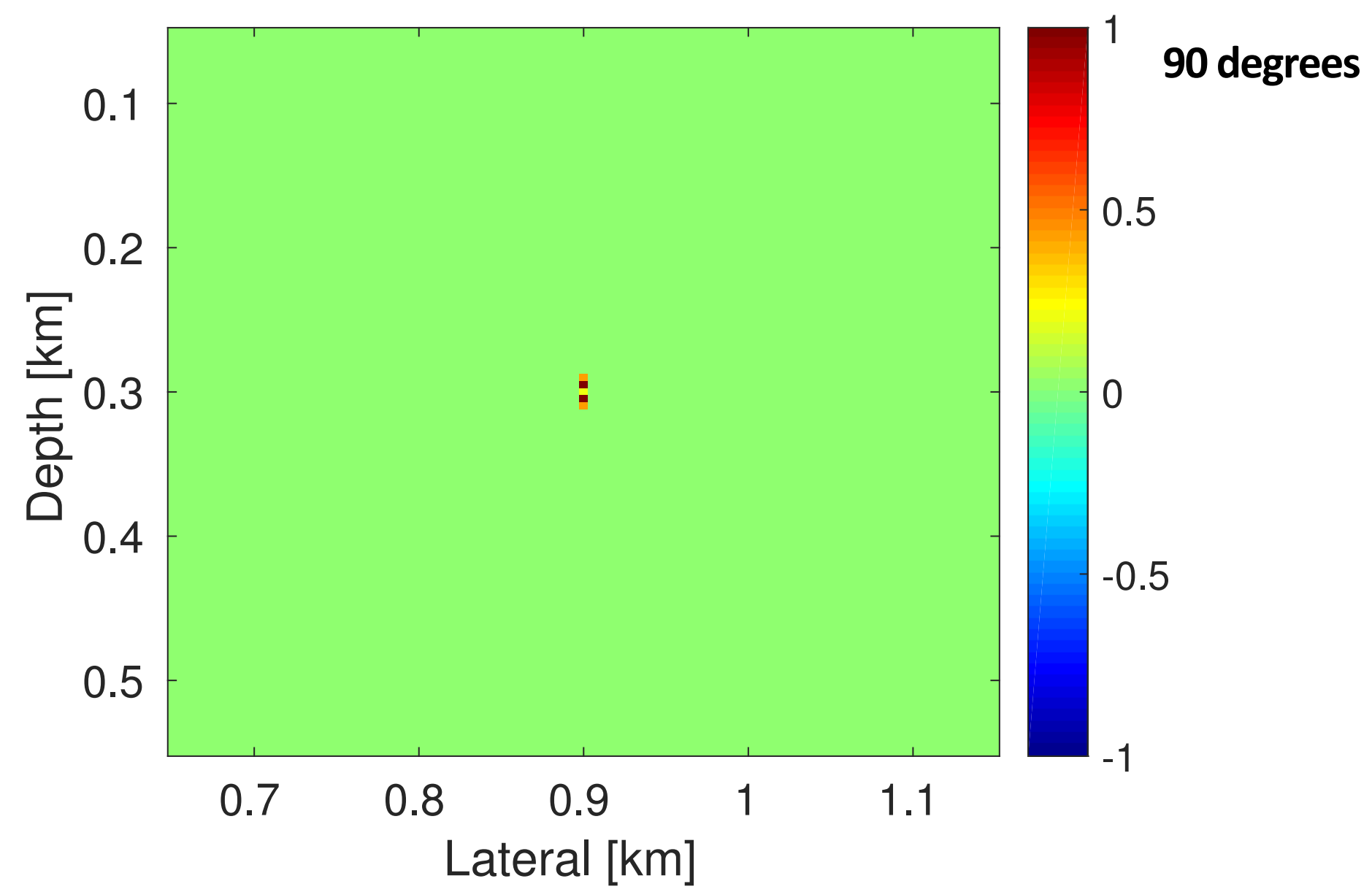


63 degrees

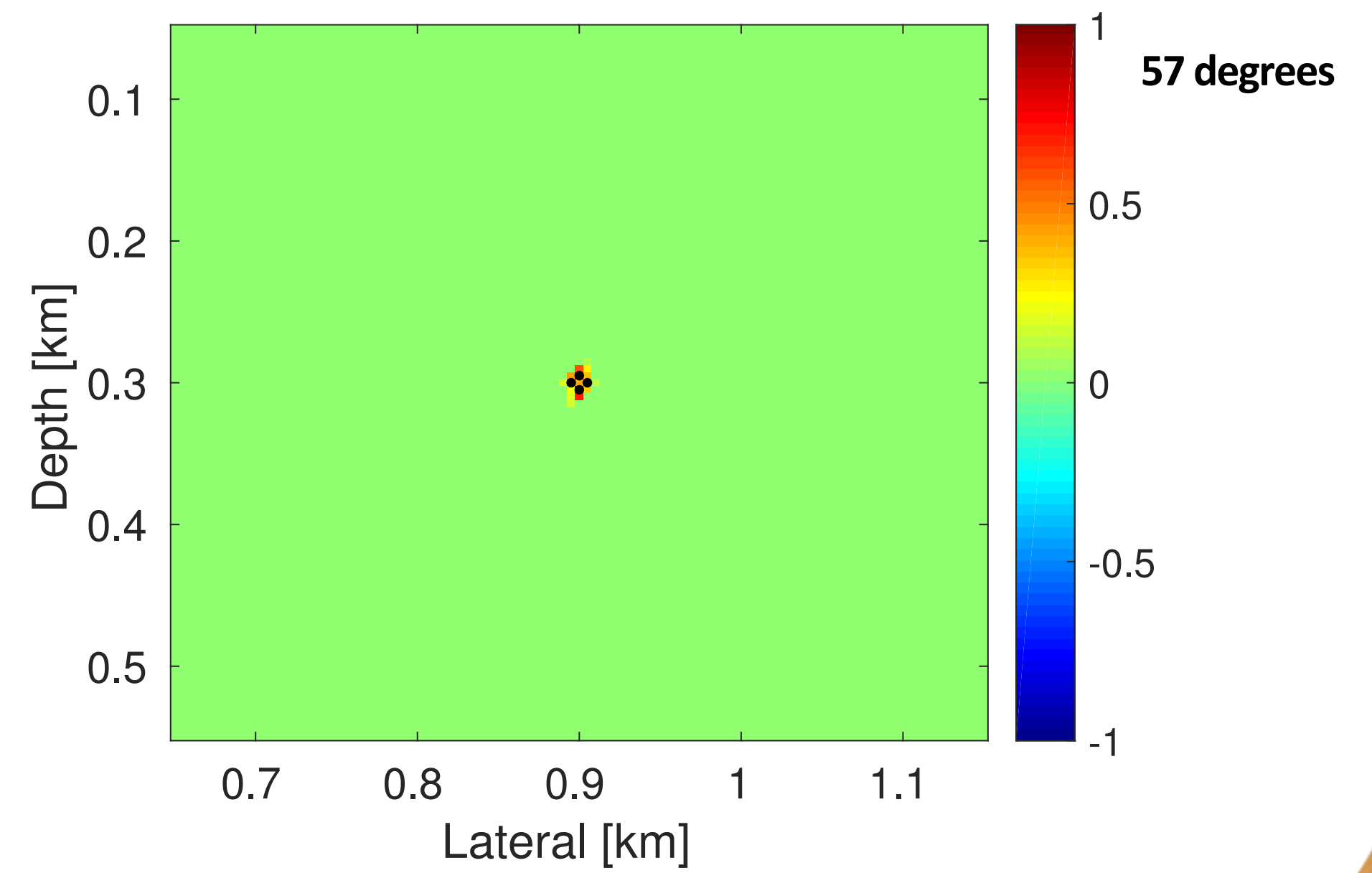
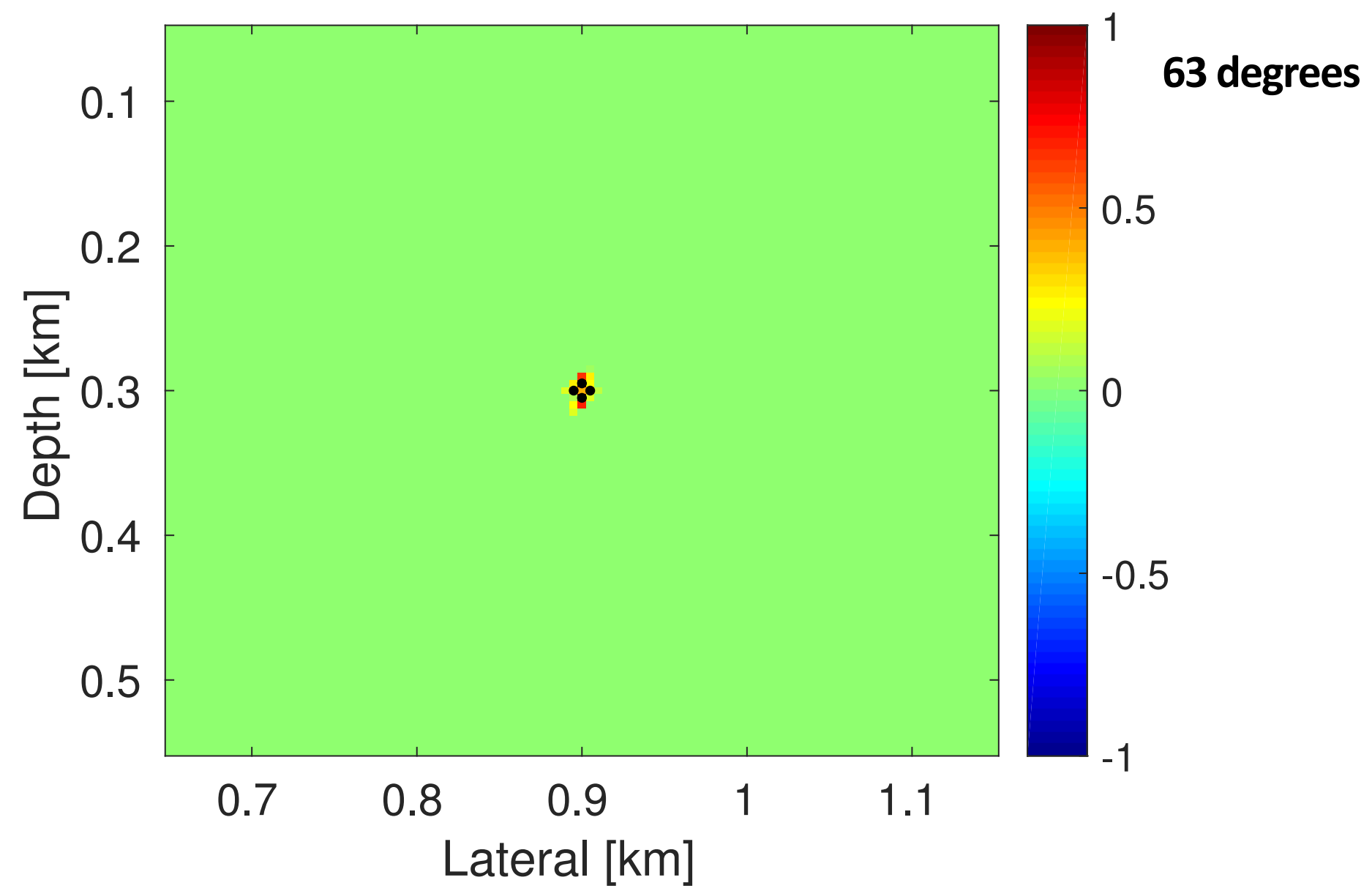
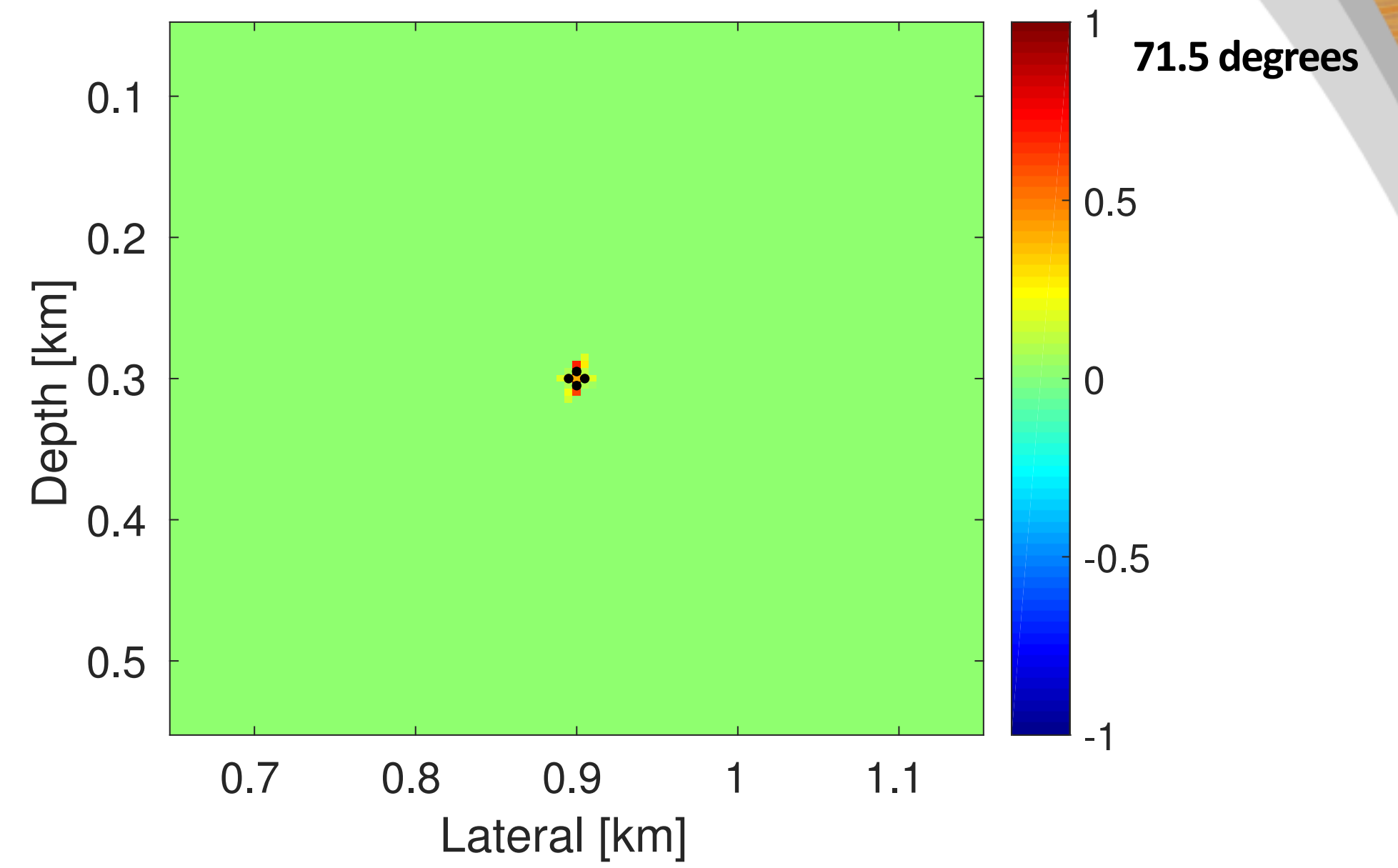
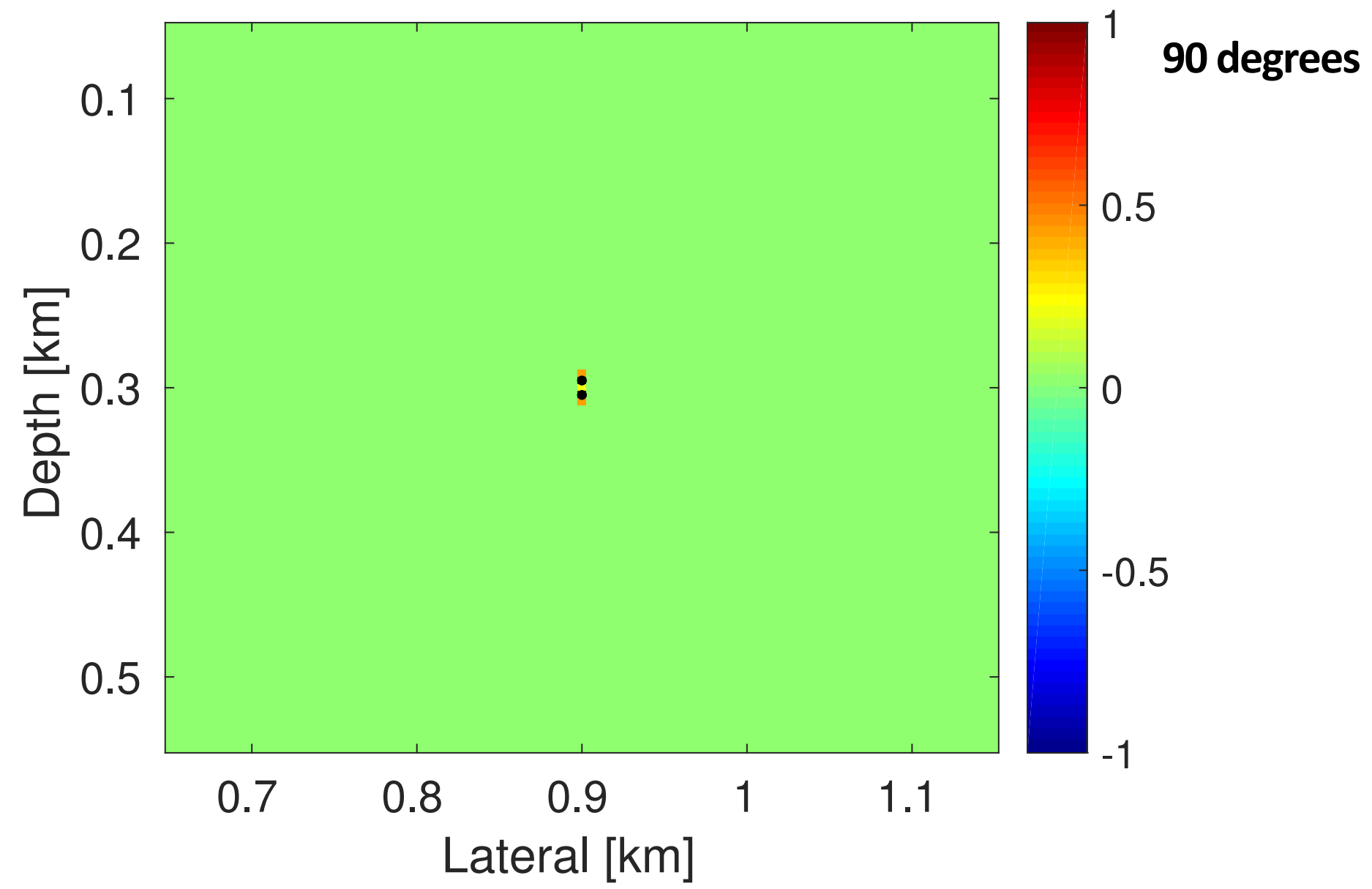


57 degrees

Estimated location



Estimated location



Conclusions

Potential applications:

- ▶ high resolution source collocation
- ▶ locate dipole sources with different directivity pattern

Works with sources of different frequencies and origin time

With zero initial guess “Sparsity-promoting” based method can estimate

- ▶ Source location
- ▶ source time function

We also demonstrated extension of our method in 2.5 D

Future work

Extension to 3D

Velocity update

Acknowledgements



The authors wish to acknowledge the SENAI CIMATEC Supercomputing Center for Industrial Innovation, with support from BG Brasil, Shell, and the Brazilian Authority for Oil, Gas and Biofuels (ANP), for the provision and operation of computational facilities and the commitment to invest in Research & Development.

Acknowledgements

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

