

Open Performance portable Seismic Imaging - OPESCI

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Seismic imaging is a hard problem...

- ▶ The objective is to see clearly through km's of rock.
- ▶ The problem is ill posed - i.e. we know we know much less than we do know.
- ▶ Arguably the most data-intensive/compute-intensive problem in the private sector.
- ▶ Very far from being a solved problem - requirements:
 - ▶ Better physics model.
 - ▶ Fast, efficient, robust PDE solvers and their adjoint.
 - ▶ Inversion/optimisation techniques.
 - ▶ Ability to manage petabytes of data.

Code modernisation and innovation

- ▶ Innovation is being stifled by the sheer size and complexity of software.
- ▶ Slow and costly to modernise/optimize code.
- ▶ Skills gap: "there are not enough HPC developers in the world to modernise our code".
- ▶ Question: How can we be agile, innovative and productive?

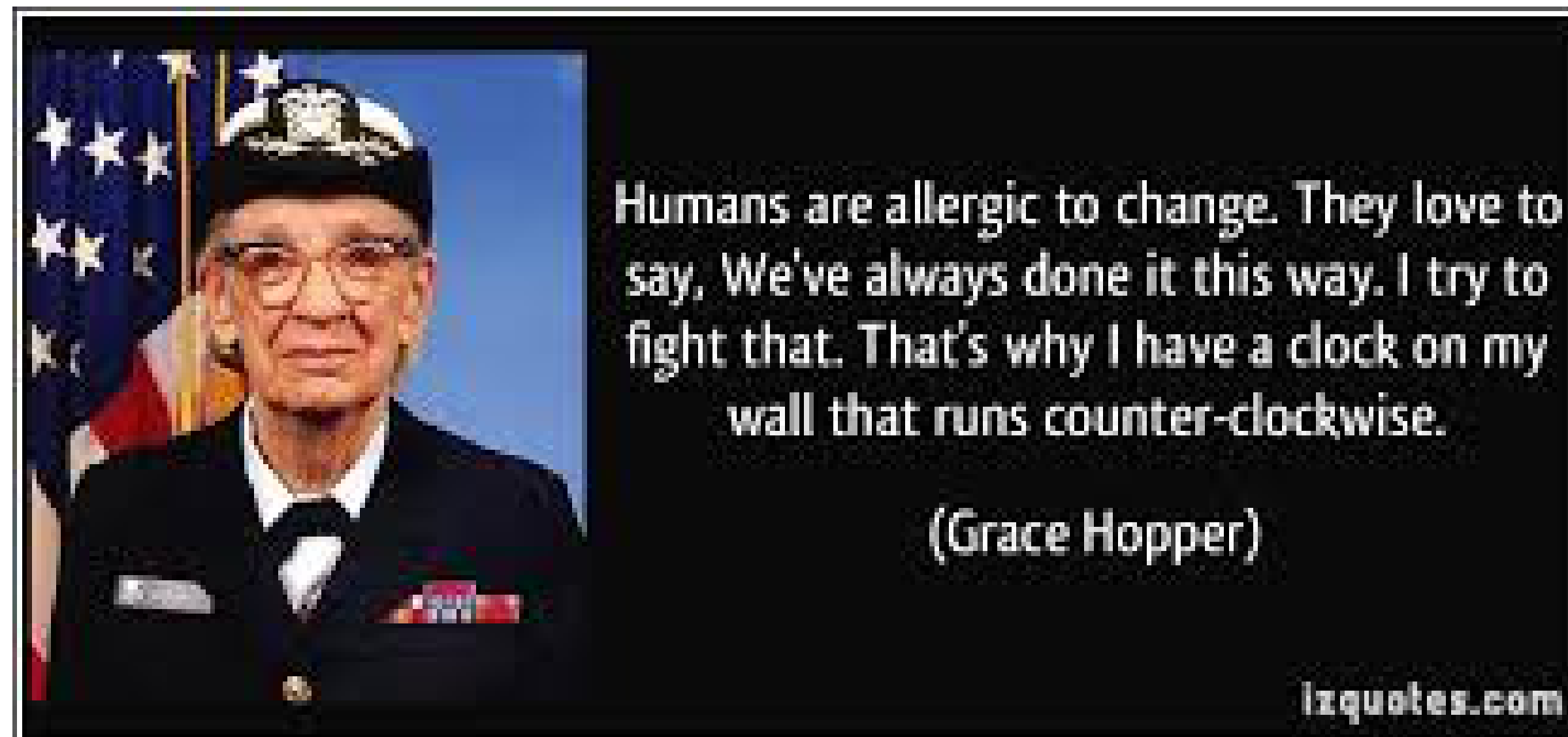


Options...

- ▶ New parallel programming languages like OpenCL can offer superior performance portability.
 - ▶ **BUT:** Part of the code modernisation challenge is the sheer quantity of legacy source code - OpenCL is far more verbose than what it is replacing.
- ▶ MATLAB highly successful high-level language.
 - ▶ **BUT:** Cannot get same performance as compiled codes; license model unattractive for HPC.
- ▶ Python (with Numpy etc) clear favorite high-level language among scientists and engineers, great at wrapping external libraries.
 - ▶ **BUT:** Pure (naive) Python code can be slow; and where do the external libraries come from?
- ▶ Julia fixes many of the performance issues present in pure Python.
 - ▶ **BUT:** Fast but still not HPC; and where do the external libraries come from?

Lesson from history

- ▶ First ever compiler was the A-0 system (Arithmetic Language version 0), written by Grace Hopper (aka Amazing Grace) in 1951 for the UNIVAC I.
- ▶ First FORTRAN compiler delivered in April 1957. This was also the first optimizing compiler - customers were reluctant to use a *high-level programming language* unless its compiler could generate code with performance comparable to that of their hand-coded assembly.



Yesterdays high-level languages are today's low-level languages

- ▶ Goal: Rather than writing software to solve **one** set of equations for **one** computer architecture; write software that can read and manipulate equations, and write software in low-level languages for different computer architectures.
- ▶ Using code generation methods millions of lines of parallel code can be generated within seconds.
- ▶ Not just trying to match performance of hand-tuned code: through automation we perform aggressive optimisations impractical for human programmers to implement by hand — and get it right first time...

Open Performance portable Seismic Imaging (OPESCI) - <http://opesci.org>

- ▶ Funding: Intel Parallel Computing Center.
- ▶ Partners: Imperial College London, SENAI CIMATEC Brazil, University of British Columbia, BG Group
- ▶ Finite element:
 - ▶ Area where we have the strongest track record - particularly in CFD (shameless plug <http://www.lcs-fast.com>)
 - ▶ Big opportunities for seismic imaging - but impacts much more of the software stack therefore...
- ▶ Finite difference:
 - ▶ Brings together a number of different (open source) software technologies and communities to develop a code generation framework for seismic imaging.

Symbolic computation is a powerful tool

Solving simple PDEs is (kind of) easy...

First-order diffusion equation:

```
for ti in range(timesteps):
    t0 = ti % 2
    t1 = (ti + 1) % 2
    for i in range(1, nx-1):
        for j in range(1, ny-1):
            uxx = (u[t0,i+1,j]-2*u[t0,i,j]+u[t0,i-1,j]) / dx2
            uyy = (u[t0,i,j+1]-2*u[t0,i,j]+u[t0,i,j-1]) / dy2
            u[t1, i, j] = u[t0, i, j] + dt * a * (uxx + uyy)
```


Symbolic computation is a powerful tool

Solving complicated PDEs is not easy!

12th-order acoustic wave equation:

```

for (int i4 = 0; i4<149; i4+=1) {
  for (int i1 = 6; i1<64; i1++) {
    for (int i2 = 6; i2<64; i2++) {
      for (int i3 = 6; i3<64; i3++) {
        u[i4][i1][i2][i3] = 6.01250601250601e-9F*(2.80896e+8F*damp[i1][i2][i3]*u[i4-2][i1
          ][i2][i3]-3.3264e+8F*m[i1][i2][i3]*u[i4-2][i1][i2][i3]+6.6528e+8F*m[i1][i2
          ][i3]*u[i4-1][i1][i2][i3]-2.12255421155556e+7F*u[i4-1][i1][i2][i3
          ]-1.42617283950617e+2F*u[i4-1][i1][i2][i3-6]+2.46442666666667e+3F*u[i4-1][
          i1][i2][i3-5]-2.11786666666667e+4F*u[i4-1][i1][i2][i3-4]+1.25503209876543e
          +5F*u[i4-1][i1][i2][i3-3]-6.3536e+5F*u[i4-1][i1][i2][i3-2]+4.066304e+6F*u[
          i4-1][i1][i2][i3-1]+4.066304e+6F*u[i4-1][i1][i2][i3+1]-6.3536e+5F*u[i4-1][
          i1][i2][i3+2]+1.25503209876543e+5F*u[i4-1][i1][i2][i3+3]-2.11786666666667e
          +4F*u[i4-1][i1][i2][i3+4]+2.46442666666667e+3F*u[i4-1][i1][i2][i3
          +5]-1.42617283950617e+2F*u[i4-1][i1][i2][i3+6]-1.42617283950617e+2F*u[i4
          -1][i1][i2-6][i3]+2.46442666666667e+3F*u[i4-1][i1][i2-5][i3
          ]-2.11786666666667e+4F*u[i4-1][i1][i2-4][i3]+1.25503209876543e+5F*u[i4-1][
          i1][i2-3][i3]-6.3536e+5F*u[i4-1][i1][i2-2][i3]+4.066304e+6F*u[i4-1][i1][i2
          -1][i3]+4.066304e+6F*u[i4-1][i1][i2+1][i3]-6.3536e+5F*u[i4-1][i1][i2+2][i3
          ]+1.25503209876543e+5F*u[i4-1][i1][i2+3][i3]-2.11786666666667e+4F*u[i4-1][
          i1][i2+4][i3]+2.46442666666667e+3F*u[i4-1][i1][i2+5][i3]-1.42617283950617e
          +2F*u[i4-1][i1][i2+6][i3]-1.42617283950617e+2F*u[i4-1][i1-6][i2][i3
          ]+2.46442666666667e+3F*u[i4-1][i1-5][i2][i3]-2.11786666666667e+4F*u[i4-1][
          i1-4][i2][i3]+1.25503209876543e+5F*u[i4-1][i1-3][i2][i3]-6.3536e+5F*u[i4
          -1][i1-2][i2][i3]+4.066304e+6F*u[i4-1][i1-1][i2][i3]+4.066304e+6F*u[i4-1][
          i1+1][i2][i3]-6.3536e+5F*u[i4-1][i1+2][i2][i3]+1.25503209876543e+5F*u[i4
          -1][i1+3][i2][i3]-2.11786666666667e+4F*u[i4-1][i1+4][i2][i3
          ]+2.46442666666667e+3F*u[i4-1][i1+5][i2][i3]-1.42617283950617e+2F*u[i4-1][
          i1+6][i2][i3))/(1.68888888888889F*damp[i1][i2][i3]+2*m[i1][i2][i3]);
      }
    }
  }
}

```

Symbolic computation is a powerful tool

- ▶ **Getting performance on modern hardware is not easy!**
 - ▶ Functioning code exists but is not optimised for current hardware
 - ▶ **Evolution vs. revolution?**
- ▶ **Domain-specific languages (DSL) make revolution easy**
 - ▶ **Separate problem definition from implementation**
 - ▶ Creates a separate of concerns between scientists and computation experts
- ▶ **Performance portability through code-generation**
 - ▶ Code is auto-generated and optimised at run-time
 - ▶ Platform-specific optimisation for target hardware



Symbolic computation is a powerful tool

- ▶ **Symbolic DSLs for solving PDEs have proven successful**

FEniCS / Firedrake - Finite element DSL packages

Velocity-stress formulation of elastic wave equation, with isotropic stress:

$$\rho \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \mathbb{T}$$

$$\frac{\partial \mathbb{T}}{\partial t} = \lambda (\nabla \cdot \mathbf{u}) \mathbb{I} + \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Weak form of equations written in UFL¹:

```
F_u = density*inner(w, (u - u0)/dt)*dx - inner(w, div(s0))*dx
solve(lhs(F_u) == rhs(F_u), u)
```

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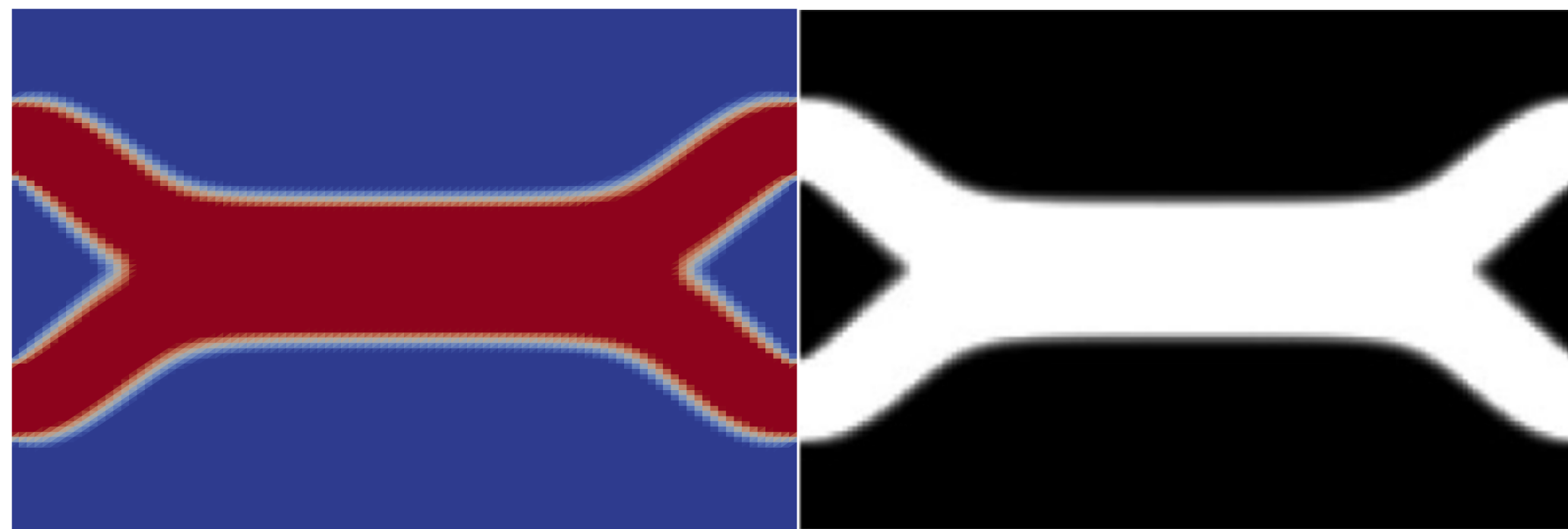
Symbolic computation is a powerful tool

- ▶ **Symbolic DSLs for solving PDEs have proven succesful**

Dolfin-Adjoint: Symbolic adjoints from symbolic PDEs¹

- ▶ Solves complex optimisation problems
- ▶ 2015 Wilkinson prize winner

Below is the optimal design of a double pipe that minimises the dissipated power in the fluid.



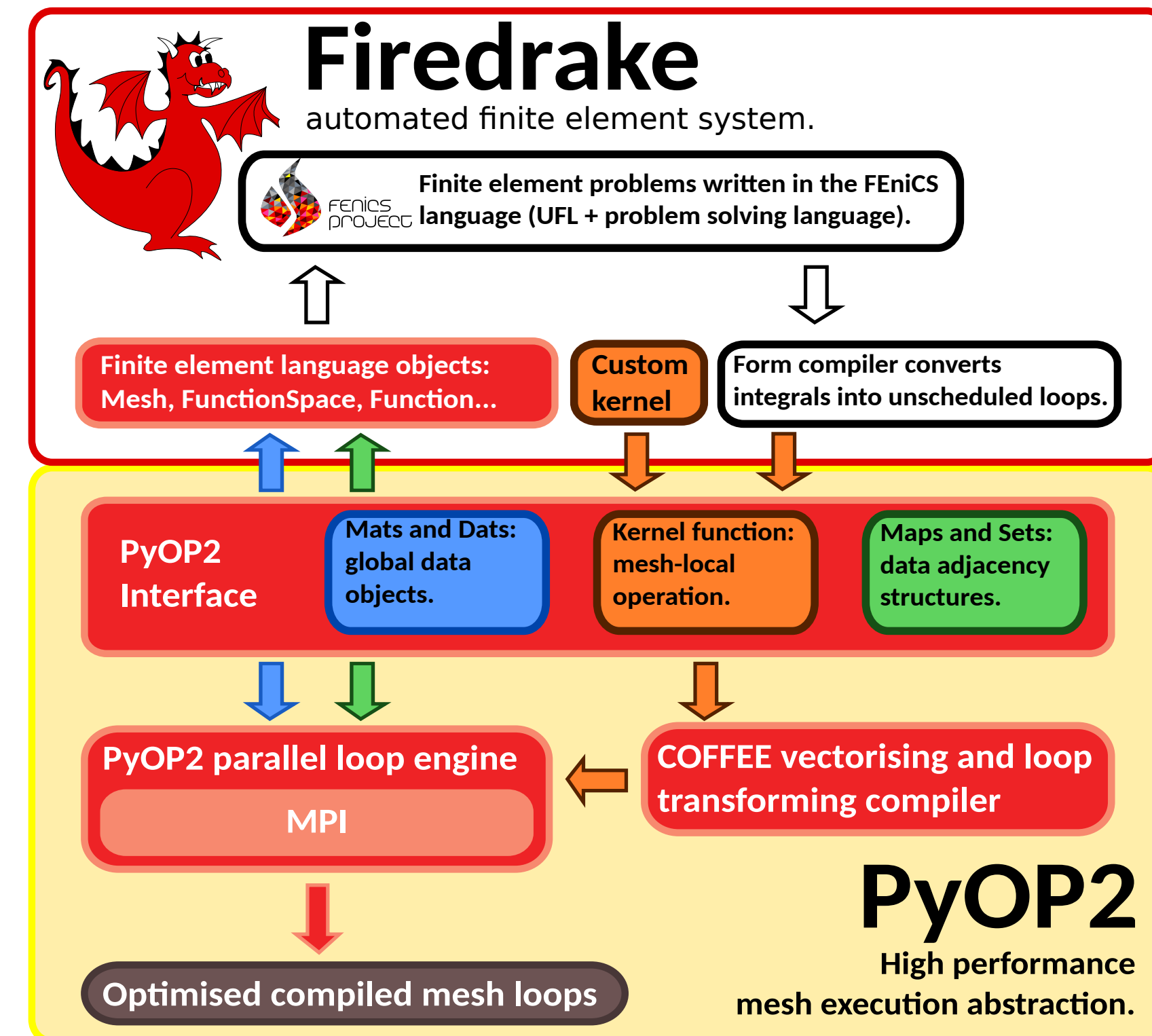
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Automating computation for finite element

Firedrake: Automated code generation for finite element

Application defines the discretised model equations in symbolic form through the Unified Form Language (UFL), a high-level domain-specific language for finite element problems that originated in the FEniCS project.

Multiple layers of abstraction allows Firedrake to utilise performance optimisations at various layers of its software stack, allowing application developers to describe the governing equations symbolically.

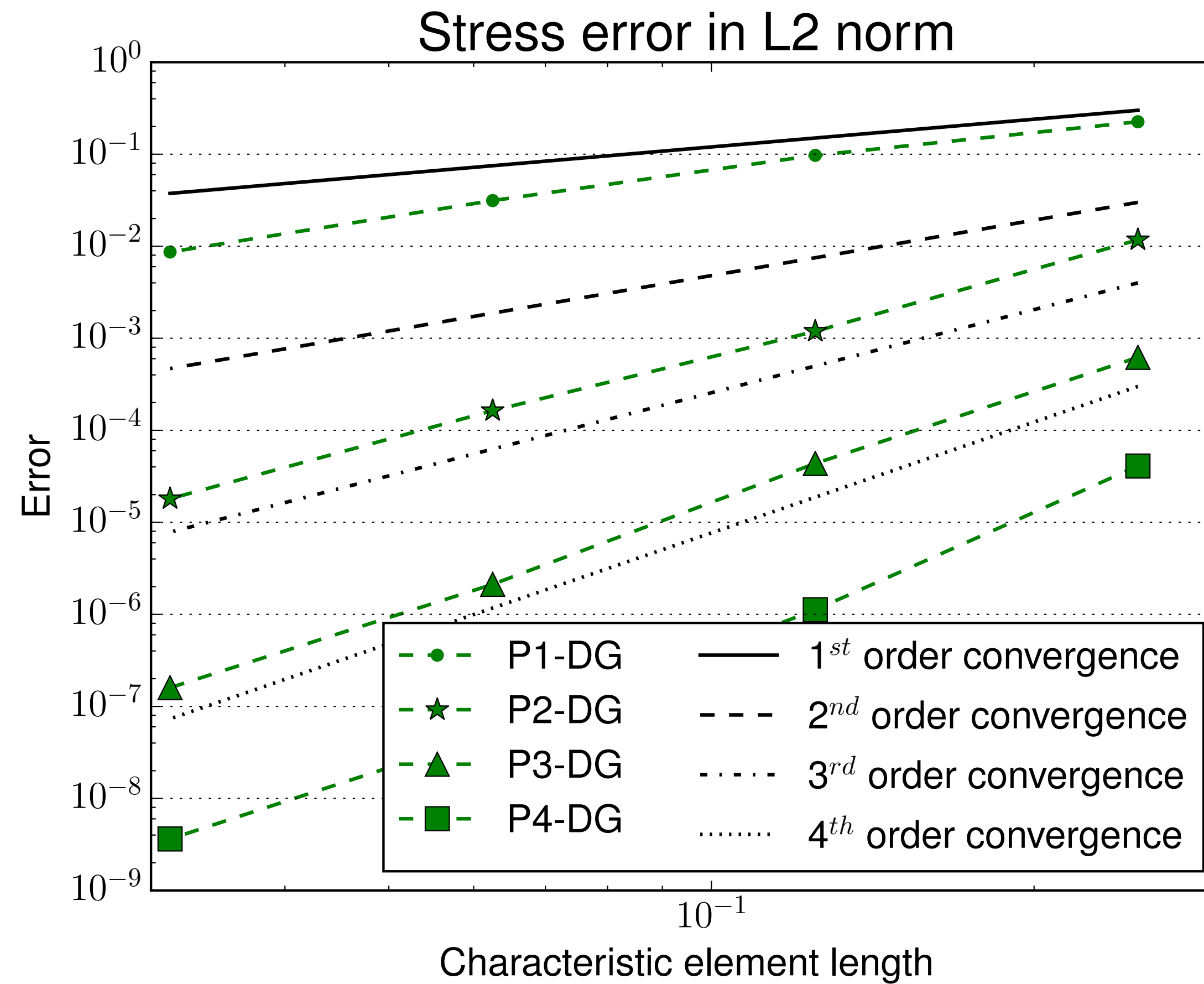


Firedrake: automating computation for finite element

Using Firedrake we implemented **Seigen** - solves elastic wave equation on unstructured meshes using discontinuous Galerkin finite element (DG-FEM) using first-order upwinding and time integration via fourth-order explicit leapfrog method.

Code verification

In general, method of manufactured solutions used to verify implementation. The test case presented here is the propagation of an eigenmode on a triangular unit meshes.

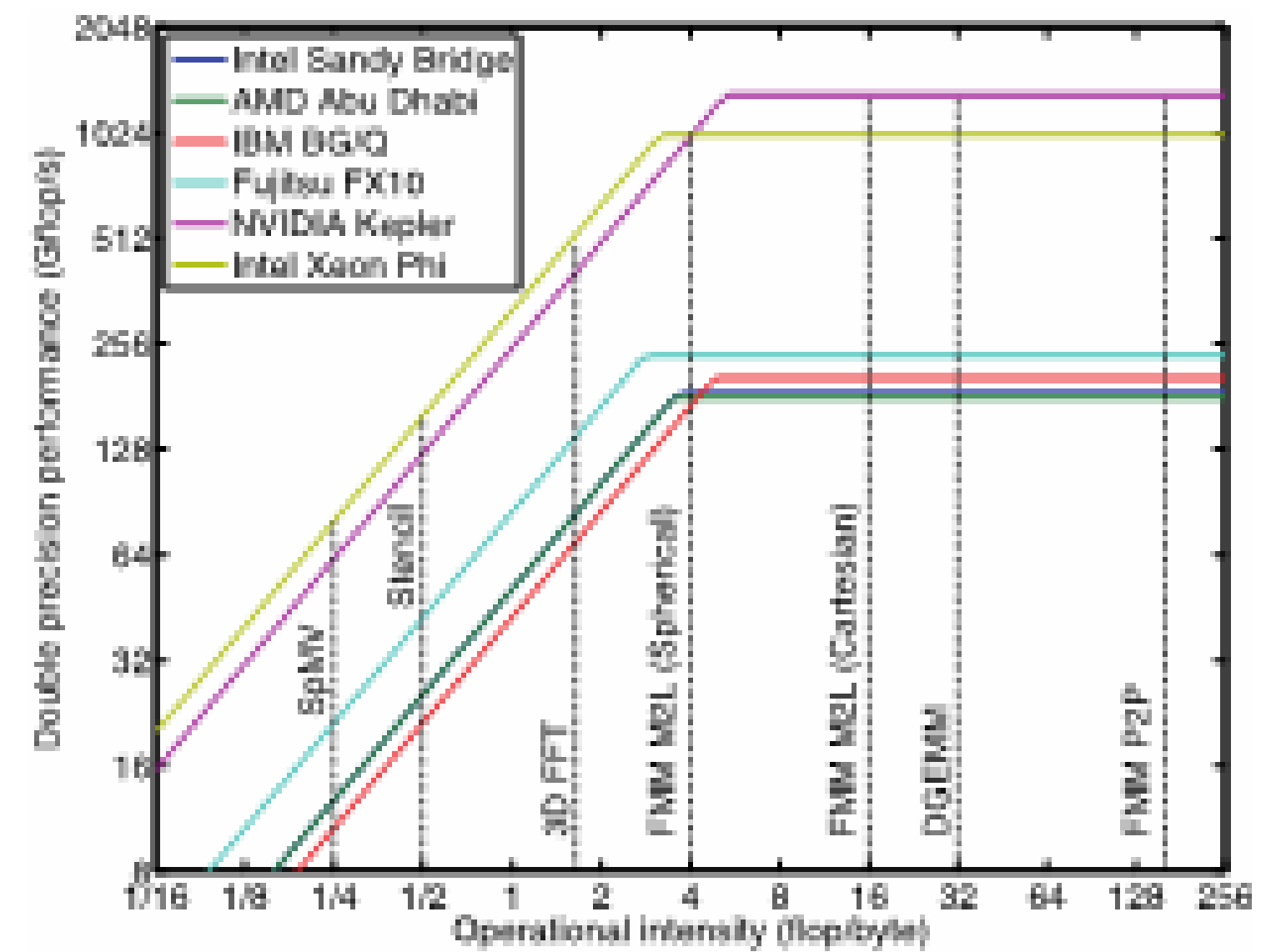


Performance results

- ▶ Defining the model equations in symbolic form allows the exploration and utilisation of various numerical schemes and discretisations with significantly less programming effort than in monolithic codes, as well as allowing various performance enhancements on multiple levels.
- ▶ Implicit and explicit solution methods can be compared directly using the same formulations of the governing equations.
- ▶ Increasing the order of the spatial discretisation requires as little as changing a single parameter, which provides close control over the trade-off between model error and computational work.
- ▶ Automated low-level optimisation of assembly kernels through COFFEE, a loop transforming compiler that uses expression rewriting to minimise FLOPs and thus increase performance.

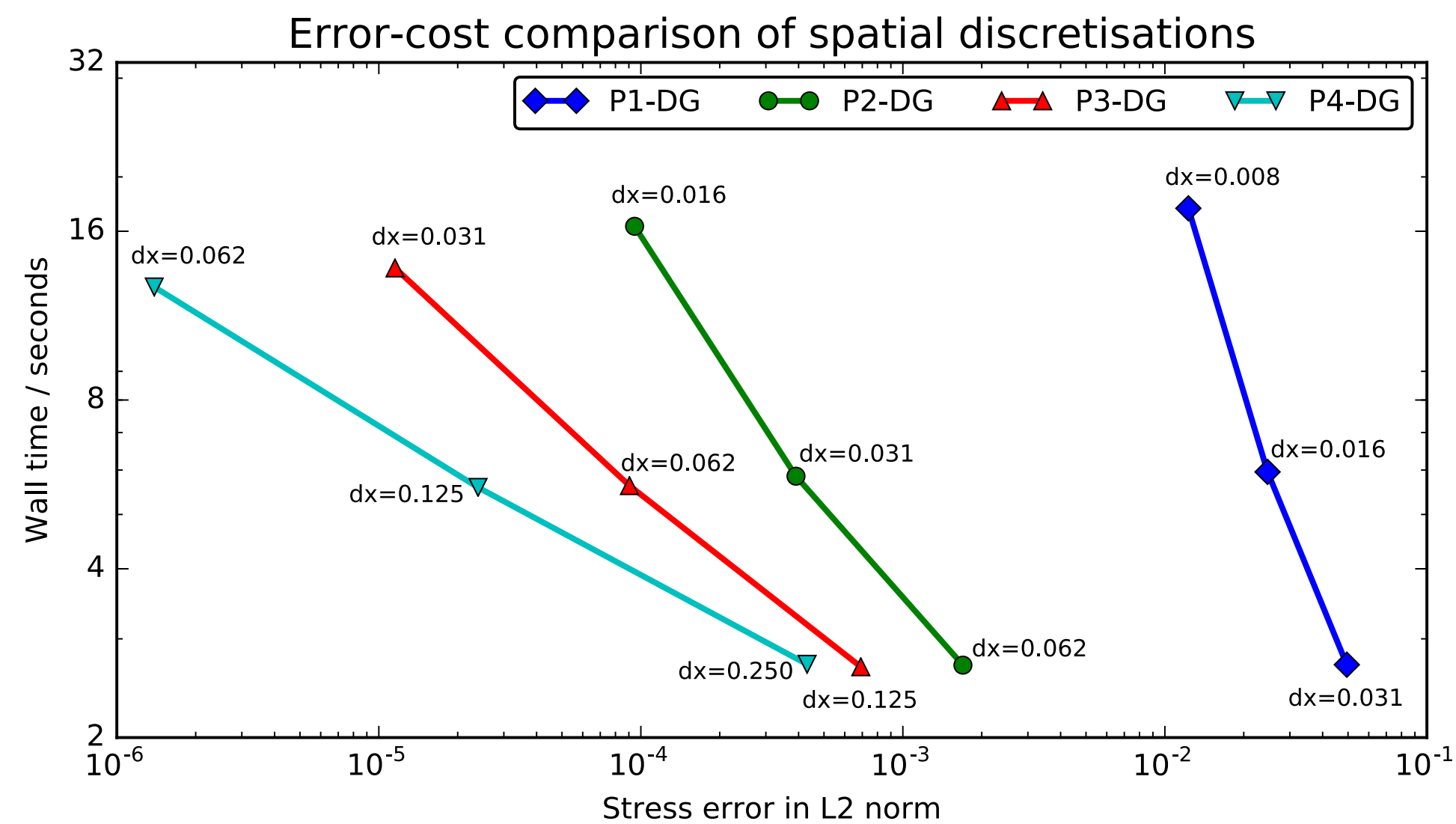
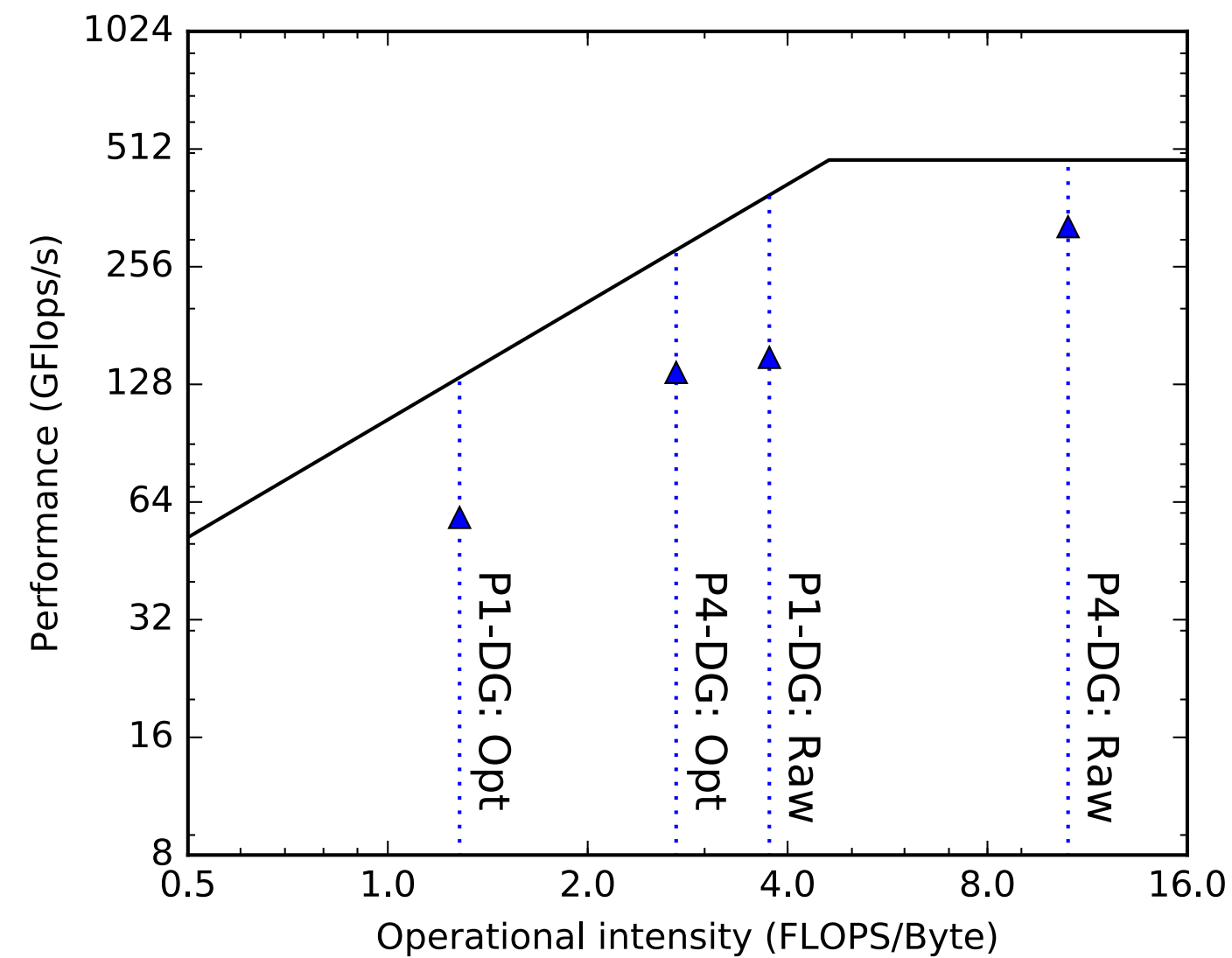
Quick revision: Roofline model - Rio Yokota's original slide

7 Dwarfs and Roofline Model



Performance results

Performance results for an auto-generated interior facet assembly kernel on P1-DG and P4-DG discretisations. The increase in FLOPs from using a higher-order method is counter-acted by compiler optimisations that reduce the operational intensity until performance is again limited by memory bandwidth.



Automating computation for finite difference

SymPy - Symbolic computation in Python

- ▶ Symbolic computer algebra system (CAS) written in pure Python¹
- ▶ *Features:*
 - ▶ Enable rapid development - hours rather than months.
 - ▶ Greatly eases the development of adjoint models.
 - ▶ High degree of automation enables extensive test coverage.
 - ▶ Complex symbolic expressions as Python object trees.
 - ▶ Symbolic manipulation routines and interfaces.
 - ▶ Convert symbolic expressions to numeric functions:
 - ▶ Python or NumPy functions
 - ▶ C or Fortran kernels
- ▶ For a great overview see [A. Meuer's talk at SciPy 2016](#)

For specialised domains generating C code is not enough!

1

SymPy - vibrant community

```

# Mixed derivative bug reported 13 April 2016.
f = Function('f')(x, y)
d2fdxdy = f.diff(x).diff(y)
pprint(as_finite_diff(d2fdxdy, wrt=x))
-f(x - 1/2, y) + f(x + 1/2, y)

# Fixed committed 29 May 2016
f = Function('f')(x, y)
d2fdxdy = f.diff(x).diff(y)
pprint(as_finite_diff(d2fdxdy, wrt=x))
-Derivative(f(x - 1/2, y), y) + Derivative(f(x + 1/2, y), y)

# Enhancement committed 01 June 2016
pprint(as_finite_diff(as_finite_diff(d2fdxdy, wrt=x), wrt=y))
f(x - 1/2, y - 1/2) - f(x - 1/2, y + 1/2) - f(x + 1/2, y - 1/2) + f(x + 1/2, y +
↪ 1/2)
dfgdx = (f/g).diff(x)
pprint(as_finite_diff(dfgdx, wrt=x))
(-f(x - 1/2, y) + f(x + 1/2, y))/g(x, y) - ((-g(x - 1/2, y) + g(x + 1/2, y)).f(x,
↪ y))/(g(x, y) * g(x, y))

# Merged into master on September 8th.

```

Todd Park's interpretation of Joy's Law: "Even if you get the best and the brightest to work for you, there will always be an infinite number of other, smarter people employed by others."

Devito - A Finite Difference DSL

Devito - A Finite Difference DSL for seismic imaging

- ▶ Aimed at creating fast high-order inversion kernels
- ▶ Design and development is driven by “real-world” problems

Devito is based on SymPy expressions

- ▶ Acoustic wave equation:

$$m \frac{\partial^2 u}{\partial t^2} + \eta \frac{\partial u}{\partial t} - \nabla^2 u = 0$$

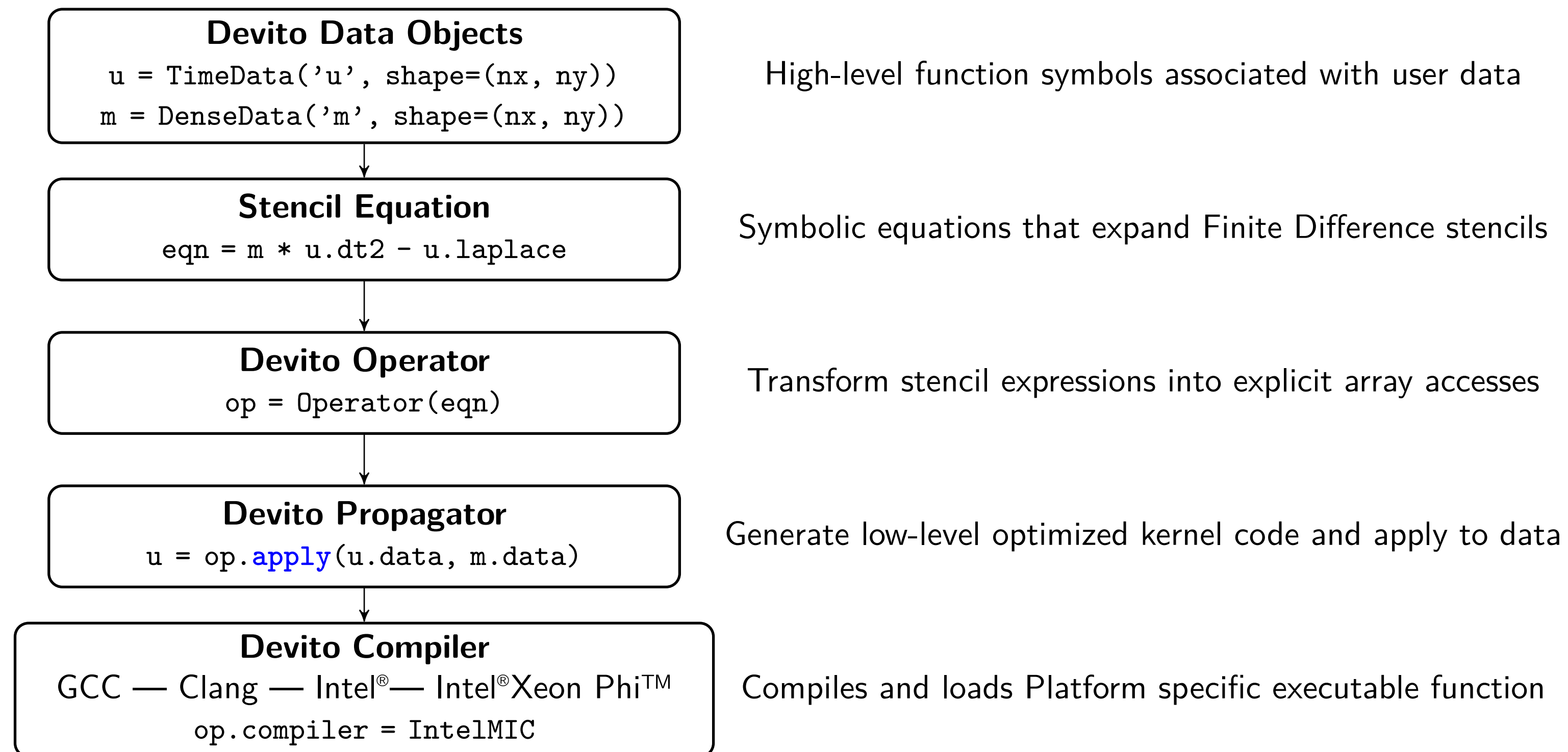
can be defined symbolically as

```
eqn = m * u.dt2 + eta * u.dt - u.laplace
```

Devito auto-generates optimised C kernel code

- ▶ OpenMP threading and vectorisation pragmas
- ▶ Cache blocking and auto-tuning
- ▶ Symbolic stencil optimisation (eg. CSE, hoisting)

Devito - A Finite Difference DSL



Devito - A Finite Difference DSL

Real-world applications need more than PDE solves

- ▶ File I/O and support for large data sets
- ▶ Non-PDE kernel code, eg. sparse point interpolation

Devito follows the principle of Graceful Degradation

- ▶ Circumvent restrictions to the high-level API by customisation
- ▶ Devito translates high-level PDE-based stencils into “matrix index” format

```
# High-level expression equivalent to f.dx2
(-2*f(x, y) + f(x - h, y) + f(x + h, y)) / h**2

# Low-level expression with explicit indexing
(-2*f[x, y] + f[x - 1, y] + f[x + 1, y]) / h**2
```

- ▶ **Allows custom functionality in auto-generated kernels**

Devito - A Finite Difference DSL

Automated code optimisations:

- ▶ OpenMP and vectorisation pragmas
- ▶ Loop blocking and auto-tuning for block size
- ▶ Automated roofline plotting for performance analysis
- ▶ Just-In-Time (JIT) compilation aids aggressive compiler optimisation.
- ▶ Choice of API's - focus on Python because of potential to integrate with TensorFlow and other data science frameworks - could equally generate API's for C, Fortran or Julia.

Symbolic optimisations:

- ▶ Common sub-expression elimination:
 - ▶ Reduces compilation time from hours to seconds for large stencils
 - ▶ Enables further factorisation techniques to reduce flops

Adjoint model in SymPy

The adjoint can often pose a major headache for developers - code generation immediately reduces the complexity.

```
# Derive stencil for adjoint model
wave_equationA = m * dtt - (dxx + dyy) - e * dt
stencilA = solve(wave_equationA, p(x, y, t-s))[0]

# Gradient calculation (performs the adjoint and cross correlation together.
def Gradient(self, rec, u):
    dt = self.dt
    grad = self._gradient_stencil.apply()[0]
    return (dt**-2)*grad

# Adjoint model as a separate call created purely for verification.
def Adjoint(self, rec):
    v = self._adjoint_stencil.apply()[0]
    return (self.srca.data, v)
```

Can rigorously verify numerical discretisation - adjoint model and gradient defined at a high level.

Example: 2D TTI wave model

Forward:

$$m \frac{d^2 p(x, t)}{dt^2} - (1 + 2\epsilon) G_{\bar{x}\bar{x}} p(x, t) - \sqrt{(1 + 2\delta)} G_{\bar{z}\bar{z}} r(x, t) = q$$

$$m \frac{d^2 r(x, t)}{dt^2} - \sqrt{(1 + 2\delta)} G_{\bar{x}\bar{x}} p(x, t) - G_{\bar{z}\bar{z}} r(x, t) = q$$

where

$$G_{\bar{x}\bar{x}} = \cos(\theta)^2 \frac{d^2}{dx^2} + \sin(\theta)^2 \frac{d^2}{dz^2} - \sin(2\theta) \frac{d^2}{dx dz}$$

$$G_{\bar{z}\bar{z}} = \sin(\theta)^2 \frac{d^2}{dx^2} + \cos(\theta)^2 \frac{d^2}{dz^2} + \sin(2\theta) \frac{d^2}{dx dz}$$

Example: 2D TTI wave model

Adjoint:

$$m \frac{d^2 p(x, t)}{dt^2} - G_{\bar{x}\bar{x}}^T((1 + 2\epsilon)p(x, t)) - G_{\bar{z}\bar{z}}^T(\sqrt{(1 + 2\delta)}r(x, t)) = q$$

$$m \frac{d^2 r(x, t)}{dt^2} - G_{\bar{x}\bar{x}}^T(\sqrt{(1 + 2\delta)}p(x, t)) - G_{\bar{z}\bar{z}}^T r(x, t) = q$$

SymPy implementation

```

dttp=as_finite_diff(p(z,x,t).diff(t,t), [t-s,t, t+s])
dttr=as_finite_diff(r(z,x,t).diff(t,t), [t-s,t, t+s])
dtp=as_finite_diff(p(z,x,t).diff(t), [t-s,t])
dtr=as_finite_diff(r(z,x,t).diff(t), [t-s,t])

dxxp=as_finite_diff(p(z,x,t).diff(x,x), [x-h, x, x+h])
dzzp=as_finite_diff(p(z,x,t).diff(z,z), [z-h, z, z+h])
dxxr=as_finite_diff(r(z,x,t).diff(x,x), [x-h, x, x+h])
dzzr=as_finite_diff(r(z,x,t).diff(z,z), [z-h, z, z+h])

dxzp = .5/(h**2)*(-2*p(z,x,t) + p(z+h,x,t) + p(z-h,x,t) - p(z-h,x+h,t) + p(z,x-h,t) - p(z+h,x-h,t) + p(z,x+h,t))
dxzr = .5/(h**2)*(-2*r(z,x,t) + r(z+h,x,t) + r(z-h,x,t) - r(z-h,x+h,t) + r(z,x-h,t) - r(z+h,x-h,t) + r(z,x+h,t))

Gxxp=cos(Th)**2 * dxxp + sin(Th)**2 * dzzp - sin(2*Th)*dxzp
Gzzr=sin(Th)**2 * dxxr + cos(Th)**2 * dzzr + sin(2*Th)*dxzr

# Forward wave equations for p and r
wavep = m * dttp - A * Gxxp - B * Gzzr - q
waver = m * dttr - B * Gxxp - Gzzr - q

stencilp =solve(wavep,p(z,x,t+s),simplify=False)[0]
tsp=lambdify(arglambp, stencilp, "numpy")

stencilr = solve(waver,r(z,x,t+s),simplify=False)[0]
tsr=lambdify(arglambr, stencilr, "numpy")

```

Automatically generated code for 3D TTI forward model

...just another afternoon for Mathias...


```

Oct 25, 16 15:19 tti_3d_so6.txt Page 1/116
#include <cassert>
#include <cstdlib>
#include <cmath>
#include <iostream>
#include <fstream>
#include <vector>
#include <string>
#include <stdio>
#include <string>
#include <inttypes.h>
#include <sys/time.h>
#include <math.h>
#include <omp.h>
struct profiler
{
    double post_stencils0;
    double post_stencils1;
    double post_stencils2;
    double loop_body;
};
extern "C" int ForwardOperator(float *m_vec, float *damp_vec, float *u_vec, flo
at *v_vec, float *epsilon_vec, float *delta_vec, float *theta_vec, float *phi_ve
c, float *src_vec, float *src_coords_vec, float *rec_vec, float *rec_coords_vec,
long i1block, long i2block, struct profiler *timings)
{
    float (*m)[70][70] = (float (*)[70][70]) m_vec;
    float (*damp)[70][70] = (float (*)[70][70]) damp_vec;
    float (*u)[70][70][70] = (float (*)[70][70][70]) u_vec;
    float (*v)[70][70][70] = (float (*)[70][70][70]) v_vec;
    float (*epsilon)[70][70] = (float (*)[70][70]) epsilon_vec;
    float (*delta)[70][70] = (float (*)[70][70]) delta_vec;
    float (*theta)[70][70] = (float (*)[70][70]) theta_vec;

```

```

Oct 25, 16 15:19 tti_3d_so6.txt Page 2/116
float (*phi)[70][70] = (float (*)[70][70]) phi_vec;
float (*src)[1] = (float (*)[1]) src_vec;
float (*src_coords)[3] = (float (*)[3]) src_coords_vec;
float (*rec)[101] = (float (*)[101]) rec_vec;
float (*rec_coords)[3] = (float (*)[3]) rec_coords_vec;
{
    int t0;
    int t1;
    int t2;
    #pragma omp parallel
    for (int i4 = 0; i4<3; i4+=1)
    {
        #pragma omp single
        {
            t0 = (i4)&3;
            t1 = (t0 + 1)&3;
            t2 = (t1 + 1)&3;
        }
        struct timeval start_loop_body, end_loop_body;
        #pragma omp master
        gettimeofday(&start_loop_body, NULL);
        {
            #pragma omp for schedule(static)
            for (int ilb = 6; ilb<64 - (58 % ilblock); ilb+=ilblock)
                for (int i2b = 6; i2b<64 - (58 % i2block); i2b+=i2block)
                    for (int il = ilb; il<ilb+ilblock; il++)
                        for (int i2 = i2b; i2<i2b+i2block; i2++)
                            #pragma omp simd aligned(m, theta, delta, phi, u, damp, v, epsilon)
                                for (int i3 = 6; i3<64; i3++)

```

```

Oct 25, 16 15:19 tti_3d_so6.txt Page 3/116
{
    float temp1 = 8.85879567828298e-1F*damp[i1][i2][i3];
    float temp3 = 2.0F*m[i1][i2][i3];
    float temp4 = 1.0F/(temp1 + temp3);
    float temp5 = 4.0F*m[i1][i2][i3];
    float temp9 = temp1 - temp3;
    float temp10 = 1.56956521739130F;
    float temp12 = 5.0000000000000e-2F;
    float temp13 = 2.91666666666667e-1F*temp12;
    float temp16 = sin(theta[i1][i2][i3]);
    float temp19 = cos(phi[i1][i2][i3]);
    float temp20 = cos(theta[i1][i2][i3]);
    float temp21 = (1.0F/60.0F)*temp12;
    float temp27 = temp12*v[t1][i1][i2][i3 - 2];
    float temp30 = temp12*v[t1][i1][i2][i3 - 1];
    float temp31 = (3.0F/4.0F)*temp30;
    float temp34 = temp12*v[t1][i1][i2][i3 + 1];
    float temp35 = (3.0F/4.0F)*temp34;
    float temp38 = temp12*v[t1][i1][i2][i3 + 2];
    float temp41 = temp21*v[t1][i1][i2][i3 + 3];
    float temp42 = sin(phi[i1][i2][i3]);
    float temp43 = temp16*temp42;
    float temp48 = temp12*v[t1][i1][i2 - 2][i3];
    float temp51 = temp12*v[t1][i1][i2 - 1][i3];
    float temp52 = (3.0F/4.0F)*temp51;
    float temp55 = temp12*v[t1][i1][i2 + 1][i3];
    float temp56 = (3.0F/4.0F)*temp55;
    float temp59 = temp12*v[t1][i1][i2 + 2][i3];
    float temp62 = temp21*v[t1][i1][i2 + 3][i3];
    float temp63 = temp16*temp19;
    float temp65 = temp12*v[t1][i1][i2][i3];

```

```

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float temp66 = (7.0F/12.0F)*temp65;
float temp67 = (1.0F/30.0F)*temp12;
float temp72 = temp12*v[t1][i1 - 1][i2][i3];
float temp75 = temp12*v[t1][i1 + 1][i2][i3];
float temp78 = temp12*v[t1][i1 + 2][i2][i3];
float temp81 = temp12*v[t1][i1 + 3][i2][i3];
float temp84 = temp21*v[t1][i1 + 4][i2][i3];
float temp87 = temp12*v[t1][i1 - 2][i2][i3];
float temp88 = (3.0F/4.0F)*temp72;
float temp89 = (3.0F/4.0F)*temp75;
float temp90 = temp21*v[t1][i1 + 3][i2][i3];
float temp91 = temp12*v[t1][i1][i2][i3 + 3];
float temp94 = temp21*v[t1][i1][i2][i3 + 4];
float temp95 = temp12*v[t1][i1][i2 + 3][i3];
float temp98 = temp21*v[t1][i1][i2 + 4][i3];
float temp99 = 2.91666666666667e-1F*temp12*(temp20*((2.0F/5.0F
)*temp30 - 4.0F/3.0F*temp34 + (1.0F/2.0F)*temp38 + temp66 - temp67*v[t1][i1][i2]
[i3 - 2] - 2.0F/15.0F*temp91 + temp94) + temp43*(2.0F/5.0F)*temp51 - 4.0F/3.0F*
temp55 + (1.0F/2.0F)*temp59 + temp66 - temp67*v[t1][i1][i2 - 2][i3] - 2.0F/15.0F
*temp95 + temp98) + temp63*(temp21*v[t1][i1 - 3][i2][i3] + (3.0F/20.0F)*temp78 -
3.0F/20.0F*temp87 + temp88 - temp89 - temp90);
float temp100 = 3.75e-1F*temp12;
float temp102 = cos(theta[i1][i2][i3 - 1]);
float temp103 = (3.0F/4.0F)*temp65;
float temp104 = -temp103;
float temp107 = temp12*v[t1][i1][i2][i3 - 3];
float temp108 = (3.0F/4.0F)*temp27;
float temp109 = (3.0F/20.0F)*temp34;
float temp110 = temp21*v[t1][i1][i2][i3 + 2];
float temp112 = sin(phi[i1][i2][i3 - 1]);
float temp113 = sin(theta[i1][i2][i3 - 1]);

```

```

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float temp114 = temp112*temp113;
float temp116 = (3.0F/20.0F)*temp12;
float temp118 = (3.0F/4.0F)*temp12;
float temp120 = temp118*v[t1][i1][i2 - 1][i3 - 1];
float temp122 = temp118*v[t1][i1][i2 + 1][i3 - 1];
float temp125 = cos(phi[i1][i2][i3 - 1]);
float temp126 = temp113*temp125;
float temp127 = (7.0F/12.0F)*temp30;
float temp129 = (2.0F/5.0F)*temp12;
float temp131 = temp129*v[t1][i1 - 1][i2][i3 - 1];
float temp132 = (4.0F/3.0F)*temp12;
float temp134 = (1.0F/2.0F)*temp12;
float temp136 = (2.0F/15.0F)*temp12;
float temp138 = 7.5e-2F*temp12;
float temp140 = cos(theta[i1][i2][i3 + 2]);
float temp141 = (3.0F/20.0F)*temp65;
float temp142 = -temp141;
float temp143 = temp12*v[t1][i1][i2][i3 + 4];
float temp146 = temp21*v[t1][i1][i2][i3 + 5];
float temp148 = sin(phi[i1][i2][i3 + 2]);
float temp149 = sin(theta[i1][i2][i3 + 2]);
float temp150 = temp148*temp149;
float temp153 = temp116*v[t1][i1][i2 - 2][i3 + 2];
float temp157 = temp116*v[t1][i1][i2 + 2][i3 + 2];
float temp159 = cos(phi[i1][i2][i3 + 2]);
float temp160 = temp149*temp159;
float temp161 = (7.0F/12.0F)*temp38;
float temp166 = temp134*v[t1][i1 + 2][i2][i3 + 2];
float temp168 = 8.33333333333333e-3F*temp12;
float temp170 = cos(theta[i1][i2][i3 - 3]);
float temp171 = (1.0F/60.0F)*temp65;

```

```

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float temp172 = -temp171;
float temp177 = temp12*v[t1][i1][i2][i3 - 5];
float temp178 = temp12*v[t1][i1][i2][i3 - 4];
float temp179 = (3.0F/20.0F)*temp30;
float temp181 = sin(phi[i1][i2][i3 - 3]);
float temp182 = sin(theta[i1][i2][i3 - 3]);
float temp183 = temp181*temp182;
float temp184 = temp21*v[t1][i1][i2 - 3][i3 - 3];
float temp190 = temp21*v[t1][i1][i2 + 3][i3 - 3];
float temp191 = cos(phi[i1][i2][i3 - 3]);
float temp192 = temp182*temp191;
float temp193 = (7.0F/12.0F)*temp107;
float temp200 = cos(theta[i1][i2][i3 + 3]);
float temp201 = (3.0F/4.0F)*temp38;
float temp202 = temp12*v[t1][i1][i2][i3 + 5];
float temp204 = temp12*v[t1][i1][i2][i3 + 6];
float temp206 = sin(phi[i1][i2][i3 + 3]);
float temp207 = sin(theta[i1][i2][i3 + 3]);
float temp209 = temp21*v[t1][i1][i2 - 3][i3 + 3];
float temp214 = -temp21*v[t1][i1][i2 + 3][i3 + 3];
float temp215 = cos(phi[i1][i2][i3 + 3]);
float temp220 = -temp136*v[t1][i1 + 3][i2][i3 + 3];
float temp222 = cos(theta[i1][i2][i3 - 2]);
float temp223 = temp21*v[t1][i1][i2][i3 + 1];
float temp225 = sin(phi[i1][i2][i3 - 2]);
float temp226 = sin(theta[i1][i2][i3 - 2]);
float temp227 = temp225*temp226;
float temp230 = -temp116*v[t1][i1][i2 - 2][i3 - 2];
float temp234 = temp116*v[t1][i1][i2 + 2][i3 - 2];
float temp236 = cos(phi[i1][i2][i3 - 2]);
float temp237 = temp226*temp236;

```

```

Oct 25, 16 15:19 tti_3d_so6.txt Page 7/116
float temp238 = (7.0F/12.0F)*temp27;
float temp240 = -temp67*v[t1][i1 - 2][i2][i3 - 2];
float temp246 = cos(theta[i1][i2][i3 + 1]);
float temp248 = sin(phi[i1][i2][i3 + 1]);
float temp249 = sin(theta[i1][i2][i3 + 1]);
float temp250 = temp248*temp249;
float temp254 = temp118*v[t1][i1][i2 - 1][i3 + 1];
float temp256 = -temp118*v[t1][i1][i2 + 1][i3 + 1];
float temp259 = cos(phi[i1][i2][i3 + 1]);
float temp260 = temp249*temp259;
float temp261 = (7.0F/12.0F)*temp34;
float temp265 = -temp132*v[t1][i1 + 1][i2][i3 + 1];
float temp270 = sin(theta[i1 - 4][i2][i3]);
float temp272 = cos(phi[i1 - 4][i2][i3]);
float temp273 = cos(theta[i1 - 4][i2][i3]);
float temp274 = sin(phi[i1 - 4][i2][i3]);
float temp279 = temp12*v[t1][i1 - 5][i2][i3];
float temp281 = temp12*v[t1][i1 - 4][i2][i3];
float temp282 = temp12*v[t1][i1 - 3][i2][i3];
float temp283 = 6.66666666666667e-2F*temp12;
float temp285 = sin(theta[i1 - 3][i2][i3]);
float temp287 = cos(phi[i1 - 3][i2][i3]);
float temp288 = cos(theta[i1 - 2][i2][i3]);
float temp289 = temp21*v[t1][i1 - 3][i2][i3 - 3];
float temp295 = sin(phi[i1 - 3][i2][i3]);
float temp296 = temp285*temp295;
float temp297 = temp21*v[t1][i1 - 3][i2 - 3][i3];
float temp303 = temp285*temp287;
float temp304 = -2.0F/15.0F*temp65;
float temp305 = (7.0F/12.0F)*temp282;
float temp306 = temp21*v[t1][i1 + 1][i2][i3];

```

```

Oct 25, 16 15:19 tti_3d_so6.txt Page 8/116
float temp307 = 2.5e-1F*temp12;
float temp309 = sin(theta[i1 - 2][i2][i3]);
float temp311 = cos(phi[i1 - 2][i2][i3]);
float temp312 = cos(theta[i1 - 2][i2][i3]);
float temp313 = -temp116*v[t1][i1 - 2][i2][i3 - 2];
float temp314 = temp116*v[t1][i1 - 2][i2][i3 + 2];
float temp315 = sin(phi[i1 - 2][i2][i3]);
float temp316 = temp309*temp315;
float temp319 = -temp116*v[t1][i1 - 2][i2 - 2][i3];
float temp323 = temp116*v[t1][i1 - 2][i2 + 2][i3];
float temp325 = temp309*temp311;
float temp326 = (1.0F/2.0F)*temp65;
float temp327 = (7.0F/12.0F)*temp87;
float temp328 = temp21*v[t1][i1 + 2][i2][i3];
float temp329 = 6.66666666666667e-1F*temp12;
float temp331 = sin(theta[i1 - 1][i2][i3]);
float temp333 = cos(phi[i1 - 1][i2][i3]);
float temp334 = cos(theta[i1 - 1][i2][i3]);
float temp335 = temp118*v[t1][i1 - 1][i2][i3 - 1];
float temp336 = temp118*v[t1][i1 - 1][i2][i3 + 1];
float temp337 = sin(phi[i1 - 1][i2][i3]);
float temp338 = temp331*temp337;
float temp342 = temp118*v[t1][i1 - 1][i2 - 1][i3];
float temp344 = temp118*v[t1][i1 - 1][i2 + 1][i3];
float temp347 = temp331*temp333;
float temp348 = -4.0F/3.0F*temp65;
float temp349 = (7.0F/12.0F)*temp72;
float temp350 = 2.0e-1F*temp12;
float temp352 = sin(theta[i1 + 1][i2][i3]);
float temp354 = cos(phi[i1 + 1][i2][i3]);
float temp355 = cos(theta[i1 + 1][i2][i3]);

```

```

Oct 25, 16 15:19 tti_3d_so6.txt Page 9/116
float temp356 = temp118*v[t1][i1 + 1][i2][i3 - 1];
float temp357 = -temp118*v[t1][i1 + 1][i2][i3 + 1];
float temp358 = sin(phi[i1 + 1][i2][i3]);
float temp359 = temp352*temp358;
float temp363 = temp118*v[t1][i1 + 1][i2 - 1][i3];
float temp365 = -temp118*v[t1][i1 + 1][i2 + 1][i3];
float temp368 = temp352*temp354;
float temp369 = (2.0F/5.0F)*temp65;
float temp370 = (7.0F/12.0F)*temp75;
float temp371 = temp12*v[t1][i1 + 4][i2][i3];
float temp374 = temp21*v[t1][i1 + 5][i2][i3];
float temp375 = 1.66666666666667e-2F*temp12;
float temp377 = sin(theta[i1 + 2][i2][i3]);
float temp379 = cos(phi[i1 + 2][i2][i3]);
float temp380 = cos(theta[i1 + 2][i2][i3]);
float temp381 = temp116*v[t1][i1 + 2][i2][i3 - 2];
float temp382 = temp116*v[t1][i1 + 2][i2][i3 + 2];
float temp383 = sin(phi[i1 + 2][i2][i3]);
float temp384 = temp377*temp383;
float temp387 = temp116*v[t1][i1 + 2][i2 - 2][i3];
float temp391 = temp116*v[t1][i1 + 2][i2 + 2][i3];
float temp392 = temp377*temp379;
float temp394 = -1.0F/30.0F*temp65;
float temp395 = (7.0F/12.0F)*temp78;
float temp396 = temp12*v[t1][i1 + 5][i2][i3];
float temp398 = temp21*v[t1][i1 + 6][i2][i3];
float temp400 = sin(phi[i1][i2 - 1][i3]);
float temp402 = sin(theta[i1][i2 - 1][i3]);
float temp403 = temp400*temp402;
float temp406 = temp12*v[t1][i1][i2 - 3][i3];
float temp407 = (3.0F/4.0F)*temp48;

```


Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 19/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 20/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 21/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 22/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 23/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 24/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 25/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 26/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

Table with 3 columns: Date/Time (Oct 25, 16 15:19), File Name (tti_3d_so6.txt), Page (Page 27/116). The content is a large block of assembly-like code with many mathematical expressions involving variables like temp51 through temp530.

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```

float temp9 = temp1 - temp3;
float temp10 = 1.56956521739130F;
float temp12 = 5.00000000000000e-2F;
float temp13 = 2.91666666666667e-1F*temp12;
float temp16 = sin(theta[i1][i2][i3]);
float temp19 = cos(phi[i1][i2][i3]);
float temp20 = cos(theta[i1][i2][i3]);
float temp21 = (1.0F/60.0F)*temp12;
float temp27 = temp12*v[t1][i1][i2][i3 - 2];
float temp30 = temp12*v[t1][i1][i2][i3 - 1];
float temp31 = (3.0F/4.0F)*temp30;
float temp34 = temp12*v[t1][i1][i2][i3 + 1];
float temp35 = (3.0F/4.0F)*temp34;
float temp38 = temp12*v[t1][i1][i2][i3 + 2];
float temp41 = temp21*v[t1][i1][i2][i3 + 3];
float temp42 = sin(phi[i1][i2][i3]);
float temp43 = temp16*temp42;
float temp48 = temp12*v[t1][i1][i2 - 2][i3];
float temp51 = temp12*v[t1][i1][i2 - 1][i3];
float temp52 = (3.0F/4.0F)*temp51;
float temp55 = temp12*v[t1][i1][i2 + 1][i3];
float temp56 = (3.0F/4.0F)*temp55;
float temp59 = temp12*v[t1][i1][i2 + 2][i3];
float temp62 = temp21*v[t1][i1][i2 + 3][i3];
float temp63 = temp16*temp19;
float temp65 = temp12*v[t1][i1][i2][i3];
float temp66 = (7.0F/12.0F)*temp65;
float temp67 = (1.0F/30.0F)*temp12;
float temp72 = temp12*v[t1][i1 - 1][i2][i3];
float temp75 = temp12*v[t1][i1 + 1][i2][i3];
float temp78 = temp12*v[t1][i1 + 2][i2][i3];

```

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```

float temp81 = temp12*v[t1][i1 + 3][i2][i3];
float temp84 = temp21*v[t1][i1 + 4][i2][i3];
float temp87 = temp12*v[t1][i1 - 2][i2][i3];
float temp88 = (3.0F/4.0F)*temp72;
float temp89 = (3.0F/4.0F)*temp75;
float temp90 = temp21*v[t1][i1 + 3][i2][i3];
float temp91 = temp12*v[t1][i1][i2][i3 + 3];
float temp94 = temp21*v[t1][i1][i2][i3 + 4];
float temp95 = temp12*v[t1][i1][i2 + 3][i3];
float temp98 = temp21*v[t1][i1][i2 + 4][i3];
float temp99 = 2.91666666666667e-1F*temp12*(temp20*((2.0F/5.0F)*temp30 - 4.0F/3.0F)*temp34 + (1.0F/2.0F)*temp38 + temp66 - temp67*v[t1][i1][i2][i3 - 2] - 2.0F/15.0F*temp91 + temp94) + temp43*((2.0F/5.0F)*temp51 - 4.0F/3.0F*temp55 + (1.0F/2.0F)*temp59 + temp66 - temp67*v[t1][i1][i2 - 2][i3] - 2.0F/15.0F*temp95 + temp98) + temp63*(temp21*v[t1][i1 - 3][i2][i3] + (3.0F/20.0F)*temp78 - 3.0F/20.0F*temp87 + temp88 - temp89 - temp90);
float temp100 = 3.75e-1F*temp12;
float temp102 = cos(theta[i1][i2][i3 - 1]);
float temp103 = (3.0F/4.0F)*temp65;
float temp104 = -temp103;
float temp107 = temp12*v[t1][i1][i2][i3 - 3];
float temp108 = (3.0F/4.0F)*temp27;
float temp109 = (3.0F/20.0F)*temp34;
float temp110 = temp21*v[t1][i1][i2][i3 + 2];
float temp112 = sin(phi[i1][i2][i3 - 1]);
float temp113 = sin(theta[i1][i2][i3 - 1]);
float temp114 = temp112*temp113;
float temp116 = (3.0F/20.0F)*temp12;
float temp118 = (3.0F/4.0F)*temp12;
float temp120 = temp118*v[t1][i1][i2 - 1][i3 - 1];
float temp122 = temp118*v[t1][i1][i2 + 1][i3 - 1];

```

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```

float temp125 = cos(phi[i1][i2][i3 - 1]);
float temp126 = temp113*temp125;
float temp127 = (7.0F/12.0F)*temp30;
float temp129 = (2.0F/5.0F)*temp12;
float temp131 = temp129*v[t1][i1 - 1][i2][i3 - 1];
float temp132 = (4.0F/3.0F)*temp12;
float temp134 = (1.0F/2.0F)*temp12;
float temp136 = (2.0F/15.0F)*temp12;
float temp138 = 7.5e-2F*temp12;
float temp140 = cos(theta[i1][i2][i3 + 2]);
float temp141 = (3.0F/20.0F)*temp65;
float temp142 = -temp141;
float temp143 = temp12*v[t1][i1][i2][i3 + 4];
float temp146 = temp21*v[t1][i1][i2][i3 + 5];
float temp148 = sin(phi[i1][i2][i3 + 2]);
float temp149 = sin(theta[i1][i2][i3 + 2]);
float temp150 = temp148*temp149;
float temp153 = temp116*v[t1][i1][i2 - 2][i3 + 2];
float temp157 = temp116*v[t1][i1][i2 + 2][i3 + 2];
float temp159 = cos(phi[i1][i2][i3 + 2]);
float temp160 = temp149*temp159;
float temp161 = (7.0F/12.0F)*temp38;
float temp166 = temp134*v[t1][i1 + 2][i2][i3 + 2];
float temp168 = 8.33333333333333e-3F*temp12;
float temp170 = cos(theta[i1][i2][i3 - 3]);
float temp171 = (1.0F/60.0F)*temp65;
float temp172 = -temp171;
float temp177 = temp12*v[t1][i1][i2][i3 - 5];
float temp178 = temp12*v[t1][i1][i2][i3 - 4];
float temp179 = (3.0F/20.0F)*temp30;
float temp181 = sin(phi[i1][i2][i3 - 3]);

```

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```

float temp182 = sin(theta[i1][i2][i3 - 3]);
float temp183 = temp181*temp182;
float temp184 = temp21*v[t1][i1][i2 - 3][i3 - 3];
float temp190 = temp21*v[t1][i1][i2 + 3][i3 - 3];
float temp191 = cos(phi[i1][i2][i3 - 3]);
float temp192 = temp182*temp191;
float temp193 = (7.0F/12.0F)*temp107;
float temp200 = cos(theta[i1][i2][i3 + 3]);
float temp201 = (3.0F/4.0F)*temp38;
float temp202 = temp12*v[t1][i1][i2][i3 + 5];
float temp204 = temp21*v[t1][i1][i2][i3 + 6];
float temp206 = sin(phi[i1][i2][i3 + 3]);
float temp207 = sin(theta[i1][i2][i3 + 3]);
float temp209 = temp21*v[t1][i1][i2 - 3][i3 + 3];
float temp214 = -temp21*v[t1][i1][i2 + 3][i3 + 3];
float temp215 = cos(phi[i1][i2][i3 + 3]);
float temp220 = -temp136*v[t1][i1 + 3][i2][i3 + 3];
float temp222 = cos(theta[i1][i2][i3 - 2]);
float temp223 = temp21*v[t1][i1][i2][i3 + 1];
float temp225 = sin(phi[i1][i2][i3 - 2]);
float temp226 = sin(theta[i1][i2][i3 - 2]);
float temp227 = temp225*temp226;
float temp230 = -temp116*v[t1][i1][i2 - 2][i3 - 2];
float temp234 = temp116*v[t1][i1][i2 + 2][i3 - 2];
float temp236 = cos(phi[i1][i2][i3 - 2]);
float temp237 = temp226*temp236;
float temp238 = (7.0F/12.0F)*temp27;
float temp240 = -temp67*v[t1][i1 - 2][i2][i3 - 2];
float temp246 = cos(theta[i1][i2][i3 + 1]);
float temp248 = sin(phi[i1][i2][i3 + 1]);
float temp249 = sin(theta[i1][i2][i3 + 1]);

```

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```

float temp250 = temp248*temp249;
float temp254 = temp118*v[t1][i1][i2 - 1][i3 + 1];
float temp256 = -temp118*v[t1][i1][i2 + 1][i3 + 1];
float temp259 = cos(phi[i1][i2][i3 + 1]);
float temp260 = temp249*temp259;
float temp261 = (7.0F/12.0F)*temp34;
float temp265 = -temp132*v[t1][i1 + 1][i2][i3 + 1];
float temp270 = sin(theta[i1 - 4][i2][i3]);
float temp272 = cos(phi[i1 - 4][i2][i3]);
float temp273 = cos(theta[i1 - 4][i2][i3]);
float temp274 = sin(phi[i1 - 4][i2][i3]);
float temp279 = temp12*v[t1][i1 - 5][i2][i3];
float temp281 = temp12*v[t1][i1 - 4][i2][i3];
float temp282 = temp12*v[t1][i1 - 3][i2][i3];
float temp283 = 6.66666666666667e-2F*temp12;
float temp285 = sin(theta[i1 - 3][i2][i3]);
float temp287 = cos(phi[i1 - 3][i2][i3]);
float temp288 = cos(theta[i1 - 3][i2][i3]);
float temp289 = temp21*v[t1][i1 - 3][i2][i3 - 3];
float temp295 = sin(phi[i1 - 3][i2][i3]);
float temp296 = temp285*temp295;
float temp297 = temp21*v[t1][i1 - 3][i2 - 3][i3];
float temp303 = temp285*temp287;
float temp304 = -2.0F/15.0F*temp65;
float temp305 = (7.0F/12.0F)*temp282;
float temp306 = temp21*v[t1][i1 + 1][i2][i3];
float temp307 = 2.5e-1F*temp12;
float temp309 = sin(theta[i1 - 2][i2][i3]);
float temp311 = cos(phi[i1 - 2][i2][i3]);
float temp312 = cos(theta[i1 - 2][i2][i3]);
float temp313 = -temp116*v[t1][i1 - 2][i2][i3 - 2];

```

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```

float temp314 = temp116*v[t1][i1 - 2][i2][i3 + 2];
float temp315 = sin(phi[i1 - 2][i2][i3]);
float temp316 = temp309*temp315;
float temp319 = -temp116*v[t1][i1 - 2][i2 - 2][i3];
float temp323 = temp116*v[t1][i1 - 2][i2 + 2][i3];
float temp325 = temp309*temp311;
float temp326 = (1.0F/2.0F)*temp65;
float temp327 = (7.0F/12.0F)*temp87;
float temp328 = temp21*v[t1][i1 + 2][i2][i3];
float temp329 = 6.66666666666667e-1F*temp12;
float temp331 = sin(theta[i1 - 1][i2][i3]);
float temp333 = cos(phi[i1 - 1][i2][i3]);
float temp334 = cos(theta[i1 - 1][i2][i3]);
float temp335 = temp118*v[t1][i1 - 1][i2][i3 - 1];
float temp336 = temp118*v[t1][i1 - 1][i2][i3 + 1];
float temp337 = sin(phi[i1 - 1][i2][i3]);
float temp338 = temp331*temp337;
float temp342 = temp118*v[t1][i1 - 1][i2 - 1][i3];
float temp344 = temp118*v[t1][i1 - 1][i2 + 1][i3];
float temp347 = temp331*temp333;
float temp348 = -4.0F/3.0F*temp65;
float temp349 = (7.0F/12.0F)*temp72;
float temp350 = 2.0e-1F*temp12;
float temp352 = sin(theta[i1 + 1][i2][i3]);
float temp354 = cos(phi[i1 + 1][i2][i3]);
float temp355 = cos(theta[i1 + 1][i2][i3]);
float temp356 = temp118*v[t1][i1 + 1][i2][i3 - 1];
float temp357 = -temp118*v[t1][i1 + 1][i2][i3 + 1];
float temp358 = sin(phi[i1 + 1][i2][i3]);
float temp359 = temp352*temp358;
float temp363 = temp118*v[t1][i1 + 1][i2 - 1][i3];

```

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```

float temp365 = -temp118*v[t1][i1 + 1][i2 + 1][i3];
float temp368 = temp352*temp354;
float temp369 = (2.0F/5.0F)*temp65;
float temp370 = (7.0F/12.0F)*temp75;
float temp371 = temp12*v[t1][i1 + 4][i2][i3];
float temp374 = temp21*v[t1][i1 + 5][i2][i3];
float temp375 = 1.66666666666667e-2F*temp12;
float temp377 = sin(theta[i1 + 2][i2][i3]);
float temp379 = cos(phi[i1 + 2][i2][i3]);
float temp380 = cos(theta[i1 + 2][i2][i3]);
float temp381 = temp116*v[t1][i1 + 2][i2][i3 - 2];
float temp382 = temp116*v[t1][i1 + 2][i2][i3 + 2];
float temp383 = sin(phi[i1 + 2][i2][i3]);
float temp384 = temp377*temp383;
float temp387 = temp116*v[t1][i1 + 2][i2 - 2][i3];
float temp391 = temp116*v[t1][i1 + 2][i2 + 2][i3];
float temp393 = temp377*temp379;
float temp394 = -1.0F/30.0F*temp65;
float temp395 = (7.0F/12.0F)*temp78;
float temp396 = temp12*v[t1][i1 + 5][i2][i3];
float temp398 = temp21*v[t1][i1 + 6][i2][i3];
float temp400 = sin(phi[i1][i2 - 1][i3]);
float temp402 = sin(theta[i1][i2 - 1][i3]);
float temp403 = temp400*temp402;
float temp406 = temp12*v[t1][i1][i2 - 3][i3];
float temp407 = (3.0F/4.0F)*temp48;
float temp408 = (3.0F/20.0F)*temp55;
float temp409 = temp21*v[t1][i1][i2 + 2][i3];
float temp410 = cos(theta[i1][i2 - 1][i3]);
float temp411 = cos(phi[i1][i2 - 1][i3]);
float temp412 = temp402*temp411;

```

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```

float temp413 = (7.0F/12.0F)*temp51;
float temp414 = temp129*v[t1][i1 - 1][i2 - 1][i3];
float temp417 = sin(phi[i1][i2 + 2][i3]);
float temp419 = sin(theta[i1][i2 + 2][i3]);
float temp420 = temp417*temp419;
float temp421 = temp12*v[t1][i1][i2 + 4][i3];
float temp424 = temp21*v[t1][i1][i2 + 5][i3];
float temp425 = cos(theta[i1][i2 + 2][i3]);
float temp426 = cos(phi[i1][i2 + 2][i3]);
float temp427 = temp419*temp426;
float temp428 = (7.0F/12.0F)*temp59;
float temp429 = temp134*v[t1][i1 + 2][i2 + 2][i3];
float temp432 = sin(phi[i1][i2 - 3][i3]);
float temp434 = sin(theta[i1][i2 - 3][i3]);
float temp435 = temp432*temp434;
float temp440 = temp12*v[t1][i1][i2 - 5][i3];
float temp441 = temp12*v[t1][i1][i2 - 4][i3];
float temp442 = (3.0F/20.0F)*temp51;
float temp443 = cos(theta[i1][i2 - 3][i3]);
float temp444 = cos(phi[i1][i2 - 3][i3]);
float temp445 = temp434*temp444;
float temp446 = (7.0F/12.0F)*temp406;
float temp449 = sin(phi[i1][i2 + 3][i3]);
float temp451 = sin(theta[i1][i2 + 3][i3]);
float temp452 = (3.0F/4.0F)*temp59;
float temp453 = temp12*v[t1][i1][i2 + 5][i3];
float temp455 = temp21*v[t1][i1][i2 + 6][i3];
float temp456 = cos(theta[i1][i2 + 3][i3]);
float temp457 = cos(phi[i1][i2 + 3][i3]);
float temp458 = -temp136*v[t1][i1 + 3][i2 + 3][i3];
float temp460 = sin(phi[i1][i2 - 2][i3]);

```

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```

float temp462 = sin(theta[i1][i2 - 2][i3]);
float temp463 = temp460*temp462;
float temp464 = temp21*v[t1][i1][i2 + 1][i3];
float temp465 = cos(theta[i1][i2 - 2][i3]);
float temp466 = cos(phi[i1][i2 - 2][i3]);
float temp467 = temp462*temp466;
float temp468 = (7.0F/12.0F)*temp48;
float temp469 = -temp67*v[t1][i1 - 2][i2 - 2][i3];
float temp472 = sin(phi[i1][i2 + 1][i3]);
float temp474 = sin(theta[i1][i2 + 1][i3]);
float temp475 = temp472*temp474;
float temp476 = cos(theta[i1][i2 + 1][i3]);
float temp477 = cos(phi[i1][i2 + 1][i3]);
float temp478 = temp474*temp477;
float temp479 = (7.0F/12.0F)*temp55;
float temp480 = -temp132*v[t1][i1 + 1][i2 + 1][i3];
float temp483 = cos(theta[i1][i2][i3 - 4]);
float temp484 = (7.0F/12.0F)*temp178;
float temp485 = sin(theta[i1][i2][i3 - 4]);
float temp487 = cos(phi[i1][i2][i3 - 4]);
float temp488 = sin(phi[i1][i2][i3 - 4]);
float temp489 = -temp67*v[t1][i1][i2 - 2][i3 - 2];
float temp490 = temp129*v[t1][i1][i2 - 1][i3 - 1];
float temp491 = -temp132*v[t1][i1][i2 + 1][i3 + 1];
float temp492 = temp134*v[t1][i1][i2 + 2][i3 + 2];
float temp493 = (3.0F/4.0F)*temp87;
float temp494 = (3.0F/20.0F)*temp75;
float temp495 = (3.0F/20.0F)*temp72;
float temp497 = sin(theta[i1 + 3][i2][i3]);
float temp499 = cos(phi[i1 + 3][i2][i3]);
float temp500 = (3.0F/4.0F)*temp78;

```


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float temp543 = (3.0F/4.0F)*temp532;
float temp544 = temp21*u[t1][i1 + 3][i2][i3];
float temp545 = temp21*u[t1][i1 - 3][i2][i3] + (3.0F/20.0F)*temp53
4 - 3.0F/20.0F*temp541 + temp542 - temp543 - temp544;
float temp546 = temp12*u[t1][i1][i2 + 3][i3];
float temp548 = temp21*u[t1][i1][i2 + 4][i3];
float temp549 = (2.0F/5.0F)*temp516 - 4.0F/3.0F*temp519 + (1.0F/2.
0F)*temp522 + temp527 - 2.0F/15.0F*temp546 + temp548 - temp67*u[t1][i1][i2 - 2][
i3];
float temp550 = (3.0F/4.0F)*temp526;
float temp551 = -temp550;
float temp552 = temp12*u[t1][i1][i2 - 3][i3];
float temp554 = (3.0F/4.0F)*temp514;
float temp555 = (3.0F/20.0F)*temp519;
float temp556 = temp21*u[t1][i1][i2 + 2][i3];
float temp557 = temp21*u[t1][i1][i2 - 4][i3] + temp551 - 3.0F/20.0
F*temp553 + temp554 + temp555 - temp556;
float temp558 = (7.0F/12.0F)*temp516;
float temp561 = temp129*u[t1][i1 - 1][i2 - 1][i3];
float temp565 = -temp132*u[t1][i1 + 1][i2 - 1][i3] + temp134*u[t1]
[i1 + 2][i2 - 1][i3] - temp136*u[t1][i1 + 3][i2 - 1][i3] + temp21*u[t1][i1 + 4][
i2 - 1][i3] + temp558 + temp561 - temp67*u[t1][i1 - 2][i2 - 1][i3];
float temp567 = temp12*u[t1][i1 - 3][i2][i3];
float temp568 = (3.0F/4.0F)*temp541;
float temp569 = (3.0F/20.0F)*temp532;
float temp570 = temp21*u[t1][i1 + 2][i2][i3];
float temp571 = temp21*u[t1][i1 - 4][i2][i3] + temp551 - 3.0F/20.0
F*temp567 + temp568 + temp569 - temp570;
float temp572 = (7.0F/12.0F)*temp530;
float temp577 = -temp132*u[t1][i1 - 1][i2 + 1][i3] + temp134*u[t1]
[i1 - 1][i2 + 2][i3] - temp136*u[t1][i1 - 1][i2 + 3][i3] + temp21*u[t1][i1 - 1][

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i2 + 4][i3] + temp561 + temp572 - temp67*u[t1][i1 - 1][i2 - 2][i3];
float temp578 = (3.0F/20.0F)*temp526;
float temp579 = -temp578;
float temp580 = temp12*u[t1][i1][i2 + 4][i3];
float temp582 = temp21*u[t1][i1][i2 + 5][i3];
float temp583 = temp21*u[t1][i1][i2 - 1][i3] + temp520 - 3.0F/4.0F
*temp546 + temp579 + (3.0F/20.0F)*temp580 - temp582;
float temp584 = (7.0F/12.0F)*temp522;
float temp588 = temp134*u[t1][i1 + 2][i2 + 2][i3];
float temp590 = temp129*u[t1][i1 - 1][i2 + 2][i3] - temp132*u[t1][
i1 + 1][i2 + 2][i3] - temp136*u[t1][i1 + 3][i2 + 2][i3] + temp21*u[t1][i1 + 4][i
2 + 2][i3] + temp584 + temp588 - temp67*u[t1][i1 - 2][i2 + 2][i3];
float temp591 = temp12*u[t1][i1 + 4][i2][i3];
float temp593 = temp21*u[t1][i1 + 5][i2][i3];
float temp594 = temp21*u[t1][i1 - 1][i2][i3] - 3.0F/4.0F*temp536 +
temp543 + temp579 + (3.0F/20.0F)*temp591 - temp593;
float temp595 = (7.0F/12.0F)*temp534;
float temp599 = temp129*u[t1][i1 + 2][i2 - 1][i3] - temp132*u[t1][
i1 + 2][i2 + 1][i3] - temp136*u[t1][i1 + 2][i2 + 3][i3] + temp21*u[t1][i1 + 2][i
2 + 4][i3] + temp588 + temp595 - temp67*u[t1][i1 + 2][i2 - 2][i3];
float temp600 = (1.0F/60.0F)*temp526;
float temp601 = -temp600;
float temp604 = temp12*u[t1][i1][i2 - 5][i3];
float temp605 = temp12*u[t1][i1][i2 - 4][i3];
float temp606 = (3.0F/20.0F)*temp516;
float temp607 = temp21*u[t1][i1][i2 - 6][i3] - temp554 + temp601 -
3.0F/20.0F*temp604 + (3.0F/4.0F)*temp605 + temp606;
float temp608 = (7.0F/12.0F)*temp553;
float temp614 = temp129*u[t1][i1 - 1][i2 - 3][i3] - temp132*u[t1][
i1 + 1][i2 - 3][i3] + temp134*u[t1][i1 + 2][i2 - 3][i3] - temp136*u[t1][i1 + 3][
i2 - 3][i3] + temp21*u[t1][i1 + 4][i2 - 3][i3] + temp608 - temp67*u[t1][i1 - 2][

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i2 - 3][i3];
float temp617 = temp12*u[t1][i1 - 5][i2][i3];
float temp618 = temp12*u[t1][i1 - 4][i2][i3];
float temp619 = (3.0F/20.0F)*temp530;
float temp620 = temp21*u[t1][i1 - 6][i2][i3] - temp568 + temp601 -
3.0F/20.0F*temp617 + (3.0F/4.0F)*temp618 + temp619;
float temp621 = (7.0F/12.0F)*temp567;
float temp627 = temp129*u[t1][i1 - 3][i2 - 1][i3] - temp132*u[t1][
i1 - 3][i2 + 1][i3] + temp134*u[t1][i1 - 3][i2 + 2][i3] - temp136*u[t1][i1 - 3][
i2 + 3][i3] + temp21*u[t1][i1 - 3][i2 + 4][i3] + temp621 - temp67*u[t1][i1 - 3][
i2 - 2][i3];
float temp628 = (3.0F/4.0F)*temp522;
float temp629 = temp12*u[t1][i1][i2 + 5][i3];
float temp630 = temp21*u[t1][i1][i2 + 6][i3];
float temp631 = -temp555 - 3.0F/4.0F*temp580 + temp600 + temp628 +
(3.0F/20.0F)*temp629 - temp630;
float temp634 = -temp136*u[t1][i1 + 3][i2 + 3][i3];
float temp635 = temp129*u[t1][i1 - 1][i2 + 3][i3] - temp132*u[t1][
i1 + 1][i2 + 3][i3] + temp134*u[t1][i1 + 2][i2 + 3][i3] + temp21*u[t1][i1 + 4][i
2 + 3][i3] + (7.0F/12.0F)*temp546 + temp634 - temp67*u[t1][i1 - 2][i2 + 3][i3];
float temp636 = (3.0F/4.0F)*temp534;
float temp637 = temp12*u[t1][i1 + 5][i2][i3];
float temp638 = temp21*u[t1][i1 + 6][i2][i3];
float temp639 = -temp569 - 3.0F/4.0F*temp591 + temp600 + temp636 +
(3.0F/20.0F)*temp637 - temp638;
float temp640 = (7.0F/12.0F)*temp536;
float temp643 = temp129*u[t1][i1 + 3][i2 - 1][i3] - temp132*u[t1][
i1 + 3][i2 + 1][i3] + temp134*u[t1][i1 + 3][i2 + 2][i3] + temp21*u[t1][i1 + 3][i
2 + 4][i3] + temp634 + temp640 - temp67*u[t1][i1 + 3][i2 - 2][i3];
float temp644 = temp21*u[t1][i1][i2 + 1][i3];
float temp645 = temp21*u[t1][i1][i2 - 5][i3] - temp517 + (3.0F/4.0

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F)*temp553 + temp578 - 3.0F/20.0F*temp605 - temp644;
float temp646 = (7.0F/12.0F)*temp514;
float temp648 = -temp67*u[t1][i1 - 2][i2 - 2][i3];
float temp650 = temp129*u[t1][i1 - 1][i2 - 2][i3] - temp132*u[t1][
i1 + 1][i2 - 2][i3] + temp134*u[t1][i1 + 2][i2 - 2][i3] - temp136*u[t1][i1 + 3][
i2 - 2][i3] + temp21*u[t1][i1 + 4][i2 - 2][i3] + temp646 + temp648;
float temp651 = temp21*u[t1][i1 + 1][i2][i3];
float temp652 = temp21*u[t1][i1 - 5][i2][i3] - temp542 + (3.0F/4.0
F)*temp567 + temp578 - 3.0F/20.0F*temp618 - temp651;
float temp653 = (7.0F/12.0F)*temp541;
float temp655 = temp129*u[t1][i1 - 2][i2 - 1][i3] - temp132*u[t1][
i1 - 2][i2 + 1][i3] + temp134*u[t1][i1 - 2][i2 + 2][i3] - temp136*u[t1][i1 - 2][
i2 + 3][i3] + temp21*u[t1][i1 - 2][i2 + 4][i3] + temp648 + temp653;
float temp656 = temp21*u[t1][i1][i2 - 2][i3] + (3.0F/20.0F)*temp54
6 - temp548 + temp550 - temp606 - temp628;
float temp657 = (7.0F/12.0F)*temp519;
float temp659 = -temp132*u[t1][i1 + 1][i2 + 1][i3];
float temp660 = temp129*u[t1][i1 - 1][i2 + 1][i3] + temp134*u[t1][
i1 + 2][i2 + 1][i3] - temp136*u[t1][i1 + 3][i2 + 1][i3] + temp21*u[t1][i1 + 4][i
2 + 1][i3] + temp657 + temp659 - temp67*u[t1][i1 - 2][i2 + 1][i3];
float temp661 = temp21*u[t1][i1 - 2][i2][i3] + (3.0F/20.0F)*temp53
6 - temp538 + temp550 - temp619 - temp636;
float temp662 = (7.0F/12.0F)*temp532;
float temp663 = temp129*u[t1][i1 + 1][i2 - 1][i3] + temp134*u[t1][
i1 + 1][i2 + 2][i3] - temp136*u[t1][i1 + 1][i2 + 3][i3] + temp21*u[t1][i1 + 1][i
2 + 4][i3] + temp659 + temp662 - temp67*u[t1][i1 + 1][i2 - 2][i3];
float temp664 = -temp116*u[t1][i1 - 4][i2 - 2][i3] + temp116*u[t1]
[i1 - 4][i2 + 2][i3] + temp118*u[t1][i1 - 4][i2 - 1][i3] - temp118*u[t1][i1 - 4]
[i2 + 1][i3] + temp21*u[t1][i1 - 4][i2 - 3][i3] - temp21*u[t1][i1 - 4][i2 + 3][i
3];
float temp665 = -2.0F/15.0F*temp530 + (1.0F/2.0F)*temp541 - 4.0F/3

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.0F*temp567 + temp600 + (2.0F/5.0F)*temp617 + (7.0F/12.0F)*temp618 - temp67*u[t1
][i1 - 6][i2][i3];
float temp666 = -temp116*u[t1][i1 - 2][i2 - 4][i3] + temp116*u[t1]
[i1 + 2][i2 - 4][i3] + temp118*u[t1][i1 - 1][i2 - 4][i3] - temp118*u[t1][i1 + 1]
[i2 - 4][i3] + temp21*u[t1][i1 - 3][i2 - 4][i3] - temp21*u[t1][i1 + 3][i2 - 4][i
3];
float temp667 = (7.0F/12.0F)*temp605;
float temp668 = (1.0F/2.0F)*temp514 - 2.0F/15.0F*temp516 - 4.0F/3.
0F*temp553 + temp600 + (2.0F/5.0F)*temp604 + temp667 - temp67*u[t1][i1][i2 - 6][
i3];
float temp669 = temp21*u[t1][i1 - 3][i2 - 3][i3];
float temp670 = -temp116*u[t1][i1 - 3][i2 - 2][i3] + temp116*u[t1][
i1 - 3][i2 + 2][i3] + temp118*u[t1][i1 - 3][i2 - 1][i3] - temp118*u[t1][i1 - 3]
[i2 + 1][i3] - temp21*u[t1][i1 - 3][i2 + 3][i3] + temp669;
float temp671 = -2.0F/15.0F*temp526;
float temp672 = (1.0F/2.0F)*temp530 - 4.0F/3.0F*temp541 + (2.0F/5.
0F)*temp618 + temp621 + temp651 - temp67*u[t1][i1 - 5][i2][i3] + temp671;
float temp673 = -temp116*u[t1][i1 + 2][i2 - 3][i3] + temp116*u[t1]
[i1 + 2][i2 - 3][i3] + temp118*u[t1][i1 - 1][i2 - 3][i3] - temp118*u[t1][i1 + 1]
[i2 - 3][i3] - temp21*u[t1][i1 + 3][i2 - 3][i3] + temp669;
float temp674 = -4.0F/3.0F*temp514 + (1.0F/2.0F)*temp516 + (2.0F/5
.0F)*temp605 + temp608 + temp644 - temp67*u[t1][i1][i2 - 5][i3] + temp671;
float temp675 = -temp116*u[t1][i1 - 2][i2 - 2][i3];
float temp676 = temp116*u[t1][i1 + 2][i2 - 2][i3];
float temp677 = temp118*u[t1][i1 - 1][i2 - 2][i3] - temp118*u[t1][
i1 + 1][i2 - 2][i3] + temp21*u[t1][i1 - 3][i2 - 2][i3] - temp21*u[t1][i1 + 3][i2
- 2][i3] + temp675 + temp676;
float temp678 = (1.0F/2.0F)*temp526;
float temp679 = -4.0F/3.0F*temp516 - 2.0F/15.0F*temp519 + (2.0F/5.
0F)*temp553 + temp556 + temp646 - temp67*u[t1][i1][i2 - 4][i3] + temp678;
float temp680 = temp118*u[t1][i1 - 1][i2 - 1][i3];

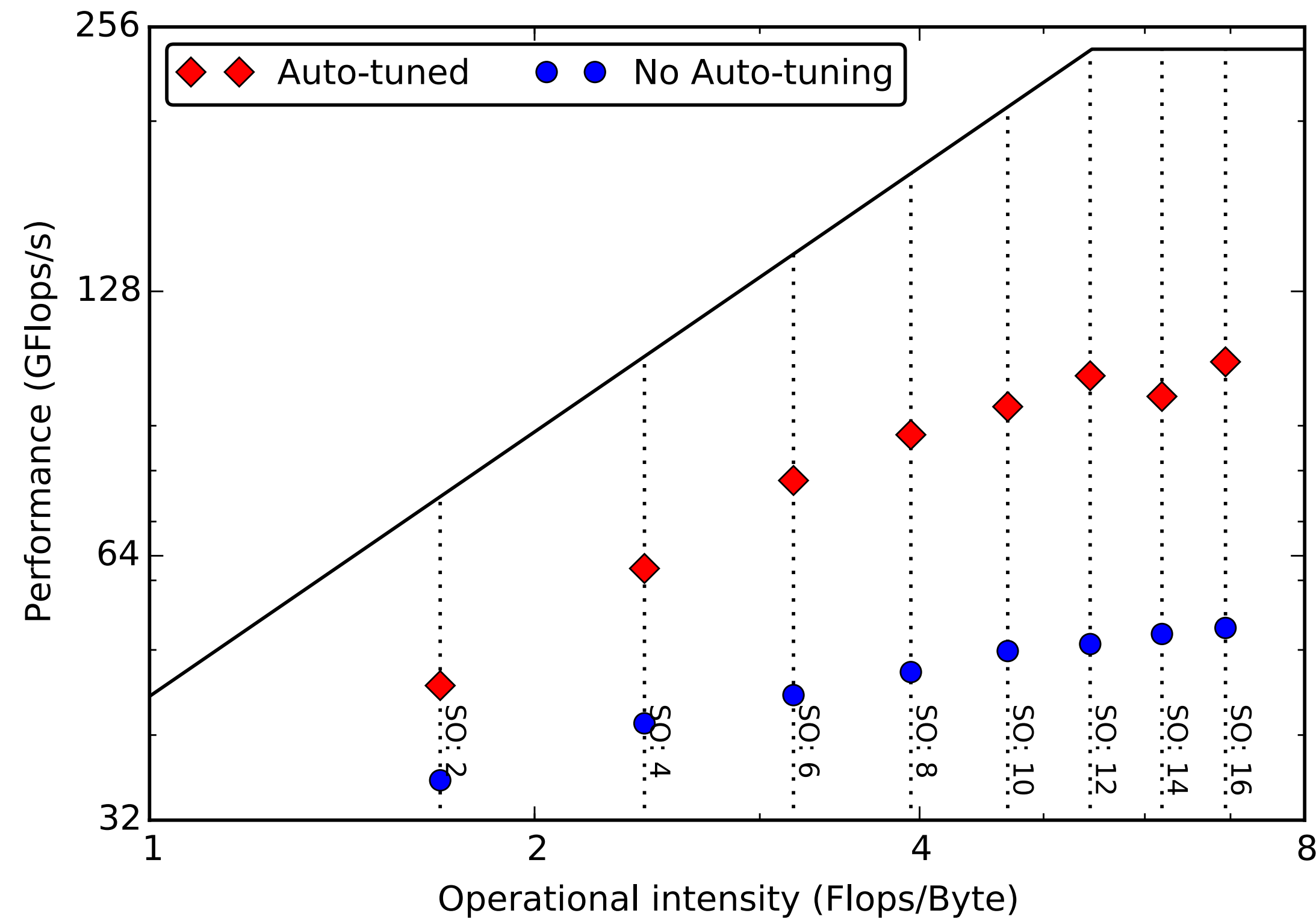
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float temp681 = temp118*u[t1][i1 + 1][i2 - 1][i3];
float temp682 = -temp116*u[t1][i1 - 2][i2 - 1][i3] + temp116*u[t1]
[i1 + 2][i2 - 1][i3] + temp21*u[t1][i1 - 3][i2 - 1][i3] - temp21*u[t1][i1 + 3][i
2 - 1][i3] + temp680 - temp681;
float temp683 = -4.0F/3.0F*temp526;
float temp684 = (2.0F/5.0F)*temp514 + (1.0F/2.0F)*temp519 - 2.0F/1
5.0F*temp522 + temp524 + temp558 - temp67*u[t1][i1][i2 - 3][i3] + temp683;
float temp685 = temp118*u[t1][i1 - 1][i2 + 1][i3];
float temp686 = -temp118*u[t1][i1 + 1][i2 + 1][i3];
float temp687 = -temp116*u[t1][i1 - 2][i2 + 1][i3] + temp116*u[t1]
[i1 + 2][i2 + 1][i3] + temp21*u[t1][i1 - 3][i2 + 1][i3] + temp21*u[t1][i1 + 3][i
2 + 1][i3] + temp685 + temp686;
float temp688 = (2.0F/5.0F)*temp526;
float temp689 = -4.0F/3.0F*temp522 + (1.0F/2.0F)*temp546 - 2.0F/15
.0F*temp580 + temp582 + temp657 - temp67*u[t1][i1][i2 - 1][i3] + temp688;
float temp690 = temp116*u[t1][i1 - 2][i2 + 2][i3];
float temp691 = temp116*u[t1][i1 + 2][i2 + 2][i3];
float temp692 = temp118*u[t1][i1 - 1][i2 + 2][i3] - temp118*u[t1][
i1 + 1][i2 + 2][i3] + temp21*u[t1][i1 - 3][i2 + 2][i3] - temp21*u[t1][i1 + 3][i2
+ 2][i3] - temp690 + temp691;
float temp693 = -1.0F/30.0F*temp526;
float temp694 = (2.0F/5.0F)*temp519 - 4.0F/3.0F*temp546 + (1.0F/2.
0F)*temp580 + temp584 - 2.0F/15.0F*temp629 + temp630 + temp693;
float temp695 = temp118*u[t1][i1 - 2][i2 - 1][i3] - temp118*u[t1][
i1 - 2][i2 + 1][i3] + temp21*u[t1][i1 - 2][i2 - 3][i3] - temp21*u[t1][i1 - 2][i2
+ 3][i3] + temp675 + temp690;
float temp696 = -4.0F/3.0F*temp530 - 2.0F/15.0F*temp532 + (2.0F/5.
0F)*temp567 + temp570 + temp653 - temp67*u[t1][i1 - 4][i2][i3] + temp678;
float temp697 = -temp116*u[t1][i1 - 1][i2 - 2][i3] + temp116*u[t1]
[i1 - 1][i2 + 2][i3] + temp21*u[t1][i1 - 1][i2 - 3][i3] - temp21*u[t1][i1 - 1][i
2 + 3][i3] + temp680 - temp685;

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float temp698 = (1.0F/2.0F)*temp532 - 2.0F/15.0F*temp534 + (2.0F/5
.0F)*temp541 + temp544 + temp572 - temp67*u[t1][i1 - 3][i2][i3] + temp683;
float temp699 = -temp116*u[t1][i1 + 1][i2 - 2][i3] + temp116*u[t1]
[i1 + 1][i2 + 2][i3] + temp21*u[t1][i1 + 1][i2 - 3][i3] - temp21*u[t1][i1 + 1][i
2 + 3][i3] + temp681 + temp686;
float temp700 = -4.0F/3.0F*temp534 + (1.0F/2.0F)*temp536 - 2.0F/15
.0F*temp591 + temp593 + temp662 - temp67*u[t1][i1 - 1][i2][i3] + temp688;
float temp701 = temp118*u[t1][i1 + 2][i2 - 1][i3] - temp118*u[t1][
i1 + 2][i2 + 1][i3] + temp21*u[t1][i1 + 2][i2 - 3][i3] - temp21*u[t1][i1 + 2][i2
+ 3][i3] - temp676 + temp691;
float temp702 = (2.0F/5.0F)*temp532 - 4.0F/3.0F*temp536 + (1.0F/2.
0F)*temp591 + temp595 - 2.0F/15.0F*temp637 + temp638 + temp693;
float temp703 = temp12*u[t1][i1][i2][i3 - 3];
float temp704 = temp12*u[t1][i1][i2][i3 - 2];
float temp705 = temp12*u[t1][i1][i2][i3 - 1];
float temp706 = (3.0F/4.0F)*temp705;
float temp707 = temp12*u[t1][i1][i2][i3 + 1];
float temp708 = (3.0F/4.0F)*temp707;
float temp709 = temp12*u[t1][i1][i2][i3 + 2];
float temp710 = temp12*u[t1][i1][i2][i3 + 3];
float temp711 = (1.0F/60.0F)*temp710;
float temp712 = temp20*temp42;
float temp713 = temp19*temp20;
float temp714 = temp12*u[t1][i1][i2][i3 + 4];
float temp715 = (1.0F/60.0F)*temp714;
float temp716 = 2.91666666666667e-1F*temp12*(-temp16*(temp527 - 1.
0F/30.0F*temp704 + (2.0F/5.0F)*temp705 - 4.0F/3.0F*temp707 + (1.0F/2.0F)*temp709
- 2.0F/15.0F*temp710 + temp715) + temp545*temp713 + temp549*temp712);
float temp717 = temp12*u[t1][i1][i2][i3 - 4];
float temp718 = (3.0F/4.0F)*temp704;
float temp719 = (3.0F/20.0F)*temp707;

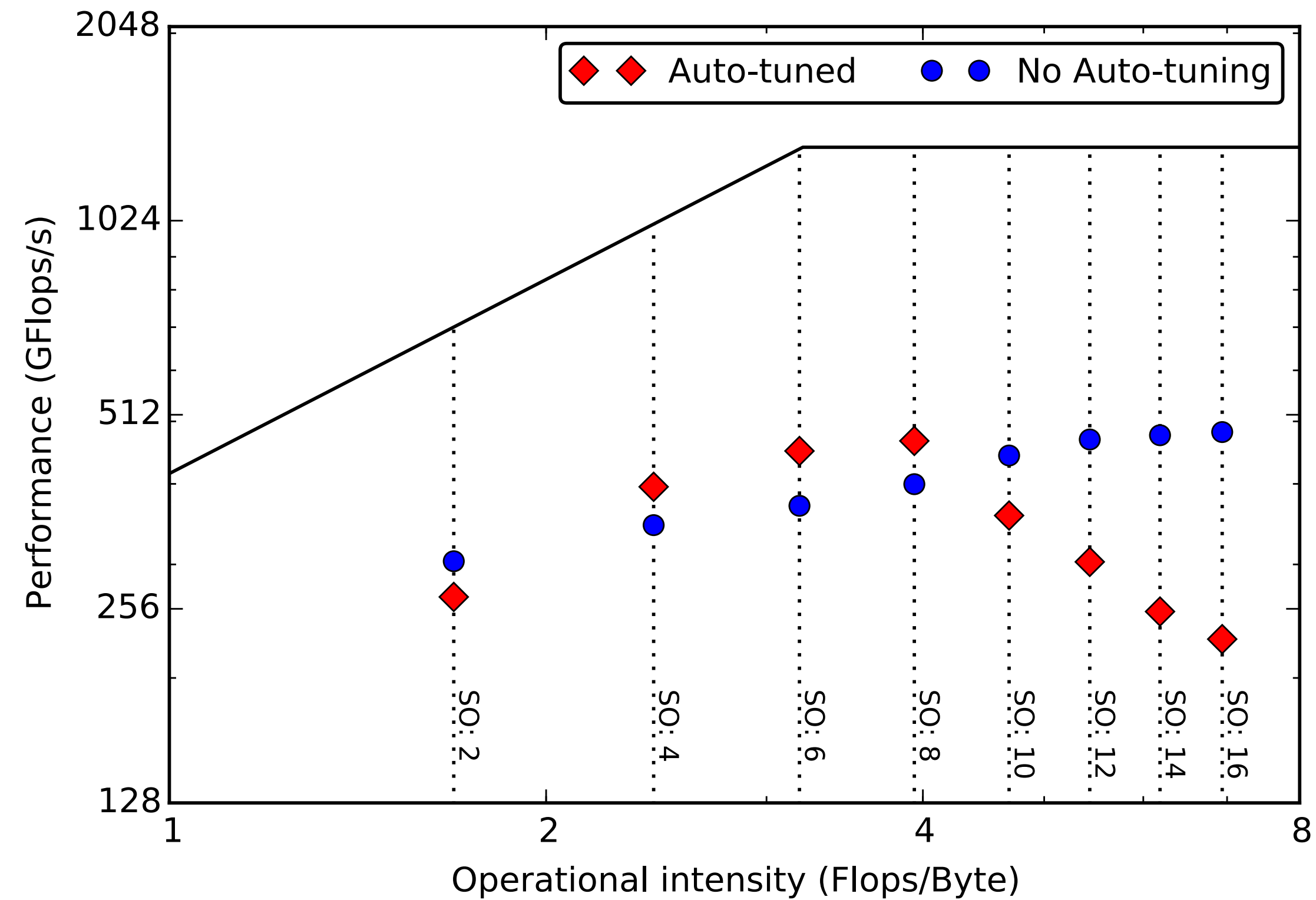
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float temp720 = (1.0F/60.0F)*temp709;
float temp721 = temp102*temp112;
float temp725 = temp118*u[t1][i1][i2 - 1][i3 - 1];
float temp727 = temp118*u[t1][i1][i2 + 1][i3 - 1];
float temp730 = temp102*temp125;
float temp731 = (7.0F/12.0F)*temp705;
float temp734 = temp129*u[t1][i1 - 1][i2][i3 - 1];
float temp738 = temp12*u[t1][i1][i2][i3 + 5];
float temp739 = (1.0F/60.0F)*temp738;
float temp740 = temp140*temp148;
float temp743 = temp116*u[t1][i1][i2 - 2][i3 + 2];
float temp747 = temp116*u[t1][i1][i2 + 2][i3 + 2];
float temp749 = temp140*temp159;
float temp750 = (7.0F/12.0F)*temp709;
float temp755 = temp134*u[t1][i1 + 2][i2][i3 + 2];
float temp757 = temp12*u[t1][i1][i2][i3 - 6];
float temp758 = temp12*u[t1][i1][i2][i3 - 5];
float temp759 = (3.0F/20.0F)*temp705;
float temp760 = temp170*temp181;
float temp761 = temp21*u[t1][i1][i2 - 3][i3 - 3];
float temp767 = temp21*u[t1][i1][i2 + 3][i3 - 3];
float temp768 = temp170*temp191;
float temp769 = (7.0F/12.0F)*temp703;
float temp775 = (3.0F/4.0F)*temp709;
float temp776 = temp21*u[t1][i1][i2][i3 + 6];
float temp778 = temp21*u[t1][i1][i2 - 3][i3 + 3];
float temp783 = -temp21*u[t1][i1][i2 + 3][i3 + 3];
float temp788 = -temp136*u[t1][i1 + 3][i2][i3 + 3];
float temp789 = (1.0F/60.0F)*temp707;
float temp790 = temp222*temp225;
float temp793 = -temp116*u[t1][i1][i2 - 2][i3 - 2];

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float temp797 = temp116*u[t1][i1][i2 + 2][i3 - 2];
float temp799 = temp222*temp236;
float temp800 = (7.0F/12.0F)*temp704;
float temp802 = -temp67*u[t1][i1 - 2][i2][i3 - 2];
float temp807 = temp246*temp248;
float temp811 = temp118*u[t1][i1][i2 - 1][i3 + 1];
float temp813 = -temp118*u[t1][i1][i2 + 1][i3 + 1];
float temp816 = temp246*temp259;
float temp817 = (7.0F/12.0F)*temp707;
float temp821 = -temp132*u[t1][i1 + 1][i2][i3 + 1];
float temp824 = temp21*u[t1][i1 - 3][i2][i3 - 3];
float temp830 = temp288*temp295;
float temp831 = temp287*temp288;
float temp832 = -temp116*u[t1][i1 - 2][i2][i3 - 2];
float temp833 = temp116*u[t1][i1 - 2][i2][i3 + 2];
float temp834 = temp312*temp315;
float temp835 = temp311*temp312;
float temp836 = temp118*u[t1][i1 - 1][i2][i3 - 1];
float temp837 = temp118*u[t1][i1 - 1][i2][i3 + 1];
float temp838 = temp334*temp337;
float temp839 = temp333*temp334;
float temp840 = temp118*u[t1][i1 + 1][i2][i3 - 1];
float temp841 = -temp118*u[t1][i1 + 1][i2][i3 + 1];
float temp842 = temp355*temp358;
float temp843 = temp354*temp355;
float temp844 = temp116*u[t1][i1 + 2][i2][i3 - 2];
float temp845 = temp116*u[t1][i1 + 2][i2][i3 + 2];
float temp846 = temp380*temp383;
float temp847 = temp379*temp380;
float temp848 = temp400*temp410;
float temp849 = temp410*temp411;

3D scalar acoustic — Intel's ivy bridge (Endeavour)

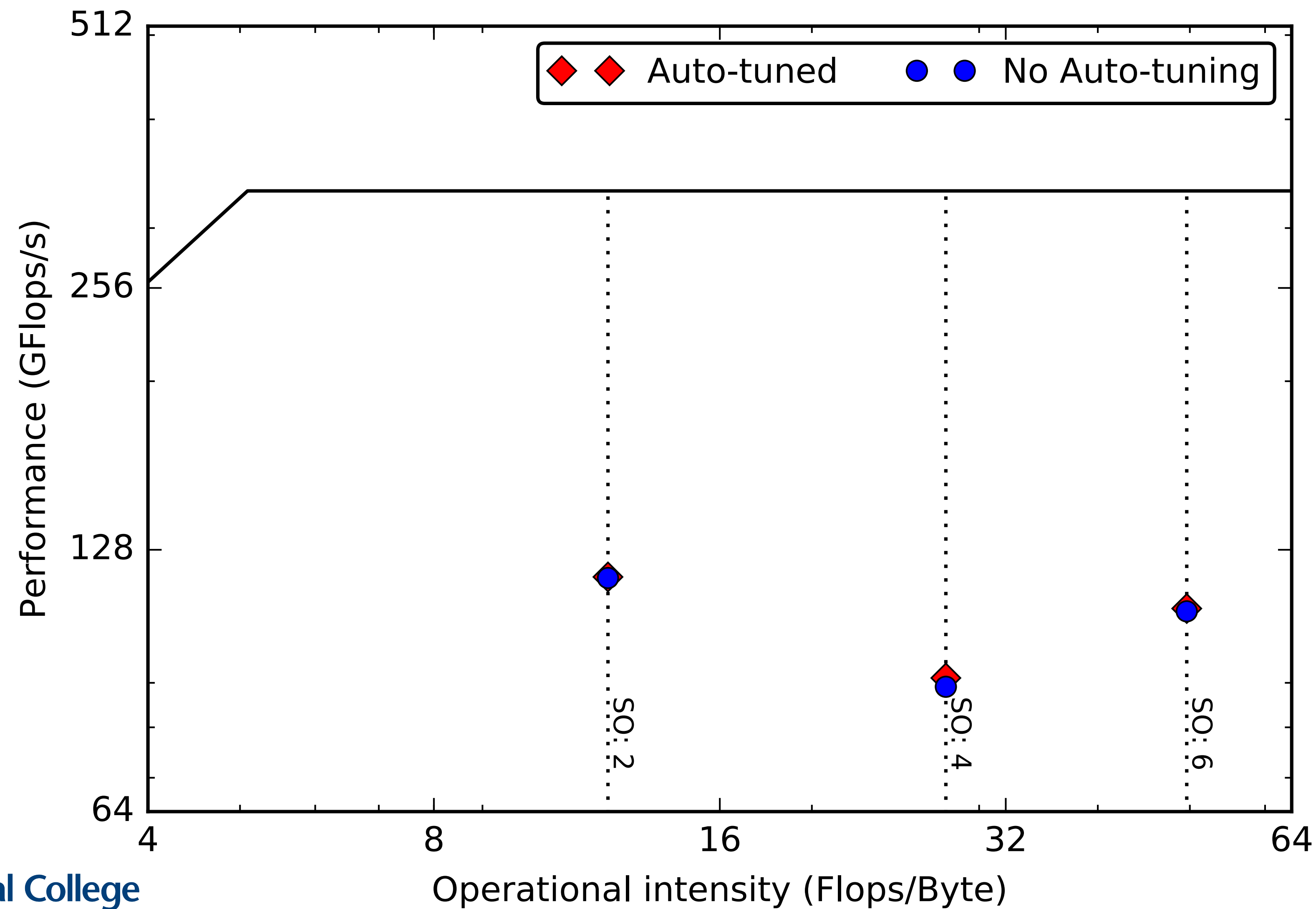


3D acoustic — Intel's KNL



Clearly optimisation strategy needs to adapt for KNL - OPESCI compiler team currently at Intel's meeting in Toulouse learning what needs to be done.

3D TTI — Intel's broadwell



3D TTI - 91 GFLOPS and counting

- ▶ Complex equation, complex implementation, extremely challenging to optimise (527, 1213, 2211 FLOPs in the 2nd, 4th, and 6th stencils).
- ▶ Symbolic engine again plays critical role:
 - ▶ Common subexpression elimination (CSE) critical **before** source is generated - we extended SymPy for this purpose.
 - ▶ Greatly speeds up code generation (critical for JIT).
 - ▶ Allows further transformations (eg factorisation, expansion).
 - ▶ Without this high level CSE TTI cannot even be compiled!
 - ▶ Replaced trigometric functions with Chebyshev polynomials of the first kind - greatly increases FLOPs and complexity of expression but results in 1.3x speed-up at order 2.

3D TTI cont...

- ▶ Future work:
 - ▶ Systematically optimise these expressions (ie to reduce the flop count), exploiting mathematical properties.
 - ▶ Not feasible to run space order 8 with the current technology - but have a solution.
 - ▶ Vectorisation is crucial.
 - ▶ Many temporaries — register spilling!

Reflections

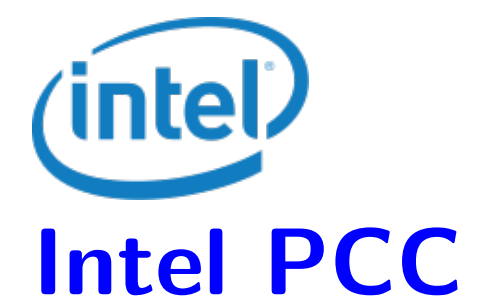
- ▶ SymPy as a DSL for finite difference is a key component. Migrate changes upstream to be supported by the wider community
- ▶ Stencil compilers are a very active area of computer science research
 - ▶ Need to maintain intermediate representation layer
 - ▶ Continue with our own stencil compiler - retain flexibility to add domain specific extensions.
 - ▶ Prototype integration with existing stencil DSL's - see where abstractions break down.
- ▶ Integration into FWI packages:
 - ▶ Unique selling points: Agility; performance; adjoint
 - ▶ Code generation allows us to generate many different API's. One target is to provide a "library" for existing C or Fortran codes. However, a high level implementation in a data flow framework such as TensorFlow is a clear topic for research.

Currently designing 3rd generation of Devito

Maintaining current generation as stable release for SINBAD. But already designing next generation of technology.

- ▶ MPI parallelism for larger models.
- ▶ Integrate with stencil and polyhedral compilers.
- ▶ Additional symbolic optimisation (factorisation, hoisting, etc.) - **critical for TTI.**
- ▶ Polyhedral compilation.
- ▶ Automated data layout optimisations.
- ▶ FLOP reduction and register pressure reduction.
- ▶ Extend feature range to facilitate more science.
- ▶ Integration of verification tools such as CodeThorn.

Thank You



Acknowledgments

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