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Recent developments in compressive sensing for time-lapse studies

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SLIM

University of British Columbia

Overview

Summary

- Timelapse (4D) & CS
- Challenges for 4D
- Recent CS extensions
- Stylized examples

Linearized Inversion of time-lapse seismic data

Conclusions

Nonlinear inversion

Future work

Felix J. Herrmann, Michael P. Friedlander, and Ozgur Yilmaz, "Fighting the Curse of Dimensionality: Compressive Sensing in Exploration Seismology", *Signal Processing Magazine, IEEE*, vol. 29, p. 88-100, 2012
Felix J. Herrmann, "Randomized sampling and sparsity: Getting more information from fewer samples", *Geophysics*, vol. 75, p. WB173-WB187, 2010

Compressive sensing *paradigm*

Find representations that reveal _structure_

- *transform*-domain *sparsity* (e.g., Fourier, curvelets, etc.)

Sample to _break_ the _structure_

- *randomized* acquisition (e.g., *jittered* sampling, time *dithering*, *encoding*, etc.)
- destroy *sparsity*

Recover _structure_ by promoting

- *sparsity* via *one*-norm *minimization*

Compressive sensing in 4D

Sampling

$$\mathbf{A}_1\mathbf{x}_1 = \mathbf{b}_1$$ ← subsampled baseline data

$$\mathbf{A}_2\mathbf{x}_2 = \mathbf{b}_2$$ ← subsampled monitor data

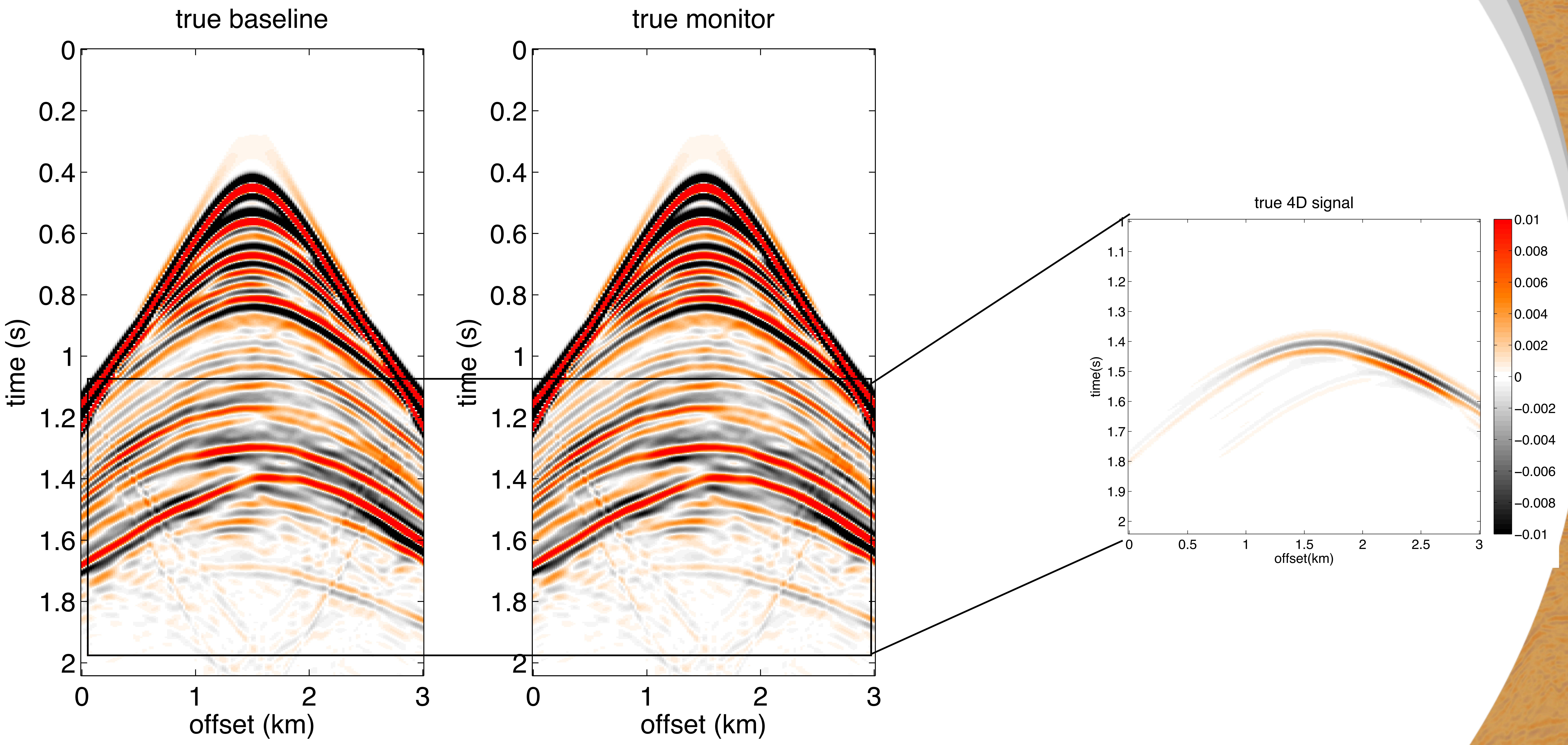
Sparsity-promoting recovery

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{b}$$

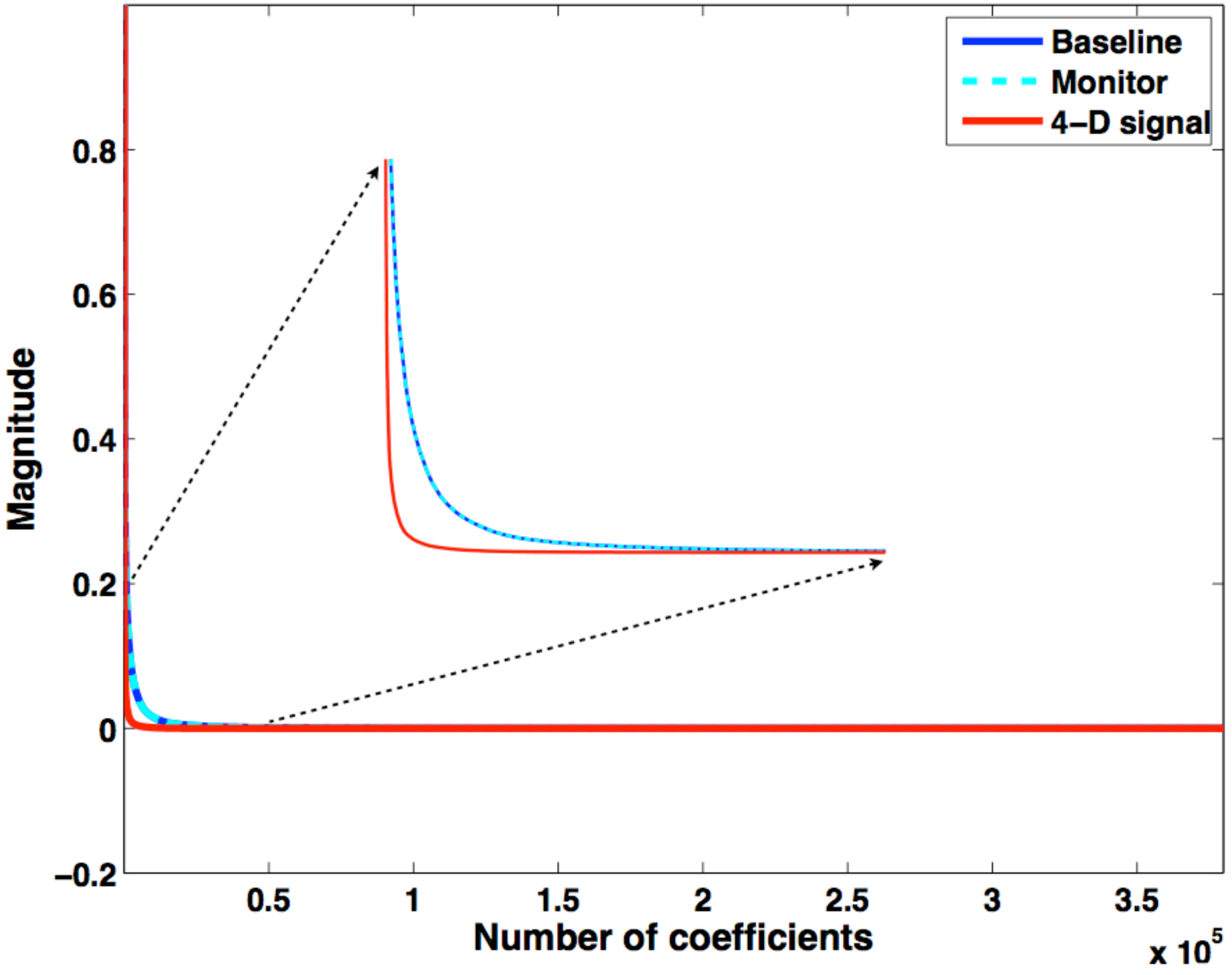
recovered data: $\tilde{\mathbf{d}} = \mathbf{S}^H\tilde{\mathbf{x}}$

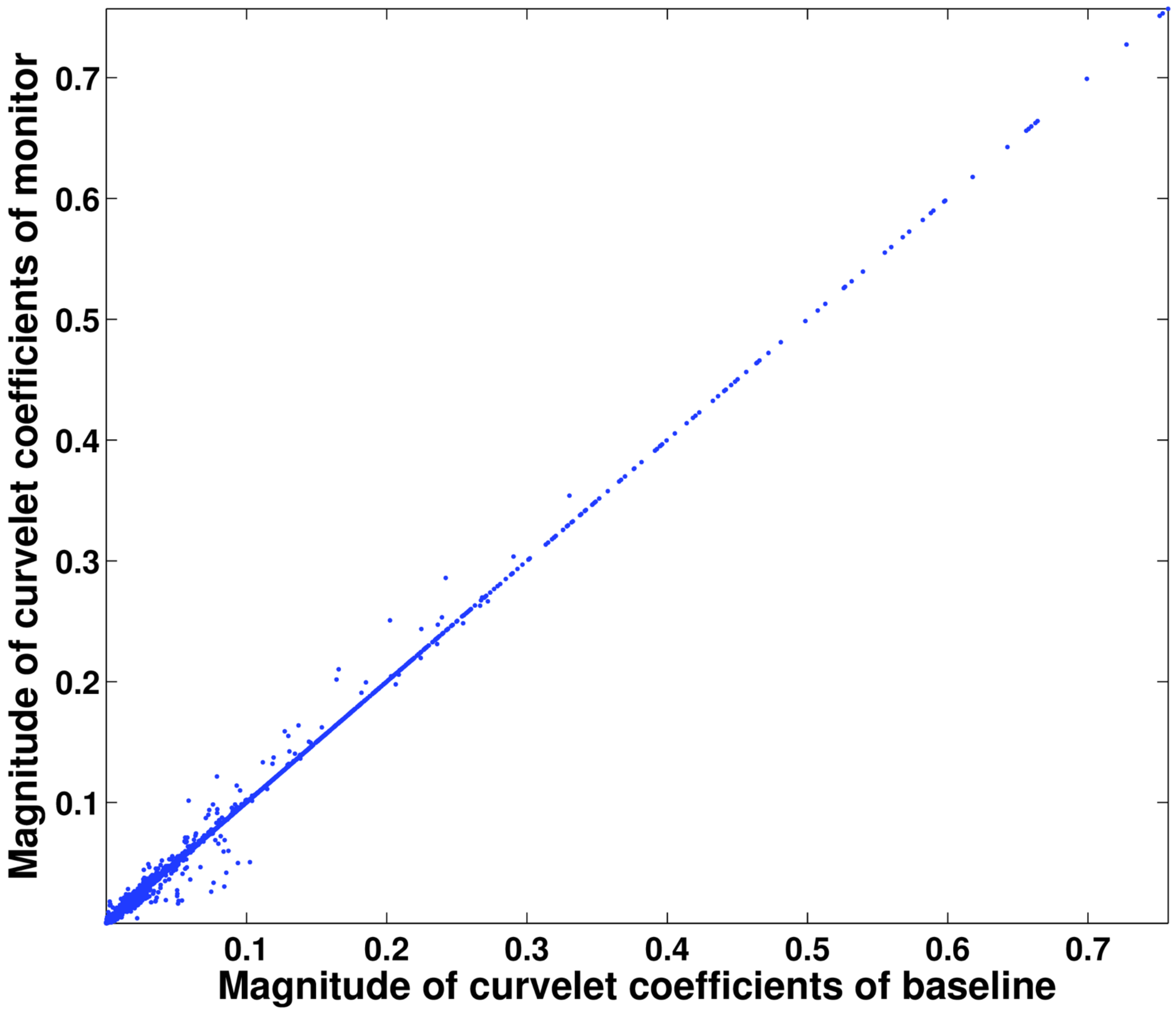
Probing time-lapse data ....

SAME Geometry – regularly & densely sampled – IDEAL but *UNREALISTIC* CASE



Structure - curvelet representation





Can we exploit the structure in the time-lapse data simultaneously ?

Dror Baron , Marco F. Duarte , Shriram Sarvotham , Michael B. Wakin , Richard G. Baraniuk. An Information-Theoretic Approach to Distributed Compressed Sensing (2005)

Distributed compressive sensing

– *joint* recovery model (JRM)

vintages

$$\mathbf{x}_1 = \color{red}{\mathbf{z}_0} + \mathbf{z}_1$$
$$\mathbf{x}_2 = \color{red}{\mathbf{z}_0} + \mathbf{z}_2$$

differences

***common* component**

$$\overbrace{\begin{bmatrix} \color{red}{\mathbf{A}_1} & \mathbf{A}_1 & \mathbf{0} \\ \color{red}{\mathbf{A}_2} & \mathbf{0} & \mathbf{A}_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \color{red}{\mathbf{z}_0} \\ \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}^{\mathbf{z}} = \overbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}}^{\mathbf{b}}$$

baseline

monitor

- **Key idea:**
  - use the fact that *different* vintages *share* common information
  - invert for *common* components & *differences* w.r.t. the *common* components with *sparse* recovery

Interpretation of the model
– w/ & w/o repetition

- **In an *ideal* world** $(\mathbf{A}_1 = \mathbf{A}_2)$
  - JRM *simplifies* to recovering the *difference* from $(\mathbf{b}_2 - \mathbf{b}_1) = \mathbf{A}_1(\mathbf{x}_2 - \mathbf{x}_1)$
  - expect *good* recovery when *difference* is *sparse*
  - ***but*** relies on "*exact*" repeatability...

- **In the *real* world** $(\mathbf{A}_1 \neq \mathbf{A}_2)$
  - no absolute *control* on *surveys*
  - *calibration* errors
  - noise...

Conventional versus proposed method

Current	Distributed Compressed Sensing
Expensive dense acquisitions for baseline or/and monitor	Cheap subsampled acquisitions for both surveys
Compute differences from baseline/monitor	Compute differences from the innovations
Effort to repeat acquisition geometry is challenging	Relaxation of repetition in view
Independent processing/inversion	Joint processing/inversion

Felix J. Herrmann and Xiang Li, “Efficient least-squares imaging with sparsity promotion and compressive sensing”, *Geophysical Prospecting*, vol. 60, p. 696-712, 2012.

Ning Tu and Felix J. Herrmann, “Fast imaging with surface related multiples by sparse inversion”, *Geophysical Journal International*, vol. 201, p. 304-317, 2014.

Application to imaging

-linearized inversion of time-lapse data

Migration

Problem formulation

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma$$

where

Linearized Demigration operator

$$\mathbf{A} = \nabla \mathrm{F}[\mathbf{m}_0, q]\mathbf{C}^H$$

$$\mathbf{b} = \mathrm{D} - \mathrm{F}[\mathbf{m}_0, q]$$

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \tilde{\mathbf{x}}$$

Migration

Dimensionality reduction

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\underline{\mathbf{A}}\mathbf{x} - \underline{\mathbf{b}}\|_2 \leq \sigma_k$$

where

$$\underline{\mathbf{A}} = (\mathrm{RM})\,\mathbf{A}$$

$$\underline{\mathbf{b}} = (\mathrm{RM})\,\mathrm{D} - \mathrm{F}[\mathbf{m}_0, \bar{q}]$$

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \tilde{\mathbf{x}}$$

SLIM

Migration

vintages

$$\mathbf{x}_1 = \mathbf{z}_0 + \mathbf{z}_1$$
$$\mathbf{x}_2 = \mathbf{z}_0 + \mathbf{z}_2$$

differences

common component

$$\overbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{A}_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}^{\mathbf{z}} = \overbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}}^{\mathbf{b}}$$

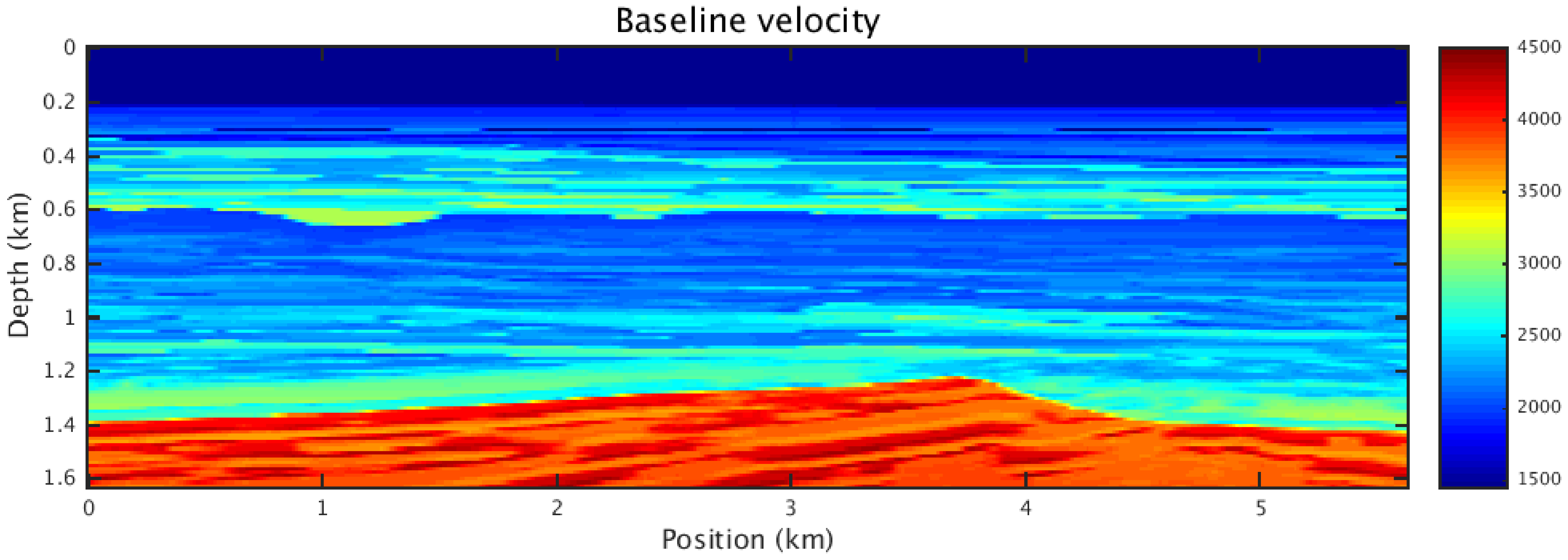
baseline

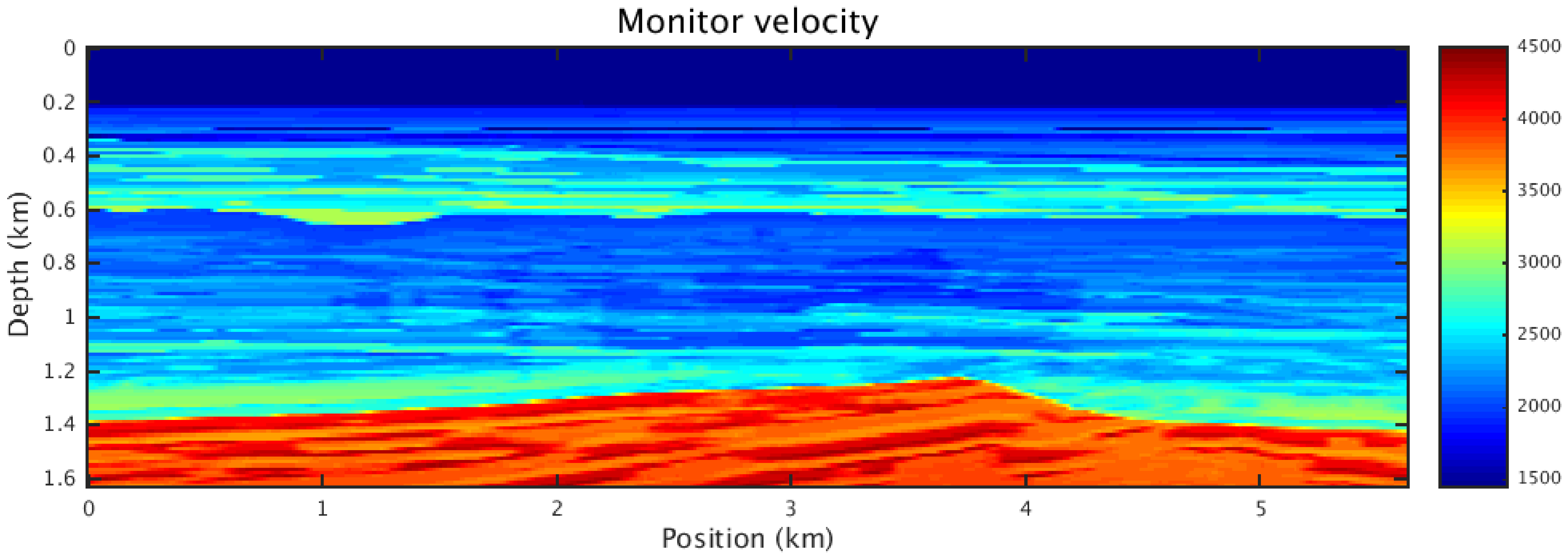
monitor

$$\mathbf{A}_1 = \mathbf{A}_1 = \nabla \mathrm{F}[\mathbf{m}_{1_0}, q_1]\mathbf{C}^H$$

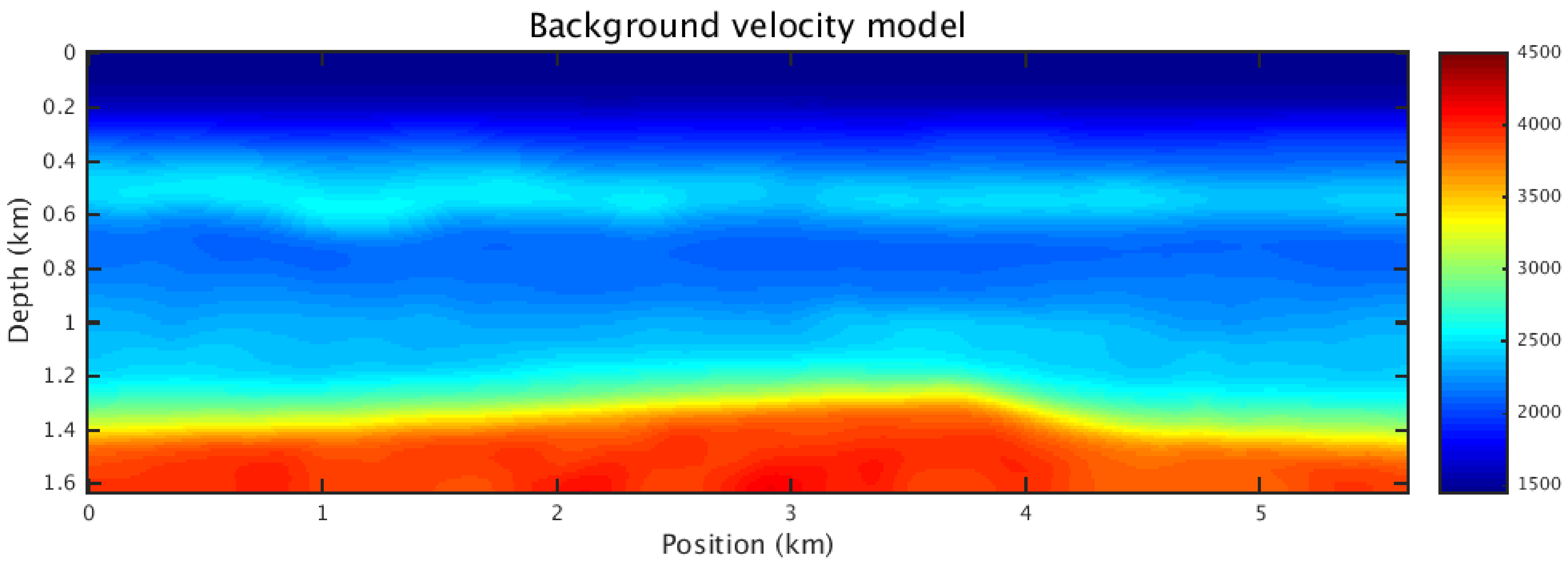
$$\mathbf{A}_2 = \mathbf{A}_2 = \nabla \mathrm{F}[\mathbf{m}_{2_0}, q_2]\mathbf{C}^H$$

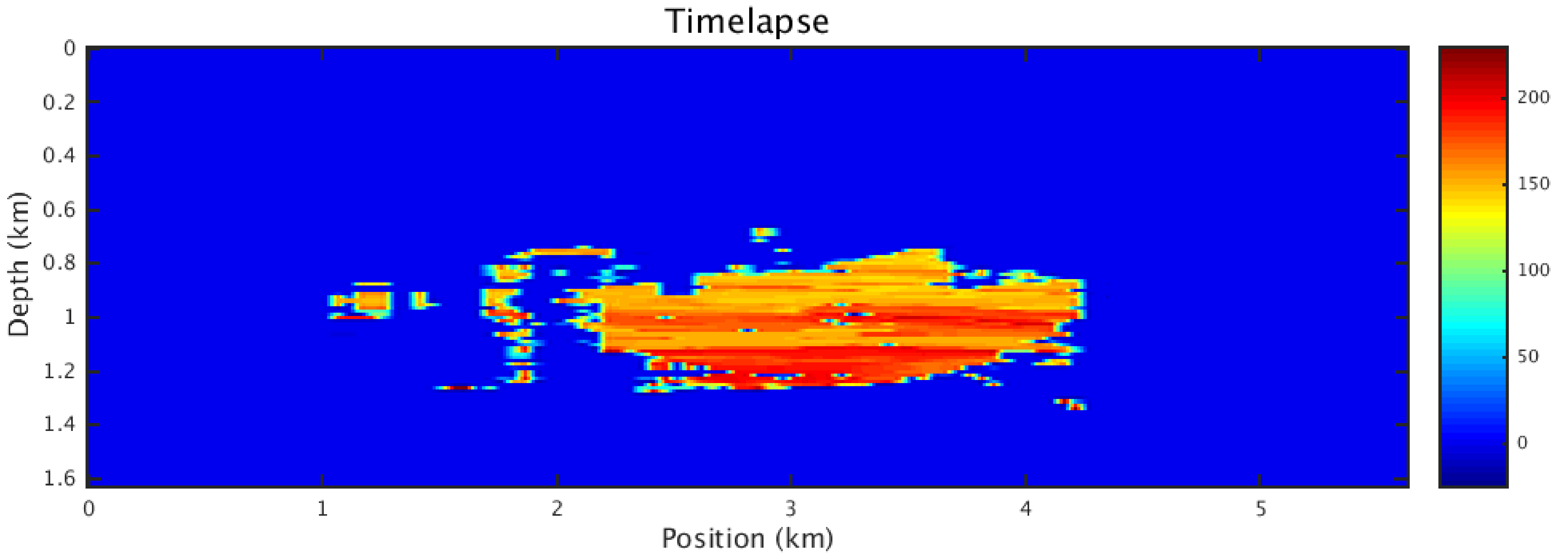
Timelapse models



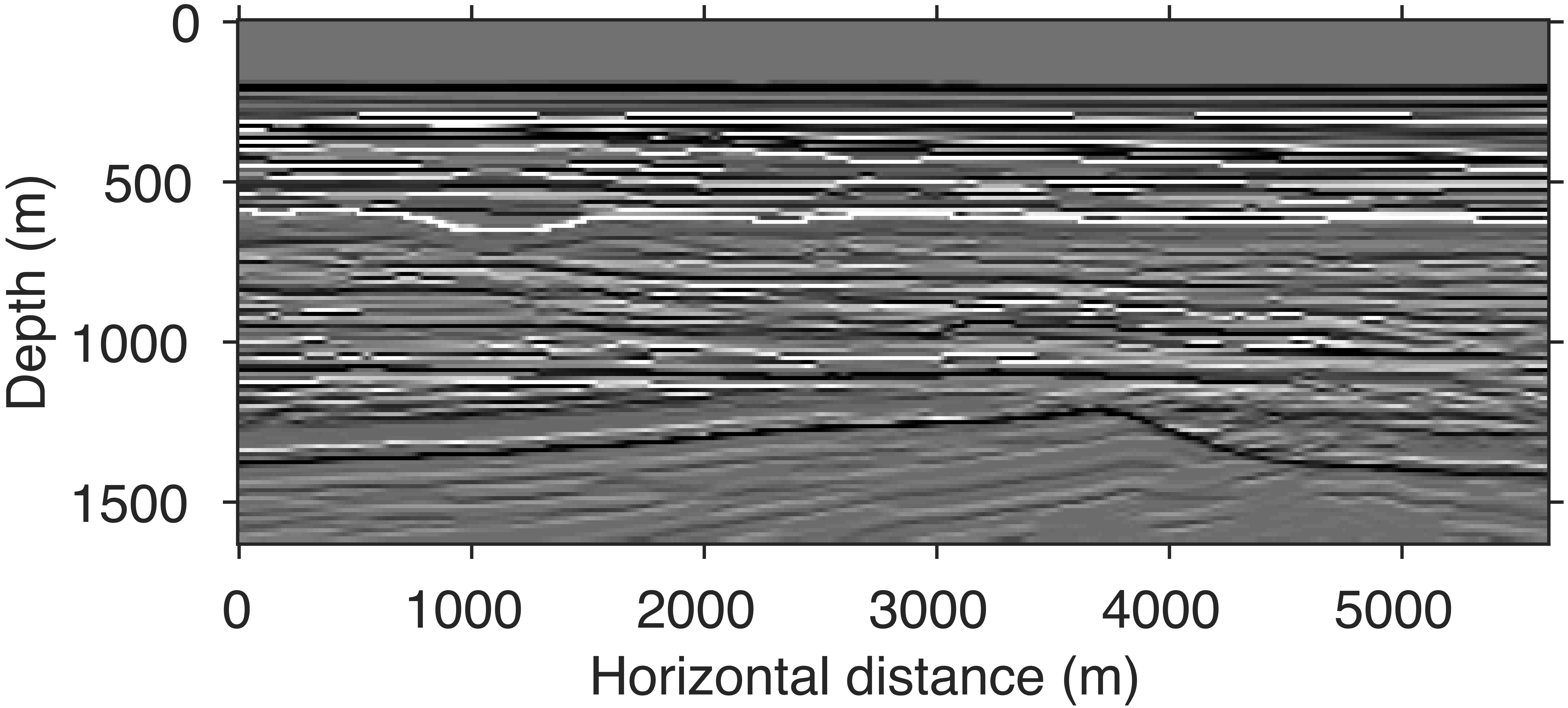


Initial/Difference

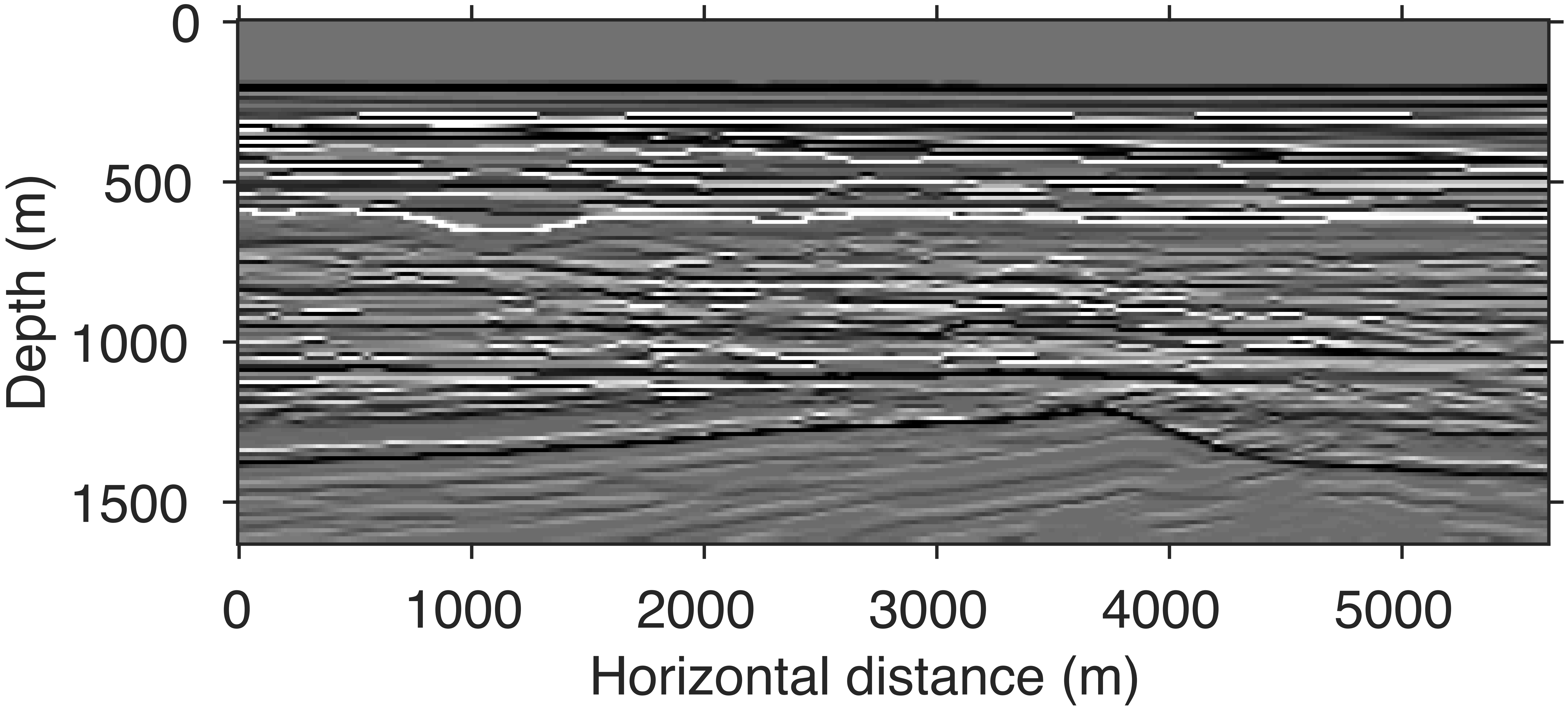




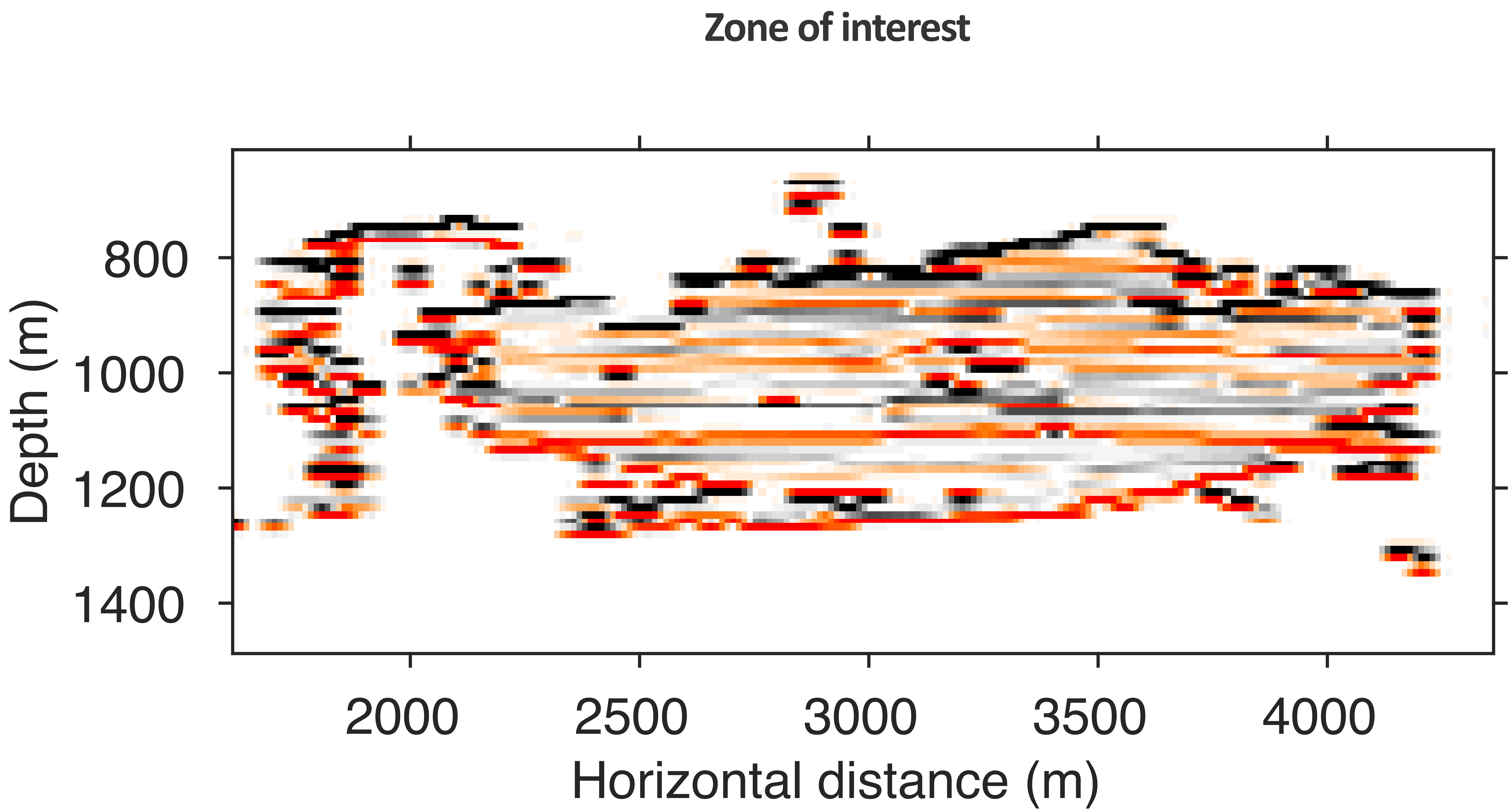
Baseline perturbation



Monitor perturbation



Time-lapse reflectivity



Migration

Modeling parameters

- 225 shots @ approx. 25m interval
- 225 receivers @ approx. 25m interval
- 120 frequencies between 5 & 35Hz for imaging
- Shot records of 4seconds
- Ricker wavelet @ 20.0Hz
- Baseline & Monitor with “different” source/receiver positions

Objective

- Imaging of baseline/monitor
- Observe and interpret changes in reflectivity
- Compare independent (IRS) and the joint method (JRM)

Migration

Use 15 randomly selected sources and all the frequencies

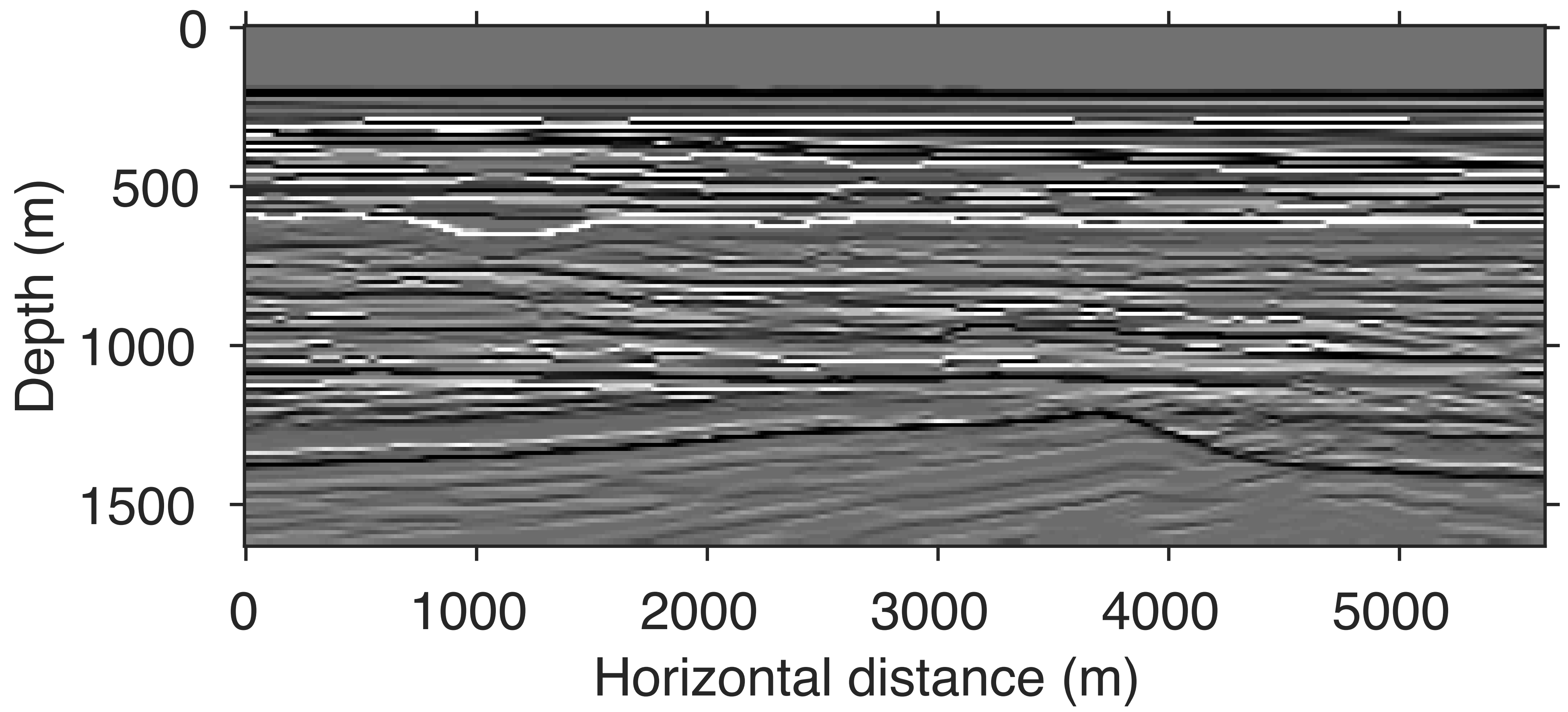
(1) Conventional RTM with data

(2) Least squares RTM

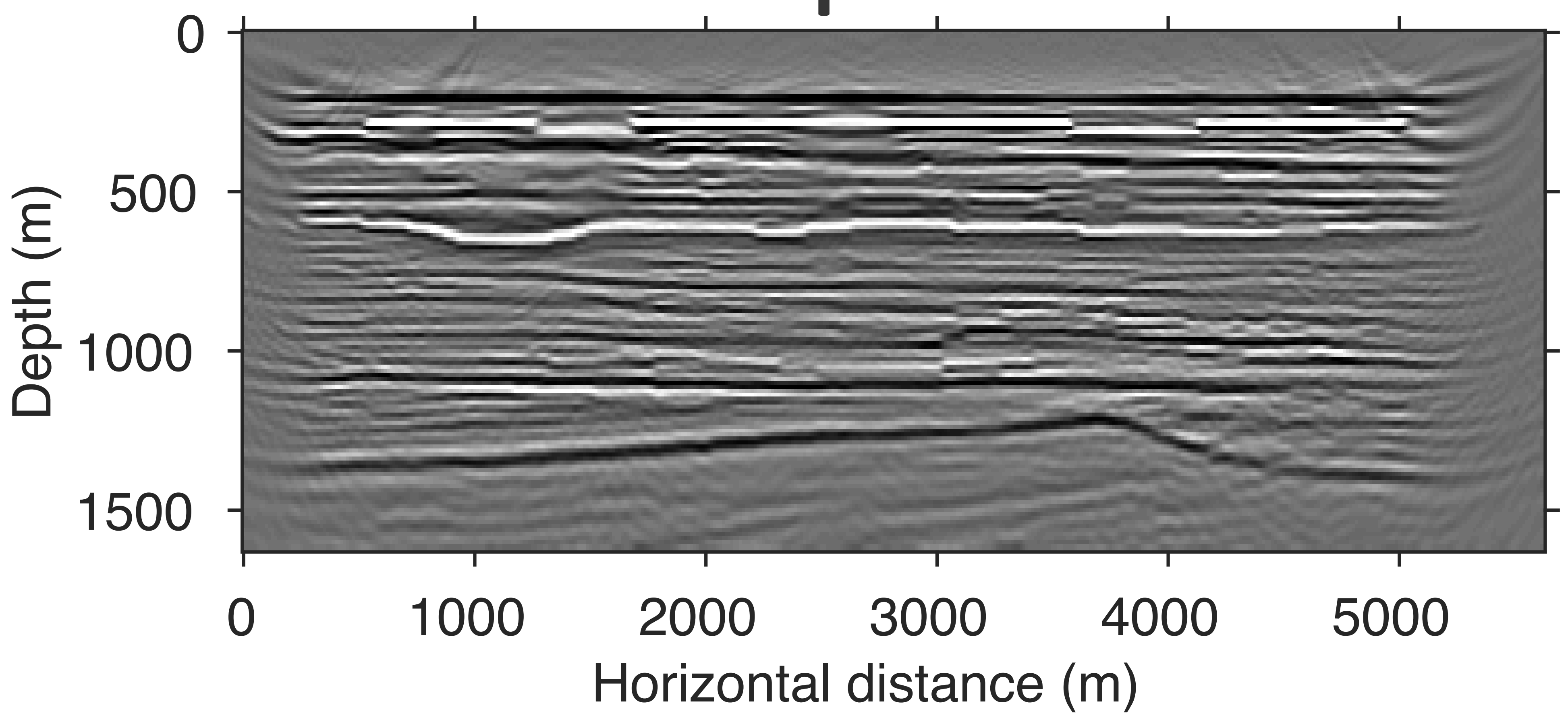
- Randomly select 15 sources and 16 frequencies at each iteration
- No renewal of sources, but frequency redraw at each iteration
- Exploit sparsity (in curvelet domain) of reflectivity
- Ricker wavelet @ 20Hz
- Fairly accurate background velocity model

Baseline Image

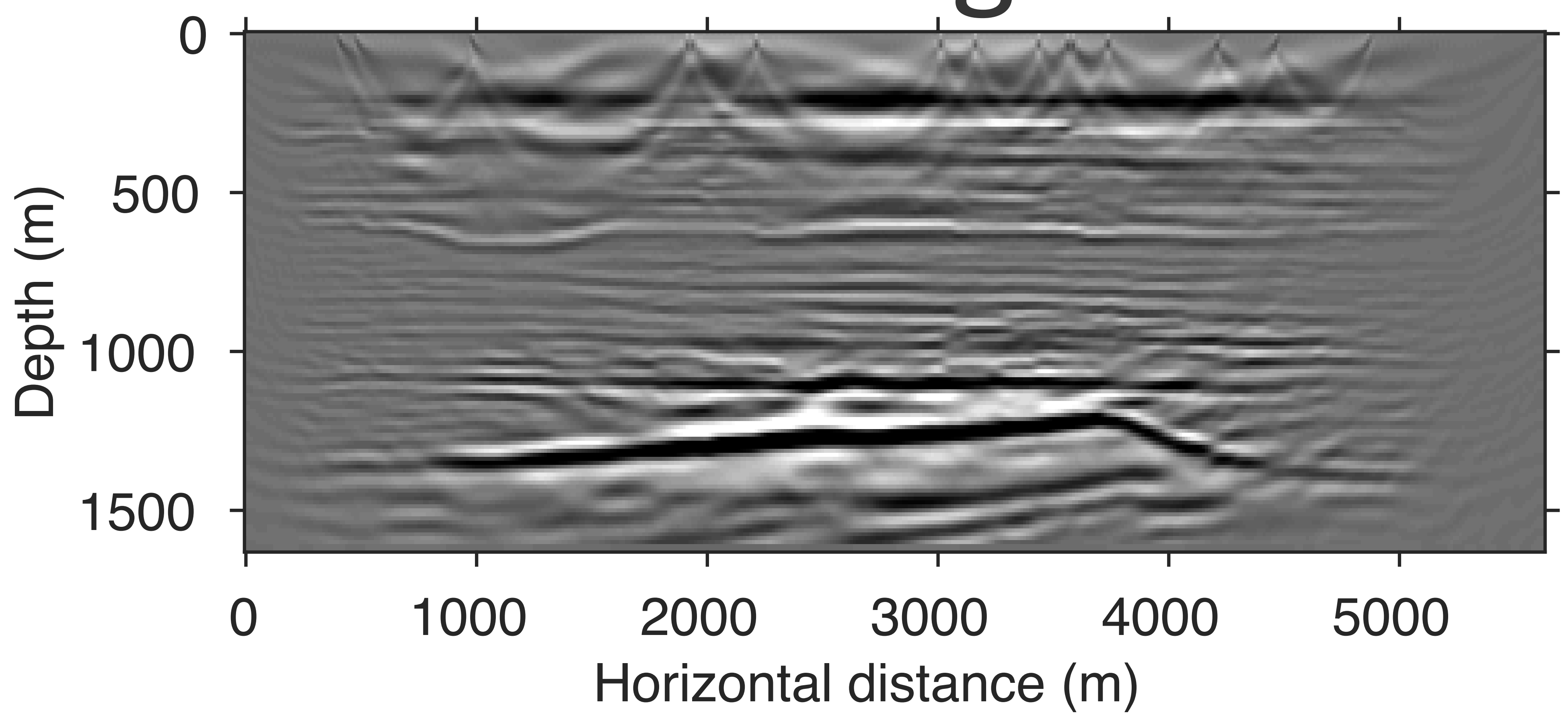
Baseline Image



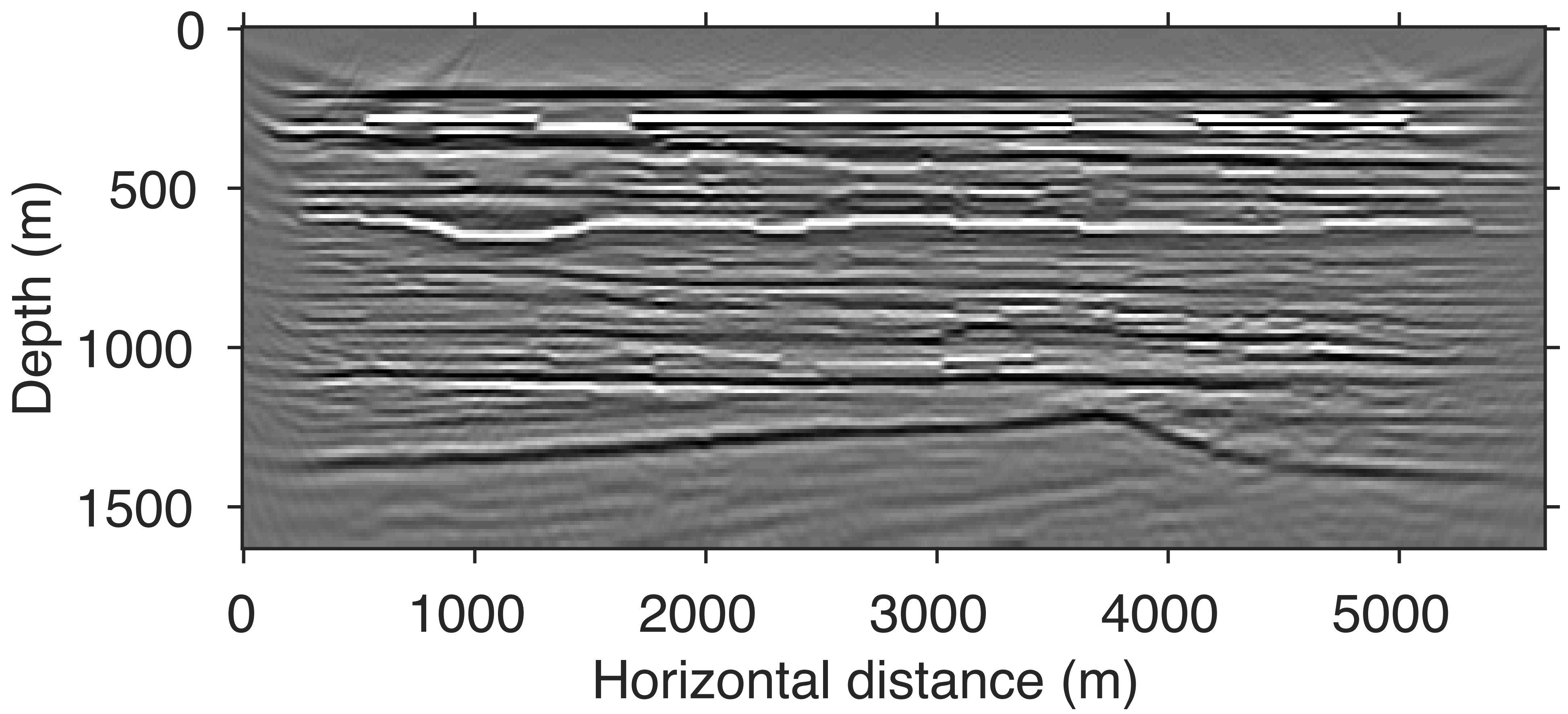
Indept. LSM



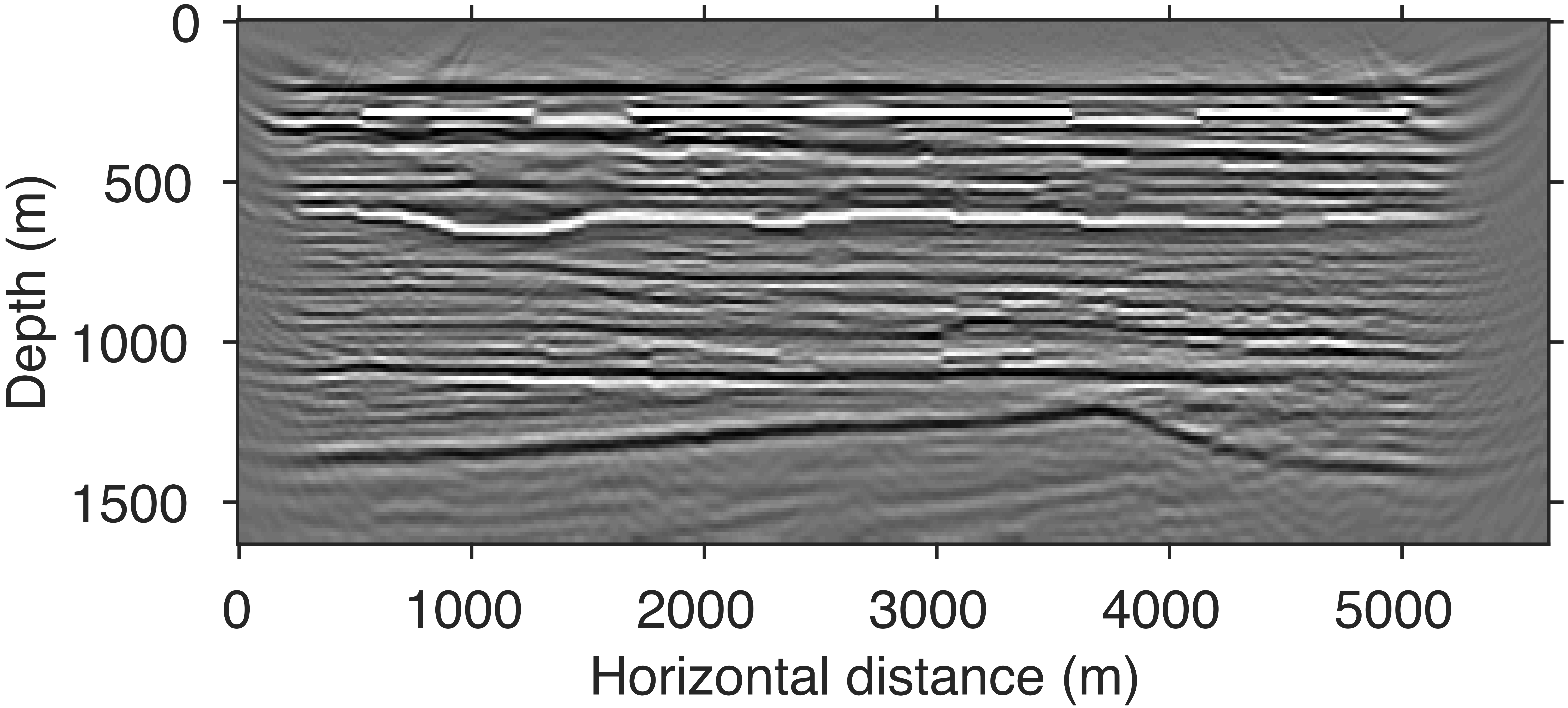
RTM Image



Joint LSM

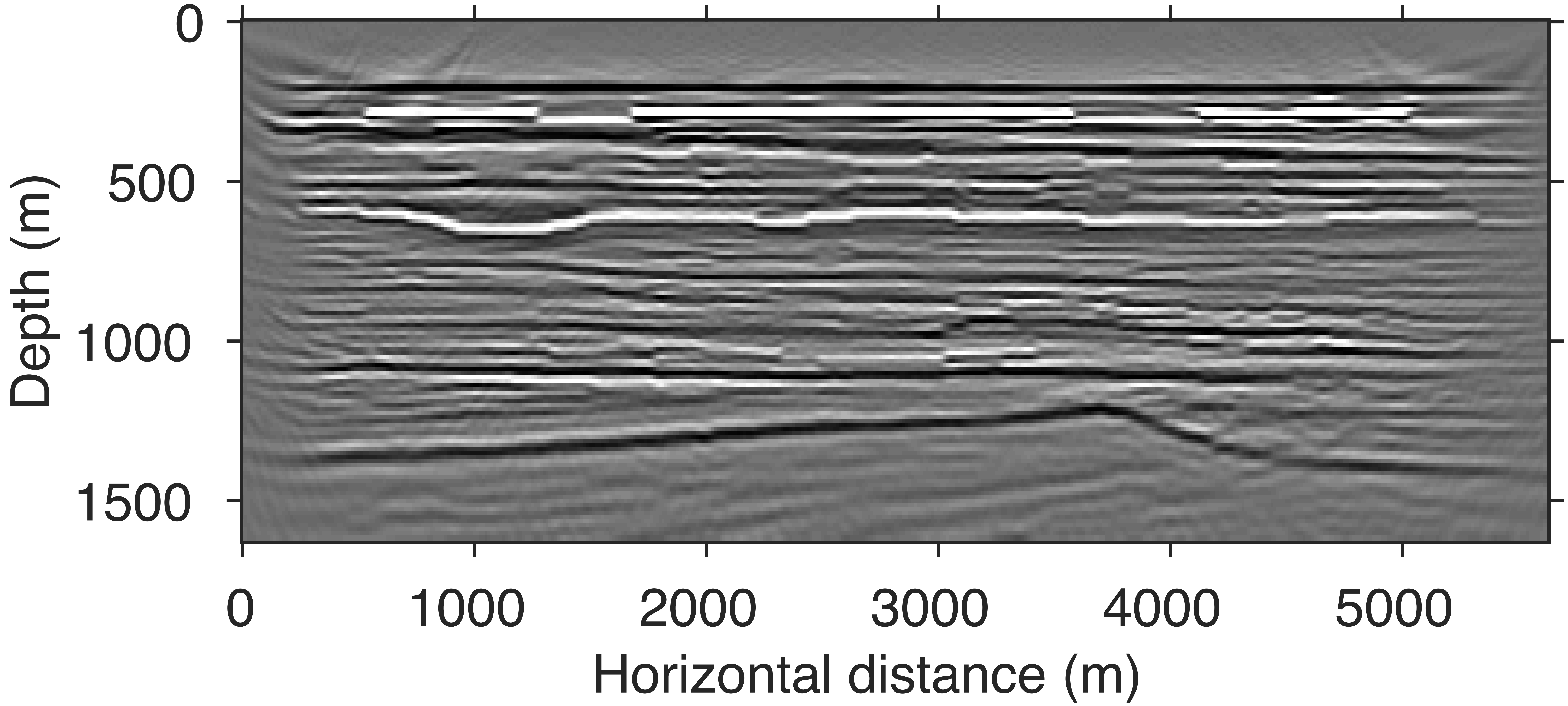


Baseline Image



Independent
LSM

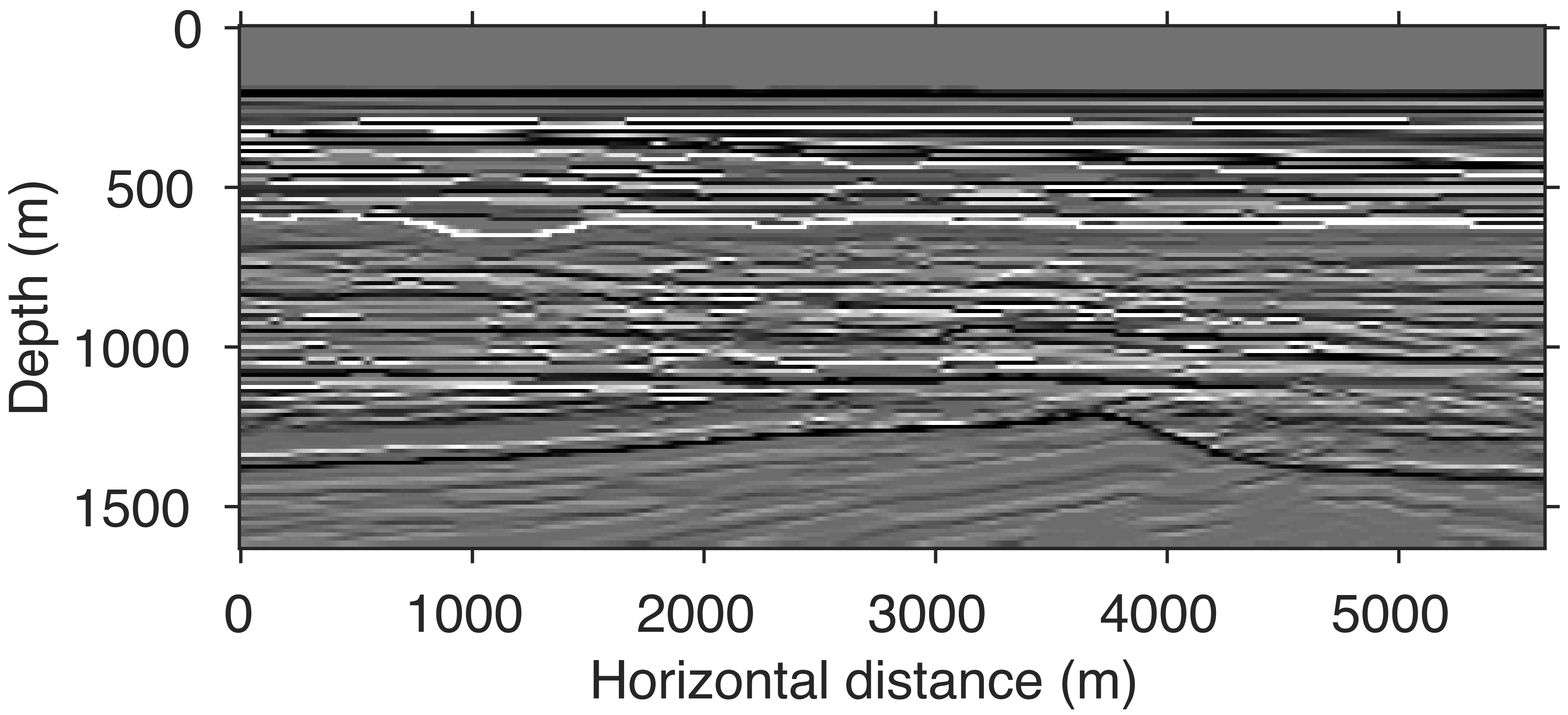
Baseline Image



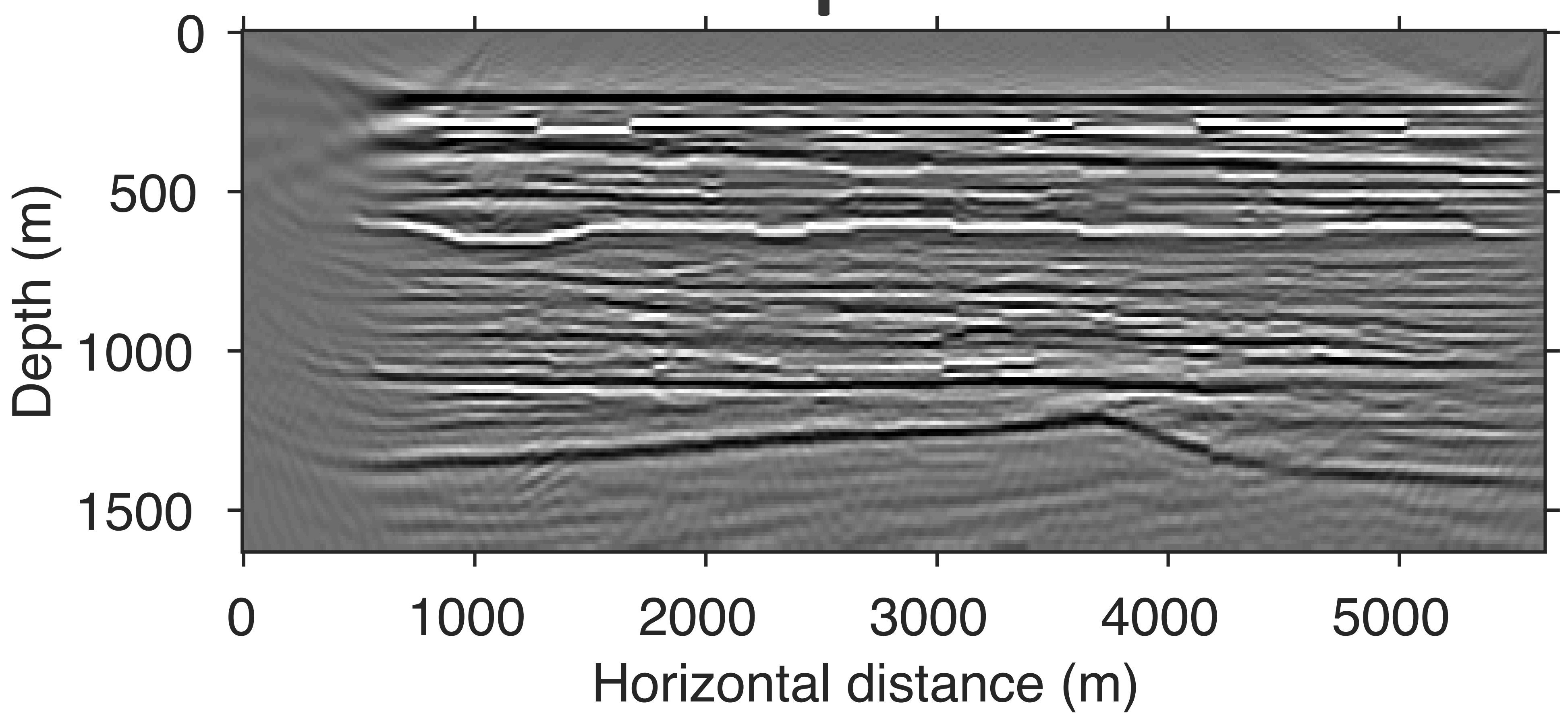
Joint
LSM

Monitor Image

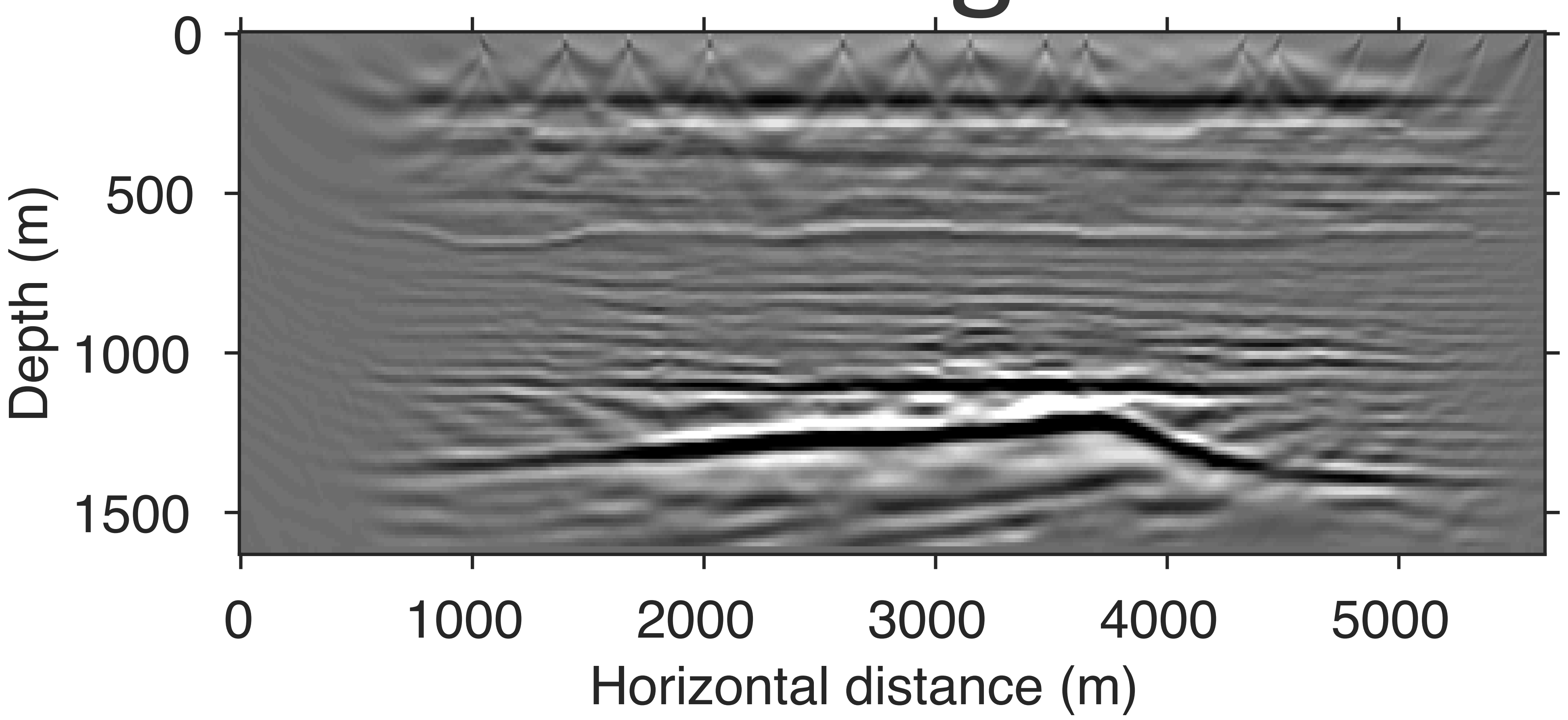
Monitor Image



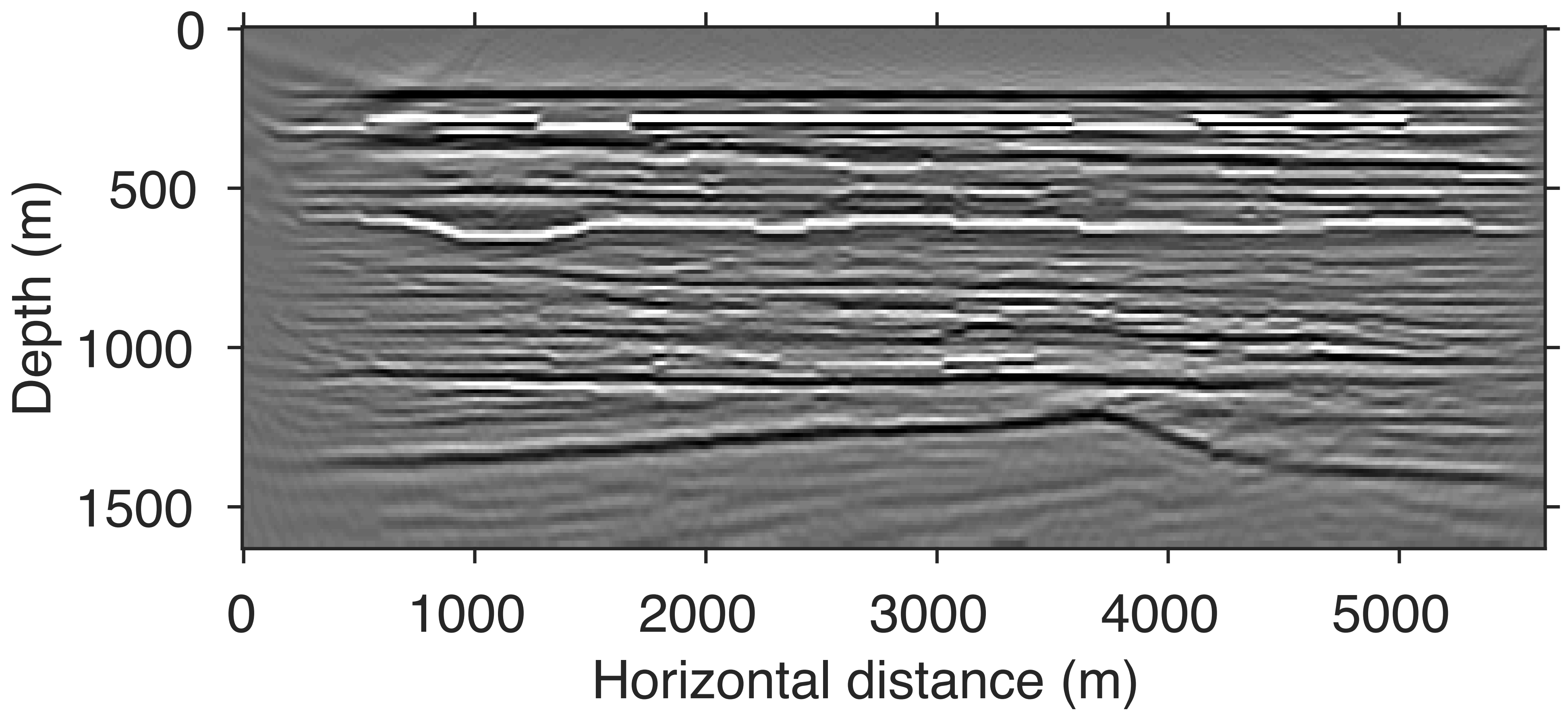
Indept. LSM



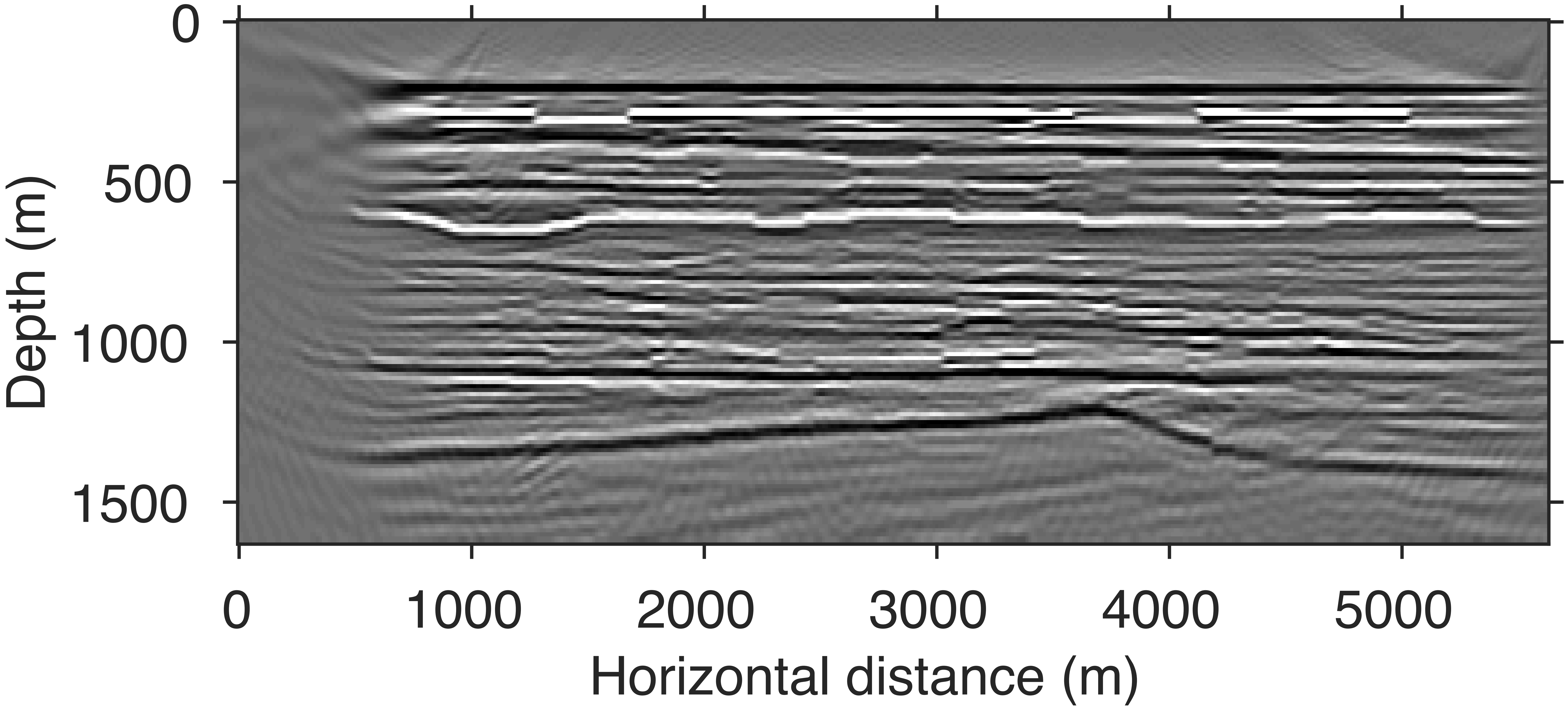
RTM Image



Joint LSM

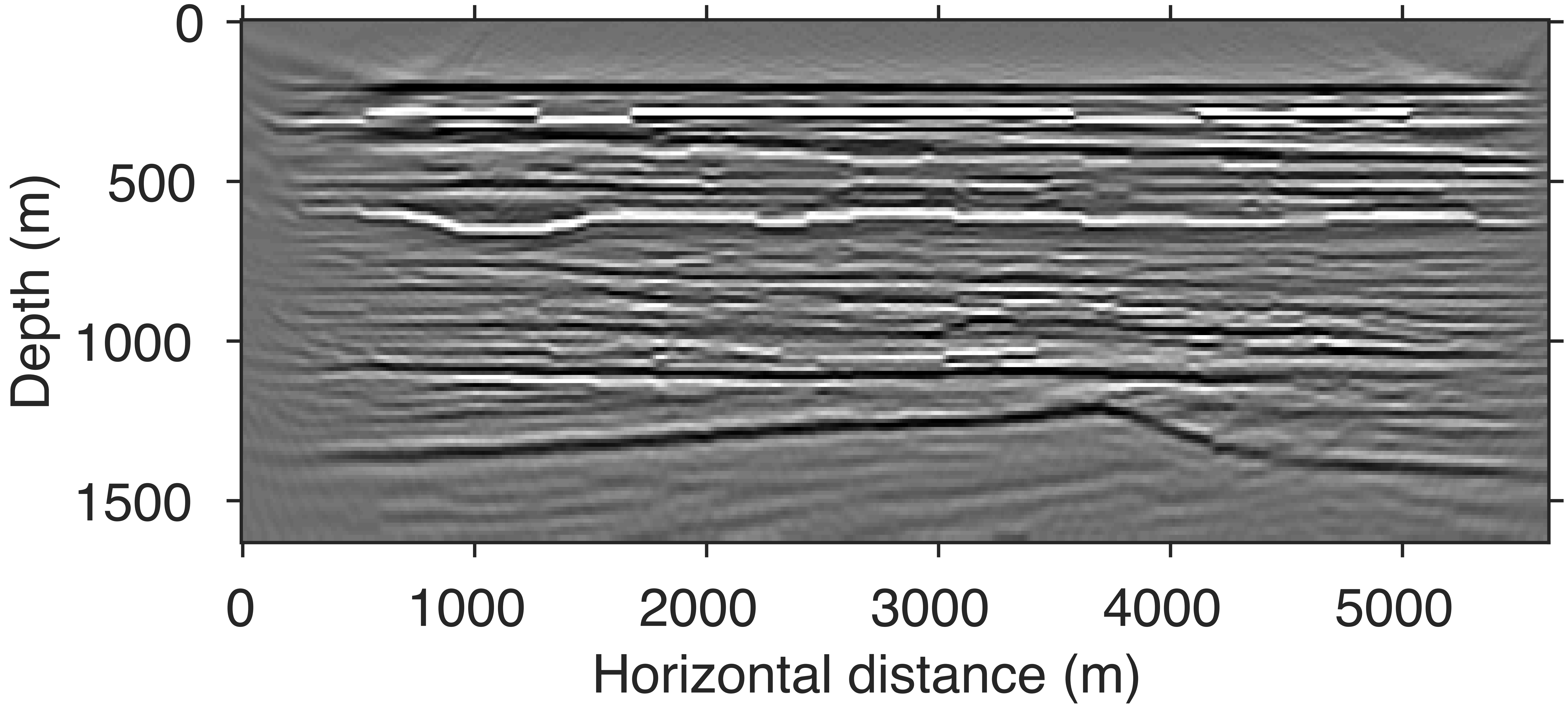


Monitor Image



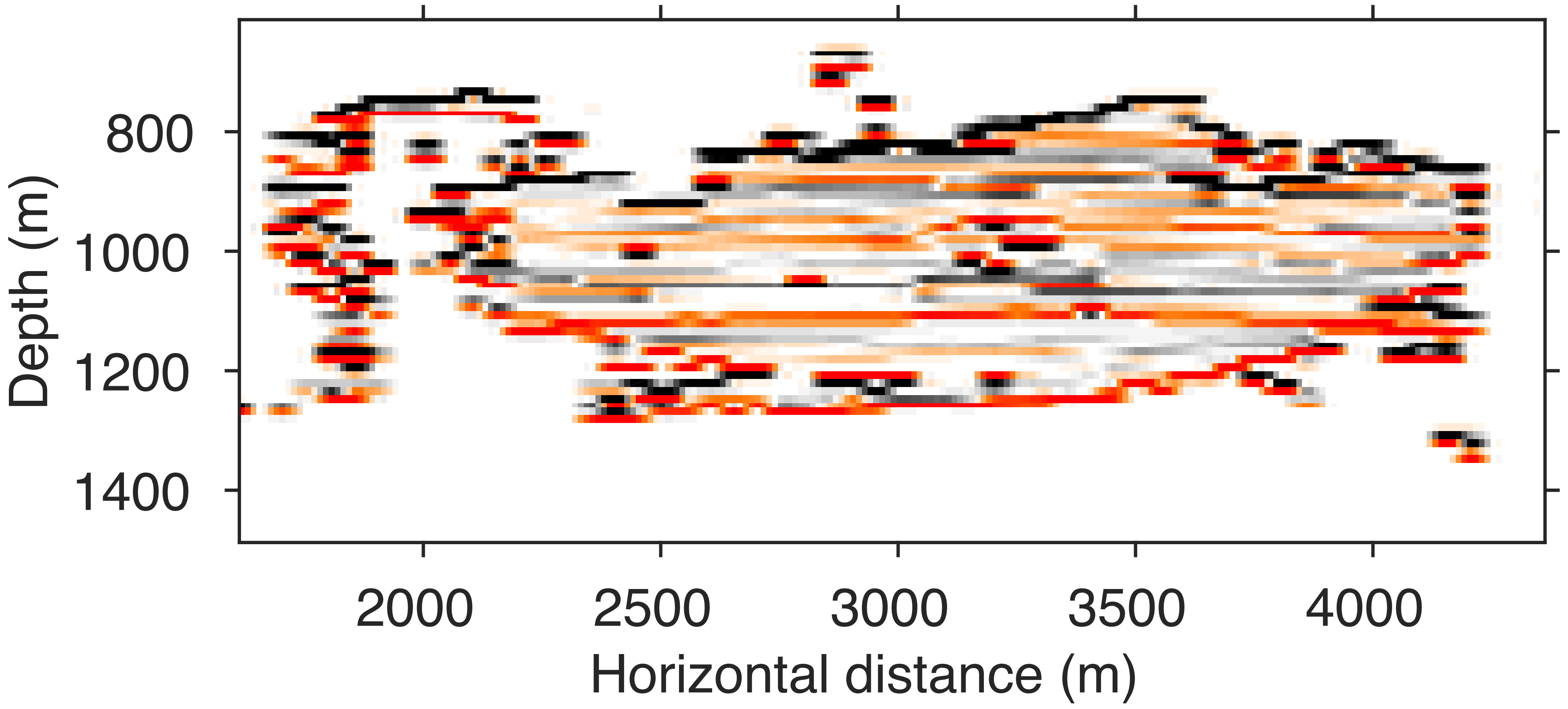
Independent LSM

Monitor Image

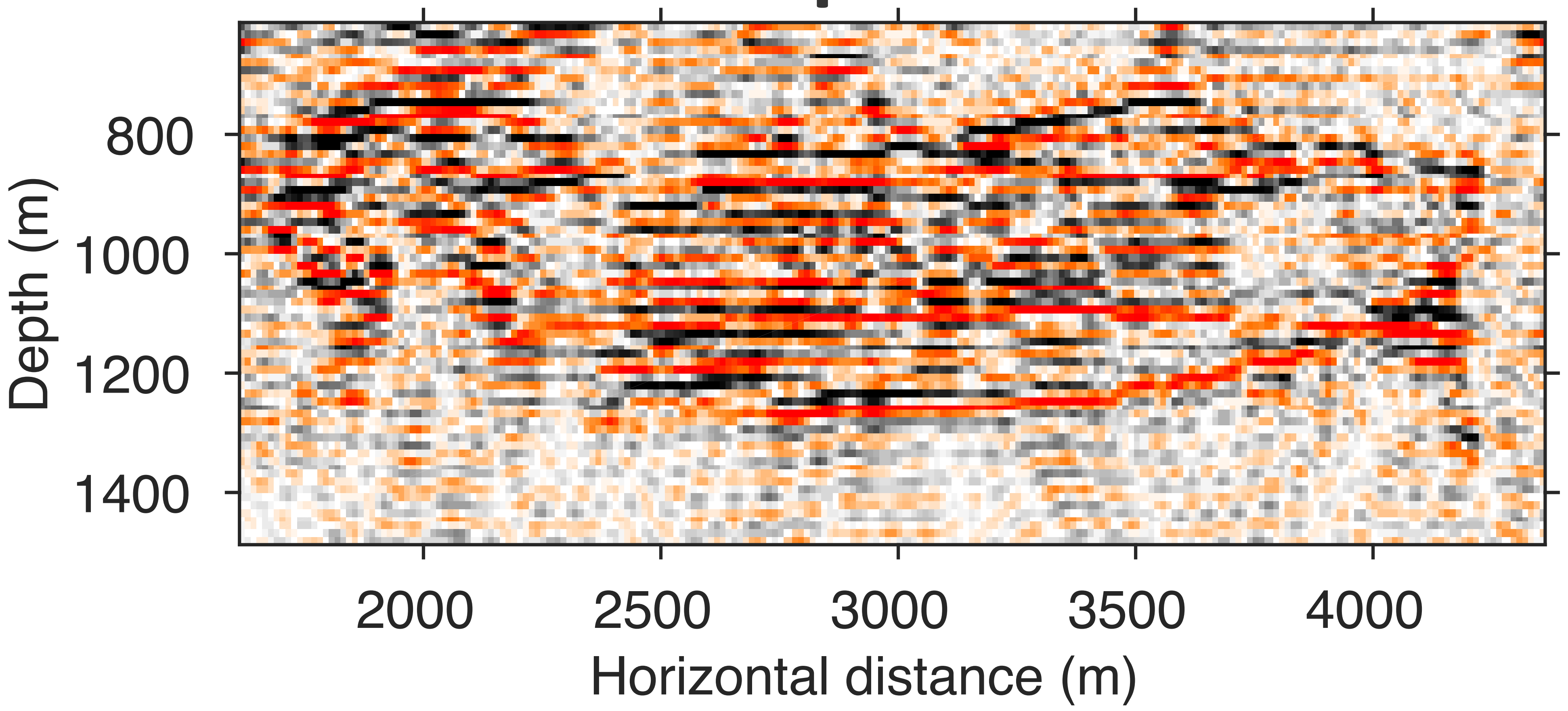


Joint
LSM

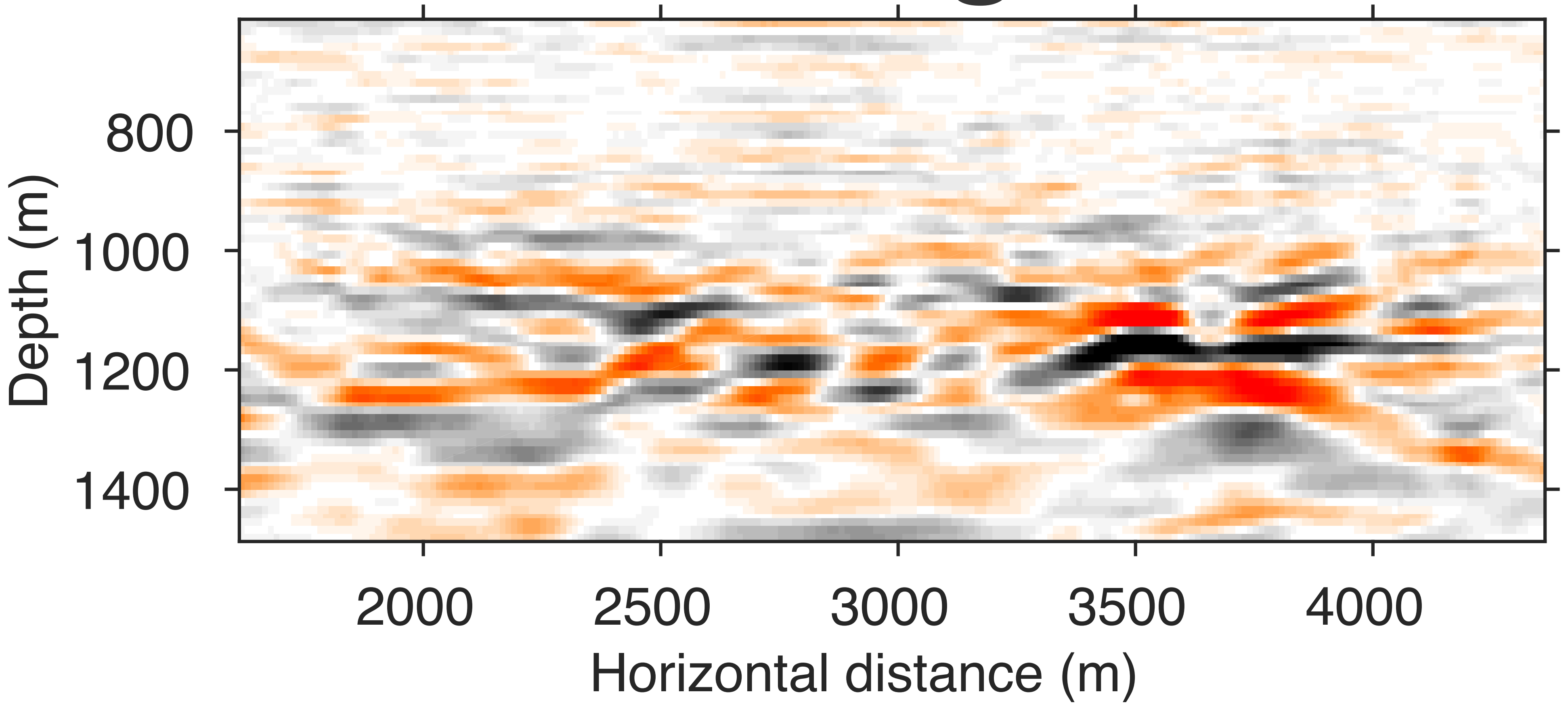
Time-lapse Image



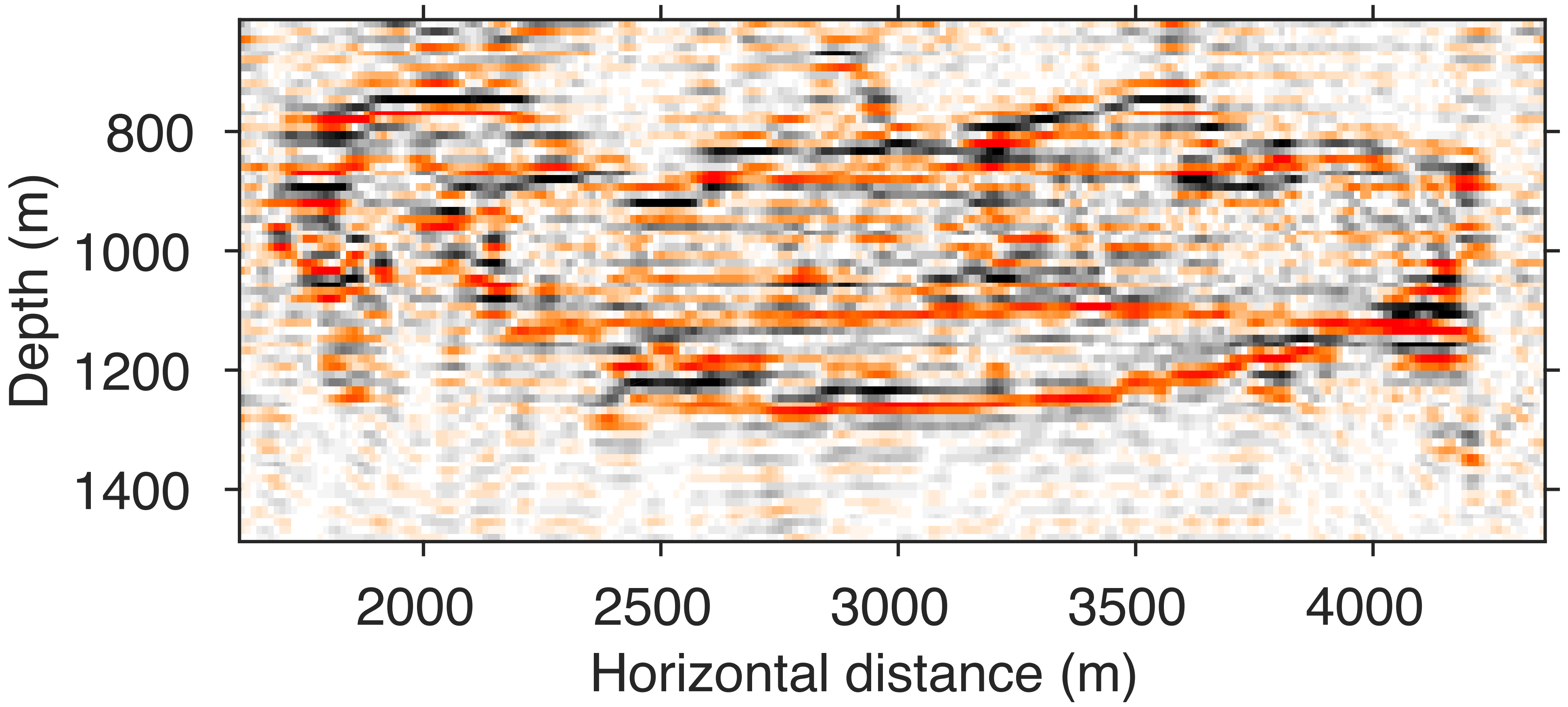
Indept. LSM



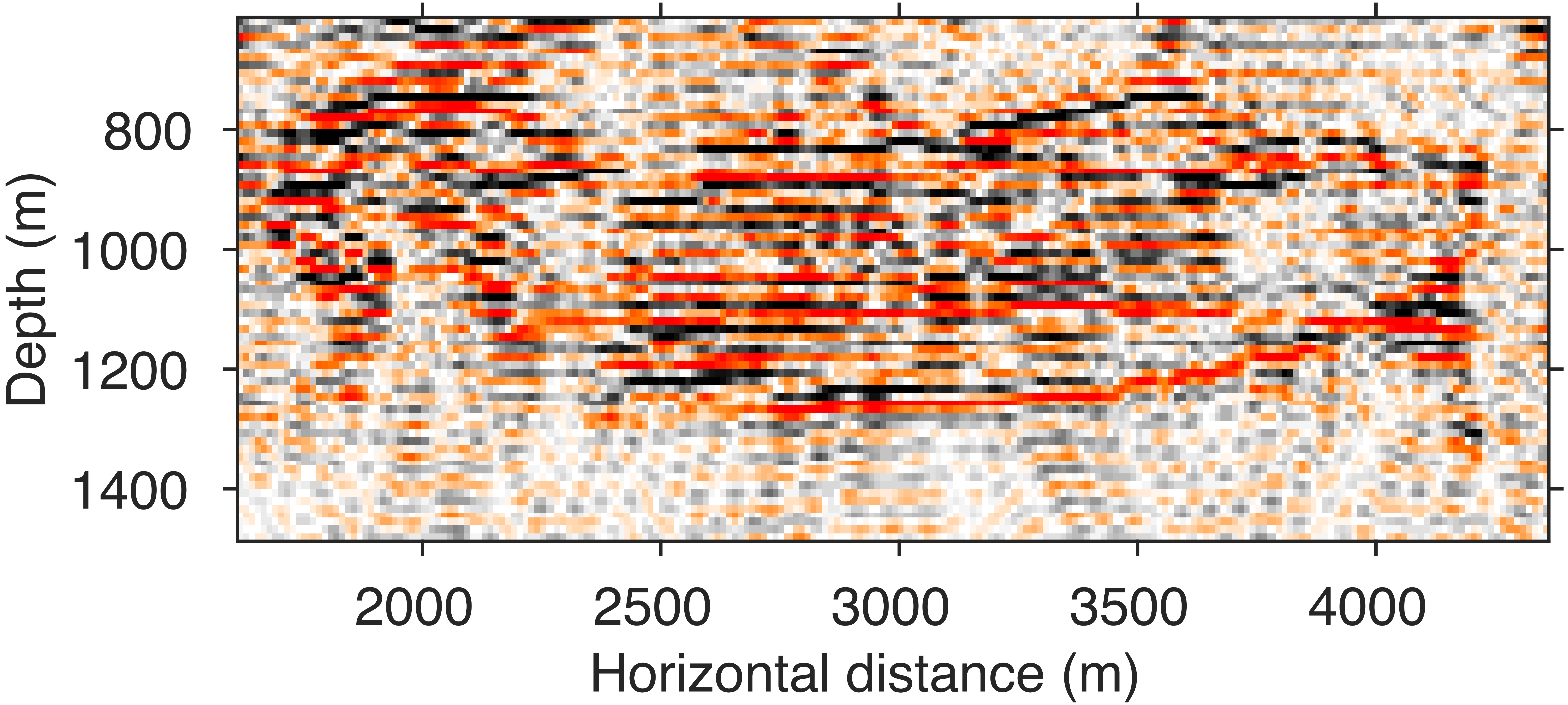
RTM Image



Joint LSM

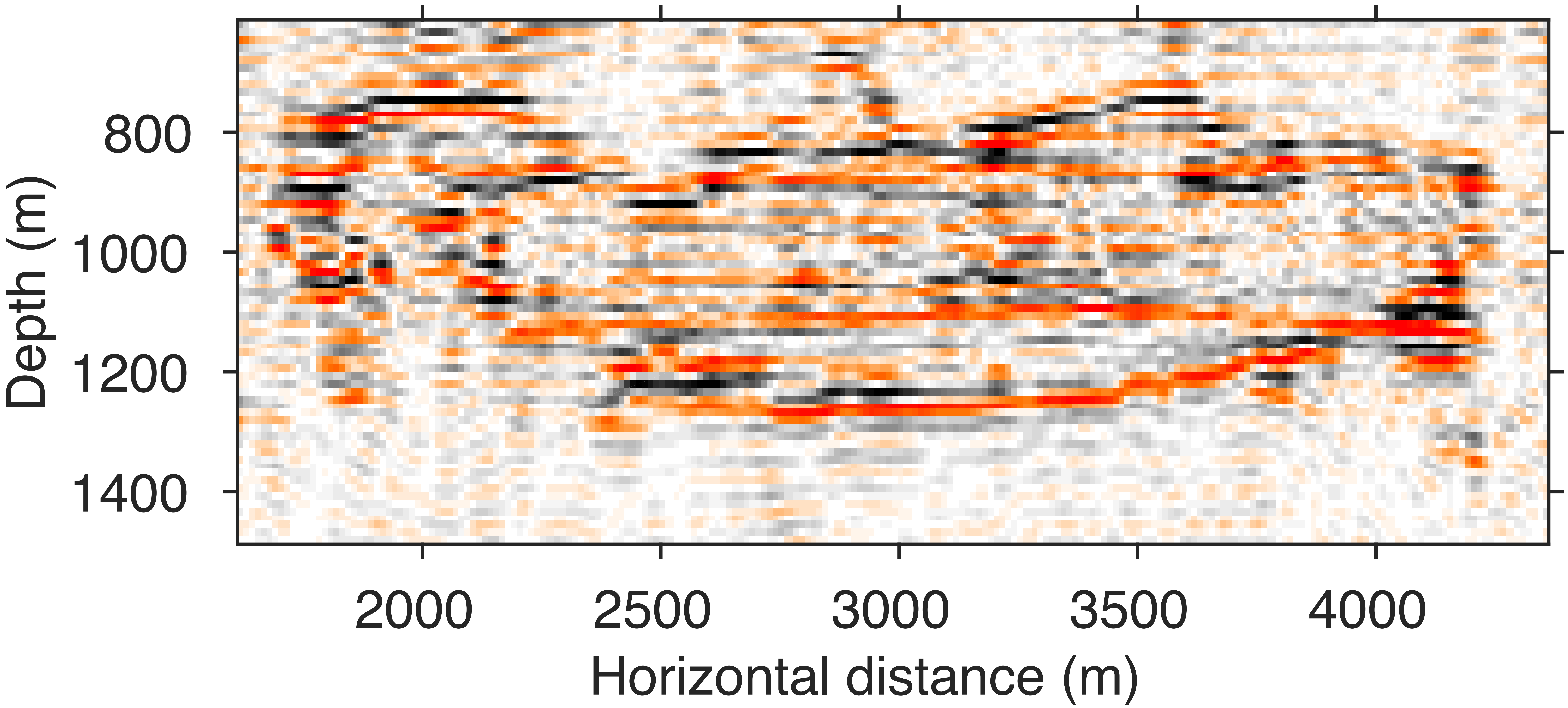


Time-lapse Image



Independent LSM

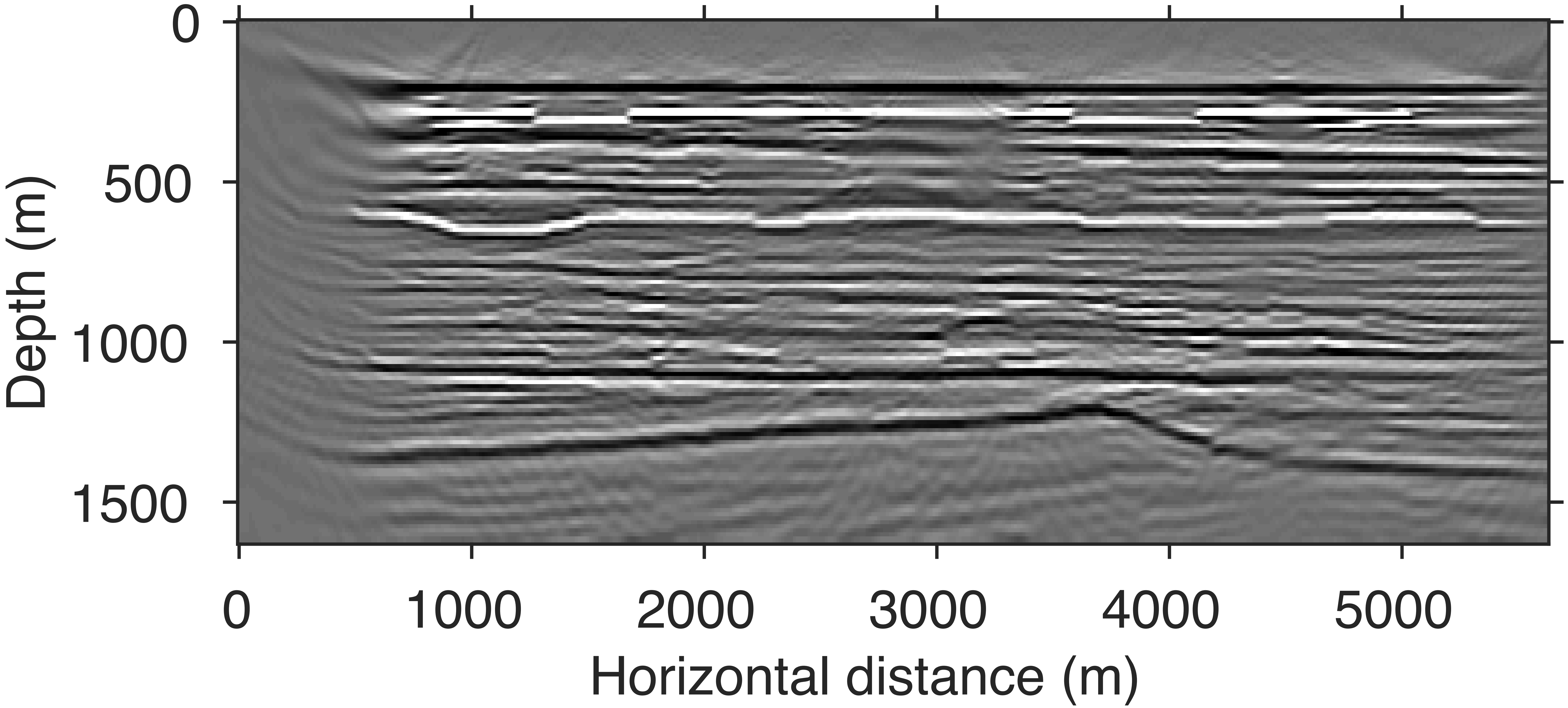
Time-lapse Image



Joint
LSM

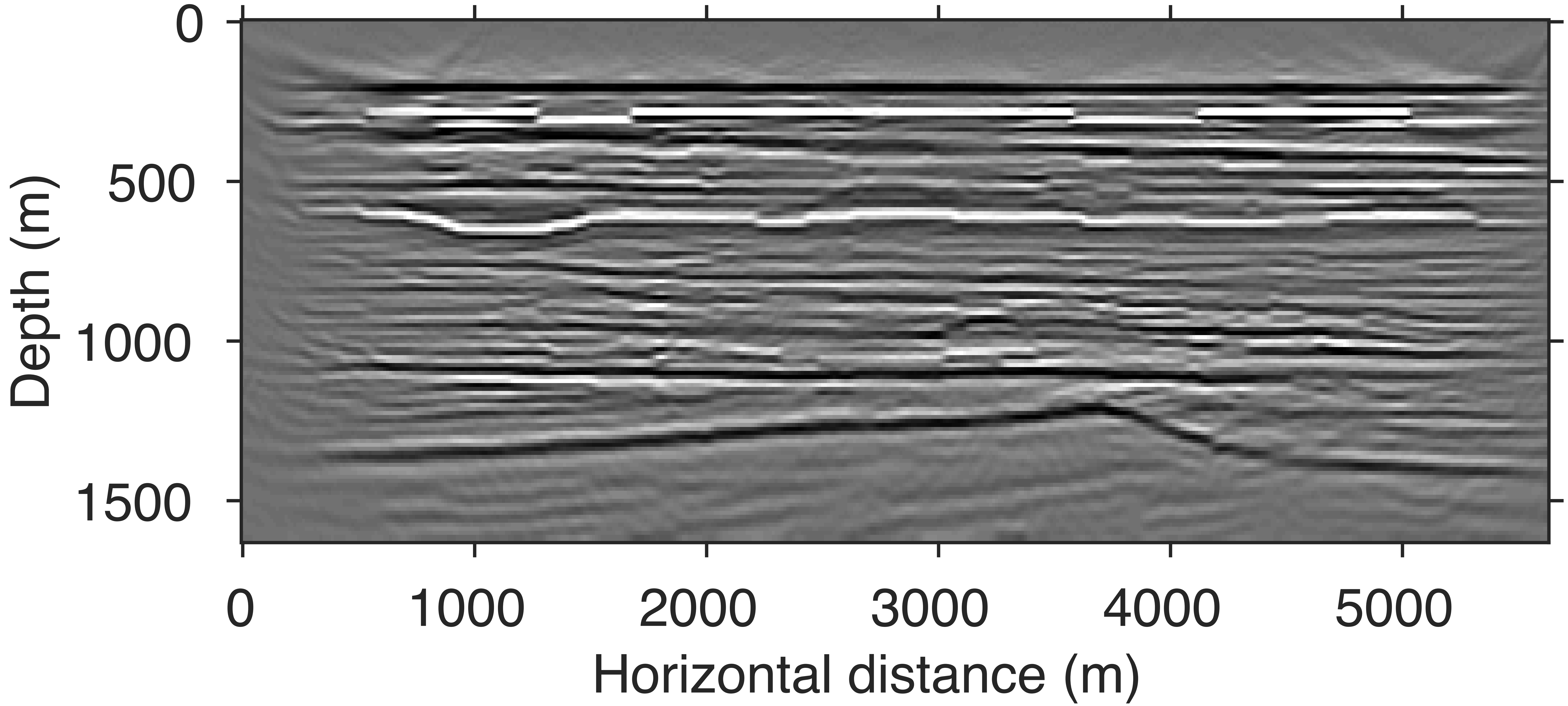
What happens when there is a gap in the monitor data?
How do we deal with the acquisition footprint?

Inversion results with 500m gap



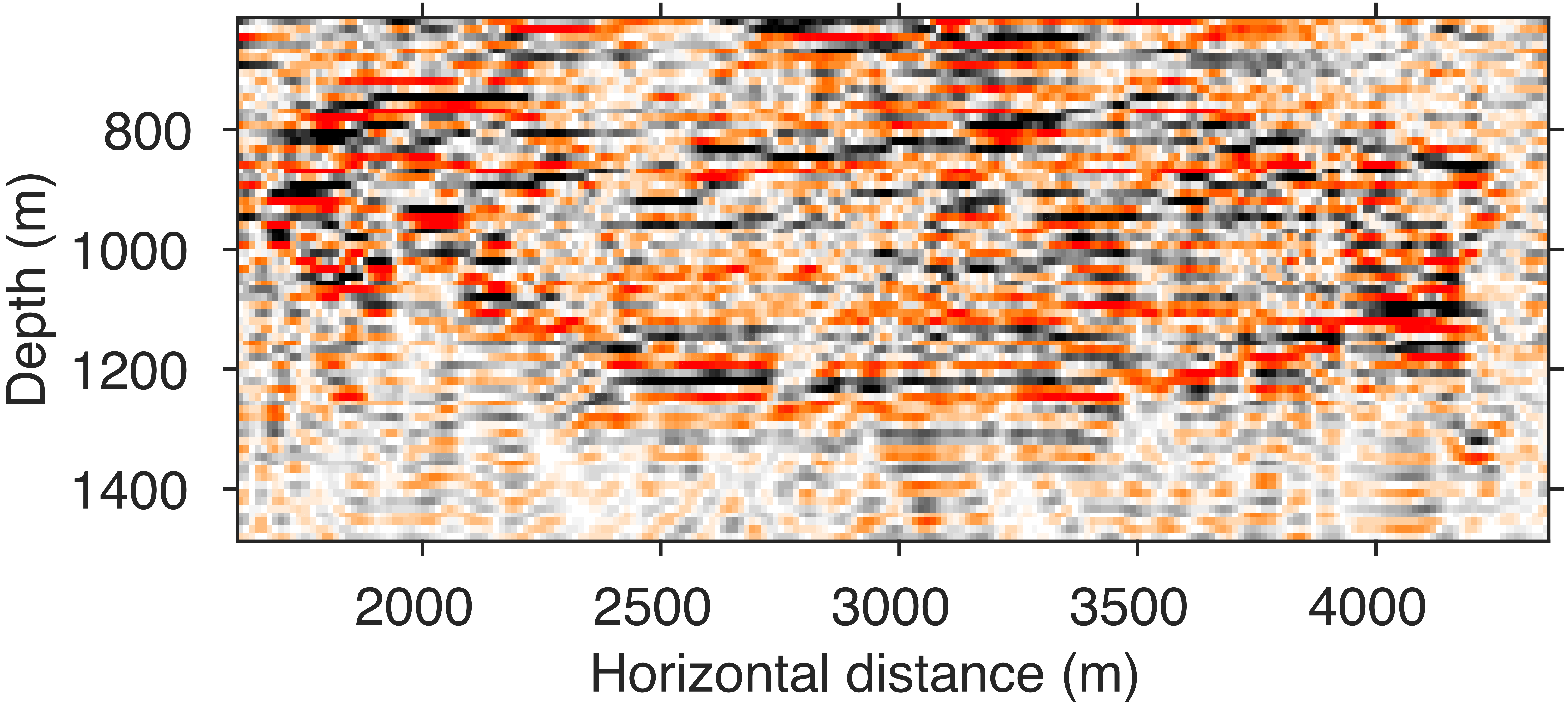
Independent LSM

Inversion results with 500m gap



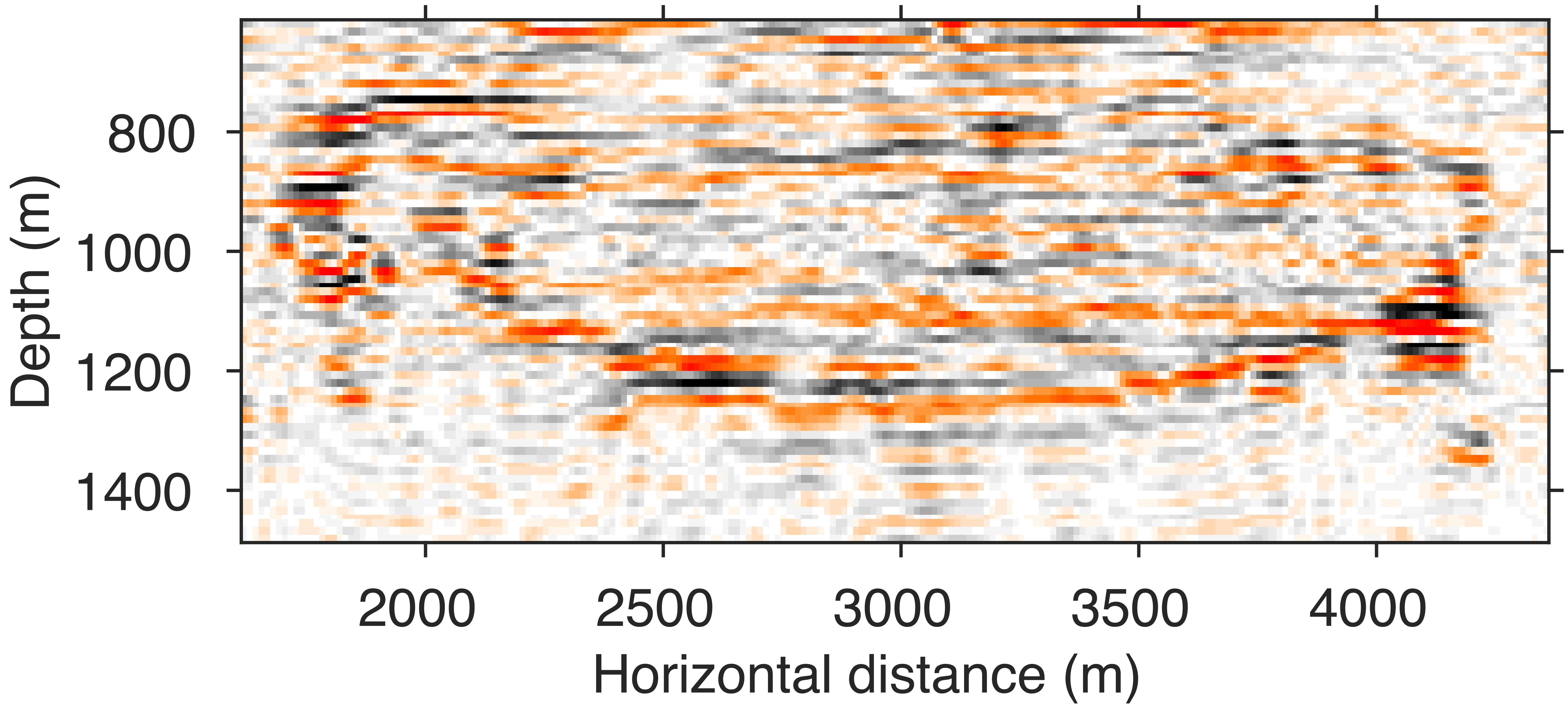
Joint
LSM

Inversion results with 500m gap



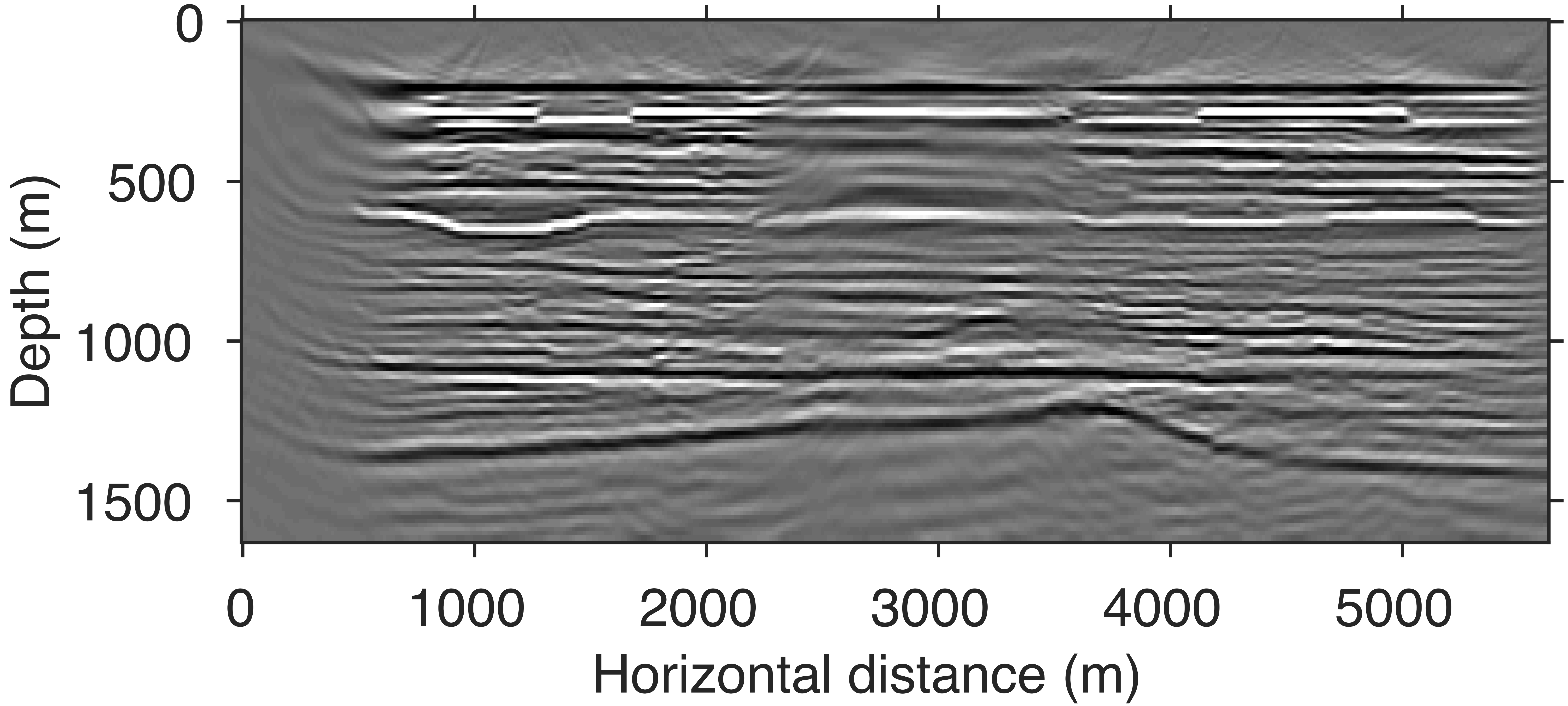
Independent LSM

Inversion results with 500m gap



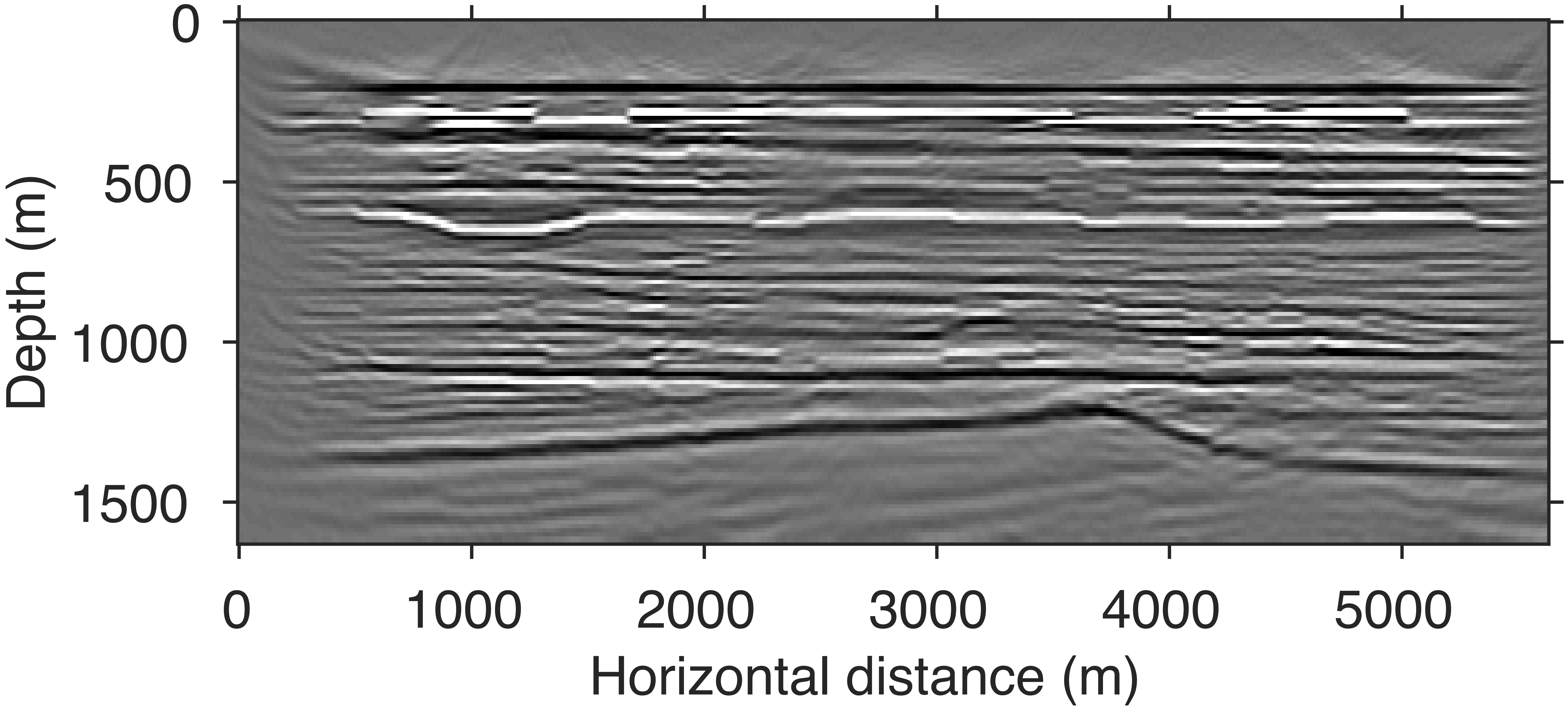
Joint
LSM

Inversion results with 1000m gap



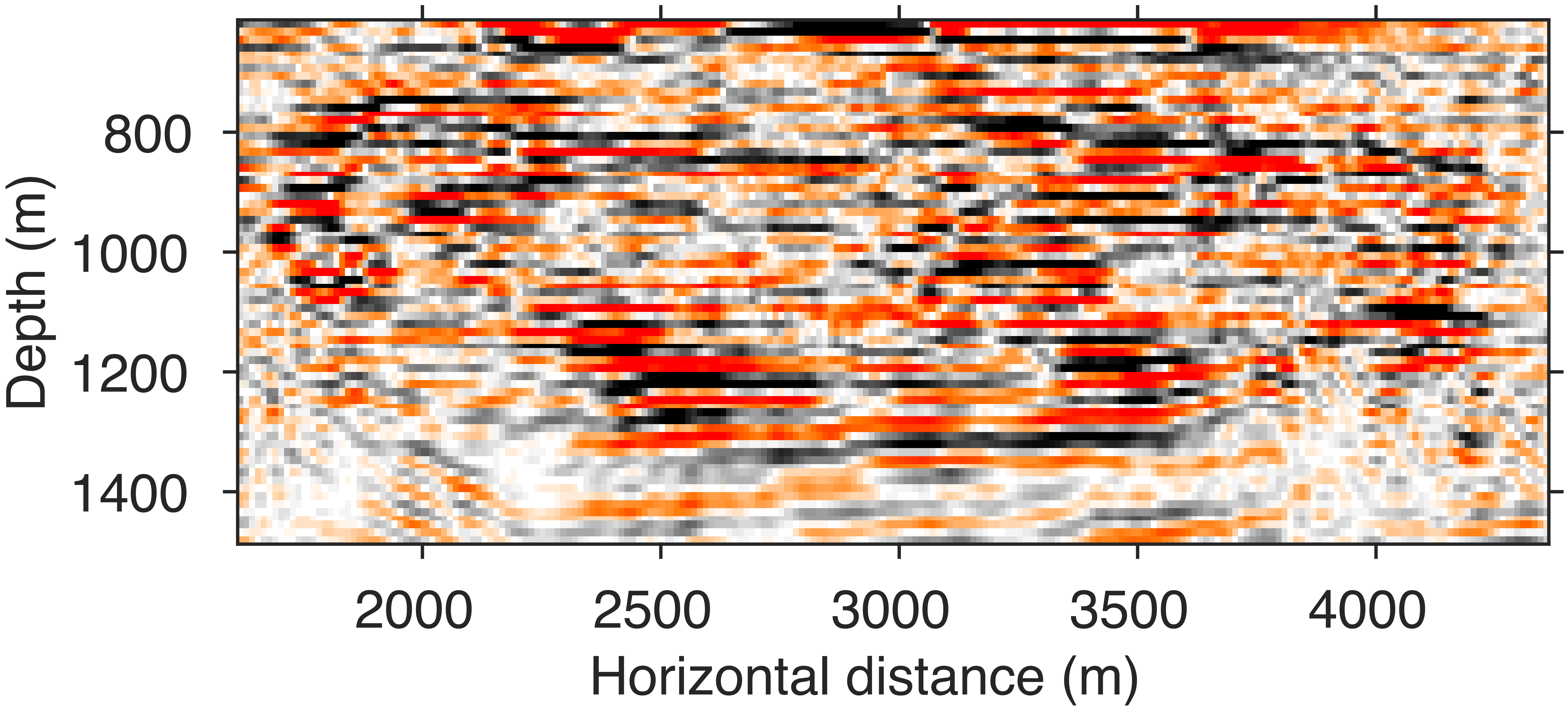
Independent
LSM

Inversion results with 1000m gap



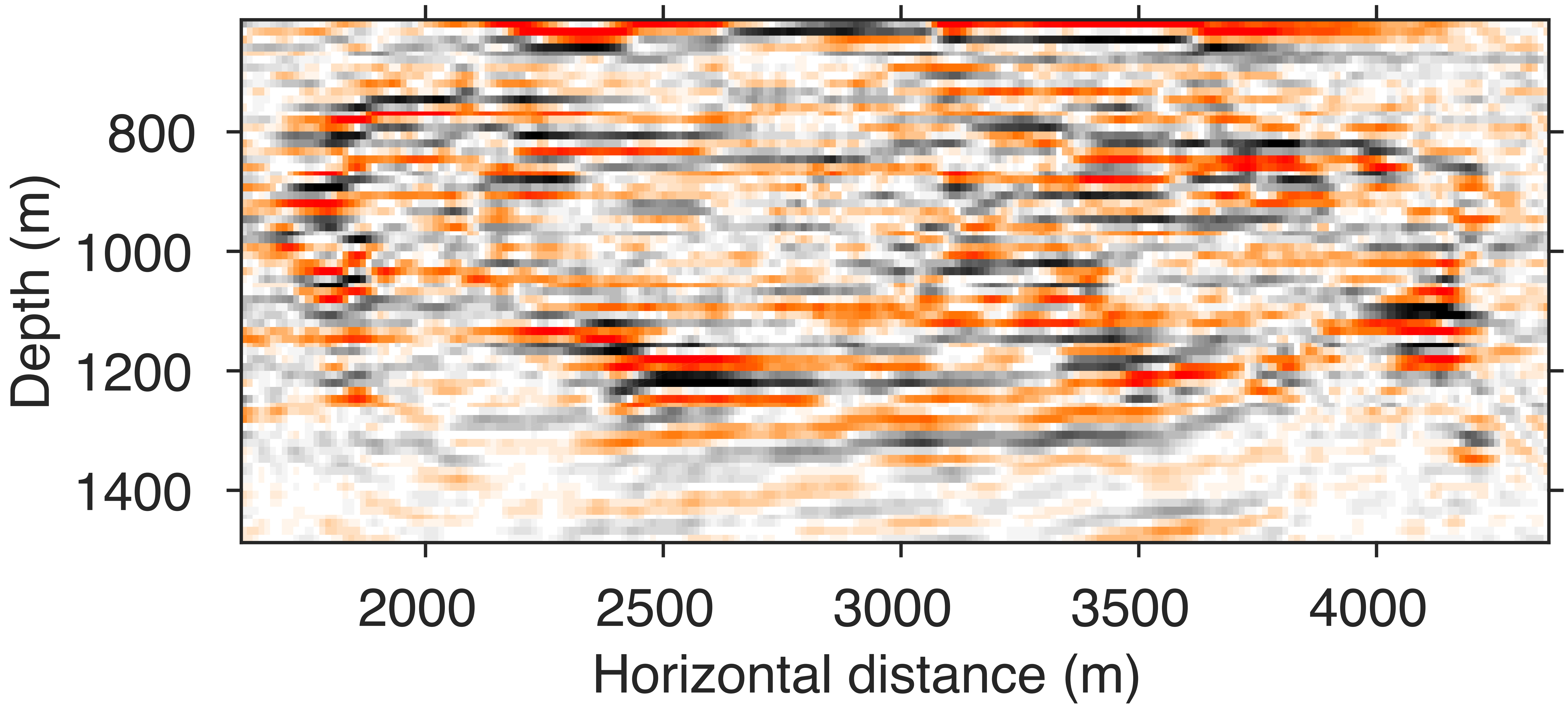
Joint
LSM

Inversion results with 1000m gap



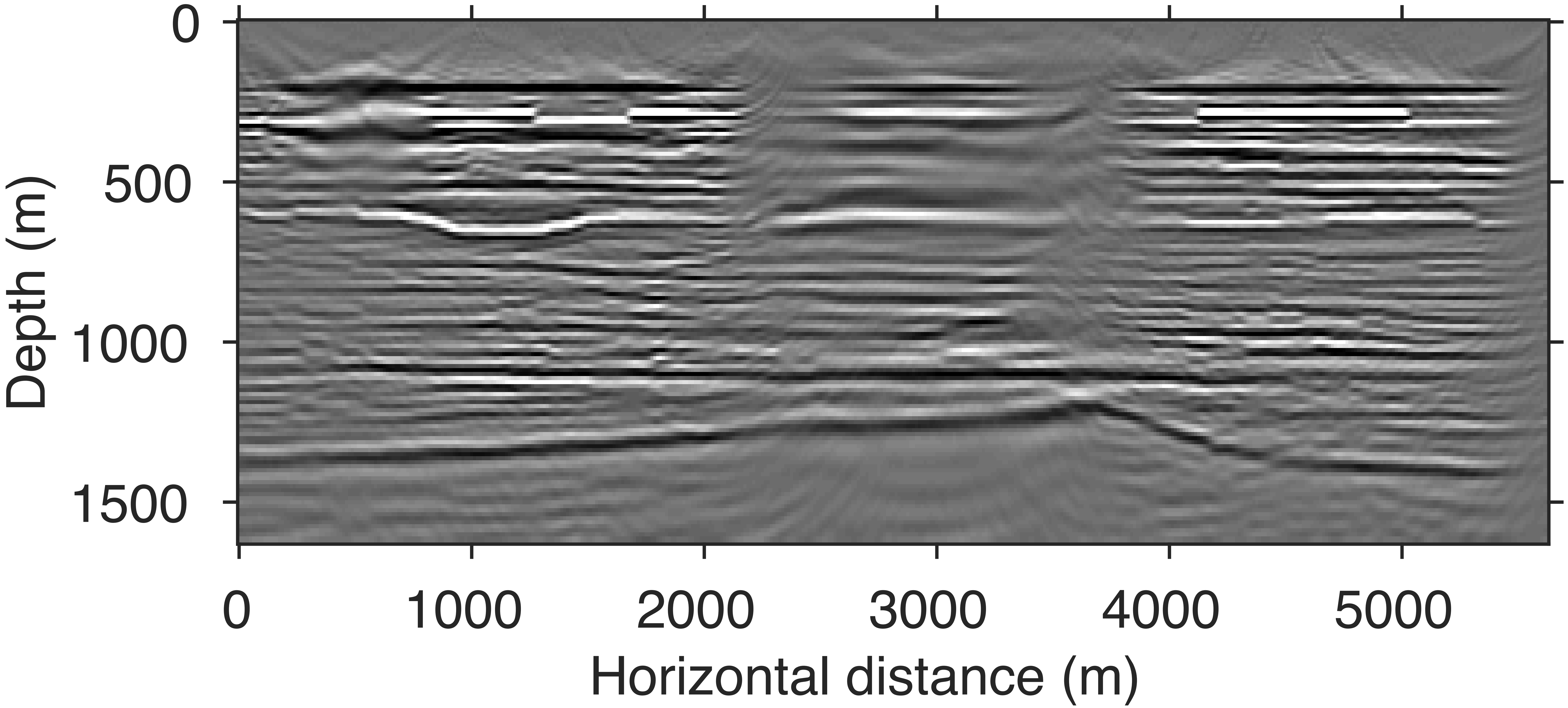
Independent LSM

Inversion results with 1000m gap



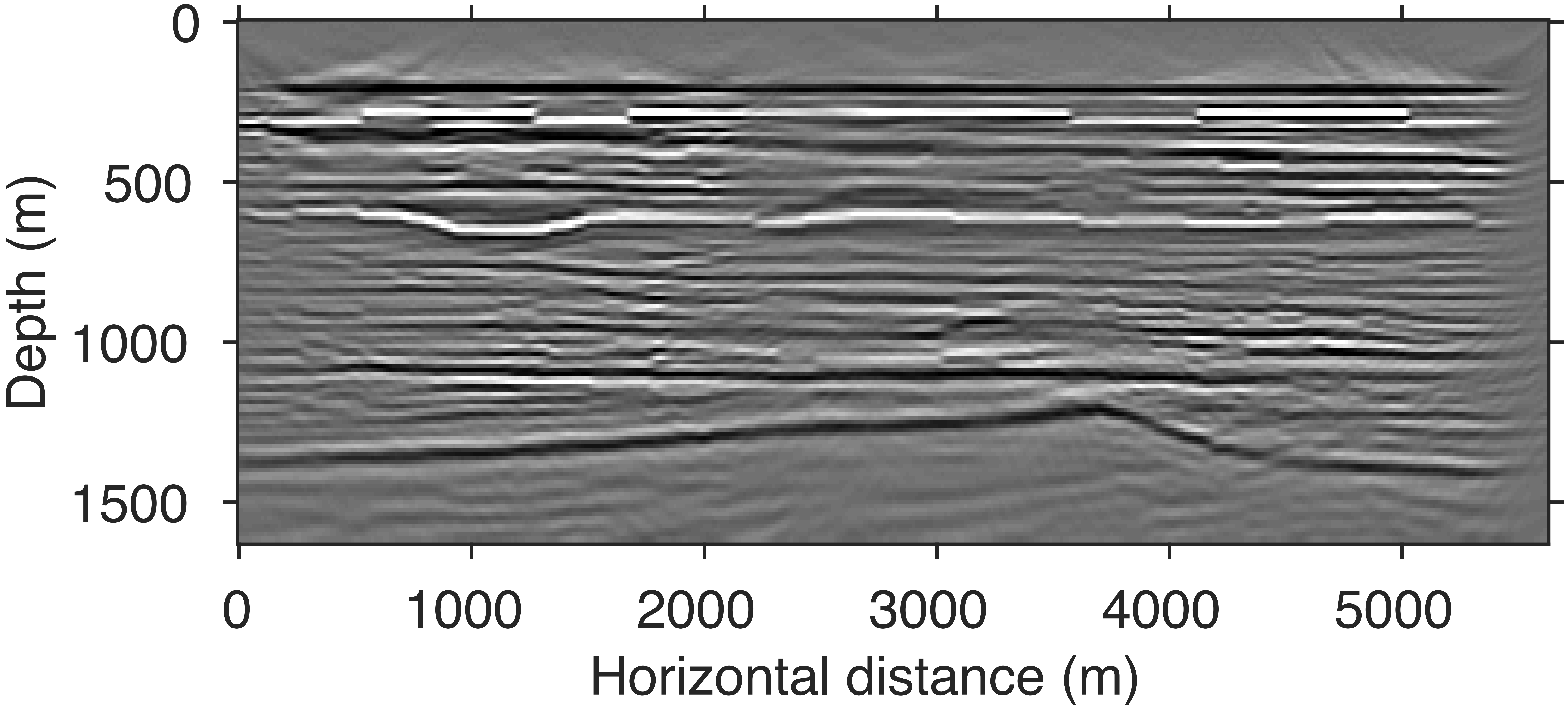
Joint LSM

Inversion results with 1500m gap



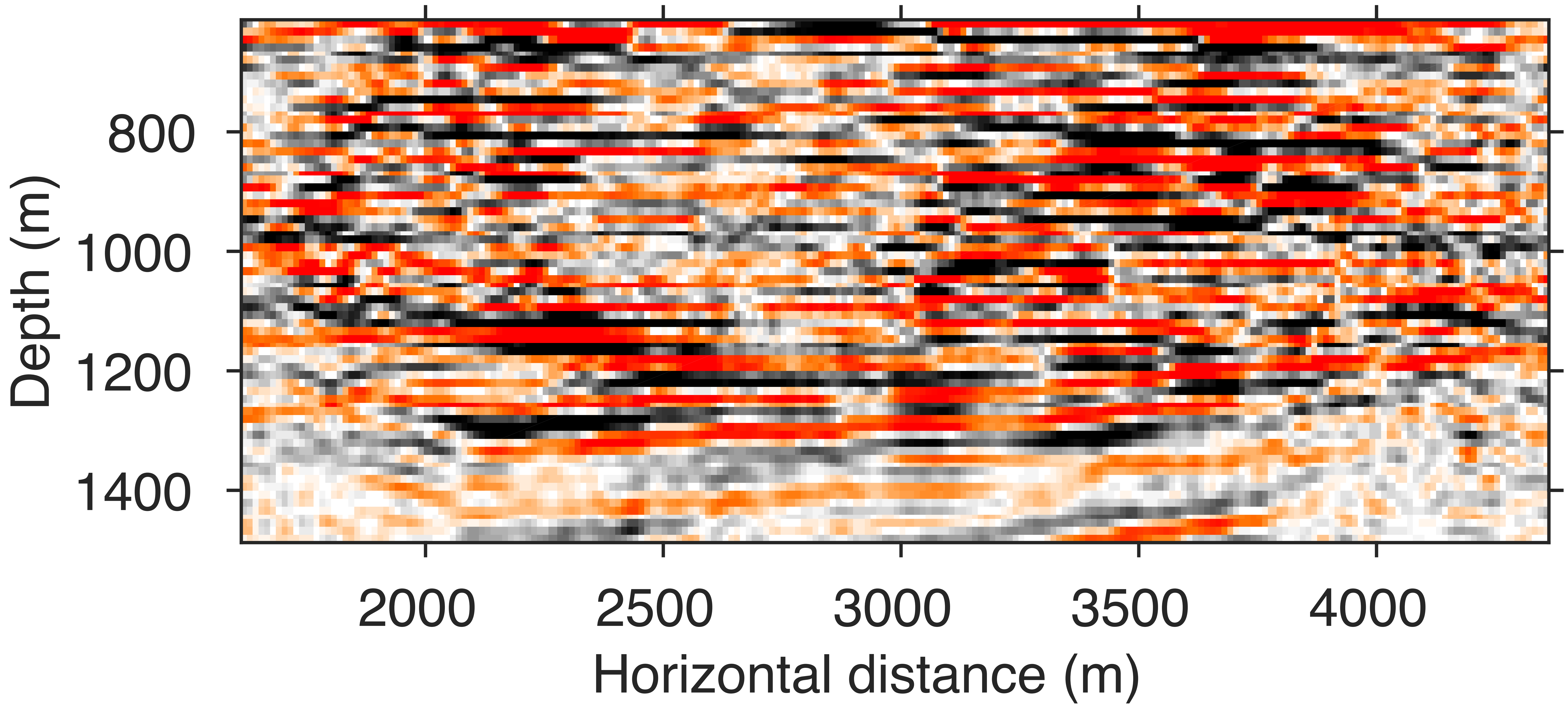
Independent LSM

Inversion results with 1500m gap



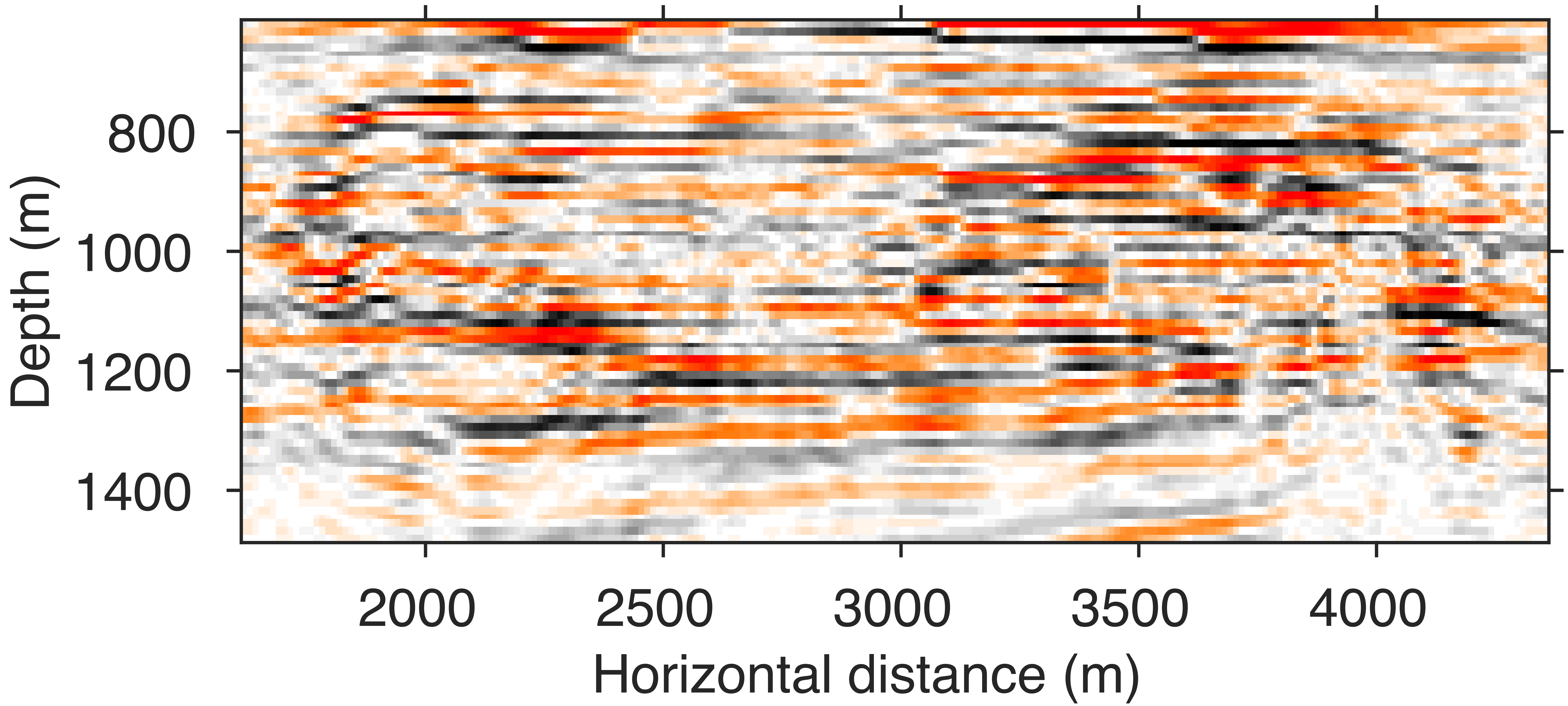
Joint
LSM

Inversion results with 1500m gap



Independent LSM

Inversion results with 1500m gap



Joint
LSM

Summary

Randomized sampling techniques may be extended to time-lapse seismic surveys and processing.

Speed-up imaging using subsets of data via sparsity-promotion.

Process time-lapse data jointly, not independently, in order to exploit the *shared* information.

Joint recovery method still fairly stable with respect to large acquisition gaps.

Provided we understand the *physics* of our model, we can reconstruct, process and interpret time-lapse vintages accurately.

Application to FWI

-Nonlinear inversion of time-lapse data

Xiang Li, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann, "Fast randomized full-waveform inversion with compressive sensing", *Geophysics*, vol. 77, p. A13-A17, 2012.

SLIM

Sparsity-promoting Gauss-Newton

$$\underset{\delta \mathbf{m}_{\mathrm{i}}}{\text{minimize}} \ \frac{1}{2}\|\mathbf{D}_{\mathrm{i}} - \boldsymbol{\mathcal{F}}(\mathbf{m}_{\mathrm{i}}^k; q_{\mathrm{i}}) - \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_{\mathrm{i}}^k; q_{\mathrm{i}})\mathbf{C}^T\mathbf{x}_{\mathrm{i}}\|_2^2 \quad \text{s.t} \quad \|\mathbf{x}_{\mathrm{i}}\|_1 < \tau_{\mathrm{i}}$$

$\mathbf{D}$: observed data

$\boldsymbol{\mathcal{F}}$: forward modelling kernel

$\mathbf{m}$: model parameters

$\nabla\boldsymbol{\mathcal{F}}$: Jacobian

q : source function

$\mathbf{C}$: curvelet transform

$\delta\mathbf{m}$: model update

$$\delta\mathbf{m}_{\mathrm{i}} = \mathbf{C}^T\mathbf{x}_{\mathrm{i}}$$

$$\mathbf{m}_{\mathrm{i}}^{k+1} = \mathbf{m}_{\mathrm{i}}^k + \delta\mathbf{m}_{\mathrm{i}}$$

Inversion with JRM

$$\tilde{\mathbf{z}} = \arg\min_{\mathbf{z}} \frac{1}{2}\|\mathbf{b} - \mathbf{A}\mathbf{z}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{z}\|_1 < \tau$$

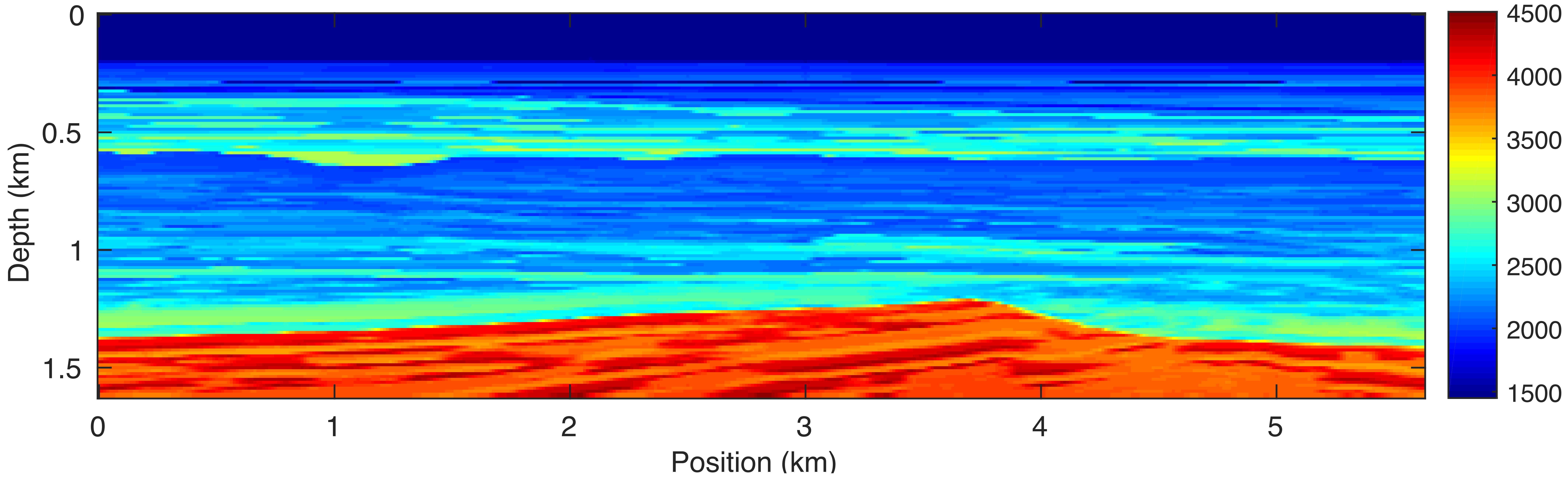
$$\mathbf{A}_{\mathrm{i}} = \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_{\mathrm{i}}^k; \bar{q}_{\mathrm{i}})\mathbf{C}^T$$

$$\mathbf{b}_{\mathrm{i}} = \bar{\mathbf{D}}_{\mathrm{i}} - \boldsymbol{\mathcal{F}}(\mathbf{m}_{\mathrm{i}}^k; \bar{q}_{\mathrm{i}})$$

$$\delta\mathbf{m}_{\mathrm{i}} = \mathbf{C}^T(\tilde{\mathbf{z}}_0 + \tilde{\mathbf{z}}_{\mathrm{i}})$$

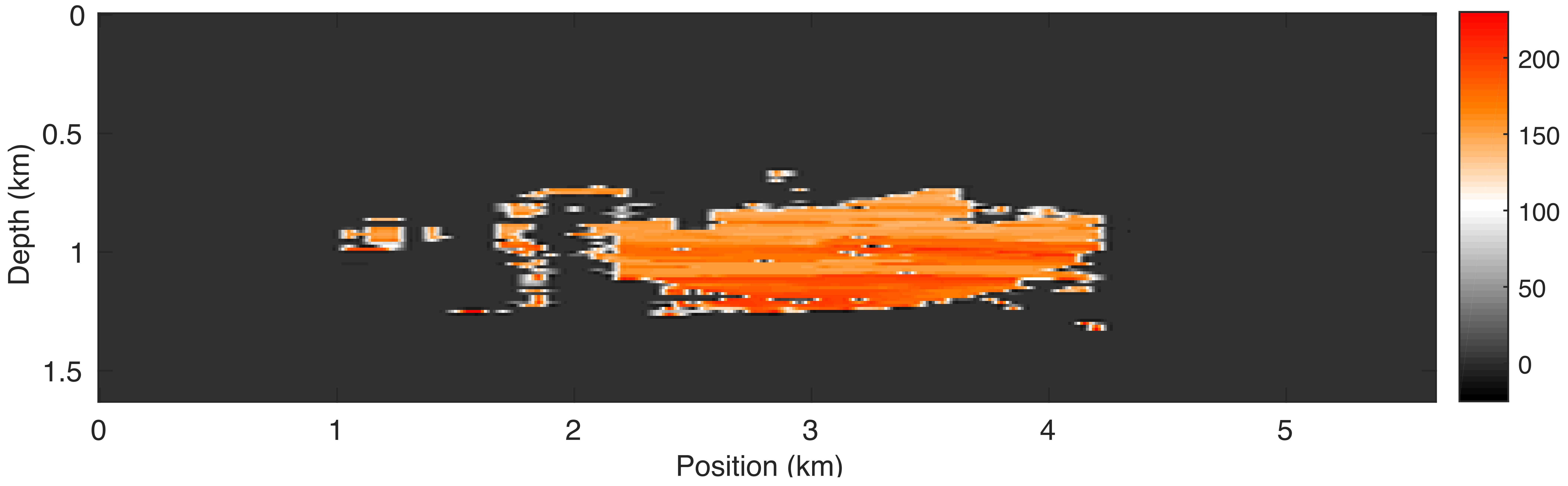
$$\mathbf{m}_{\mathrm{i}}^{k+1} = \mathbf{m}_{\mathrm{i}}^k + \delta\mathbf{m}_{\mathrm{i}}$$

Baseline



Depth (km)
0
0.5
1
1.5
0
1
2
3
4
5
Position (km)
4500
4000
3500
3000
2500
2000
1500

Difference



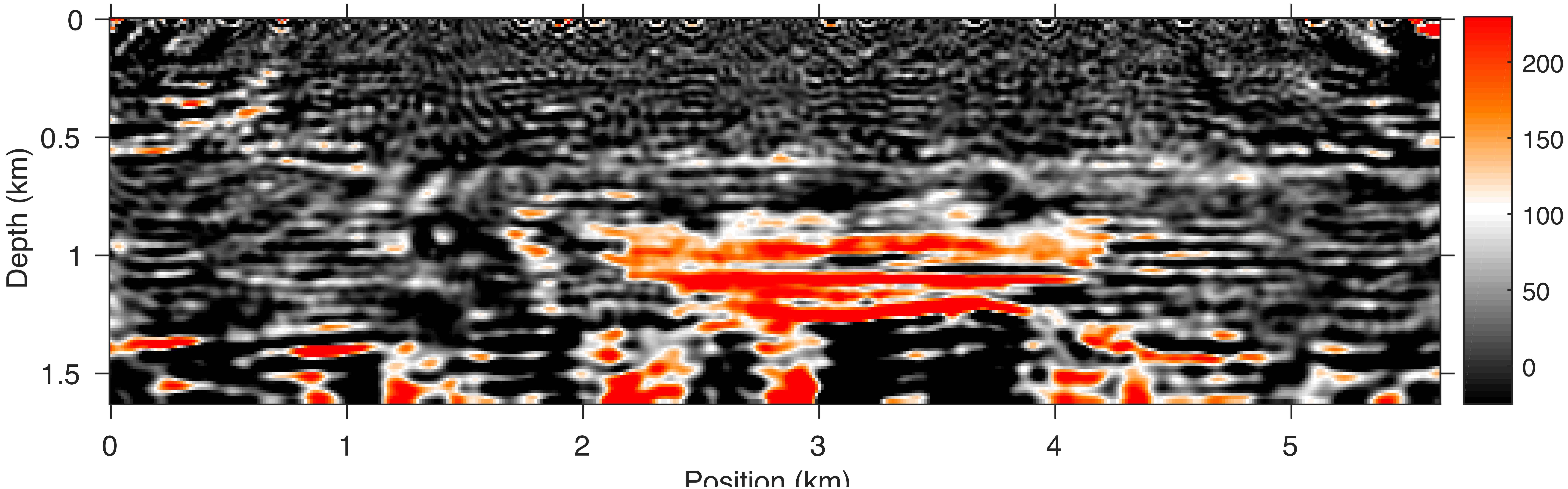
Set-up

Modeling parameters
38 shots (150m spacing)
113 receivers (50m spacing)
Different acquisition geometry
Ricker wavelet @ 12Hz

Modified Gauss-Newton
Assume good starting model
Draw randomly selected shots @ every iteration
Started inversion @ 3Hz to 20Hz
8 frequencies per band
10 Gauss-Newton subproblems
10 iterations per subproblem

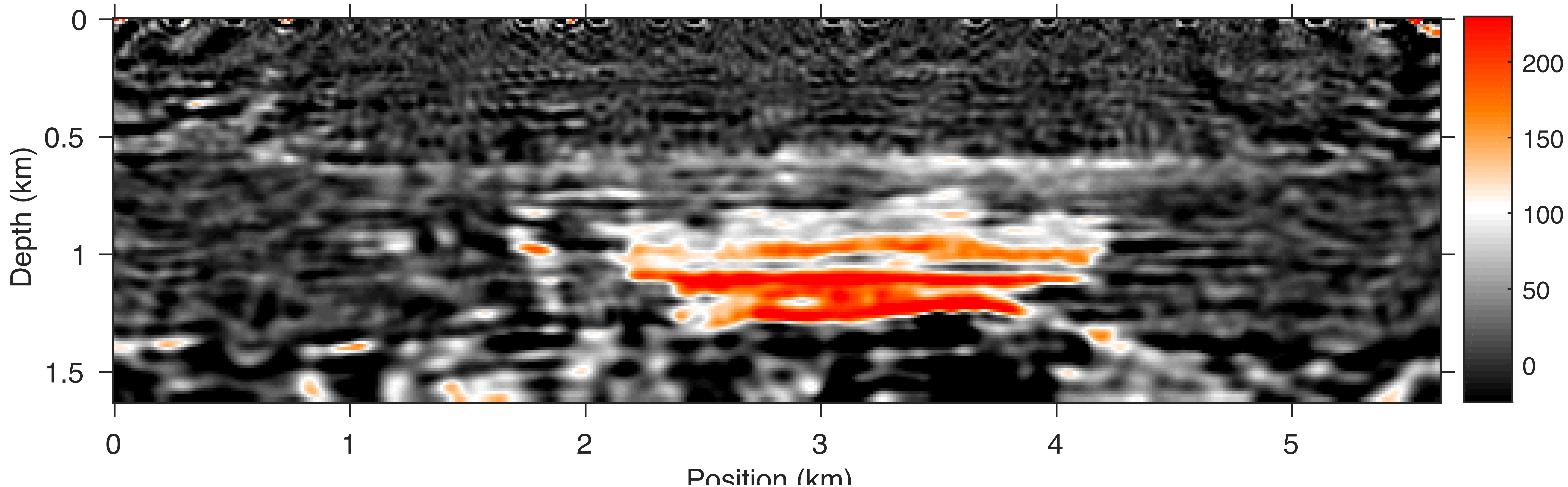
Independent inversion

SNR = -4.5dB



Joint inversion

SNR = 1.4 dB



Summary

With subsampled time-lapse data, we can perform 4D FWI using a modified Gauss-Newton inversion combined with JRM.

Significant attenuation of artifacts in the time-lapse difference model obtained from joint inversion, giving improved signal to noise ratio

Recommend using the JRM inverted models for subsequent migration.

Future work

Timelapse imaging with multiples

Asymmetric acquisition geometry

Multiple vintages

3-D linear/non-linear inversion of time-lapse data set

Acknowledgements

Thank you for your attention!

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