Deterministic Matrix Completion: Applications to Seismic Trace Interpolation

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Motivation

- acquisition challenges
  - missing data

- exploit *low-rank* structure of seismic data
  - *SVD-free* matrix completion (2D & 3D)

- need analysis
  - consider *general* subsampling schemes
  - reconstruction guarantees
  - how should we subsample?

Contributions

- Quantification of subsampling
  - measure "spectral gap"
  - computationally simple

$$\frac{\sigma_2}{\sigma_1}$$

- Application to seismic interpolation
  - optimally design acquisition
  - tools for 3D data

Outline

- Current Work
  - matrix completion analysis
  - seismic trace interpolation

- Deterministic Matrix Completion
  - spectral gap
  - implications for seismic data

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Matrix Completion Literature

Given a matrix $\mathbf{X} \in \mathbb{R}^{n\times m}$ of rank $r \ll \min(m,n)$, we exploit its low dimensional structure to recover $\mathbf{X}$ from limited and noisy samples via

$$\underset{\mathbf{Y}}{\text{minimize}} \|\mathbf{Y}\|_* \text{ subject to } \|P_\Omega(\mathbf{Y}) - \mathbf{b}\|_F \leq \epsilon,$$

where $\mathbf{b}_{i,j} = P_\Omega(\mathbf{X})_{i,j} = \begin{cases} \mathbf{X}_{i,j} & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$

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subsampling scheme

Matrix Completion Literature

Typical Assumptions:

Suppose $|\Omega|$ entries of $\mathbf{X}$ are observed with locations sampled uniformly at random...

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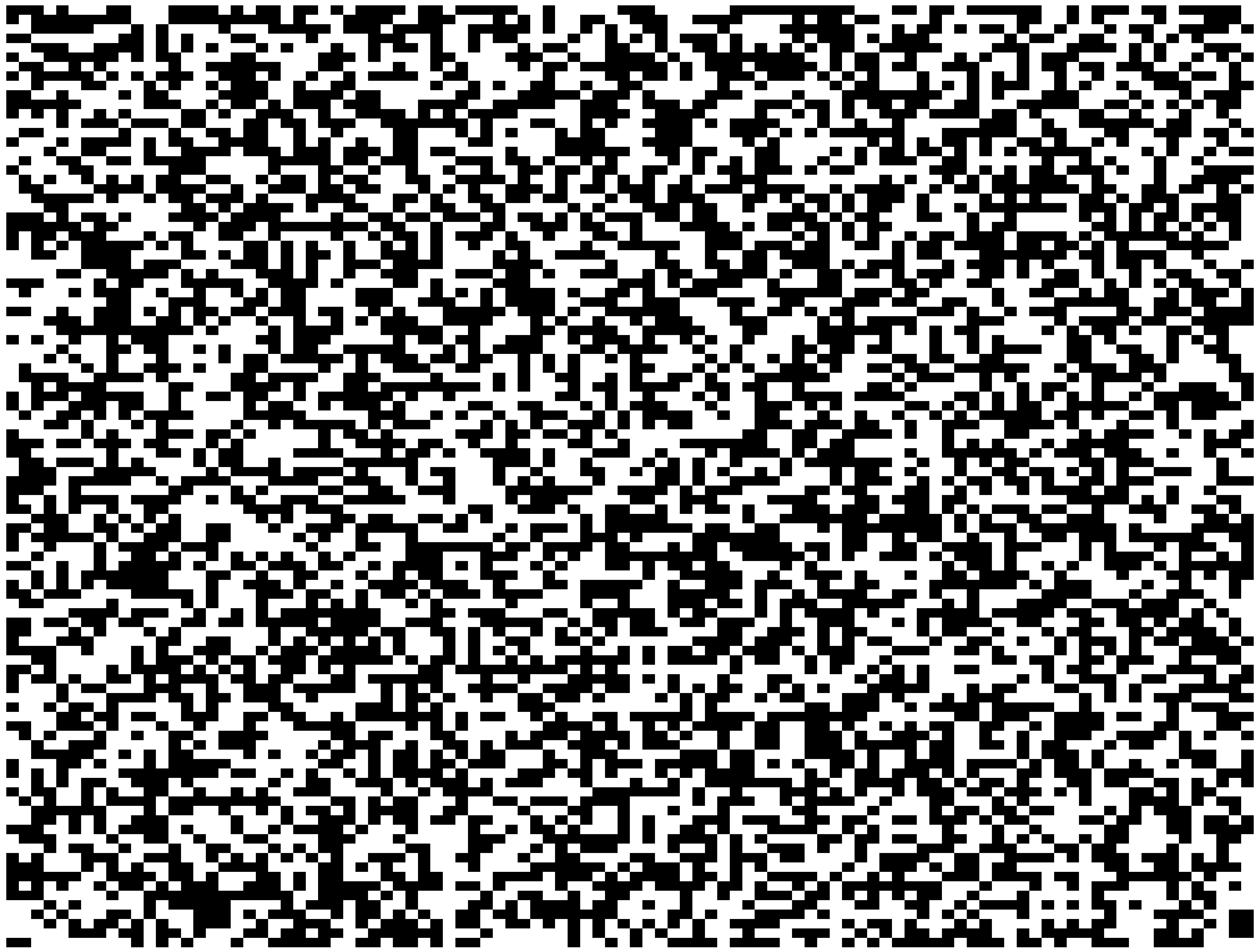
consider the sampling mask $$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$$

Matrix Completion Literature

Assumed sampling mask: Not practical

$$A =$$



■ $= 0$

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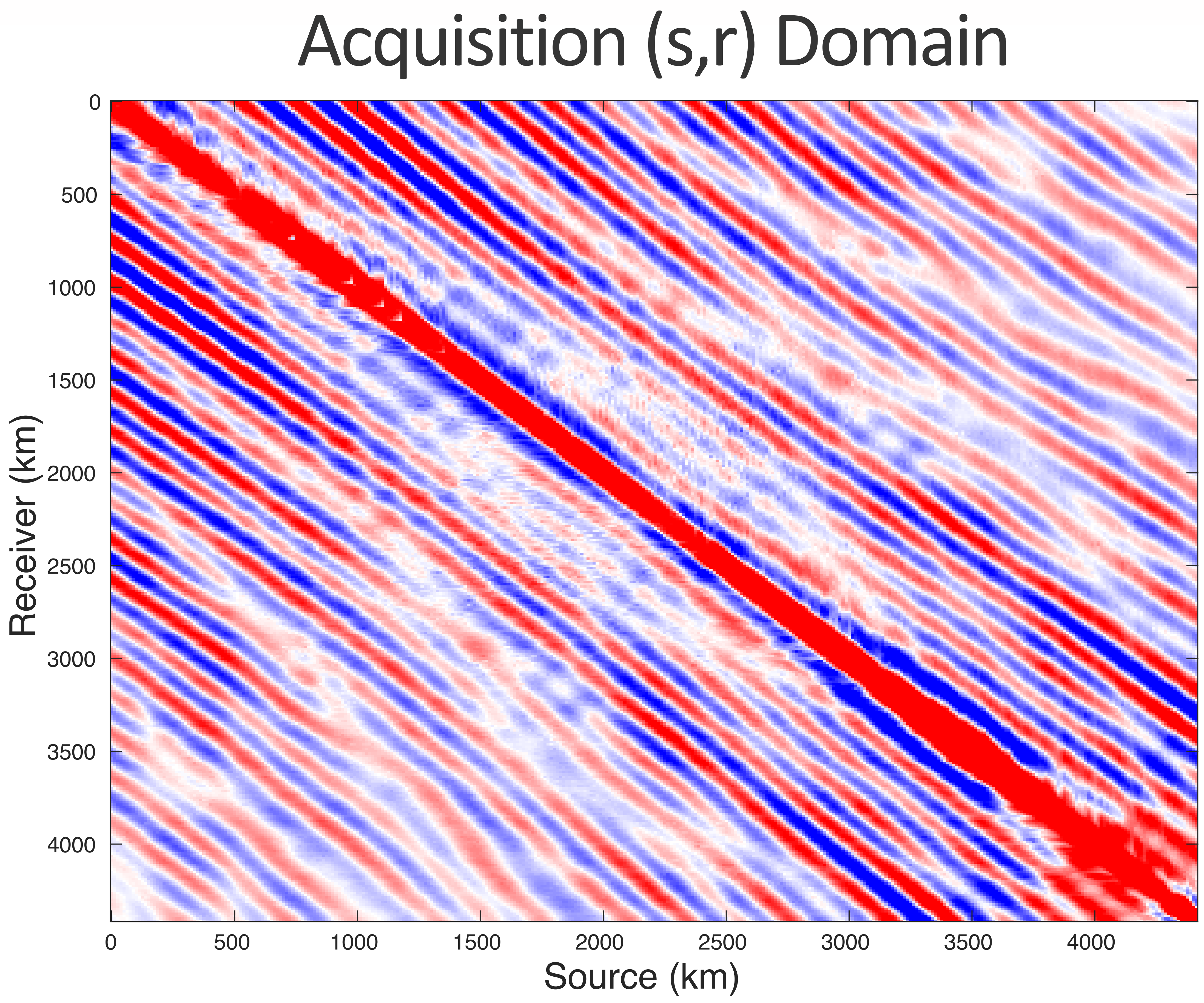
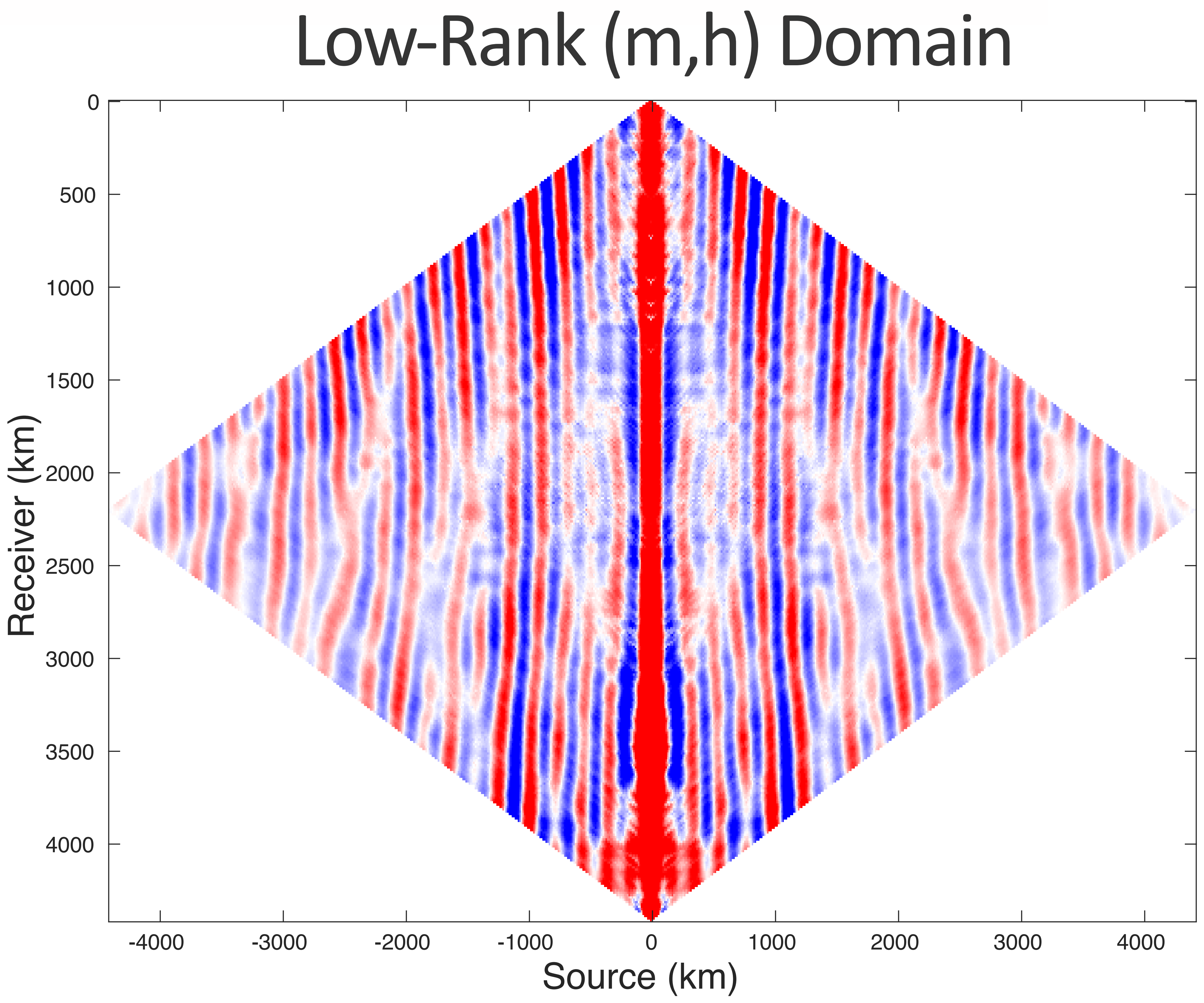
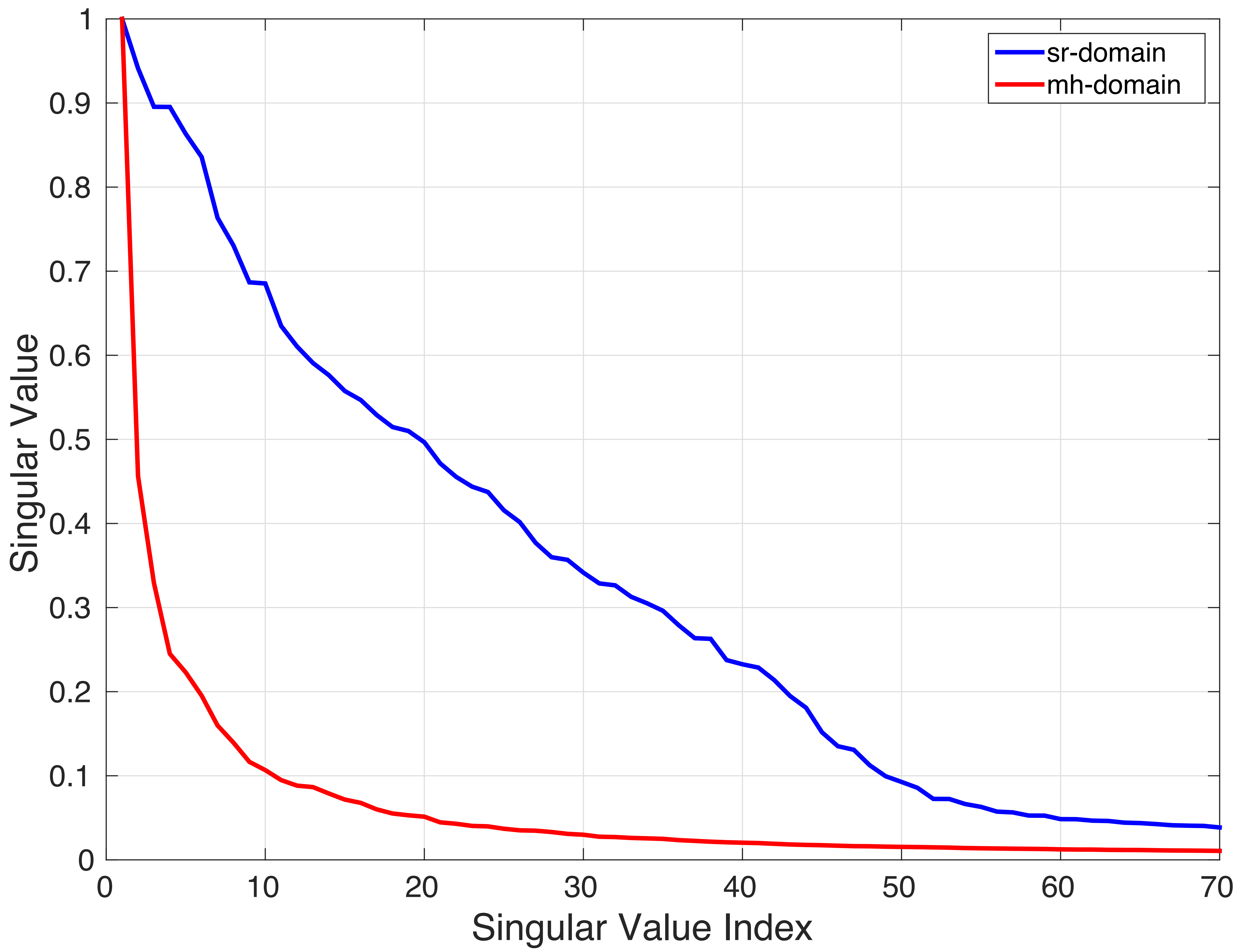
2D Low-rank structure





Singular value decay

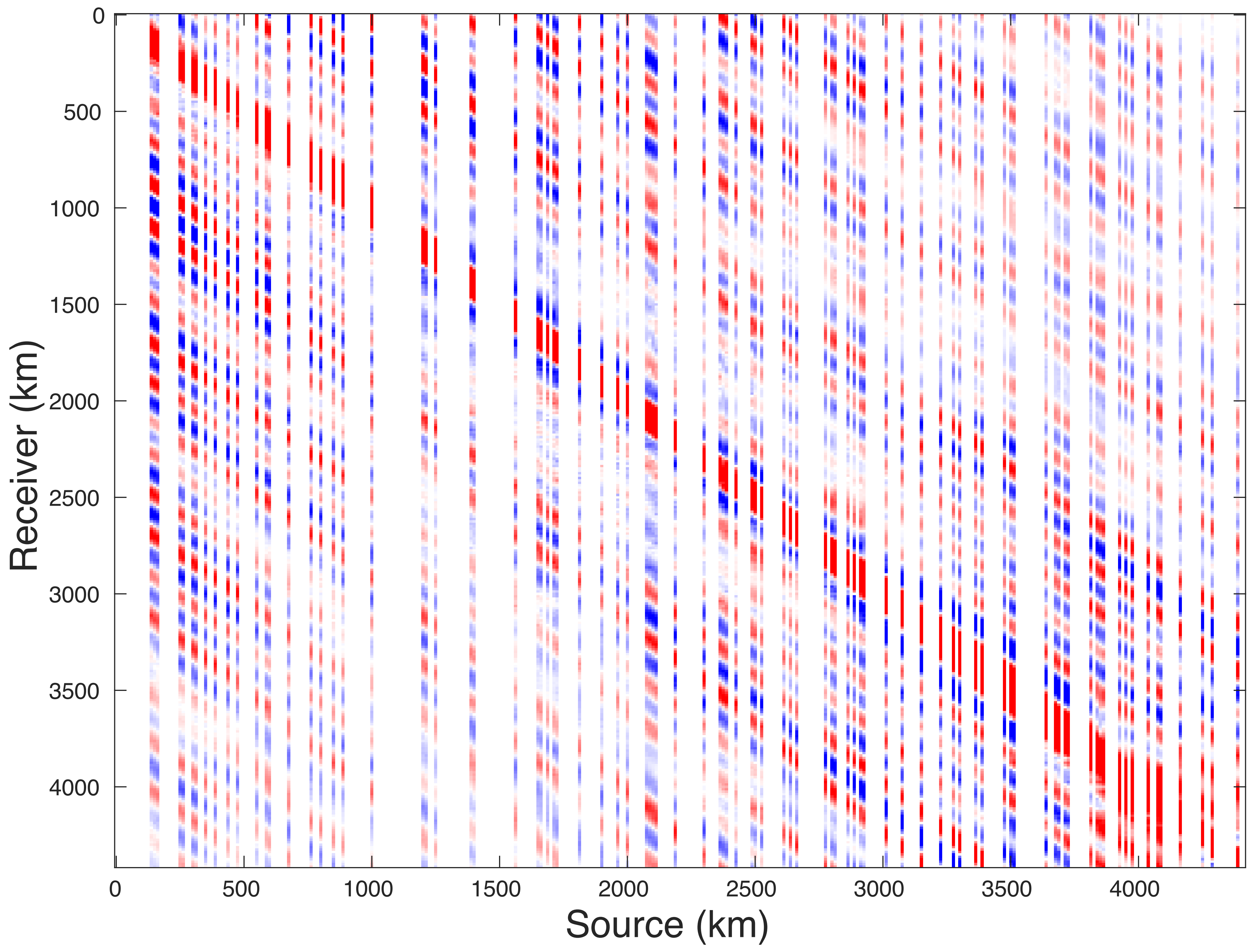
s-r domain vs m-h domain



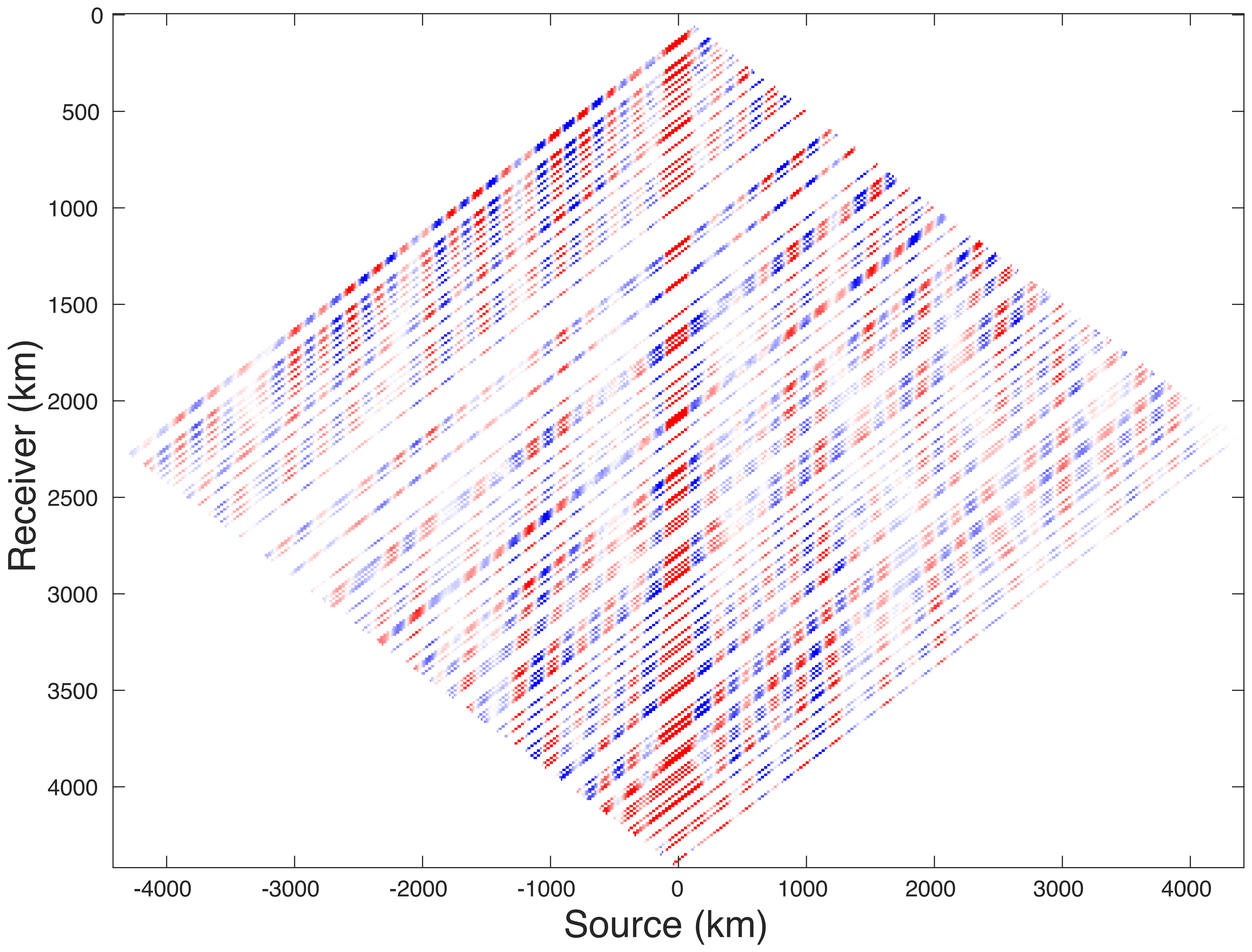
2D Seismic Subsampling

Missing Sources

Acquisition (s,r) Domain

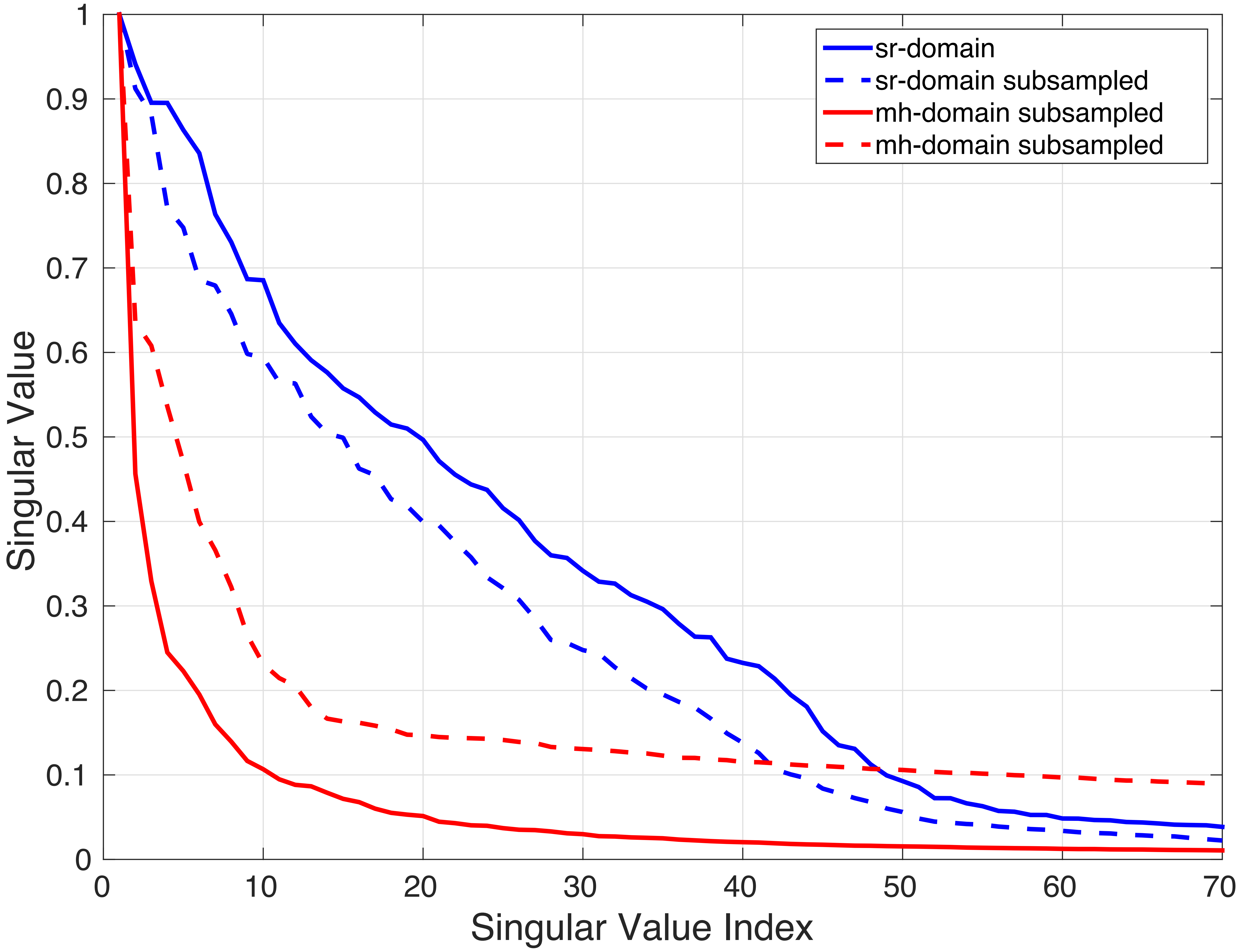


Low-Rank (m,h) Domain

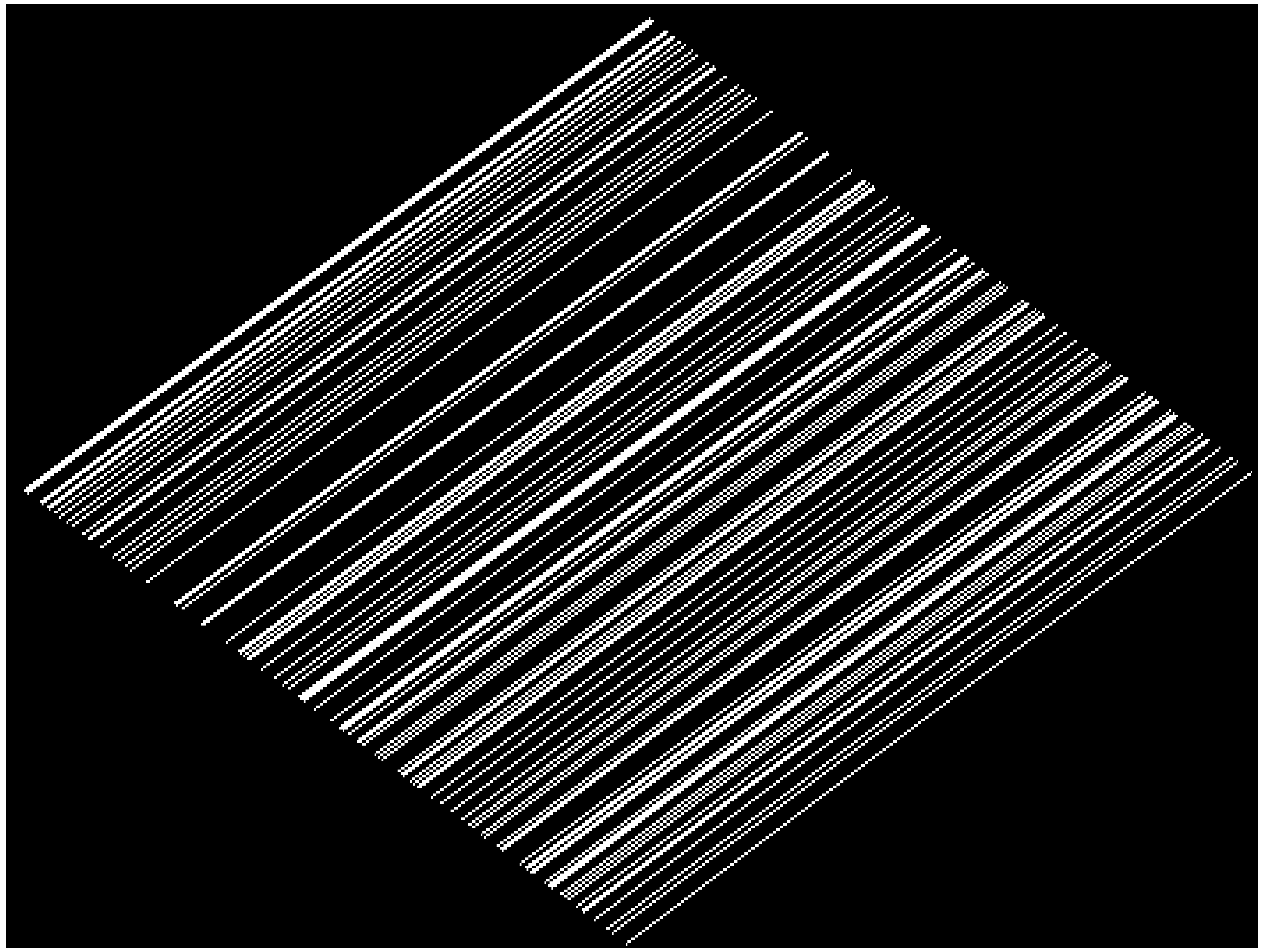


Singular value decay

full data vs subsampled data



2D Seismic Masks



3D Seismic Data Interpolation

- Consider a 3D seismic survey with coordinates (src x, src y, rec x, rec y, time)

- Take a Fourier transform in time and restrict ourselves to
a single frequency slice.

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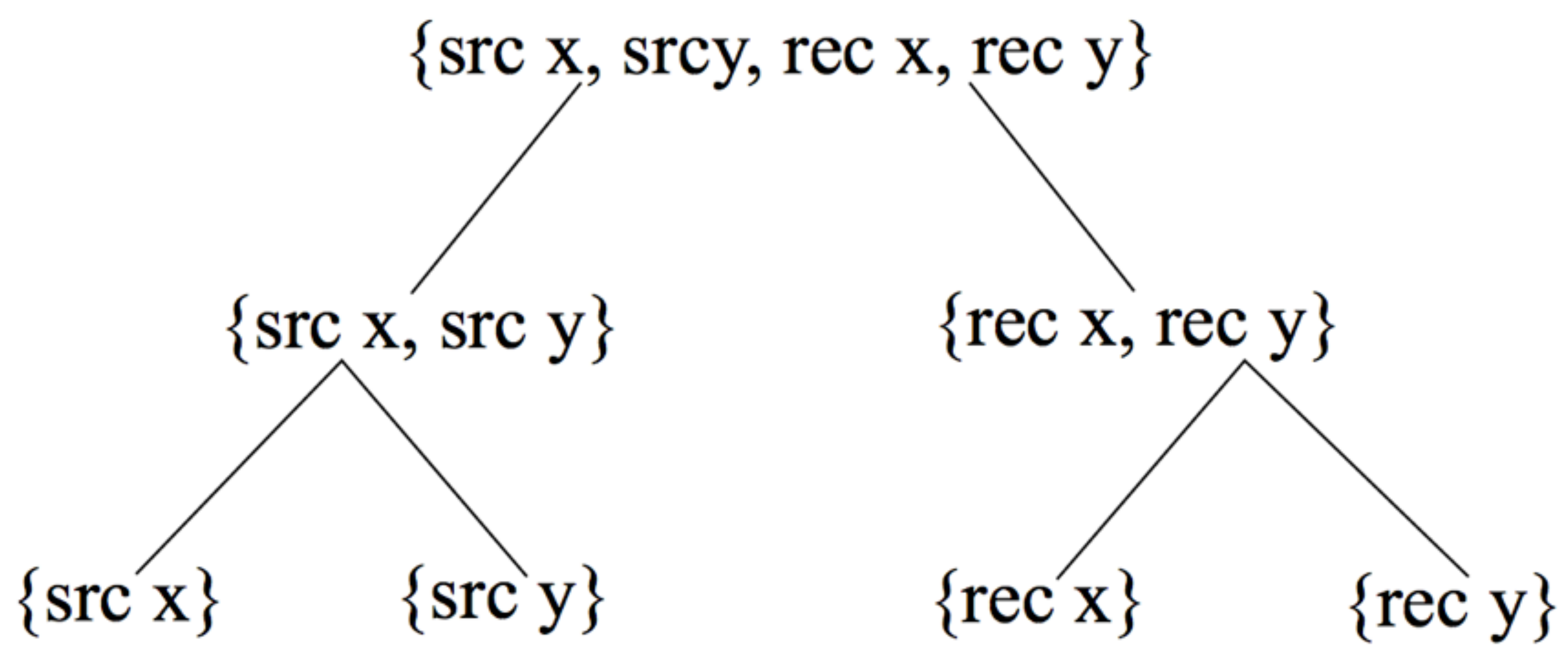
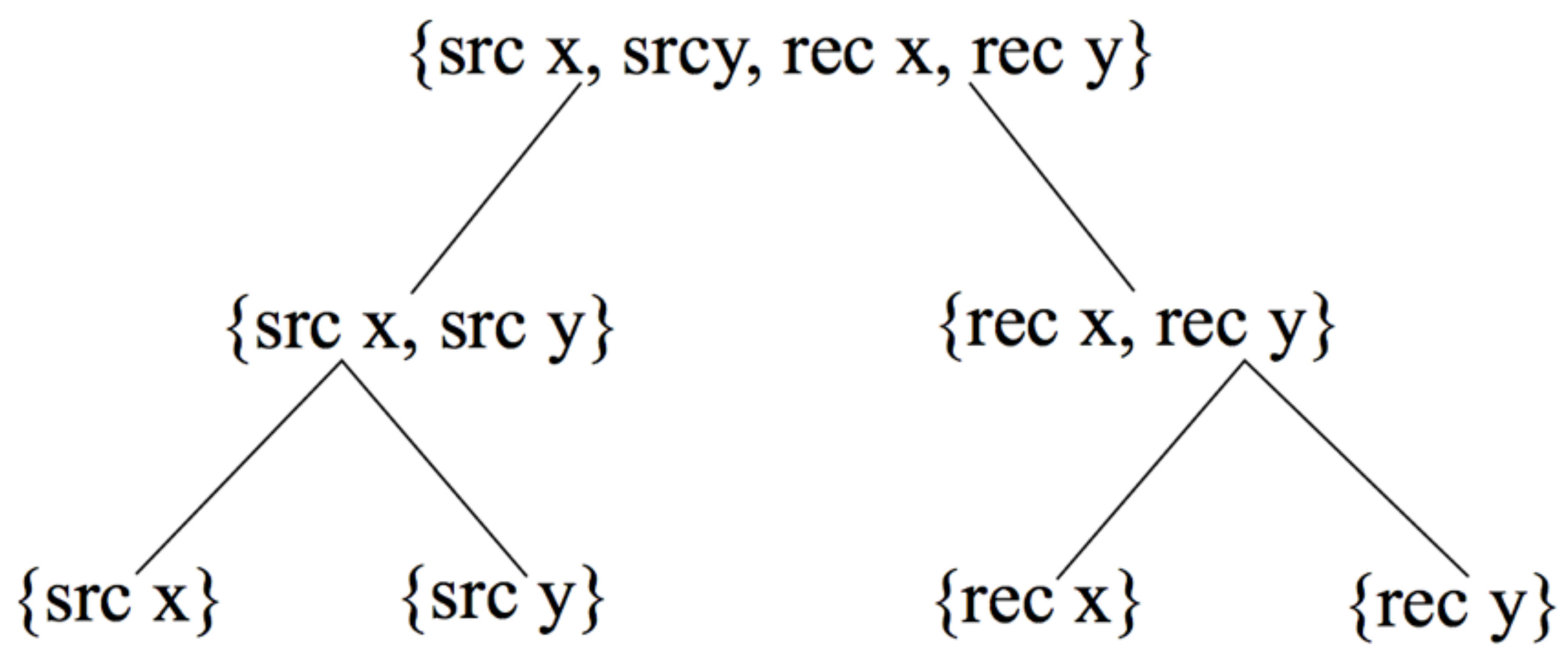
- Take a Fourier transform in time and restrict ourselves to
a single frequency slice.

- Many options on how to matricize

3D Data: "Canonical Form"



3D Data: “Canonical Form”

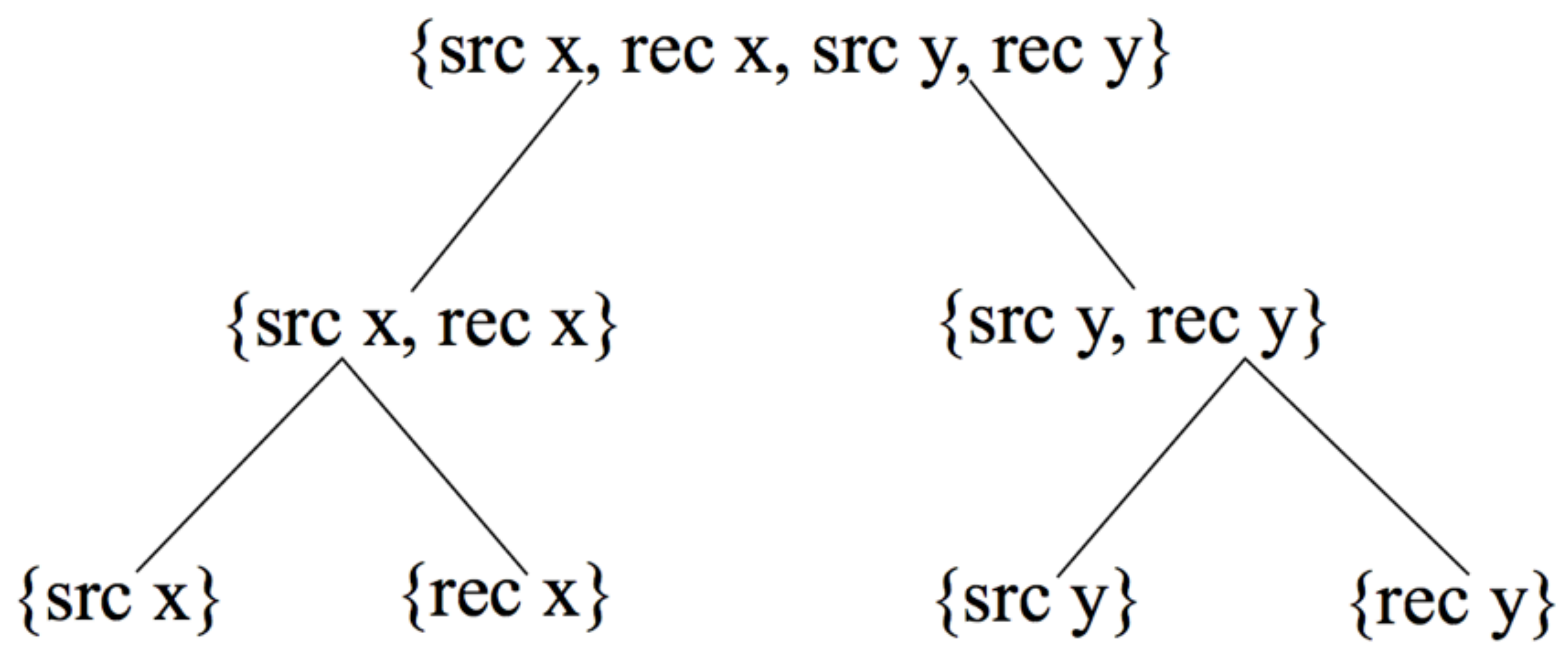


Similar to 2D case

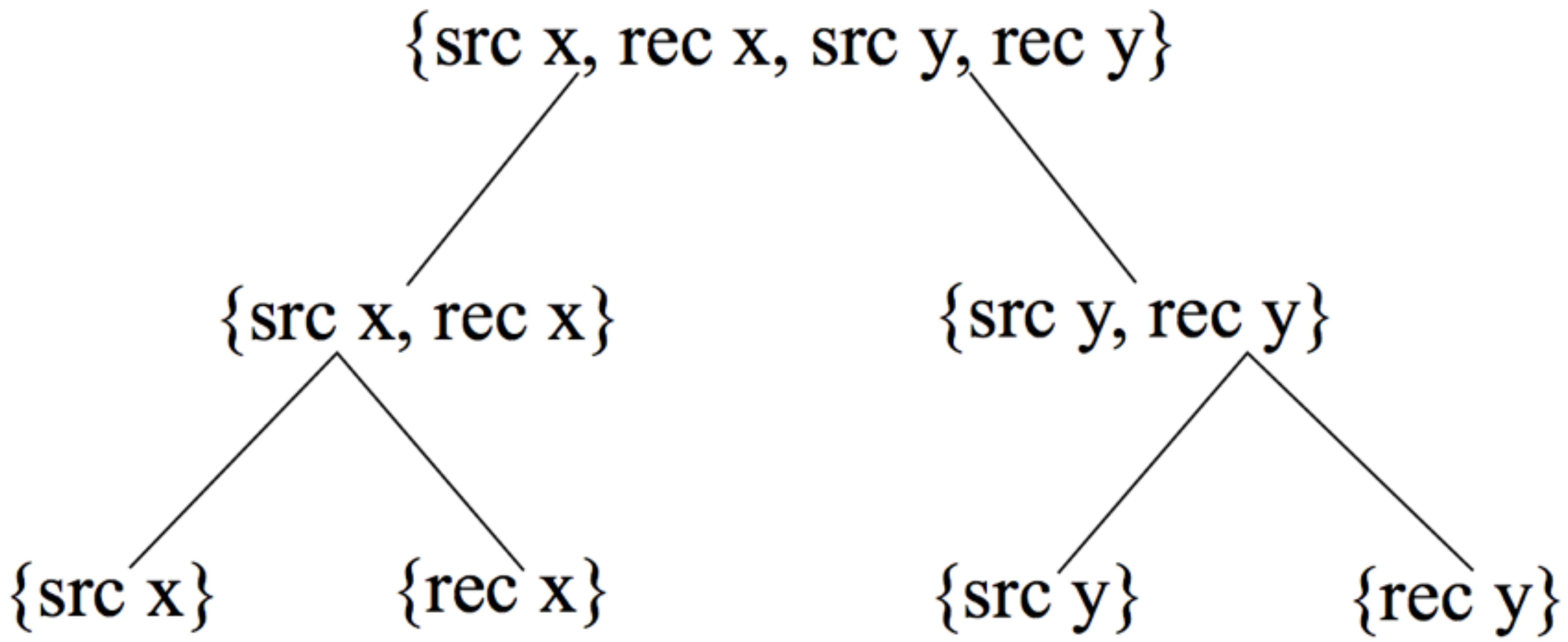
Go to (m,h)-domain

missing diagonals

3D Data: "Non-Canonical Form"



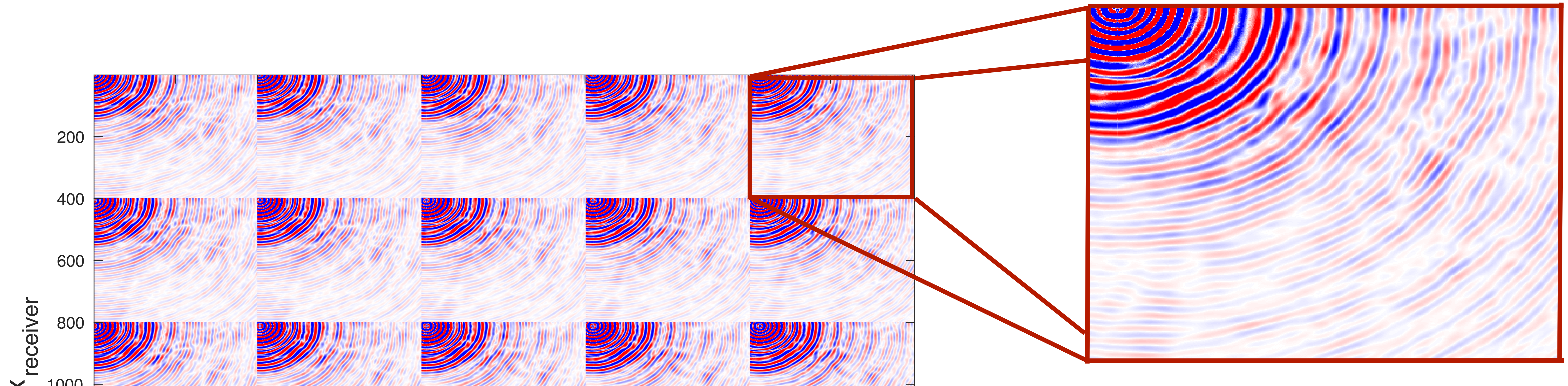
3D Data: "Non-Canonical Form"

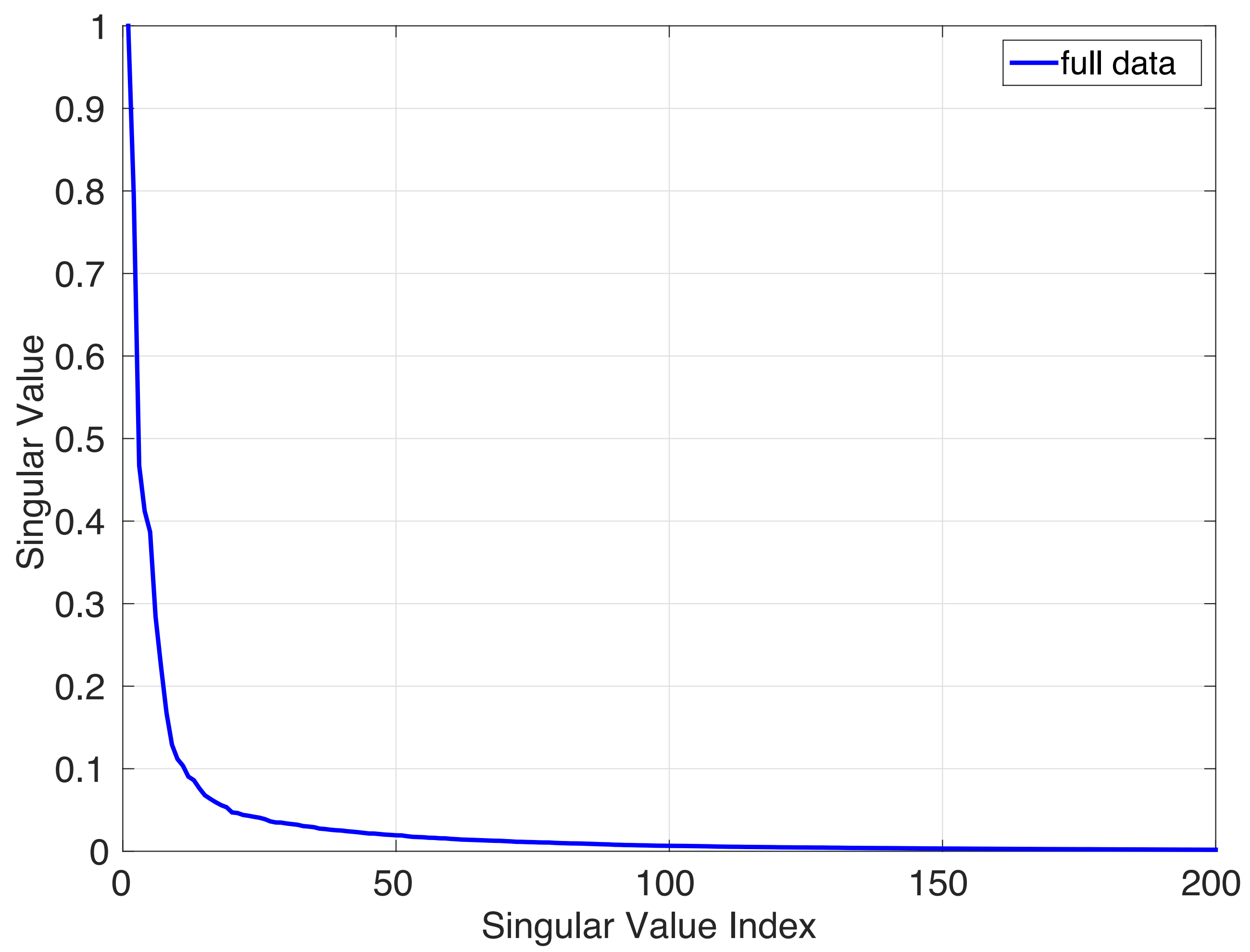


Option 1: (rec,rec) - form

Make receiver by receiver blocks

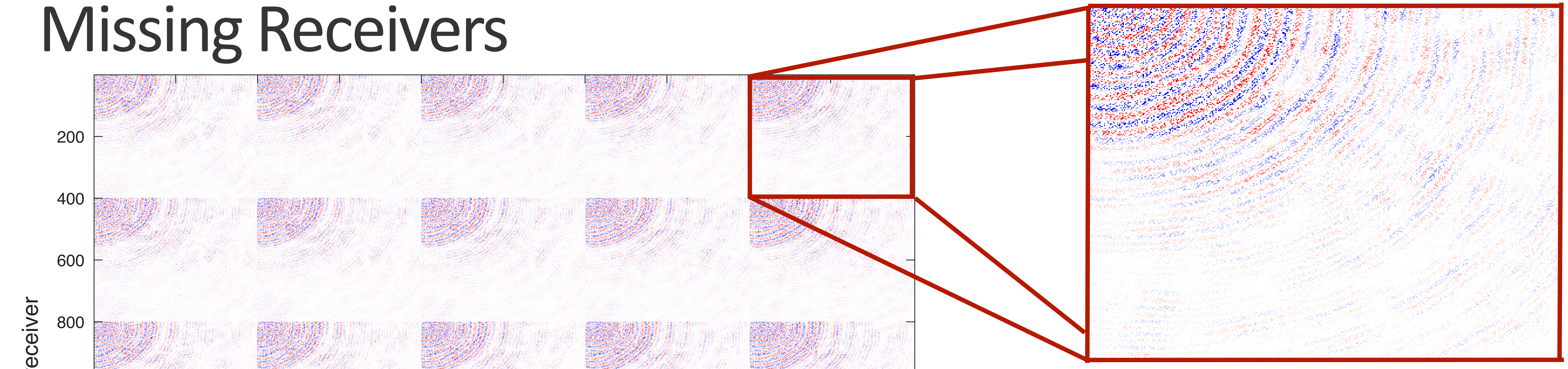
“Non-Canonical” - (rec,rec) form

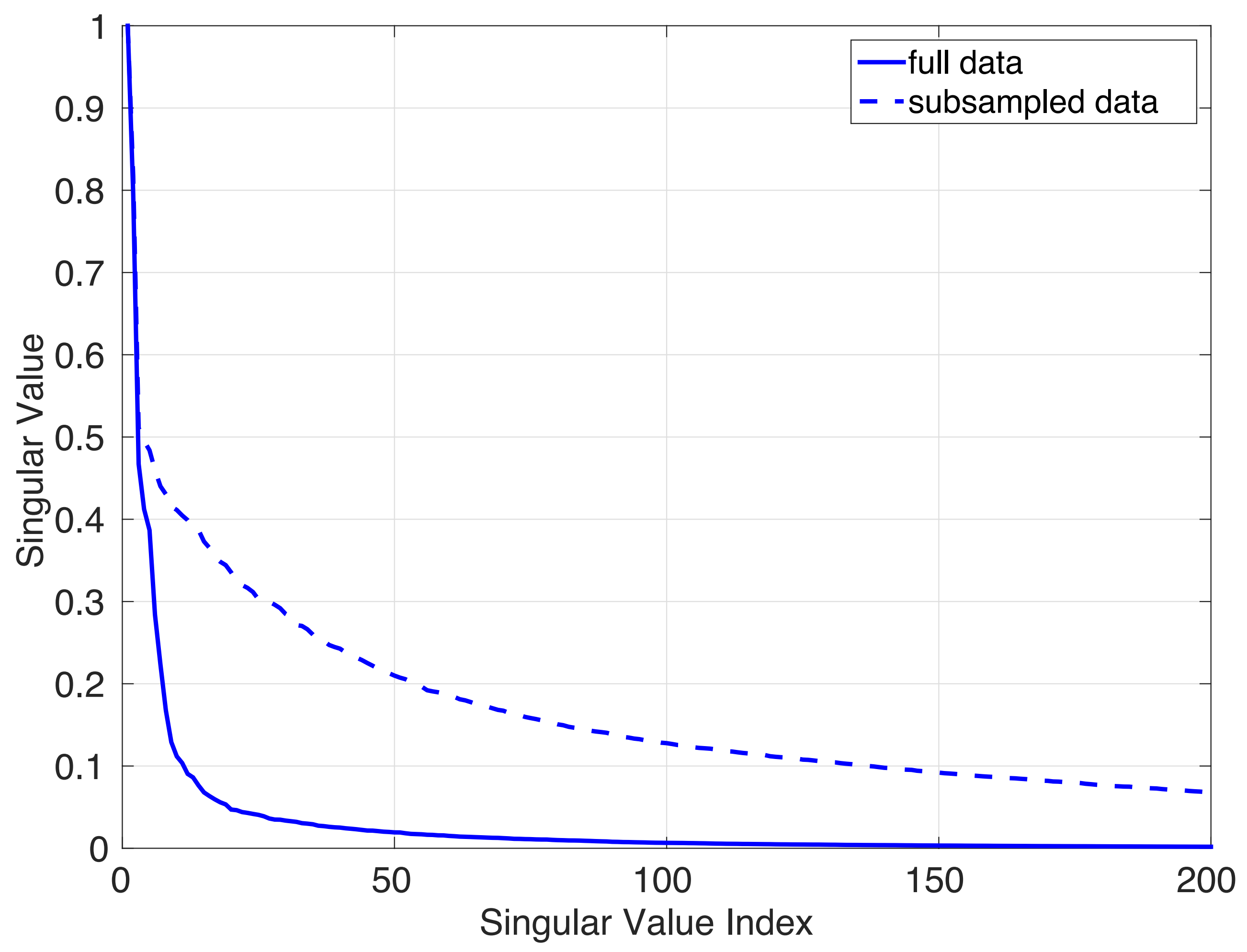




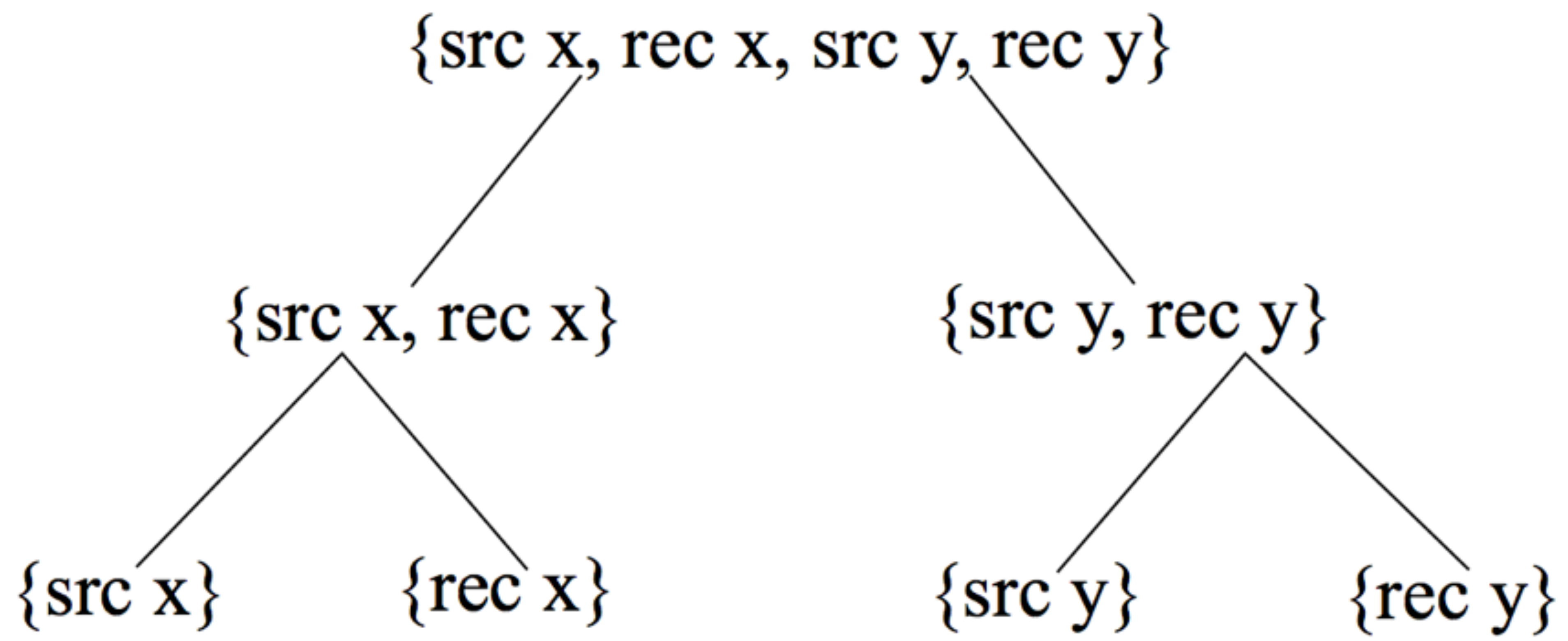
“Non-Canonical” - (rec,rec) form

Missing Receivers





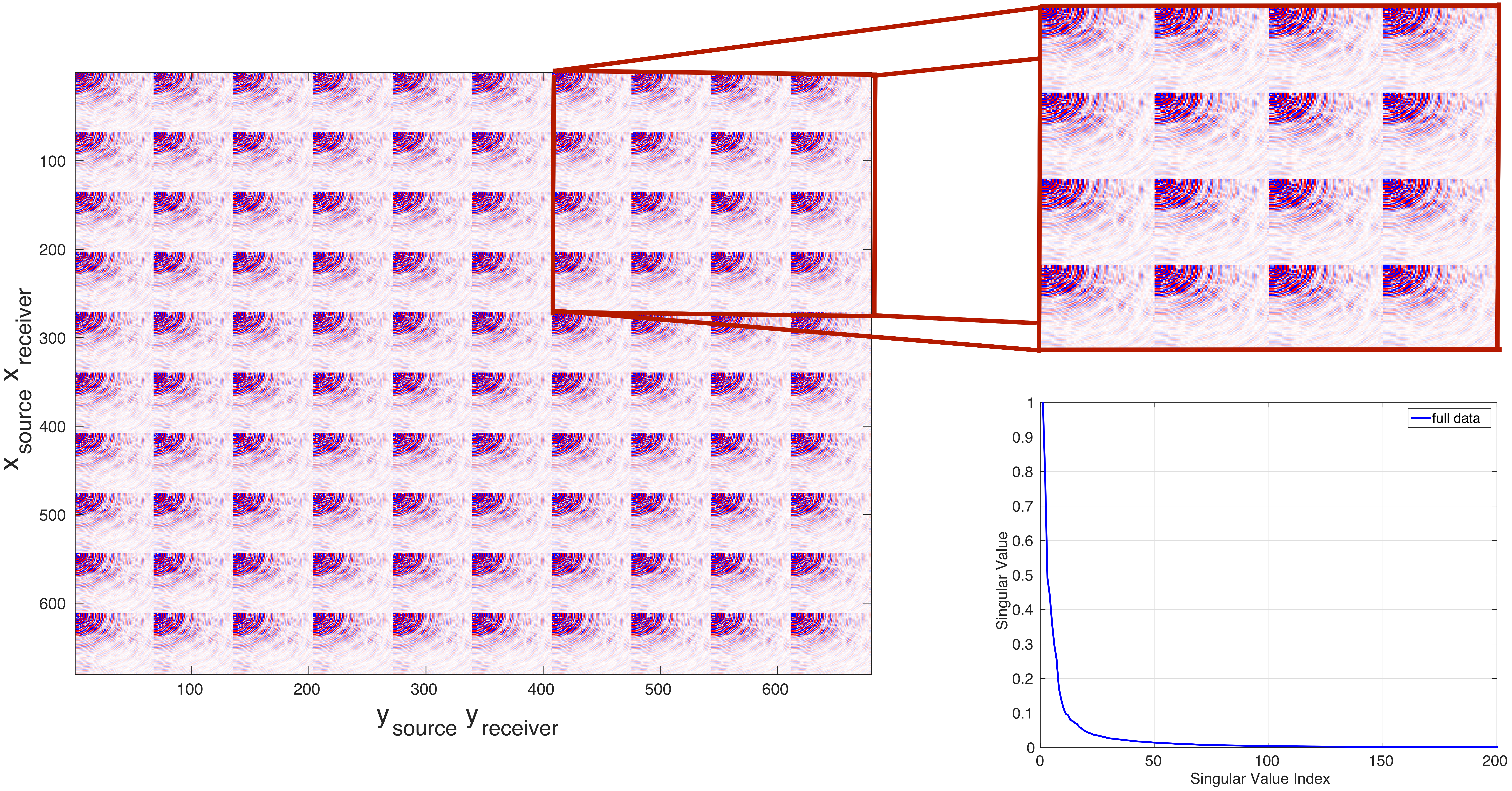
3D Data: “Non-Canonical Form”



Option 2: (src,src) - form

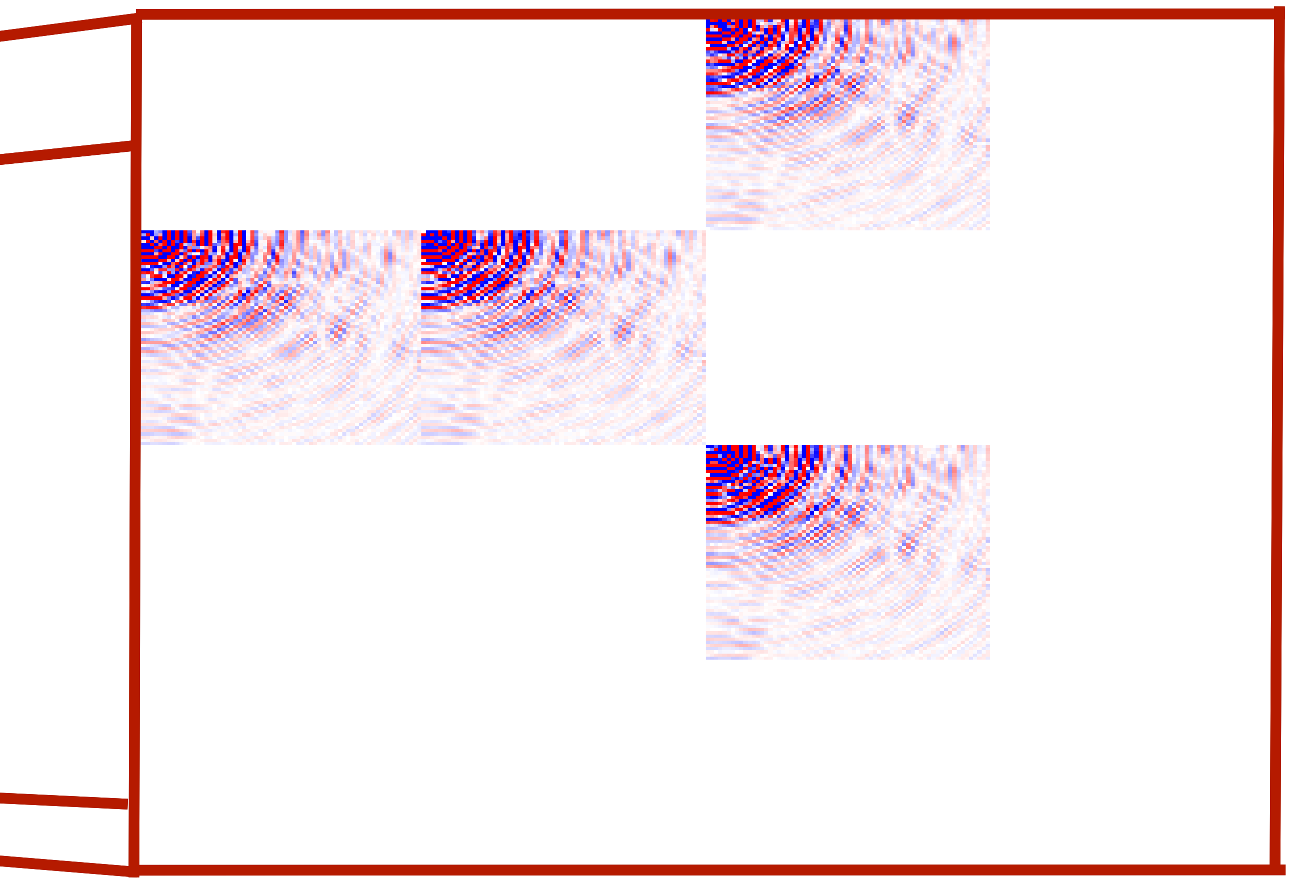
Make source by source blocks

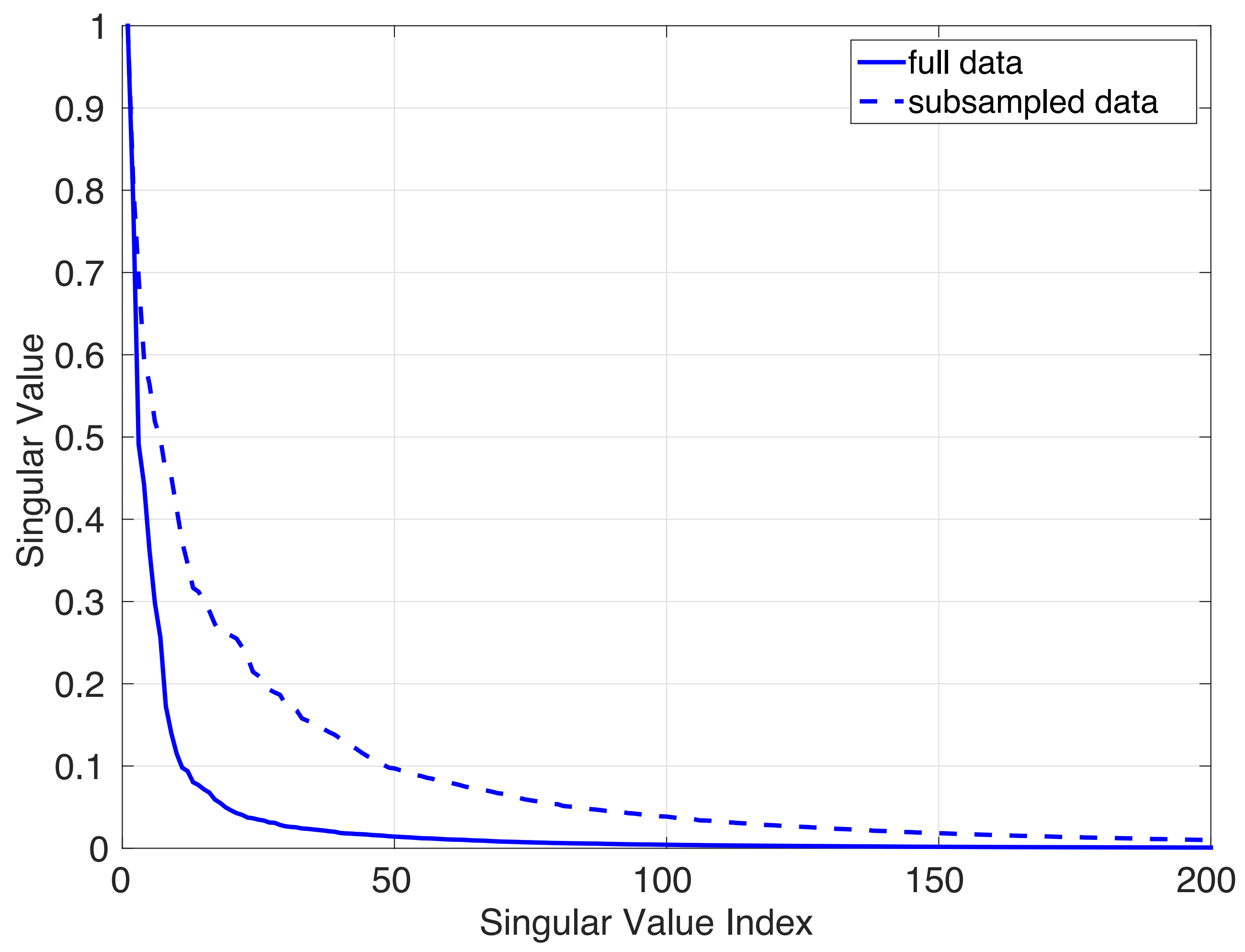
“Non-Canonical” - (src,src) form



“Non-Canonical” - (src,src) form

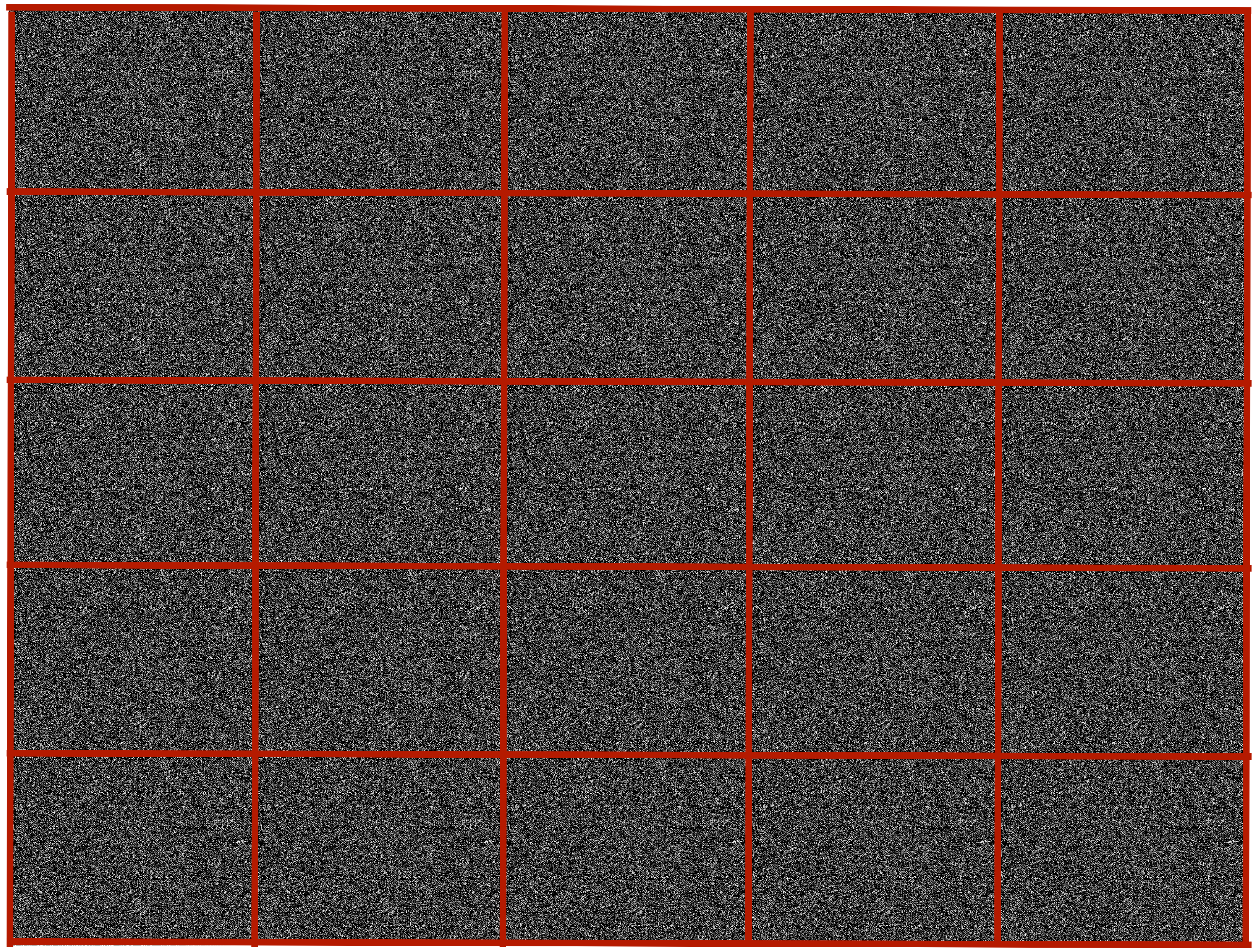
Missing Receivers



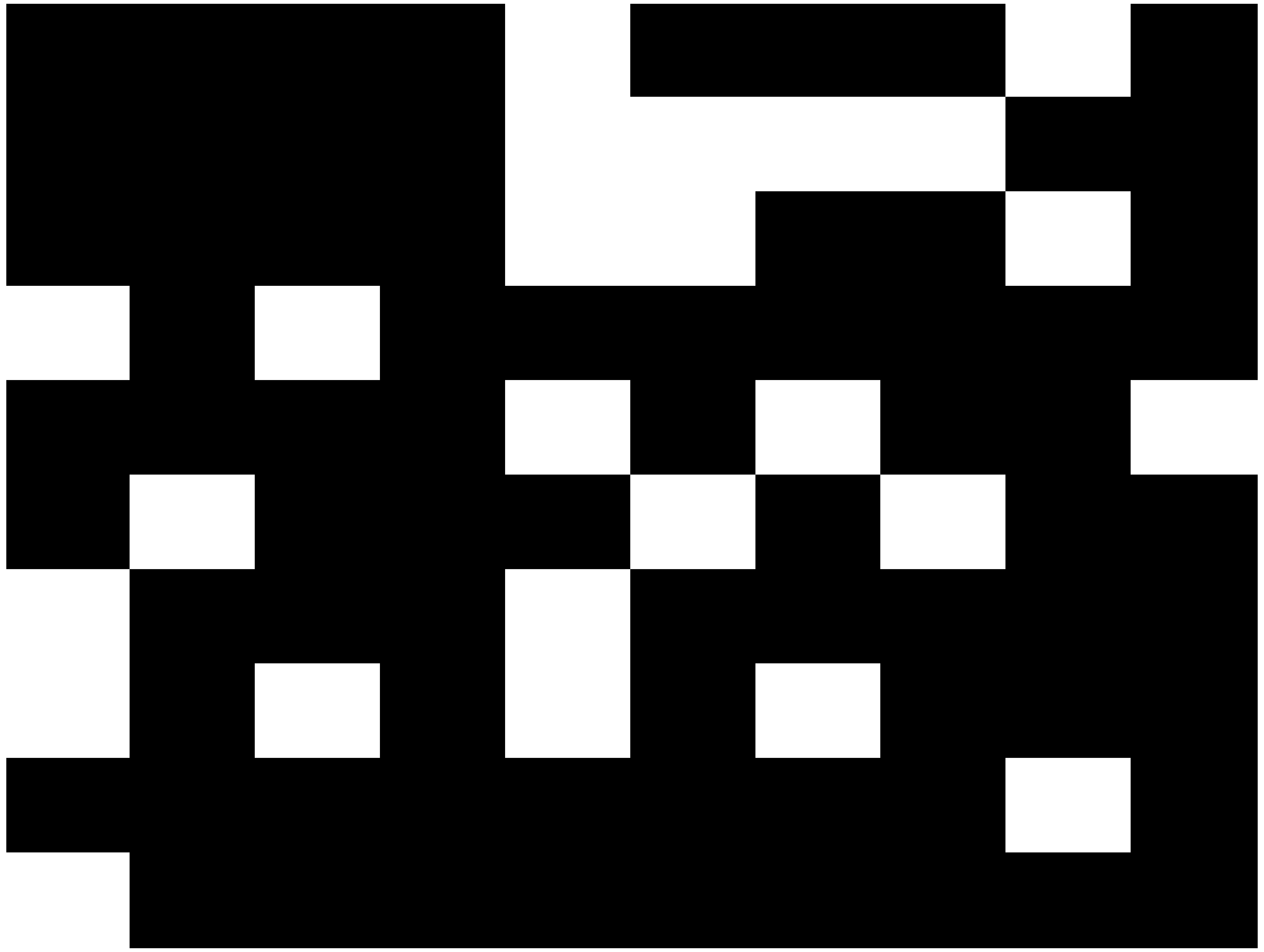


3D Seismic Masks

(rec,rec) - form



(src,src) - form



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  - seismic trace interpolation
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  - spectral gap
  - implications for seismic data

How should we subsample?

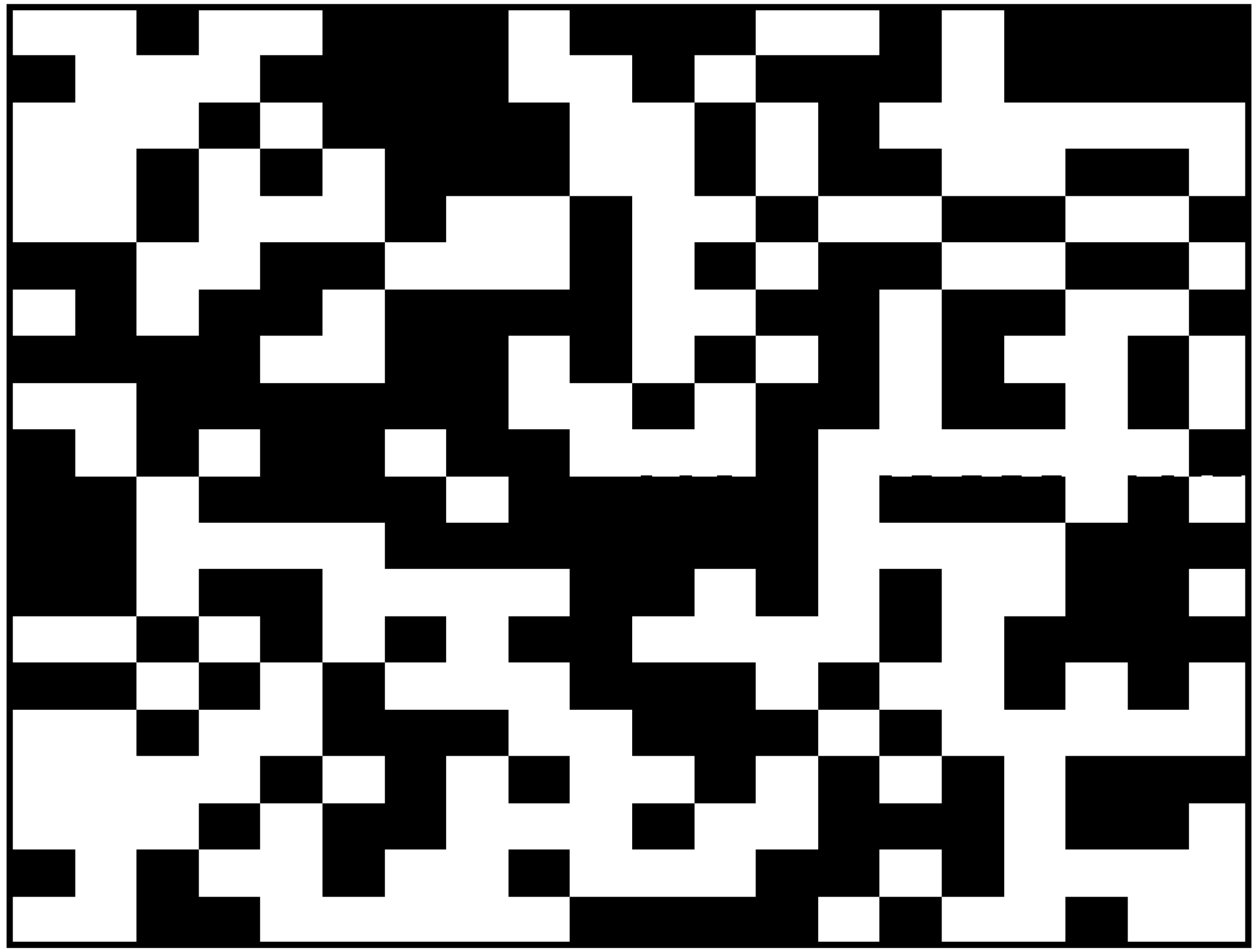
Consider our sampling mask

$$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$$

What determines if A is good for matrix completion?

Example: Ideal Mask

Samples chosen uniformly at random

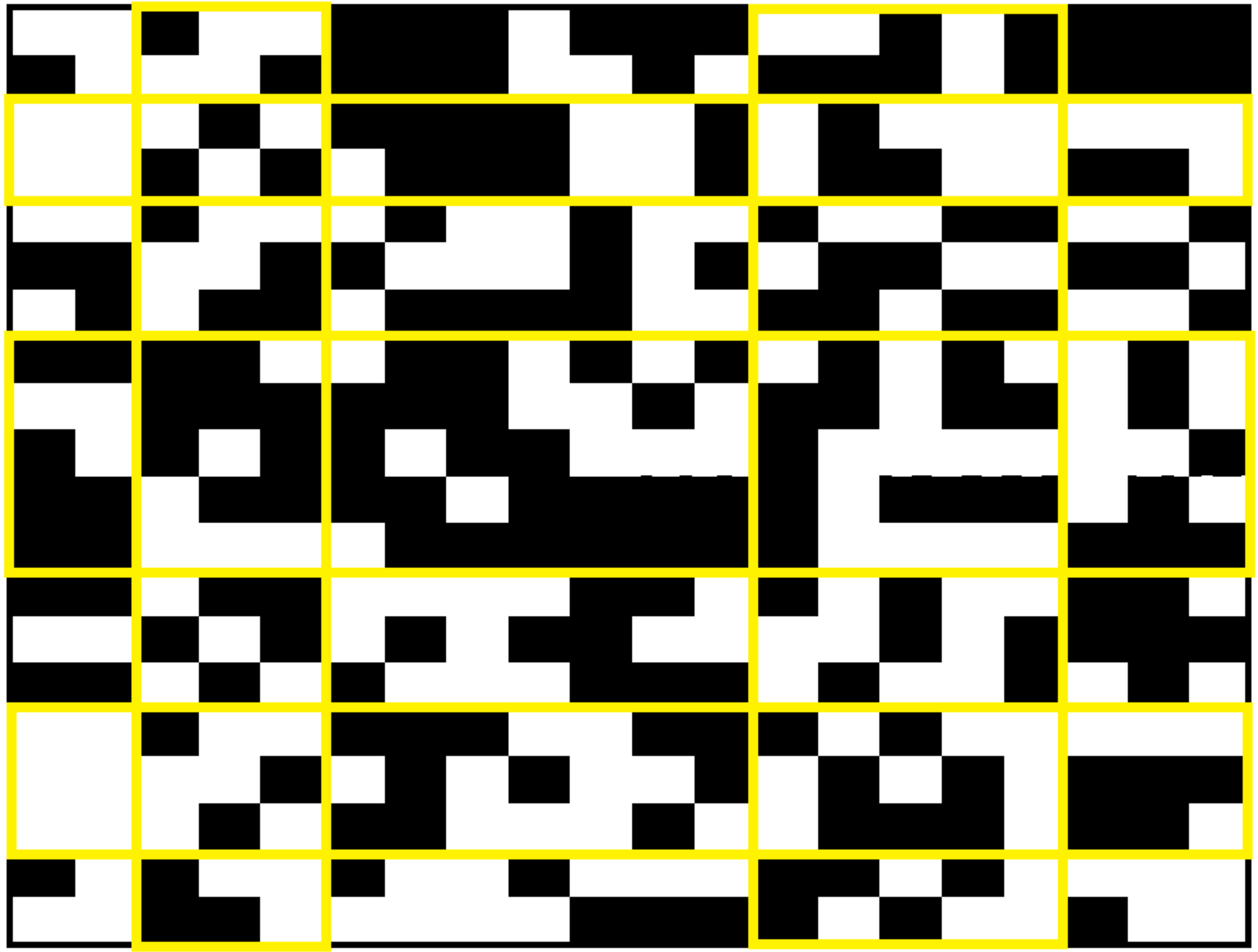
$A =$ 

■ $= 0$

□ $= 1$

Example: Ideal Mask

Choose any sub matrix

$A =$ 

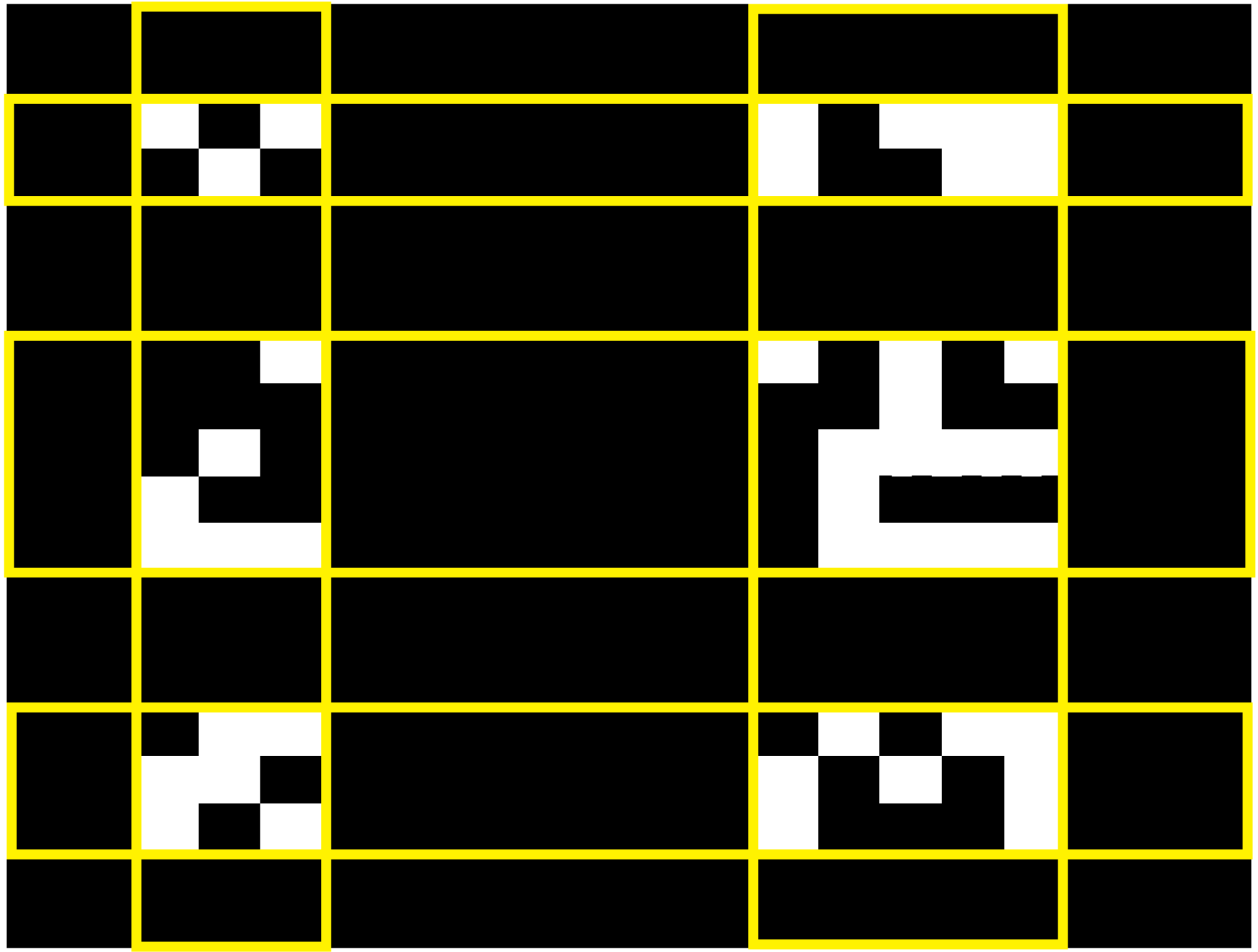
$\blacksquare = 0$

$\square = 1$

Example: Ideal Mask

All sub matrices are nicely sampled!

$A =$



$\blacksquare = 0$

$\square = 1$

Bhojanapalli, Jain. "Universal Matrix Completion" ICML 2014.

Spectral Gap

Consider the gap between the two largest singular values of A

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 & \text{if small spectral gap} \\ << 1 & \text{if large spectral gap} \end{cases}$$

where σ_i is the i-th largest singular value of A

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Spectral Gap

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 & \text{if small spectral gap} \\ << 1 & \text{if large spectral gap} \end{cases}$$

From graph theory literature:

A with Large Spectral Gap $\Longrightarrow$ all “sub matrices” are nicely sampled

$\Longrightarrow$ better results for matrix completion

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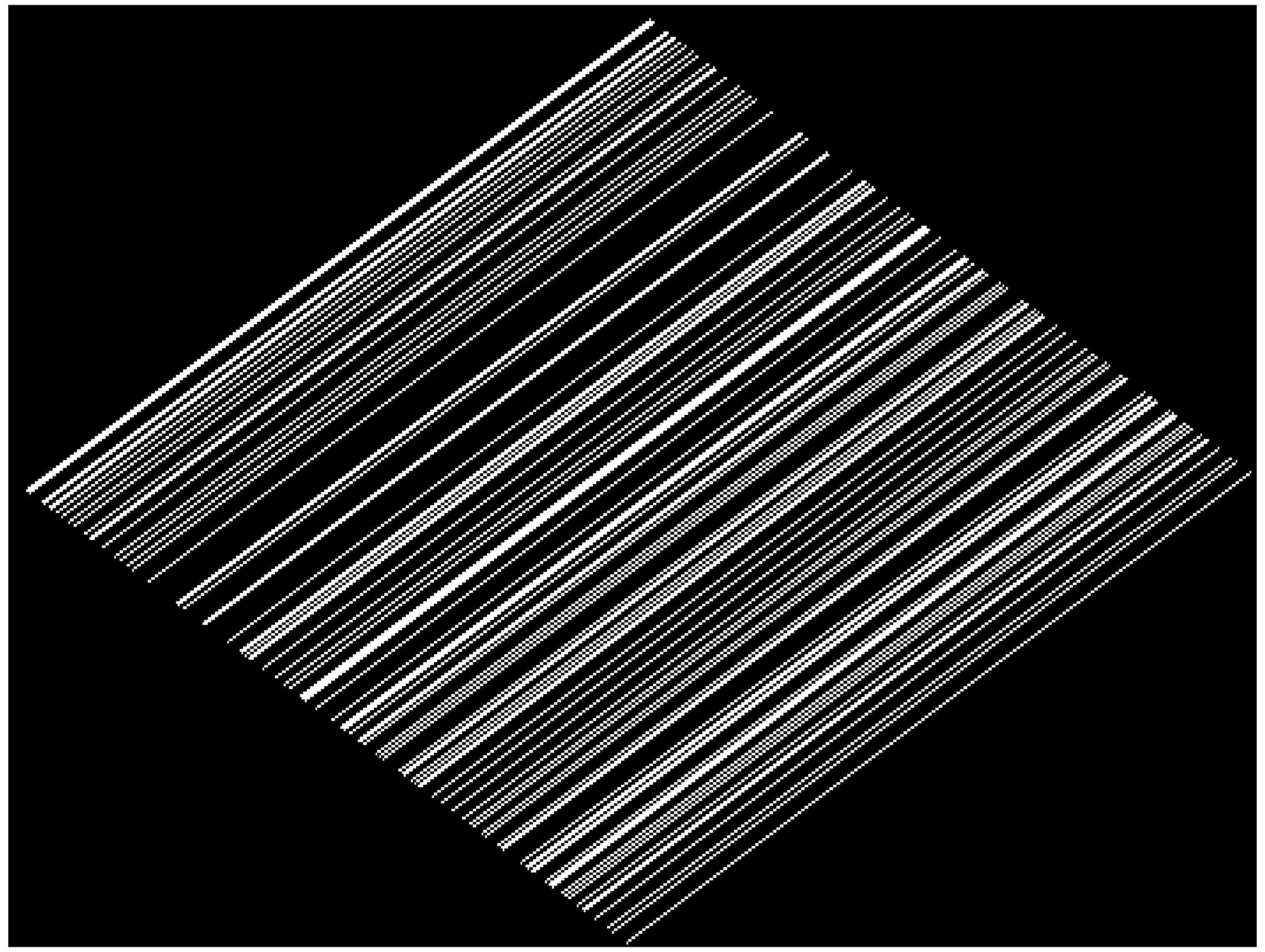
2D Interpolation Experiments

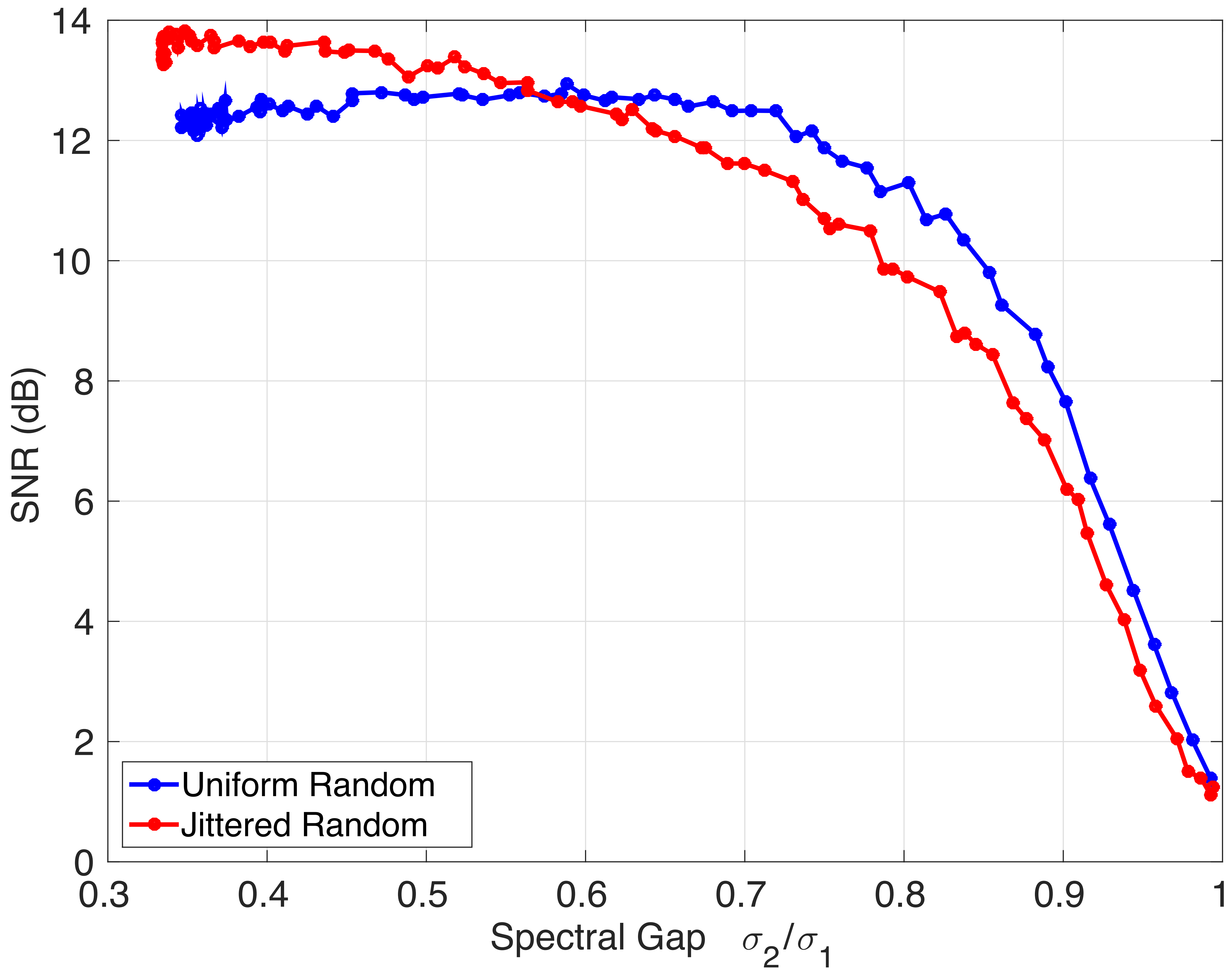
Generate 2D seismic Masks with increasing spectral gap

plot correlation with reconstruction SNR

Uniform random vs Jittered random



2D Interpolation Experiments: 75% missing sources



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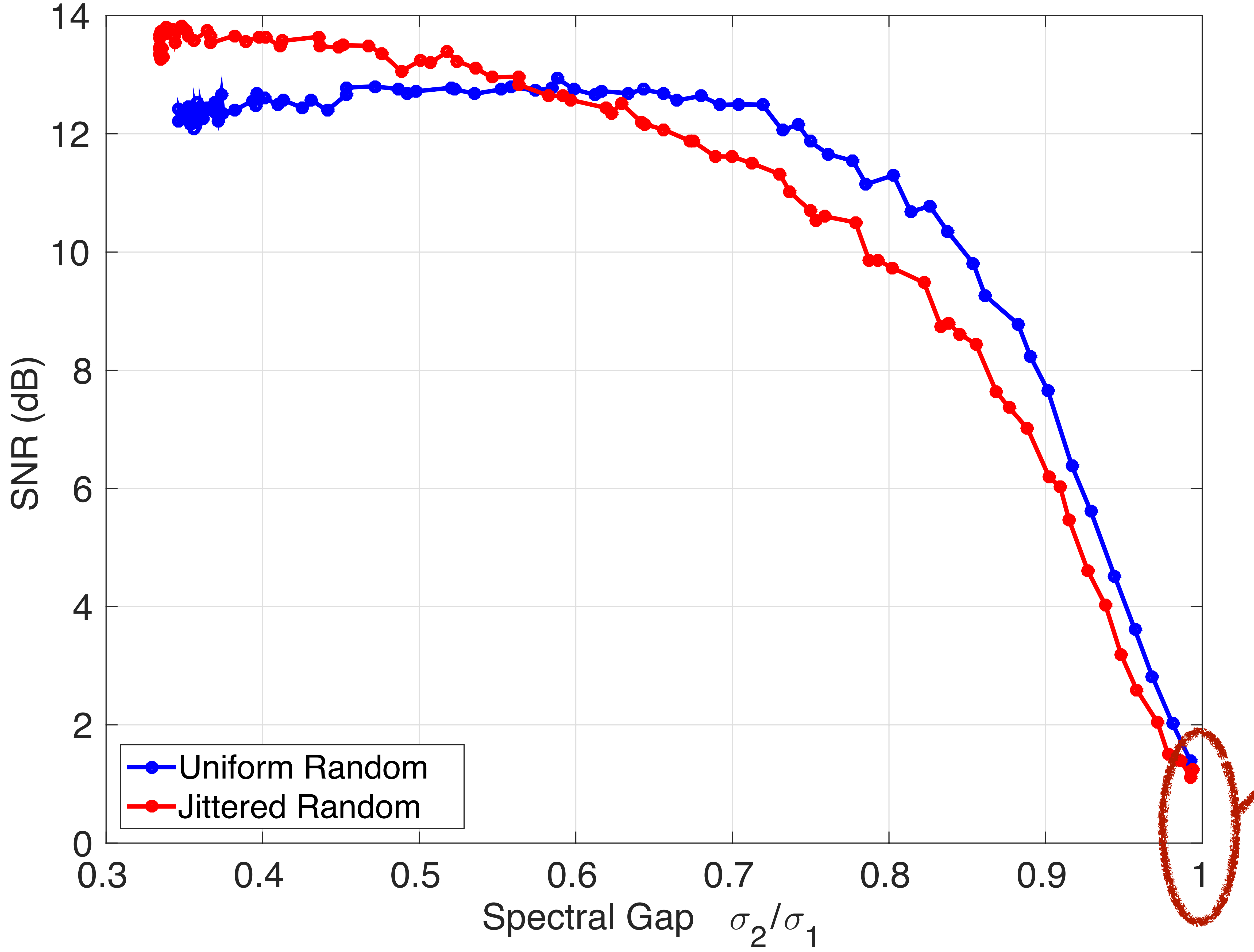
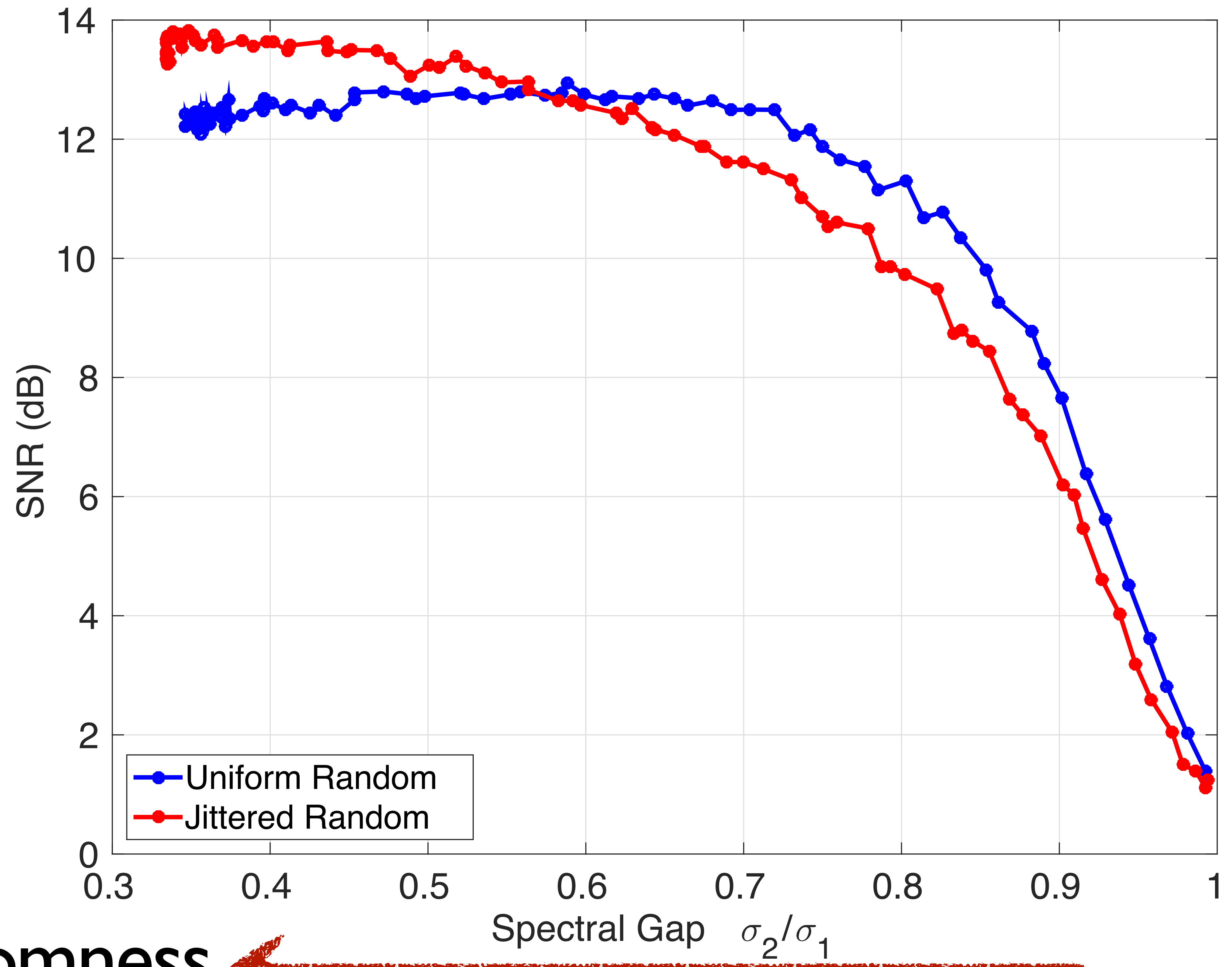


periodic mask

2D Interpolation Experiments: 75% missing sources



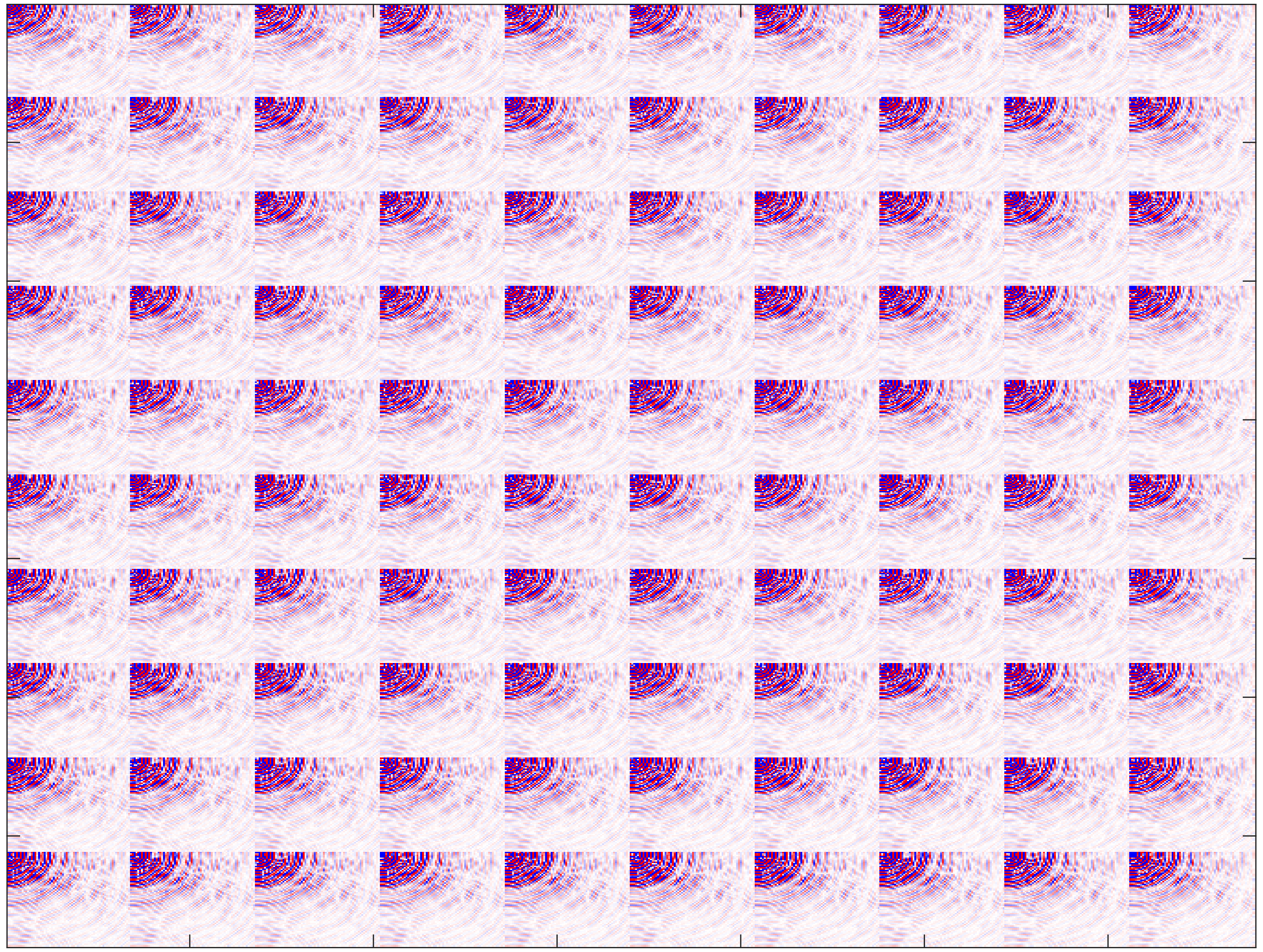
increasing randomness

R. Kumar, et al. "Efficient matrix completion for seismic data reconstruction" Geophysics 2014.

3D Interpolation Experiments

Matrix Completion with Windowing

- Full matrix is too large



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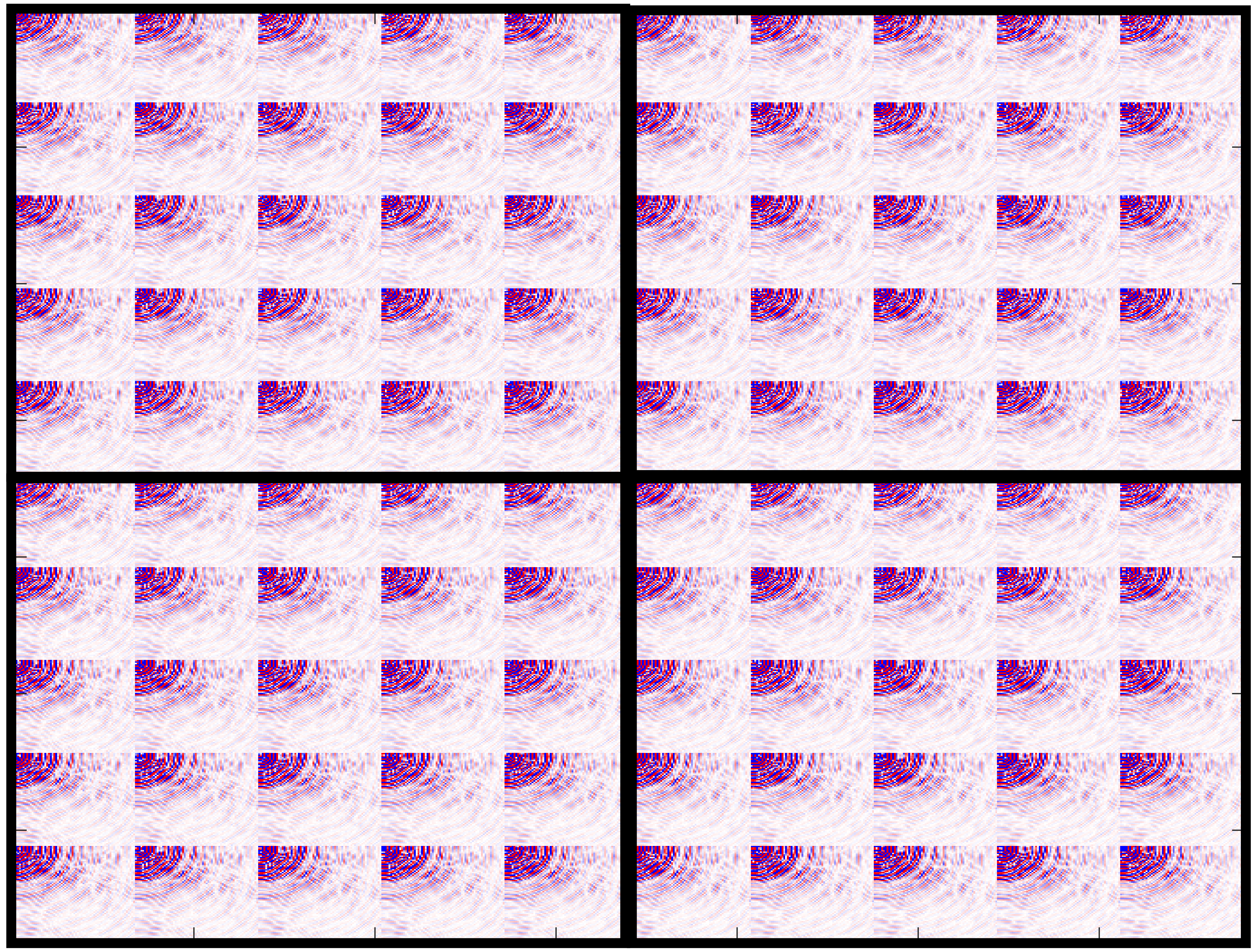
3D Interpolation Experiments

Matrix Completion with Windowing

- Full matrix is too large
- Split volume into smaller windows
- Solve in parallel



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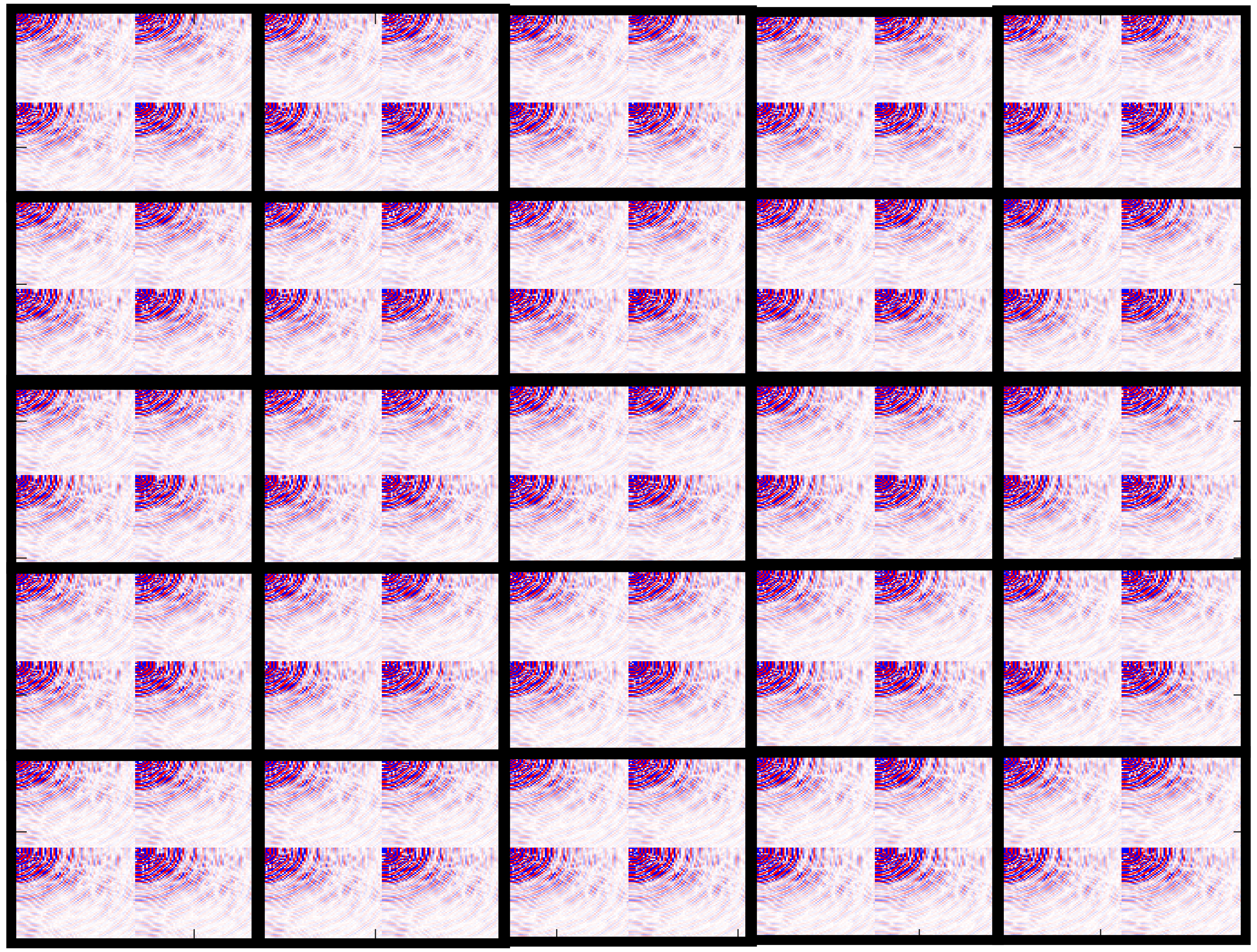
3D Interpolation Experiments

Matrix Completion with Windowing

- (rec, rec) or (src, src) form?
- What size windows?



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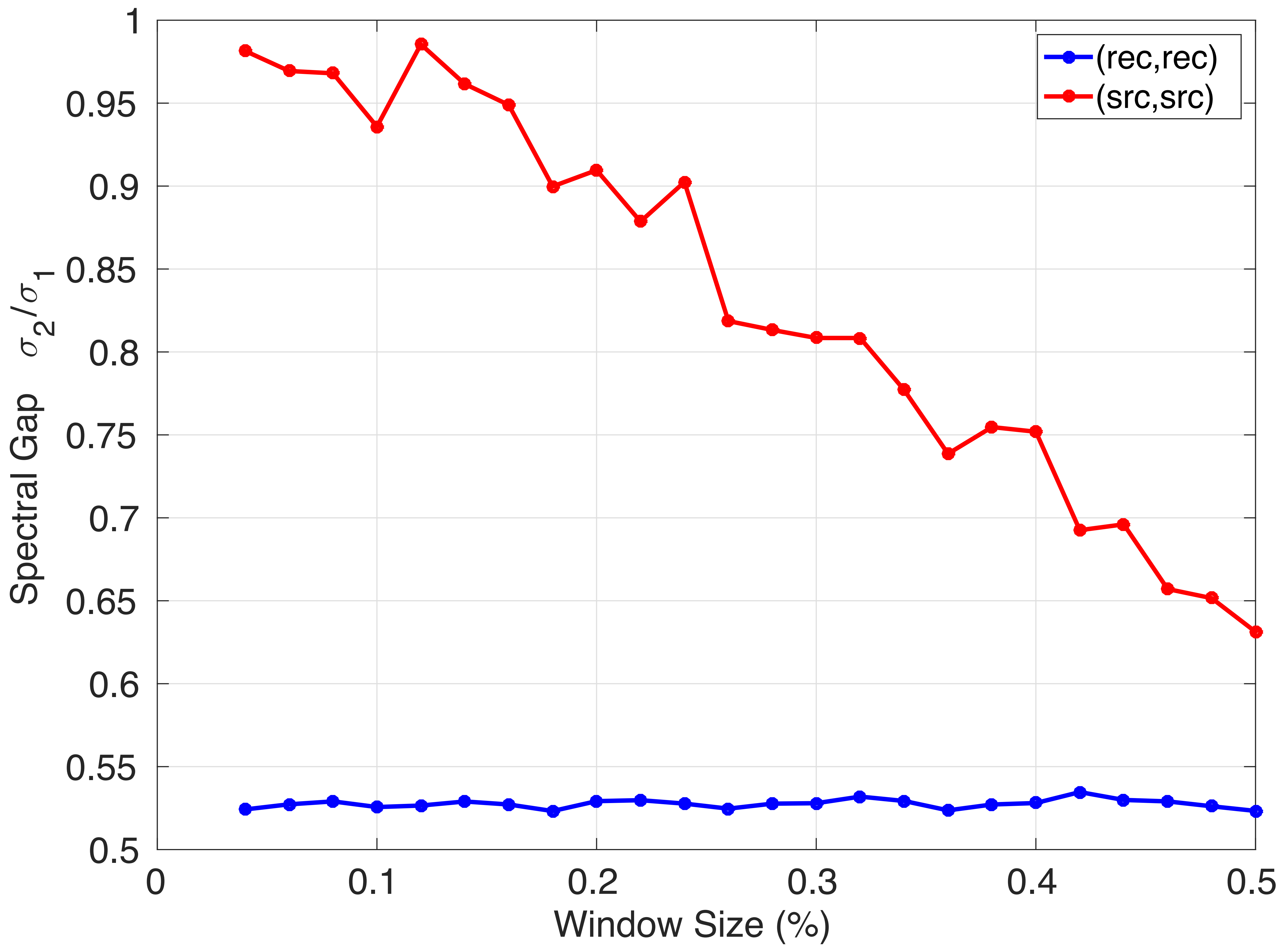
3D Interpolation Experiments: 75% missing receivers

Average spectral gap for various window sizes



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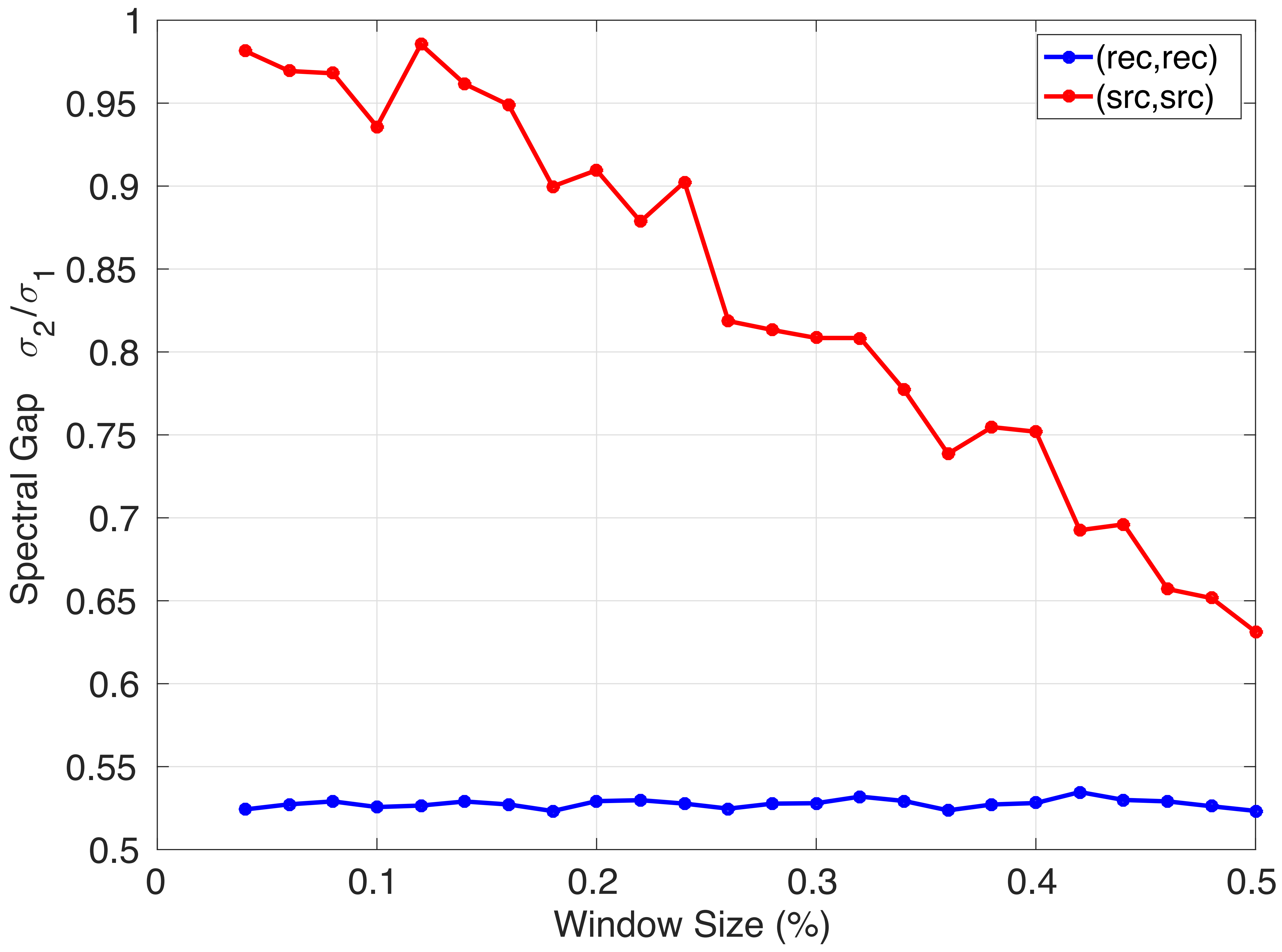
R. Kumar, et al. "Efficient matrix completion for seismic data reconstruction" Geophysics 2014.

3D Interpolation Experiments: 75% missing receivers

Average spectral gap for various window sizes

- (rec,rec) is best
- mask stability as window size decreases



Conclusion

- Good understanding of how to subsample

- Simple procedure to quantify acquisition design
  - compute only σ_1, σ_2 of sampling mask
  - critical tools for 3D data

Future work

- Further analysis of spectral gap quantification
    - Reconstruction Error bounds
    - Use theory to design optimal acquisition schemes
    - Suggestions?

Acknowledgements

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