Fast "online" migration with Compressive Sensing

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Motivation

Push from processing to inversion exposes challenges w.r.t.

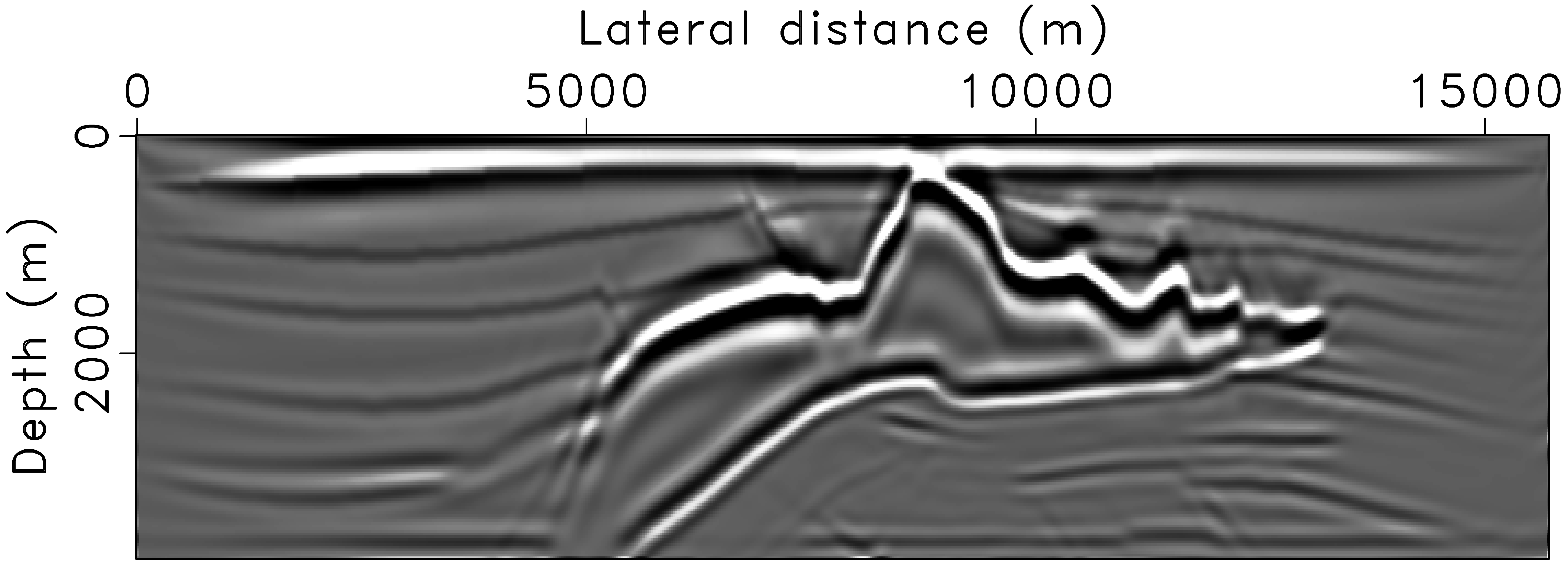
- handling IO for larger and larger datasets
- computational resources needed by wave-equation based inversions

Sparsity-promoting inversions:

- produce hifi/high-resolution results
- but require too many computations & passes through the data (IO), and
- are algorithmically complex

Stifles uptake by industry...

Inversion vs processing
– reverse-time migration (RTM)



RTM imaging via **adjoint**, high-pass filtered to remove low-wavenumber RTM artifacts

Felix J. Herrmann and Xiang Li, "Efficient least-squares imaging with sparsity promotion and compressive sensing", *Geophysical Prospecting*, vol. 60, p. 696-712, 2012
Ning Tu and Felix J. Herrmann, "Fast imaging with surface-related multiples by sparse inversion", *Geophysical Journal International*, vol. 201, p. 304-317, 2015

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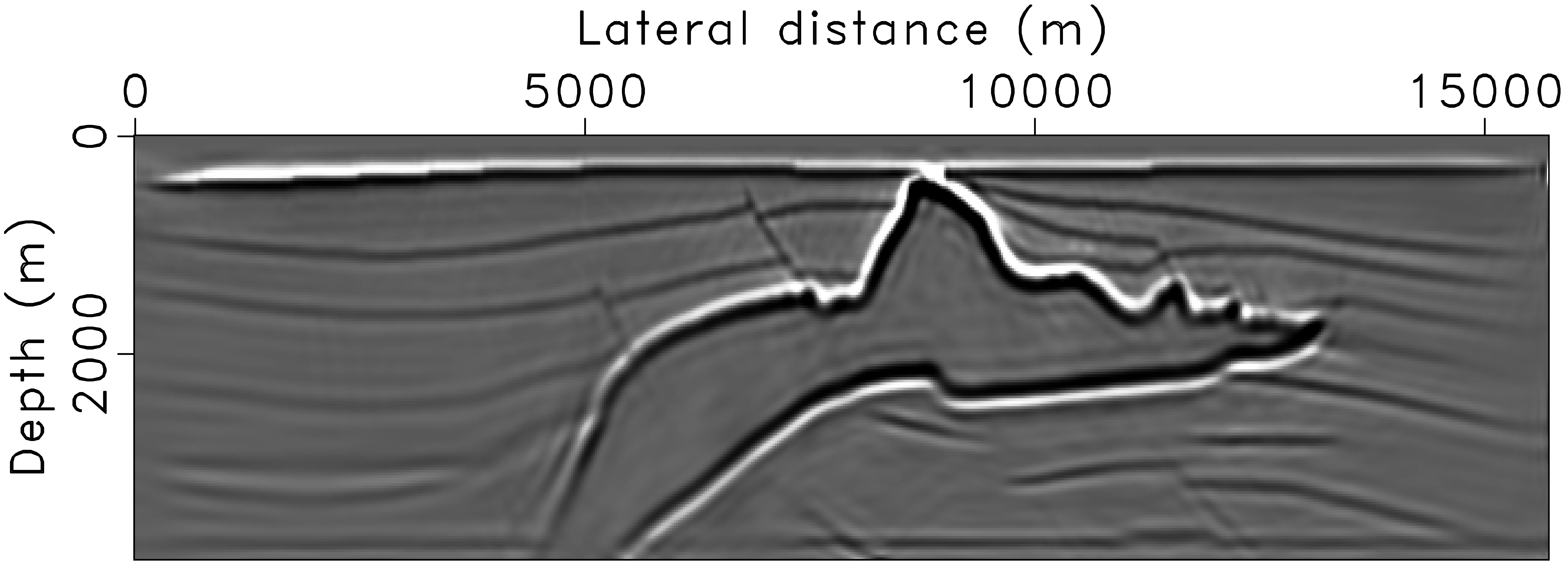
Inversion vs processing
– sparsity-promoting least-squares migration (SPLSM)



SPLSM image via **inversion**, # of wave-equation solves roughly equals 1 RTM w/ all data

Contributions

New “online” scheme that provably inverts large-scale problems by

- working on small randomized subsets of data (e.g. shots) only
- making the objective strongly convex by thresholding the dual variable

Extremely simple “three liner” implementation that

- limits # of passes through data & offers flexible parallelism
- is easily extendible to include e.g. on-the-fly source estimation & multiples

Application areas include:

- least-squares migration & AVA

[Shen et. al. '01]

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Sparsity promotion

Basis Pursuit (BP):

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \|\mathbf{x}\|_1 \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$

- undergirds most sparse recovery problems & compressive sensing (CS)
- designed for underdetermined systems
- needs many iterations

[Daubechies ,03; Figueiredo and Nowak, '03;Yin et al. , '08; Beck and Teboulle, '09']

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ISTA

– Iterative Shrinkage Thresholding Algorithm

1. $\textbf{for } k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{x}_k - t_k \mathbf{A}^*(\mathbf{A}\mathbf{x}_k - \mathbf{b}_k)$
3. $\quad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
4. $\textbf{end for}$

*where $S_\lambda(x) = \mathrm{sign}(x) \cdot \max(|x| - \lambda, 0)$ is soft thresholding and t_k are step lengths

- simple but converges slowly, especially for λ small
- BP corresponds to non-trivial limit $\lambda \to 0^+$
- requires (complicated) continuation strategies for λ

Gilles Hennenfent, Ewout van den Berg, Michael P. Friedlander, and Felix J. Herrmann, "**New insights into one-norm solvers from the Pareto curve**", *Geophysics*, vol. 73, p. A23-A26, 2008.

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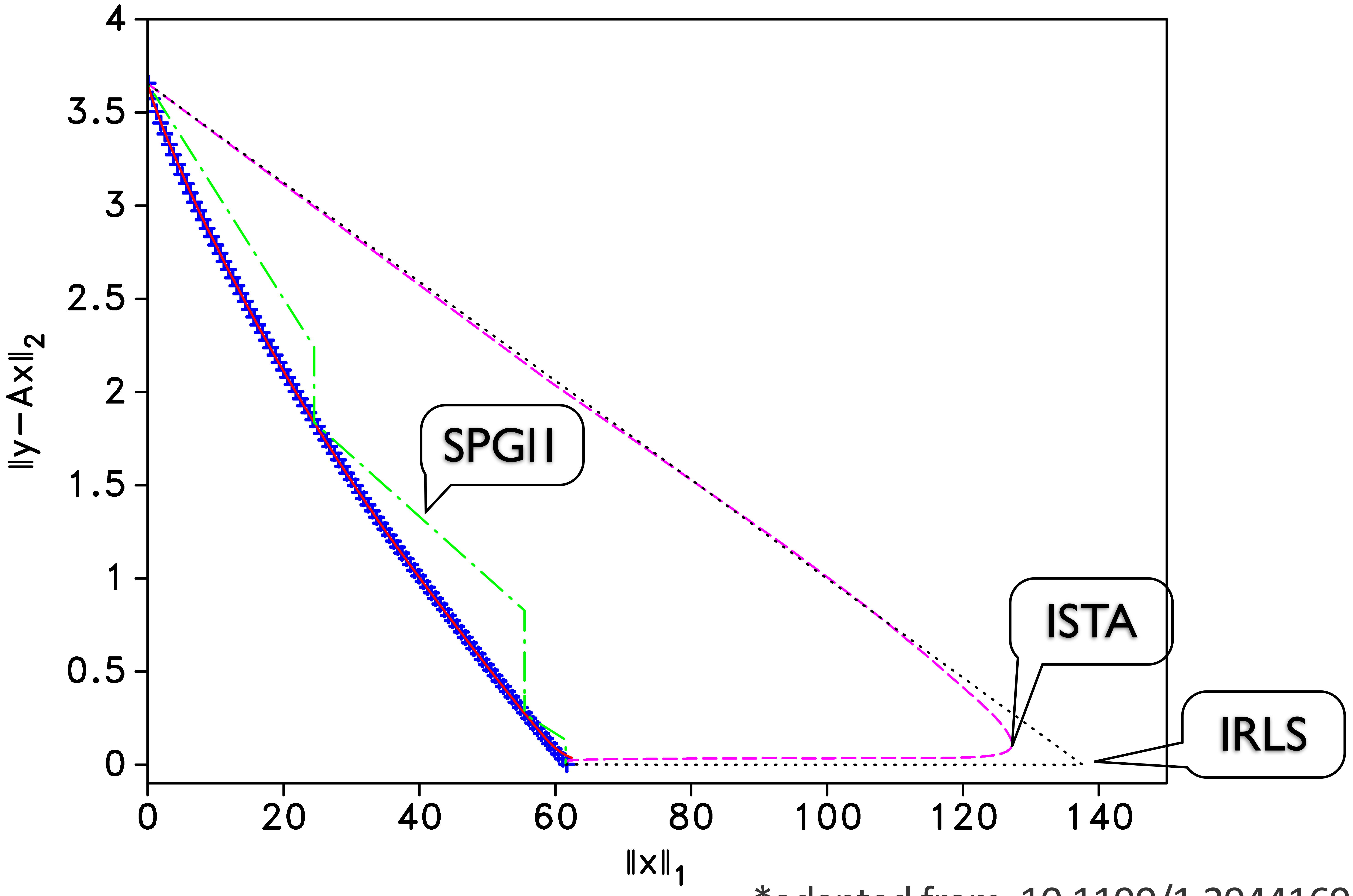
Solution paths



*adapted from 10.1190/1.2944169

Ewout van den Berg and Michael P. Friedlander, "Probing the Pareto frontier for basis pursuit solutions", *SIAM Journal on Scientific Computing*, vol. 31, p. 890-912, 2008

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Observations

Contributions from "optimizers" yielded robust solvers such as SPGl1

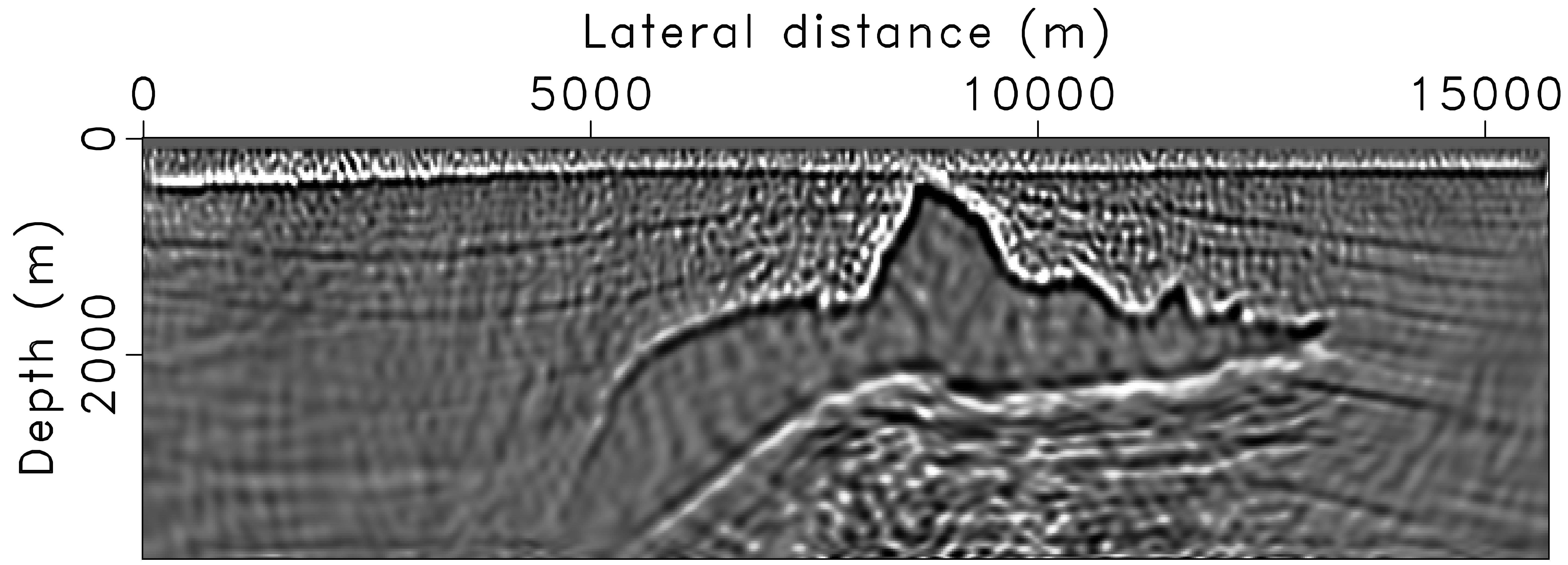
- relatively fast because of continuation methods that relax the constraint
- black boxes with clever state-of-the-art "tricks"

But, their

- convergence is too slow for realistic seismic problems w/ expensive matvecs & IO
- implementation is rather complicated & somewhat inflexible
- design is not optimized for overdetermined problems

SPLSM w/ CS

– slow convergence

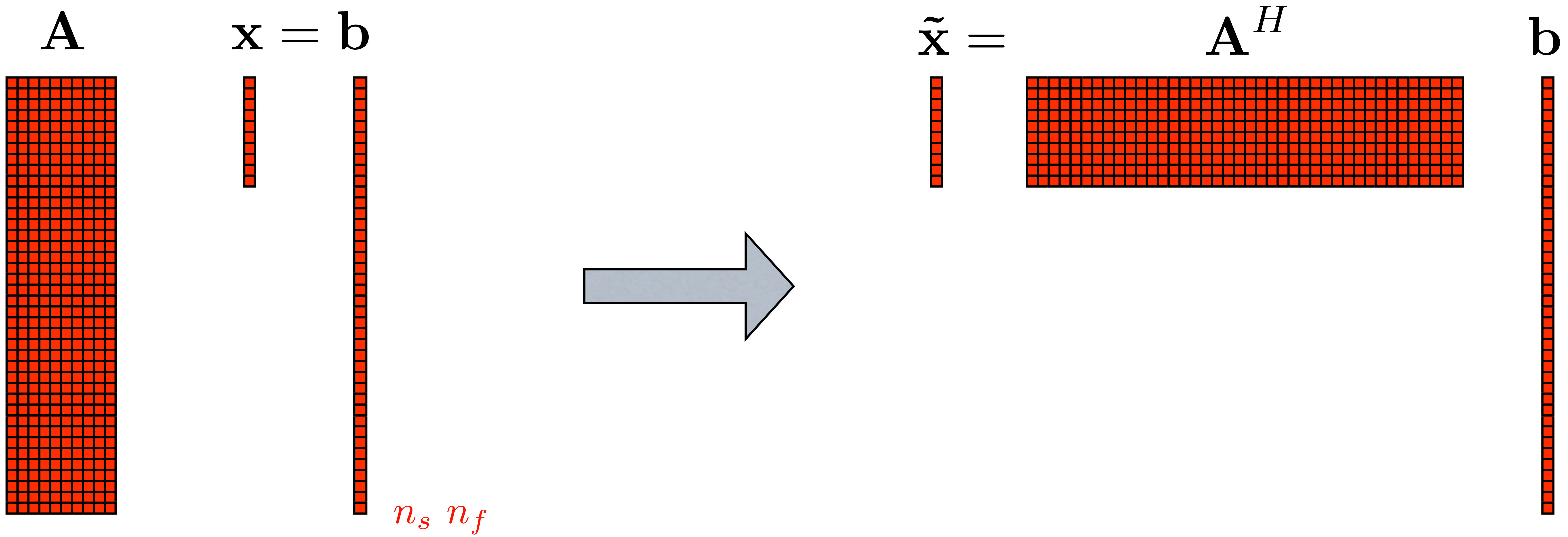


SPLSM image via **inversion** w/ **fixed** randomized simultaneous shots and in the presence of modelling errors

Migration

Seismic problems are

- often overdetermined
- often “inverted” by applying the (scaled) adjoint (e.g. migration)



Least-squares inversion

Consistent & inconsistent overdetermined systems can be solved by

$$\underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{2}\|\mathbf{A}\mathbf{x}-\mathbf{b}\|_2^2$$

which requires

- multiple matrix-free actions of $\{\mathbf{A},\ \mathbf{A}^H\}$
- multiple paths through the data (= many wave-equation solves), and
- does not exploit structure in $\mathbf{x}$

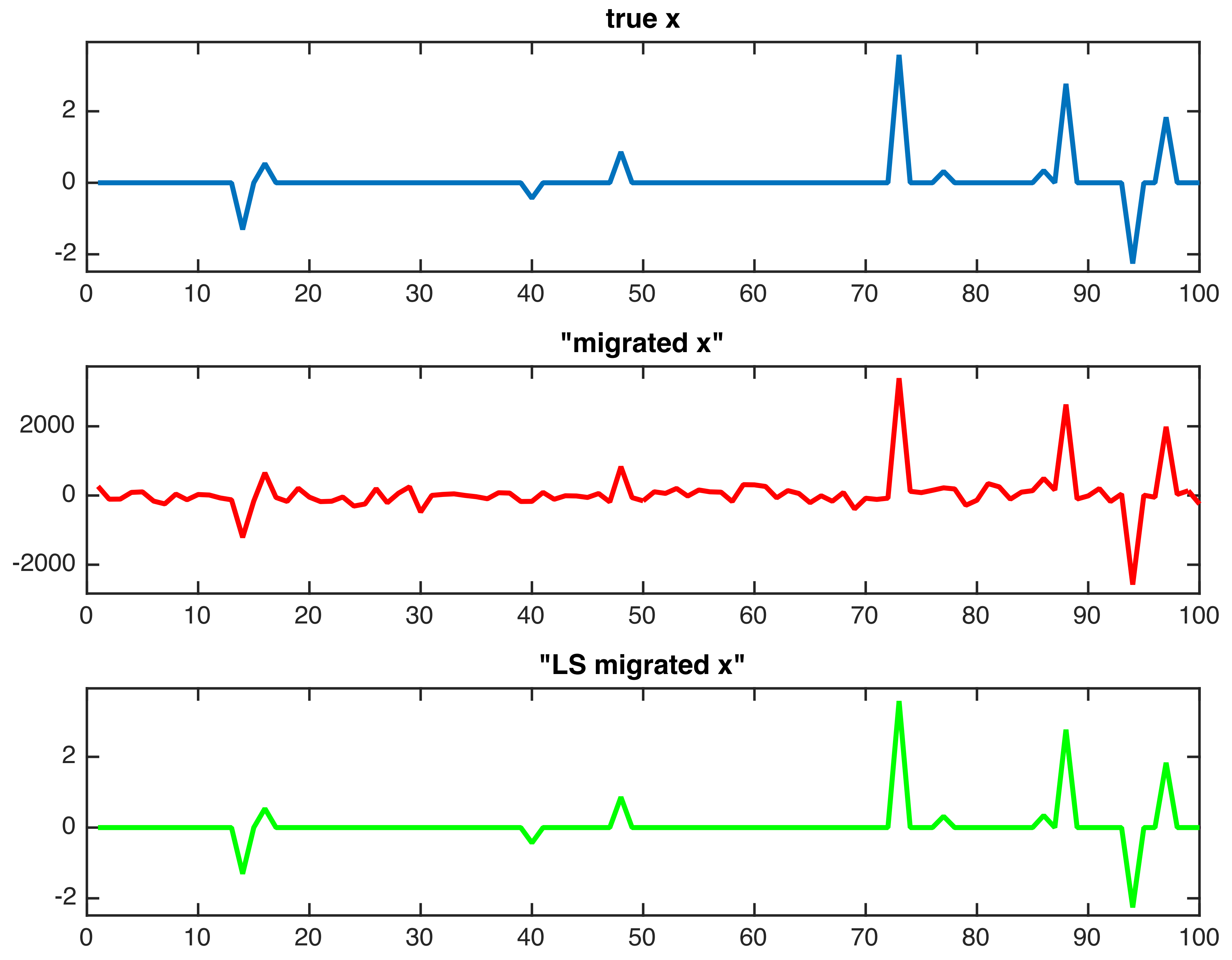
Example – noise-free

```
m=1000;  % Number of rows
n=100;   % number of columns
nnz=10;  % Number of nonzeros

x0 = zeros(n,1);
x0(randperm(n,nnz))= randn(nnz,1); % Sparse vector
A = randn(m,n);                    % Tall system
b = A*x0;                          % data

xcor = A'*b;                       % "Migrate image"
xls  = lsqr(A,b);                  % "LS-migrated
image"
lsqr converged at iteration 12 to a solution with
relative residual 9e-07.
```

12 passes through data



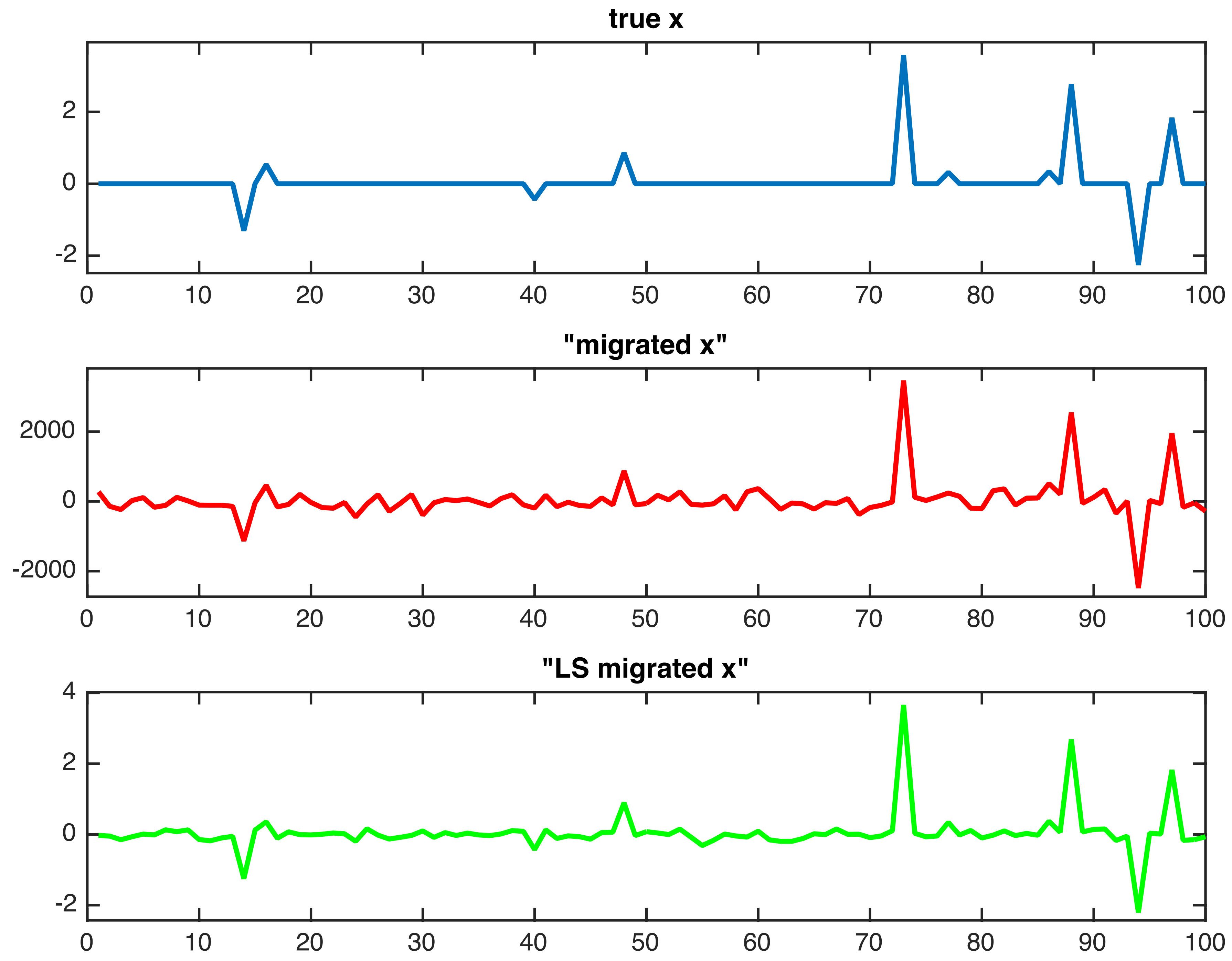
Example – noisy

```
m=1000;  % Number of rows
n=100;   % number of columns
nnz=10;  % Number of nonzeros

x0 = zeros(n,1);
x0(randperm(n,nnz))= randn(nnz,1); % Sparse vector
A = randn(m,n);                    % Tall system
b = A*x0;                          % data
b = b+0.5*std(b)*randn(m,1);       % noisy data

xcor = A'*b;                       % "Migrate image"
xls  = lsqr(A,b);                  % "LS-migrated
image"
lsqr converged at iteration 12 to a solution with
relative residual 0.44.
```

imprint of noise



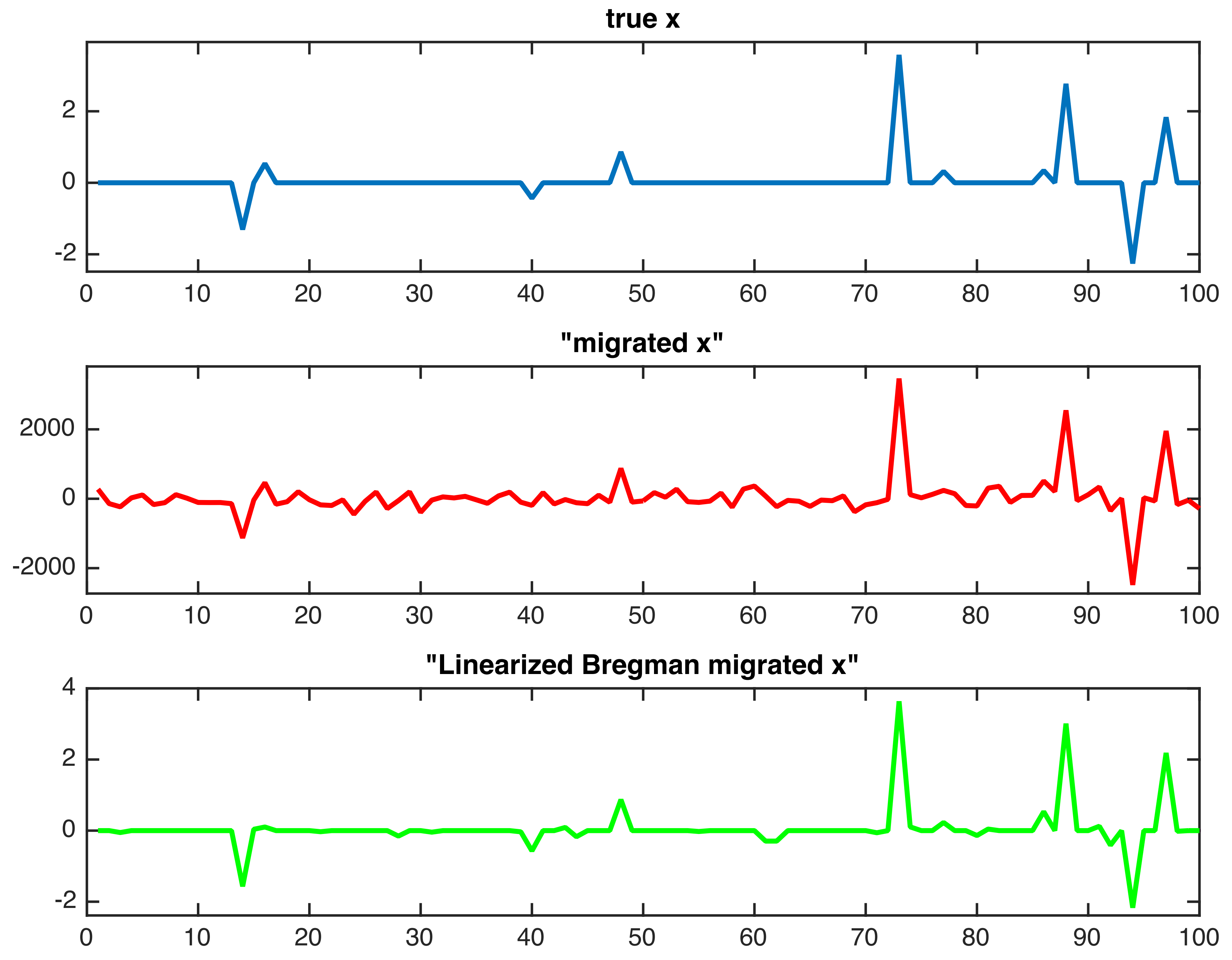
Example
– proposed method

```
for k=1:niter

    inds = randperm(m);
    rk  =inds(1:batch);
    Ark = A(rk,:);
    brk = b(rk);

    tk = norm(Ark*xk-brk)^2/norm(Ark'*(Ark*xk-brk))^2;
    zk = zk-tk*Ark'*(Ark*xk-brk);
    xk = sign(zk).*max(abs(zk)-lambda,0)

end
```



Fast randomized least squares

Hot topic in “big data” and randomized algorithms

- sketching techniques that randomly sample rows & solve [Li, Nguyên & Woodruff, ’14]

$$\underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{RM}\left(\mathbf{Ax} - \mathbf{b}\right)\|_2^2$$

- randomized preconditioning, e.g. w/ QR factorization on reduced system [Avron et. al., ’10]
- randomized Kaczmarz [Strohmer & Vershynin,’09; Zouzias & Freris, ’13]

These do not exploit structure (e.g. sparsity) & may require infeasible storage.

Felix J. Herrmann and Xiang Li, "Efficient least-squares imaging with sparsity promotion and compressive sensing", *Geophysical Prospecting*, vol. 60, p. 696-712, 2012
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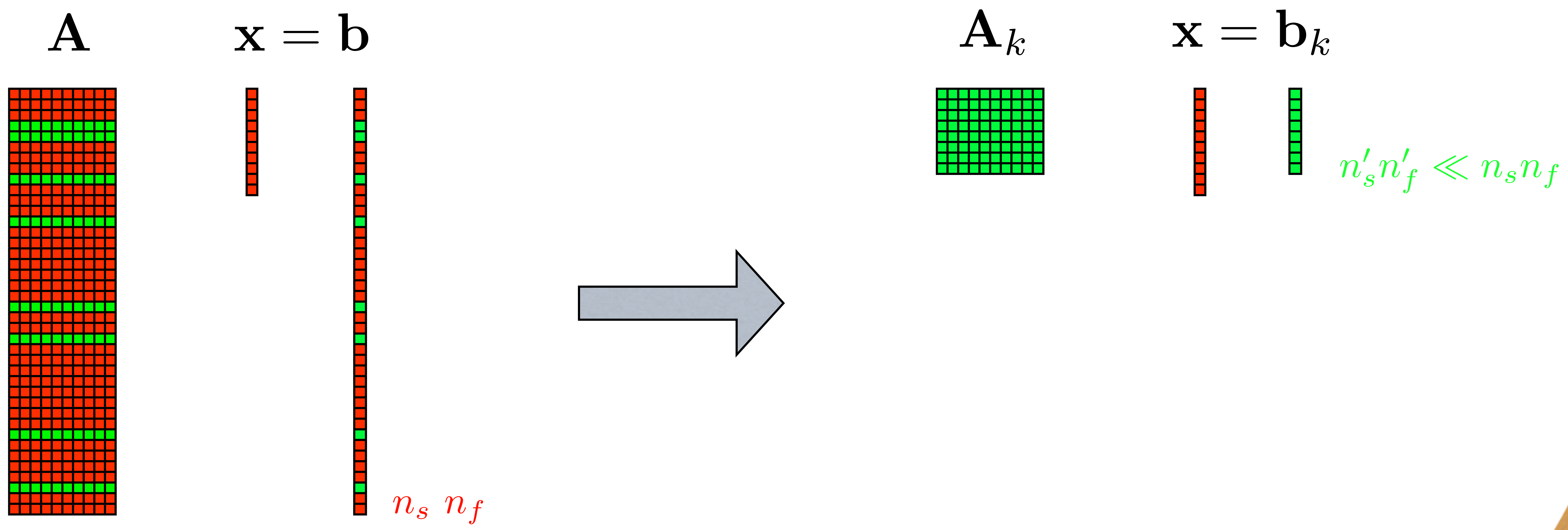
Leveraging the fold & threshold
– Randomized Iterative Shrinkage Thresholding Algorithm (RISTA)

Work /w for each iteration w/ independent randomized subsets of rows only

- simultaneous sourcing/phase encoding
- compressive sensing



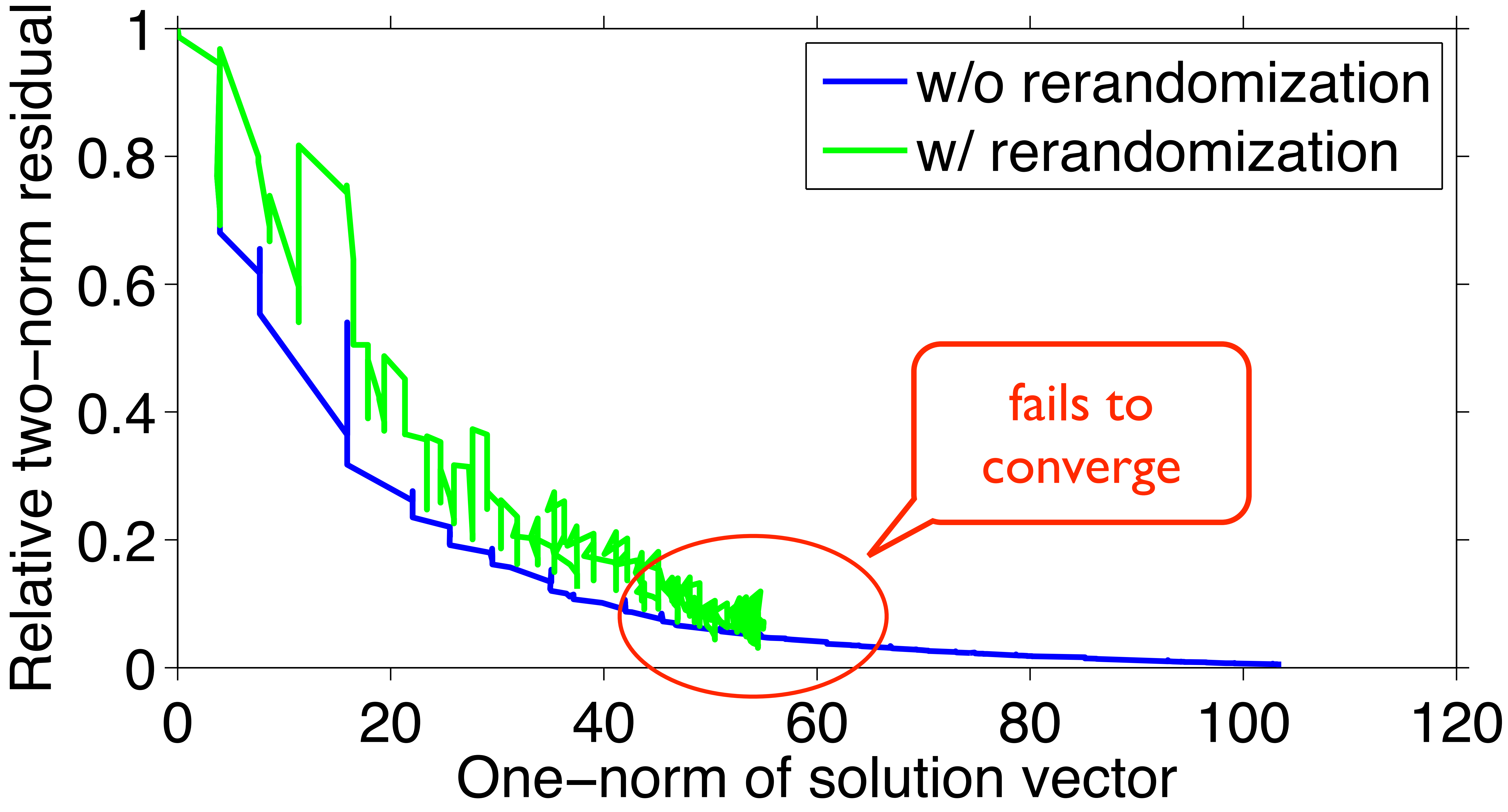
RISTA
– Randomized Iterative Shrinkage Thresholding Algorithm

1. **for** $k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{x}_k - t_k \mathbf{A}_k^* (\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)$
3. $\quad \mathbf{x}_{k+1} = S_{\lambda_k}(\mathbf{z}_{k+1})$
4. **end for**

*where $S_\lambda(x) = \mathrm{sign}(x) \cdot \max(|x| - \lambda, 0)$ is soft thresholding and t_k are step lengths

- relates to delicate "approximate" message passing theory [Montanari, '09]
- reduces IO & works on "small" subsets of (block) rows in parallel
- only converges for special $\{\mathbf{A},\ \mathbf{A}^H\}$ and tuned λ_k's
- havocs continuation strategies & does not converge

Solution path



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Relaxed sparsity objective

Consider $\lambda \to \infty$

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2 \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \end{array}$$

- strictly convex objective known as “elastic” net in machine learning
- corresponds to Basis Pursuit for “large enough” λ
- corresponds to [Lorentz et. al.,‘14]
  - sparse Kaczmarz for single-row $\mathbf{A}_k$’s
  - linearized Bregman for full $\mathbf{A}$’s

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RISKA

– Randomized IS Kaczmarz Algorithm w/ linearized Bregman

1. **for** $k = 0, 1, \cdots$
2. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}_k^* (\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)$
3. $\mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
4. **end for**

*where $t_k = \frac{\|\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k\|^2}{\|\mathbf{A}_k^*(\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)\|^2}$ are the step lengths

- exceedingly simple flexible “three line” algorithm
- gradient descend on the dual problem, which provably converges
- total different role for λ

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RISKA
– Randomized IS Kaczmarz Algorithm w/ linearized Bregman

1. **for** $k = 0, 1, \cdots$
2. $\qquad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}_k^*(\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)$
3. $\qquad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
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*where $t_k = \frac{\|\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k\|^2}{\|\mathbf{A}_k^*(\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)\|^2}$ are the step lengths

- exceedingly simple flexible “three line” algorithm
- gradient descend on the dual problem, which provably converges
- total different role for λ

Felix J. Herrmann and Xiang Li, “Efficient least-squares imaging with sparsity promotion and compressive sensing”, *Geophysical Prospecting*, vol. 60, p. 696-712, 2012
Felix J. Herrmann, “Accelerated large-scale inversion with message passing”, in *SEG Technical Program Expanded Abstracts*, 2012, vol. 31, p. 1-6.

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RISTA
– Randomized Iterative Shrinkage Thresholding Algorithm

1. **for** $k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{x}_k - t_k \mathbf{A}_k^* (\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k)$
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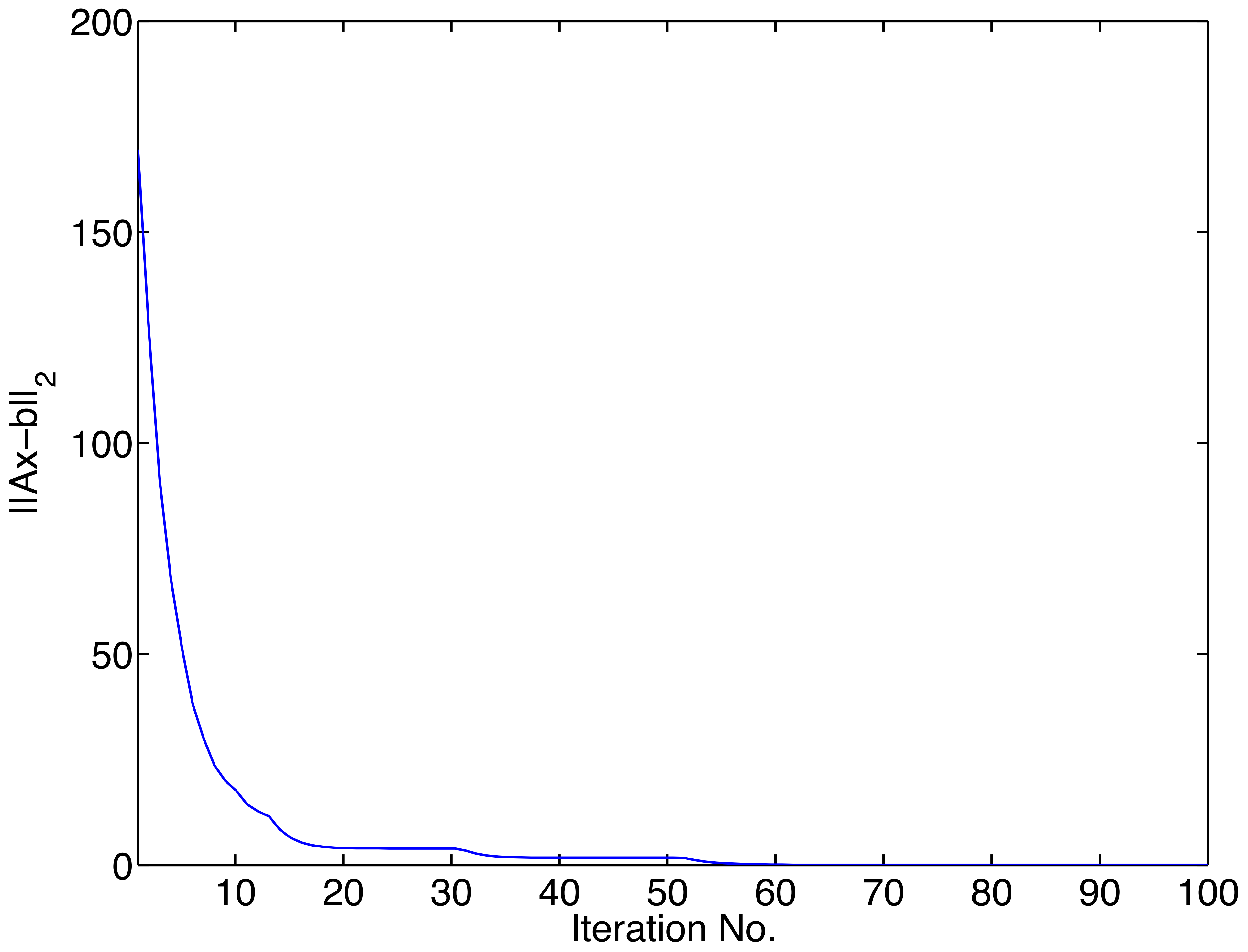
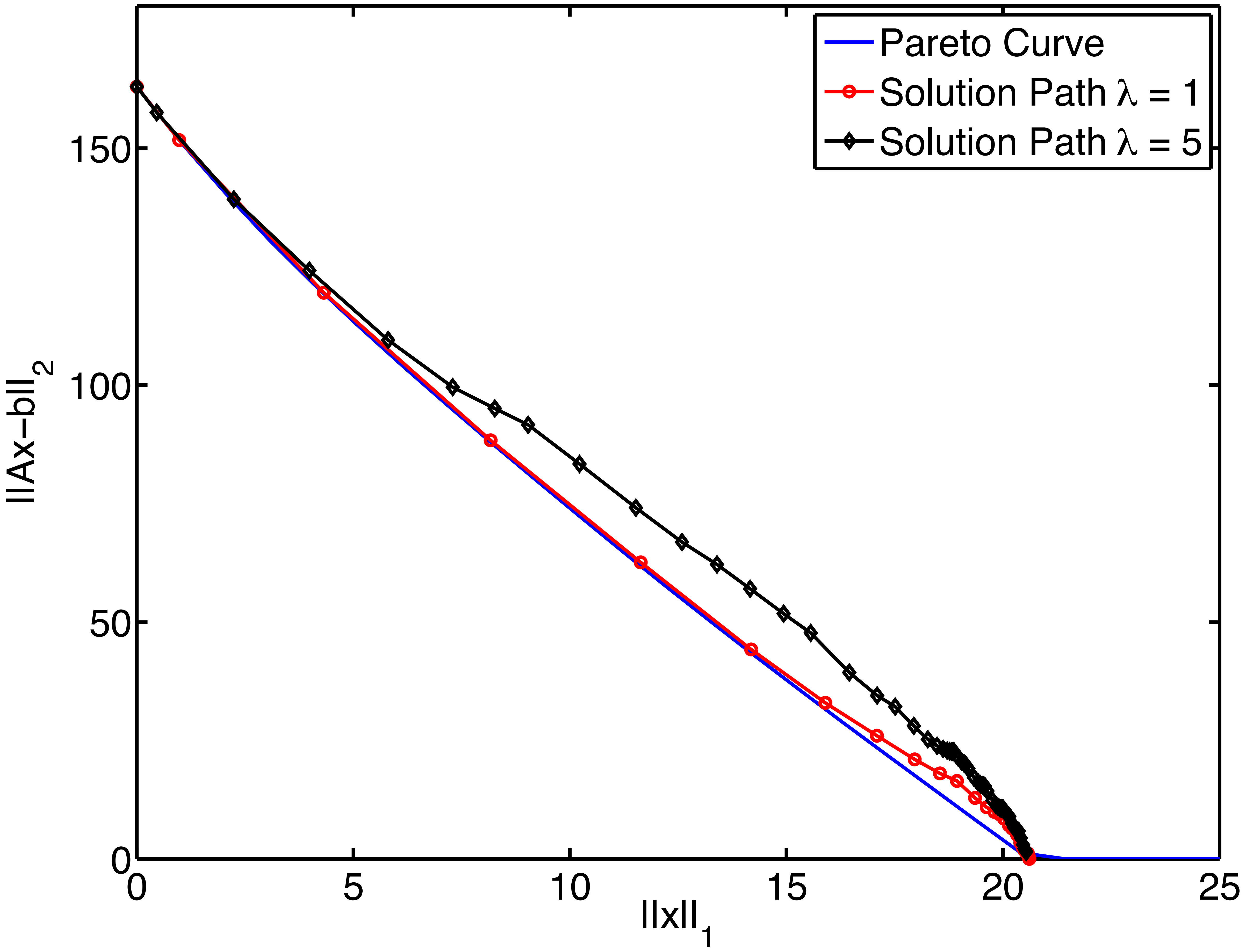
- relates to delicate “approximate” message passing theory [Montanari, ’09]
- reduces IO & works on “small” subsets of (block) rows in parallel
- only converges for special $\{\mathbf{A},\ \mathbf{A}^H\}$ and tuned λ_k’s
- havocs continuation strategies

Converges



Solution paths



Extension

– inconsistent systems

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2 \\ \text{subject to} \quad & \|\mathbf{A}\mathbf{x} - \mathbf{b}\| \leq \sigma \end{aligned}$$

via projections onto norm balls

$$\begin{array}{ll} 1. & \textbf{for } k = 0, 1, \cdots \\ 2. & \quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}_k^* \mathcal{P}_\sigma(\mathbf{A}_k \mathbf{x}_k - \mathbf{b}_k) \\ 3. & \quad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1}) \\ 4. & \textbf{end for} \end{array}$$

*where $\mathcal{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k) = \max\{0, 1 - \frac{\sigma}{\|\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k\|}\} \cdot (\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$

Osher, S., Mao, Y., Dong, B., Yin, W.: Fast Linearized Bregman iteration for compressive sensing and sparse denoising. Commun. Math. Sci. 8, 93–111 (2010)

Role of threshold

$\lambda \to \infty$

- solution corresponds to BP (or BPDN)
- difficult to solve (like $\lambda \to 0^+$ for ISTA)
- thresholded components first step guaranteed to be in support

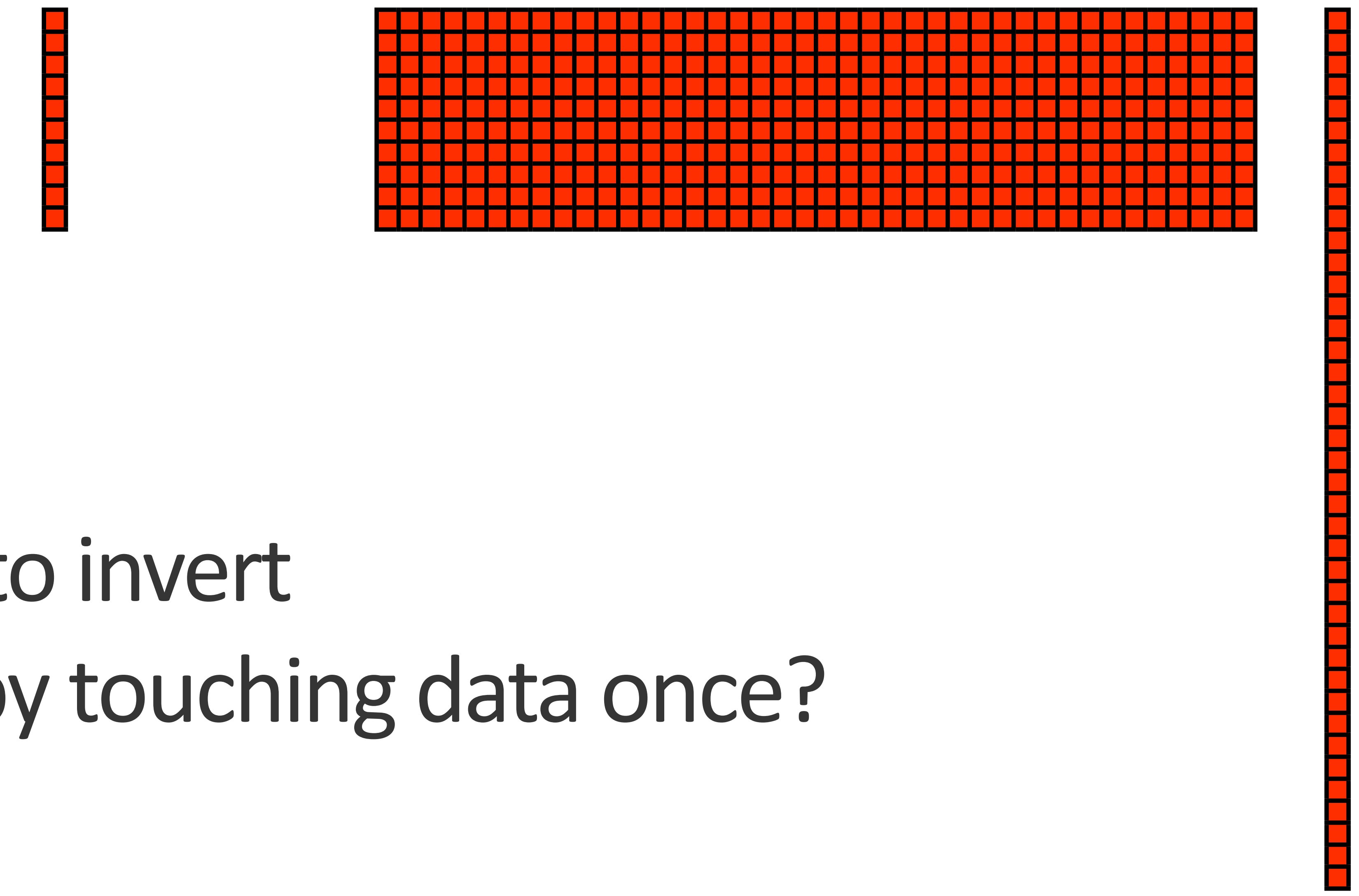
$1 \ll \lambda \ll \infty$

- iterations "auto tune" and do not wander off too far from optimal Pareto curve
- when threshold too large RISTA still makes progress
- room for acceleration w/ kicking techniques

Application

Least-squares (RTM) migration:

$$\delta \mathbf{m} = \sum_{ij} \nabla \mathbf{F}_{ij}^{H}(\mathbf{m}_0, \mathbf{q}_{ij}) \delta \mathbf{d}_{ij}$$



- too expensive to invert
- can we invert by touching data once?

Fast SPLSM w/ CS

– w/ randomized source subsets

$$\underset{\mathbf{x}}{\text{minimize}} \quad \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2$$
$$\text{subject to} \quad \sum_{ij} \|\nabla \mathbf{F}_{ij}(\mathbf{m}_0, \mathbf{q}_{ij})\mathbf{C}^*\mathbf{x} - \delta\mathbf{d}_{ij}\| \leq \sigma$$

By iterating

1. $\textbf{for } k = 0, 1, \cdots$
2. $\quad \Omega \in [1 \cdots n_f],\ \Sigma \in [1 \cdots n_s] \text{ for } \#\Omega \ll n_f,\ \#\Sigma \ll n_s$
3. $\quad \mathbf{A}_k = \{\nabla \mathbf{F}_{ij}(\mathbf{m}_0, \bar{\mathbf{q}}_{ij})\mathbf{C}^*\}_{i\in\Omega, j\in\Sigma} \text{ with } \bar{\mathbf{q}}_{ij} = \sum_{l=1}^{n_s} w_l \mathbf{q}_{i,l}$
4. $\quad \mathbf{b}_k = \{\delta\bar{\mathbf{d}}_{ij}\}_{i\in\Omega, j\in\Sigma} \text{ with } \delta\bar{\mathbf{d}}_{ij} = \sum_{l=1}^{n_s} w_l \delta\mathbf{d}_{i,l}$
5. $\quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}_k^* \mathcal{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$
5. $\quad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
6. $\textbf{end for}$

Fast SPLSM w/ CS
– experimental setup

Data:

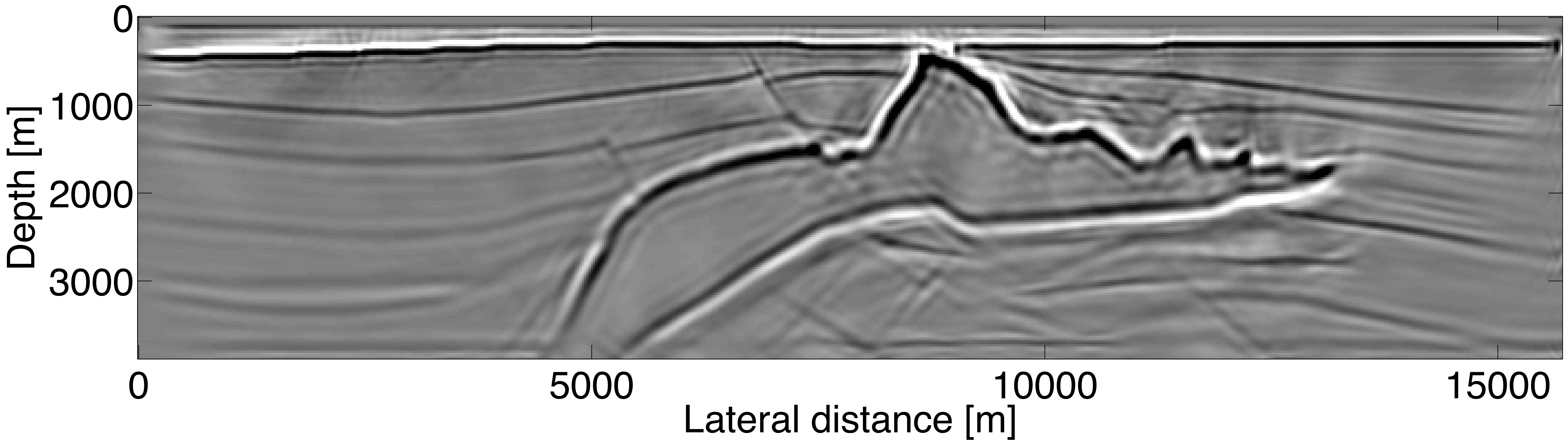
- 320 sources and receivers
- 72 frequency slices ranging from 3 – 12 Hz
- $\delta \mathbf{d} = \mathbf{F}(\mathbf{m}) - \mathbf{F}(\mathbf{m}_0)$, generated with separate modeling engine

Experiments:

- one pass through the data with different batch/block sizes
- simultaneous vs sequential shots
- choose λ according to $\max\left(t_1 \cdot \mathbf{A}_1^* \mathbf{b}_1\right)$ and number of iterations
- no source estimation – use correct source for linearized inversions

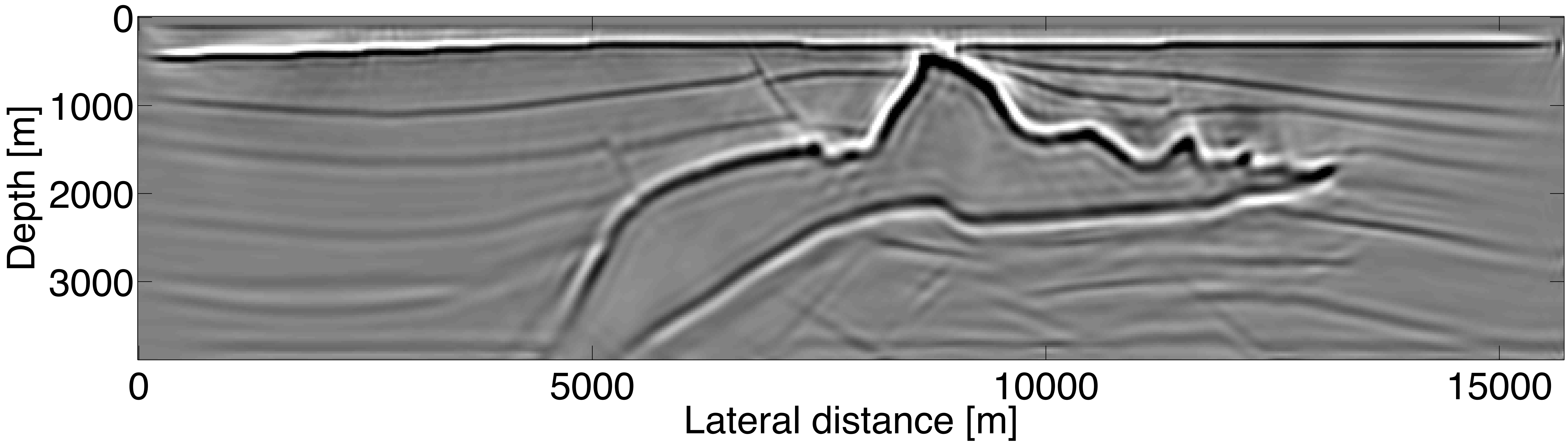
Fast SPLSM w/ CS

– 360 iterations, each w/ 8 frequencies/sim. shots



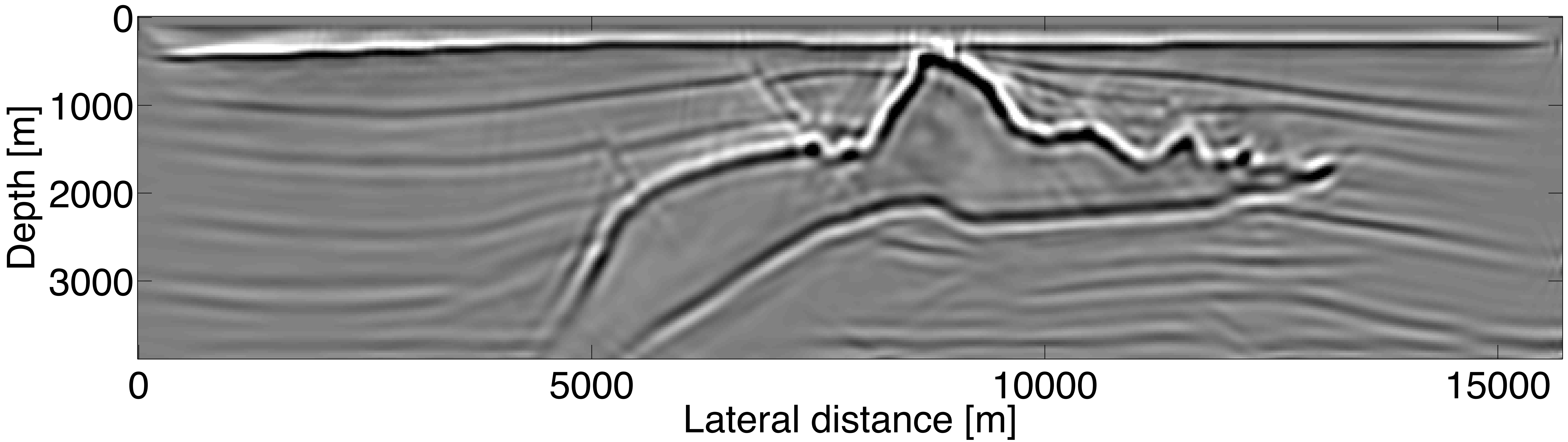
Fast SPLSM w/ CS

– 90 iterations, each w/ 16 frequencies/sim. shots

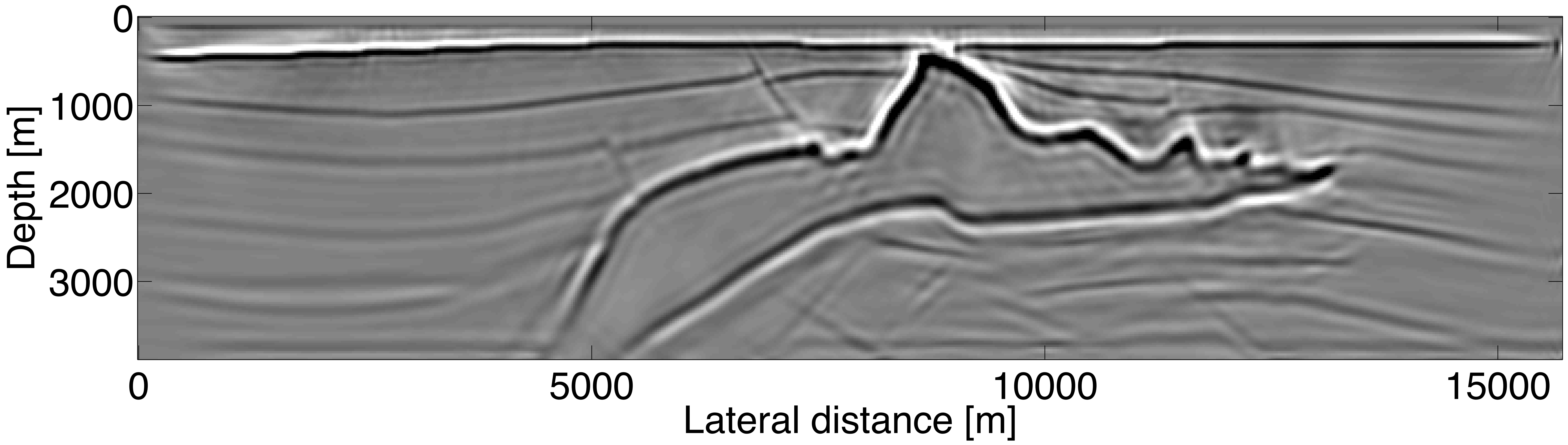


Fast SPLSM w/ CS

– 23 iterations, each w/ 32 frequencies/sim. shots

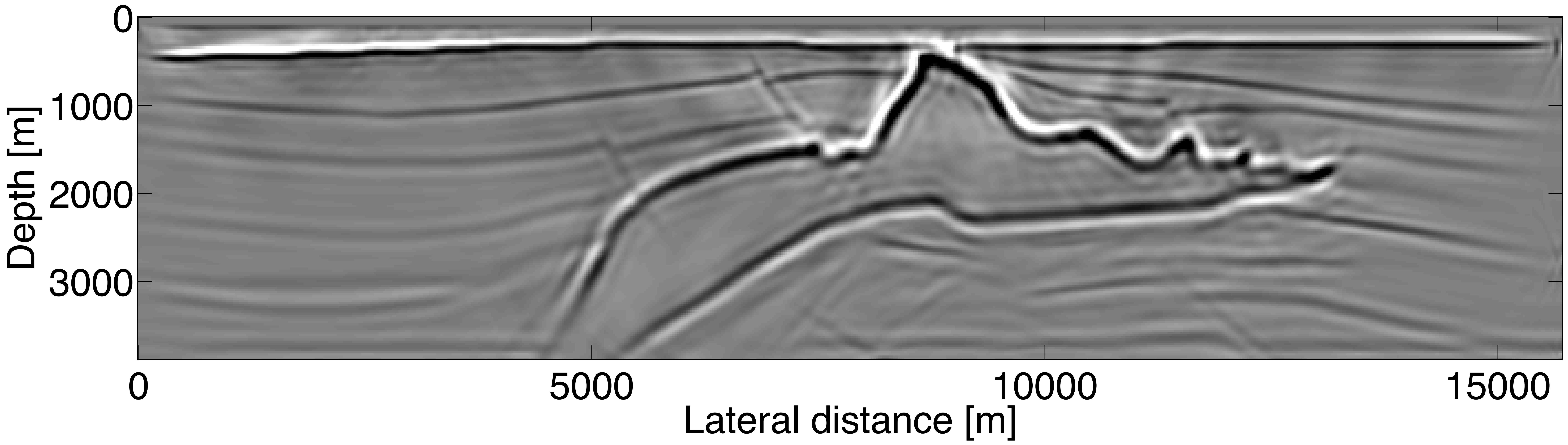


Fast SPLSM w/ CS
– 90 iterations, each w/ 16 frequencies/sim. shots



Fast SPLSM w/ CS

– 90 iterations, each w/ 16 frequencies/sequential shots



Ning Tu, Aleksandr Y. Aravkin, Tristan van Leeuwen, Tim T.Y. Lin, and Felix J. Herrmann, "Source estimation with multiples—fast ambiguity-resolved seismic imaging". 2015

Fast SPLSM w/ CS

– on-the-fly source estimation

$$\underset{\mathbf{x}}{\text{minimize}} \quad \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2$$

$$\text{subject to} \quad \sum_{ij}\|\nabla\mathbf{F}_{ij}(\mathbf{m}_0, \mathbf{q}_{ij})\mathbf{C}^*\mathbf{x} - \delta\mathbf{d}_{ij}\| \leq \sigma$$

By iterating

1. **for** $k = 0, 1, \cdots$
2. $\Omega \in [1 \cdots n_f],\ \Sigma \in [1 \cdots n_s]$ for $\#\Omega \ll n_f,\ \#\Sigma \ll n_s$
3. $\mathbf{A}_k = \{\nabla\mathbf{F}_{ij}(\mathbf{m}_0, s_i\bar{\mathbf{q}}_{ij})\mathbf{C}^*\}_{i\in\Omega, j\in\Sigma}$ with $\bar{\mathbf{q}}_{ij} = \sum_{l=1}^{n_s} w_l \mathbf{q}_{i,l}$
4. $\mathbf{b}_k = \{\delta\bar{\mathbf{d}}_{ij}\}_{i\in\Omega, j\in\Sigma}$ with $\delta\bar{\mathbf{d}}_{ij} = \sum_{l=1}^{n_s} w_l \delta\mathbf{d}_{i,l}$
5. $s_i = \frac{\sum_{j\in\Sigma}\langle\delta\bar{\mathbf{d}}_{i,j}, \nabla\mathbf{F}[\mathbf{m}_0, \bar{\mathbf{q}}_j]\mathbf{C}^*\mathbf{x}\rangle}{\sum_{j\in\Sigma}\langle\nabla\mathbf{F}[\mathbf{m}_0, \bar{\mathbf{q}}_j]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \bar{\mathbf{q}}_j]\mathbf{C}^*\mathbf{x}\rangle},\ \mathbf{A}_k = \{\nabla\mathbf{F}_{ij}(\mathbf{m}_0, s_i\bar{\mathbf{q}}_{ij})\mathbf{C}^*\}_{i\in\Omega, j\in\Sigma}$
6. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k\mathbf{A}_k^*\mathcal{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$
7. $\mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
8. **end for**

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Fast SPLSM w/ source estimation
– experimental setup

Data:

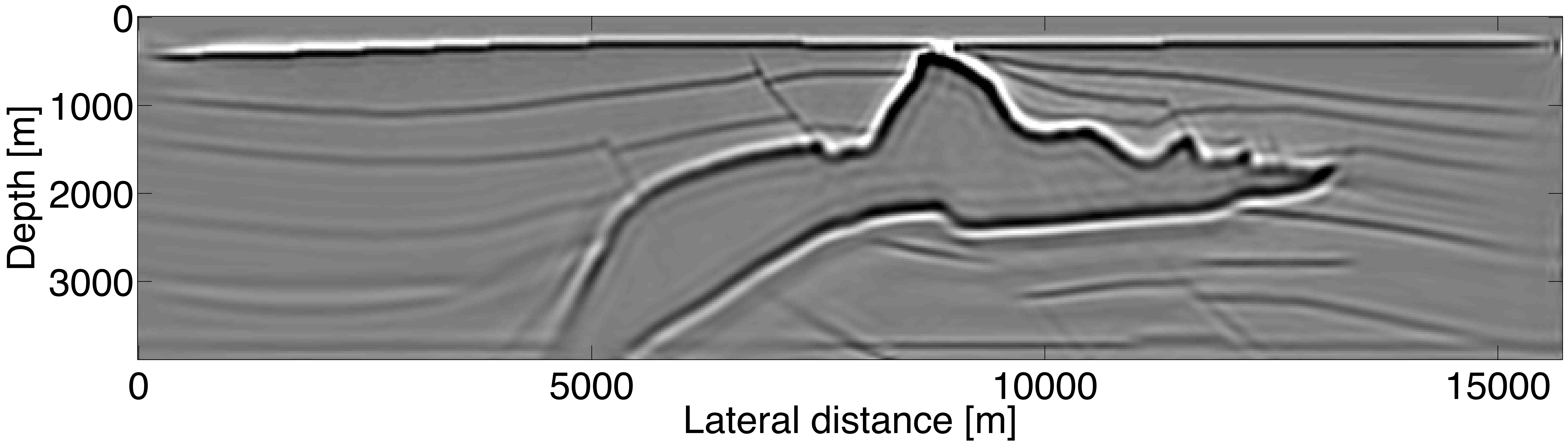
- 320 sources and receivers
- 72 frequency slices ranging from 3 - 12 Hz
- $\delta\mathbf{d} = \nabla\mathbf{F}\delta\mathbf{m}$ inverse crime data

Experiments:

- one pass through the data with the same block size & different frequncy-shot ratios
- simultaneous sources
- choose λ according to $\max\left(t_1 \cdot \mathbf{A}_1^*\mathbf{b}_1\right)$
- source estimation with delta Dirac as initial guess
- estimated source scaled w.r.t. true source

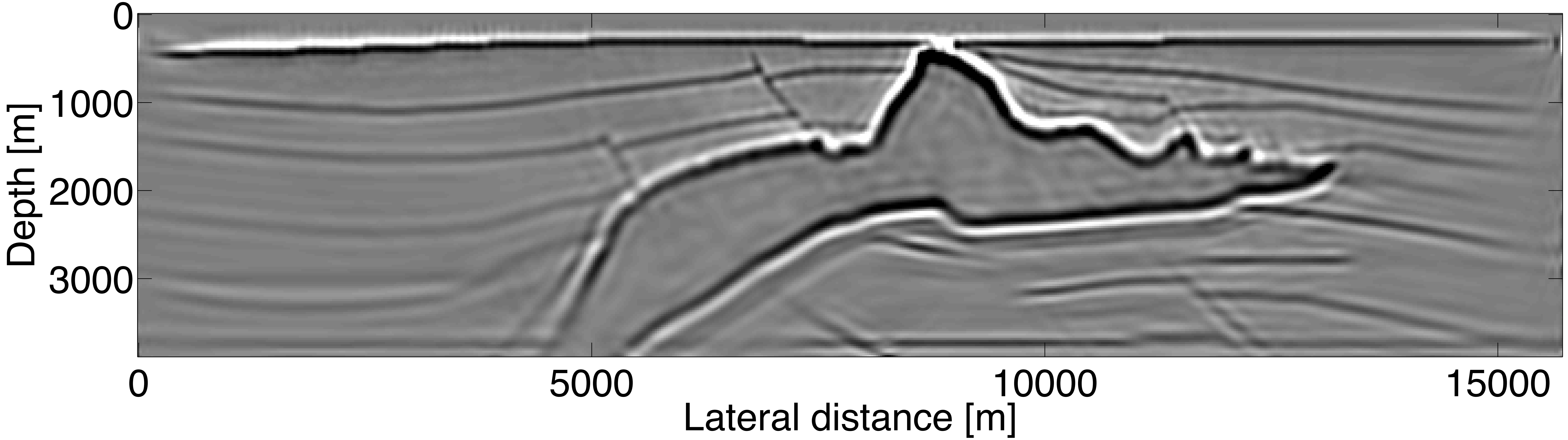
Fast SPLSM w/ source estimation

– 80 iterations, each w/ 72 frequencies/4 sim. shots & true source



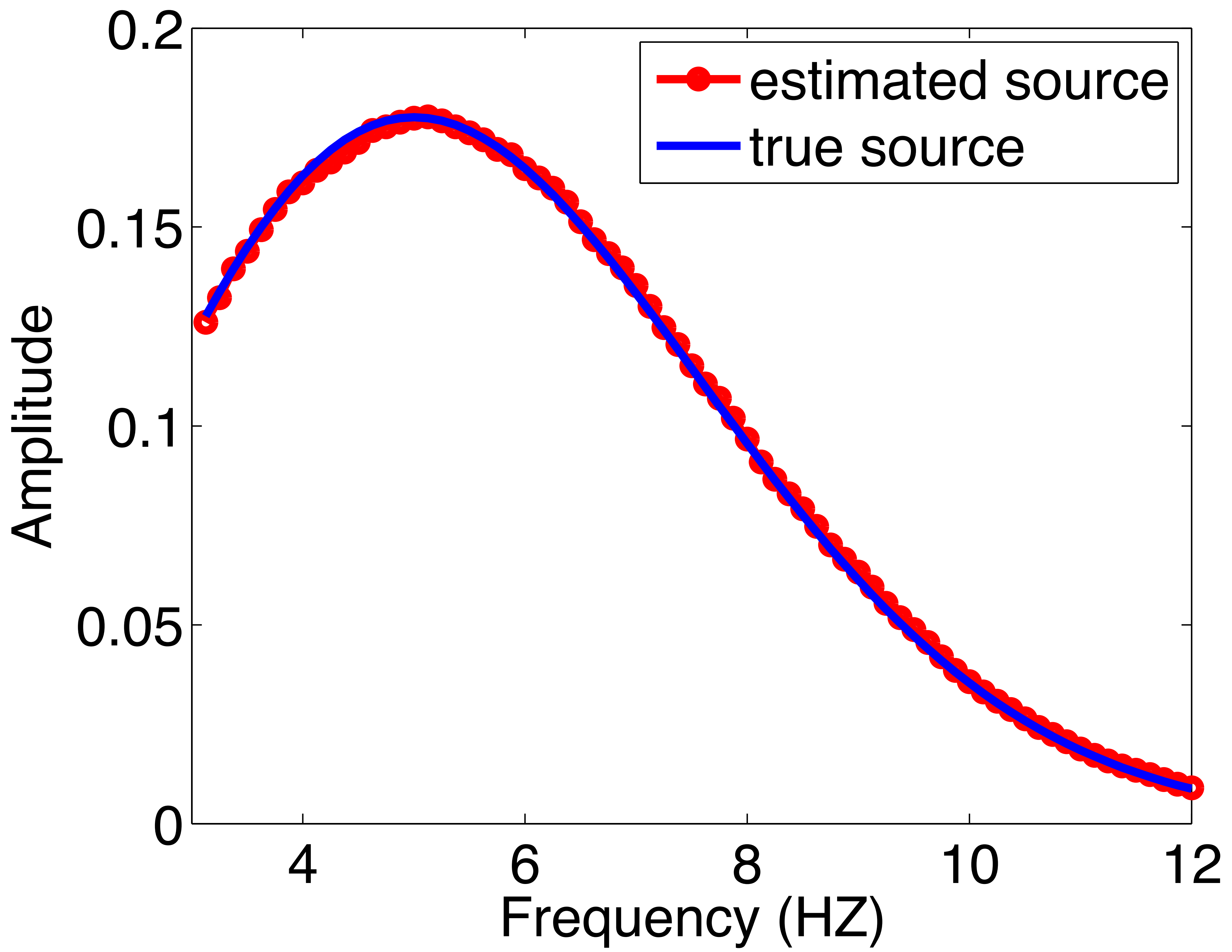
Fast SPLSM w/ source estimation

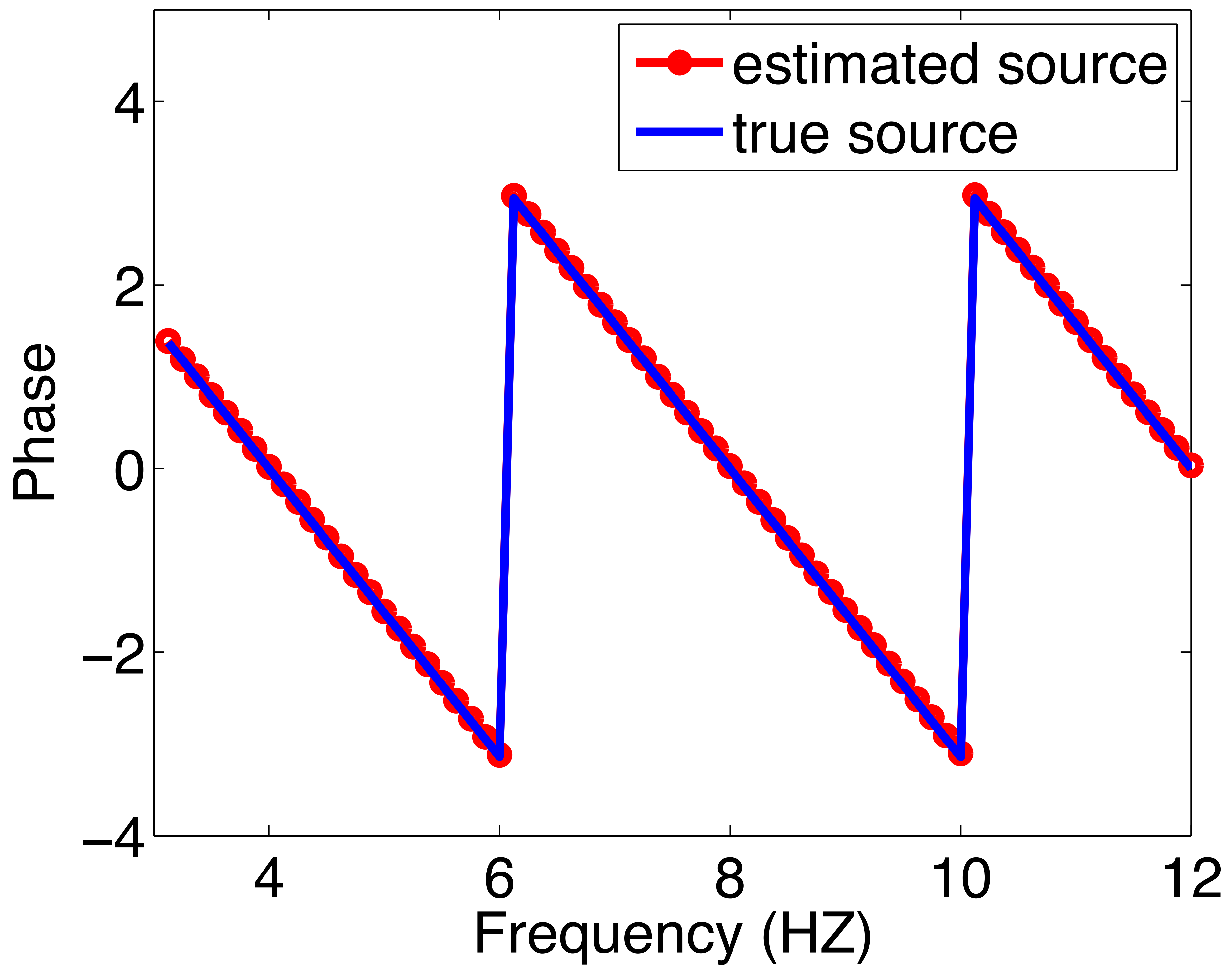
– estimated source



Fast SPLSM w/ source estimation

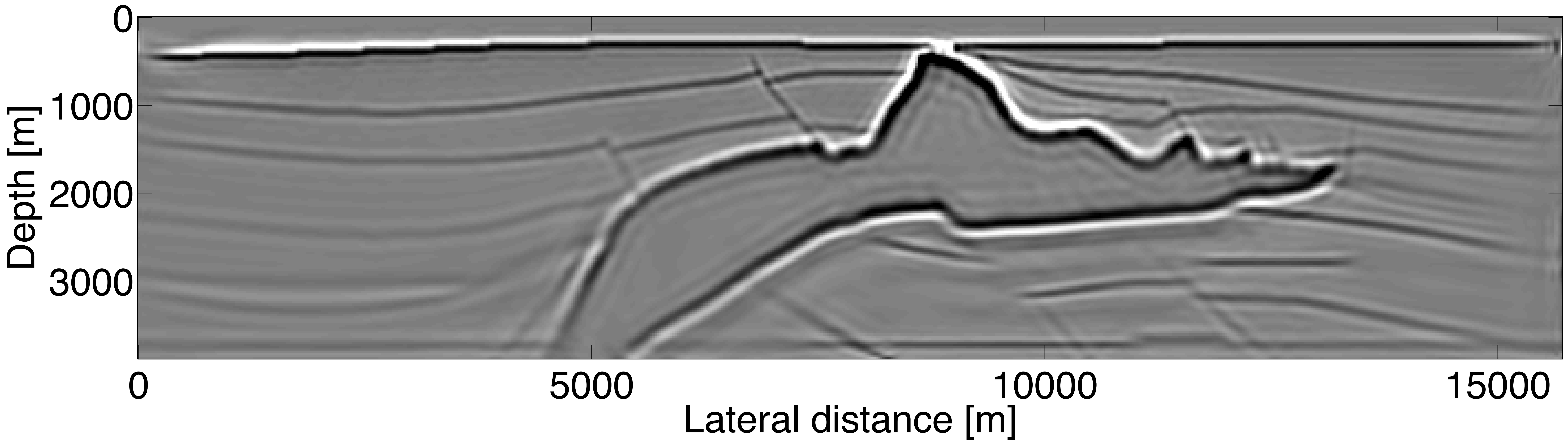
– estimated source





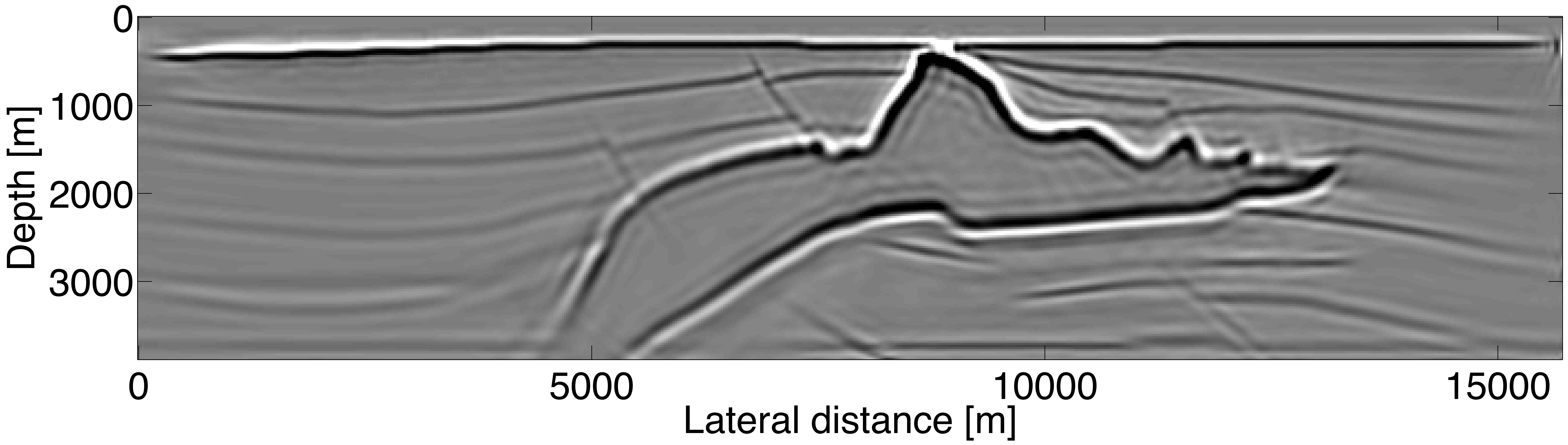
Fast SPLSM w/ source estimation

– 90 iterations, each w/ 16 frequencies/16 sim. shots w/ true source

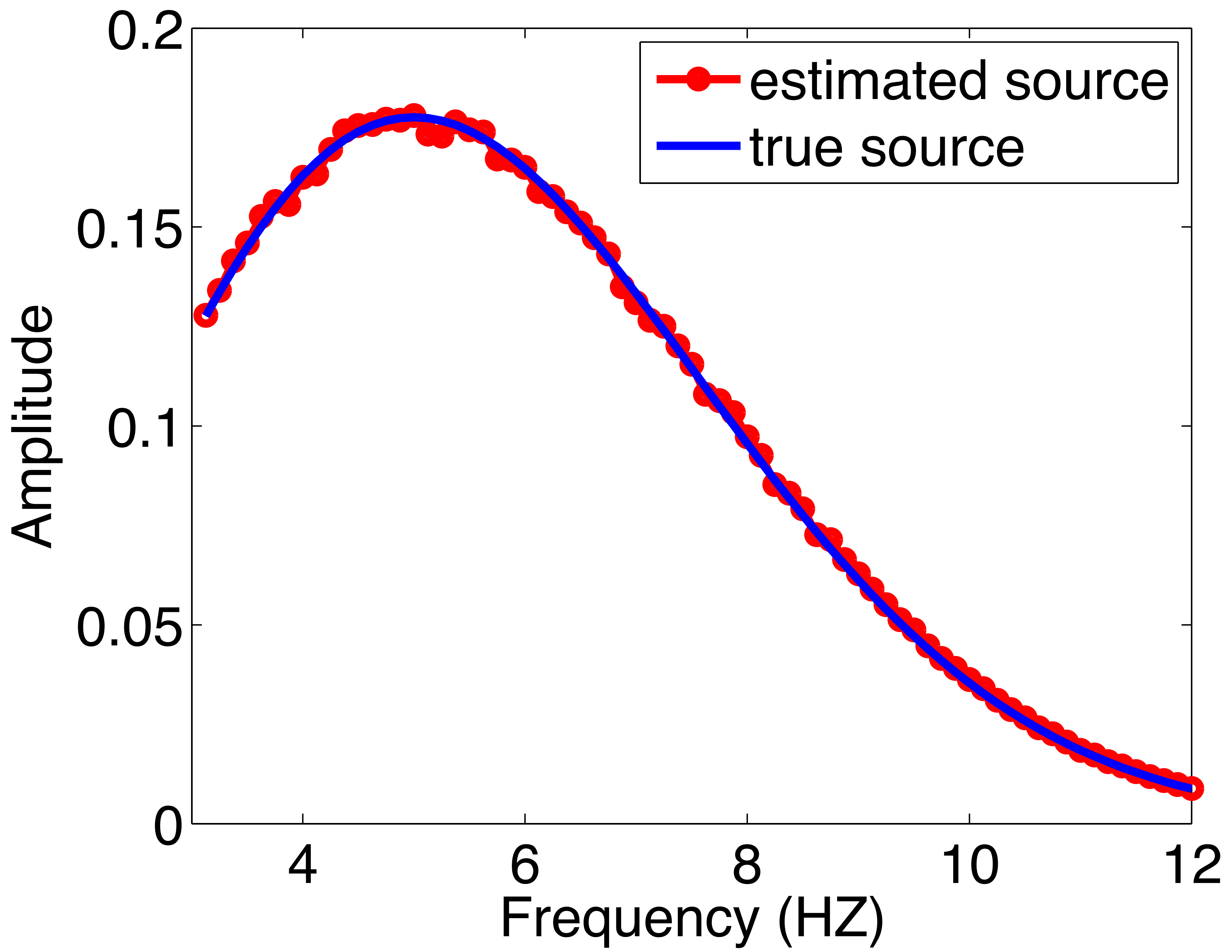


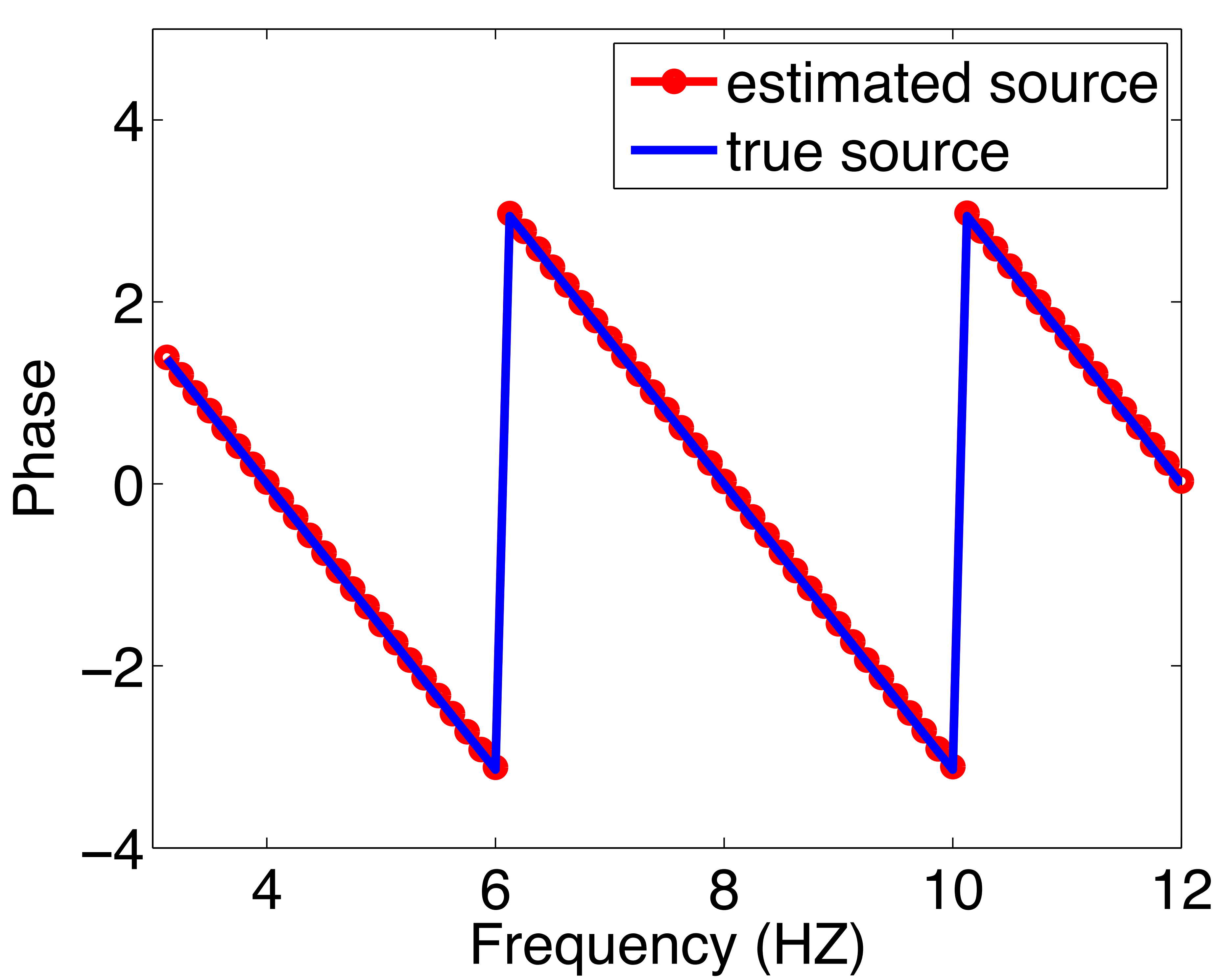
Fast SPLSM w/ source estimation

– estimated source



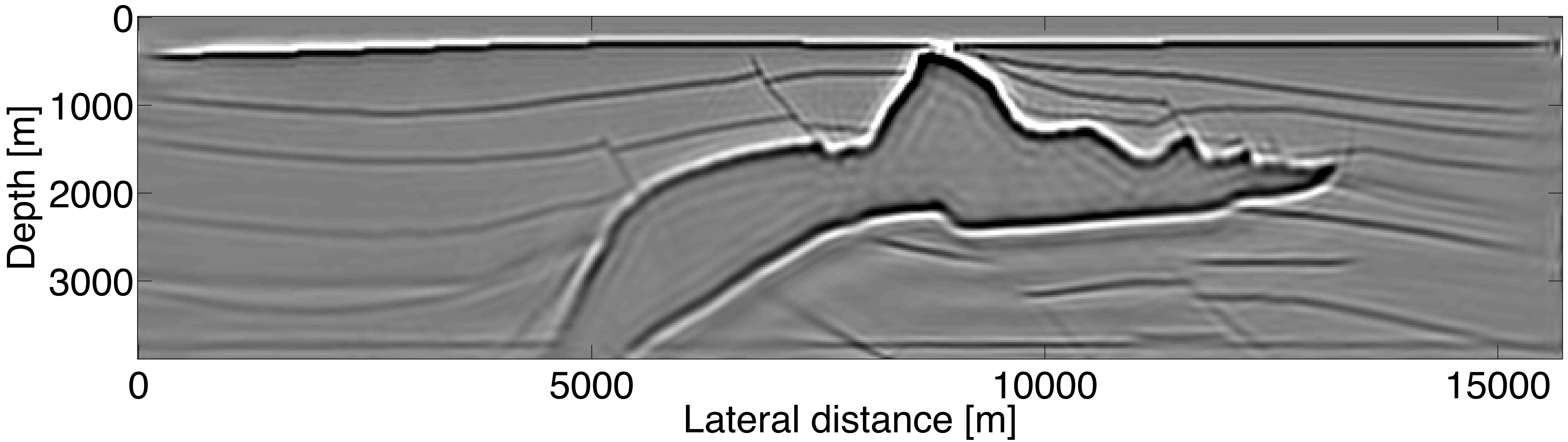
Fast SPLSM w/ source estimation
– estimated source





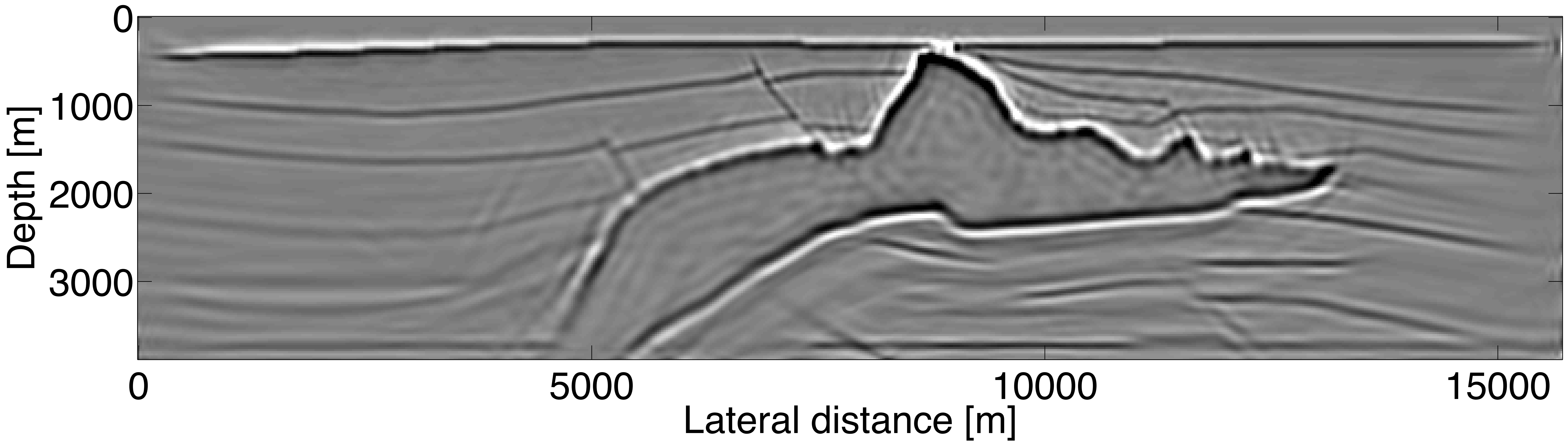
Fast SPLSM w/ source estimation

– 90 iterations, each w/ 4 frequencies/64 sim. shots w/ true source



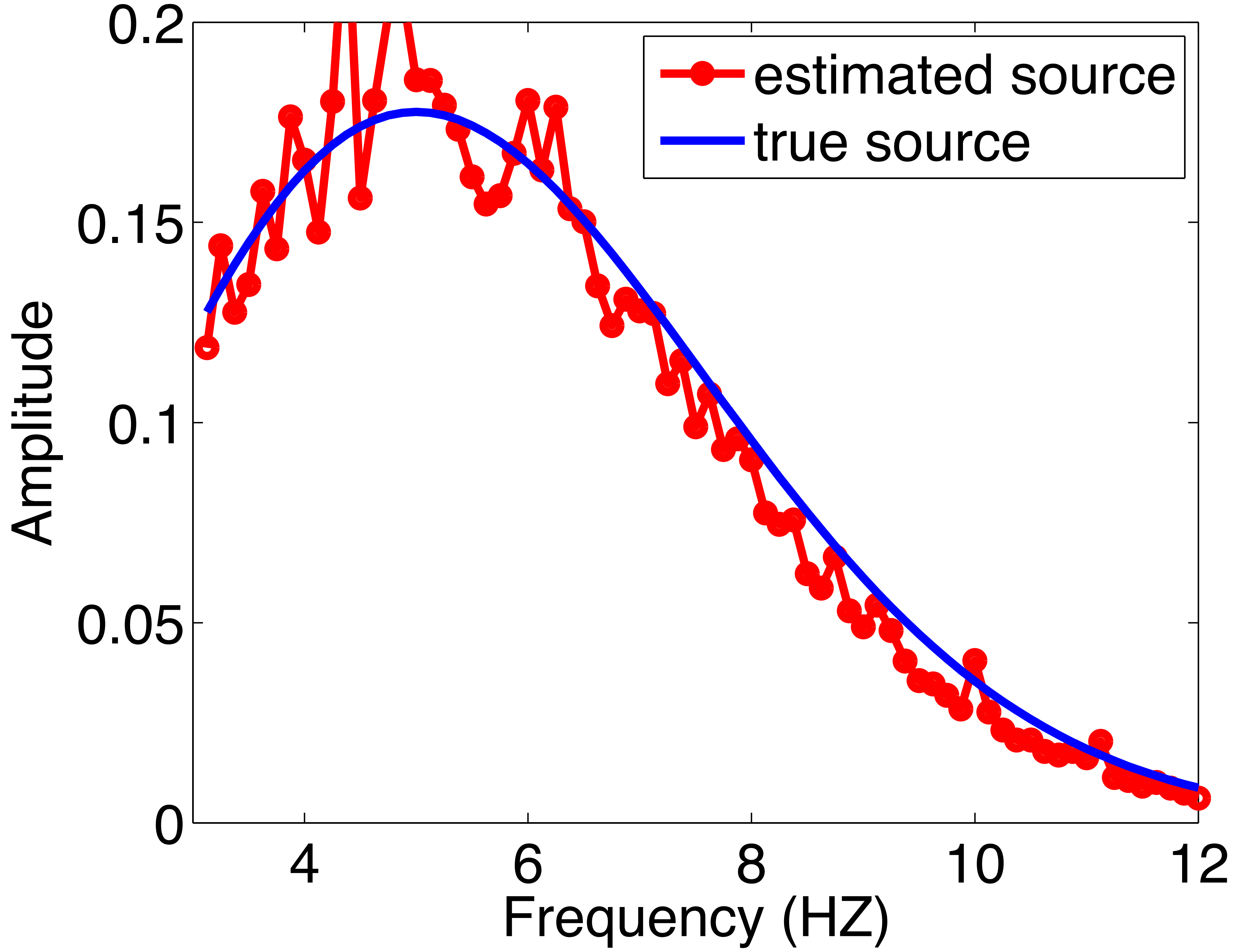
Fast SPLSM w/ source estimation

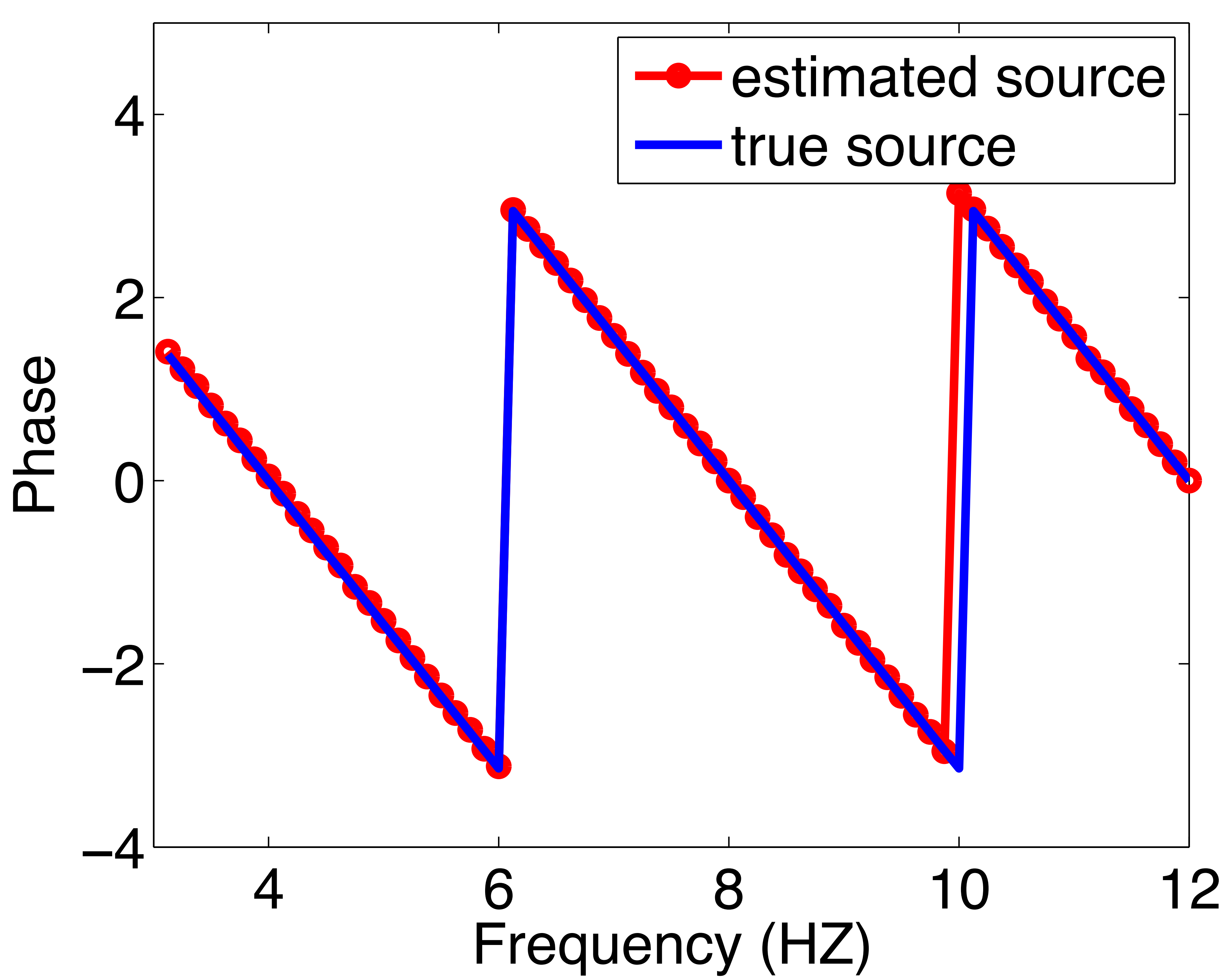
– estimated source



Fast SPLSM w/ source estimation

– estimated source





Observations

Inversions can be carried out at cost (= batch size X # iterations) of ~1 RTM

For known source function:

- quality is best for intermediate batch size & # of iterations
- results for randomly selected sources are of similar quality
- offers flexibility for parallelism

For unknown source function:

- source function is best estimated when # of frequencies is not too low
- quality is similar to cases where the source function is known

Ning Tu and Felix J. Herrmann, "Fast imaging with surface-related multiples by sparse inversion", *Geophysical Journal International*, vol. 201, p. 304-317, 2015

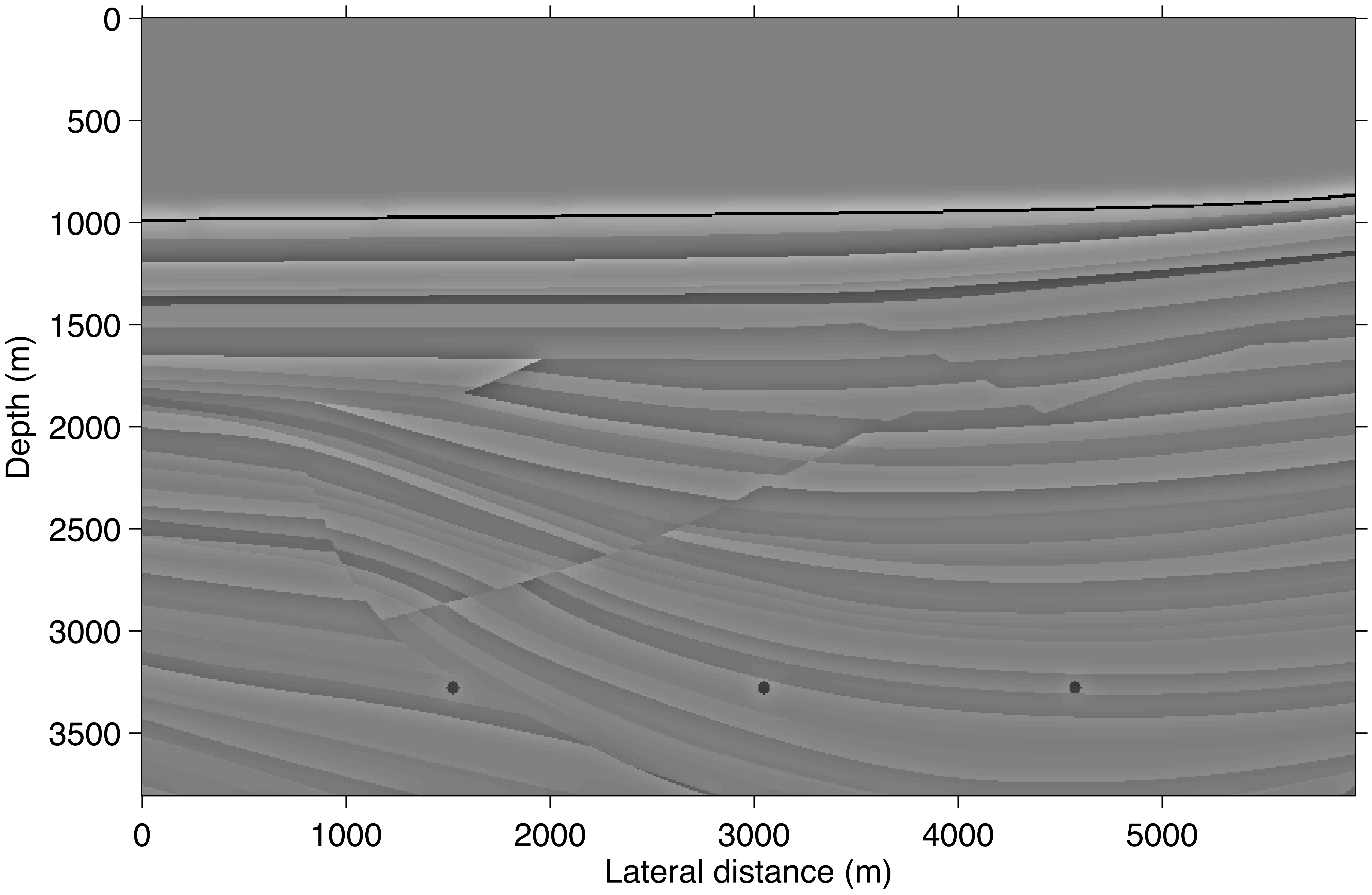
SLIM

Extension
– imaging w/ surface-related multiples

Incorporate predictor of surface-related multiples via areal sources

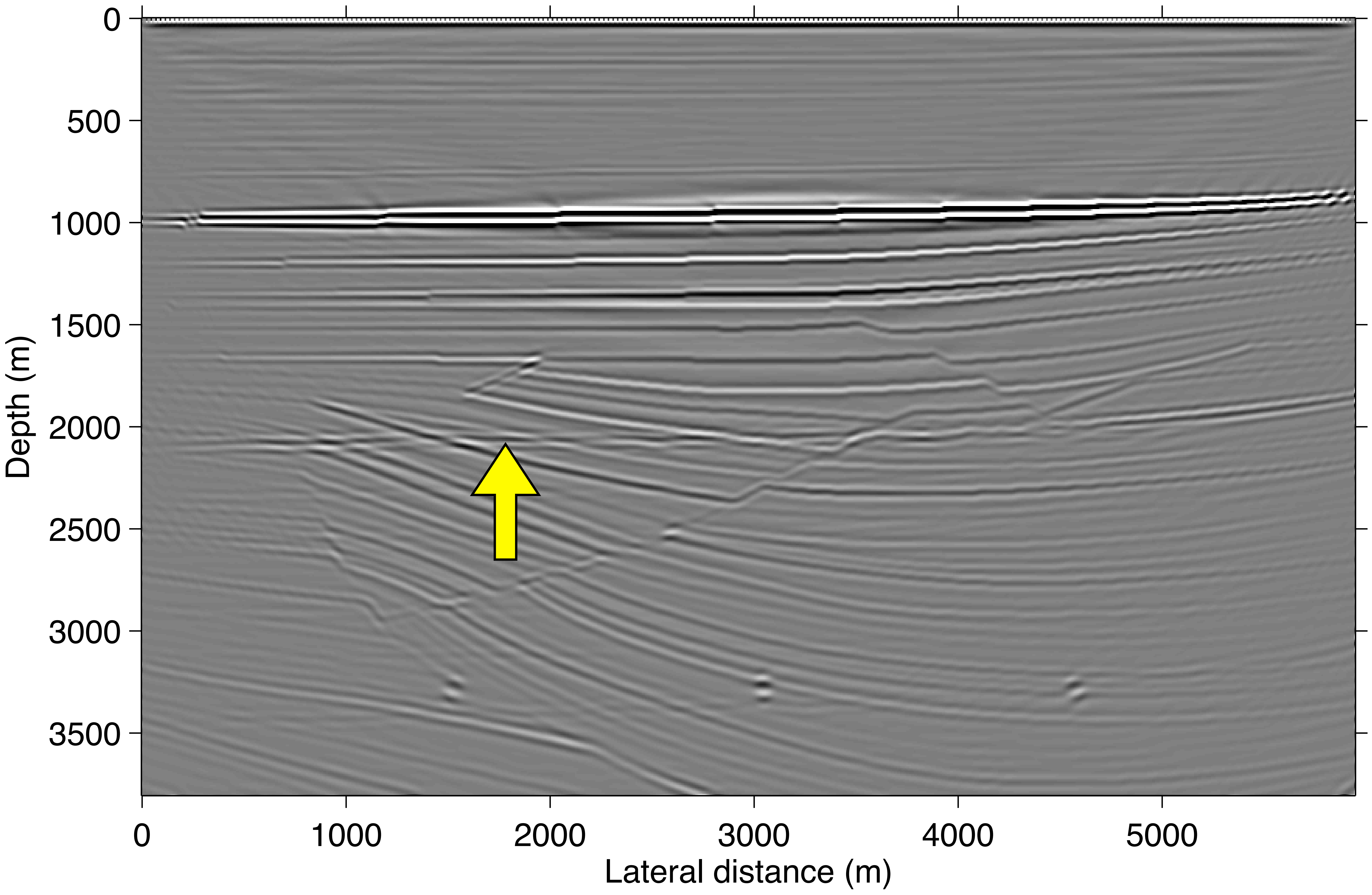
$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i \in \Omega} \sum_{j \in \Sigma} \|\delta \bar{\mathbf{d}}_{i,j} - \nabla \mathbf{F}[\mathbf{m}_0, s_i \bar{\mathbf{q}}_j - \delta \bar{\mathbf{d}}_{i,j}] \mathbf{C}^* \mathbf{x}\|_2^2$$

True image

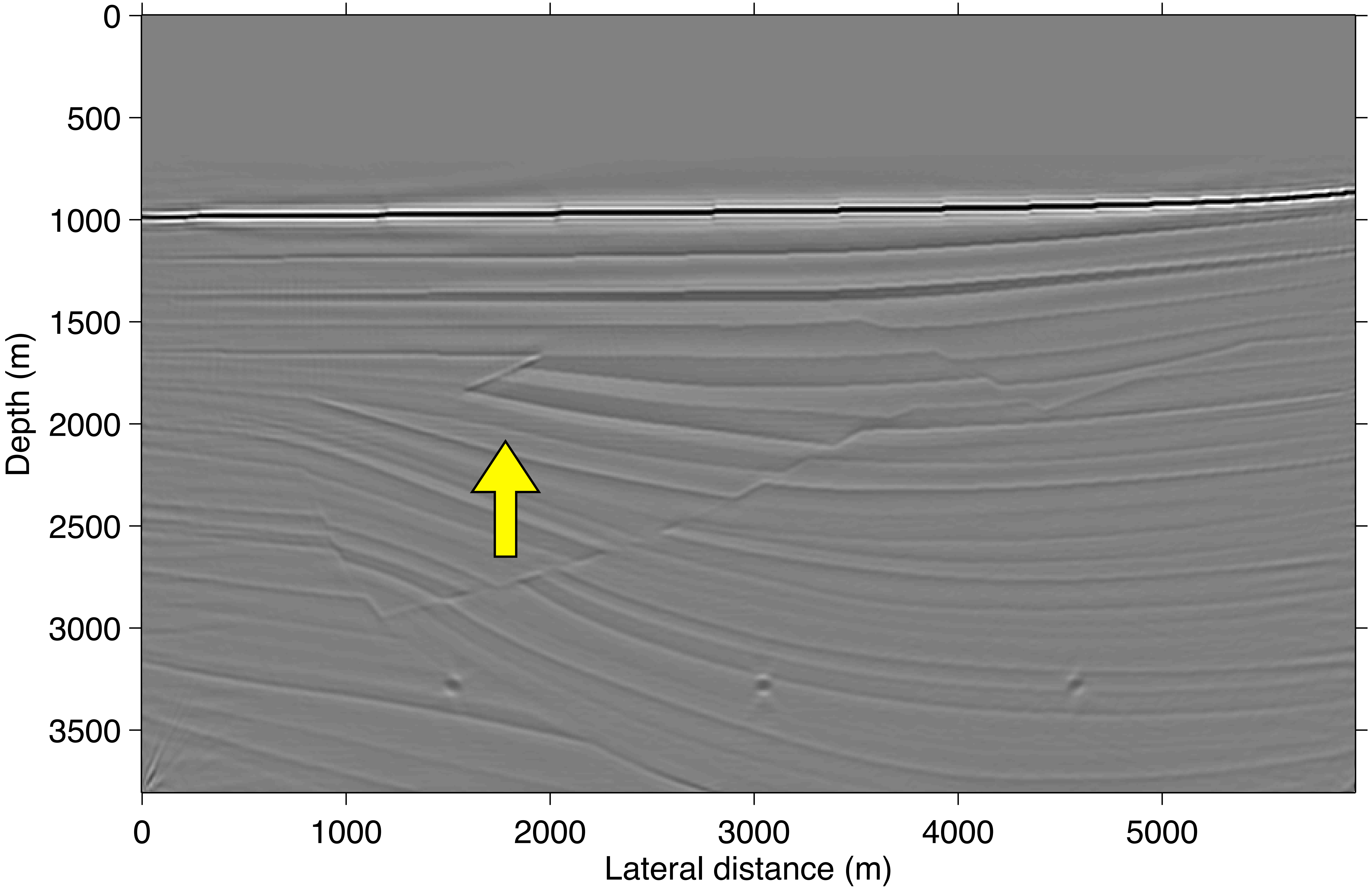


0
500
1000
1500
2000
2500
3000
3500
Depth (m)
0
1000
2000
3000
4000
5000
Lateral distance (m)

RTM w/ multiples

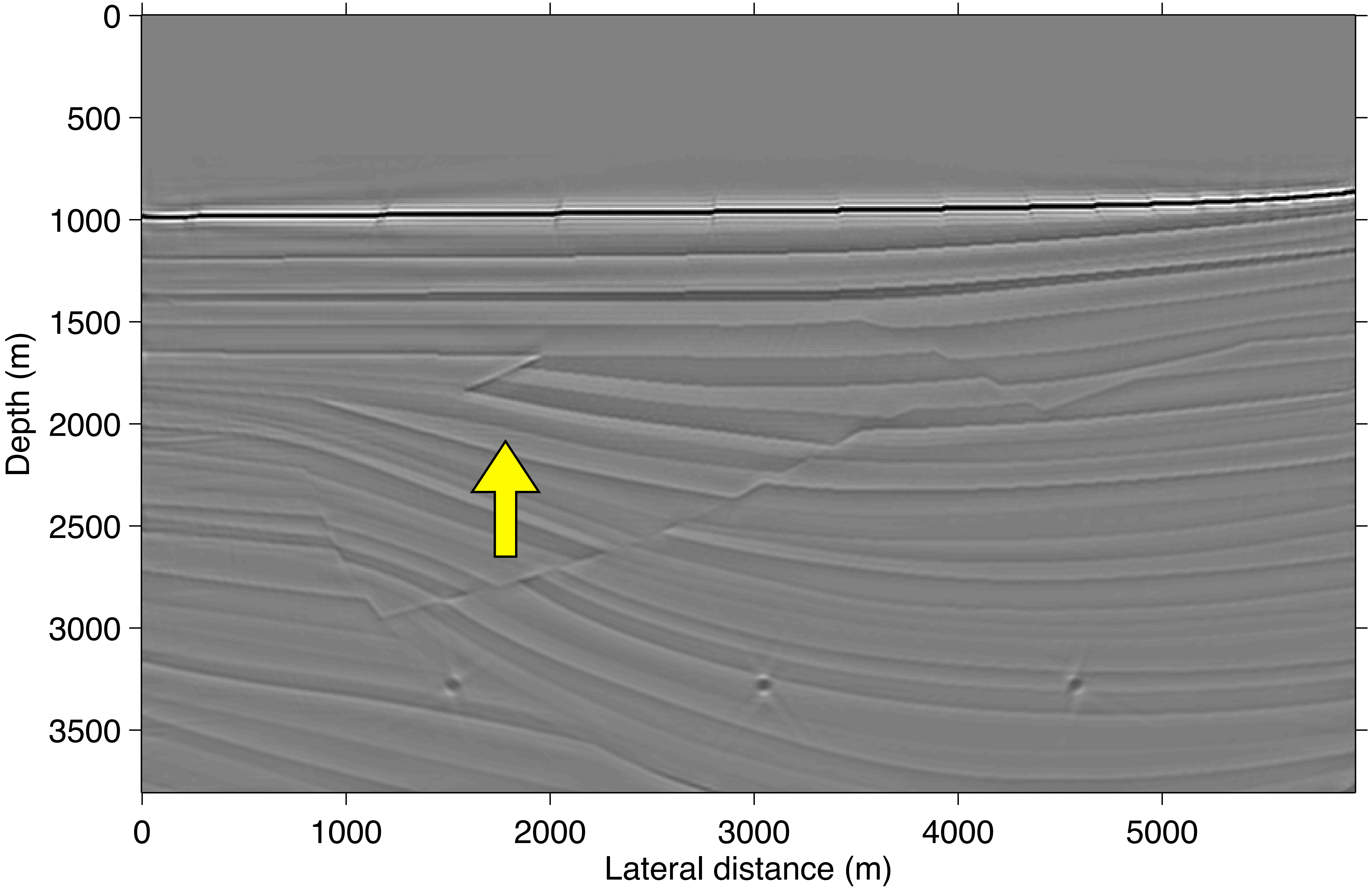


Fast SPLSM w/ multiples by SPGl1



Simulation cost **~1 RTM** using all the data

Fast SPLSM w/ multiples by **RISKA**



Simulation cost **~1 RTM** using all the data

Bottom line
– what you need

Access to $\{\mathbf{A},\ \mathbf{A}^H\}$ or $\{\mathbf{A}^H,\ \mathbf{A}^H\mathbf{A}\}$

- migration, demigration or migration, Gauss-Newton Hessian
- norms for residual & gradient

Ability to subsample data

- randomized supershots or randomly selected shots in RTM
- or randomized traces (source/receiver) pairs in Kirchhoff migration

Some idea of max entry of $\mathbf{A}_k^*\mathbf{b}_k$

Conclusions & extensions

Algorithm:

- simple, converges & has very few tuning parameters
- offers maximal flexibility for
  - implementations that strike a balance between data- and model-space parallelism
  - extensions such as source estimation & imaging w/ multiples
  - other overdetermined problems such as AVO
- gets hifi/high-resolution images touching the data only once

Simple structure also offers flexibility to do

- adaptive sampling
- on-line recovery while randomized data streams in

John “Ernie” Esser (May 19, 1980 – March 8, 2015)

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

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