# Source estimation for wavefield-reconstruction inversion and its application to Chevron blind test data

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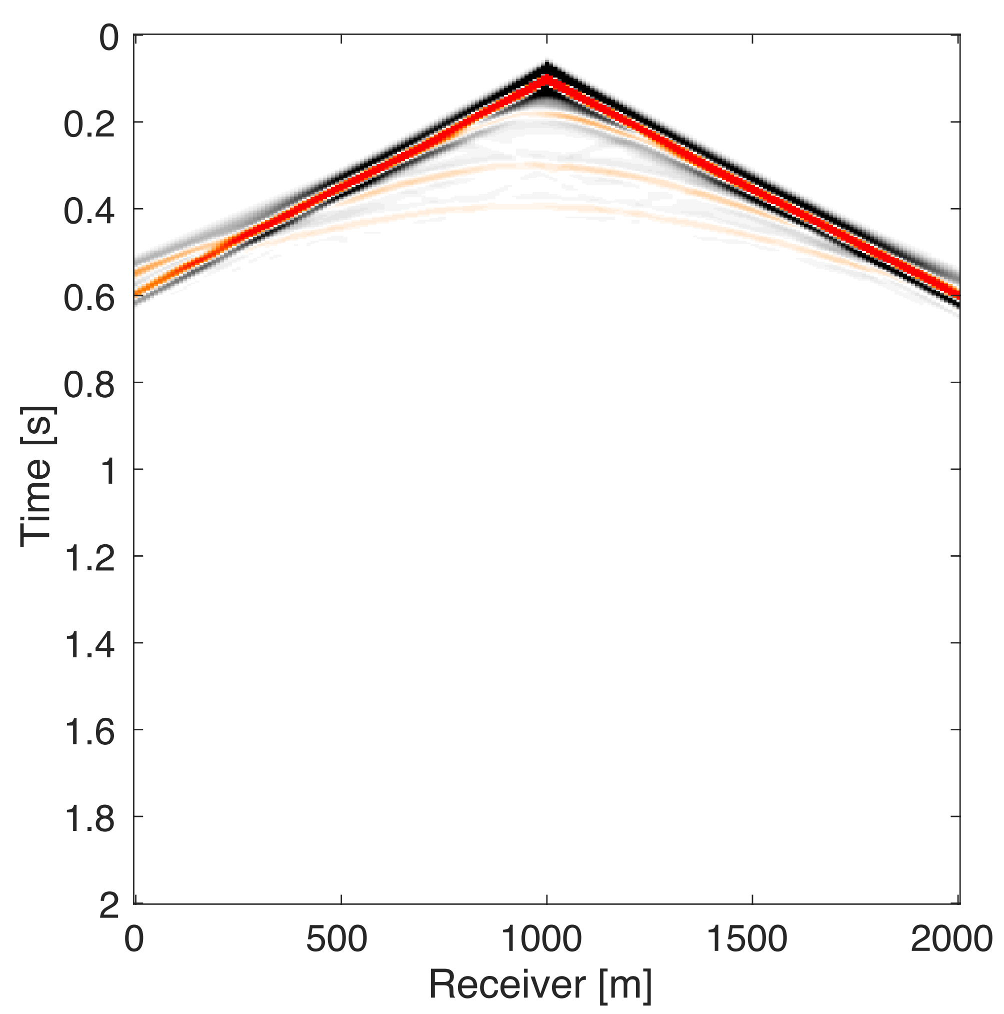
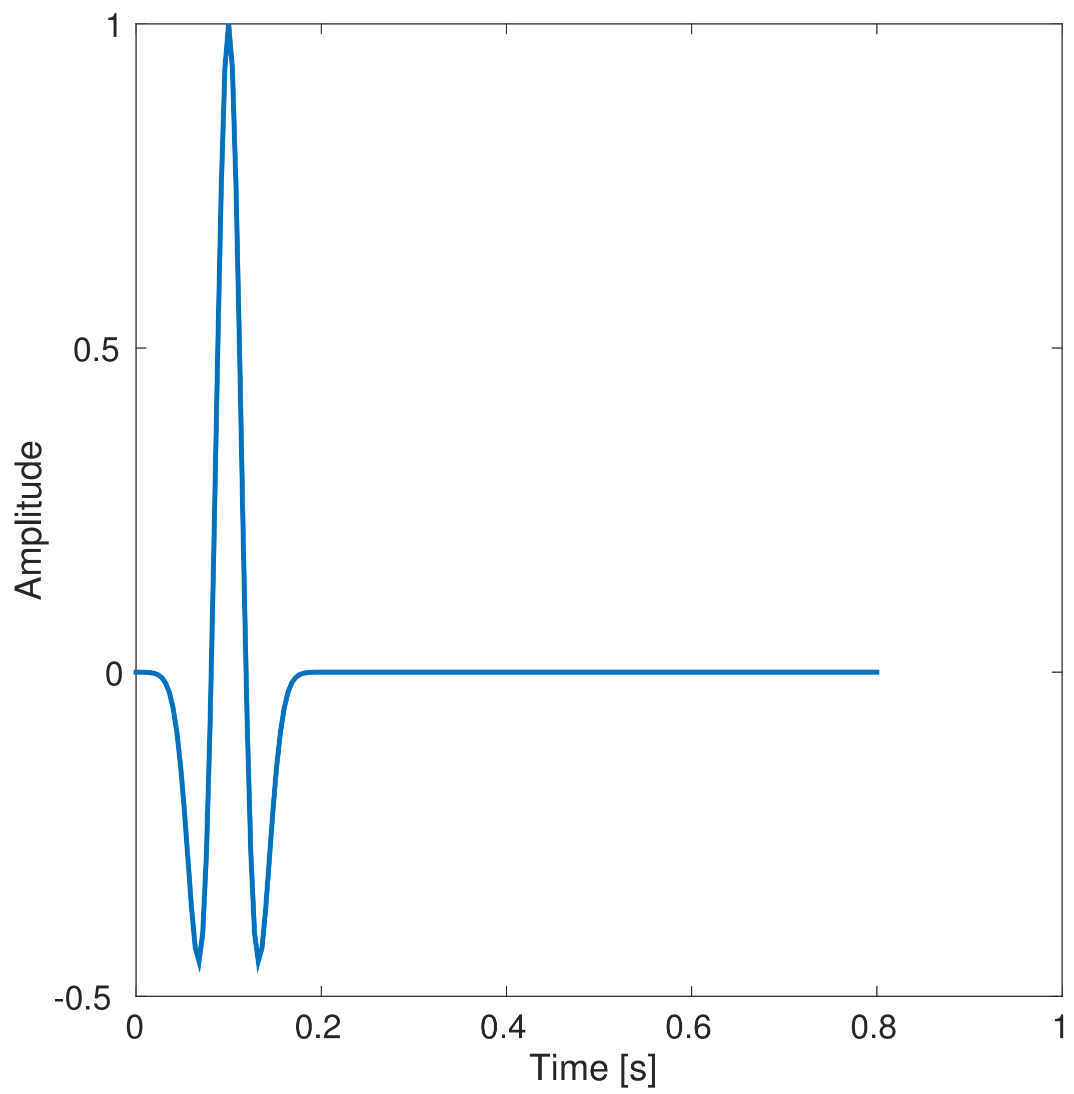
# Motivation



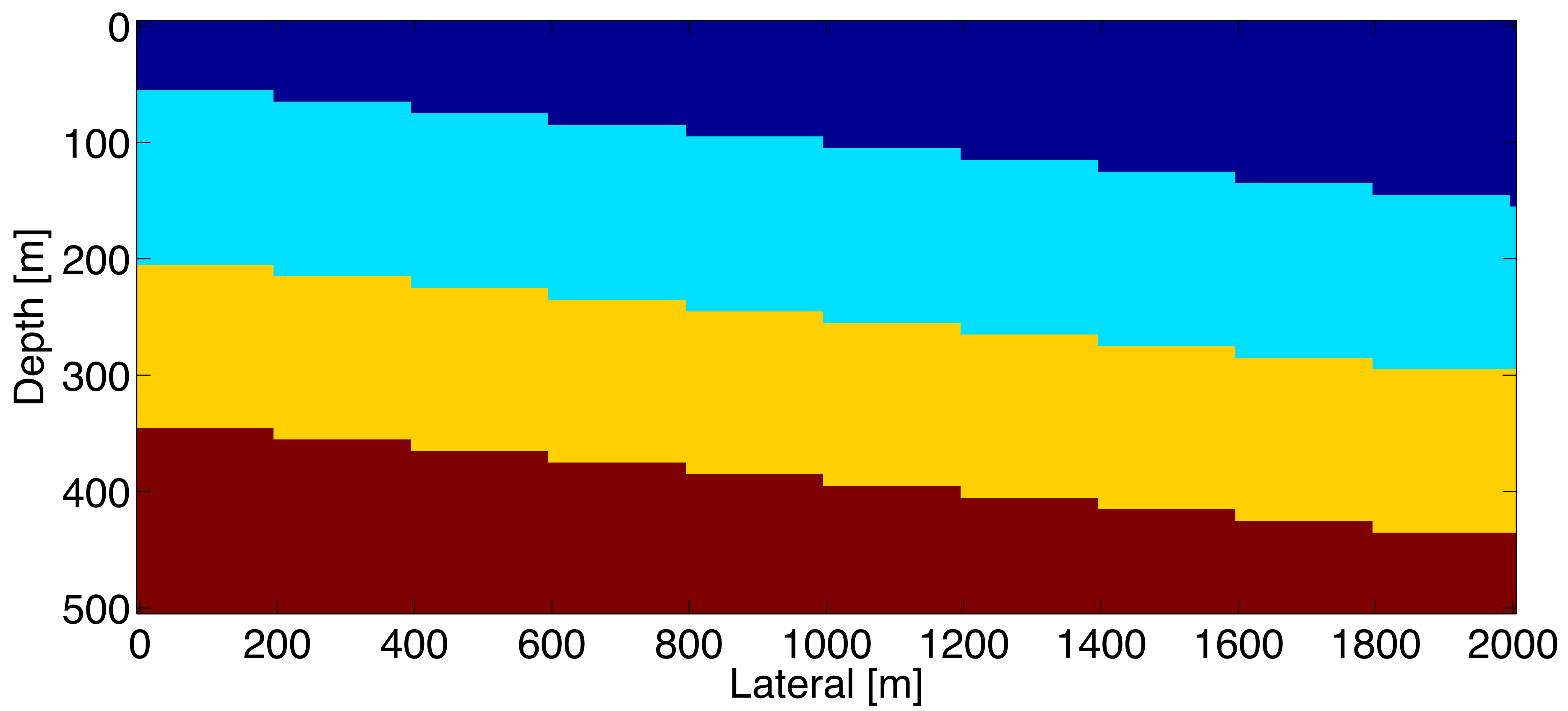

Data

=



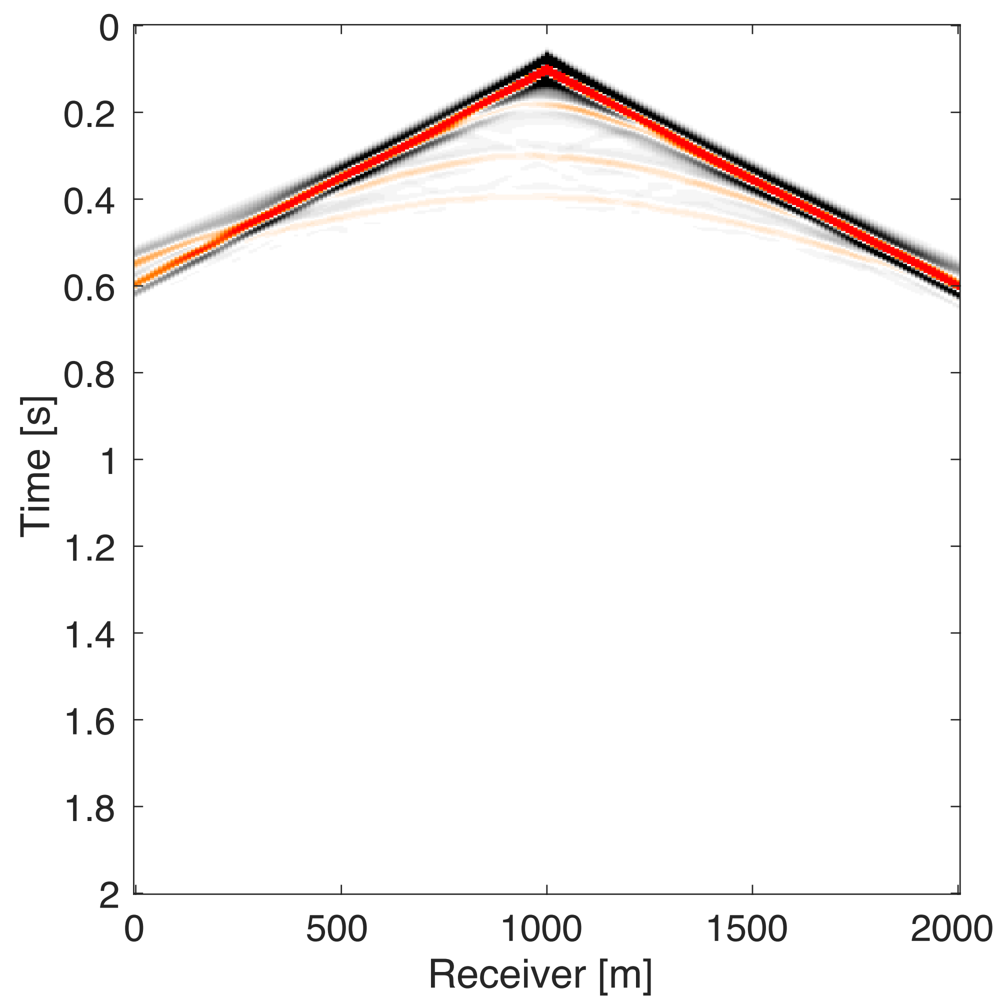

Source wavelet

*



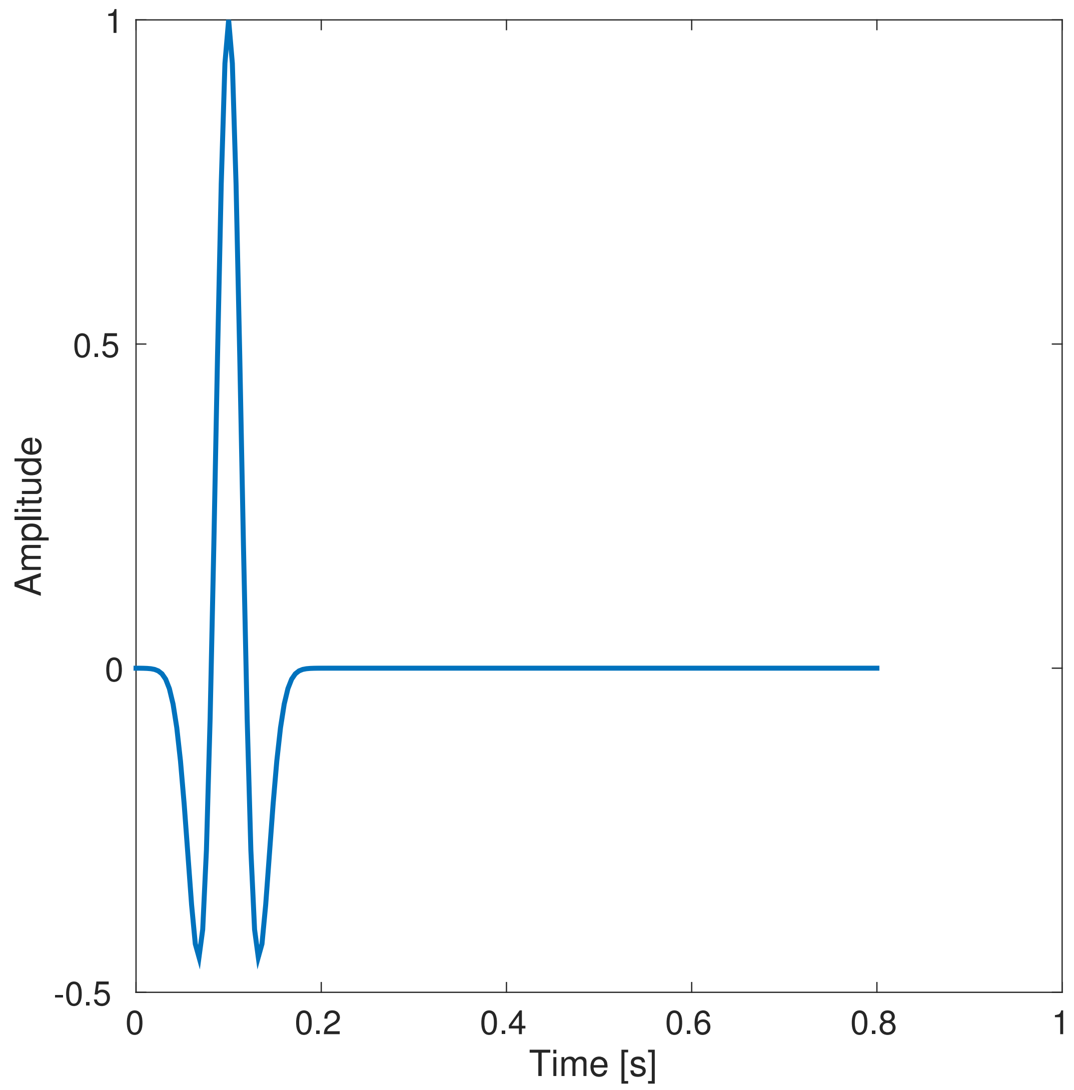

Velocity model

# Motivation



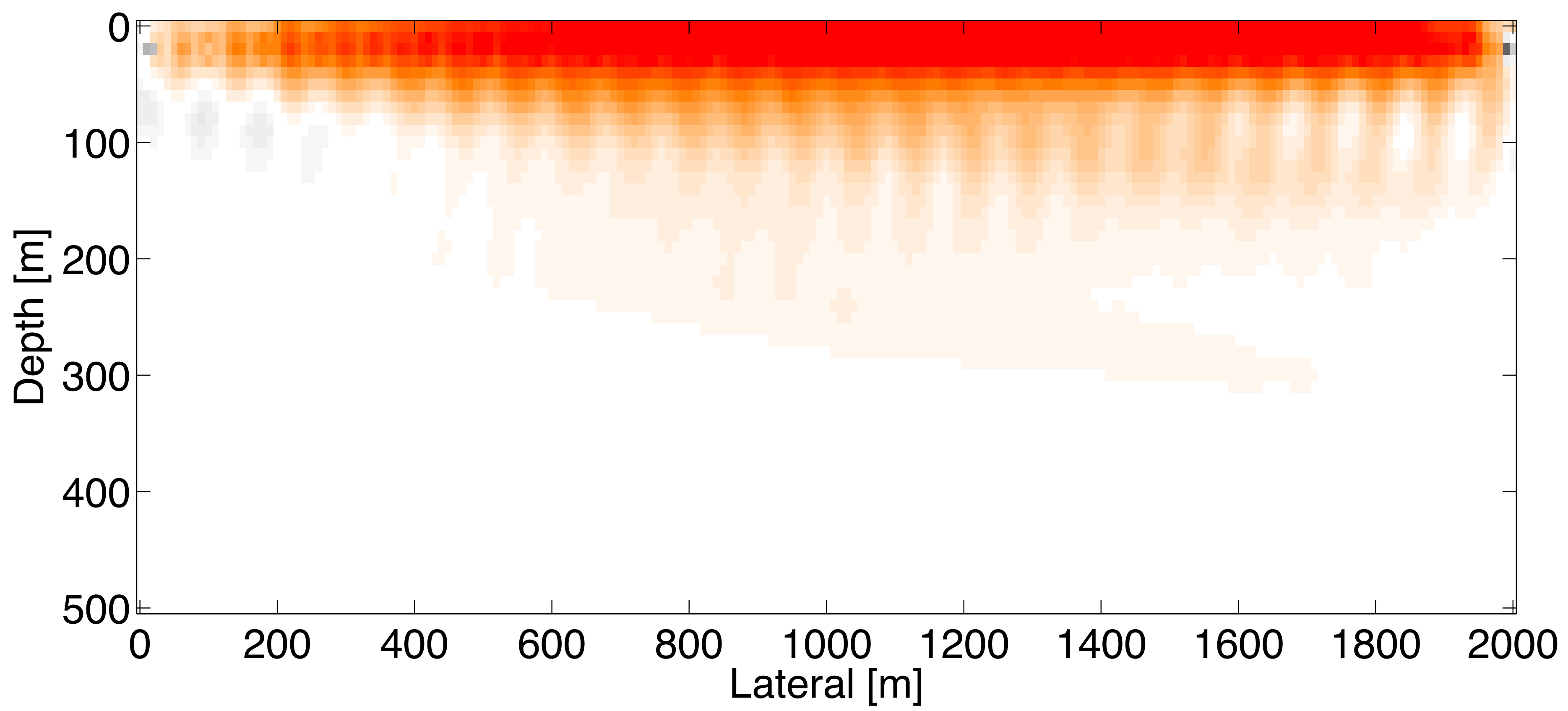

Data

\+



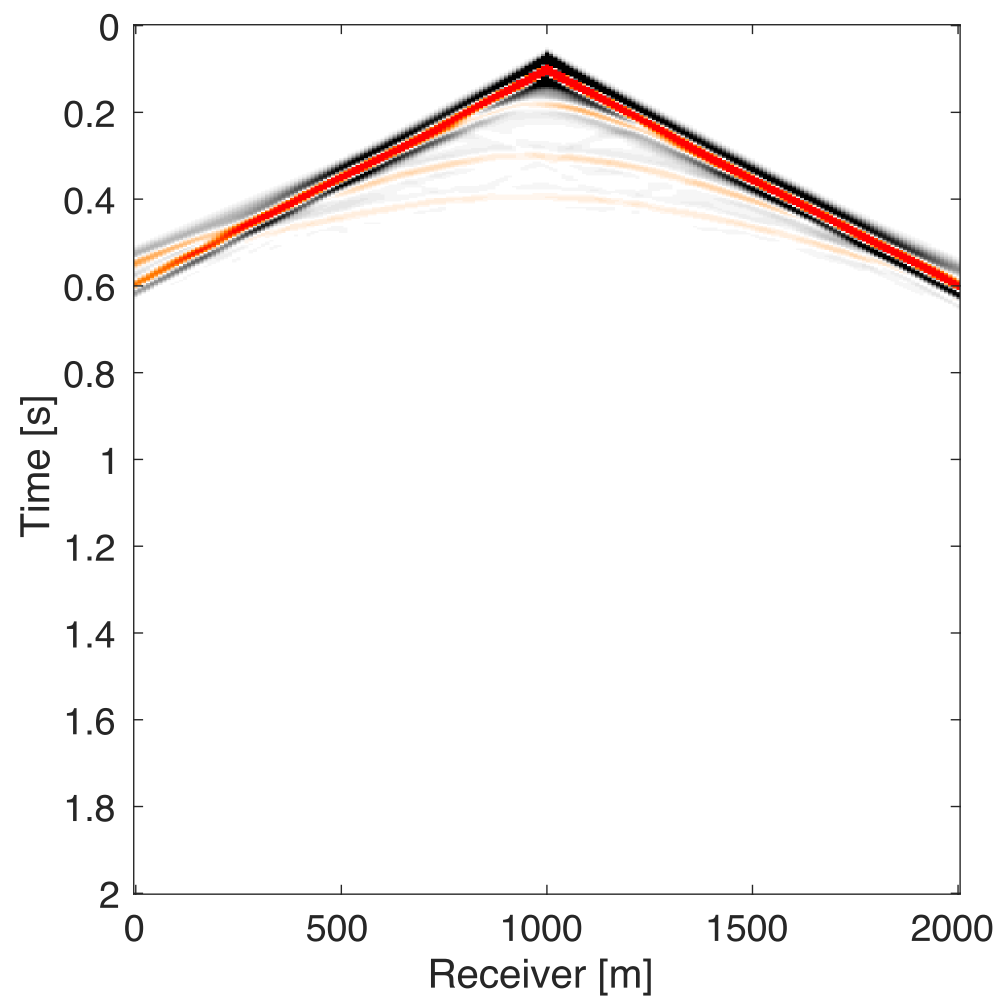

Correct source wavelet

→

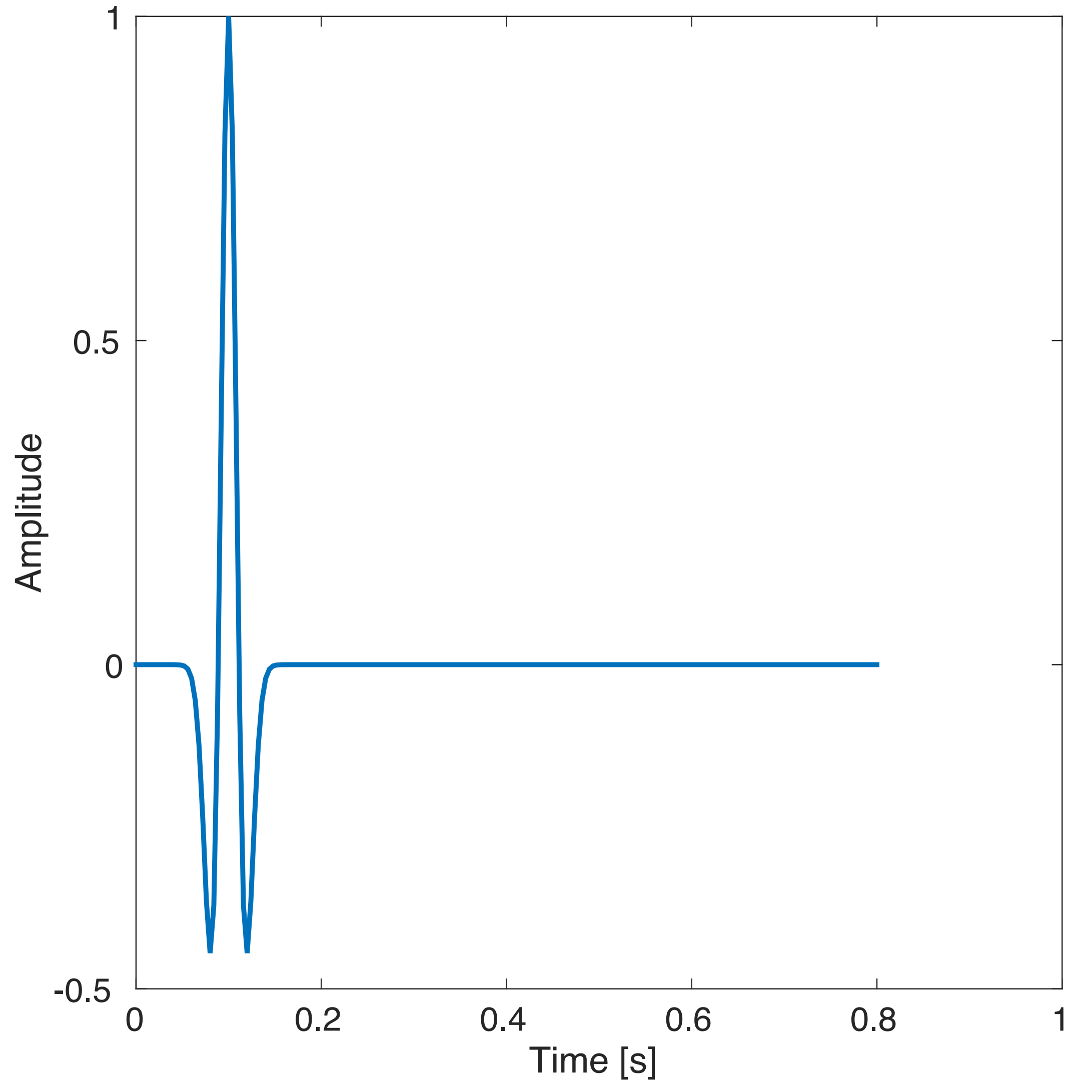



Correct gradient

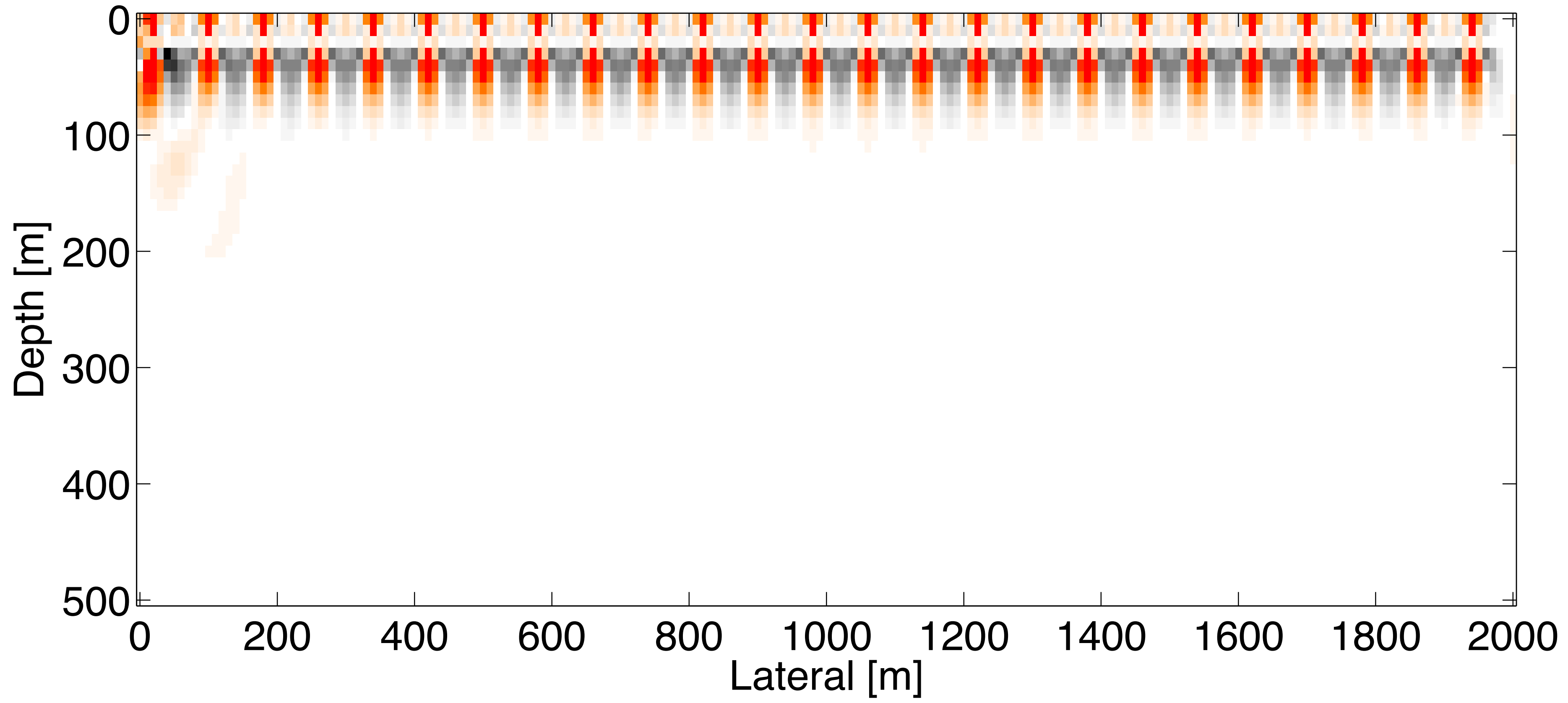
# Motivation




Data



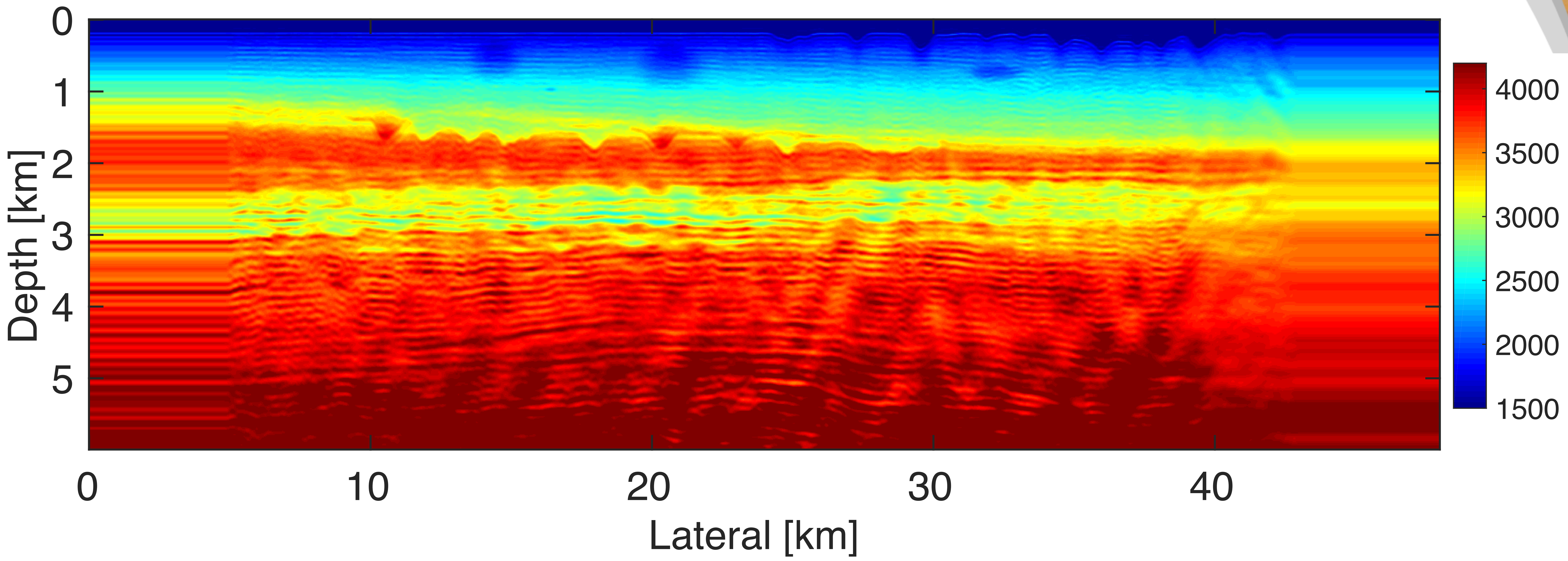

Wrong source wavelet




Wrong gradient

# Chevron blind test data

## — Wavefield-reconstruction inversion with source estimation



[Tarantola, 1984]

[J. Virieux and S. Operto, 2009 ]

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## Full-waveform inversion

Original problem:

$$\underset{\mathbf{u},\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

$$\text{subject to } \mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} = \mathbf{q}_{k,l},$$

where,

$\mathbf{u}_{k,l}$ – Wavefield of the $k$th shot at $l$th frequency

$\mathbf{d}_{k,l}$ – Observed data of the $k$th shot at $l$th frequency

$\mathbf{q}_{k,l}$ – Source of the $k$th shot at $l$th frequency

$\mathbf{A}_{k,l}$ – Helmholtz of the $k$th shot at $l$th frequency

$\mathbf{P}_k$ – Receiver projection operator of the $k$th shot

$\mathbf{m}$ – Squared-slowness

[Tarantola, 1984]

[J. Virieux and S. Operto, 2009 ]

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# Full-waveform inversion

Reduced/adjoint-state method:

$$\underset{\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

with the gradient given by

$$\mathbf{g} = \sum_{k,l} \mathbf{u}_{k,l}^* \frac{\partial \mathbf{A}_{k,l}^*}{\partial \mathbf{m}} \mathbf{v}_{k,l}$$

$$\mathbf{u}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l}$$

$$\mathbf{v}_{k,l} = \mathbf{A}_{k,l}^{-*}(\mathbf{m})\mathbf{P}_k^*\mathbf{r}_{k,l}$$

$$\mathbf{r}_{k,l} = \mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1}\mathbf{q}_{k,l} - \mathbf{d}_{k,l}$$

2 PDE solves are required !

[van Leeuwen, T and Herrmann, F J , 2013]
[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]
[Golub, G H and Pereyra, V , 1973]

# Wavefield-reconstruction inversion

Joint optimization problem:

$$\underset{\mathbf{u},\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

Eliminating $\mathbf{u}$ w/ variable projection:

$$\overline{\mathbf{u}} = \arg\min_{\mathbf{u}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

[van Leeuwen, T and Herrmann, F J , 2013]

[Golub, G and Pereyra, V, 1973]

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# Wavefield-reconstruction inversion

Corresponds to solving the following augmented system:

$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \overline{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

with the gradient

**1 augmented system solves is required !**

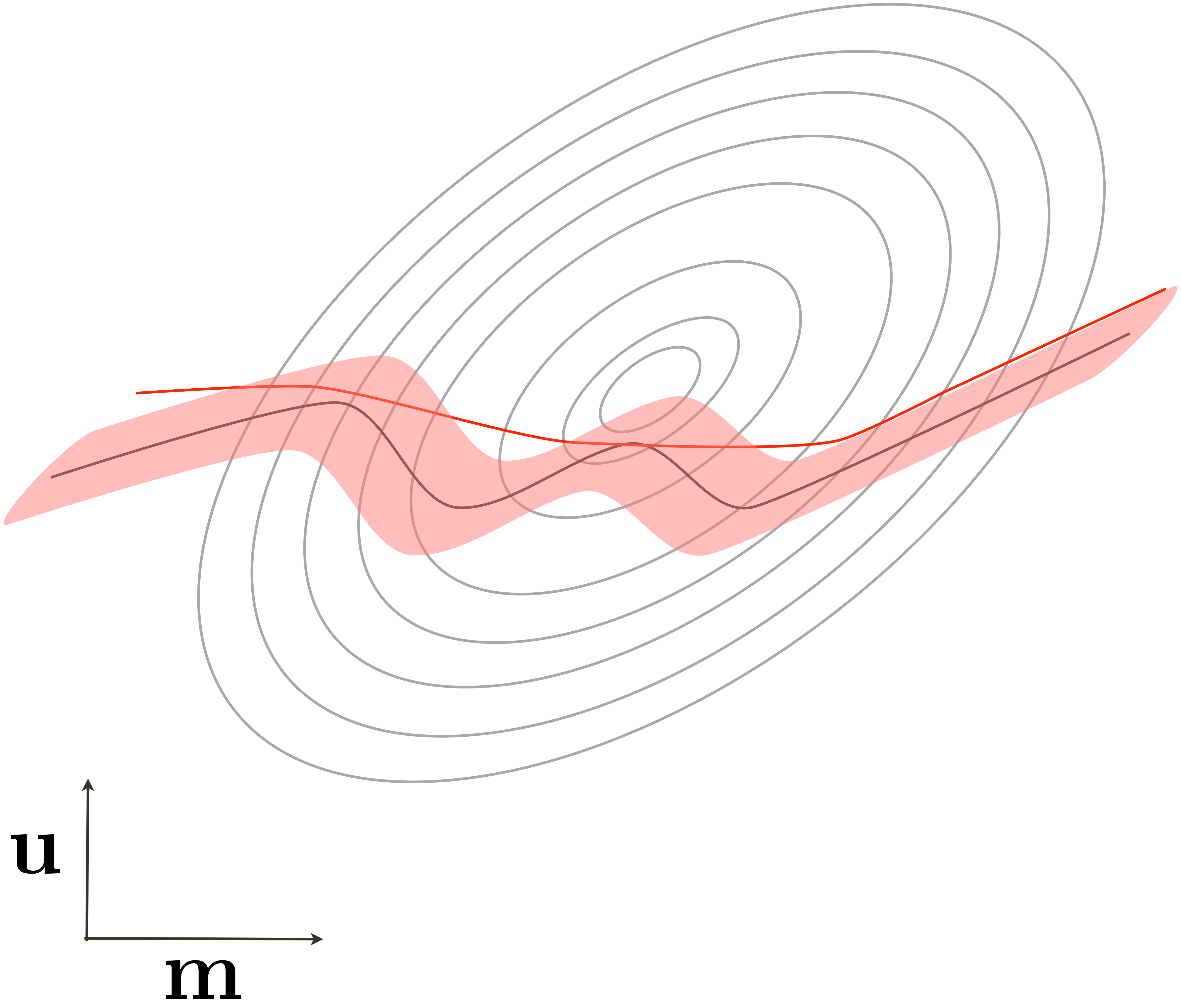
$$\mathbf{g} = \sum_{k,l} \overline{\mathbf{u}}^*_{k,l} \frac{\partial \mathbf{A}^*_{k,l}}{\partial \mathbf{m}} \overline{\mathbf{v}}_{k,l}$$

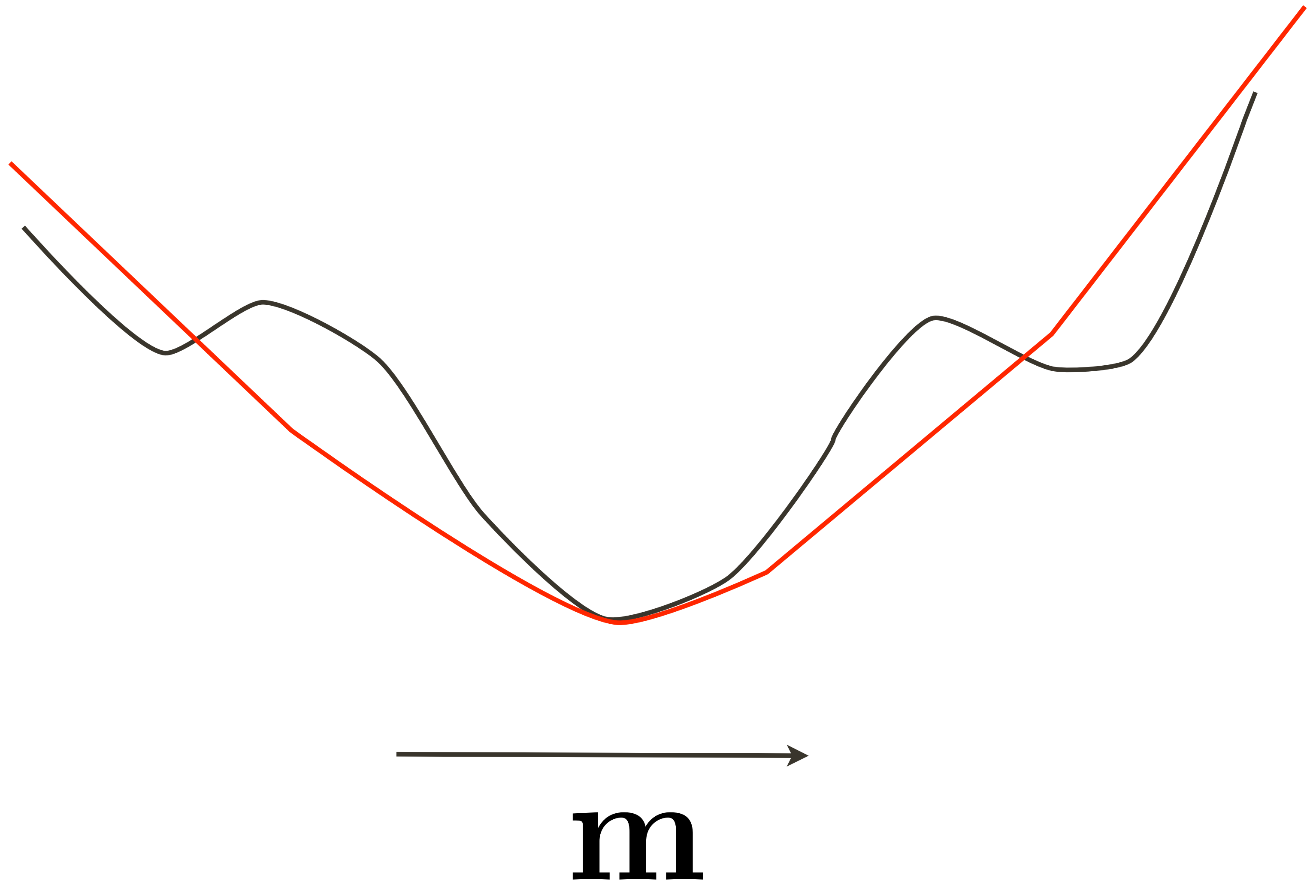
$$\overline{\mathbf{v}}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m}) \overline{\mathbf{u}}_{k,l} - \mathbf{q}_{k,l}$$

# WRI vs. FWI

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B,  Herrmann, F J  and van Leeuwen, T, 2014]

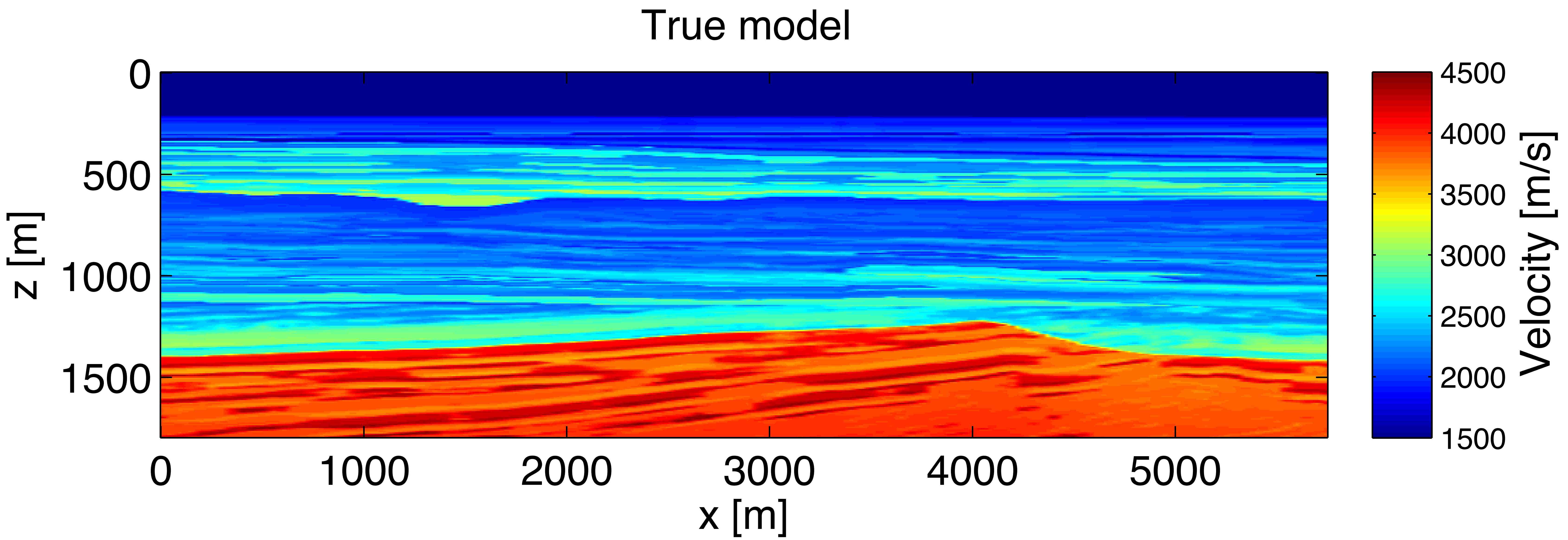


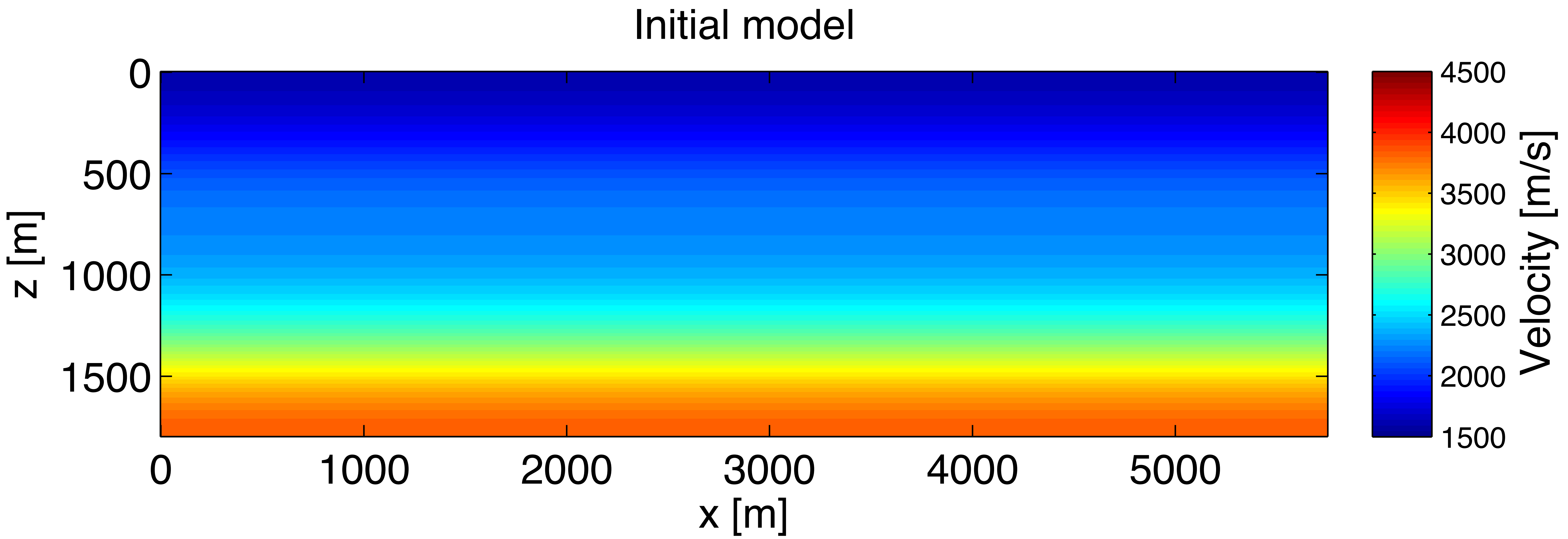



# True & initial model

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]

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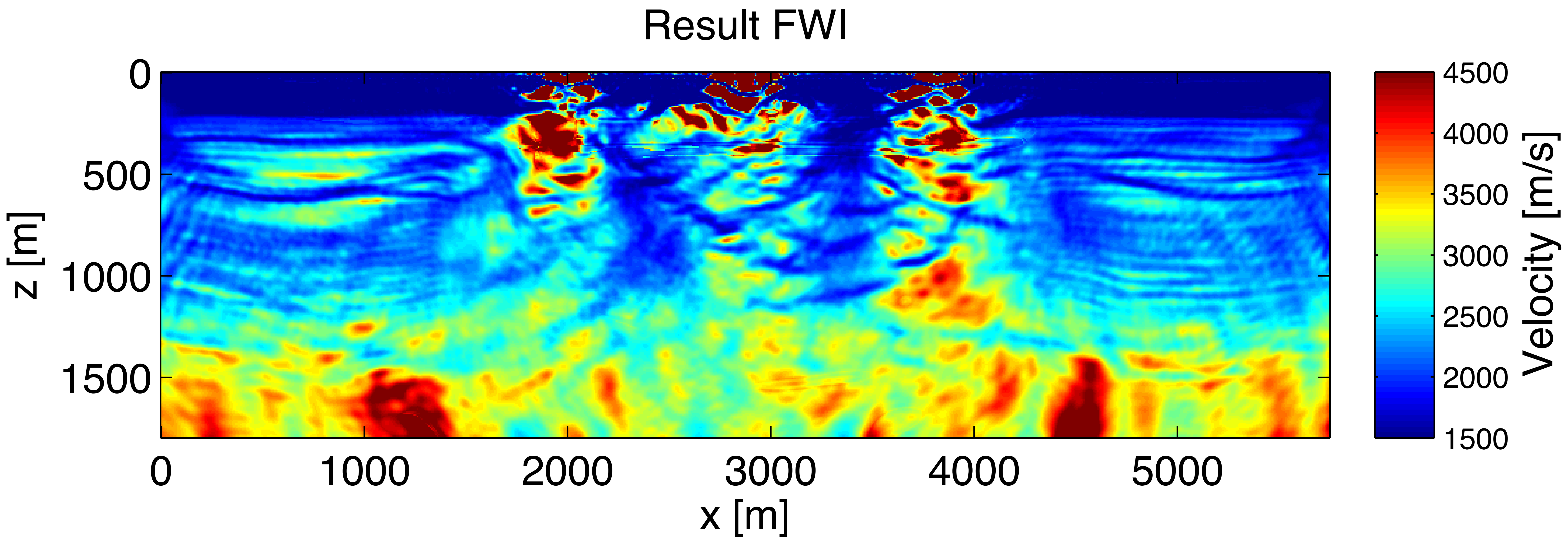
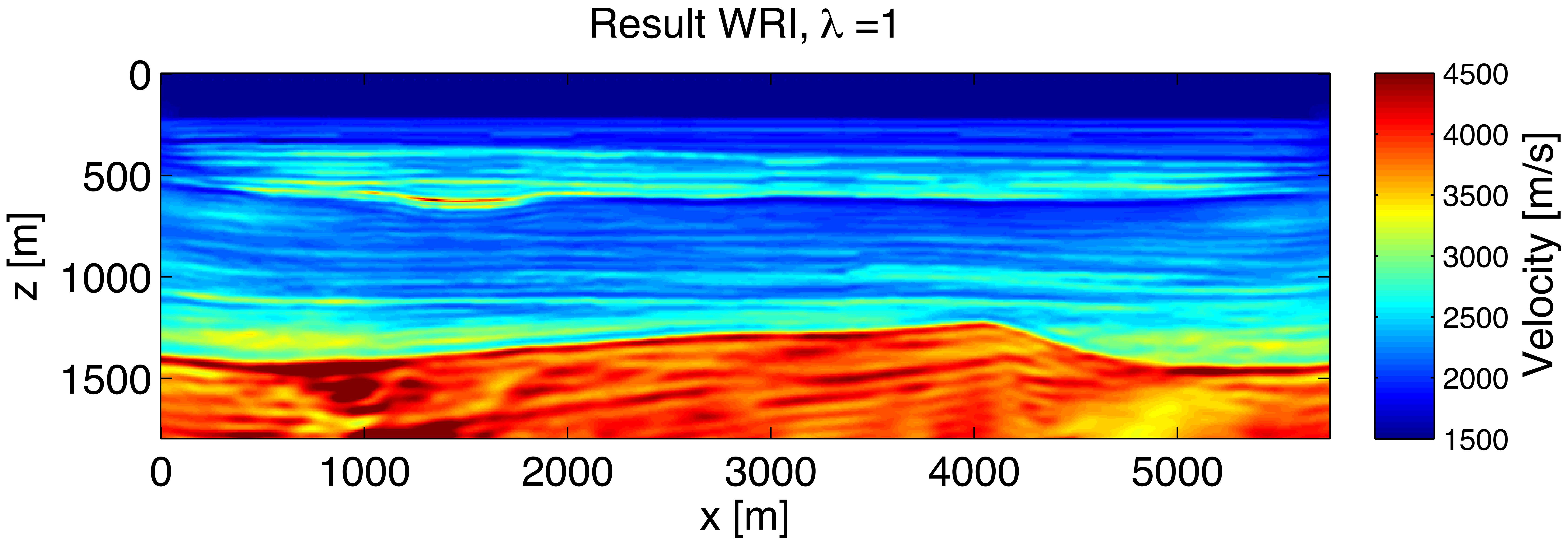
# FWI vs WRI

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]



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# FWI vs WRI

| FWI | WRI |
| --- | --- |
| Strong nonlinearity | Weaker nonlinearity |
| 2 PDE solves for each gradient | 1 data augmented system solve for each gradient |
| Sensitive to the initial model | Less sensitive to the initial model |
| Dense Gauss-Newton Hessian | Approximately diagonal Hessian |

# WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u},\mathbf{m},\alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

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# WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u},\mathbf{m},\alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

Eliminate $\mathbf{u}$ and $\alpha$ jointly w/ variable projection:

$$[\overline{\mathbf{u}}, \overline{\alpha}] = \arg\min_{\mathbf{u},\alpha} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m})\mathbf{u}_{k,l} - \alpha_{k,l}\mathbf{e}_{k,l}\|_2^2$$

# WRI with source estimation

Corresponds to solving the following augmented system:

$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} & -\lambda \mathbf{e}_{k,l} \\ \mathbf{P}_k & 0 \end{pmatrix} \begin{pmatrix} \overline{\mathbf{u}}_{k,l} \\ \overline{\alpha}_{k,l} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{k,l} \end{pmatrix}$$

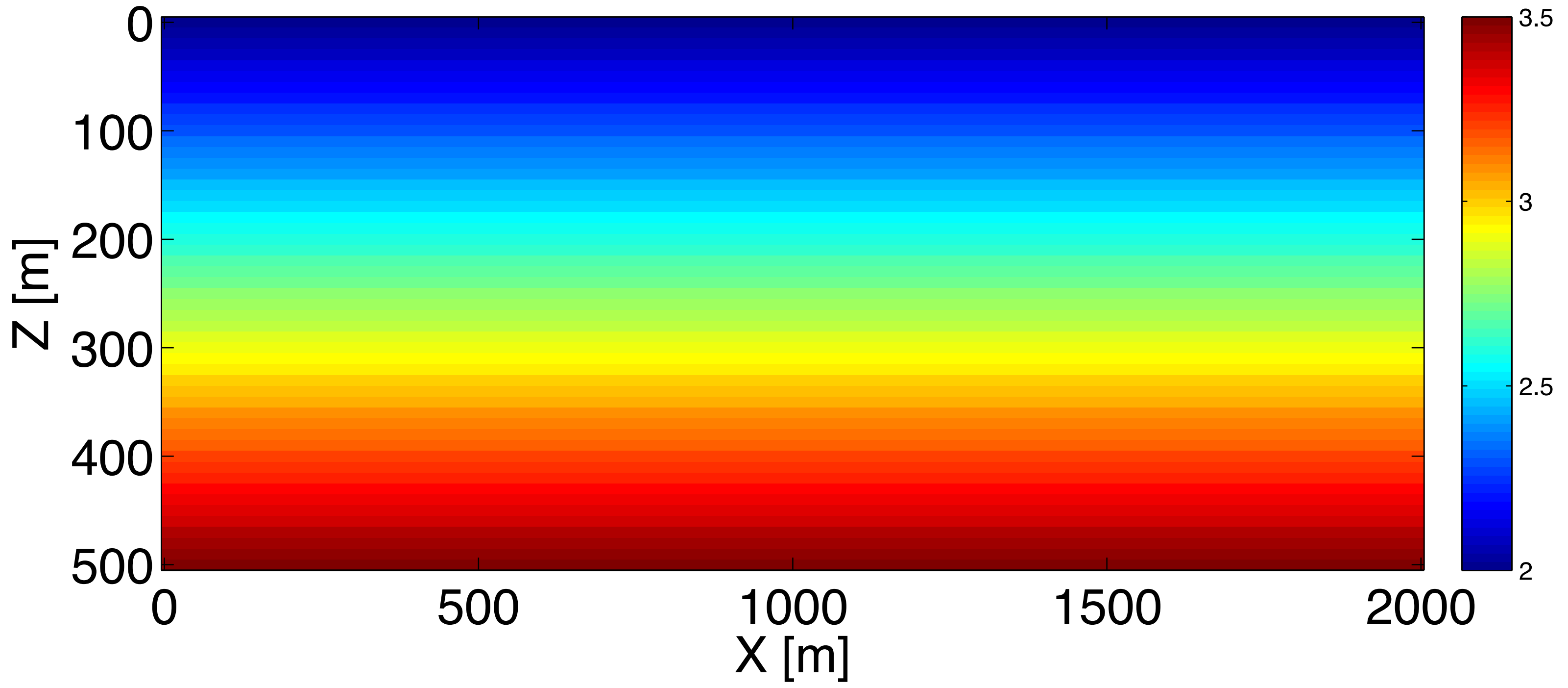
Cf. original augmented system:

**Full column rank!**

**No additional computational cost!**

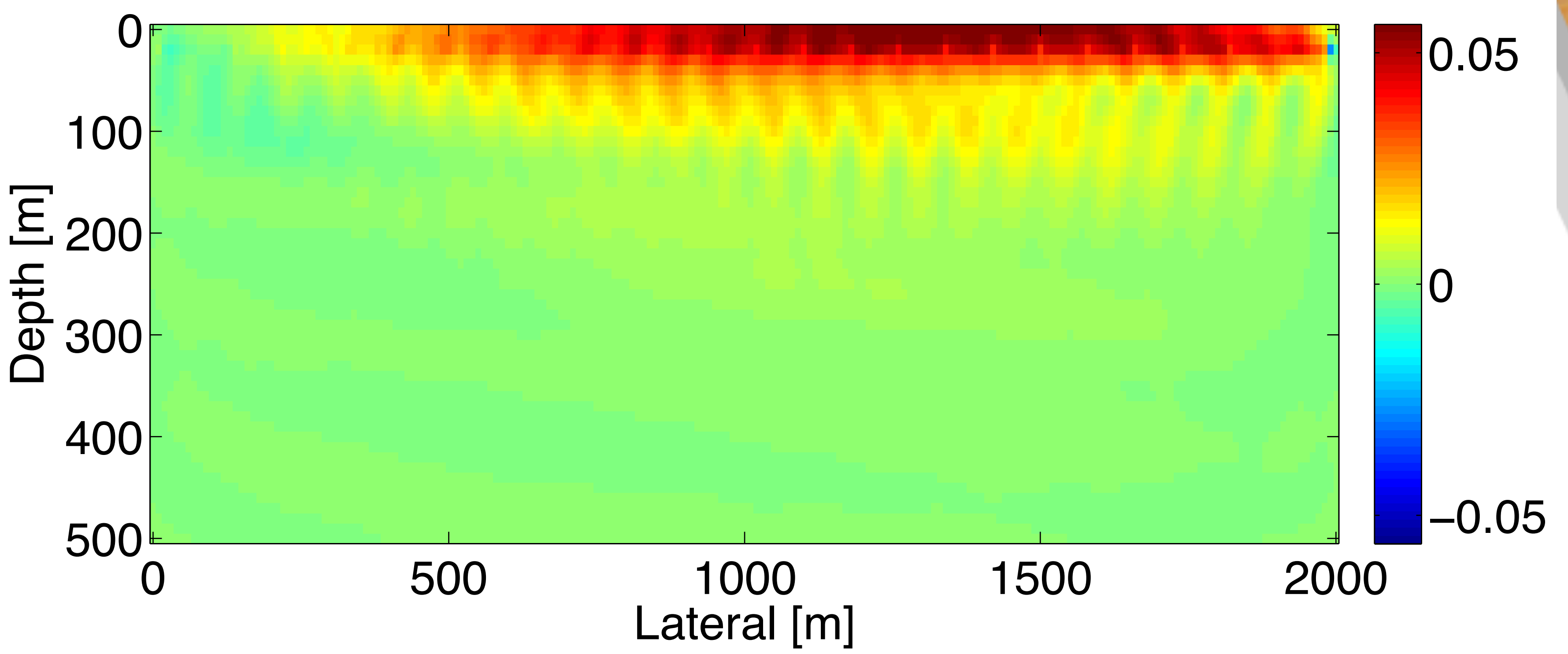
$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \overline{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

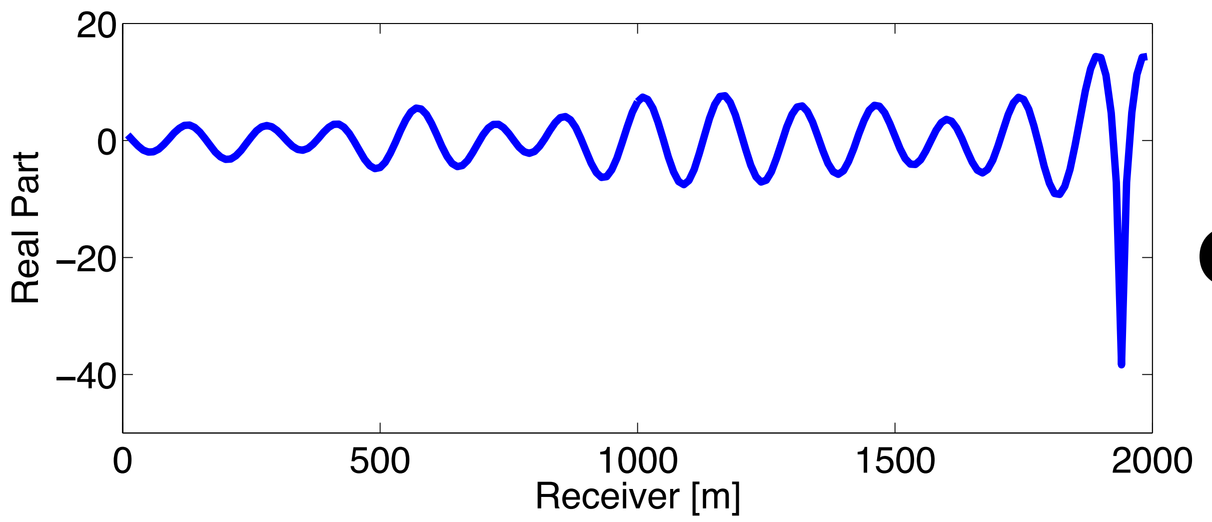
# WRI with source estimation




$\mathbf{m}$

$\mathbf{g}$

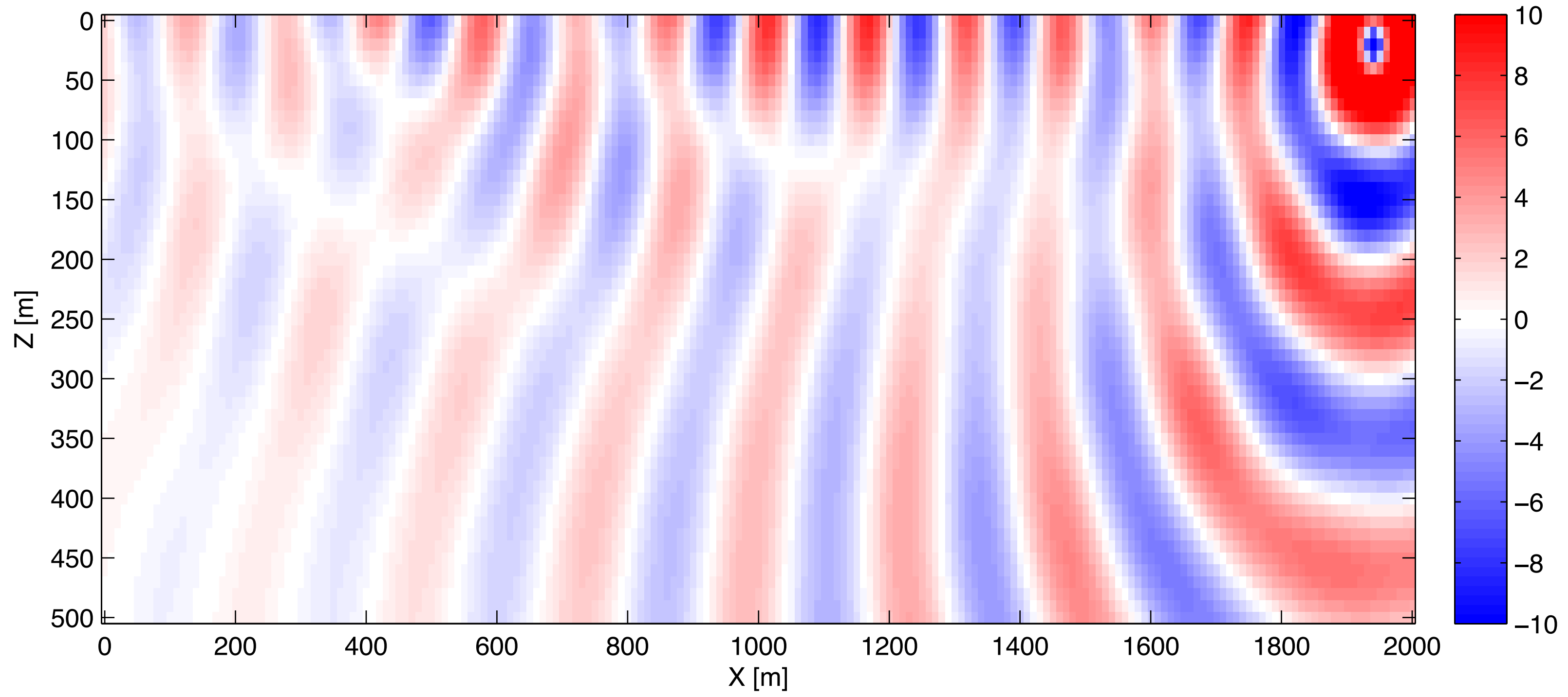



$\mathbf{d_{obs}}$

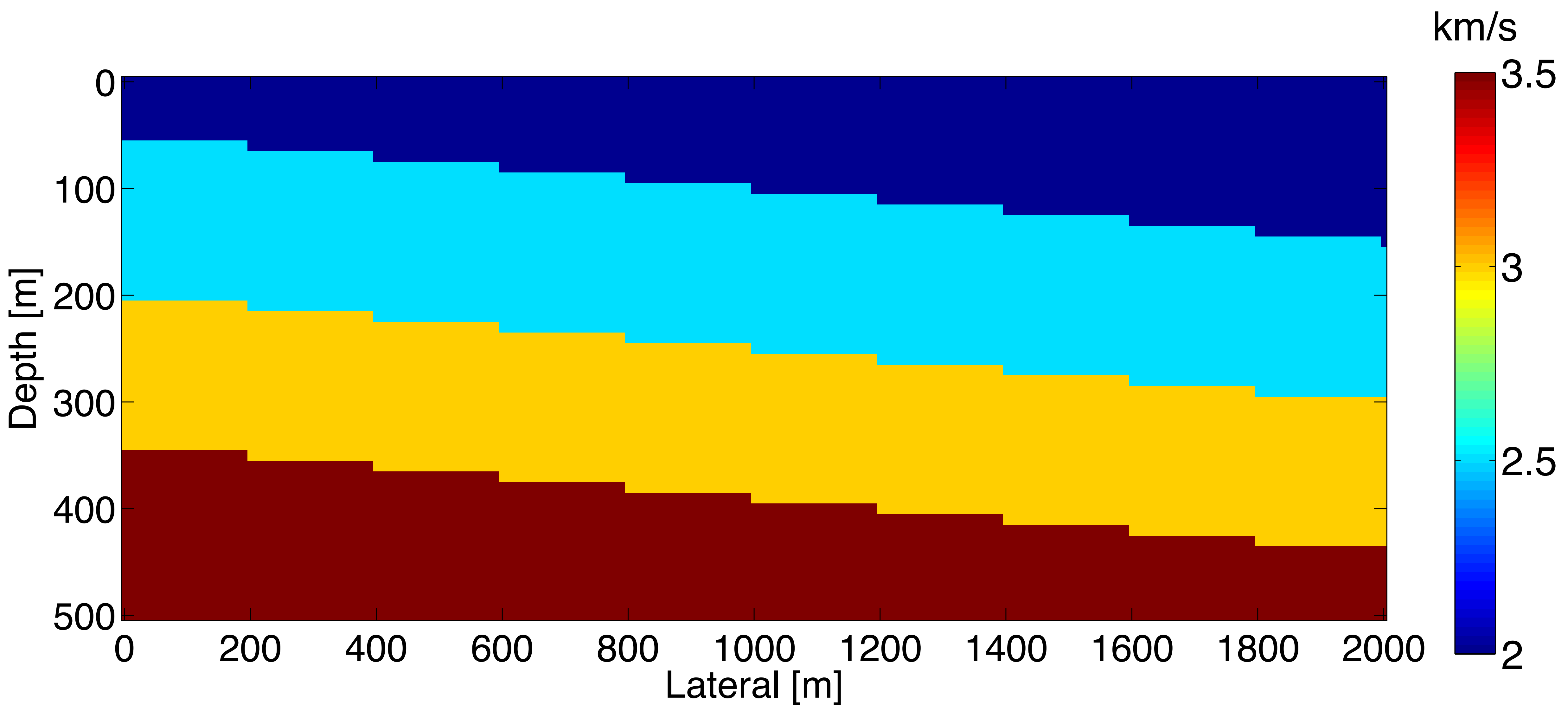
$$\begin{pmatrix} \lambda \mathbf{A} & -\lambda \mathbf{e} \\ \mathbf{P} & 0 \end{pmatrix} \begin{pmatrix} \overline{\mathbf{u}} \\ \overline{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d_{obs}} \end{pmatrix}$$

$\overline{\alpha}$ and

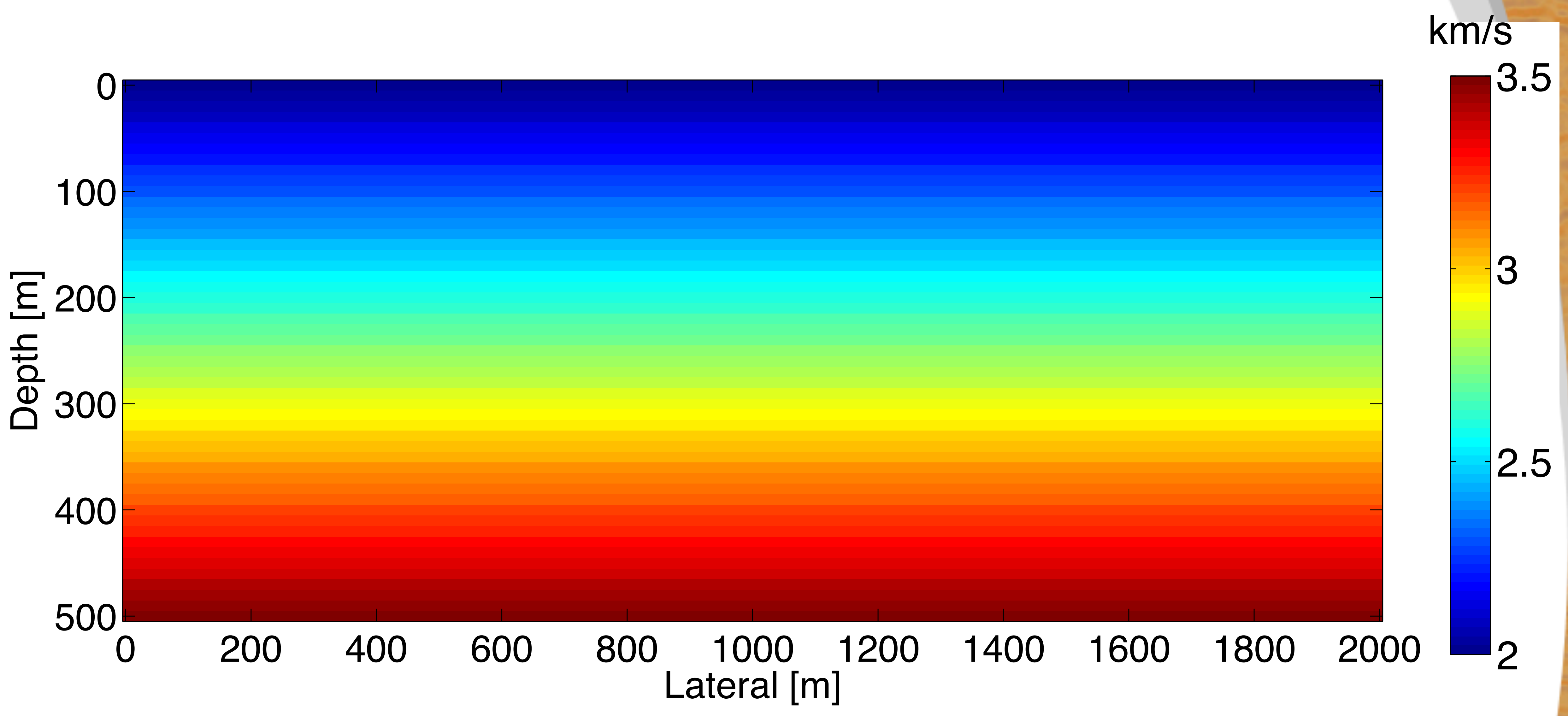



$\overline{\mathbf{u}}$

# Synthetic example

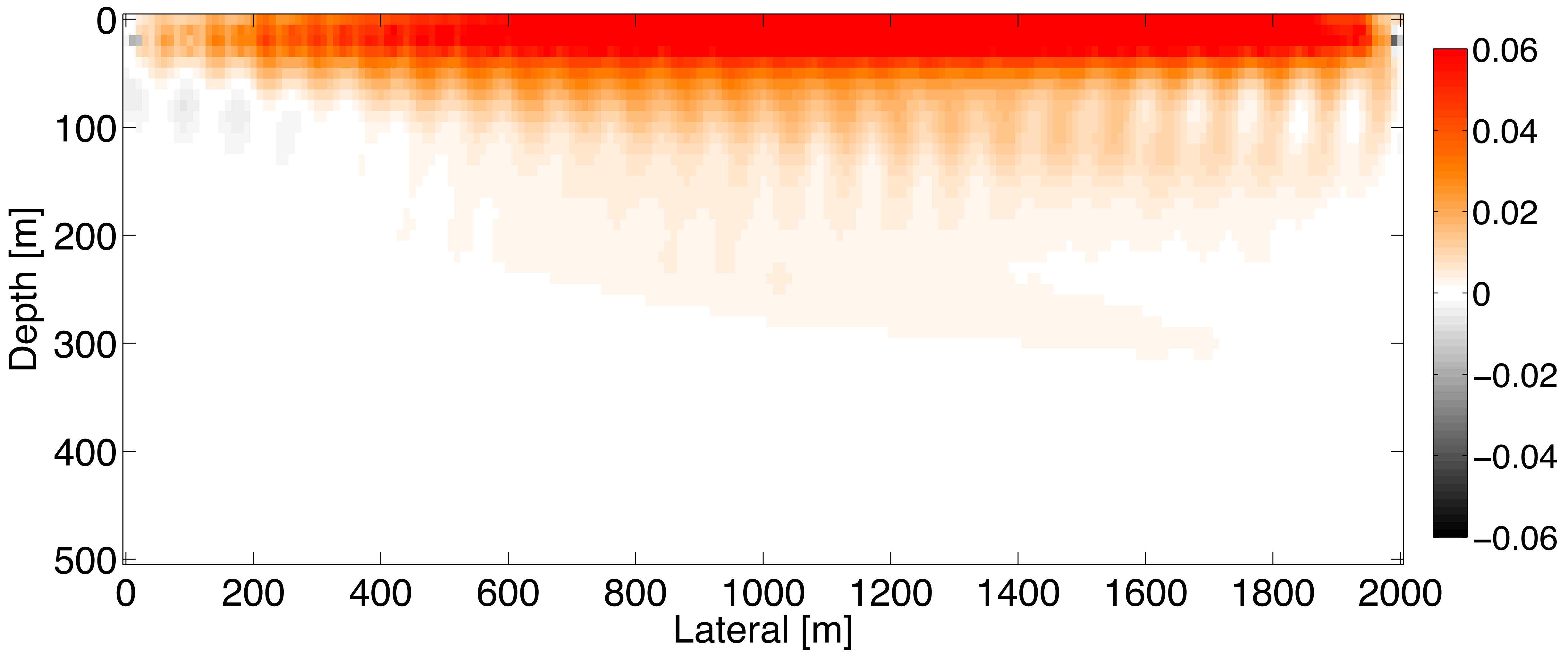



**True Model**

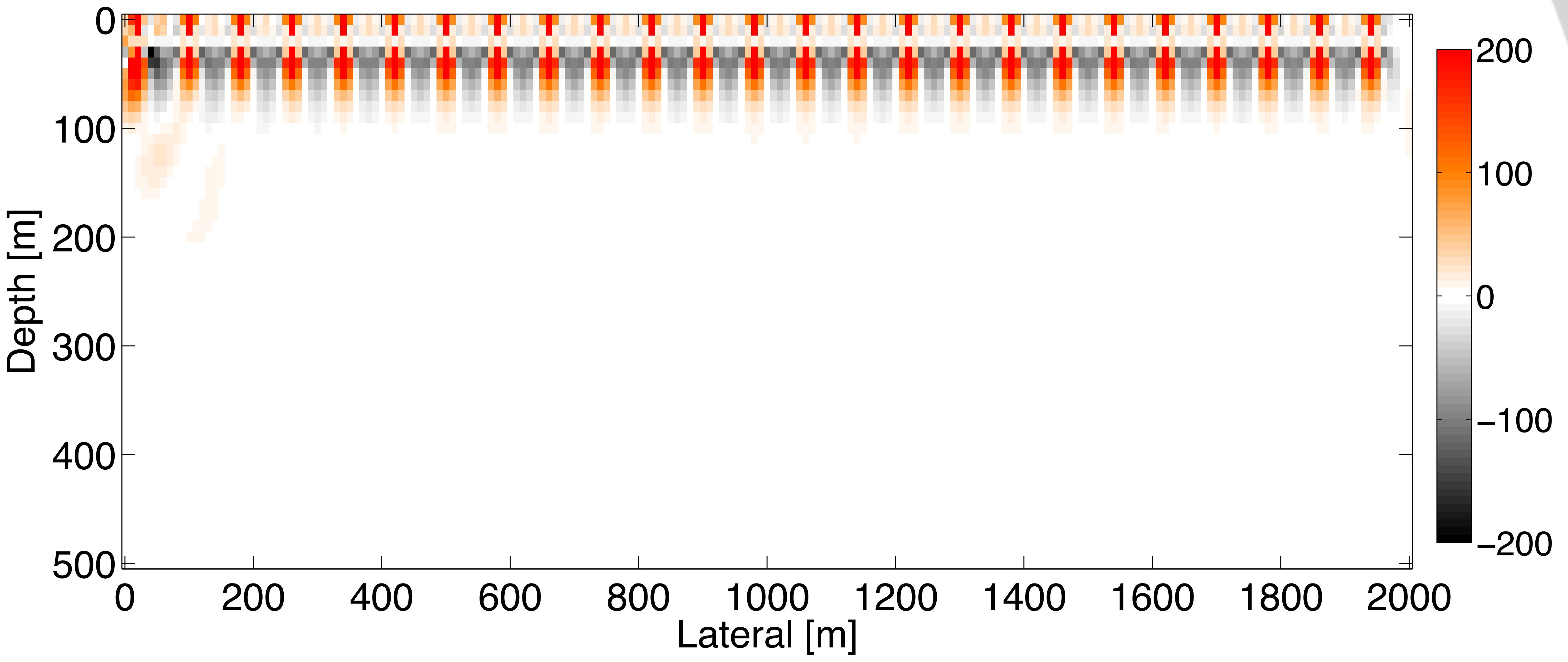



**Initial Model**

# Gradient comparison

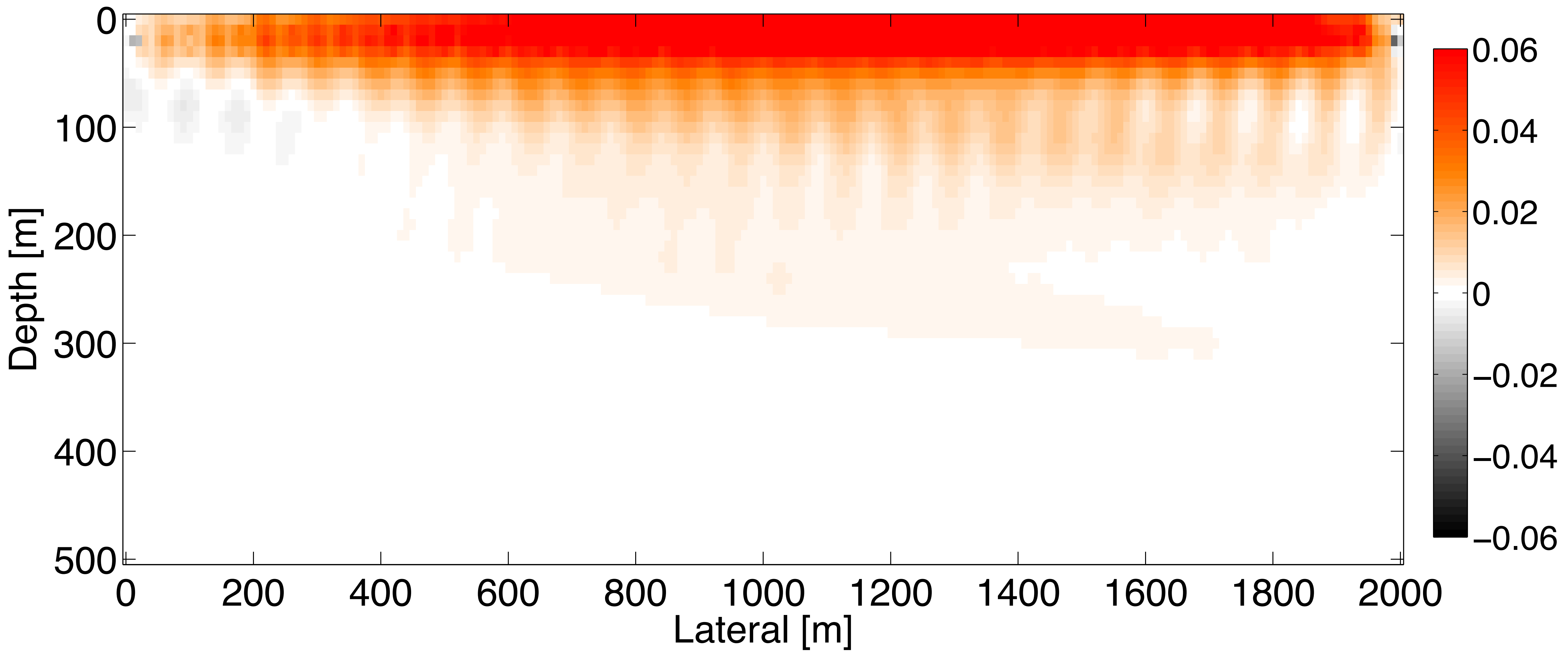



**Gradient with true source wavelet**

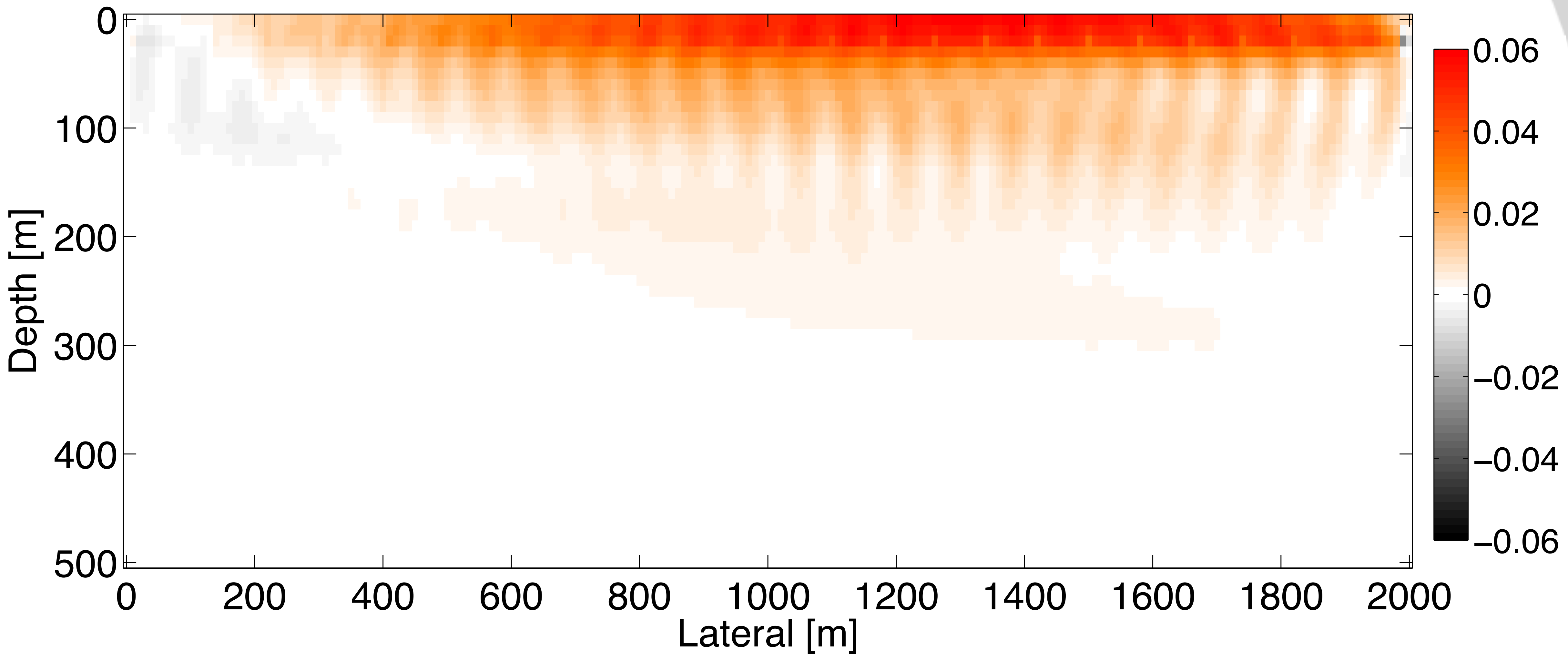



**Gradient with wrong source wavelet**

# Gradient comparison

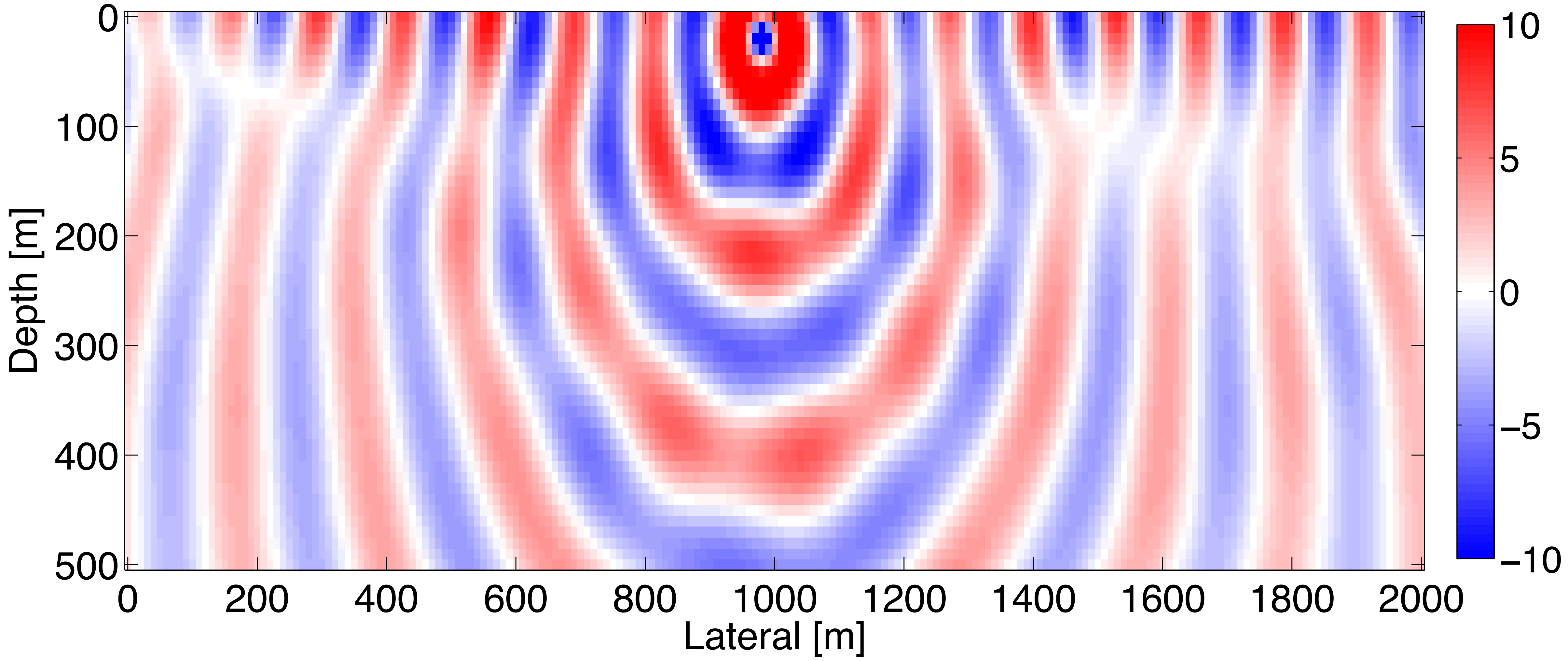



Gradient with true source wavelet

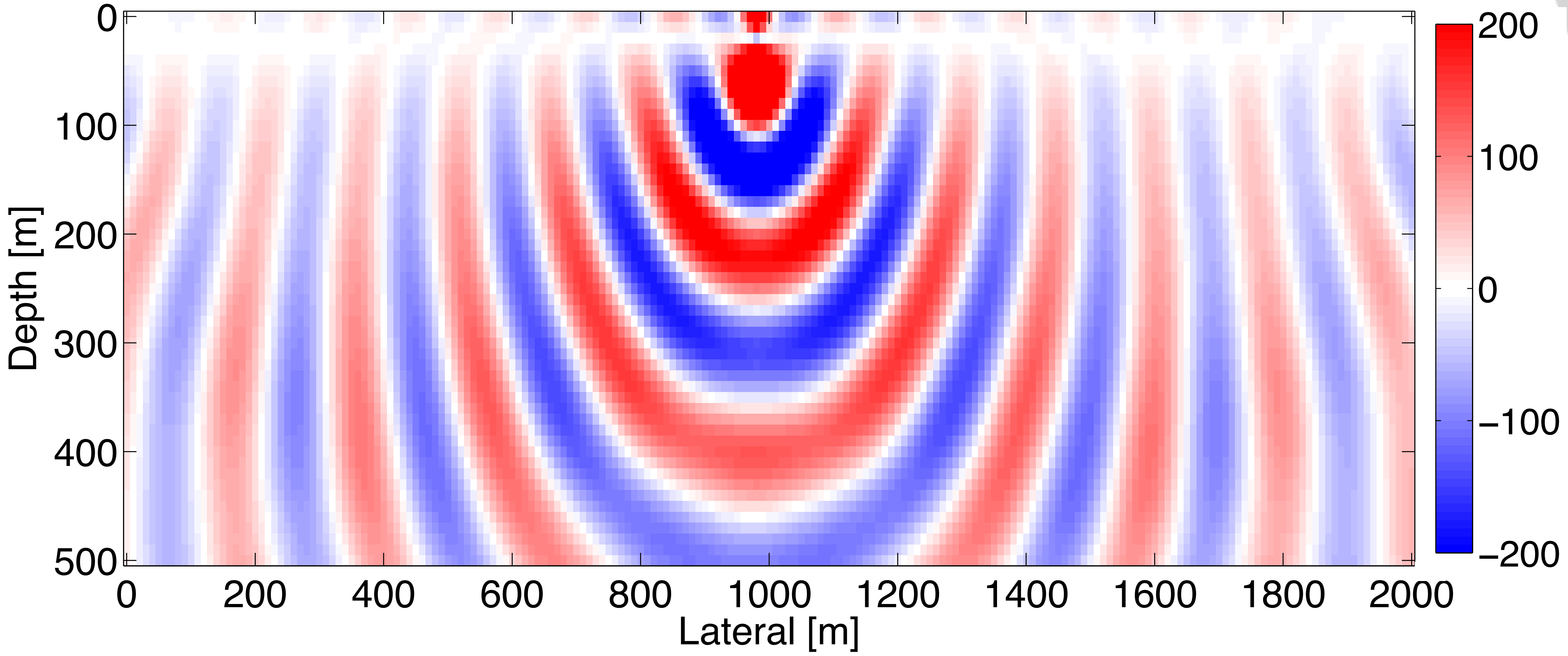



Gradient with estimated source wavelet

# Wavefield comparison

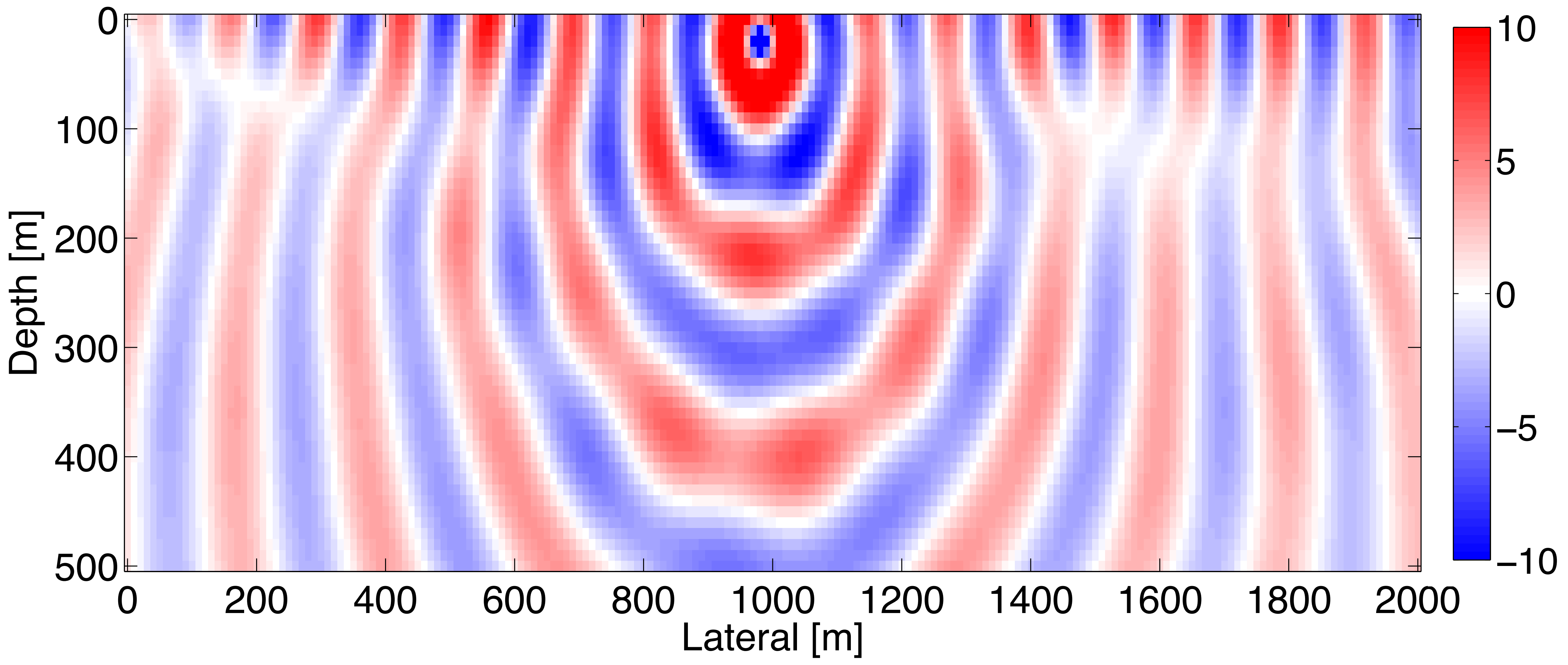


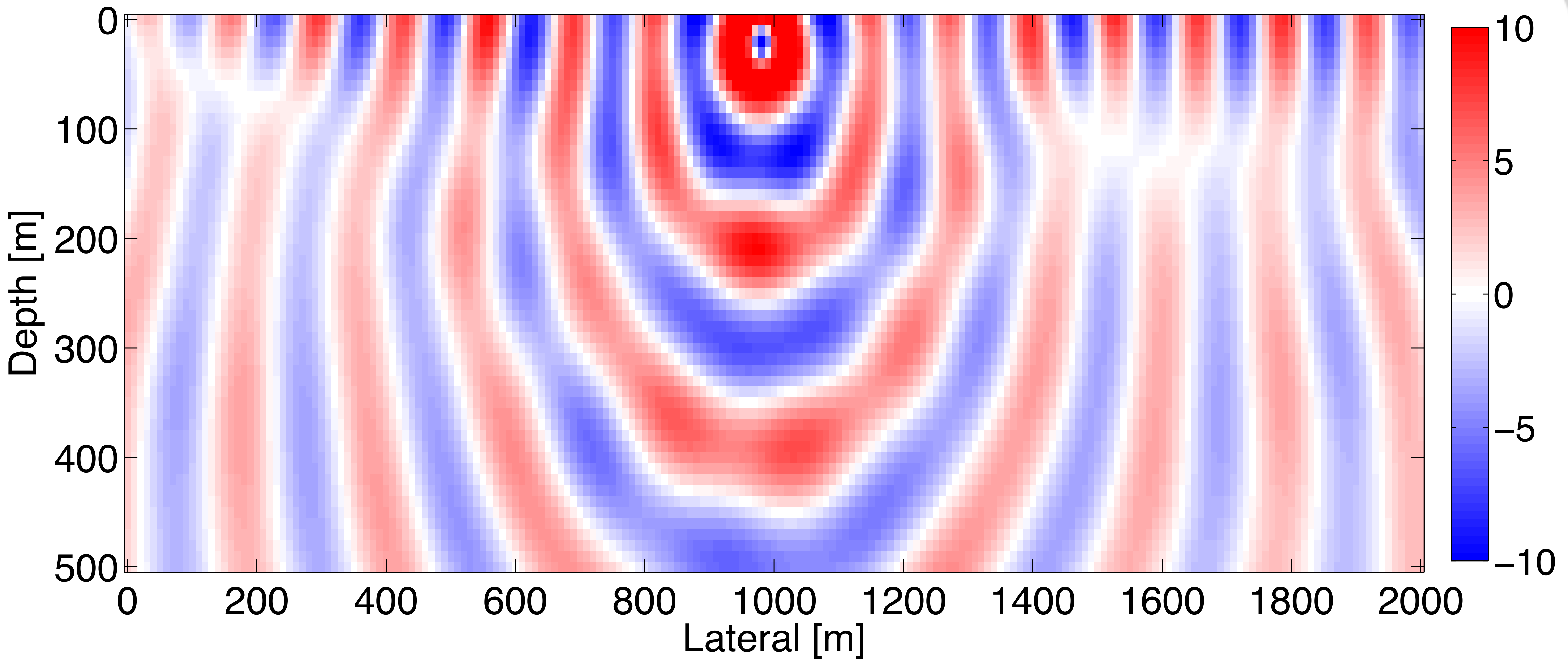

Wavefield with true source wavelet




Wavefield with wrong source wavelet

# Wavefield comparison

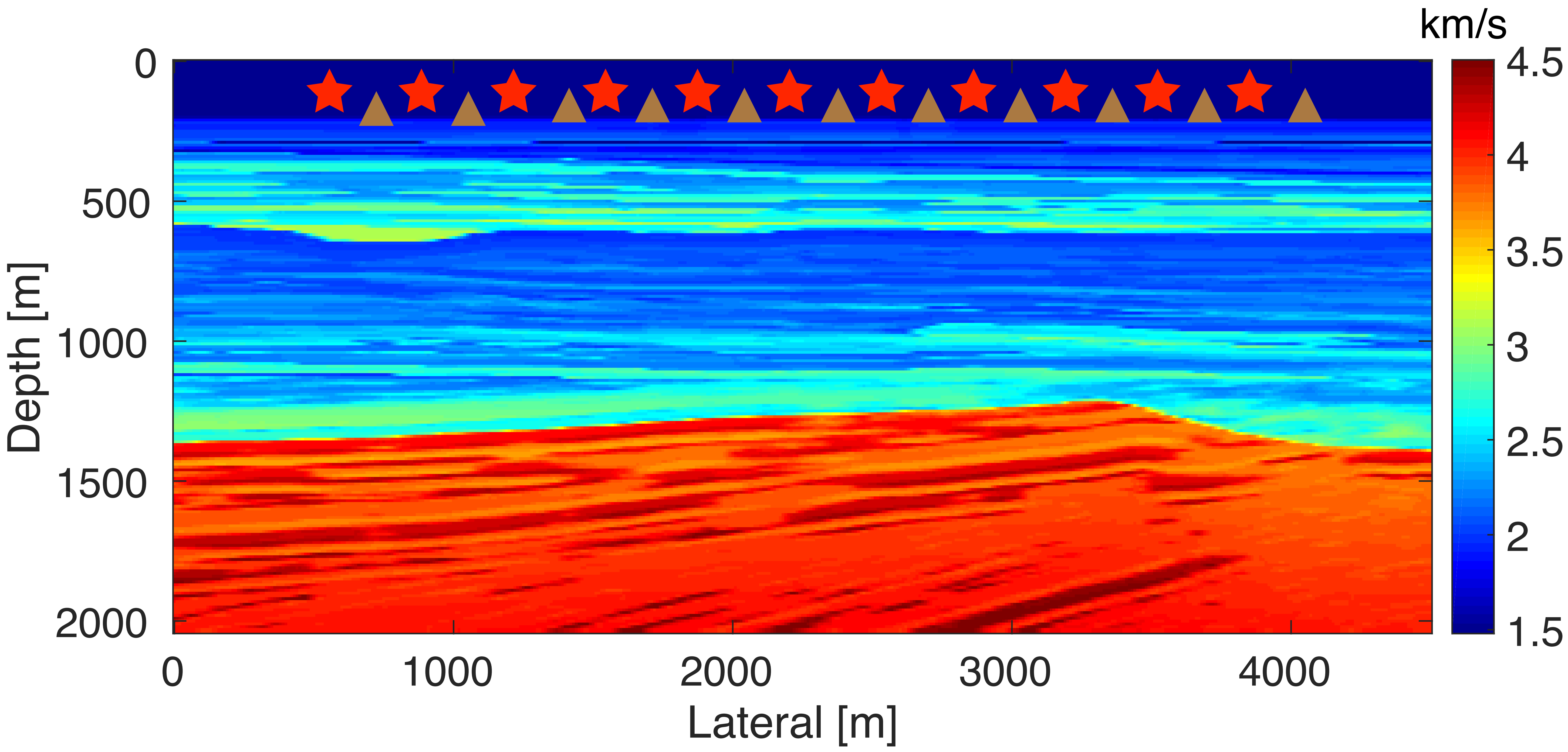



**Wavefield with true source wavelet**

**Wavefield with estimated source wavelet**

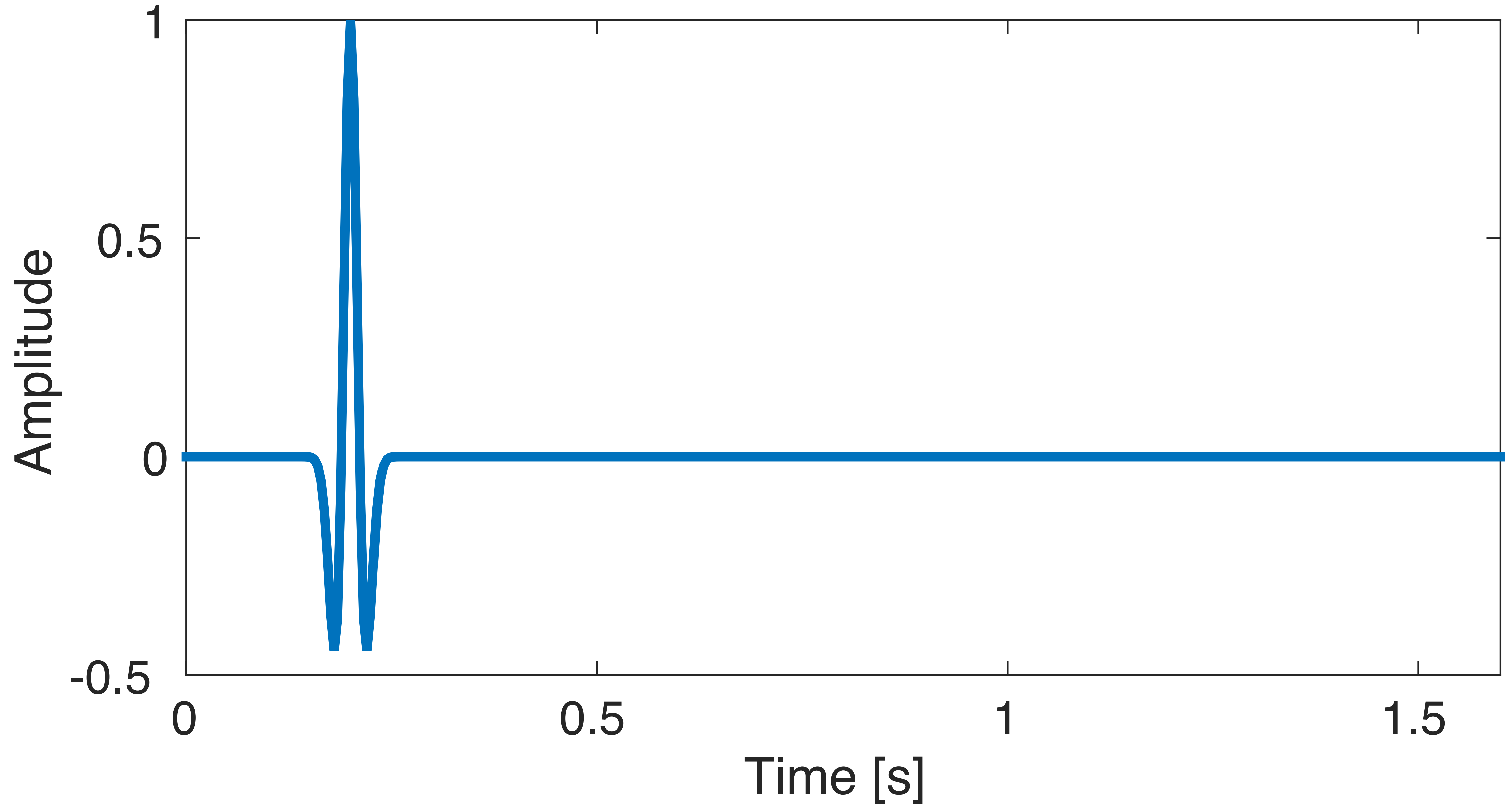
# BG model



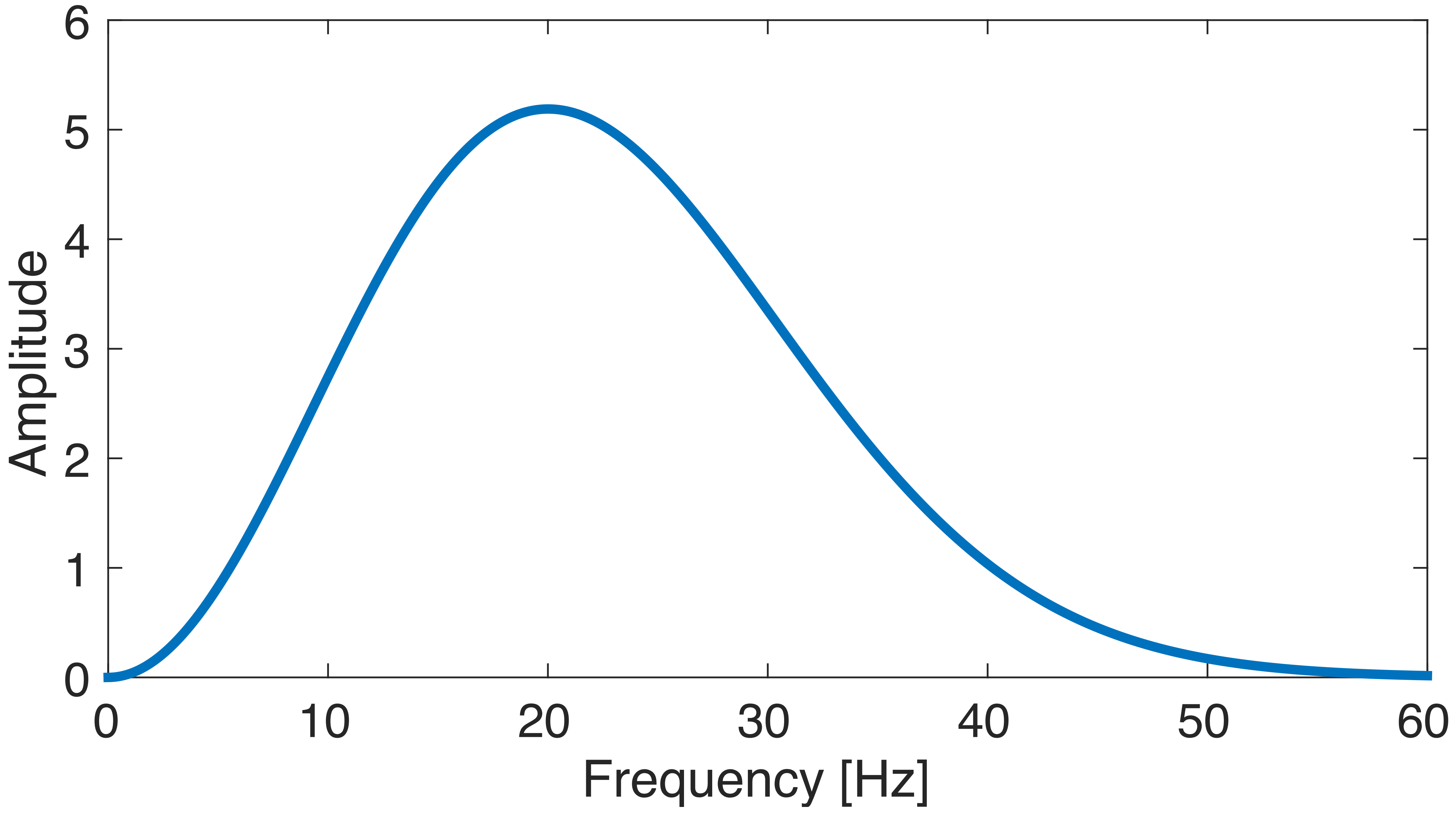

**Modeling information:**
**Model size:** 2000m x 4500m
**Source spacing:** 50m
**Receiver spacing:** 10m
**Fixed spread 4.5km**
**Frequency :** 2~31 Hz

**Inversion information:**
**Optimization Solver:** Gauss-Newton
**Iterations per frequency band:** 21
**Batch size:** 15

# Source wavelet

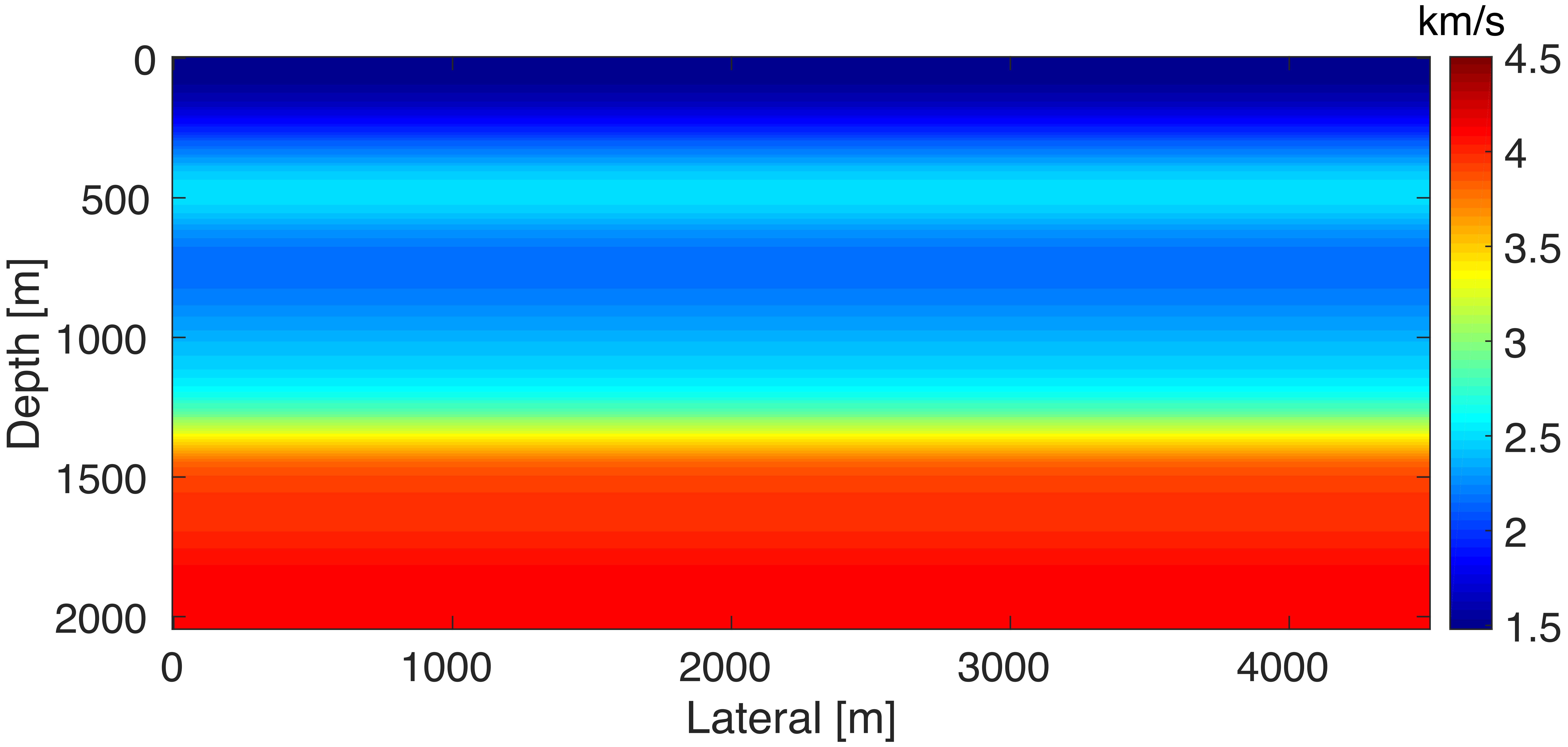



Source Wavelet

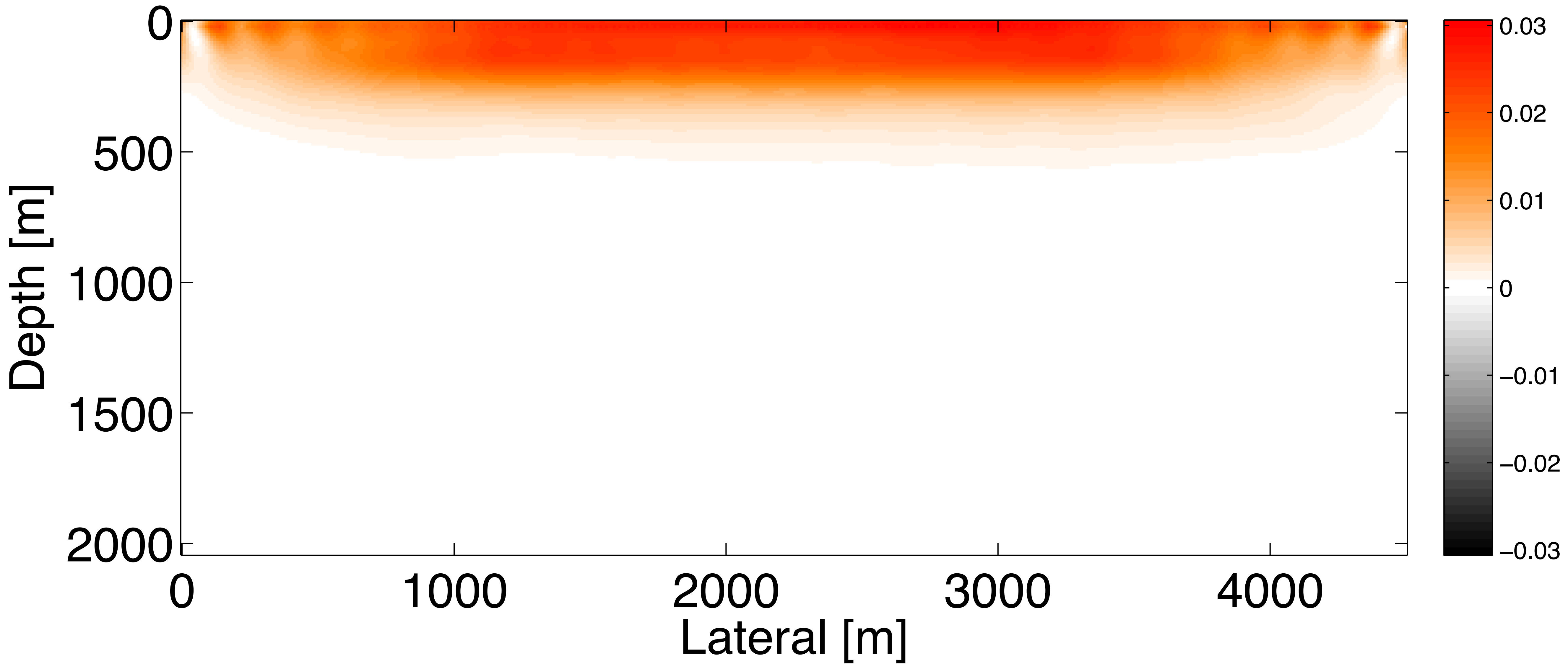



Spectrum

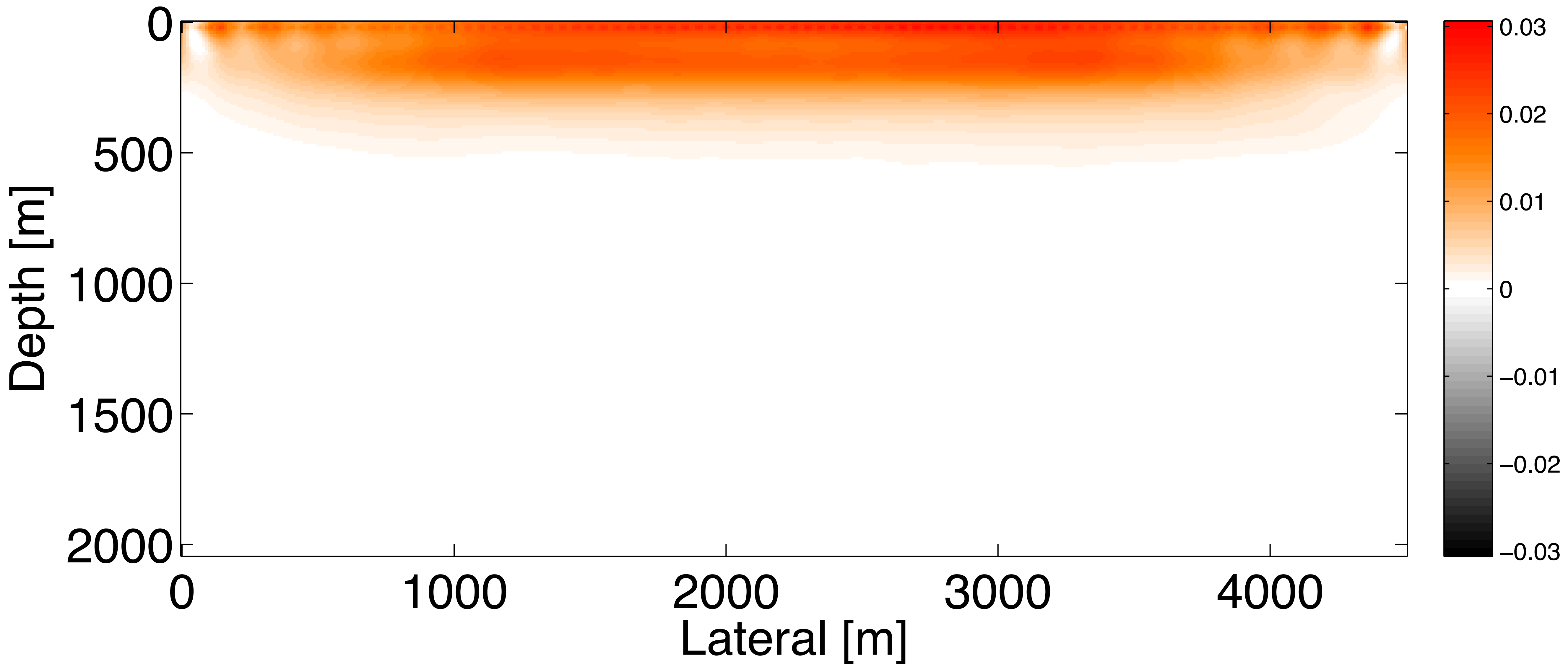
# Initial model



# First gradient comparison

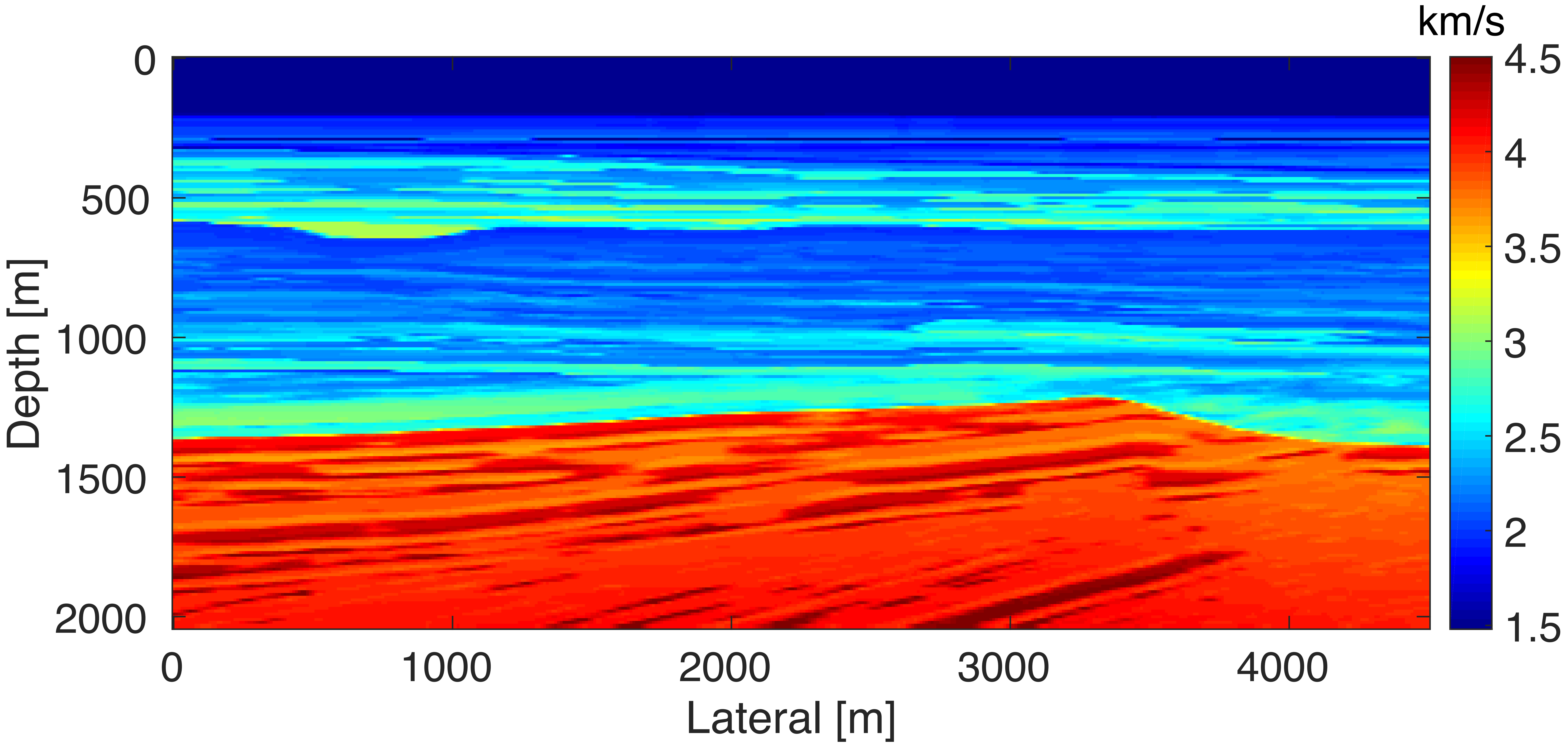



Gradient with true source wavelet



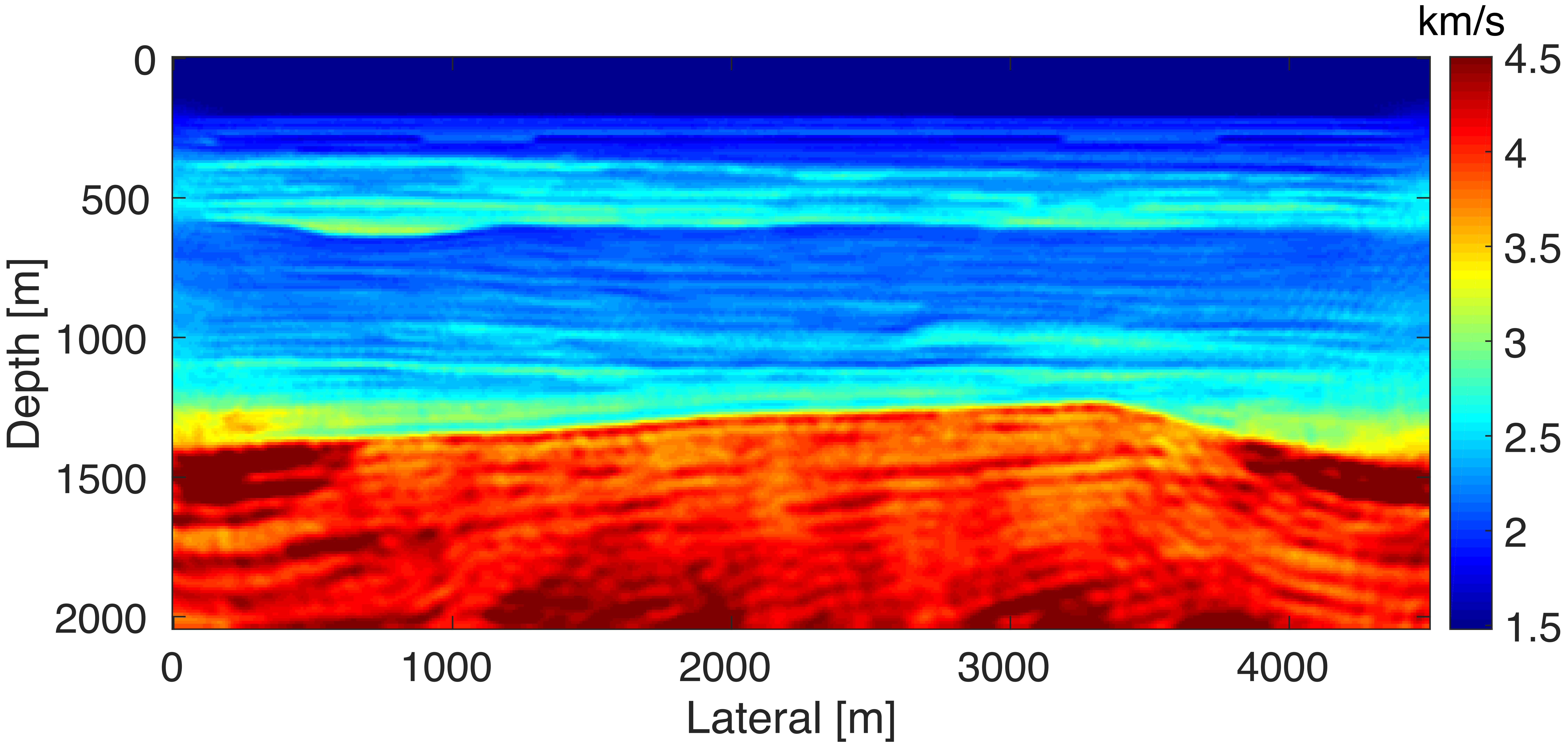

Gradient with estimated source wavelet

# True Model

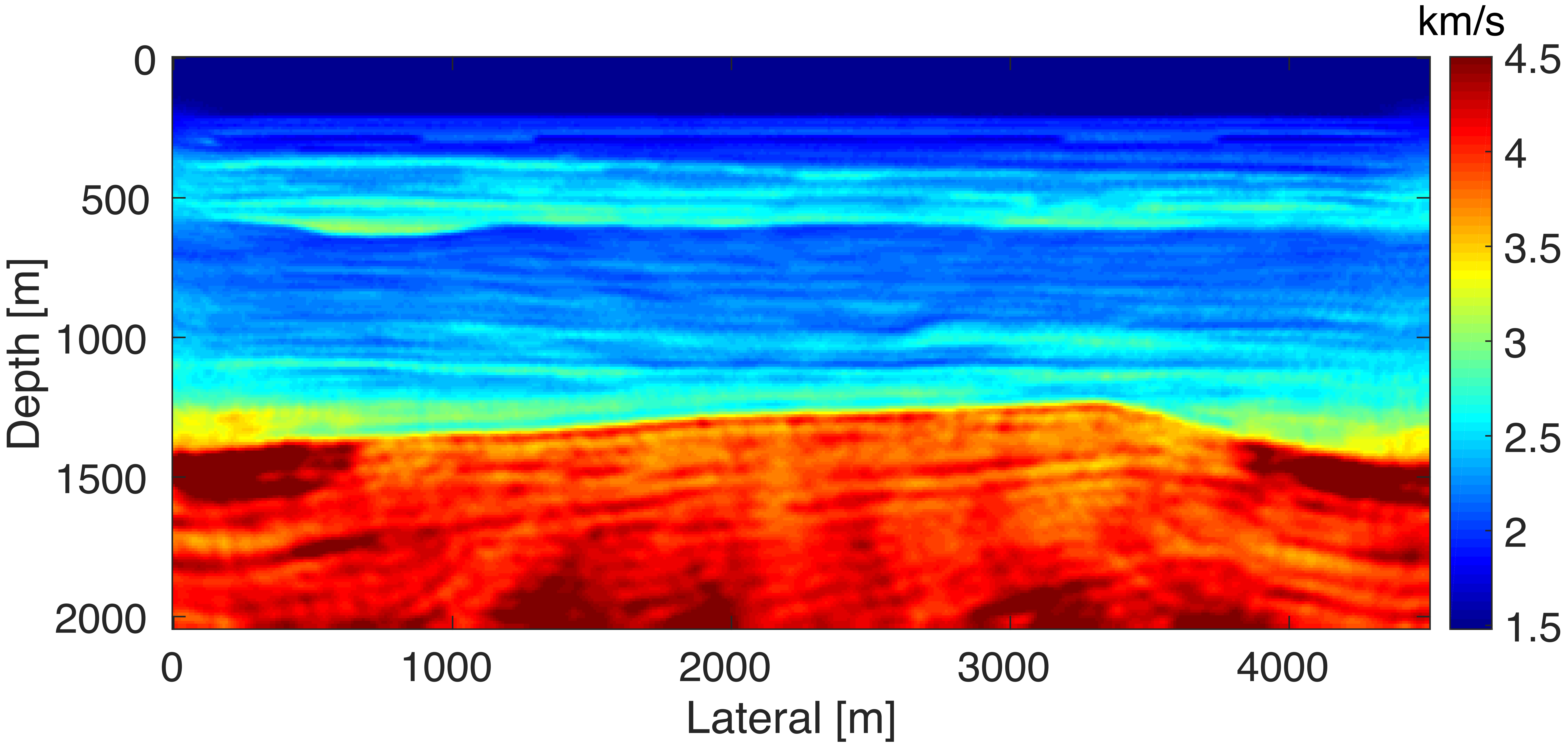


km/s
4.5
4
3.5
3
2.5
2
1.5
0
500
1000
1500
2000
Depth [m]
0
1000
2000
3000
4000
Lateral [m]

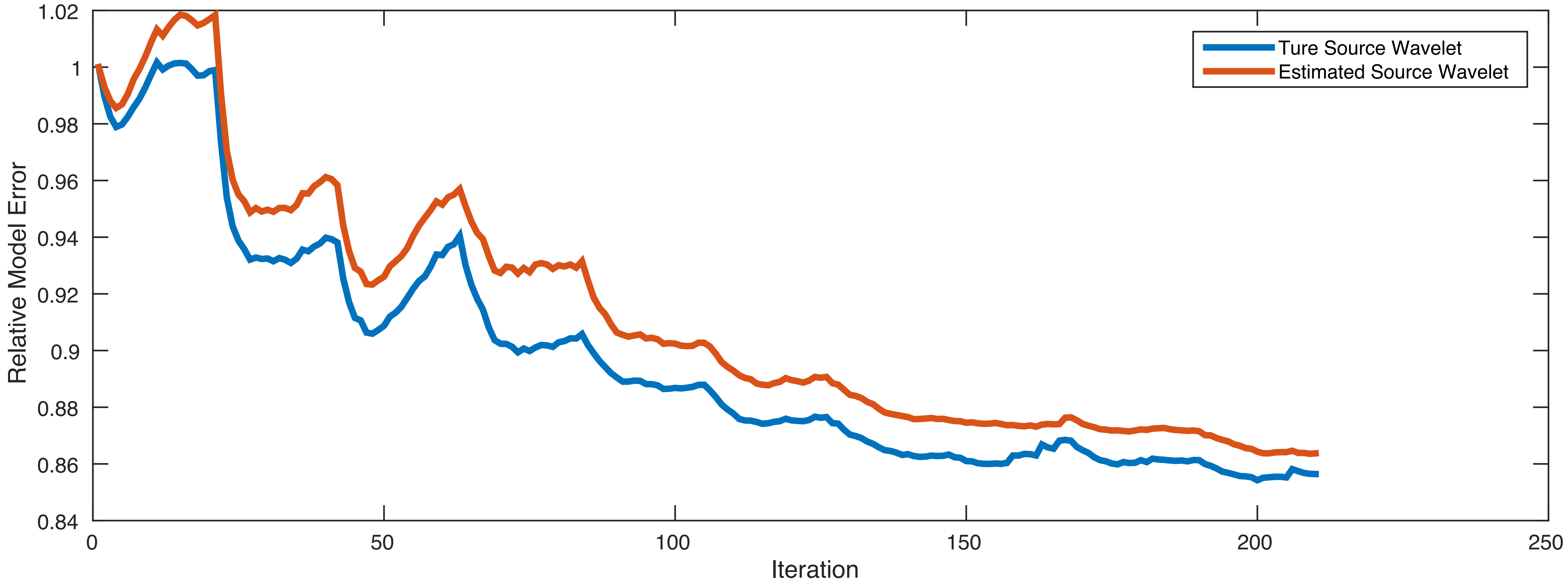
# Result with true source wavelet



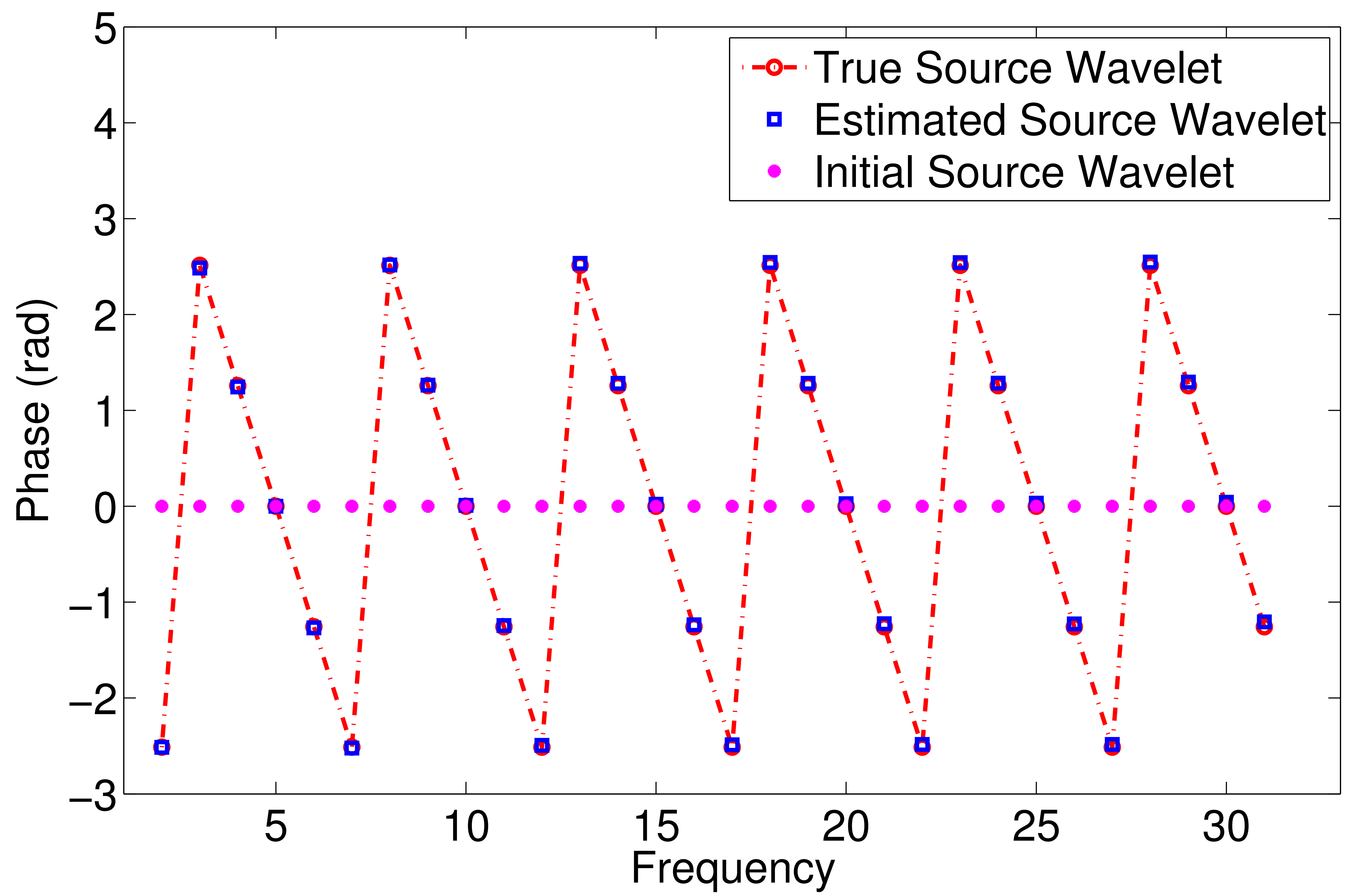
# Result with estimated source wavelet



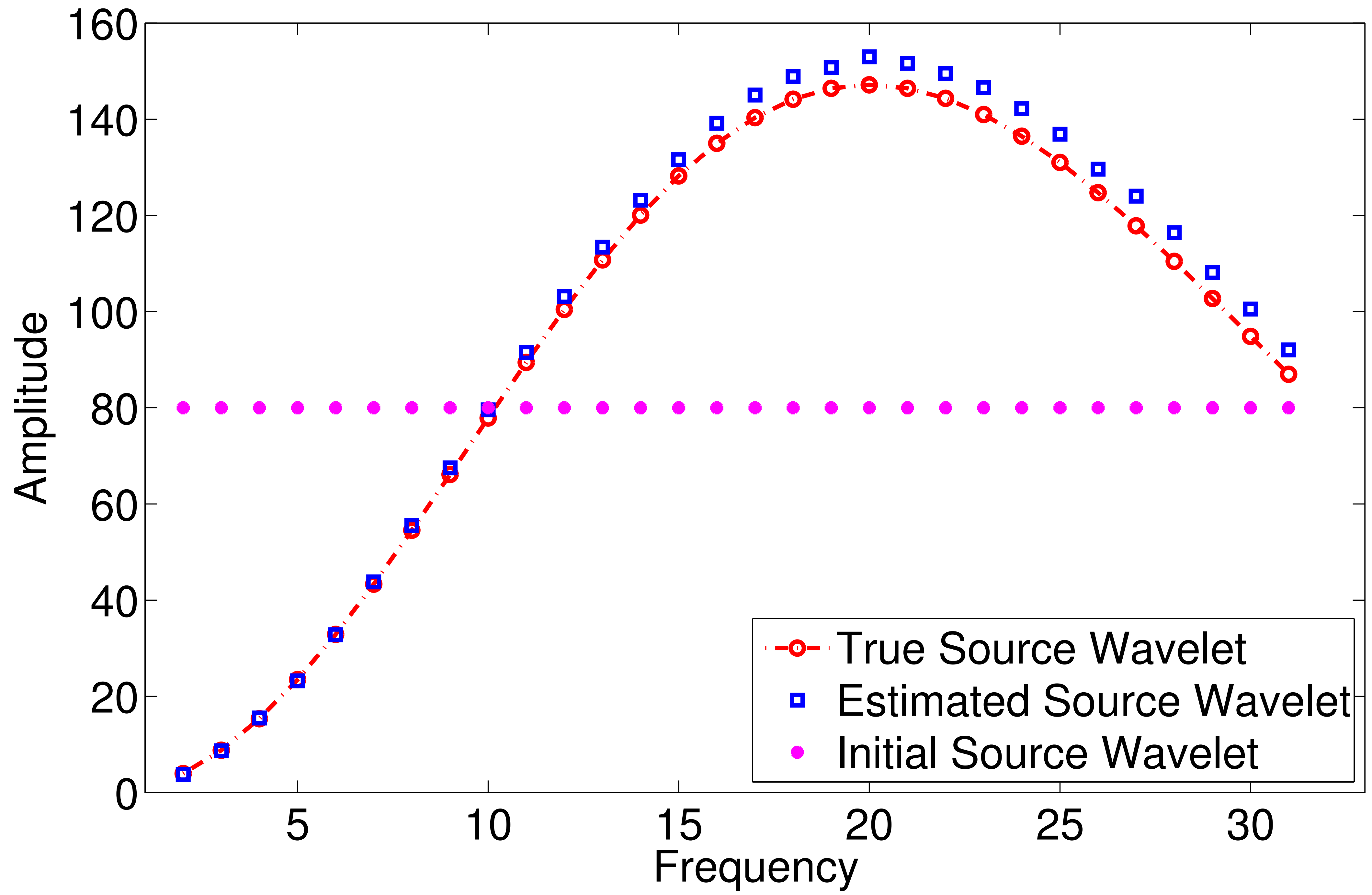
# Relative model-error comparison



# Source wavelet comparison

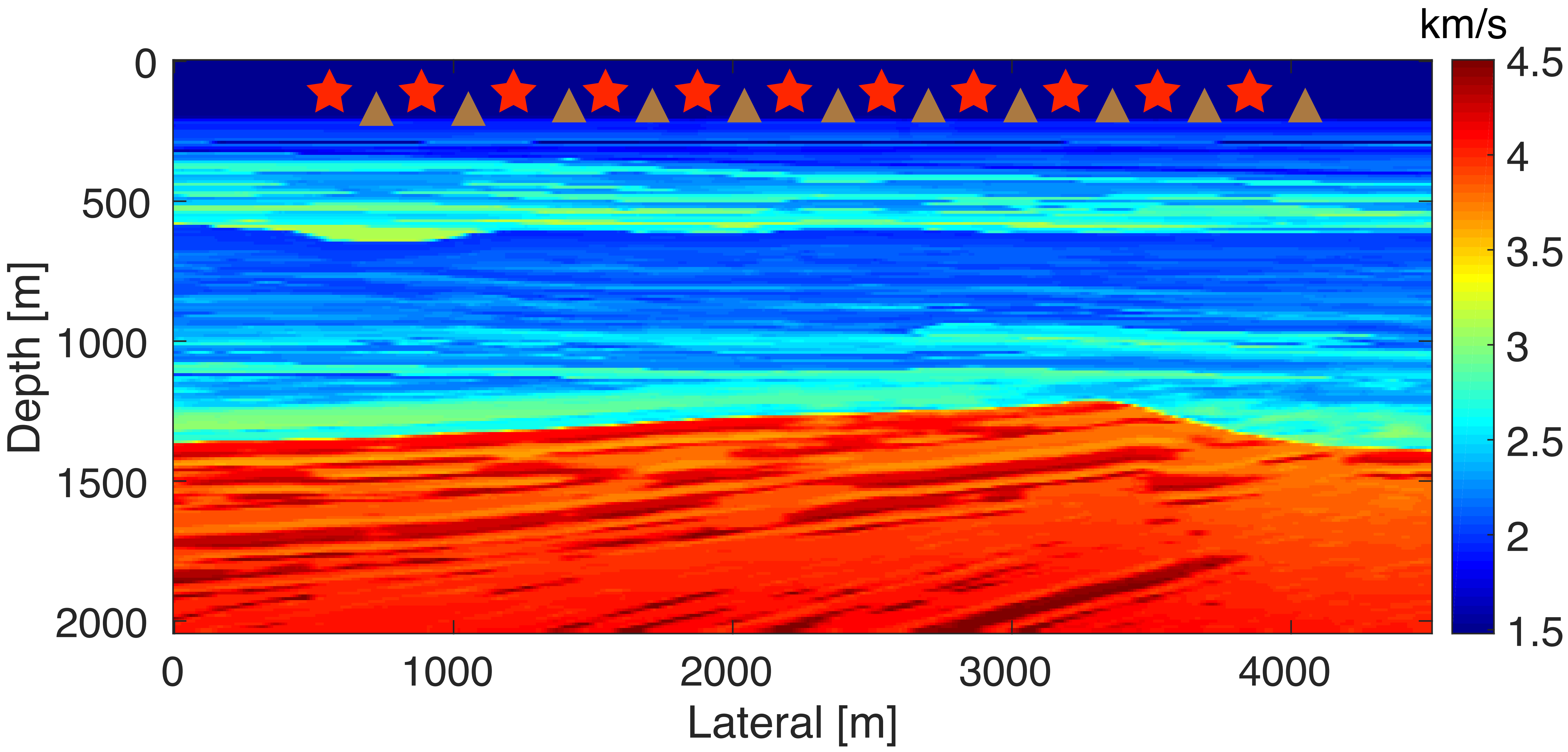



**Phase**




**Amplitude**

# BG model




**Modeling information:**
**Model size:** 2000m x 4500m
**Source spacing:** 50m
**Receiver spacing:** 10m
**Fixed spread 4.5km**

# Non-inversion crime Example

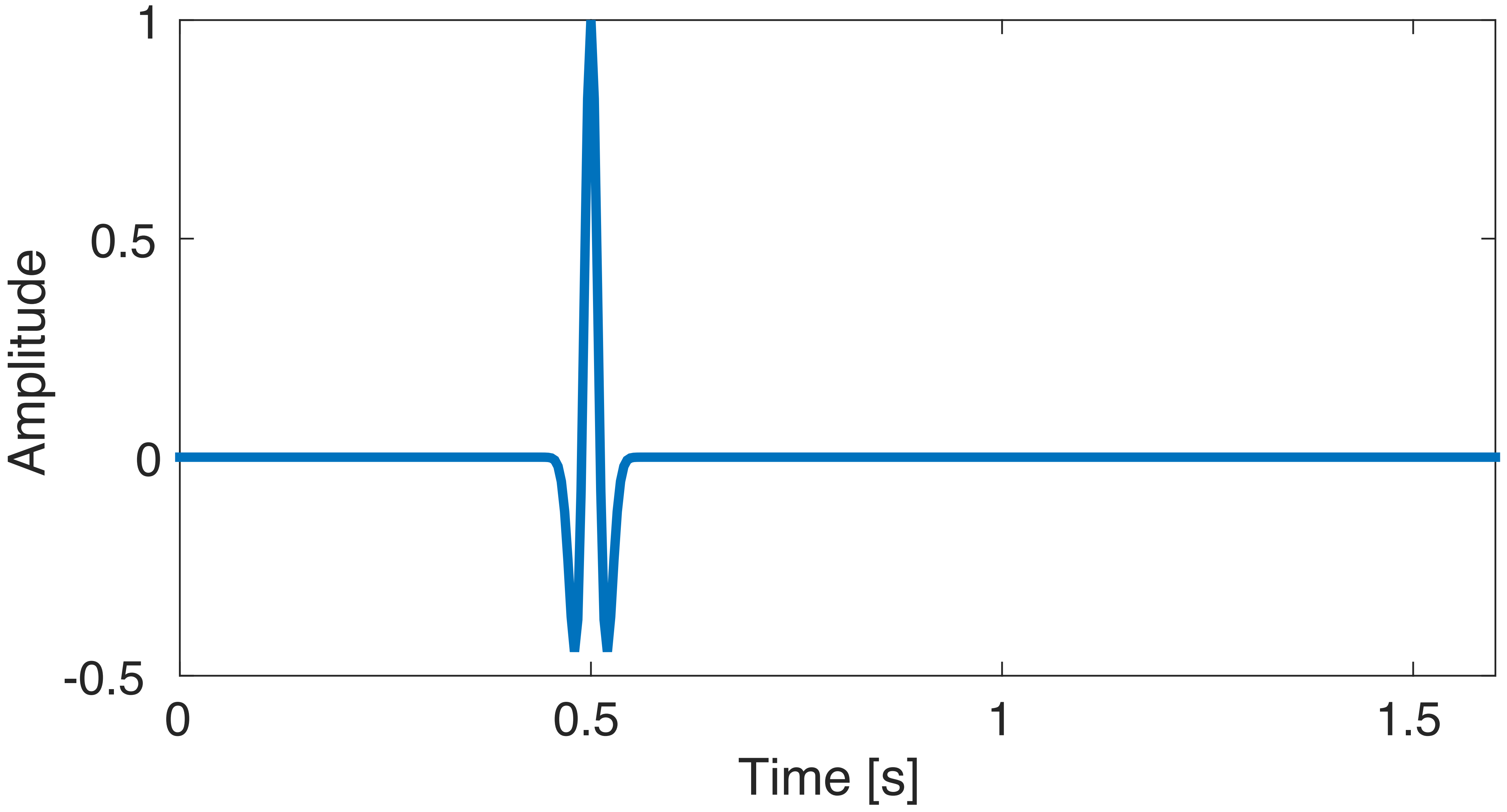
Observed Data information:

- Time domain data by iWAVE
- Grid size: 5m
- Source wavelet: 20Hz Ricker wavelet

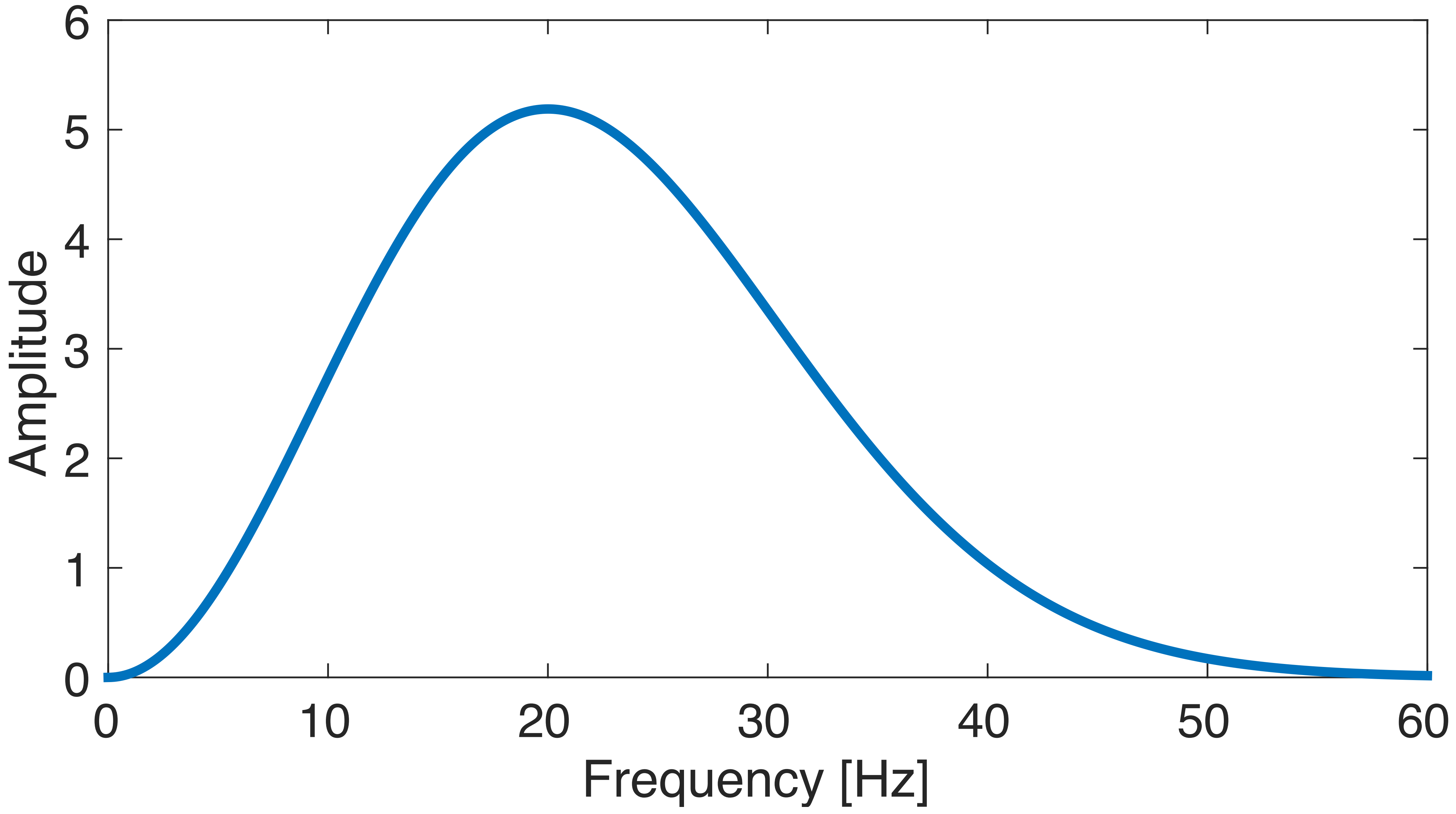
Inversion strategy:

- Frequency: 2-29Hz
- Grid size: 10m (2-20 Hz), and 6m (20-29 Hz)
- Gauss-Newton method with 20 iterations per frequency band

# Source wavelet

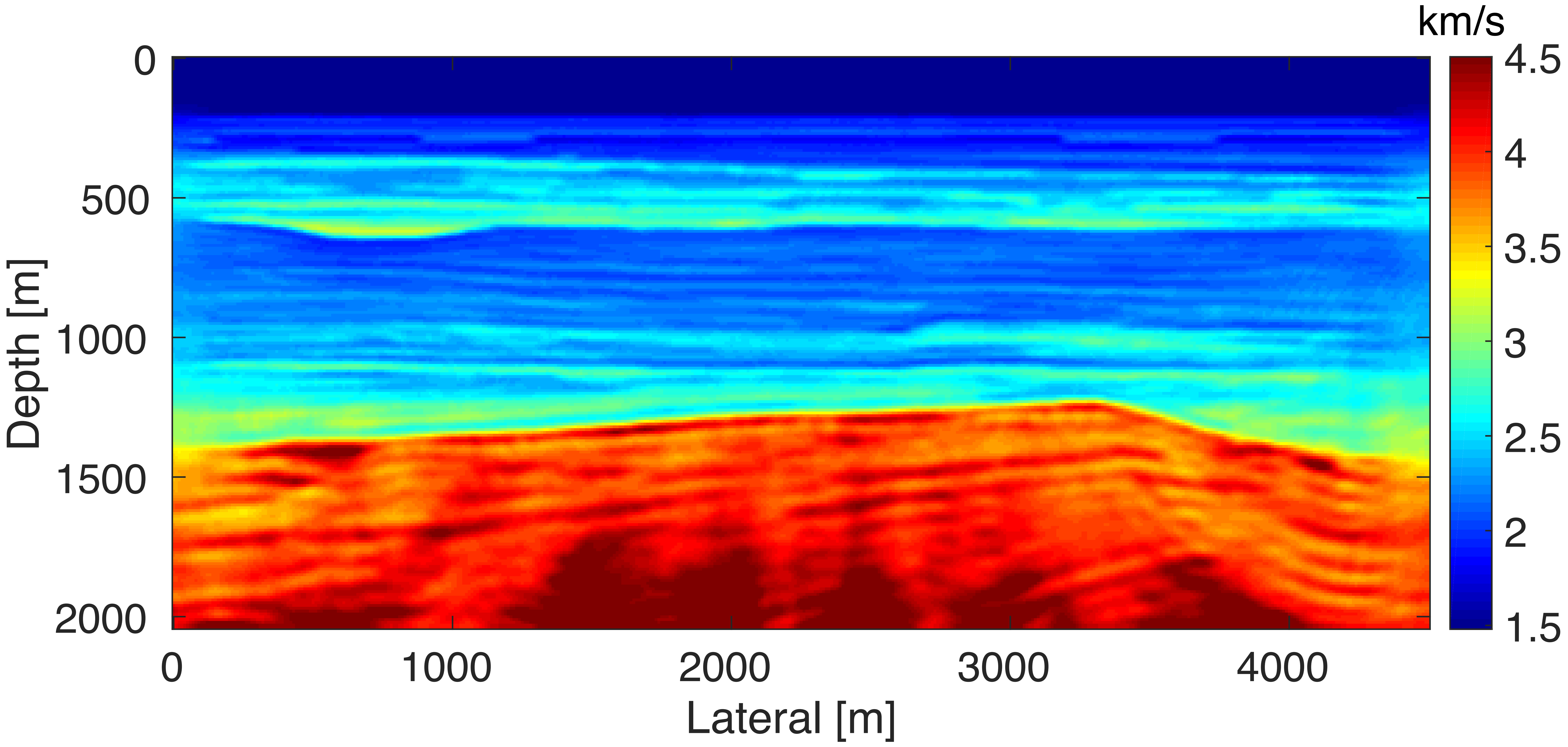



Source wavelet

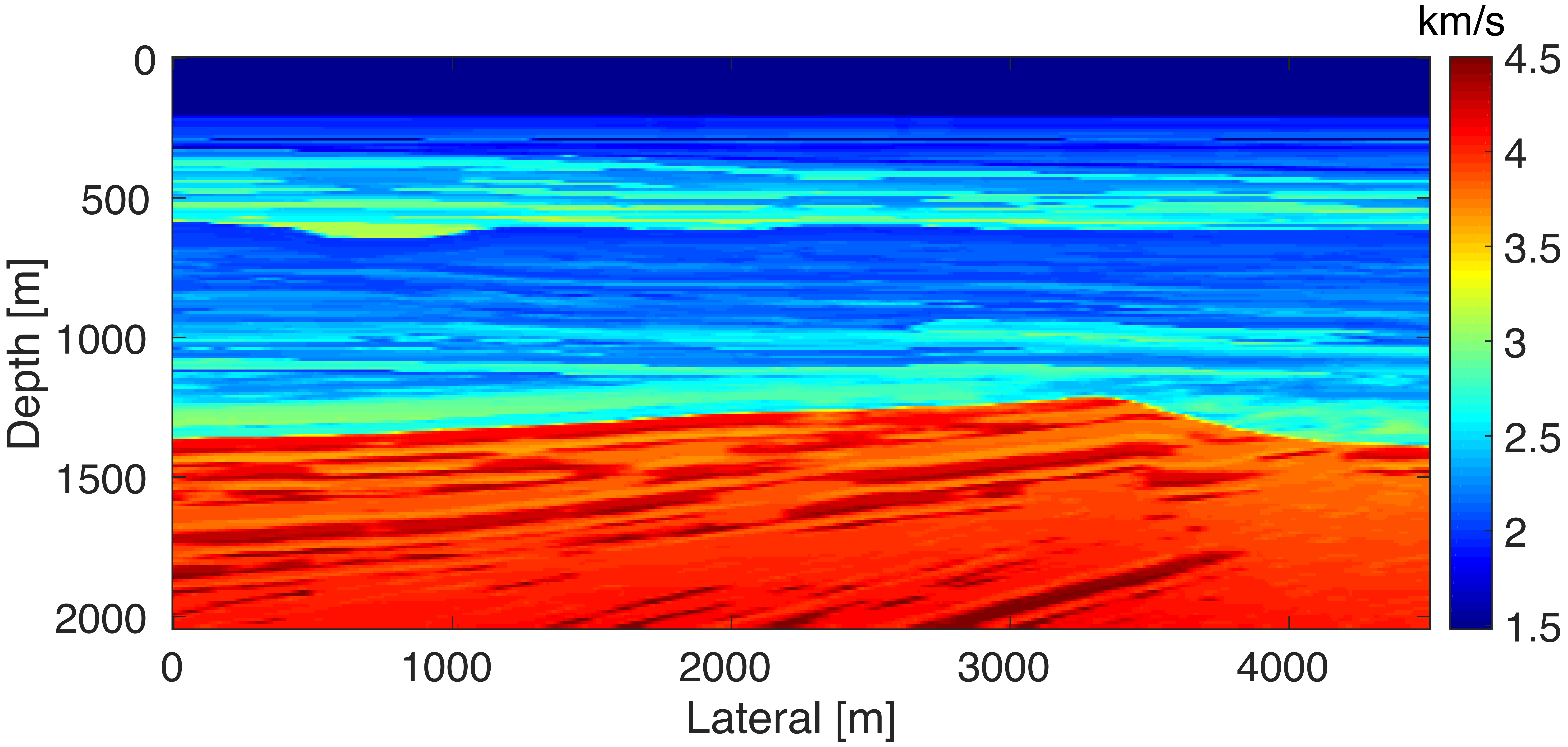



Spectrum

# Inversion result

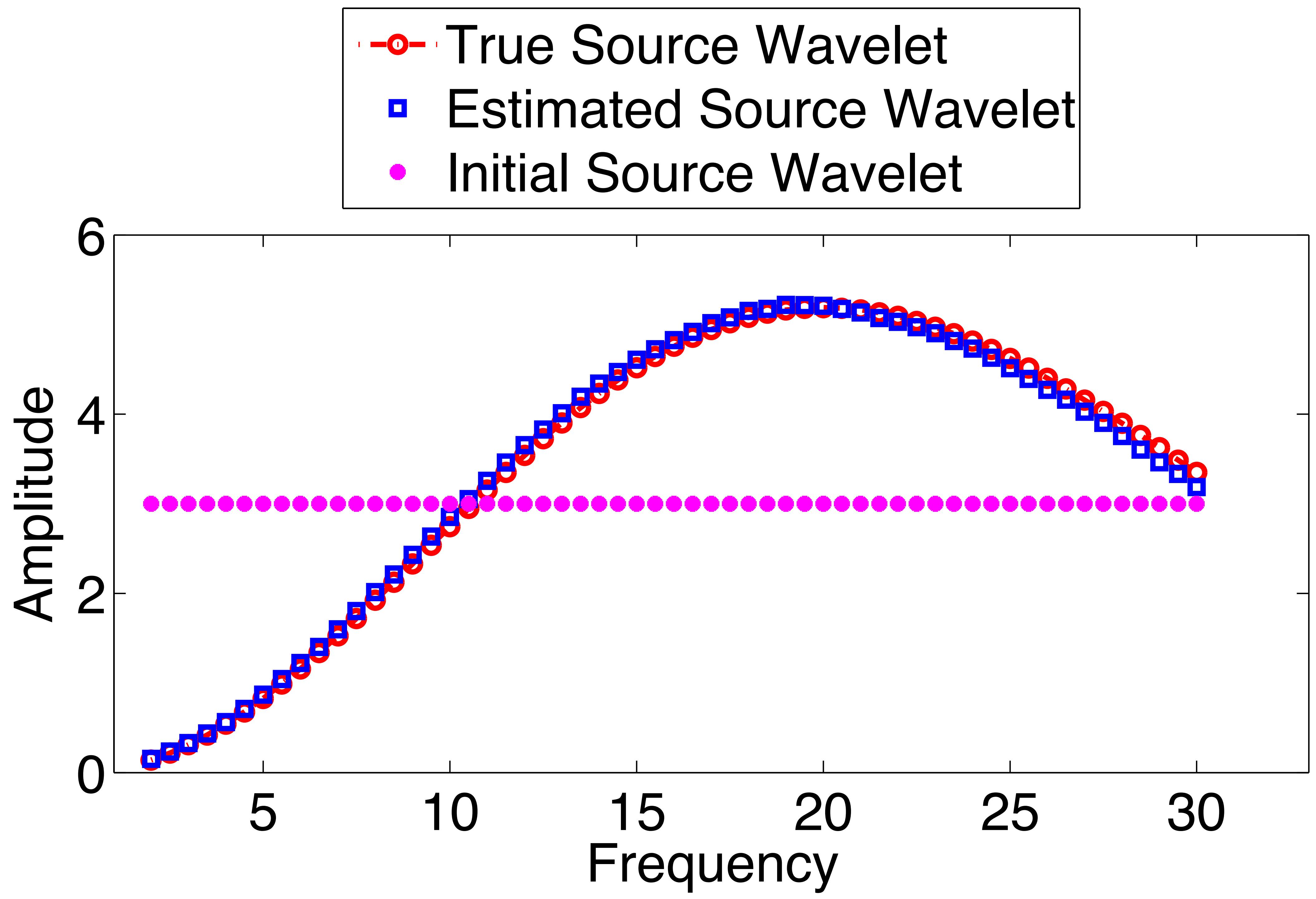


# True Model

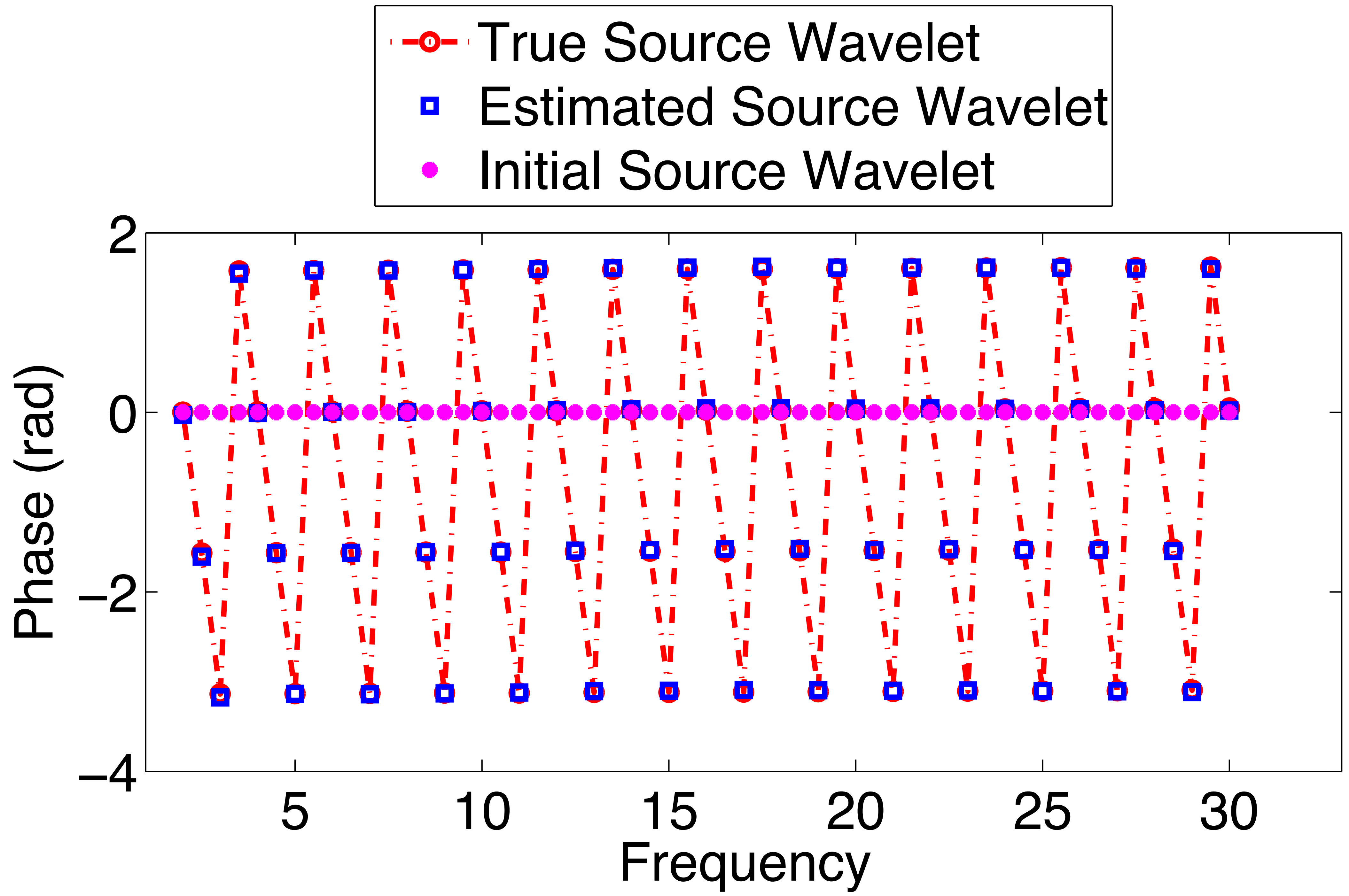


km/s
4.5
4
3.5
3
2.5
2
1.5
0
500
1000
1500
2000
Depth [m]
0
1000
2000
3000
4000
Lateral [m]

# Source wavelet comparison

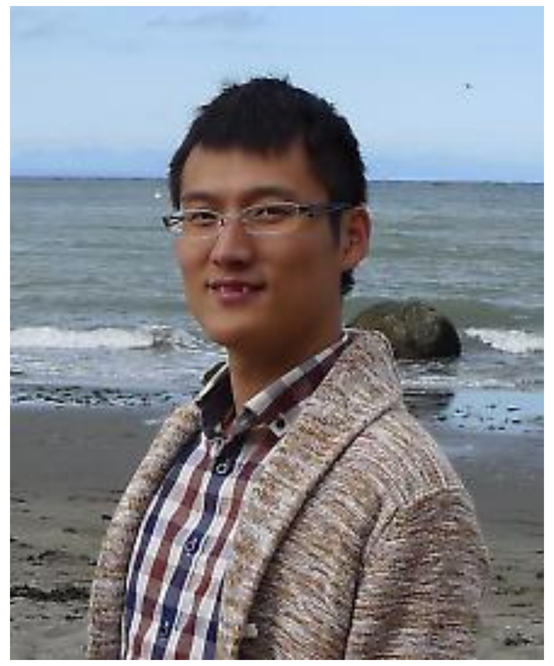



**Amplitude**

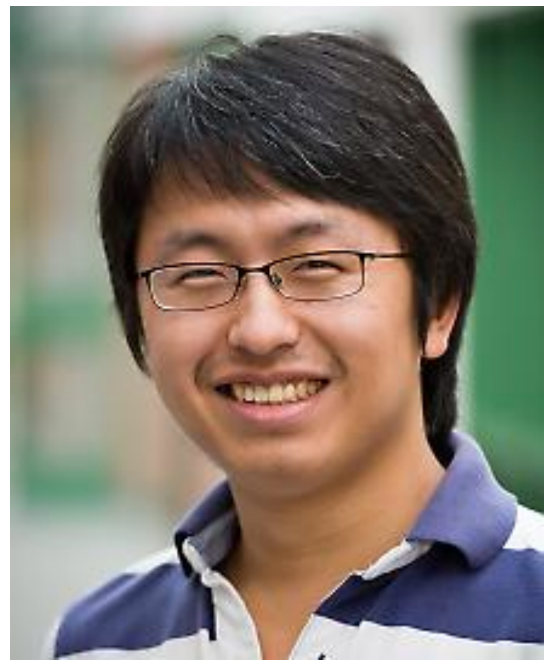



**Phase**

# Chevron blind test data



Zhilong Fang



Xiang Li

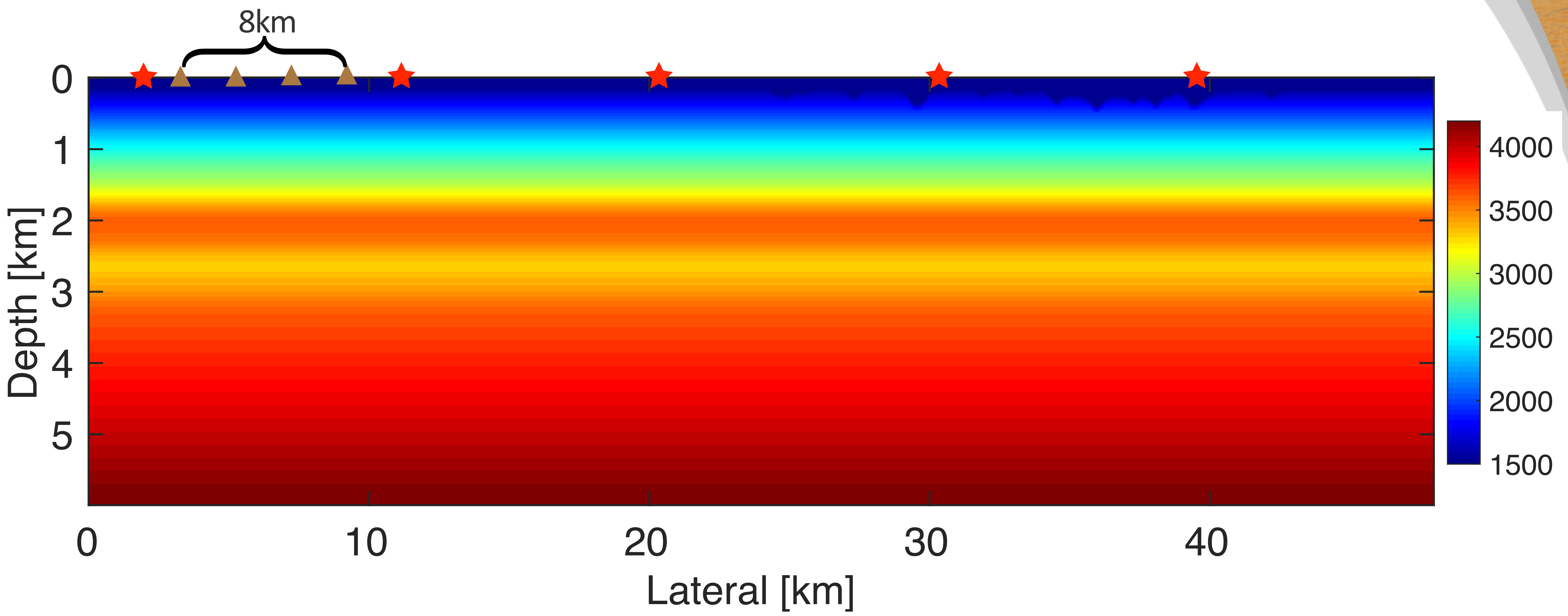
Bas Peters

Brendan Smithyman

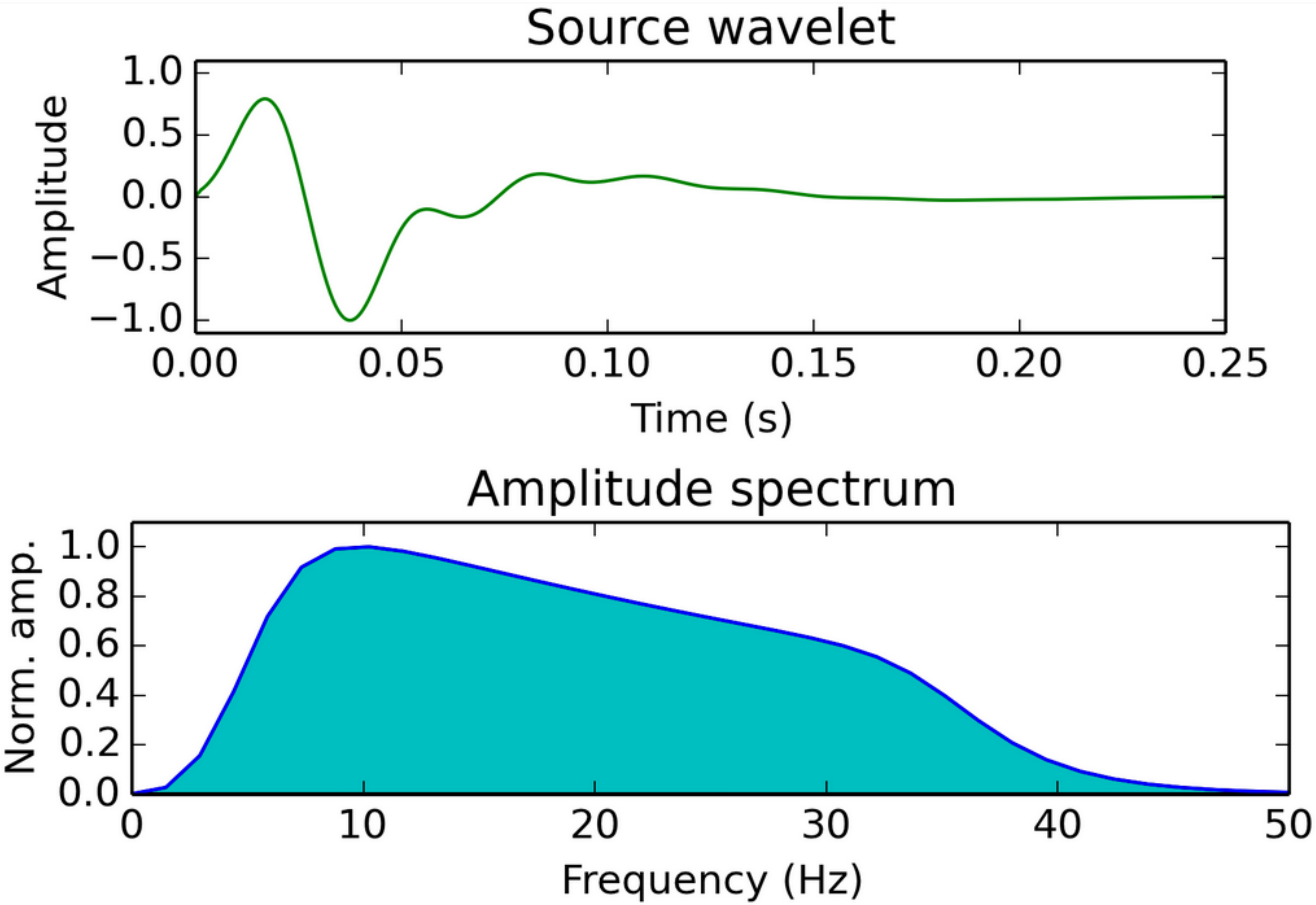
Mengmeng Yang

Felix J. Herrmann

# Chevron blind test data



# Chevron blind test data



# Chevron blind test data

**Data-set information:**

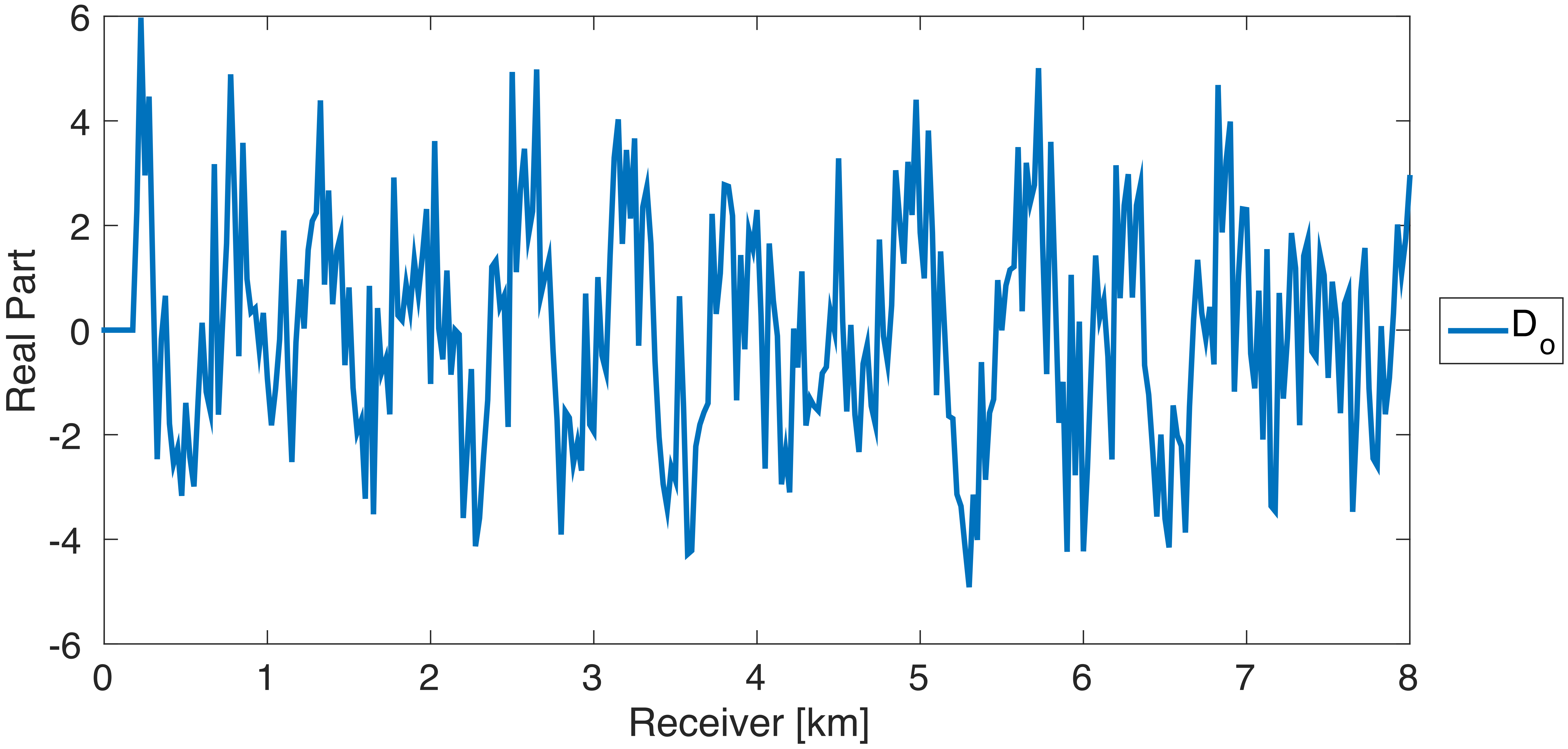
1. 1600 shots: ds = 25 m, Source depth = 15 m;
2. 321 hydrophone recs/shot: dr = 25 m, Receiver depth = 15 m;
3. Maximum offset = 8000 m;
4. Record time = 8.0 s, sample rate 4 ms;
5. Vp water = constant = 1510 m/s;
6. With free surface multiples present in the data;
7. Isotropic Elastic.

# Chevron blind test data

**Inversion strategy:**

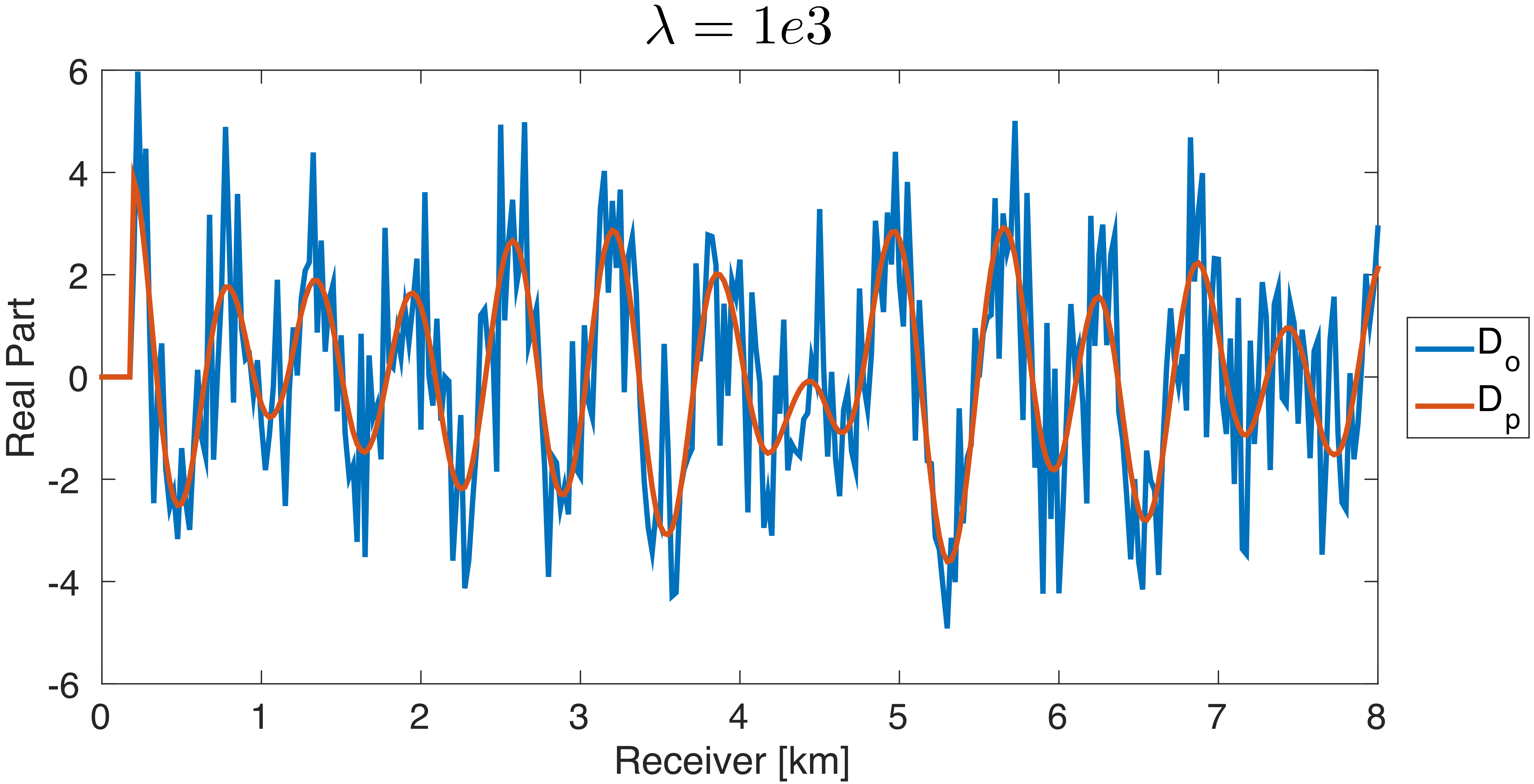
1. Frequency domain WRI with Source estimation;
2. Frequency bands: [3:0.2:5]Hz, [3:0.2:7]Hz, [3:0.2:9]Hz, [3:0.2:11]Hz, [3:0.2:19]Hz;
3. Batch sizes of random frequency subsets: 3, 6, 10, 10, 15;
4. Batch size of random source subsets: 300;
5. Optimization solver: l-BFGS with 20 iterations per frequency band;
6. 4 passes of WRI at frequency 3-11 Hz and 1 pass to 19 Hz;
7. Grid size: 20m for 3-11Hz and 12m for 3-19Hz;
8. No pre-processing !!!

# Data comparison
## — 3 Hz Data of 800th shot

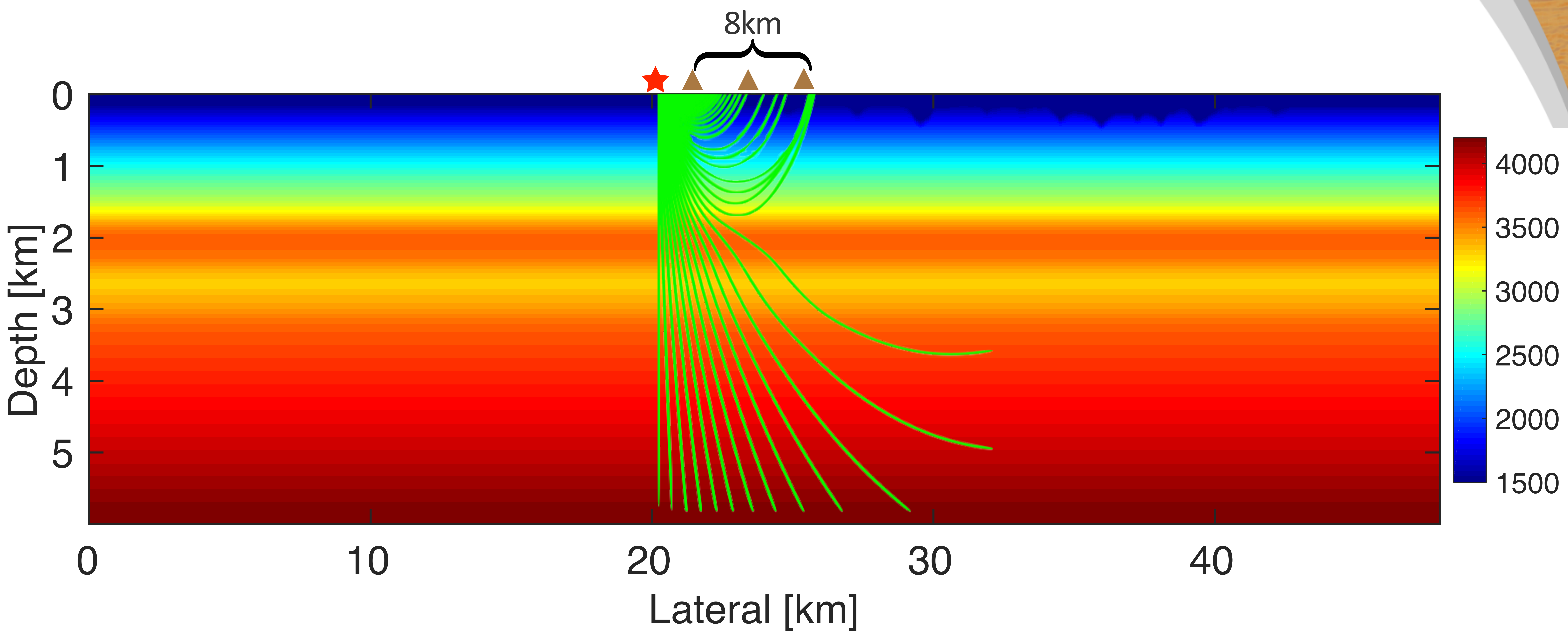


# Data comparison

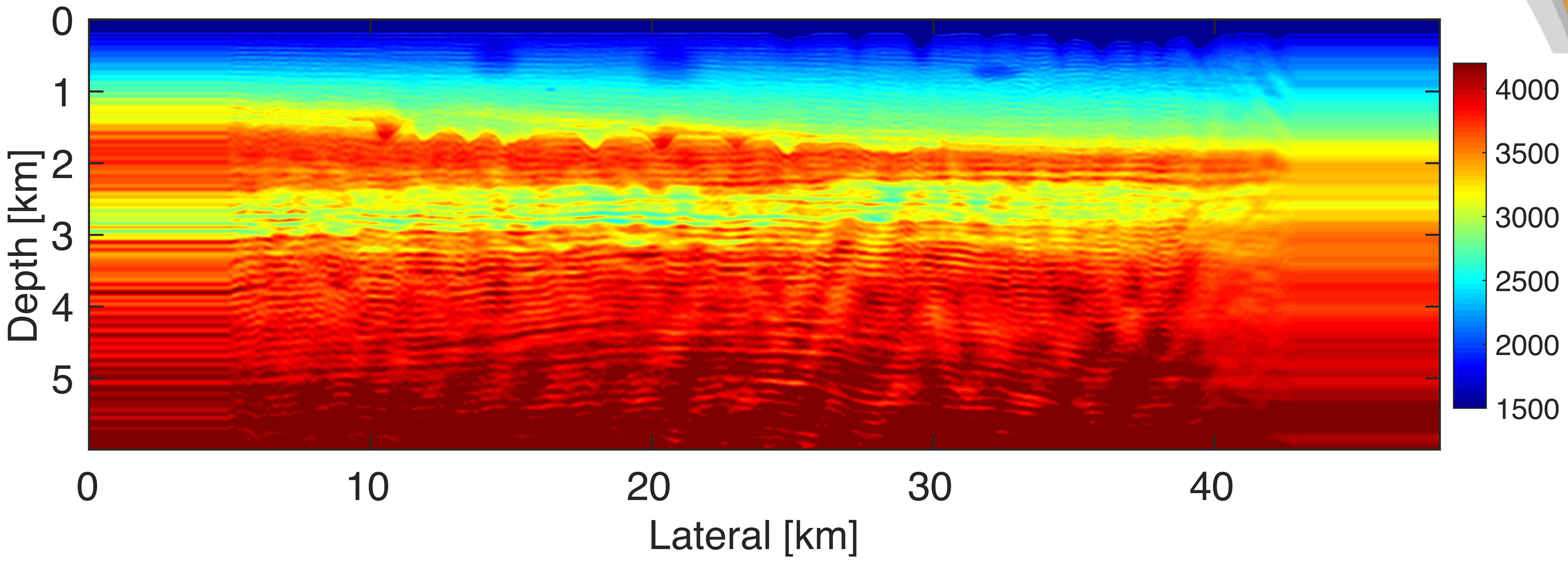
## — 3 Hz Data of 800th shot



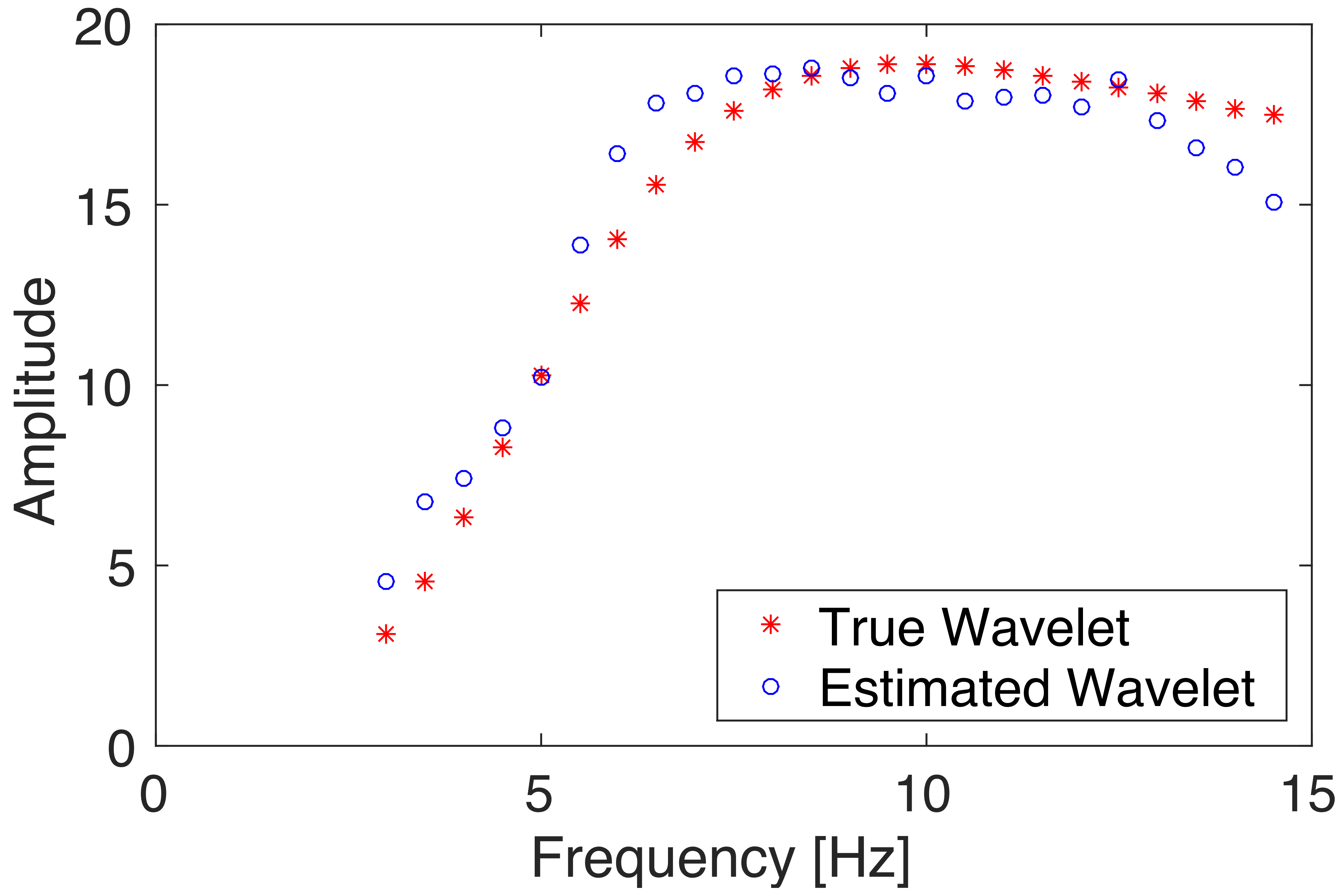
# Initial model



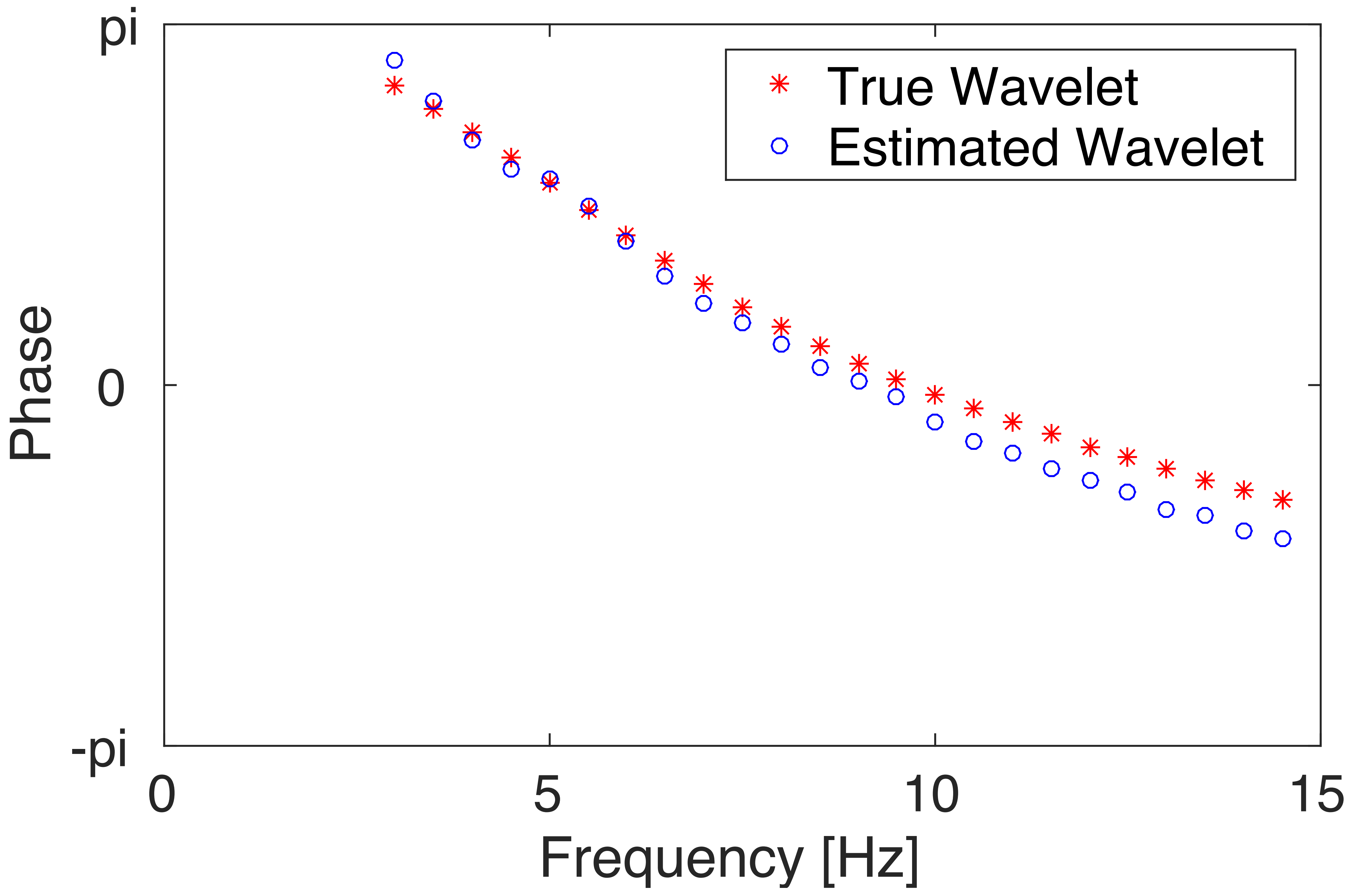
# Inversion result



# Source wavelet comparison

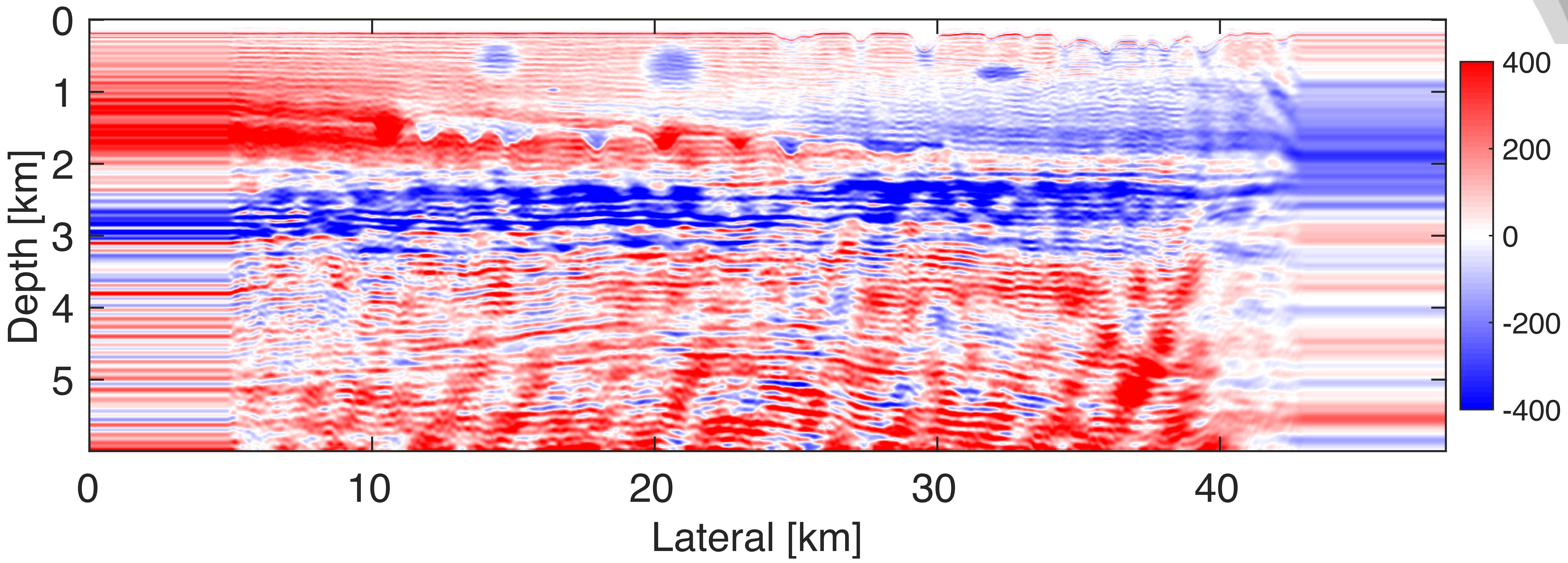



**Amplitude**

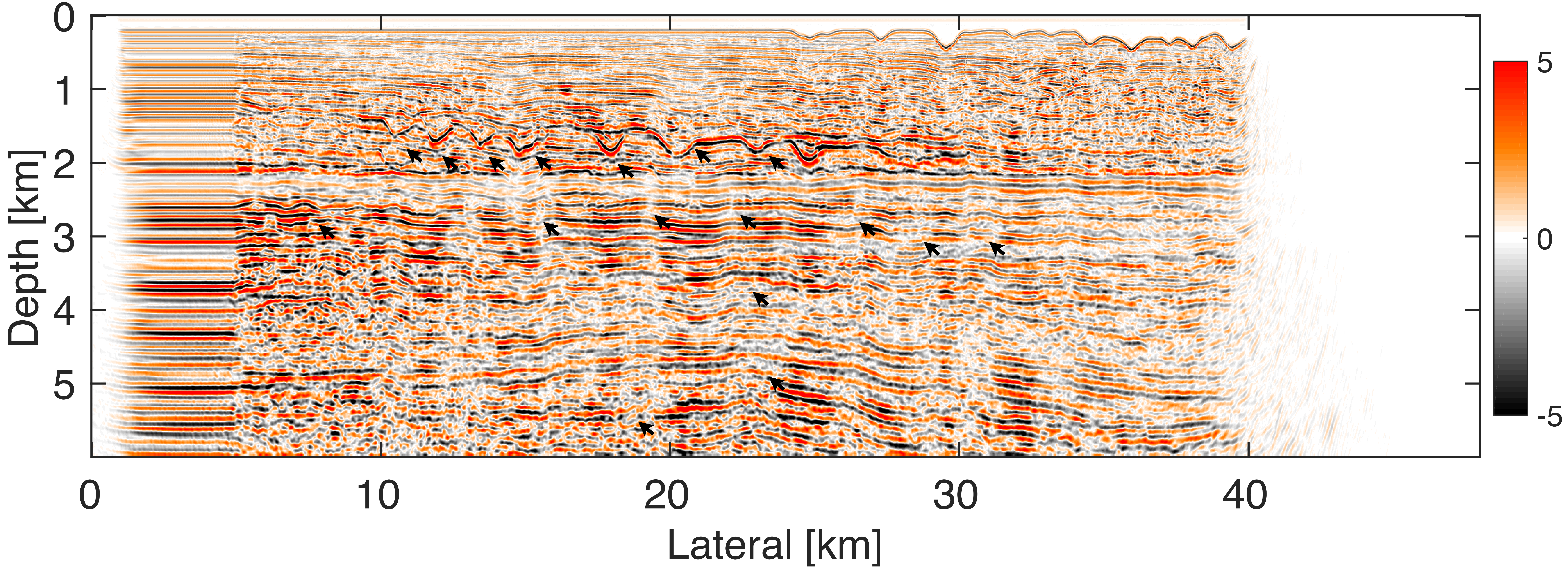



**Phase**

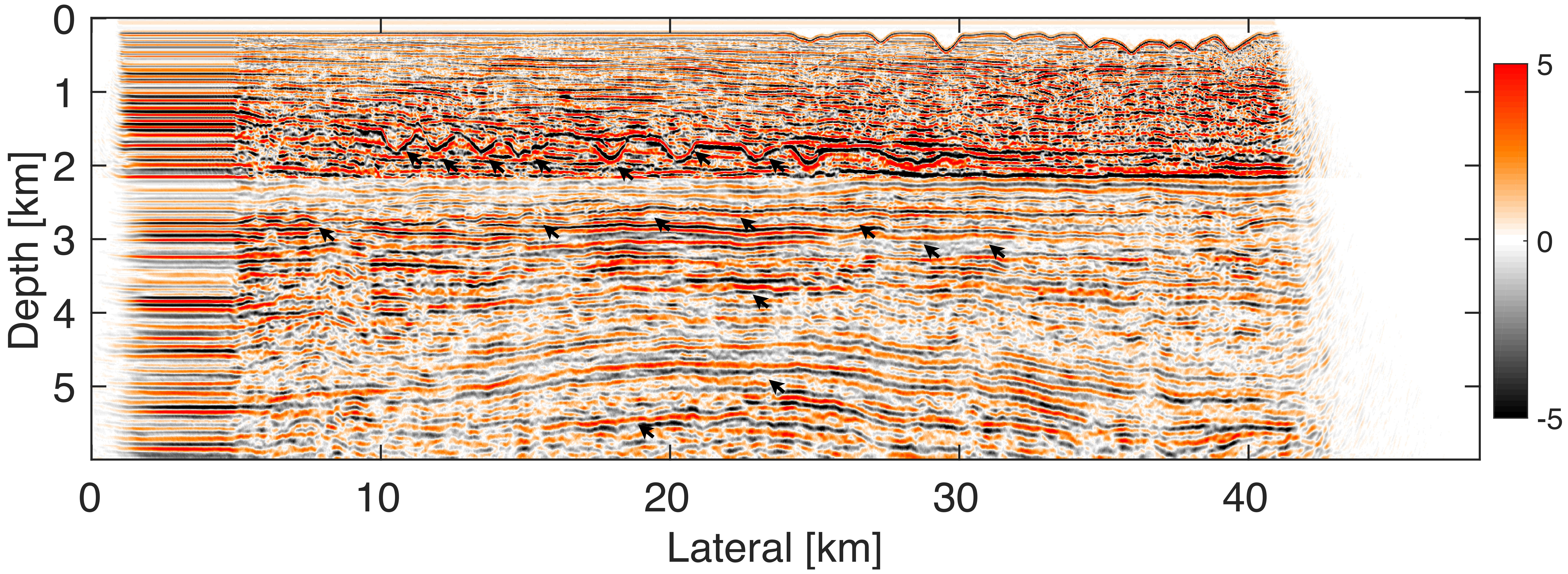
# Model update



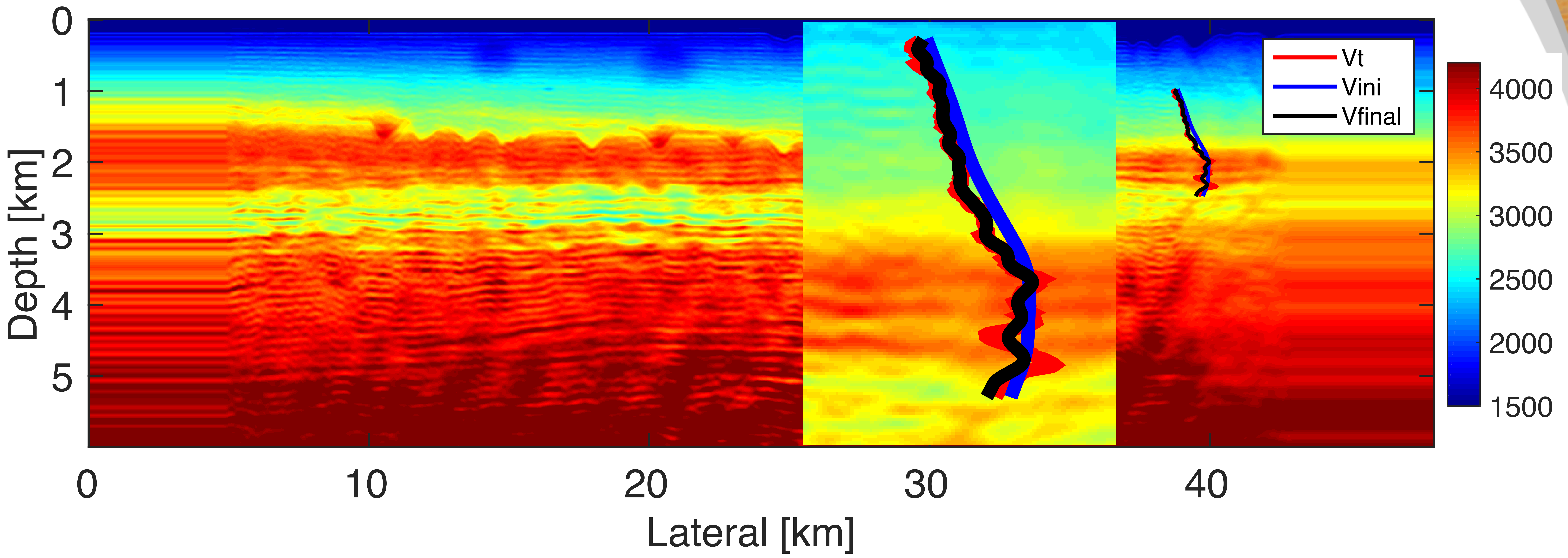
# Kirchhoff migration
## —Initial model



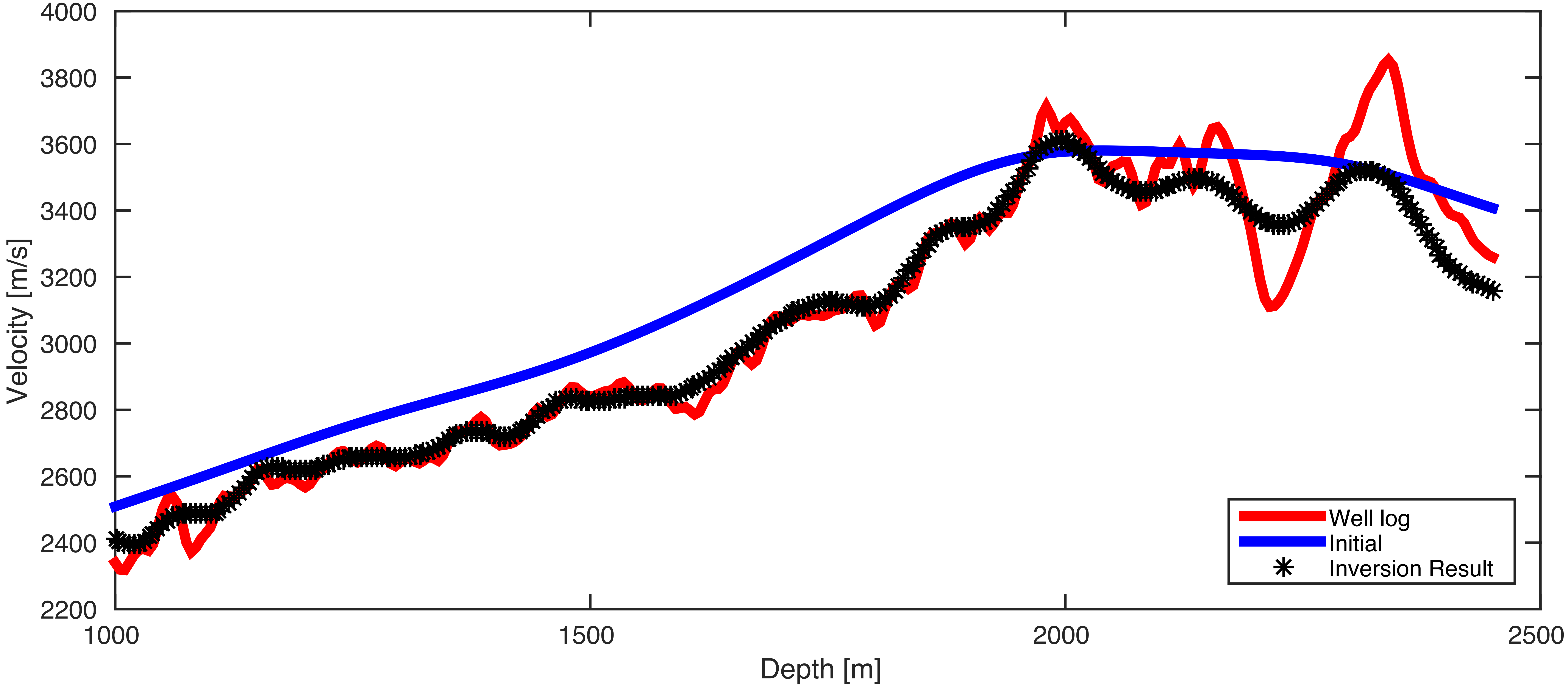
# Kirchhoff migration
## —Inversion result



# Well-log comparison



# Well-log comparison



# Conclusions

1. Using the variable projection method, we can estimate the source wavelet for the WRI.
   - Synthetic BG model

2. Source estimation enhances the robustness of WRI for field seismic data.
   - Chevron blind test data

## Future work

1. Incorporate smoothing / sparse constraint to the optimization problem to obtain a higher resolution result.

2. Source estimation for the uncertainty quantification of WRI.

# Acknowledgements

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SINBAD

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