Fast imaging with surface-related multiples:
- shallow water multiples

Mengmeng Yang, Ning Tu, Felix J. Herrmann

SLIM
University of British Columbia

Lu S, Whitmore D N, Valenciano A A, et al. Separated-wavefield imaging using primary and multiple energy[J]. The Leading Edge, 2015, 34(7): 770-778.
Lu S, Whitmore D, Valenciano A, et al. Enhanced subsurface illumination from separated wavefield imaging[J]. First Break, 2014, 32(11): 87-92.

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Motivation

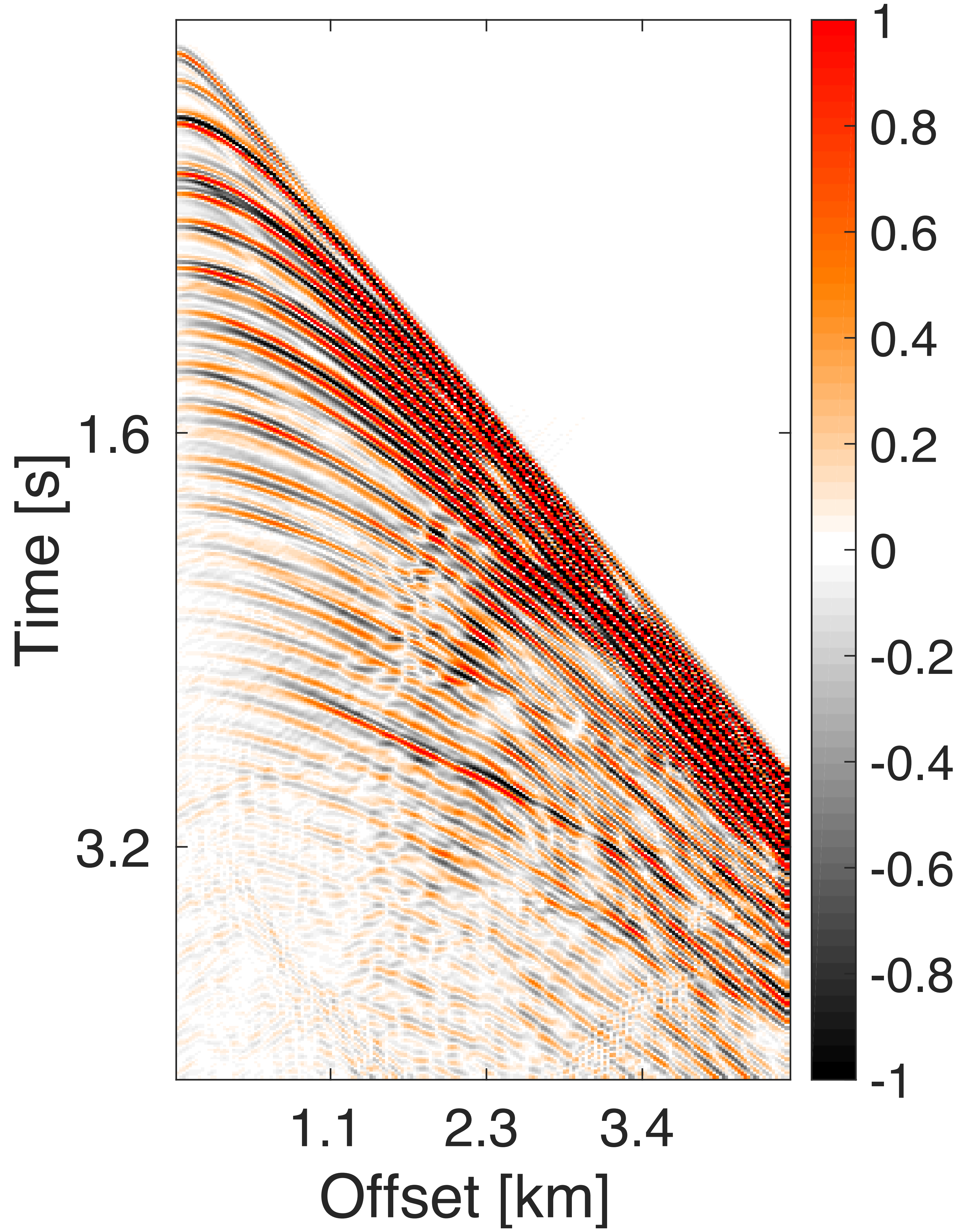
Processing challenges:

- estimation of primaries from shallow water multiples
- failure of SRME
- expense of EPSI

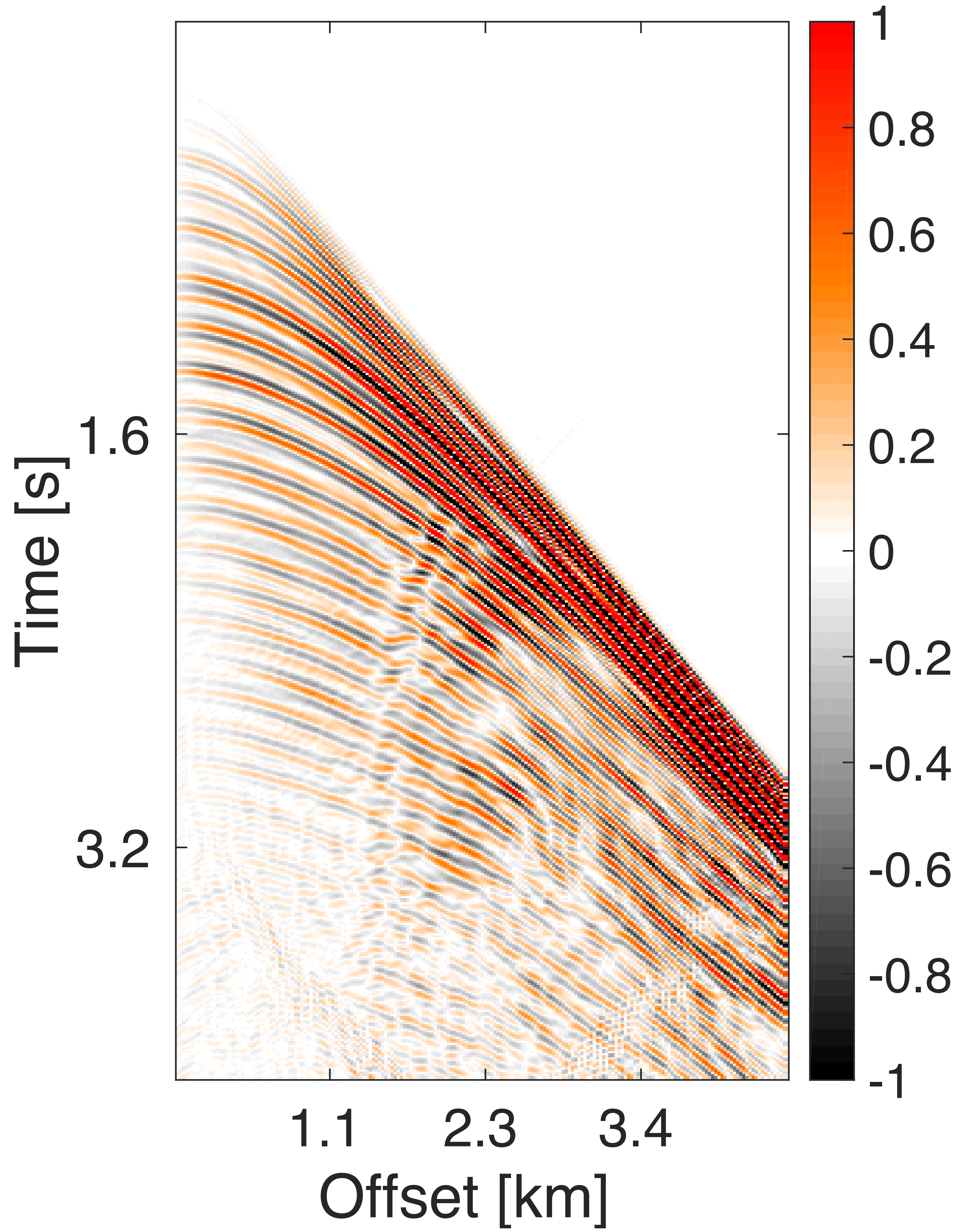
Imaging challenges:

- gaps in the survey lead to insufficient illumination to image the shallow layers

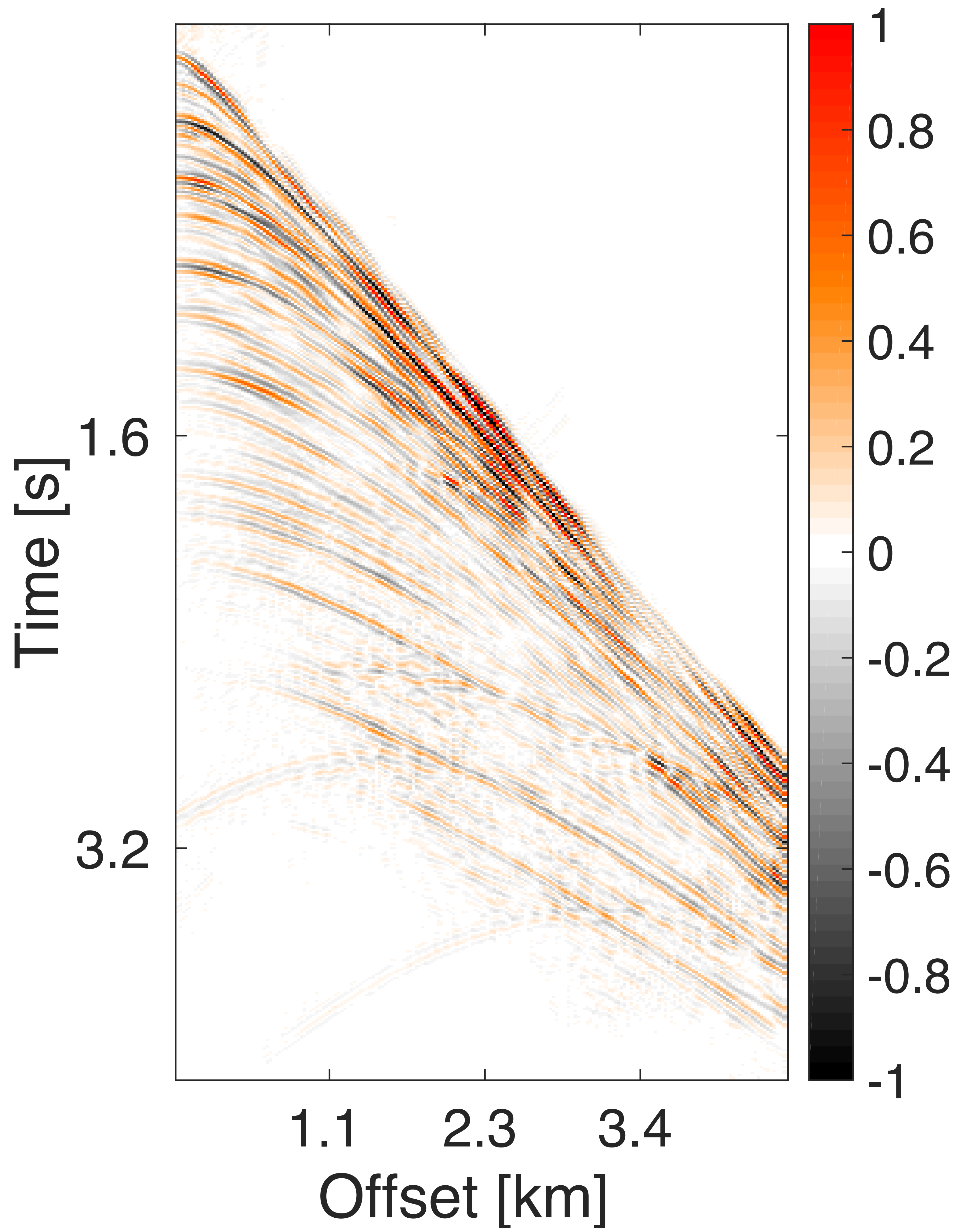
Total data



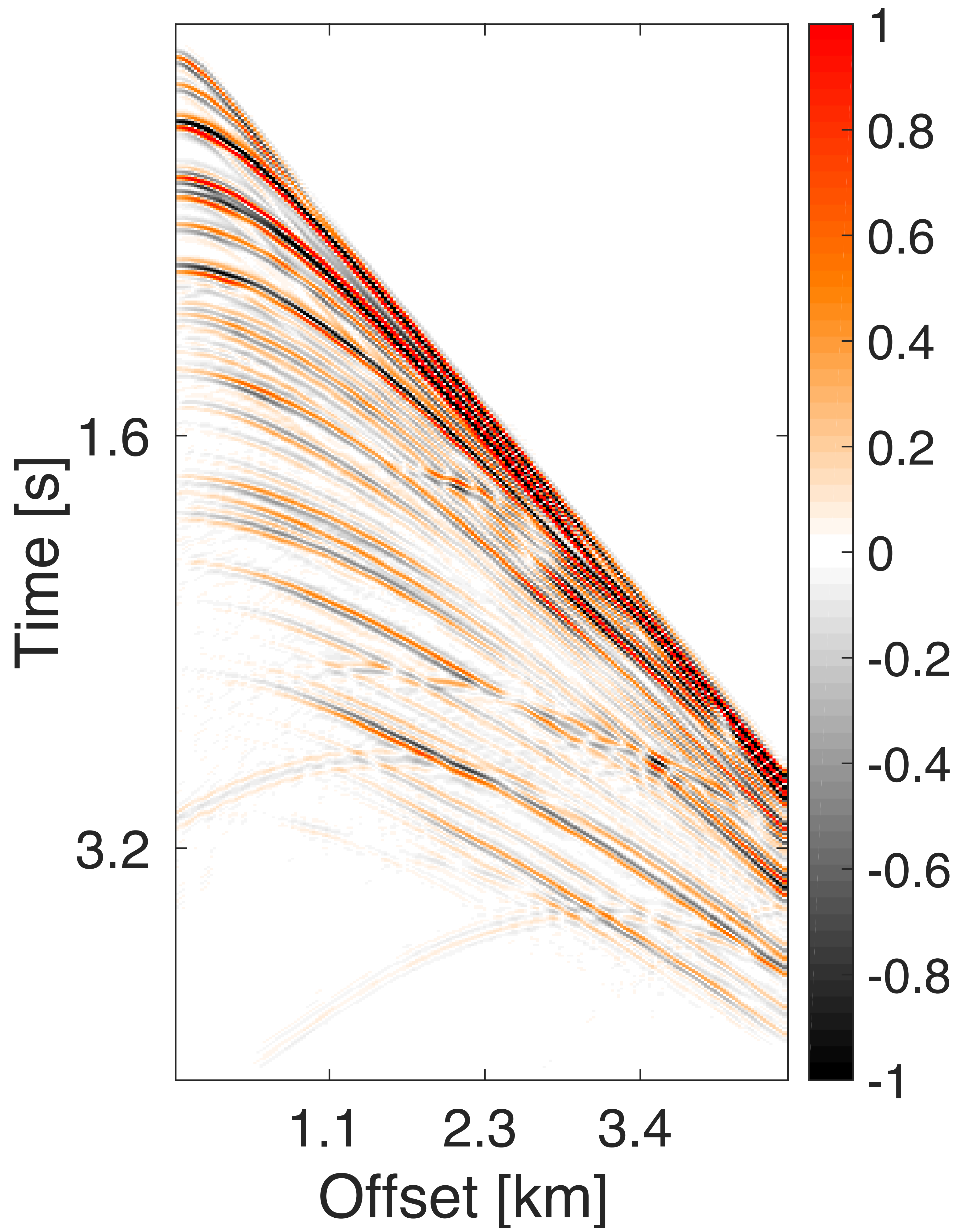
Multiples – synthesized, exact



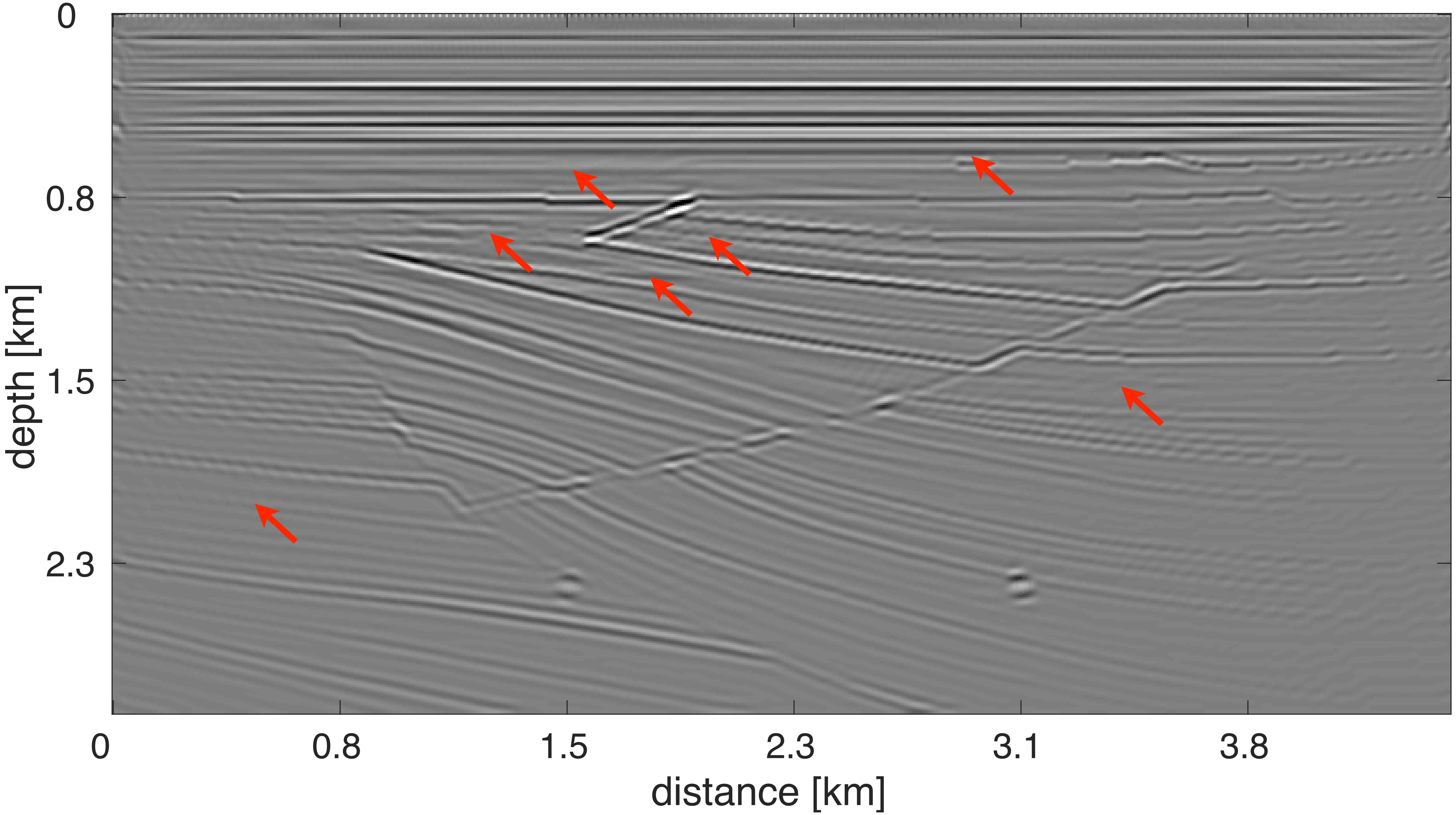
Primaries – w/ SRME-type adaptive subtraction



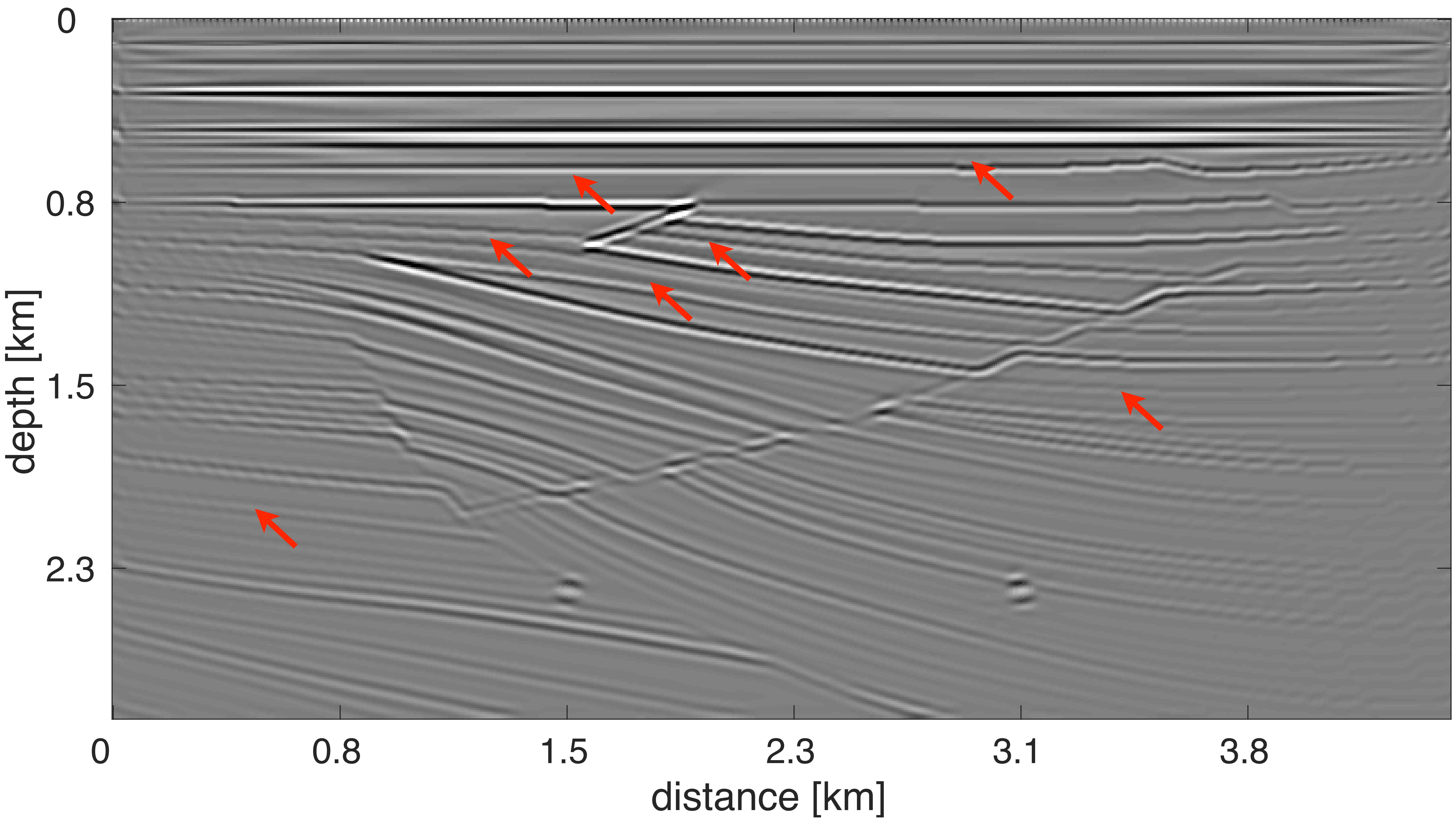
True synthesized primaries



RTM of SRME-predicted primaries
– after adaptive subtraction of multiples from total data & Laplacian filtering



RTM of true primaries
– after Laplacian filtering



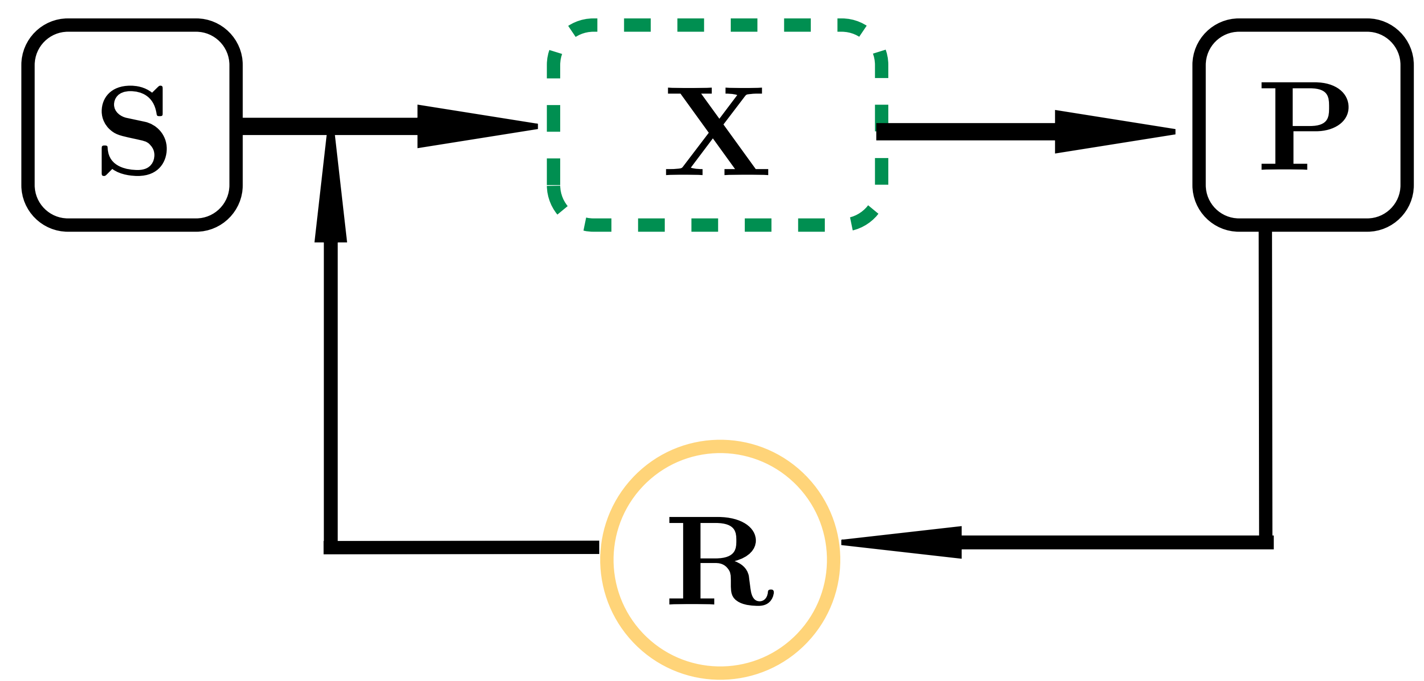
Solution

Map surface-related multiples directly into the image

- can be derived from the SRME relation
- can be carried out computationally efficiently
- exploits extra illumination from multiples
- can be easily implemented via linearized Bregman projection (LBP)

SRME relation

$$\mathbf{P}_i = \mathbf{X}_i(\mathbf{S}_i + \mathbf{R}_i\mathbf{P}_i)$$



$\mathbf{P}$: total up-going wavefield

$\mathbf{X}$: surface-free dipole Green's function

$\mathbf{S}$: point-source wavefield $= \omega_i I$

$\mathbf{R}$: surface reflectivity

i : frequency index

Tu, N. & Herrmann, F. J. "Least-squares migration of full wavefield with source encoding", 74th EAGE Conference & Exhibition incorporating SPE EUROPEC 2012

Tu, N. & Herrmann, F. J. , "Fast imaging with surface-related multiples by sparse inversion", Geophysical journal International, vol. 201, p. 304-317,2015

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Eliminating dense matrix-matrix products

SRME relation & wave-equation solver

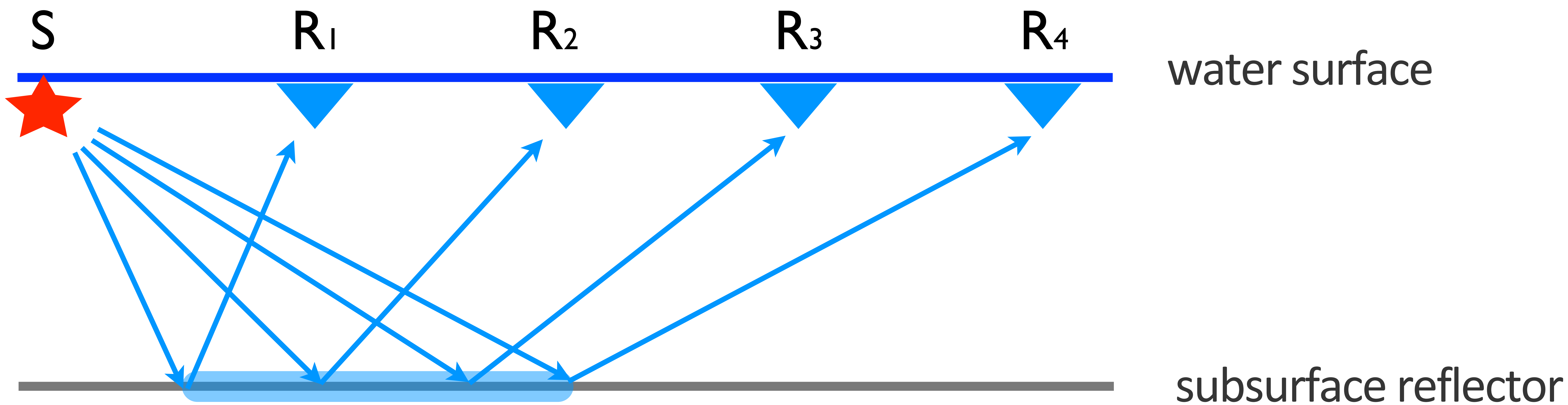
Linearized modelling Combined with free-surface physics:

$$
\begin{aligned}
\mathbf{P}_i &\approx \nabla \mathbf{F}_i[\mathbf{m_0}, \delta\mathbf{m}, \mathbf{I}](\mathbf{S}_i - \mathbf{P}_i) \\
&= \nabla \mathbf{F}_i[\mathbf{m_0}, \delta\mathbf{m}](\mathbf{D}_s^*\mathbf{I})(\mathbf{S}_i - \mathbf{P}_i) \\
&= \nabla \mathbf{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*(\mathbf{S}_i - \mathbf{P}_i)) \\
&= \nabla \mathbf{F}_i[\mathbf{m}_0, \mathbf{S}_i - \mathbf{P}_i]
\end{aligned}
$$

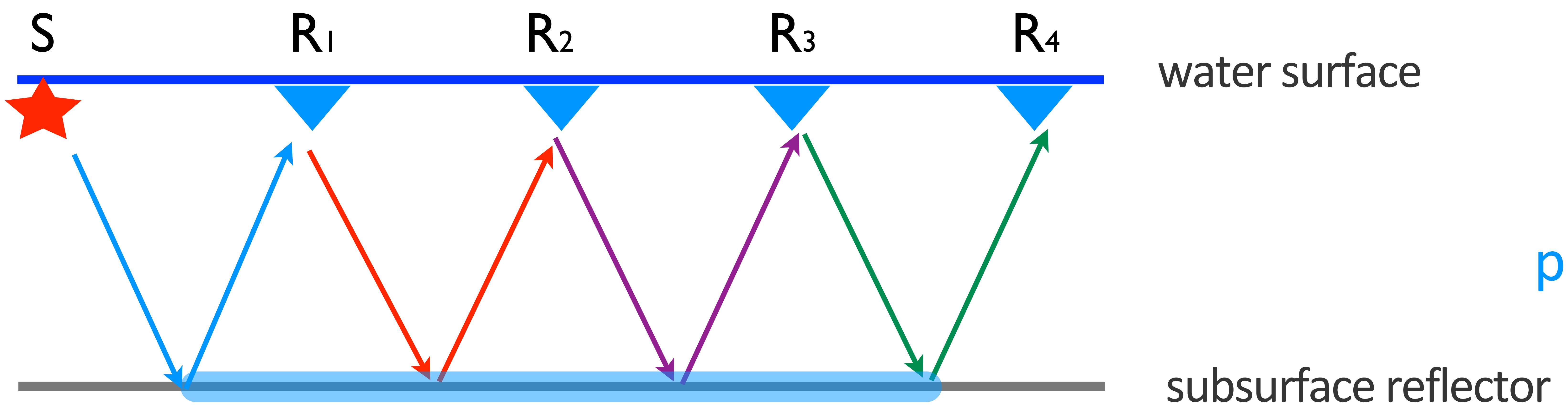
→ Dense matrix-matrix products

→ Wave-equation solves with total downgoing data injected as "areal" source

Imaging w/ multiples



Illumination by
primaries



Illumination by
primaries+***multiples***

Sparsity-promoting inversion via LBP

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \lambda \|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2 \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$

- strongly convex objective function by adding 2-norm term
- for $\lambda \to \infty$ solves BP problem

Lorenz, Dirk A.; Wenger, Stephan; A sparse Kaczmarz solver and a linearized Bregman method for online compressed sensing. arXiv:1403.7543

Felix J. Herrmann,Ning Tu and Ernie Esser, “Fast ‘online’ migration with Compressive Sensing”, EAGE Annual Conference Proceeding, 2015, vol. 60, p. 696-712, 2012

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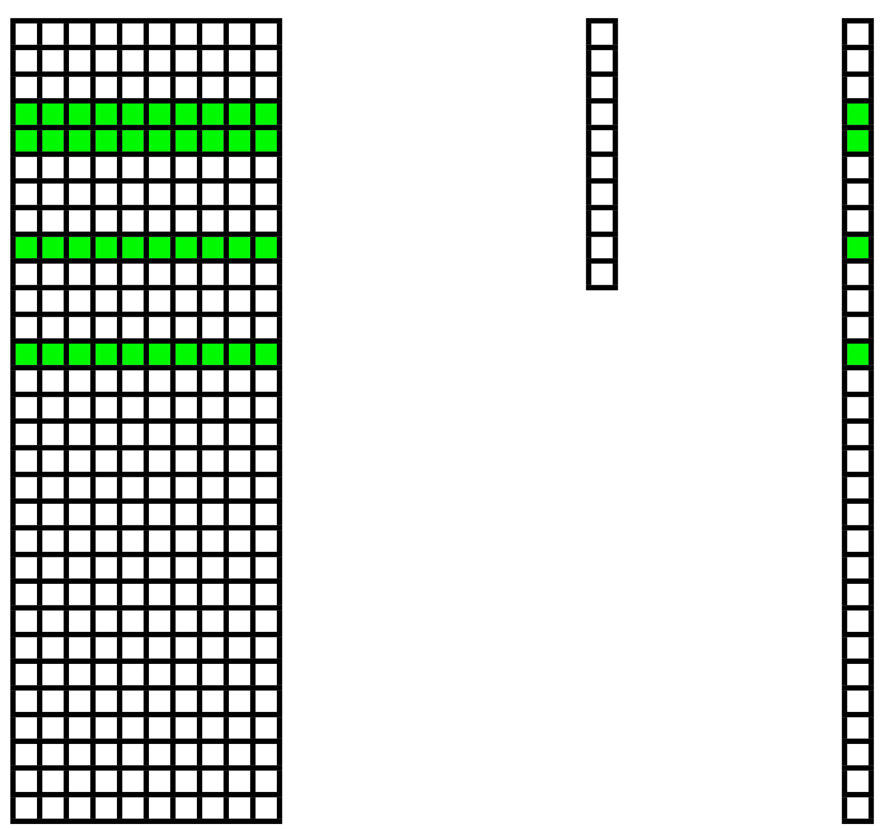
LBP via randomized subsampling

Randomized subsets of $\mathbf{A}, \mathbf{b}$ for linearized Bregman method:

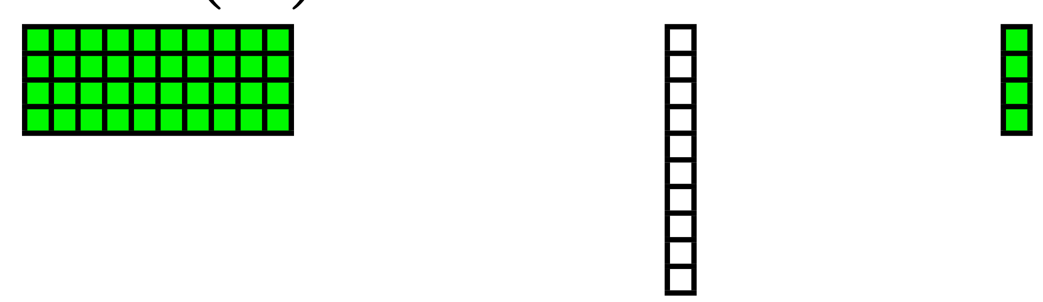
1. **for** $k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}^*_{r(k)}(\mathbf{A}_{r(k)}\mathbf{x}_k - \mathbf{b}_{r(k)})$
3. $\quad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
4. **end for**

$\mathbf{A} \quad \mathbf{x} = \mathbf{b}$



$n_s \; n_f$

$\mathbf{A}_{r(k)} \quad \mathbf{x} = \mathbf{b}_{r(k)}$



$n_s' n_f' \ll n_s n_f$

Sparse seismic imaging with multiples

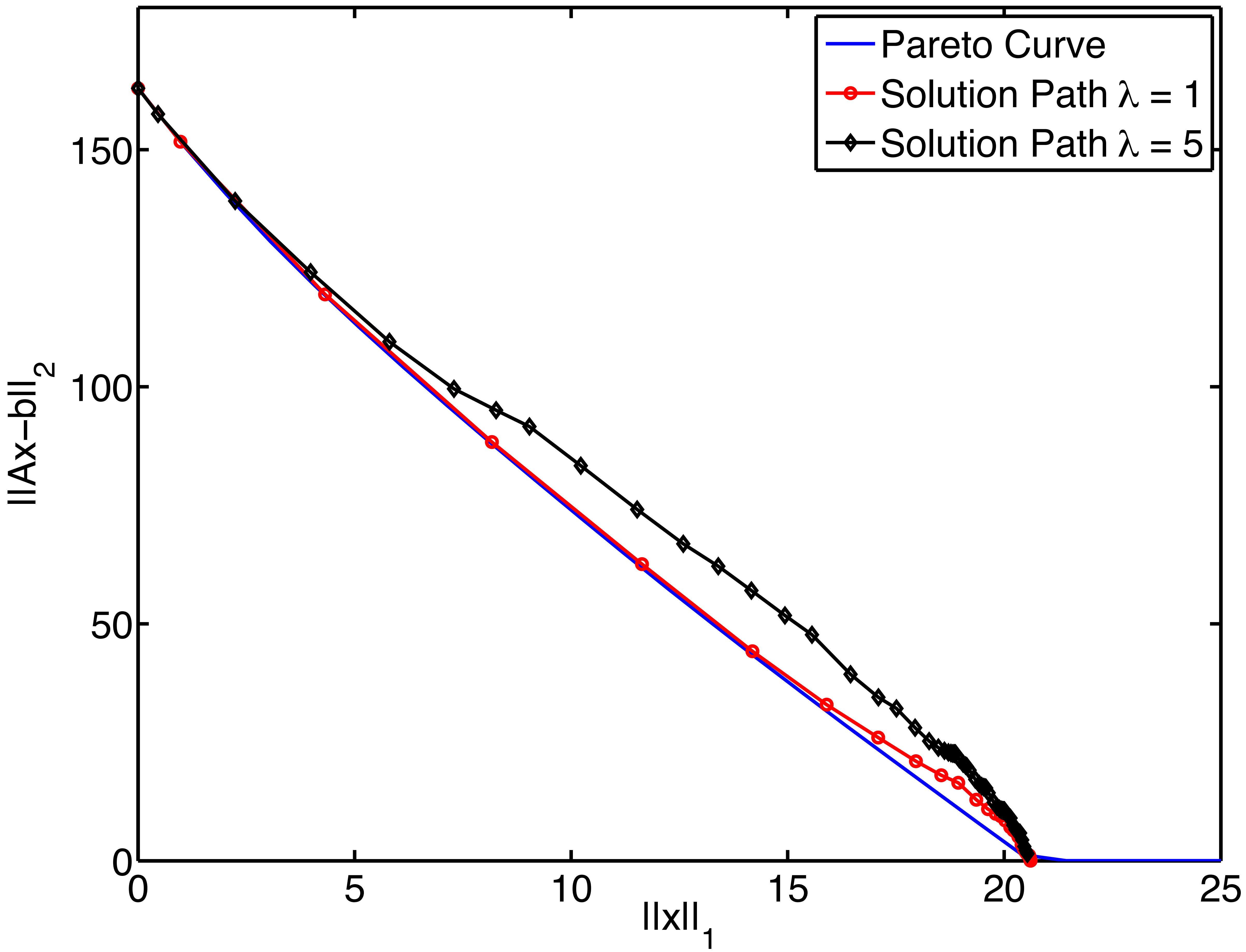
$$\underset{\mathbf{x}}{\text{minimize}} \quad \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|^2$$

$$\text{subject to} \quad \sum_{ij} \|\underline{\mathbf{p}}_{ij} - \nabla\mathbf{F}_{ij}(\mathbf{m}_0, \underline{\mathbf{S}}_{ij} - \underline{\mathbf{P}}_{ij})\mathbf{C}^*\mathbf{x}\| \leq \sigma$$

areal source

1. $\textbf{for } k = 0, 1, \cdots$
2. $\quad \Omega \in [1 \cdots n_f],\ \Sigma \in [1 \cdots n_s] \text{ for } \#\Omega \ll n_f,\ \#\Sigma \ll n_s$
3. $\quad \mathbf{A}_k = \{\nabla\mathbf{F}_{ij}(\mathbf{m}_0, \bar{\mathbf{q}}_{ij} - \underline{\mathbf{P}}_{ij})\mathbf{C}^*\}_{i\in\Omega, j\in\Sigma} \text{ with } \bar{\mathbf{q}}_{ij} = \sum_{l=1}^{n_s} w_l \mathbf{q}_{i,l}$
5. $\quad \mathbf{b}_k = \{\underline{\mathbf{p}}_{ij}\}_{i\in\Omega, j\in\Sigma}$
7. $\quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{A}_k^* \mathcal{P}_\sigma(\mathbf{A}_k\mathbf{x}_k - \mathbf{b}_k)$
8. $\quad \mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
9. $\textbf{end for}$

Solution path



Sparse seismic imaging with multiples using LBP

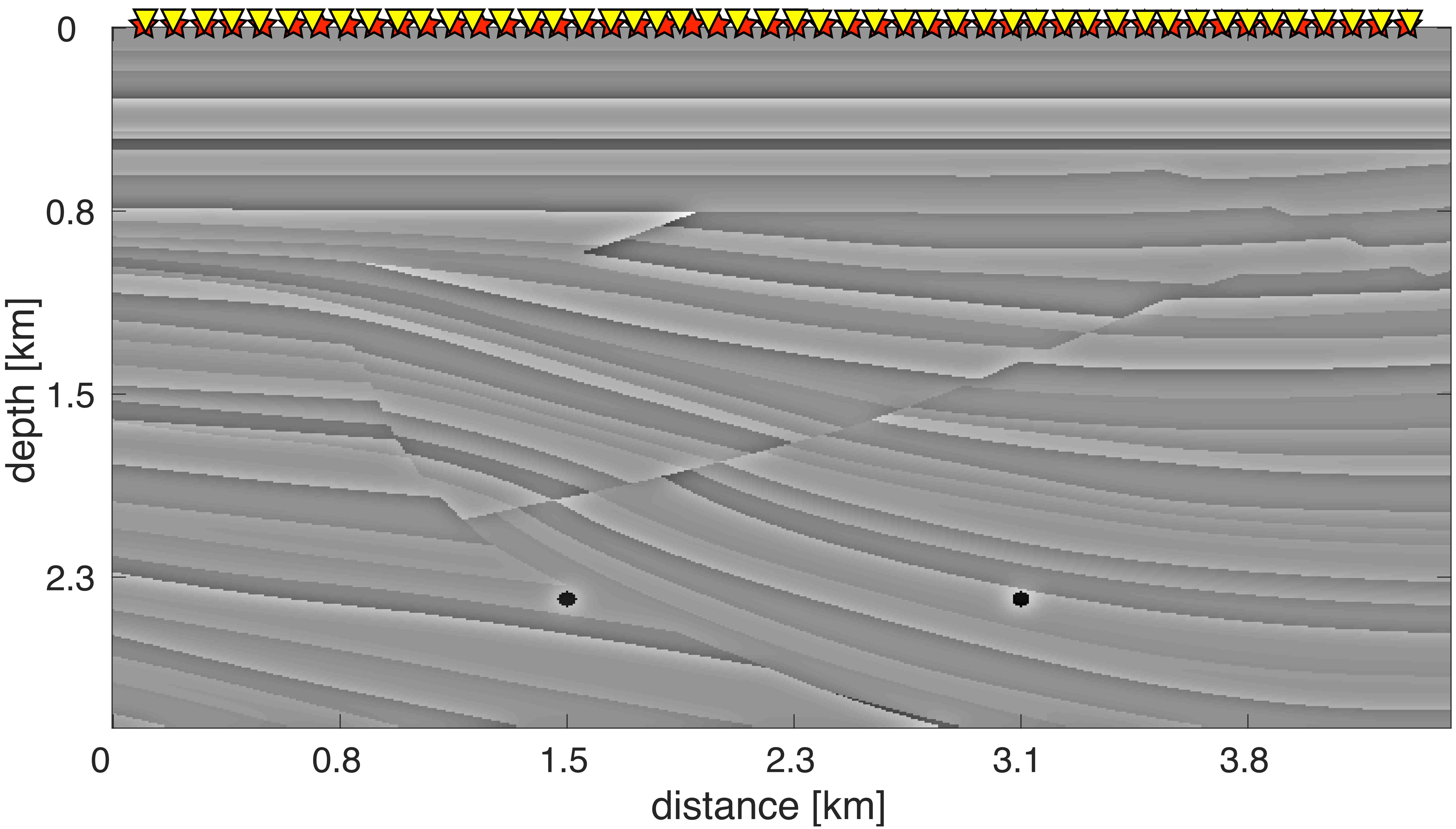
Data:

- 197 sources and receivers
- 311 frequency up to 38 Hz
- synthetic linearized data

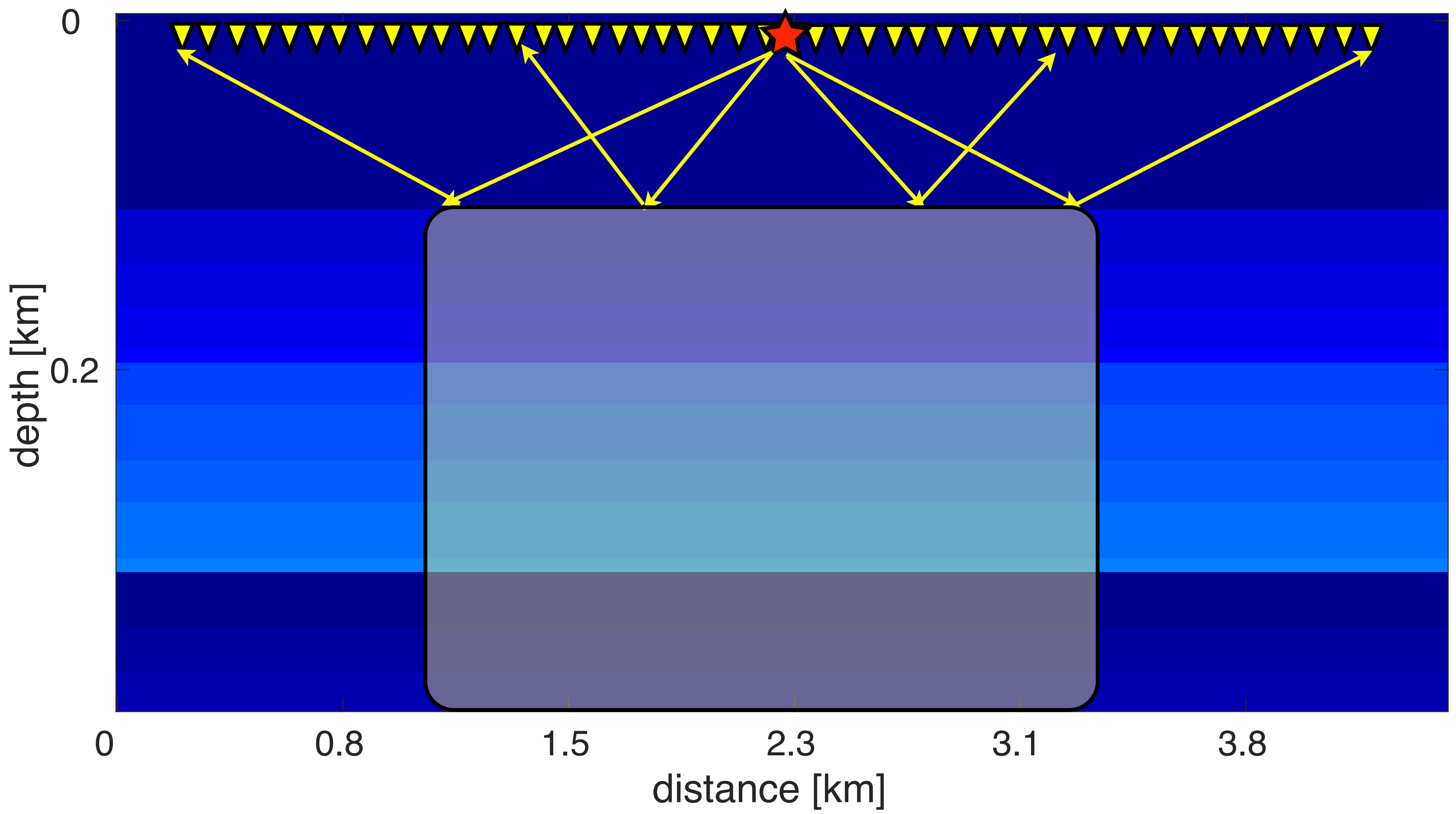
Experiments:

- one pass through the data with batch sizes 1% data
- randomized subset of shots & frequencies
- true source wavelet

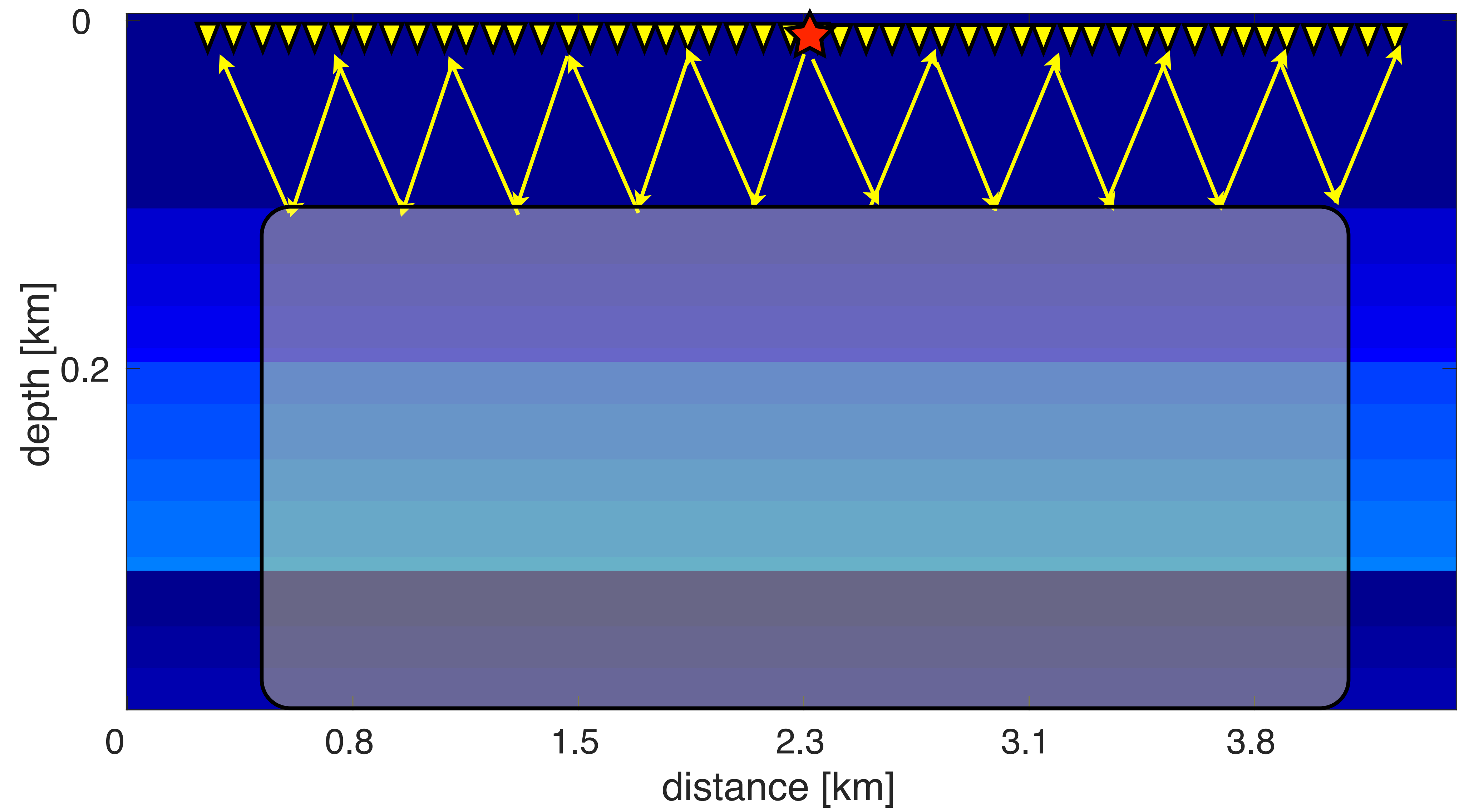
True model perturbation



Illumination comparison

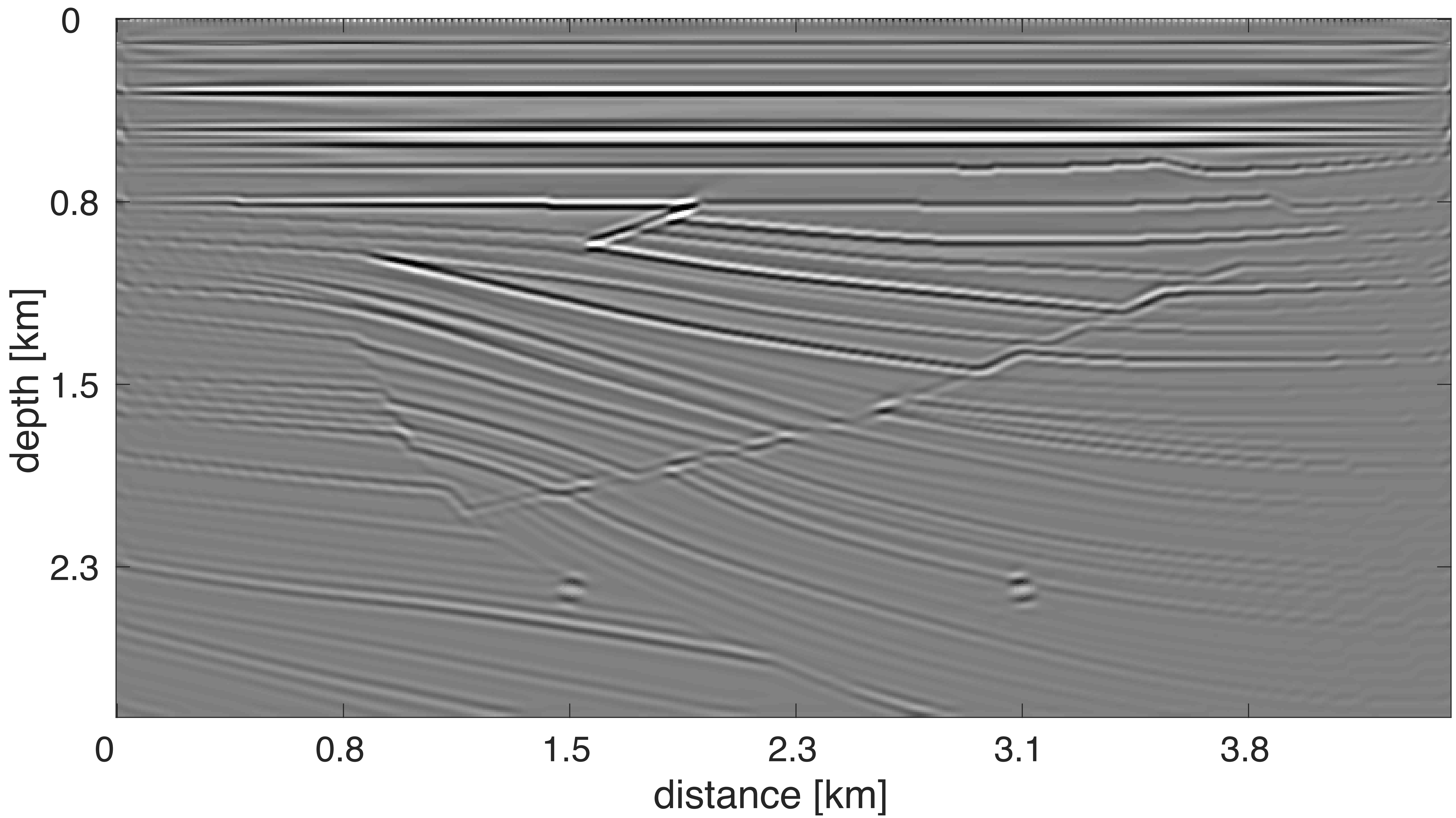


Primaries illumination area

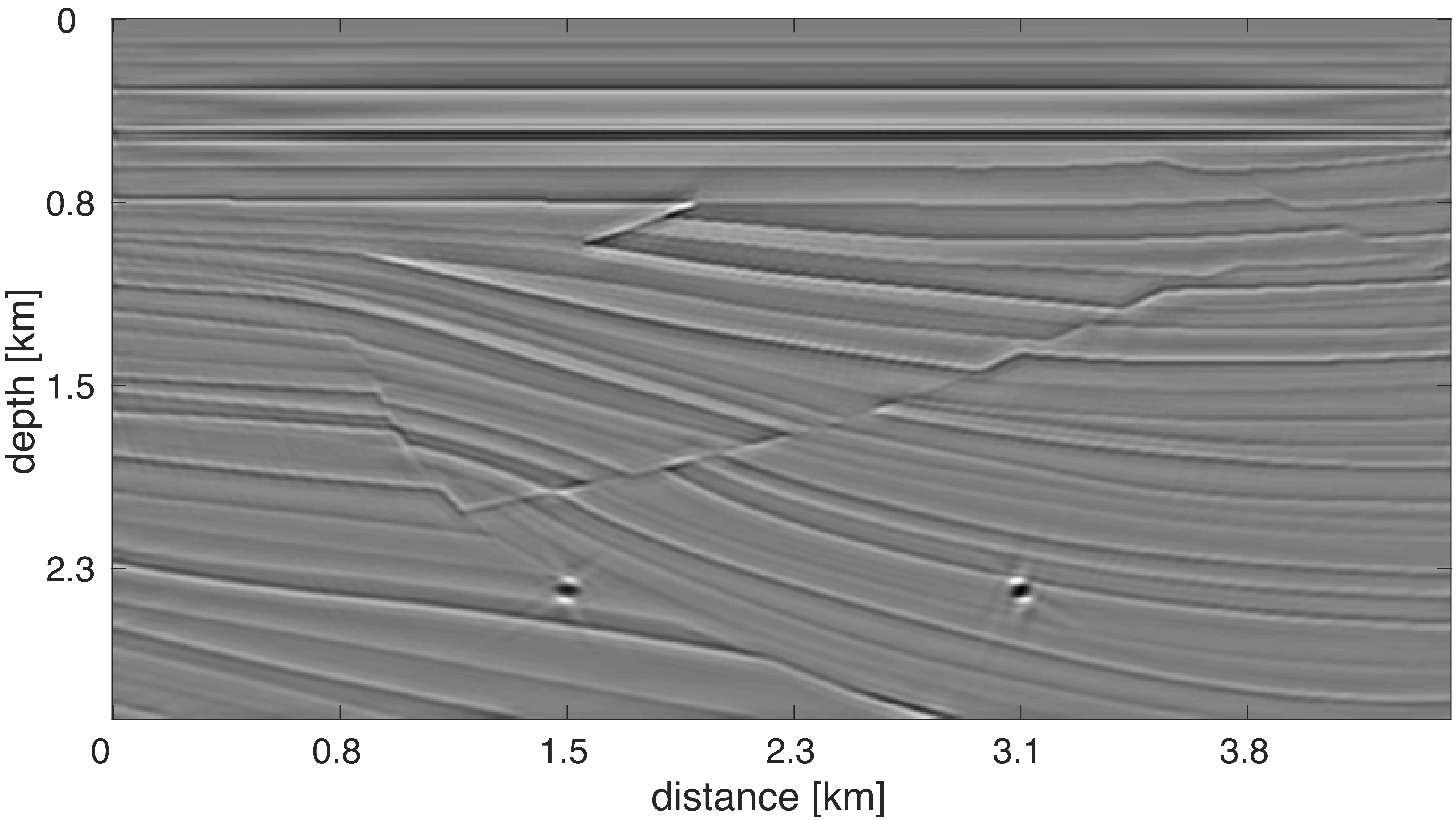


Multiples illumination area

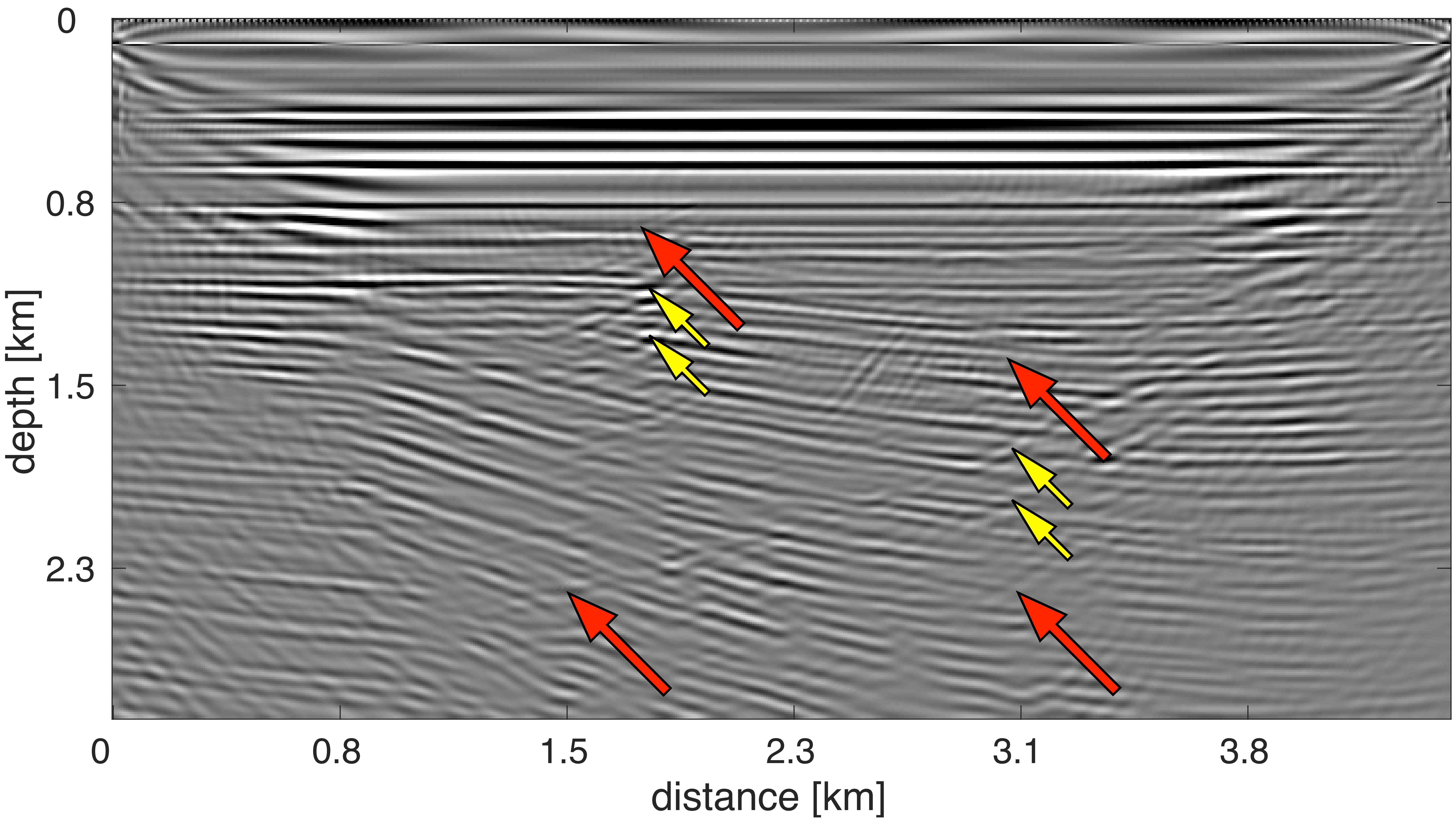
RTM of primaries
after Laplacian filtering



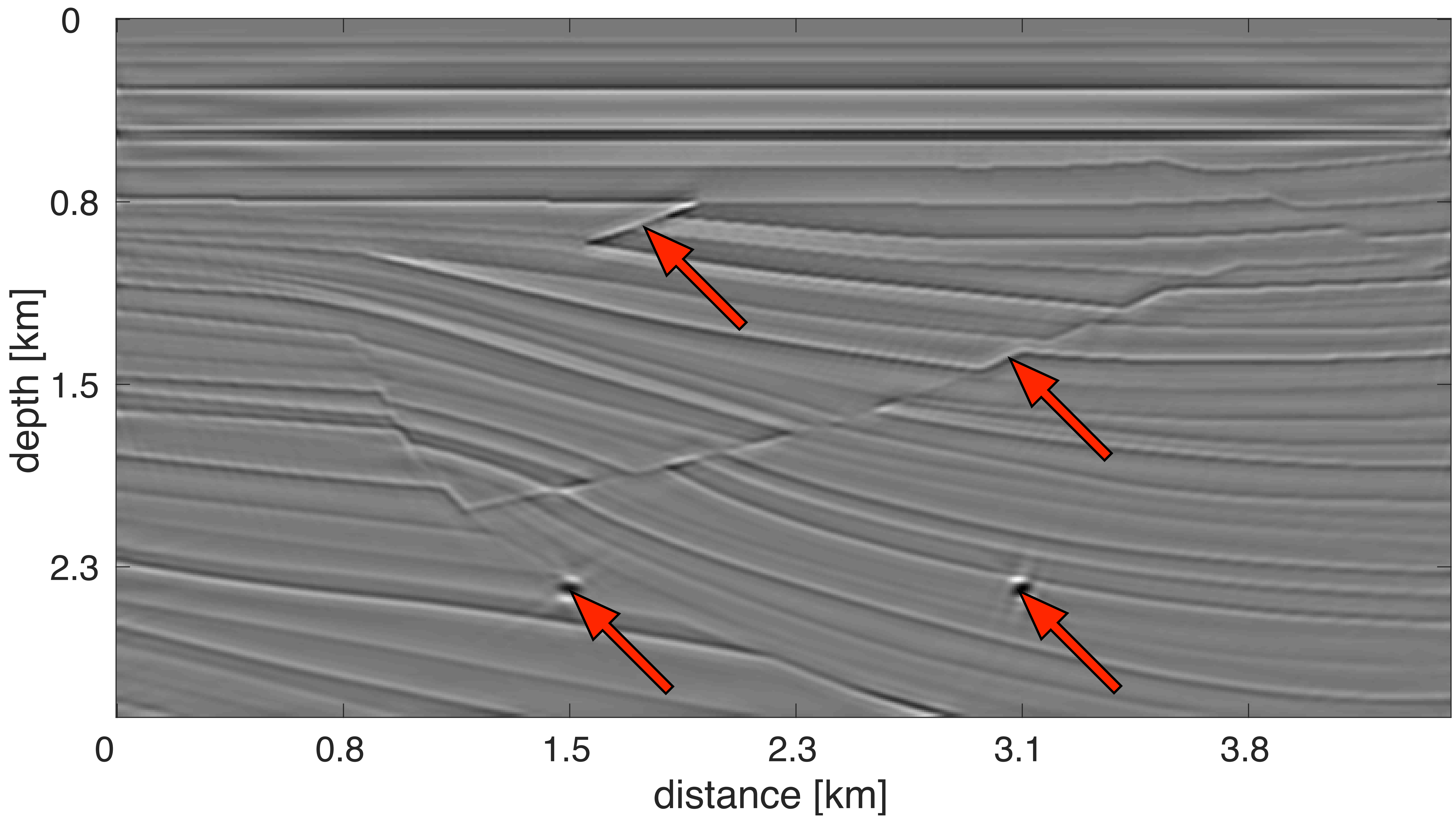
Inversion of primaries



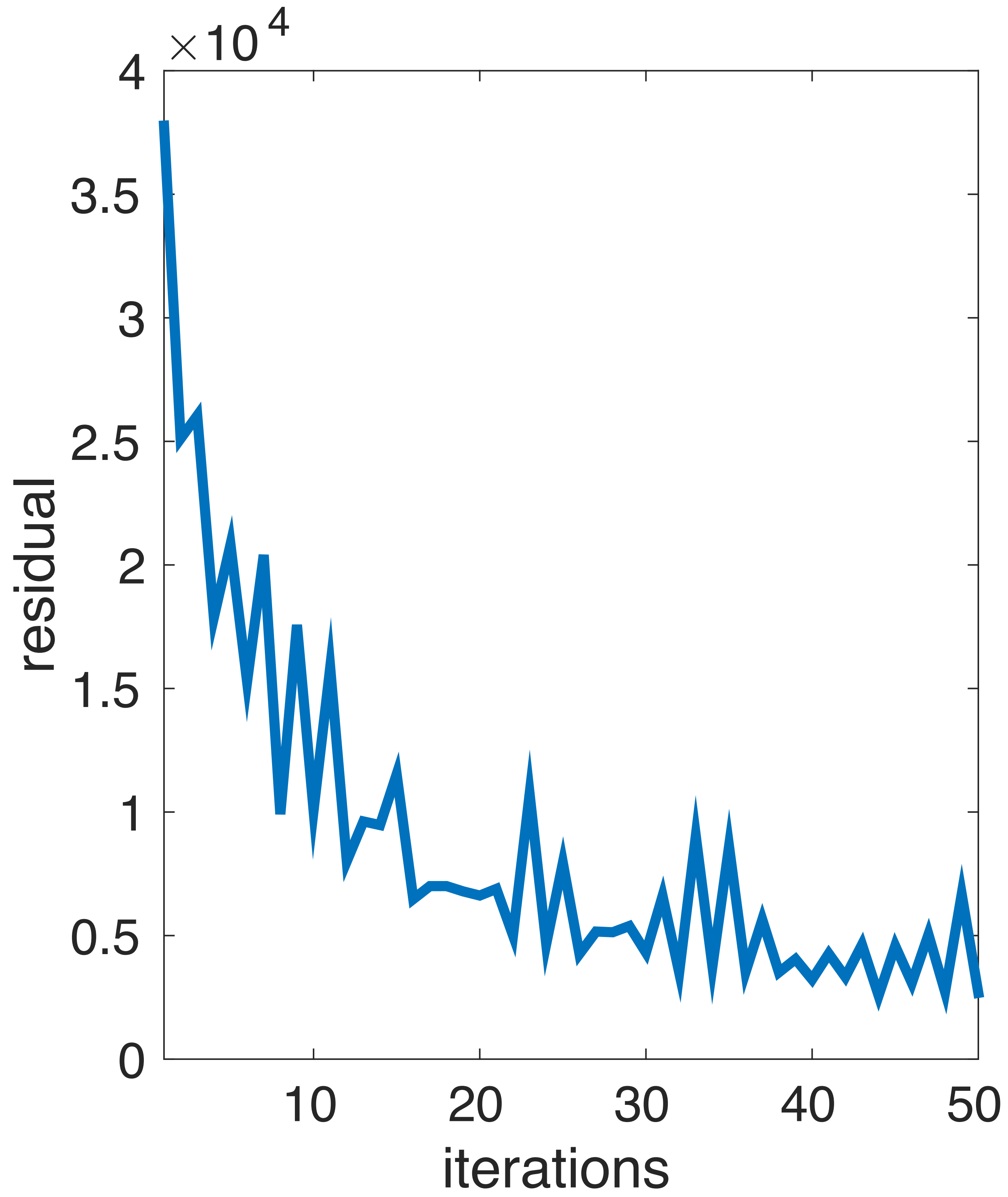
RTM of primaries + multiples
after Laplacian filtering



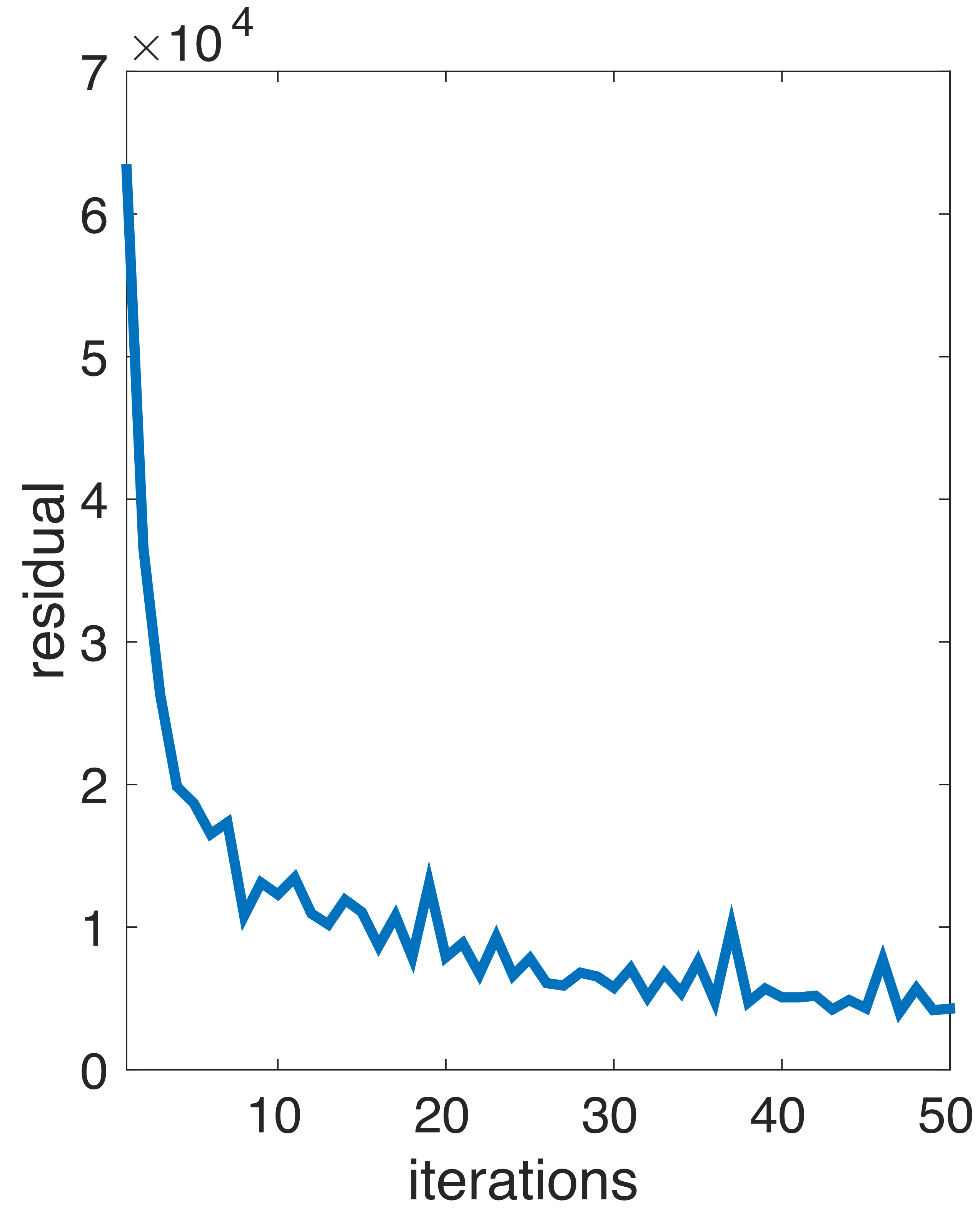
Inversion of primaries + multiples



Convergence path

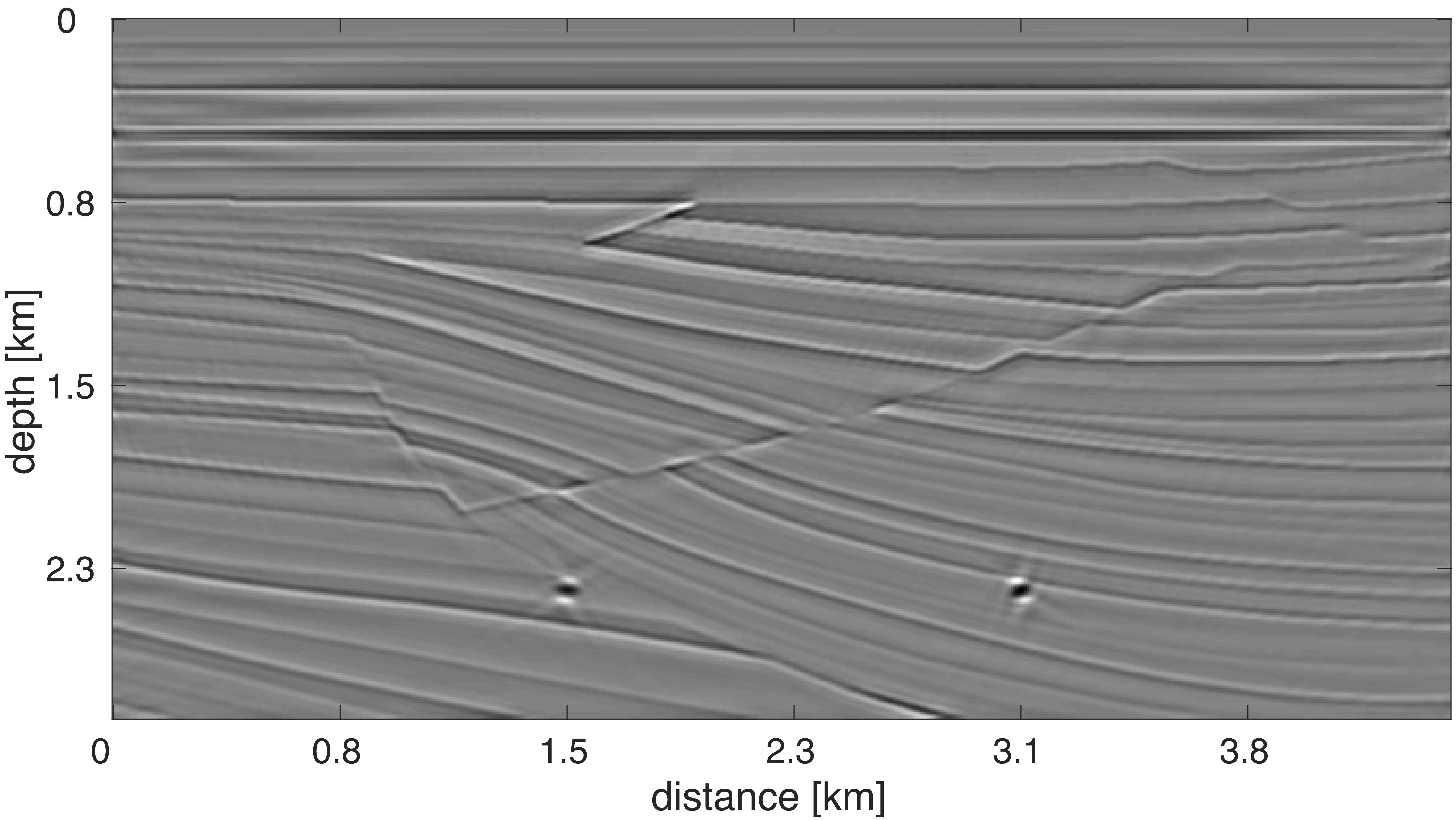


residual of Primaries inversion

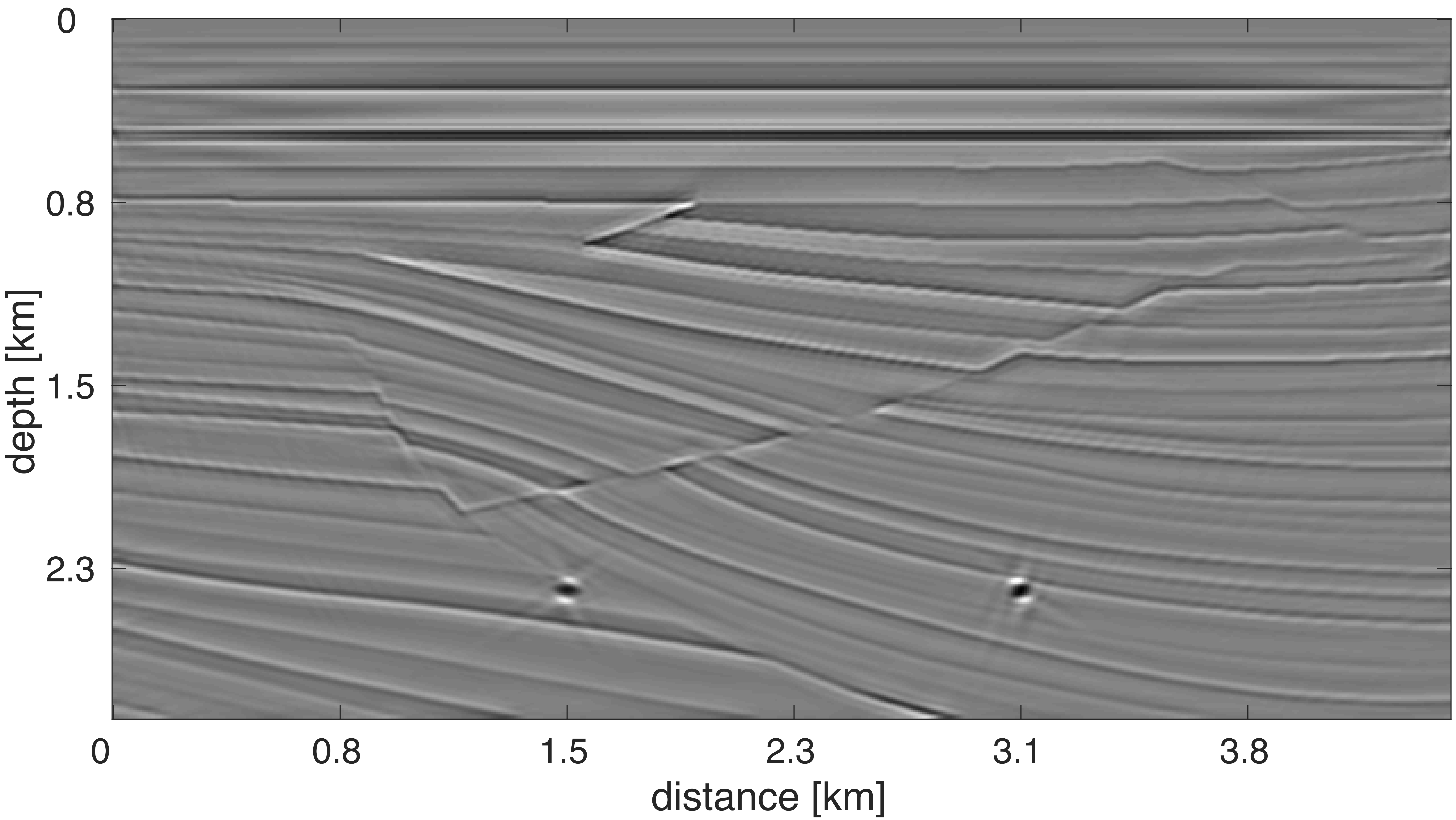


residual of Primaries+Multiples inversion

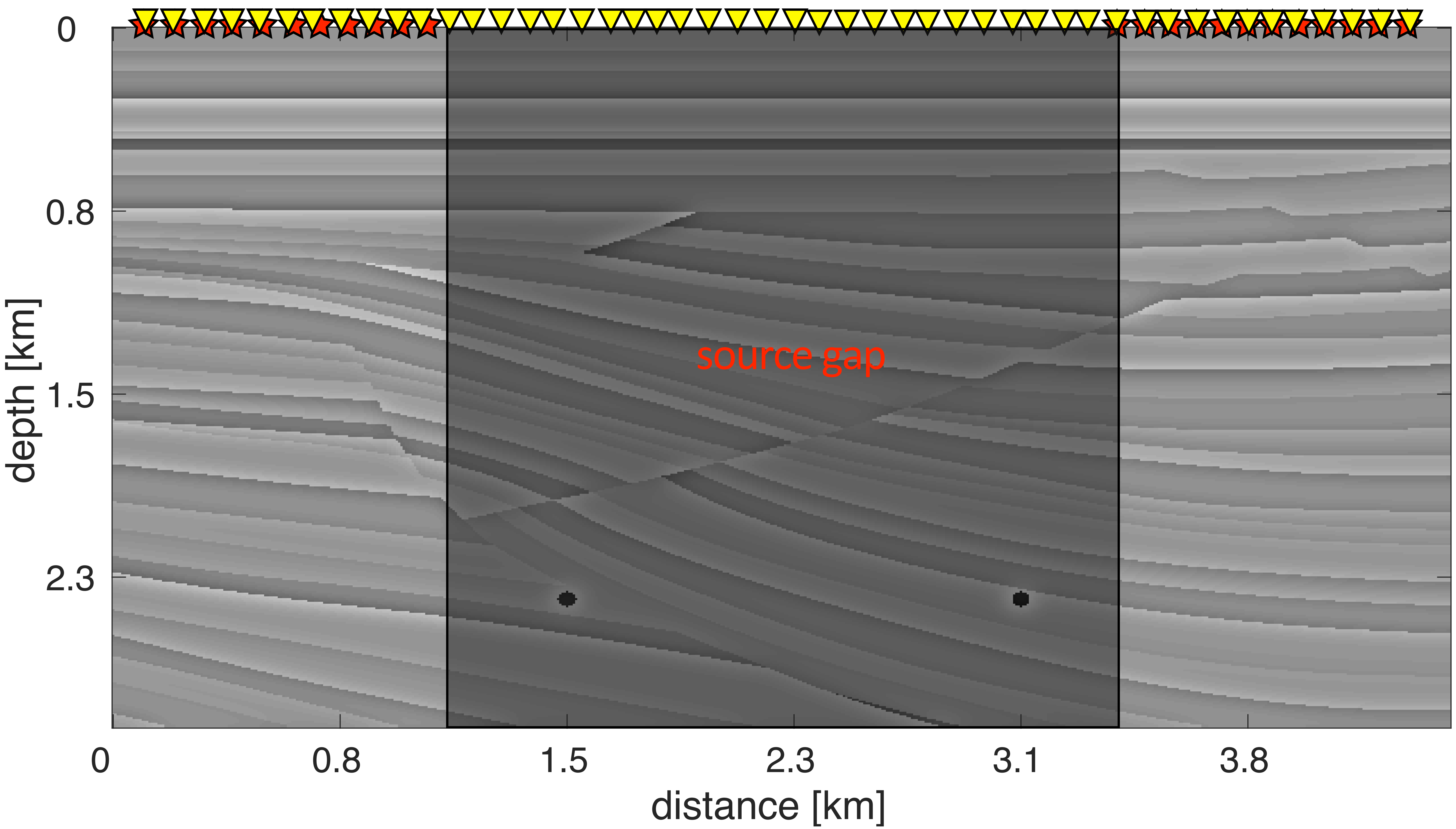
Inversion of primaries + multiples



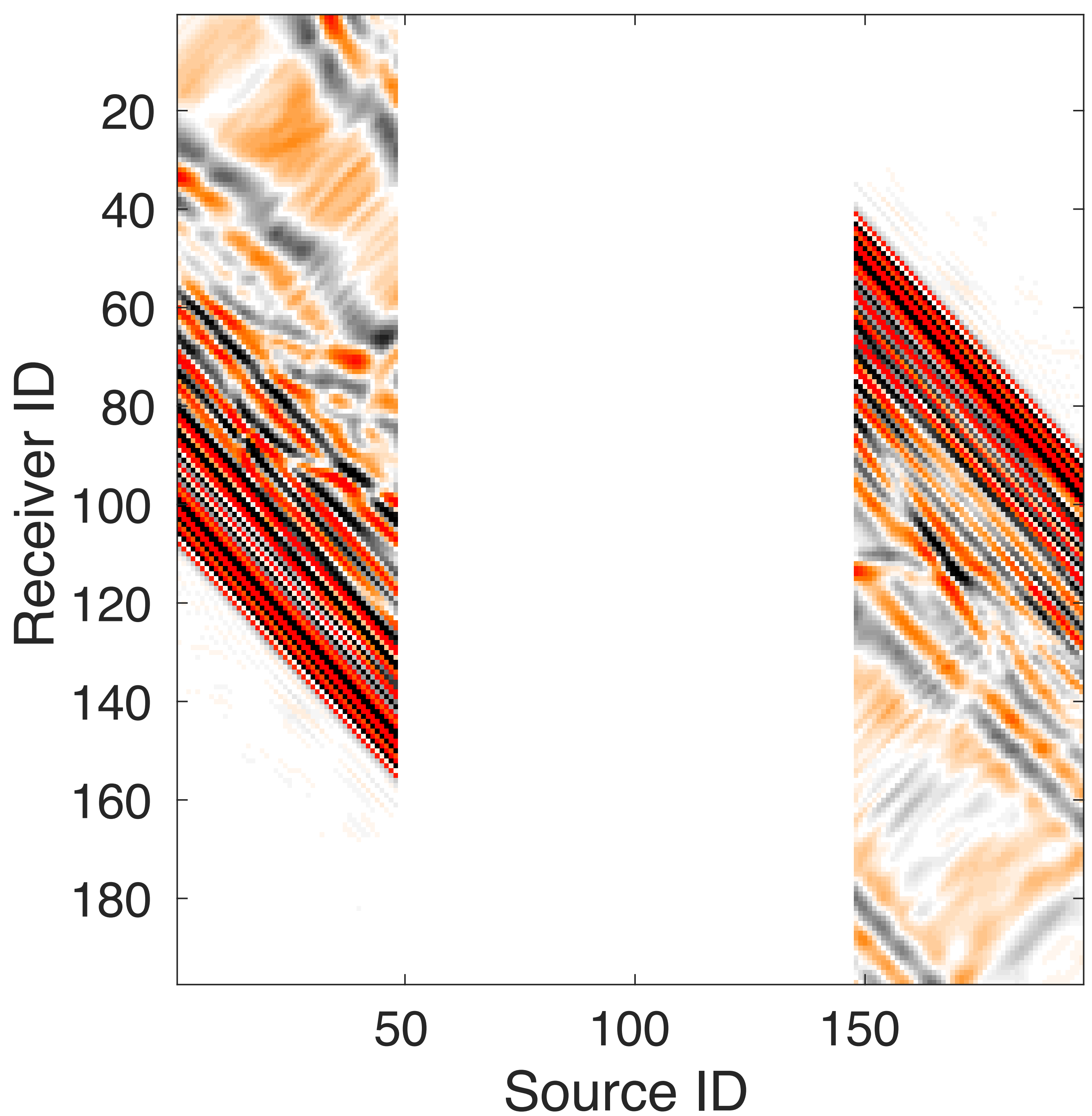
Inversion of primaries



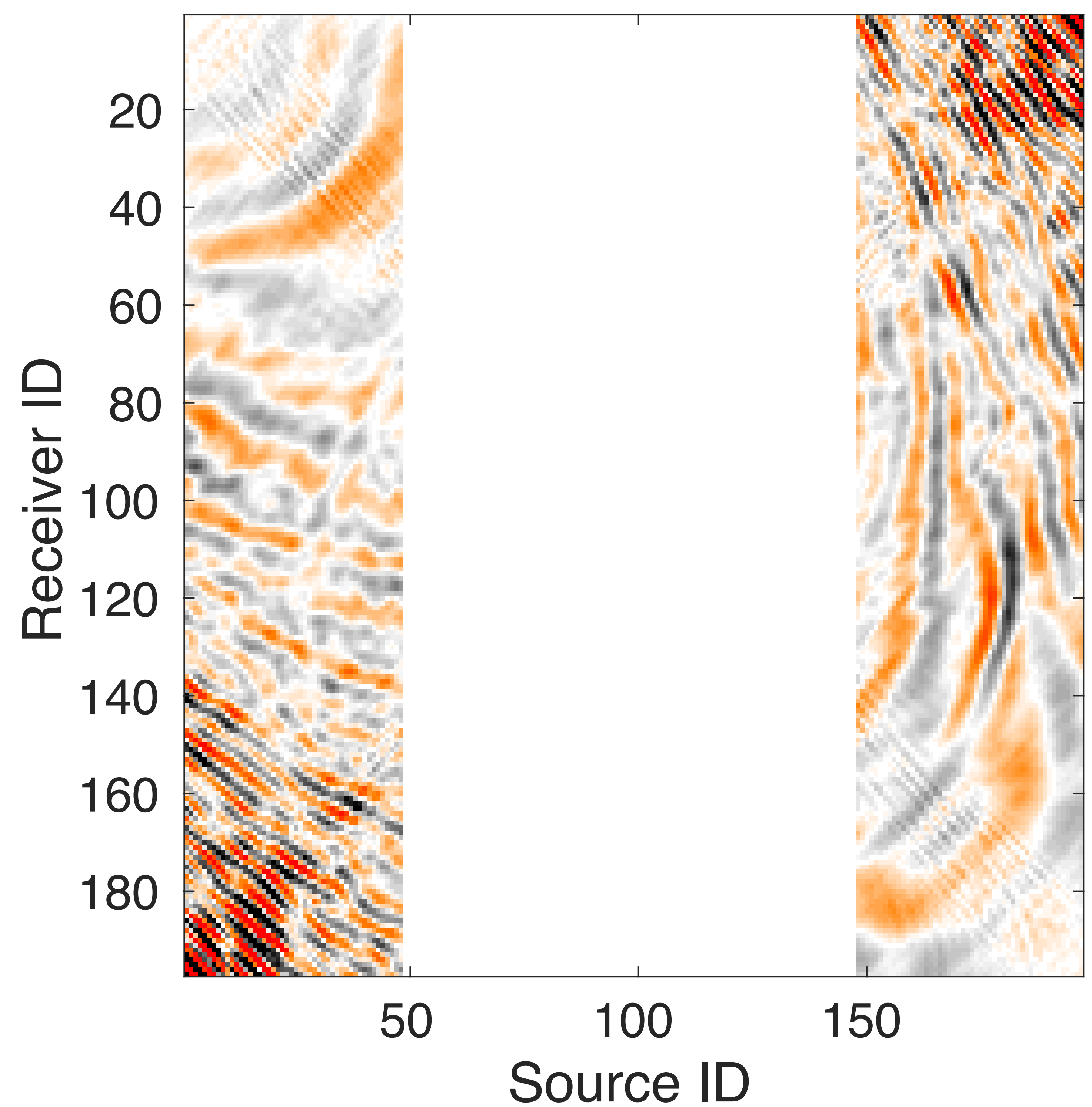
Acquisition with large source gap at the center



Acquired data: time slices



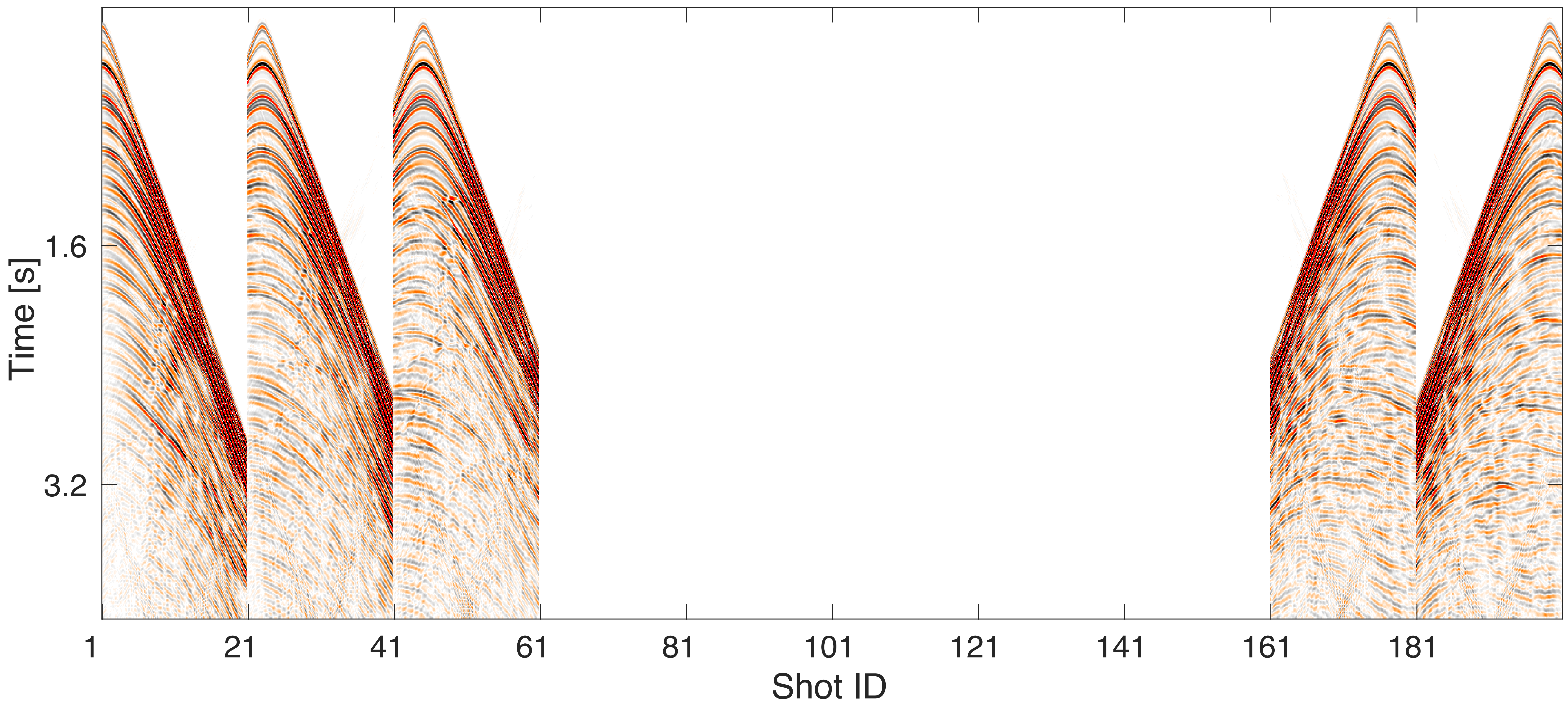
time slice at t = 1.6 s



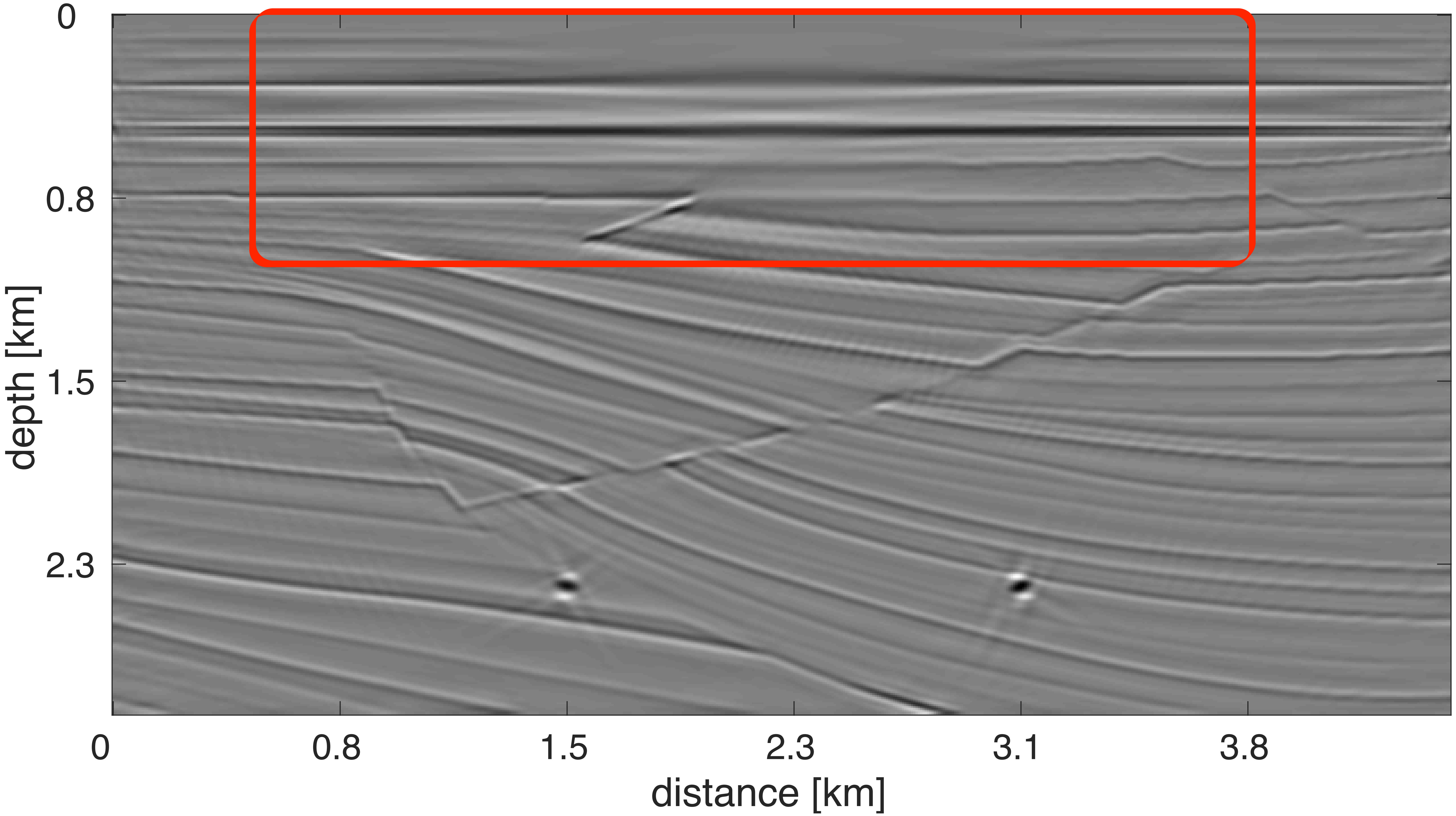
time slice at t = 3.2 s

Acquired data: shot gathers

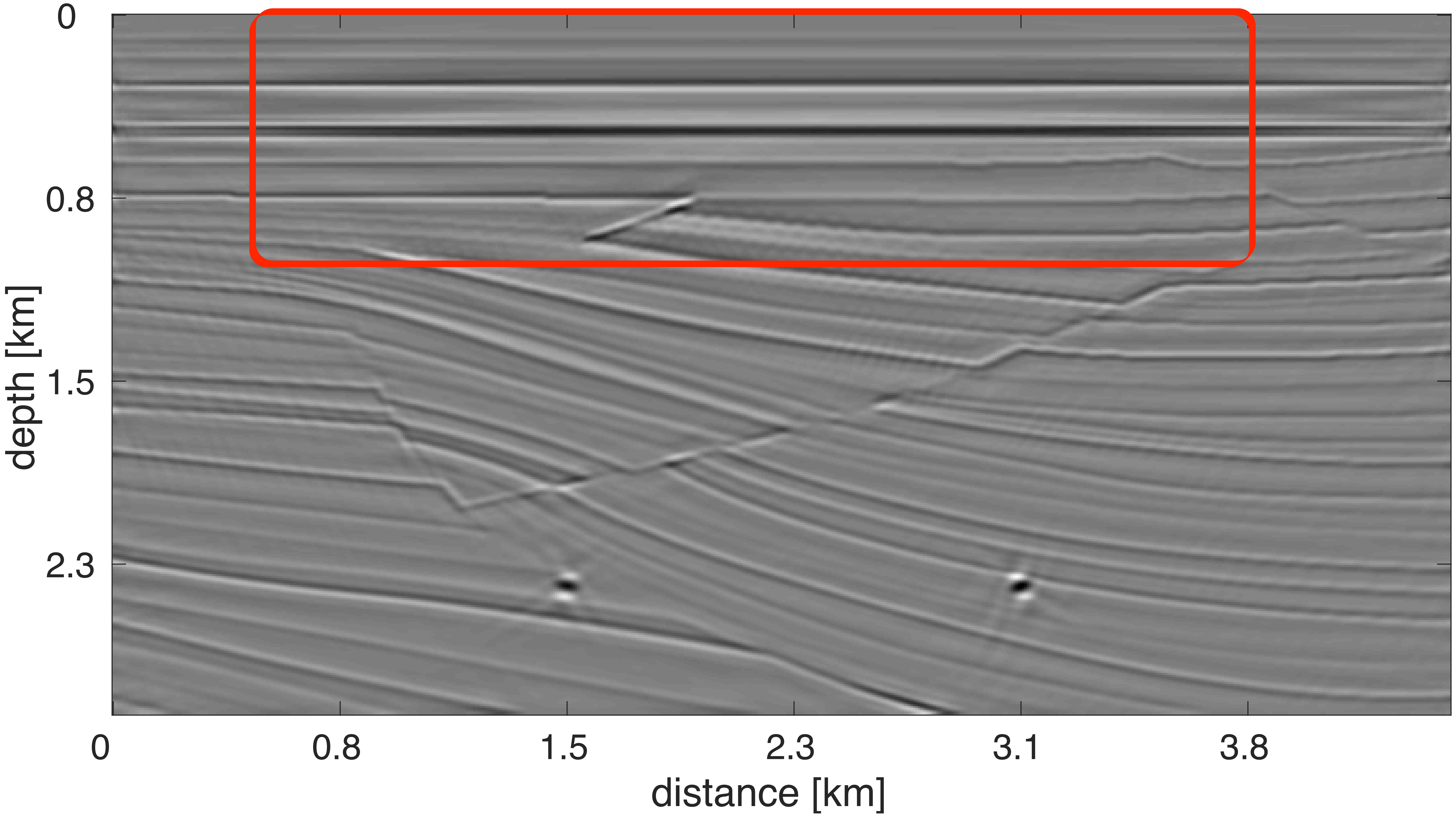
one out of 20 shot gathers are shown



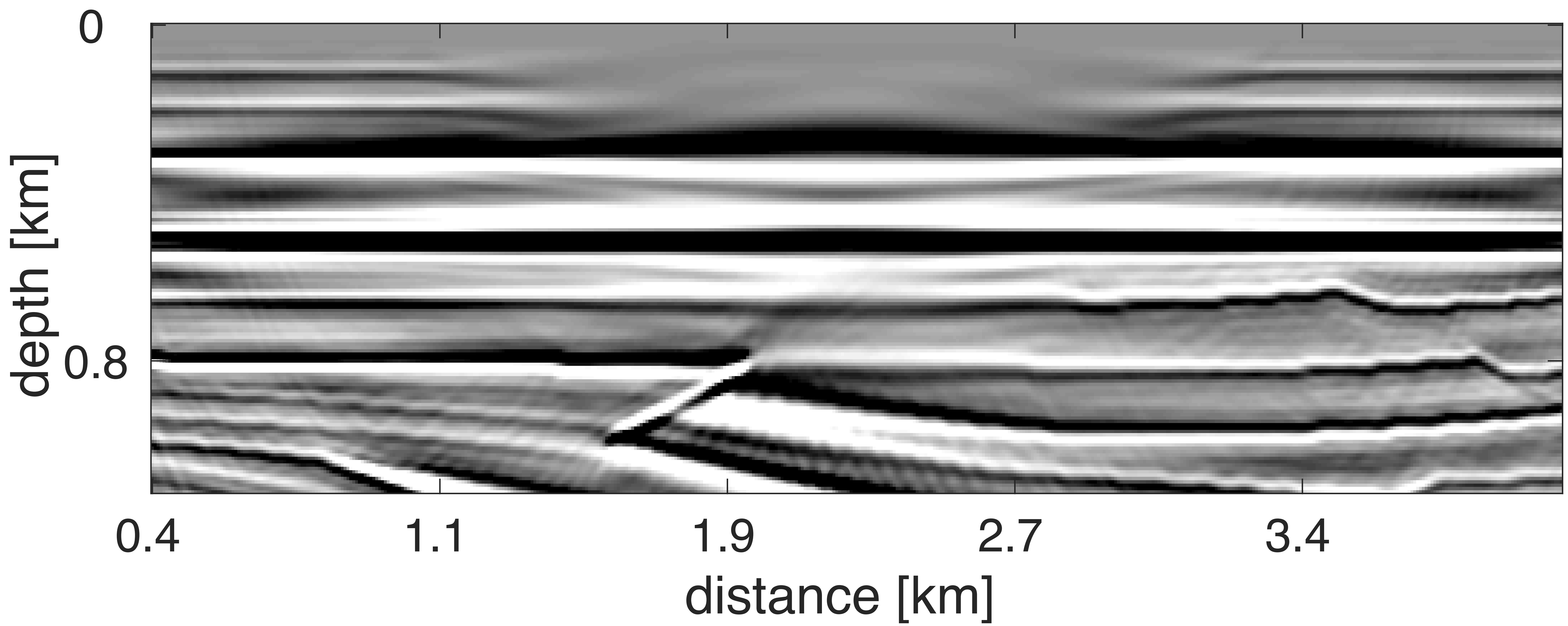
Inversion of primaries



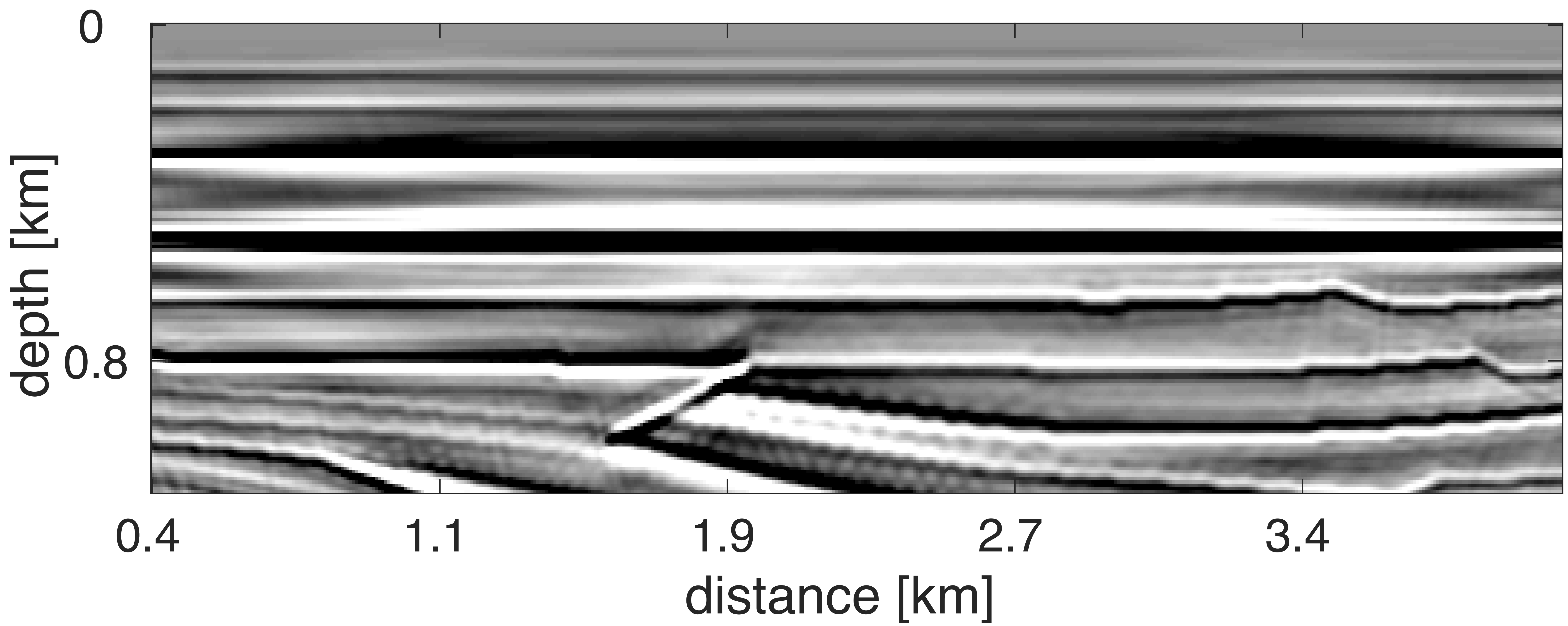
Inversion of primaries + multiples



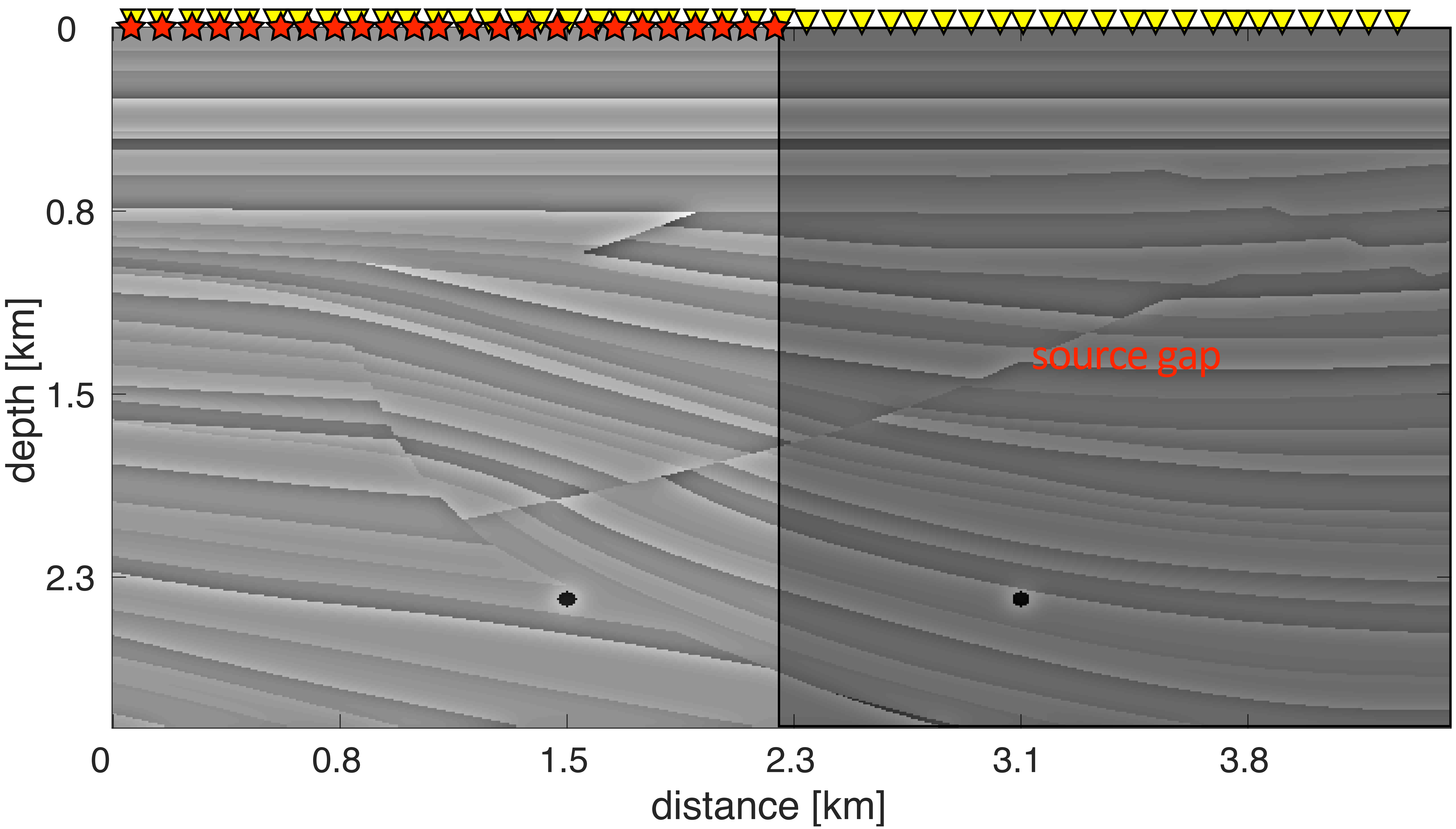
Inversion of primaries: the shallow part



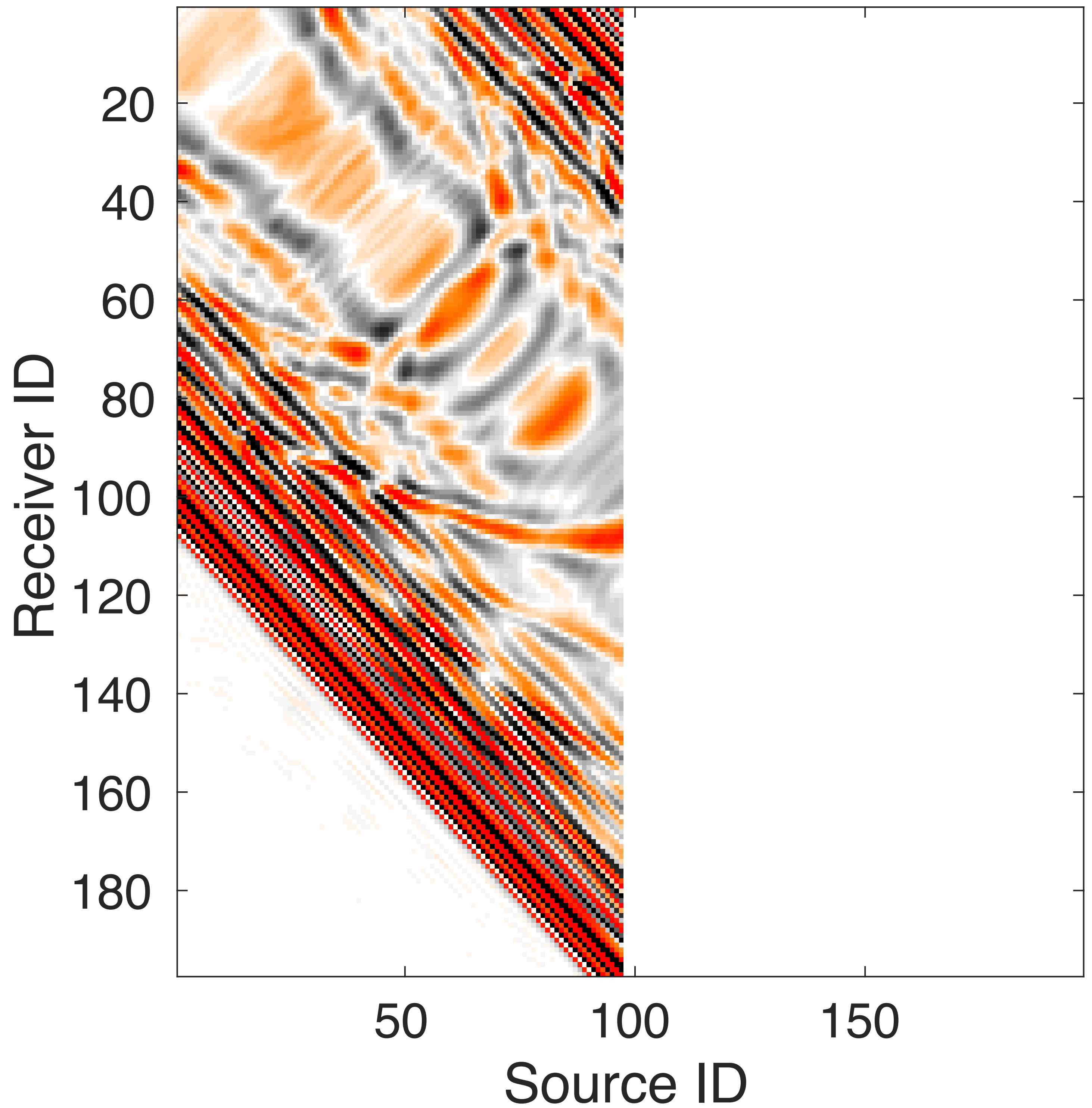
Inversion of primaries+multiples: the shallow part



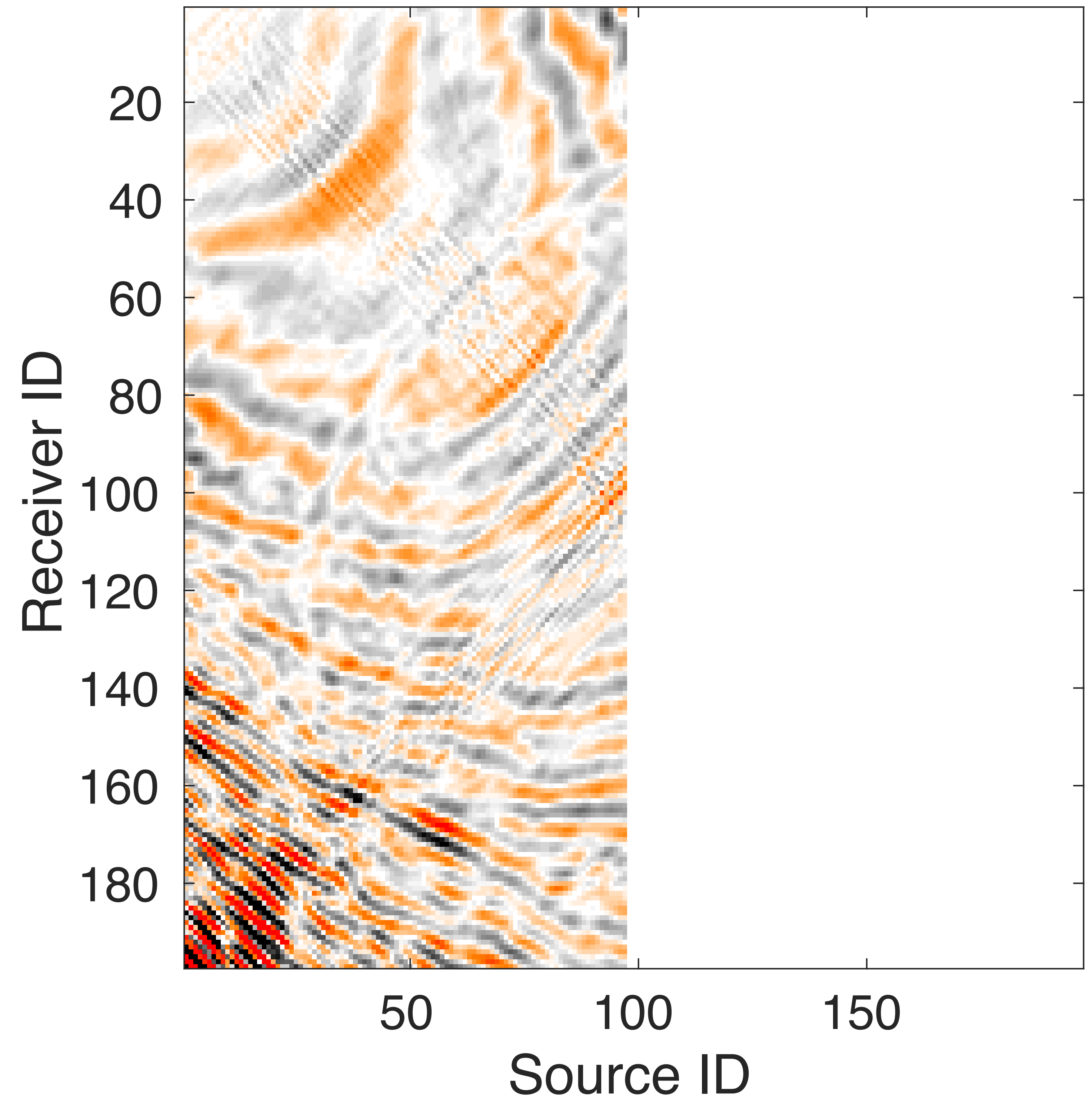
Acquisition with sources only on one side



Acquired data: time slices

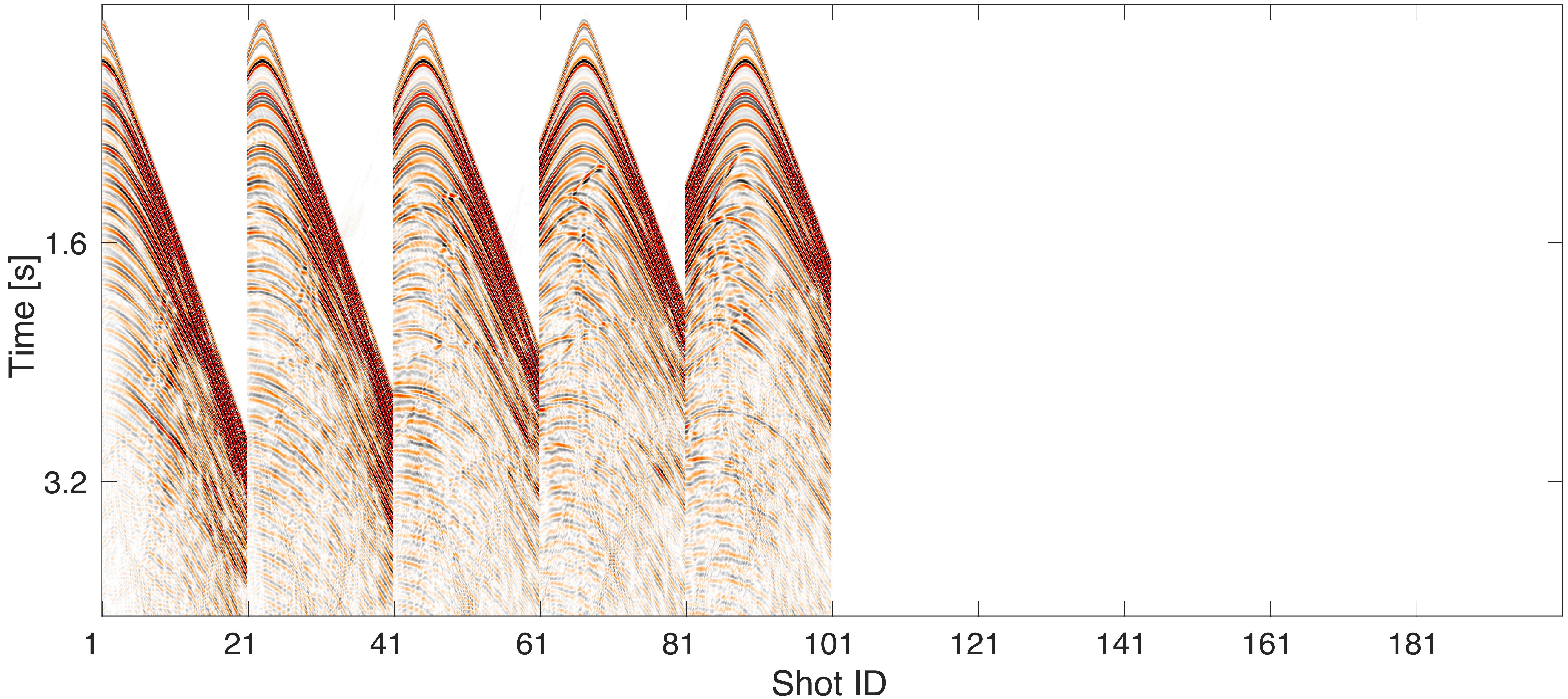


time slice at t = 1.6 s

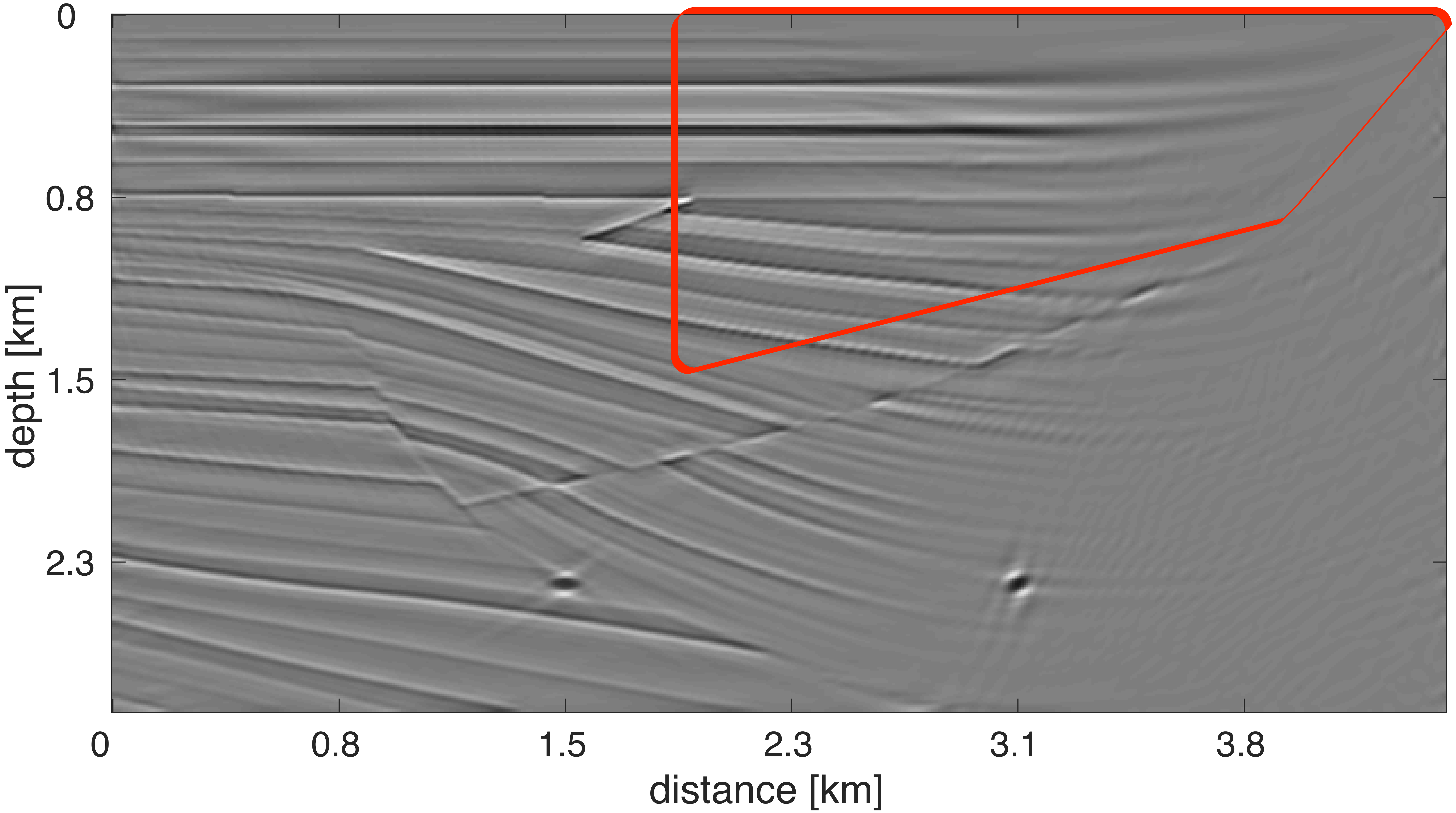


time slice at t = 3.2 s

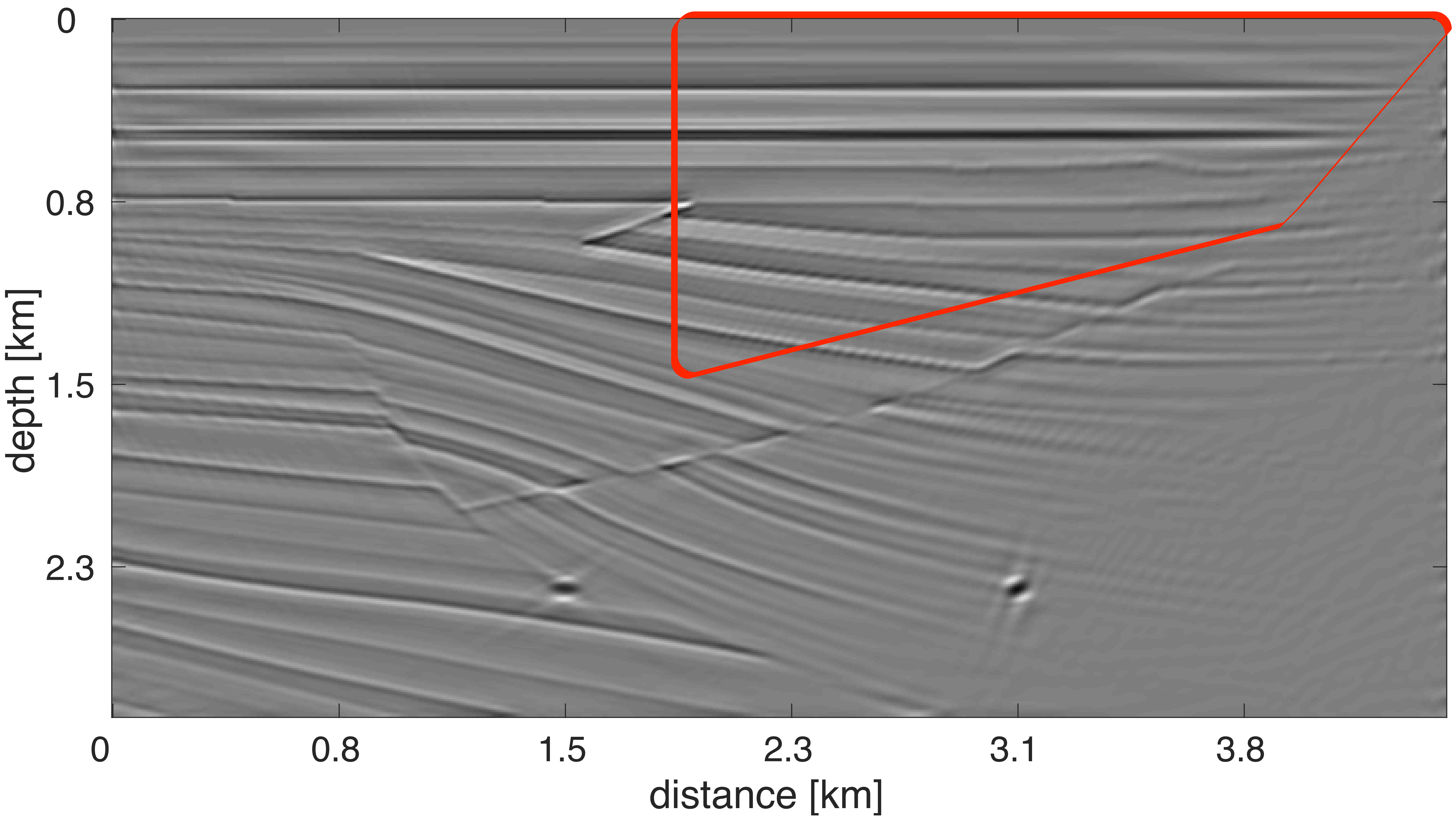
Acquired data: shot gathers
[every 20 shot gathers are shown]



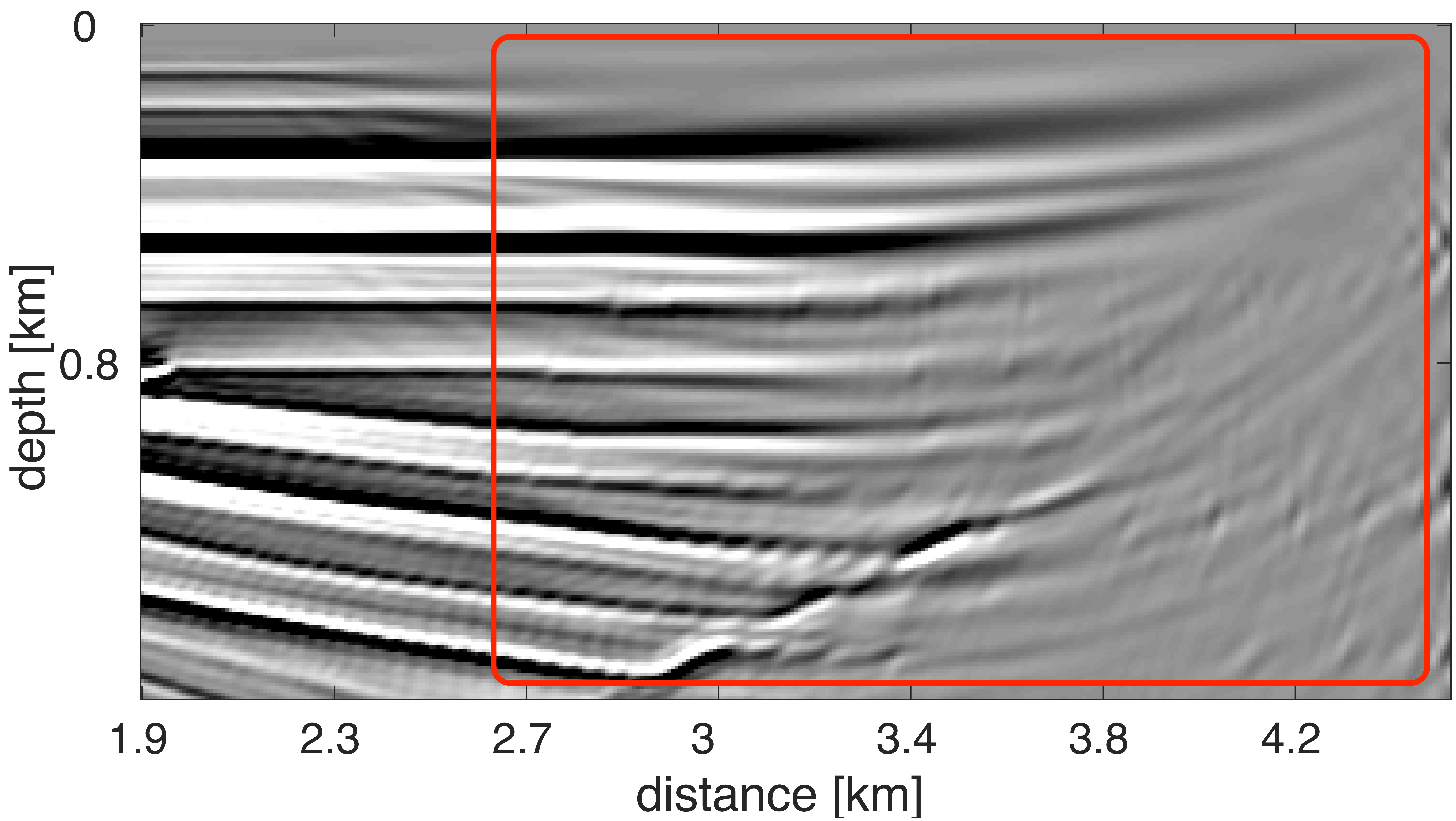
Inversion of primaries



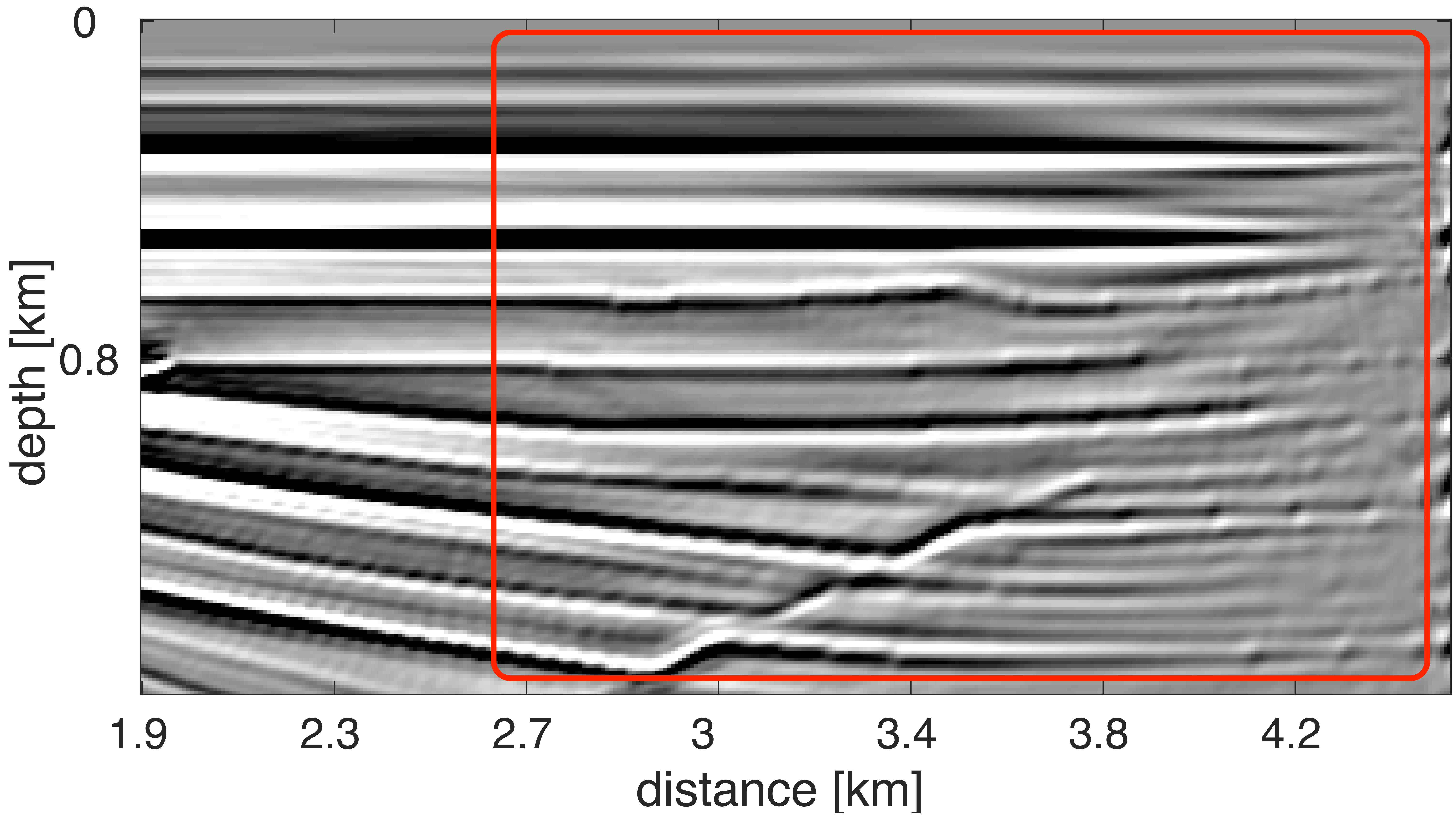
Inversion of primaries + multiples



Zoomed in: inversion of primaries



Zoomed in: inversion of primaries + multiples



Conclusions

Imaging with primary and multiples works well for shallow water surface-related multiples

Inversion can be carried out computationally efficiently via randomized sampling

Linearized Bregman leads to convergent algorithm

Bonus points of multiples act as secondary sources improving the illumination of shallow layers

Improved illumination of the shallow subsurface is essential for assessment of hazards

Future Work

To take density perturbations into account

Apply the method to ocean bottom data

Time-domain implementation in 3D

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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