Time-domain FWI in TTI media

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SLIM
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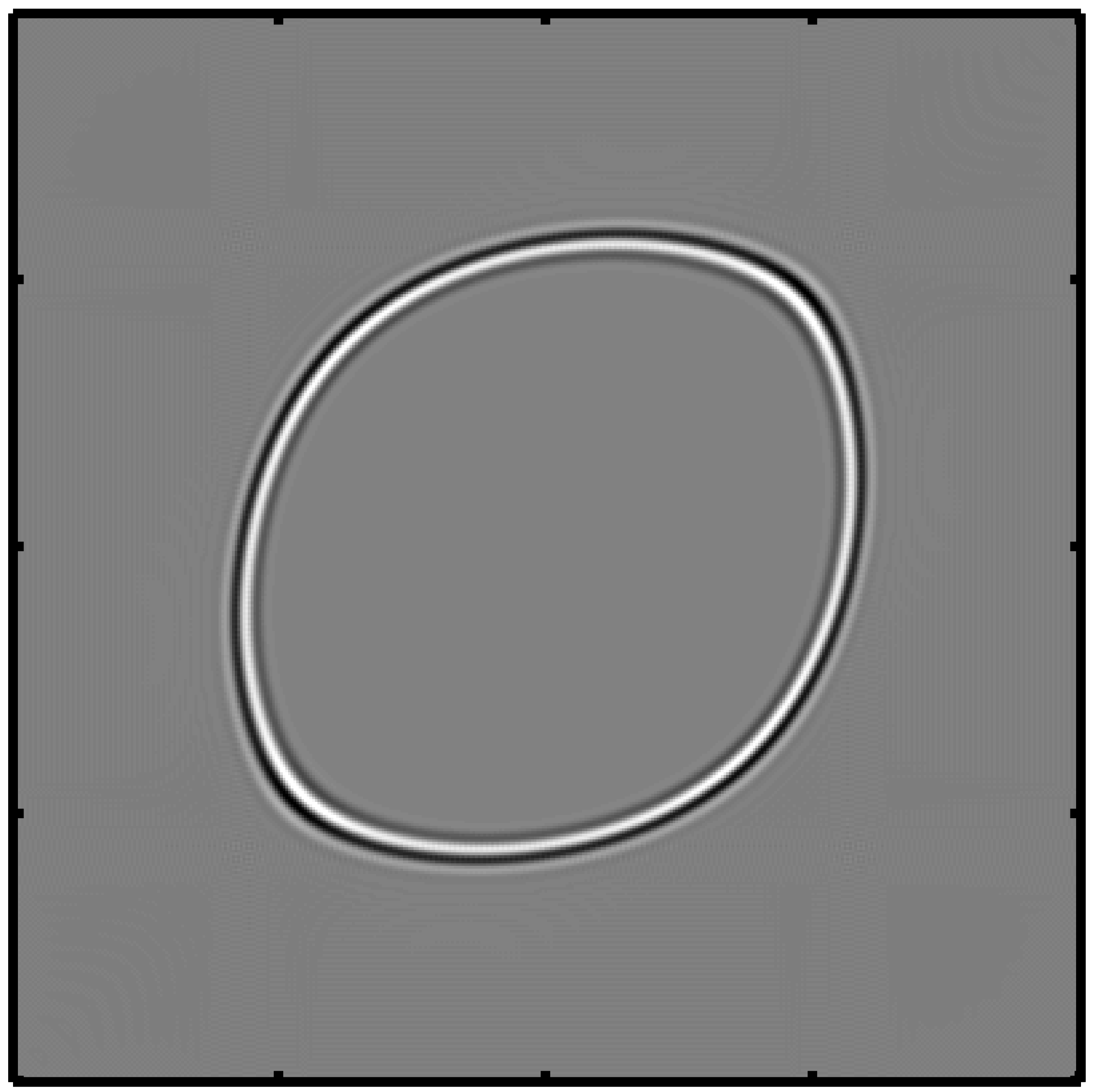
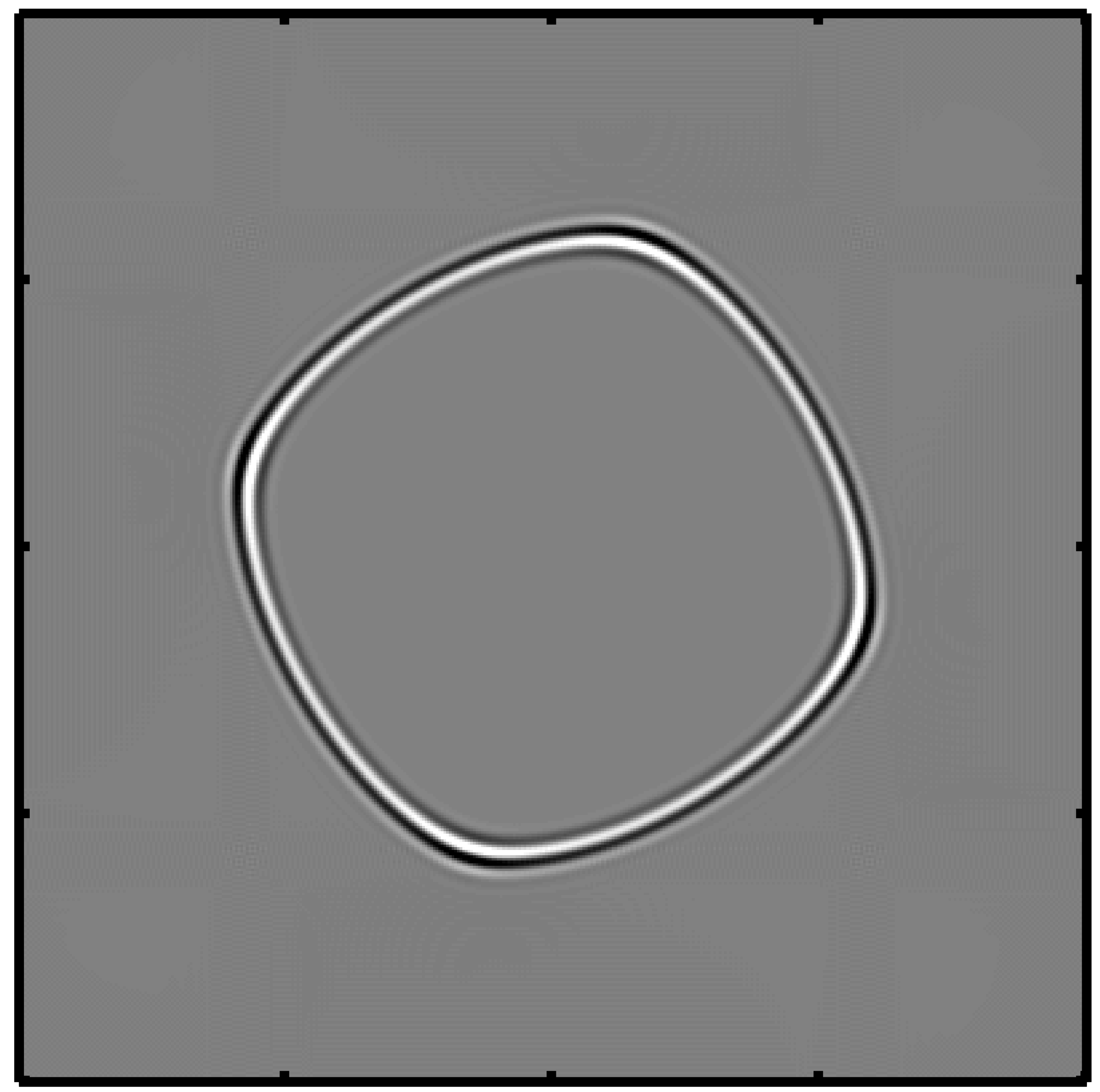
Introduction

Seismic inversion with field data: requires anisotropic modeling!

ε=0.25, δ=0, θ=45^o



ε=0, δ=0.4, θ=30^o



Introduction

Anisotropy is an issue in field data applications and caused by

- thin sedimentary layers below signal wavelength
- fractures, faults
- rock textures itself
- influences especially travel times for long offsets

So far all SLIM modeling codes are acoustic or visco-elastic

- primary goal: extend existing modeling/inversion codes to include spacially varying anisotropy
- secondary goal: multi-parameter FWI

(Backus, 1962, Long-wave elastic anisotropy produced by horizontal layering)

Recap of our time-domain modeling engine

Acoustic, isotropic wave equation in continuous form

$$m\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = q$$

- discretized with 2nd order leap frog in time
- 2nd or 4th order FD method for the Laplacian

Recap of our time-domain modeling engine

Matrix based discretized wave equation:

$$\mathbf{A}_1\mathbf{u}^{k+1} + \mathbf{A}_2\mathbf{u}^{k} + \mathbf{A}_3\mathbf{u}^{k-1} = \mathbf{q}^{k-1}$$

with:

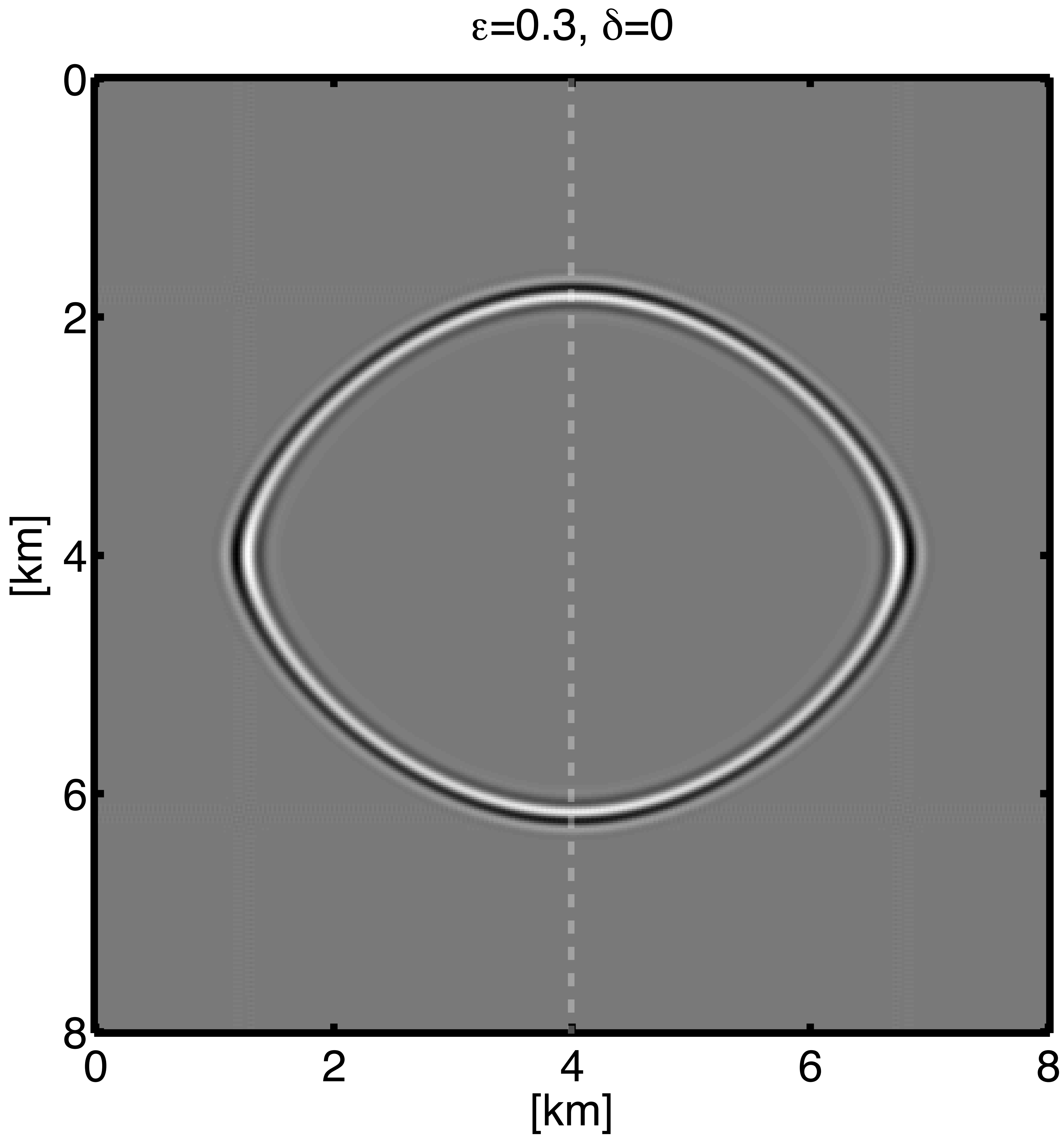
$$\mathbf{A}_1 = \mathrm{diag}(\frac{\mathbf{m}}{\triangle t^2})$$

$$\mathbf{A}_3 = \mathrm{diag}(\frac{\mathbf{m}}{\triangle t^2})$$

$$\mathbf{A}_2 = -\mathbf{L} - 2\mathrm{diag}(\frac{\mathbf{m}}{\triangle t^2})$$

$\mathbf{q}^k$: Source wave field at time step k

Extension to anisotropy



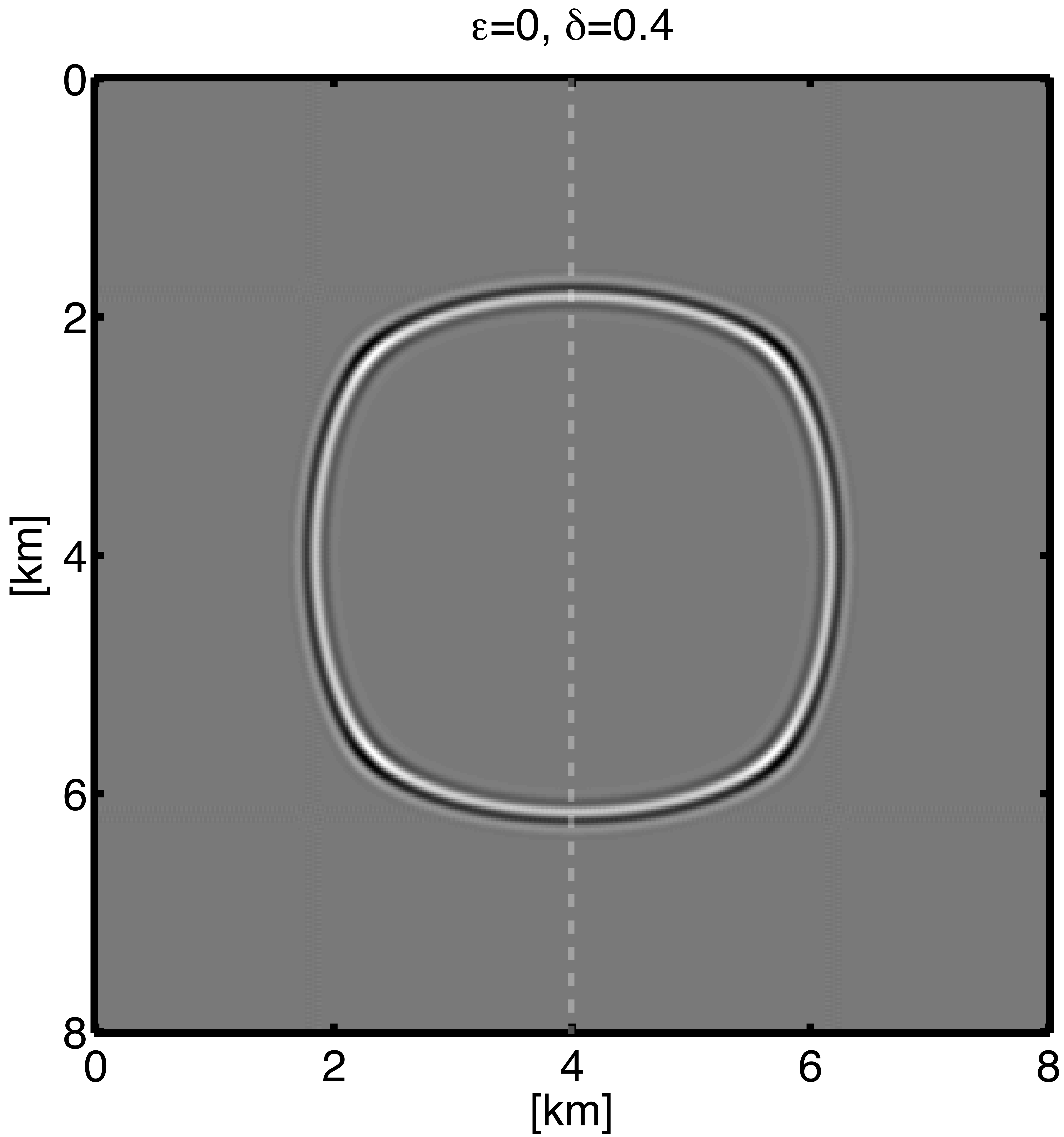
Physical interpretation ϵ :

$$v_{p_x} = v_{p_z}\sqrt{1+2\epsilon}$$

- ratio of vertical velocity v_{p_z} to horizontal velocity v_{p_h}

(Tsvankin, 2001, Seismic Signatures and Analysis of Reflection Data in Anisotropic Media)

Extension to anisotropy



Physical interpretation δ:

$$v_{p_n} = v_{p_z}\sqrt{1+2\delta}$$

- angular dependence of v_{p_z}
- related to NMO velocity v_{p_n}

(Tsvankin, 2001, Seismic Signatures and Analysis of Reflection Data in Anisotropic Media)

Tilted TI pure P-wave equation

2D anisotropic wave equation with PS methods

$$
\begin{aligned}
\frac{\partial U}{\partial t^2} =& c_{xx}\mathcal{F}^{-1}\{-k_x^2\bar{U}\} + c_{zz}\mathcal{F}^{-1}\{-k_z^2\bar{U}\} + c_{xz}\mathcal{F}^{-1}\{-k_xk_z\bar{U}\} \\
&+ c_{xxxx}\mathcal{F}^{-1}\left\{\frac{k_x^4}{k_x^2+k_z^2}\bar{U}\right\} + c_{zzzz}\mathcal{F}^{-1}\left\{\frac{k_z^4}{k_x^2+k_z^2}\bar{U}\right\} \\
&+ c_{xxxz}\mathcal{F}^{-1}\left\{\frac{k_x^3k_z}{k_x^2+k_z^2}\bar{U}\right\} + c_{xzzz}\mathcal{F}^{-1}\left\{\frac{k_xk_z^3}{k_x^2+k_z^2}\bar{U}\right\} \\
&+ c_{xxzz}\mathcal{F}^{-1}\left\{\frac{k_x^2k_z^2}{k_x^2+k_z^2}\bar{U}\right\}
\end{aligned}
$$

k_x: spatial wavenumber in x-direction
$\bar{U}$: wavefield in frequency domain
$\mathcal{F}$: 2D Fourier transform

(Chunlei Chu, Brian K. Macy and Phil D. Anno, 2011)

Tilted TI pure P-wave equation

Model parameters contained in $c_{xx}, c_{zz}, c_{xxxx}, \dots$

- parametrization in terms of velocities: v_z, v_h, v_{NMO}
- alternatively with Thomson parameters: v_z, ϵ, δ

Tilted TI pure P-wave equation

Fully discretized TTI equation with $(\mathbf{m}, \boldsymbol{\epsilon}, \boldsymbol{\delta}, \boldsymbol{\theta})$ parametrization

$$\mathrm{diag}(\frac{\mathbf{m}}{\Delta t^2})\Big(\mathbf{u}^{n+1} - 2\mathbf{u}^{n} + \mathbf{u}^{n-1}\Big) - \mathbf{L}\mathbf{u}^{n} = \mathbf{q}^{n+1}$$

→ same parametrization as for isotropic modeling, only that

$$\begin{aligned}\mathbf{L} = \Big\{ & \mathrm{diag}(\mathbf{c}_{xx})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_1) + \mathrm{diag}(\mathbf{c}_{zz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_2) \\ & + \mathrm{diag}(\mathbf{c}_{xz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_3) + \mathrm{diag}(\mathbf{c}_{xxzz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_4) \\ & + \mathrm{diag}(\mathbf{c}_{xxxx})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_5) + \mathrm{diag}(\mathbf{c}_{zzzz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_6) \\ & + \mathrm{diag}(\mathbf{c}_{xxxz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_7) + \mathrm{diag}(\mathbf{c}_{xzzz})\mathbf{F}^*\mathrm{diag}(\mathbf{k}_8) \Big\}\mathbf{F}\end{aligned}$$

Forward Modeling

Test the forward modeling code with comparison to analytic travel times under weak anisotropy assumption

$$\phi = \vartheta + \left(\delta + 2(\epsilon - \delta)\sin^2\vartheta\right)\sin 2\vartheta$$

ϕ: group angle

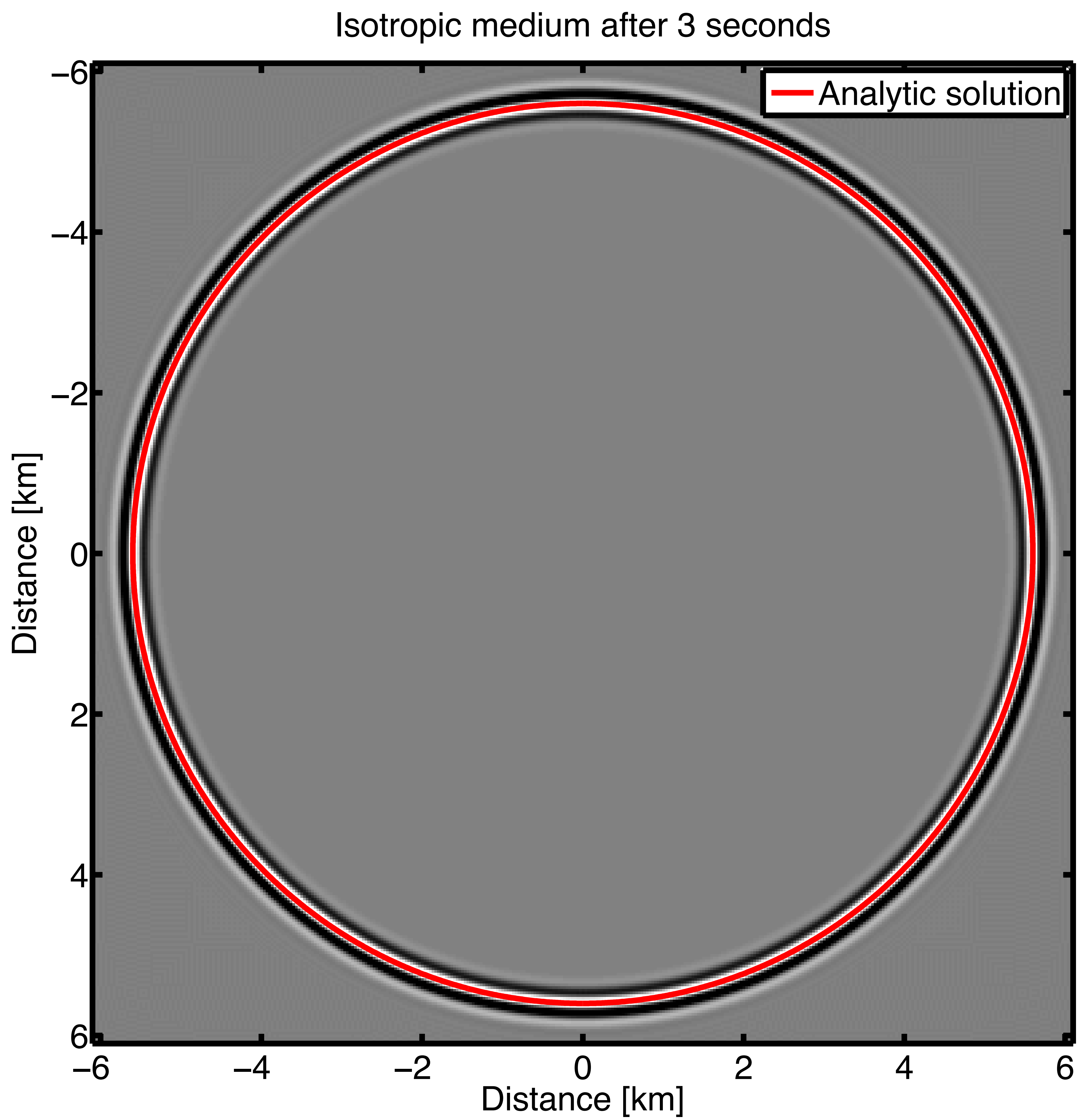
ϑ: phase angle

$$v_{group} = v_{pz}\left(1 + \delta\sin^2\phi + (\epsilon - \delta)\sin^4\phi\right)$$

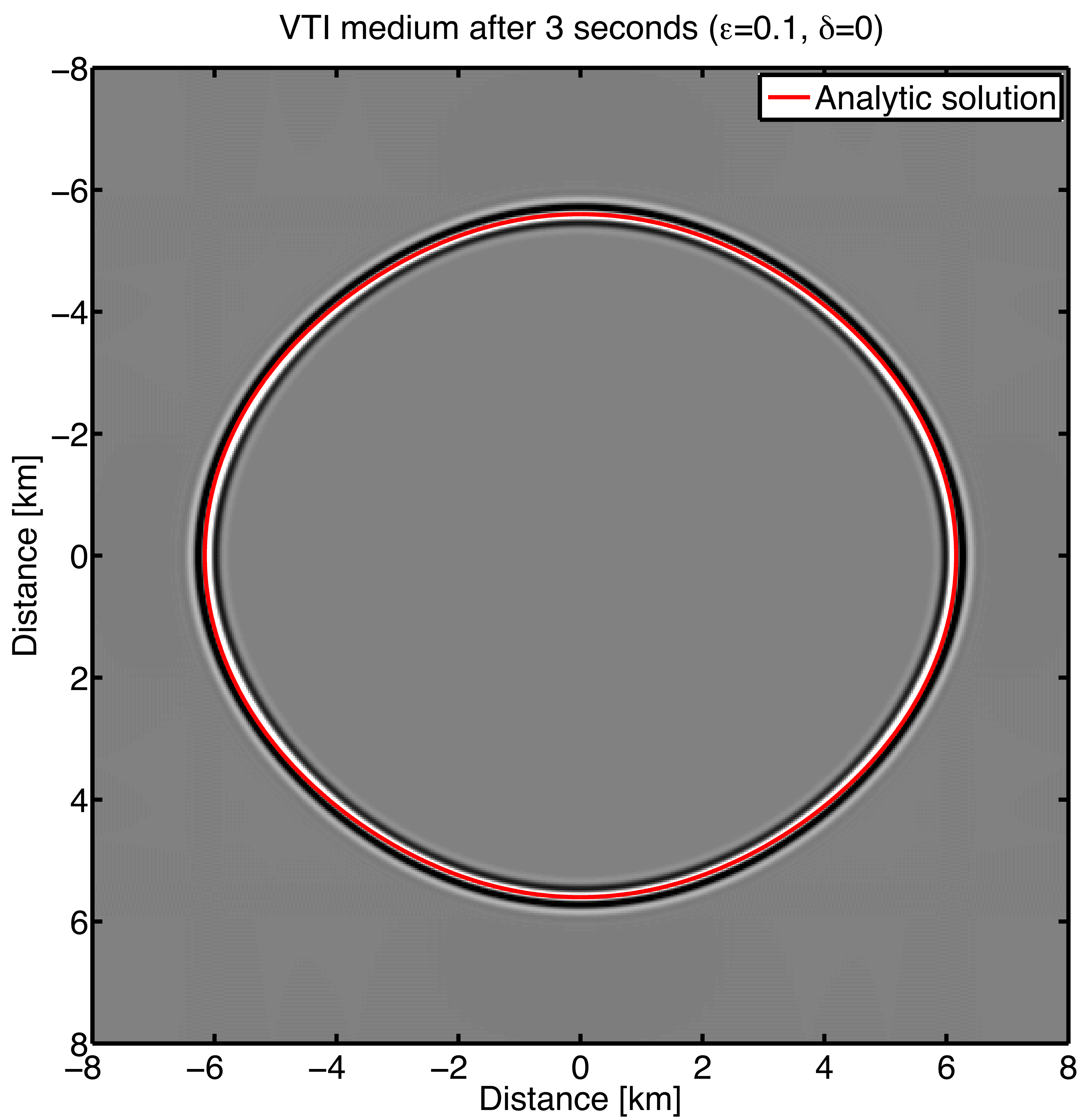
→ Calculate wavefronts in homogeneous medium from group velocity and angle.

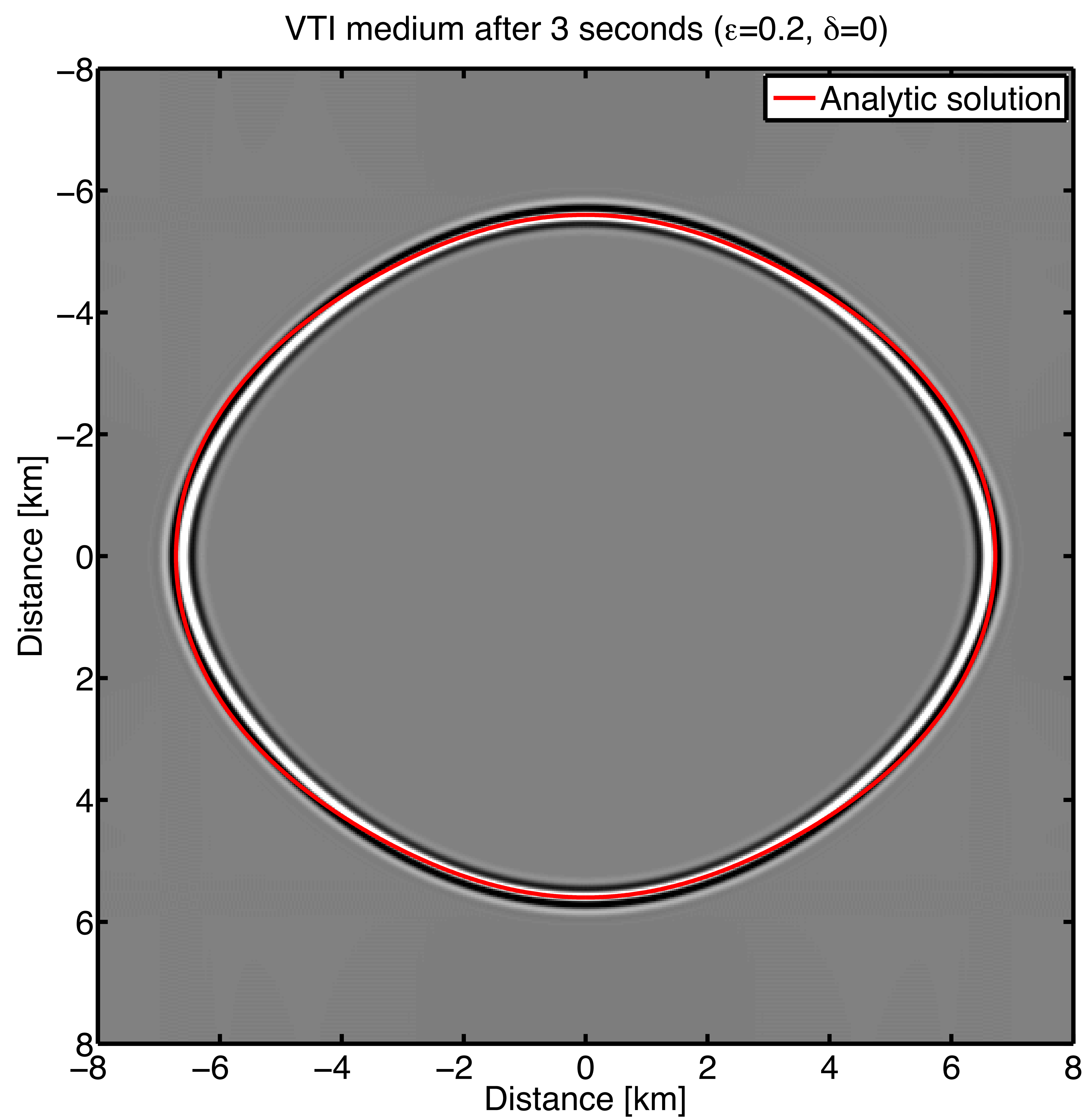
(Tsvankin, 2012, Seismic signatures and analysis of reflection data in anisotropic media: SEG, 3rd edition)

Forward Modeling

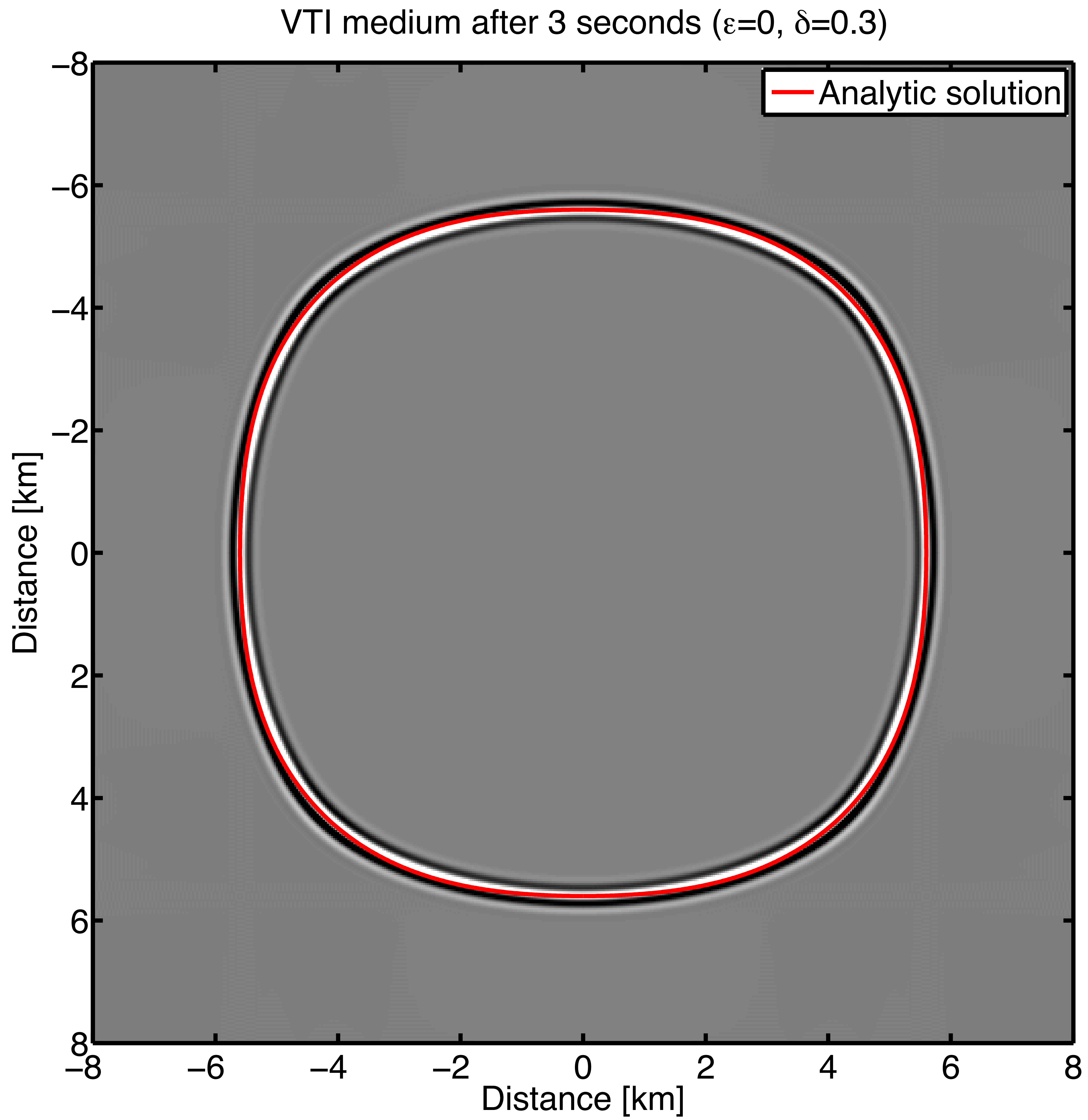


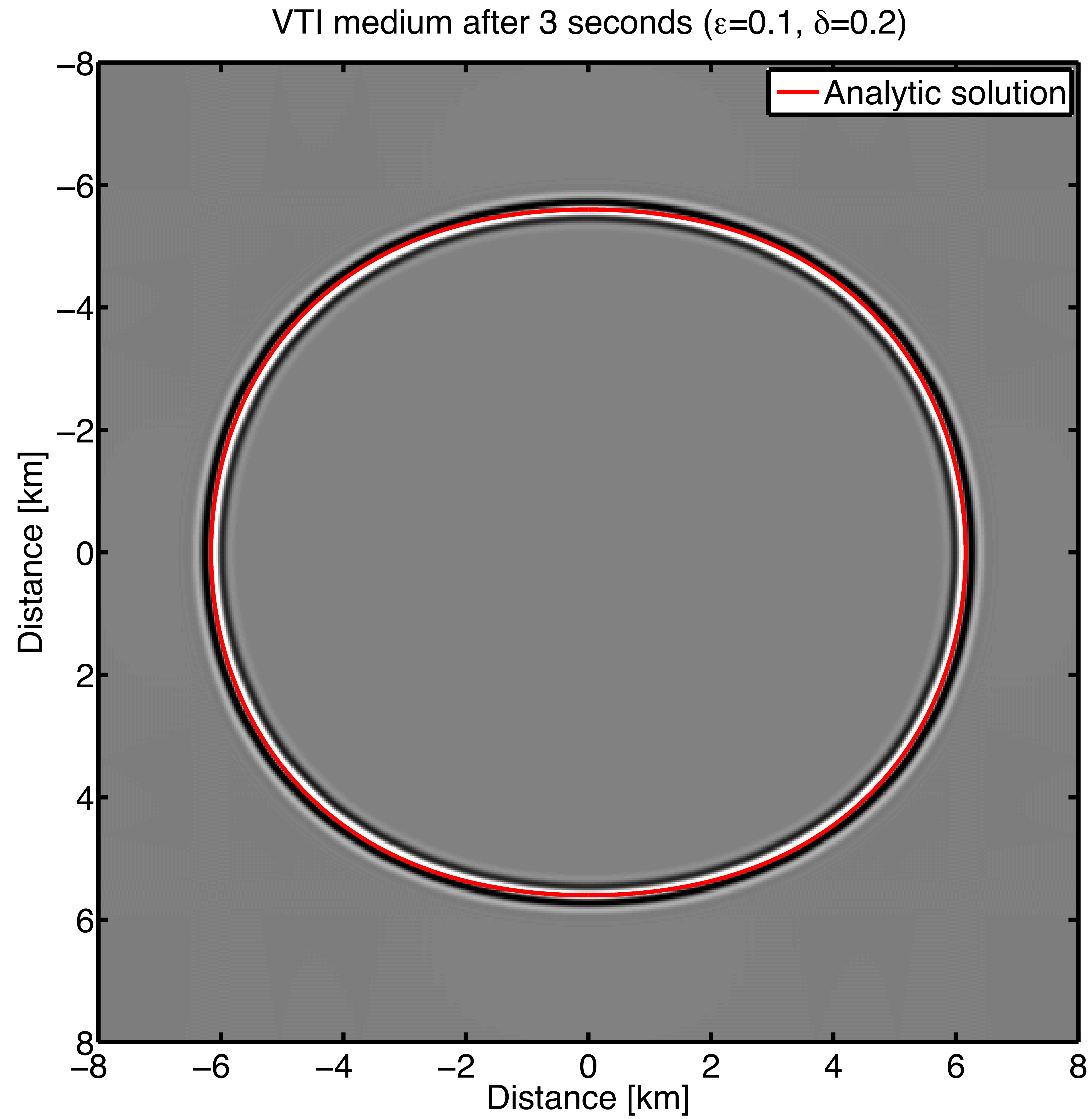
Forward Modeling





Forward Modeling





Forward Modeling

One code for both isotropic and anisotropic modeling and inversion

- both modes use the same functions and operators
- only Laplacian changes for anisotropy
- easy to use via function overloading

$$\texttt{data} = \texttt{Gen\_data}(\texttt{m0}, \texttt{params}, \texttt{q})$$

$$\texttt{data} = \texttt{Gen\_data}(\texttt{m0}, \texttt{params}, \texttt{q}, \texttt{ani})$$

isotropic modeling with FD Laplacian

anisotropic modeling with PS Laplacian

Sensitivities

How does a change in $\delta \mathbf{m}/\delta \epsilon$ affect the data?

$$\mathbf{J_m} = \frac{\partial \mathbf{u}}{\partial \mathbf{m}} = -\mathcal{A}(\mathbf{m}, \boldsymbol{\epsilon})^{-1} \Big(\frac{\partial \mathcal{A}(\mathbf{m}, \boldsymbol{\epsilon})}{\partial \mathbf{m}} \mathbf{u} \Big)$$

$$\mathbf{J}_\epsilon = \frac{\partial \mathbf{u}}{\partial \boldsymbol{\epsilon}} = -\mathcal{A}(\mathbf{m}, \boldsymbol{\epsilon})^{-1} \Big(\frac{\partial \mathcal{A}(\mathbf{m}, \boldsymbol{\epsilon})}{\partial \boldsymbol{\epsilon}} \mathbf{u} \Big)$$

(Haber, 2013)

Sensitivities

Partial derivative of modeling operator w.r.t. slowness:

$$\frac{\partial}{\partial \mathbf{m}}\mathbf{A}_1 = \frac{\partial}{\partial \mathbf{m}}\mathbf{A}_3 = \mathrm{diag}(\frac{1}{\triangle t^2})$$

$$\frac{\partial}{\partial \mathbf{m}}\mathbf{A}_2 = -2\mathrm{diag}(\frac{1}{\triangle t^2})$$

- Laplacian drops out
- partial derivative corresponds to discrete 2nd order time derivative

Sensitivities

Partial derivative of modeling operator w.r.t. epsilon:

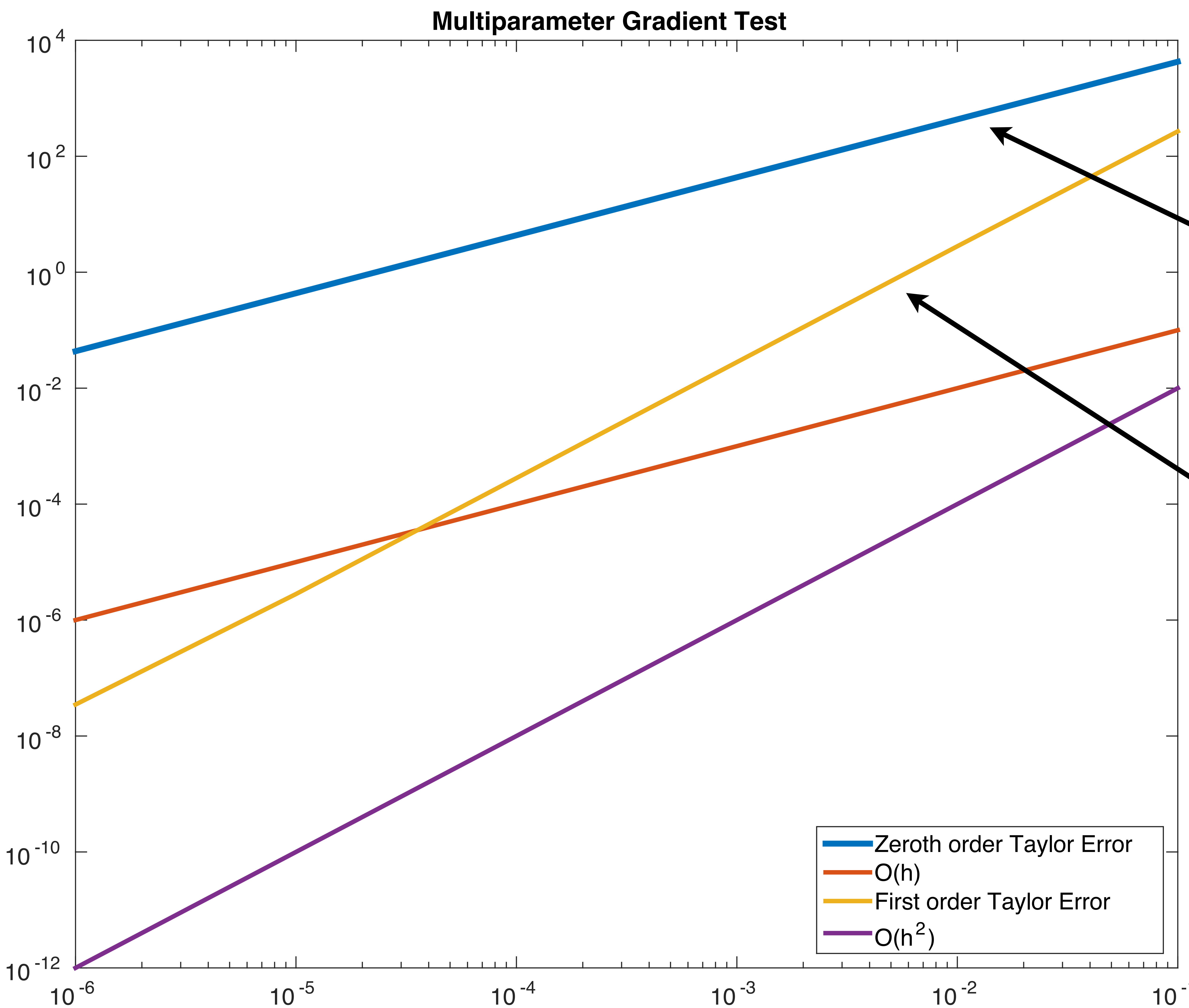
$$\frac{\partial}{\partial \boldsymbol{\epsilon}}\mathbf{A}_1 = \frac{\partial}{\partial \boldsymbol{\epsilon}}\mathbf{A}_3 = 0$$

$$\frac{\partial}{\partial \boldsymbol{\epsilon}}\mathbf{A}_2 = -\frac{\partial \mathbf{L}}{\partial \boldsymbol{\epsilon}}$$

- requires partial derivative of the Laplacian w.r.t. epsilon

Gradient Test

Ensure 2nd order convergence of Taylor expansion



$$||\mathbf{F}(\mathbf{m}_0 + h \cdot \delta\mathbf{m}, \boldsymbol{\epsilon}_0 + h \cdot \delta\boldsymbol{\epsilon}) - \mathbf{F}(\mathbf{m}_0, \boldsymbol{\epsilon}_0)||$$

$$||\mathbf{F}(\mathbf{m}_0 + h \cdot \delta\mathbf{m}, \boldsymbol{\epsilon}_0 + h \cdot \delta\boldsymbol{\epsilon}) - \mathbf{F}(\mathbf{m}_0, \boldsymbol{\epsilon}_0) - h \cdot \mathbf{J}_{\mathbf{m}}\delta\mathbf{m} - h \cdot \mathbf{J}_{\epsilon}\delta\boldsymbol{\epsilon}||$$

FWI Gradient

FWI least squares objective function

$$\Phi(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{n_s} \left\|\mathbf{d}_i - \mathbf{P_r}\mathcal{A}(\mathbf{m})^{-\mathbf{1}}\mathbf{q}_i\right\|_2^2$$

$\mathbf{d_i}$: i_{th} observed shot record

$\mathbf{P_r}$: Restriction operator

$\mathbf{q_i}$: i_{th} source

FWI Gradient

Gradient w.r.t. squared slowness

$$\mathbf{J}_{\mathbf{m}}^{T}\delta\mathbf{d} = -\sum_{i=1}^{n_s}\Big(\frac{\partial\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})}{\partial\mathbf{m}}\mathbf{u}_i\Big)^{T}\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})^{-T}\mathbf{P_r}^{T}\delta\mathbf{d}_i$$

FWI Gradient

Gradient w.r.t. squared slowness

$$\mathbf{J}_{\mathbf{m}}^{T}\delta\mathbf{d} = -\sum_{i=1}^{n_s}\Big(\frac{\partial\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})}{\partial\mathbf{m}}\mathbf{u}_i\Big)^{T}\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})^{-T}\mathbf{P_r}^{T}\delta\mathbf{d}_i$$

Gradient w.r.t. epsilon (computed on the fly)

$$\mathbf{J}_{\boldsymbol{\epsilon}}^{T}\delta\mathbf{d} = -\sum_{i=1}^{n_s}\Big(\frac{\partial\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})}{\partial\boldsymbol{\epsilon}}\mathbf{u}_i\Big)^{T}\mathcal{A}(\mathbf{m},\boldsymbol{\epsilon})^{-T}\mathbf{P_r}^{T}\delta\mathbf{d}_i$$

FWI Gradient

The FWI gradients have to pass the adjoint test:

- we only compute actions of $\mathbf{J}, \mathbf{J}^T$, never the matrices itself
- to ensure they are true adjoints, the migration/demigration operators need to satisfy

$$||\delta \mathbf{d}^T \mathbf{J} \delta \mathbf{m} - \delta \mathbf{m}^T \mathbf{J}^T \delta \mathbf{d}|| < \epsilon$$

Synthetic example: FWI on BG model

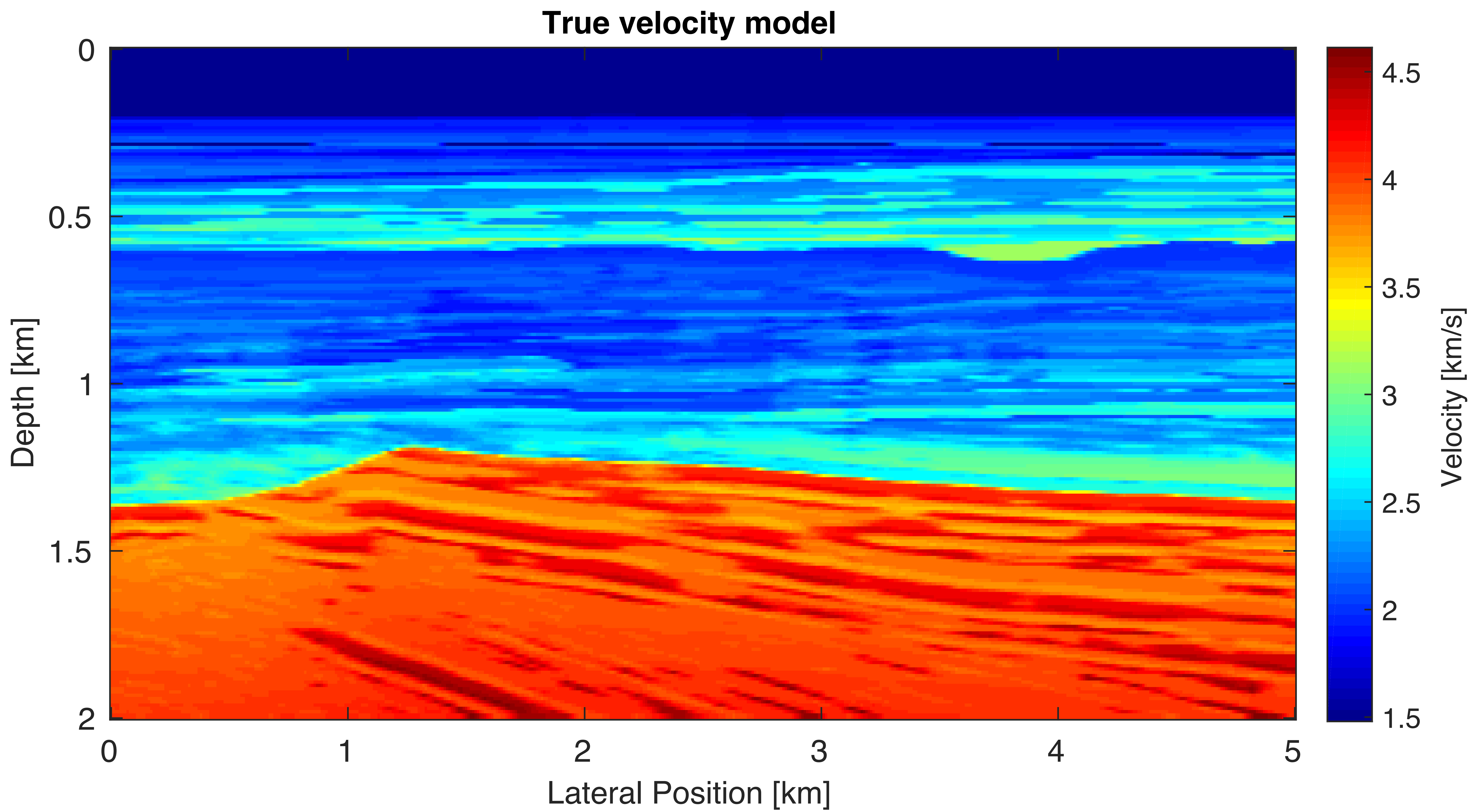
Generate data with anisotropic modeling

- invert for squared slowness only: with correct and wrong anisotropy
- invert for squared slowness and epsilon

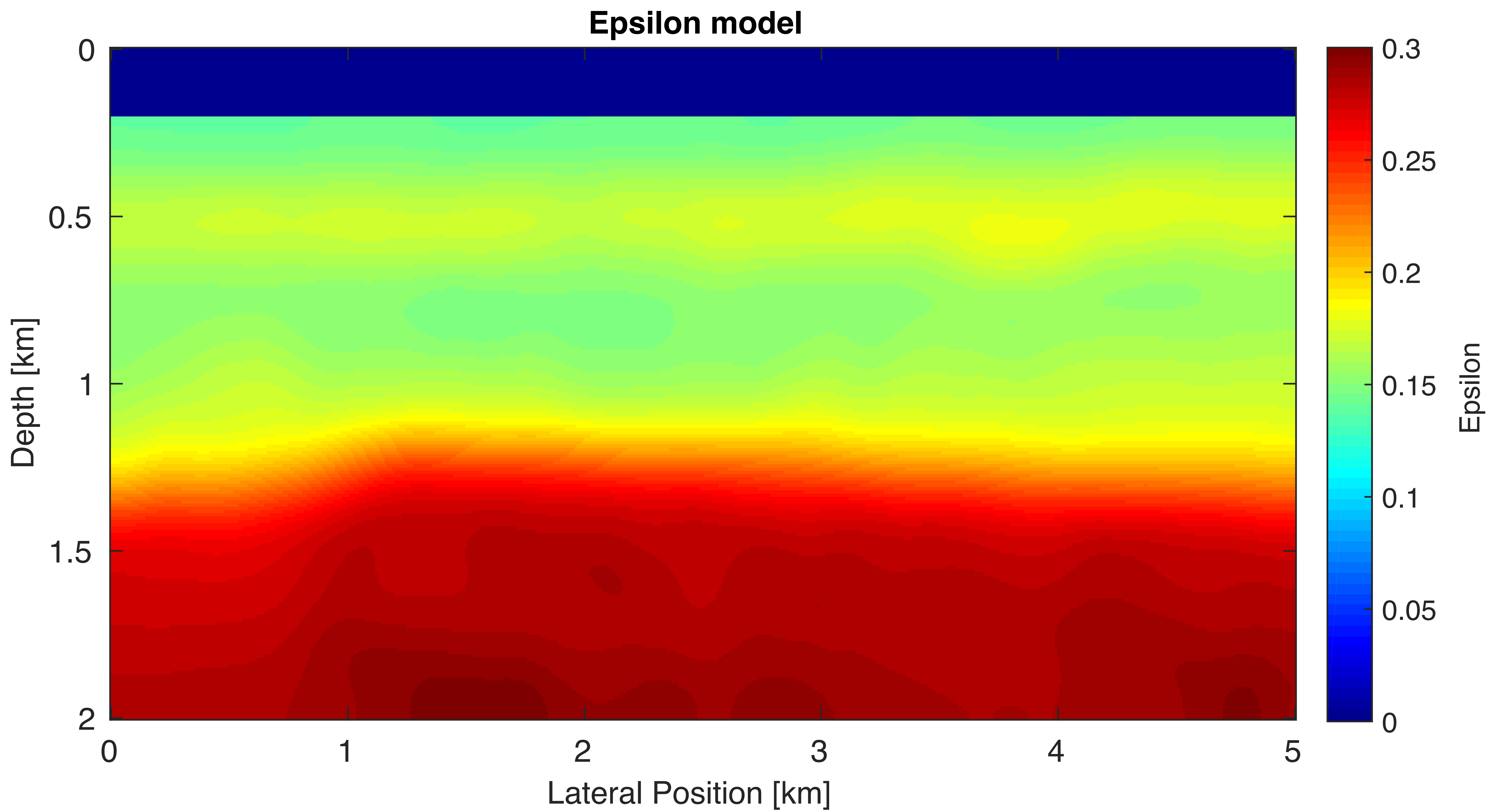
Setup:

- 501 receivers (10 m distance), 99 sources (50 m distance)
- 2.4 s recording time (601 samples)
- data generated with 15 Hz peak frequency

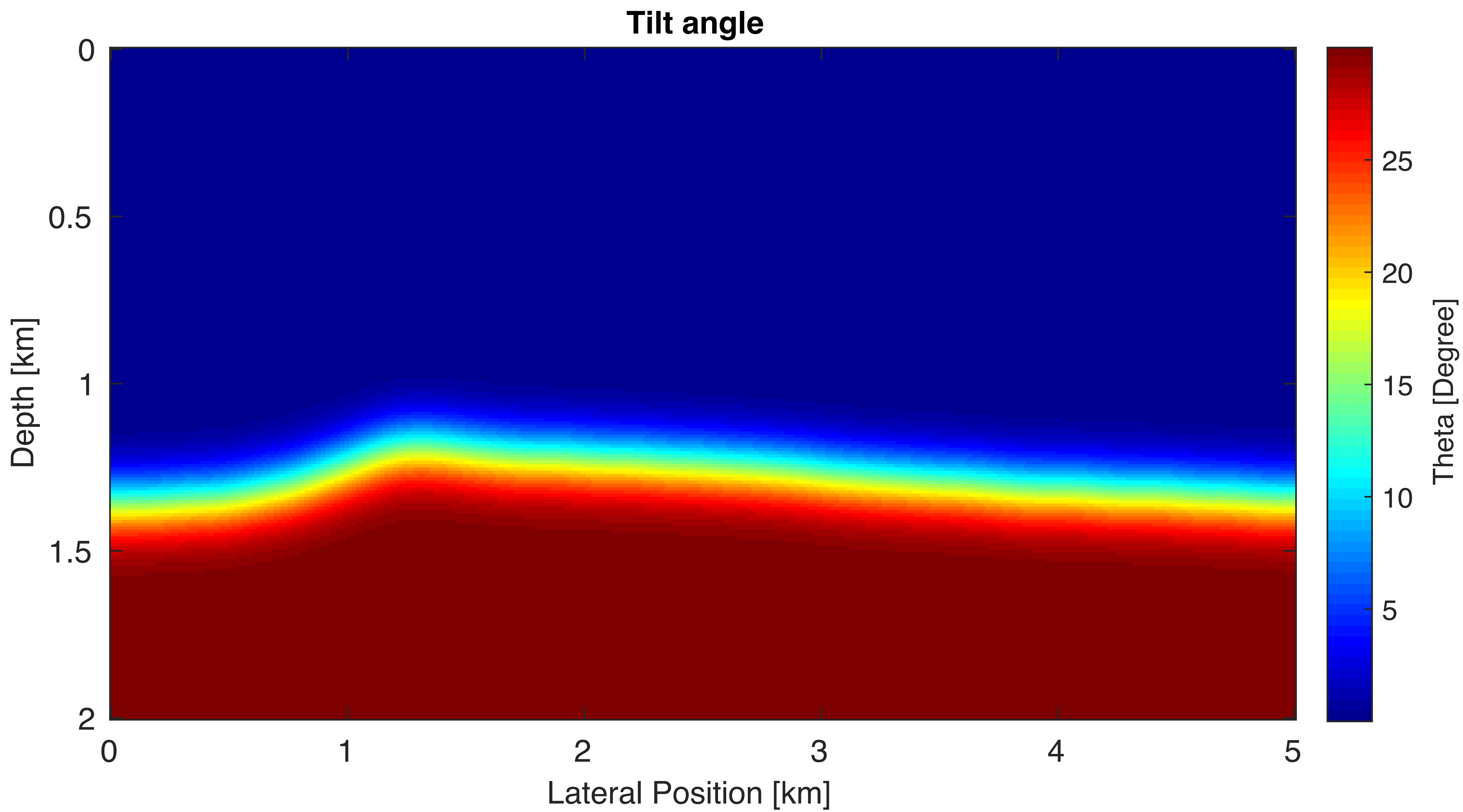
Subsurface Models



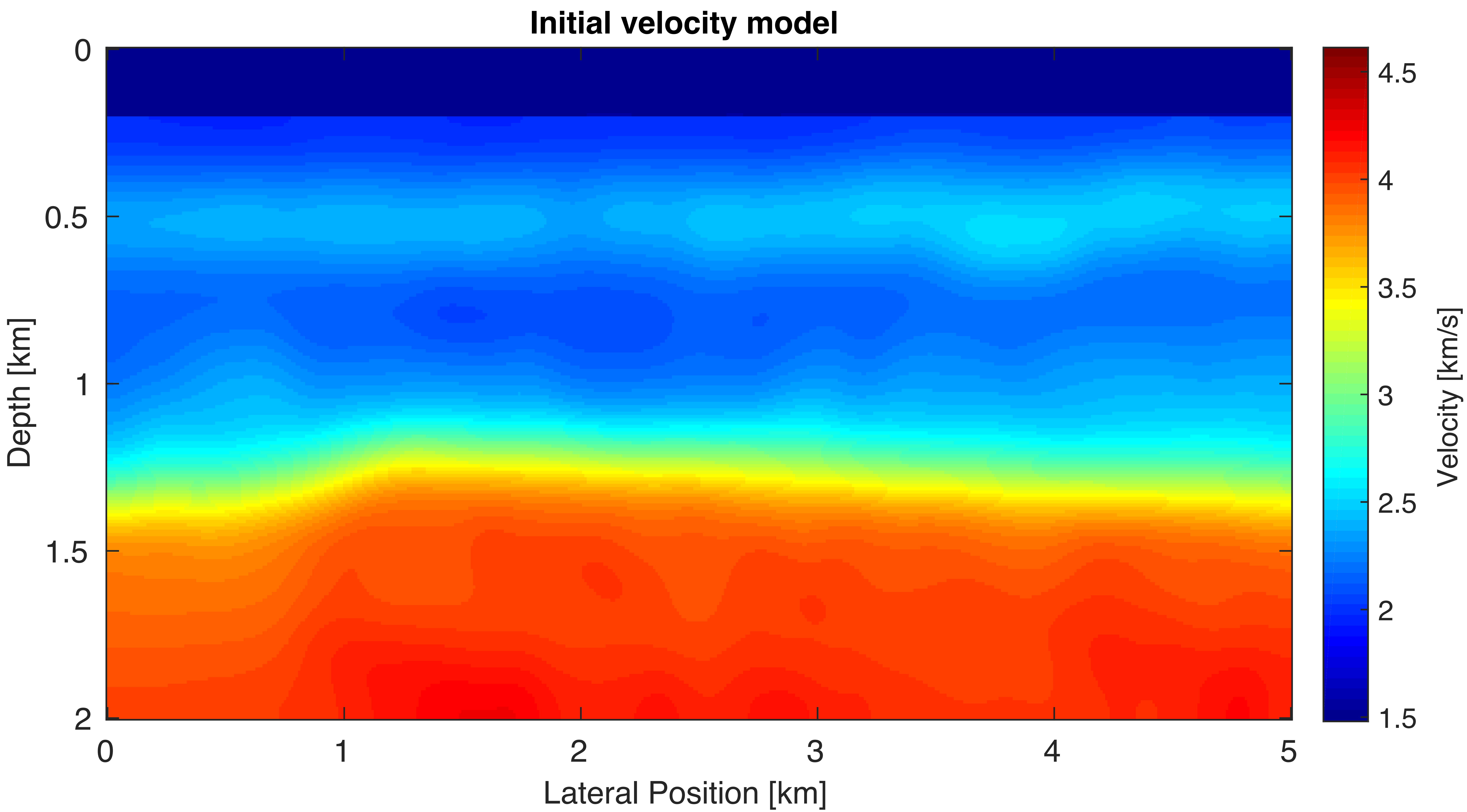
Subsurface Models



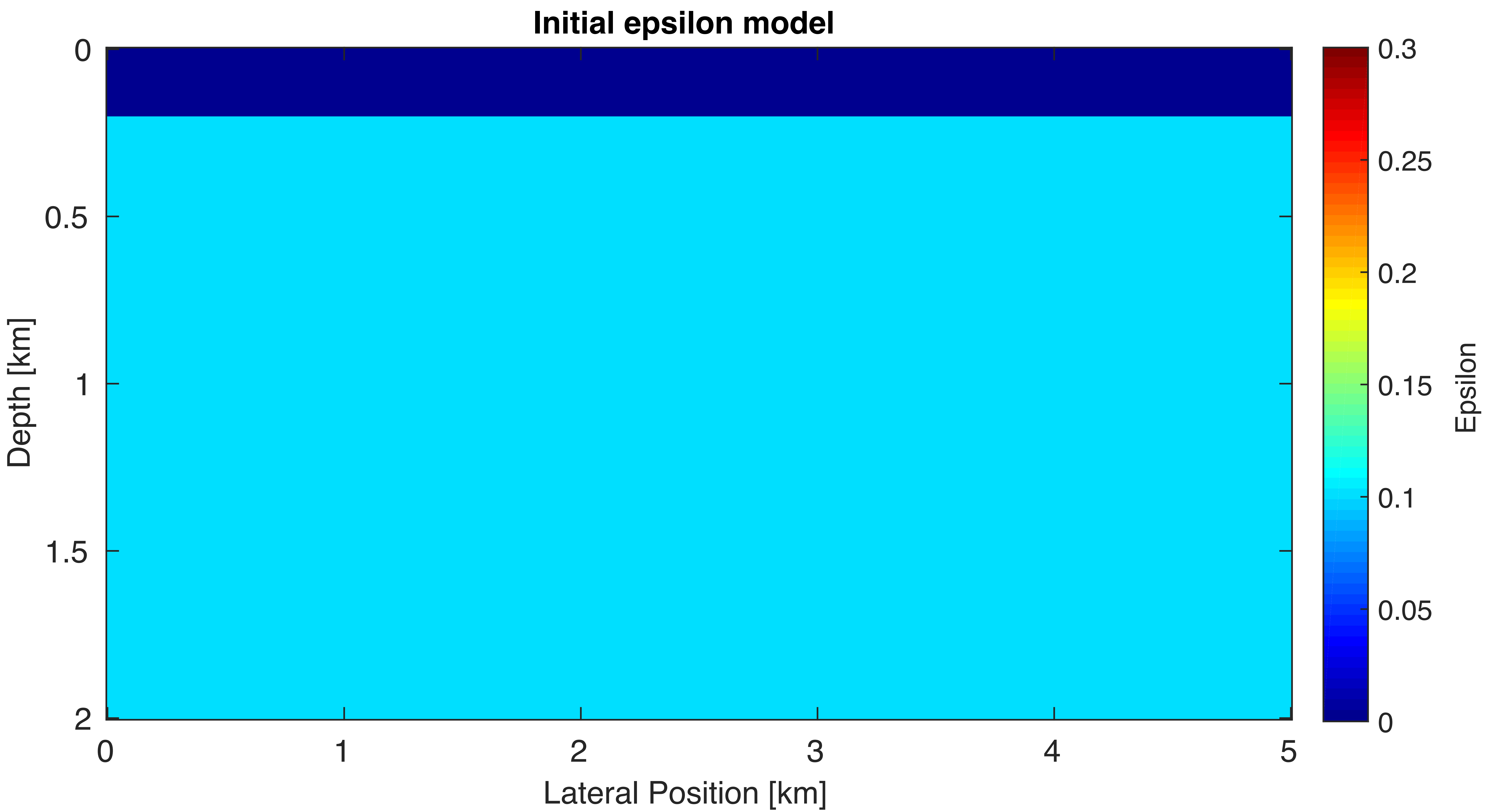
Subsurface Models



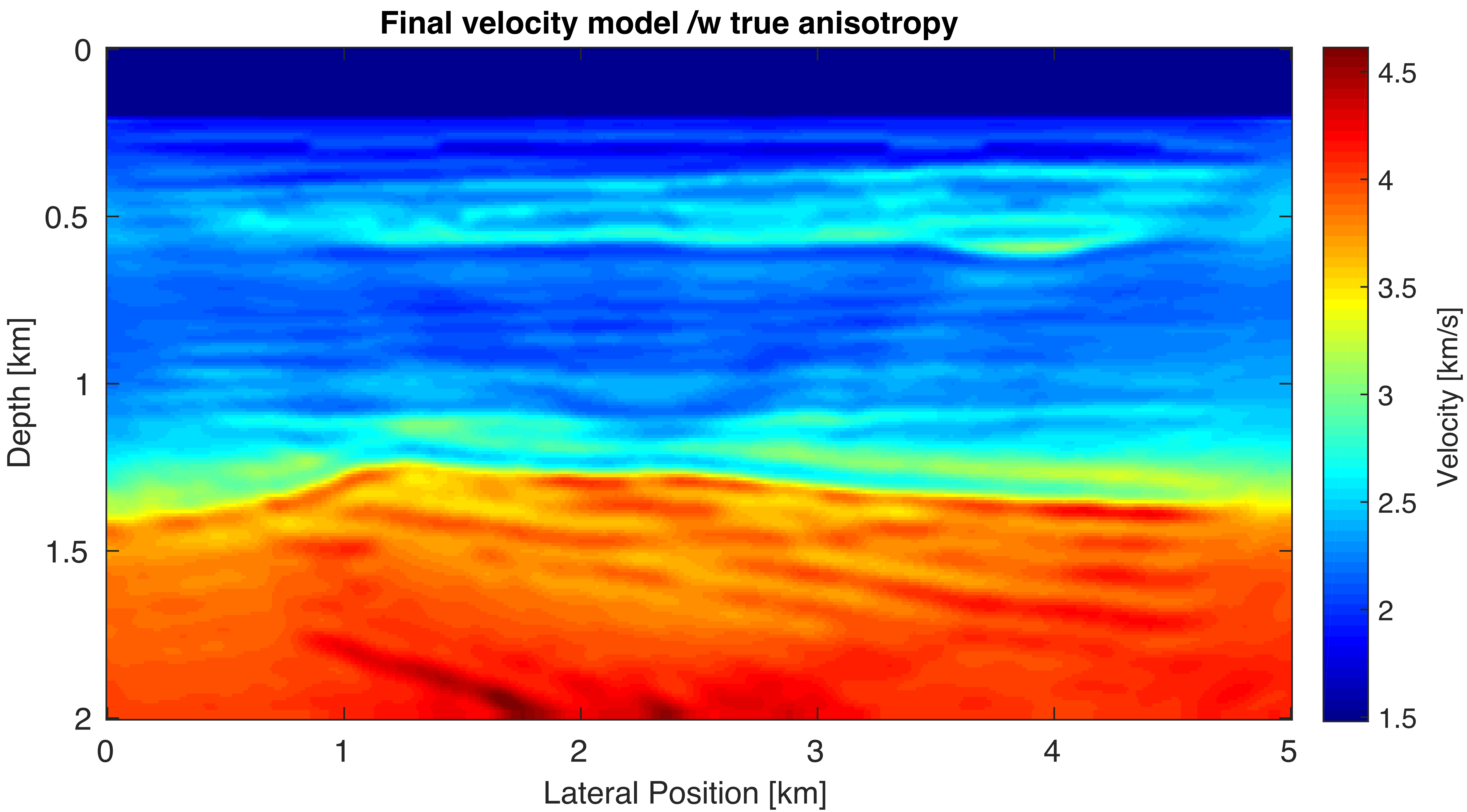
Subsurface Models



Subsurface Models

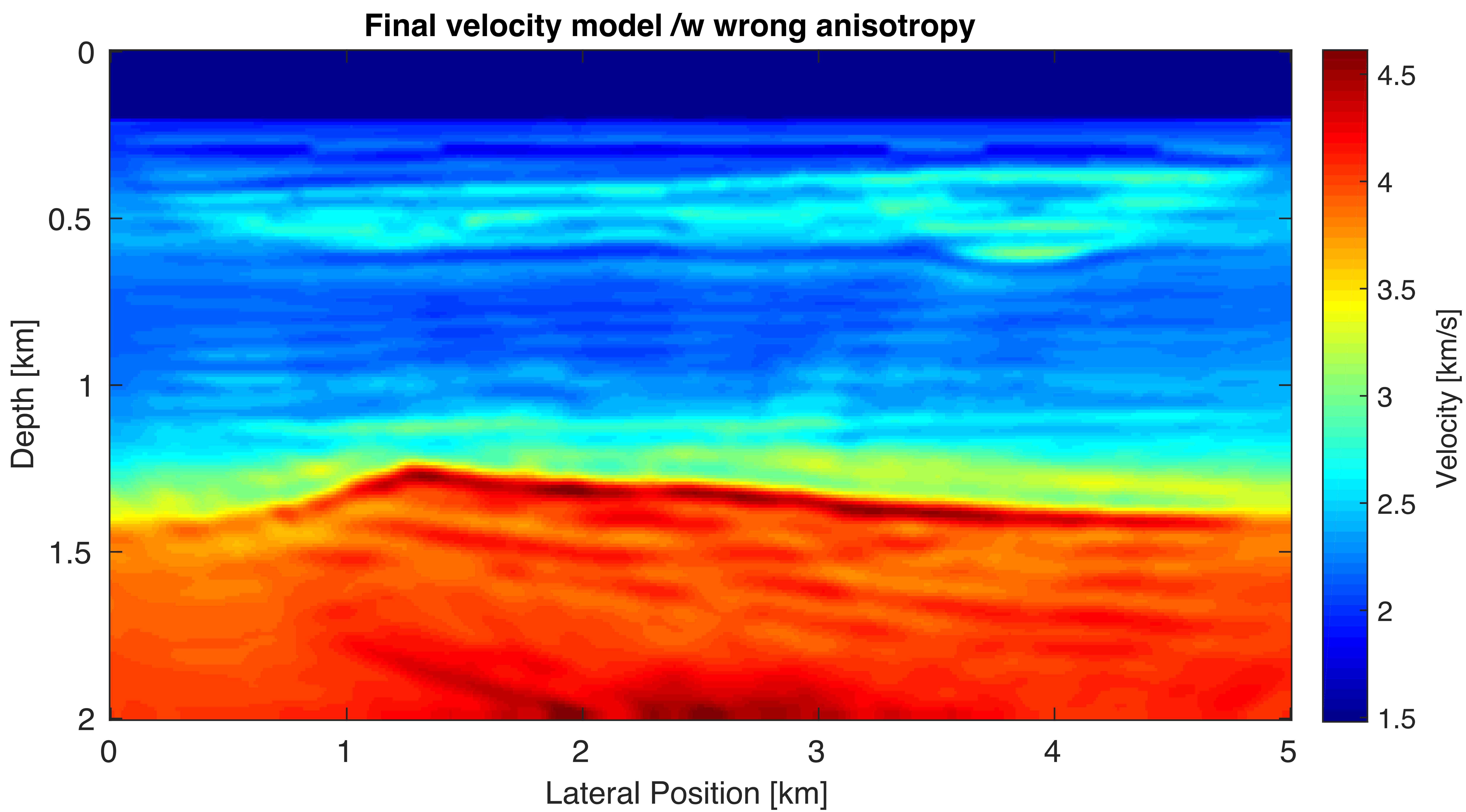


Results

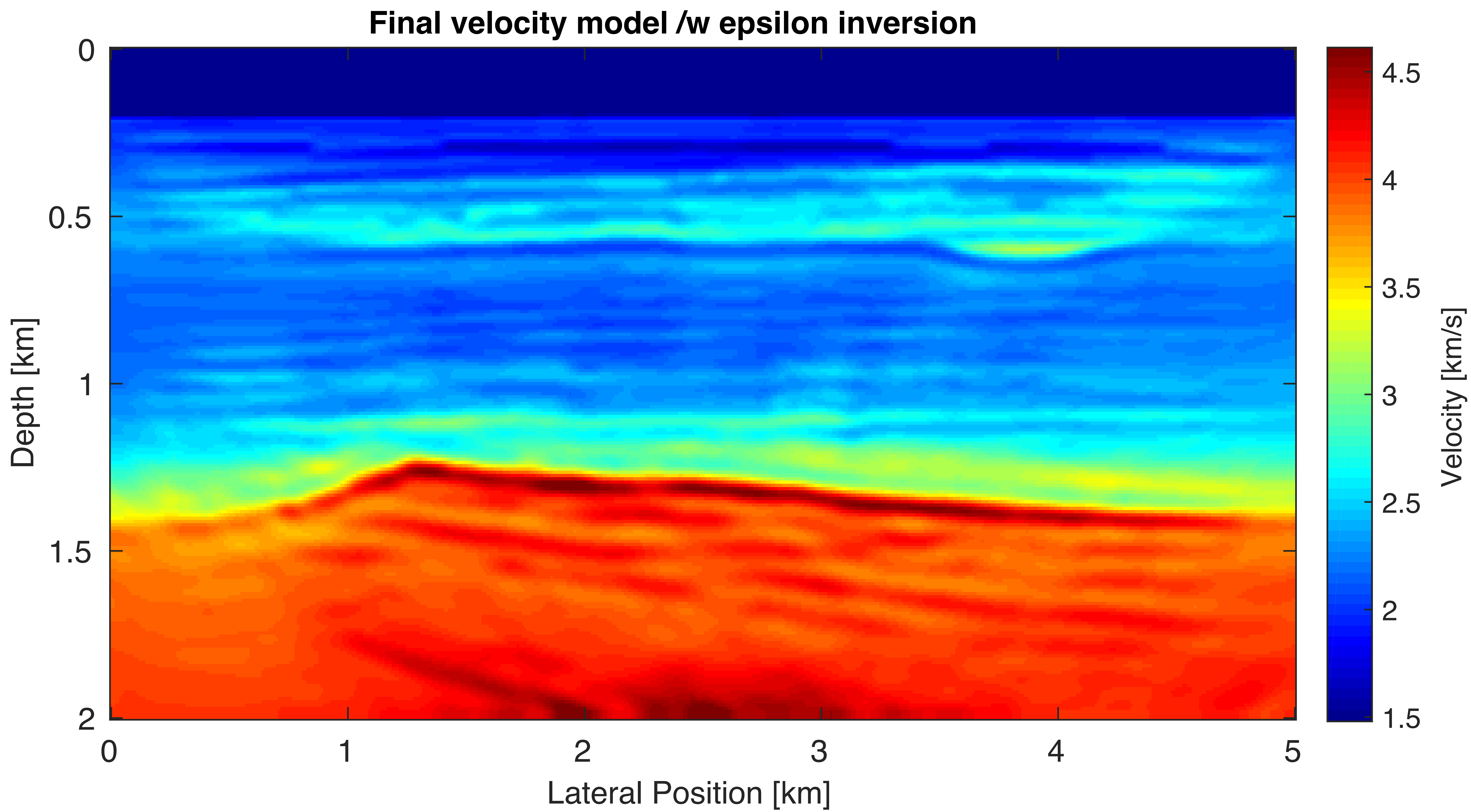


Final velocity model /w true anisotropy
0
0.5
1
1.5
2
Depth [km]
0
1
2
3
4
5
Lateral Position [km]
4.5
4
3.5
3
2.5
2
1.5
Velocity [km/s]

Results

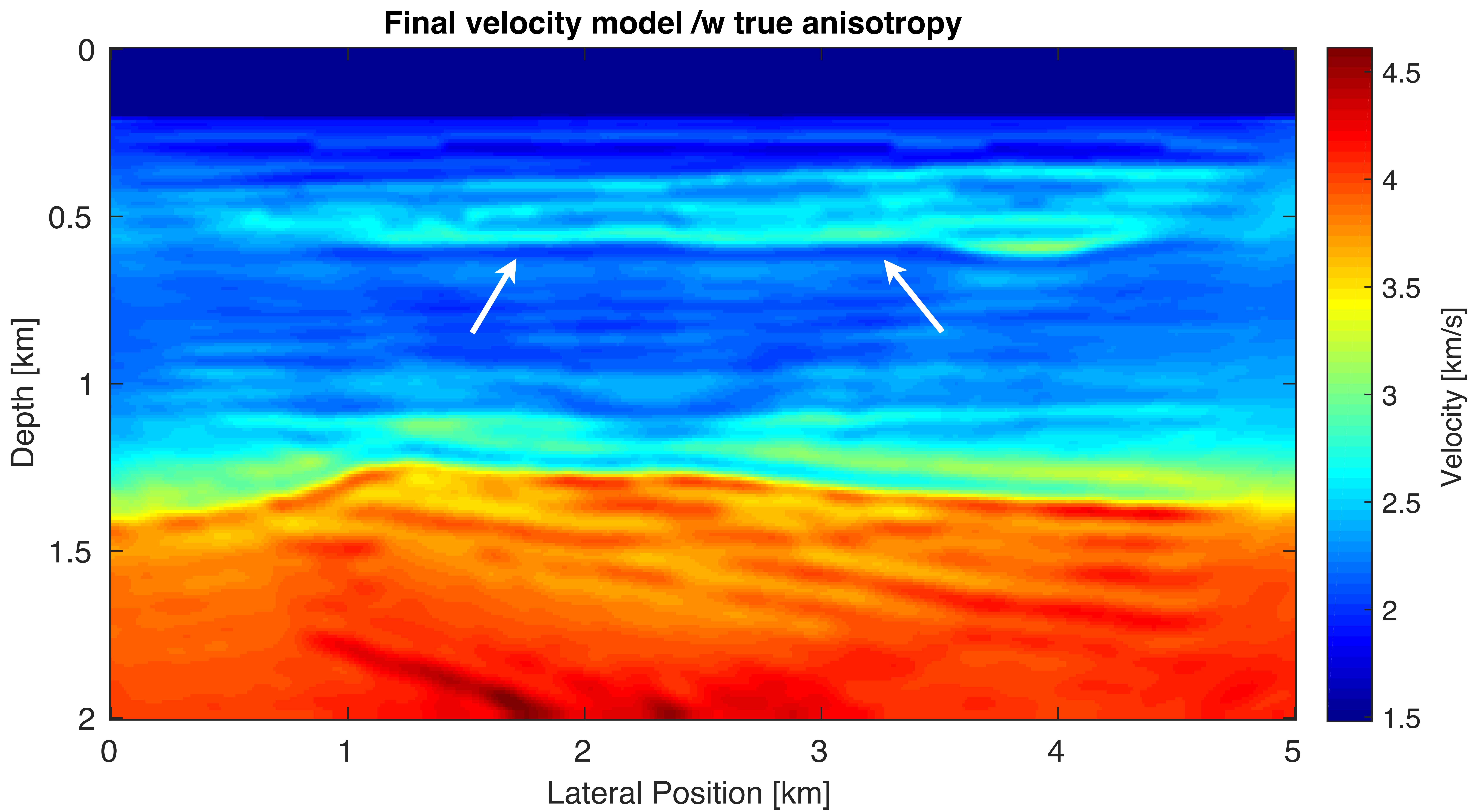

Final velocity model /w wrong anisotropy
Depth [km]
0
0.5
1
1.5
2
Lateral Position [km]
0
1
2
3
4
5
Velocity [km/s]
4.5
4
3.5
3
2.5
2
1.5

Results



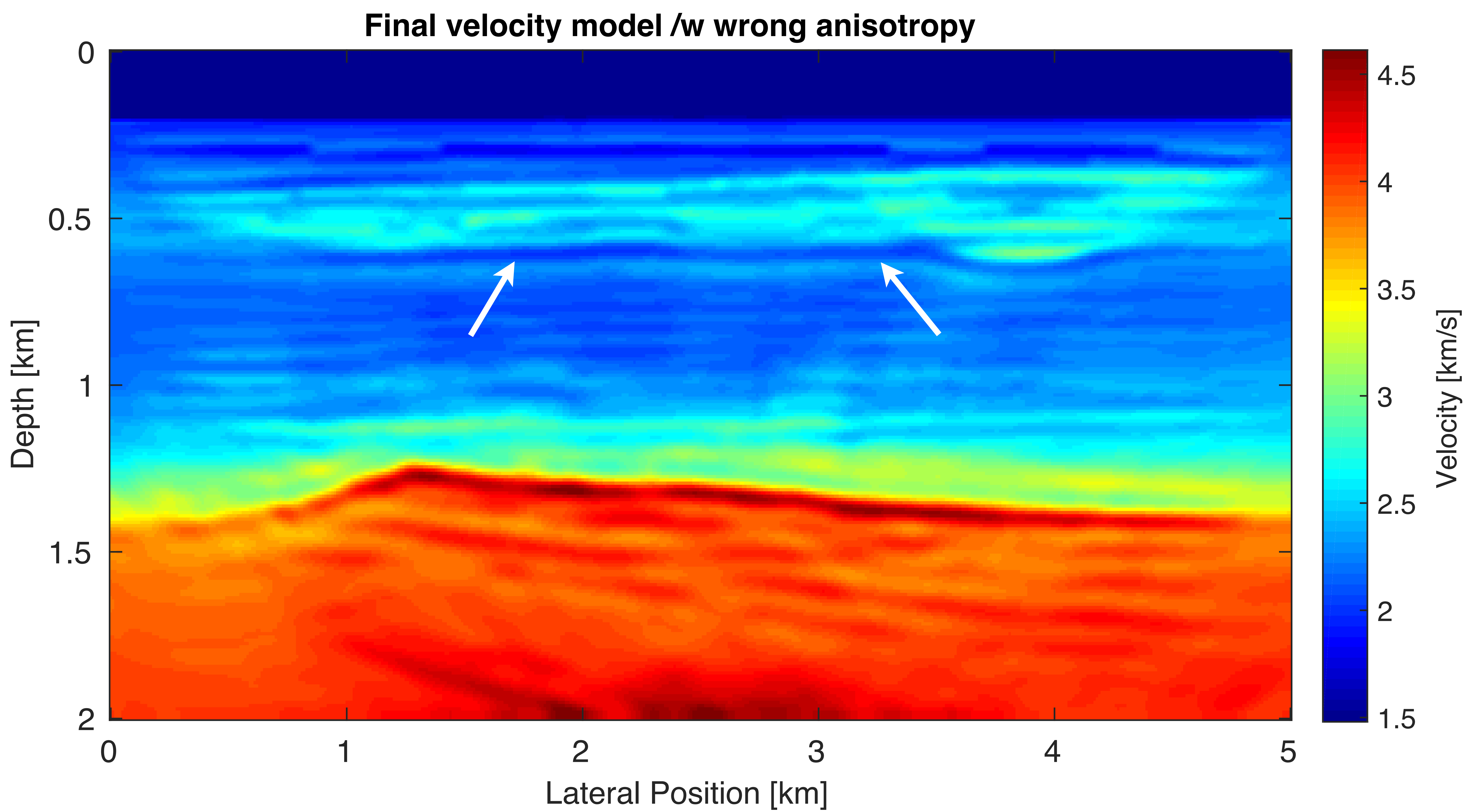
Final velocity model /w epsilon inversion
Depth [km]
0
0.5
1
1.5
2
0
1
2
3
4
5
Lateral Position [km]
Velocity [km/s]
4.5
4
3.5
3
2.5
2
1.5

Results



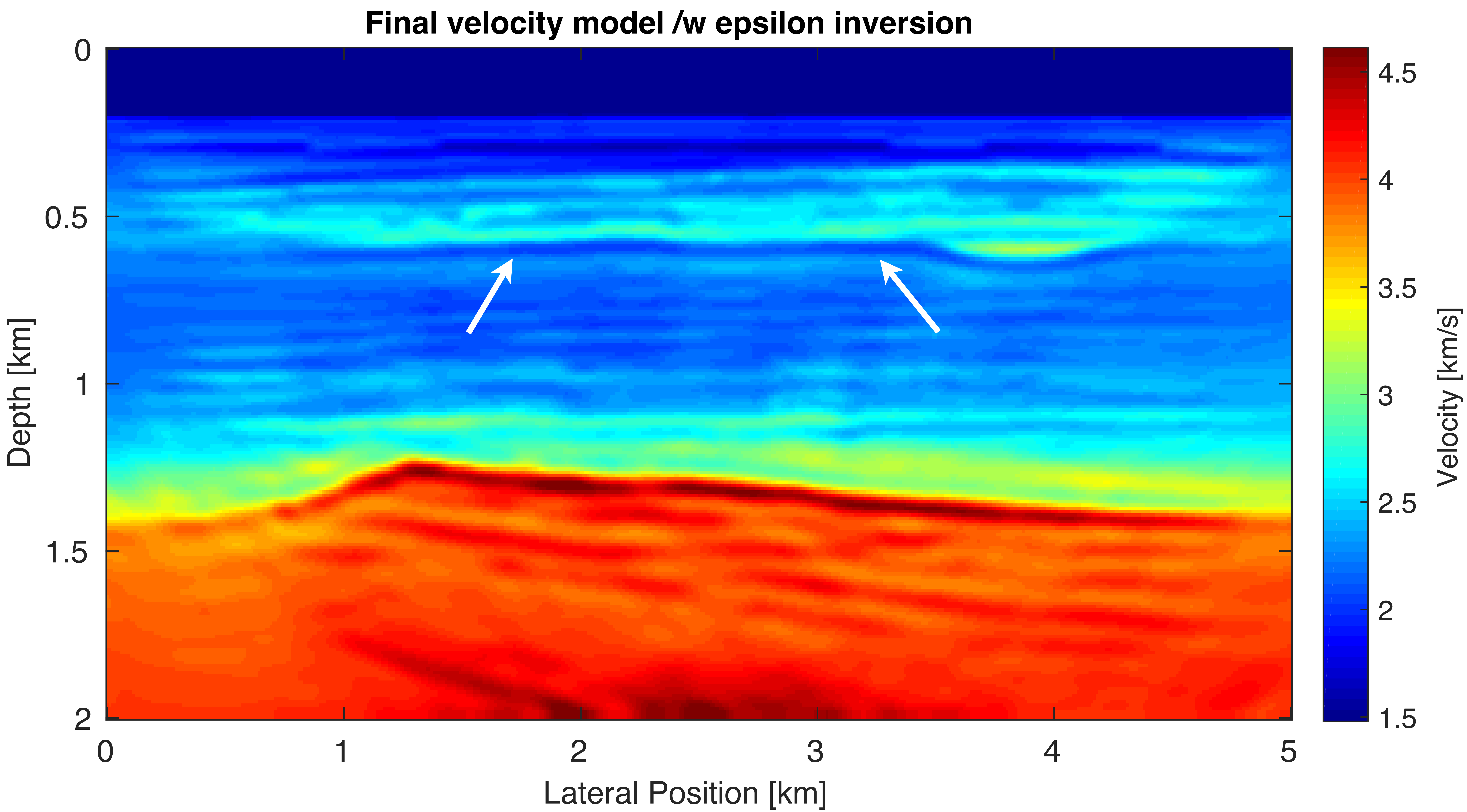
Final velocity model /w true anisotropy
0
0.5
1
1.5
2
Depth [km]
0
1
2
3
4
5
Lateral Position [km]
4.5
4
3.5
3
2.5
2
1.5
Velocity [km/s]

Results



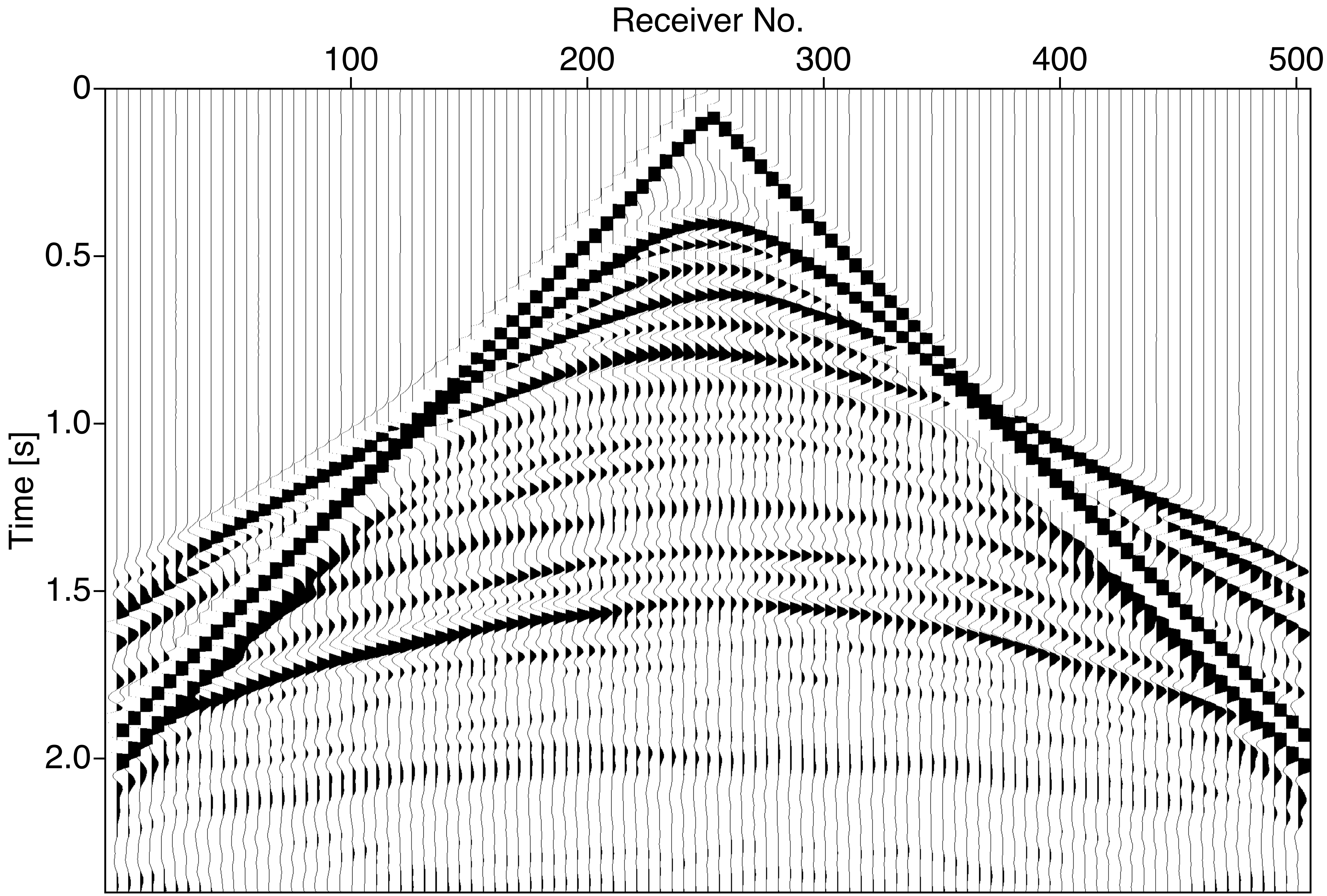
Final velocity model /w wrong anisotropy
Depth [km]
0
0.5
1
1.5
2
Lateral Position [km]
0
1
2
3
4
5
Velocity [km/s]
4.5
4
3.5
3
2.5
2
1.5

Results



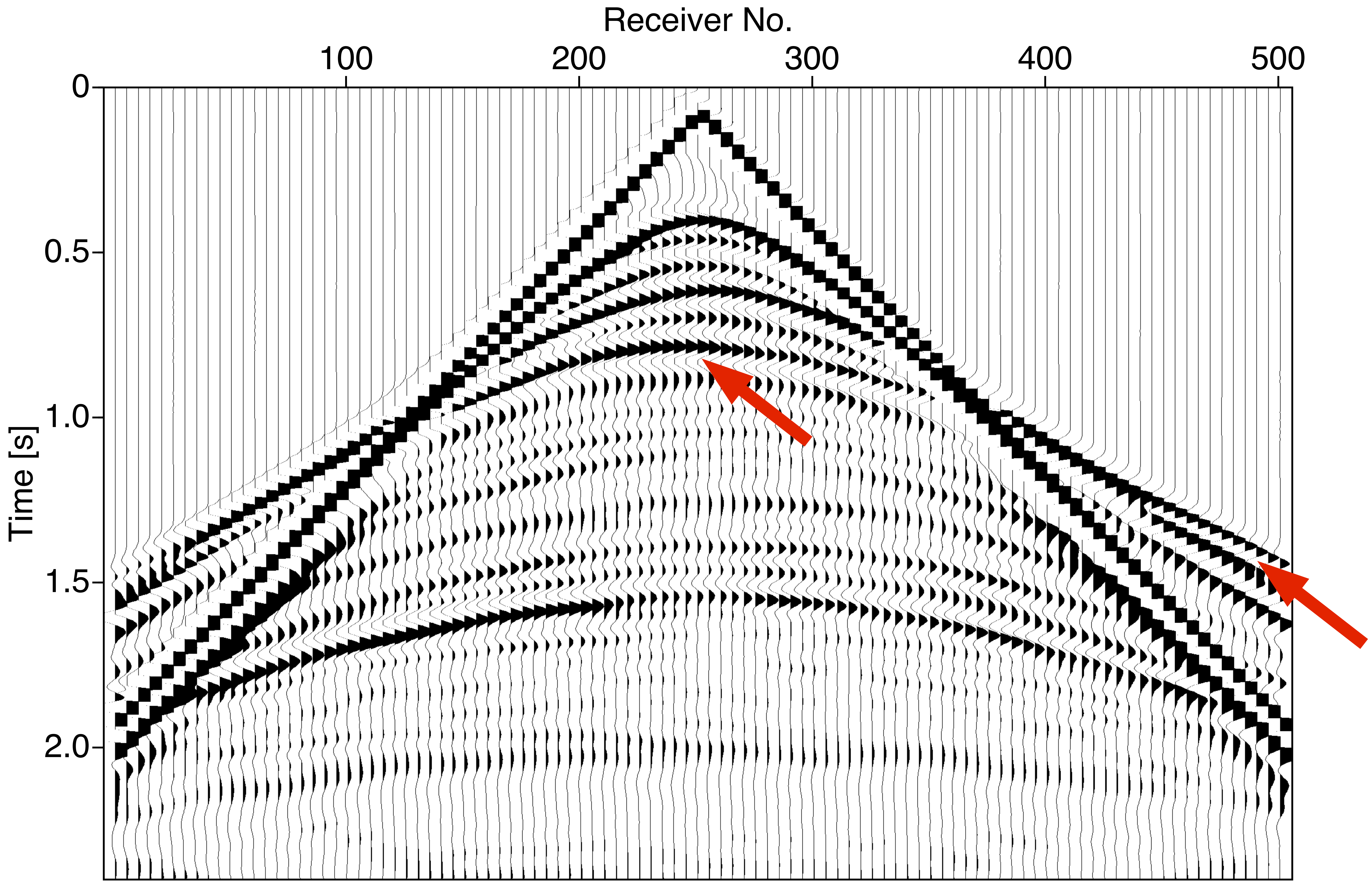
Final velocity model /w epsilon inversion
0
0.5
1
1.5
2
Depth [km]
0
1
2
3
4
5
Lateral Position [km]
4.5
4
3.5
3
2.5
2
1.5
Velocity [km/s]

Shot records



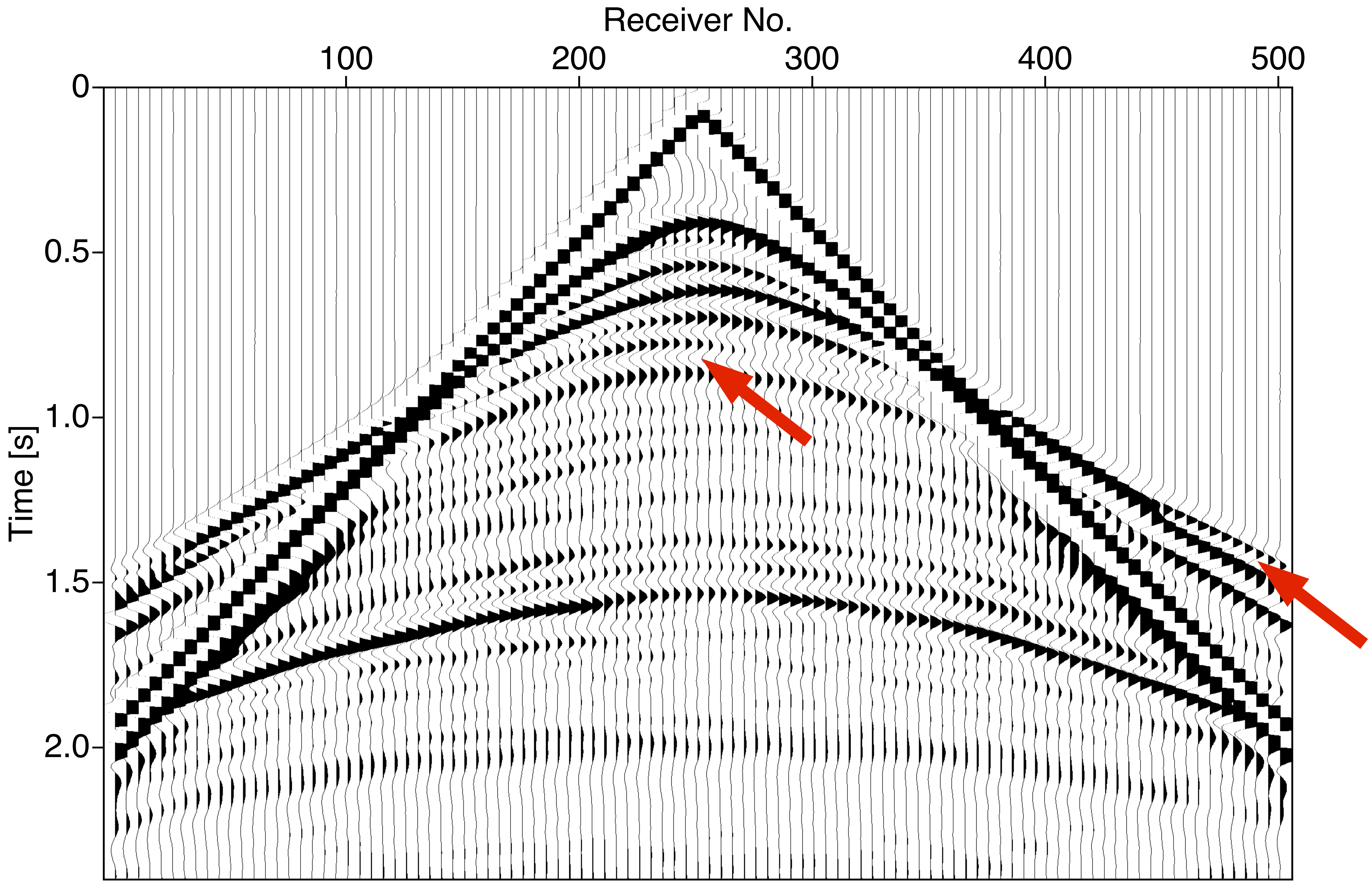
Observed shot record

Shot records



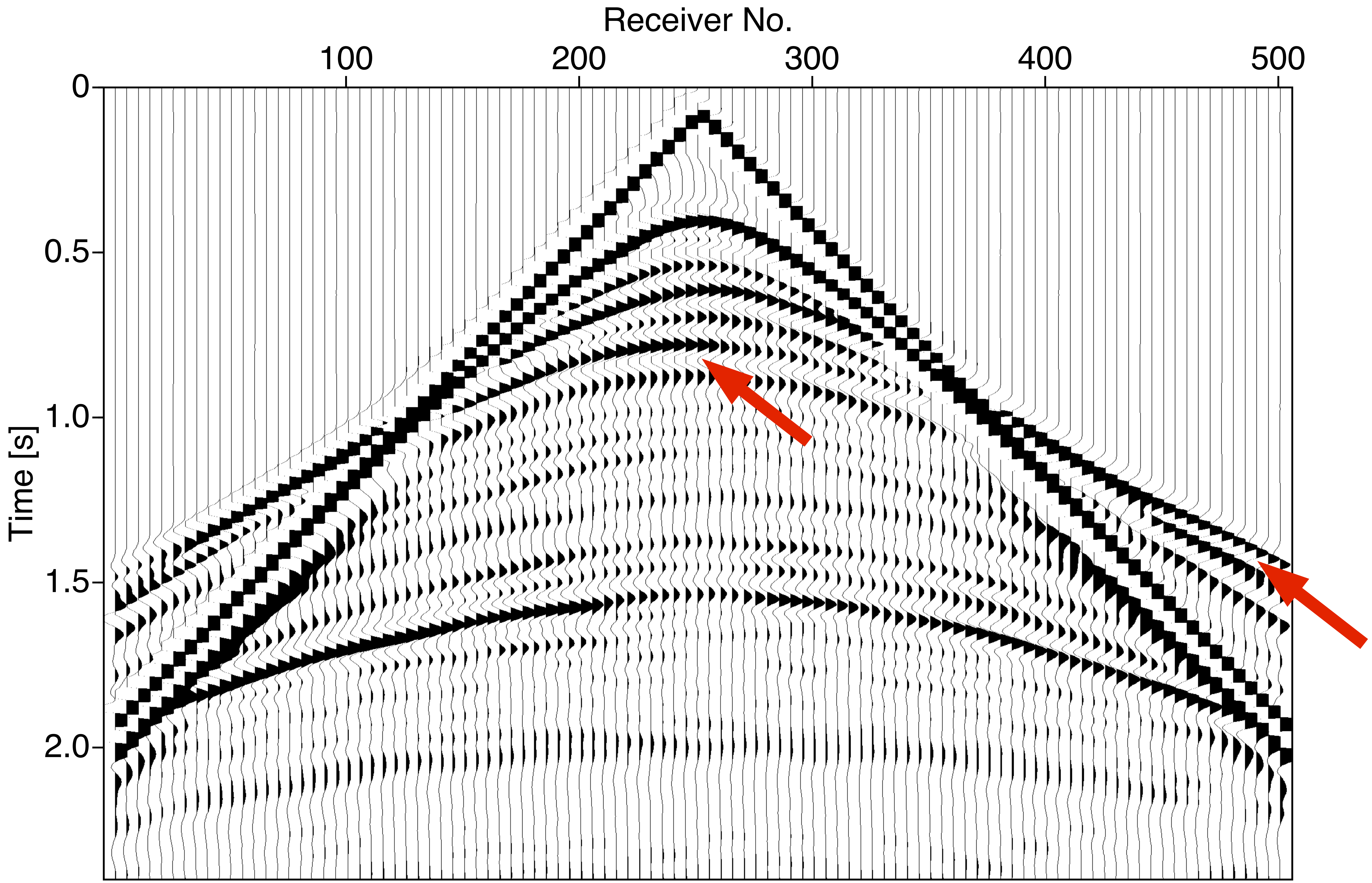
Final shot record /w correct anisotropy

Shot records



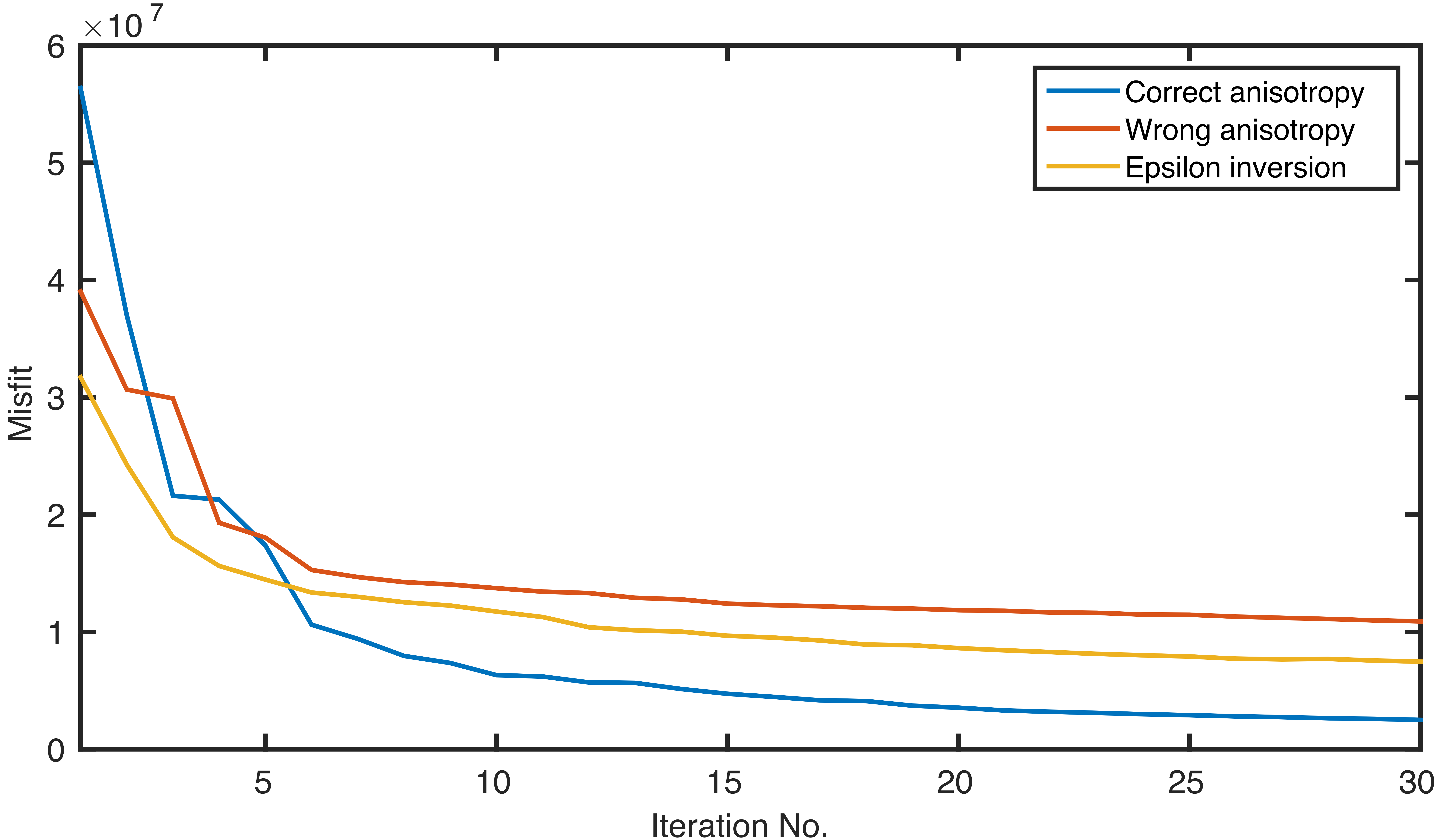
Final shot record /w wrong anisotropy

Shot records



Final shot record /w epsilon inversion

Misfit



×10^7
6
5
4
3
2
1
0
Misfit
5
10
15
20
25
30
Iteration No.
Correct anisotropy
Wrong anisotropy
Epsilon inversion

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Alternative derivative operators

Pseudo-spectral methods accurate but expensive

- 8 FFTs for 2D, 22 for 3D in general case

Possible alternative: Eigen-decomposition Pseudo-Spectral (EPS) method

- derivative operator as integral operator

$$Lf(x) = \int_a^b K(x,y) f(y) dy$$

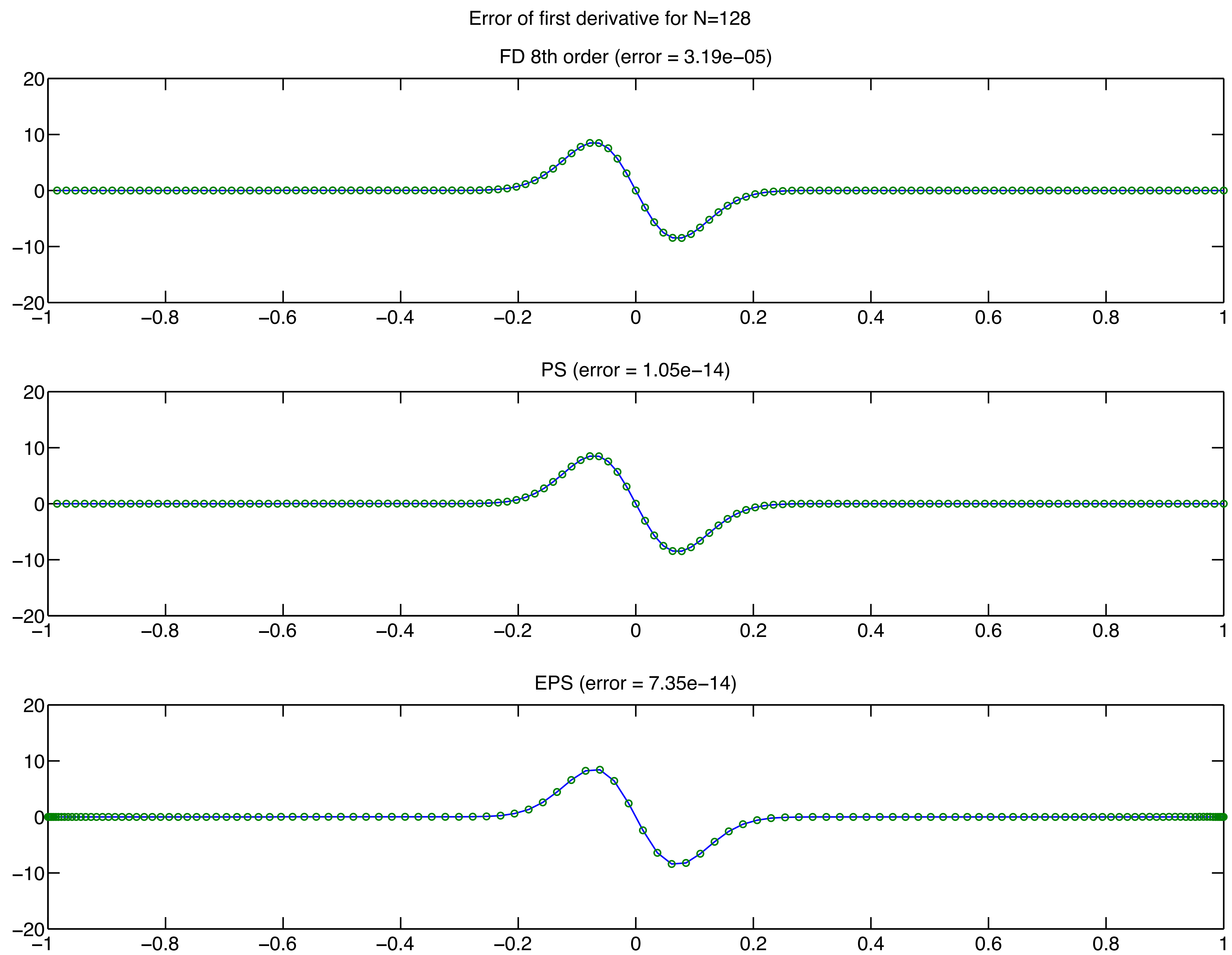
- Kernel function $K(x,y)$ constructed from eigenfunctions of derivative operator

(Sandberg and Wojciechowskit, 2011, The EPS method: A new method for constructing pseudospectral derivative operators)

Alternative derivative operators

- Example for 1D first derivative operator
- Test function:

$$f(x) = e^{-100x^2}$$



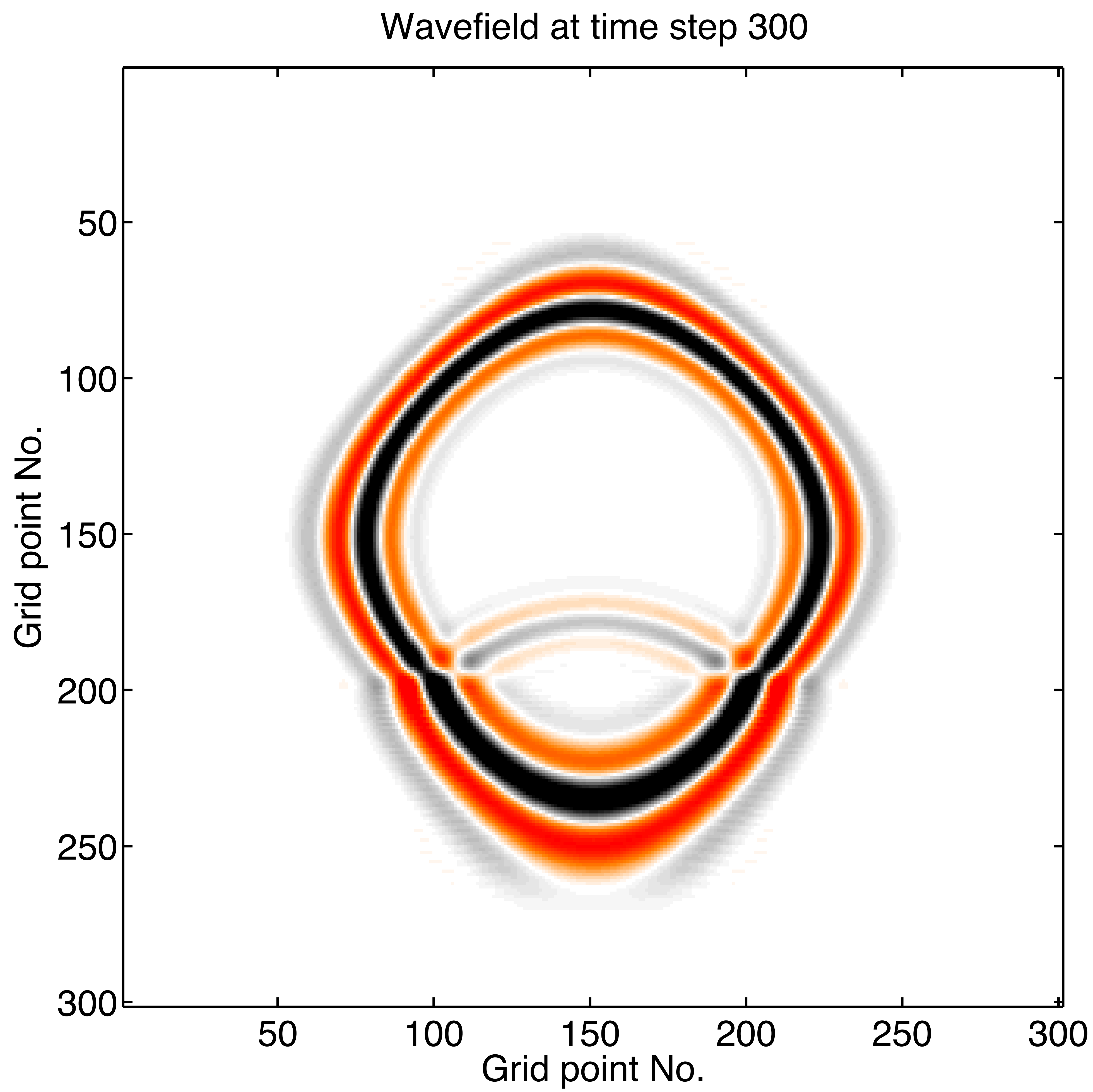
Alternative derivative operators

2D derivative operators:

$$\mathbf{L_x} = \mathbf{D_{xx}} \otimes \mathbf{I_{zz}}$$

$$\mathbf{L} = \mathbf{L_x} + \mathbf{L_z}$$

Model 2D wave equation (non regular grid)



Alternative derivative operators

EPS derivative operator:

- small norm ➡ allows larger time steps + close to optimal spatial sampling
- accuracy in range of machine precision
- is a dense matrix
- requires special algorithms to be applied efficiently (e.g. partitioned low rank representation)

(Sandberg and Wojciechowskit, 2011, The EPS method: A new method for constructing pseudospectral derivative operators)

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Conclusion

Time-domain modeling and inversion code with matrix based operators:

- simple extension to anisotropy (2D TTI with PS method)
- exact Jacobians and adjoint Jacobians
- run existing codes for RTM, LSRTM, FWI in anisotropic mode by passing additional argument with Thomsen parameters
- first steps towards multi-parameter FWI

Outlook

Current issues:

- PS method is computationally expensive
- crosstalk/low epsilon sensitivity in multi-parameter FWI

Future steps

- construction of EPS operators for mixed wavenumber terms required in TTI equation + fast application algorithms (HSS etc.)
- alternate wave equation parametrization
- anisotropic LSRTM, FWI on field data sets

Acknowledgements

Thank you for your attention!

https://www.slim.eos.ubc.ca

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