Improving Full-Waveform Inversion with Spectral Extrapolation

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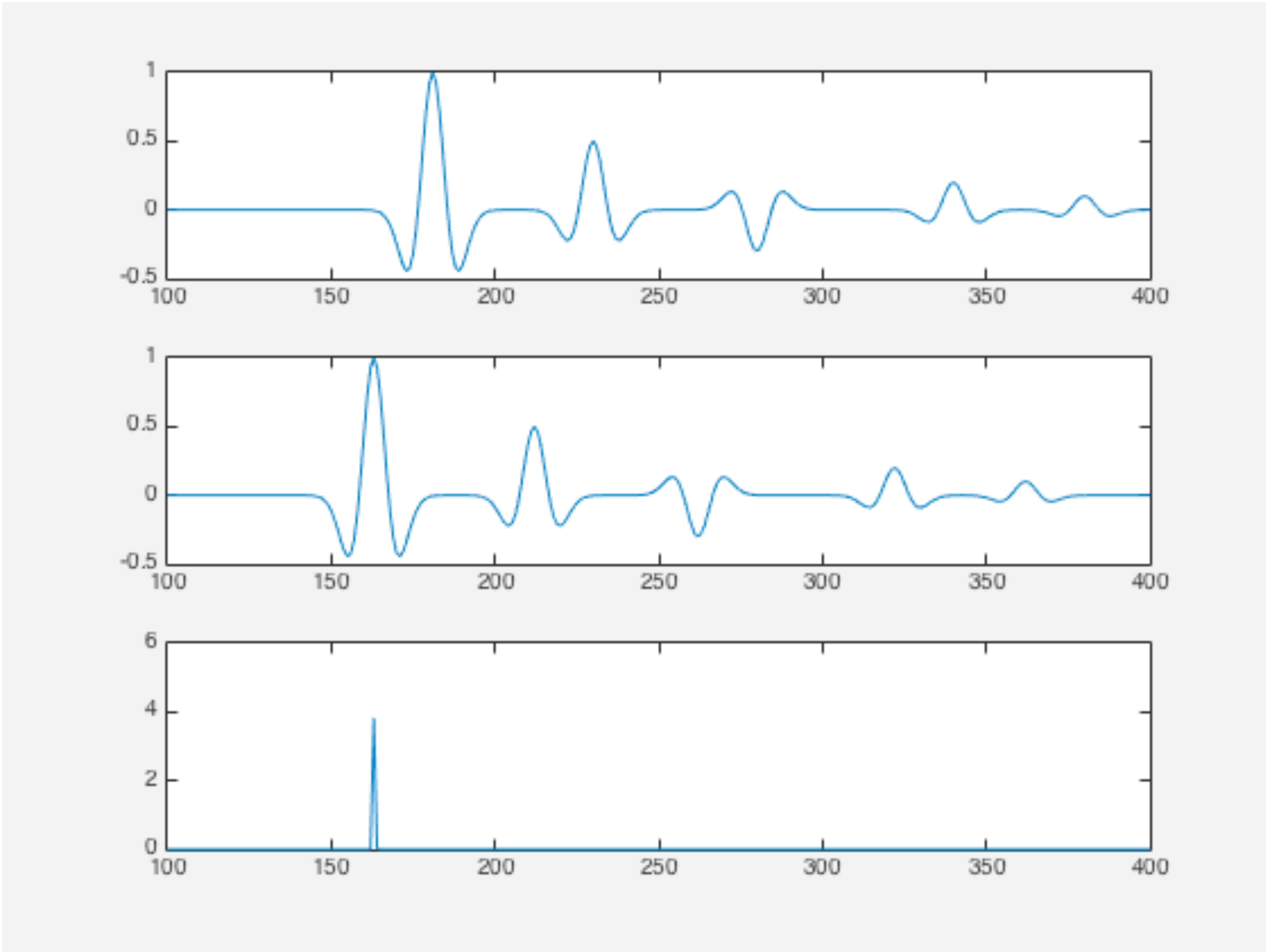
Outline

- Motivation
- The proposed method
- Numerical results
- Conclusions

Motivation

Challenges in FWI:

- High frequency data introduces abundant local minima.
- Field data lacks low frequency information, or the low frequency data is very noisy.



Local minima in the objective function

Motivation

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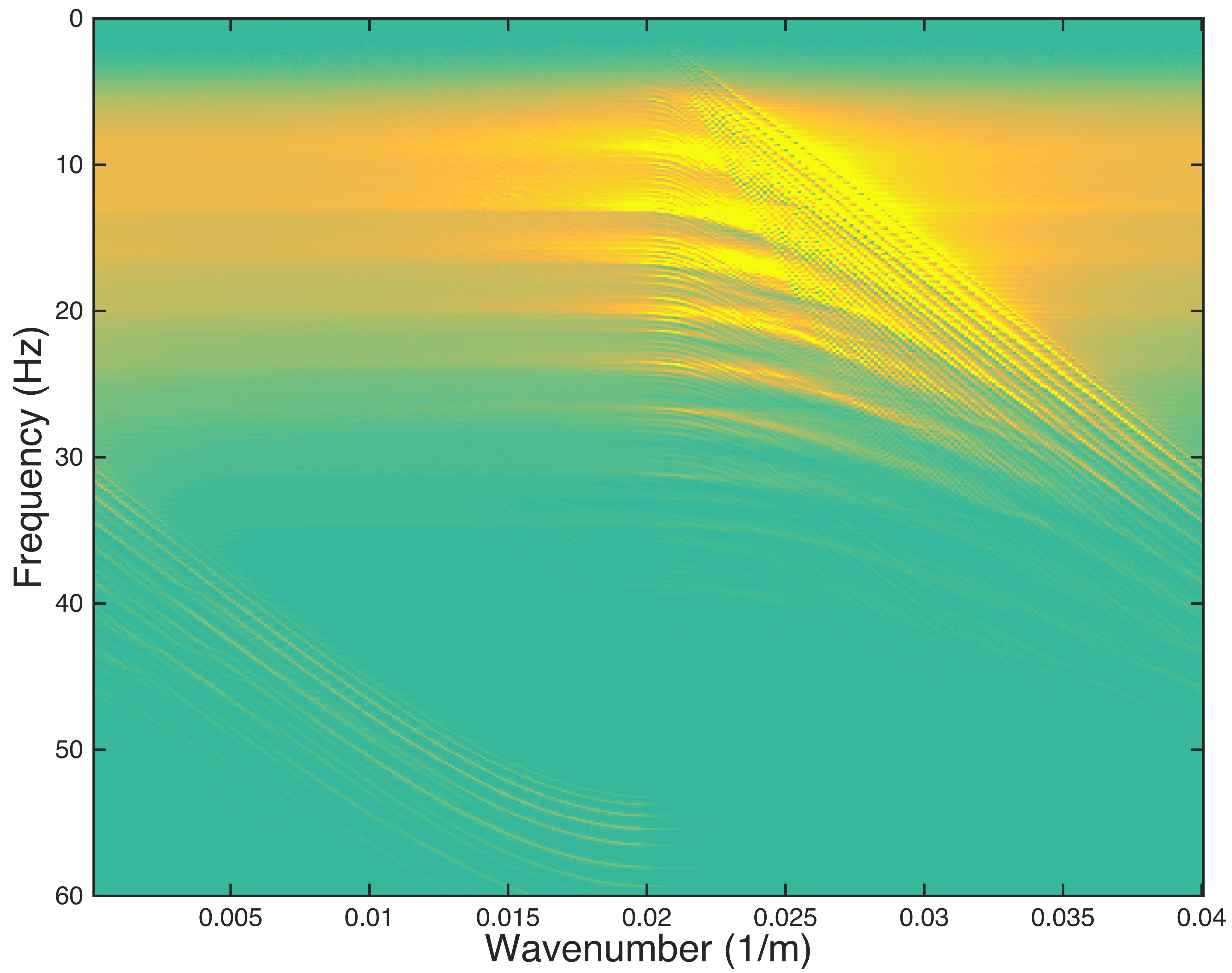
A shot gather in the f-k domain Chevron (2014)



Motivation

Challenges in FWI:

- High frequency data introduces abundant local minimum.
- Field data lacks low frequency information, or the low frequency data is very noisy.

Our goal:

To use the mid-band clean data to extrapolate towards the low frequencies.

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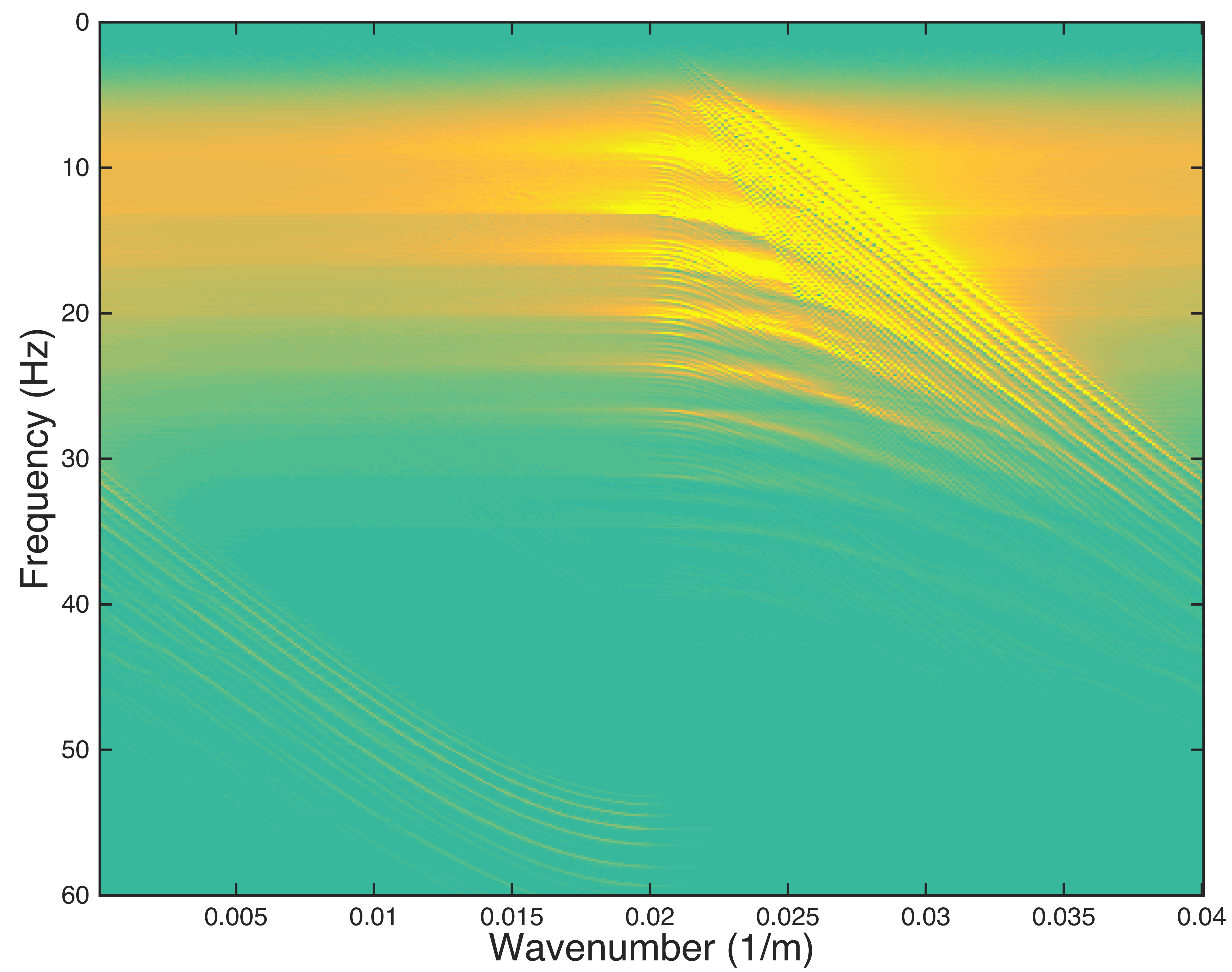

Problem description

Convolution model:

$$d(t) = w(t) * G(t)$$

Trace Source Green's function (unknown)

Assume:

$$G(t) = \sum_{i=1}^{s} a_i \delta_{t_i}(t)$$

$$\hat{G}(\omega) = \sum_{i=1}^{s} a_i e^{2\pi i t_i \omega}$$

s: number of events

Classical methods

Two classical methods to identify sums of exponentials:

- Multiple Signal Classification (MUSIC)
- Linear Programming (LP)

Other approaches:

- Autoregressive method (AR) [Lines and Clayton, 1977]
- Minimum Entropy Method (MEM) [Wiggins, R. A., 1978]

Classical methods

Two classical methods to identify sums of exponentials:

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Classical methods

Multiple signal classification (MUSIC):

- separates signal from white Gaussian noise
- uses an eigenspace method
- finds the frequencies by identifying peaks in the signal-noise correlation function
- needs prior knowledge of the number of frequencies

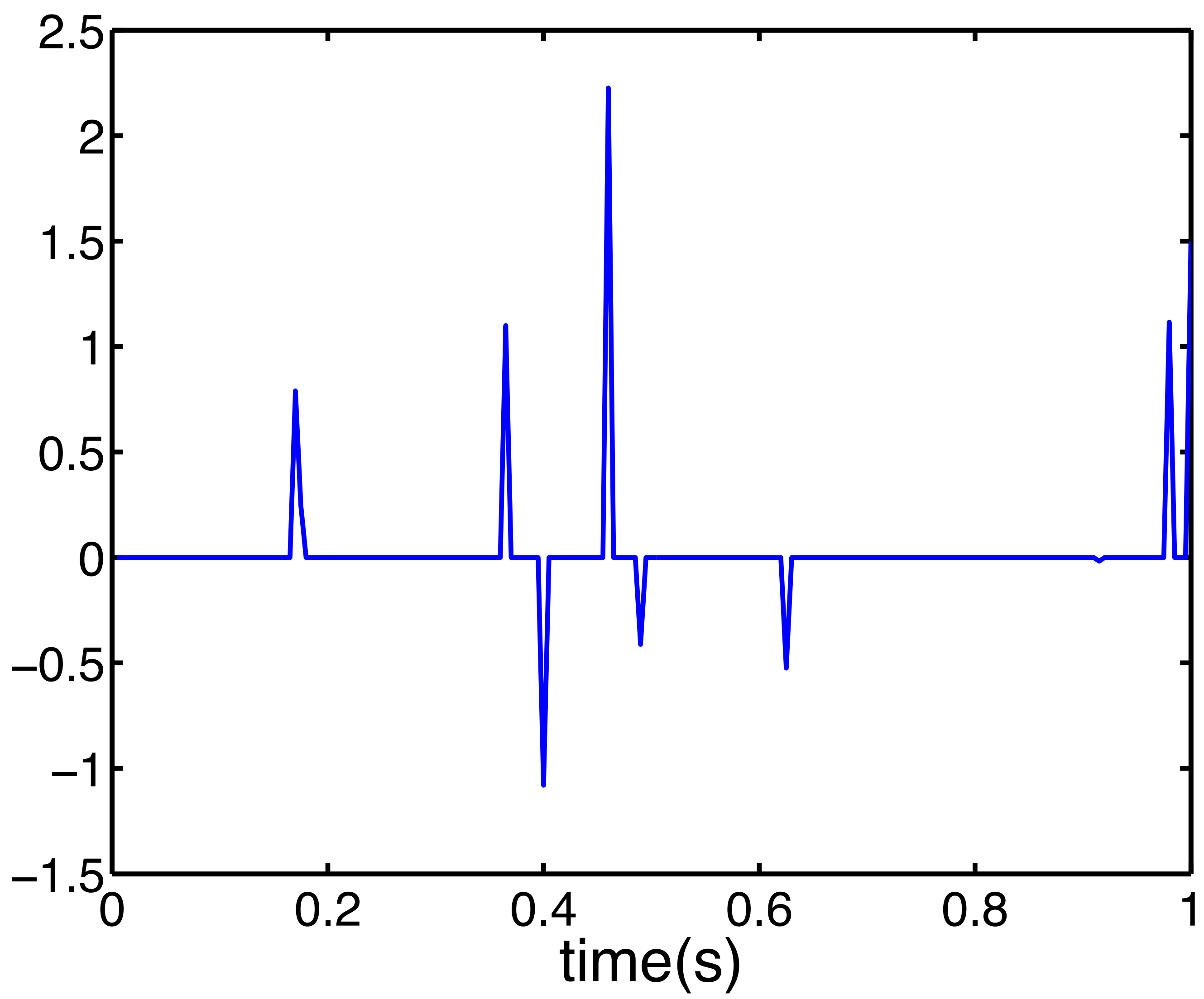
Classical methods

Linear Programming (Basis Pursuit):

$$\min \|G(t)\|_1 \;\; \text{subject to } d = w * G$$

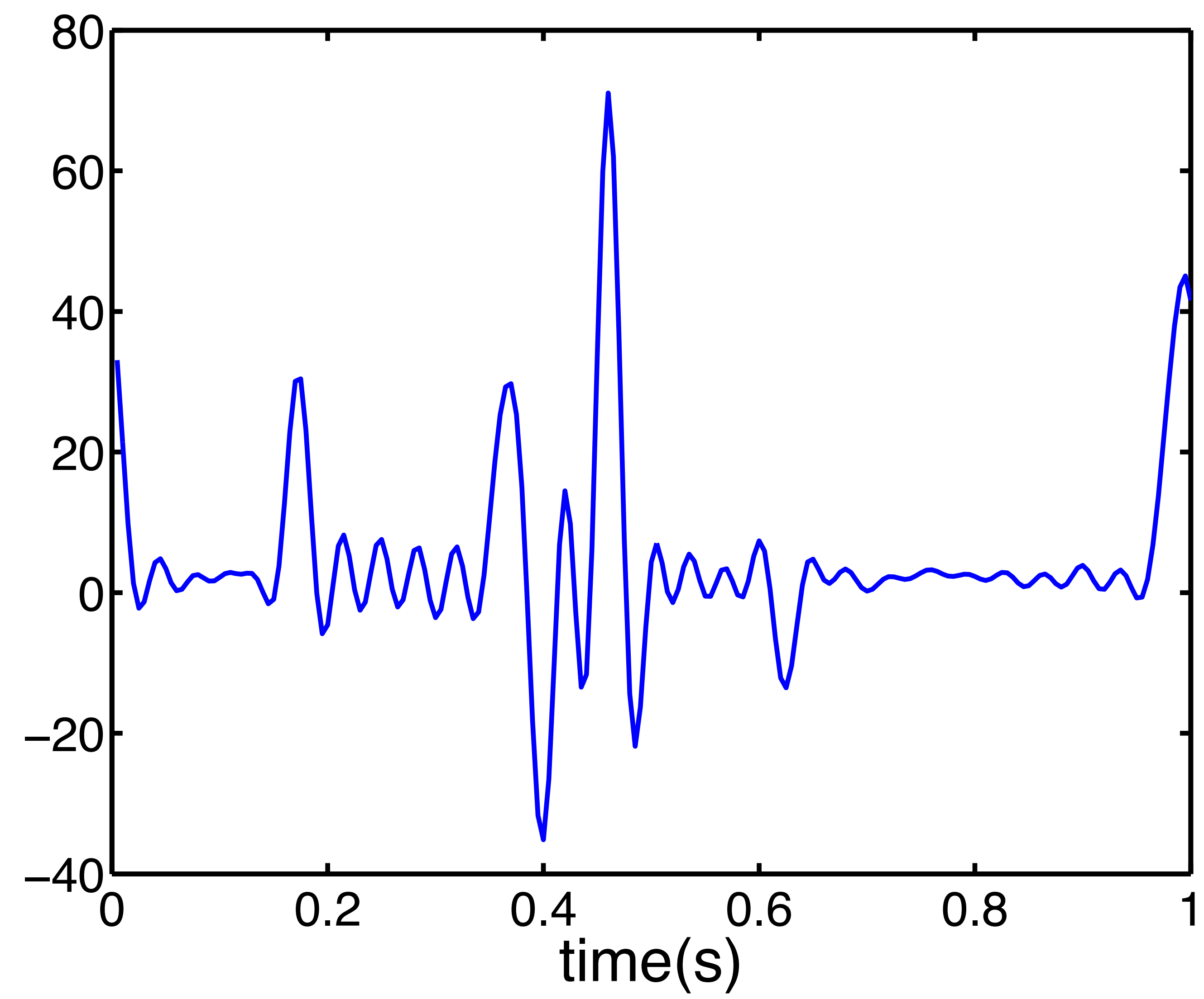
- No auxiliary parameter
- Works best when a random set of frequency coefficients are known (Compressed Sensing)
- Works under certain assumptions when only the low frequency is known (Super-resolution) [Candes and Fernandez-Granda, 2012]

Comparison of MUSIC and L1 – Toy example



Original wide-band signal x

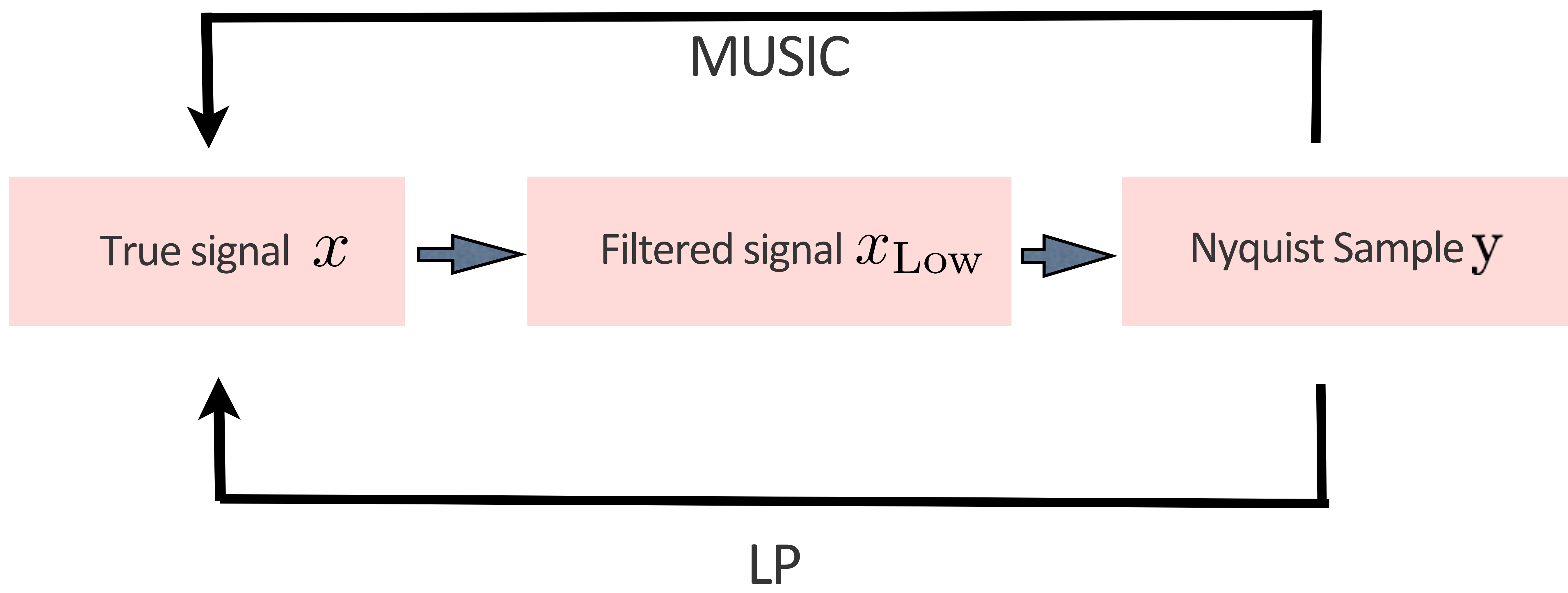
0-100Hz



Low pass filtered signal x_{Low}

0-30Hz

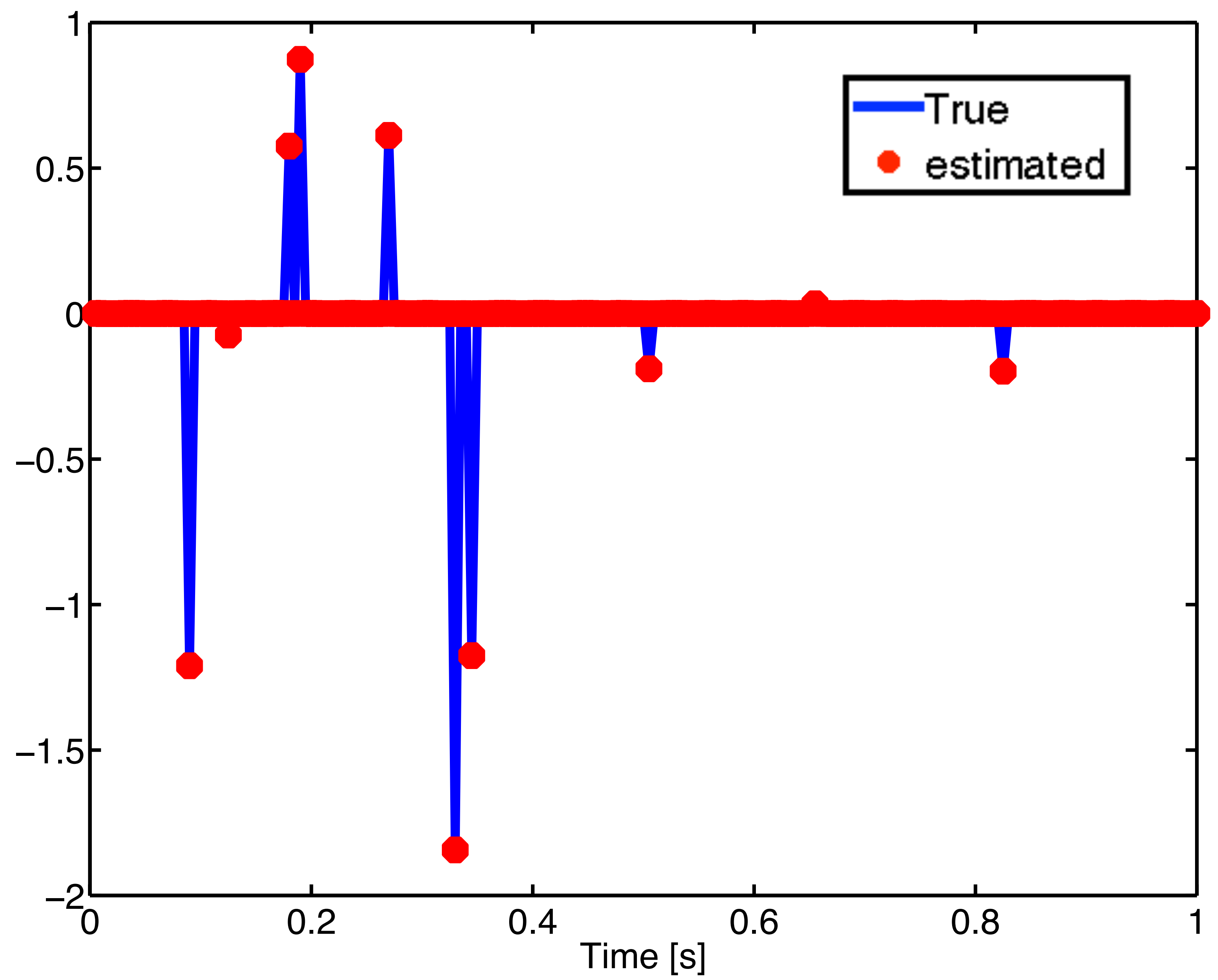
Comparison of MUSIC and L1 – Toy example



Recovery via LP

Necessary condition for accurate recovery

- Spikes of opposite signs cannot be too close.



LP returns an accurate recovery when spikes with opposite signs have enough separations.

Recovery via LP

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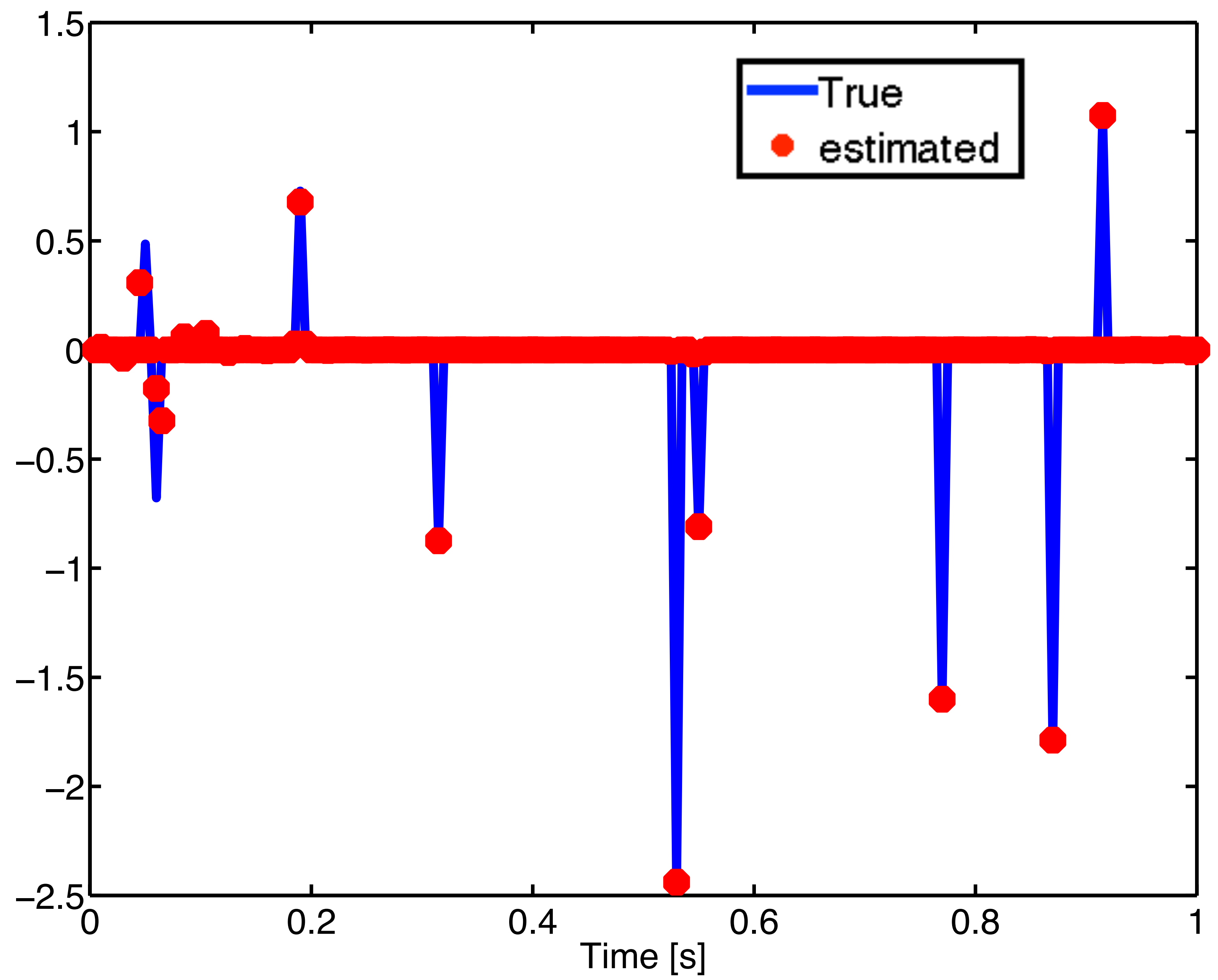


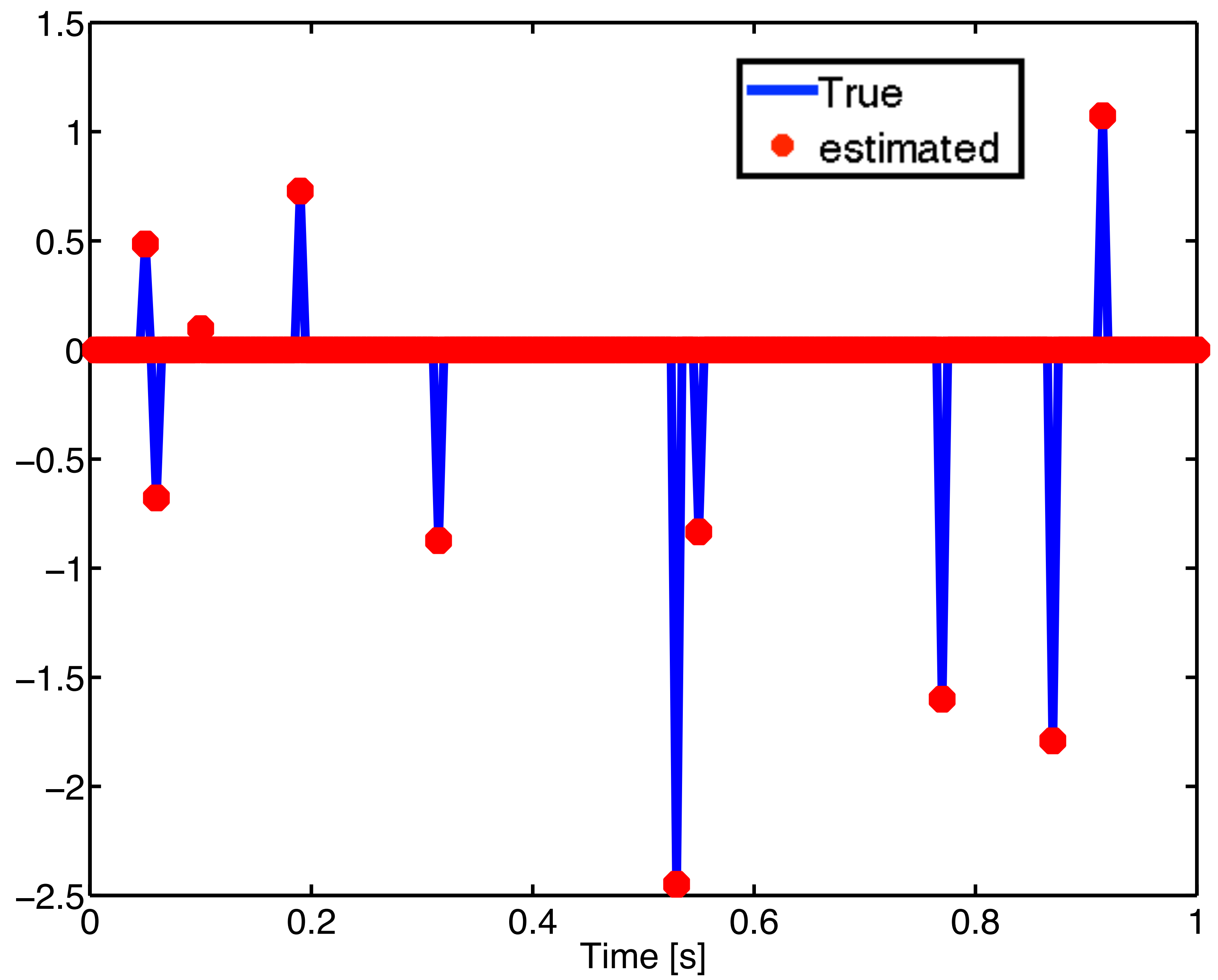
LP returns an inaccurate recovery when spikes with opposite signs do not have enough separations.

Recovery via MUSIC

MUSIC works for clean data ubiquitously as long as the number of measurements obeys $m \geq 2s + 1$.

number of samples

number of spikes

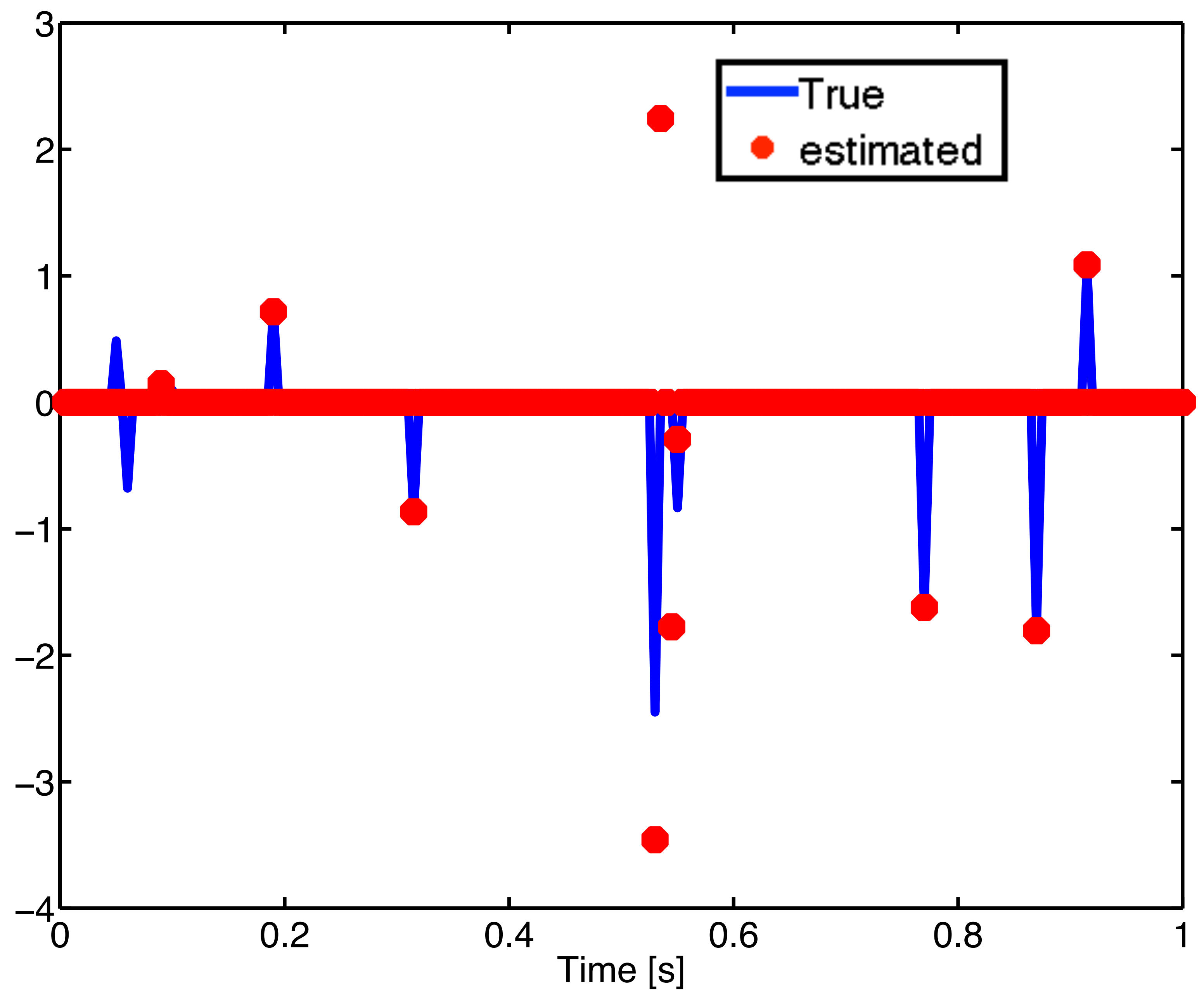


MUSIC accurately recovers the previous signal where LP failed.

Recovery via MUSIC (noisy data SNR=30dB)

MUSIC is not very stable to noise:

SNR=30dB random Gaussian noise

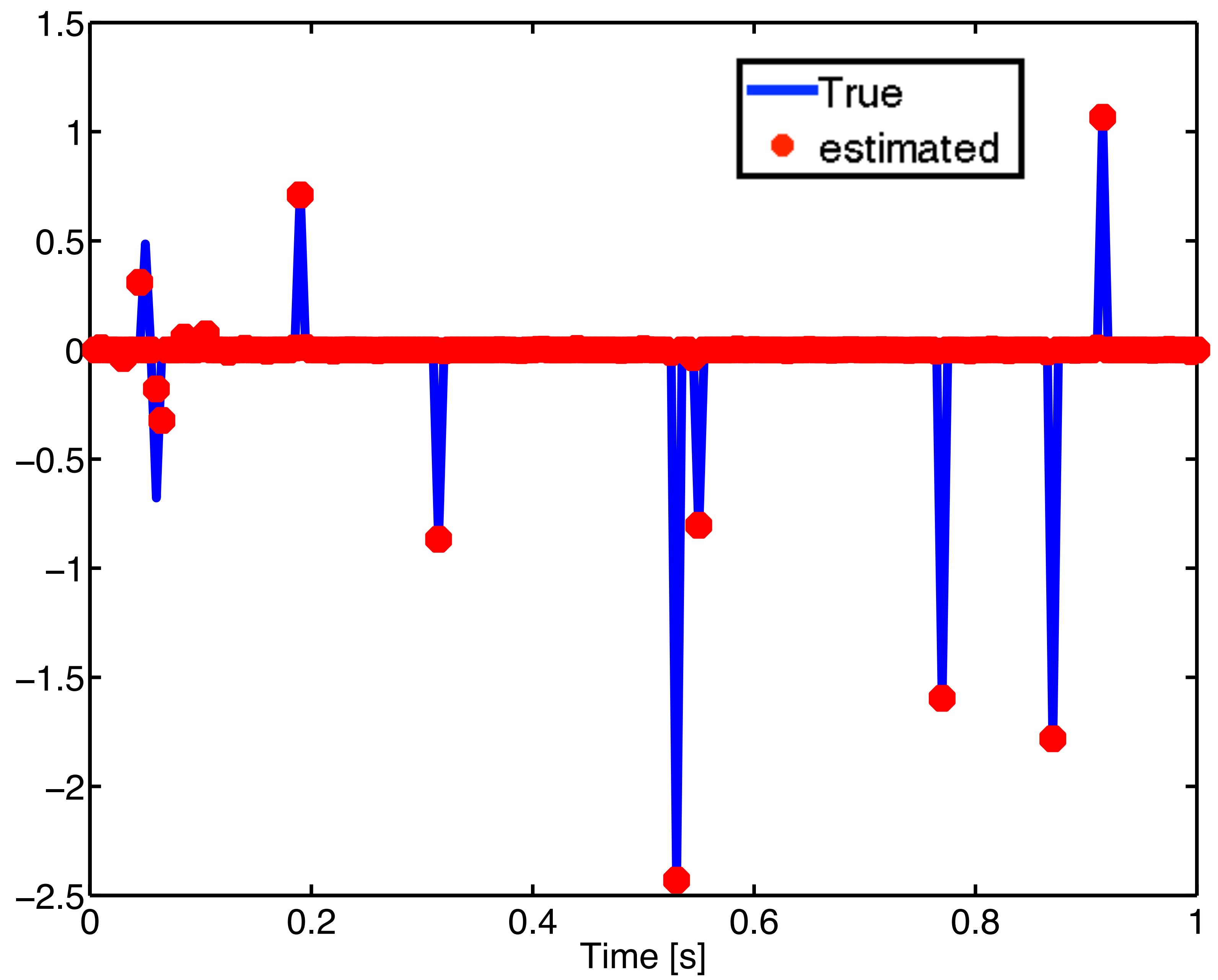


Noise has greatly affected the support detection of MUSIC.

Recovery via LP (noisy data SNR=30dB)

LP is more stable to noise:

SNR=30dB random Gaussian noise

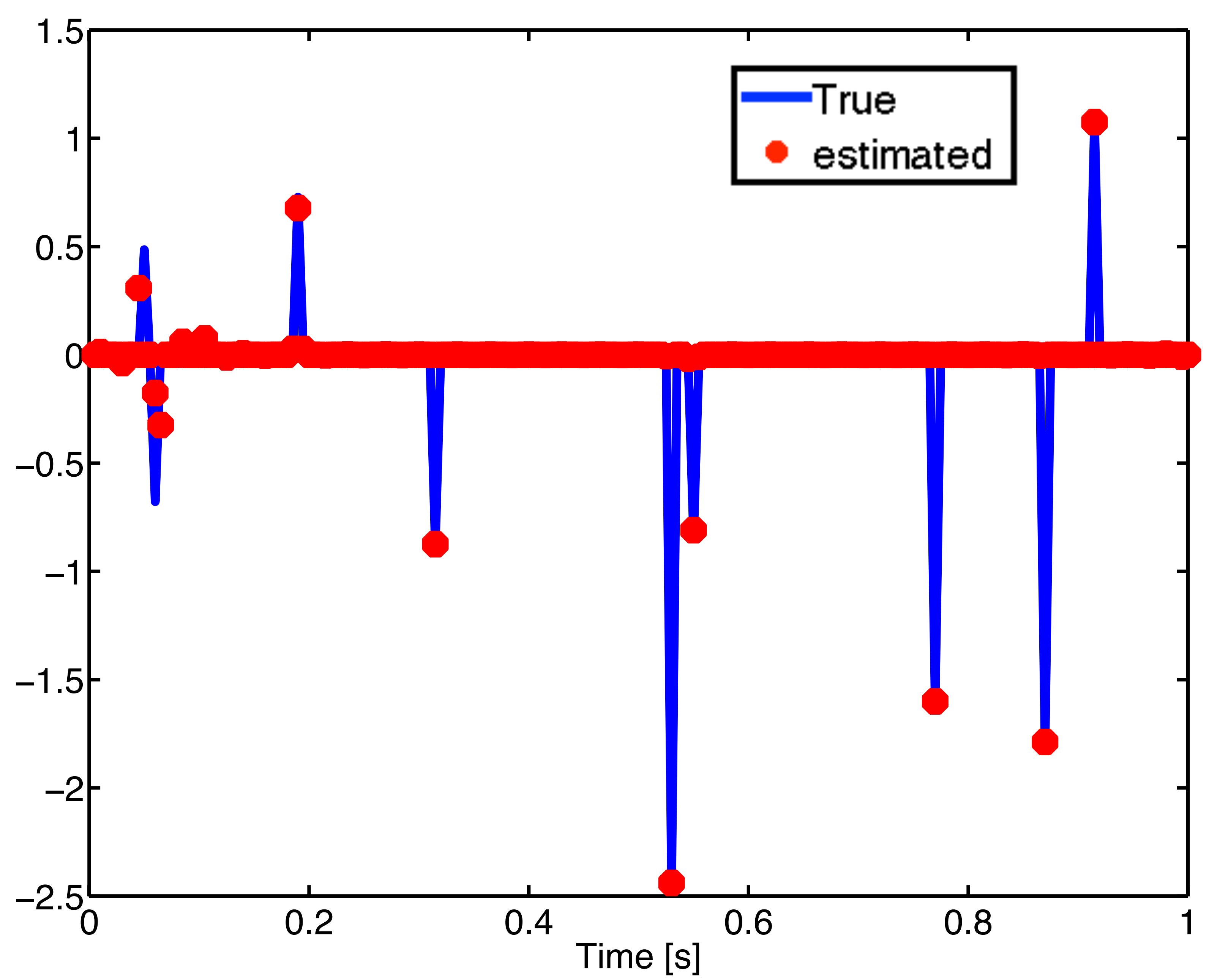


Recovery of LP from noisy data is similar to that of the clean data.

Recovery via LP (clean)

LP is more stable to noise:

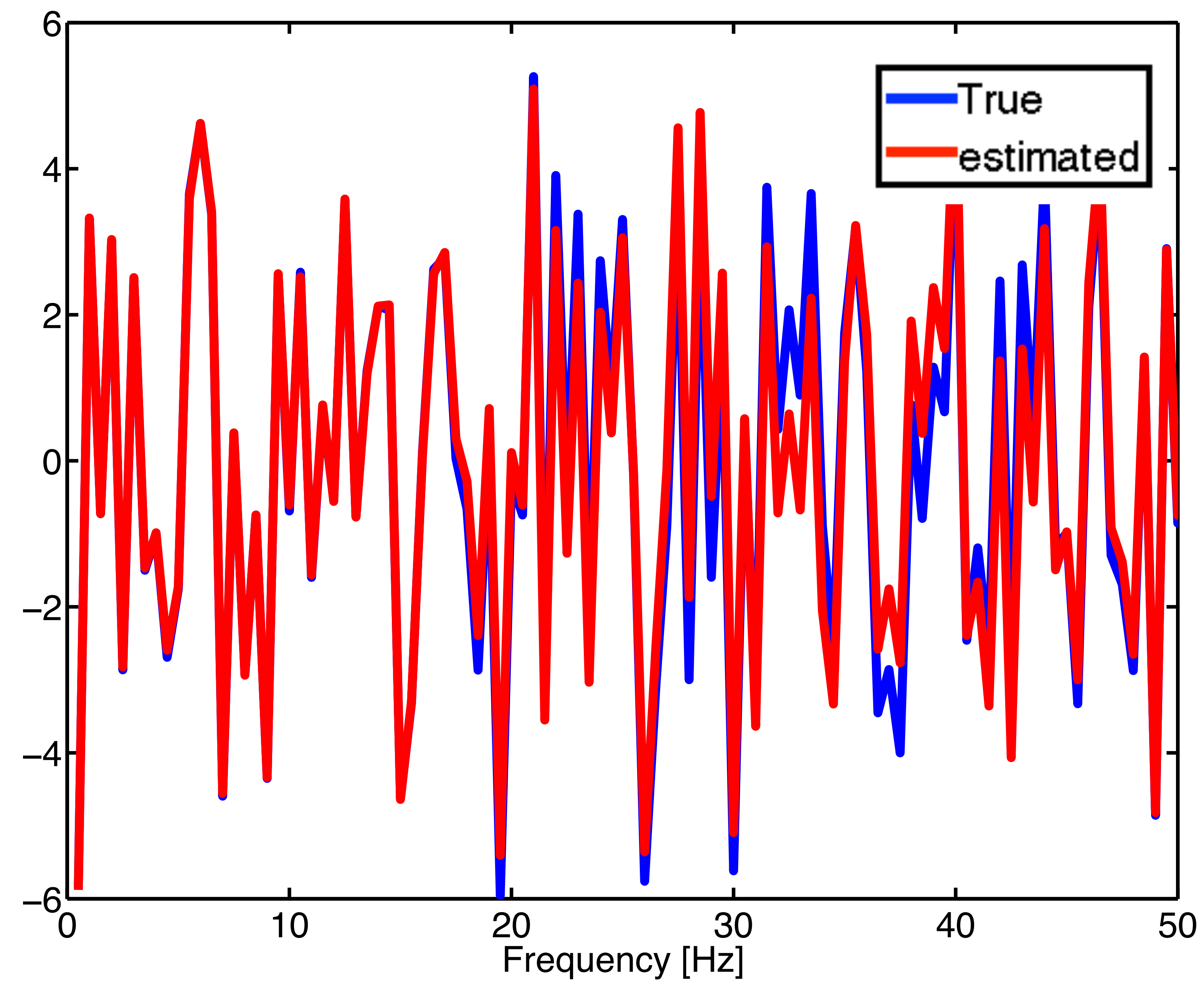
Clean data



Recovery via LP (noisy data SNR=30dB)

LP is more stable to noise:

Spectrum of the recovery

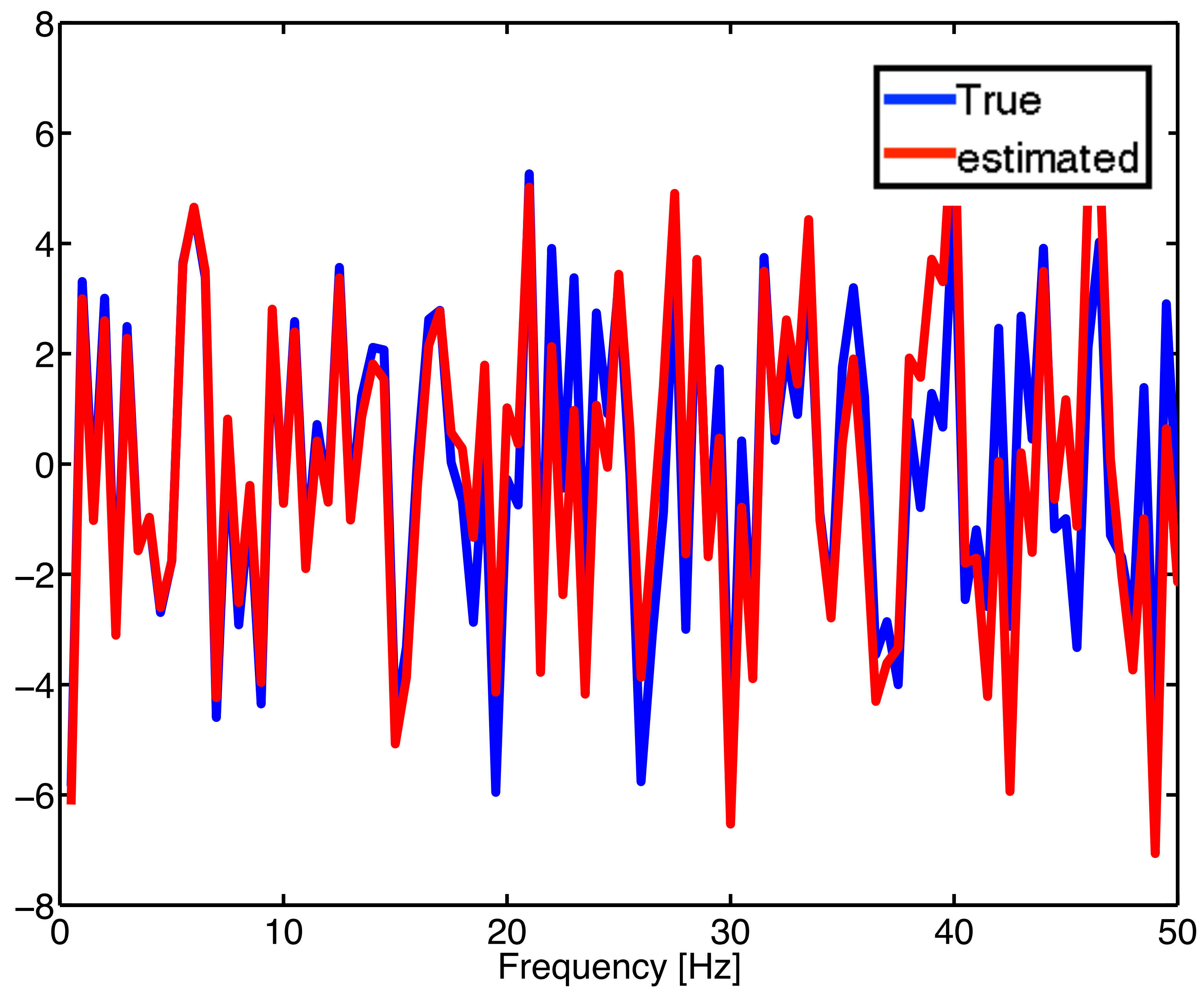


Error mostly comes from the unresolvable nearby spikes.

Recovery via MUSIC (noisy data SNR=30dB)

MUSIC is not stable to noise:

Spectrum of the recovery

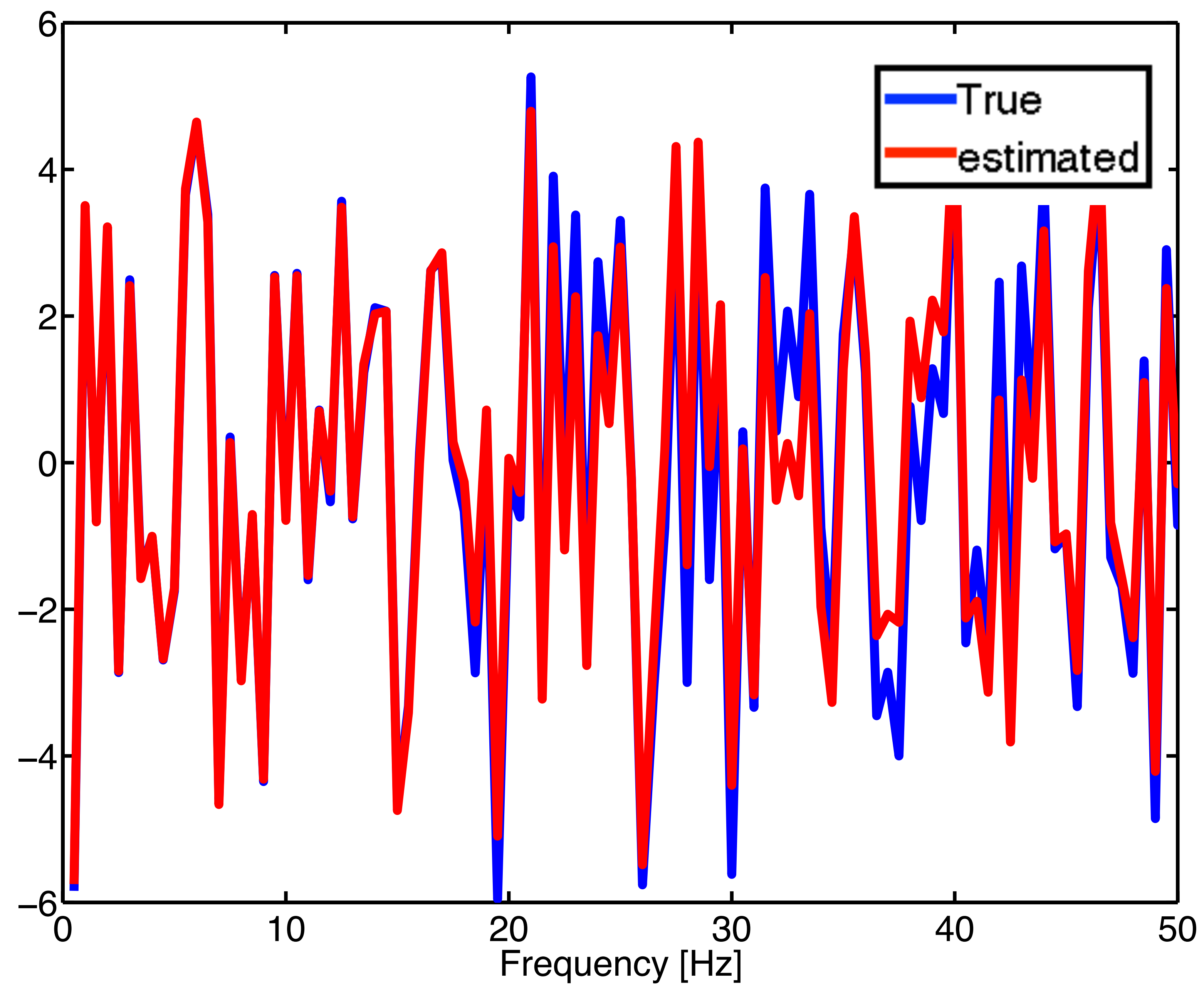


Error mostly arises from noise.

Recovery via LP (noisy data SNR=25dB)

LP is more stable to noise:

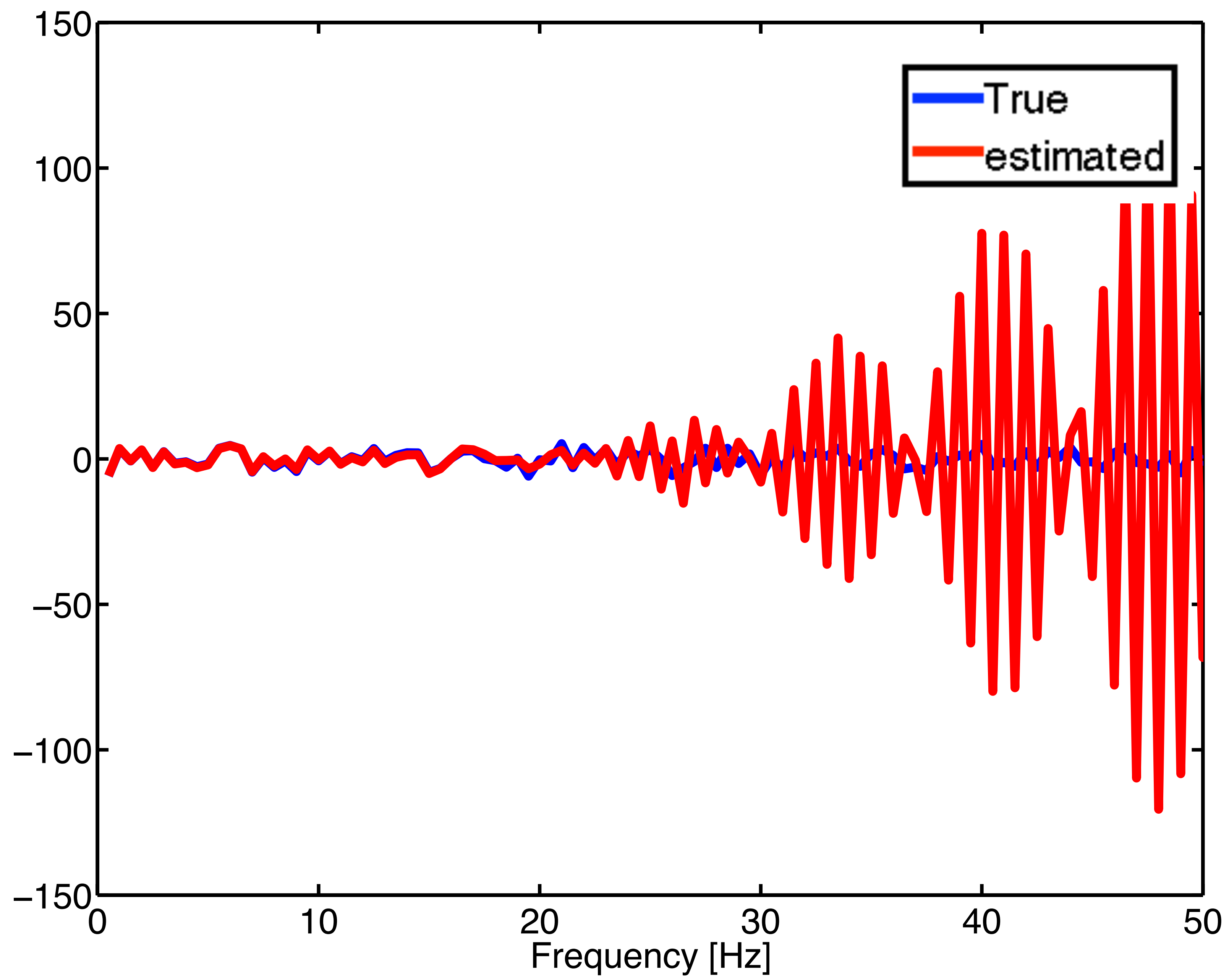
Spectrum of the recovery



Recovery via MUSIC (noisy data SNR=25dB)

MUSIC is not stable to noise:

Spectrum of the recovery



Comparison between MUSIC and L1

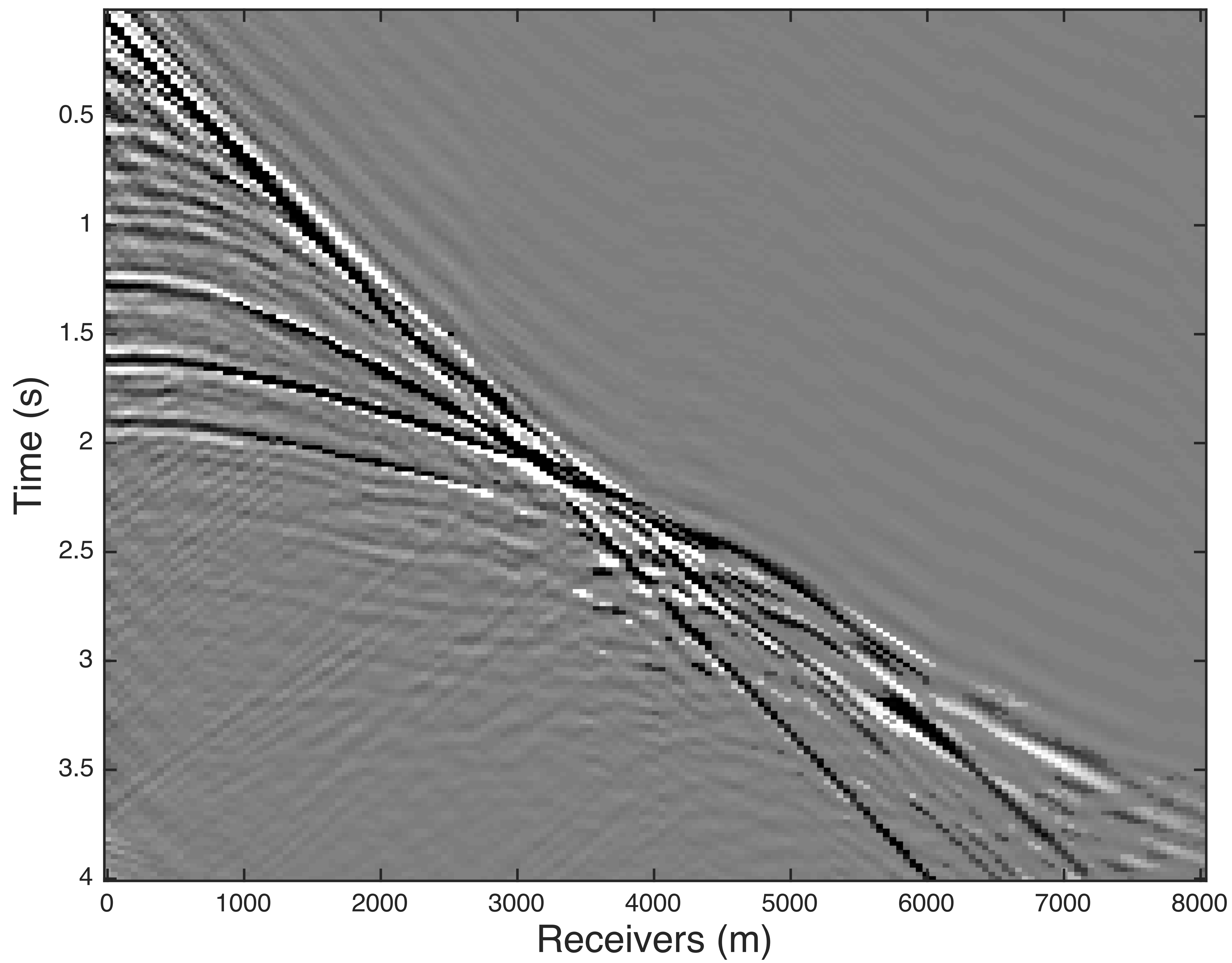
	Complexity	Number of Frequency samples	Parameter sensitivity	Noise resiliance	Need the Separation of Spikes condition
MUSIC	$O(ns^2)$	$2s + 1$	high	low	no
L_1	superlinear in n	$s \log n$	no auxiliary parameter needed	high	yes

The proposed method

A curvelet based L_1 -minimization:

- The Green's function of a single trace is sparse
- The Green's function of a shot record is sparse under the 2D Curvelet transform.
- The two sparsities can be imposed by using a joint L1 minimization objective.
- The resulting optimization problem is convex.

A visualization of the 2D Green's function



The proposed method

Let C be the 2D Curvelet transform, we solve the joint sparsity problem:

$$\hat{G} = \arg\min_{\mathbf{G}} \|\mathbf{G}W\|_{1,1} + \lambda\|C\mathbf{G}W\|_{1,1}$$
$$\text{subject to } \|\hat{w} \odot F_\Omega \mathbf{G} - \mathbf{d}_\Omega\| \leq \epsilon,$$
$$\text{for } \Omega = \{\omega_1, ..., \omega_N\}$$

- $\mathbf{G} = [G_1, ..., G_{nr}]$: Green's function of a common source
- F_Ω : partial Fourier matrix.
- $\|C\mathbf{G}\|_{1,1}$: ℓ_1 norm of the vectorization.
- W: : a trace balancing matrix

Low Frequency Extrapolation

Step 1: Estimate the Green's function.

Step 2: Find the phase of the low frequency data

$$\mathbf{d}_{extra} = F_{\Omega_{extra}} \hat{G}$$

Step 3: Find the correct magnitude

- Source estimation
- Estimating the scaling factor using information of the initial guess, i.e., Match the energy of $\mathbf{d}_{extra} := \mathcal{F}_{\text{extra}} \mathbf{u}$ to $\mathbf{d}_0 = \mathcal{F}_{\text{extra}} A(m_0)$

[S. Becker, J. Bobin, and E, Candes (2009)]

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Algorithm:

The NESTA algorithm

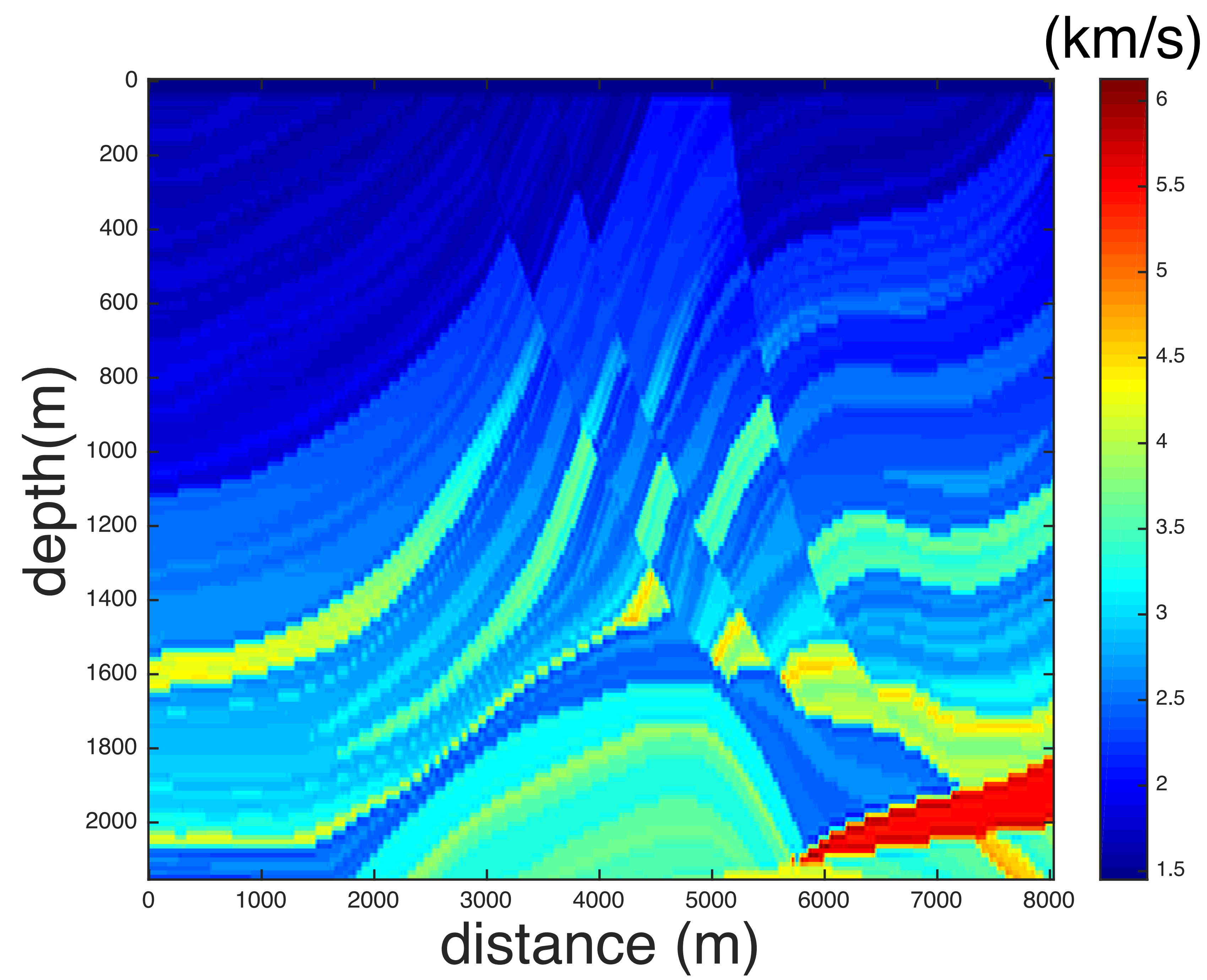
- solves the general L1 analysis problem

$$\min_x \|\Omega x\|_1 \text{ subject to } \|Mx - y\|_2 \leq \epsilon.$$

- is a modification of the Nesterov's algorithm;
- each iteration only involves applying matrix-vector product of Ω, Ω^T, M, M^T
- convergence rate approximately $O\left(\frac{1}{t^2}\right)$, where t is the number of iterations .

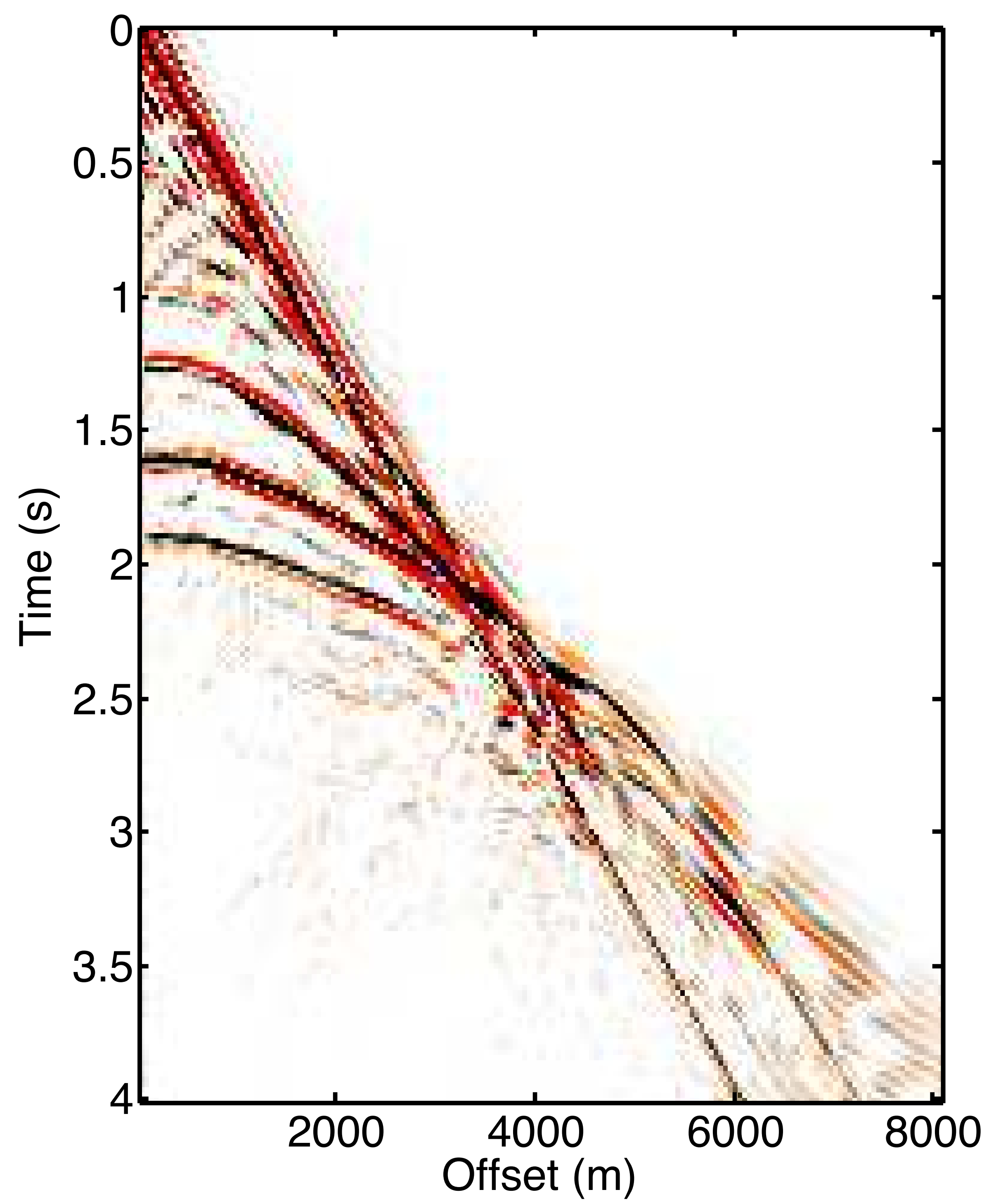
Numerical Example

- 10Hz Ricker wavelet
- 5-15Hz data
- source spacing: 240m
- receiver spacing: 48m
- maximum offset: 8km
- Grid size: 12m
- model size: 180×670

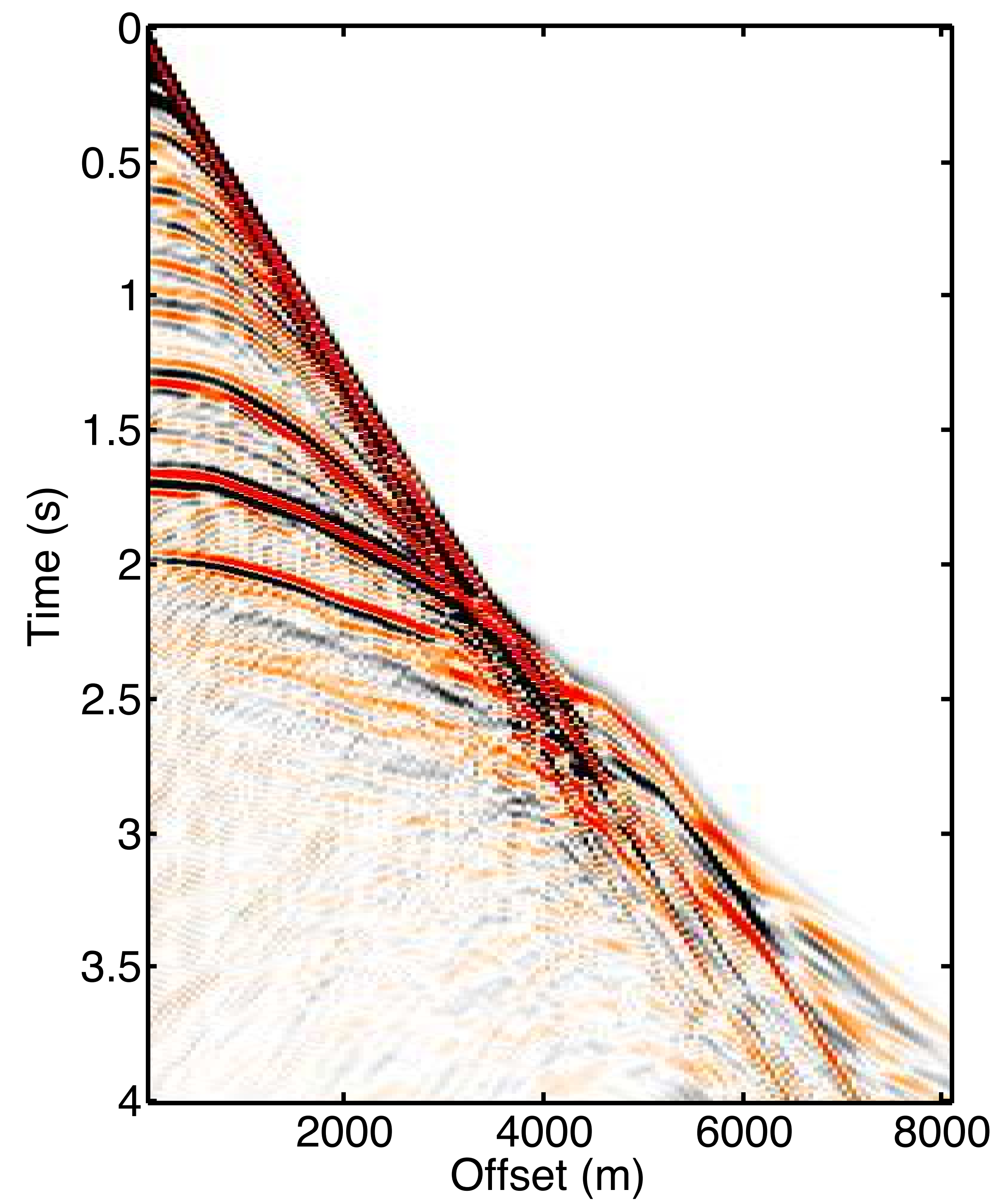


Deconvolution result with 5-15Hz data

Estimated Green's function



Shot Record



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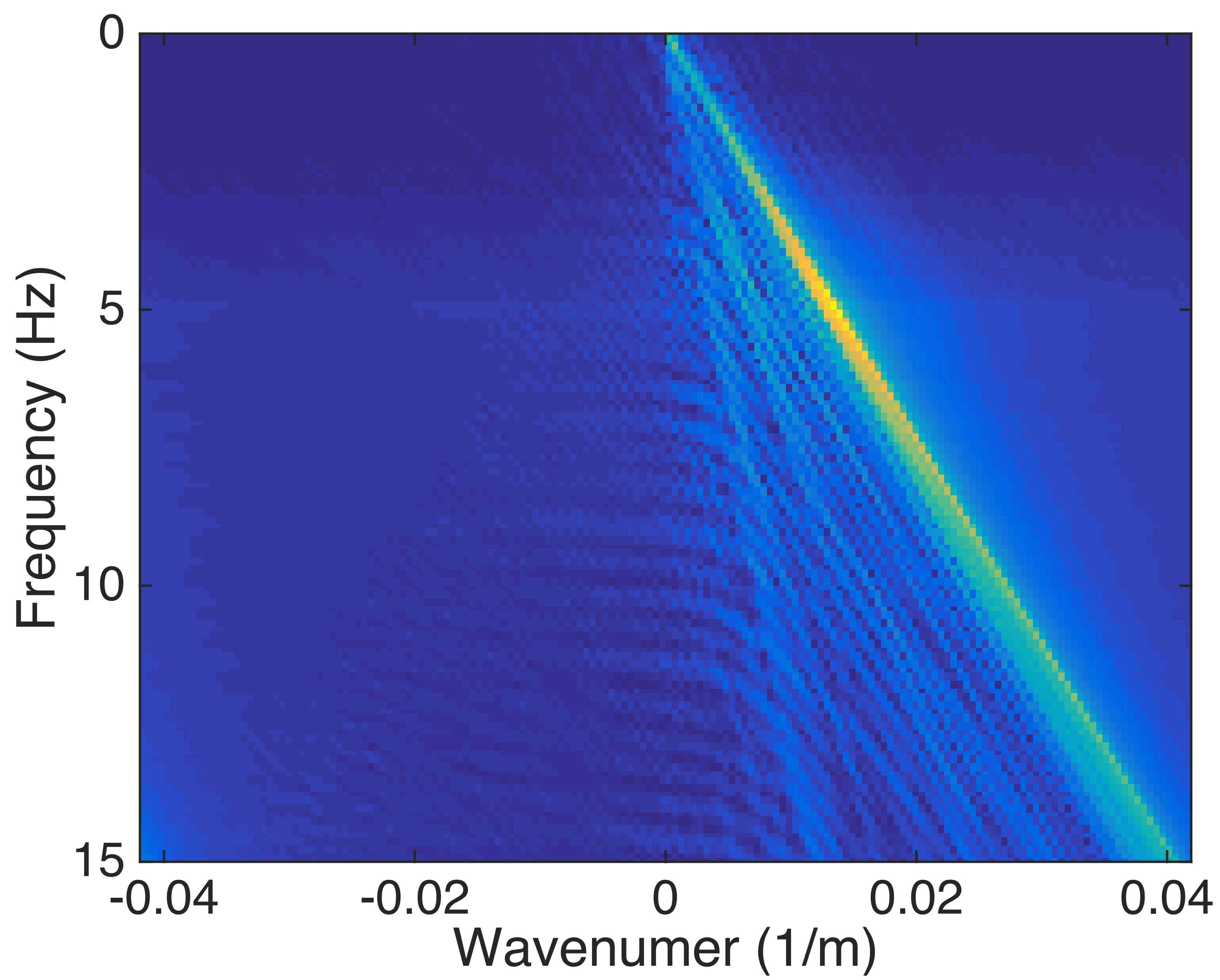

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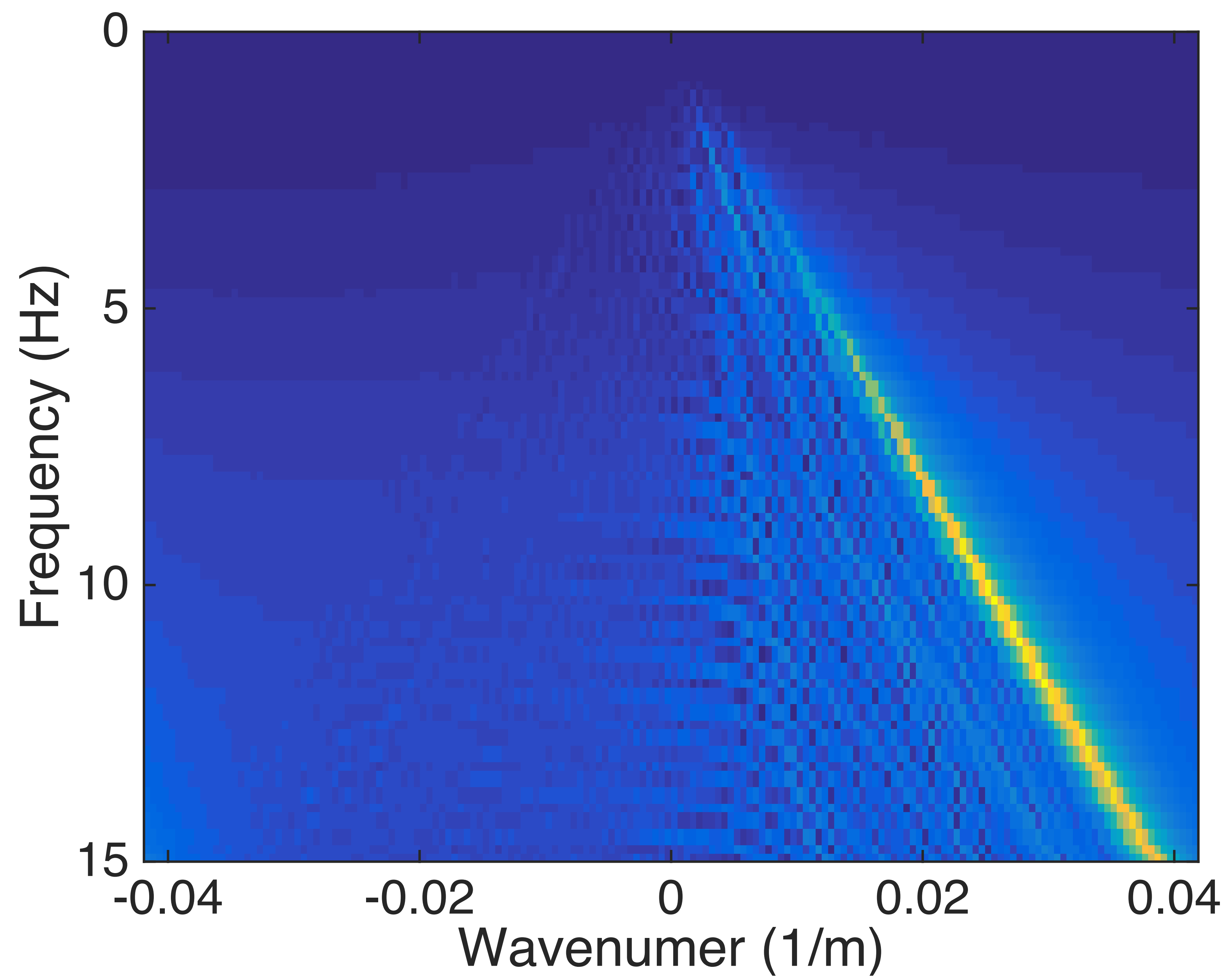
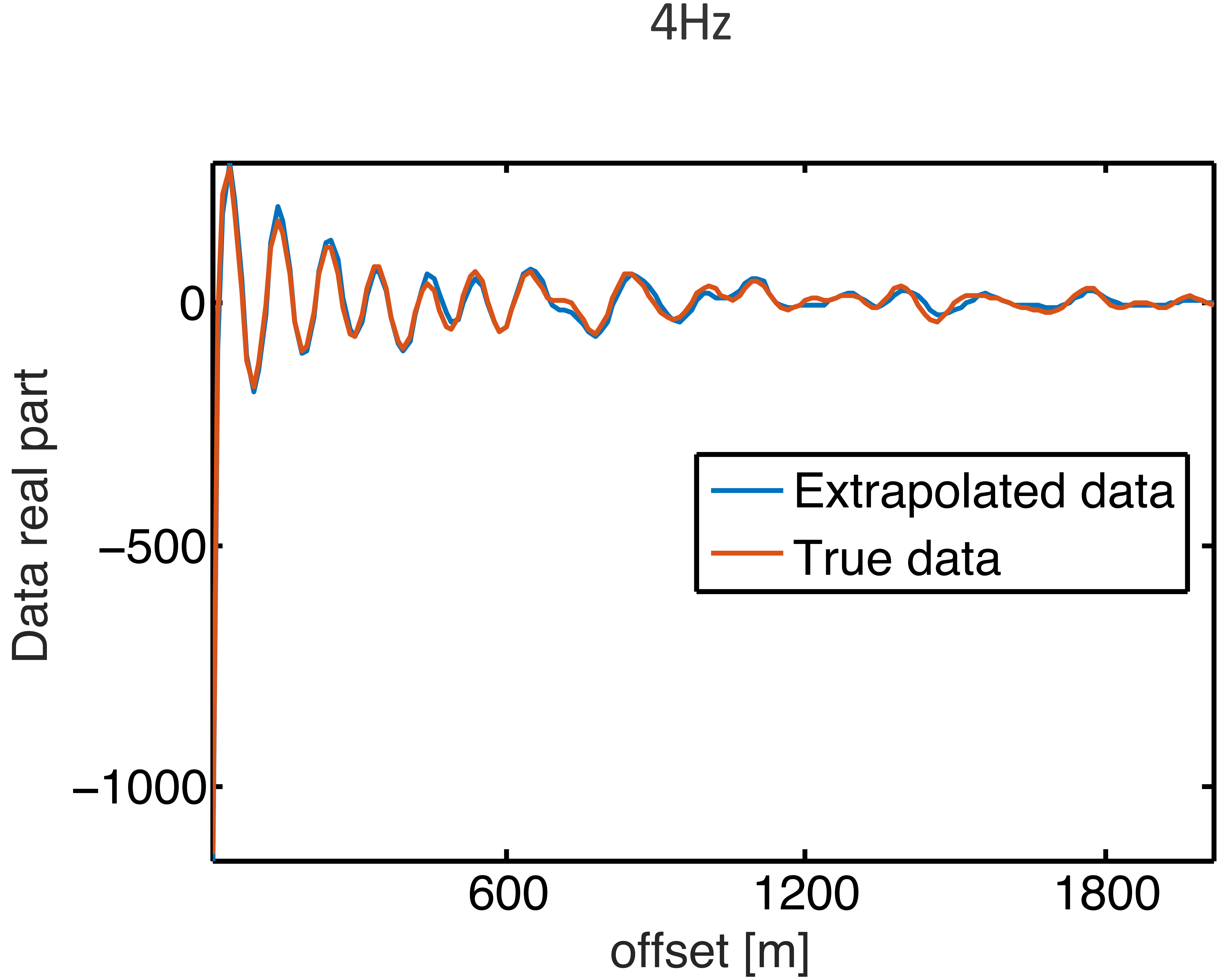
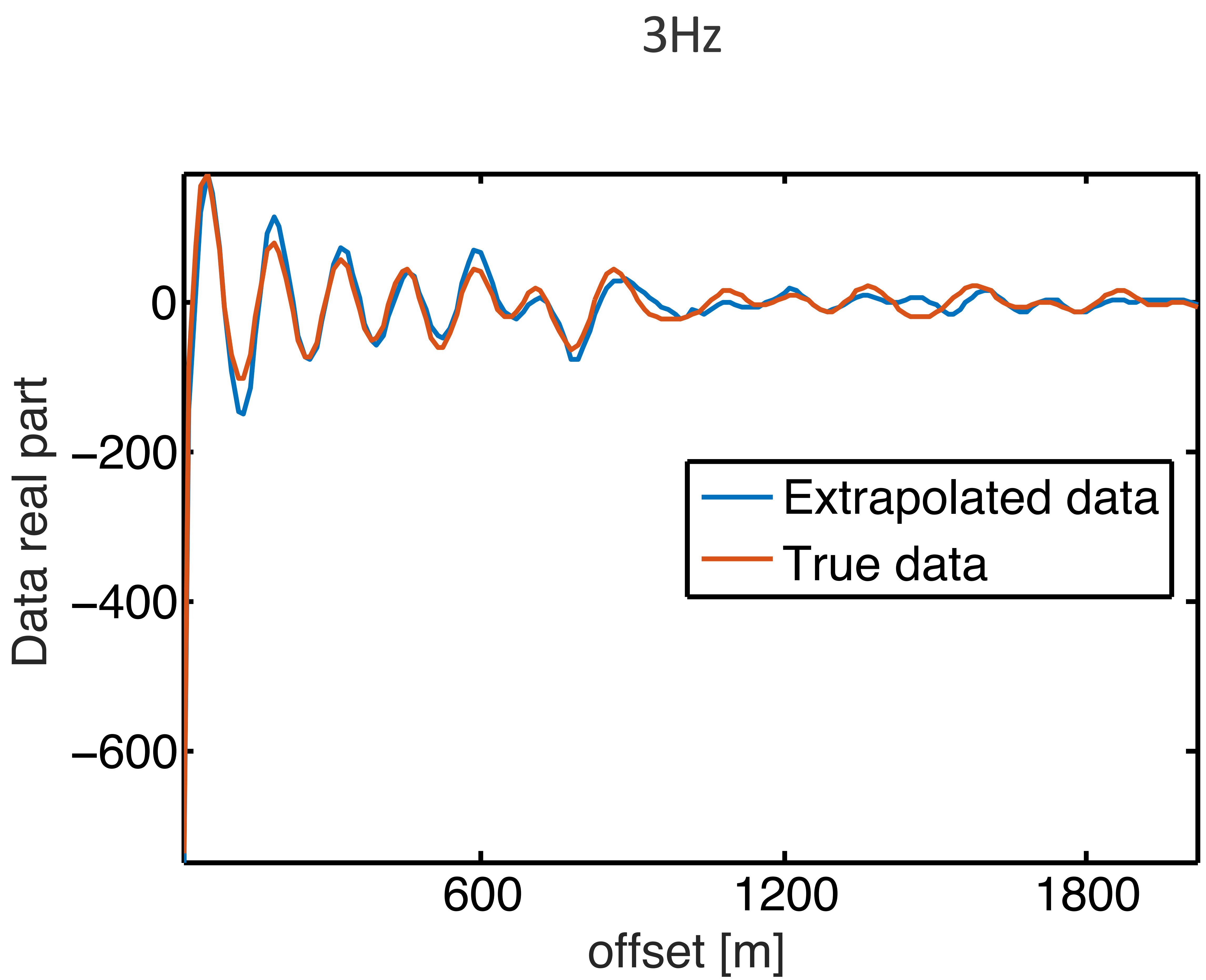


Data extraplation result
from clean 5-15Hz data



Data extrapolation result
from clean 5-15Hz data



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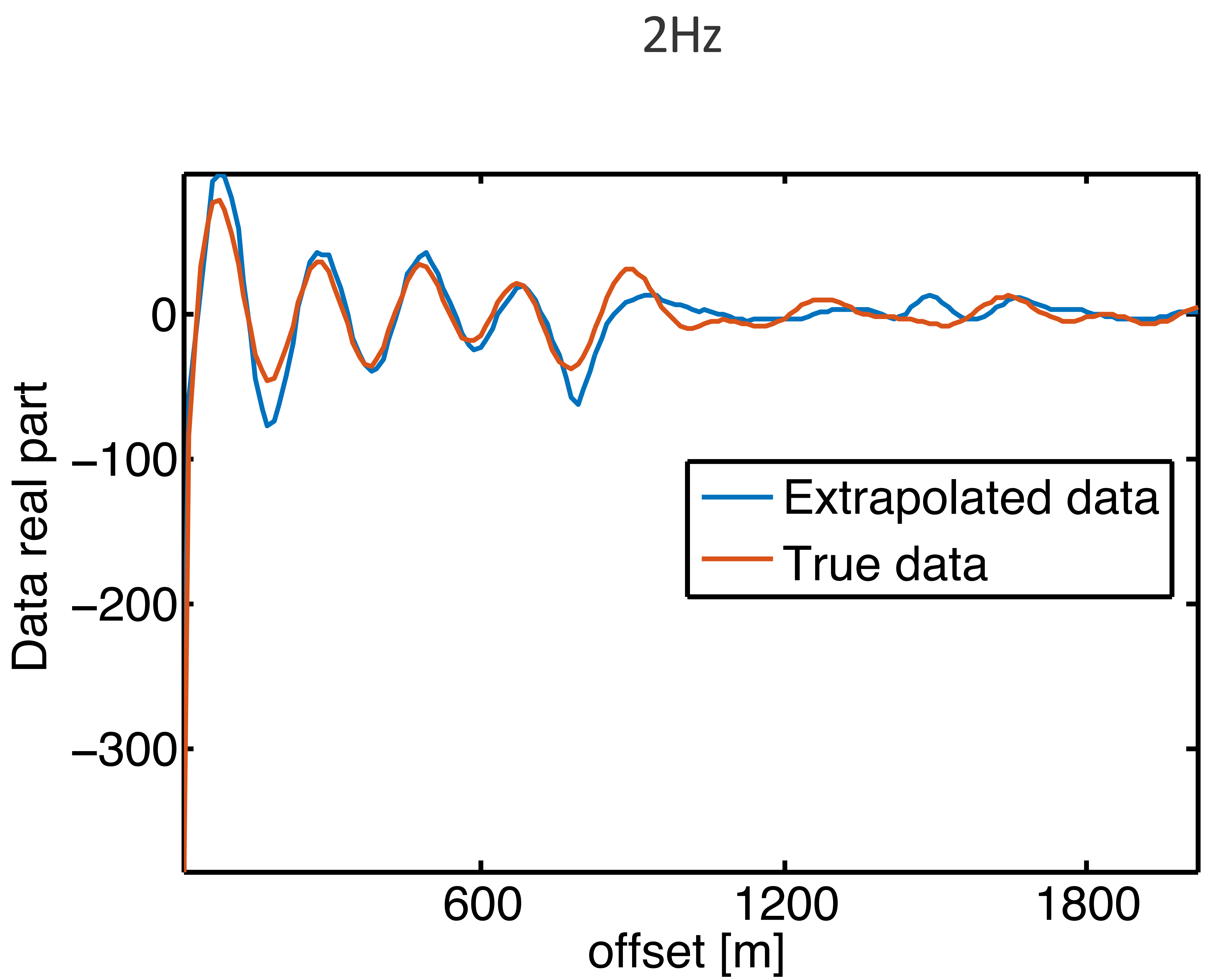
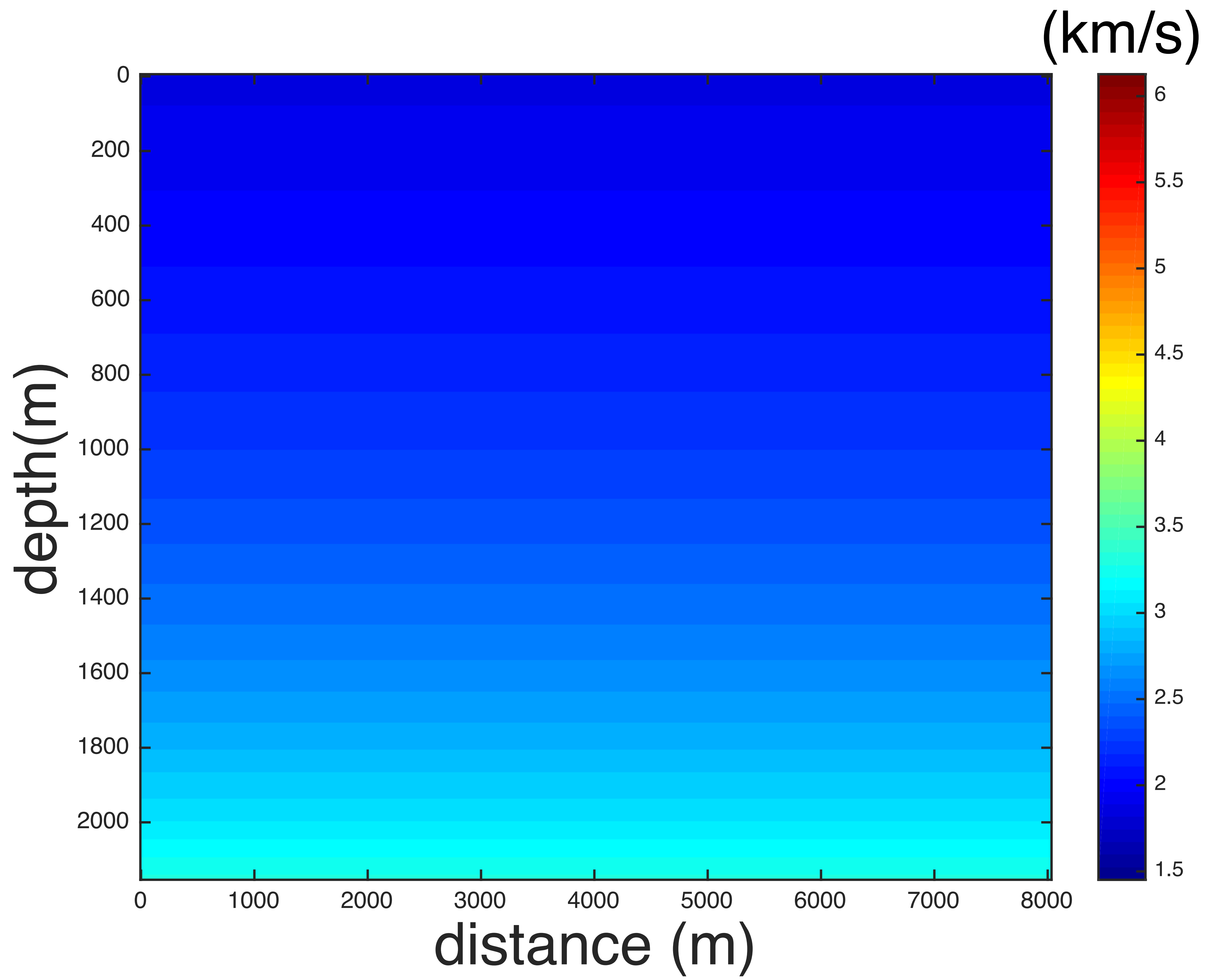
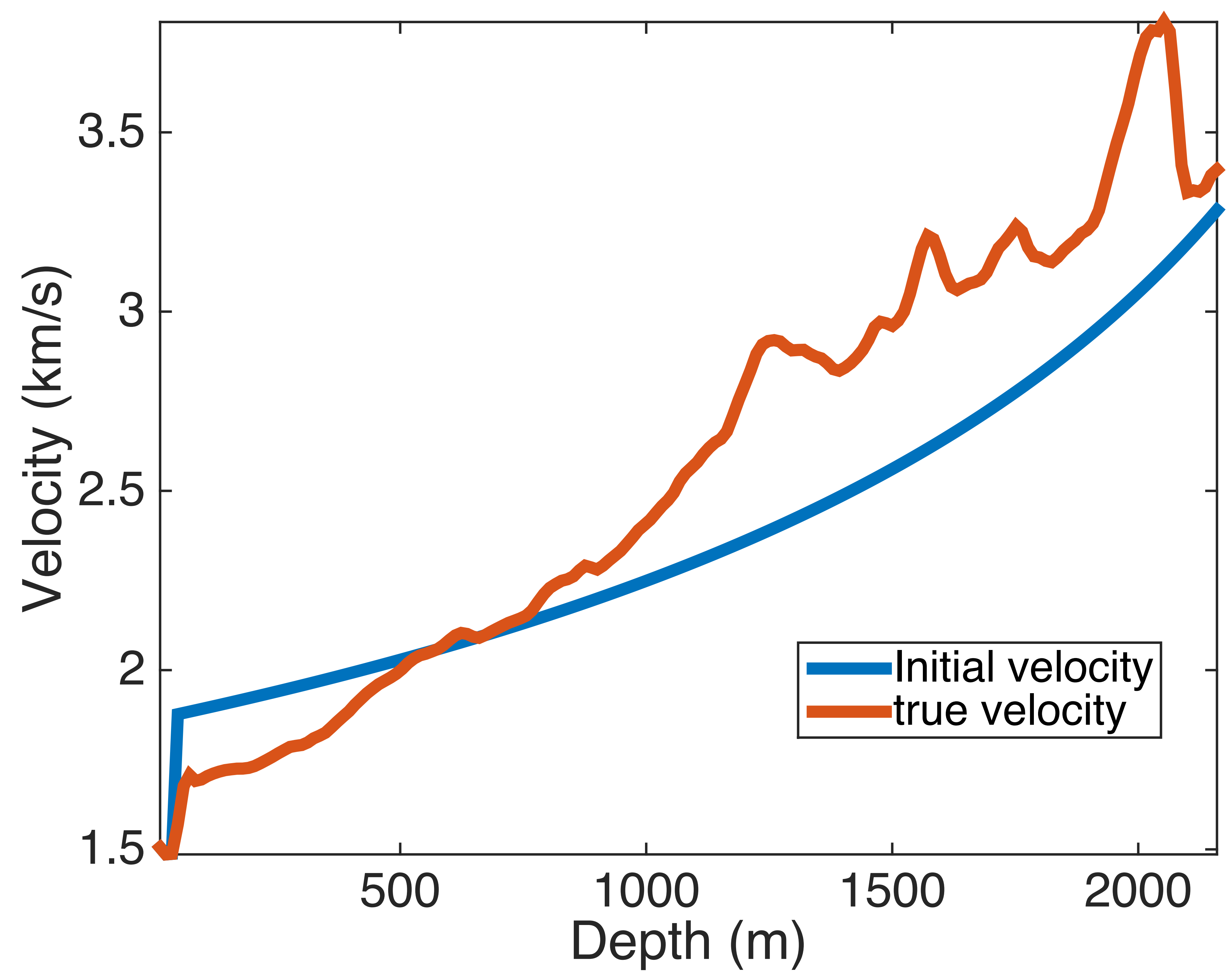


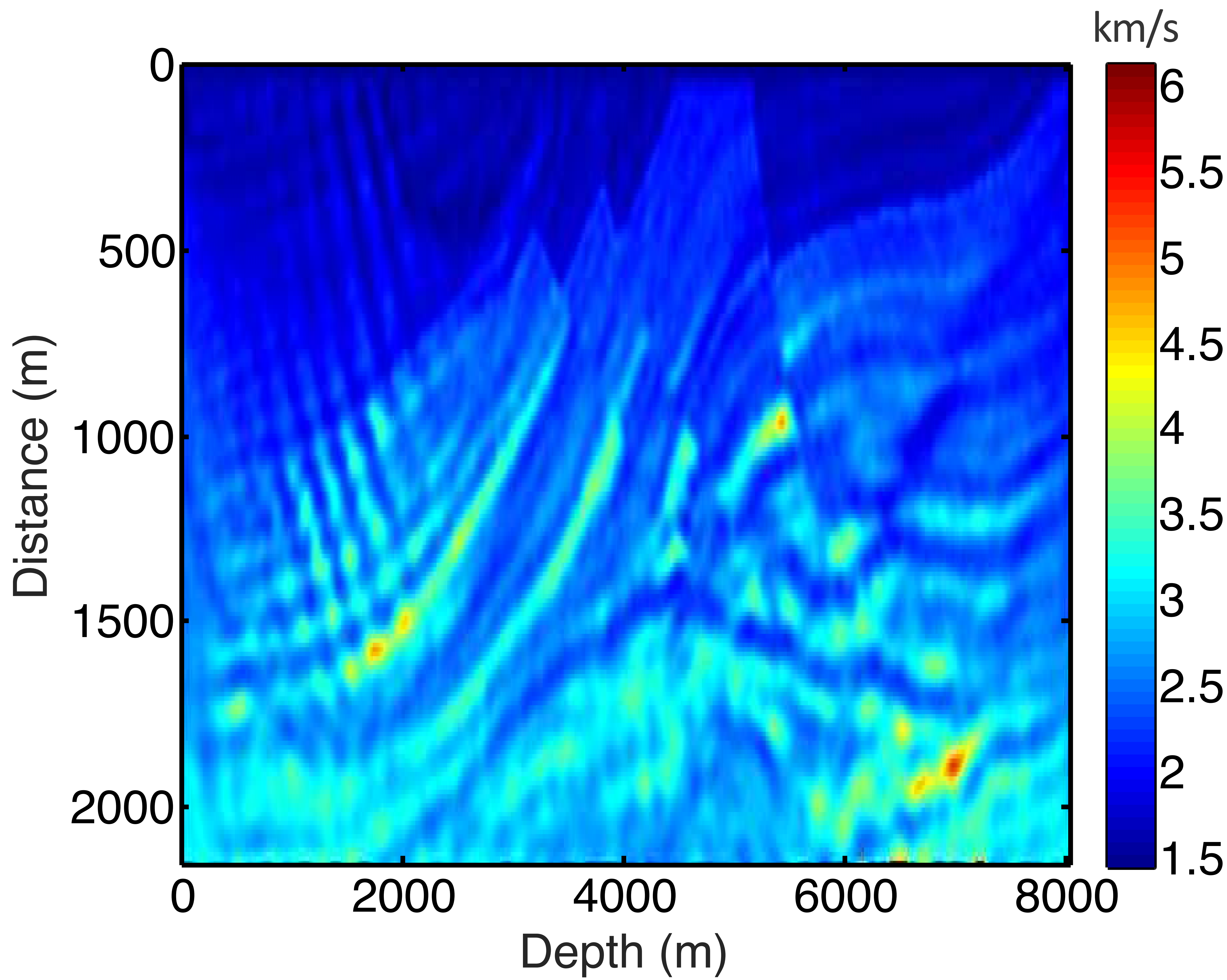
Initial guess





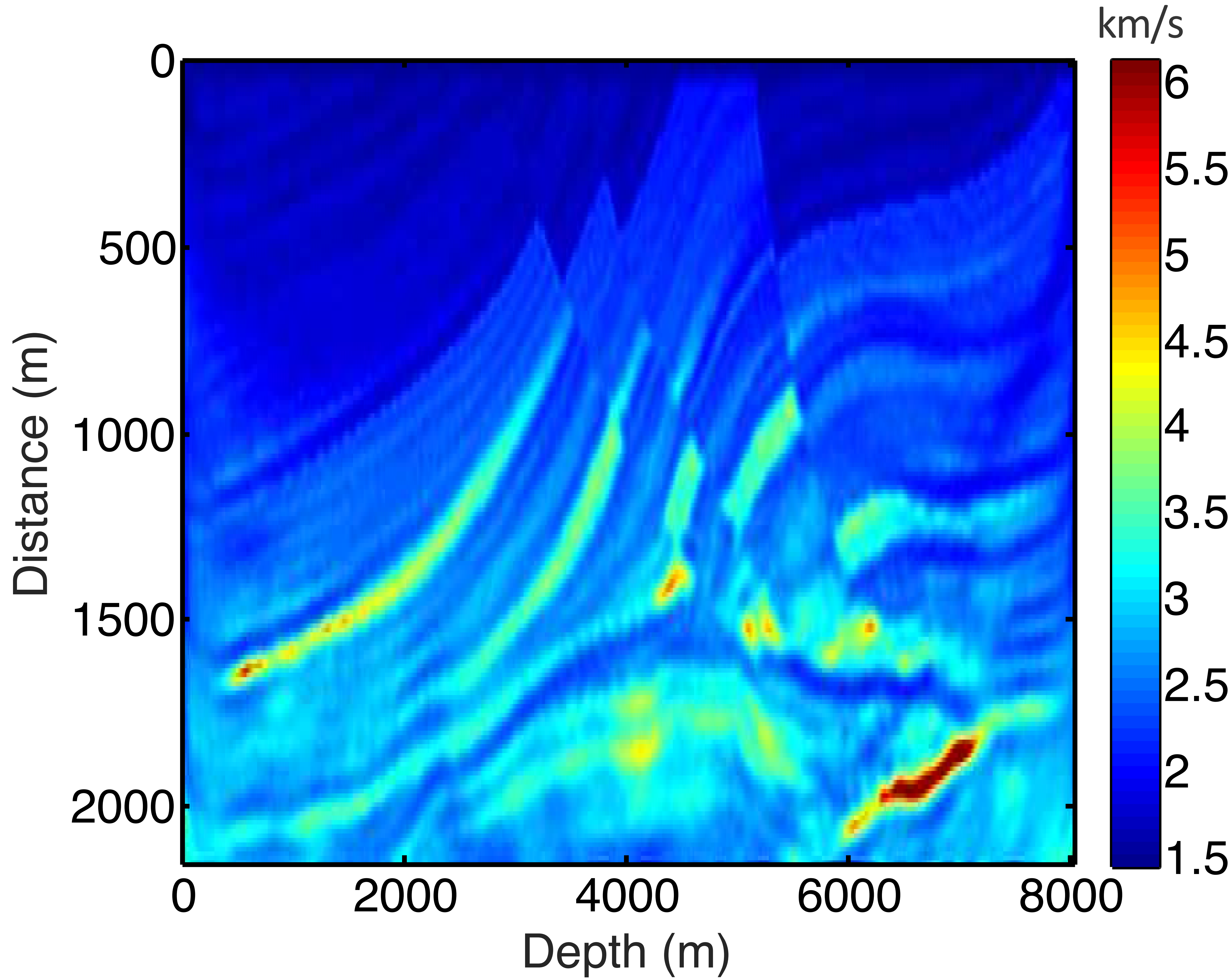
Recovery of FWI

using 5-15Hz clean data

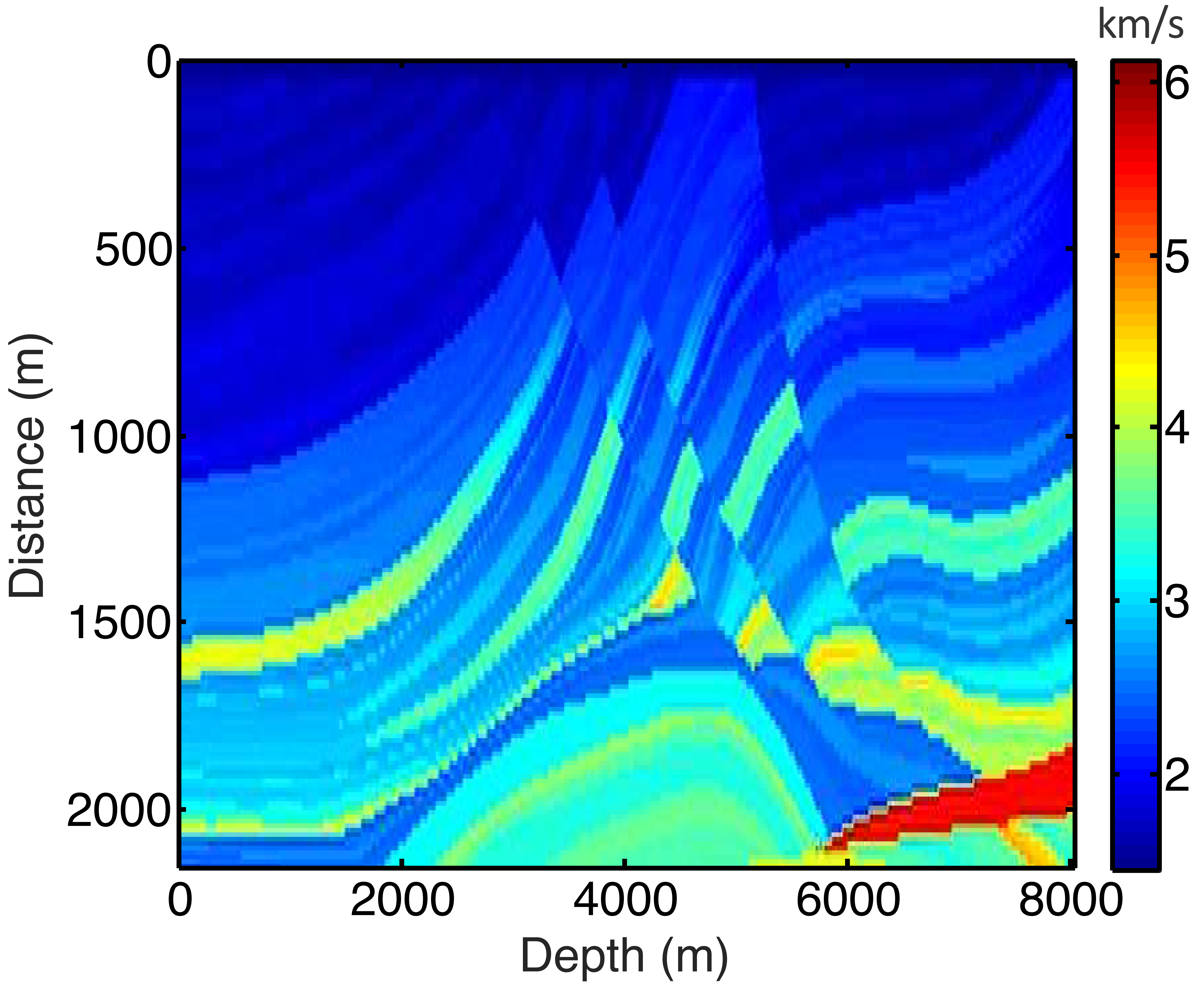


Recovery of FWI

using 5-15Hz clean data + 3-5Hz extrapolated data

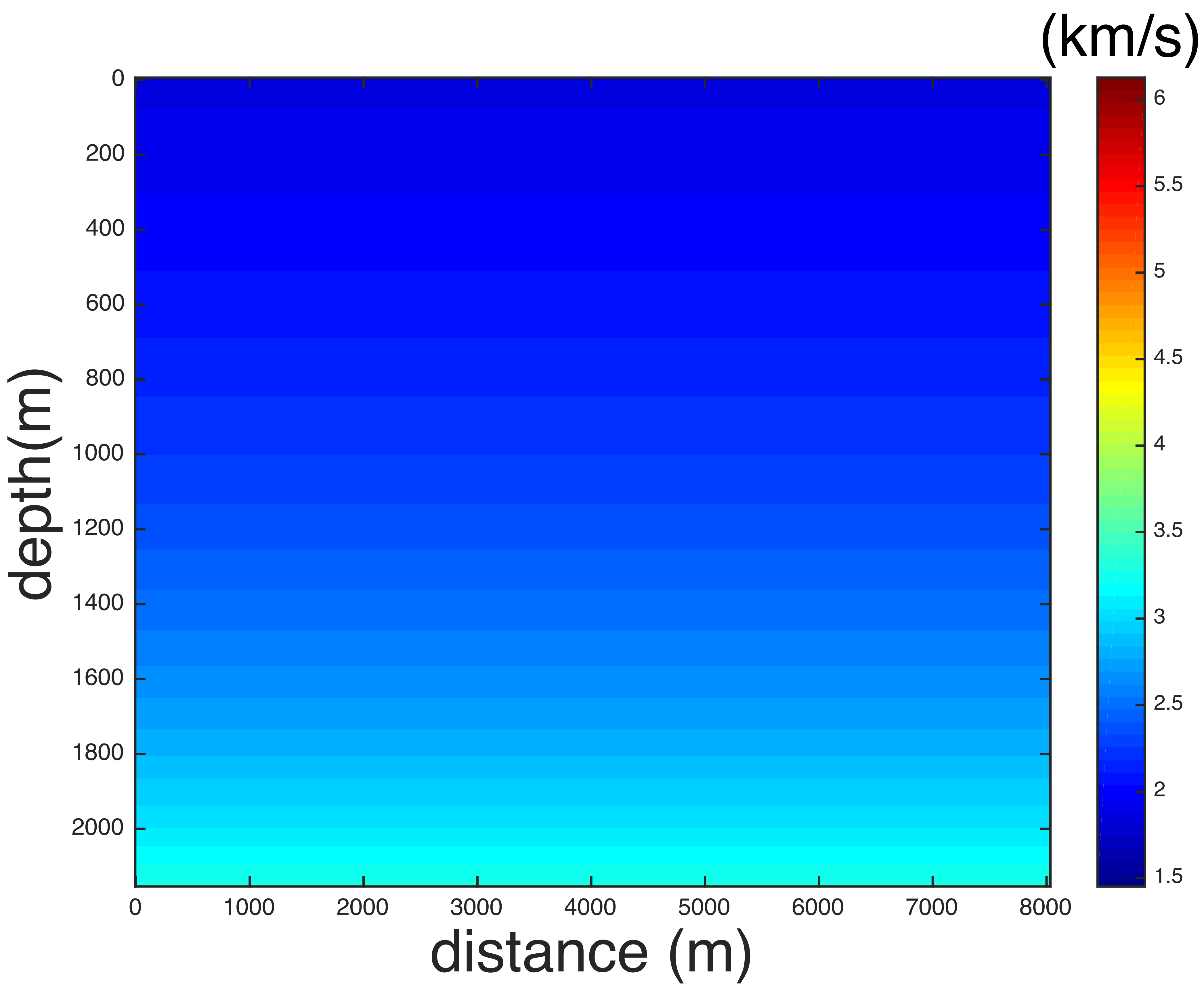


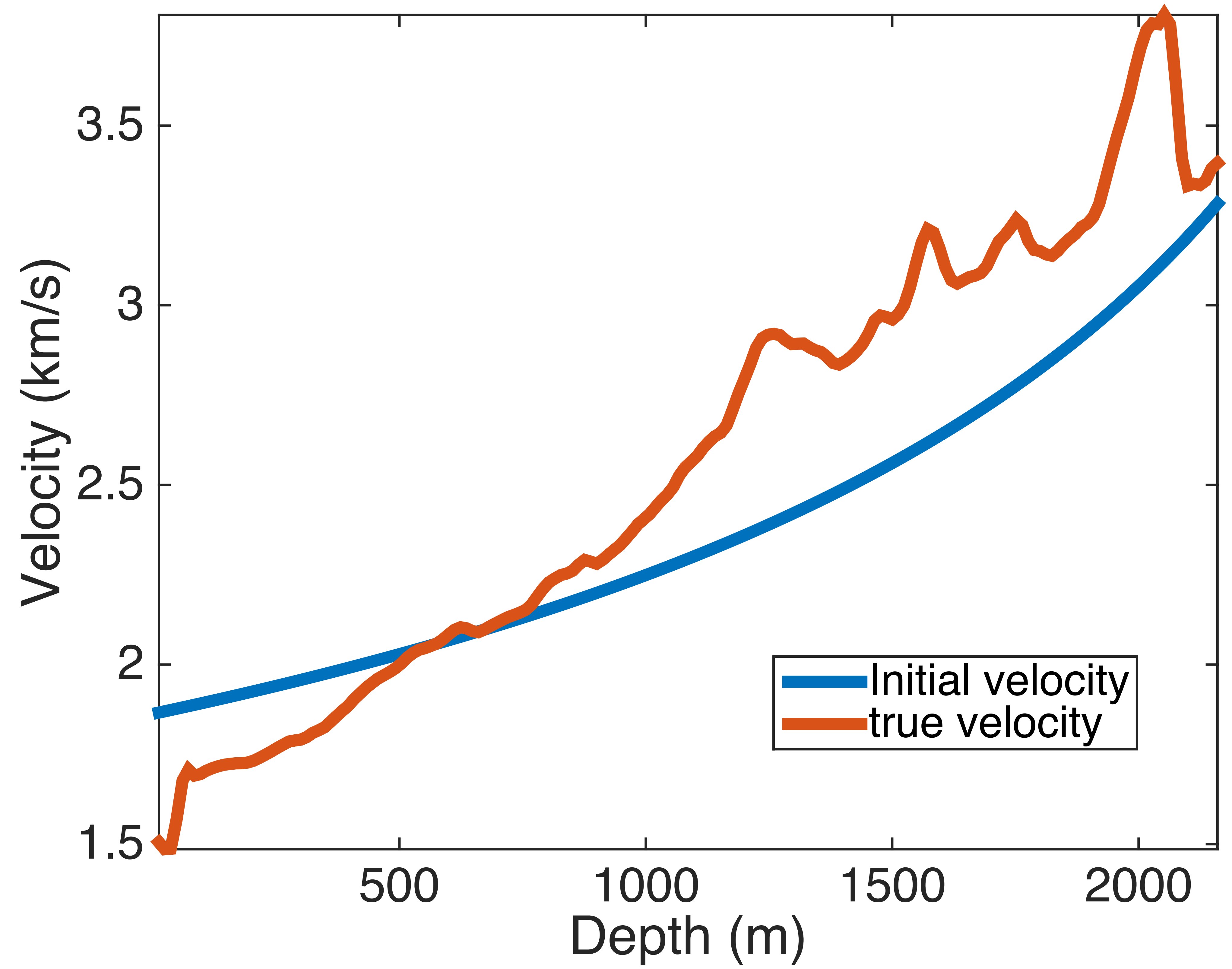
True model



km/s
0
500
1000
1500
2000
Distance (m)
0
2000
4000
6000
8000
Depth (m)
6
5
4
3
2

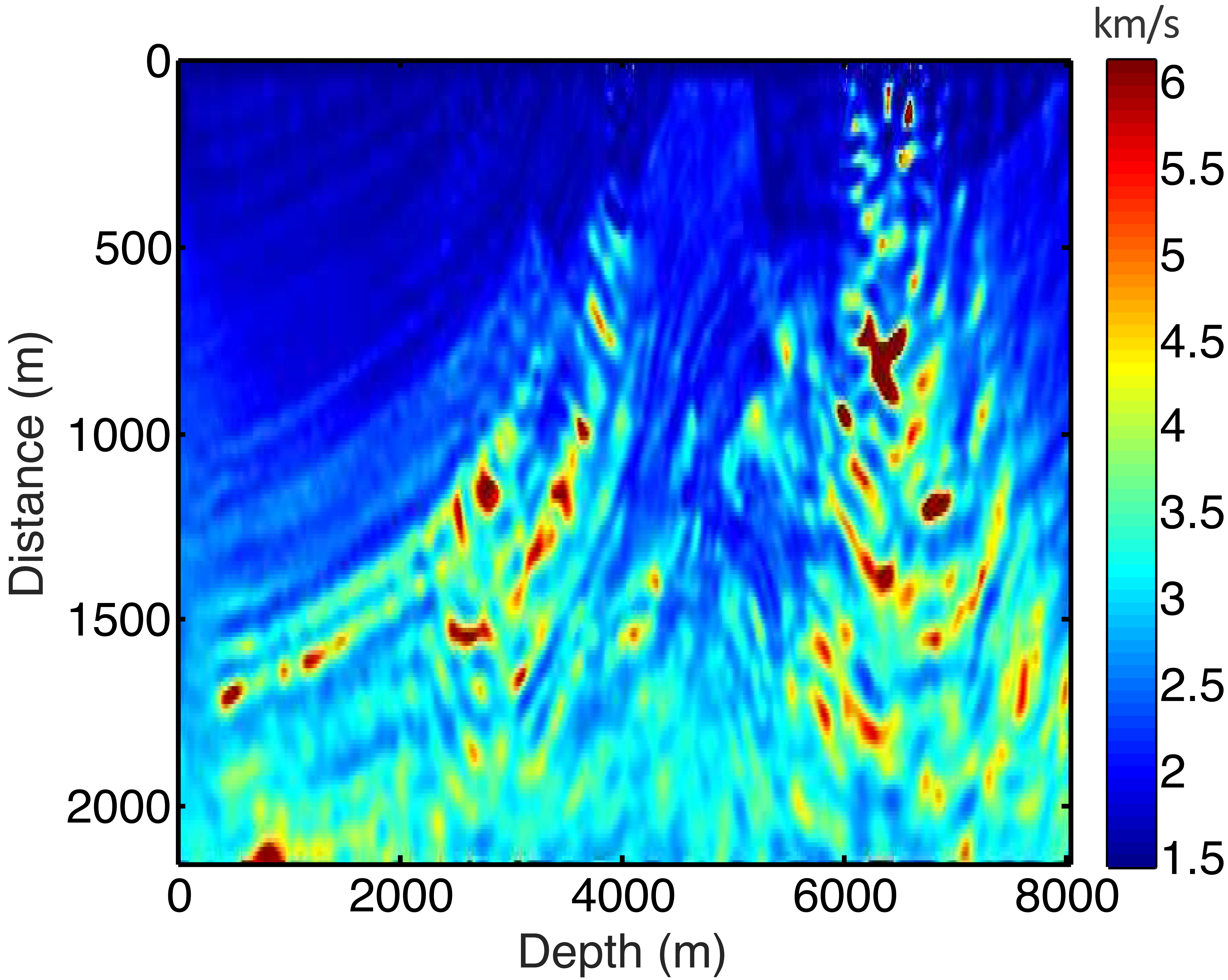
Wrong initial guess at shallow layer





Recovery of FWI

using 5-15Hz clean data



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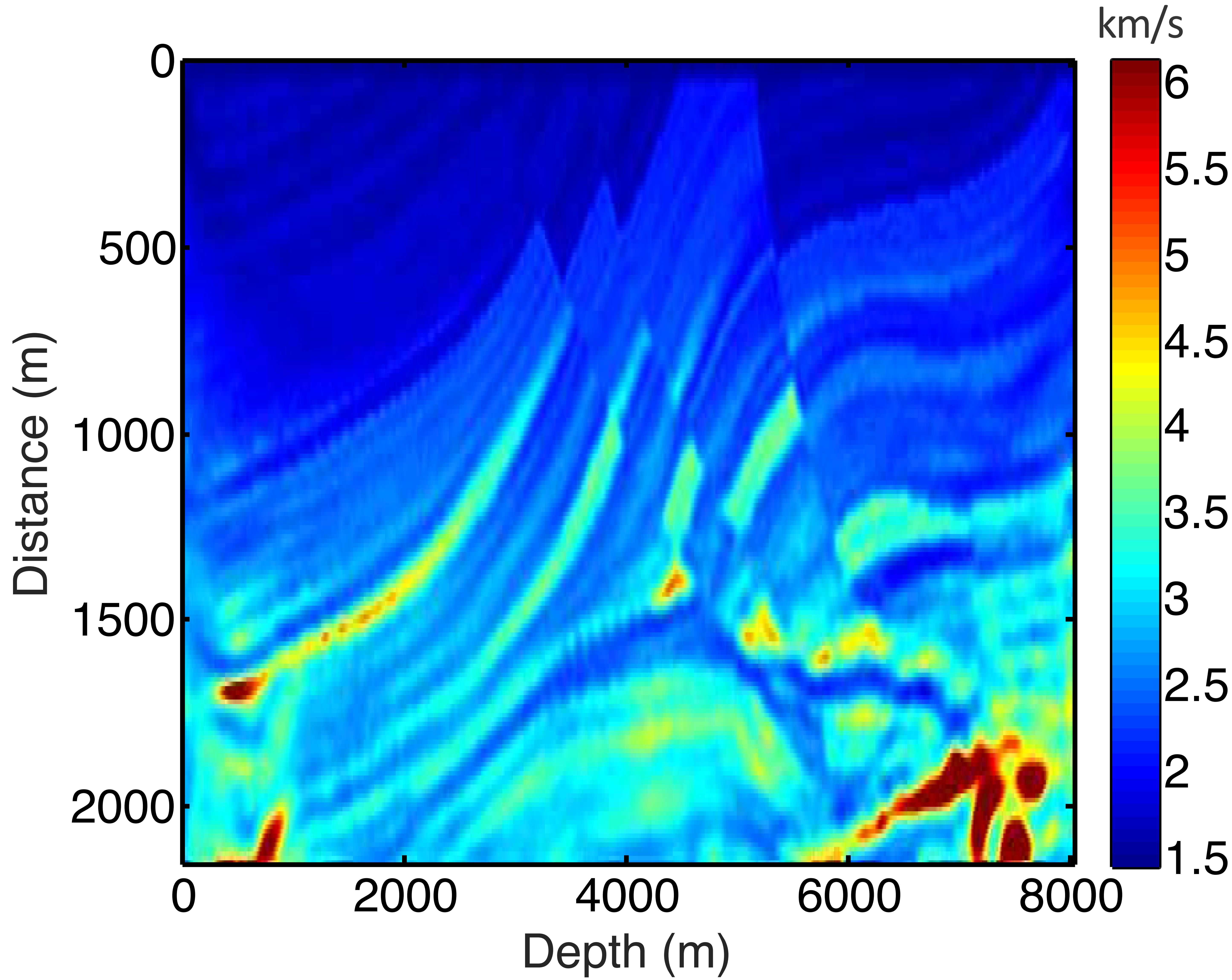

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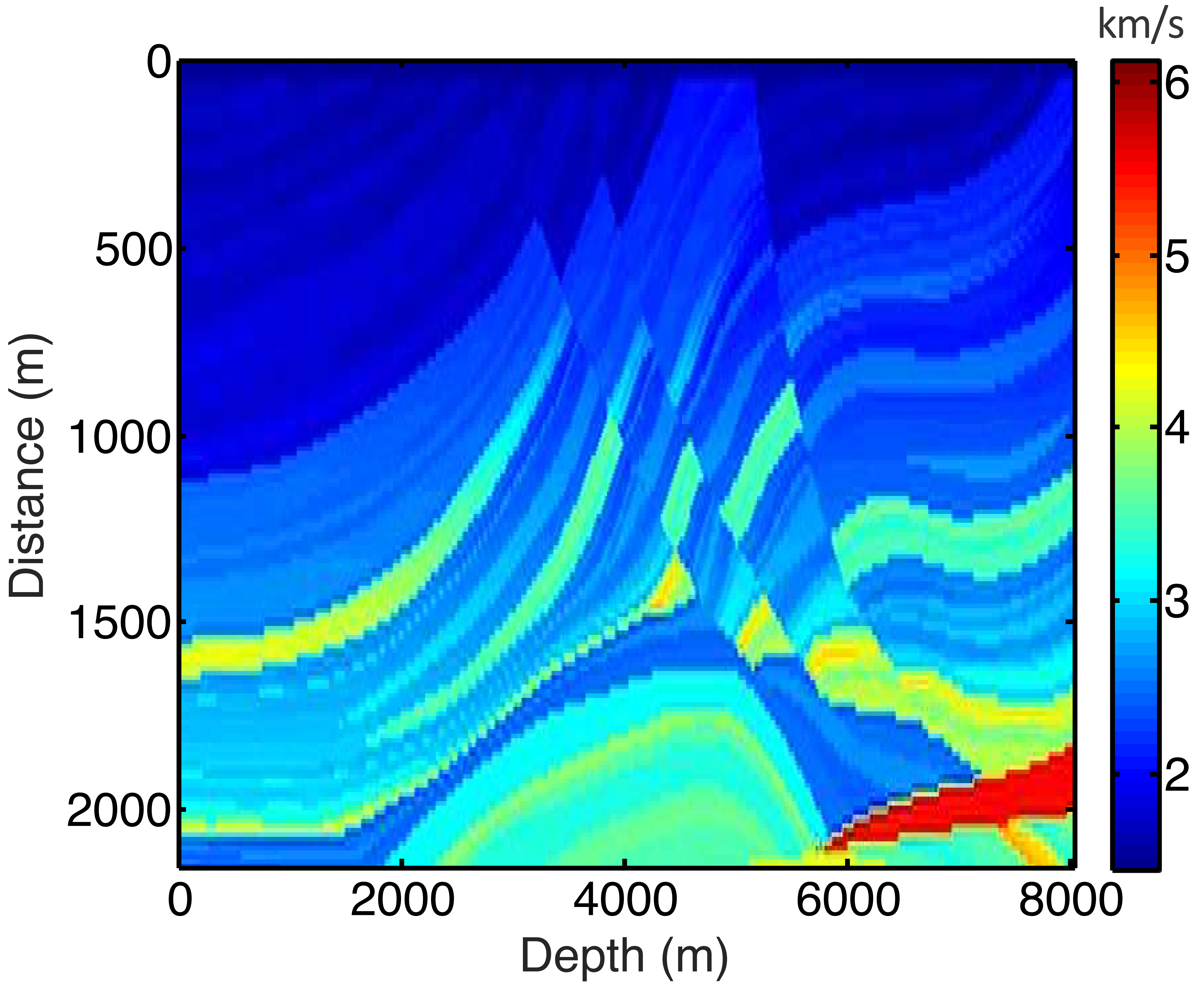
km/s
6
5
4
3
2
0
500
1000
1500
2000
Distance (m)
0
2000
4000
6000
8000
Depth (m)

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Conclusion

- We demonstrated that the LP approach can be used as a spectrum extrapolation method.
- The 2D Curvelet regularization makes the extrapolation more stable.

Limitations:

- The extrapolated data is always noisy.
- The algorithm may break in very complex model.
- The result may be inaccurate in dispersive medium.

Future directions

- Increase the stability of the algorithm by exploiting the noise distribution in seismic data (e.g., the SNR is changing with frequencies)
- Replace the 2D Curvelet transform on a shot gather by a 3D Curvelet transfrom on the 3D data cube.
- Test the method on real data.

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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