Source collocation using the method of Linearized Bregman

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2015 SINBAD Fall Consortium meeting



with help from Mathias Louboutin

SLIM
University of British Columbia

Motivation

Benefits of source localization and/or signature estimation

- locate sources in microseismic
- Identify & utilize interfering sources from nearby surveys
- Remove coherent noise (in the range of the wave equation)
- “Blind” source deblending
- Deghosting ...

Existing methods

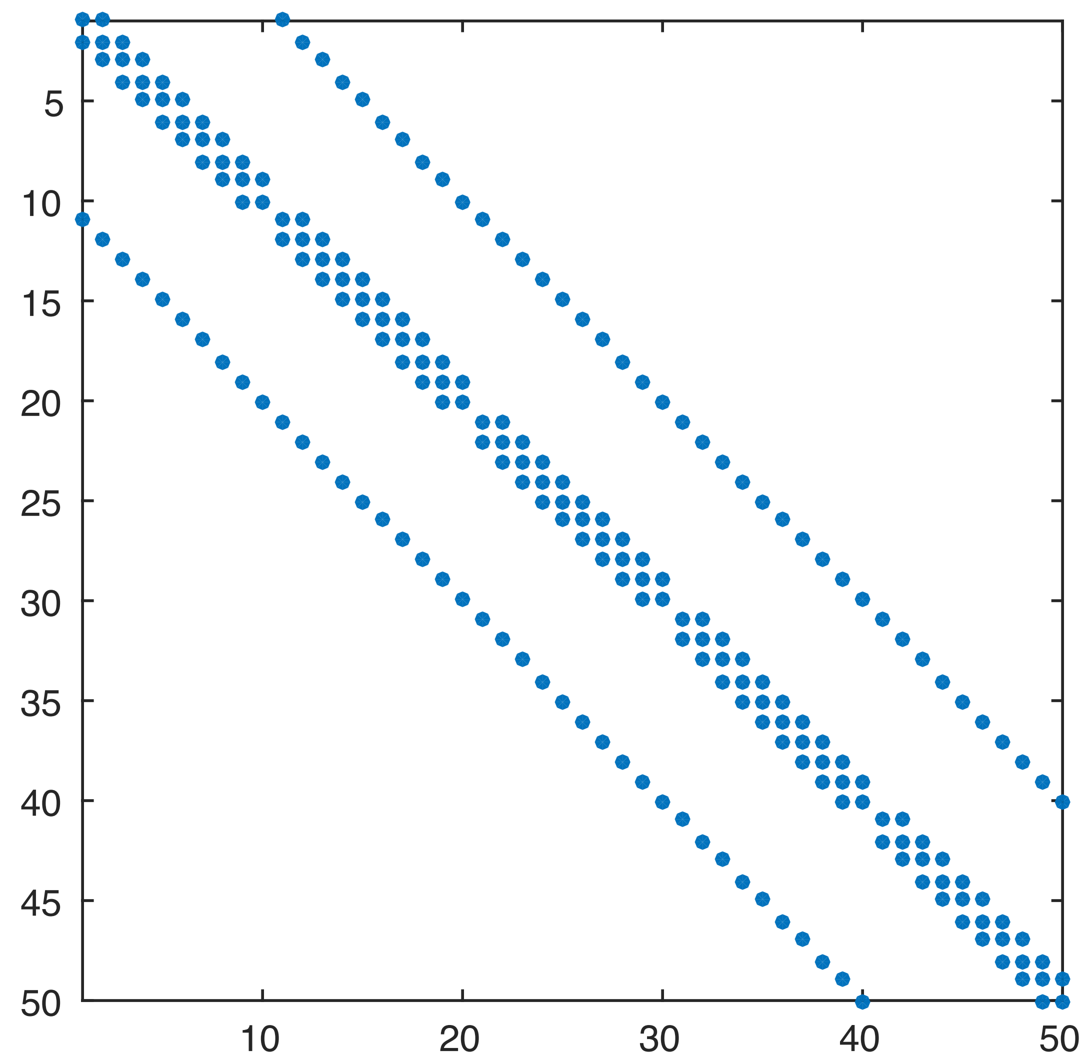
- RTM based method for micro-seismic source imaging **[N.Nakata et. al., 2015; G.C Beroza, 2015; B. Artman et. al., 2010; J. Sun et. al., 2015]**
- Full-Waveform inversion based non-convex optimization **[K.Kaderli et. al., 2015]**

Our goal:

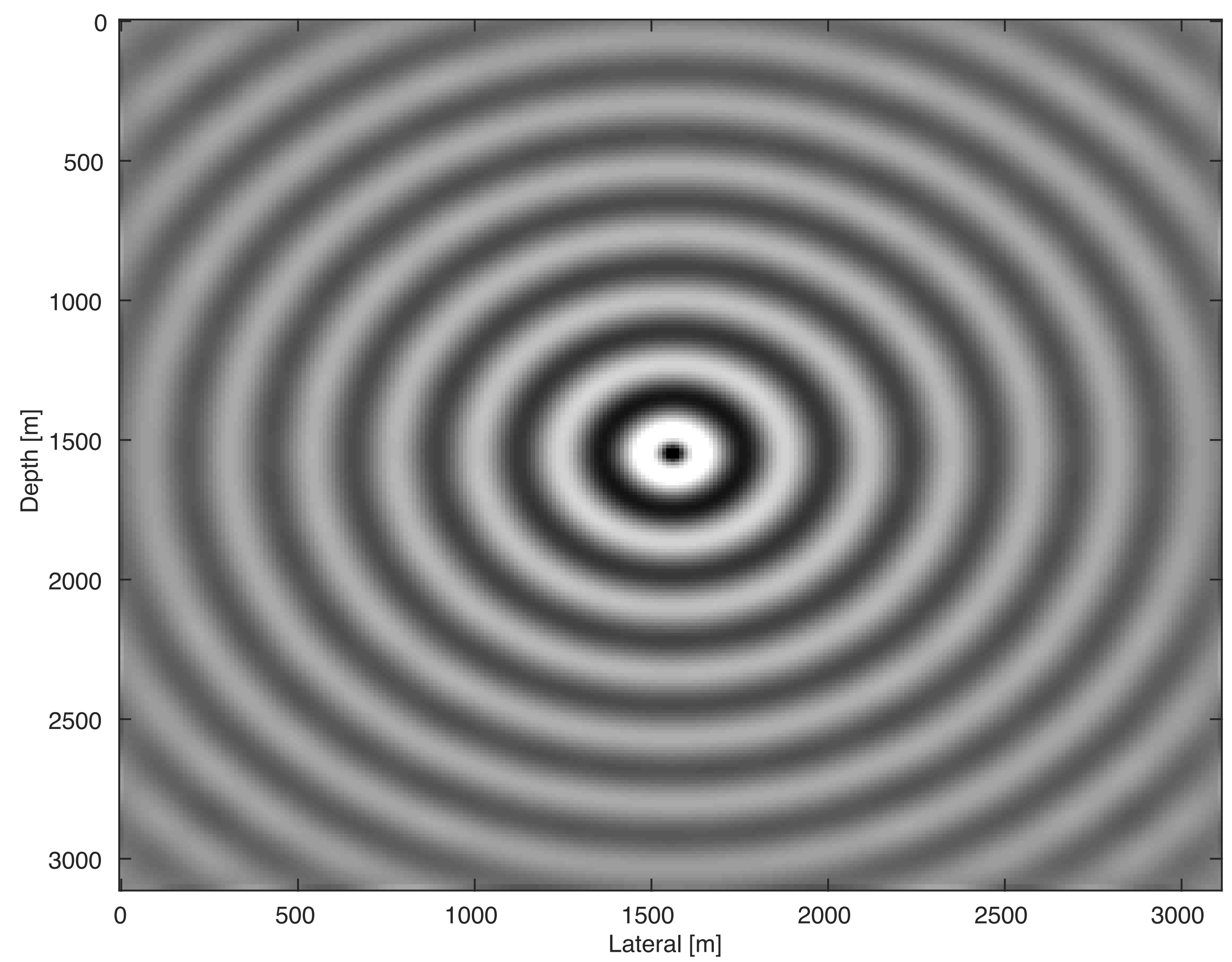
Find the source location and signature simultaneously by solving a tractable convex optimization problem.

[Kitic, S., Albera, L., Bertin, N., and Gribonval, R., 2015]

Motivation: wave equation as sparsity promoting transform

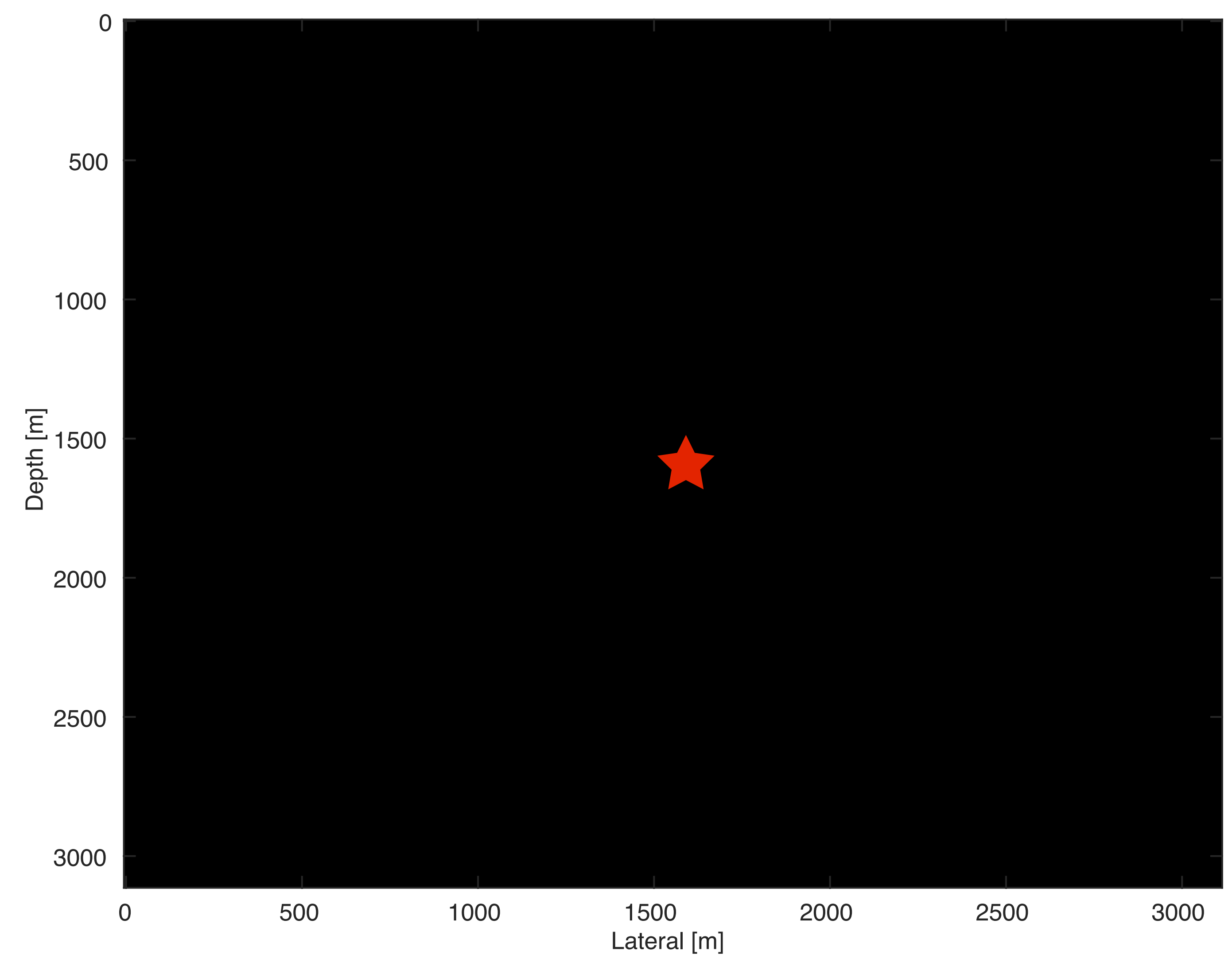


Time harmonic forward modeling operator: **A**



Wavefield: **u**

=



Source: **q**

[Kitic, S., Albera, L., Bertin, N., and Gribonval, R., 2015]
[van Leeuwen, T and Herrmann, F J , 2013]

Methodology: Cosparse regularization

Time domain Forward operator

Receiver Projector

$$\underset{\mathbf{u}}{\text{minimize}} \;\; F_1\left(\mathbf{Au}\right) + F_2\left(\mathbf{Pu} - \mathbf{d}\right)$$

[Kitic, S., Albera, L., Bertin, N., and Gribonval, R., 2015]
[van Leeuwen, T and Herrmann, F J , 2013]

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Sparsity inducing penalty

Data fidelity

[Kitic, S., Albera, L., Bertin, N., and Gribonval, R., 2015]
[van Leeuwen, T and Herrmann, F J , 2013]

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Receiver Projector

$$\underset{\mathbf{u}}{\text{minimize}} \;\; F_1\left(\mathbf{Au}\right) + F_2\left(\mathbf{Pu} - \mathbf{d}\right)$$

Sparsity inducing penalty

Data fidelity

Assume: source is sparse in space but not in time!

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Methodology: Cosparse regularization

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Receiver Projector

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Sparsity inducing penalty

Data fidelity

Assume: source is sparse in space but not in time!

$$F_1 = \|\cdot\|_{1,2}$$

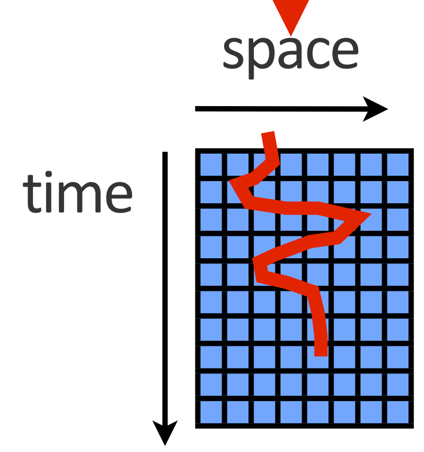
Methodology: Cosparse regularization

Time domain Forward operator

Receiver Projector

$$\underset{\mathbf{u}}{\text{minimize}}\ F_1\left(\boxed{\mathbf{Au}}\right) + F_2\left(\mathbf{Pu} - \mathbf{d}\right)$$

Sparsity inducing penalty



Data fidelity

$$\|[v_1, v_2, ..., v_n]\|_{1,2} = \sum_{i=1}^{n} \|v_i\|_2$$

Assume: source is sparse in space but not in time!

$$F_1 = \|\cdot\|_{1,2}$$

Methodology: Cosparse regularization

Time domain Forward operator

Receiver Projector

$$\underset{\mathbf{u}}{\text{minimize}} \; F_1\left(\mathbf{Au}\right) + F_2\left(\mathbf{Pu} - \mathbf{d}\right)$$

Sparsity inducing penalty

Data fidelity

Assume: “source-side” noise is well spread both in time and space!

Methodology: Cosparse regularization

Time domain Forward operator

Receiver Projector

$$\underset{\mathbf{u}}{\text{minimize}} \; F_1\left(\mathbf{Au}\right) + F_2\left(\mathbf{Pu} - \mathbf{d}\right)$$

Sparsity inducing penalty

Data fidelity

Assume: noise is spread both in time and space

$$F_2 = \| \cdot \|_F^2$$

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Methodology: Cosparse regularization

They solve both for the source location & signature from

$$\underset{\mathbf{u}}{\text{minimize}}\ \ \lambda\|\mathbf{Au}\|_{1,2} + \|\mathbf{Pu} - \mathbf{d}\|_F^2$$

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Methodology: Cosparse regularization

It is proposed to solve both the source location and signature from

$$\underset{\mathbf{u}}{\text{minimize}} \;\; \lambda \|\mathbf{A}\mathbf{u}\|_{1,2} + \|\mathbf{P}\mathbf{u} - \mathbf{d}\|_F^2$$

We solve a slight variant

$$\underset{\mathbf{u}}{\text{minimize}} \;\; \lambda \|\mathbf{A}\mathbf{u}\|_{1,2} + \frac{1}{2}\|\mathbf{A}\mathbf{u}\|_{2,2}^2$$

$$\text{subject to } \|\mathbf{P}\mathbf{u} - \mathbf{d}\|_F^2 \leq \epsilon$$

Methodology: Linearized Bregman iteration

The proposed method: Linearized Bregman with an $L_{1,2}$ projection

- L_1 norm regularization is well studied.
- is faster than the ADMM for this specific problem where
  - the Forward operator is ill-conditioned
  - computing the inverse of the Forward operator is cheap

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Source collocation – w/ Linearized Bregman

$$\underset{\mathbf{q}}{\text{minimize}} \quad \lambda\|\mathbf{q}\|_{1,2} + \frac{1}{2}\|\mathbf{q}\|_F^2$$

$$\text{subject to} \qquad \mathbf{P}\mathbf{A}^{-1}\mathbf{q} = \mathbf{d}$$

1. **for** $k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{P}^*(\mathbf{P}\mathbf{u}_k - \mathbf{d})$
3. $\quad \mathbf{u}_{k+1} = \mathbf{A}^{-1}\text{Prox}_{\lambda\|\cdot\|_{1,2}}(\mathbf{A}\mathbf{z}_{k+1})$
4. **end for**

*where $t_k = \frac{\|\mathbf{P}\mathbf{u}_k - \mathbf{d}\|^2}{\|(\mathbf{P}\mathbf{A}^{-1})*(\mathbf{P}\mathbf{u}_k - \mathbf{d})\|^2}$ are the step lengths

*where $\text{Prox}_{\lambda\|\cdot\|_{1,2}}(x) := \min_t \lambda\|t\|_{1,2} + \frac{1}{2}\|x - t\|_F^2$ is the proximal mapping of the l_{12} norm

[Lorenz, D.A., Schöpfer, F., and Wenger, S., 2013]

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Source collocation – w/ Linearized Bregman

$$\underset{\mathbf{q}}{\text{minimize}} \quad \lambda\|\mathbf{q}\|_{1,2} + \frac{1}{2}\|\mathbf{q}\|_F^2$$

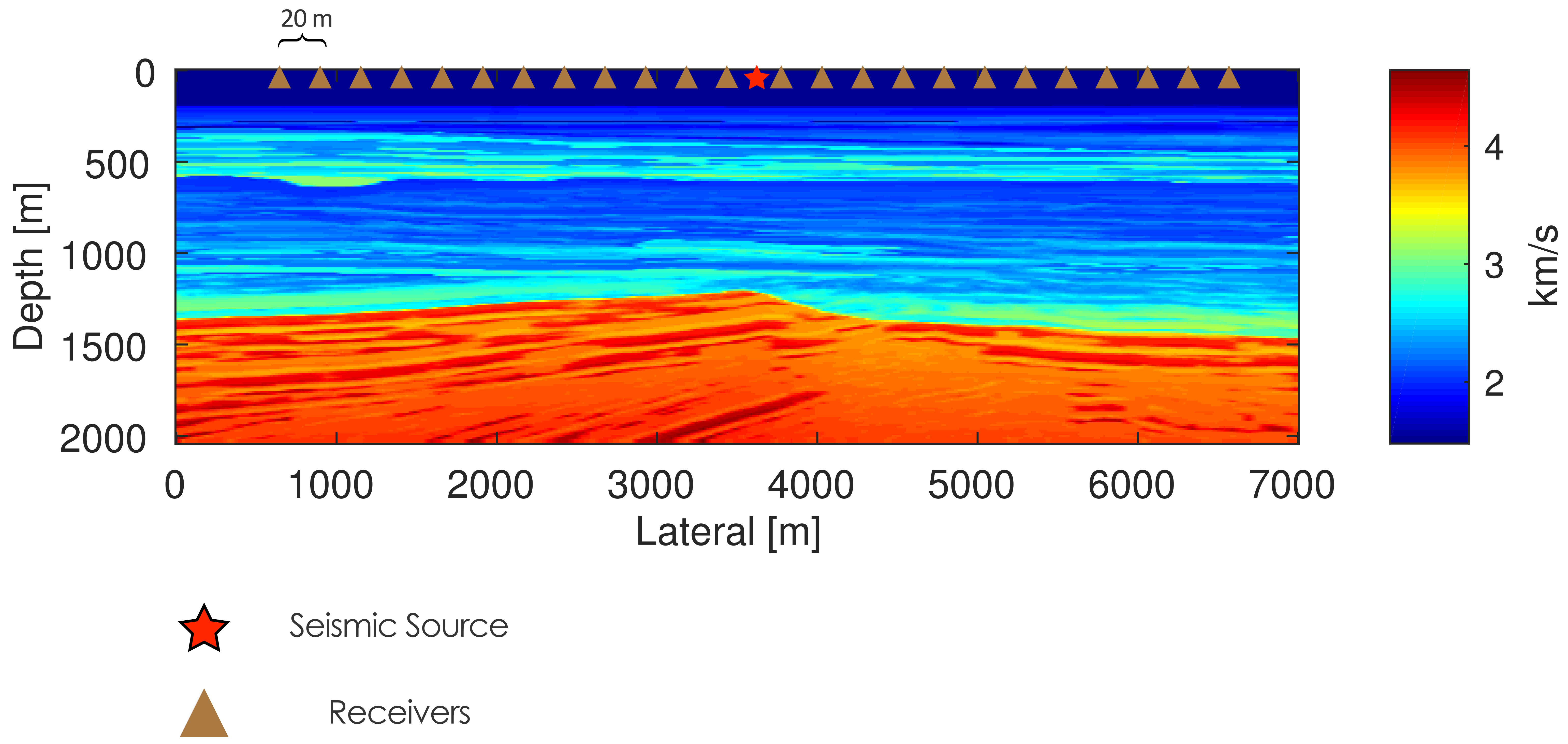
$$\text{subject to} \qquad \|\mathbf{P}\mathbf{A}^{-1}\mathbf{q} - \mathbf{d}\| \leq \sigma$$

1. **for** $k = 0, 1, \cdots$
2. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{P}^* \mathcal{P}_\sigma(\mathbf{P}\mathbf{u}_k - \mathbf{d})$
3. $\mathbf{u}_{k+1} = \mathbf{A}^{-1}\text{Prox}_{\lambda\|\cdot\|_{1,2}}(\mathbf{A}\mathbf{z}_{k+1})$
4. **end for**

*where $\mathcal{P}_\sigma(\mathbf{P}\mathbf{u}_k - \mathbf{d}) = \max\{0, 1 - \frac{\sigma}{\|\mathbf{P}\mathbf{u}_k - \mathbf{d}\|}\} \cdot (\mathbf{P}\mathbf{u}_k - \mathbf{d})$

What happens if we have noise free single source data?

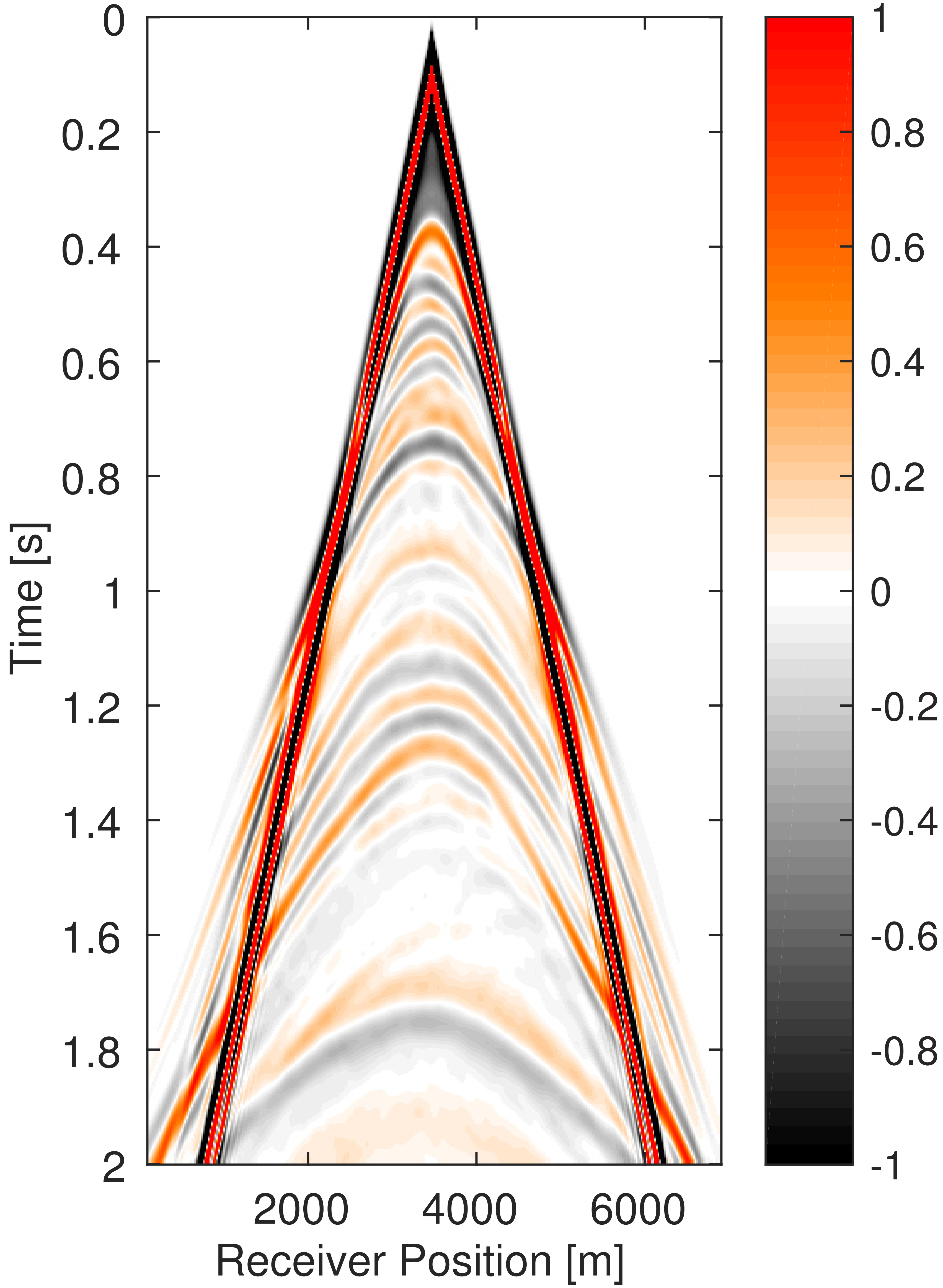
Experimental setup



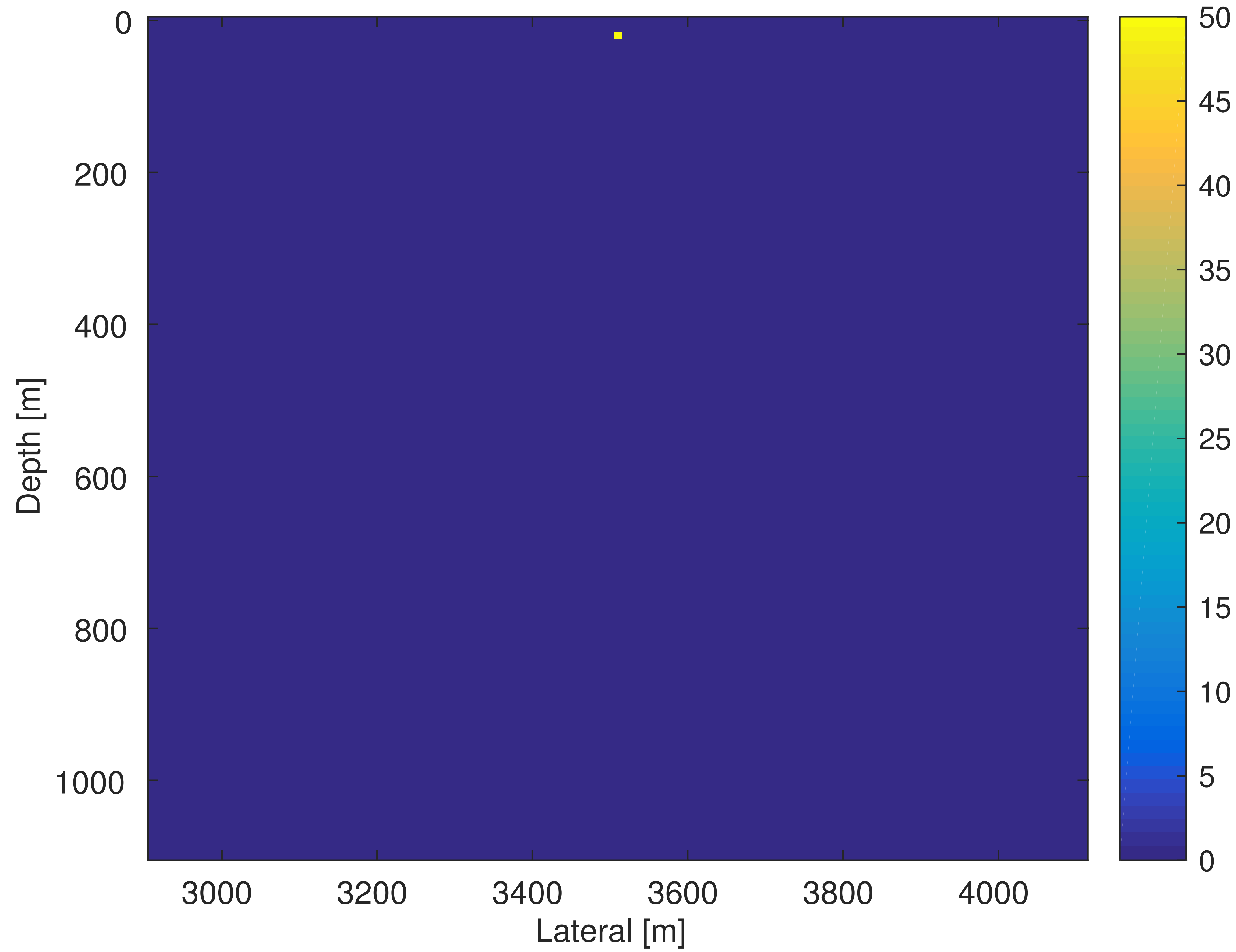
Modeling information:

Model: BG Compass model
Model size: 2040m x 7000m
Grid spacing: 10m
Receiver spacing: 20m
Source depth: 20m
Receiver depth: 20m
Fixed spread: 6.8km
Sampling interval: 1ms
Recording length: 2s
Peak frequency : 10 Hz

Lambda : 100

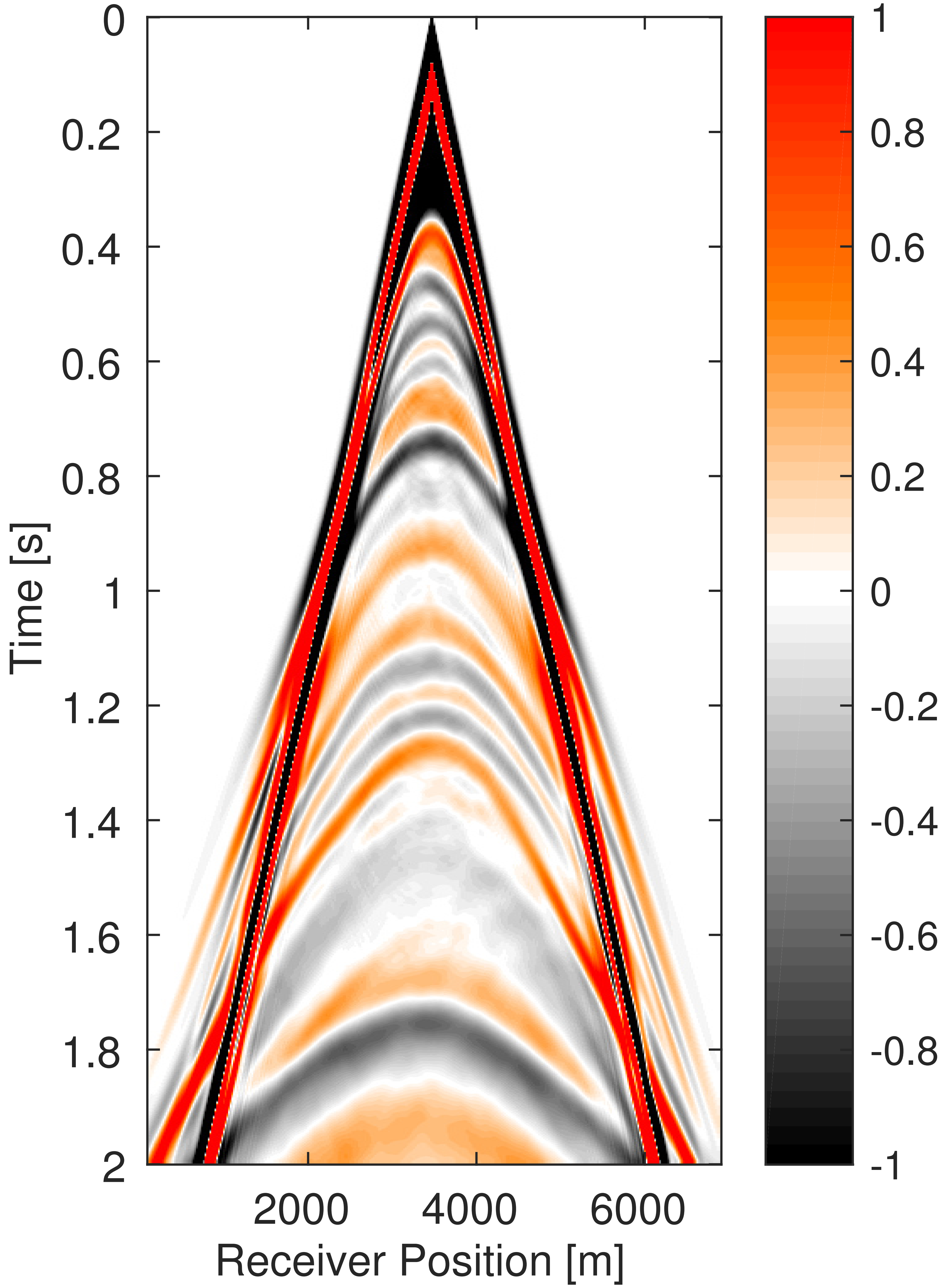


True data

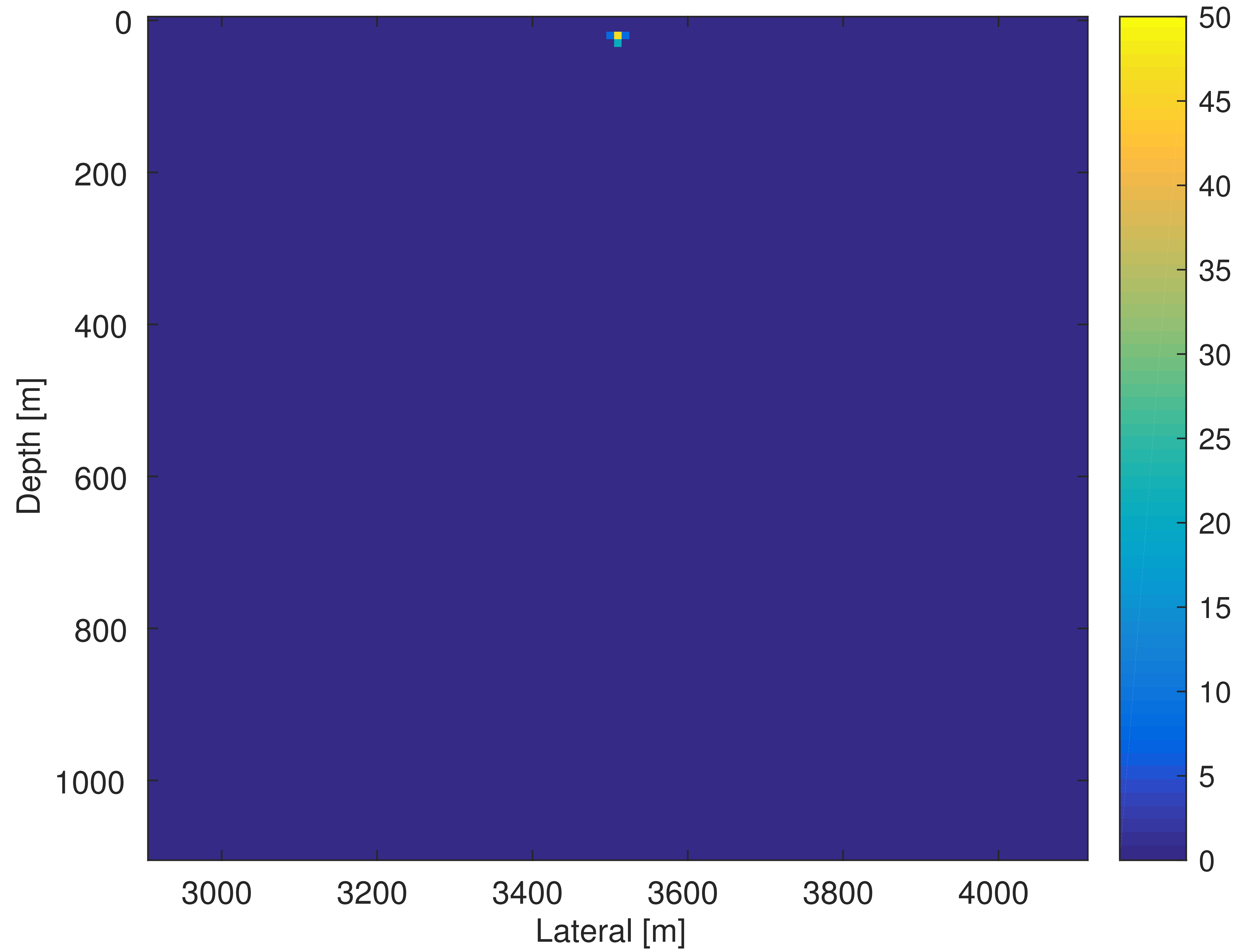


True source location (zoomed)

Sum of the absolute value of source wavelet along time



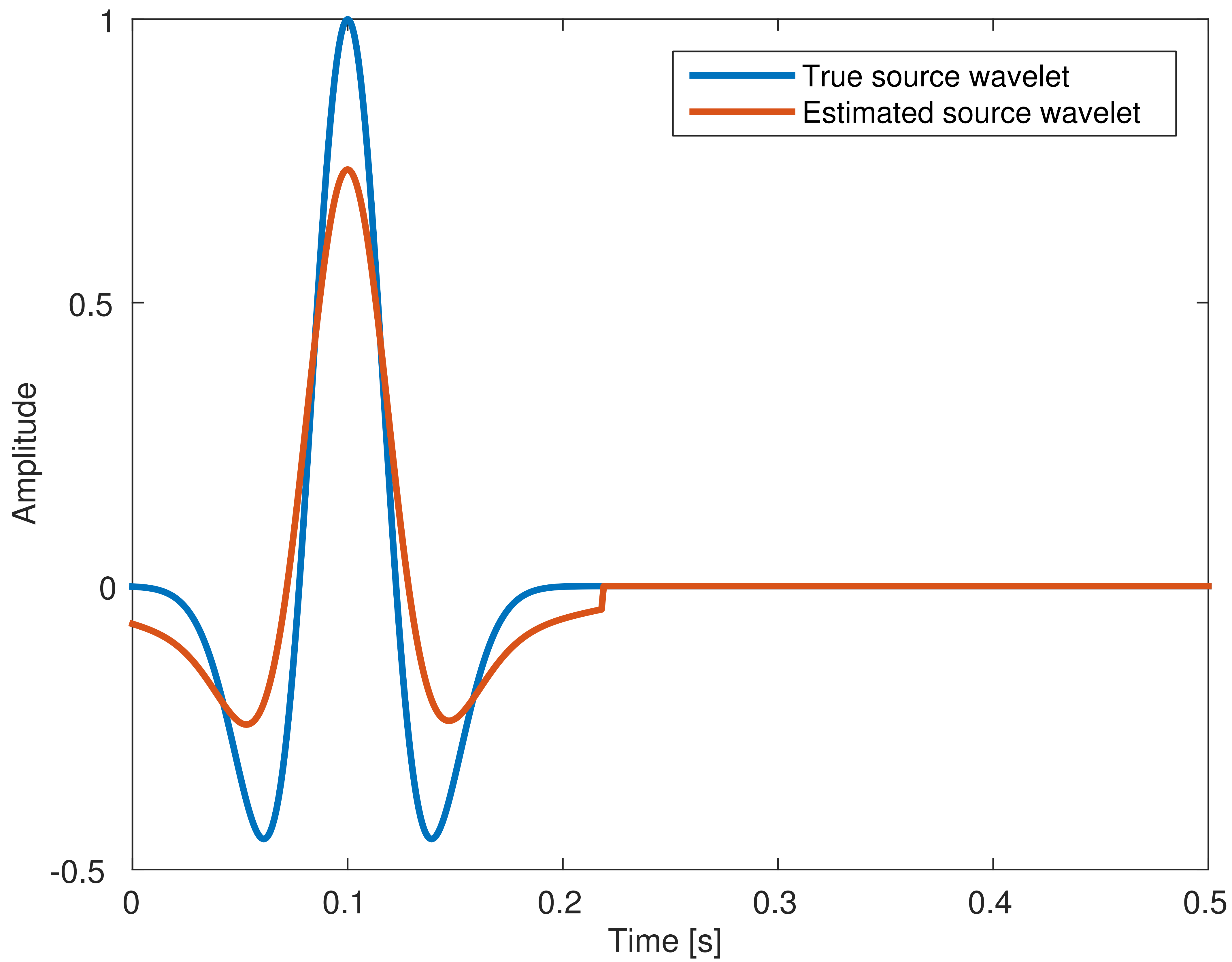
Estimated shot record



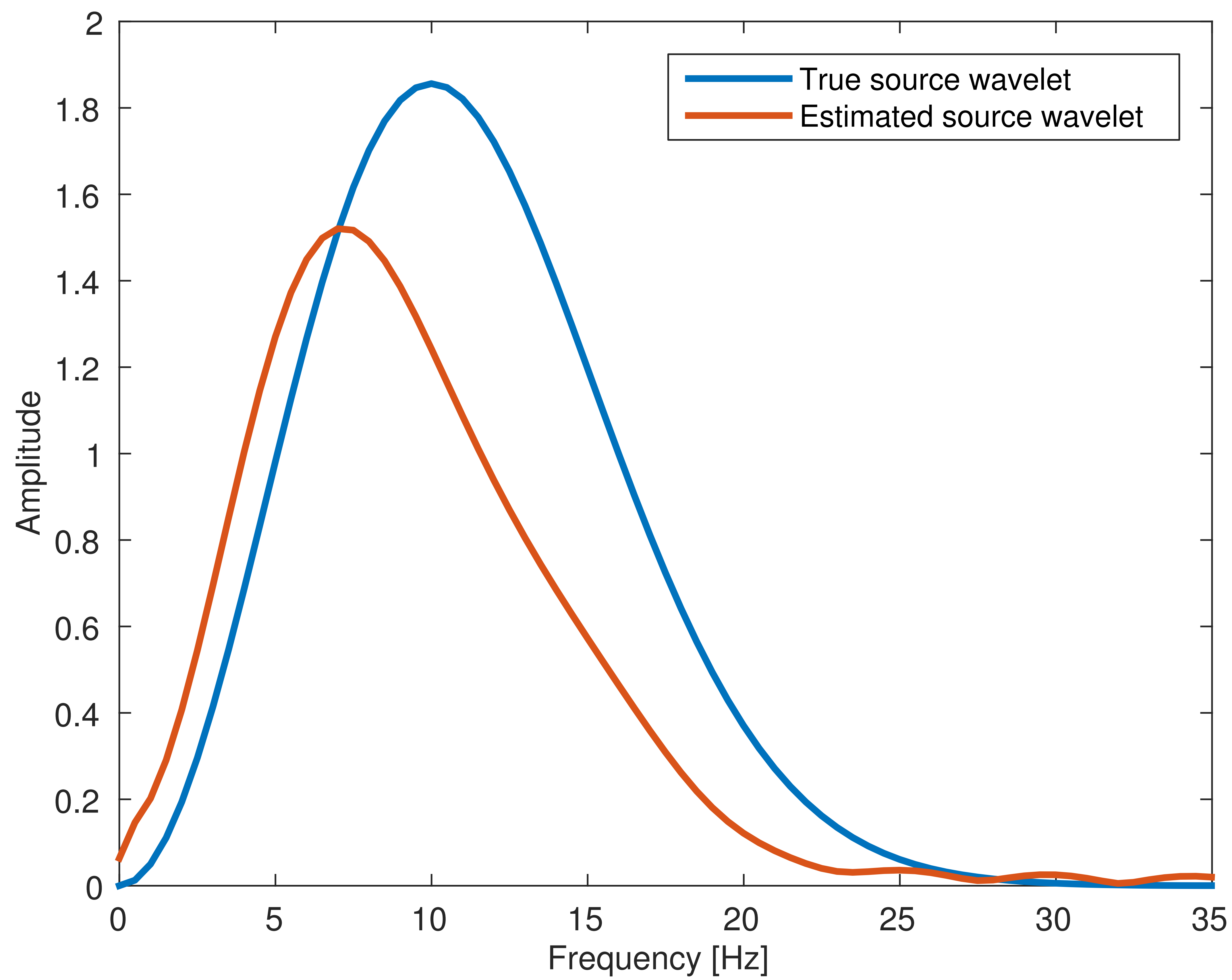
Estimated source location (zoomed)

Sum of the absolute value of source wavelet along time

Source wavelet comparison

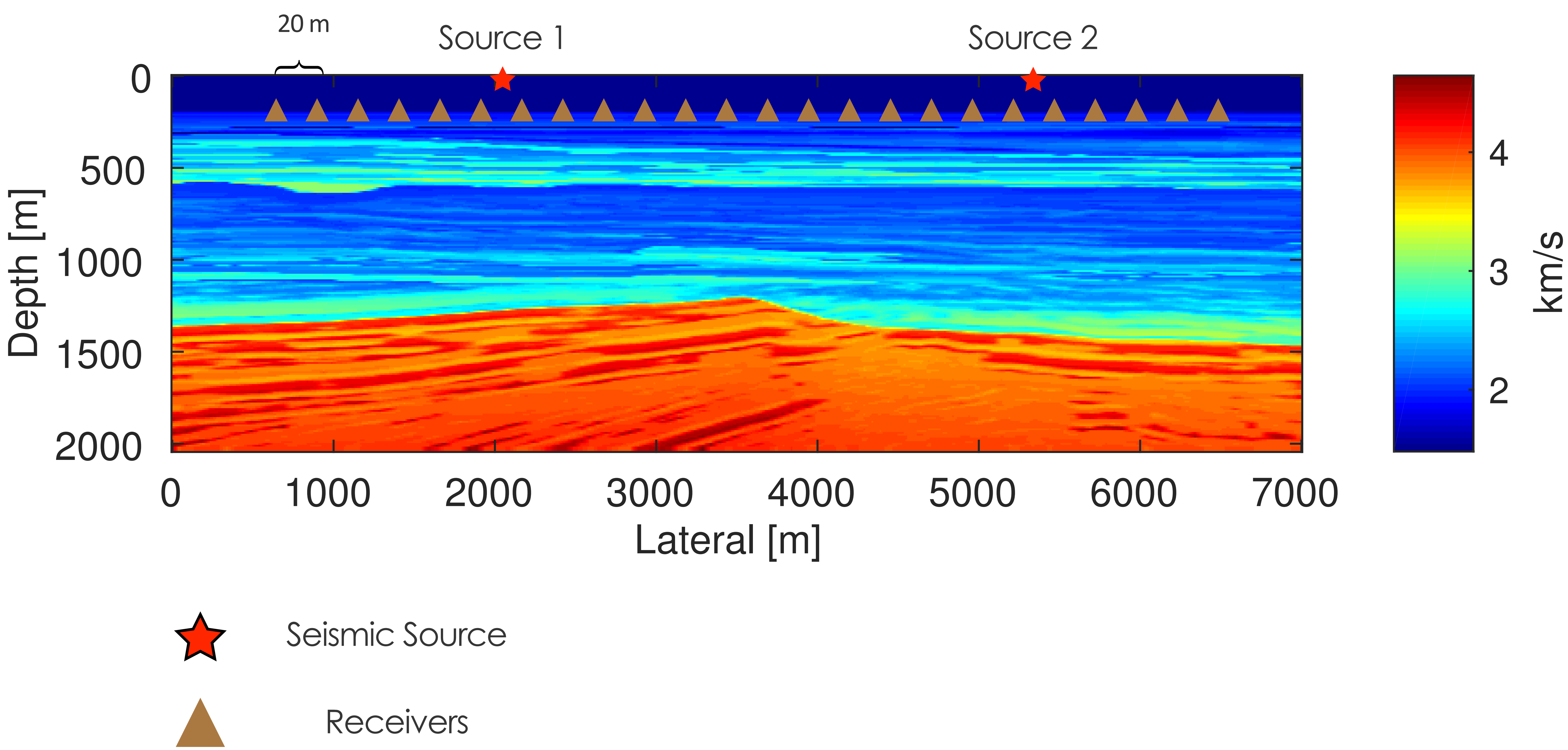


Spectrum



What happens if we have noise free shot record with simultaneous sources firing at different times?

Experimental setup

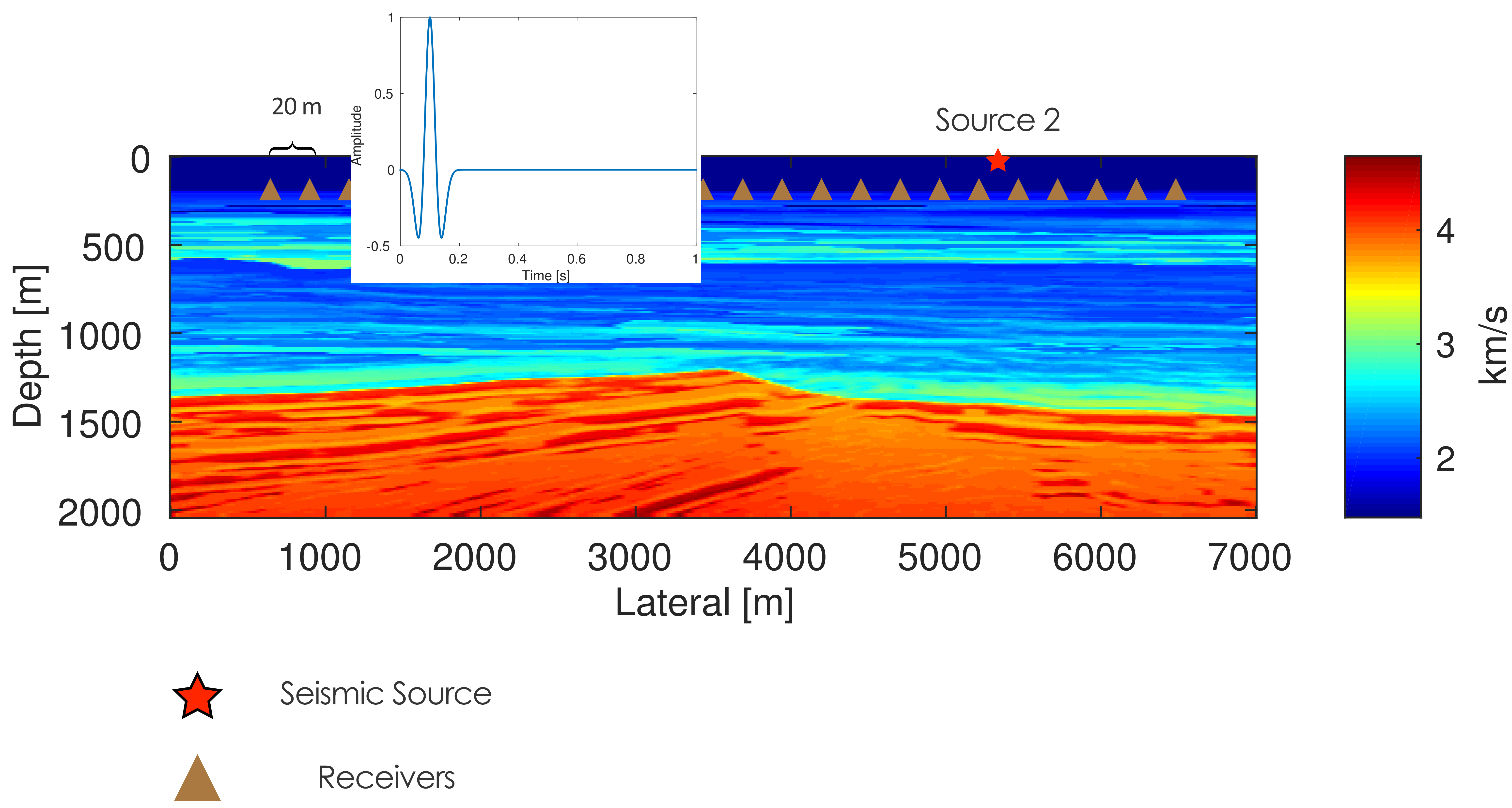


Modeling information:

Model: BG Compass model
Model size: 2040m x 7000m
Grid spacing: 10m
Receiver spacing: 20m
Source depth: 20m
Receiver depth: 200m
Fixed spread: 6.8km
Sampling interval: 1ms
Recording length: 2s
Peak frequency : 10 Hz

Lambda : 100

Experimental setup

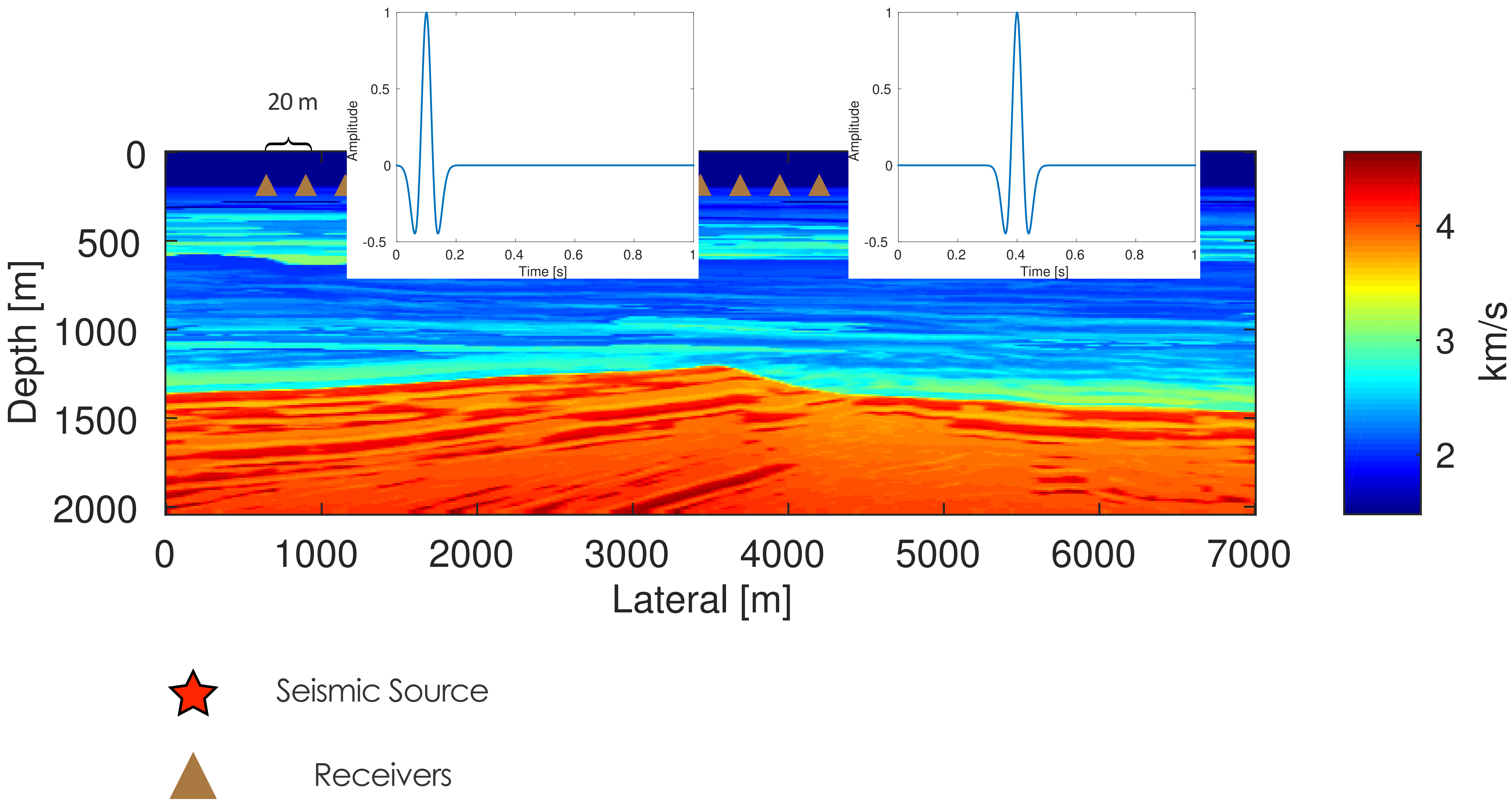


Modeling information:

Model: BG Compass model
Model size: 2040m x 7000m
Grid spacing: 10m
Receiver spacing: 20m
Source depth: 20m
Receiver depth: 200m
Fixed spread: 6.8km
Sampling interval: 1ms
Recording length: 2s
Peak frequency : 10 Hz

Lambda : 100

Experimental setup



Modeling information:

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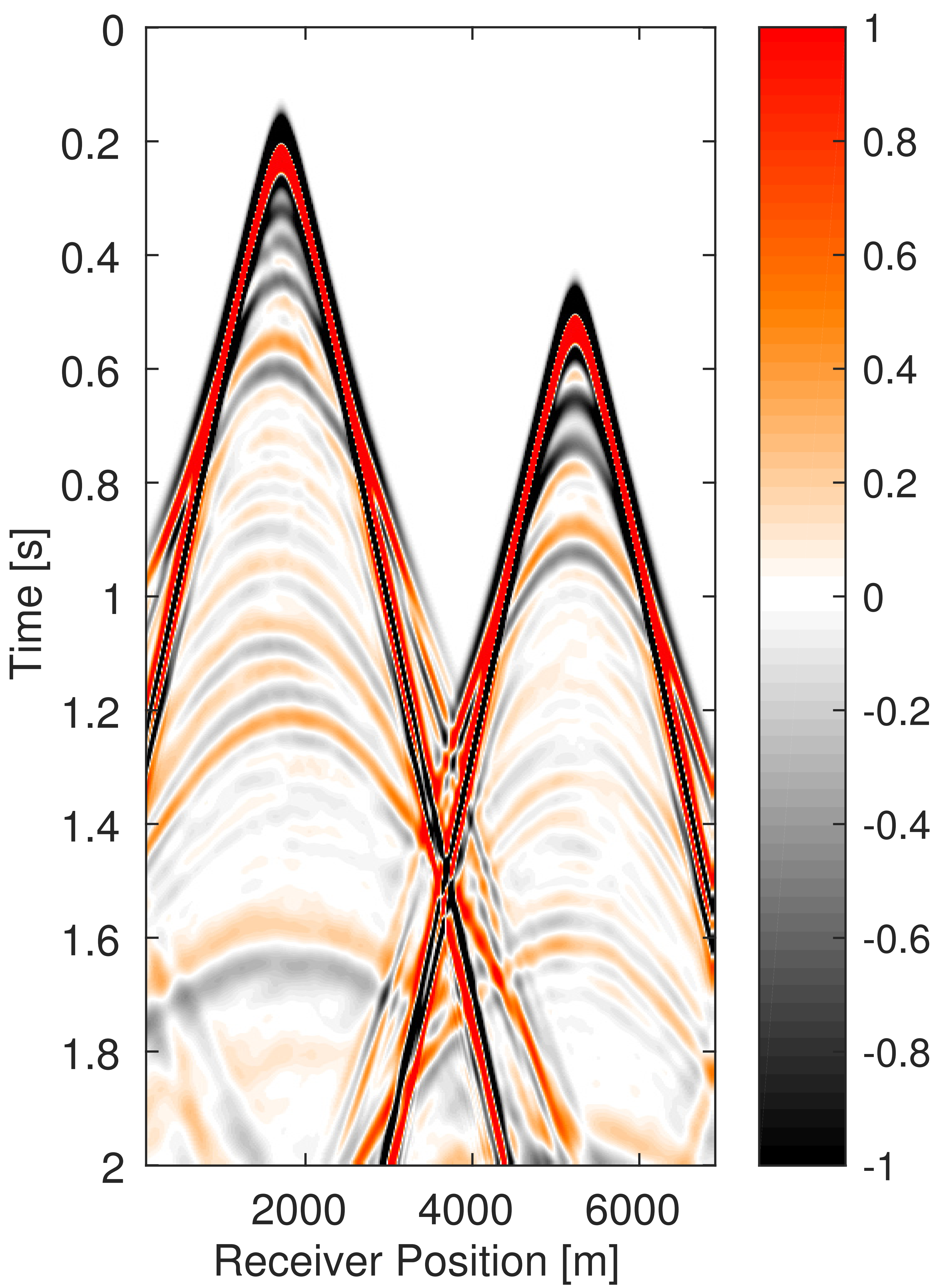
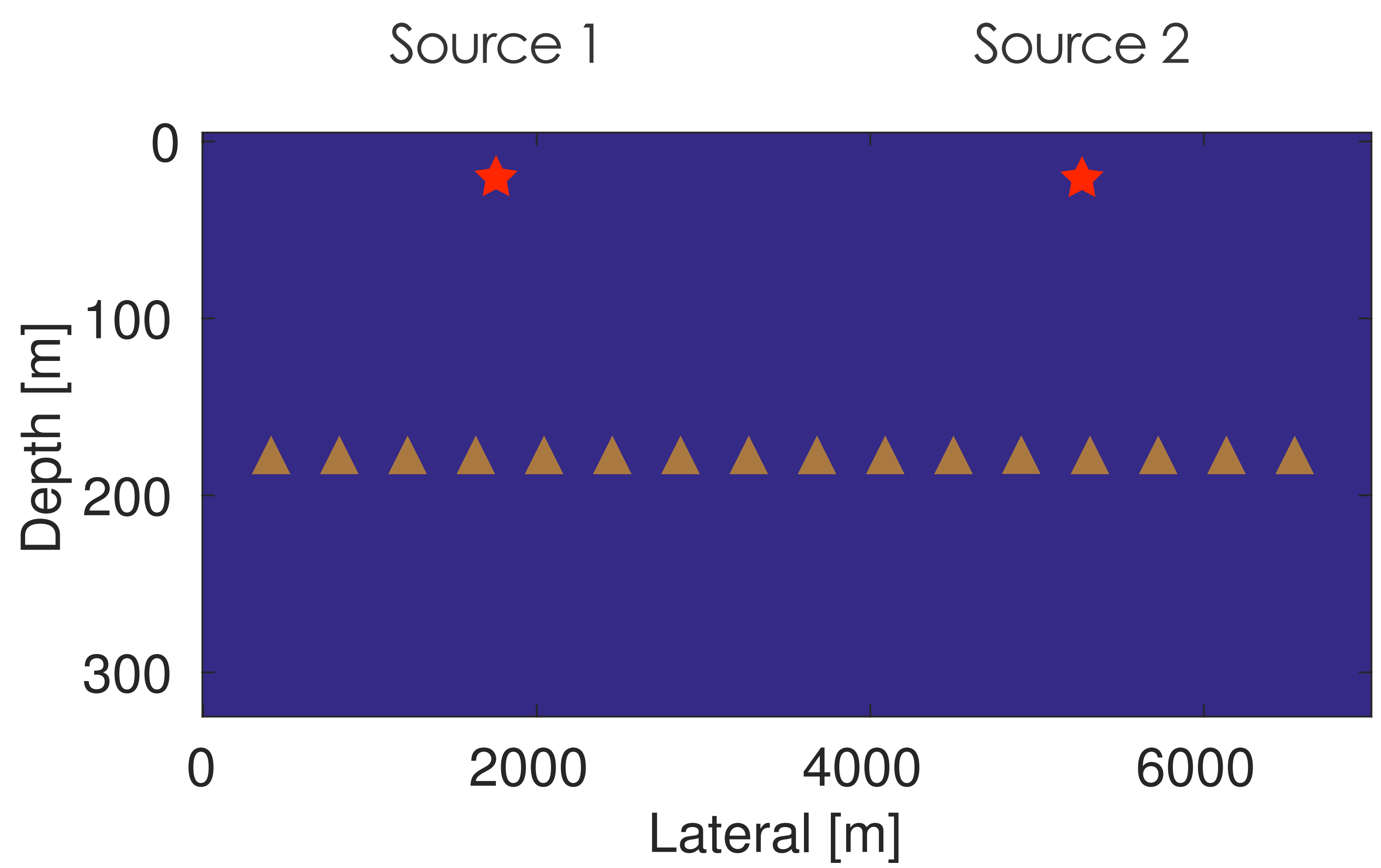
Grid spacing: 10m
Receiver spacing: 20m
Source depth: 20m
Receiver depth: 200m
Fixed spread: 6.8km
Sampling interval: 1ms
Recording length: 2s
Peak frequency : 10 Hz

Lambda : 100

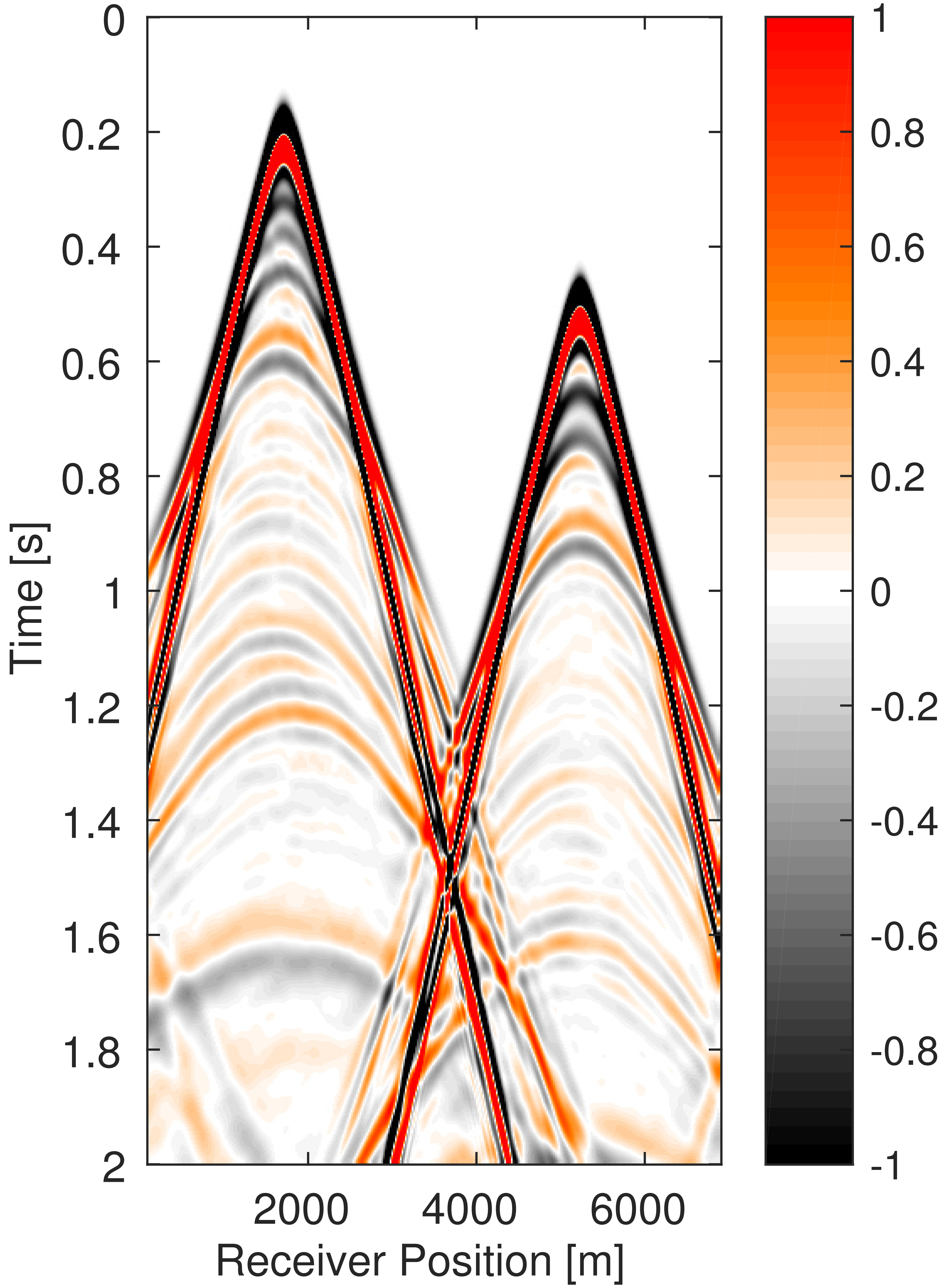


True data

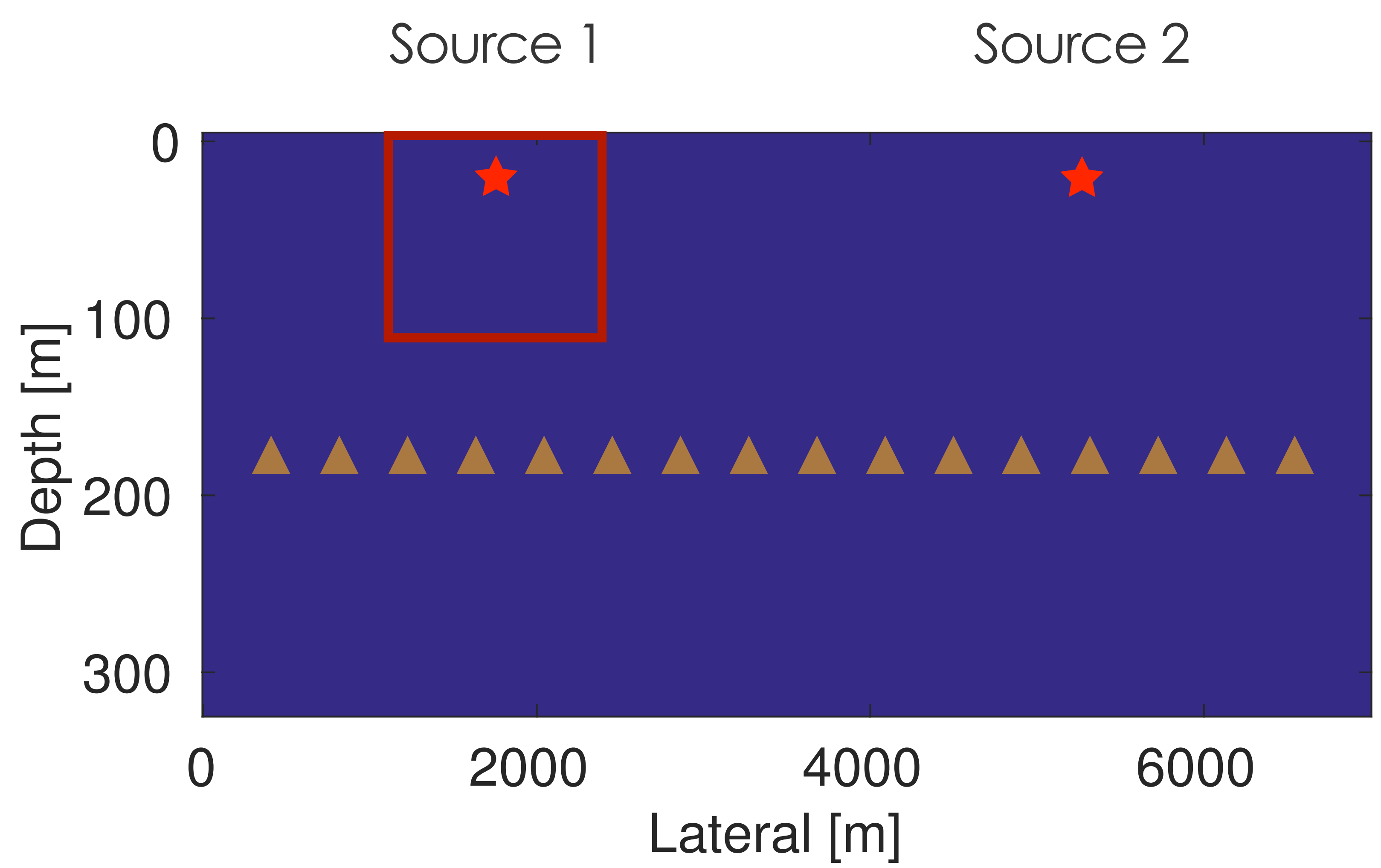


True source location

Sum of the absolute value of source wavelet along time

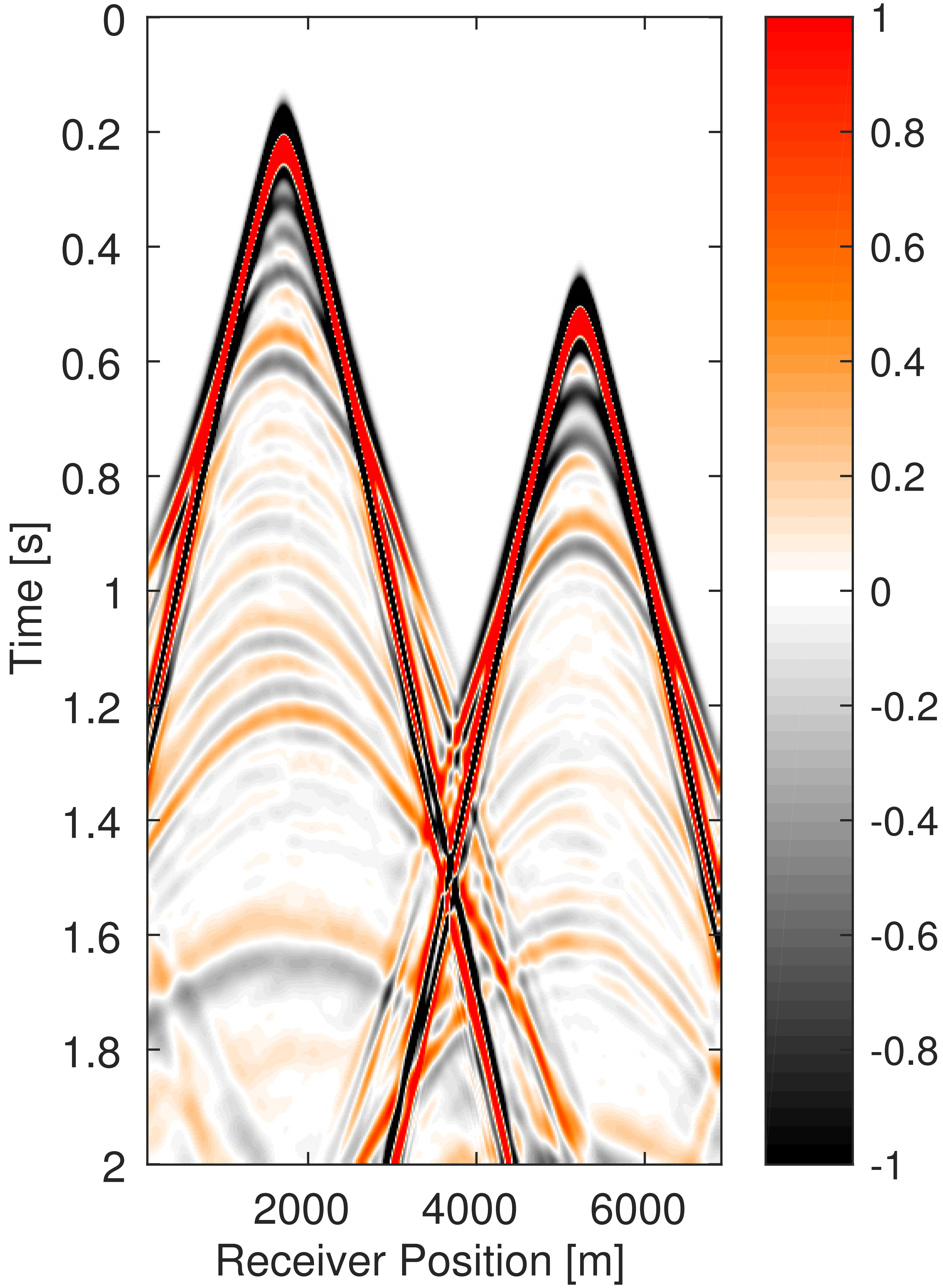


True data

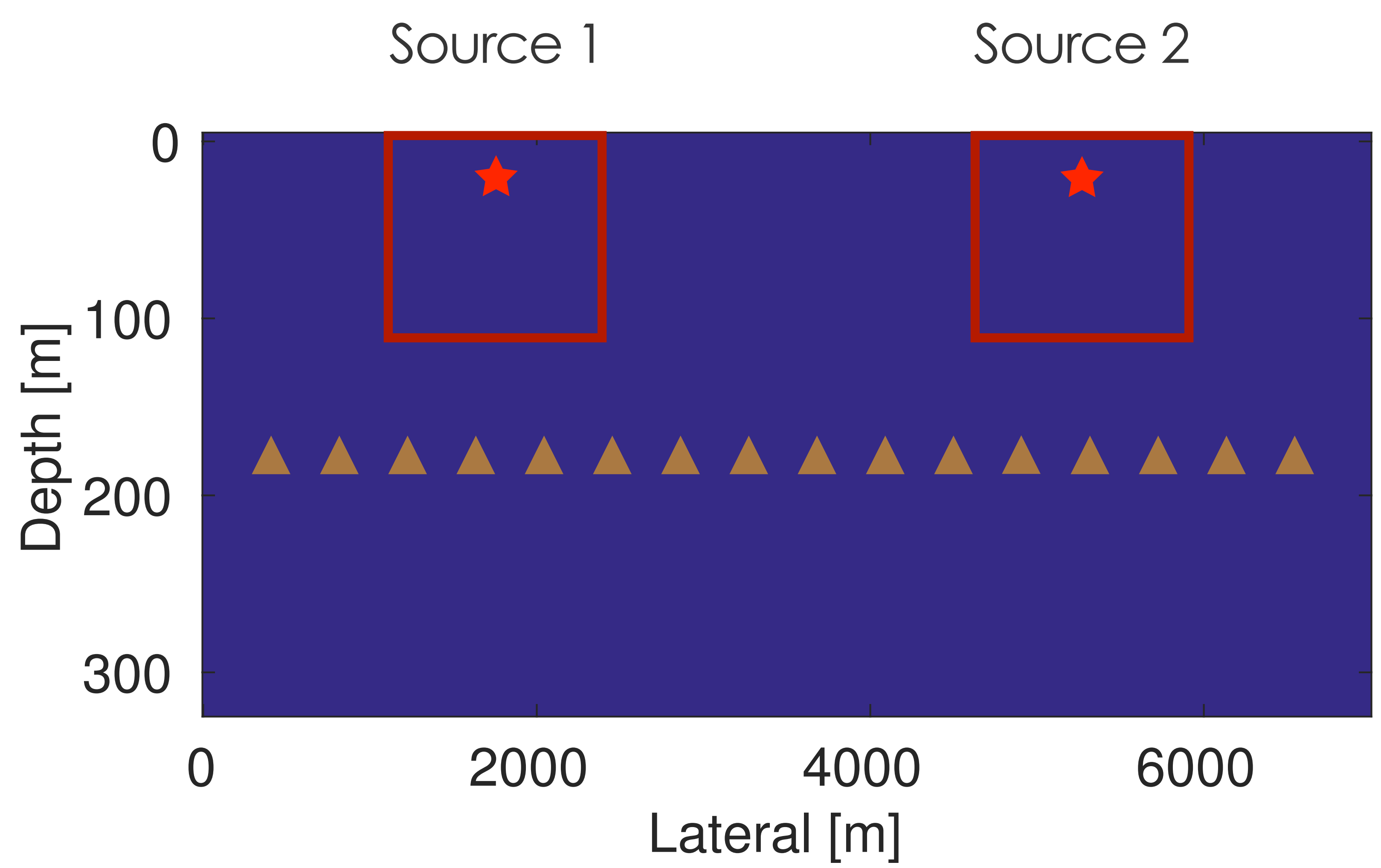


True source location

Sum of the absolute value of source wavelet along time

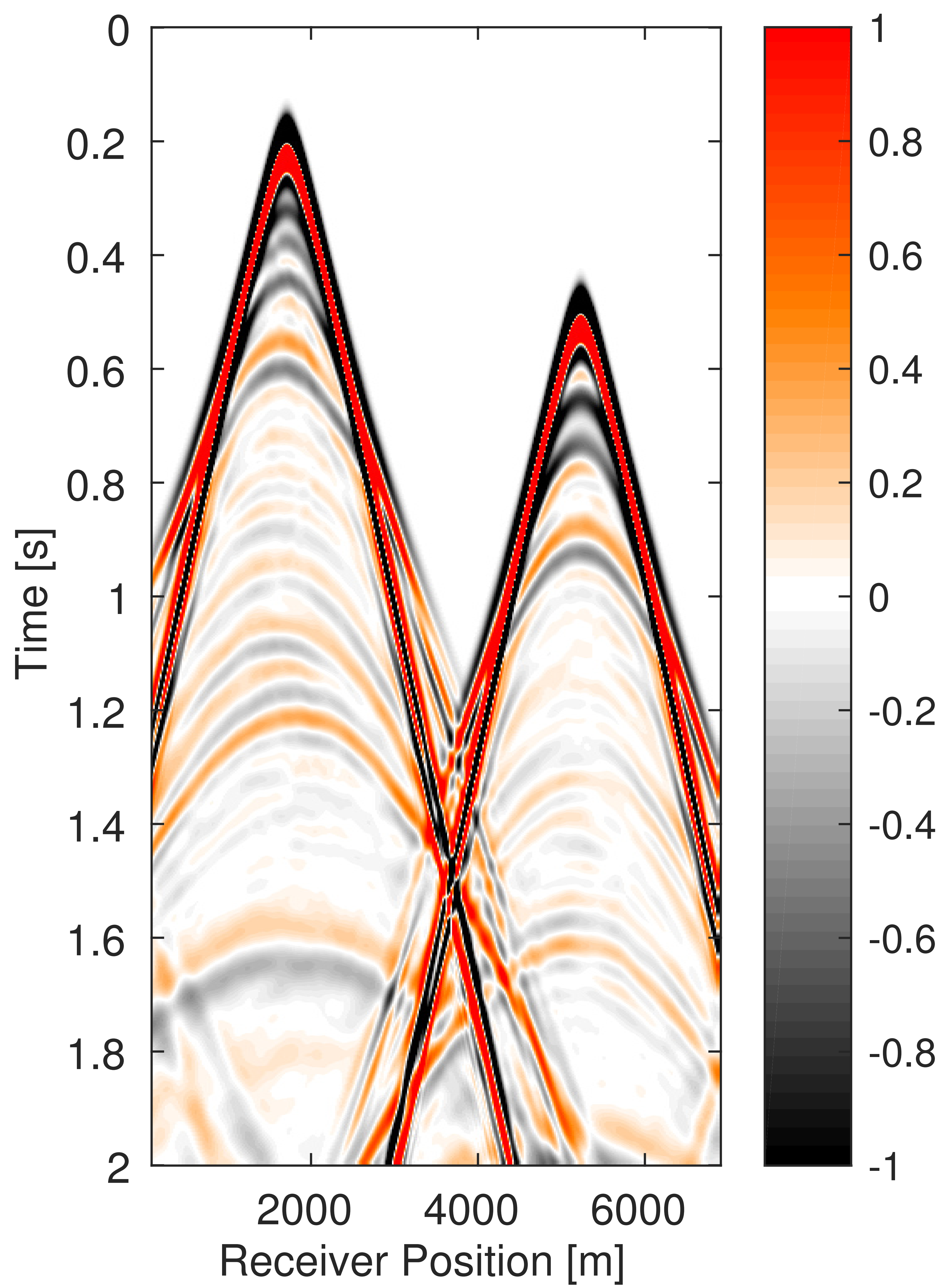


True data

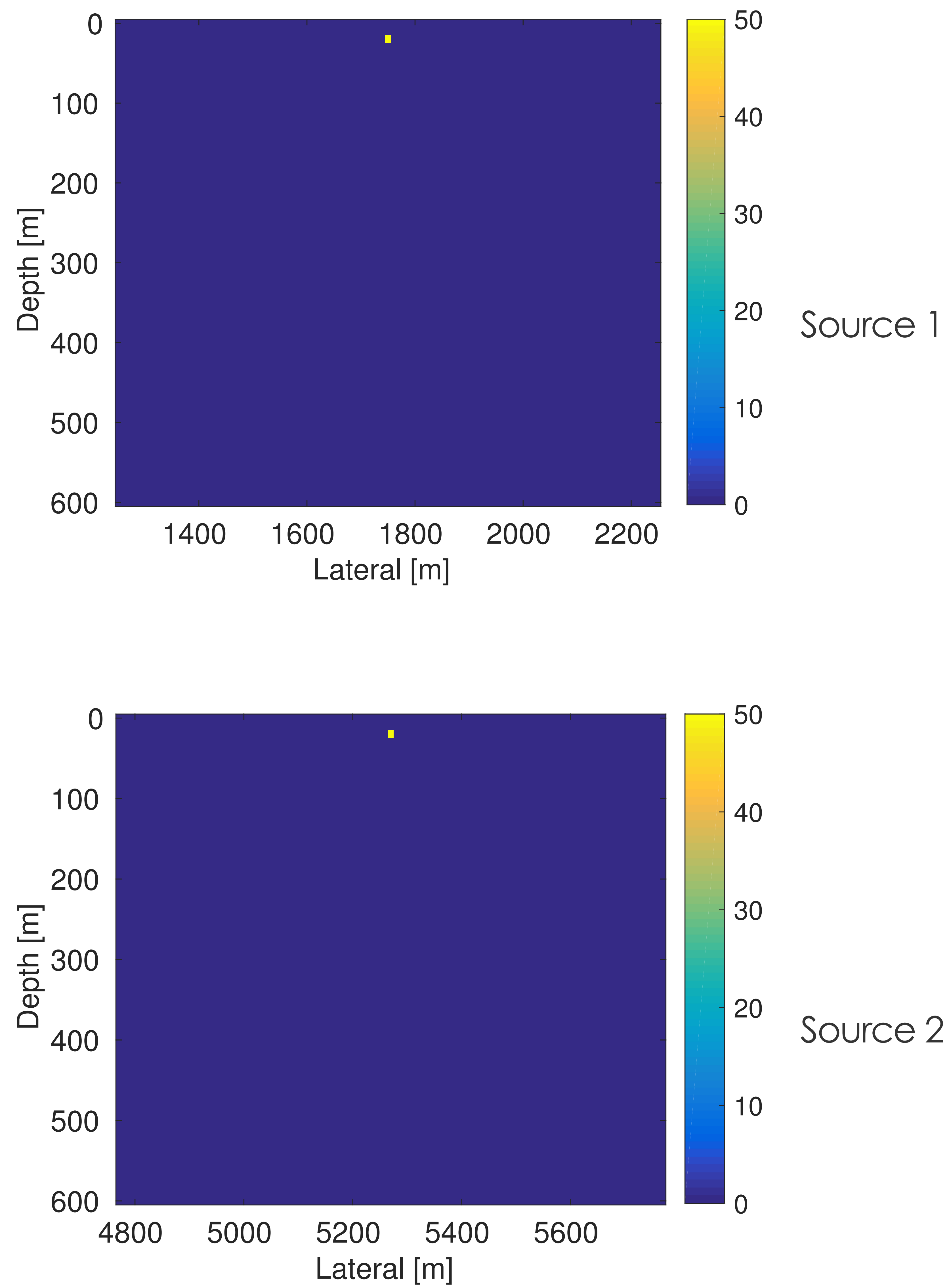


True source location

Sum of the absolute value of source wavelet along time

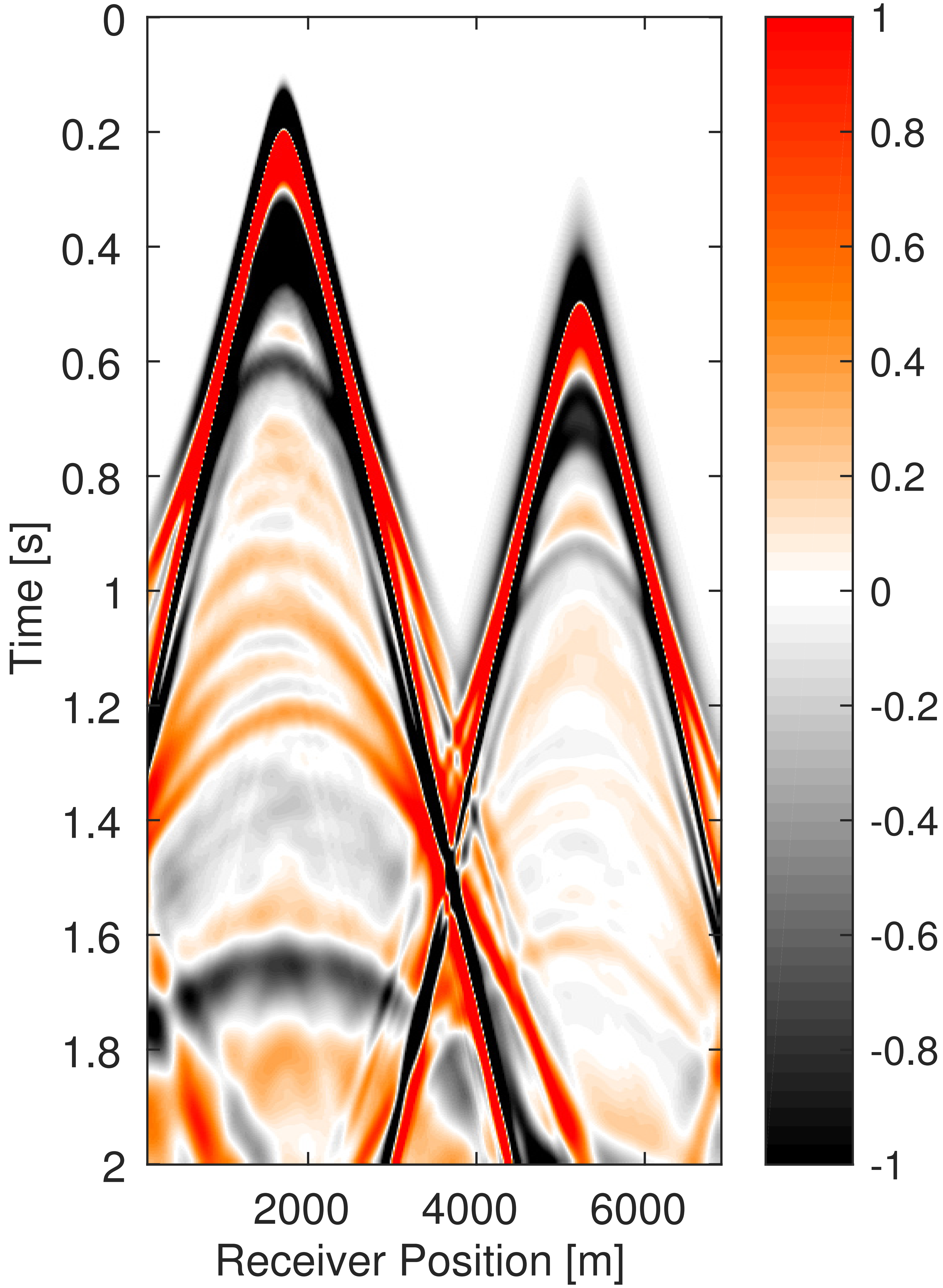


True data

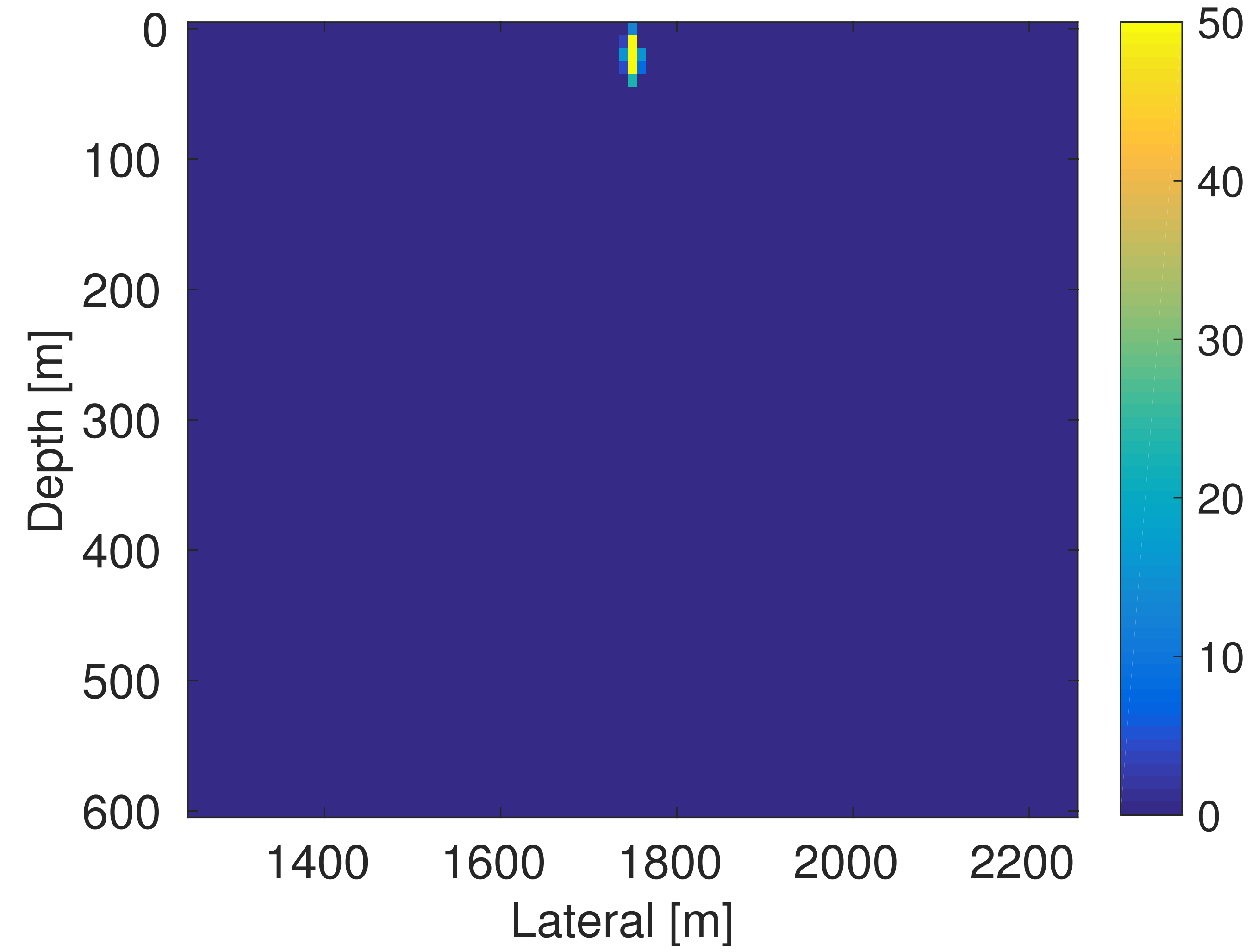


True source location (zoomed)

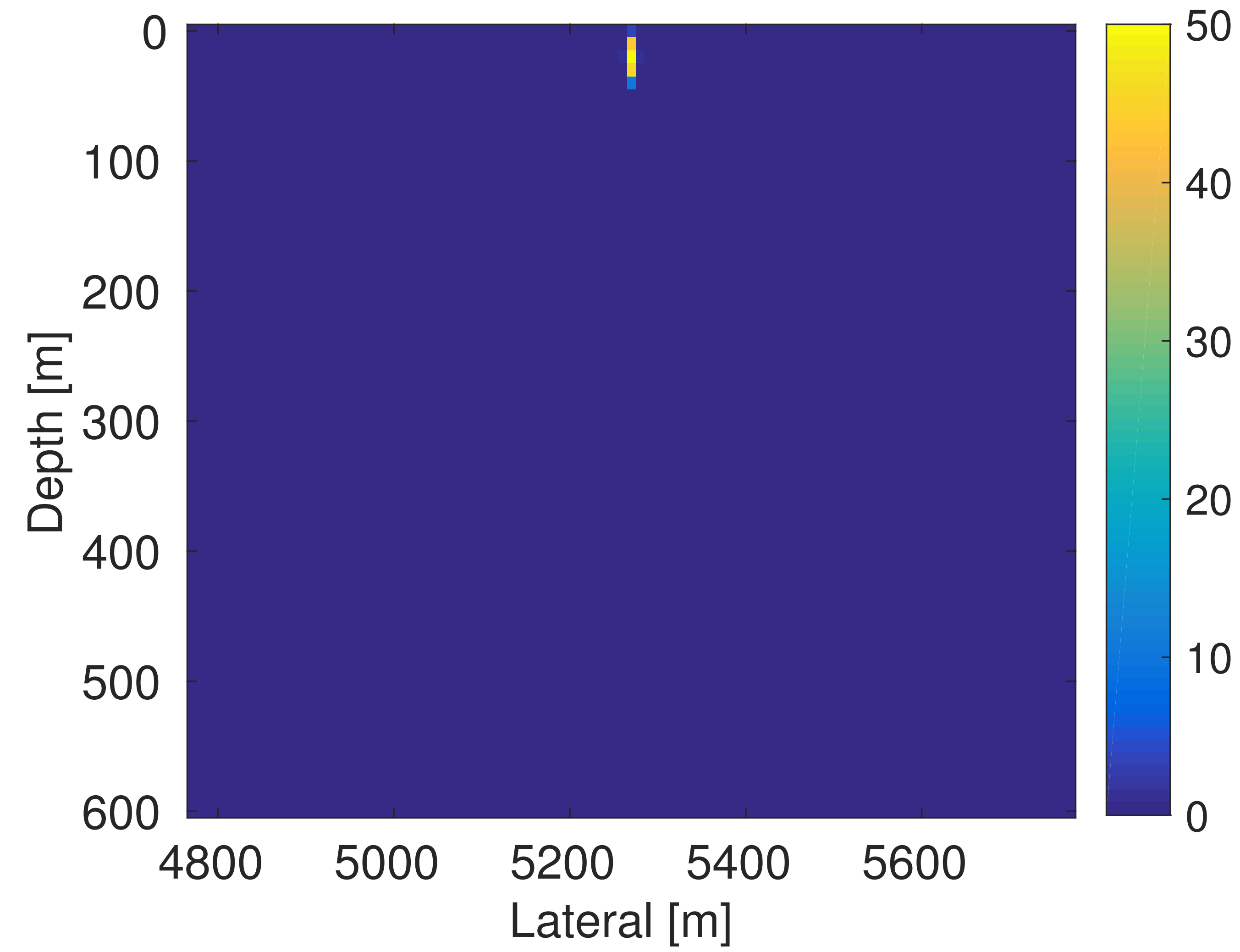
Sum of the absolute value of source wavelet along time



Estimated shot record (Scaled down by factor of 3.5)



Source 1



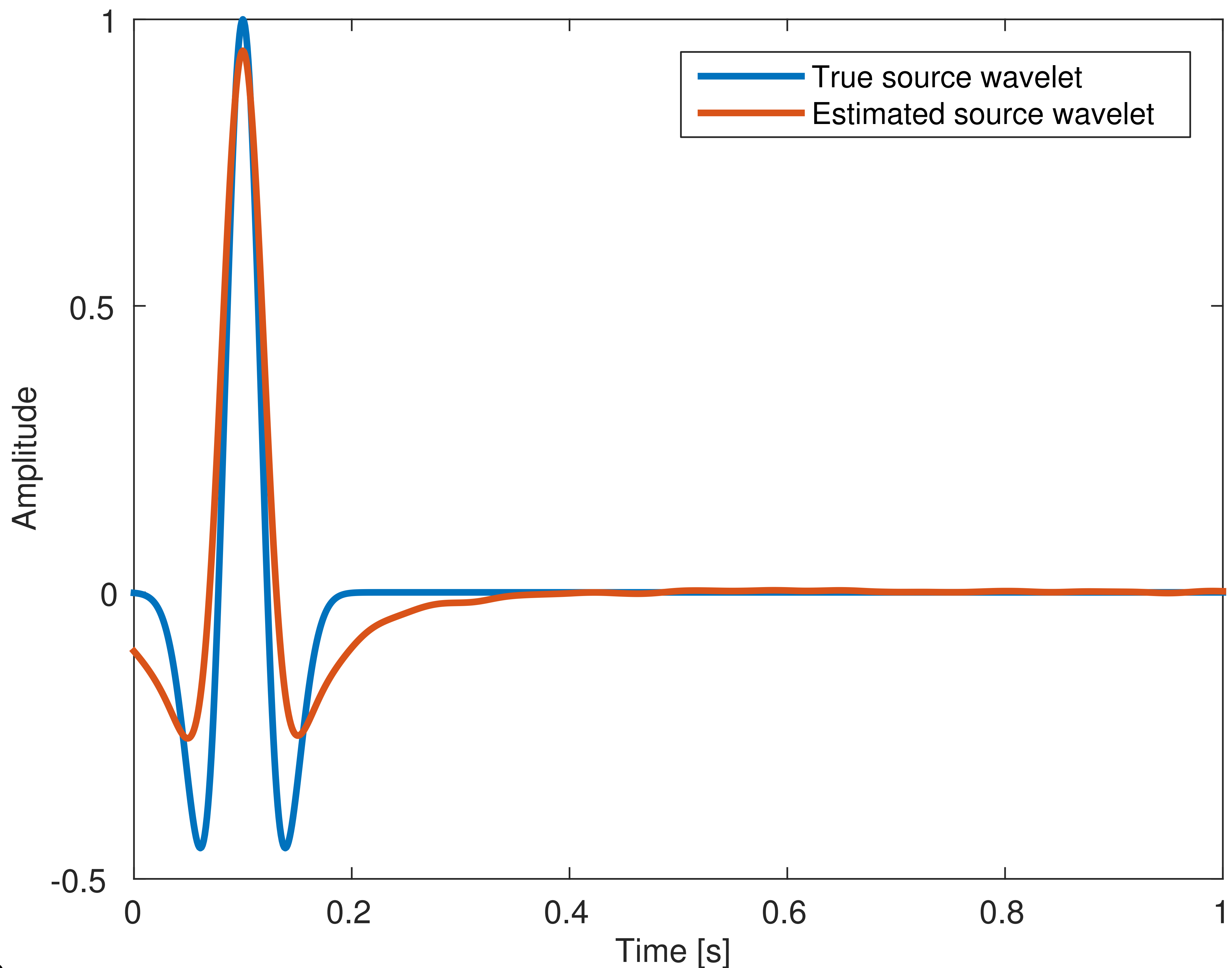
Source 2

Estimated source location (zoomed)

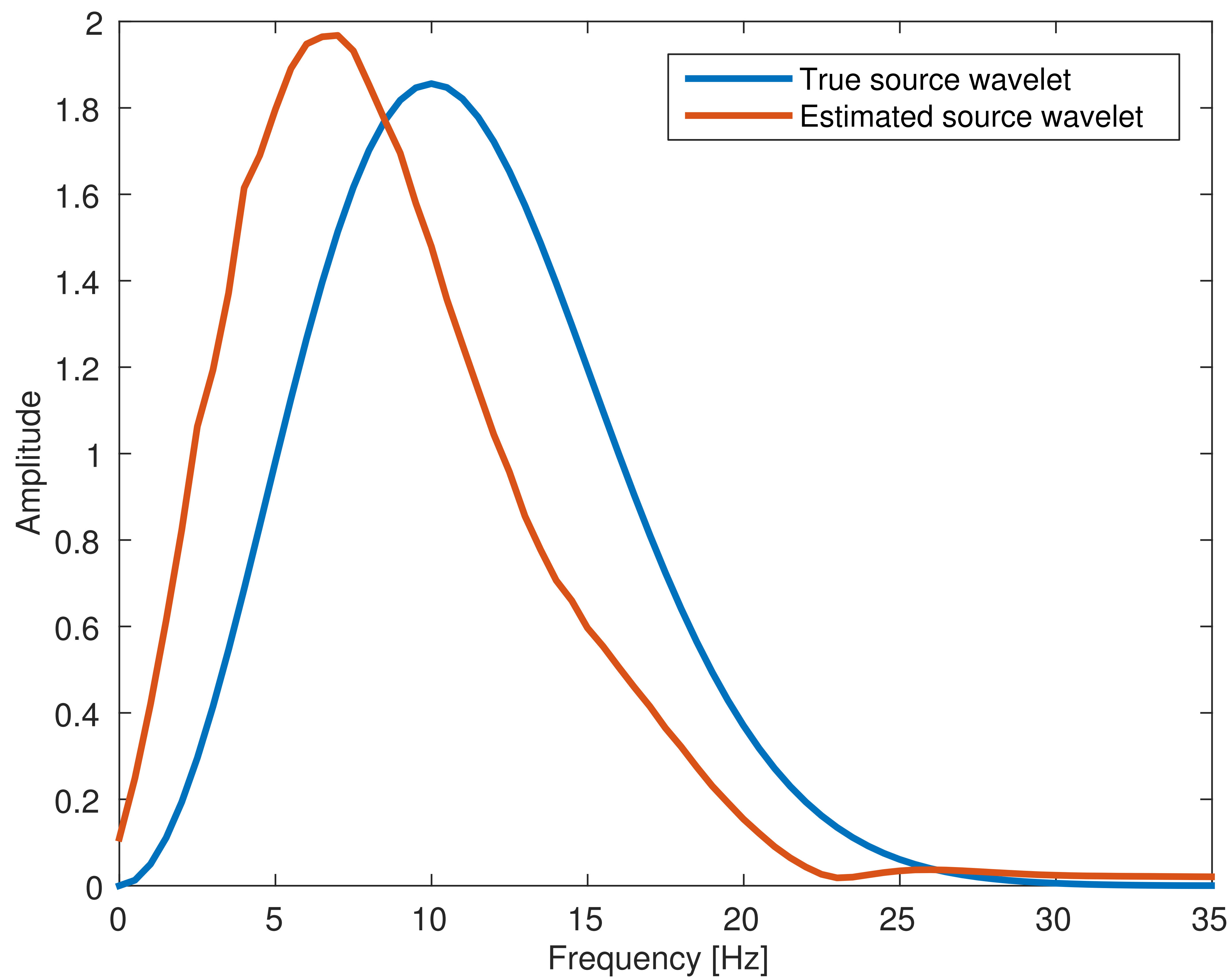
Sum of the absolute value of source wavelet along time

Source wavelet comparison

Source 1

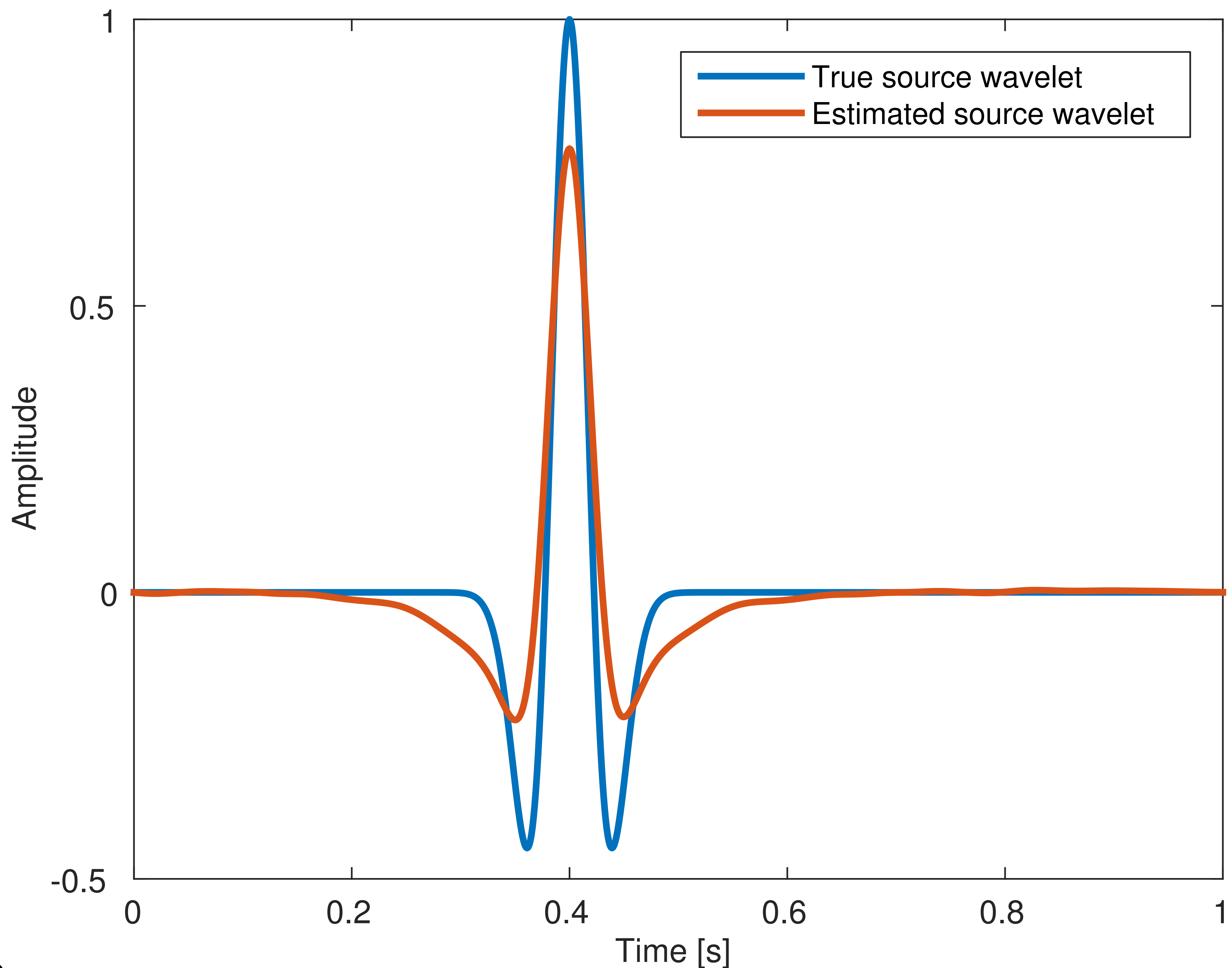


Spectrum

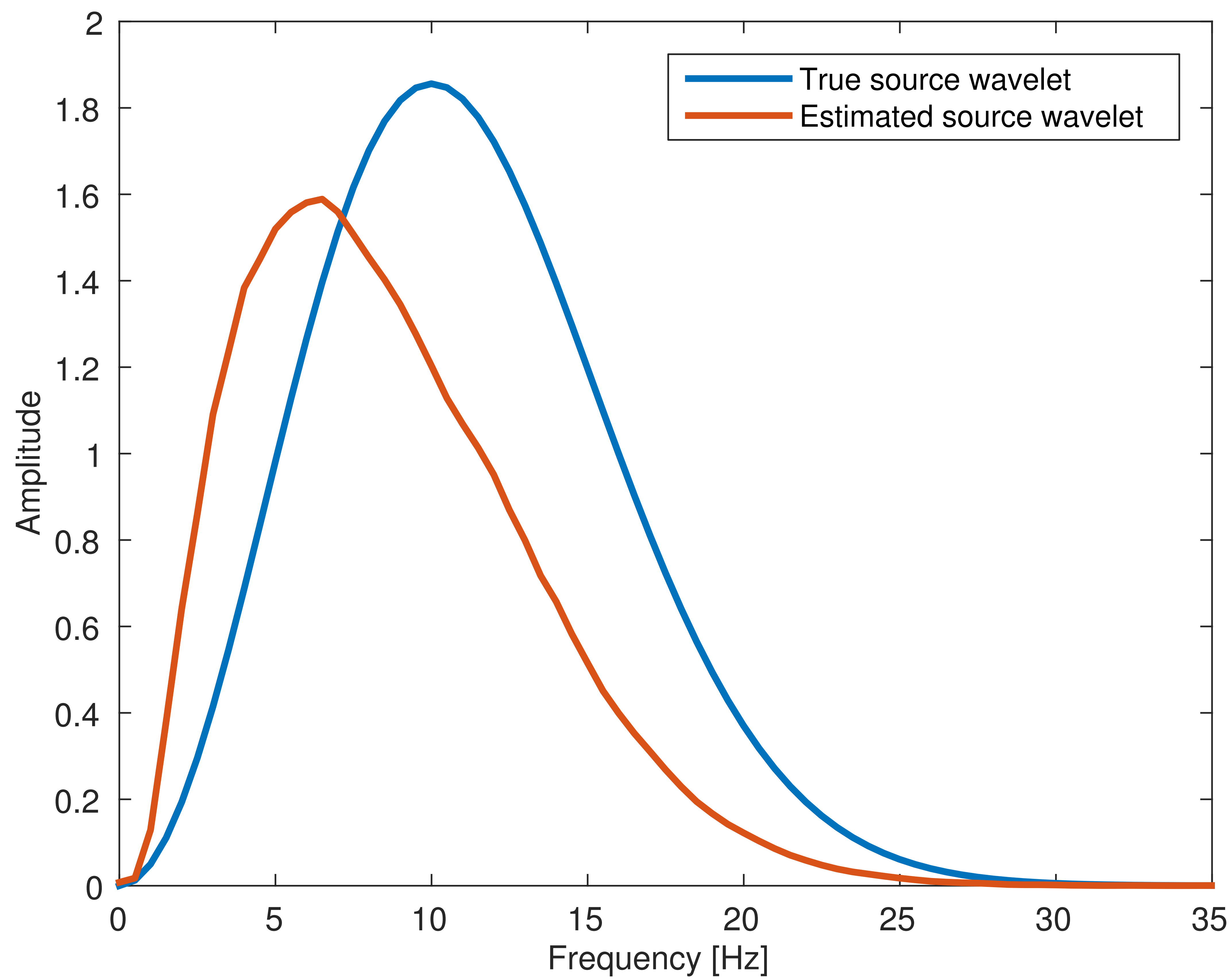


Source wavelet comparison

Source 2



Spectrum



Potential application: Noise removal

Future extension: Wave equation based denoising

- Seismic data challenges
  - contaminated with wind generated noise originating from the surface
  - seismic interference

$$\mathbf{A}(\mathbf{m})\mathbf{u} = \mathbf{q} + \eta$$

- η: spatially distributed wind noise source

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Source collocation – w/ Linearized Bregman

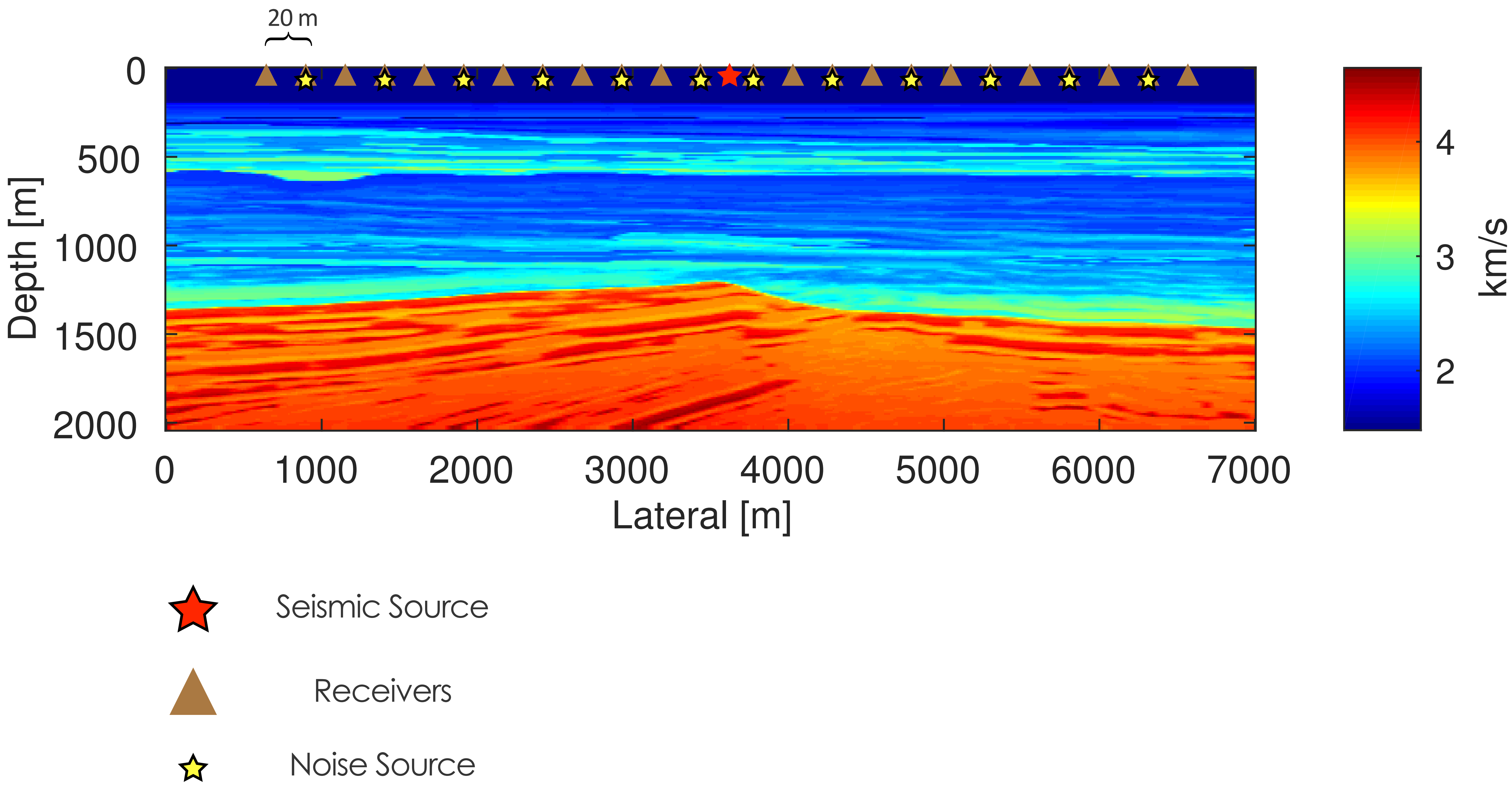
$$\underset{\mathbf{q}}{\text{minimize}} \quad \lambda\|\mathbf{q}\|_{1,2} + \frac{1}{2}\|\mathbf{q}\|_F^2$$

$$\text{subject to} \quad \|\mathbf{P}\mathbf{A}^{-1}\mathbf{q} - \mathbf{d}\| \leq \sigma$$

1. **for** $k = 0, 1, \cdots$
2. $\quad \mathbf{z}_{k+1} = \mathbf{z}_k - t_k \mathbf{P}^* \mathcal{P}_\sigma(\mathbf{P}\mathbf{u}_k - \mathbf{d})$
3. $\quad \mathbf{u}_{k+1} = \mathbf{A}^{-1}\text{Prox}_{\lambda\|\cdot\|_{1,2}}(\mathbf{A}\mathbf{z}_{k+1})$
4. **end for**

*where $\mathcal{P}_\sigma(\mathbf{P}\mathbf{u}_k - \mathbf{d}) = \max\{0, 1 - \frac{\sigma}{\|\mathbf{P}\mathbf{u}_k - \mathbf{d}\|}\} \cdot (\mathbf{P}\mathbf{u}_k - \mathbf{d})$

Experimental setup



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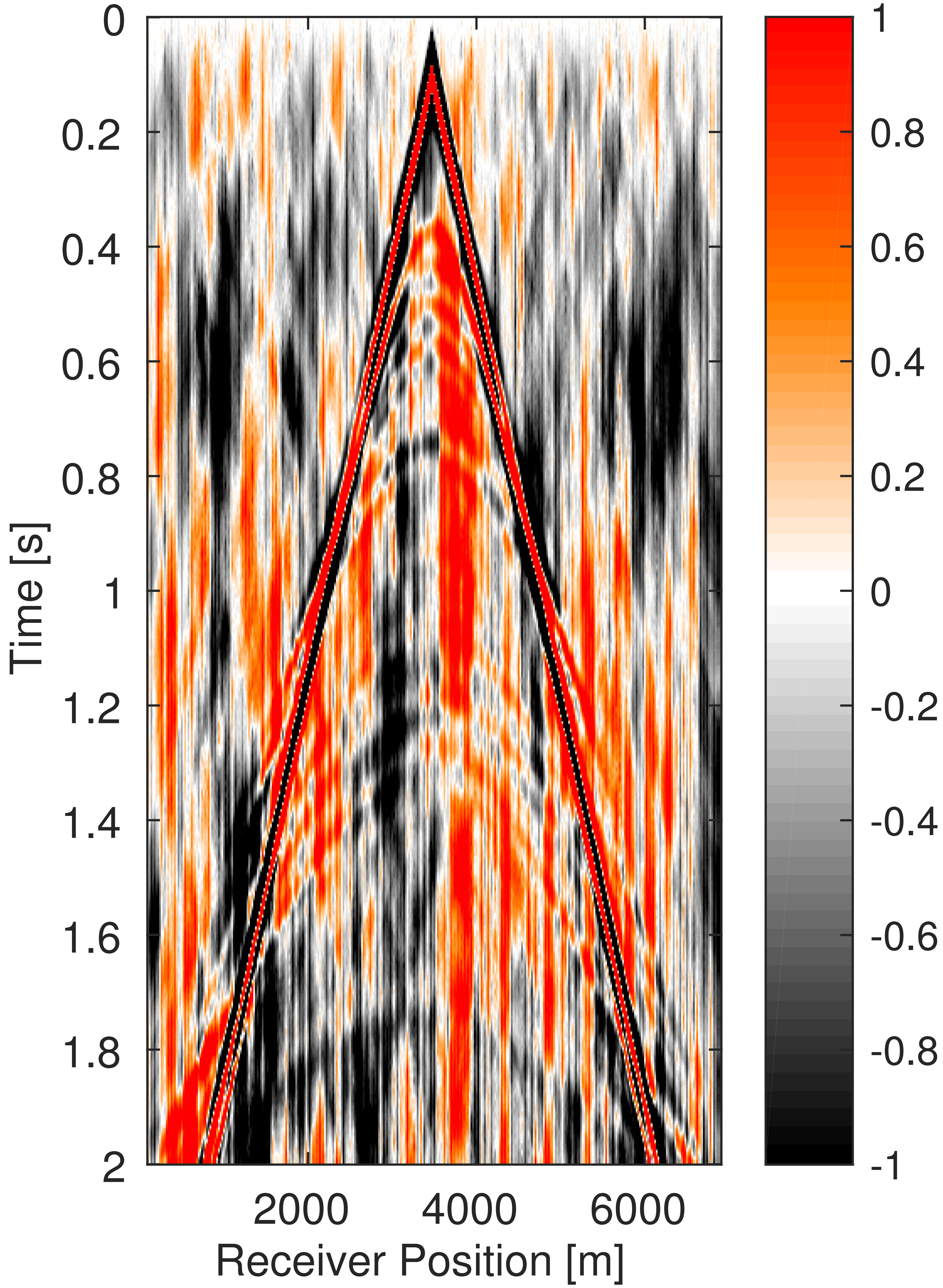
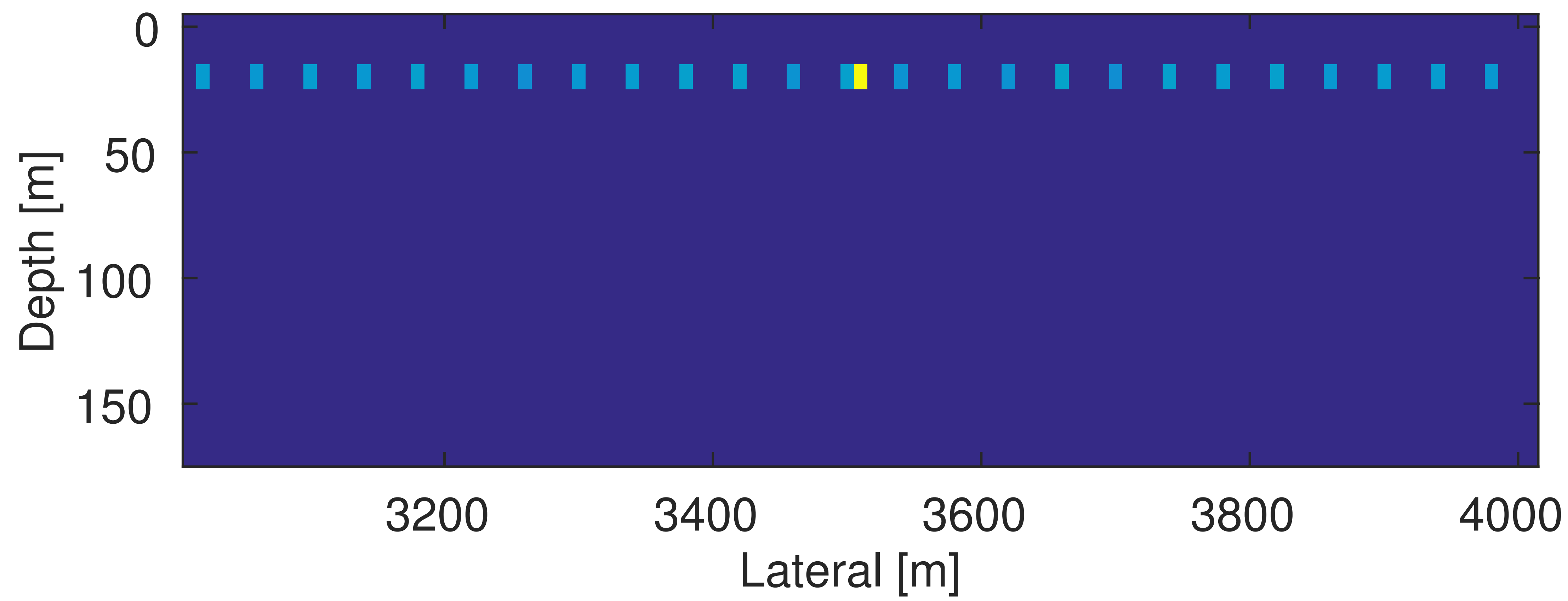
Source depth: 20m
Receiver depth: 20m
Fixed spread: 6.8km
Sampling interval: 1ms
Recording length: 2s
Peak frequency : 10 Hz
Noise type : Brownian noise

Lambda : 100

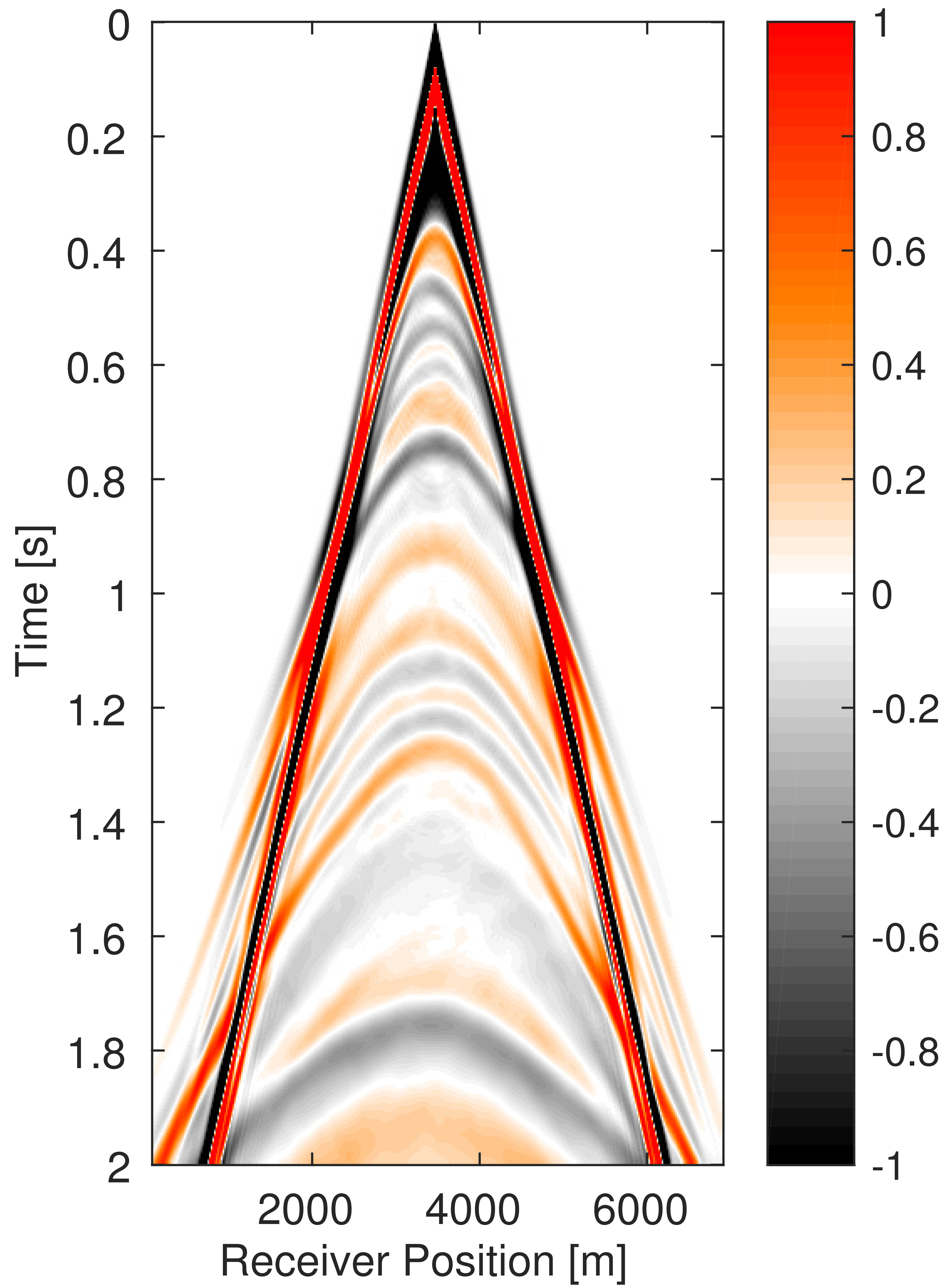


Observed data, SNR = 1.65

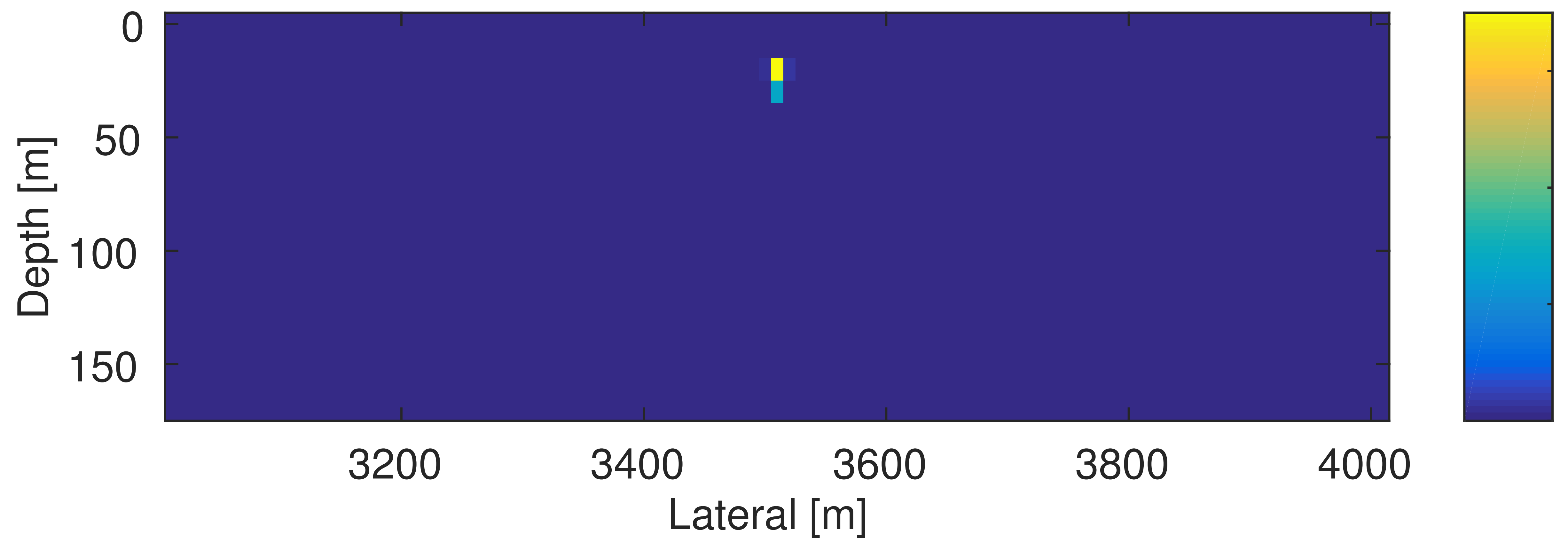


True source location + Noise source locations (zoomed)

Sum of the absolute value of source wavelet along time

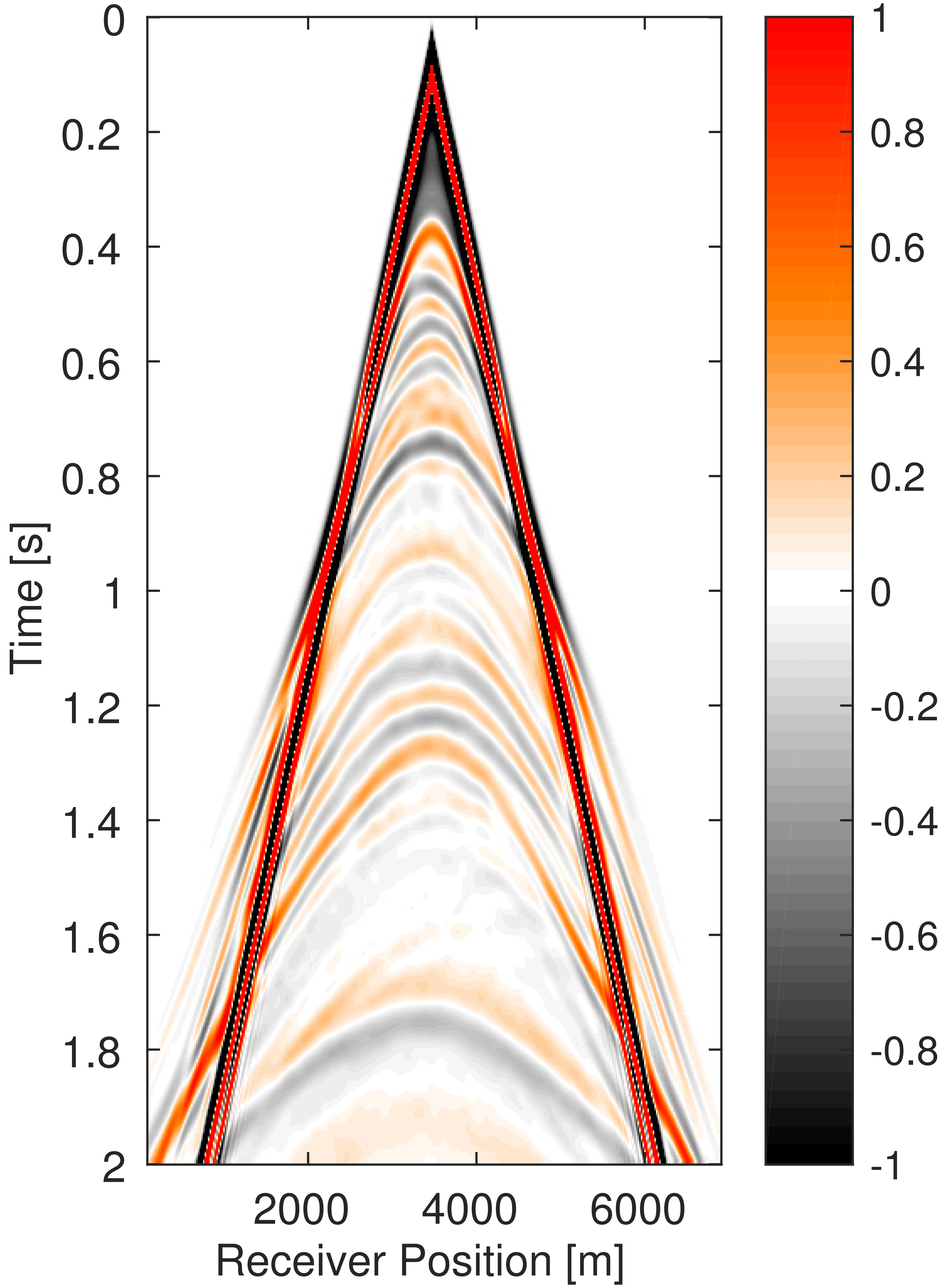


Estimated shot record, SNR = 8.10

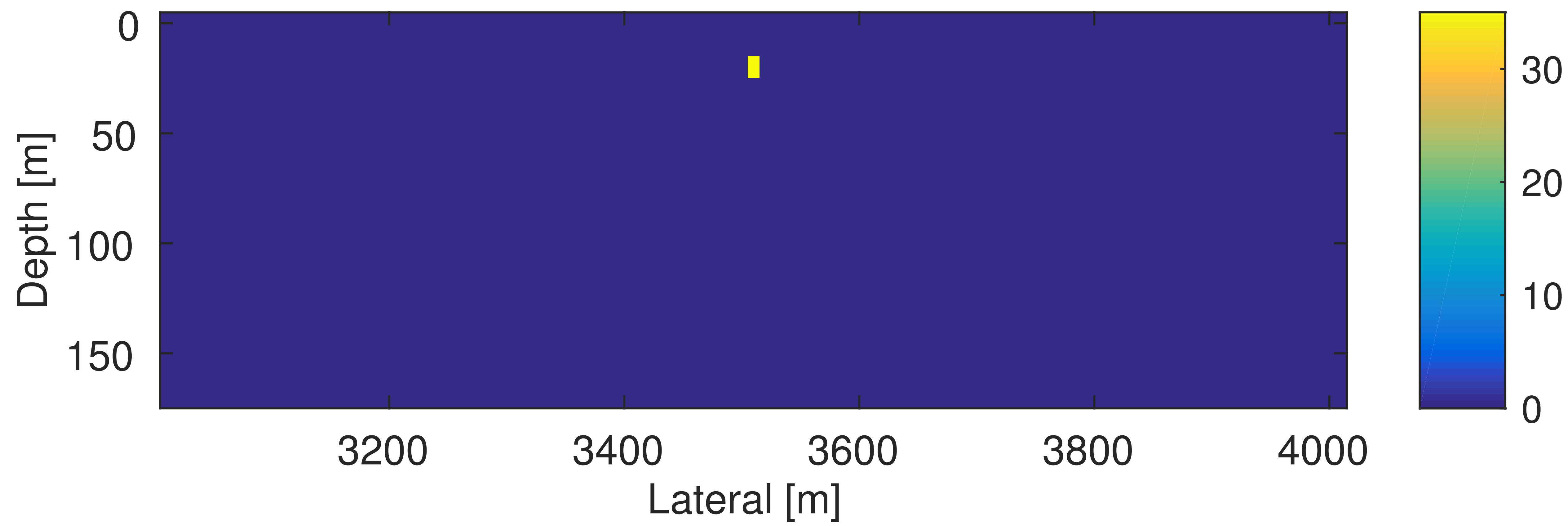


Estimated source location (zoomed)

Sum of the absolute value of source wavelet along time

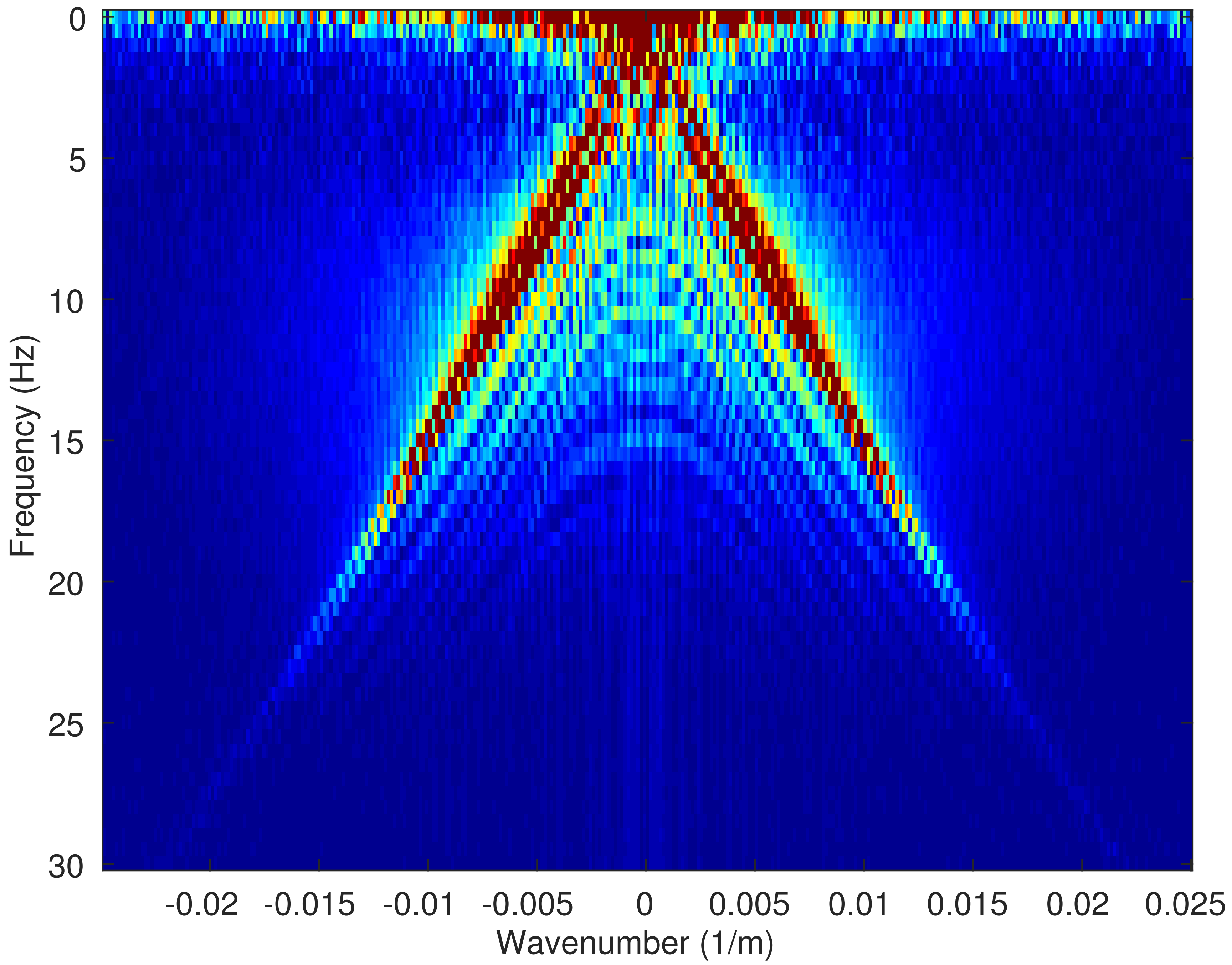


True shot record

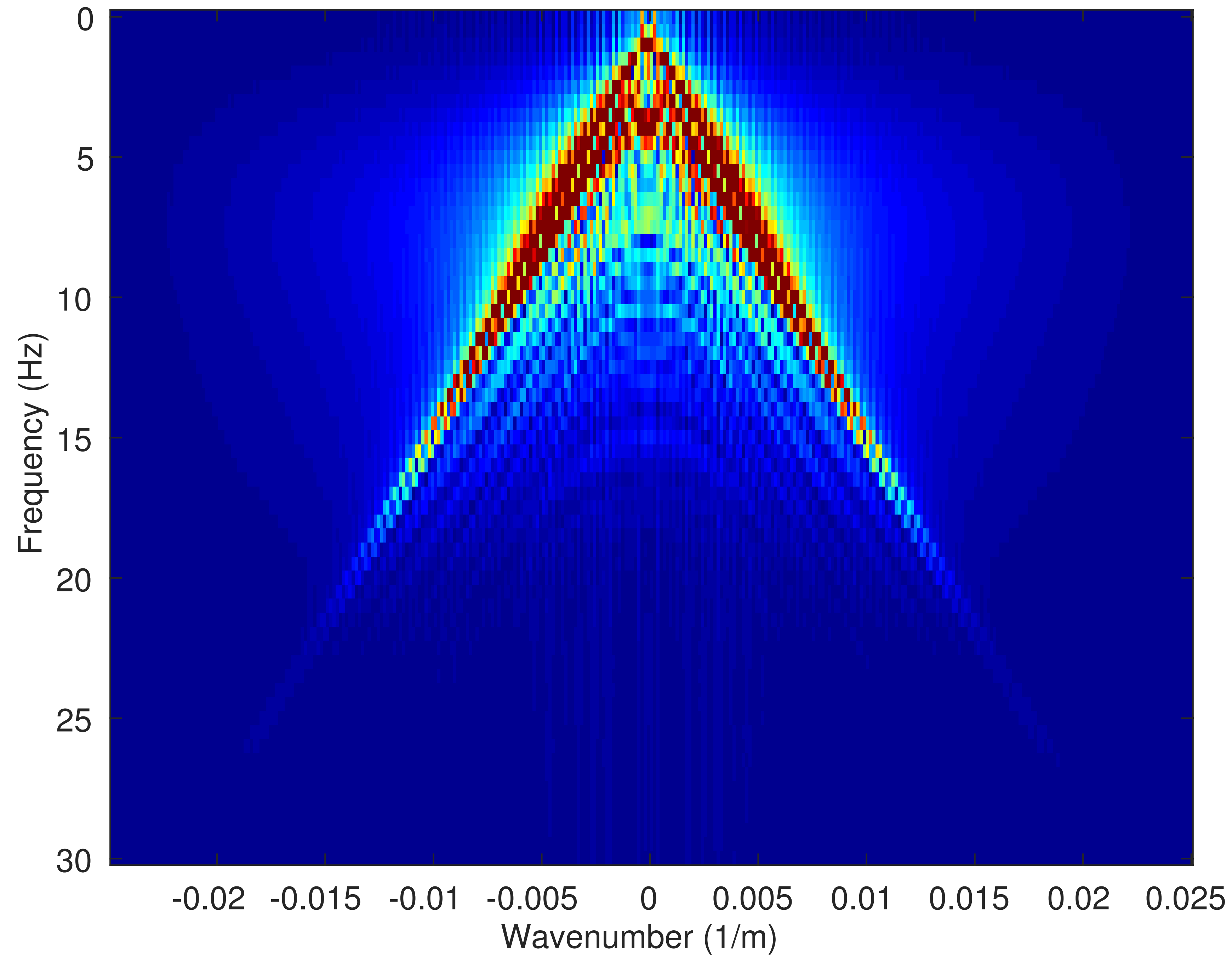


True source location (zoomed)

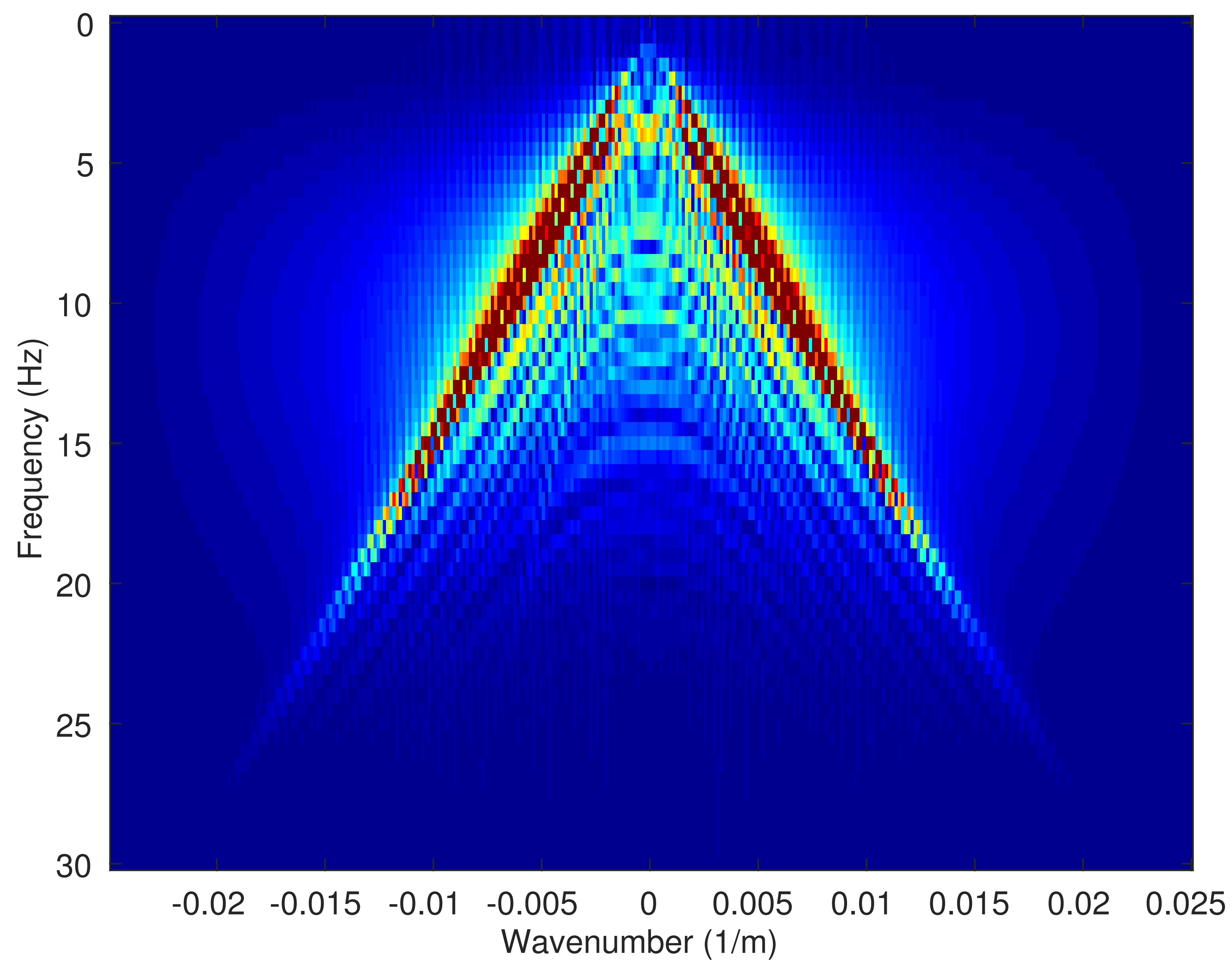
Sum of the absolute value of source wavelet along time



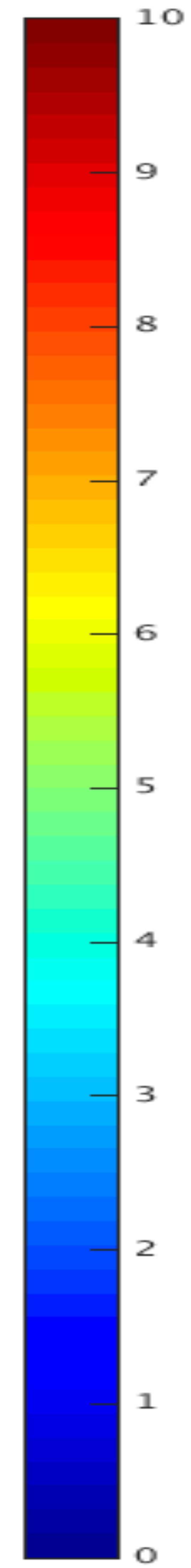
Noisy data



Estimated data

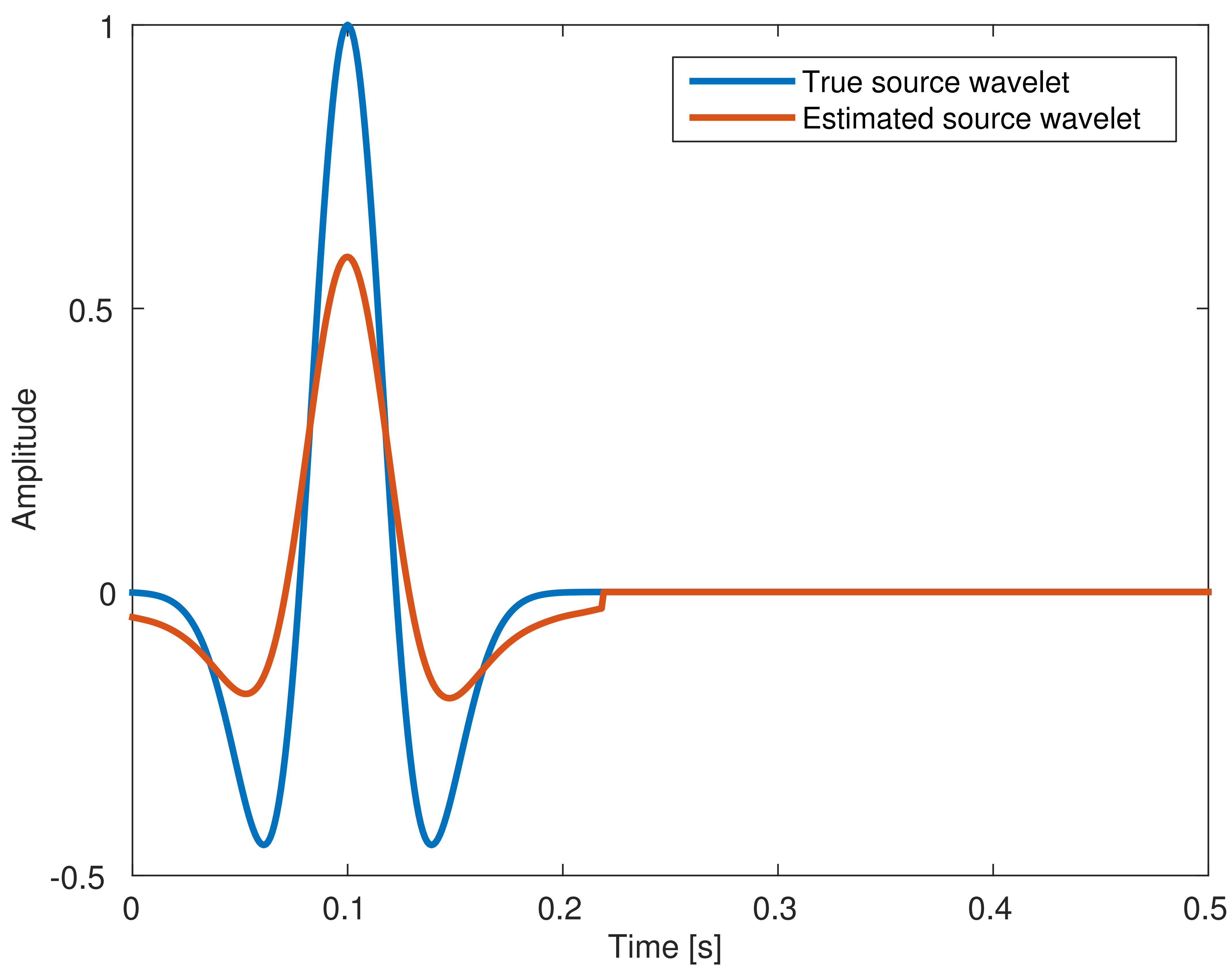


True data

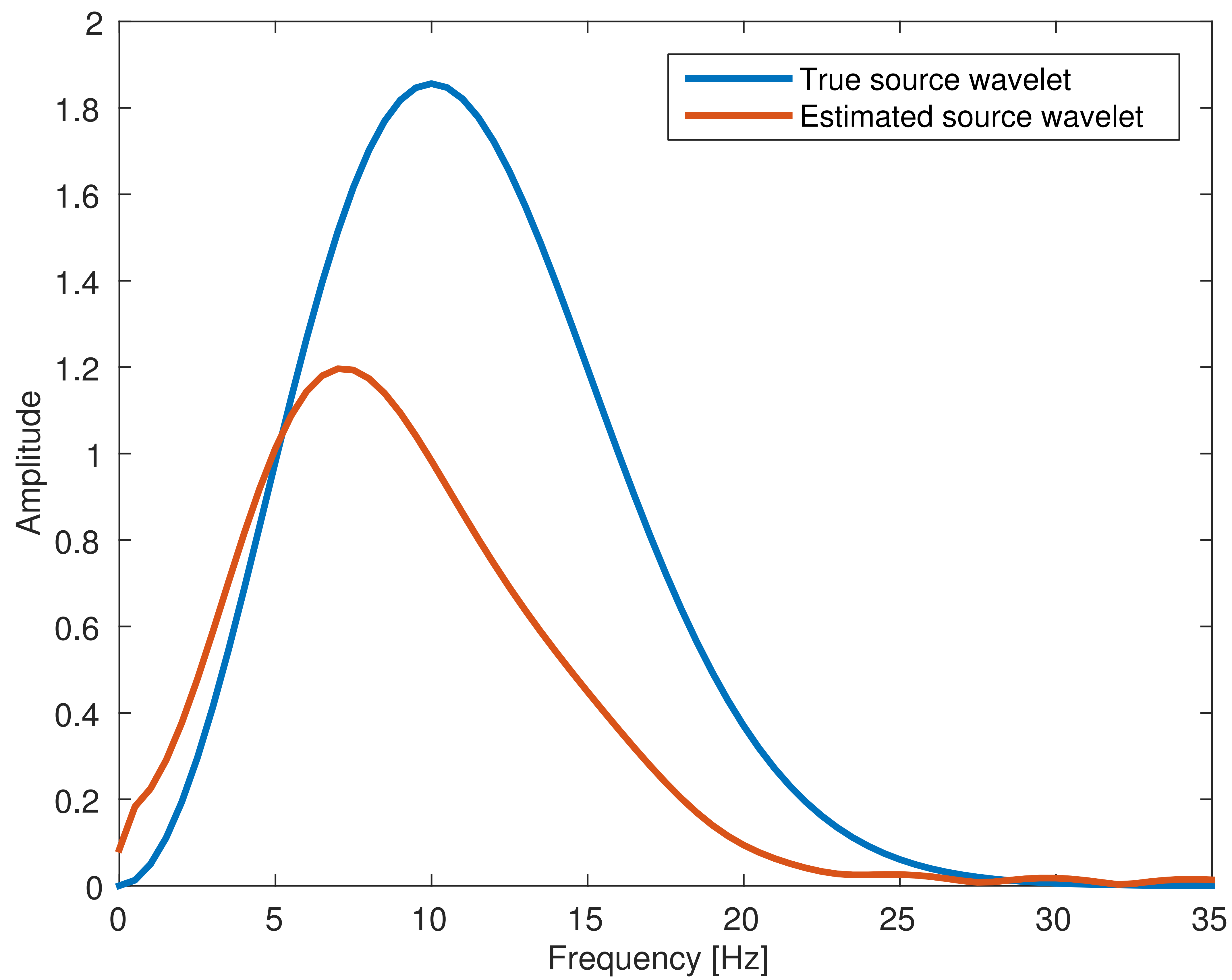


FK spectrum

Source wavelet comparison



Spectrum



SLIM

Conclusions

- Using the method of Linearized Bregman, source locations(both known and unknown) can be estimated
  - For sources firing at different times
  - For unknown velocity model

- Potential application
  - Noise removal

- Algorithm is simple, converges and has very few tuning parameters

Future work

- Implement for noisy data scenario in case of unknown velocity model
- 3D implementation

Acknowledgements

Thank you for your attention!

SINBAD

NSERC CRSNG

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