

Comparative study of time-lapse FWI approaches

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Motivation

Need for accurate migration velocity models for time-lapse seismic analysis e.g. NRMS measure

FWI in time-lapse might address some of the issues faced in 4D seismic

- acquisition geometry repeatability
- water column statics
- complex overburden changes

Main message

Review of current time-lapse FWI approaches

Present our joint time-lapse FWI method

- that leverages the shared information from the vintages
- that is relatively robust w.r.t to the starting model
- that is fairly stable in the presence of large acquisition gaps
- that can be extended to multiple surveys
- that can be easily implemented

Full-waveform inversion

Problem

$$\underset{\mathbf{m}, \alpha}{\text{minimize}} \frac{1}{2} \|\mathbf{d} - \alpha \mathcal{F}[\mathbf{m}]\|_2^2$$

\mathbf{d} : observed data

\mathcal{F} : forward modelling kernel

α : source wavelet

\mathbf{m} : model parameters

Assume known source wavelet

Full-waveform inversion

Problem

$$\underset{\mathbf{m}}{\text{minimize}} \frac{1}{2} \|\mathbf{d} - \mathcal{F}[\mathbf{m}]\|_2^2$$

\mathbf{d} : observed data
 \mathcal{F} : forward modelling kernel
 \mathbf{m} : model parameters

Standard FWI

$$\underset{\mathbf{m}}{\text{minimize}} \frac{1}{2} \|\mathbf{d} - \mathcal{F}[\mathbf{m}]\|_2^2$$

Initialization, iteration $k = 0$: \mathbf{m}_k

Compute gradient : $\delta\mathbf{m}$

Update model- iteration @ $k + 1$: $\mathbf{m}_{k+1} = \mathbf{m}_k + \delta\mathbf{m}$

Linearization + sparsity on update

$$\tilde{\mathbf{x}}^k = \arg \min_{\mathbf{x}} \frac{1}{2} \left\| \mathbf{d} - \underbrace{\mathcal{F}[\mathbf{m}^k; \mathbf{Q}]}_{\mathbf{b}} - \underbrace{\nabla \mathcal{F}[\mathbf{m}^k; \mathbf{Q}] \mathbf{C}^T}_{\mathbf{A}} \mathbf{x} \right\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 < \tau$$

\mathbf{Q} = source

\mathbf{C} = curvelet

model update: $\mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{C}^T \tilde{\mathbf{x}}^k$

Linearization + sparsity on update

$$\tilde{\mathbf{x}}^k = \arg \min_{\mathbf{x}} \frac{1}{2} \left\| \mathbf{d} - \underbrace{\mathcal{F}[\mathbf{m}^k; \mathbf{Q}]}_{\mathbf{b}} - \underbrace{\nabla \mathcal{F}[\mathbf{m}^k; \mathbf{Q}] \mathbf{C}^T}_{\mathbf{A}} \mathbf{x} \right\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 < \tau$$

\mathbf{Q} = source

\mathbf{C} = curvelet

*Computationally
very expensive!!!
with all the data*

model update: $\mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{C}^T \tilde{\mathbf{x}}^k$

Fast randomized inversion

$$\tilde{\mathbf{x}}^k = \arg \min_{\mathbf{x}} \frac{1}{2} \left\| \underbrace{\underline{\mathbf{d}} - \mathcal{F}[\mathbf{m}^k; \underline{\mathbf{Q}}]}_{\underline{\mathbf{b}}} - \underbrace{\nabla \mathcal{F}[\mathbf{m}^k; \underline{\mathbf{Q}}]}_{\underline{\mathbf{A}}} \mathbf{C}^T \mathbf{x} \right\|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 < \tau$$

$$\underline{\mathbf{Q}} = \mathbf{W} \mathbf{Q}$$

$$\underline{\mathbf{d}} = \mathbf{W} \mathbf{d}$$

Fast computations!!!

\mathbf{W} : matrix that encodes simultaneous or randomly selected shots

$$\text{model update:} \quad \mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{C}^T \tilde{\mathbf{x}}^k$$

Time-lapse FWI approaches

Time-lapse FWI approaches

Parallel difference

Start with similar initial model, given observed data : $\mathbf{m}_0, \mathbf{d}_1, \mathbf{d}_2$

Invert for baseline and monitor separately : $\mathbf{m}_1, \mathbf{m}_2$

Estimate timelapse model : $d\mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1$

Sequential difference

Start with baseline data and initial model : $\mathbf{m}_0, \mathbf{d}_1$

Invert for baseline : \mathbf{m}_1

Inversion of \mathbf{d}_2 using \mathbf{m}_1 as starting model : \mathbf{m}_2

Estimate timelapse model : $d\mathbf{m} = \mathbf{m}_2 - \mathbf{m}_1$

Watanabe et al., 2004; Denli and Huang, 2009; Zheng et al., 2011;
Asnaashari et al., 2012; Raknes et al., 2013)

Time-lapse FWI approaches

Double difference or Differential FWI

$$\text{minimize } \Delta \mathbf{d} := (\mathbf{d}_2 - \mathbf{d}_1) - (\mathcal{F}[\mathbf{m}_2] - \mathcal{F}[\mathbf{m}_1])$$

Start with baseline data and initial model : $\mathbf{m}_0, \mathbf{d}_1$

Invert for baseline : \mathbf{m}_1

Construct composite data : $\widetilde{\mathbf{d}}_2 = \mathbf{d}_2 - \mathbf{d}_1 + \mathcal{F}[\mathbf{m}_1]$

Replace \mathbf{d}_2 with $\widetilde{\mathbf{d}}_2$ obtain : $\widetilde{\mathbf{m}}_2$

Estimate timelapse model : $d\mathbf{m} = \widetilde{\mathbf{m}}_2 - \mathbf{m}_1$

Watanabe et al., 2004; Denli and Huang, 2009; Zheng et al., 2011;
Asnaashari et al., 2012; Raknes et al., 2013)

Time-lapse FWI approaches

Double difference or Differential FWI

$$\text{minimize } \Delta \mathbf{d} := (\mathbf{d}_2 - \mathbf{d}_1) - (\mathcal{F}[\mathbf{m}_2] - \mathcal{F}[\mathbf{m}_1])$$

Relies on accurately repeating the acquisition

Not quite conducive when the vintage noise are highly uncorrelated

Time-lapse joint FWI approaches

Robust joint FWI with TV regularization

$$\alpha \|\mathbf{M}_1 \mathcal{F}[\mathbf{m}_1] - \mathbf{d}_1\|_2^2 + \beta \|\mathbf{M}_2 \mathcal{F}[\mathbf{m}_2] - \mathbf{d}_2\|_2^2 + \quad (1)$$

$$\gamma \|(\mathbf{M}_2^s \mathcal{F}[\mathbf{m}_2] - \mathbf{M}_1^s \mathcal{F}[\mathbf{m}_1]) - (\mathbf{M}_2 \mathbf{d}_2 - \mathbf{M}_1 \mathbf{d}_1)\|_2^2 + \quad (2)$$

$$\alpha_1 \|\mathbf{W}_1 \mathbf{R}_1(\mathbf{m}_1 - \mathbf{m}_1^{prior})\|_1 + \quad (3)$$

$$\beta_1 \|\mathbf{W}_2 \mathbf{R}_2(\mathbf{m}_2 - \mathbf{m}_2^{prior})\|_1 + \quad (4)$$

$$\delta \|\mathbf{W} \mathbf{R}(\mathbf{m}_2 - \mathbf{m}_1 - \Delta \mathbf{m}^{prior})\|_1 + \quad (5)$$

Our parallel versus joint inversion approach

Parallel FWI

Parallel inversion (uses the **fast** randomized inversion technique based on CS)

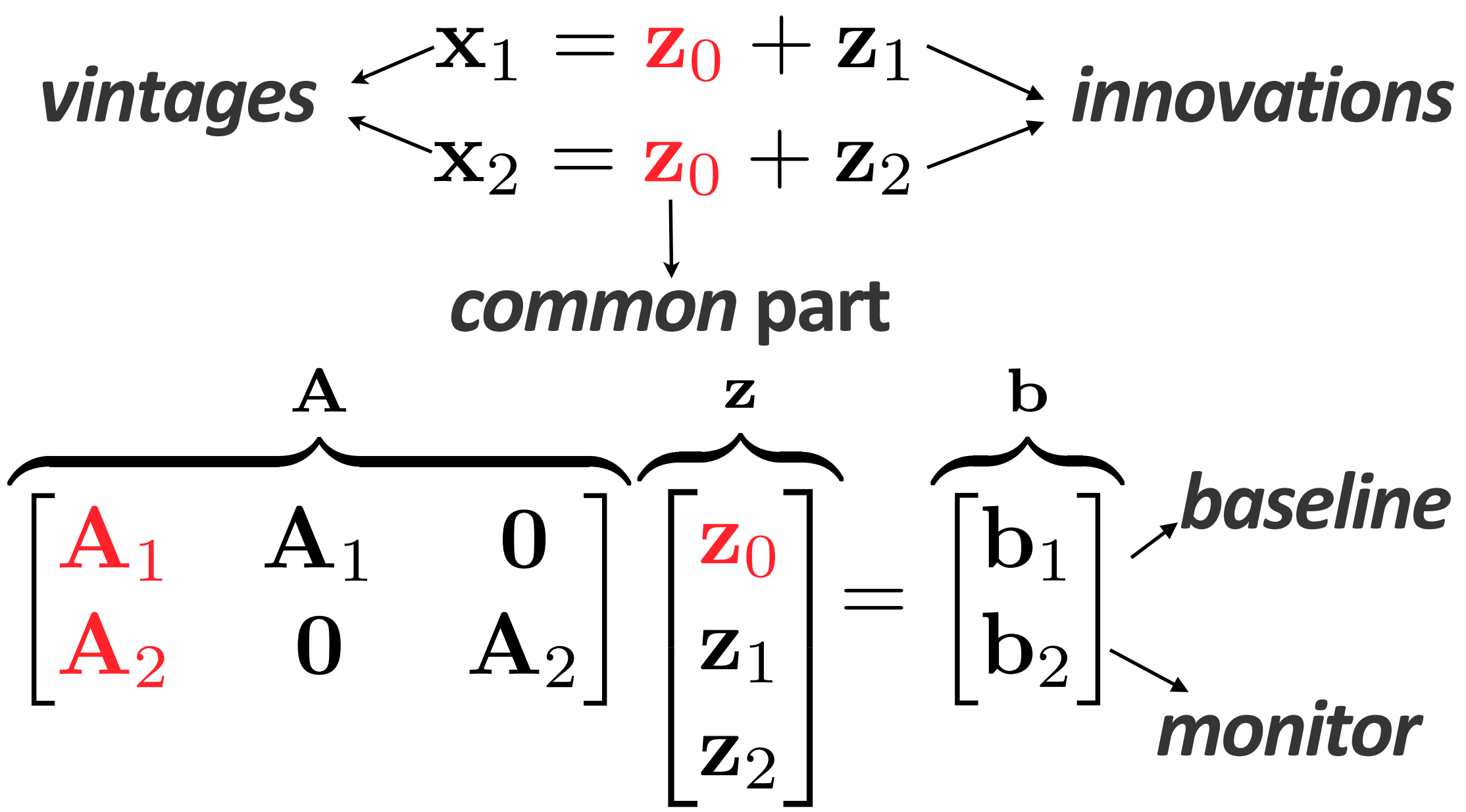
for $i = 1, 2$

$$\tilde{\mathbf{x}}_i^k = \arg \min_{\mathbf{x}_i} \frac{1}{2} \left\| \underbrace{\mathbf{d}_i^k - \mathcal{F}(\mathbf{m}_i^k)}_{\mathbf{b}_i} - \underbrace{\nabla \mathcal{F}(\mathbf{m}_i^k) \mathbf{C}^T}_{\mathbf{A}_i} \mathbf{x}_i \right\|_2 \quad \text{s.t.} \quad \|\mathbf{x}_i\|_1 < \tau_i^k$$

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T \tilde{\mathbf{x}}_i^k$$

Objective: Invert for baseline, monitor; difference = baseline-monitor

Distributed compressive sensing – joint recovery model (JRM)



- ▶ Decompose vintage into common and innovations
- ▶ Timelapse vintages share a lot of common information
- ▶ DCS exploits the common or shared information
- ▶ Invert for common component and innovations

$$\tilde{\mathbf{z}} = \arg \min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{s.t.} \quad \mathbf{b} = \mathbf{Az}$$

Joint FWI

with distributed compressed sensing

$$\tilde{\mathbf{z}}_k = \arg \min_{\mathbf{z}_k} \frac{1}{2} \|\mathbf{b}_k - \mathbf{A}_k \mathbf{z}_k\|_2^2 \quad \text{s.t.} \quad \|\mathbf{z}_k\|_1 < \tau^k$$

$$\mathbf{b}_k = \begin{bmatrix} \mathbf{d}_1^k - \mathcal{F}(\mathbf{m}_1^k) \\ \mathbf{d}_2^k - \mathcal{F}(\mathbf{m}_2^k) \end{bmatrix}$$

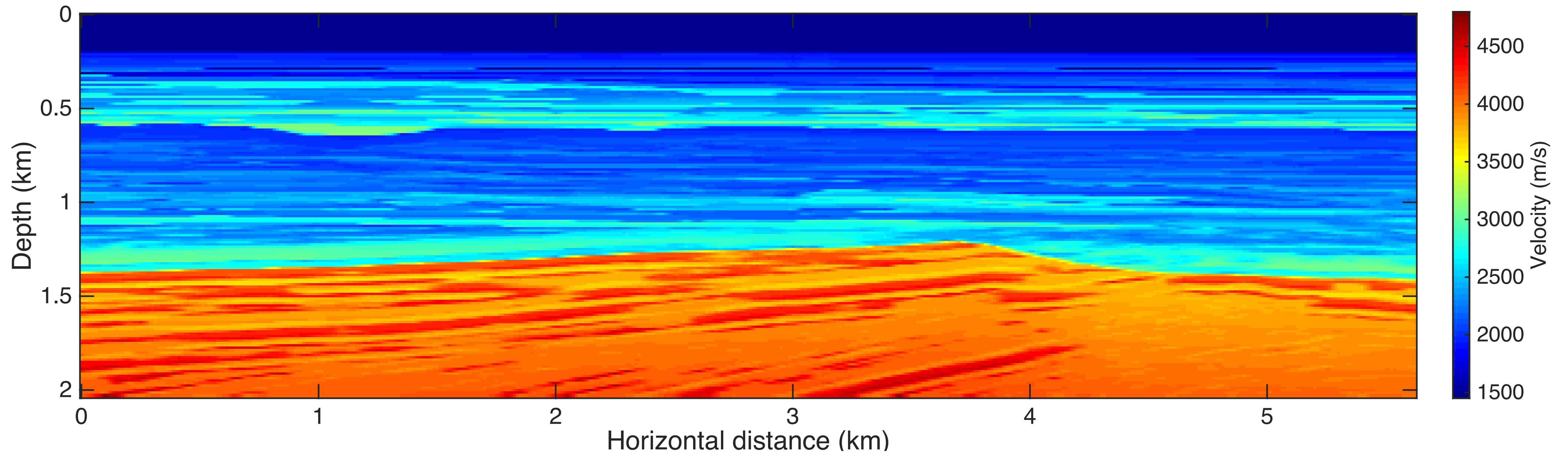
$$\mathbf{A}_k = \begin{bmatrix} \nabla \mathcal{F}(\mathbf{m}_1^k) \mathbf{C}^T & \nabla \mathcal{F}(\mathbf{m}_1^k) \mathbf{C}^T & \mathbf{0} \\ \nabla \mathcal{F}(\mathbf{m}_2^k) \mathbf{C}^T & \mathbf{0} & \nabla \mathcal{F}(\mathbf{m}_2^k) \mathbf{C}^T \end{bmatrix}$$

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{z}_0^k \\ \mathbf{z}_1^k \\ \mathbf{z}_2^k \end{bmatrix}$$

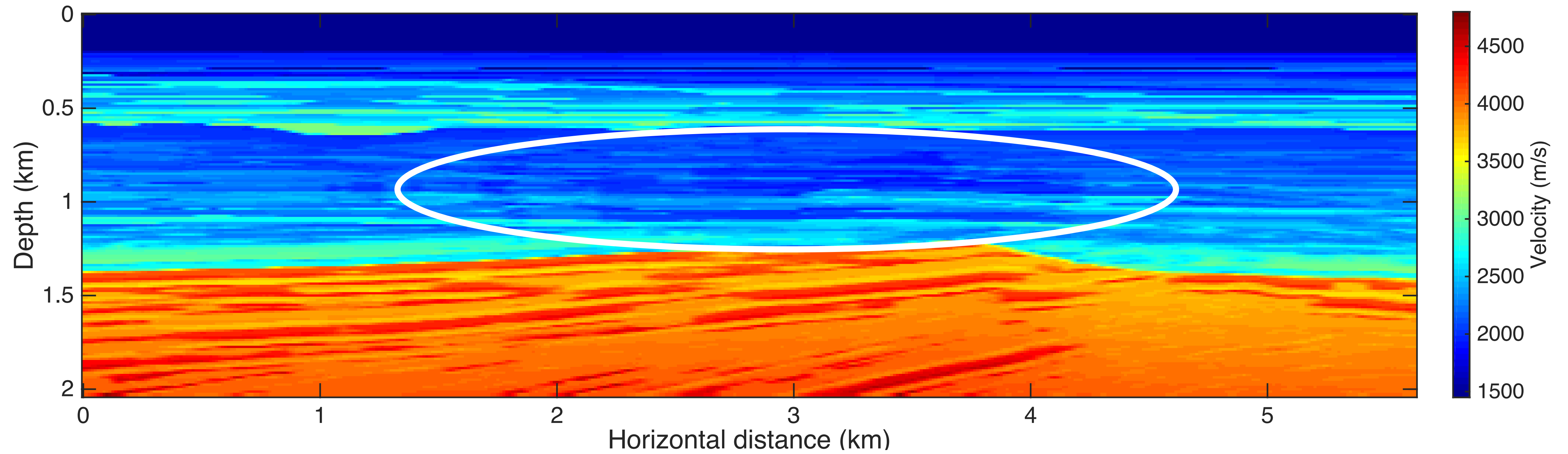
$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T (\tilde{\mathbf{z}}_0^k + \tilde{\mathbf{z}}_i^k)$$

Synthetic example

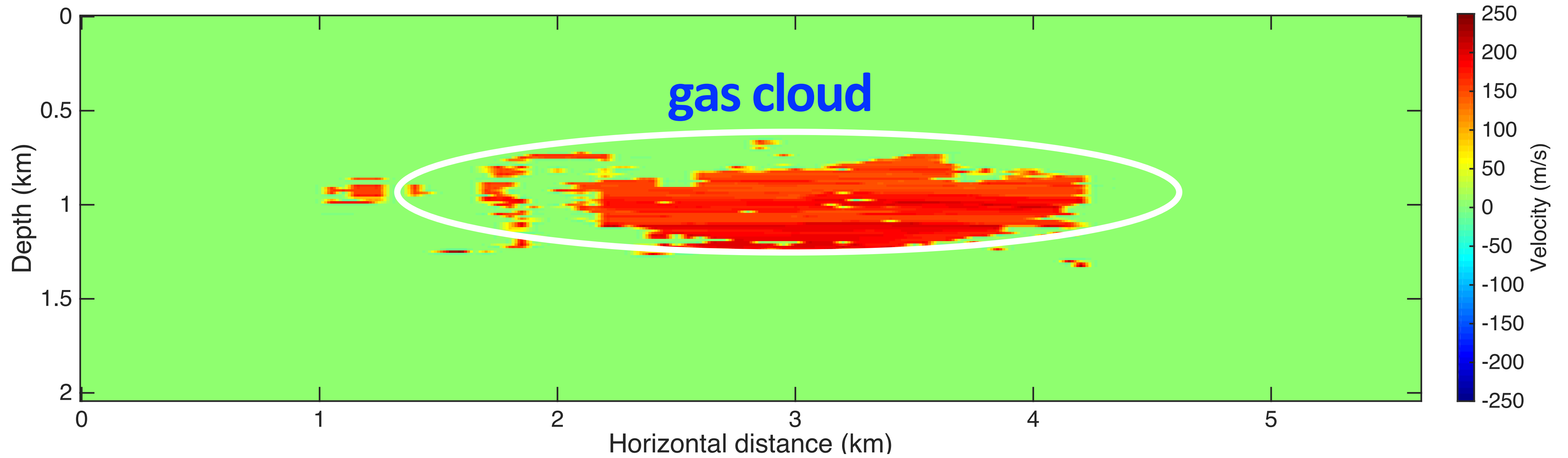
True baseline



True monitor



True time-lapse



Parallel/Sequential/Joint inversion

Given a good starting model:

- ▶ assuming similar acquisition geometry
- ▶ assuming different acquisition geometry

Given a poor starting model

In the presence of large acquisition gap in the monitor survey

In noisy environment

Parallel/Sequential/Joint inversion

Given a good starting model:

- ▶ assuming similar acquisition geometry
- ▶ assuming different acquisition geometry

Given a poor starting model

In the presence of large acquisition gap in the monitor survey

In noisy environment

Experiment : part 1

Assuming similar geometry
“good” starting model

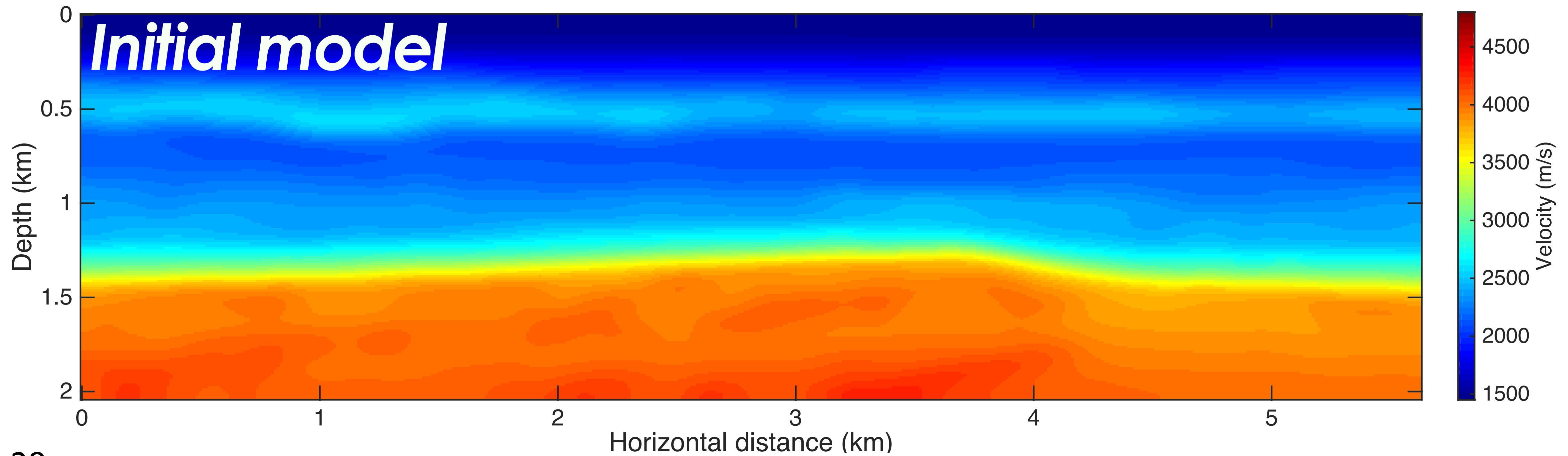
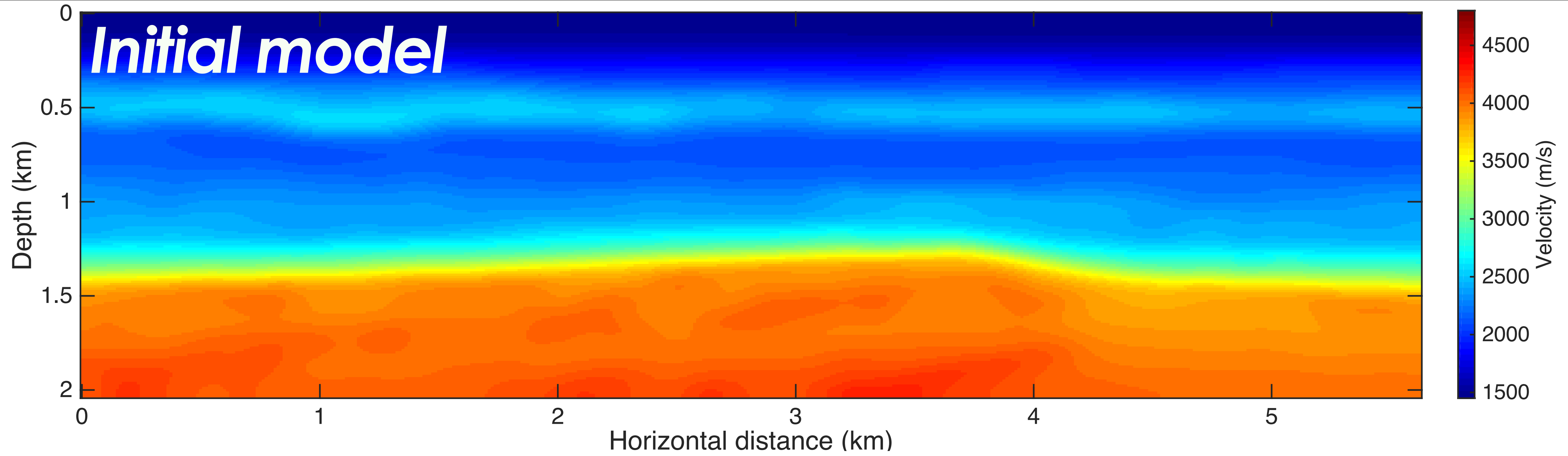
Modeling/Inversion parameters

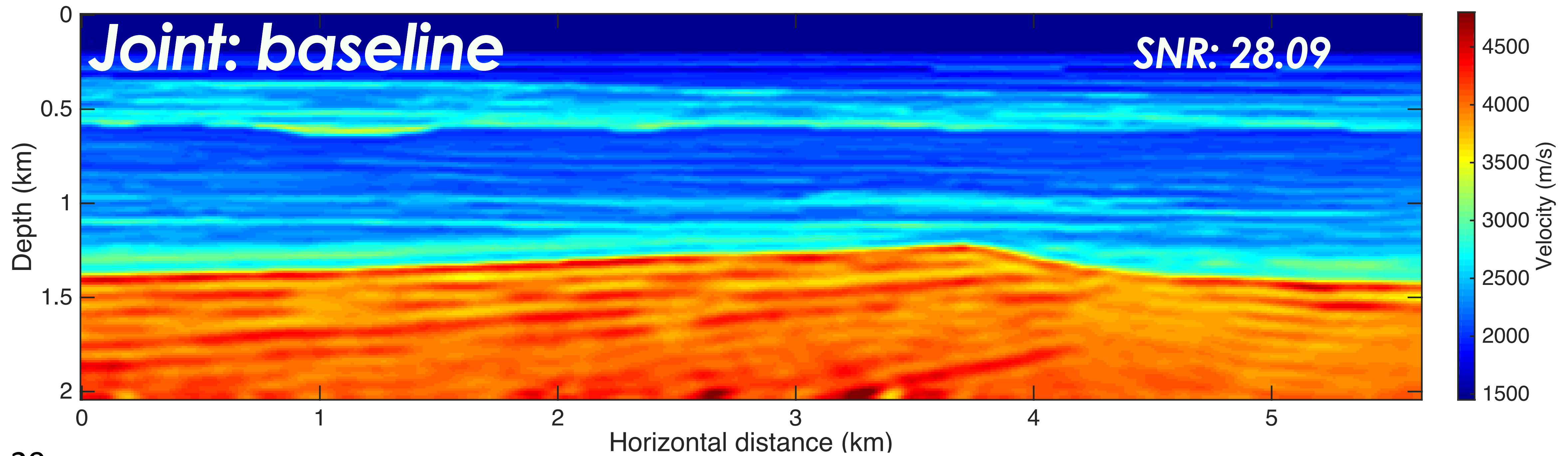
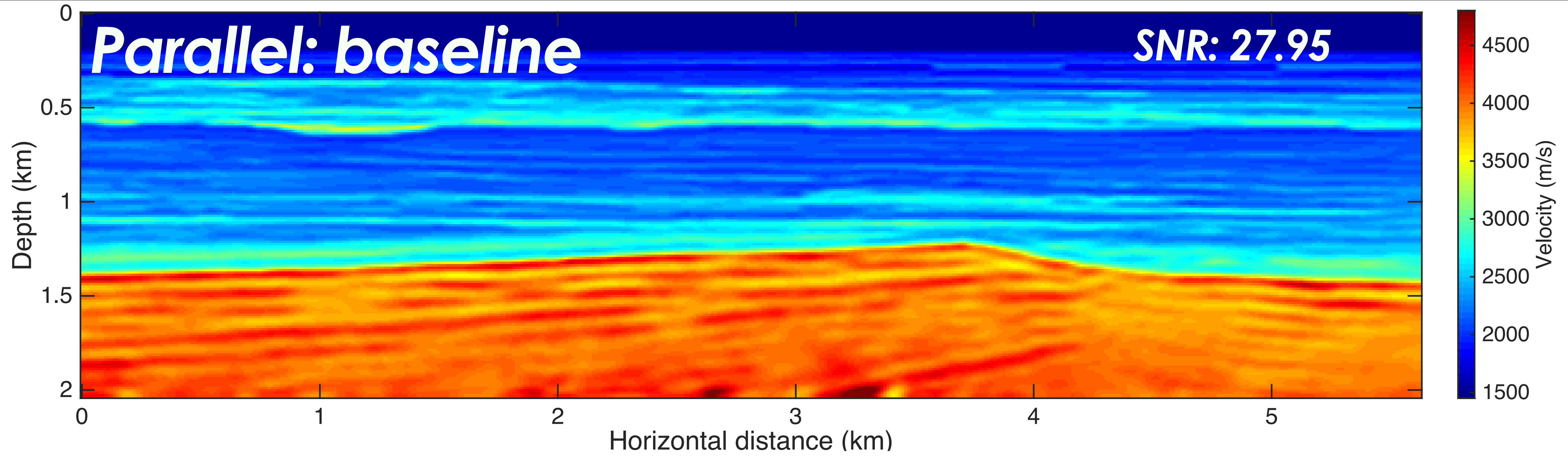
Data simulation:

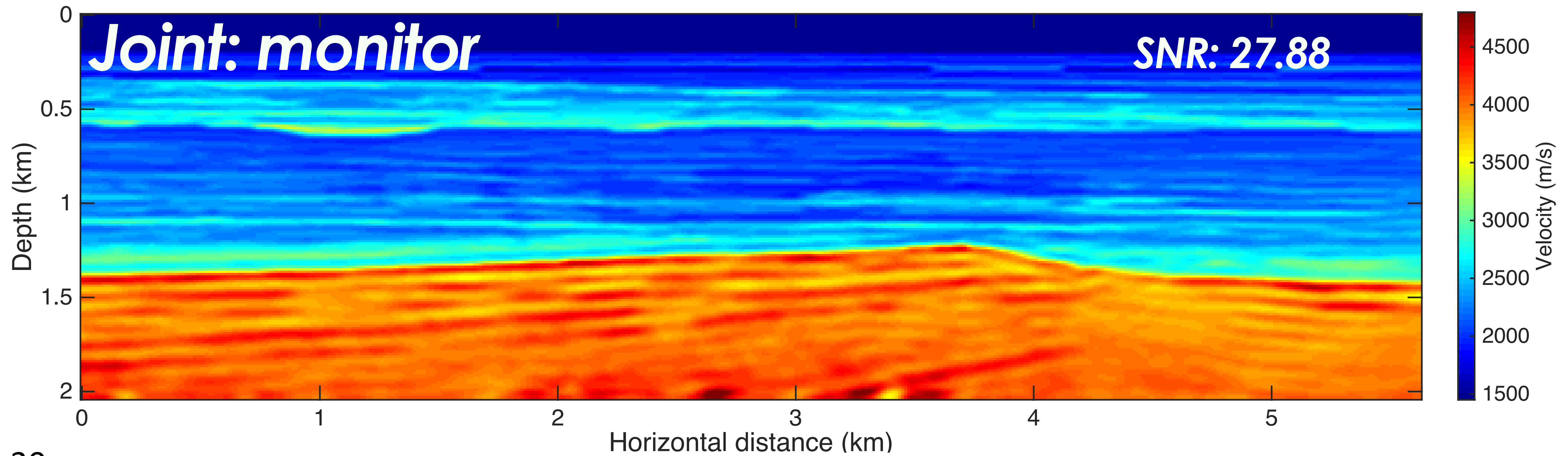
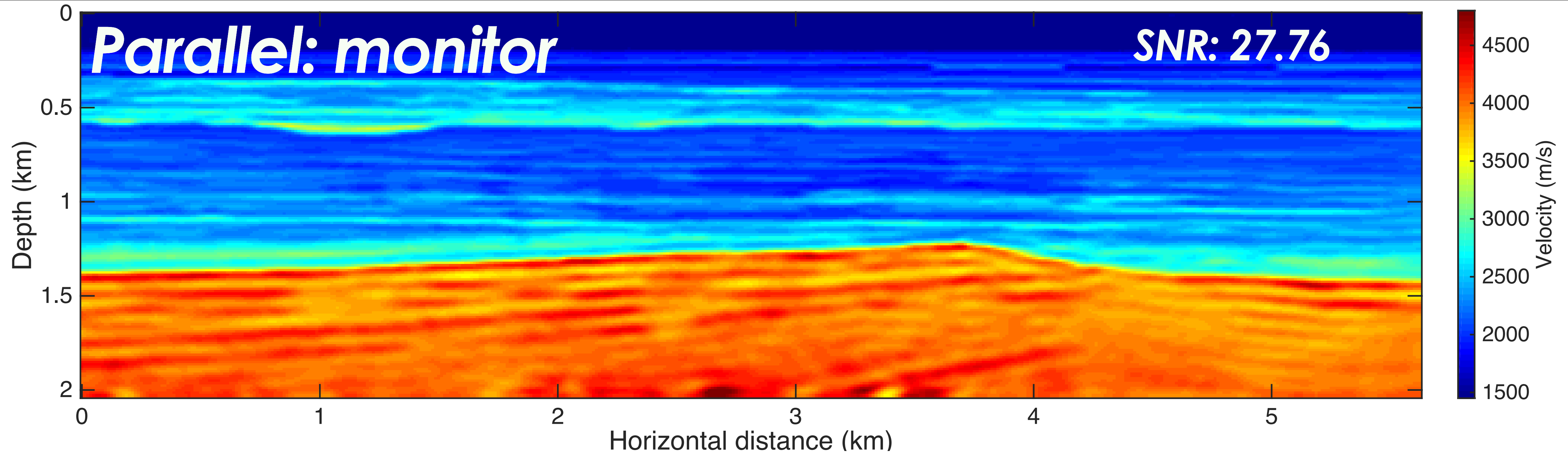
- ▶ Models ocean bottom seismic acquisition
- ▶ 23 sources @ 250m, 113 receivers @ 50m spacing
- ▶ Ricker source wavelet @ 12Hz peak frequency
- ▶ 80 frequencies between 3 and 22.5Hz

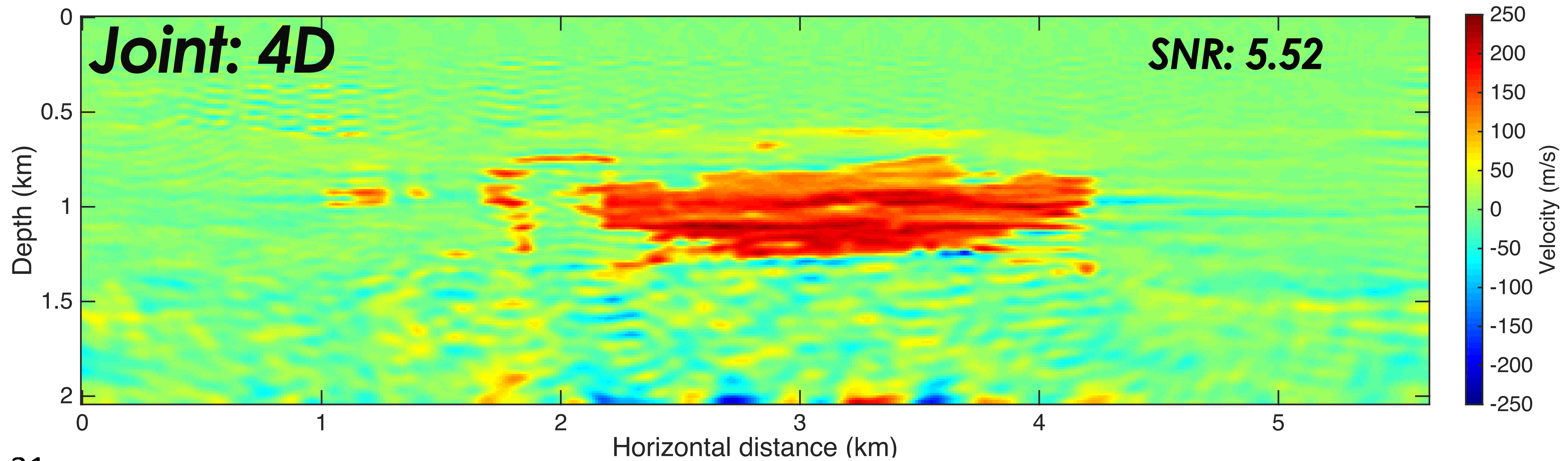
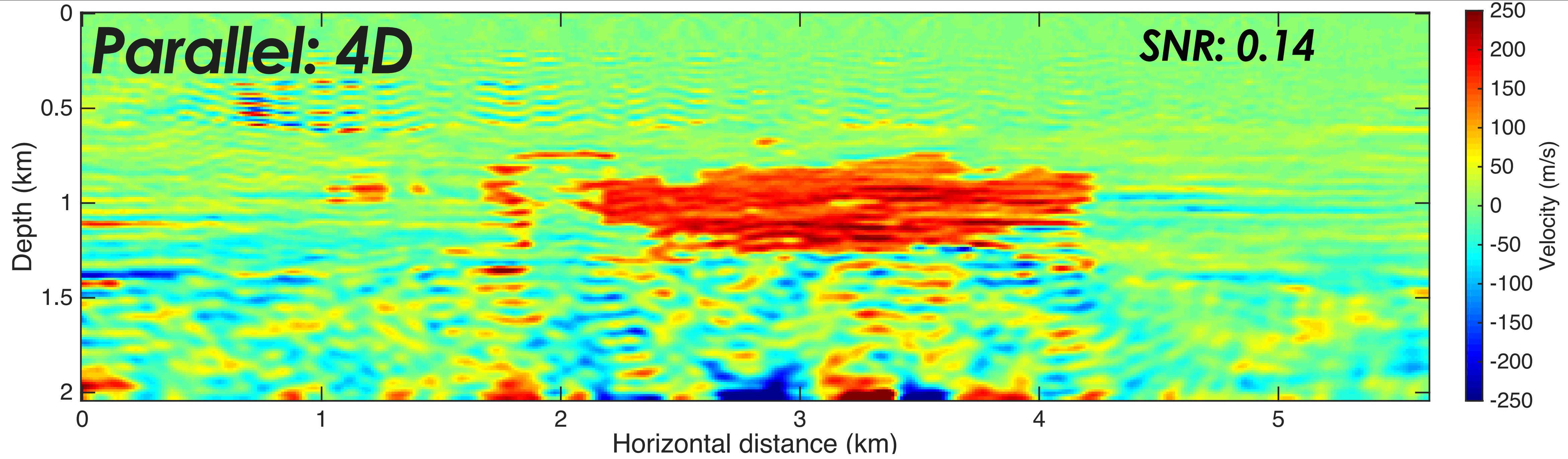
Inversion:

- ▶ 16 frequencies per batch, 9 batches in total with overlapping frequencies
- ▶ Maximum of 150 iterations of spgl1 per frequency batch



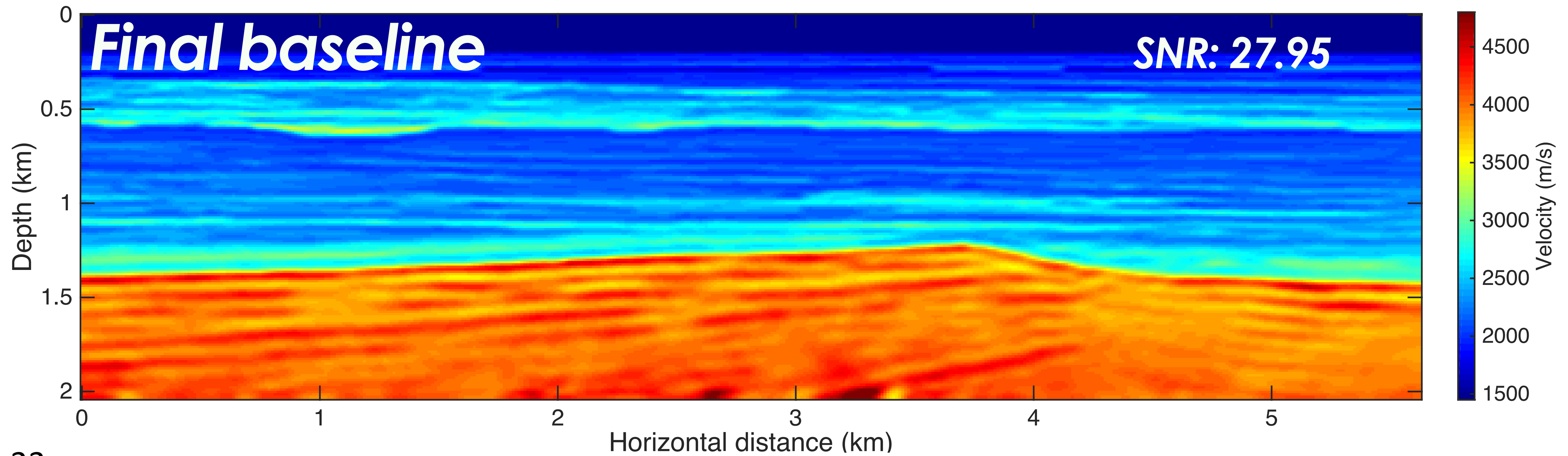
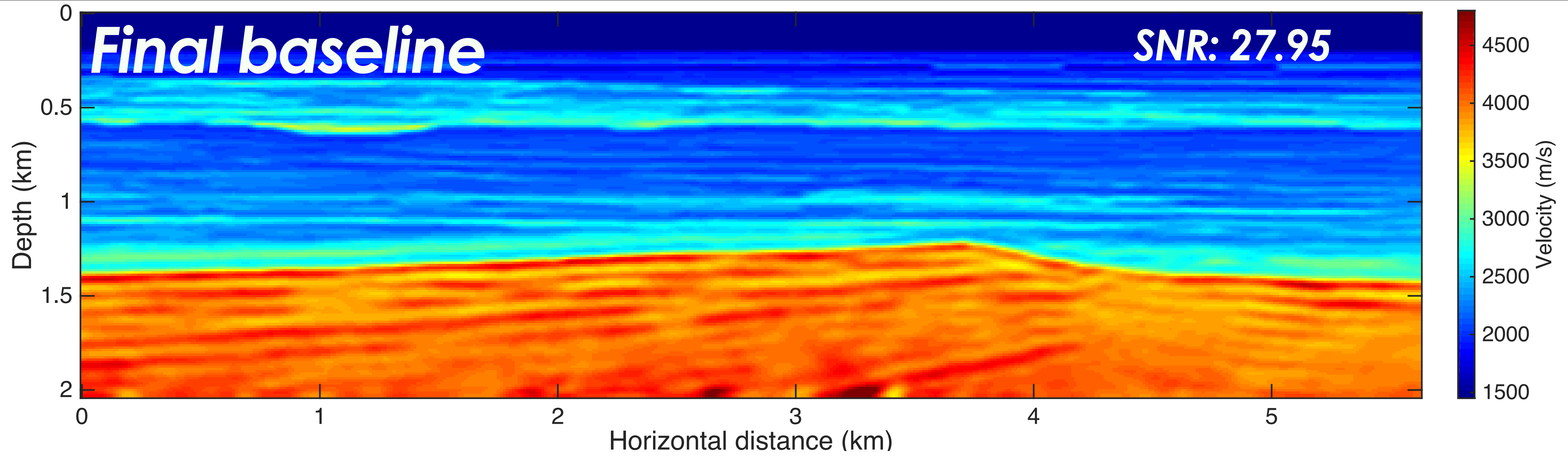


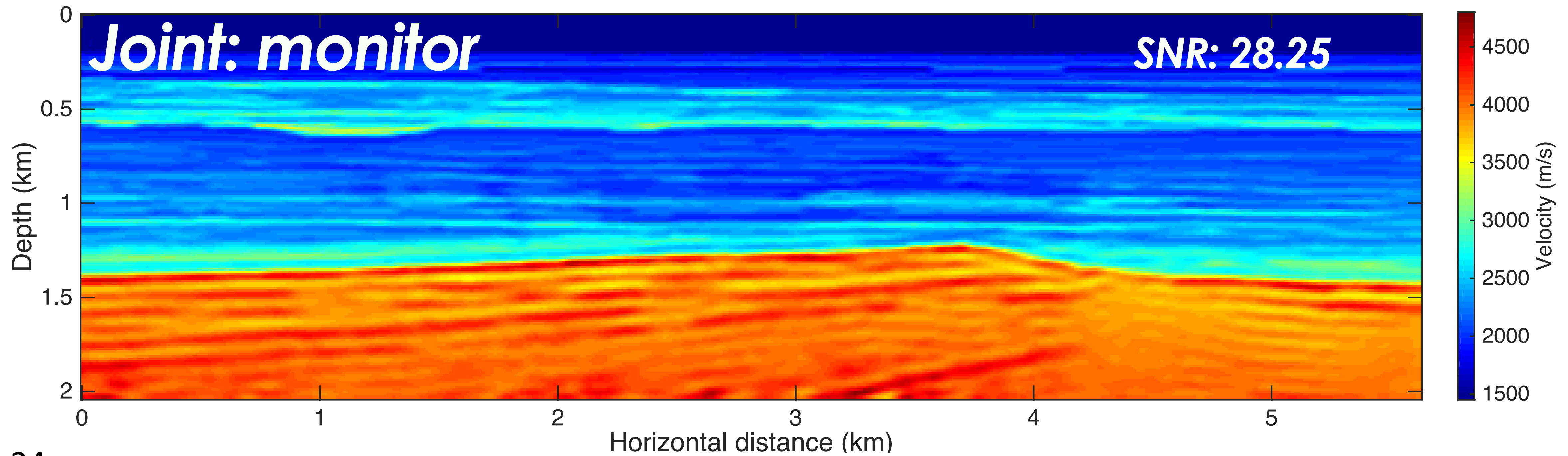
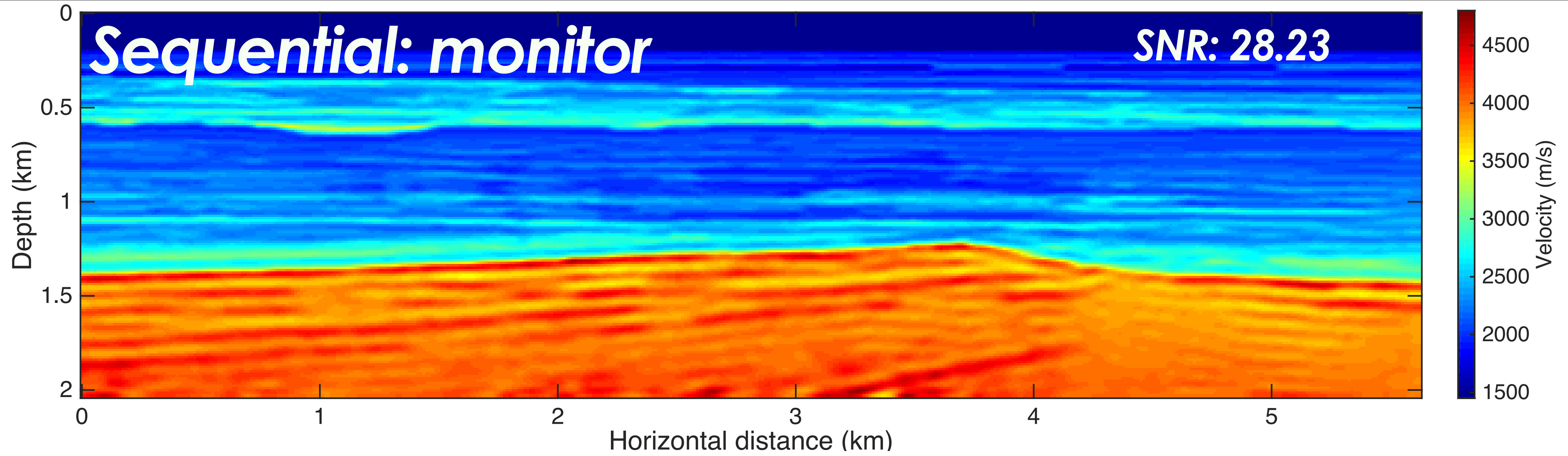


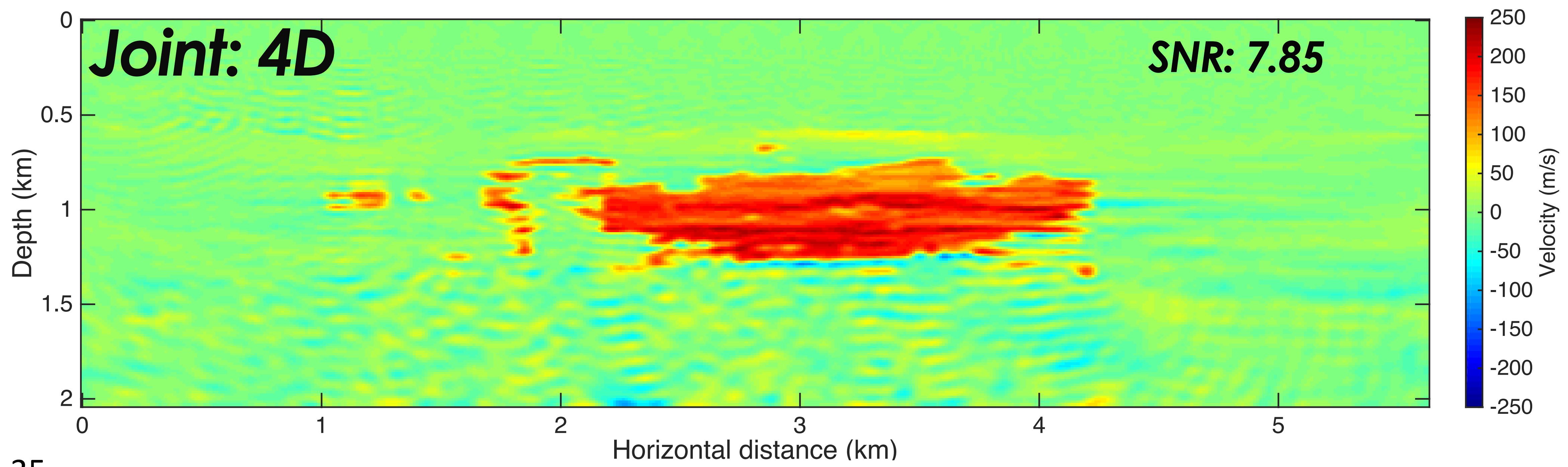
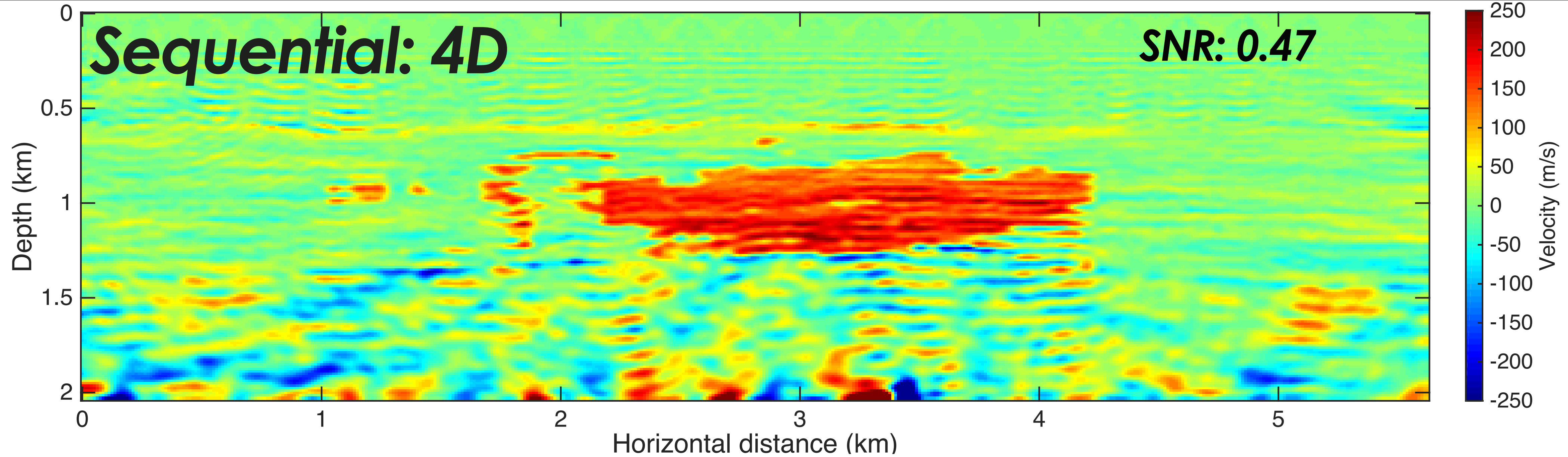


Sequential inversion

Start monitor inversion with the baseline inversion result

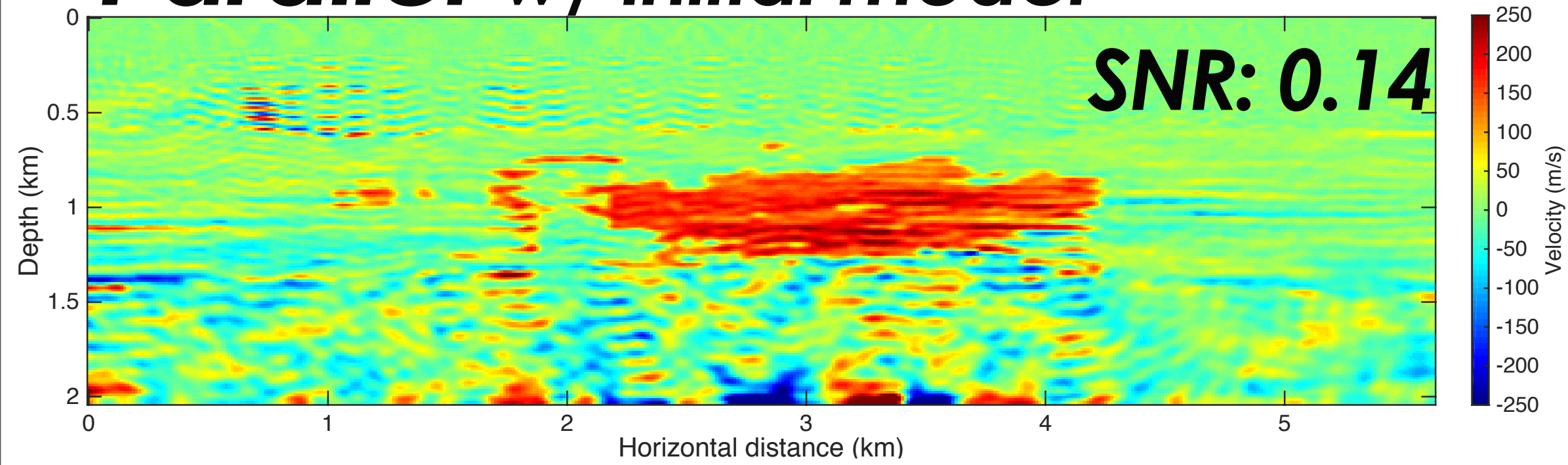




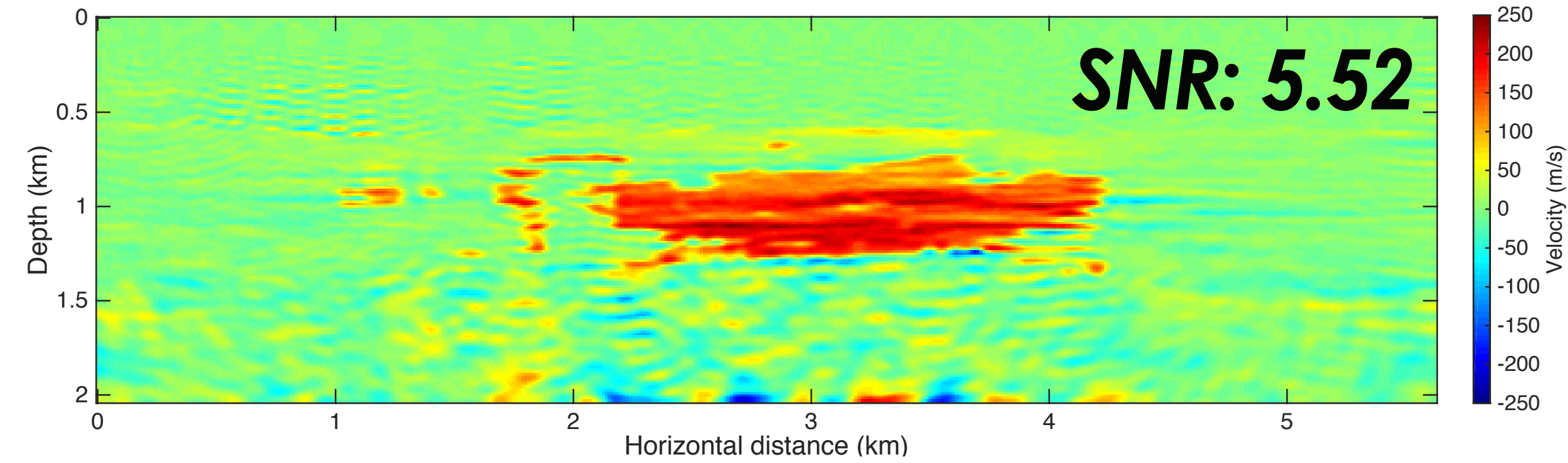


Time-lapse results

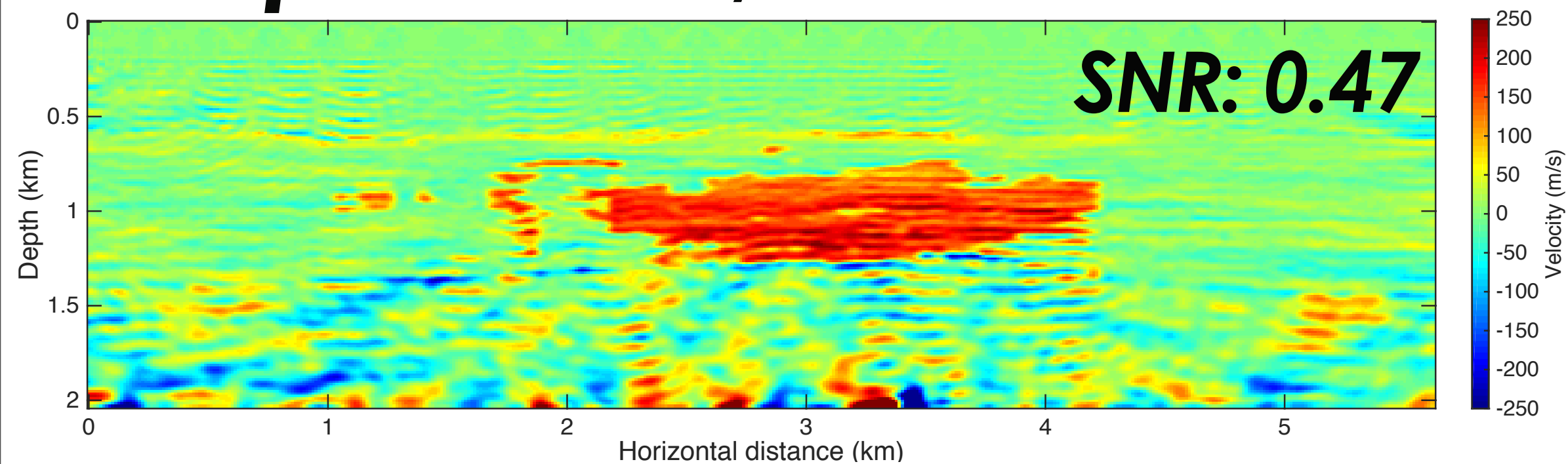
Parallel w/ initial model



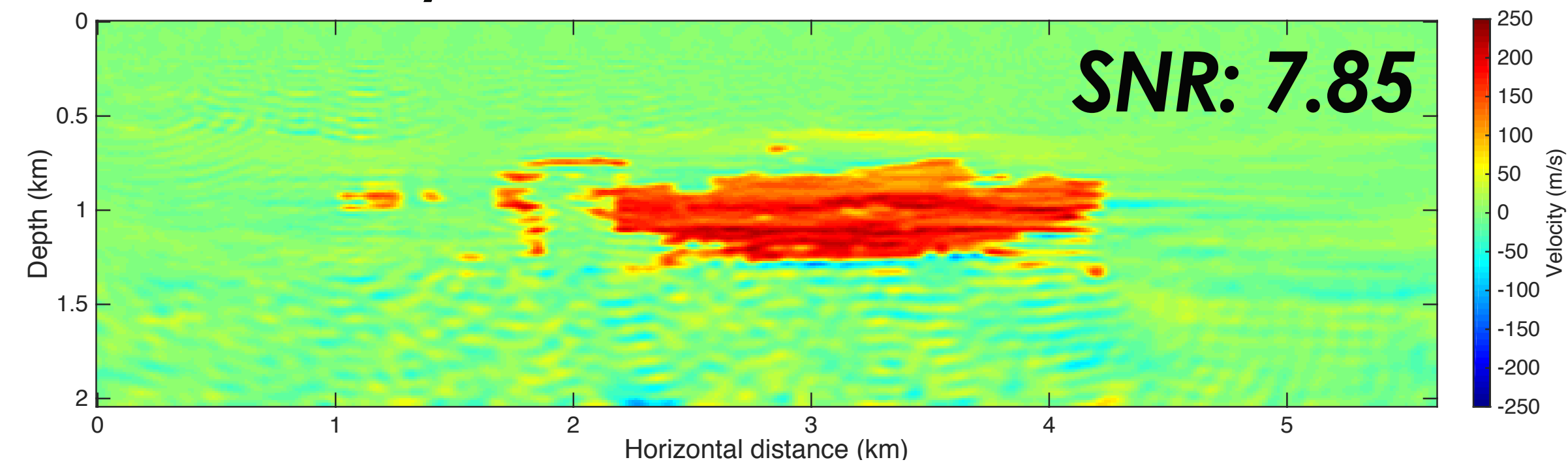
Joint w/ initial model



Sequential w/ inverted base

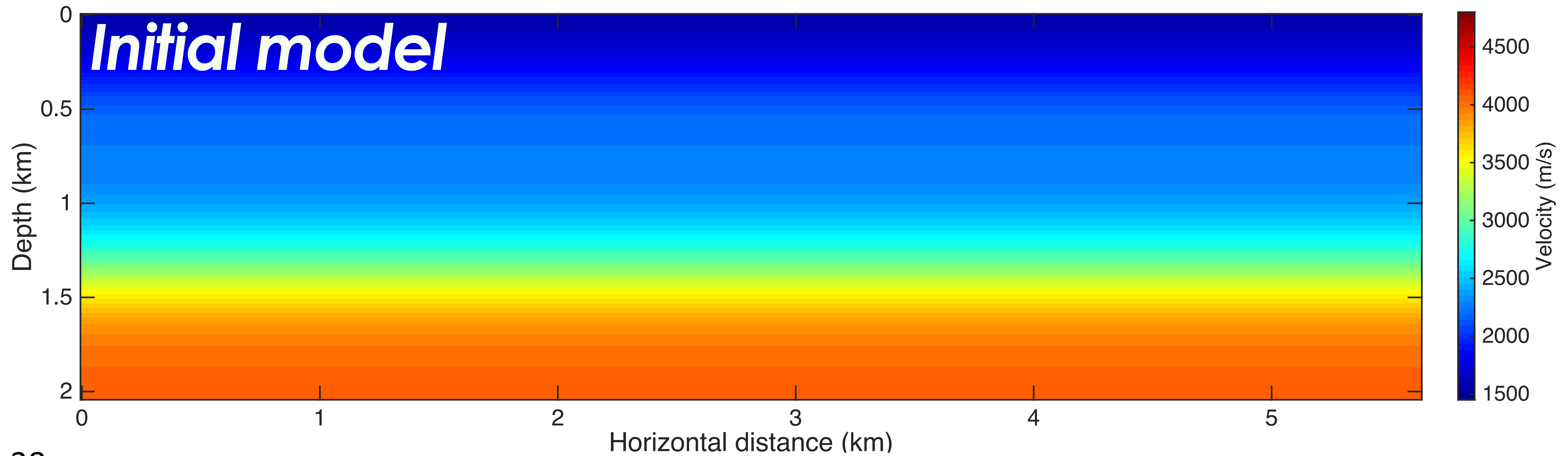
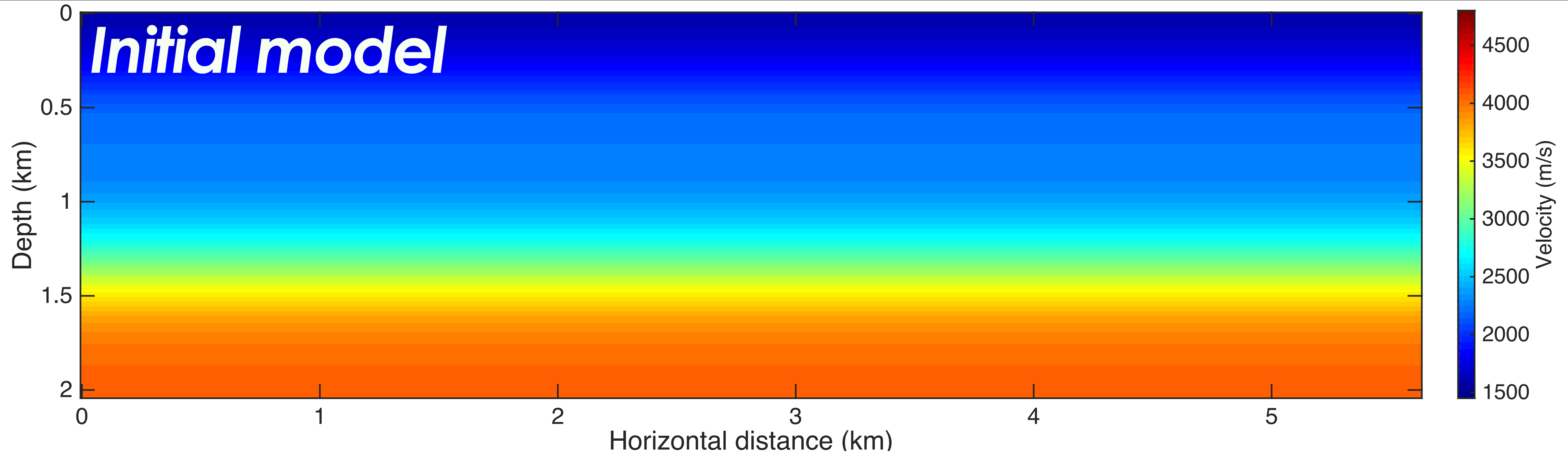


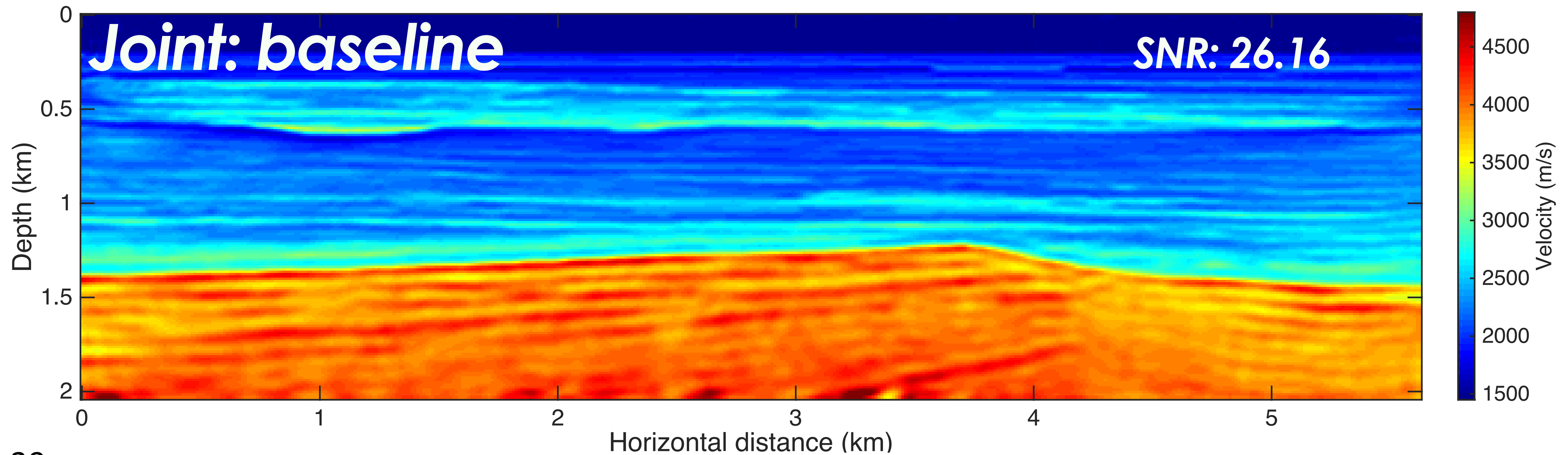
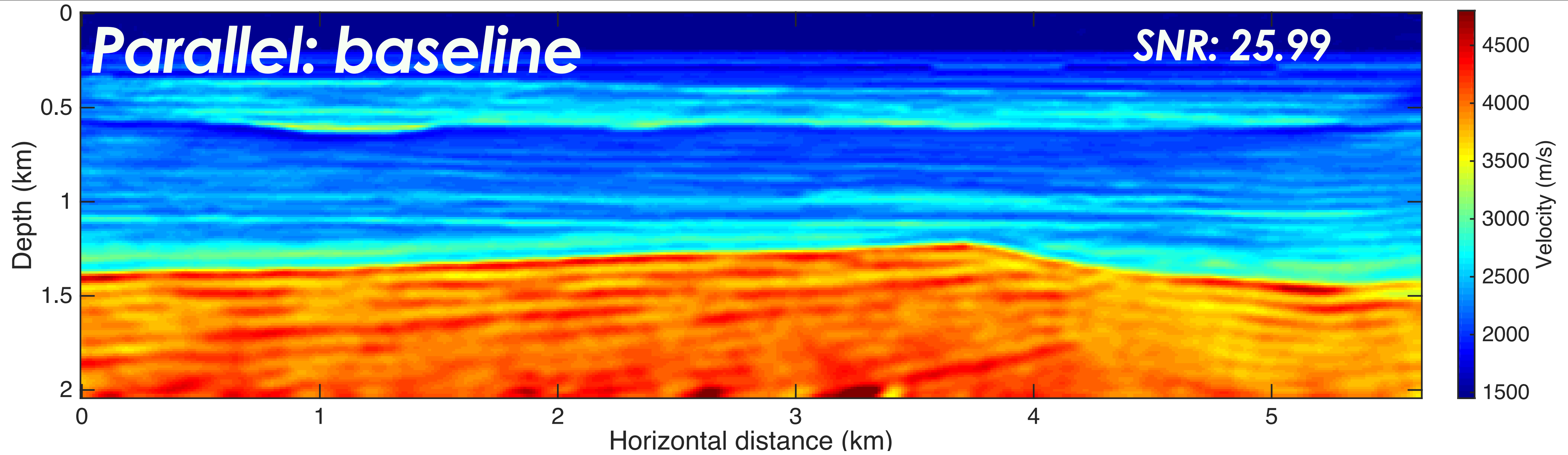
Joint w/ inverted base

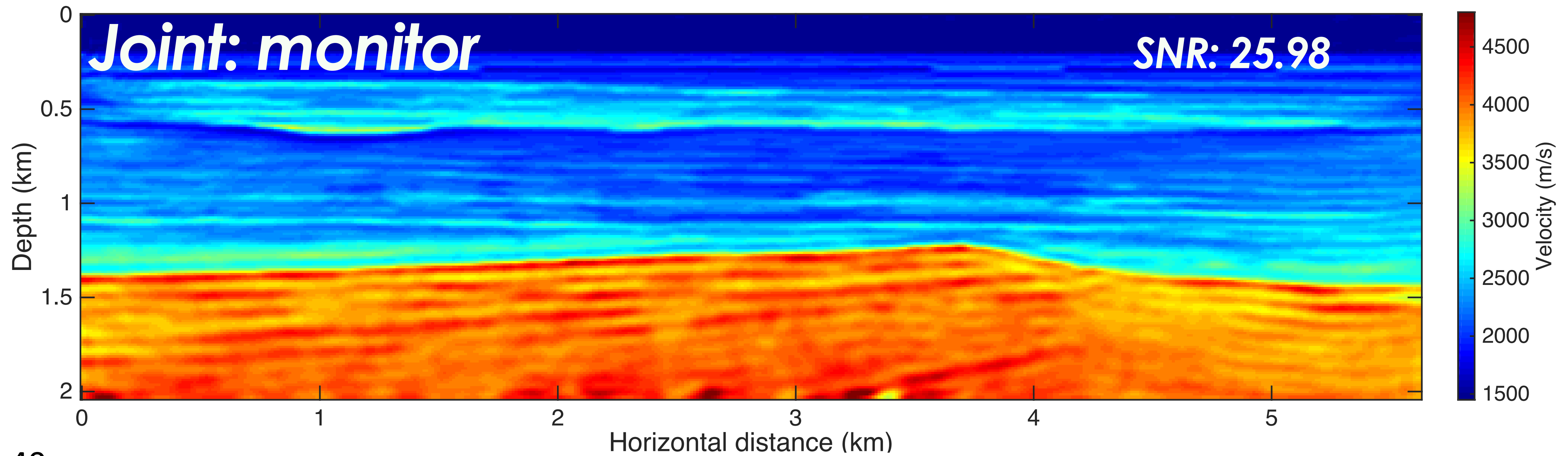
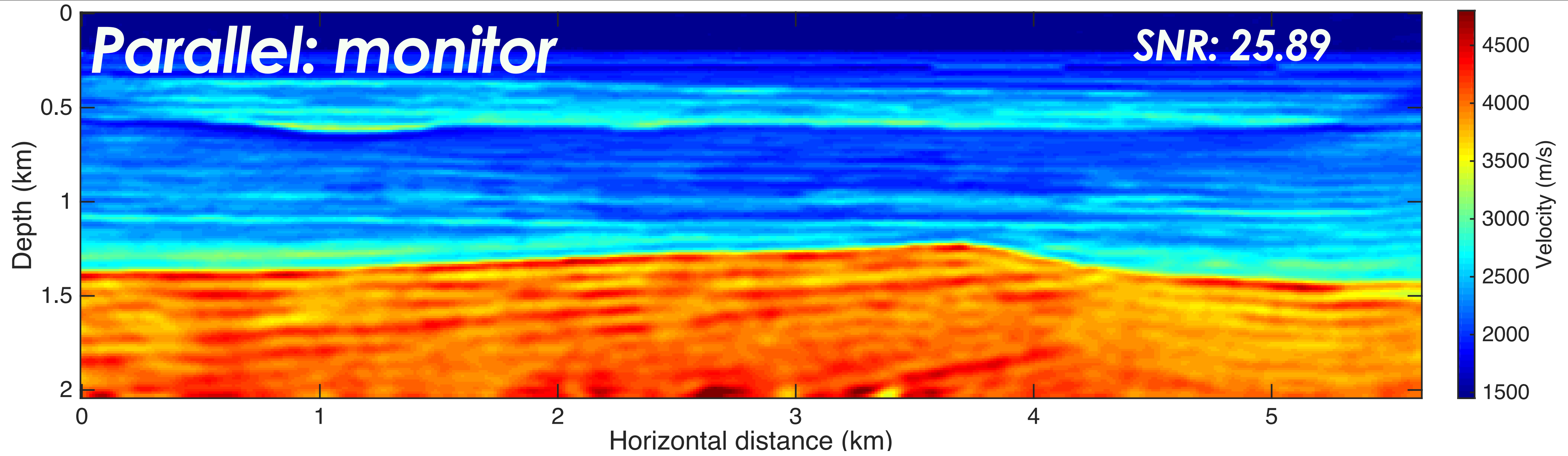


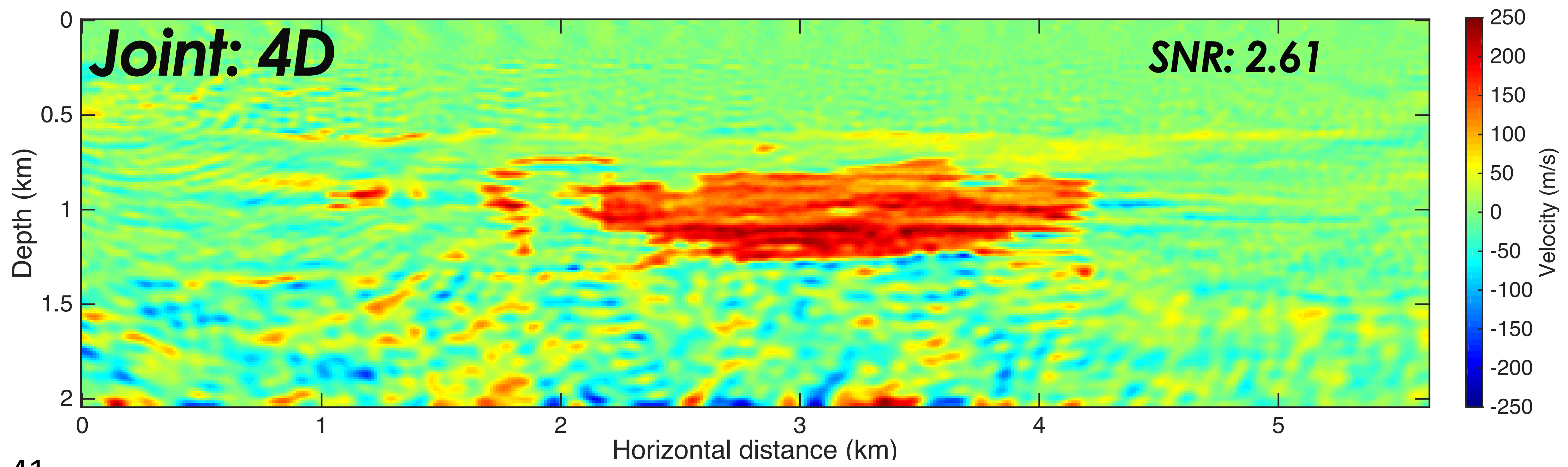
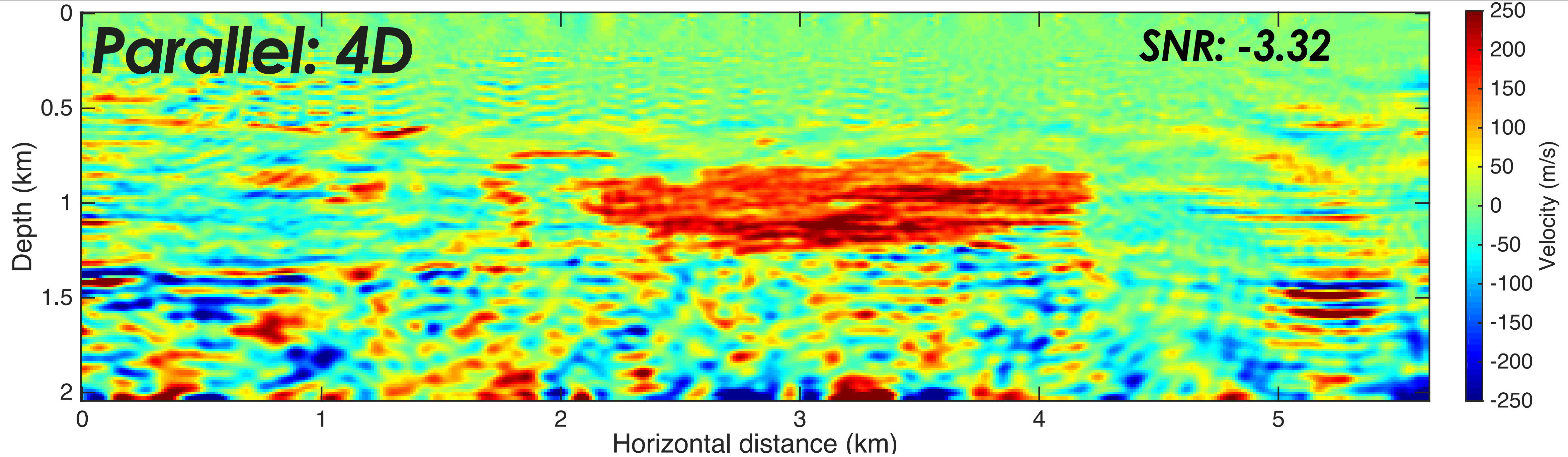
Experiment : part 2

Assuming similar geometry
“poor” starting model



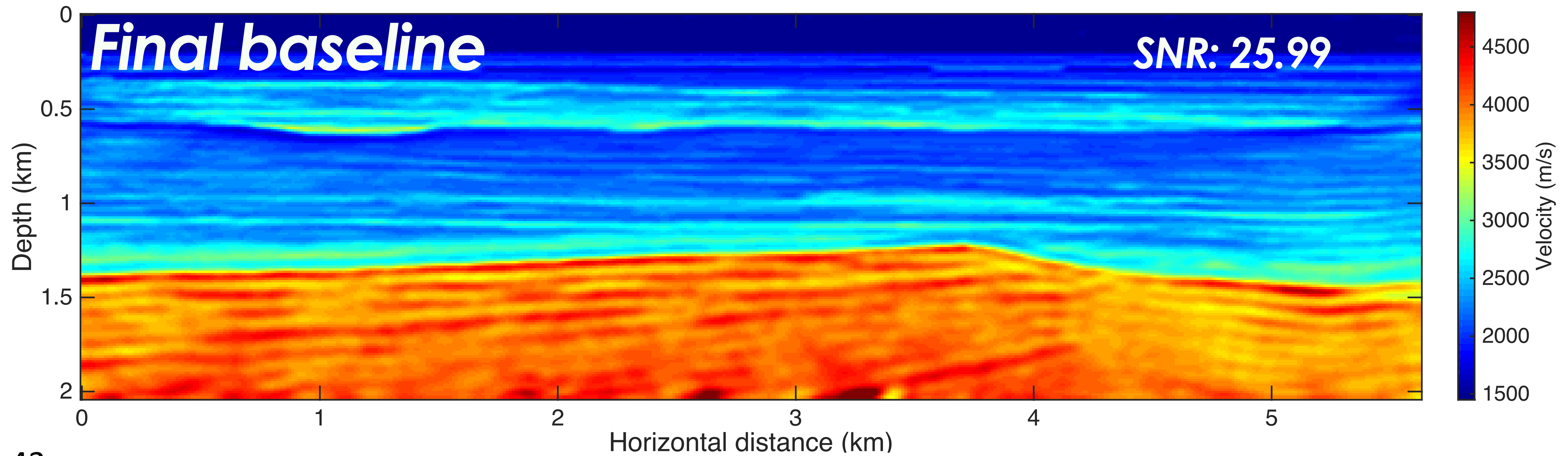
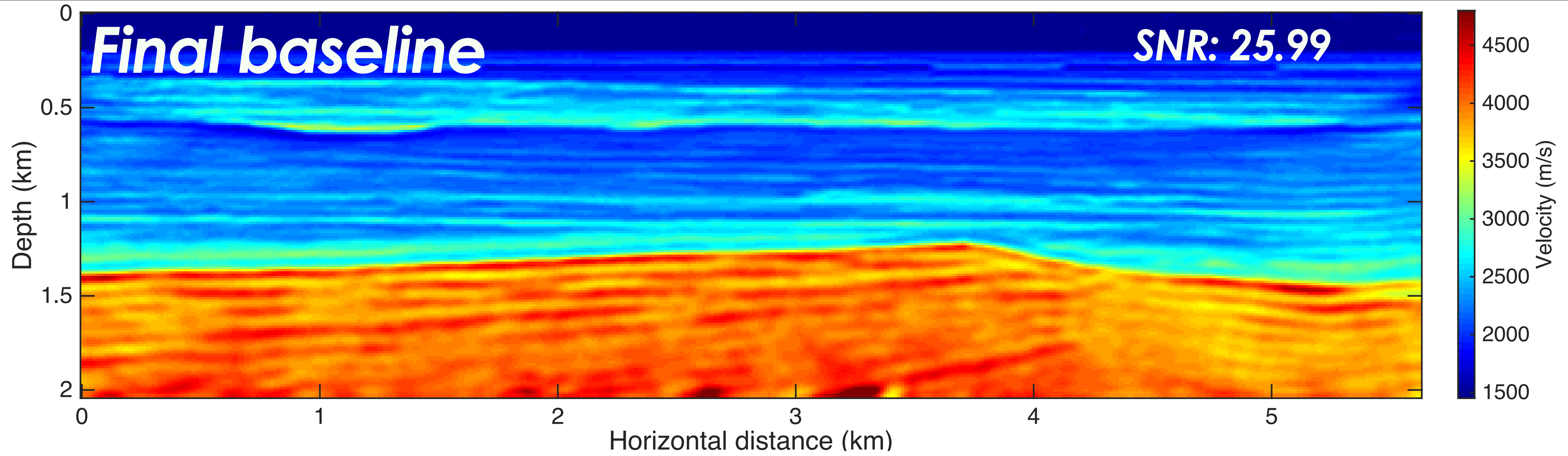


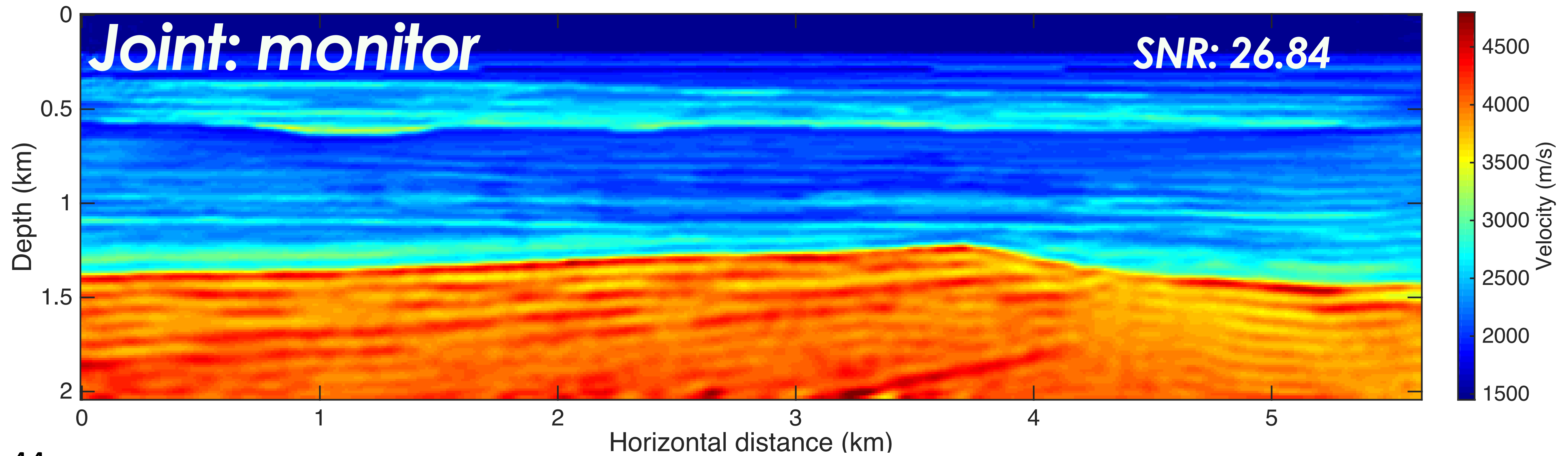
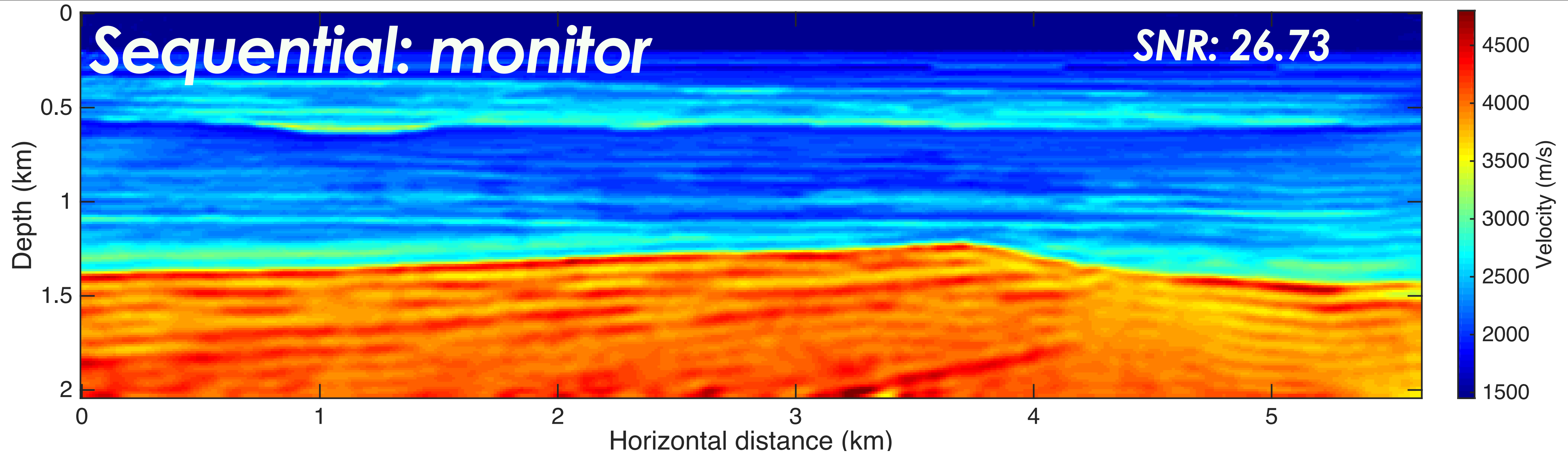


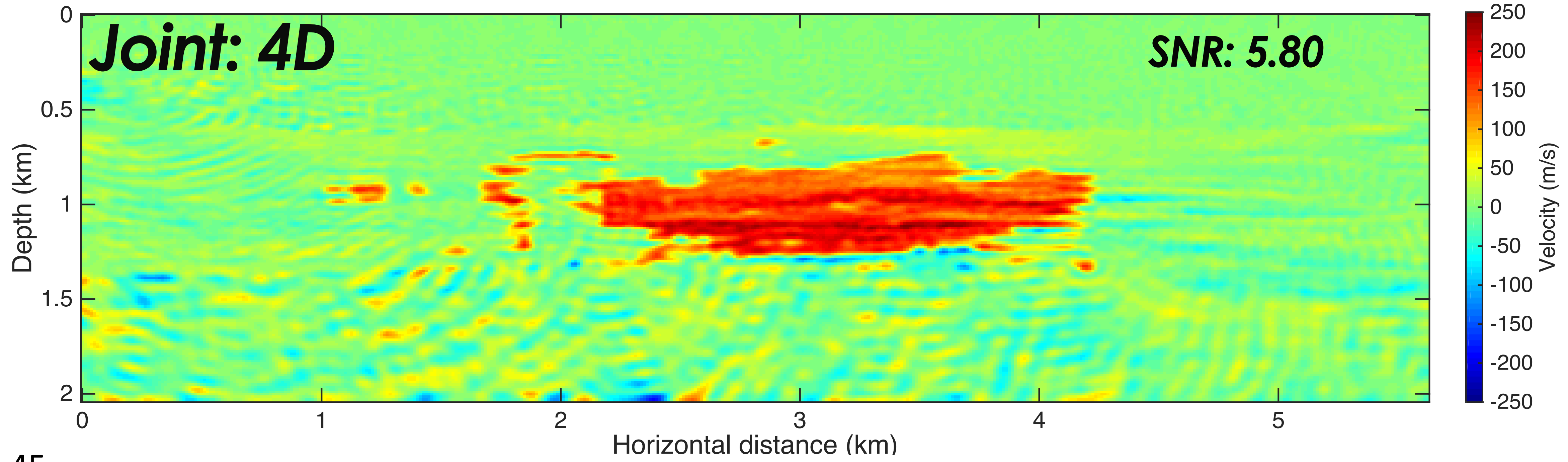
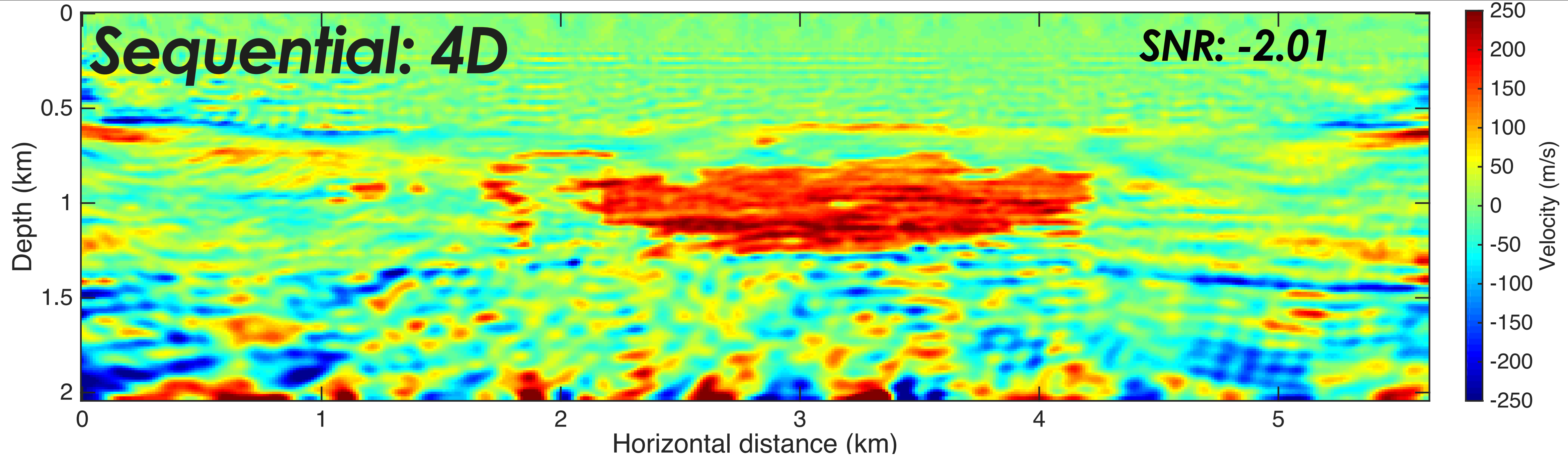


Sequential inversion

Start monitor inversion with the baseline inversion result

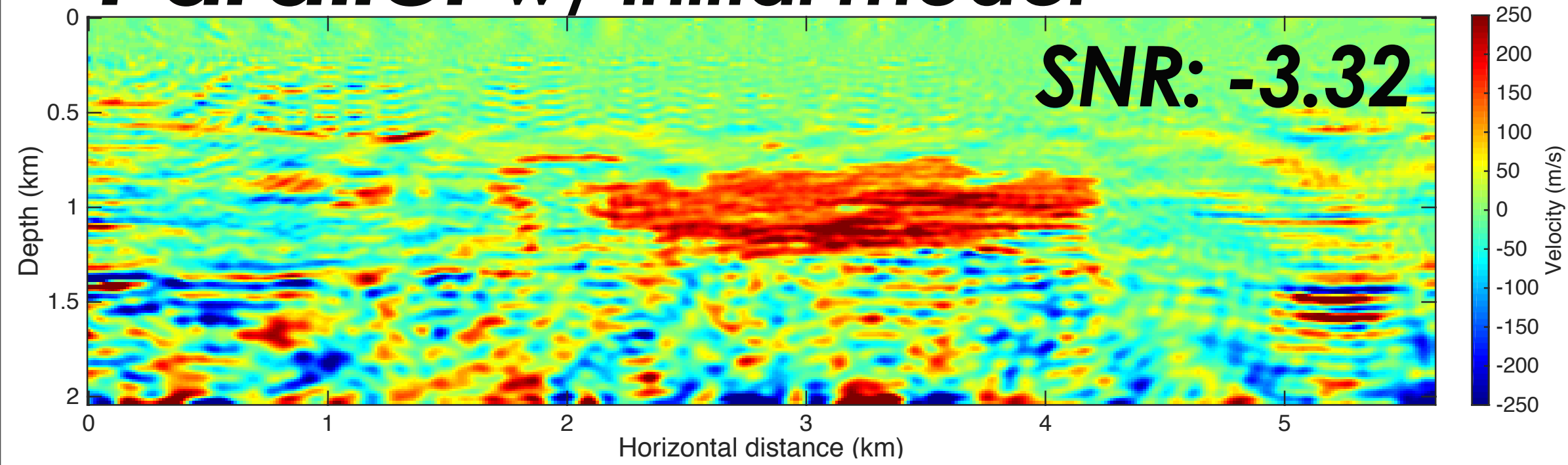




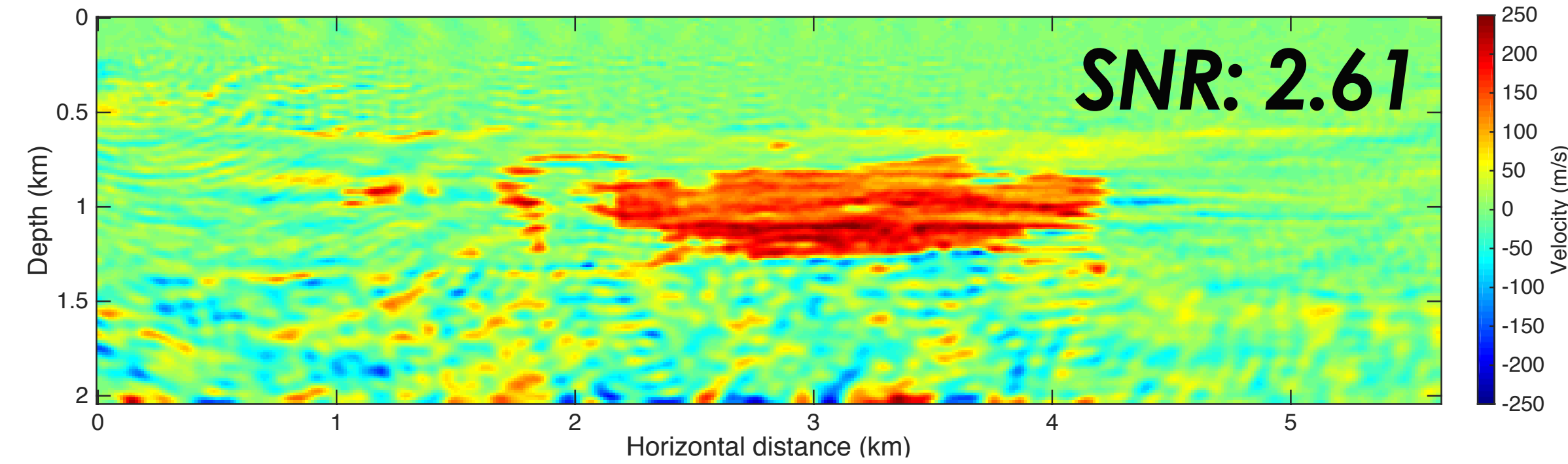


Time-lapse results

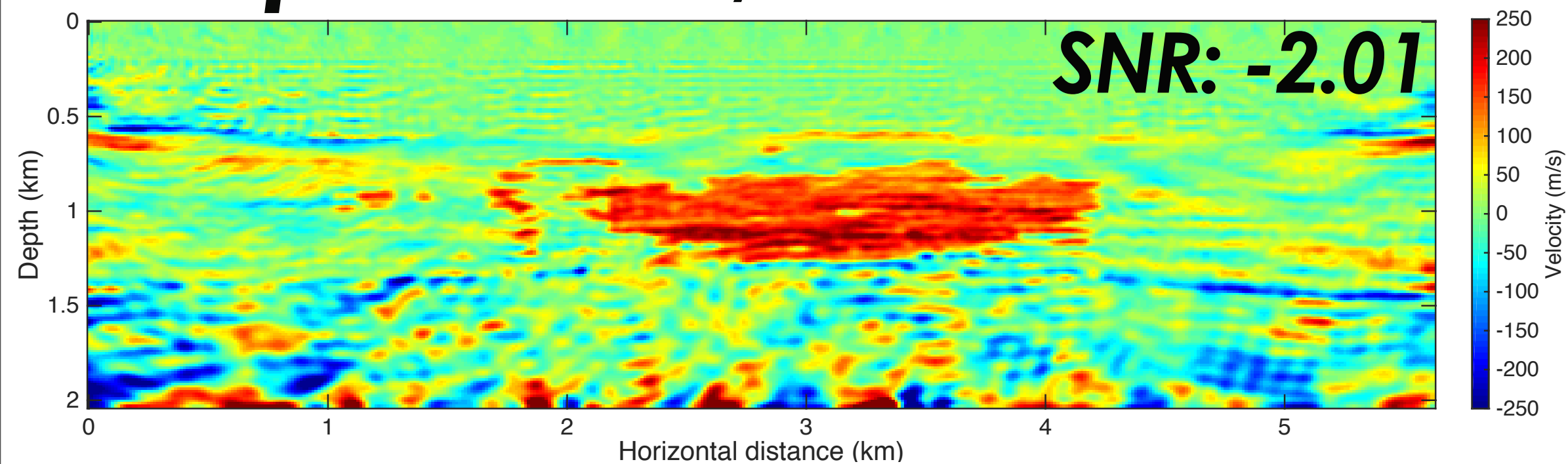
Parallel w/ initial model



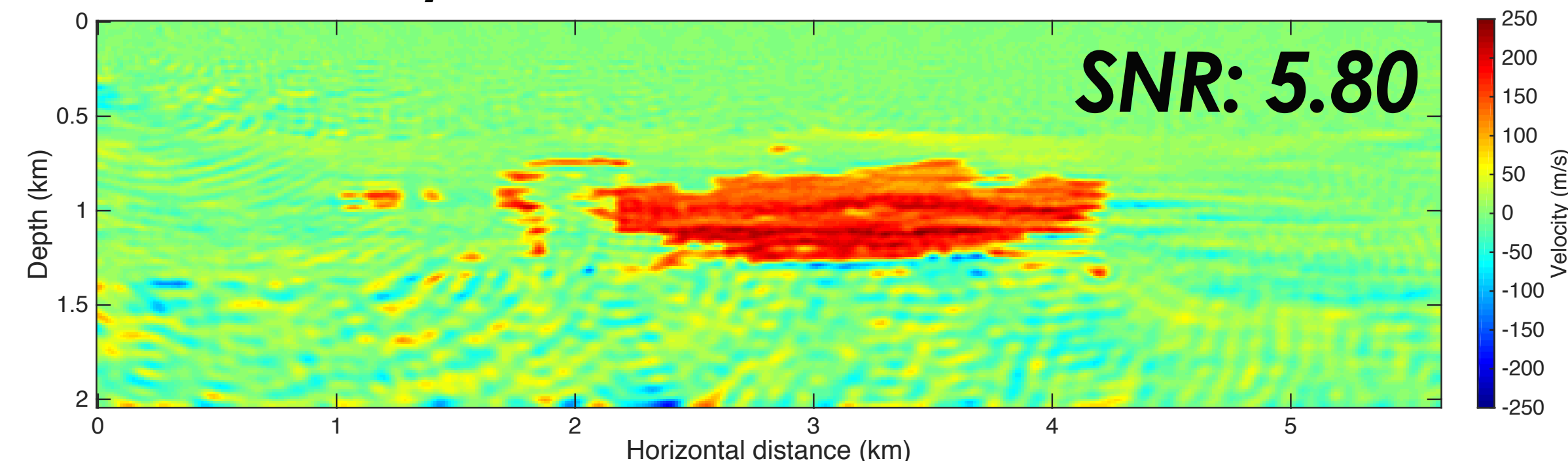
Joint w/ initial model



Sequential w/ inverted base



Joint w/ inverted base



Observations

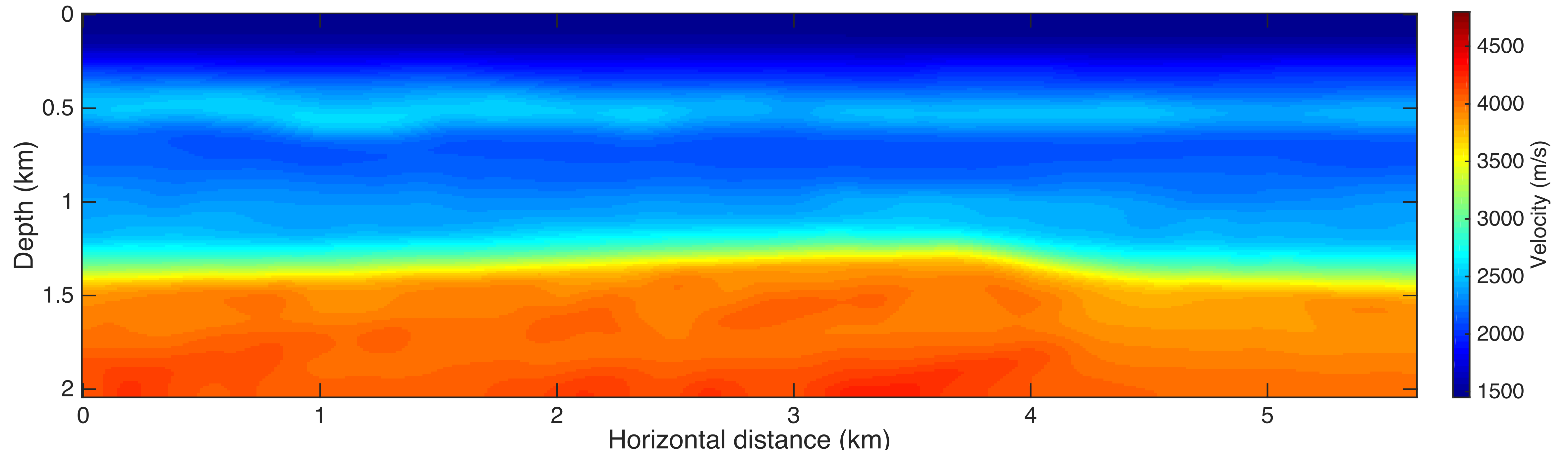
A good initial model drives the inversion results for the vintages and time-lapse model

Sequential FWI is better than **parallel** FWI, however **joint** inversion with JRM is better than both approaches

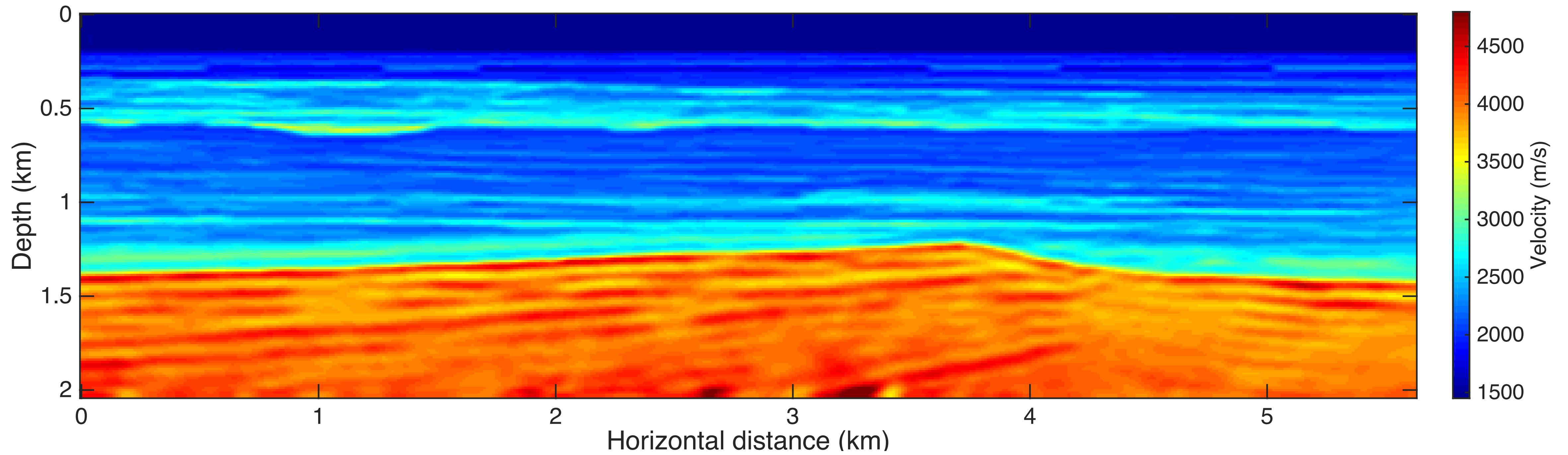
Significant attenuation of the artifacts in the time-lapse model using JRM, which exploits the shared information in time-lapse

*Assuming **accurate** baseline inversion result,
and **monitor** data has a large **acquisition gap***

Initial model

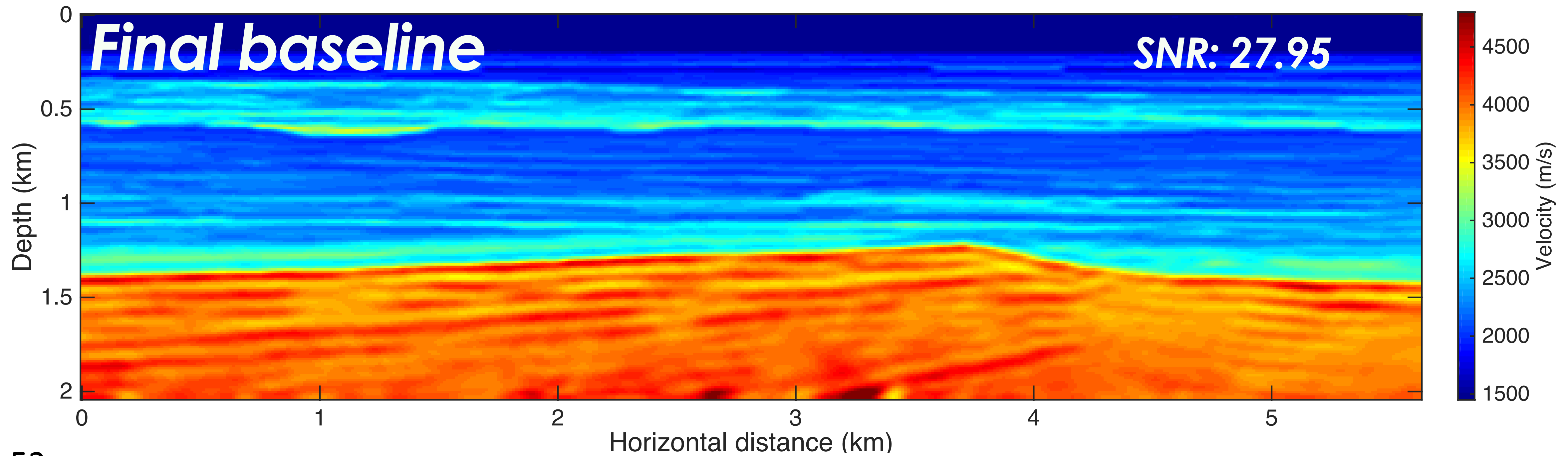
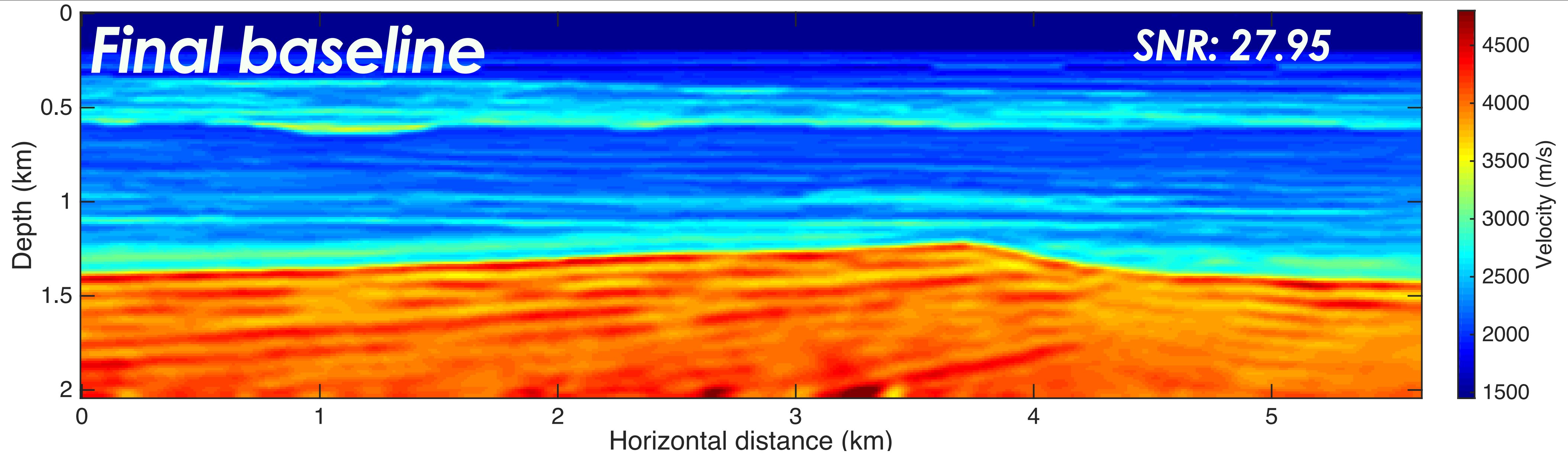


FWI baseline → *monitor*

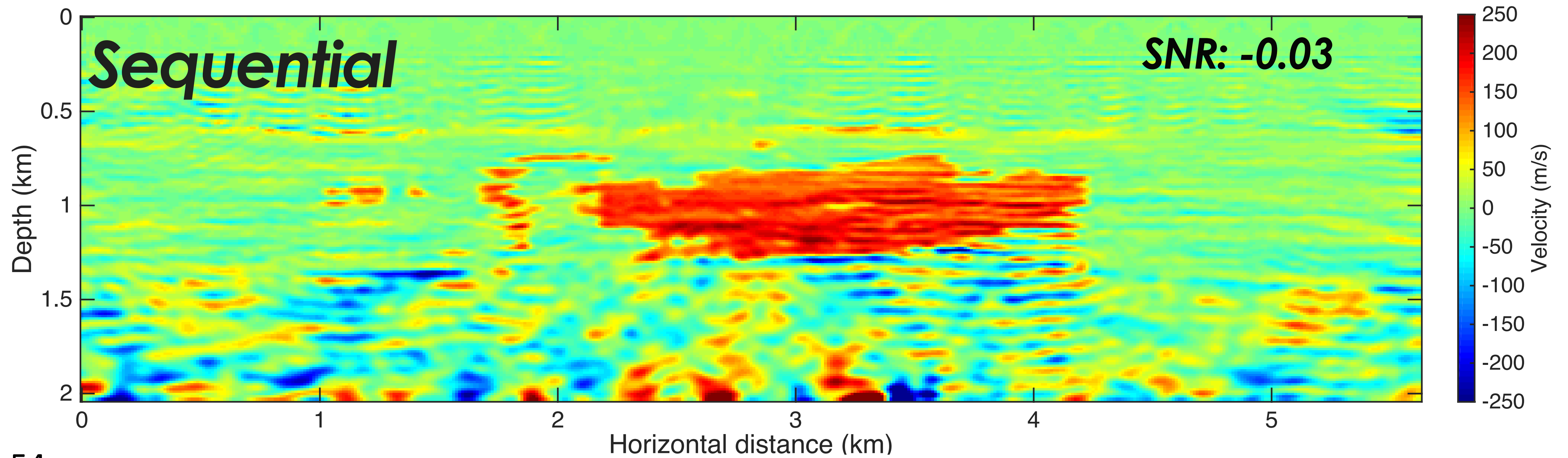
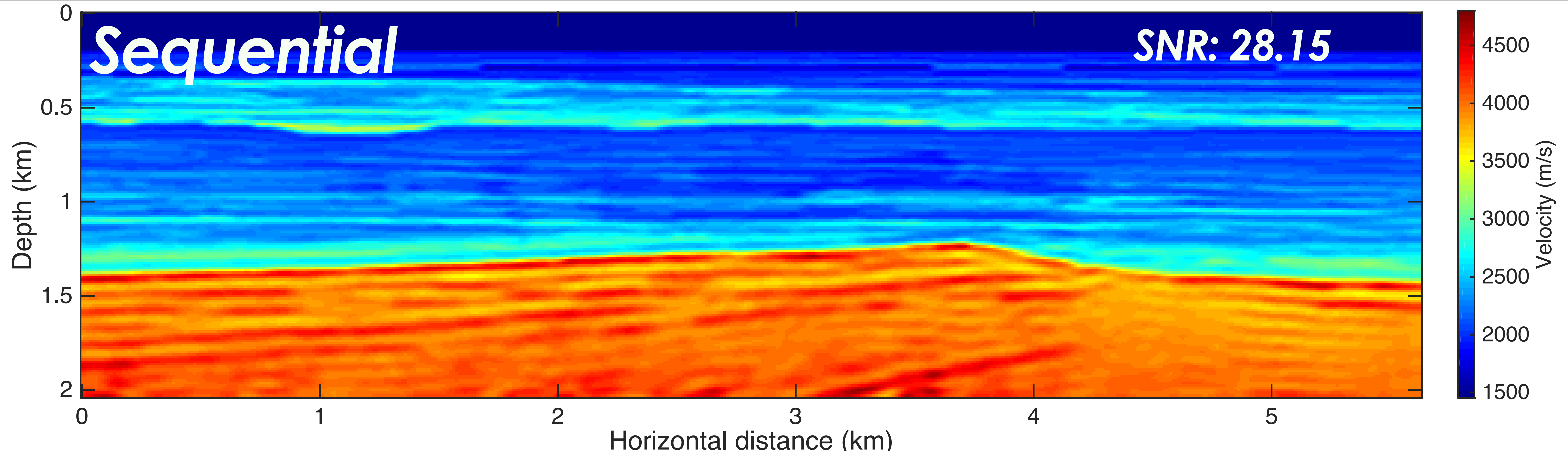


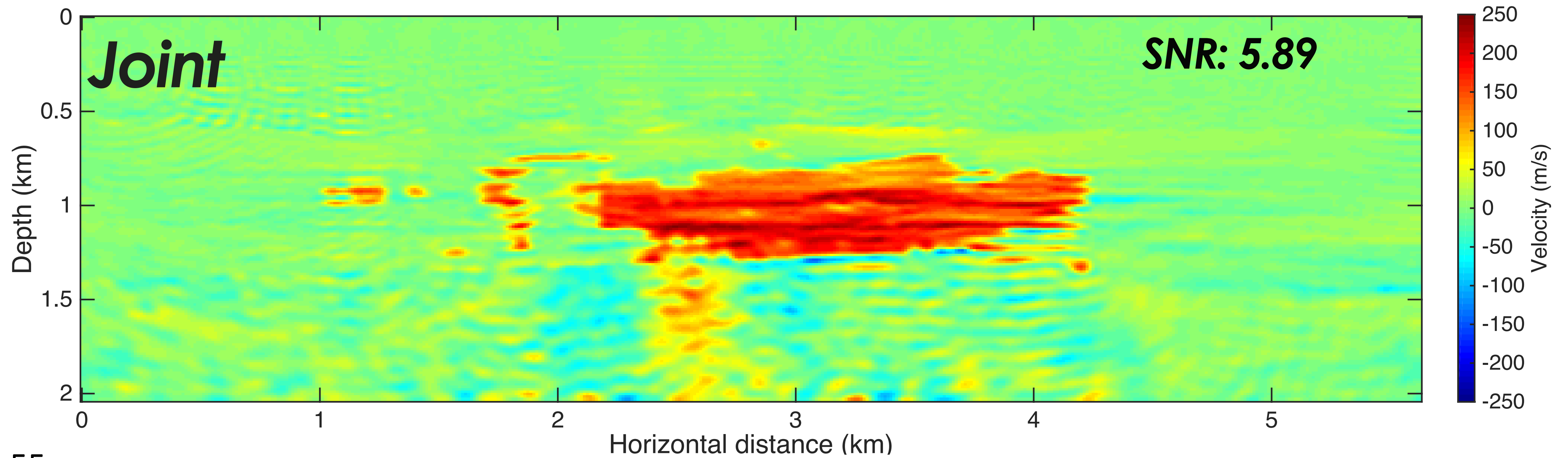
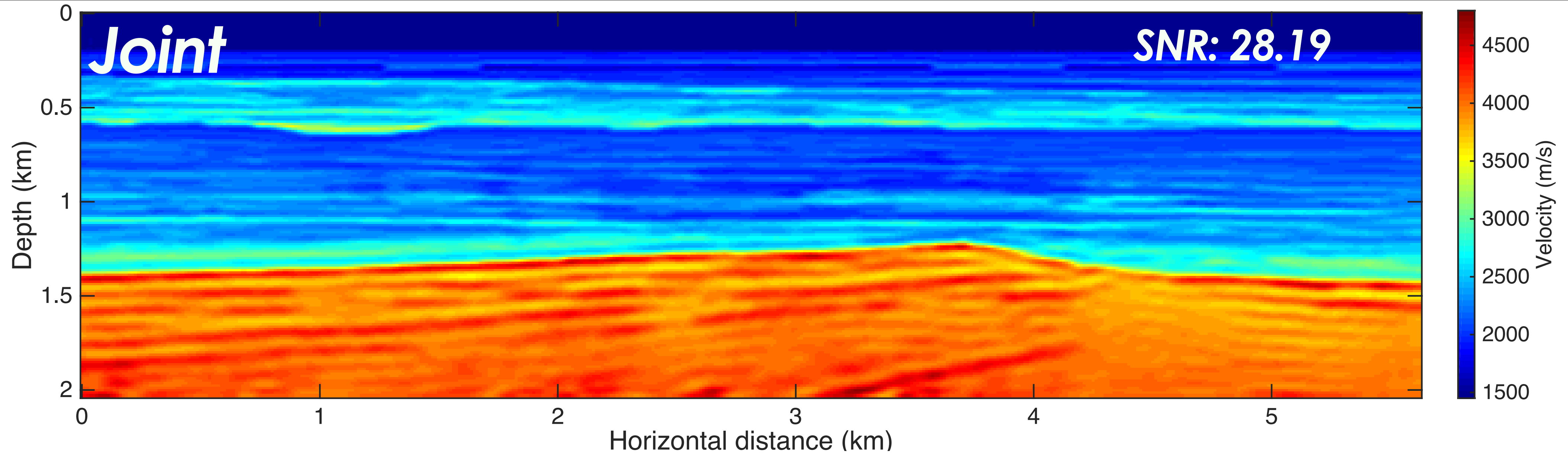
Monitor



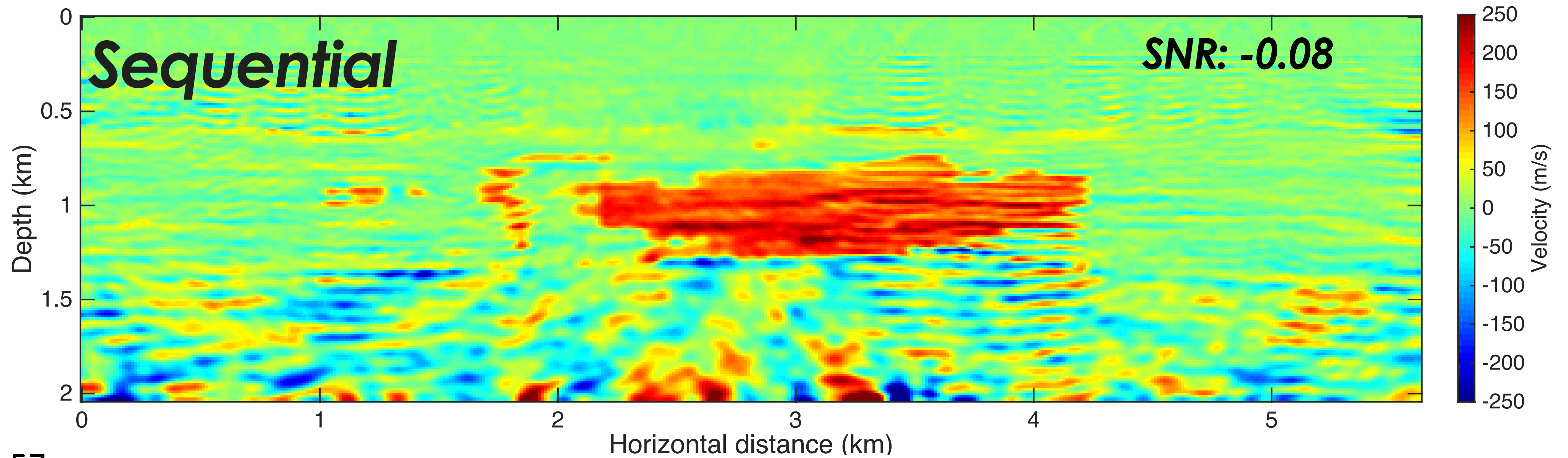
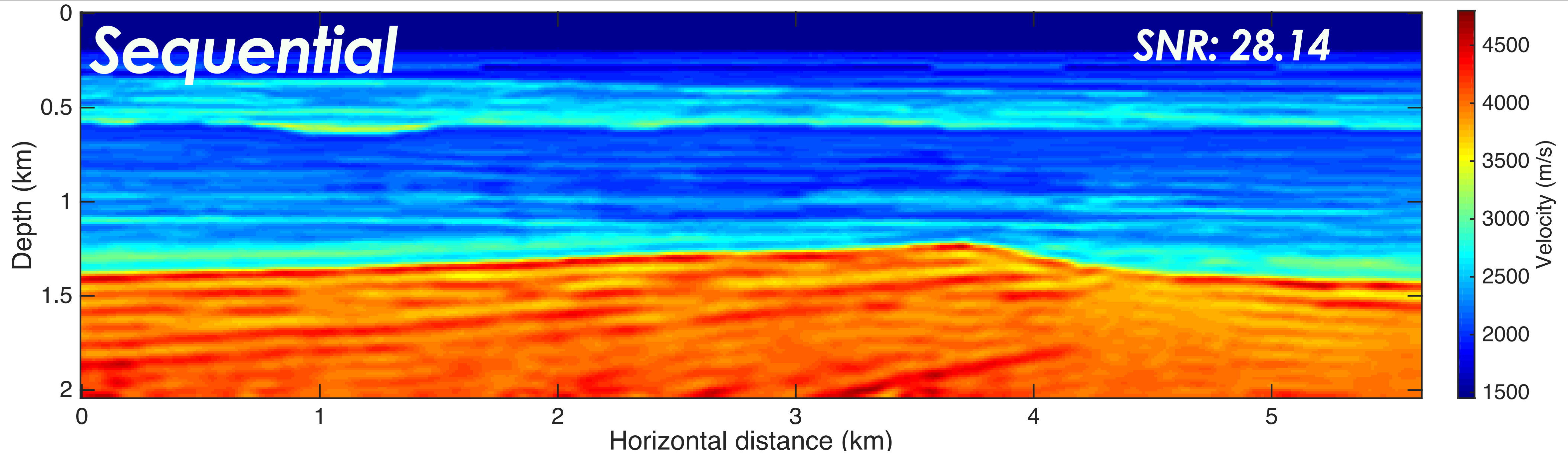


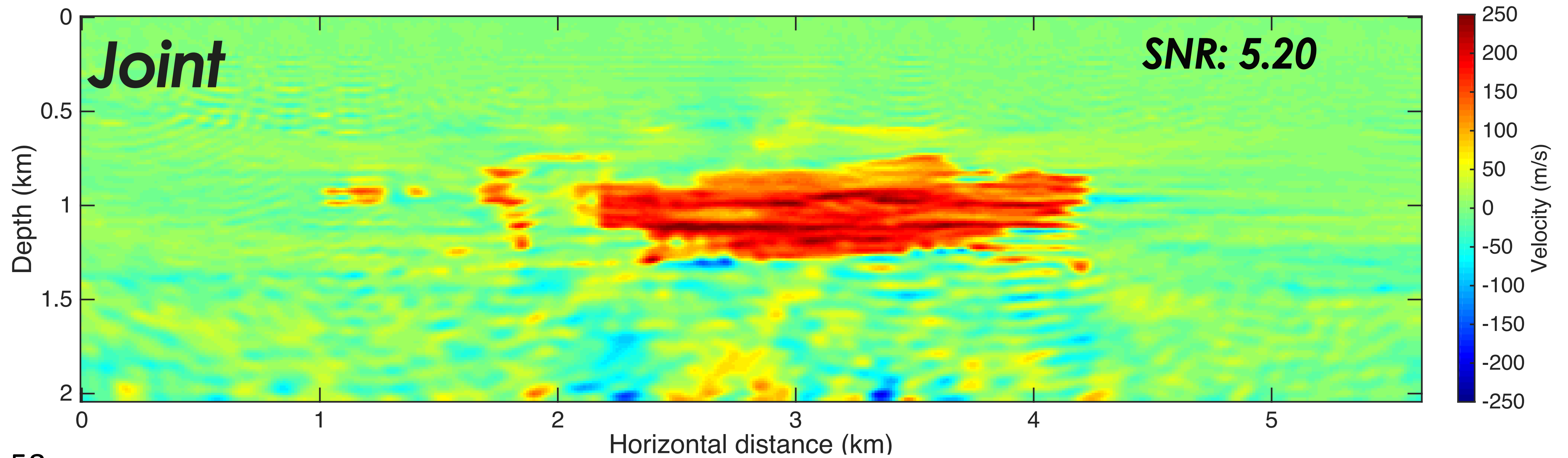
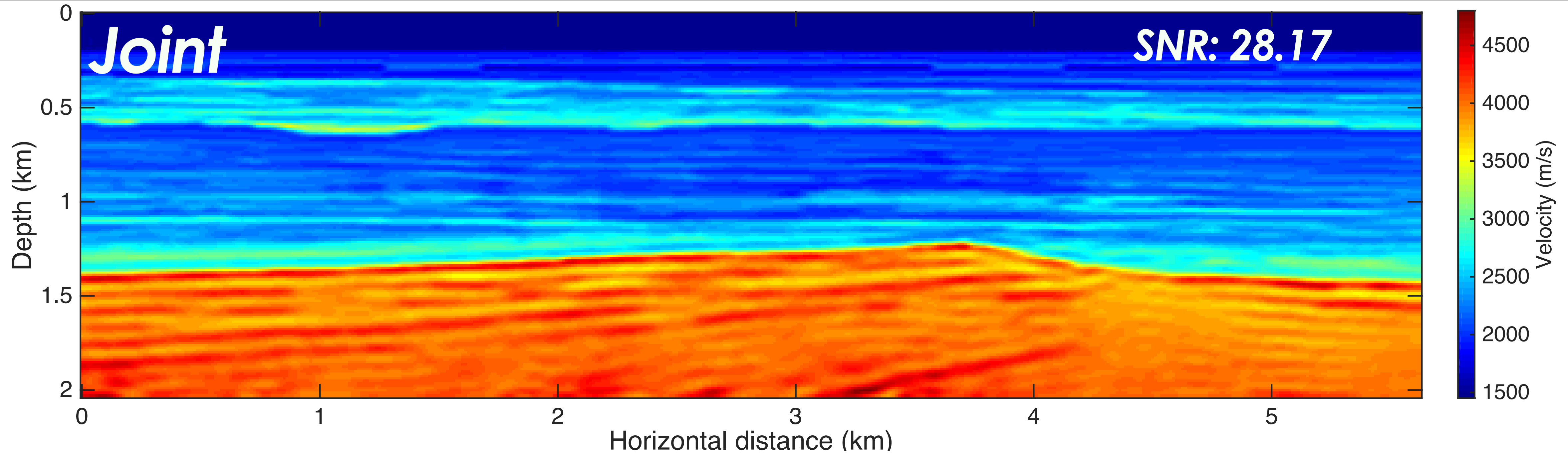
*Assuming **1000m** gap in the monitor*





*Assuming **1500m** gap in the monitor*

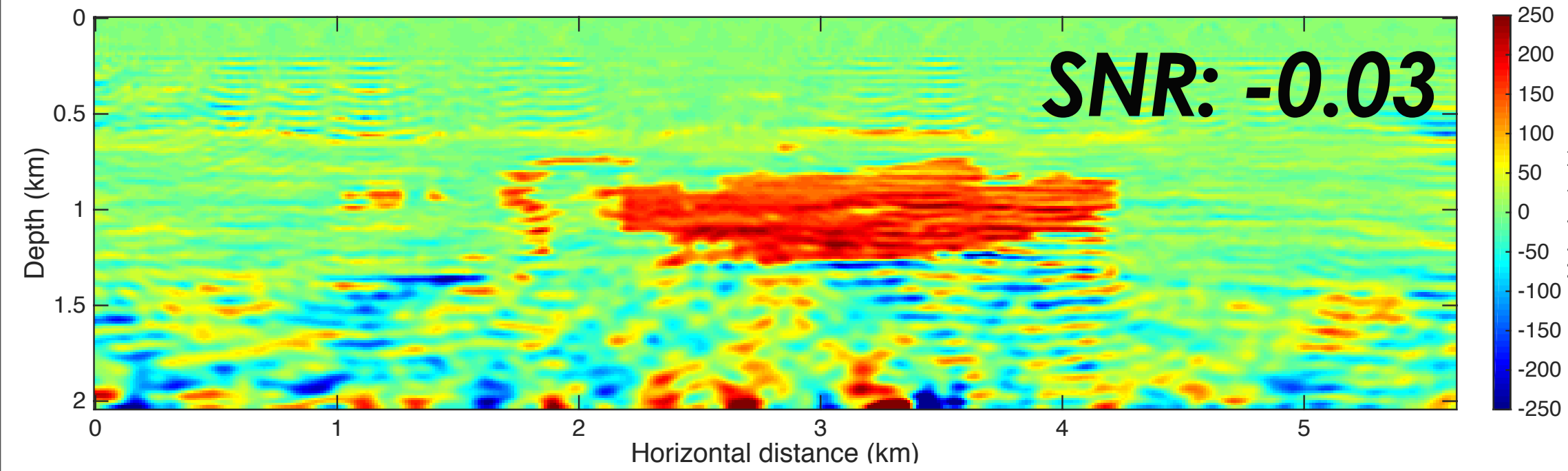




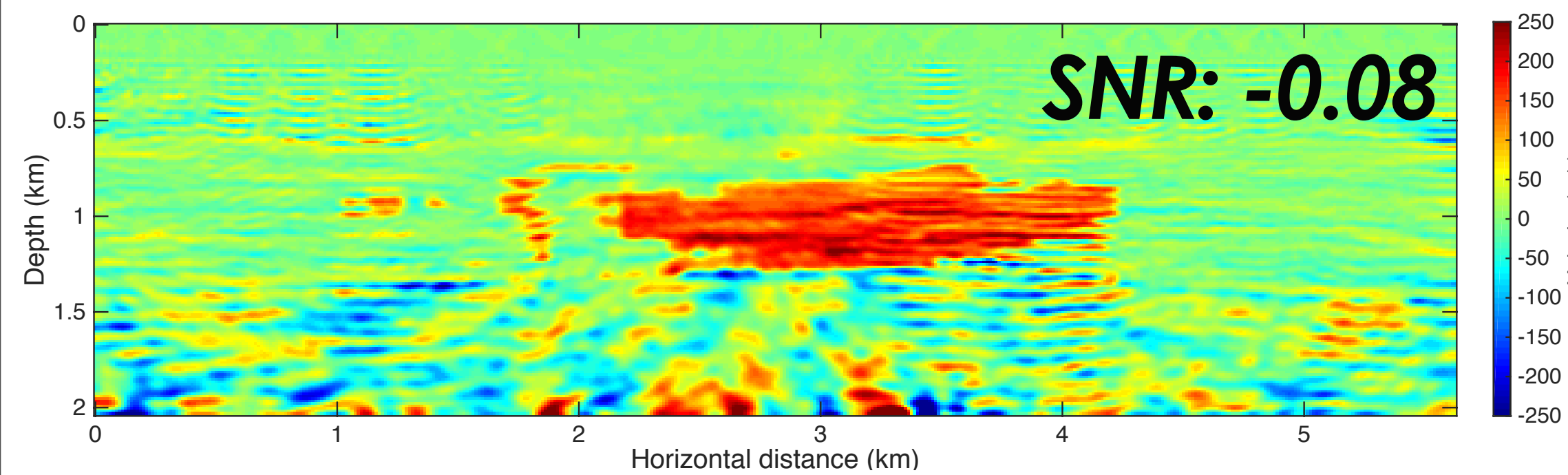
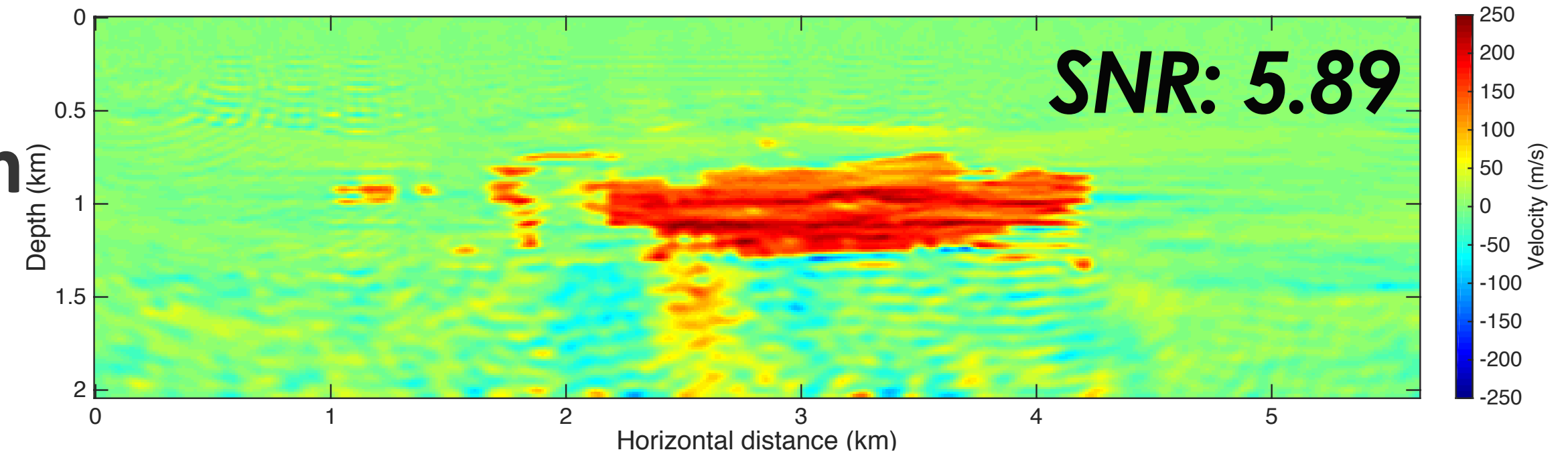
Time-lapse results

Sequential

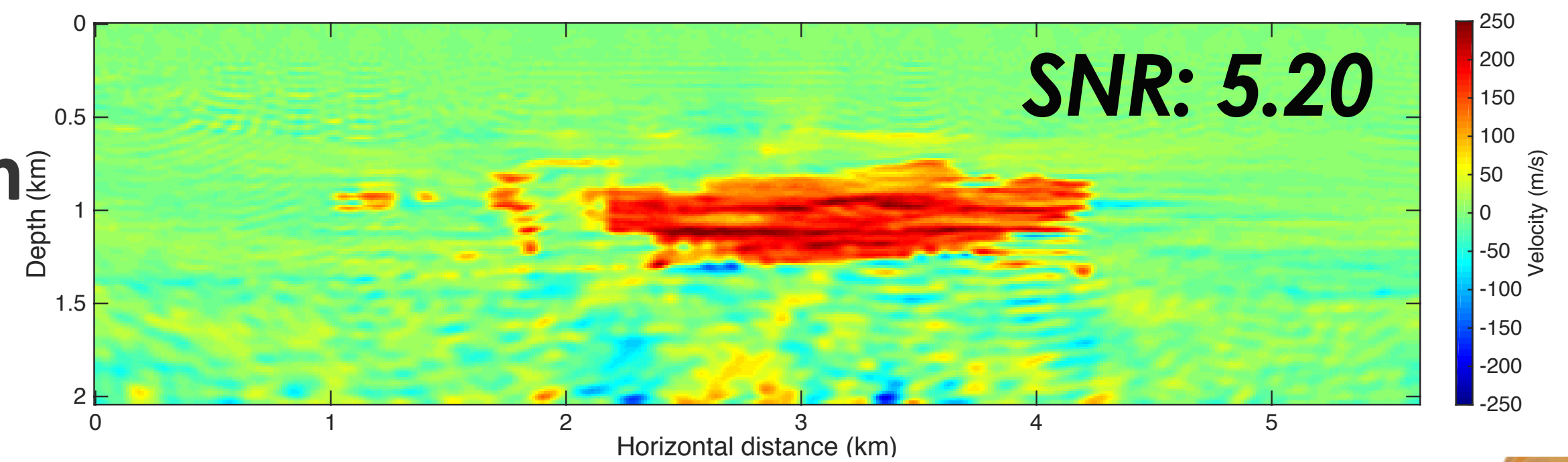
Joint



**1000m
gap**



**1500m
gap**



Conclusions

Parallel or/and sequential FWI on time-lapse data is more prone to errors in the time-lapse difference.

Joint inversion based on DCS gives better time-lapse models.

Larger acquisition gaps adversely affect the time-lapse difference.

Joint inversion with DCS can minimize the errors in time-lapse models caused by inaccurate initial models or/and missing data caused by large acquisition gaps.

Future work

- ▶ Extension to multiple vintages
- ▶ Explore the role of noise/other factors unaccounted for
- ▶ Implementation on a more realistic 3D-synthetic time-lapse data/
Field data

Acknowledgements

Thank you for your attention !!!



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