Universal Matrix Completion: Applications to Seismic Data Acquisition

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Universal Matrix Completion: Applications to Seismic Data Acquisition

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Motivation

- Acquisition challenges
  - missing data (subsampled or coverage holes)

- Exploit *low-rank* structure of seismic data
  - *SVD-free* matrix completion (2D & 3D)

- Need analysis
  - how should we subsample?
  - reconstruction guarantees

Contributions

- Quantification of subsampling
  - measure "spectral gap"
  - computationally simple

$$\frac{\sigma_2}{\sigma_1}$$

- Applications to seismic data acquisition
  - optimally design acquisition
  - tools for 3D data

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Outline

- Current Work
  - matrix completion analysis
  - seismic trace interpolation

- Universal Matrix Completion
  - spectral gap
  - applications for seismic data

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Matrix Completion Literature

Given a matrix $\mathbf{X} \in \mathbb{R}^{n \times m}$ of rank $r \ll \min(m, n)$, we exploit its low dimensional structure to recover $\mathbf{X}$ from limited and noisy samples via

$$\underset{\mathbf{Y}}{\text{minimize}} \|\mathbf{Y}\|_* \text{ subject to } \|P_\Omega(\mathbf{Y}) - \mathbf{b}\|_F \leq \epsilon,$$

where $\mathbf{b}_{i,j} = P_\Omega(\mathbf{X})_{i,j} = \begin{cases} \mathbf{X}_{i,j} & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$

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subsampling scheme

Matrix Completion Literature

Typical assumptions:

Suppose $|\Omega|$ entries of $\mathbf{X}$ are observed with locations sampled uniformly at random…

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Matrix Completion Literature

Typical assumptions:

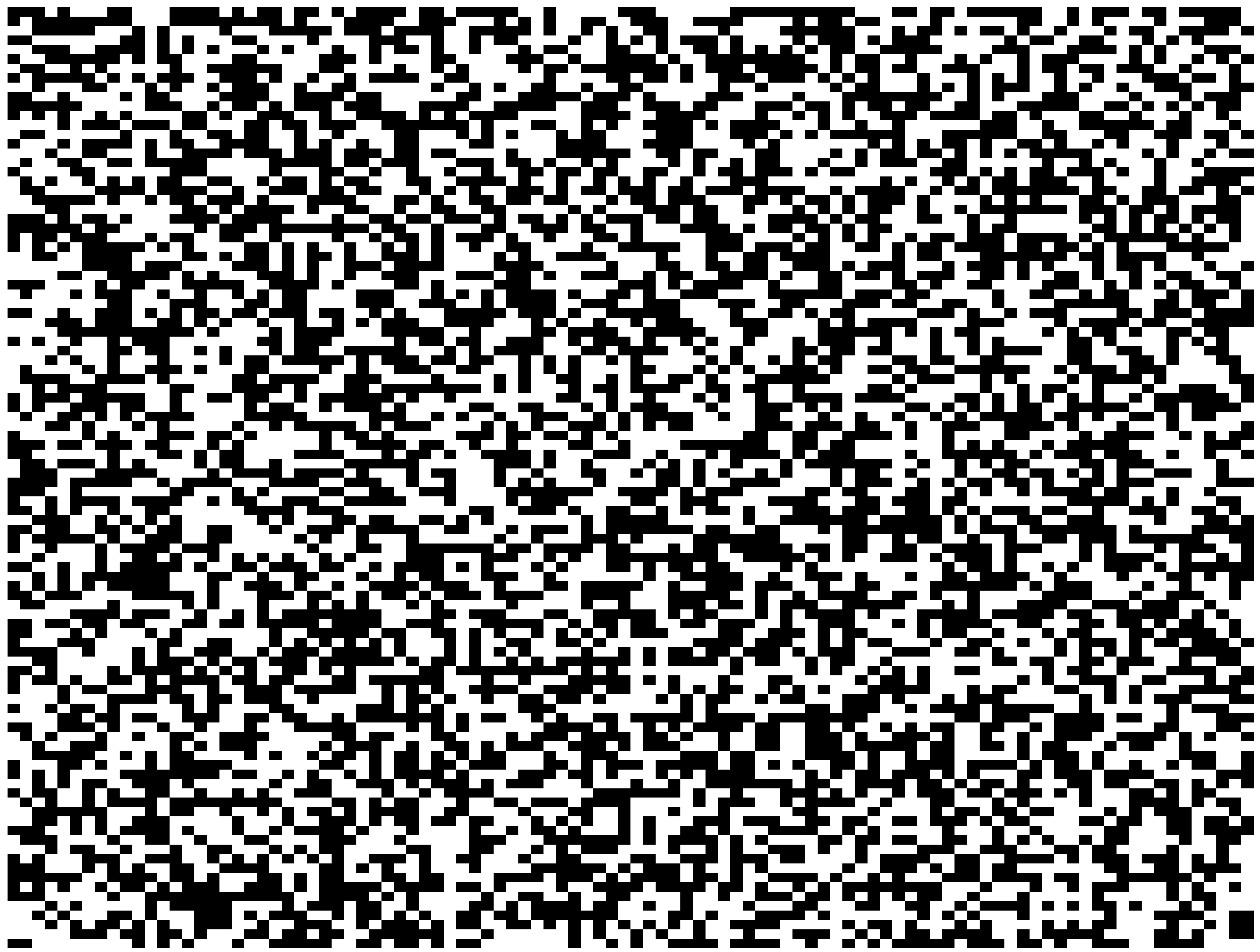
Suppose $|\Omega|$ entries of $\mathbf{X}$ are observed with locations sampled uniformly at random...

Consider the sampling mask
$$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$$

Matrix Completion Literature

Assumed sampling mask: not practical

$$A =$$



$\blacksquare = 0$

$\square = 1$

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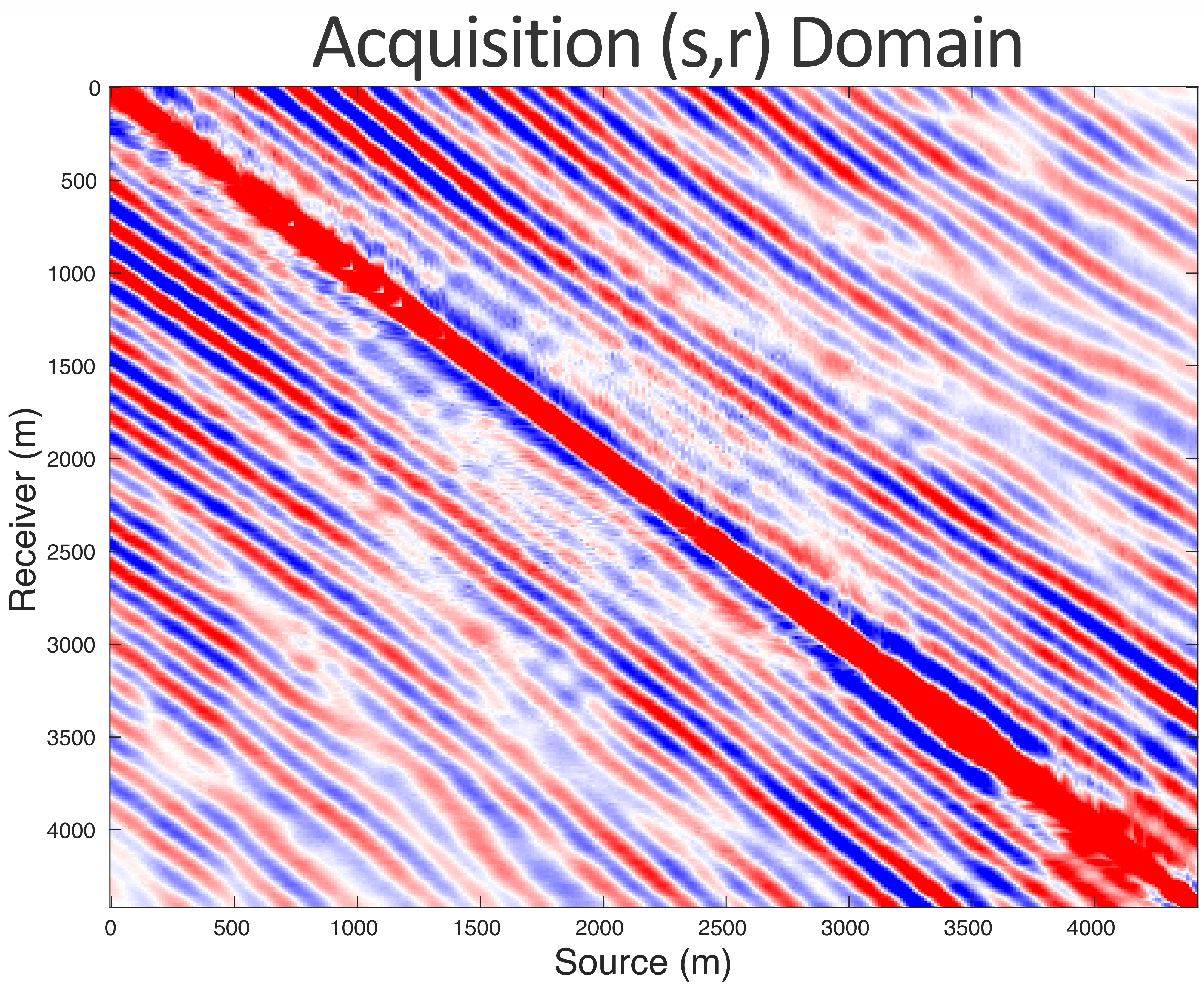
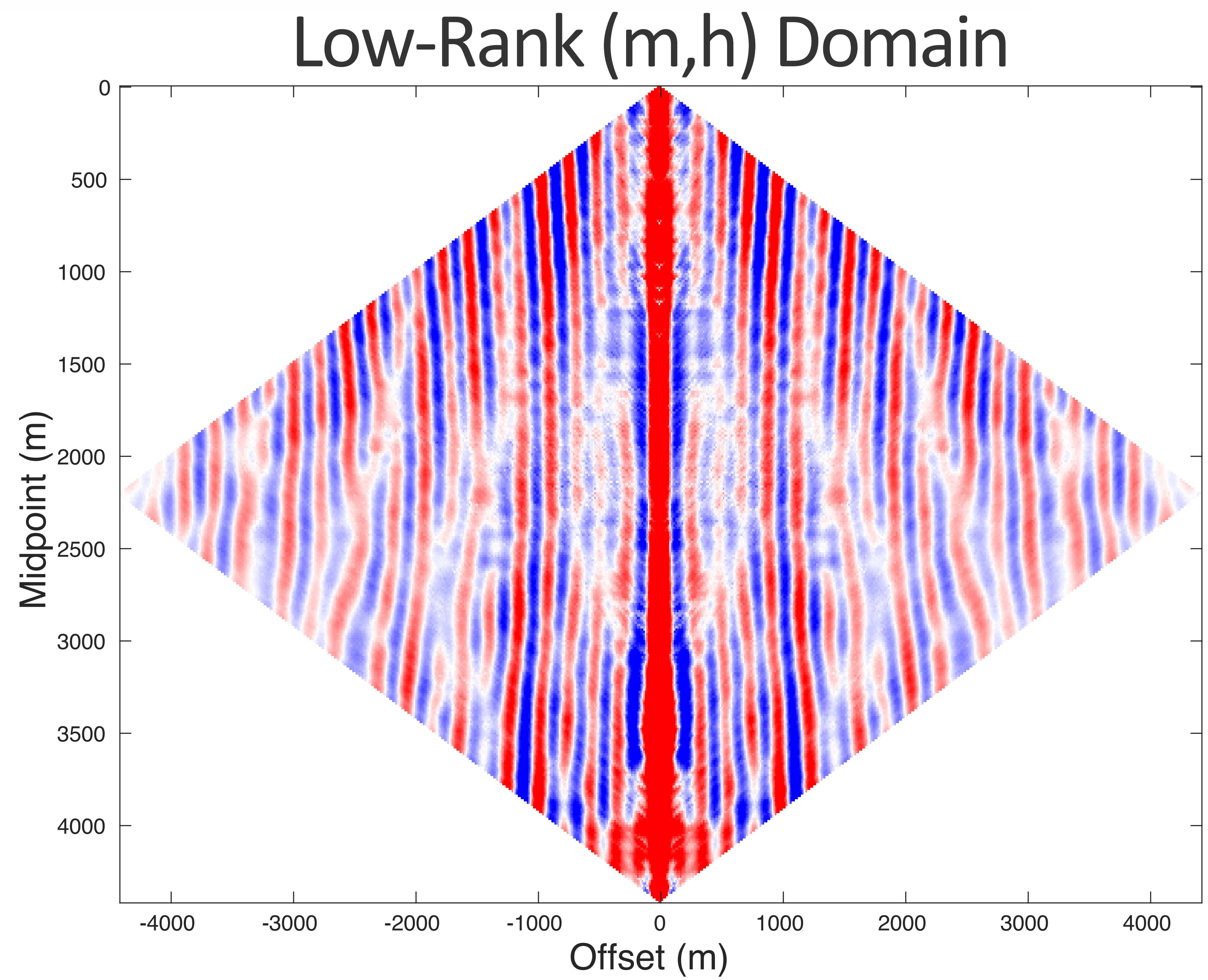
2D seismic data low-rank structure

- Consider a 2D seismic survey with coordinates (src x, rec x, time)

- Take a Fourier transform in time and restrict ourselves to a single frequency slice.

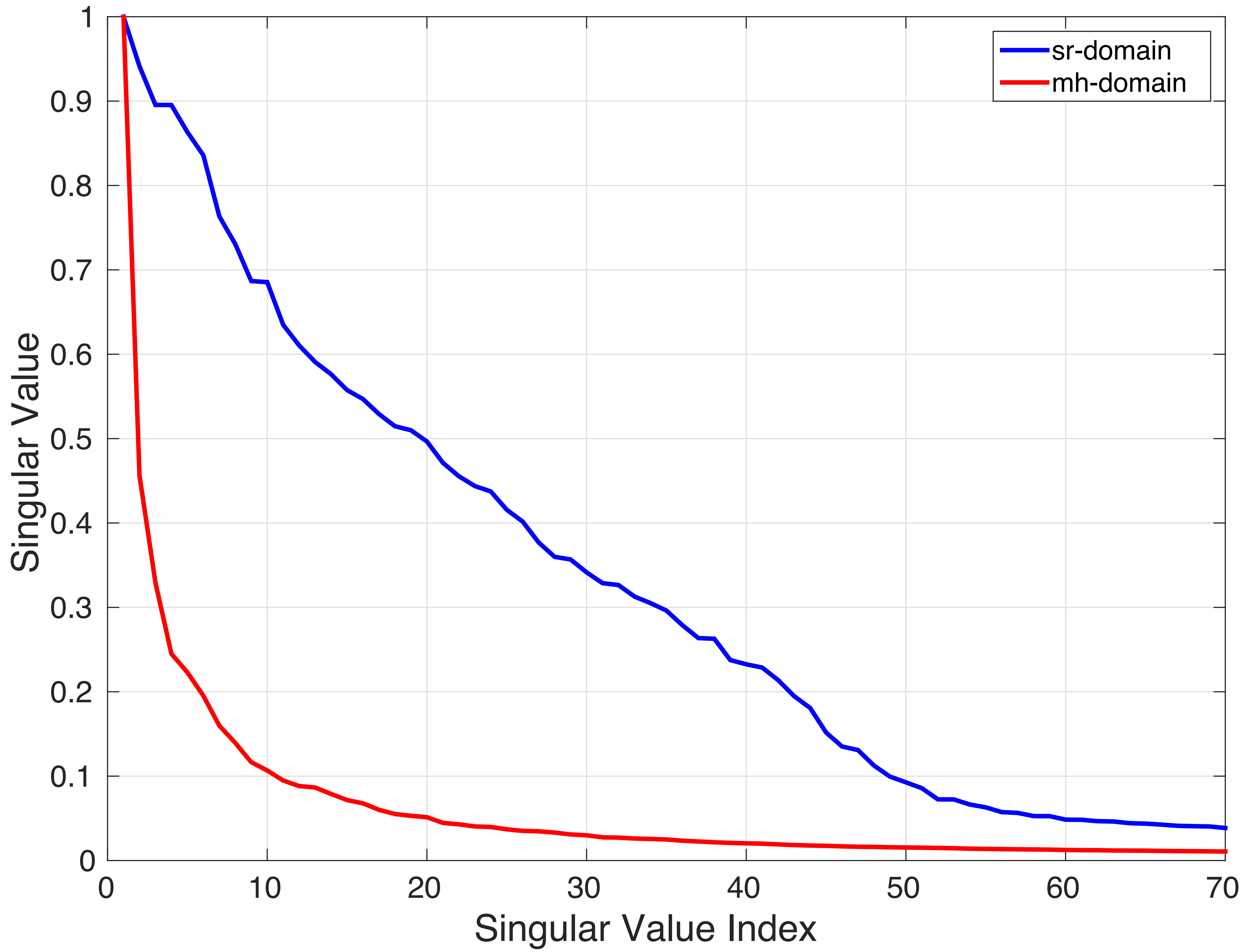
2D Low-rank structure





Singular value decay

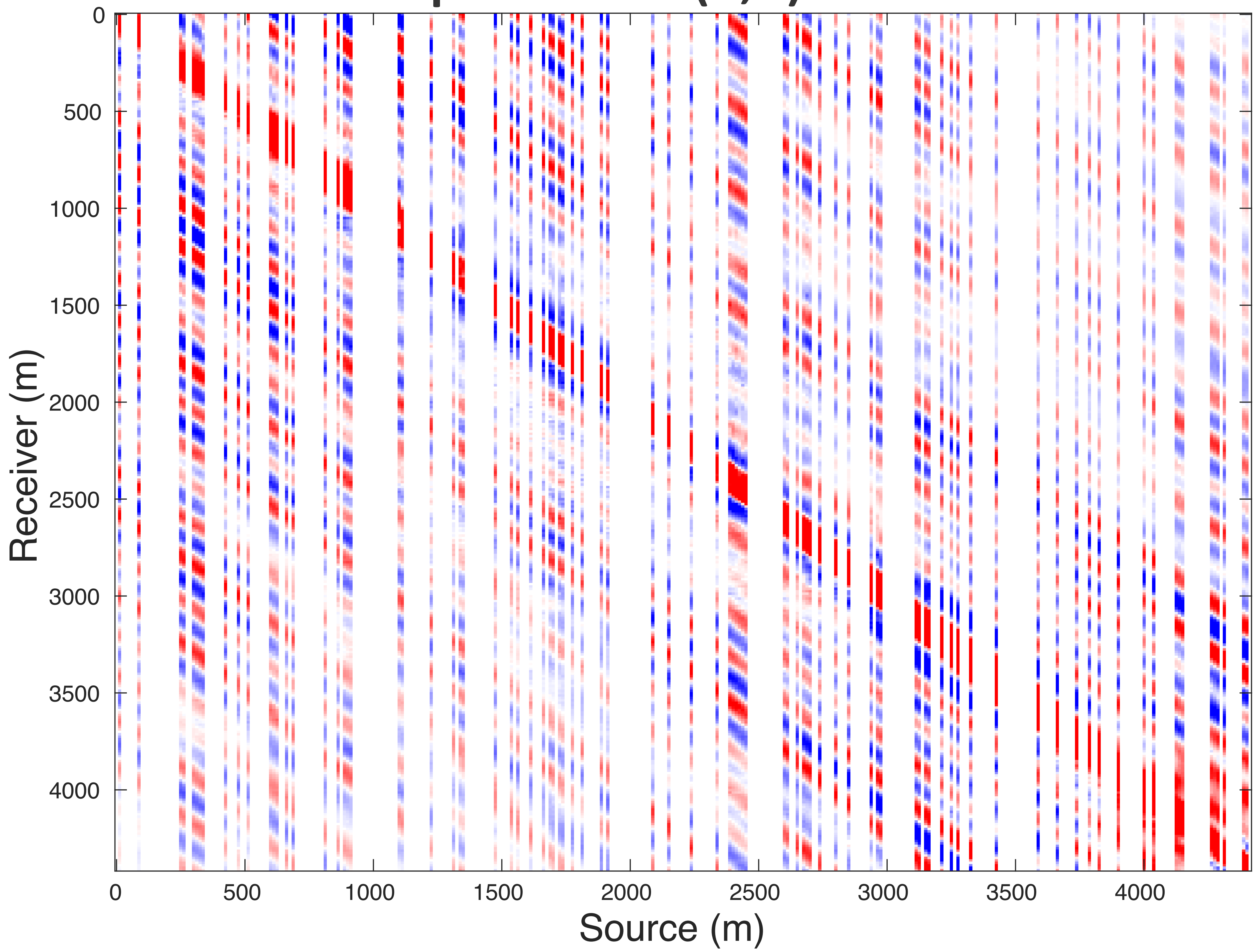
s-r domain vs m-h domain



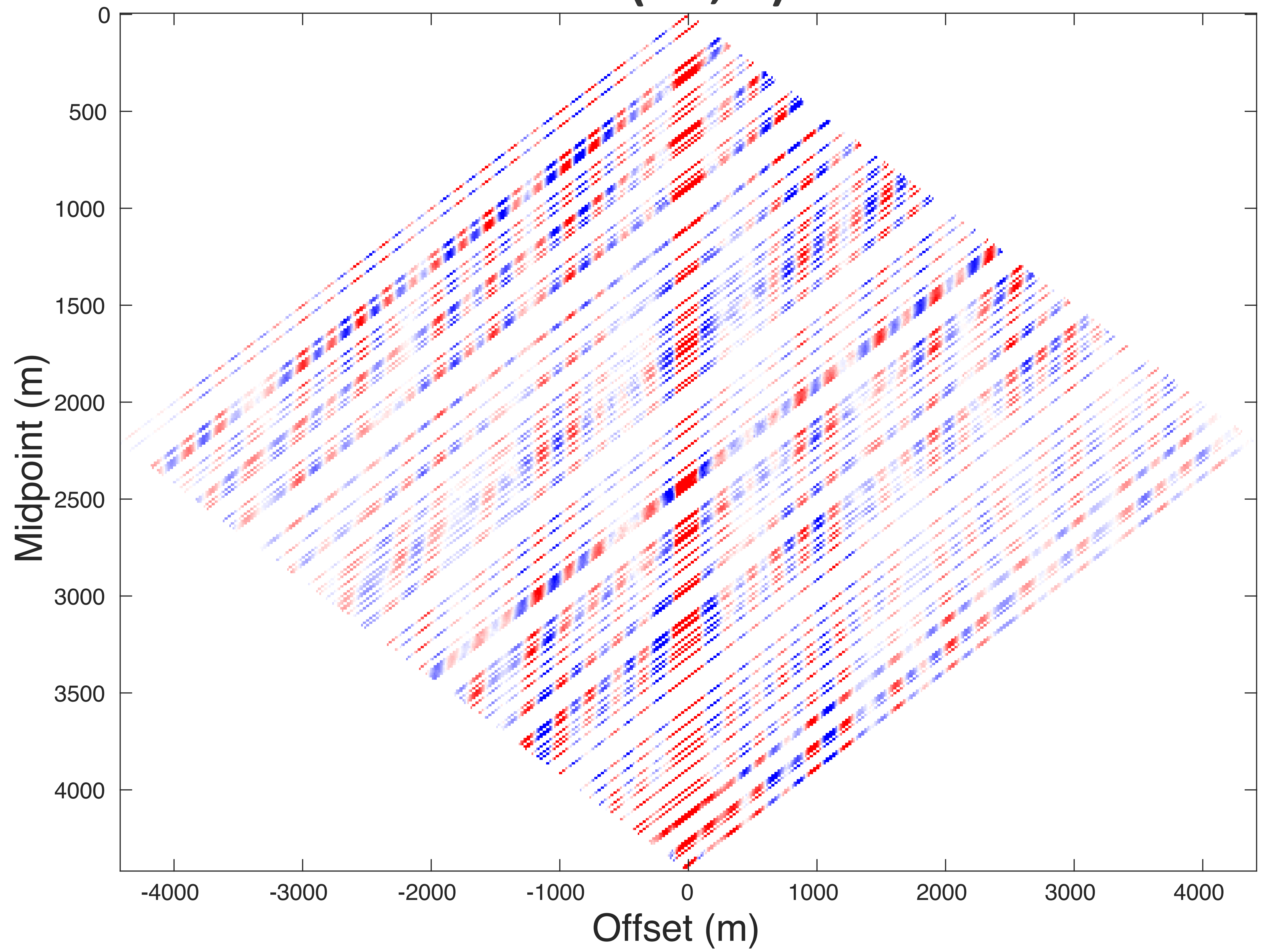
2D Seismic Subsampling

Missing Sources

Acquisition (s,r) Domain

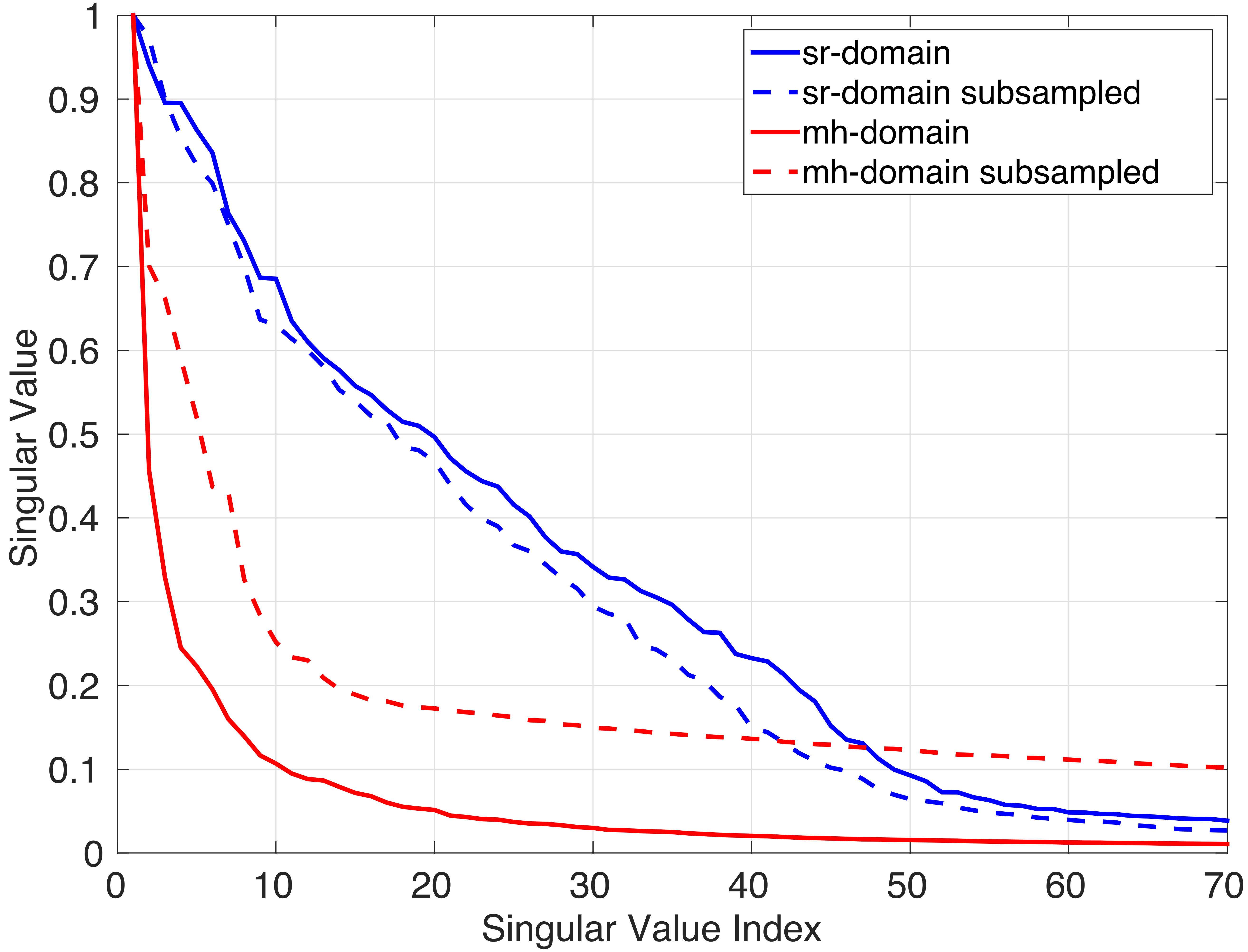


Low-Rank (m,h) Domain

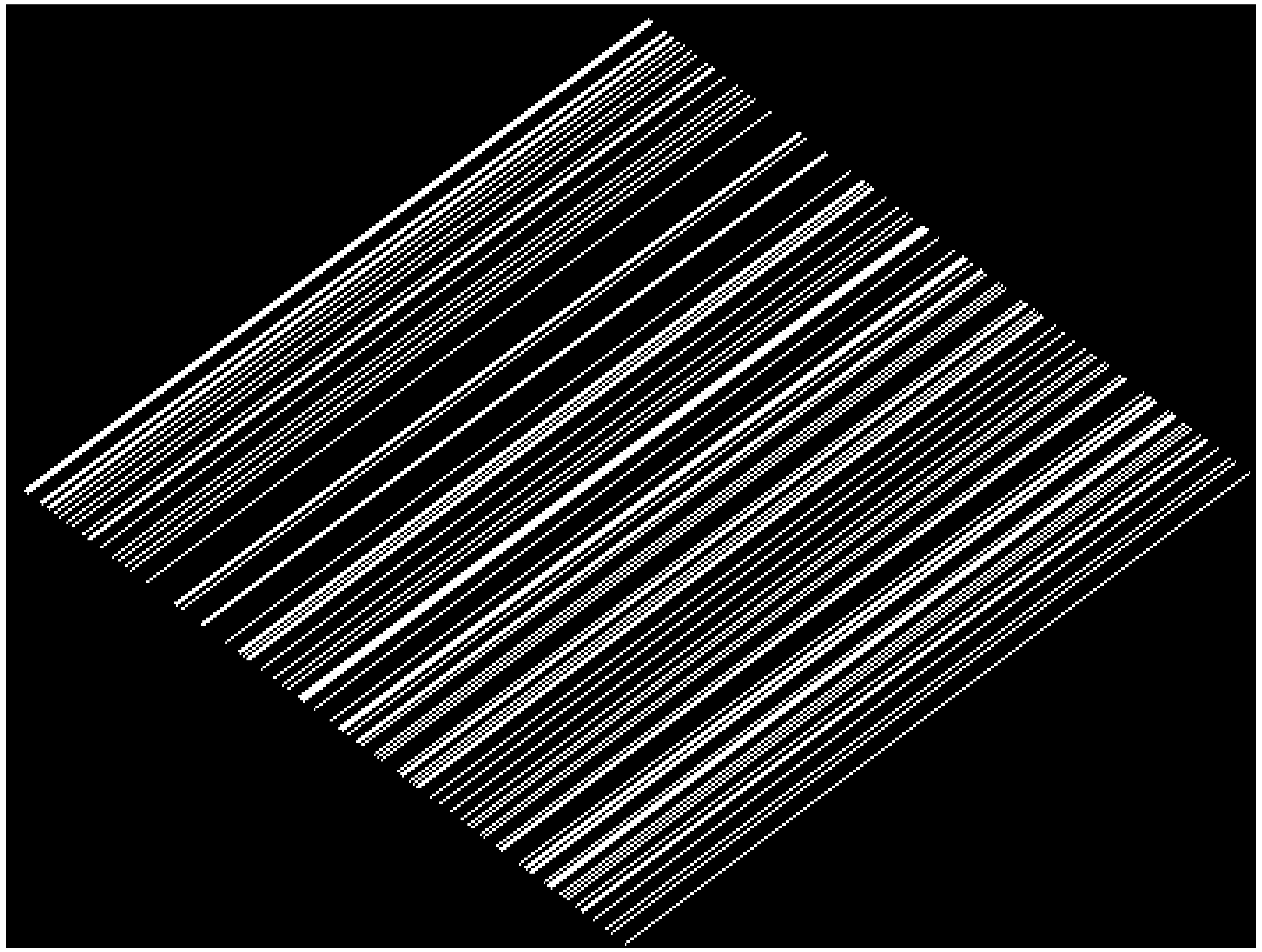


Singular value decay

full data vs subsampled data



2D Seismic Masks



3D Seismic Data Interpolation

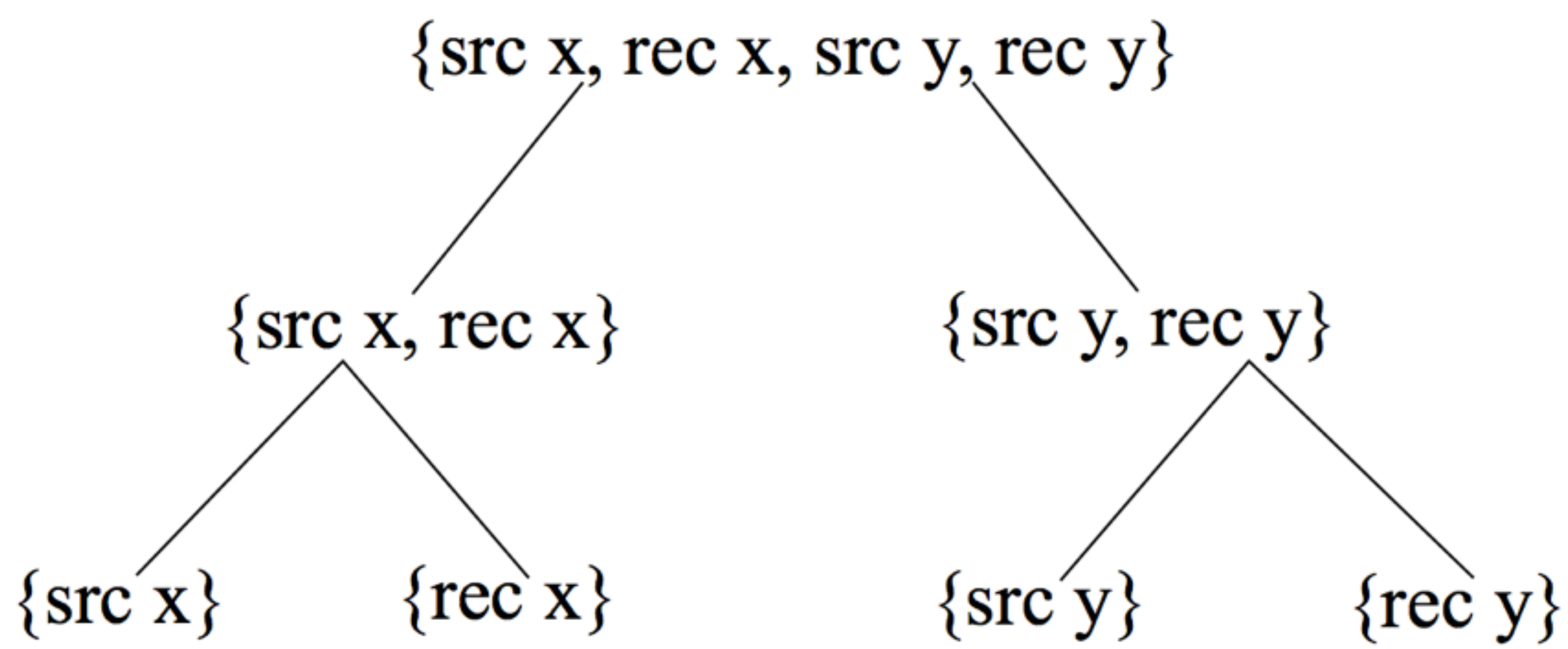
- Consider a 3D seismic survey with coordinates (src x, src y, rec x, rec y, time)

- Take a Fourier transform in time and restrict ourselves to a single frequency slice.

3D Seismic Data Interpolation

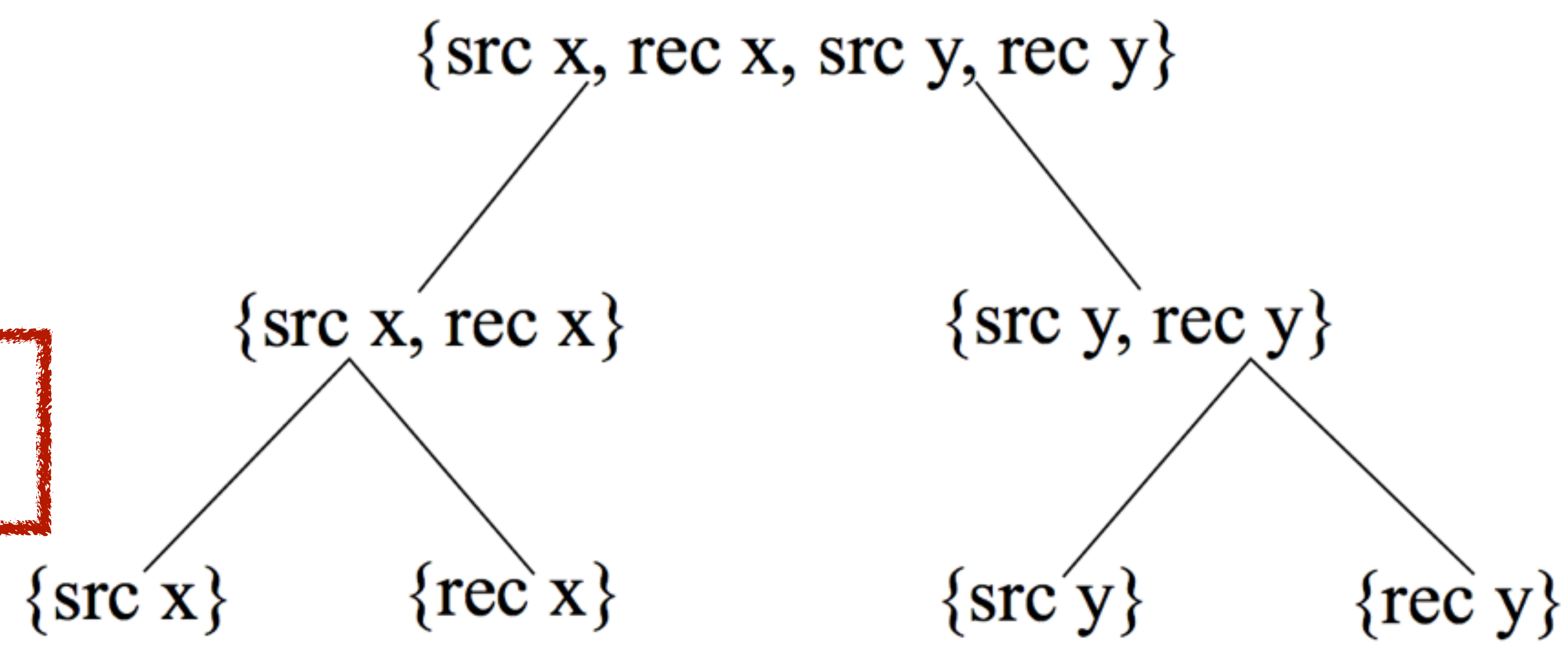
- Consider a 3D seismic survey with coordinates (src x, src y, rec x, rec y, time)
- Take a Fourier transform in time and restrict ourselves to a single frequency slice.
- Many options on how to matricize

3D Data: Matricized



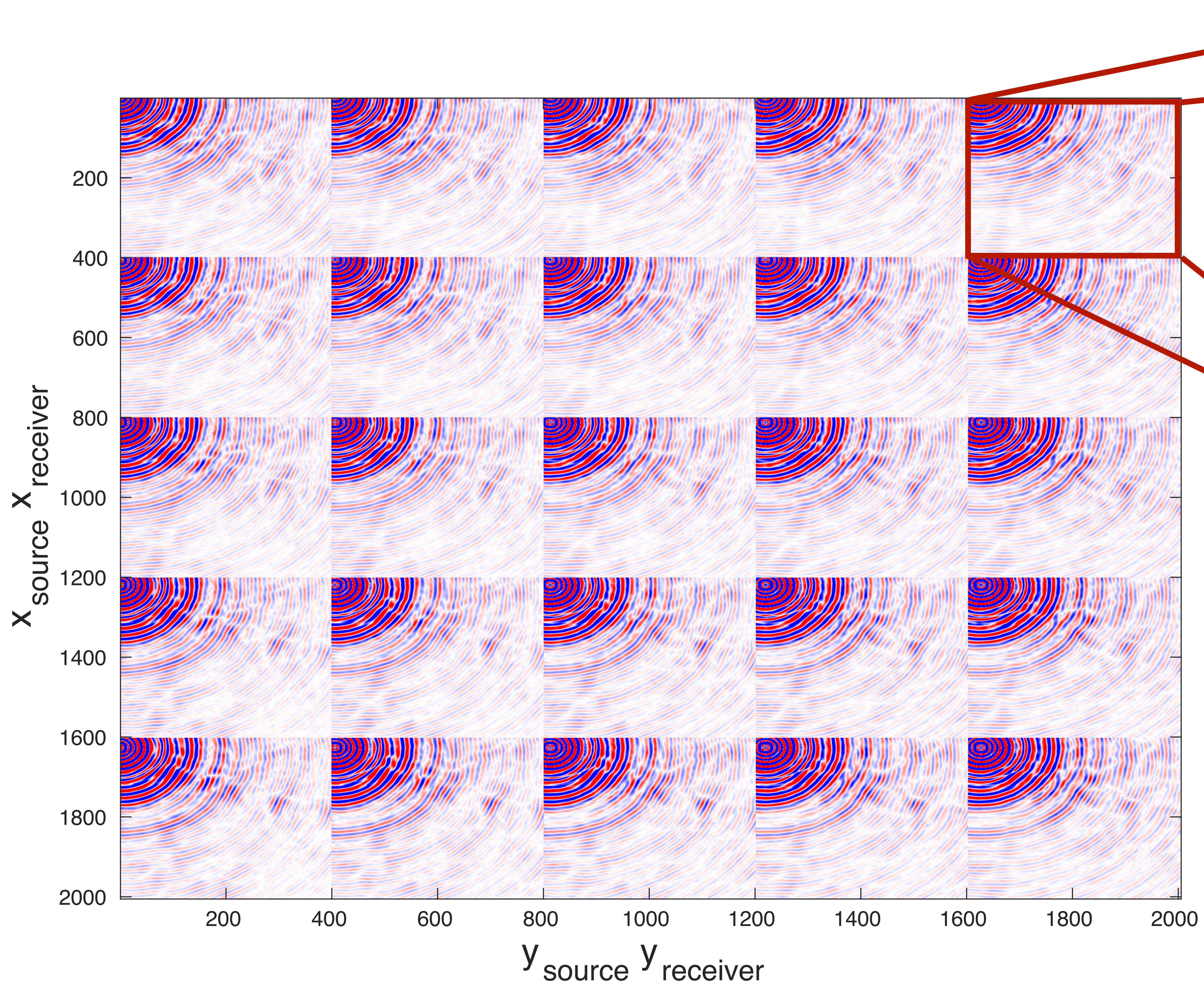
3D Data: Matricized

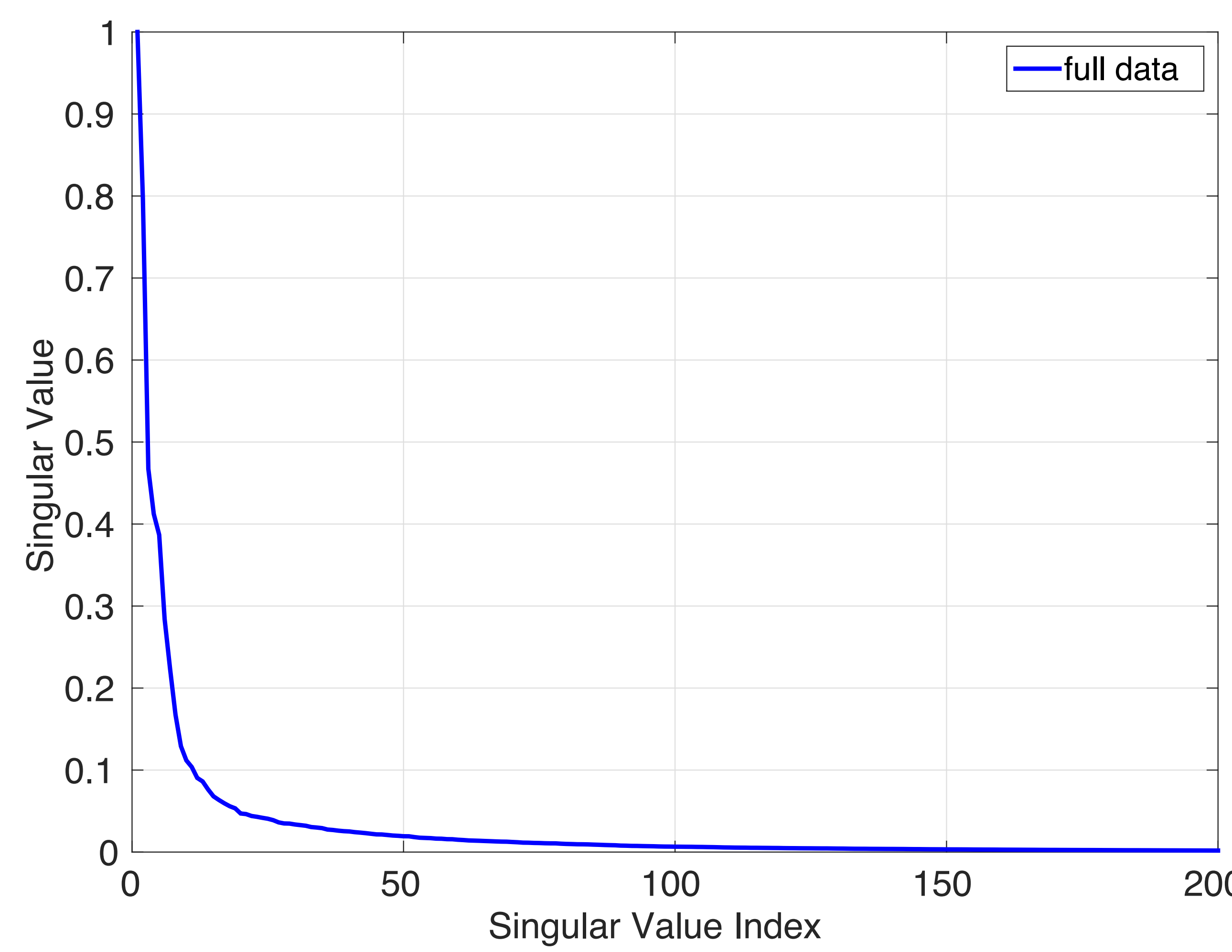
Option 1: (rec,rec) - form



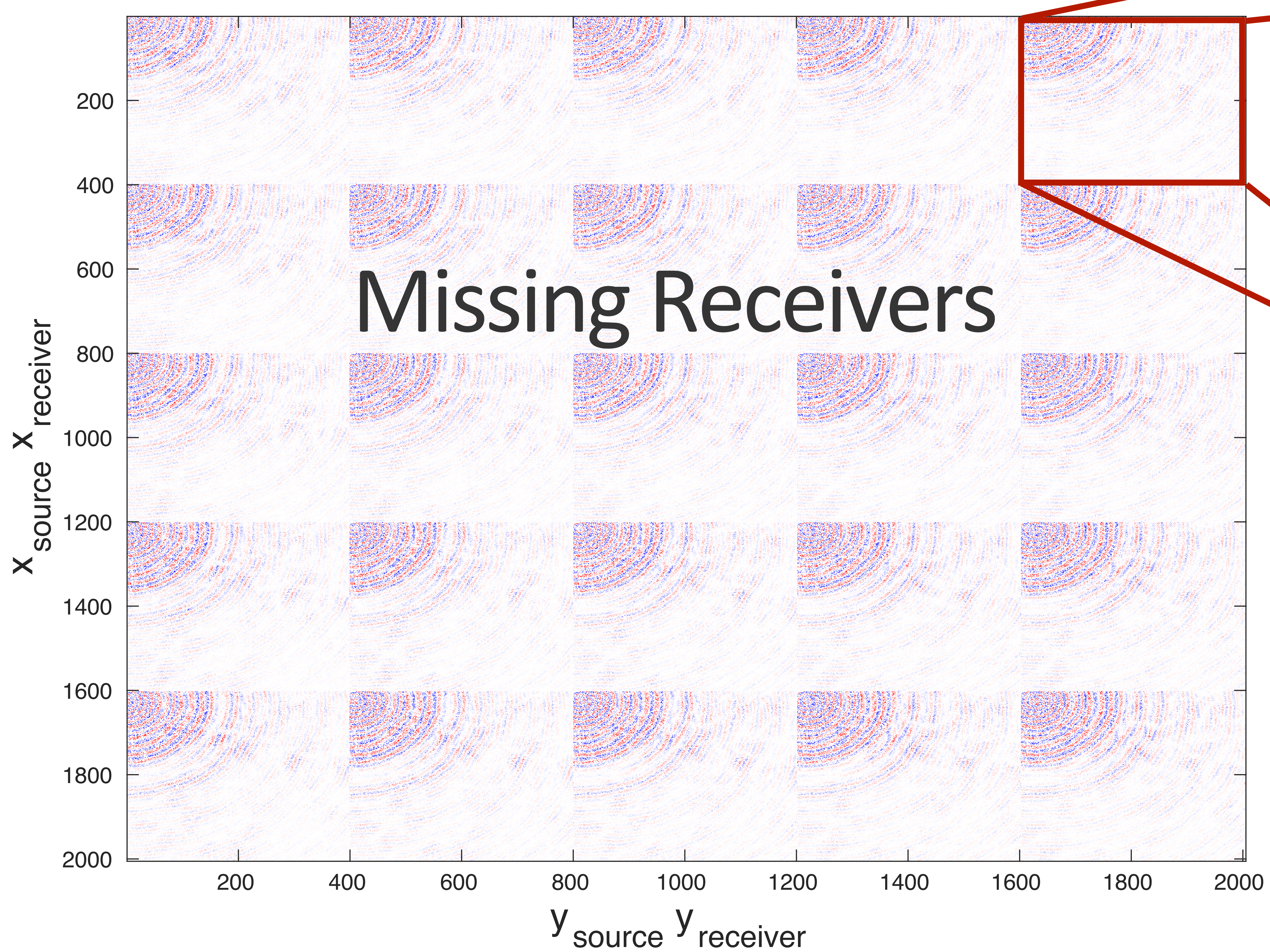
Make receiver by receiver blocks

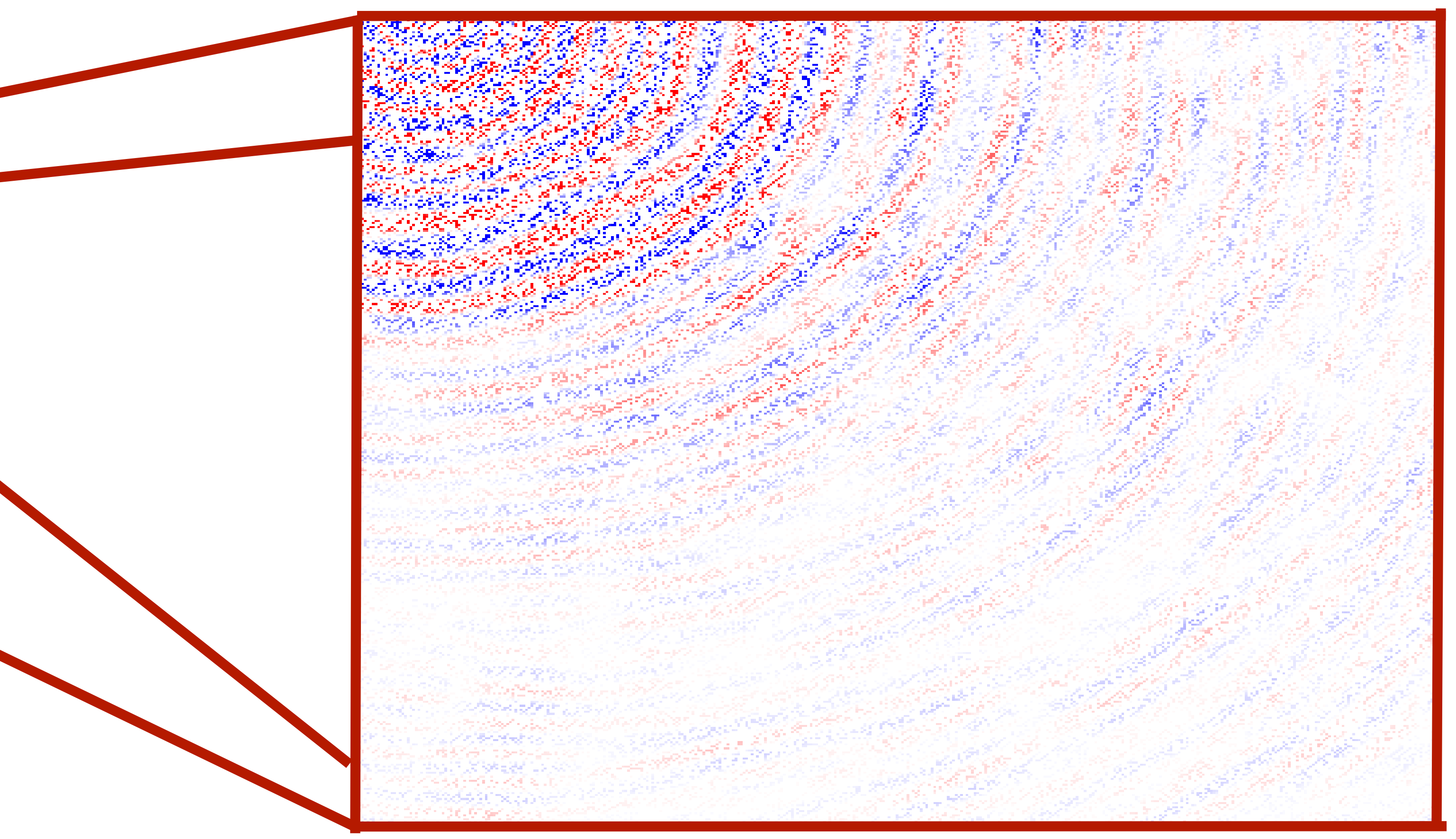
3D Data Matricized - (rec,rec) form

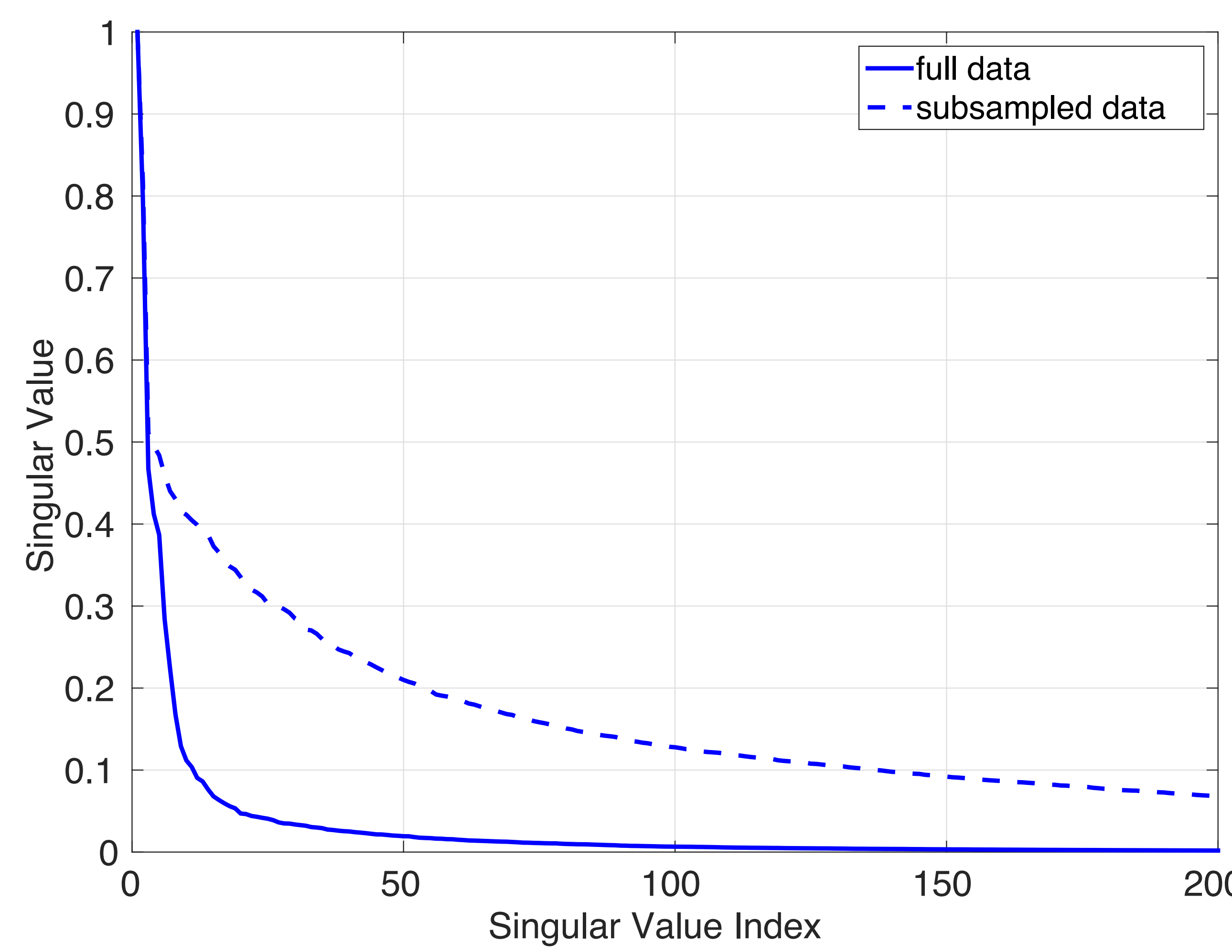




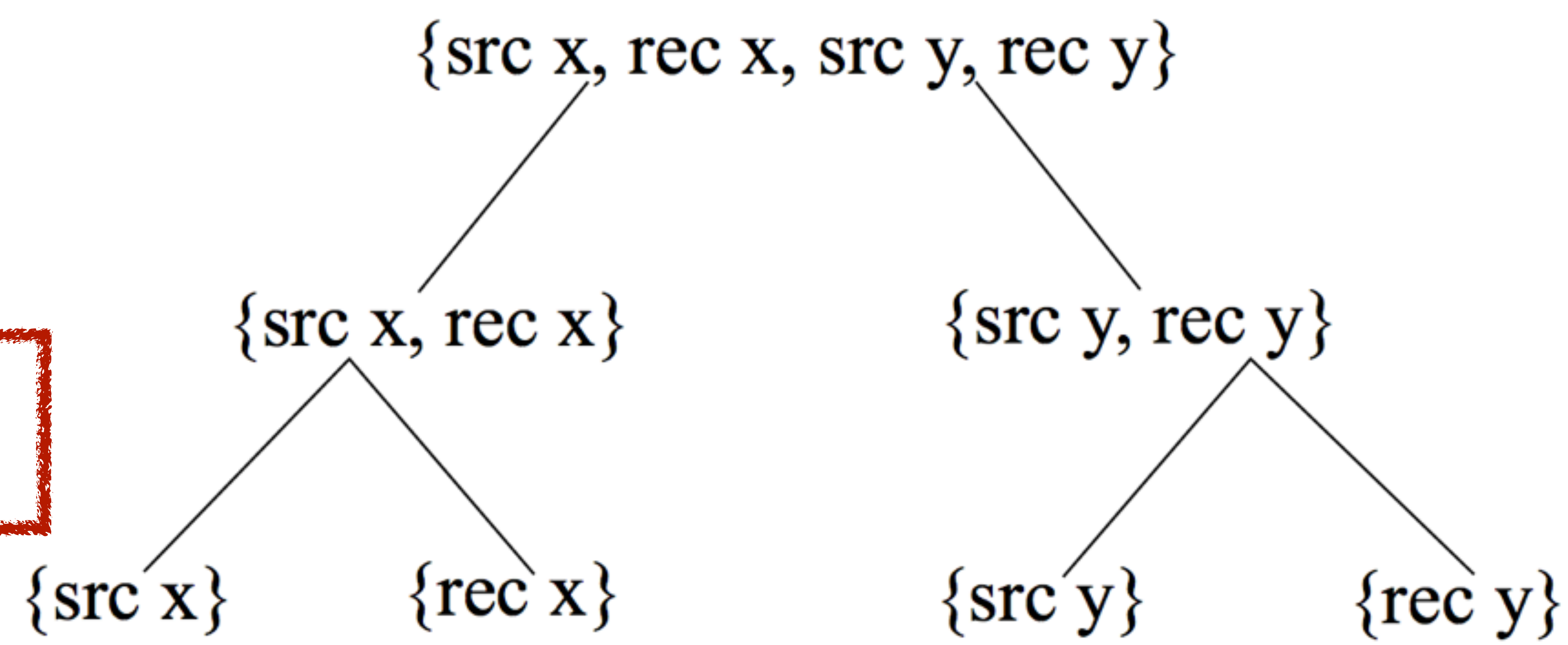
3D Data Matricized - (rec,rec) form







3D Data: Matricized



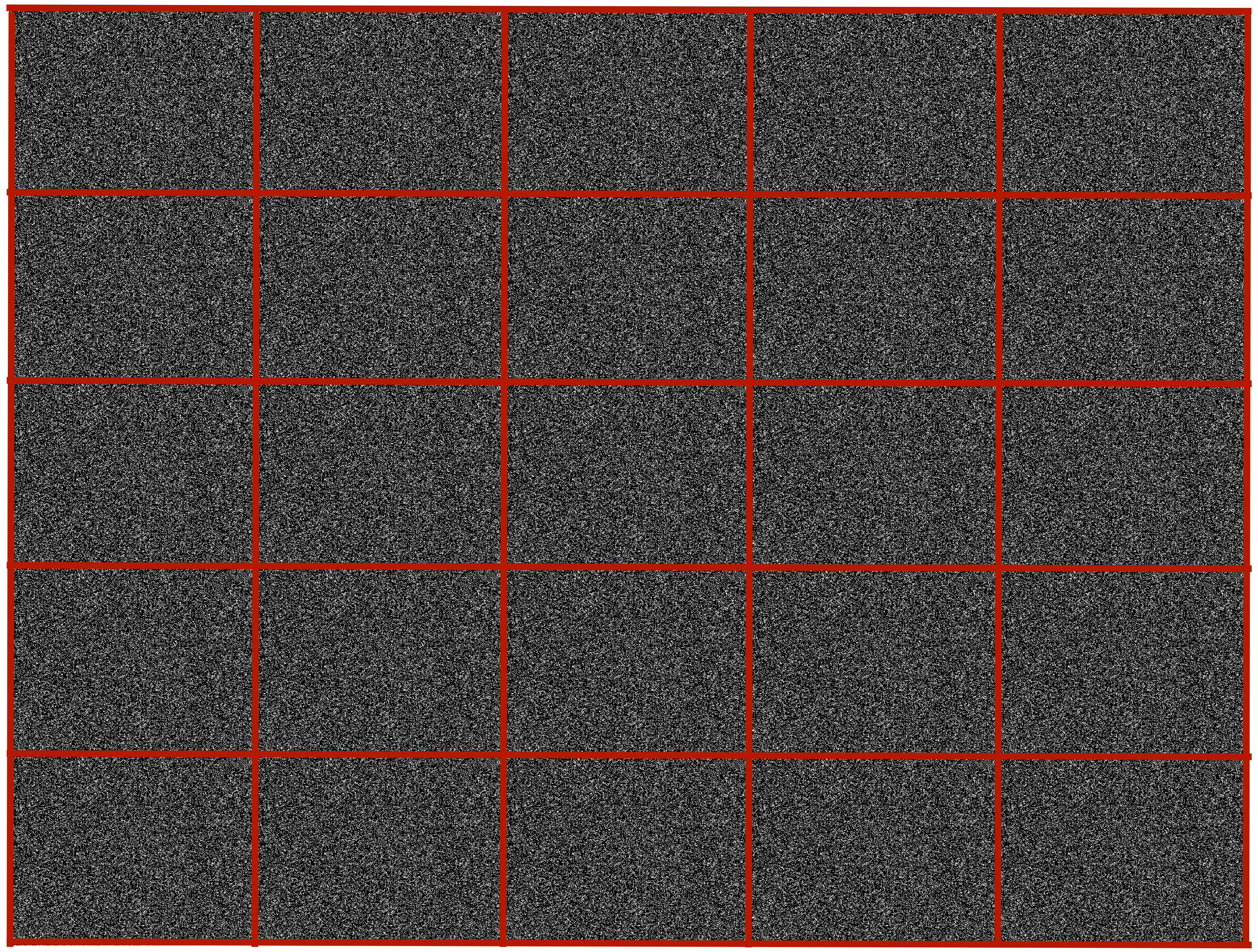
Option 2: (src,src) - form

Make source by source blocks

(similar low-rank structure as before)

3D Seismic Masks

(rec,rec) - form



Many other options on how to matricize

Outline

- Current Work
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How should we subsample?

Consider our sampling mask

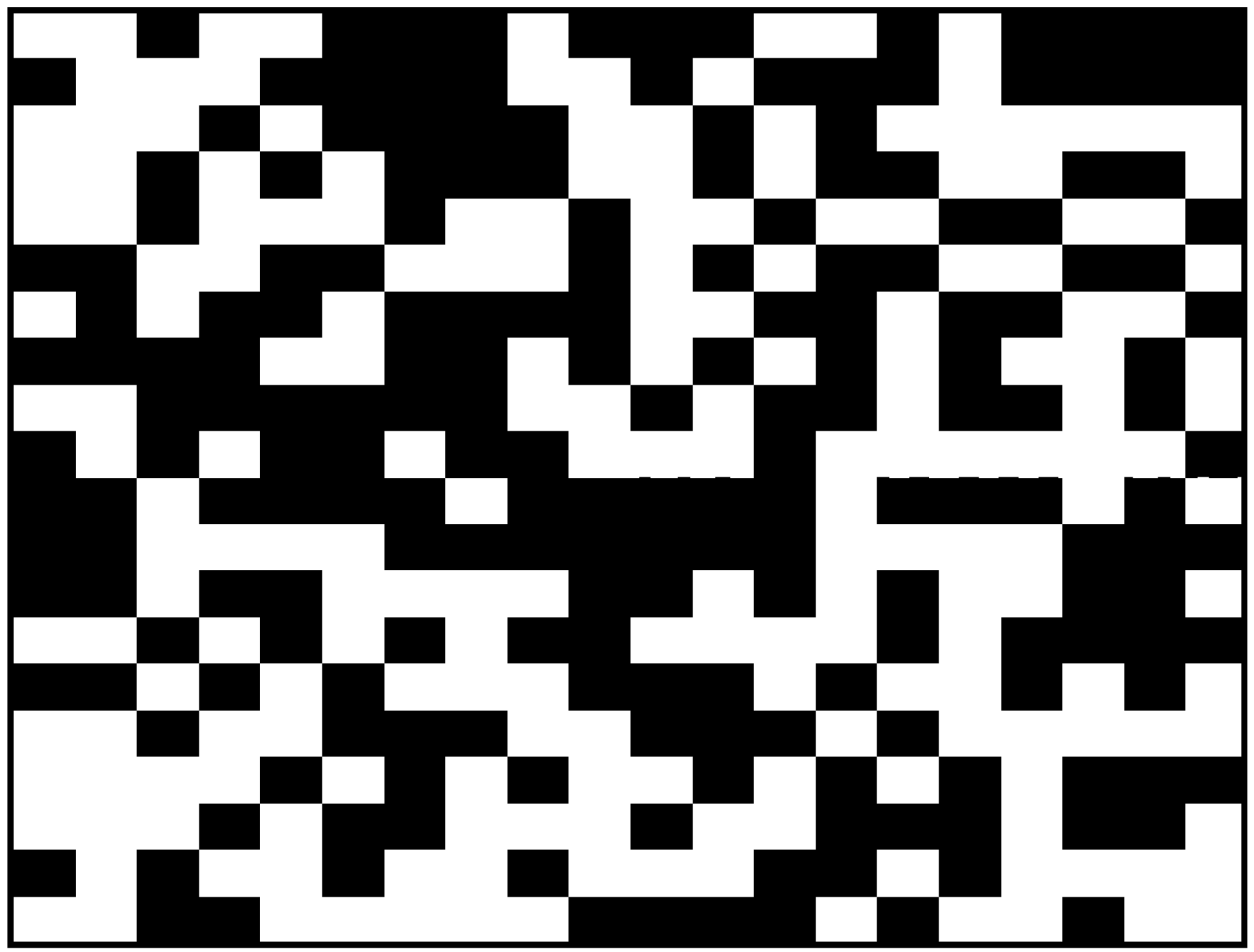
$$A_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{otherwise} \end{cases}$$

What determines if A is good for matrix completion?

Example: Ideal Mask

Samples chosen uniformly at random

$$A =$$



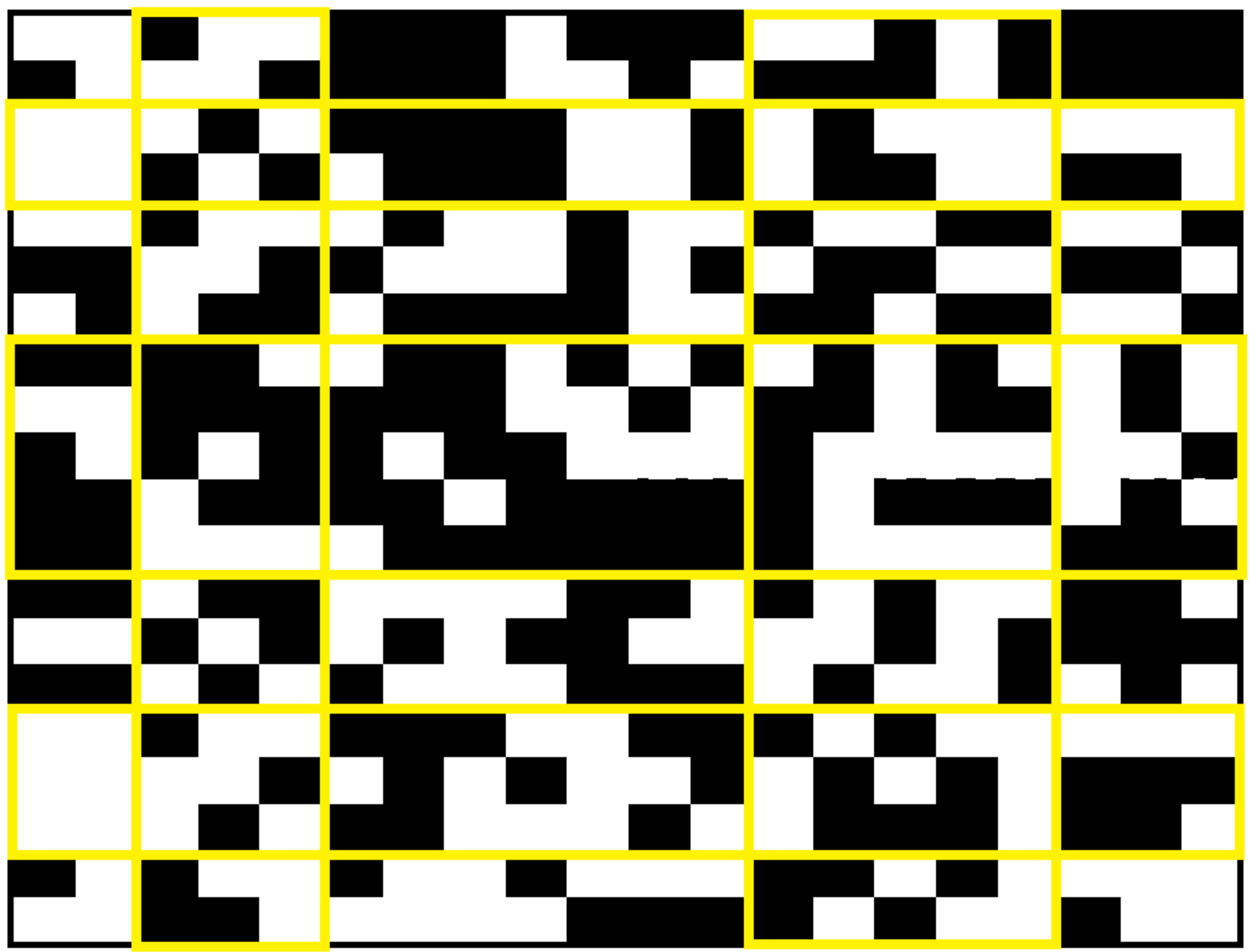
$\blacksquare = 0$

$\square = 1$

Example: Ideal Mask

Choose any sub matrix

$A =$



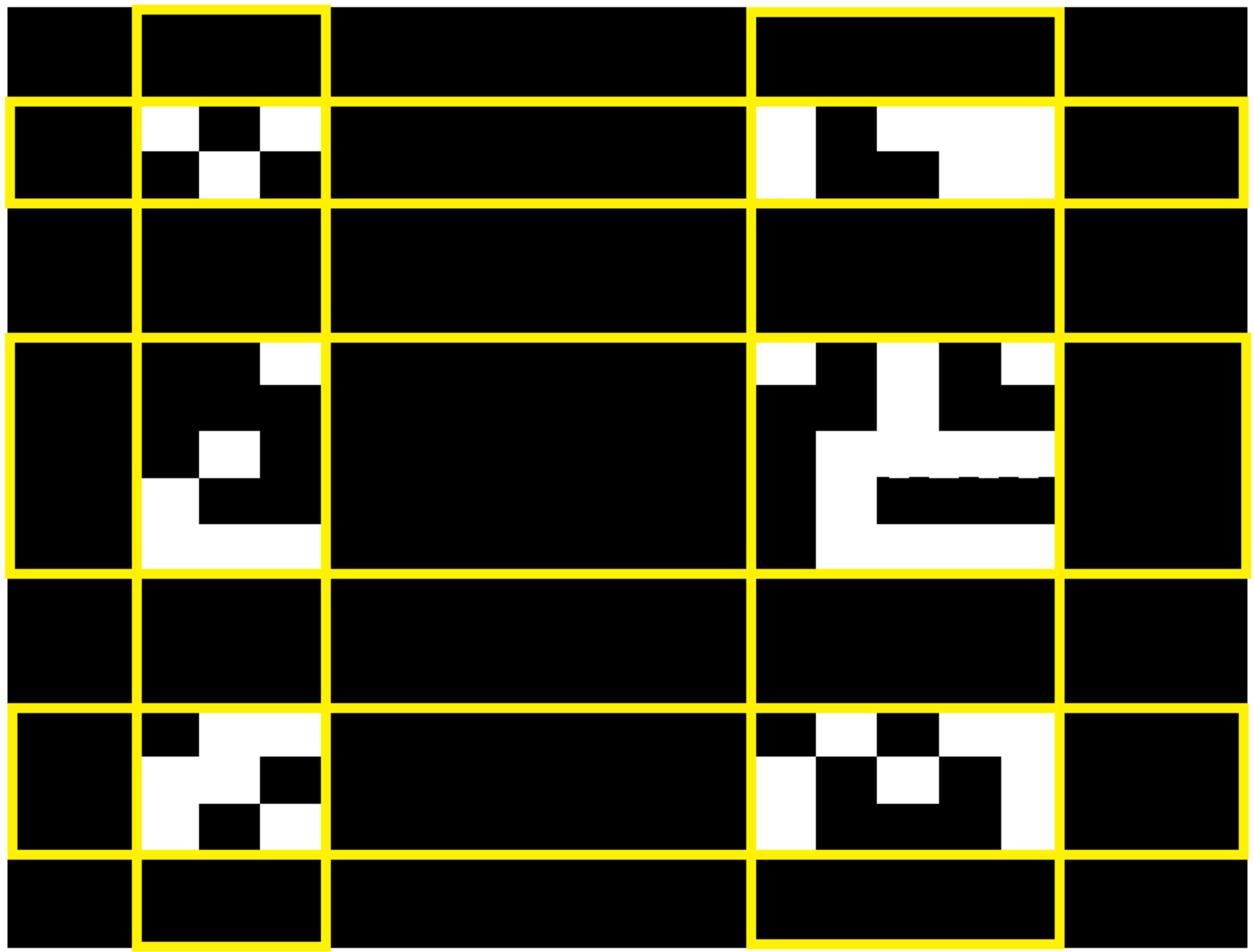
$\blacksquare = 0$

$\square = 1$

Example: Ideal Mask

All sub matrices are nicely sampled!

$$A =$$



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Bhojanapalli, Jain. "Universal Matrix Completion" ICML 2014.

Spectral Gap

Consider the gap between the two largest singular values of A

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 & \text{if small spectral gap} \\ << 1 & \text{if large spectral gap} \end{cases}$$

where σ_i is the i-th largest singular value of A

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Spectral Gap

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 \ \text{ if small spectral gap} \\ << 1 \ \text{ if large spectral gap} \end{cases}$$

From graph theory literature:

A with Large Spectral Gap $\Longrightarrow$ all “sub matrices” are nicely sampled

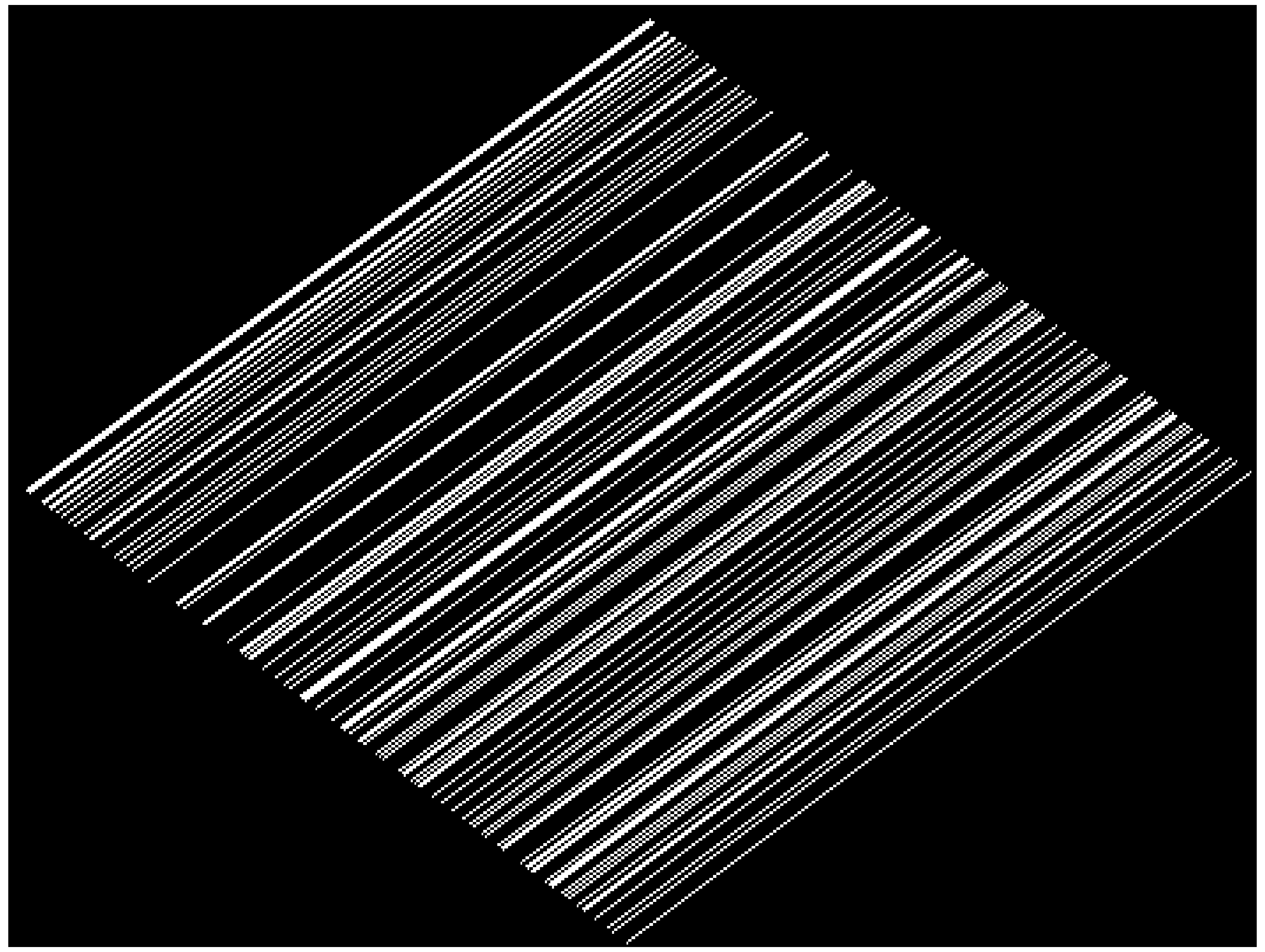
$\Longrightarrow$ better results for matrix completion

2D Interpolation Experiments

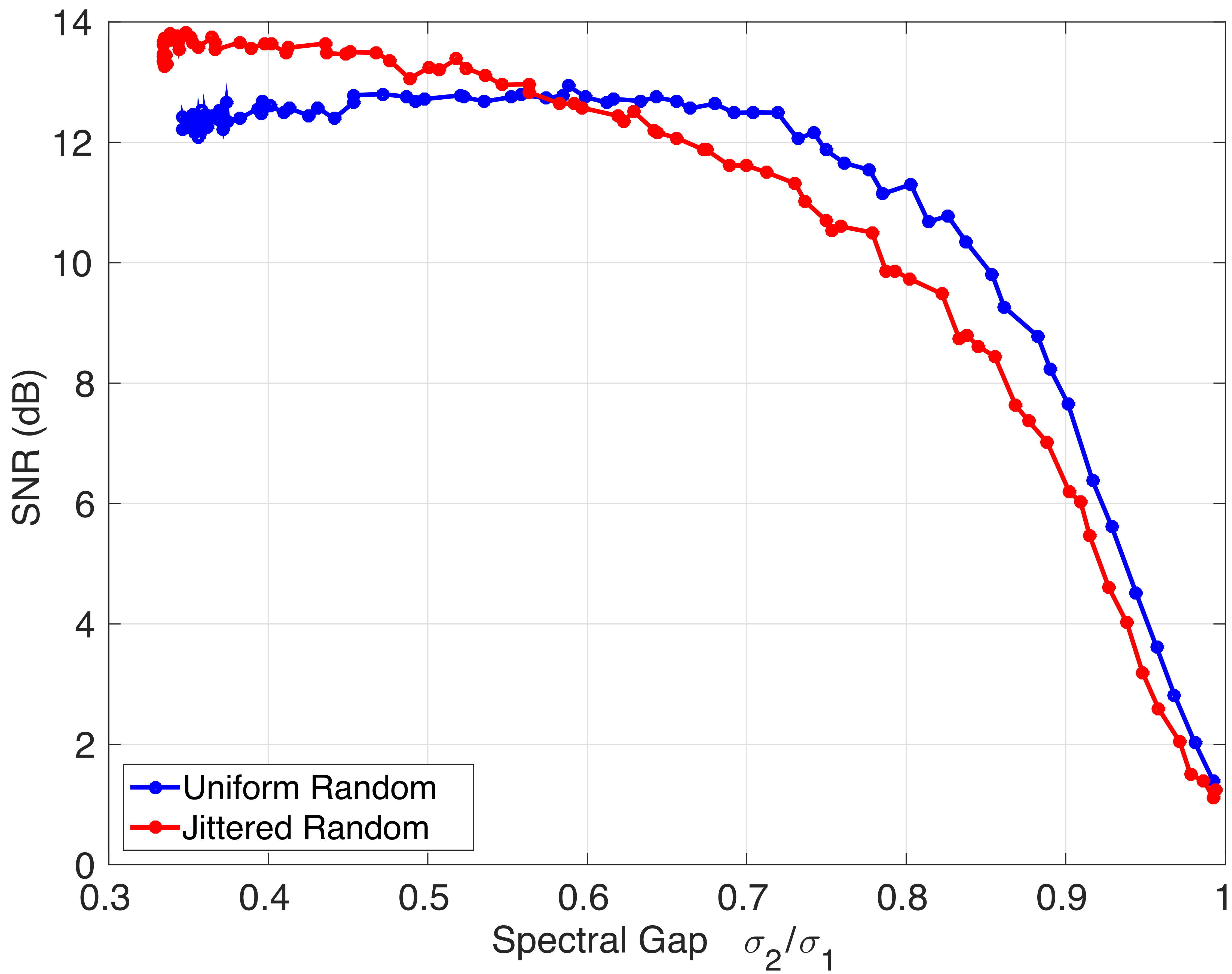
Generate 2D seismic Masks with increasing spectral gap

plot correlation with reconstruction SNR

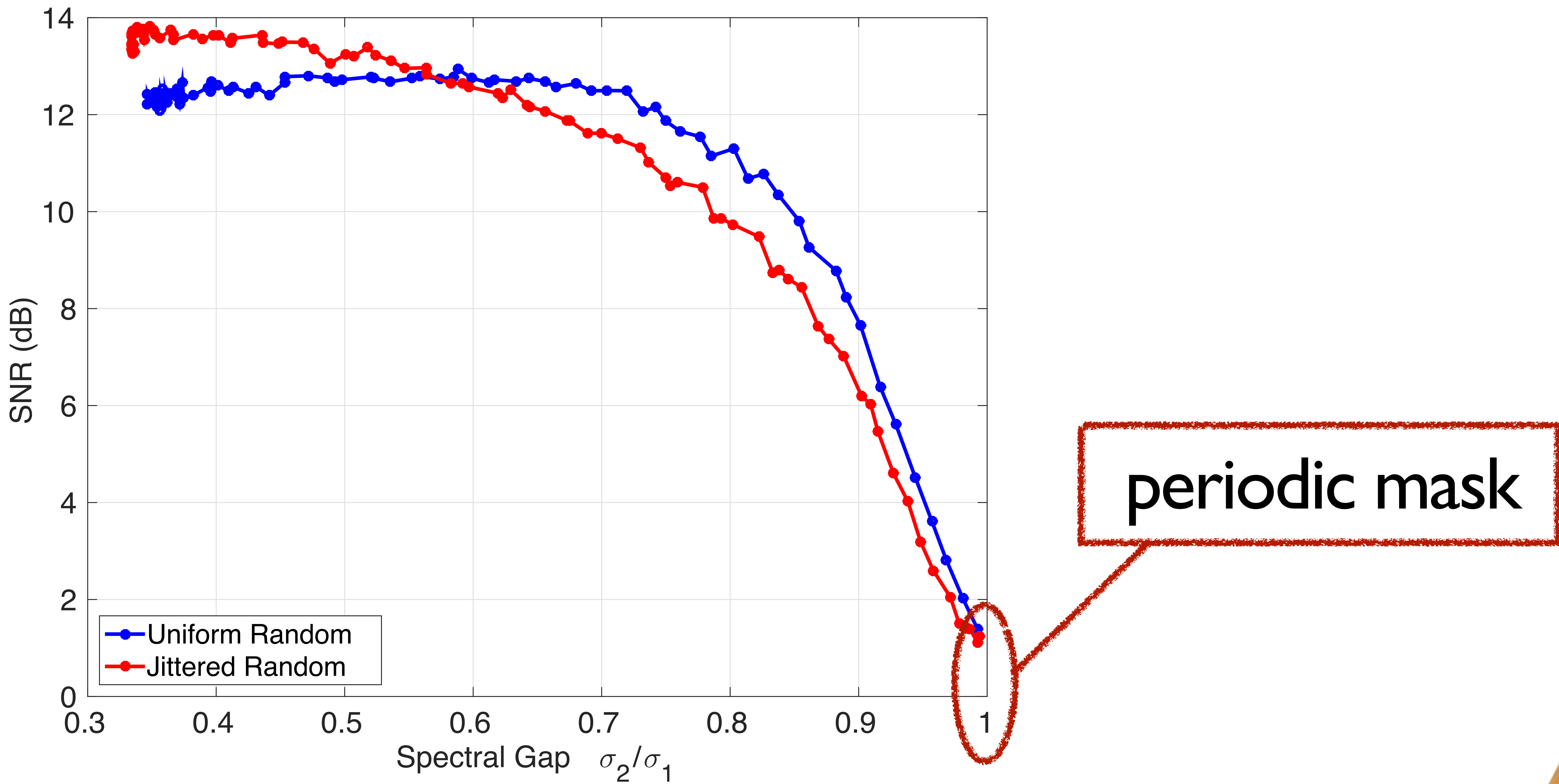
Uniform random vs Jittered random



2D Interpolation Experiments: 75% missing sources



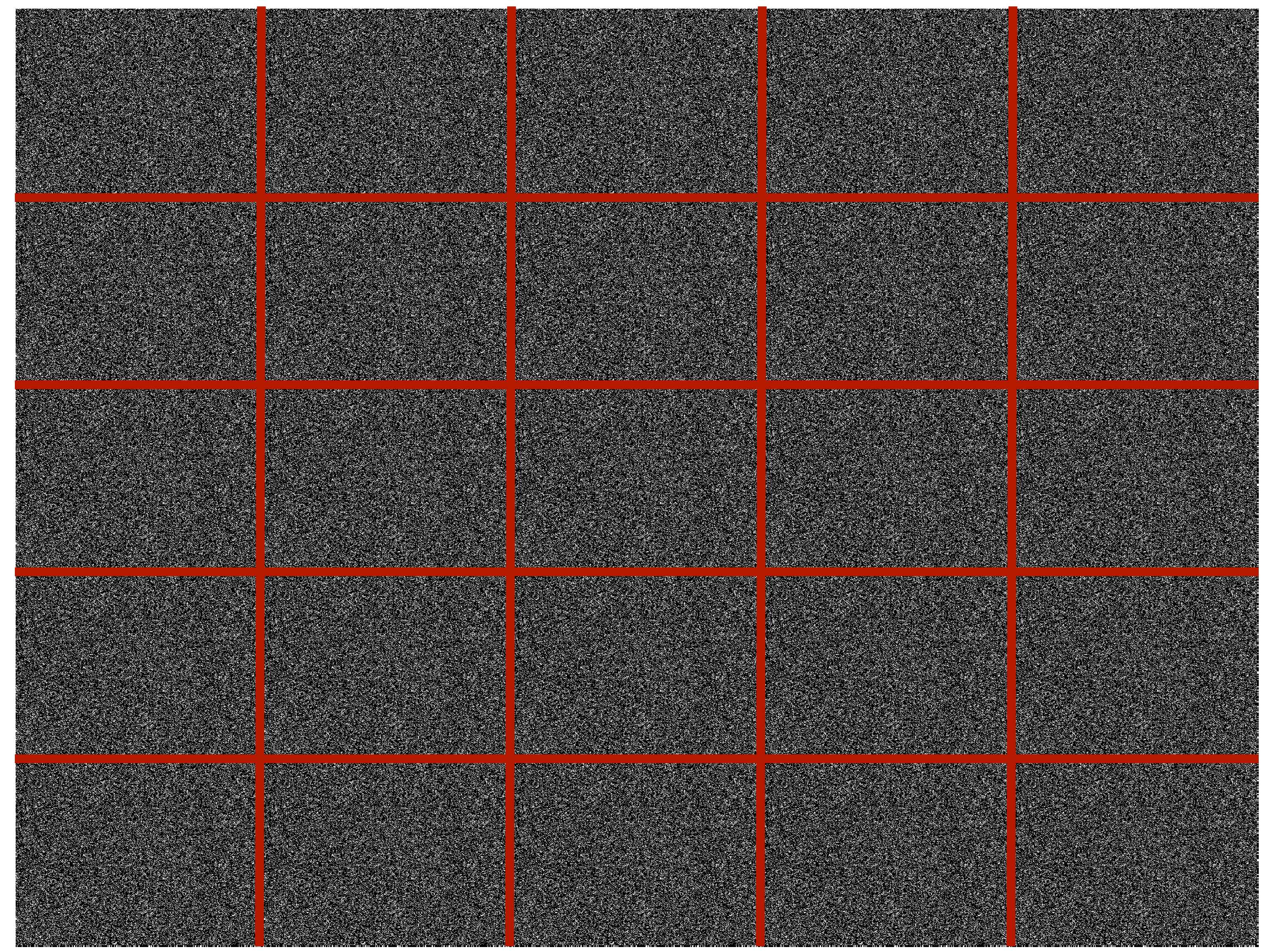
2D Interpolation Experiments: 75% missing sources



3D Interpolation Experiments

Generate 3D seismic Masks with increasing spectral gap.

Plot correlation with reconstruction SNR.



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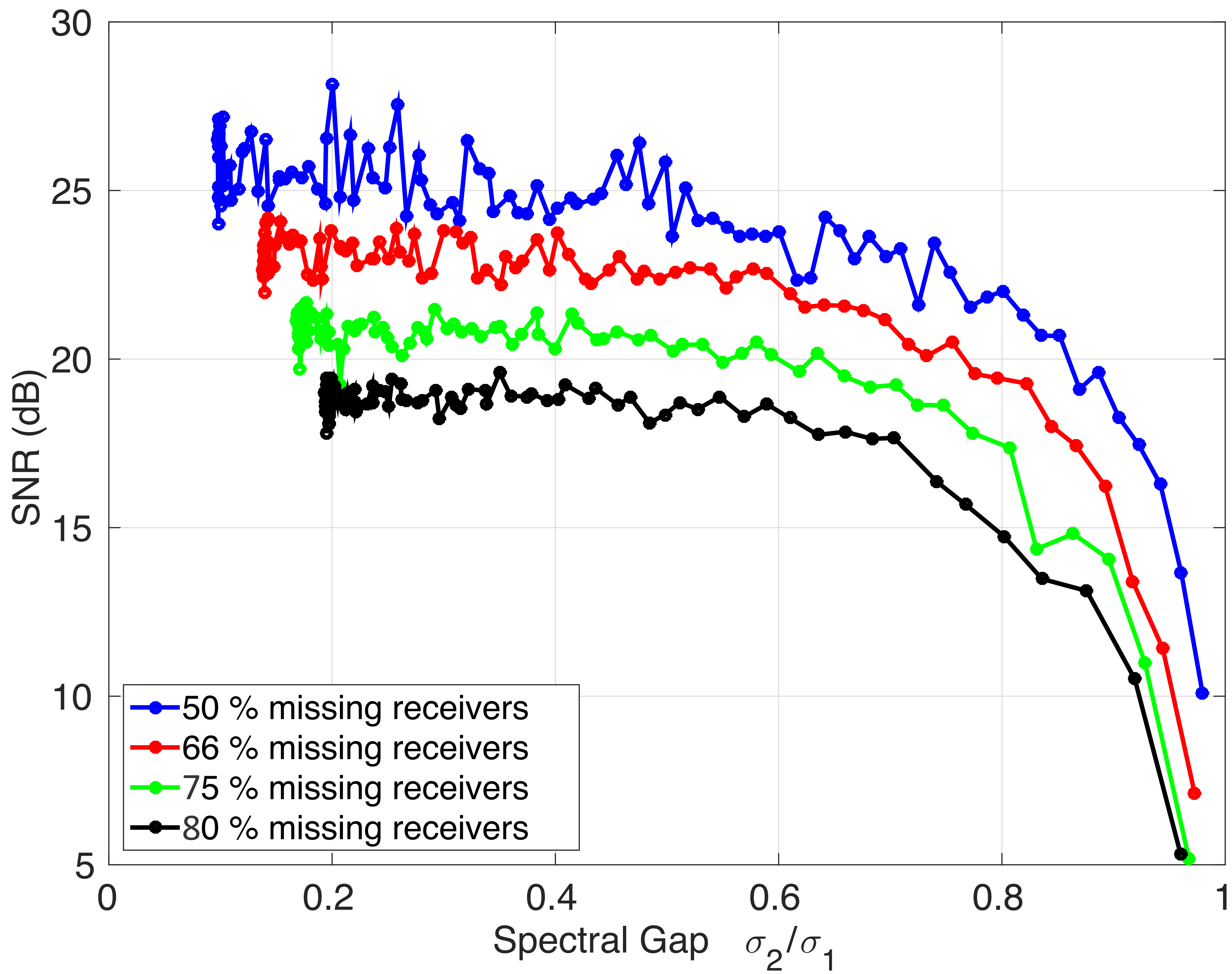
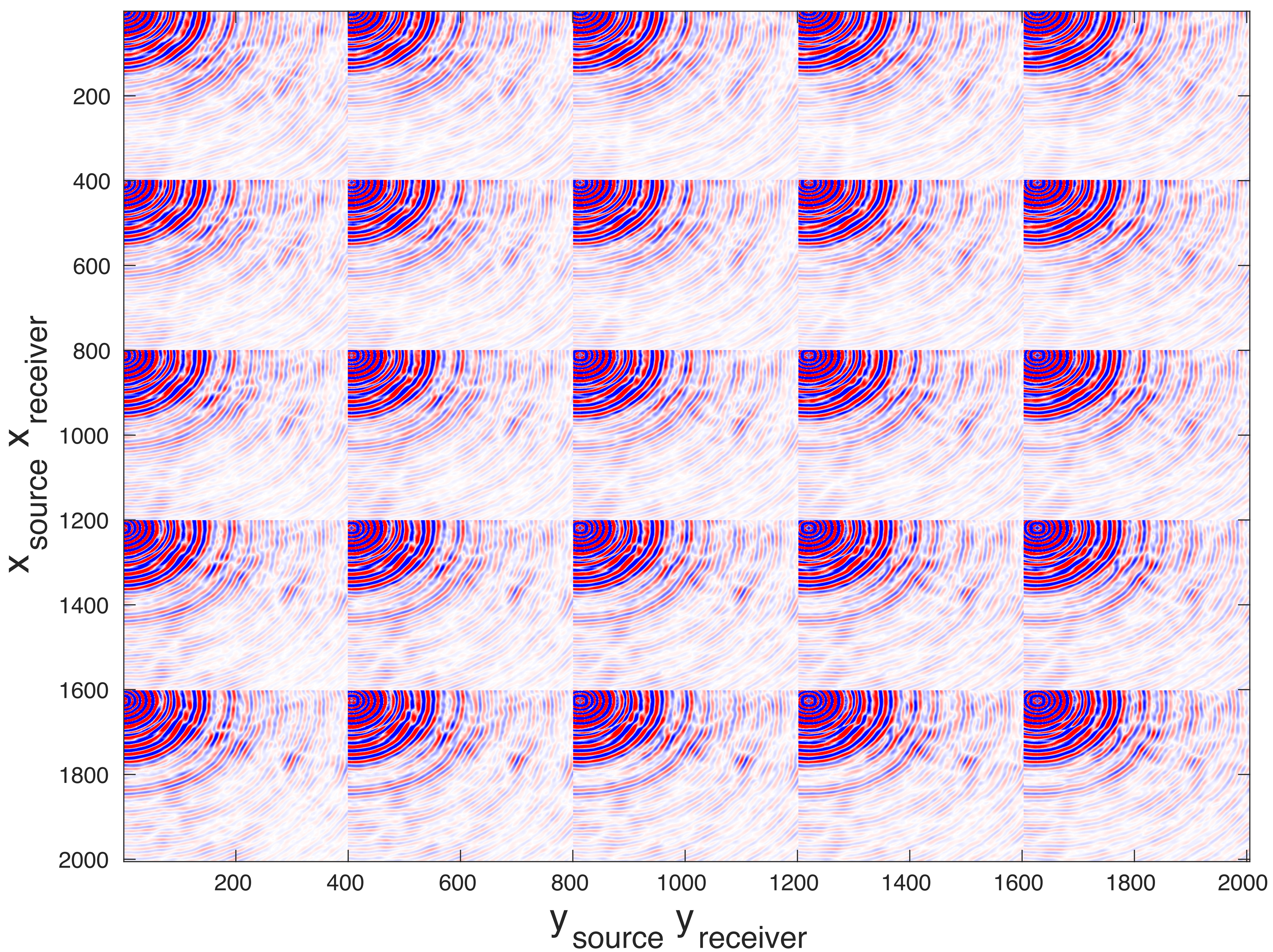


3D Interpolation Example

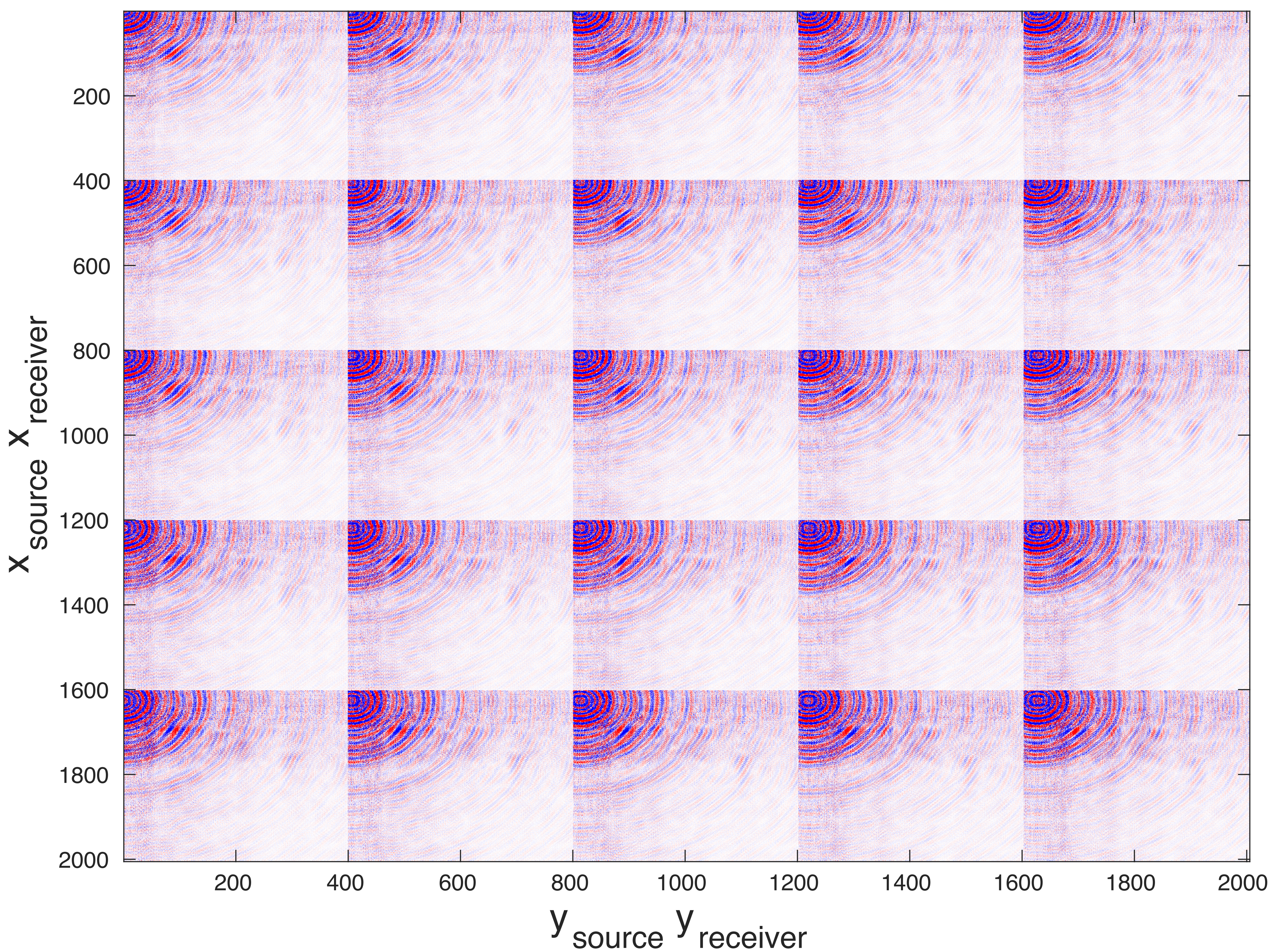


Fully Sampled Data

Size: 2005 x 2005

Remove 75 % of Receivers

3D Interpolation Example: Bad Recovery

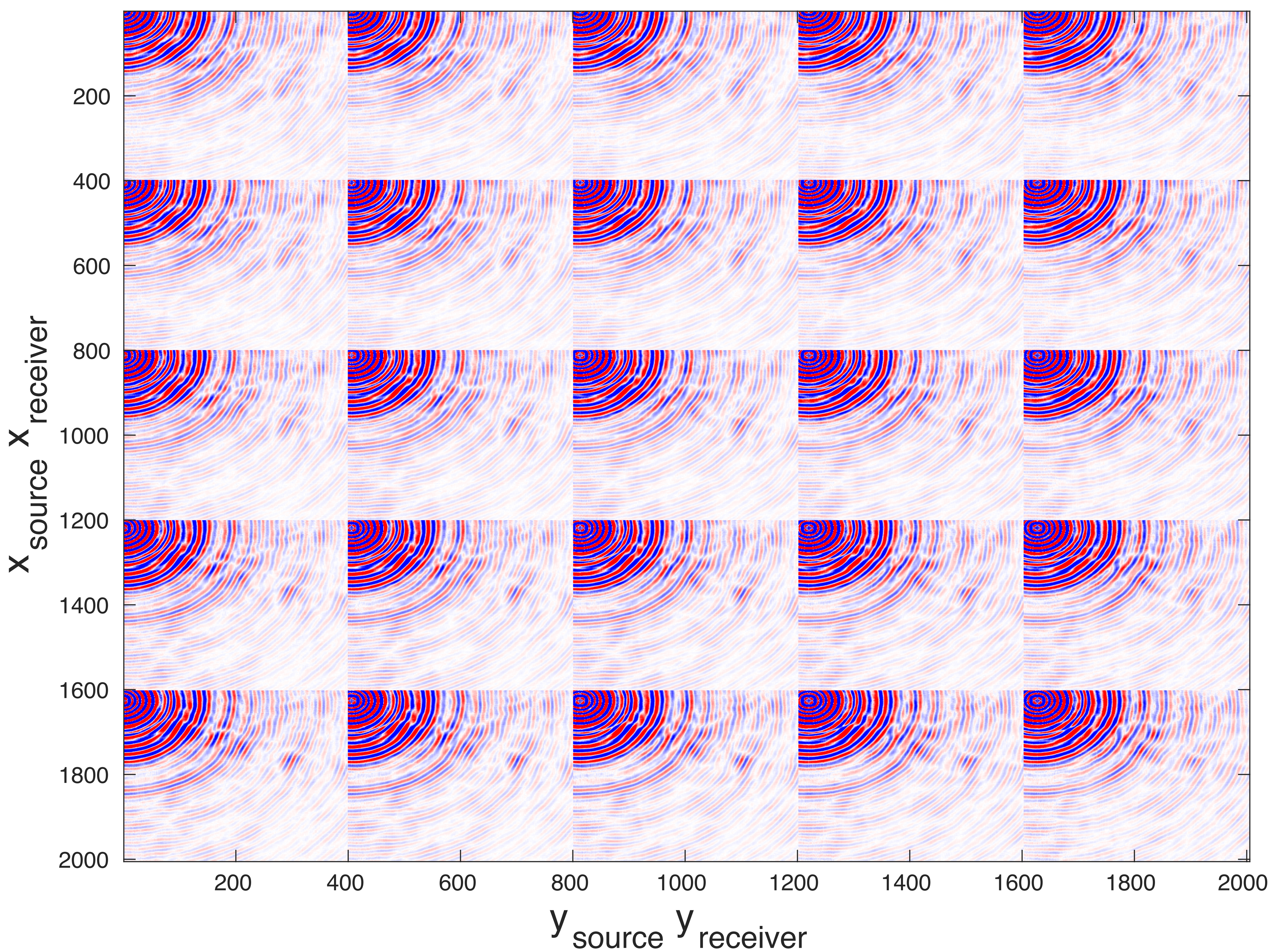


Reconstruction

SNR: 3.5 dB

$\frac{\sigma_2}{\sigma_1}$: .9828

3D Interpolation Example: Good Recovery



Reconstruction

SNR: 20.7 dB

$\frac{\sigma_2}{\sigma_1}$: .1796

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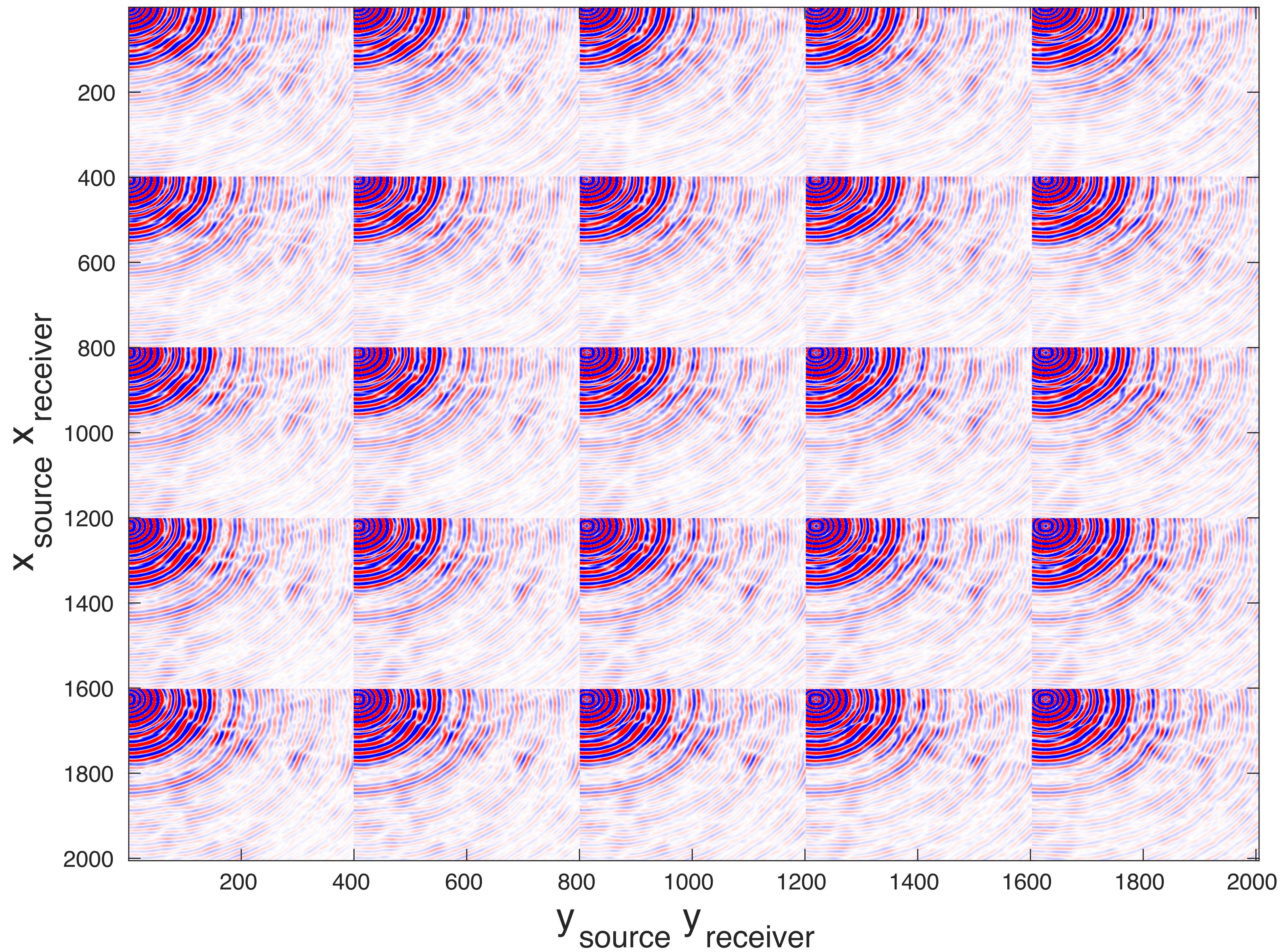
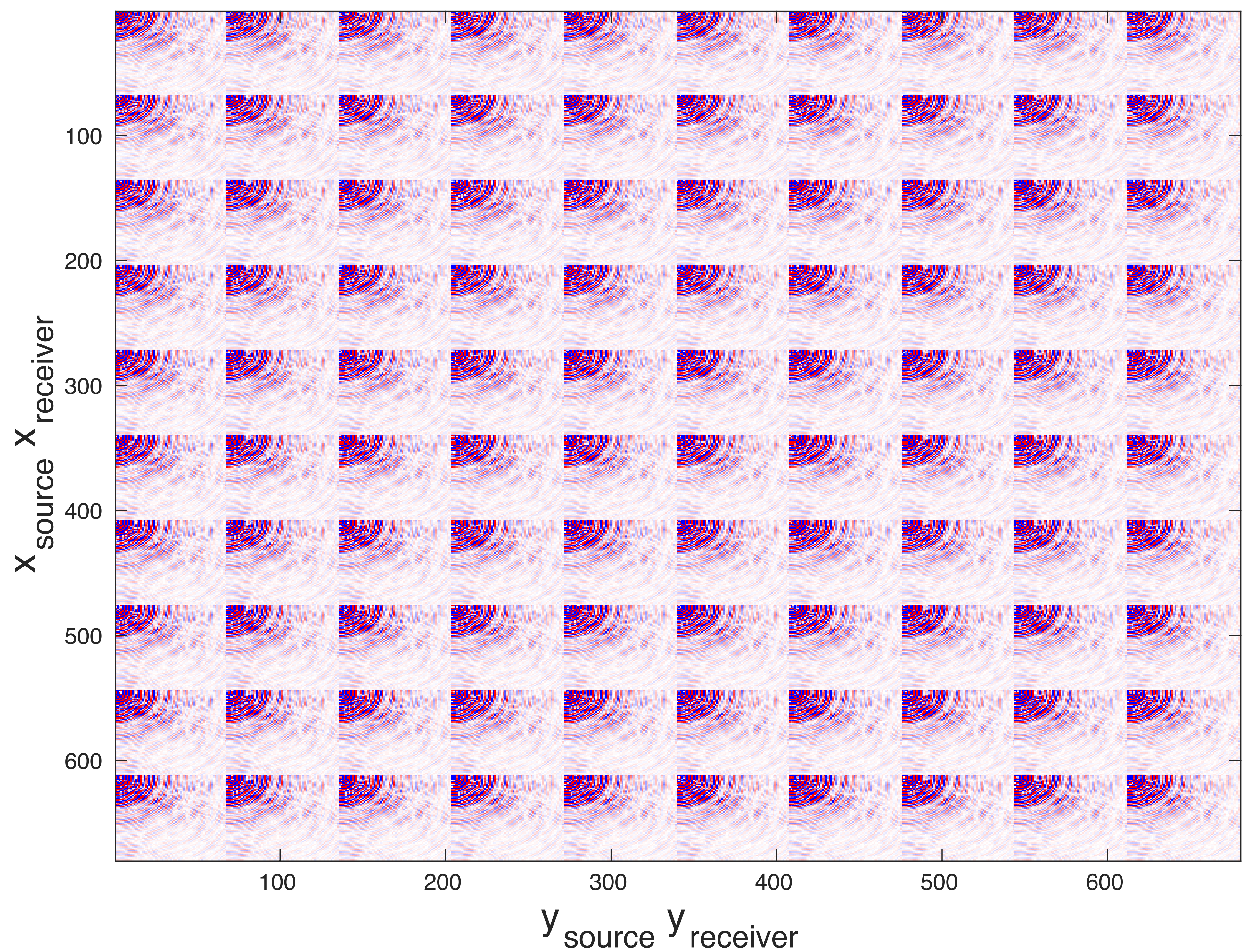
How to Matricize?



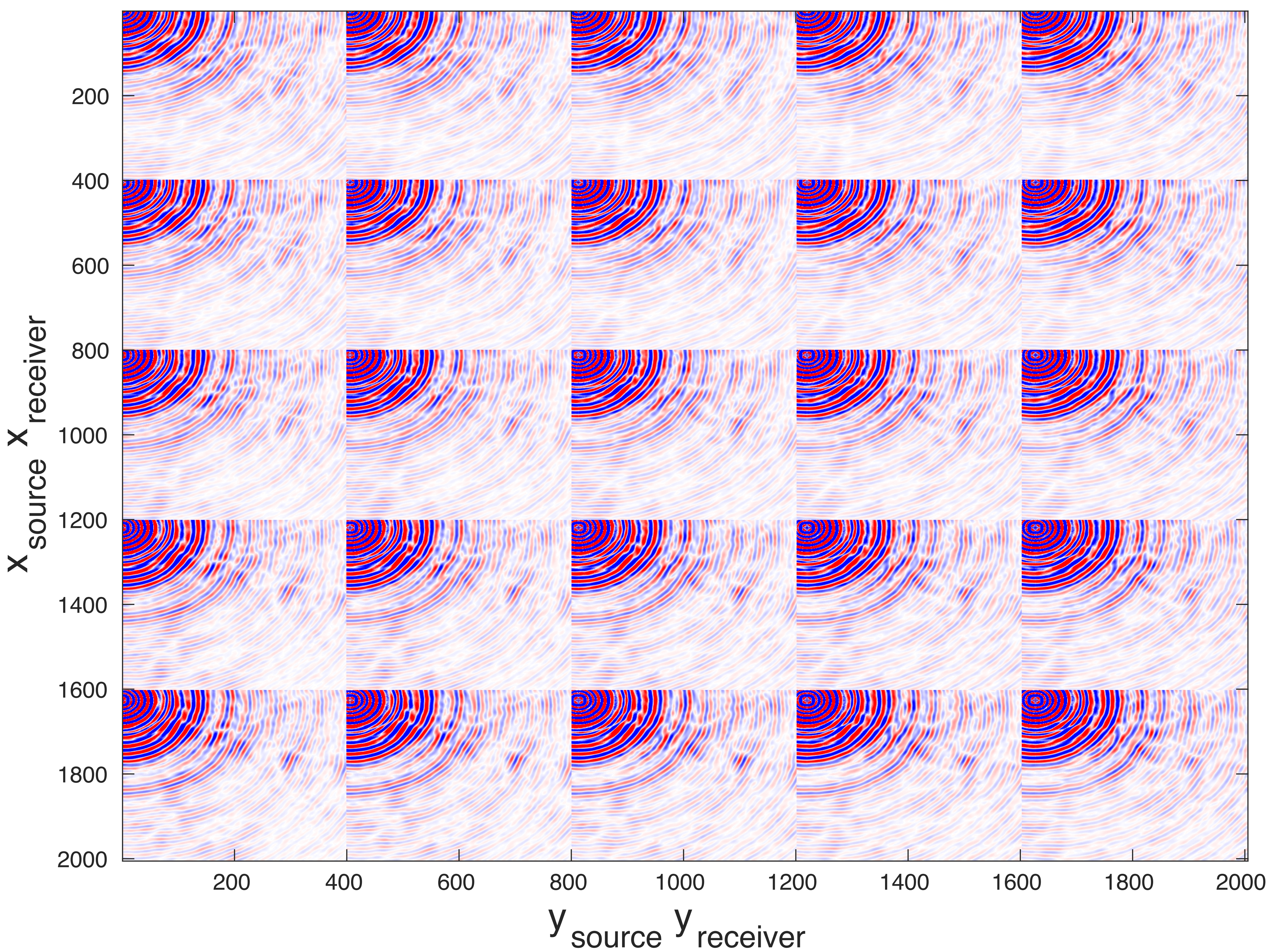
receiver by receiver blocks

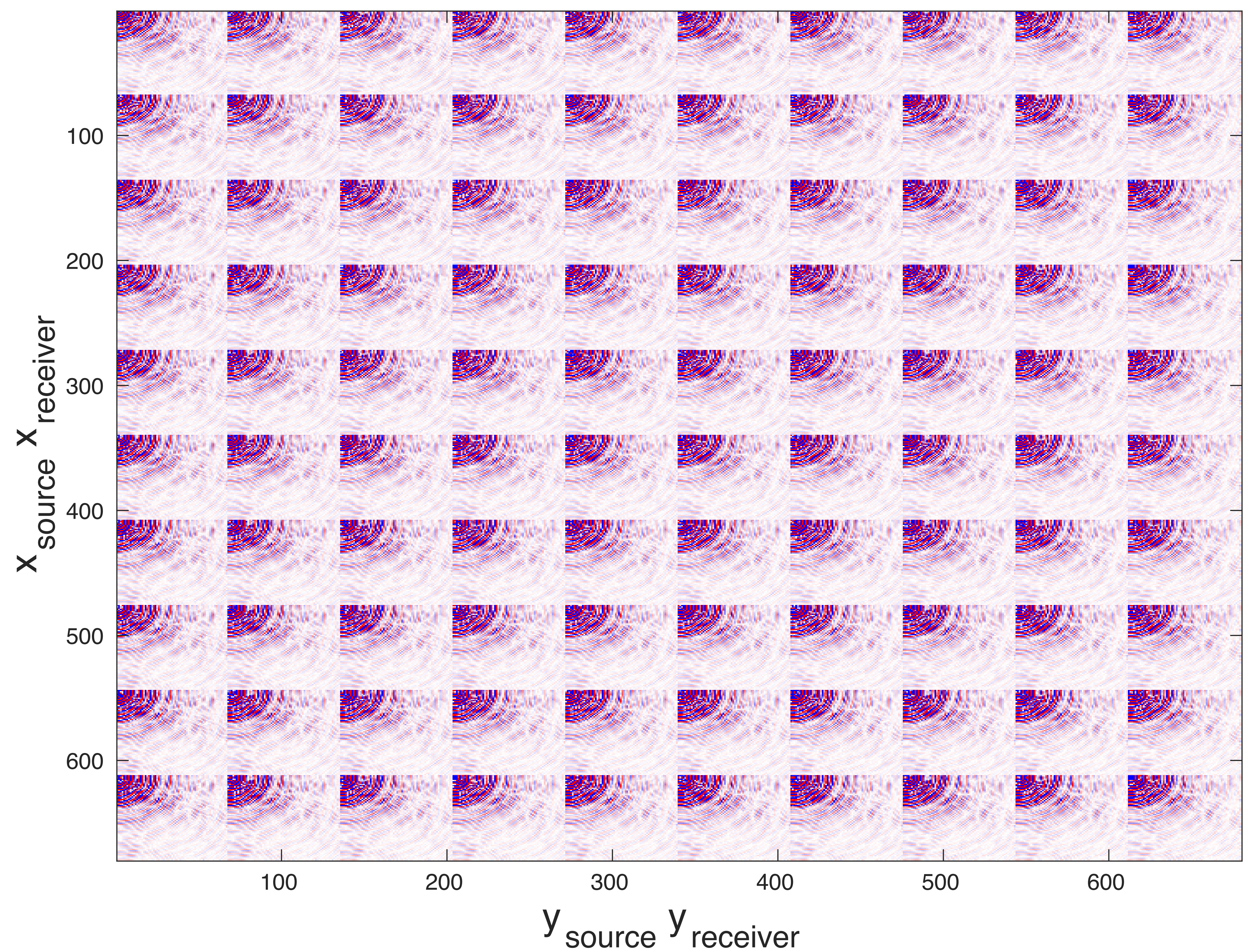
How to Matricize?



source by source blocks

How to Matricize?

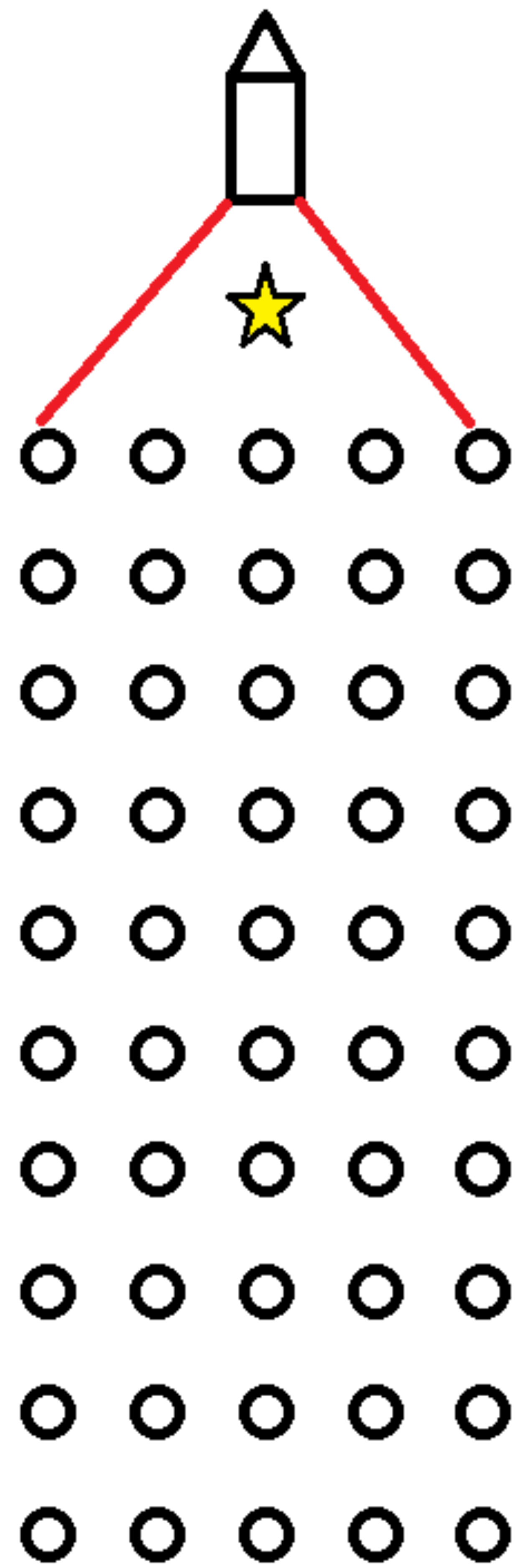




Use spectral gap to decide which matricization works best for given subsampling

Infill Management

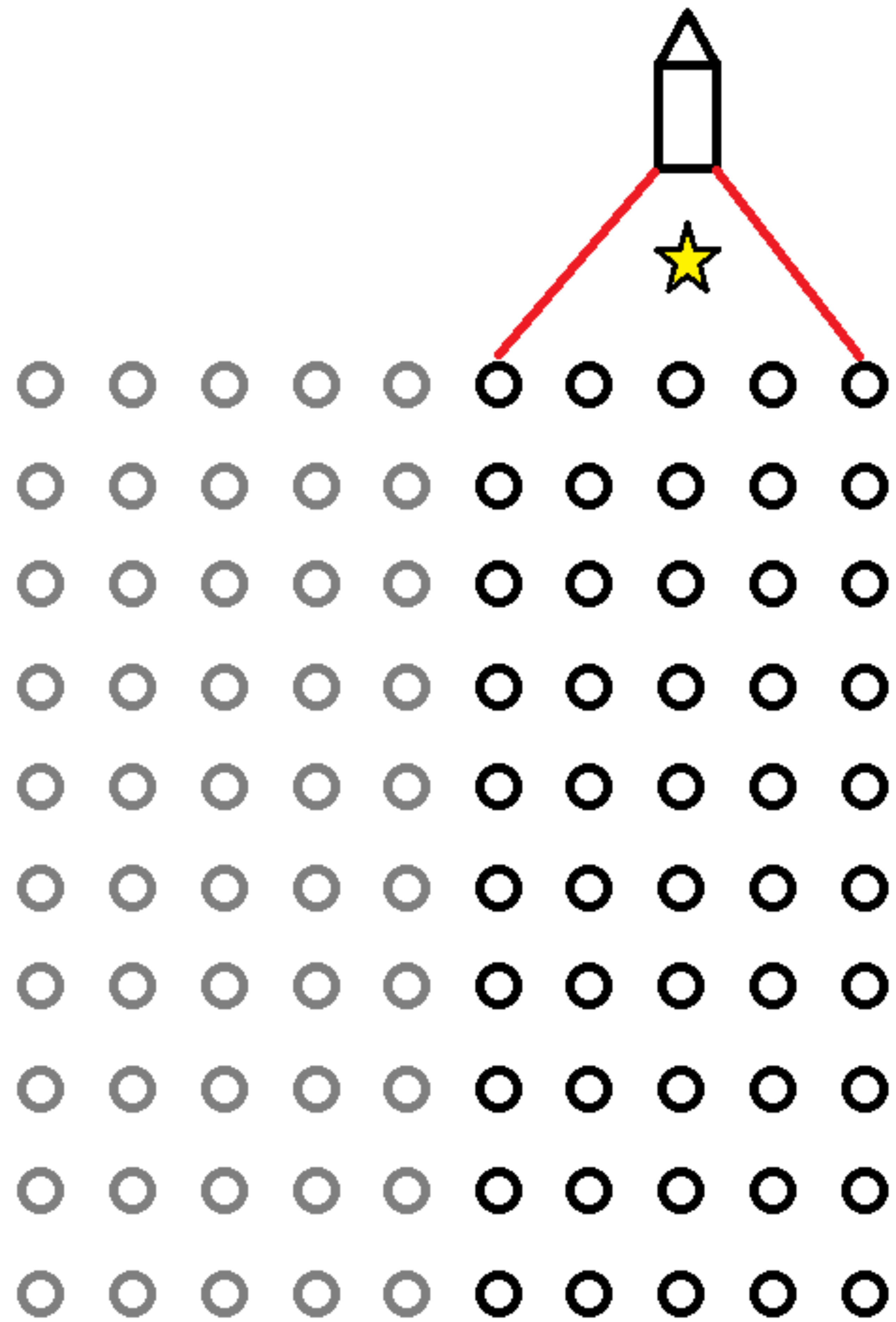
First acquisition pass



Infill Management

Second acquisition pass

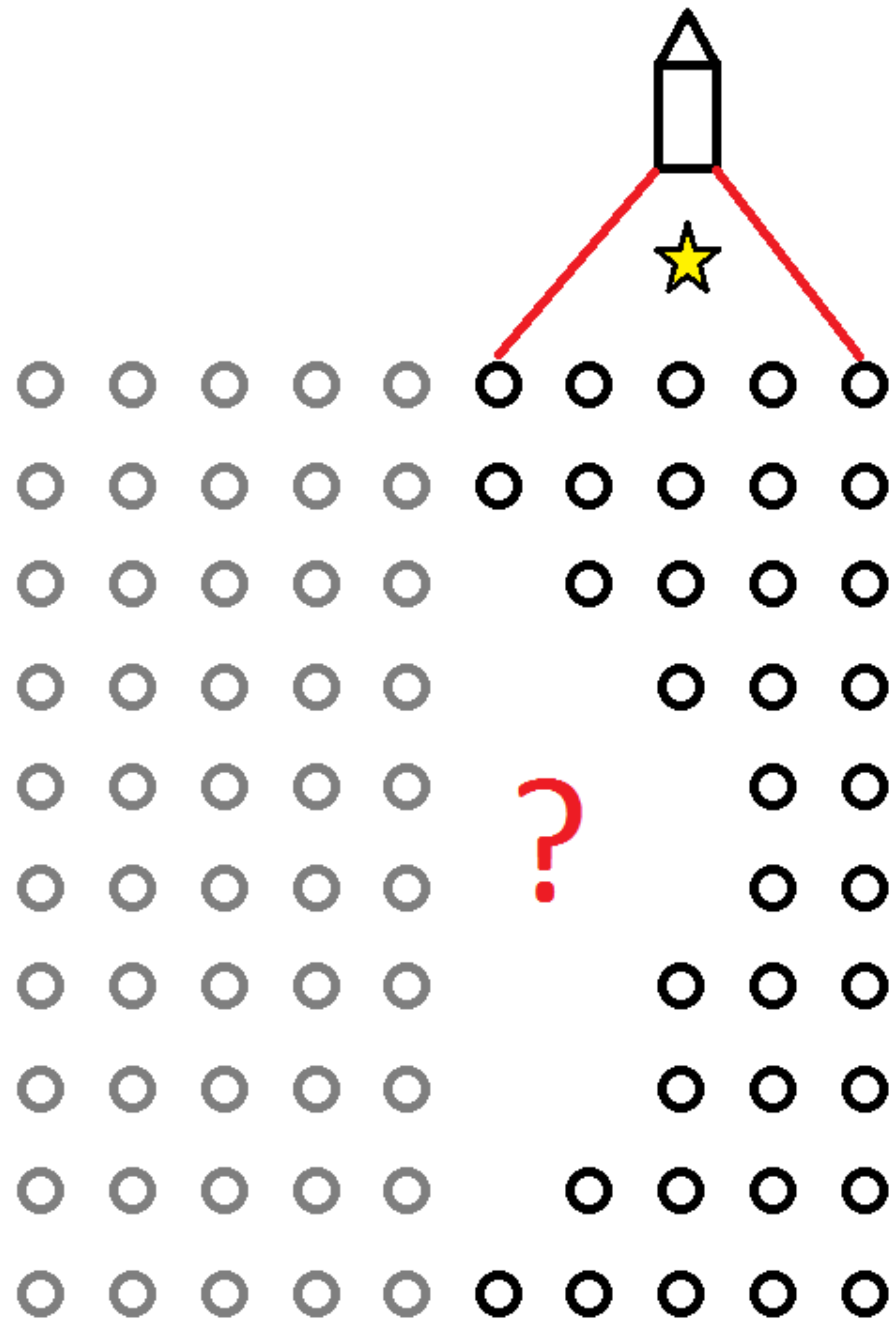
Desired sampling



Infill Management

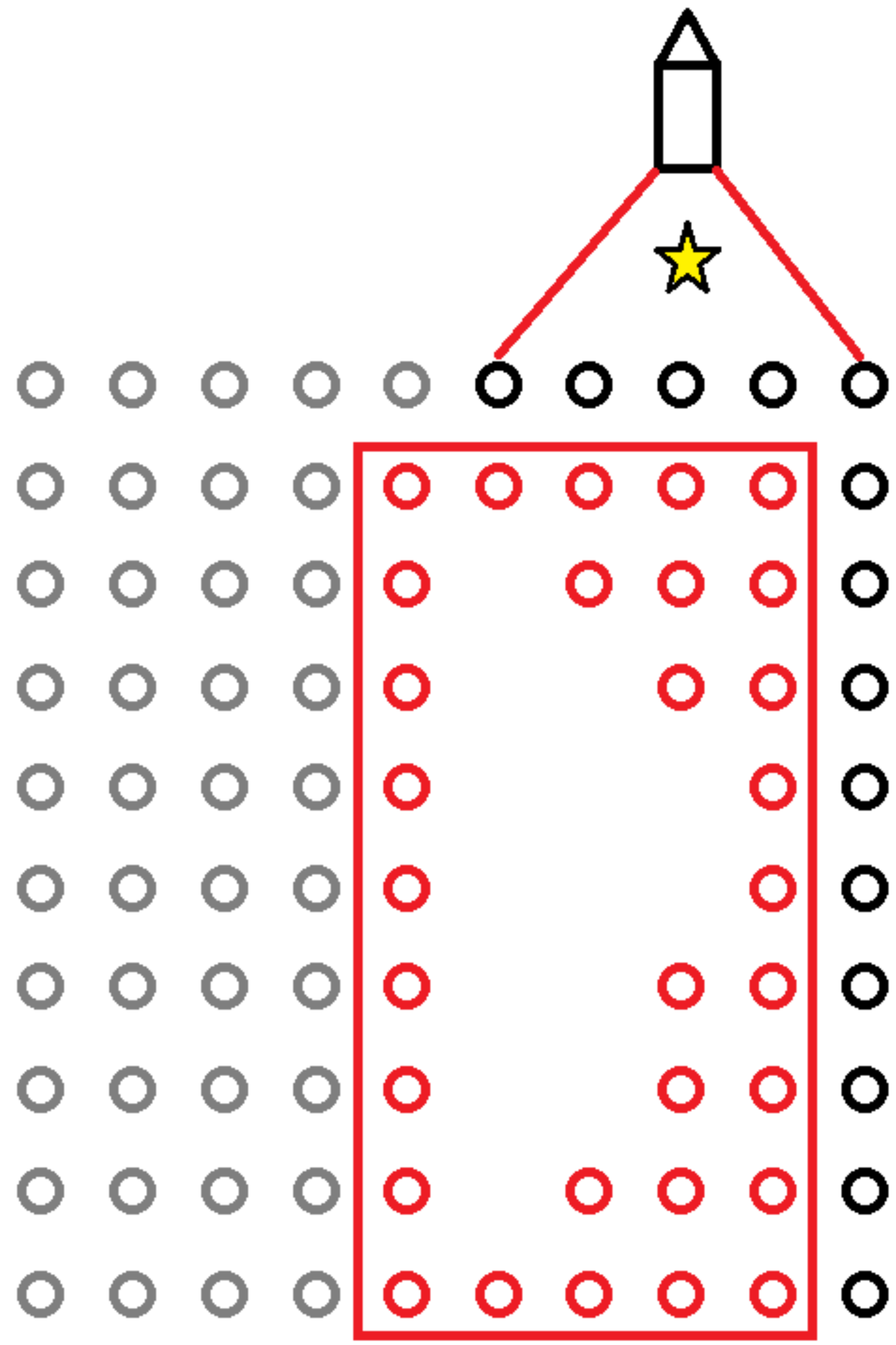
Coverage Hole:

additional infill acquisition?



Infill Management

Form local mask:

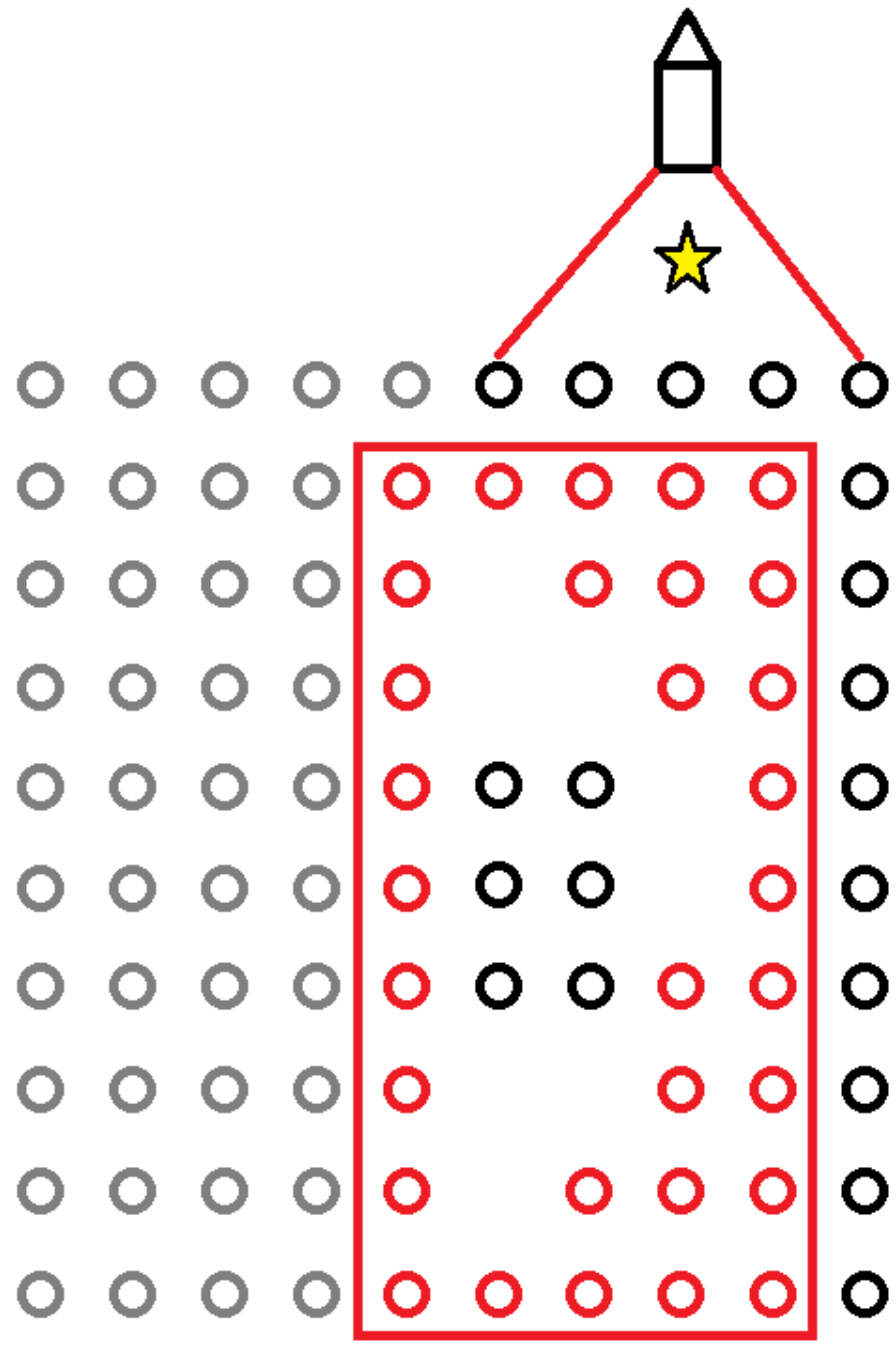


use spectral gap
as decision tool

Infill Management

Additional infill acquisition:

use spectral gap to minimize sampling



Conclusion

- Good understanding of how to subsample

- Simple procedure to quantify acquisition design
  - compute only σ_1, σ_2 of sampling mask
  - useful tools for 3D data

Future work

- Further analysis of spectral gap quantification
  - Reconstruction Error bounds
  - Generalize analysis to other measurement operators
  - Suggestions?

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Acknowledgements

Thank you for your attention!

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NSERC CRSNG

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