

# Affordable omnidirectional image volumes extension to 3D

Rajiv Kumar, Tristan van Leeuwen and Felix J. Herrmann



SLIM   
University of British Columbia

# Motivation

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## AVA analysis

- ▶ local geological dip estimation

## Velocity analysis

## Targeted imaging

## Creation of subsurface offset image volumes

# Motivation

Computation of full-subsurface offset volumes is prohibitively expensive in 3D (storage & computation time)

Can not form full  $E$  but *action* on (random) vectors allows us to get information from *all* or *subsets* of *subsurface points*

# Extended images

Given two-way wave equations, source and receiver wavefields are defined as

$$\begin{aligned}H(\mathbf{m})U &= P_s^T Q \\ H(\mathbf{m})^* V &= P_r^T D\end{aligned}$$

where

$H(\mathbf{m})$  : discretization of the Helmholtz operator

$Q$  : source

$D$  : data matrix

$P_s, P_r$  : samples the wavefield at the source and receiver positions

$\mathbf{m}$  : slowness

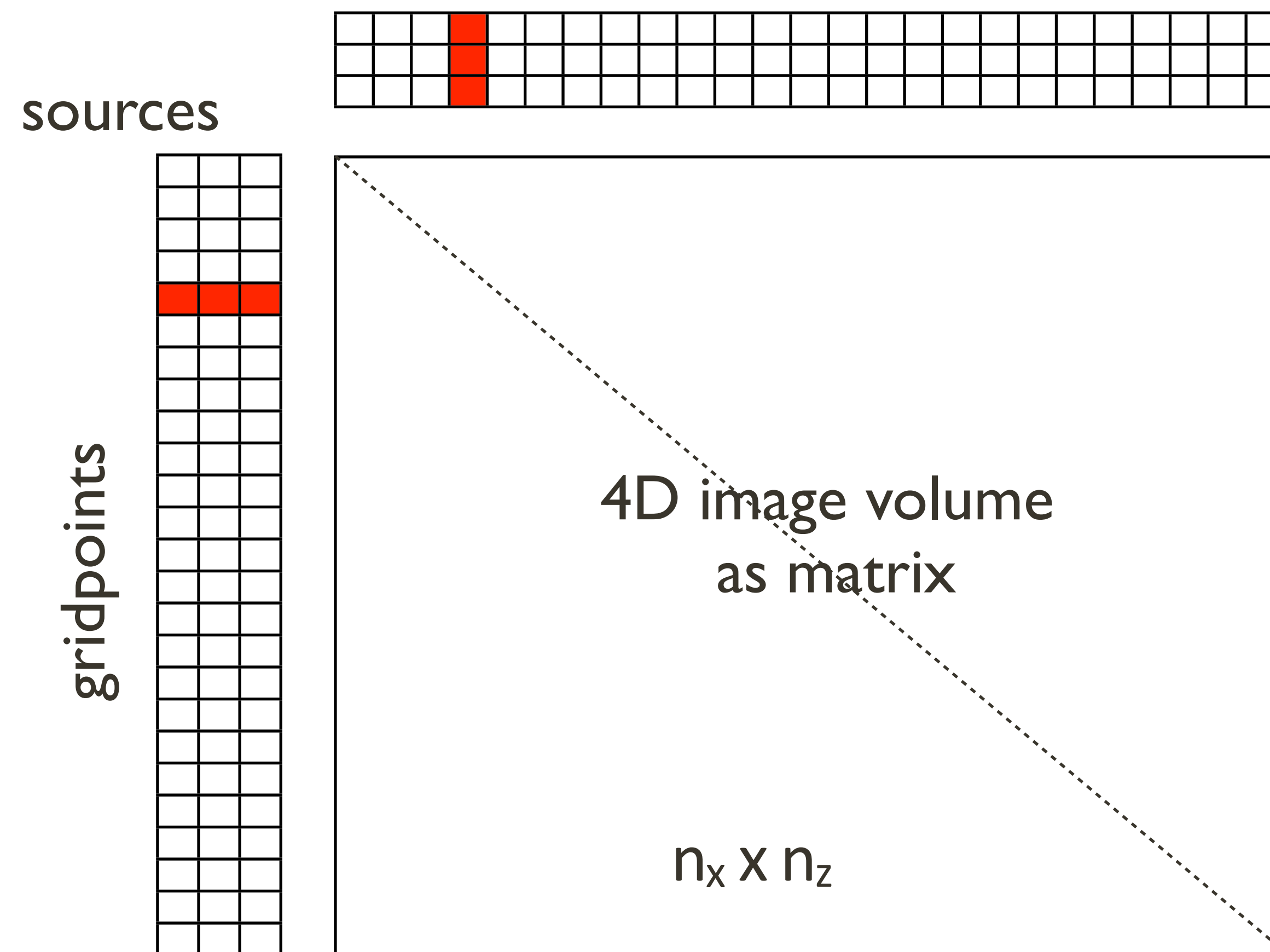
# Extended images

Organize wavefields in monochromatic data *matrices* where each *column* represents a *common* shot gather

Express image volume *tensor* for *single* frequency as a *matrix*

$$E = UV^*$$

# Extended images



# Extended images

Too expensive to compute (*storage and computational time*)

Instead, *probe* volume with *tall* matrix  $W = [\mathbf{w}_1, \dots, \mathbf{w}_l]$

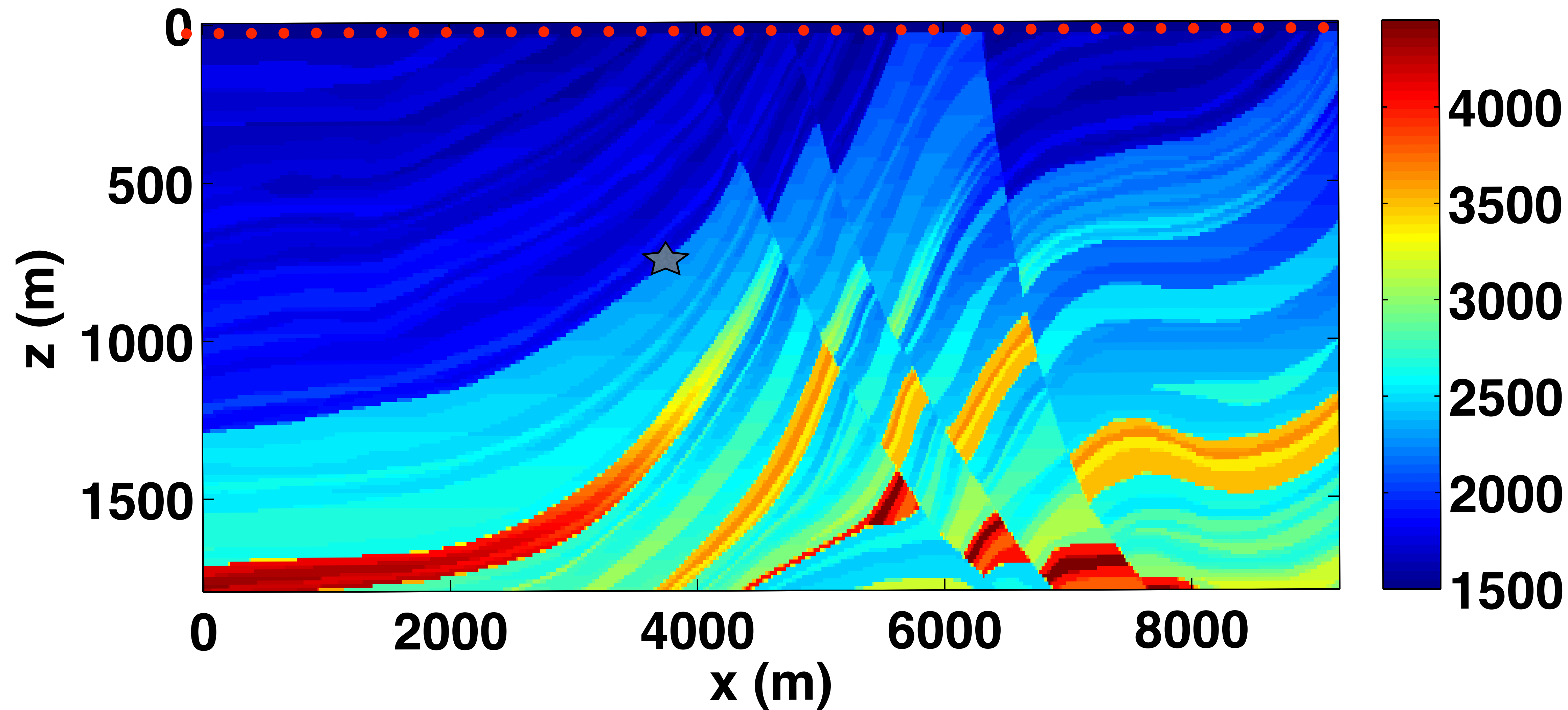
$$\tilde{E} = EW = H^{-1} P_s^T Q D^* P_r H^{-1} W$$

where  $\mathbf{w}_i = [0, \dots, 0, 1, 0, \dots, 0]$  represents *single* scattering points

# Extended images

Marmousi model

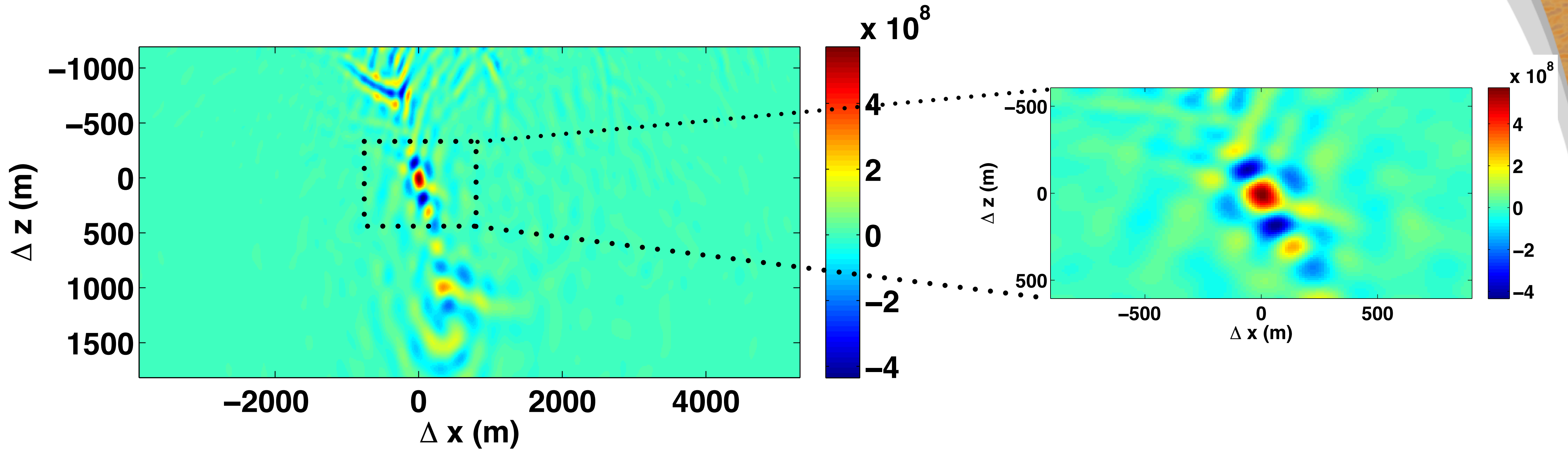
-  Common image point
-  Source / Receiver location





# Extended images

common image point gather, 3- 30 Hz



$\Delta x$  : **Horizontal offset**

$\Delta z$  : **Vertical offset**

# Take-away message

## Computational costs

Full subsurface offset extended images:

|              | # of PDE solves | “flops for correlations” |
|--------------|-----------------|--------------------------|
| conventional | $2N_s$          | $N_s \times N_h$         |
| mat-vecs     | $2N_x$          | $N_s \times N_r$         |

$N_s$  – # of sources

$N_r$  – # of receivers

$N_h$  – # of subsurface offsets

$N_x$  – # of sample points

# Take-away message

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**We win when  $N_x < N_s$  !**

# Applications in 2D

# AVA analysis

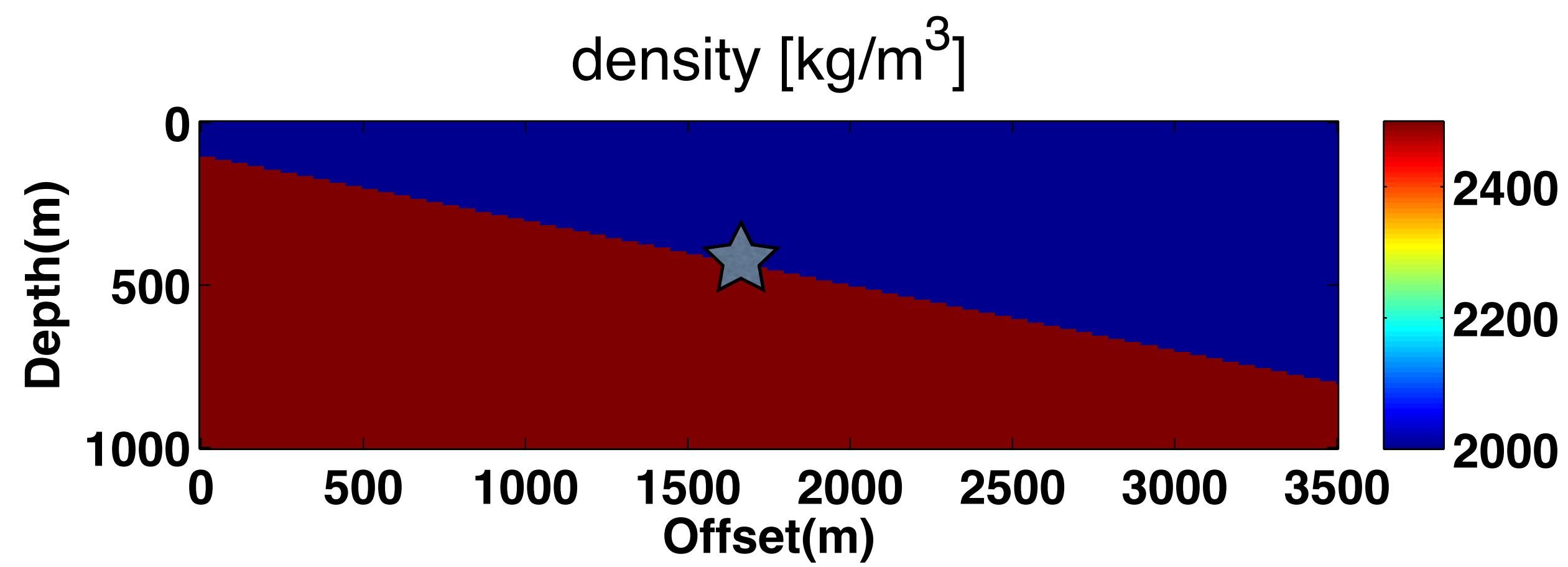
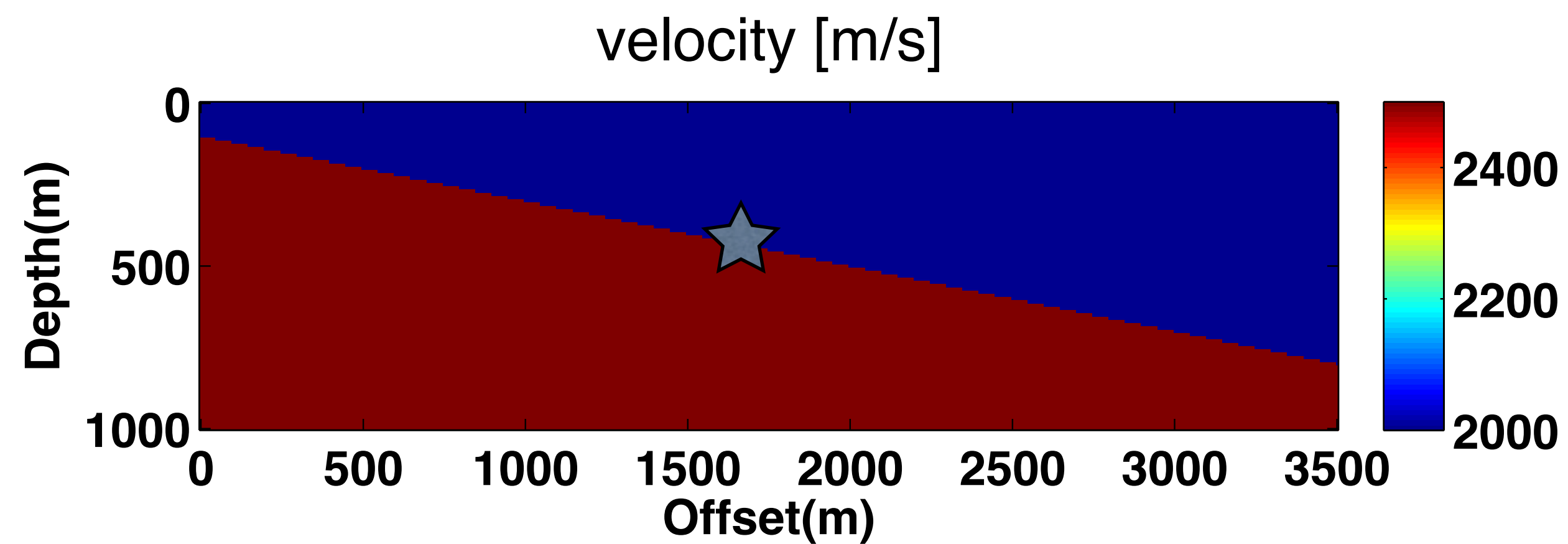
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Reservoir characterization

Local geological dip estimation

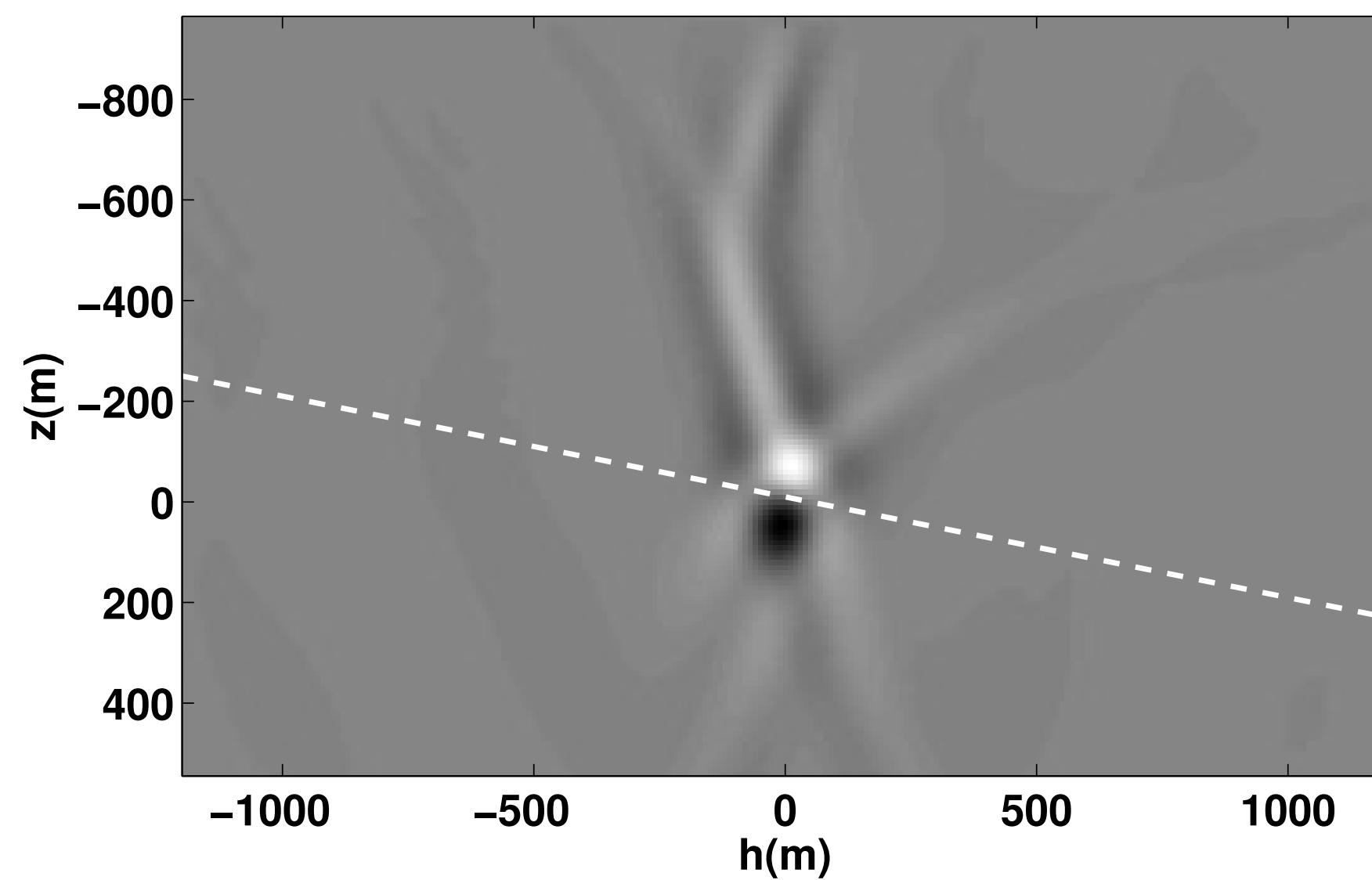
# Dip-angle gathers

## dipping layer model



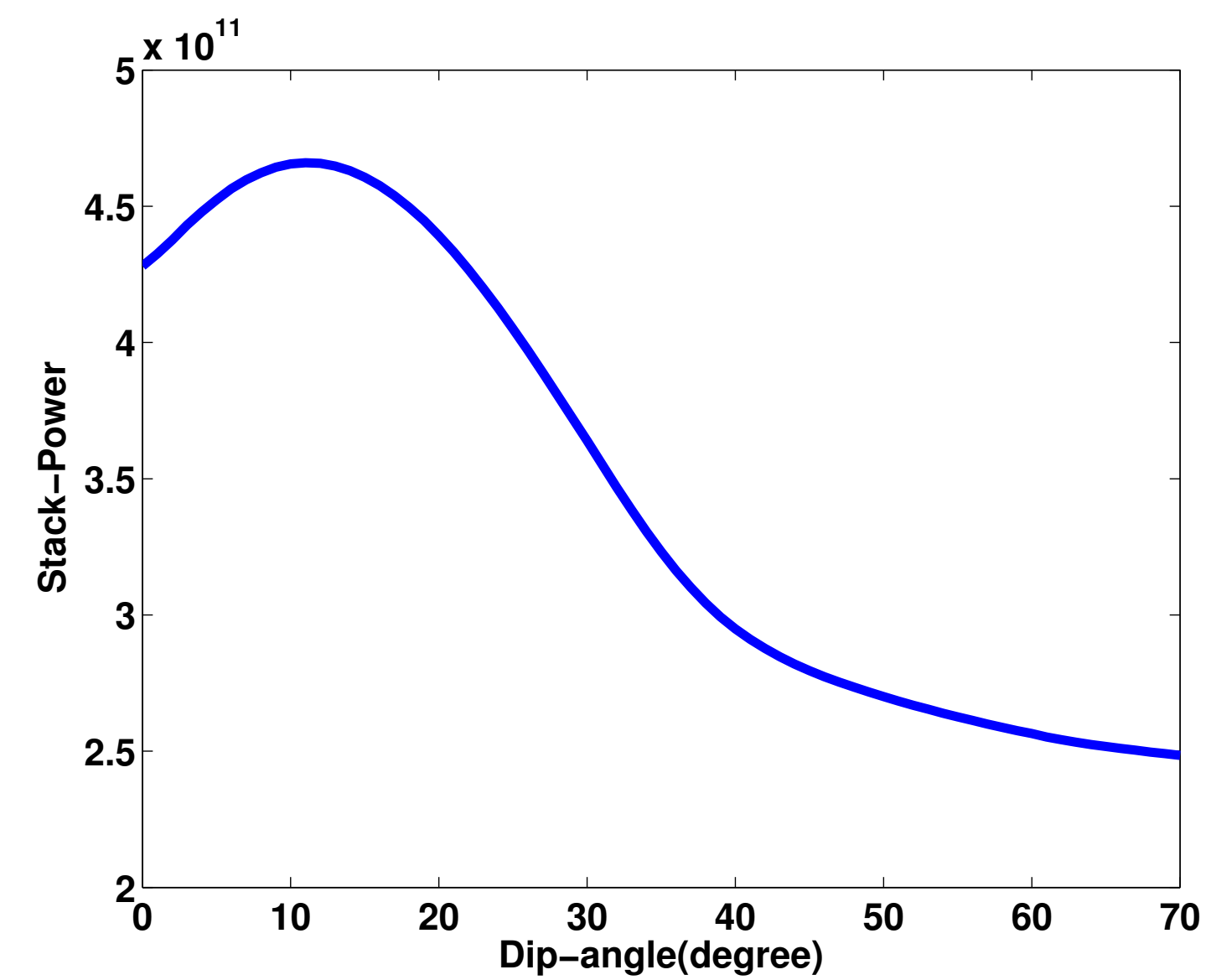
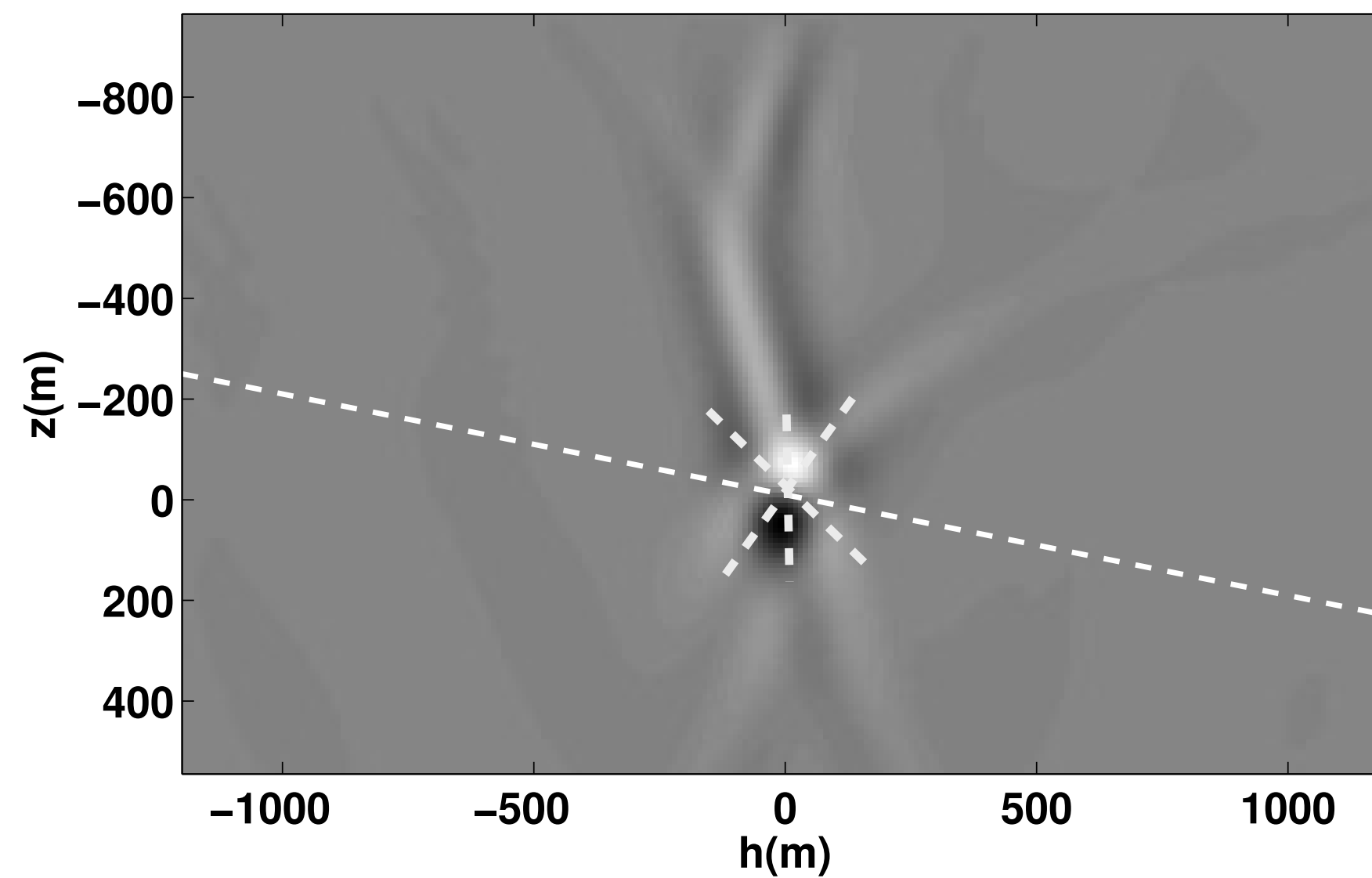
# Dip angle gathers

Dip can be detected by measuring the *stack-power* normal to the dip



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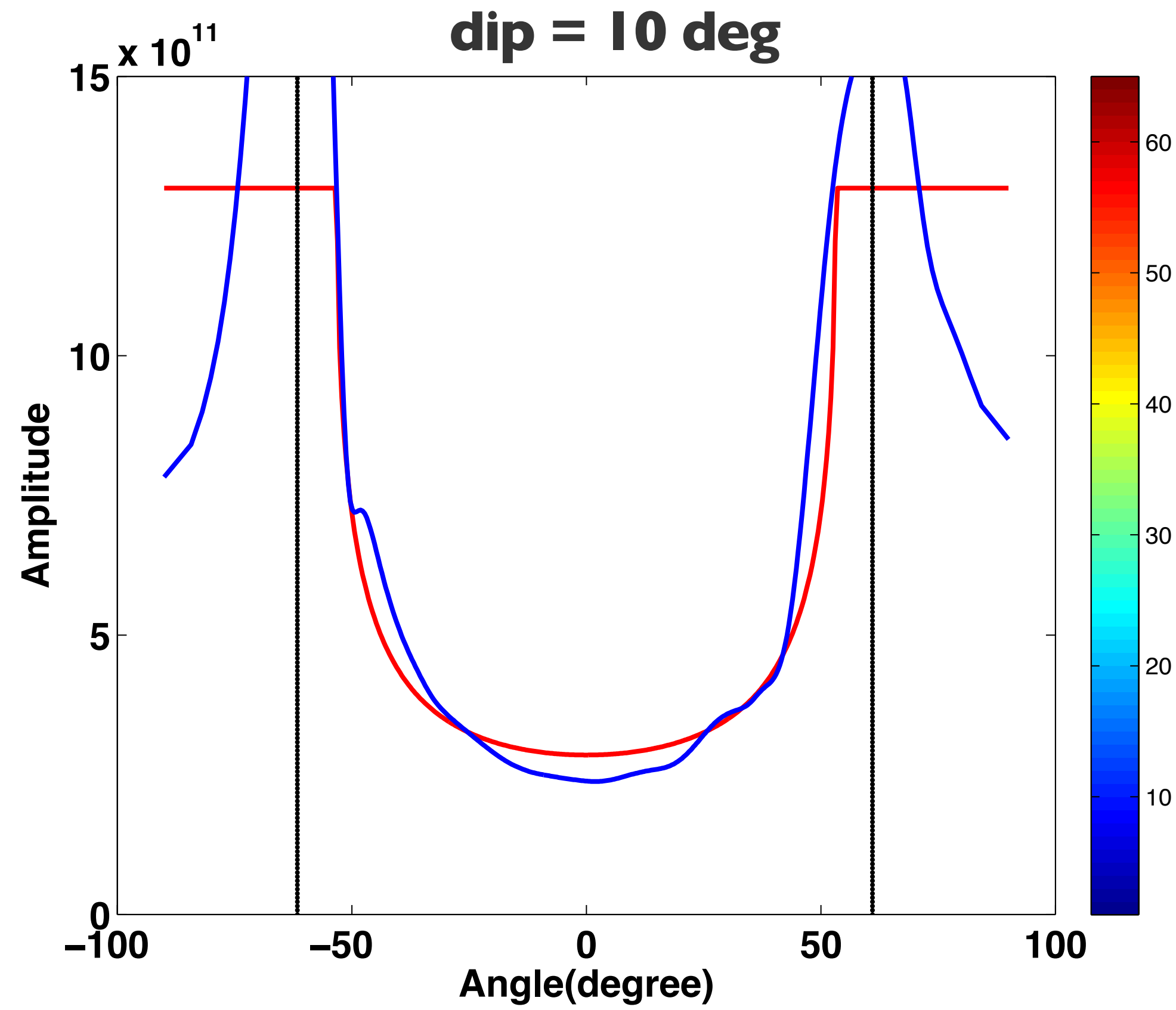
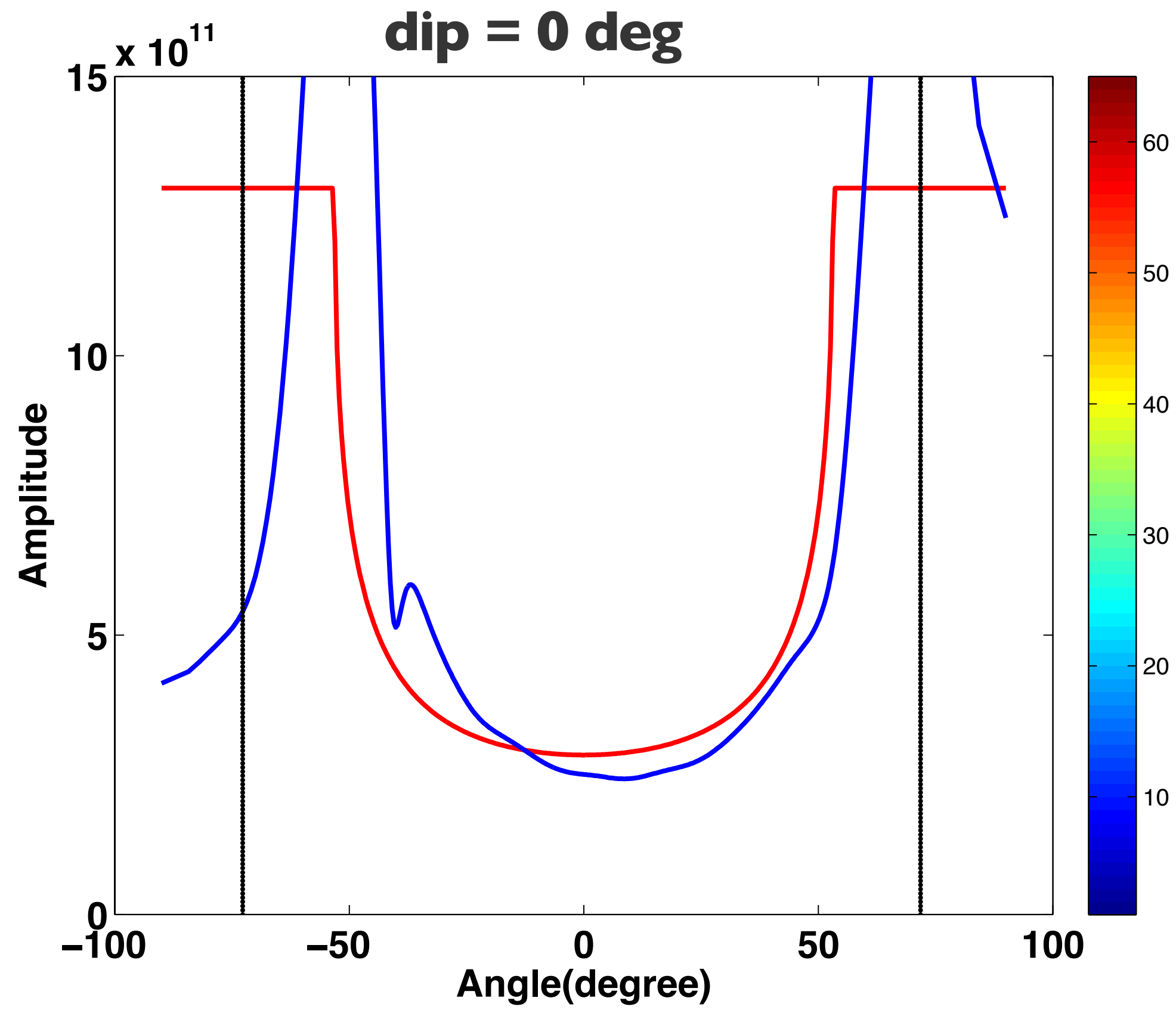




# AVA

## dipping layer model

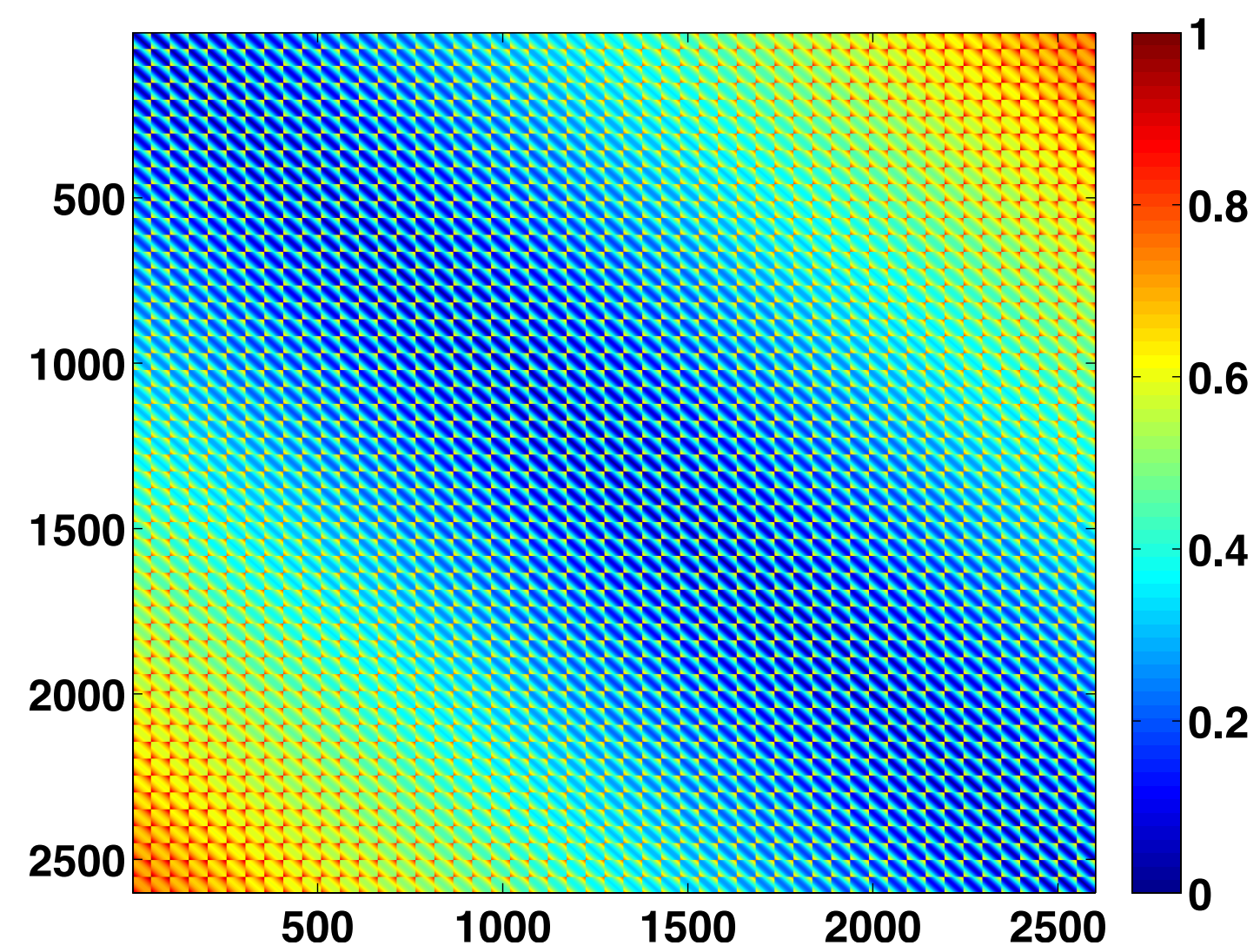
- Zoeppritz equation
- Predicted response



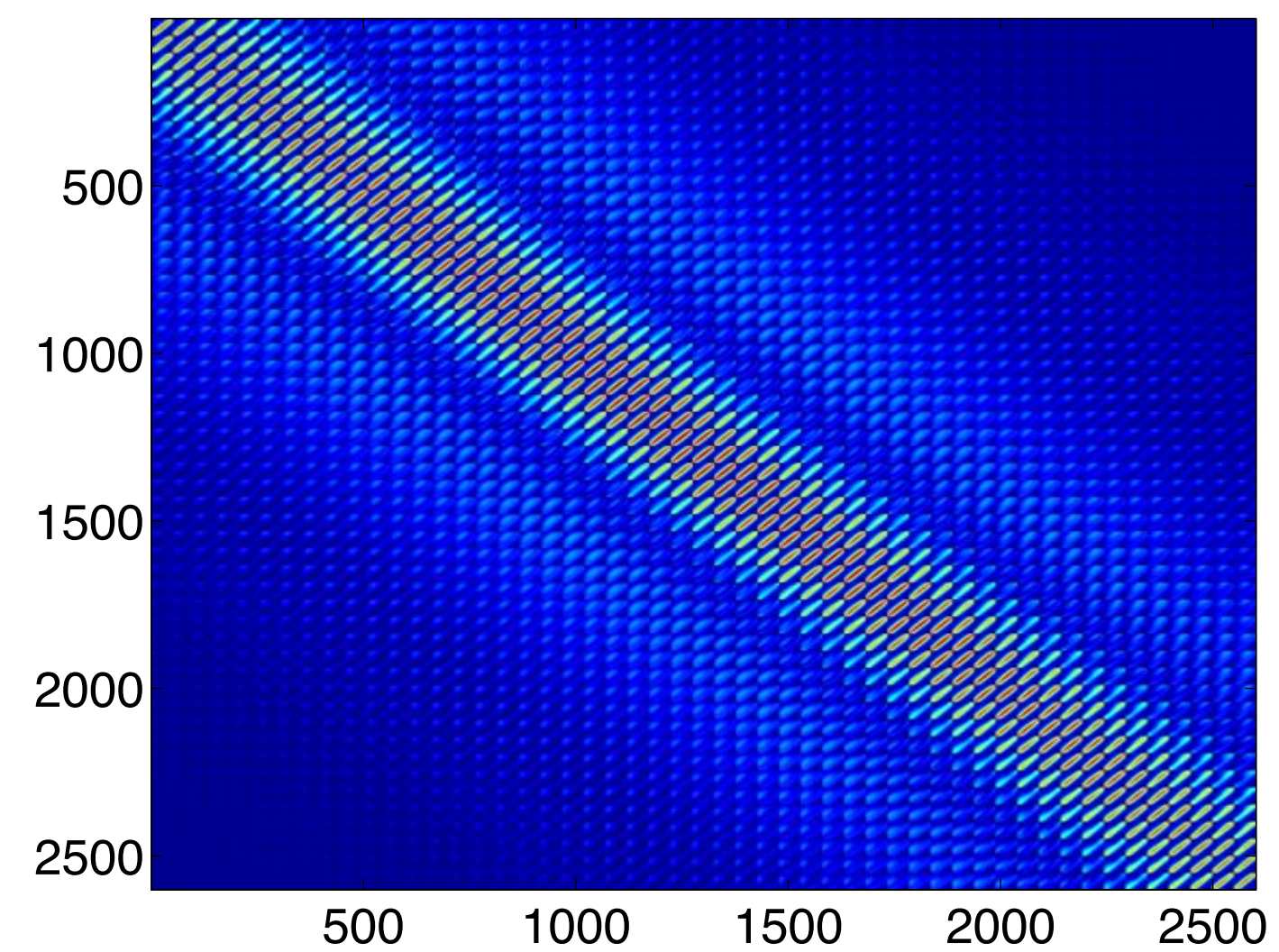
Biondo & Symes, '04 , Symes 2008, Sava & Vasconcelos, '11

# WEMVA

conventional approach



• \*



$h$

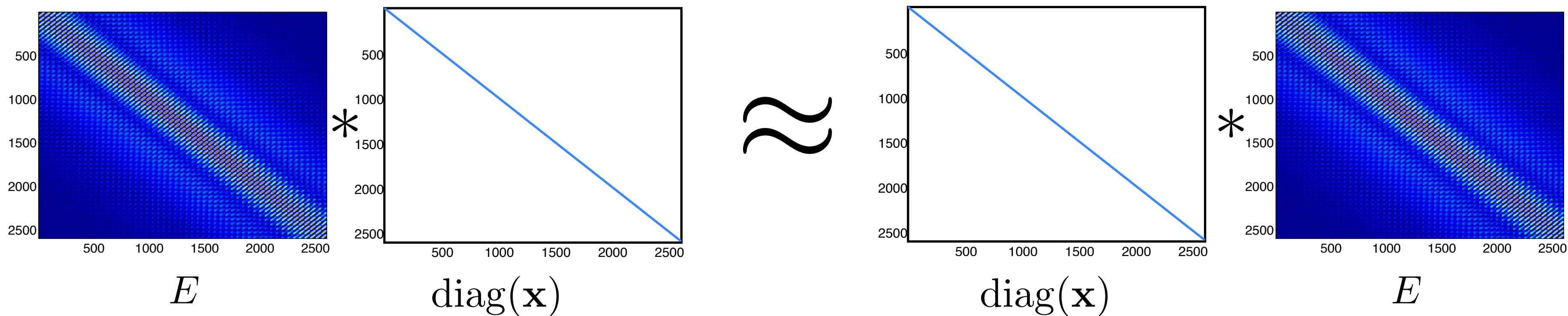
$E$

• \* stand for element-wise multiplication

# Focusing

propose method approach

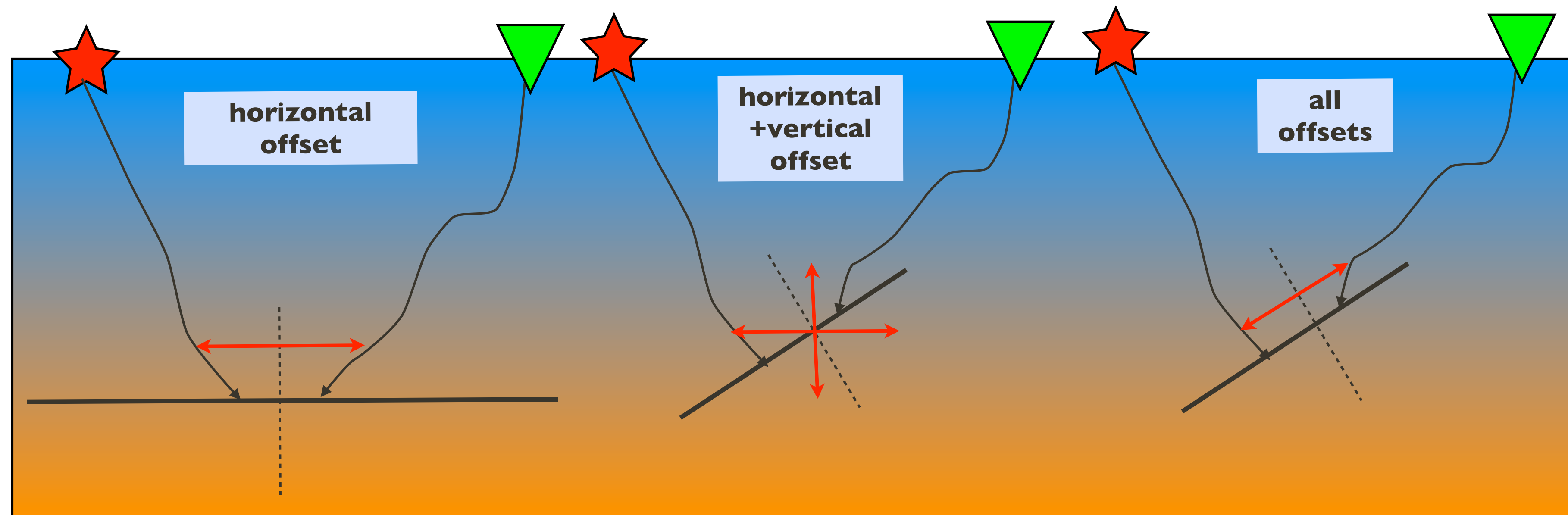
$$E \text{diag}(\mathbf{x}) \approx \text{diag}(\mathbf{x}) E$$



 **matrix-matrix multiplication**

# Focusing

where  $x$  represents horizontal, vertical or all offset.



# Fast WEMVA w/ randomized probing

Measure the error in some norm

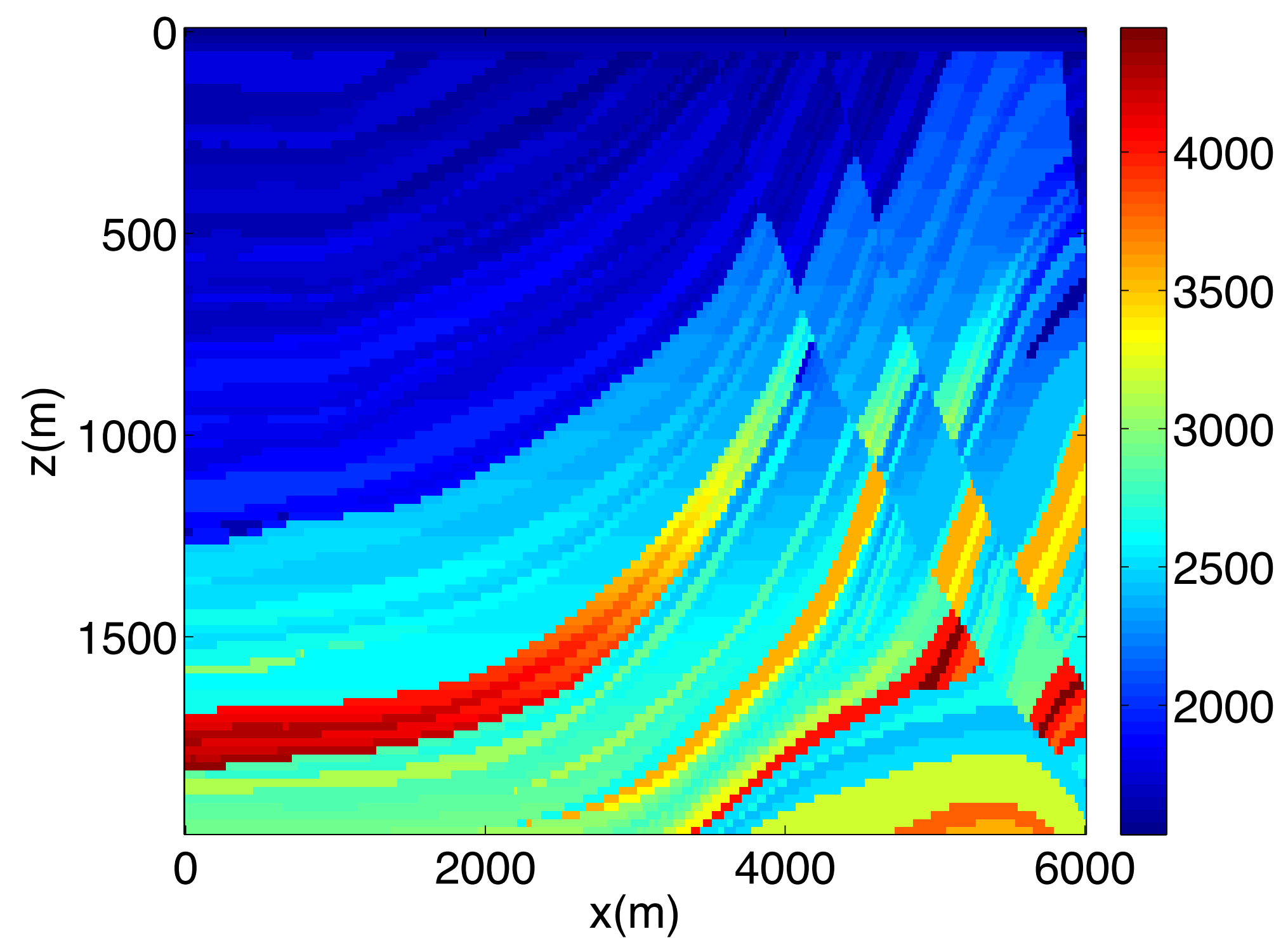
$$\min_{\mathbf{m}} \|\mathbf{E}(\mathbf{m})\text{diag}(\mathbf{x}) - \text{diag}(\mathbf{x})\mathbf{E}(\mathbf{m})\|_?^2$$

The *Frobenius* norm can be estimated via randomized trace estimation : [Avron and Toledo, 2011](#)

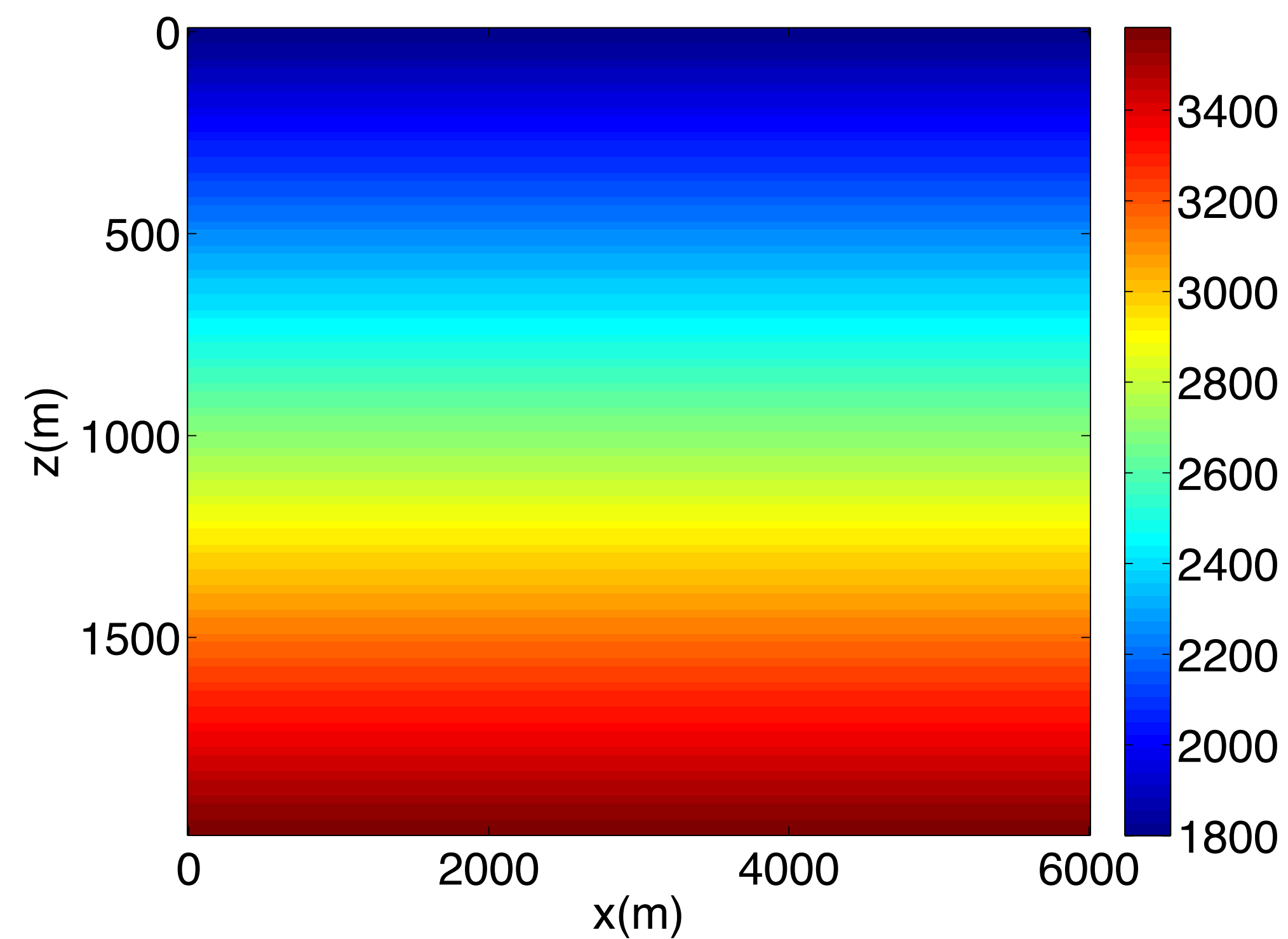
$$\begin{aligned} \|A\|_F^2 &= \text{trace}(A^T A) \\ &\approx \sum_{i=1}^K \mathbf{w}_i^T A^T A \mathbf{w}_i = \sum_{i=1}^K \|A \mathbf{w}_i\|_2^2 \end{aligned}$$

where  $\sum_{i=1}^K \mathbf{w}_i \mathbf{w}_i^T \approx I$

# Randomized probing reflection data



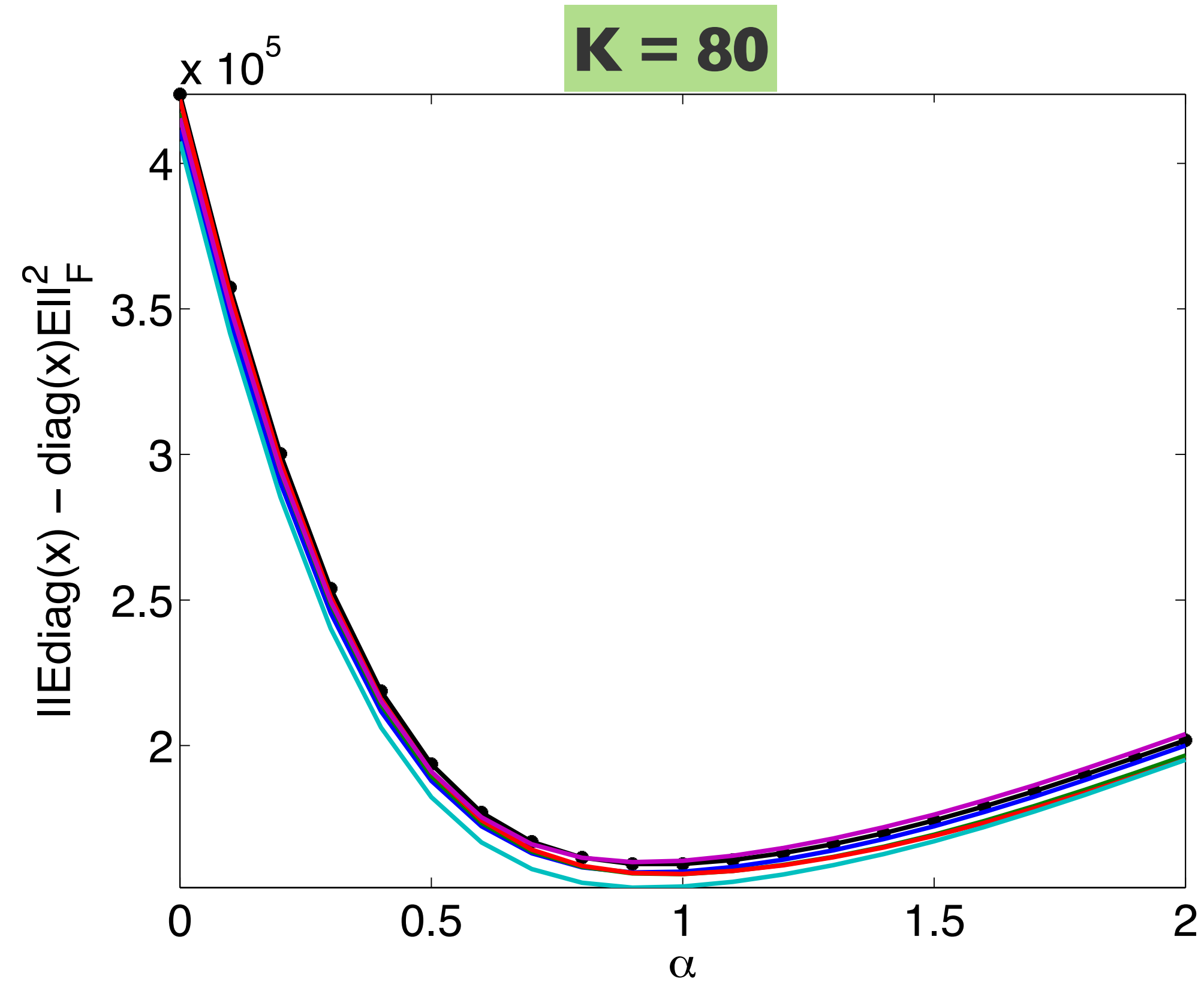
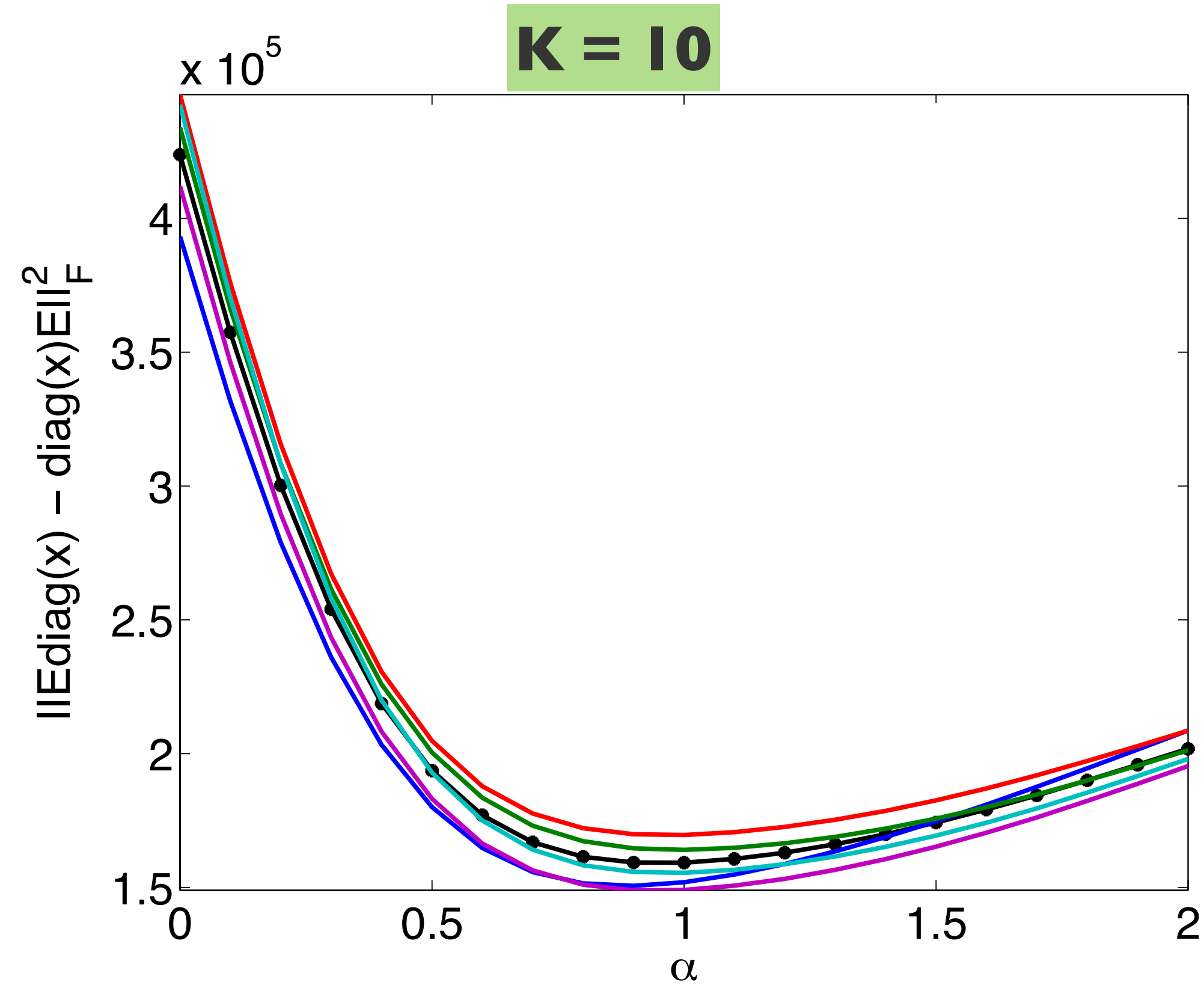
**true model**



**initial model**

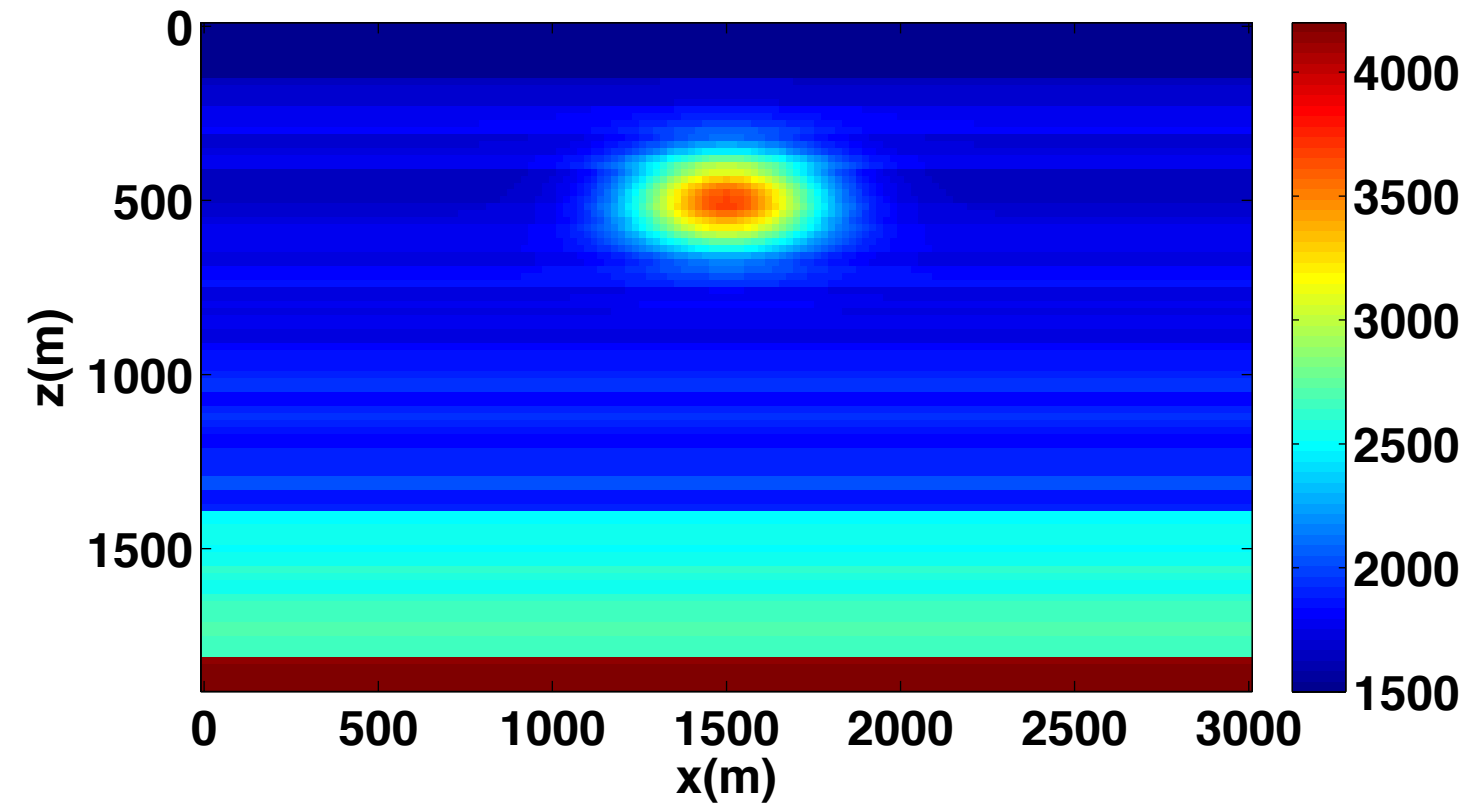
# Randomized probing reflection

- **Exact**
- **different color represents different random realization**

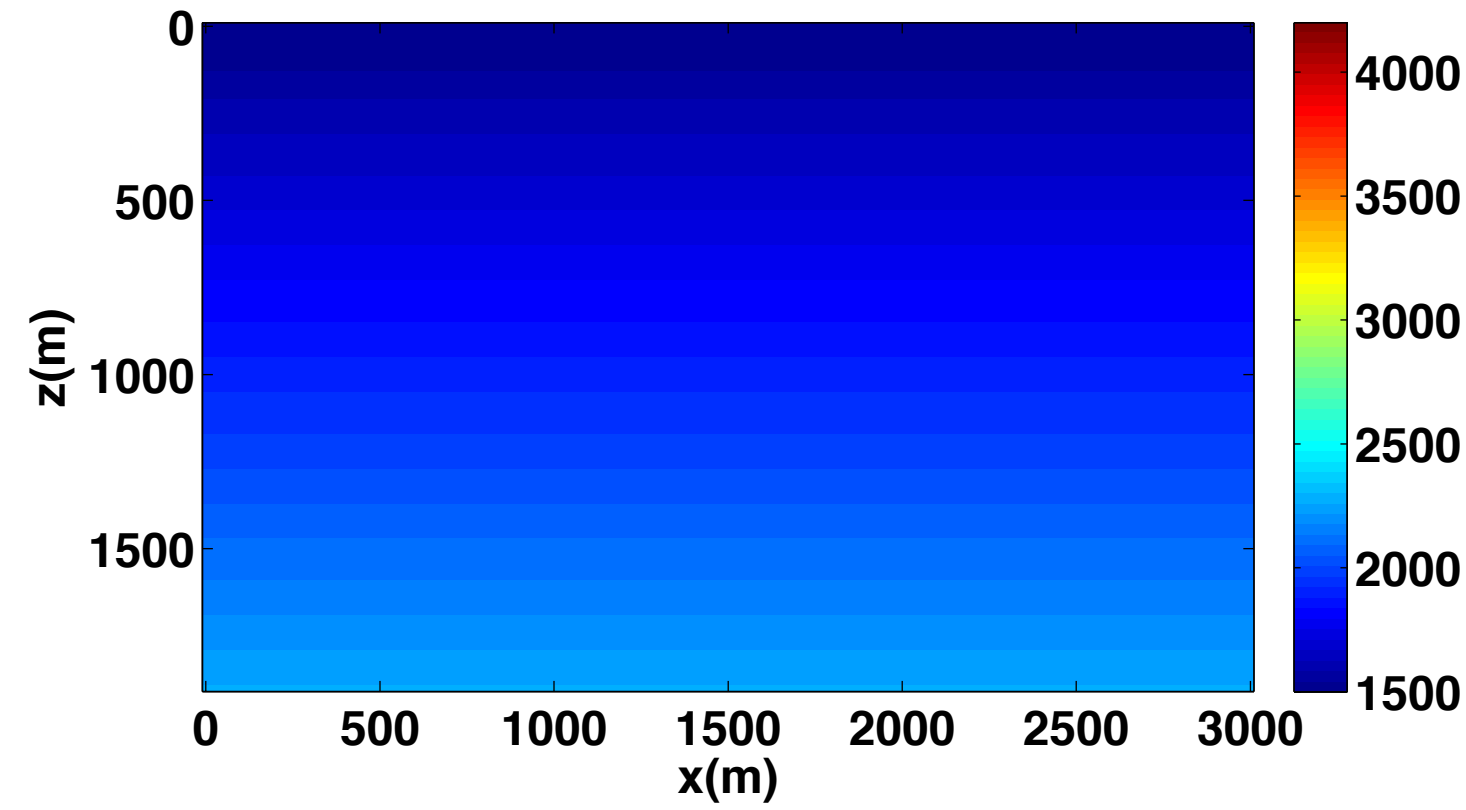


# Lens Model

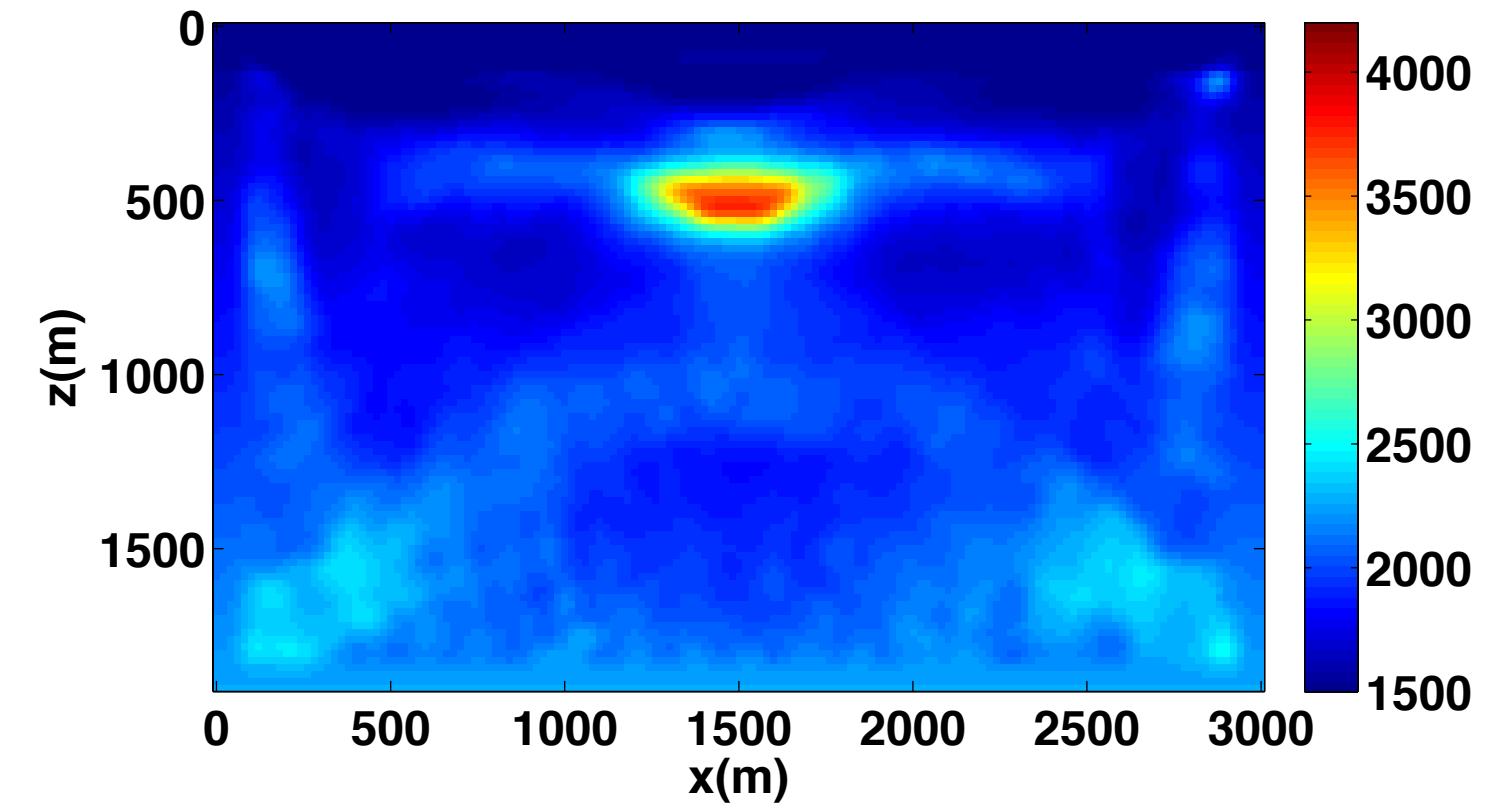
**True Model**



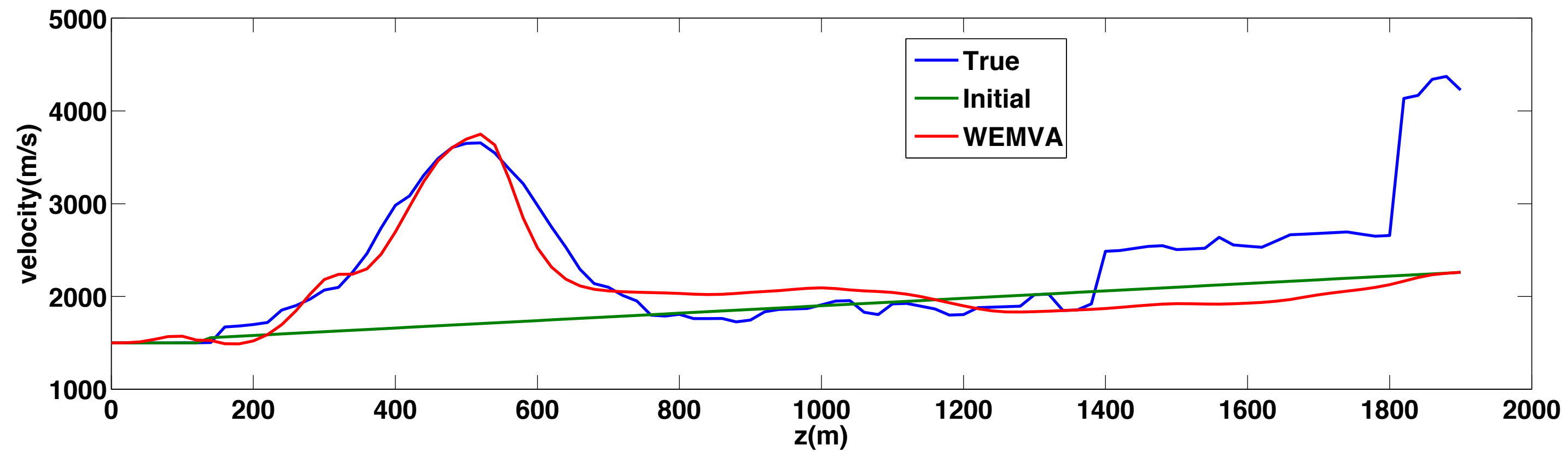
**Initial Model**



**WEMVA**



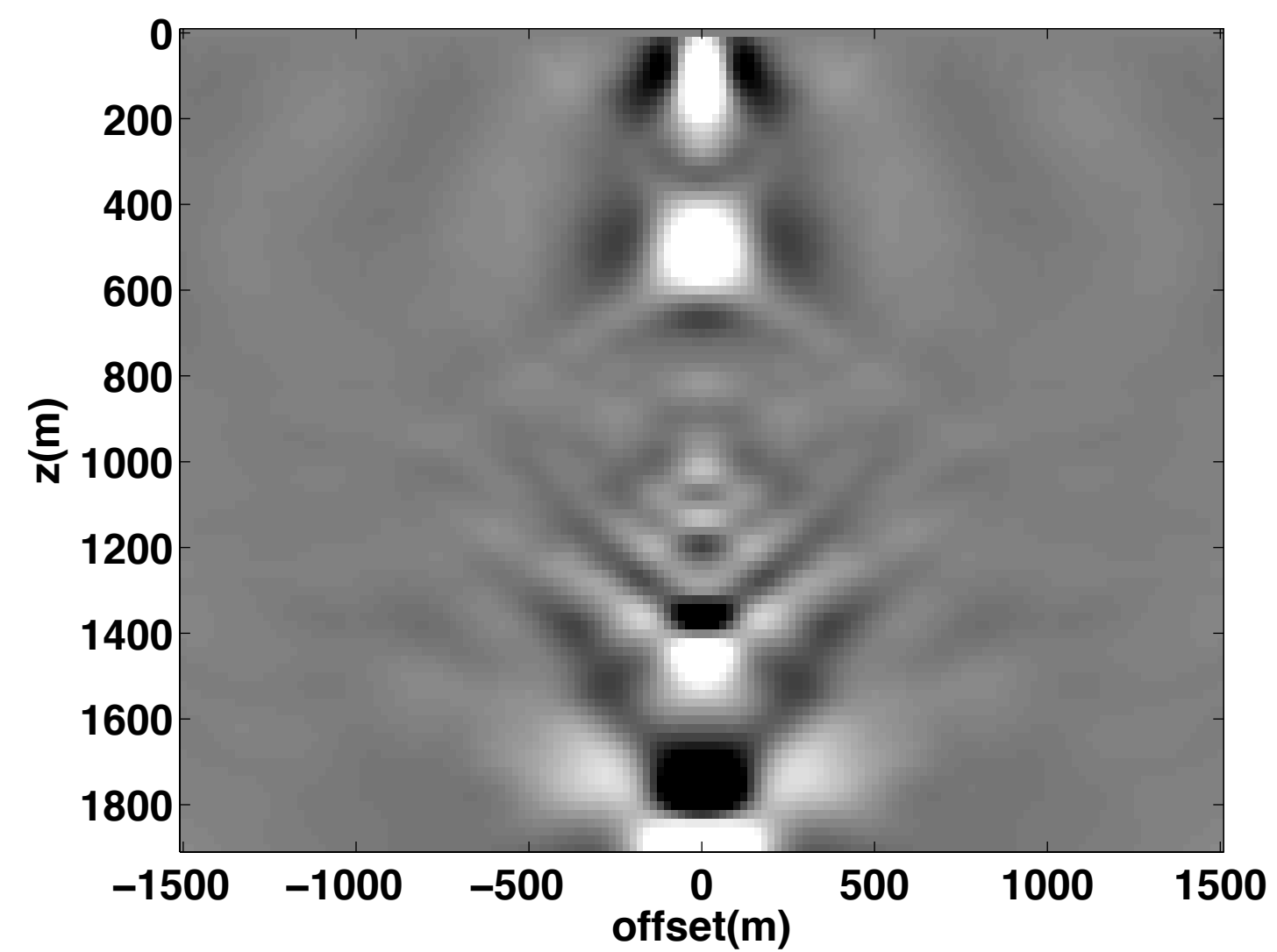
**Vertical Trace**



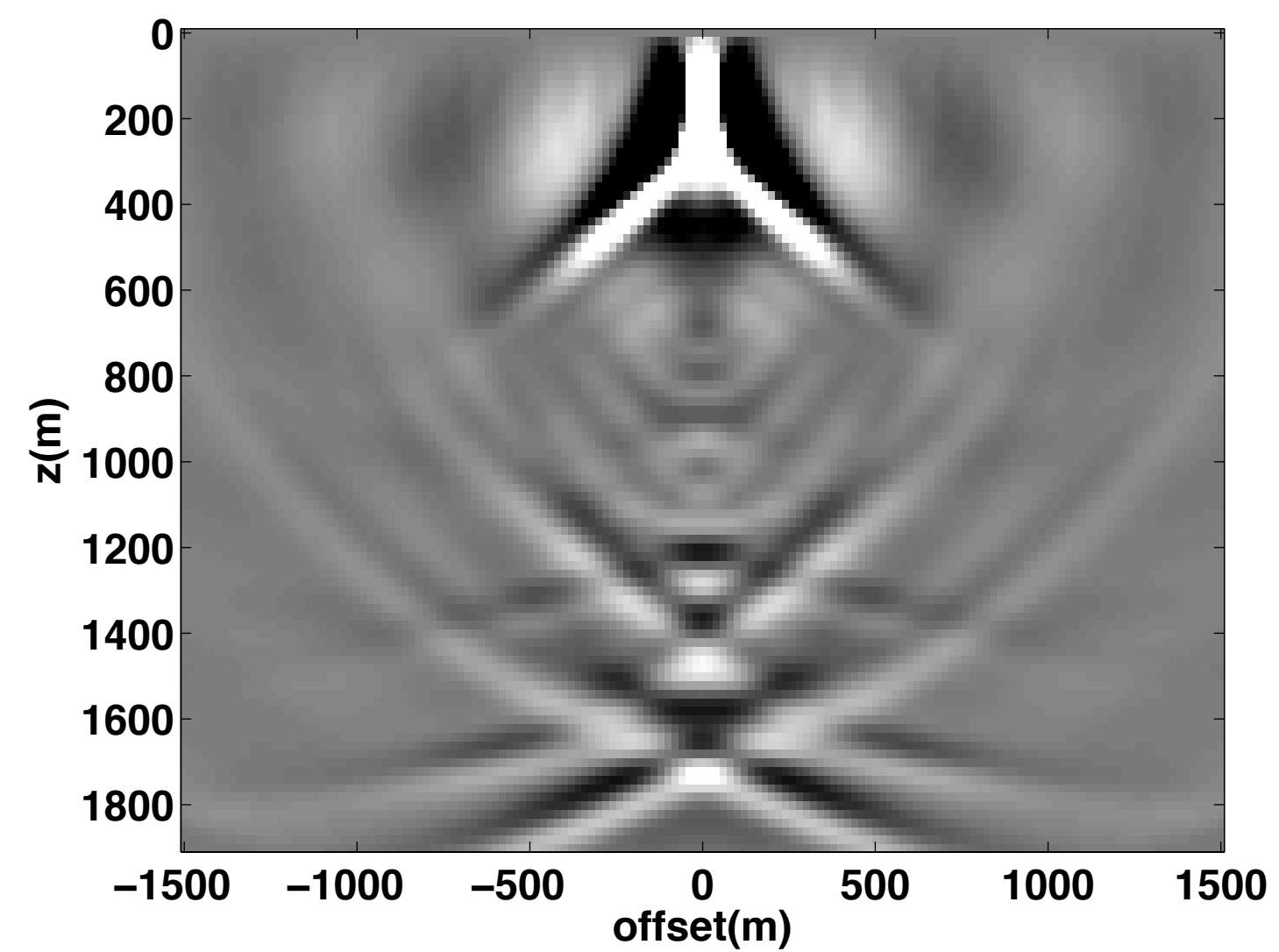


# Lens Model

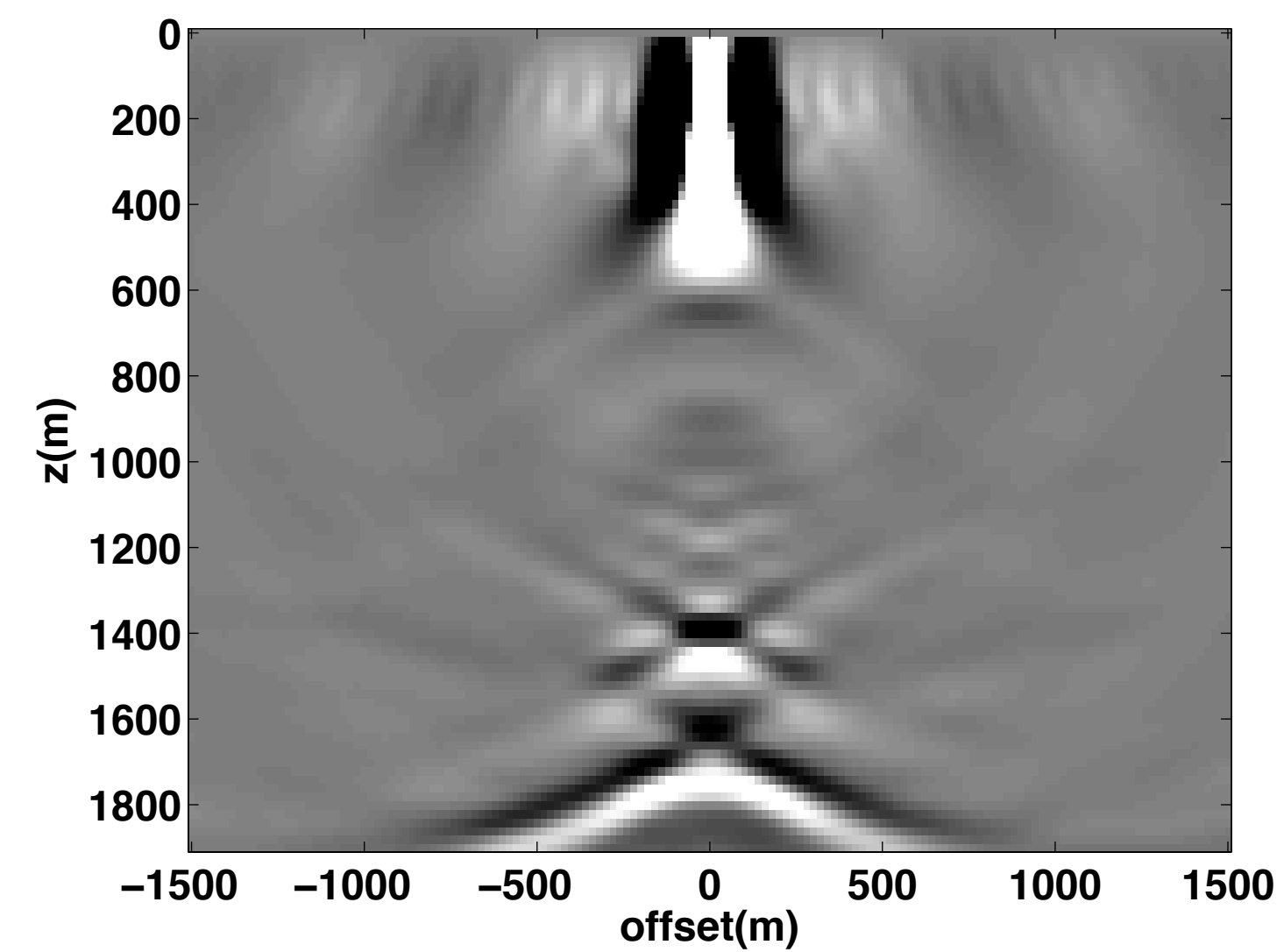
## common image gathers



**true model**



**initial model**



**WEMVA**

# Targeted Imaging

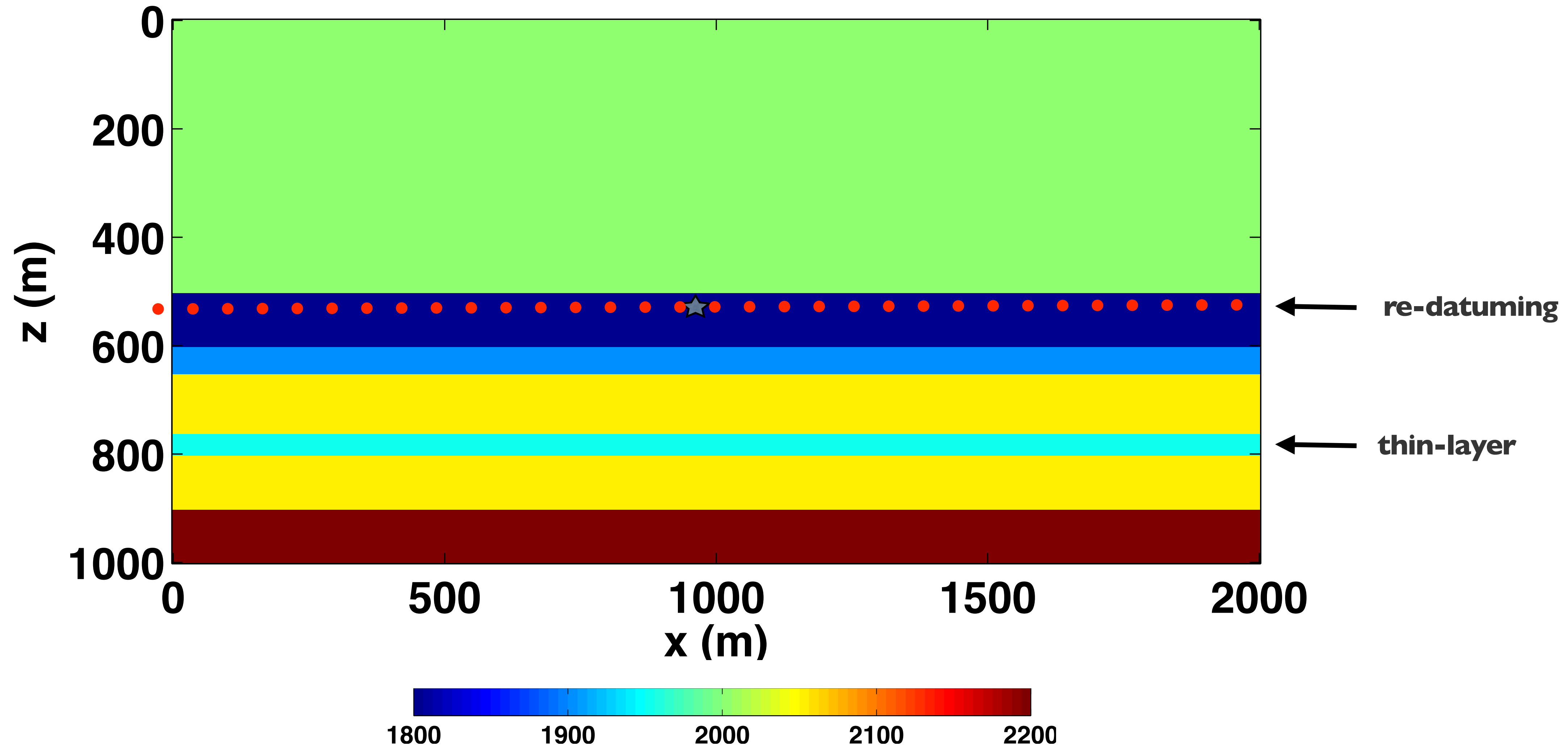
Image only reservoir area

Re-datum data above the reservoir

Mitigate overburden artifacts

# Layer model imaging thin-layer

-  Common image point
-  Virtual Receiver location



# Experimental details

101 source (20 m spacing) , 201 receivers (10 m spacing)

3-60 Hz

split-spread acquisition

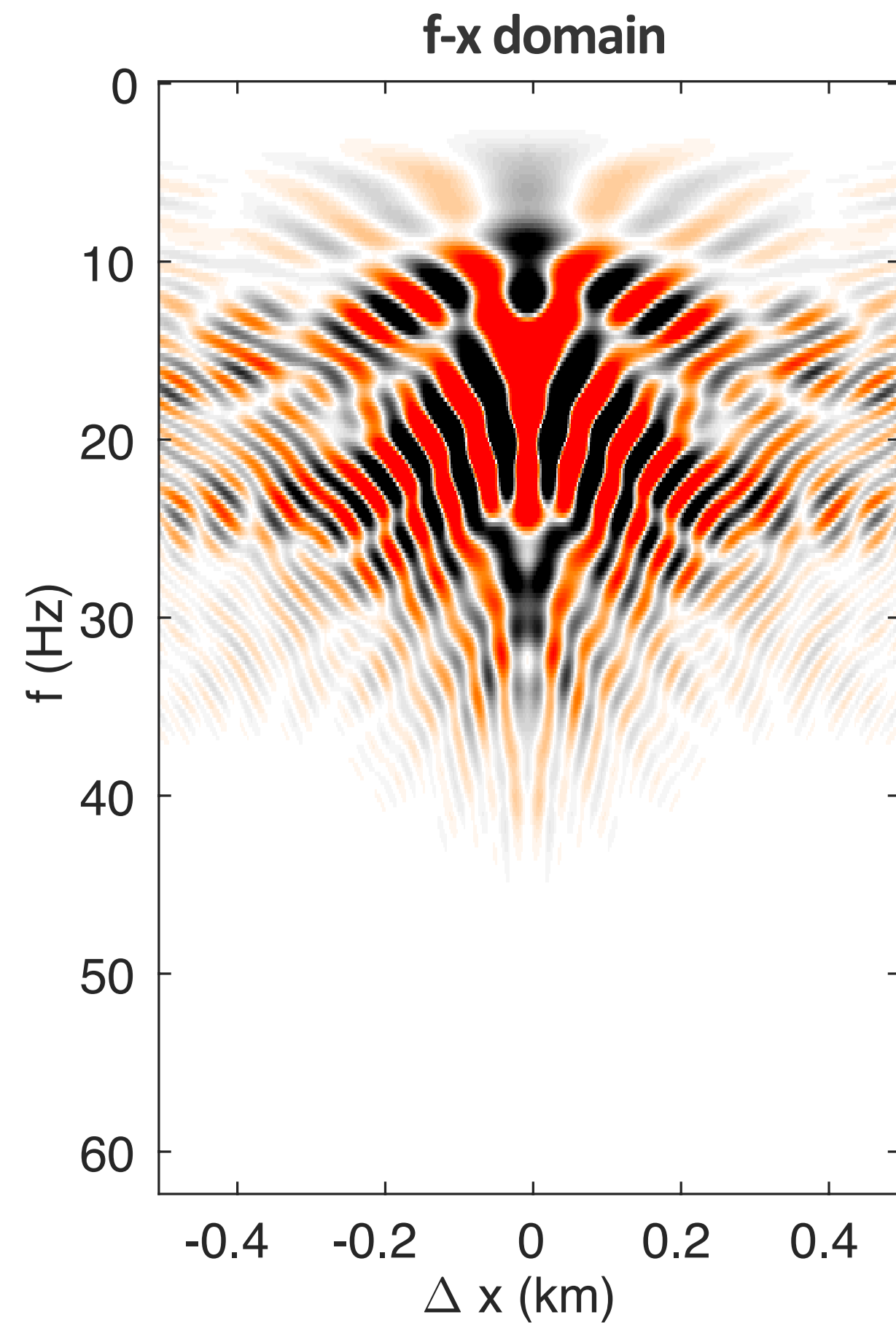
recording length 4s, sampling interval 4ms

peak frequency 20 Hz

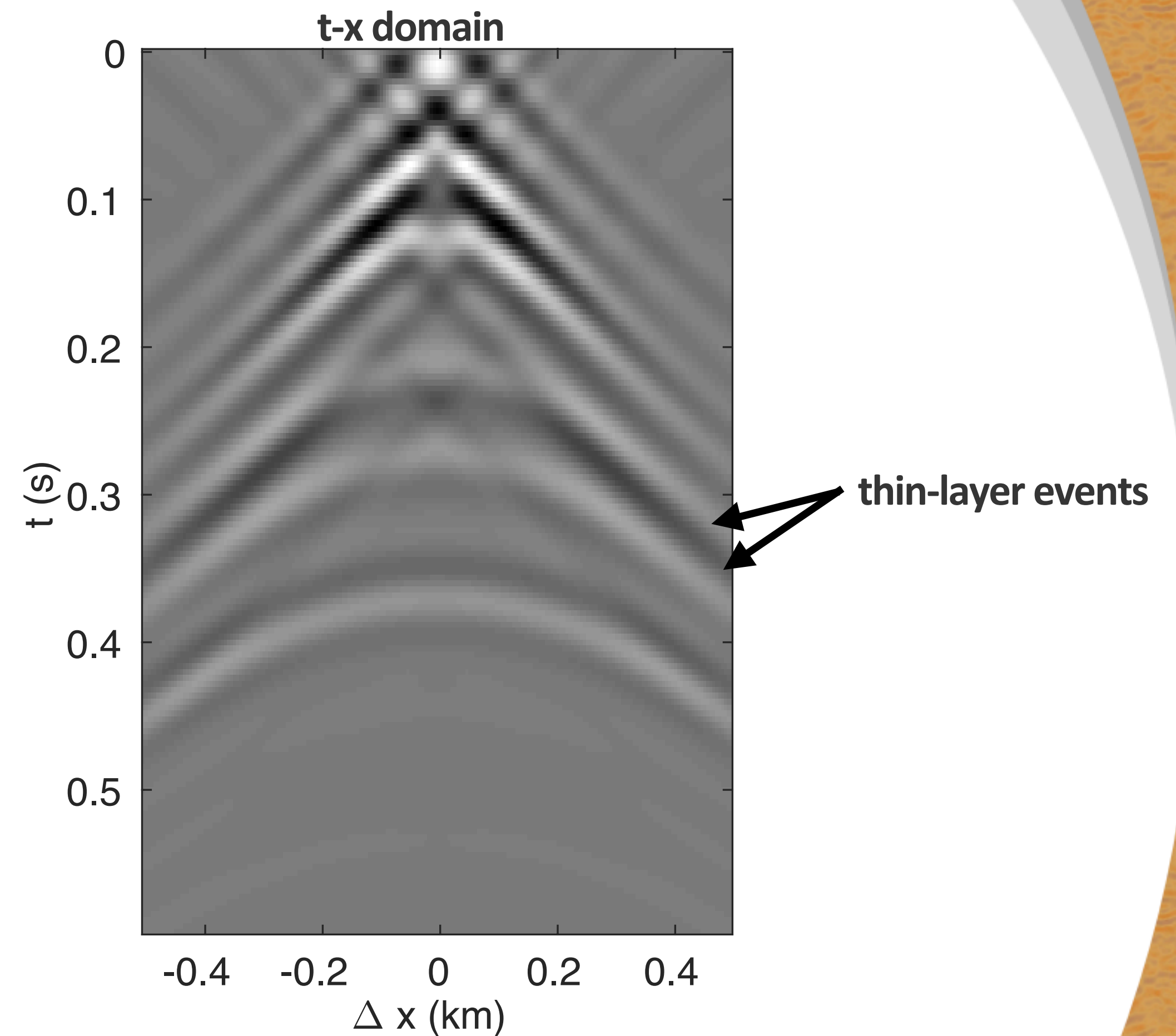
1 node, 10 workers

# Layer model

common-image gather along  $z = 500$

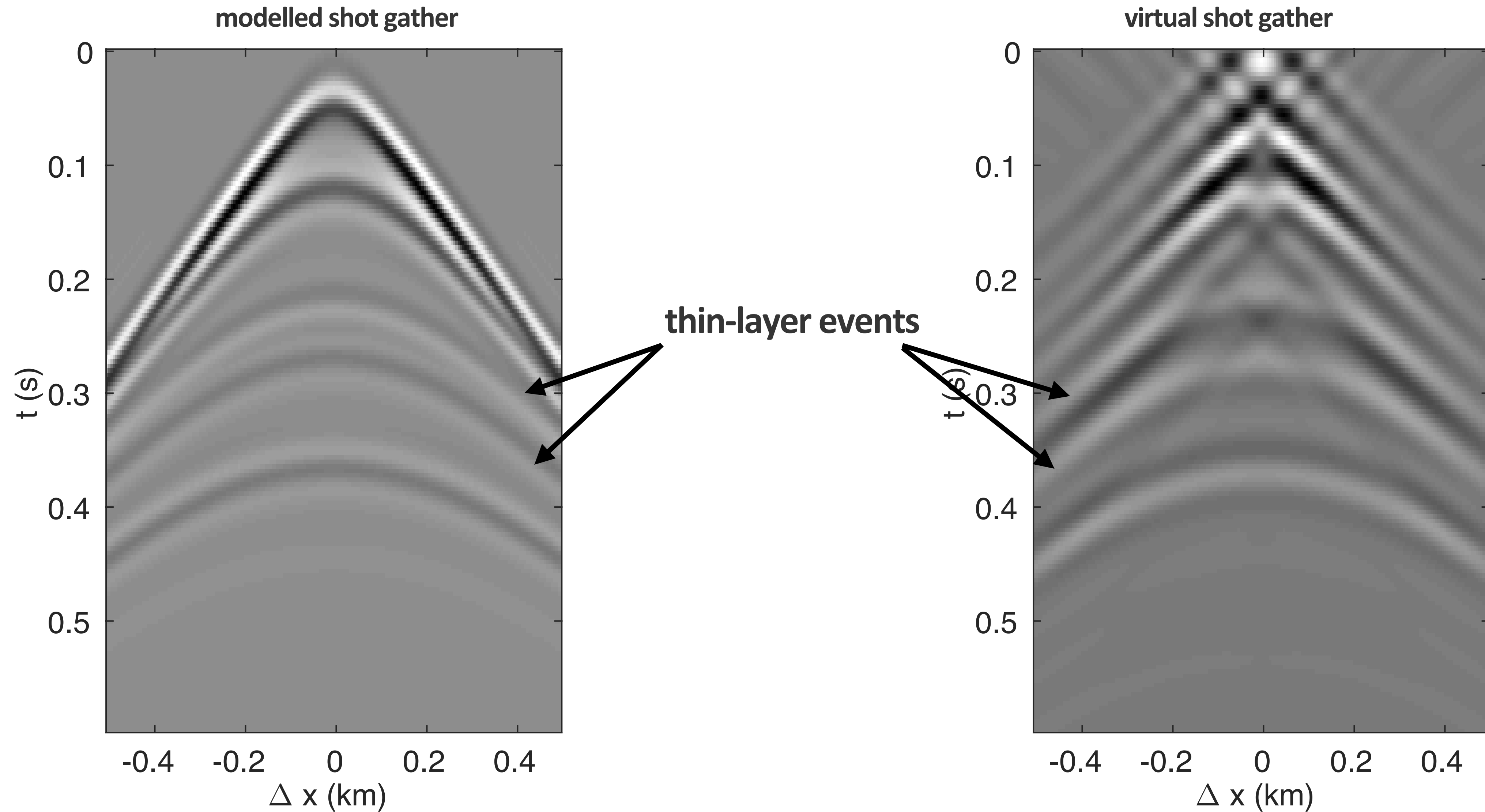


Inverse FFT



**computational time ~ 4 min (proposed) v/s 10 hrs (classical)**

# Layer model comparison (modelled versus virtual source)



# Extension to 3D

# Impediments

*No direct solver for Helmholtz*

- ▶ *iterative solver*

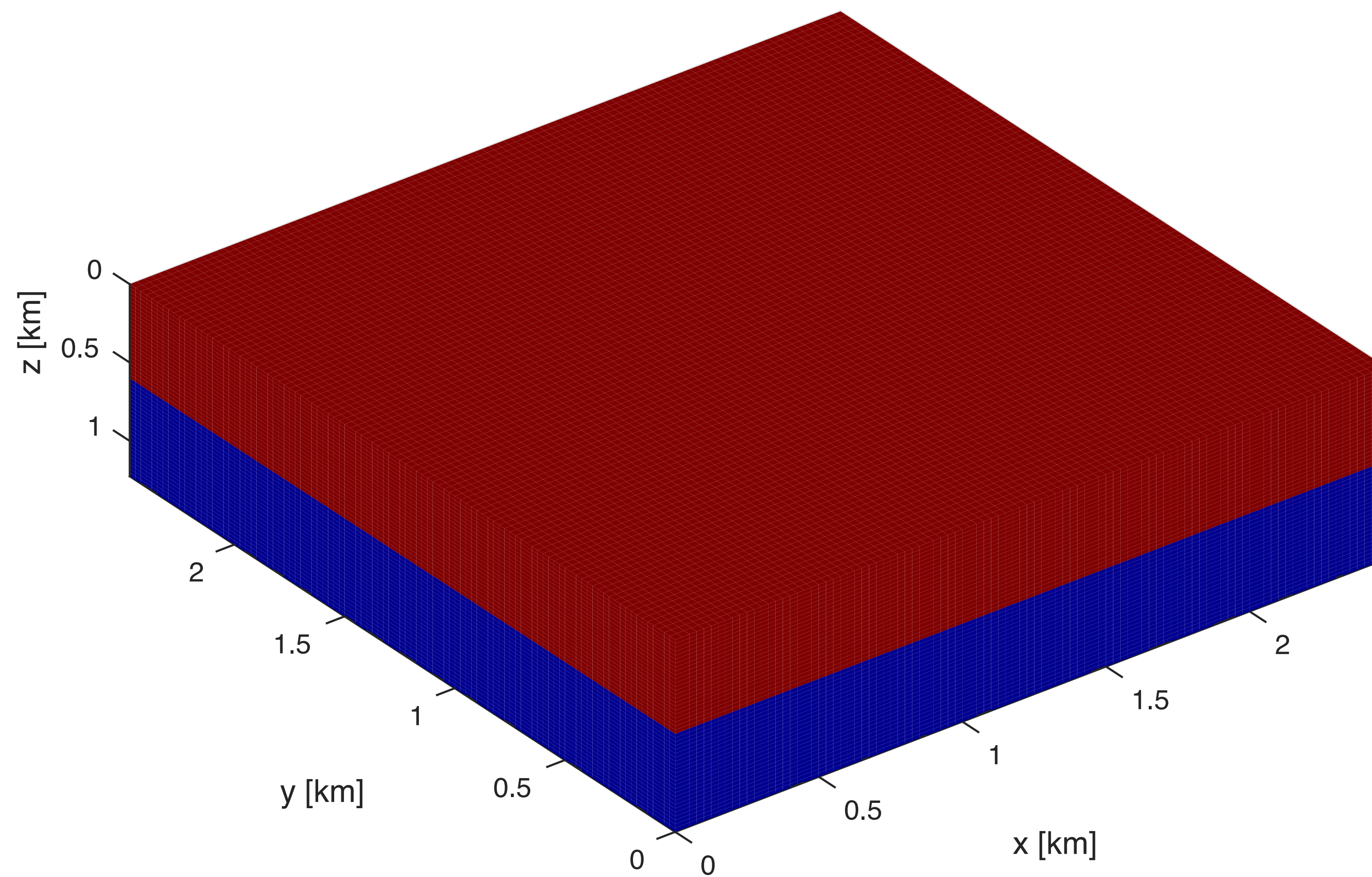
Limited budget

- ▶ in terms of # of wave-equation to solve

Practically impossible to form full subsurface image volume in 3D!



# I-Layer model



# Experimental details

2500 sources (50 m spacing) , 625 receivers (100 m spacing)

5-15 Hz , 0.5 Hz sampling

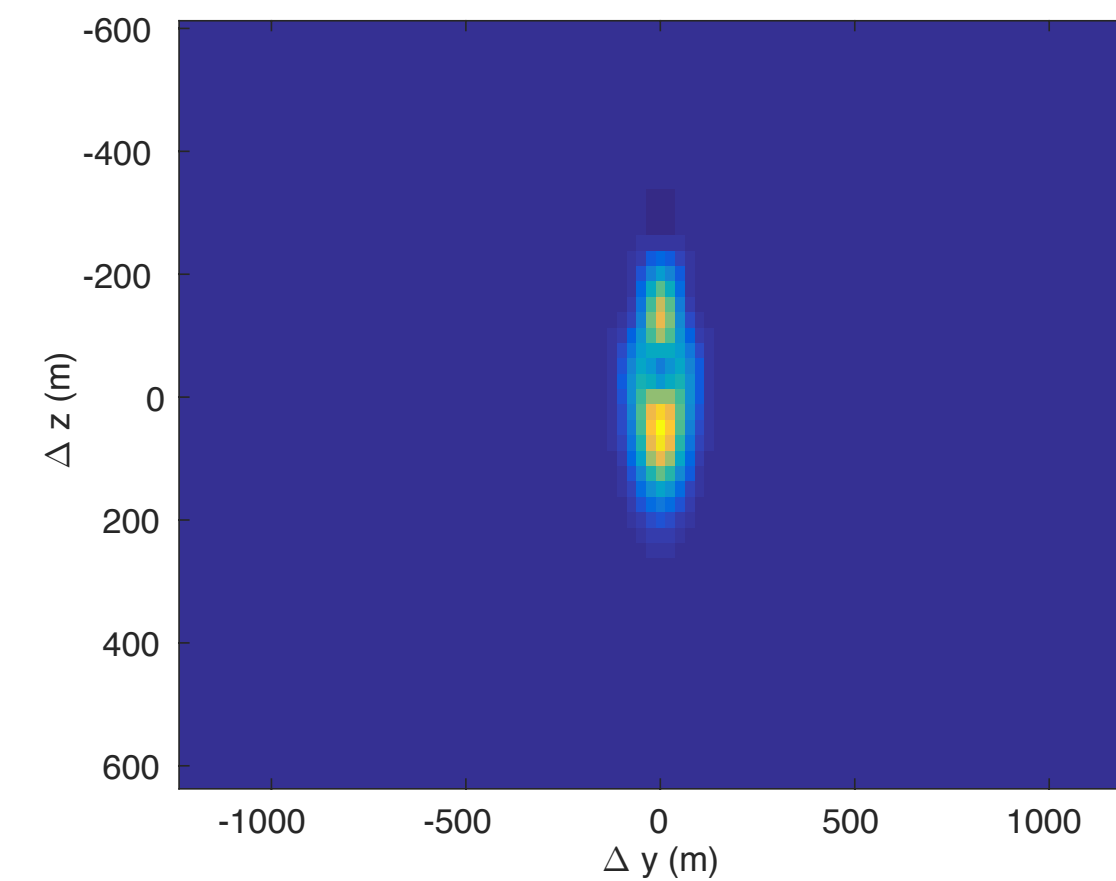
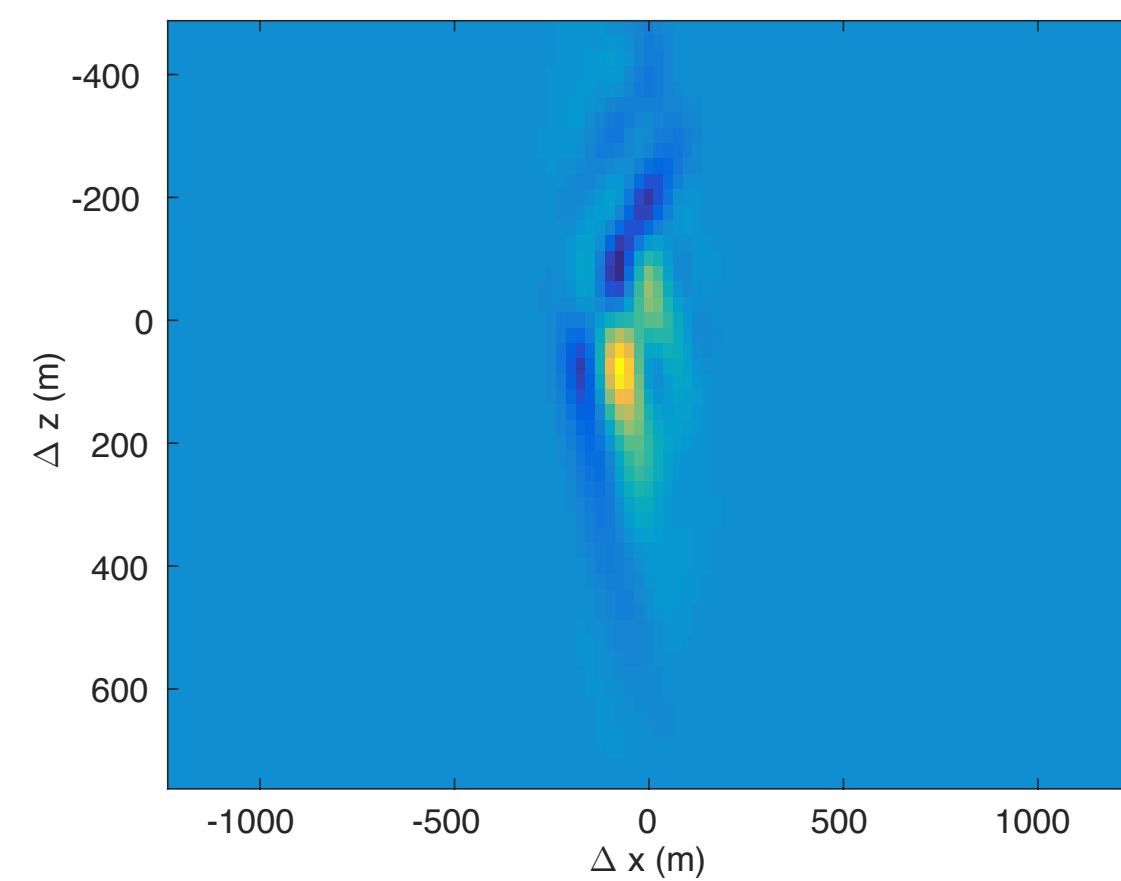
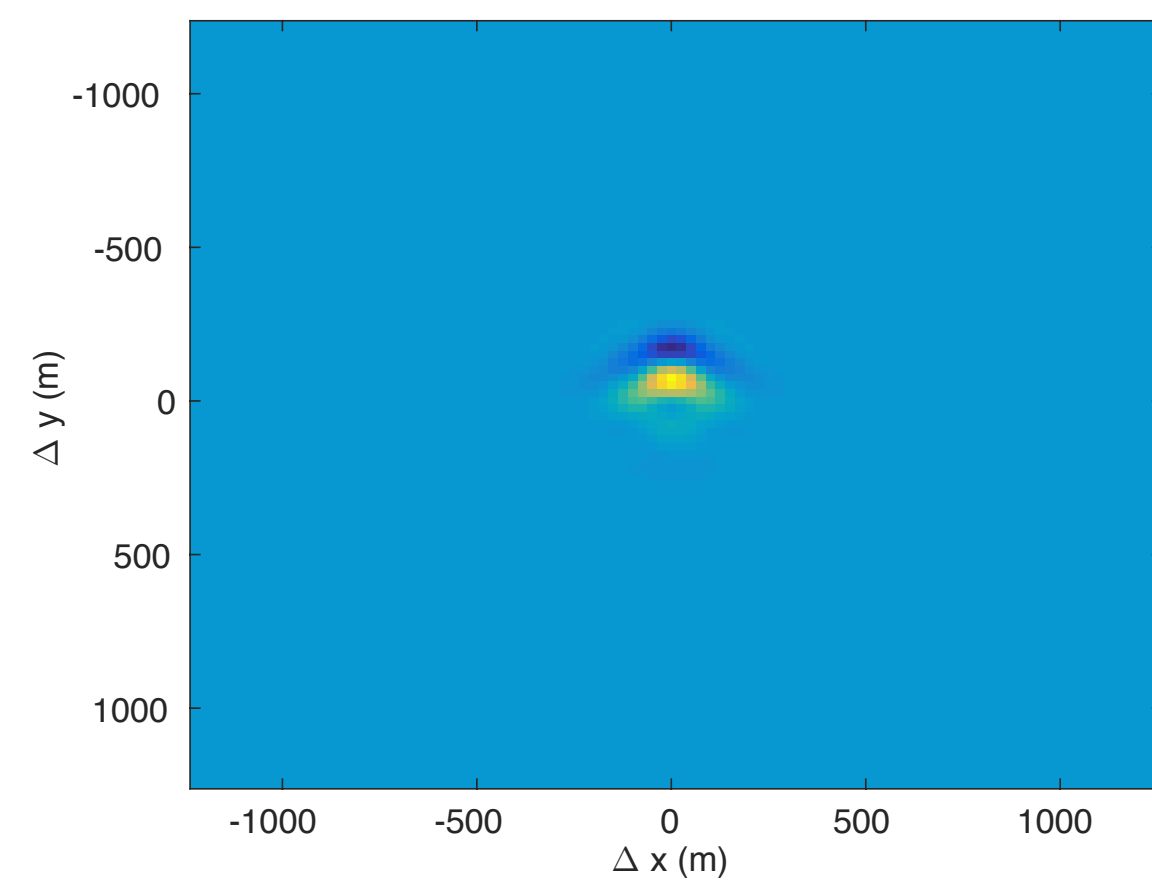
ocean-bottom acquisition node

peak frequency 10 Hz

3 node, 7 workers

# I-Layer model

## common image gather



**computational time ~ 45 min (proposed) v/s 47 days (classical)**

# Conclusions

*Probing*s make computation of full subsurface-offset image gathers in 3D affordable

Enabler for

- automatic velocity model building (WEMVA)
- forming of densely sampled subsurface image volumes
  - Least-squares extended imaging (reduce artifacts)
- AVO/AVP analyses
- targeted imaging

# Future Work

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Incorporate *free-surface multiple* in 3D

Extension to WEMVA in 3D

Extension to time-stepping

Target imaging in more complex environment

# Acknowledgements

Thank you for your attention !

<https://www.slim.eos.ubc.ca/>



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