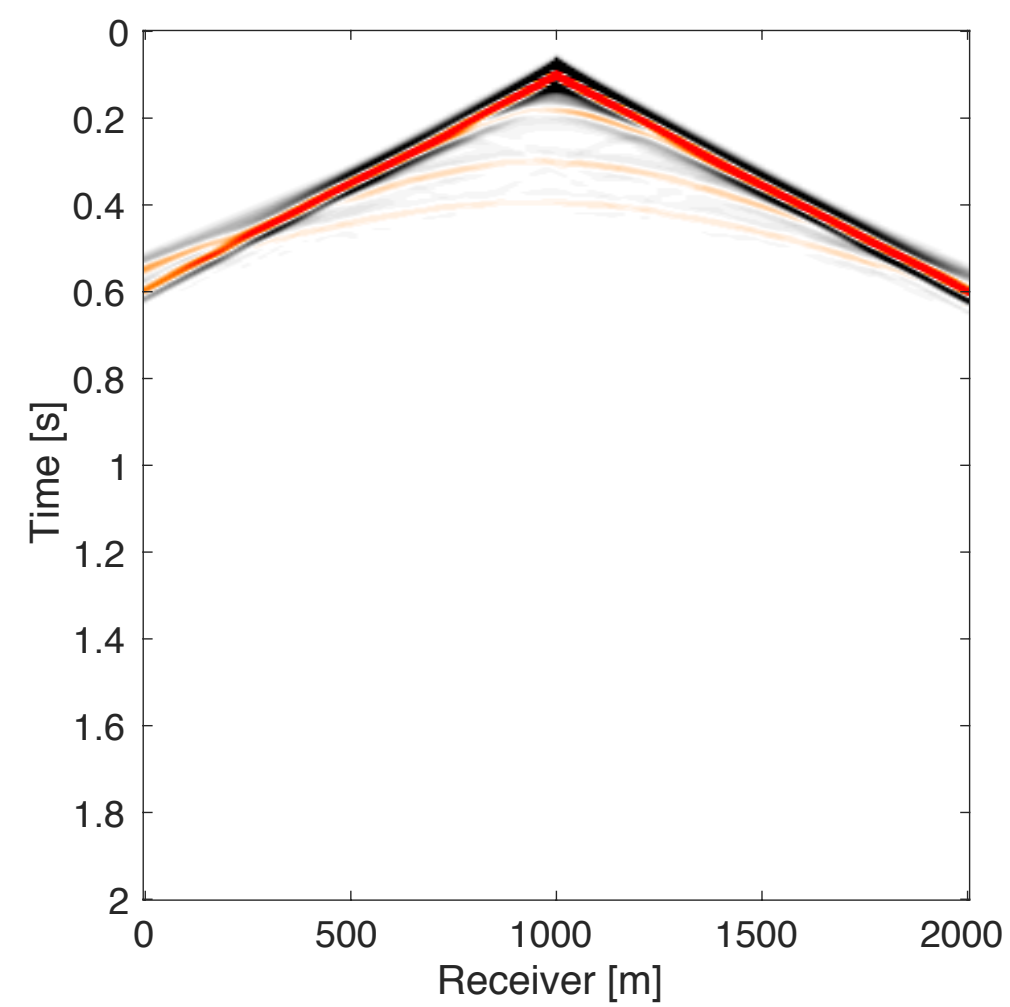


# Wavefield-reconstruction inversion with source estimation and minimum smoothness constraint - application to the Chevron 2014 dataset

Zhilong Fang, Bas Peters and Felix J. Herrmann

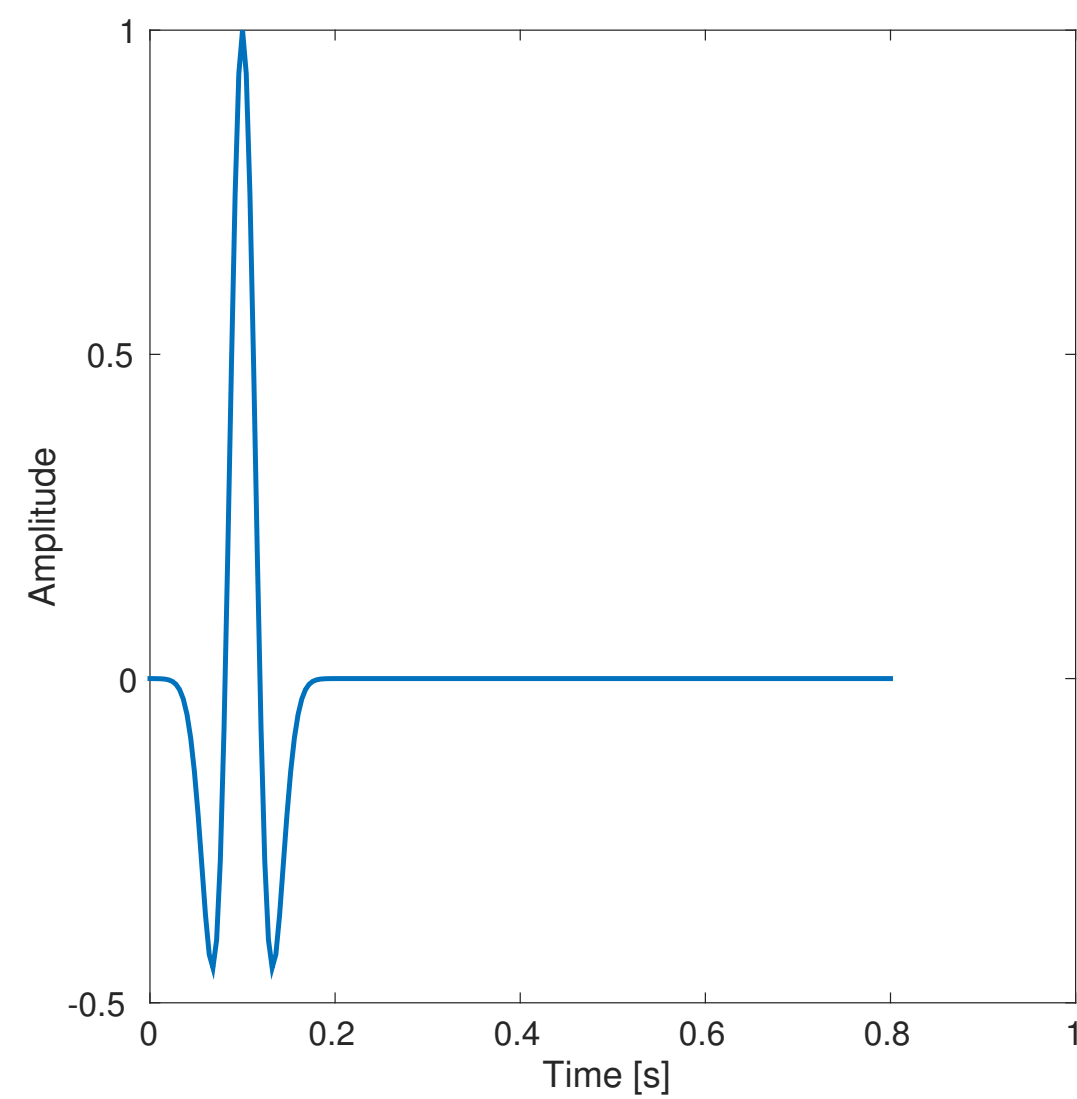


# Motivation



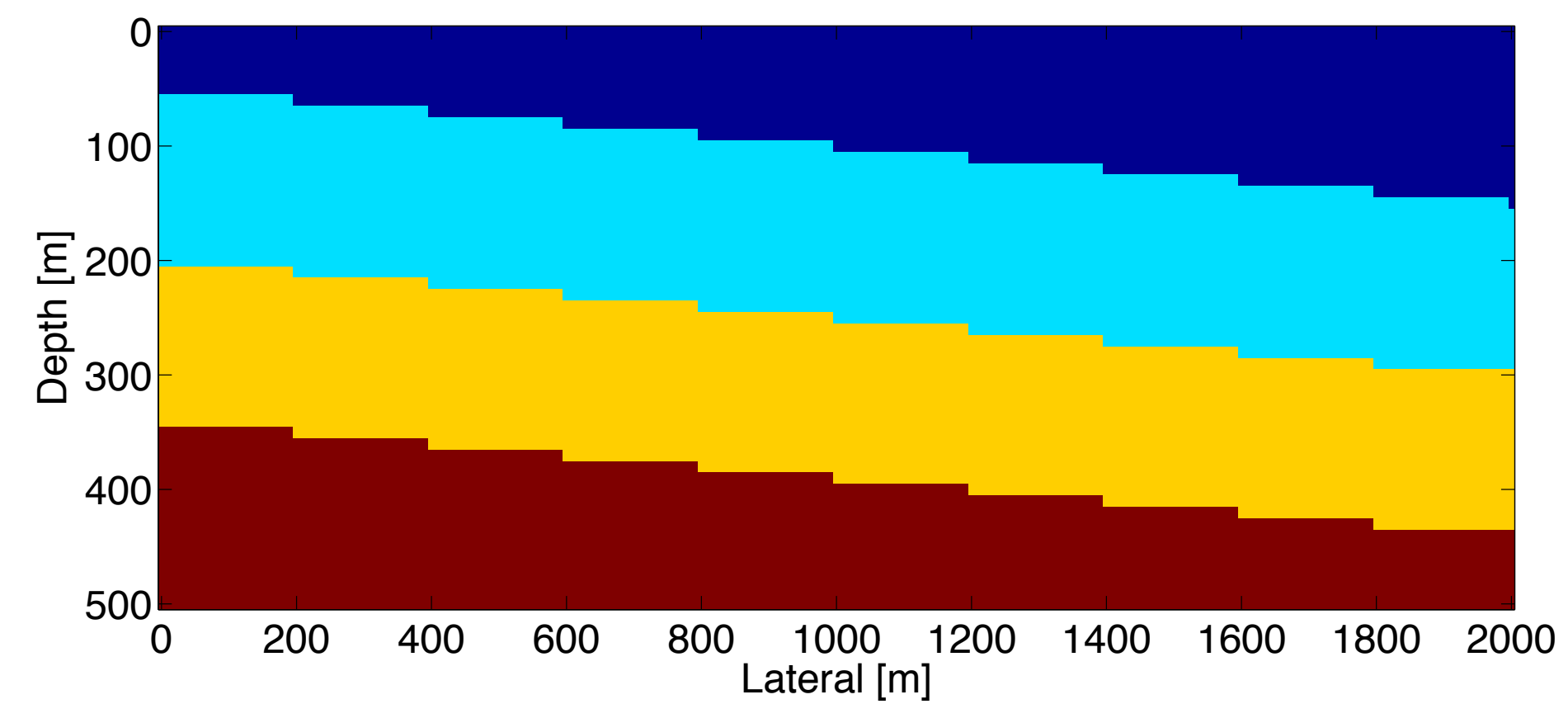
Data

=



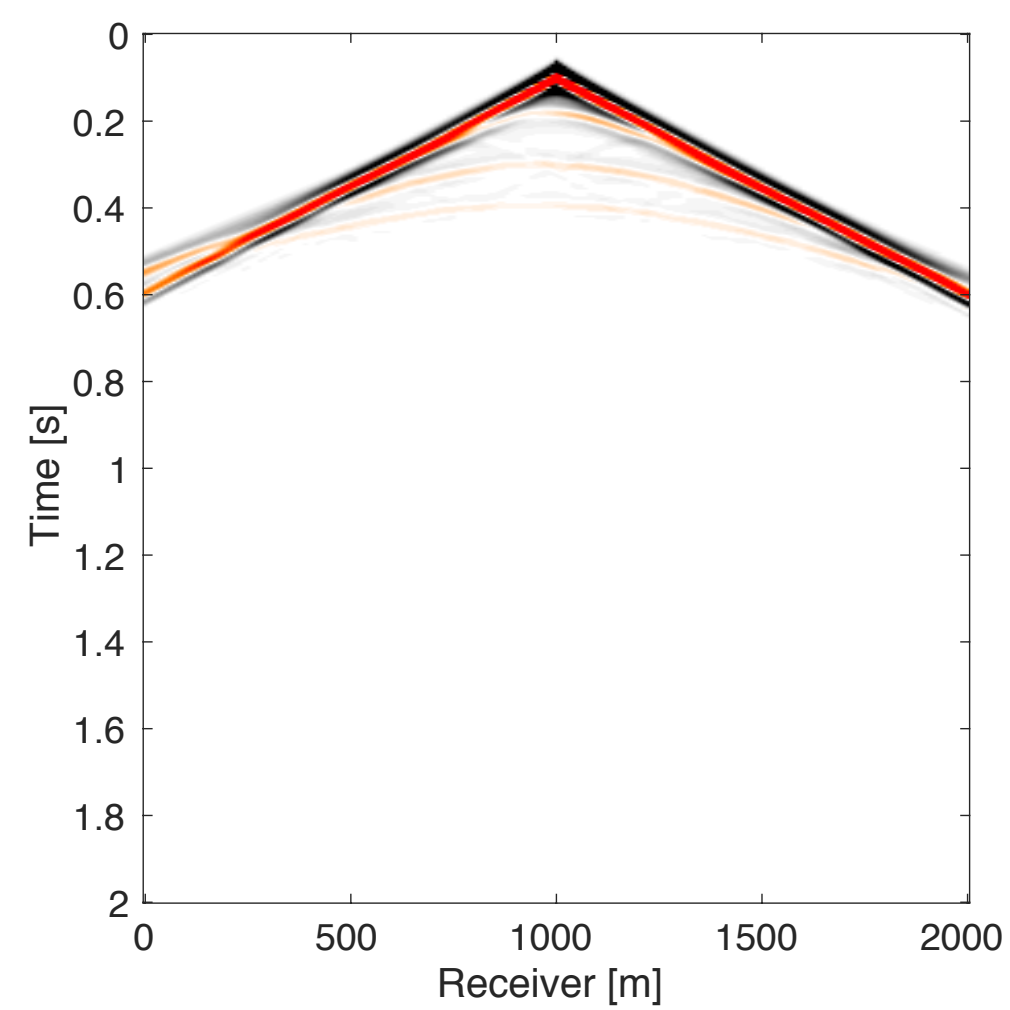
Source wavelet

\*



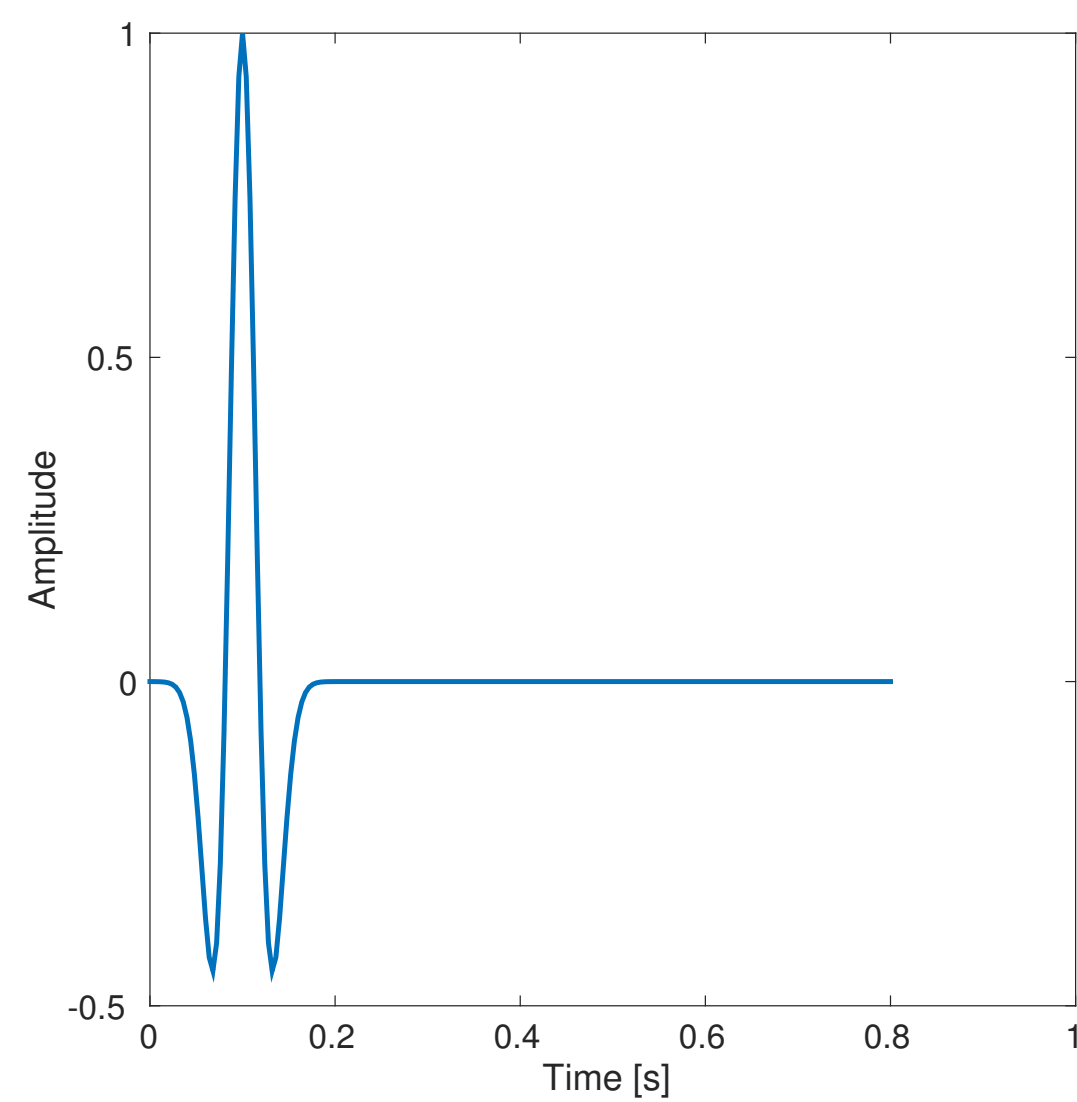
Velocity model

# Motivation

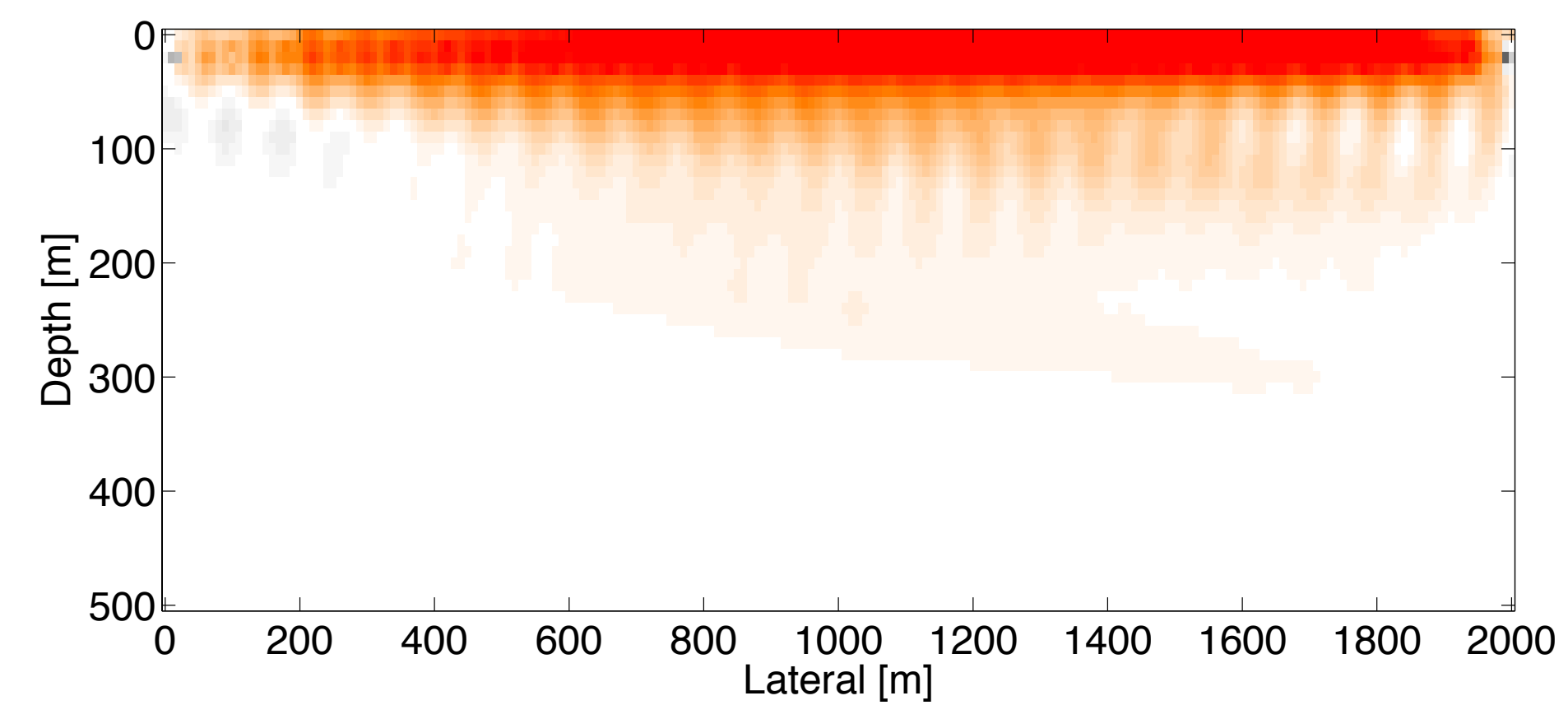
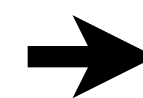


Data

+

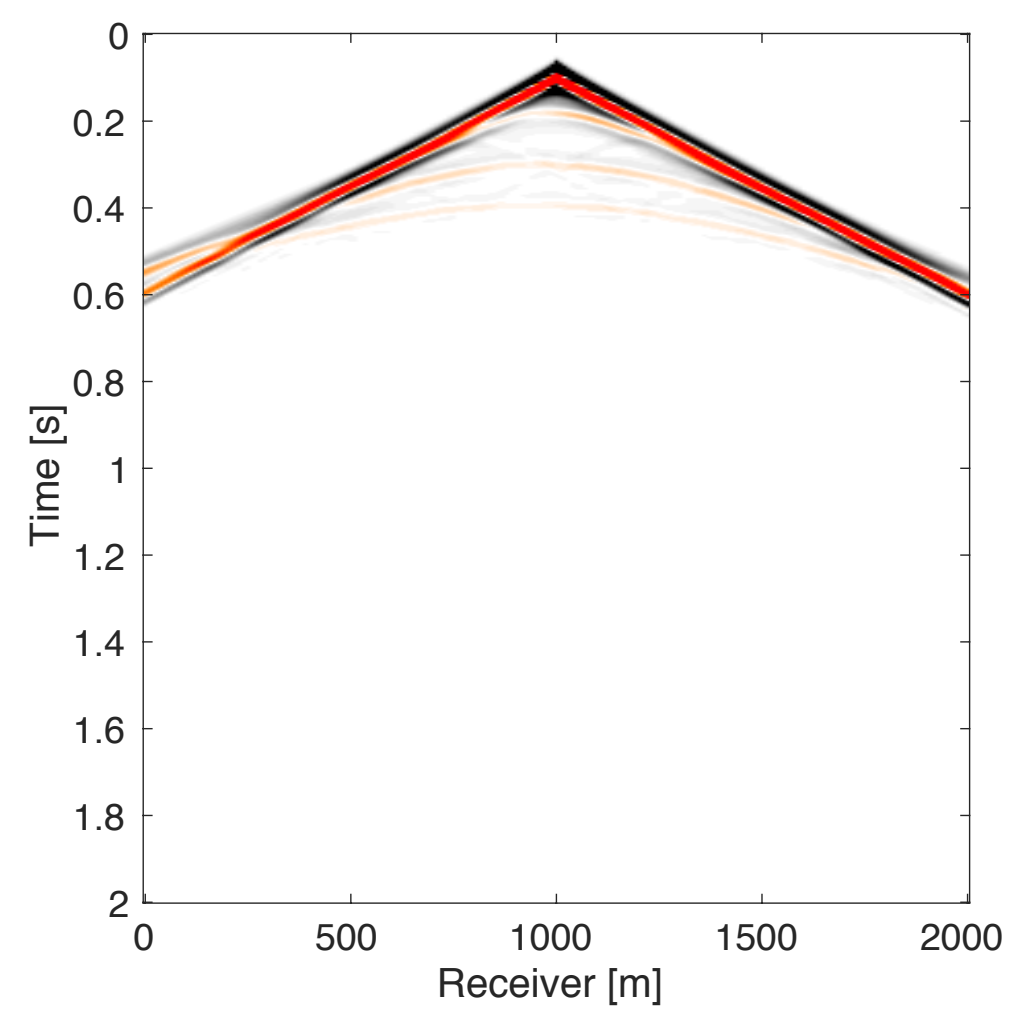


Correct source wavelet



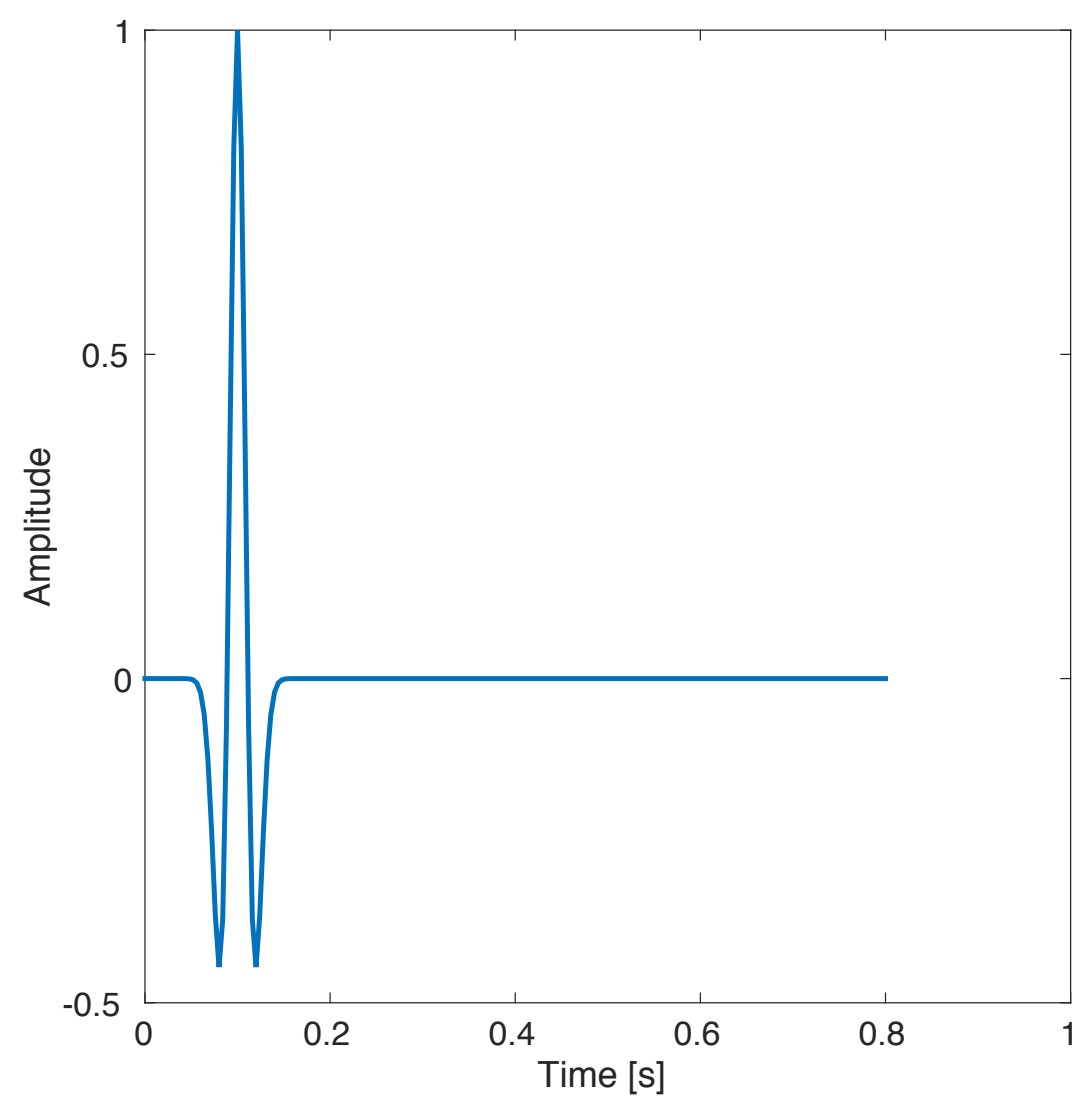
Correct gradient

# Motivation

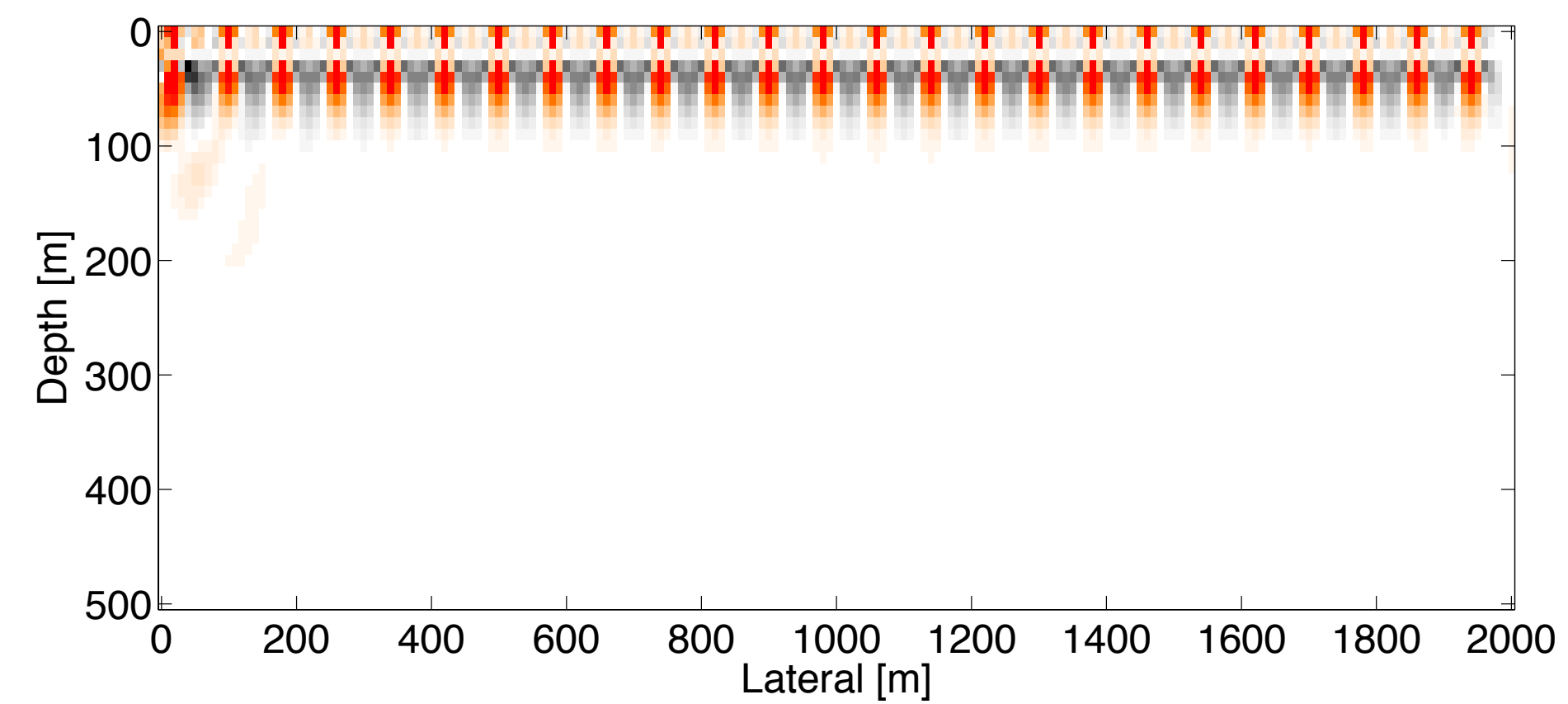
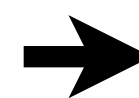


Data

+



Wrong source wavelet

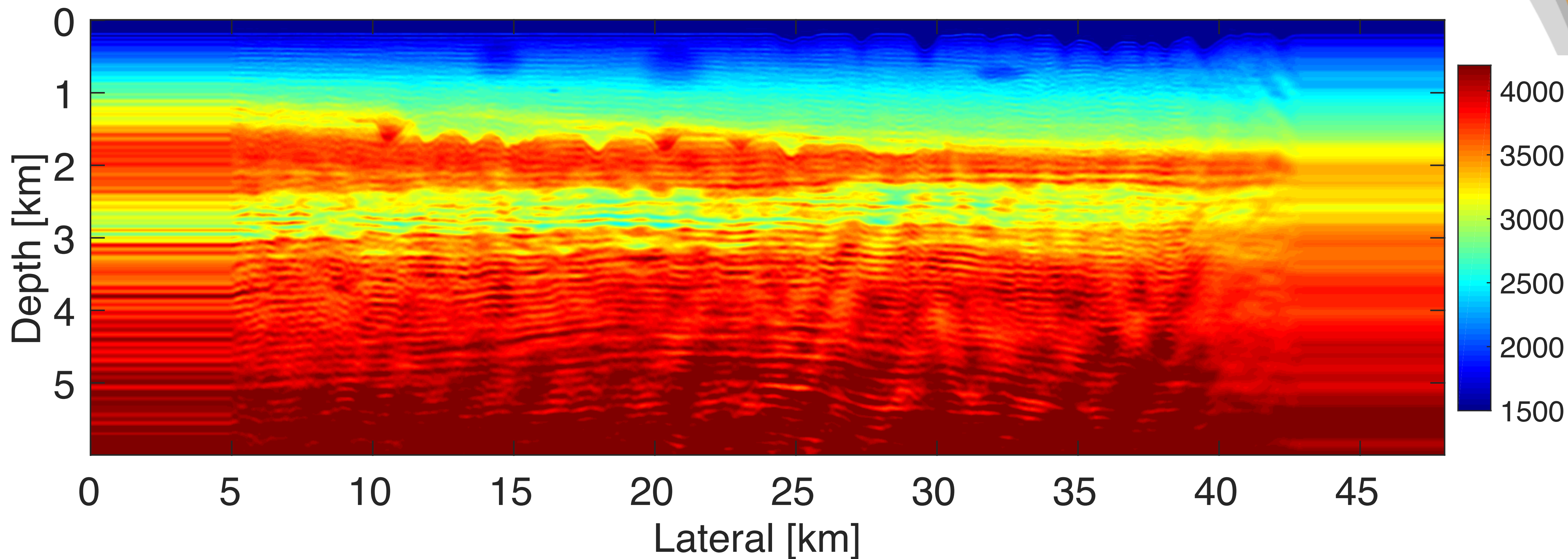


Wrong gradient



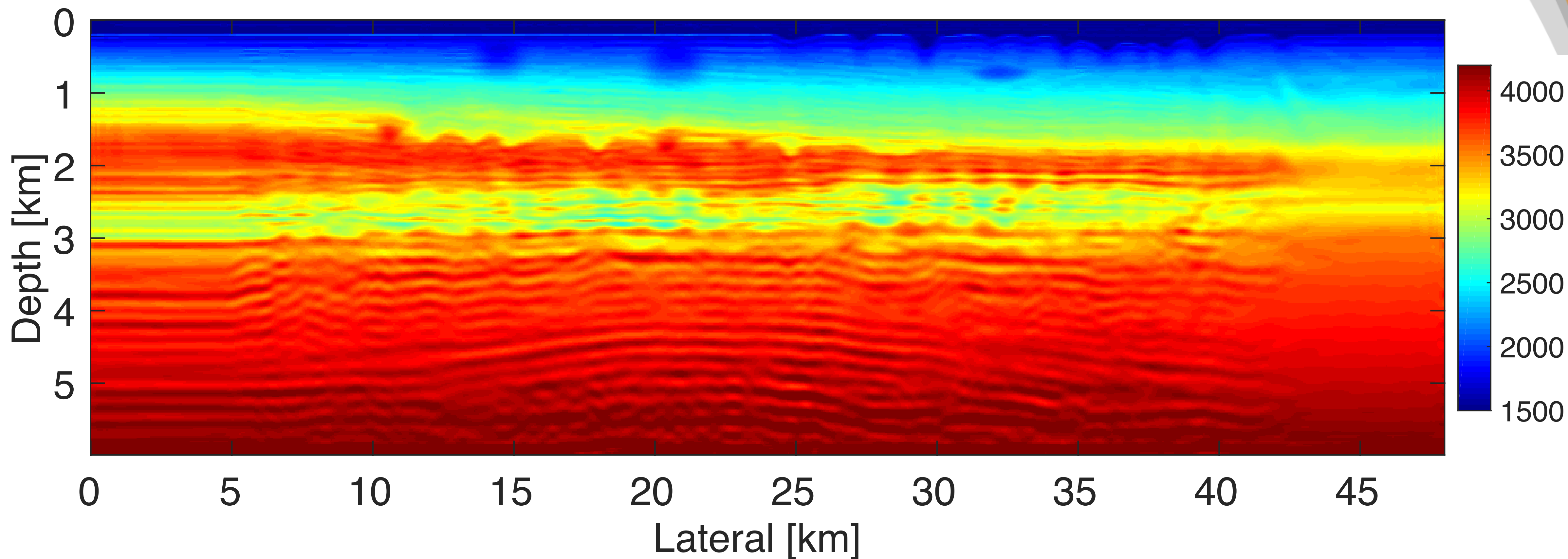
# Chevron blind test data

— Wavefield-reconstruction inversion with source estimation  
(presented in the EAGE 2015)



# Chevron blind test data

— Wavefield-reconstruction inversion with source estimation and minimum smoothness constraint





# Full-waveform inversion

Original problem:

$$\underset{\mathbf{u}, \mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

$$\text{subject to } \mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} = \mathbf{q}_{k,l},$$

where,

$\mathbf{u}_{k,l}$  – Wavefield of the  $k$ th shot at  $l$ th frequency

$\mathbf{d}_{k,l}$  – Observed data of the  $k$ th shot at  $l$ th frequency

$\mathbf{q}_{k,l}$  – Source of the  $k$ th shot at  $l$ th frequency

$\mathbf{A}_{k,l}$  – Helmholtz of the  $k$ th shot at  $l$ th frequency

$\mathbf{P}_k$  – Receiver projection operator of the  $k$ th shot

$\mathbf{m}$  – Squared-slowness

# Full-waveform inversion

Reduced/adjoint-state method:

$$\underset{\mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1} \mathbf{q}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

with the gradient given by

$$\mathbf{g} = \sum_{k,l} \mathbf{u}_{k,l}^* \frac{\partial \mathbf{A}_{k,l}^*}{\partial \mathbf{m}} \mathbf{v}_{k,l}$$

$$\mathbf{u}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m})^{-1} \mathbf{q}_{k,l}$$

$$\mathbf{v}_{k,l} = \mathbf{A}_{k,l}^{-*}(\mathbf{m}) \mathbf{P}_k^* \mathbf{r}_{k,l}$$

$$\mathbf{r}_{k,l} = \mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1} \mathbf{q}_{k,l} - \mathbf{d}_{k,l}$$

**2 PDE solves are required !**



## Wavefield-reconstruction inversion

Joint optimization problem:

$$\underset{\mathbf{u}, \mathbf{m}}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

Eliminate  $\mathbf{u}$  w/ variable projection:

$$\bar{\mathbf{u}} = \arg \min_{\mathbf{u}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \mathbf{q}_{k,l}\|_2^2$$

## Wavefield-reconstruction inversion

Corresponds to solving the following augmented system:

$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \bar{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

with the gradient

**1 augmented system solves is required !**

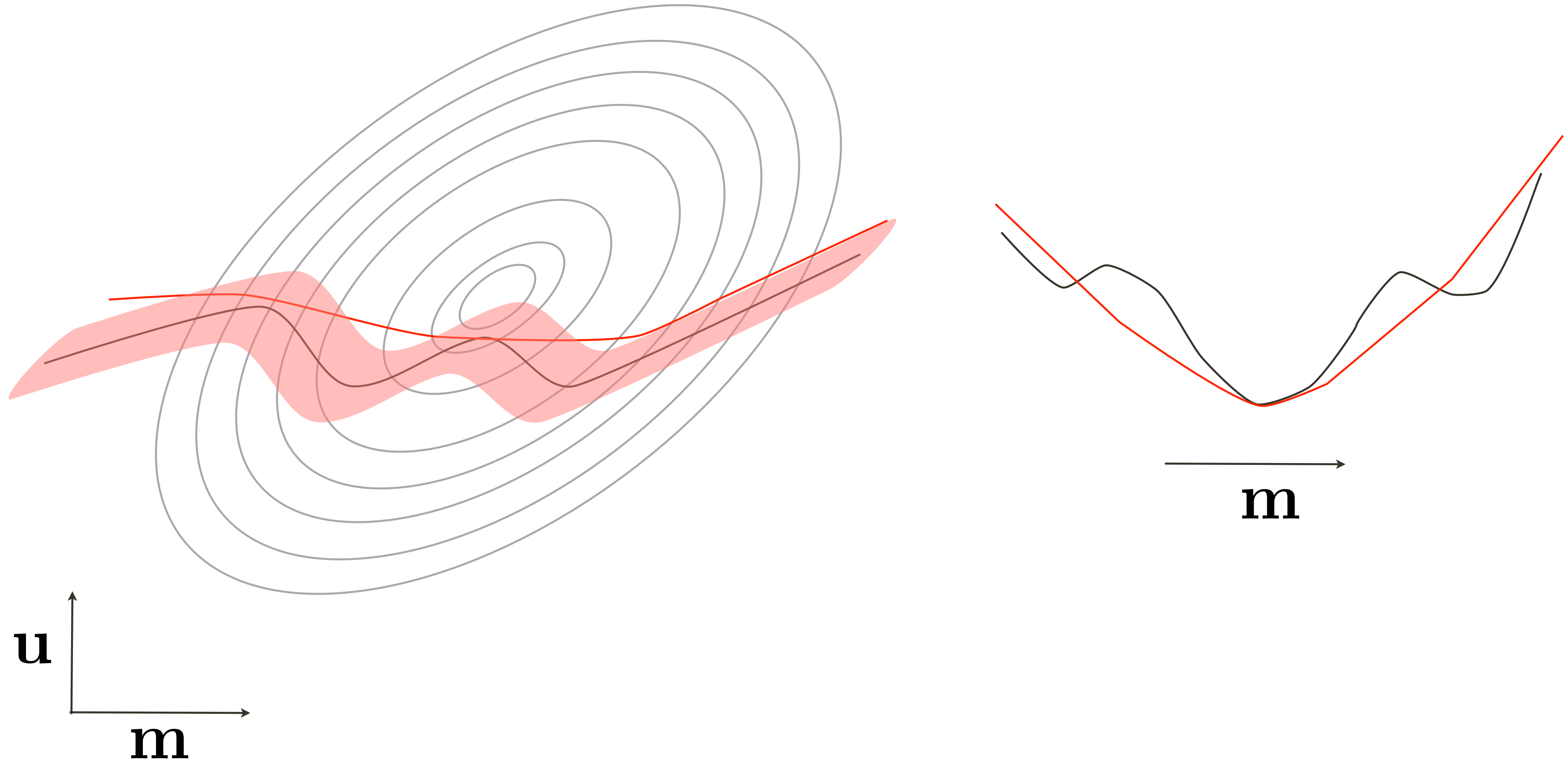
$$\mathbf{g} = \sum_{k,l} \bar{\mathbf{u}}_{k,l}^* \frac{\partial \mathbf{A}_{k,l}^*}{\partial \mathbf{m}} \bar{\mathbf{v}}_{k,l}$$

$$\bar{\mathbf{v}}_{k,l} = \mathbf{A}_{k,l}(\mathbf{m}) \bar{\mathbf{u}}_{k,l} - \mathbf{q}_{k,l}$$

# WRI vs. FWI

[van Leeuwen, T and Herrmann, F J , 2013]

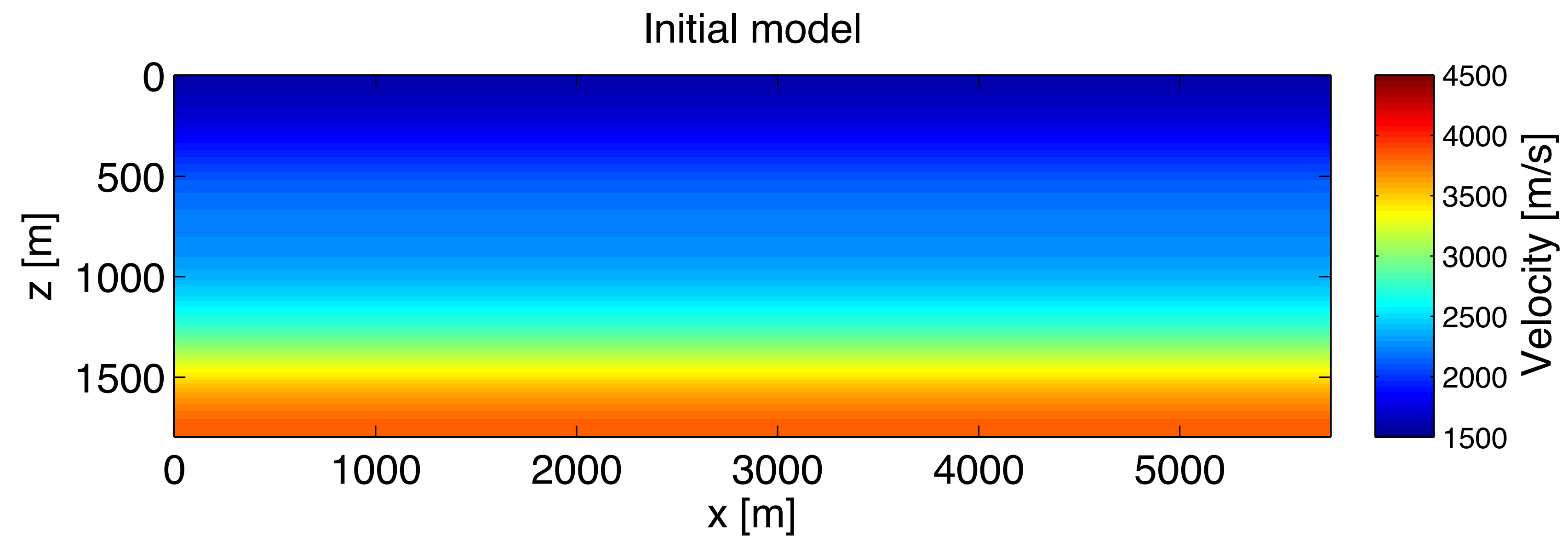
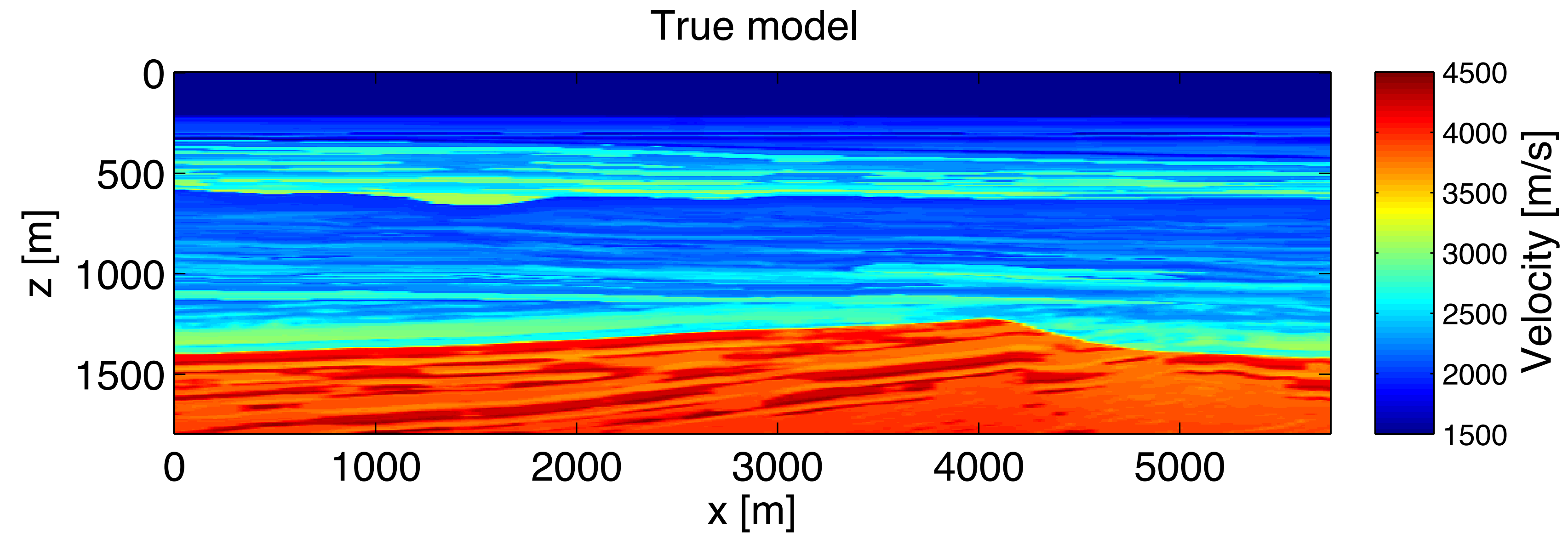
[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]



# True & initial model

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]

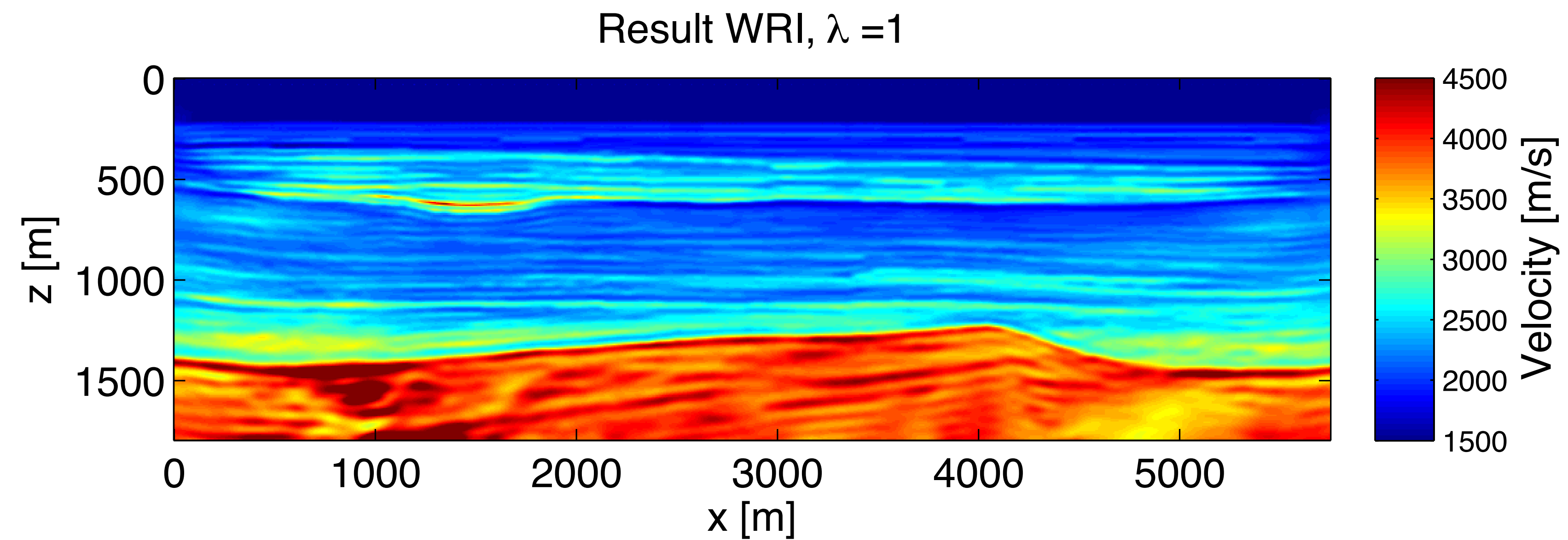
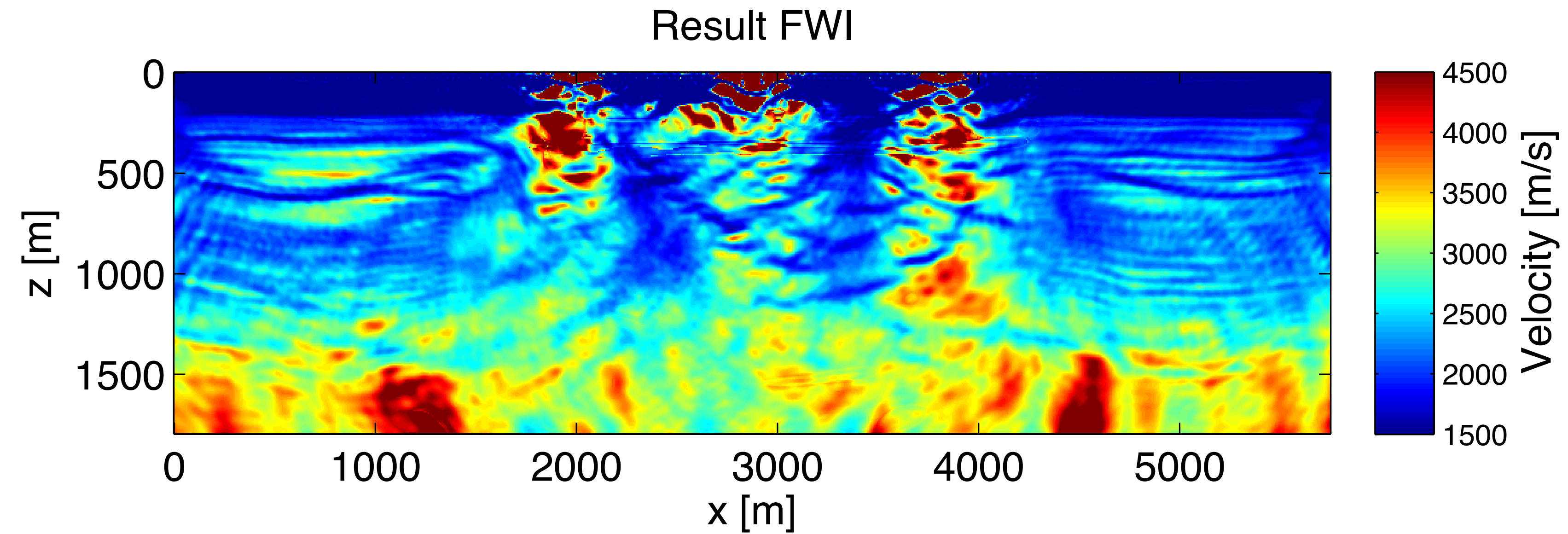




# FWI vs WRI

[van Leeuwen, T and Herrmann, F J , 2013]

[Peters, B, Herrmann, F J and van Leeuwen, T, 2014]



## WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u}, \mathbf{m}, \alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

## FWI with source estimation

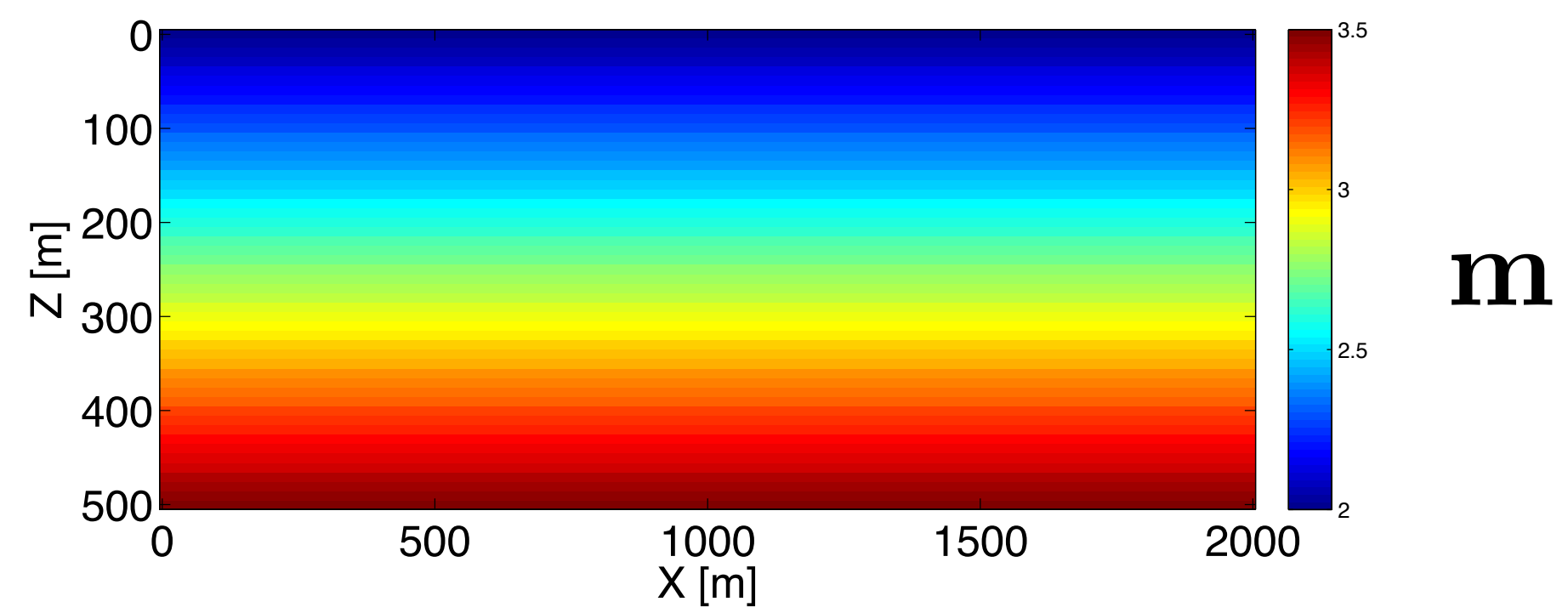
Joint optimization problem:

$$\underset{\mathbf{m}, \alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1} \alpha_{k,l} \mathbf{e}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

Eliminate  $\alpha$  w/ variable projection:

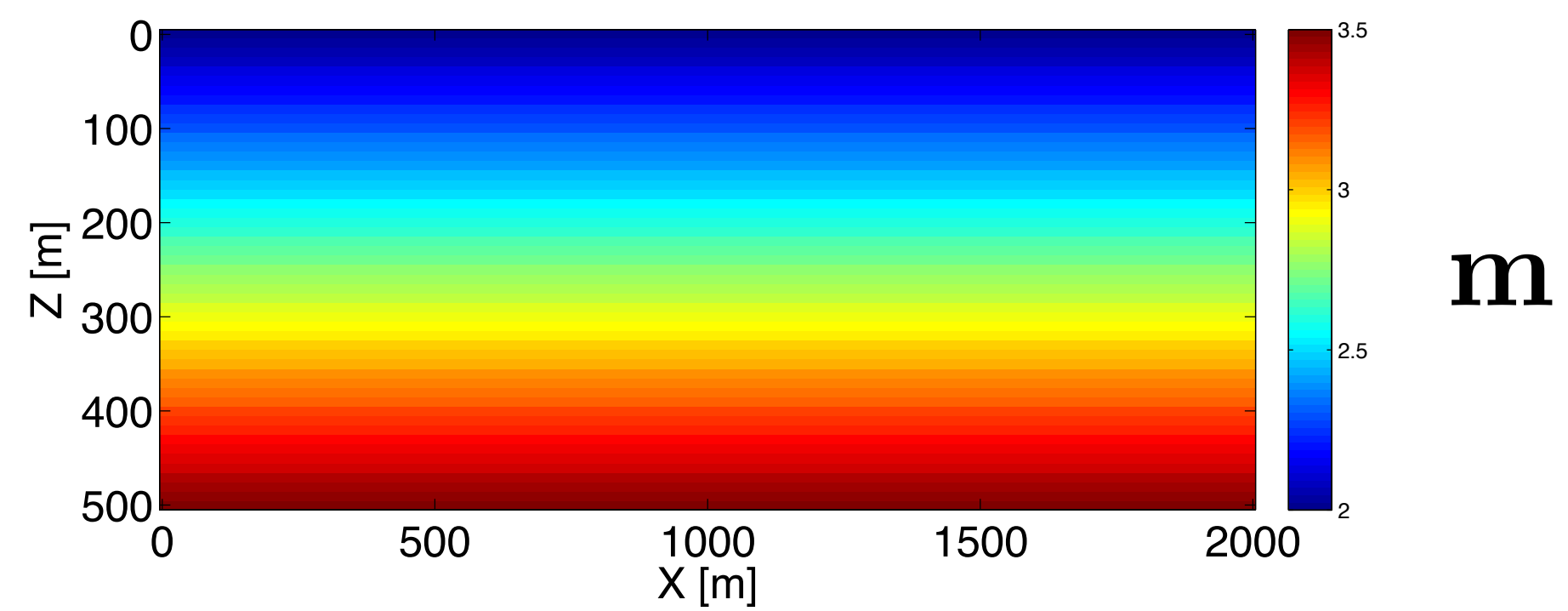
$$\bar{\alpha} = \arg \min_{\alpha} \sum_{k,l} \|\mathbf{P}_k \mathbf{A}_{k,l}(\mathbf{m})^{-1} \alpha_{k,l} \mathbf{e}_{k,l} - \mathbf{d}_{k,l}\|_2^2$$

# FWI with source estimation





# FWI with source estimation

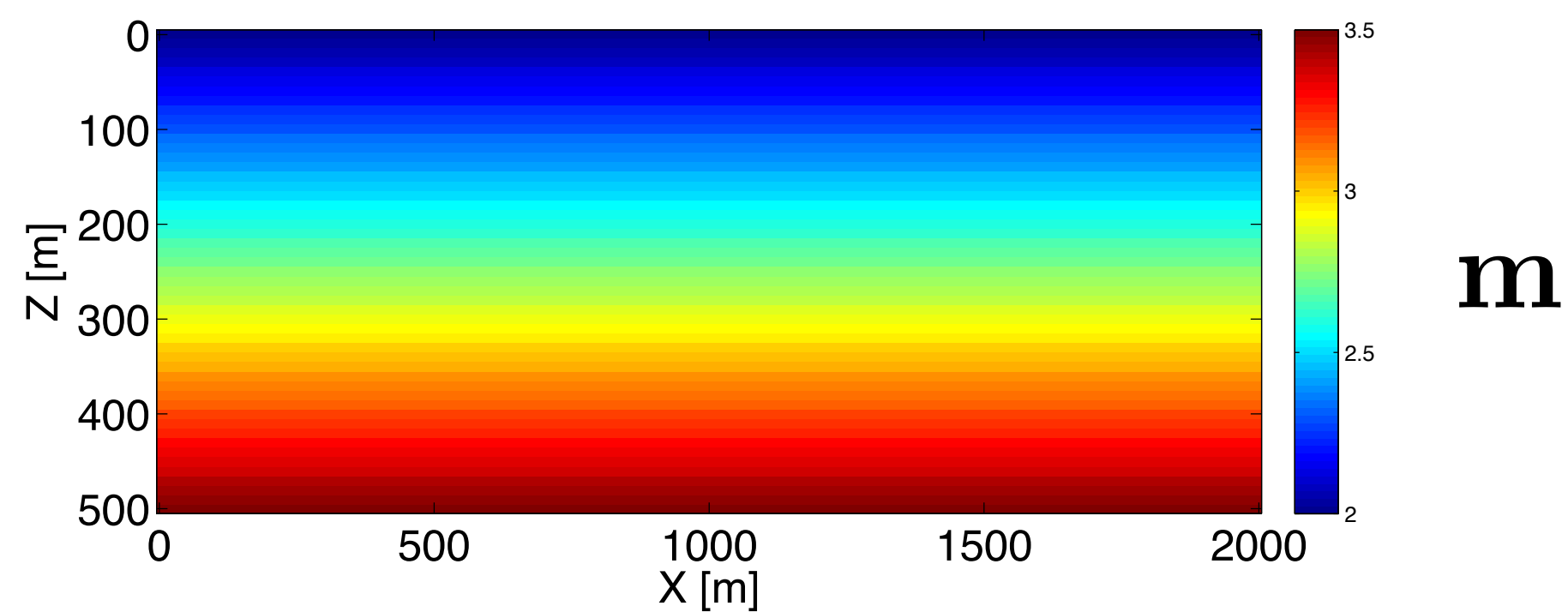


$$\mathbf{u} = \mathbf{A}(\mathbf{m})^{-1} \mathbf{e}$$

↓

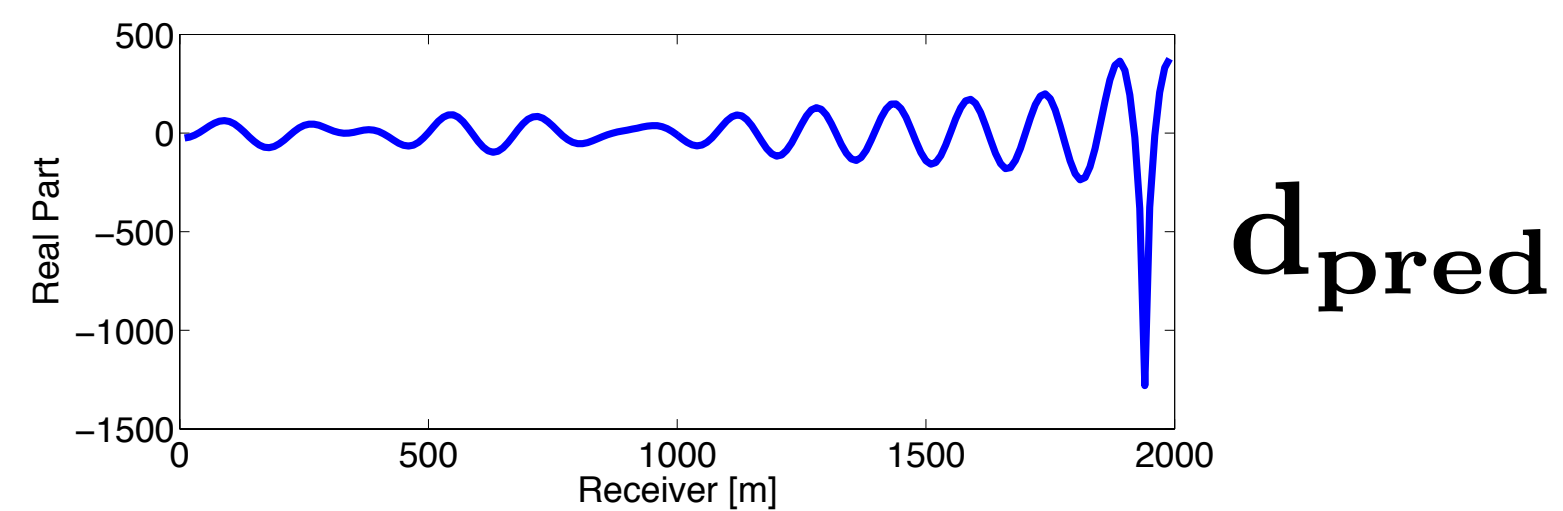
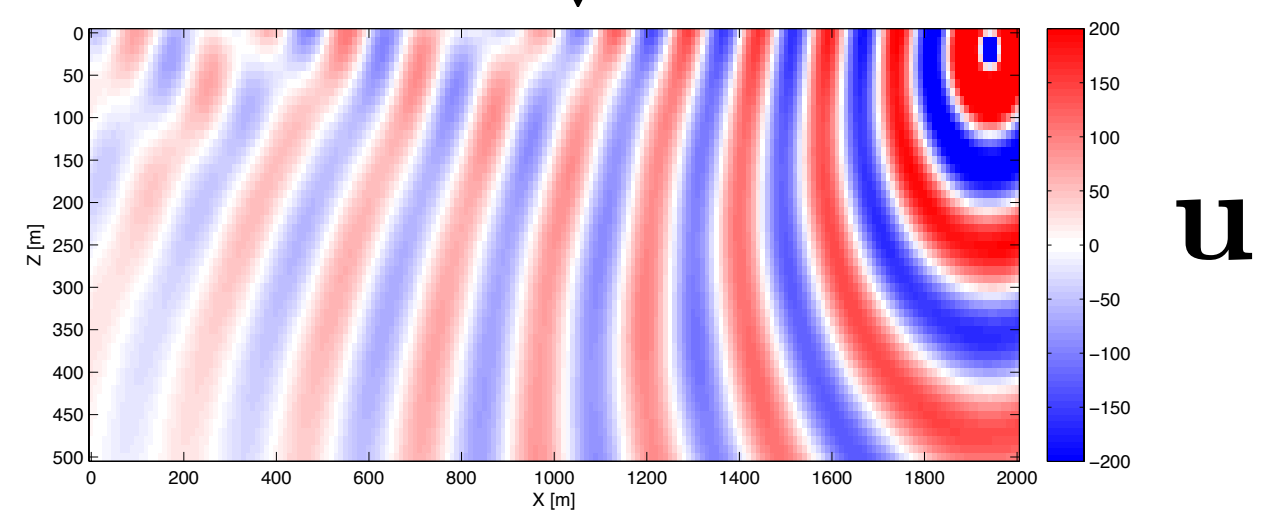
$$\mathbf{d}_{\text{pred}} = \mathbf{P} \mathbf{u}$$

# FWI with source estimation

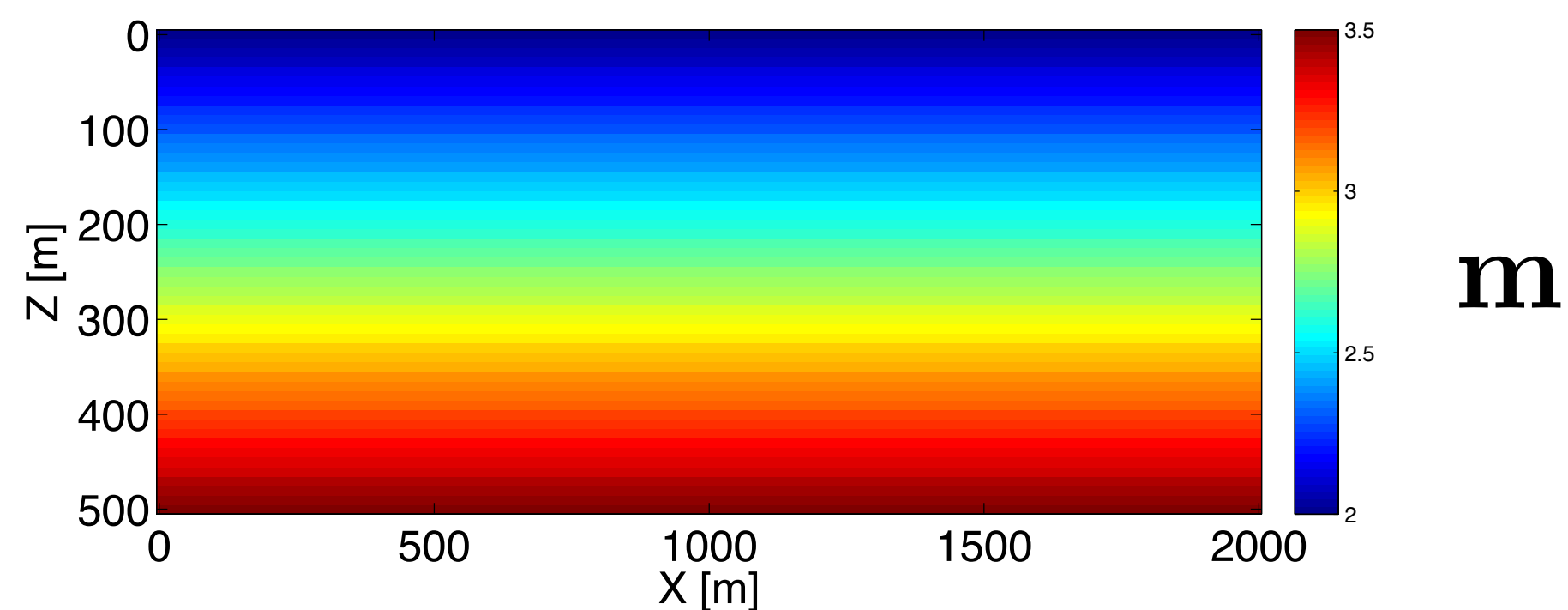


$$\mathbf{u} = \mathbf{A}(\mathbf{m})^{-1} \mathbf{e}$$

$$\mathbf{d}_{\text{pred}} = \mathbf{P} \mathbf{u}$$

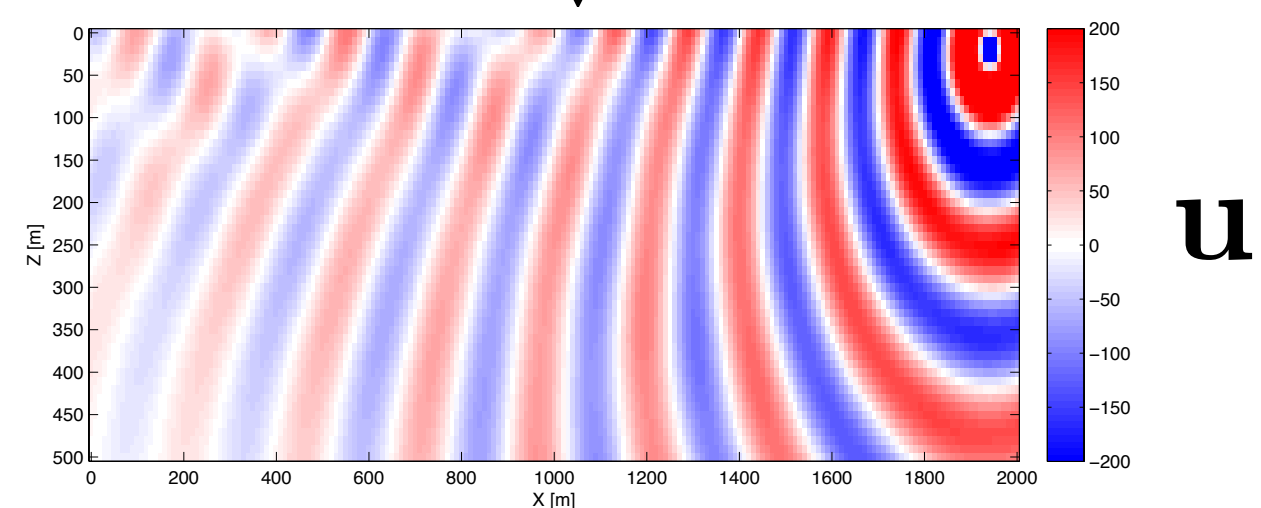


# FWI with source estimation

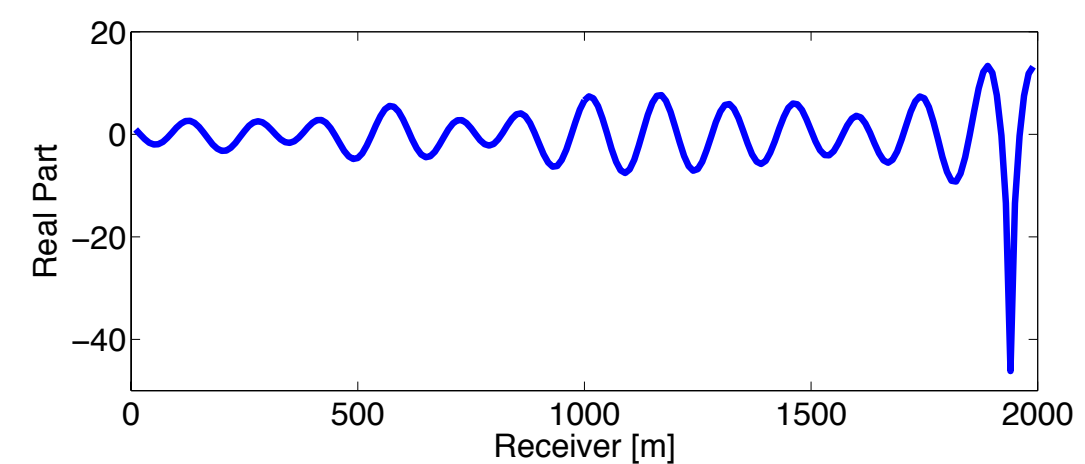
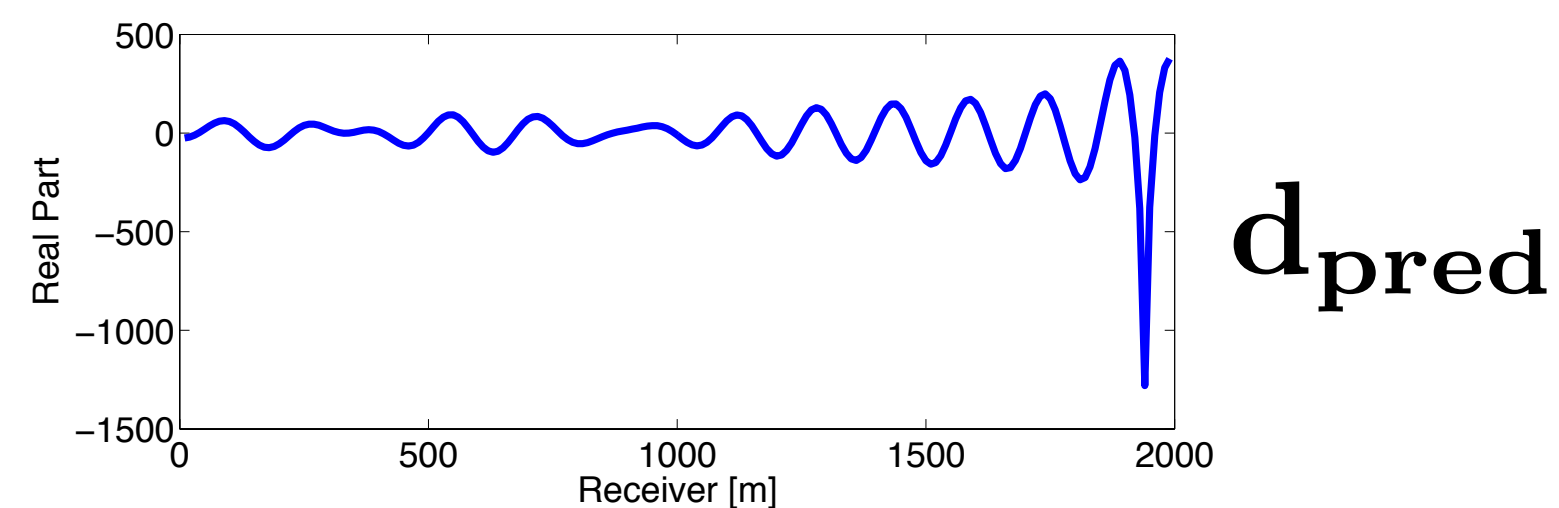


$$\mathbf{u} = \mathbf{A}(\mathbf{m})^{-1} \mathbf{e}$$

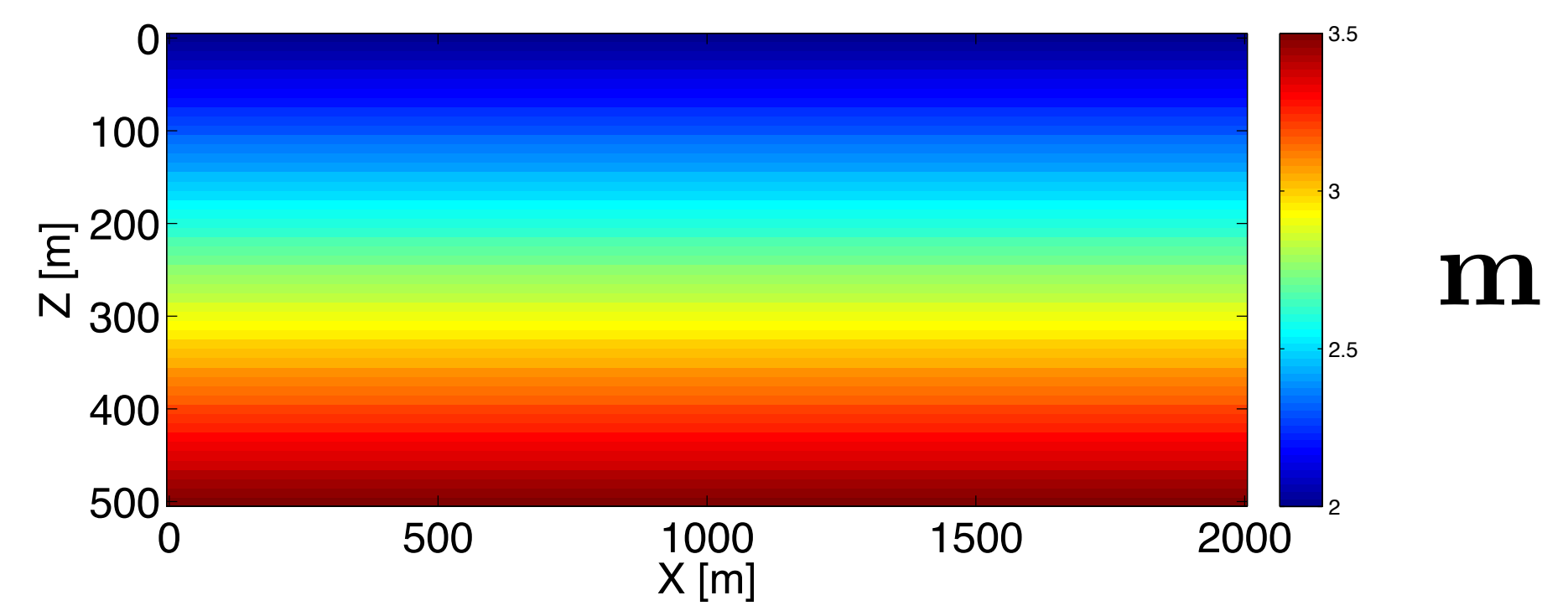
$$\mathbf{d}_{\text{pred}} = \mathbf{P} \mathbf{u}$$



$$\bar{\alpha} = \frac{\mathbf{d}_{\text{pred}}^T \mathbf{d}_{\text{obs}}}{\mathbf{d}_{\text{pred}}^T \mathbf{d}_{\text{pred}}}$$

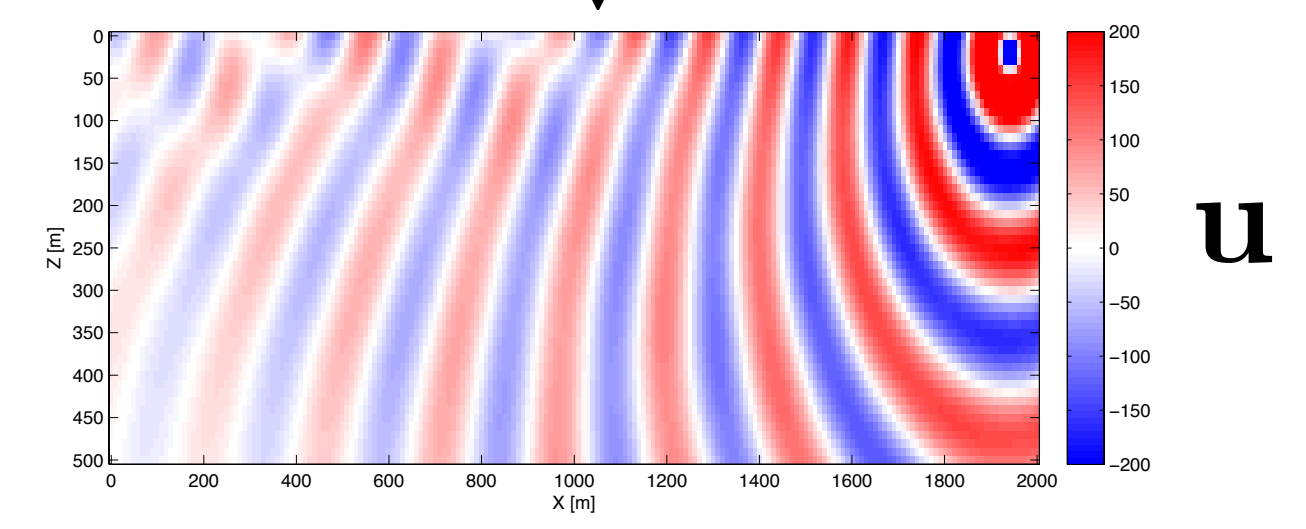


# FWI with source estimation

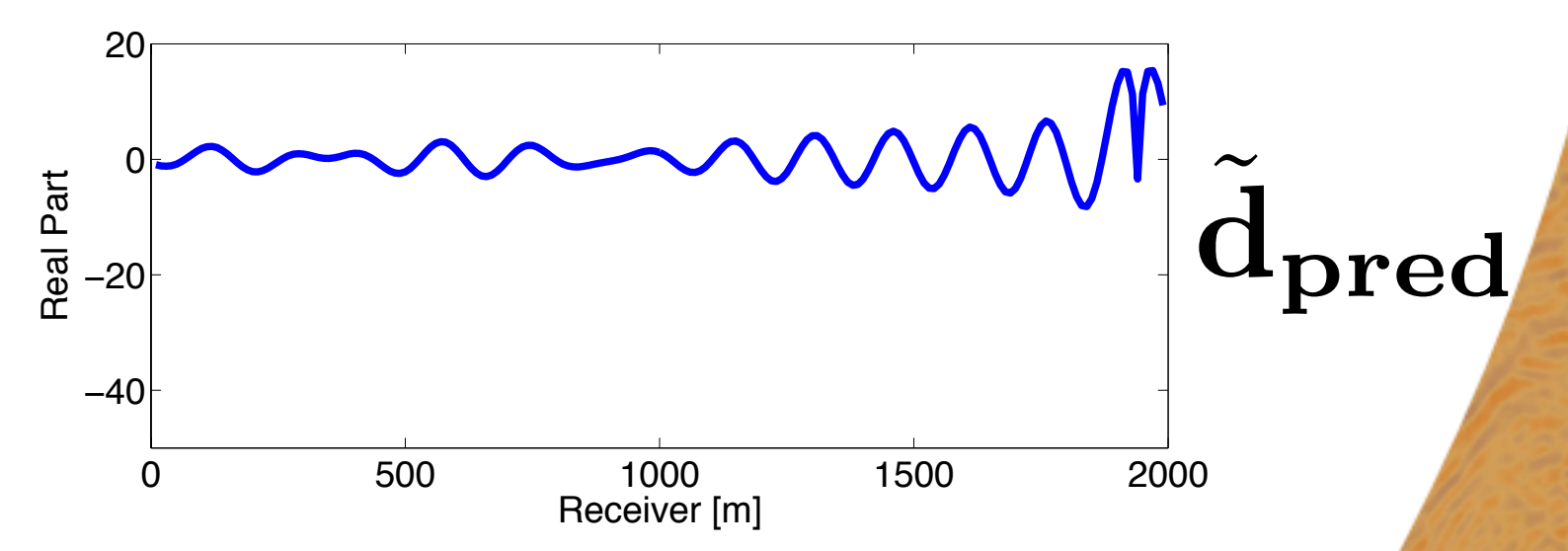
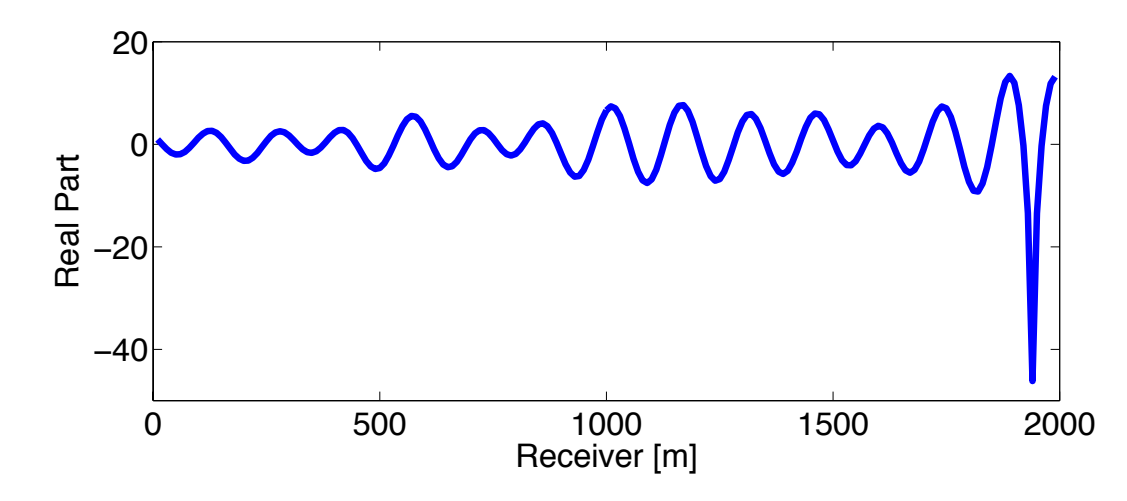
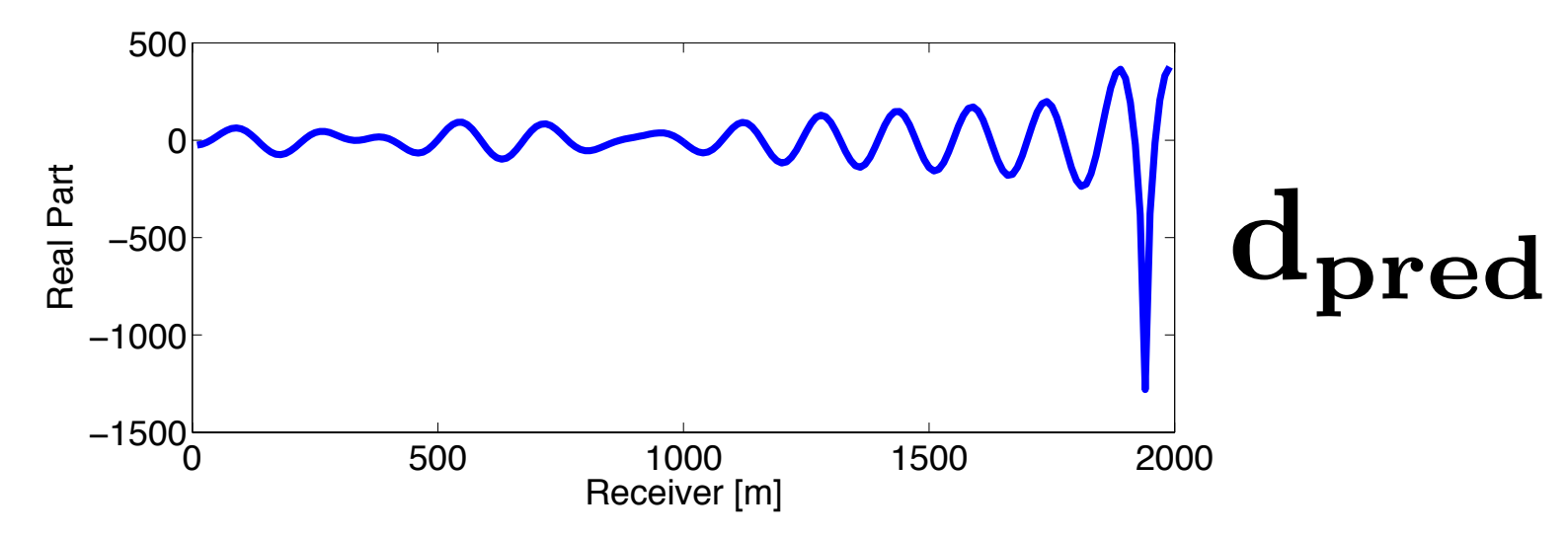
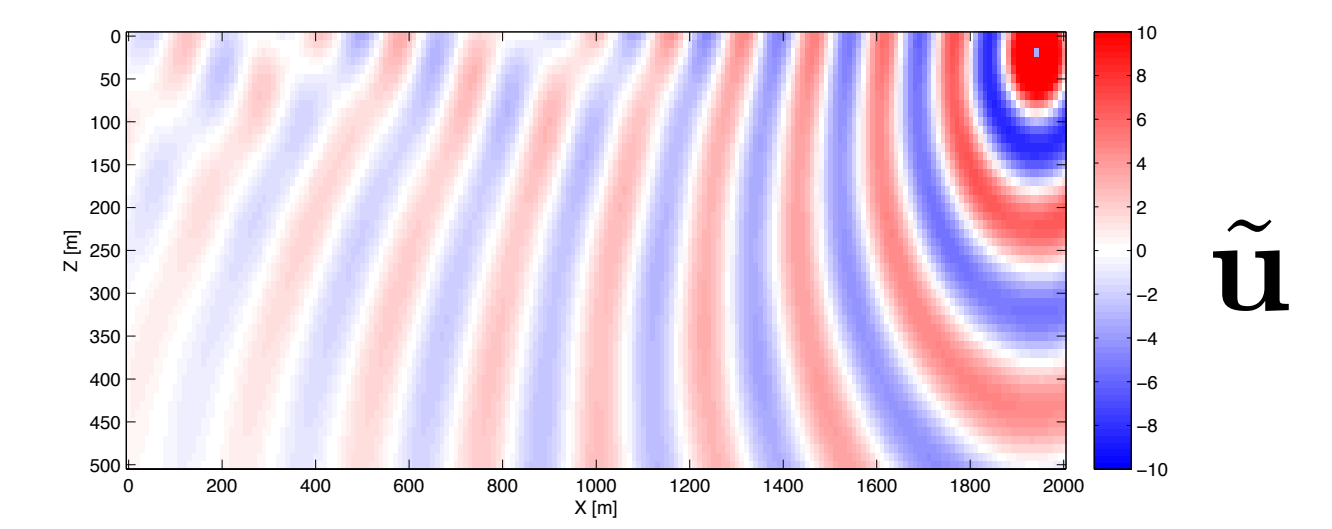


$$\mathbf{u} = \mathbf{A}(\mathbf{m})^{-1} \mathbf{e}$$

$$\mathbf{d}_{\text{pred}} = \mathbf{P} \mathbf{u}$$



$$\bar{\alpha} = \frac{\mathbf{d}_{\text{pred}}^T \mathbf{d}_{\text{obs}}}{\mathbf{d}_{\text{pred}}^T \mathbf{d}_{\text{pred}}}$$









## WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u}, \mathbf{m}, \alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

## WRI with source estimation

Triple parameters optimization problem:

$$\underset{\mathbf{u}, \mathbf{m}, \alpha}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

Eliminate  $\mathbf{u}$  and  $\alpha$  jointly w/ variable projection:

$$[\bar{\mathbf{u}}, \bar{\alpha}] = \arg \min_{\mathbf{u}, \alpha} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$



## WRI with source estimation

Corresponds to solving the following augmented system:

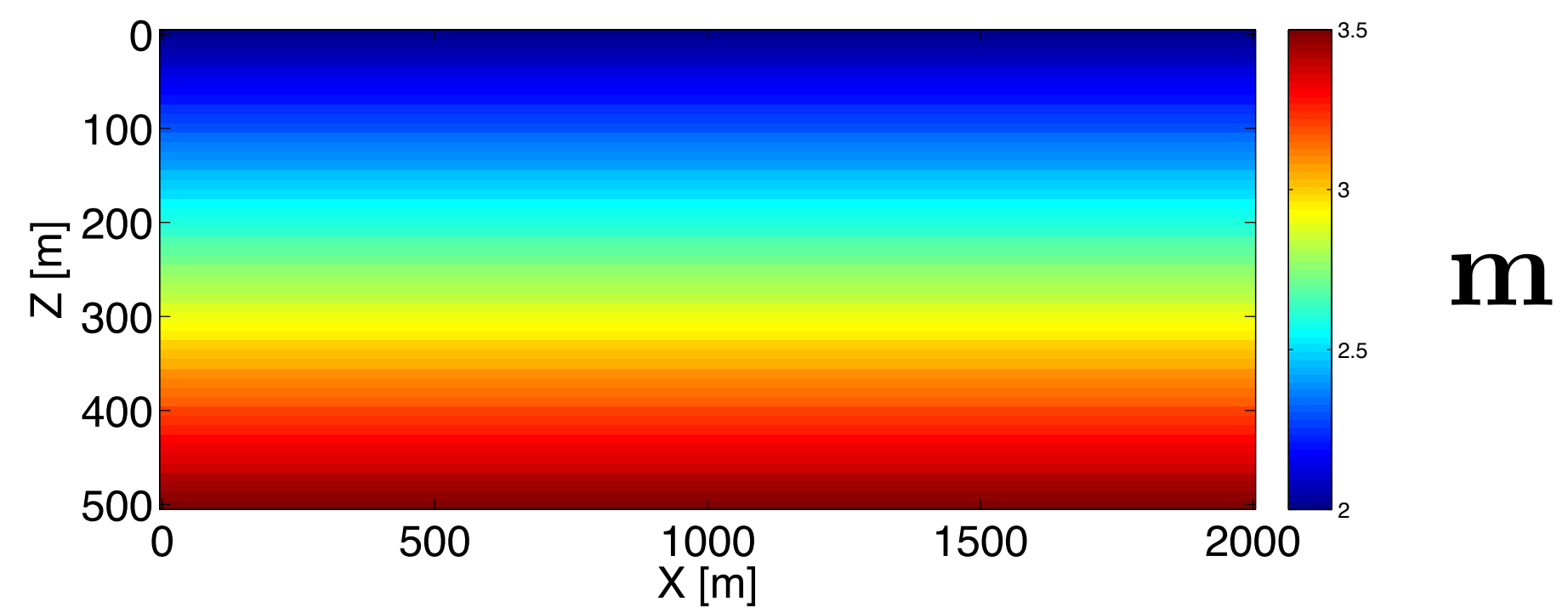
$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} & -\lambda \mathbf{e}_{k,l} \\ \mathbf{P}_k & 0 \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}}_{k,l} \\ \bar{\alpha}_{k,l} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{k,l} \end{pmatrix}$$

Cf. original augmented system:

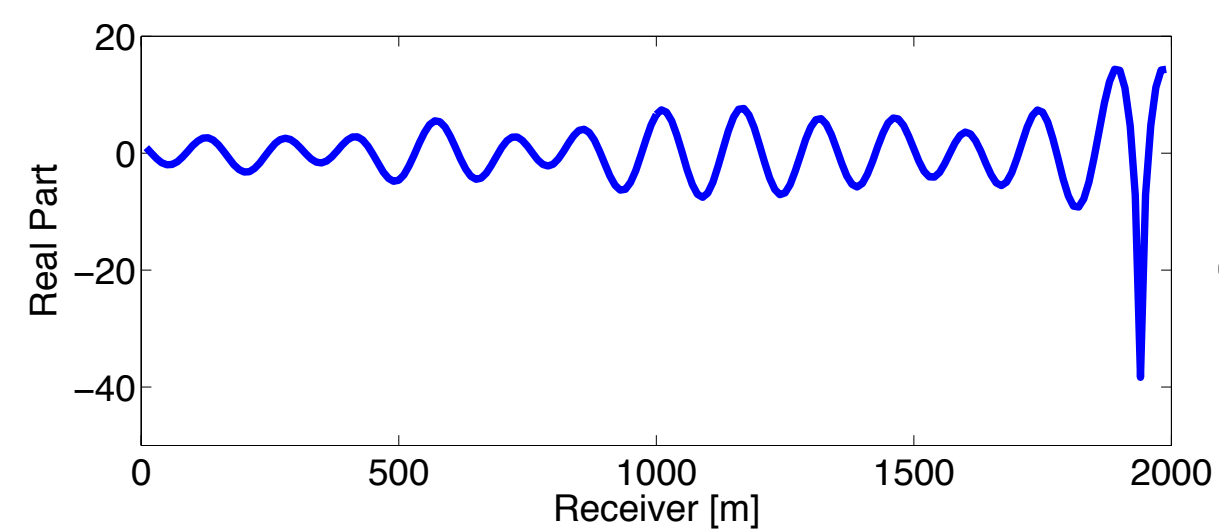
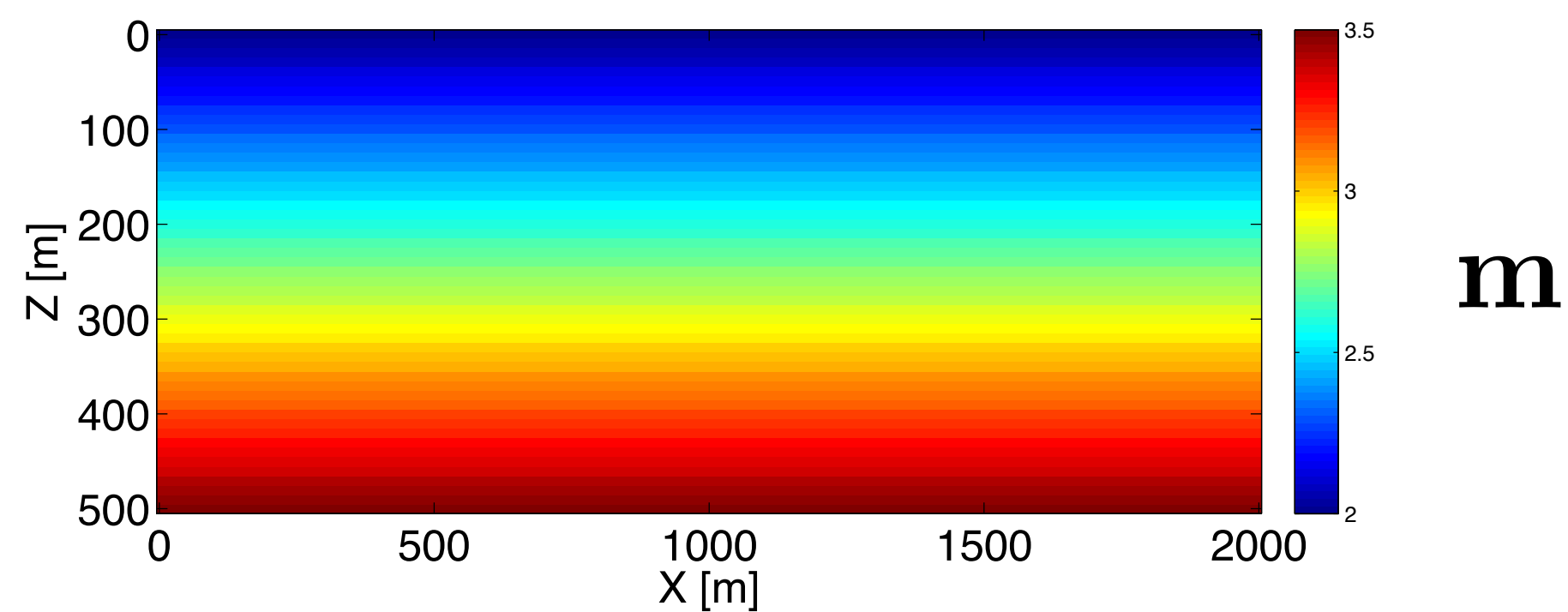
$$\begin{pmatrix} \lambda \mathbf{A}_{k,l} \\ \mathbf{P}_k \end{pmatrix} \bar{\mathbf{u}}_{k,l} = \begin{pmatrix} \lambda \mathbf{q}_{k,l} \\ \mathbf{d}_{k,l} \end{pmatrix}$$

**Full column rank!**  
**No additional computational cost!**

# WRI with source estimation

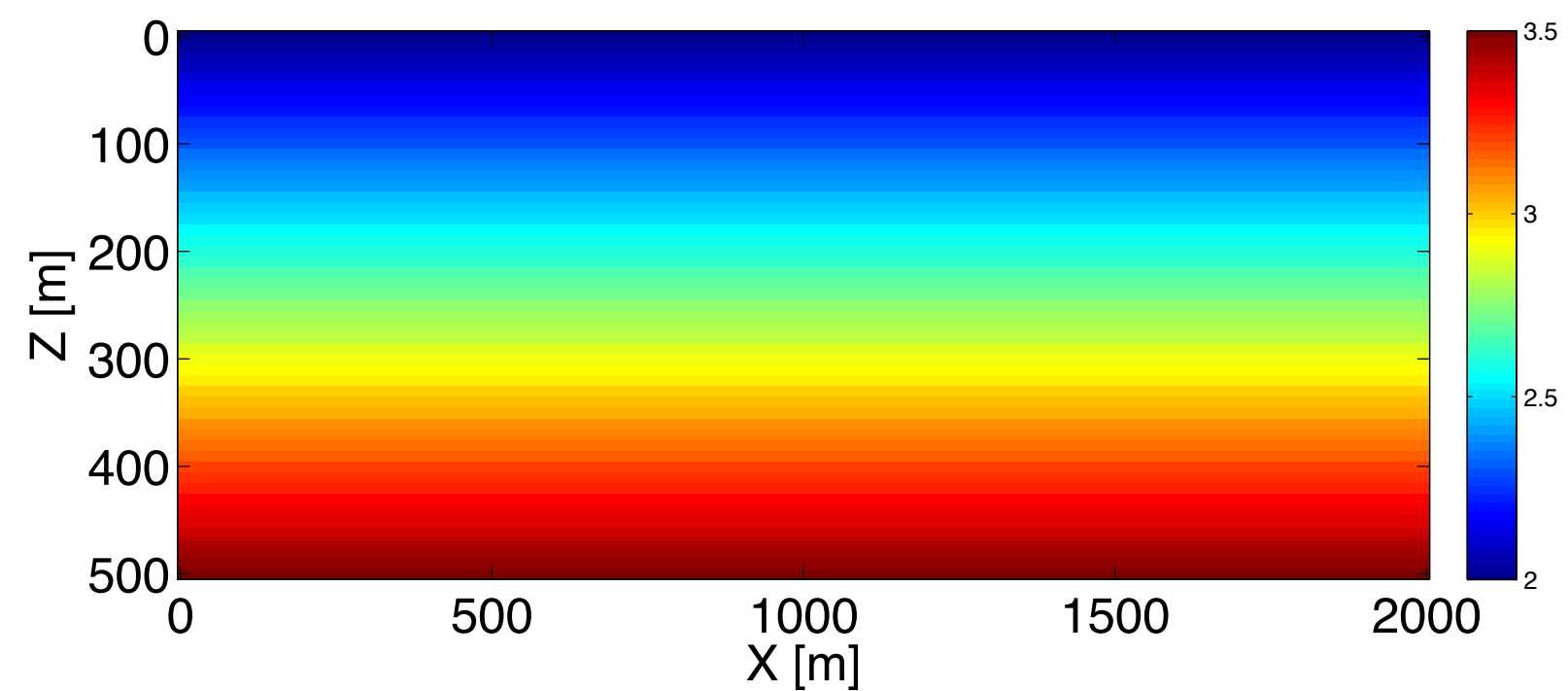


# WRI with source estimation

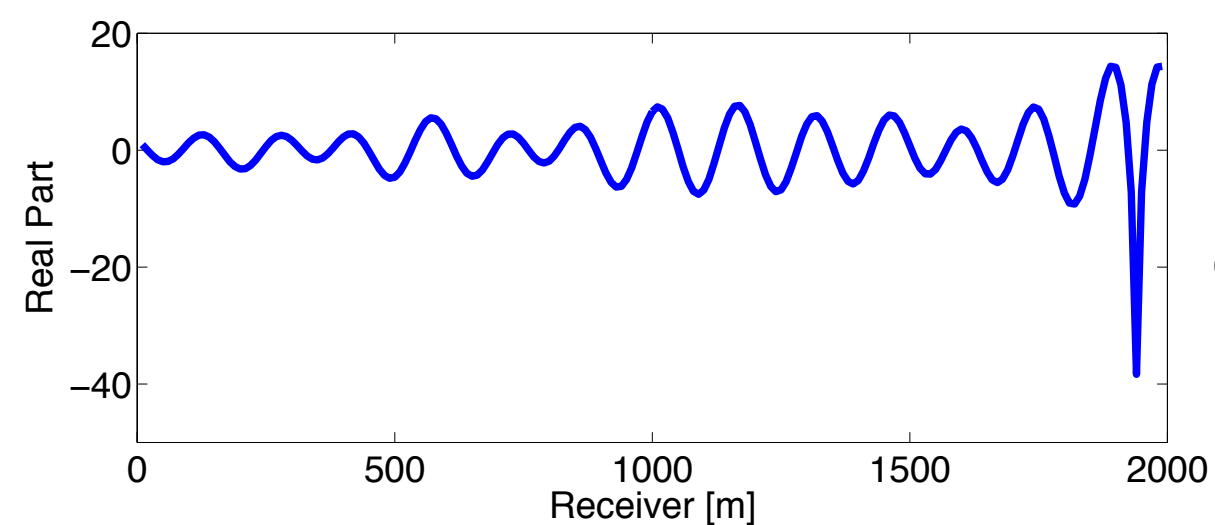


$$\begin{pmatrix} \lambda \mathbf{A} & -\lambda \mathbf{e} \\ \mathbf{P} & 0 \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}} \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{\text{obs}} \end{pmatrix}$$

# WRI with source estimation



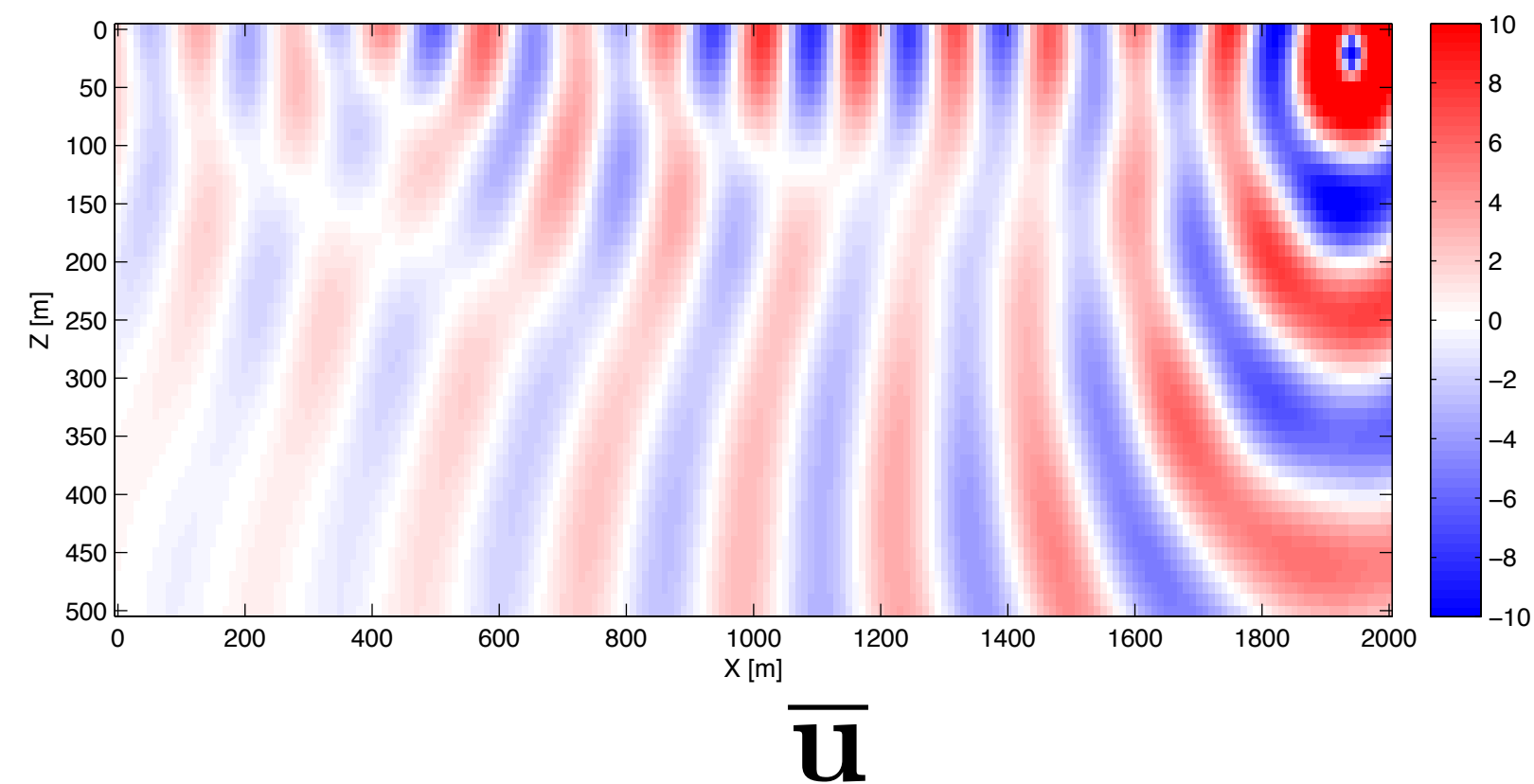
**m**



**d<sub>obs</sub>**

$$\begin{pmatrix} \lambda \mathbf{A} & -\lambda \mathbf{e} \\ \mathbf{P} & 0 \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}} \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{\text{obs}} \end{pmatrix}$$

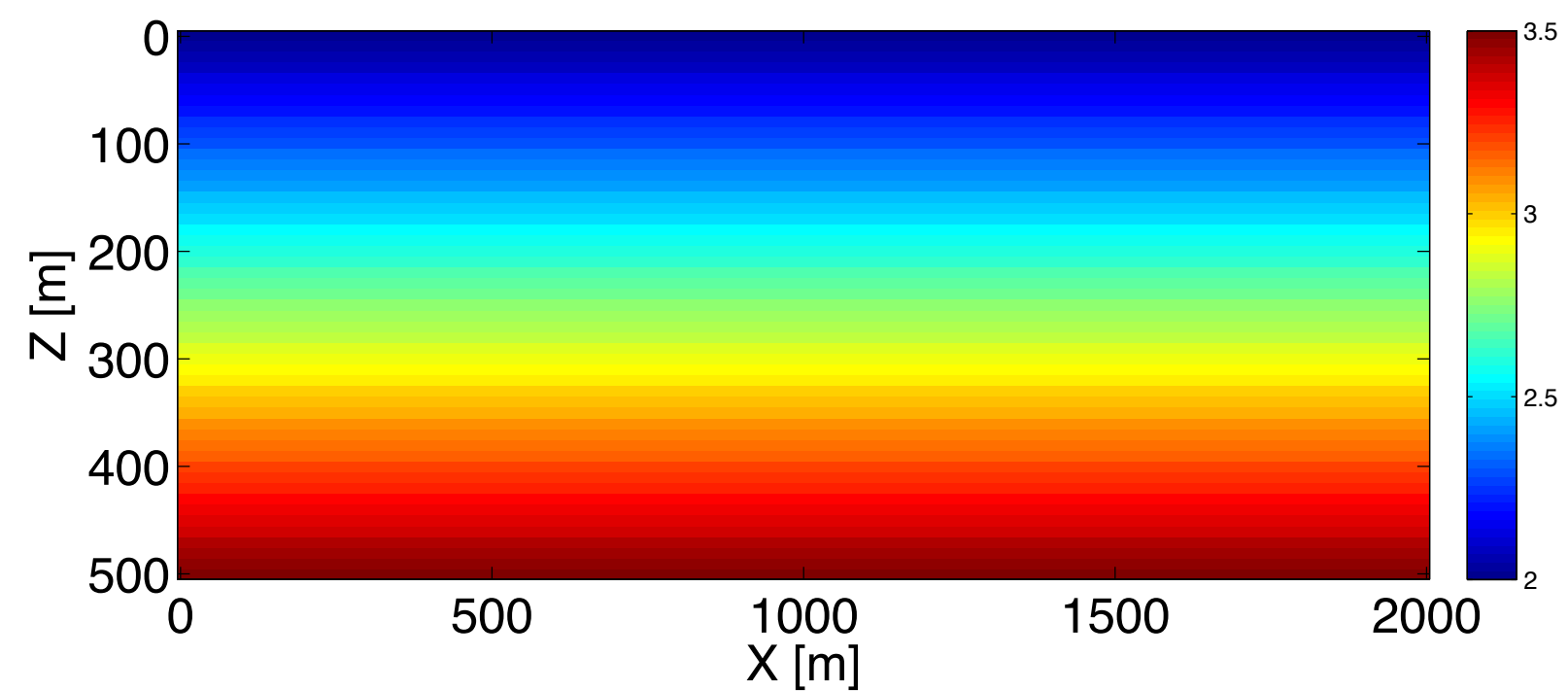
**$\bar{\alpha}$  and**



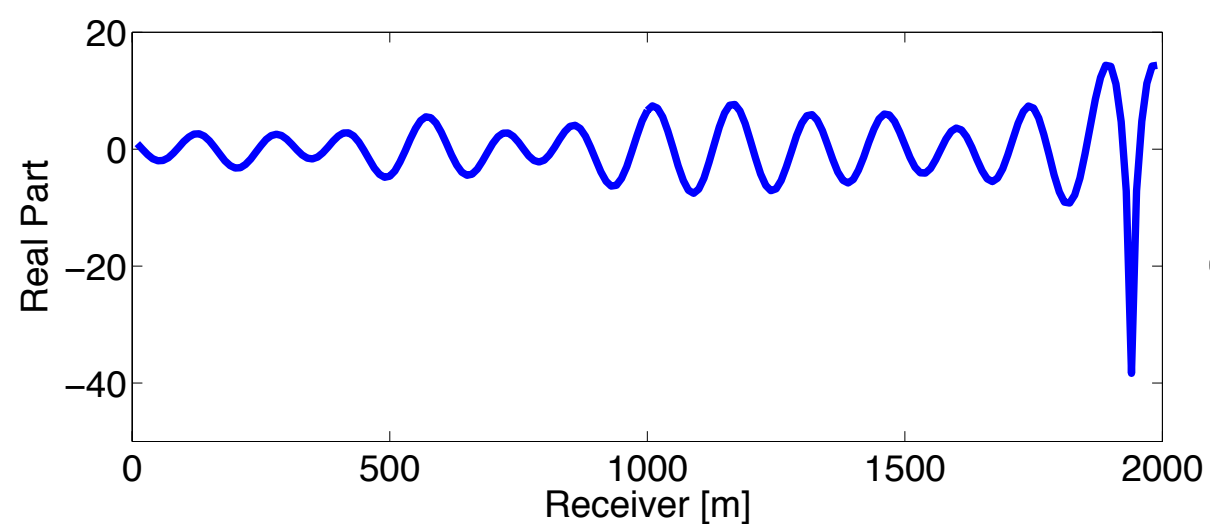
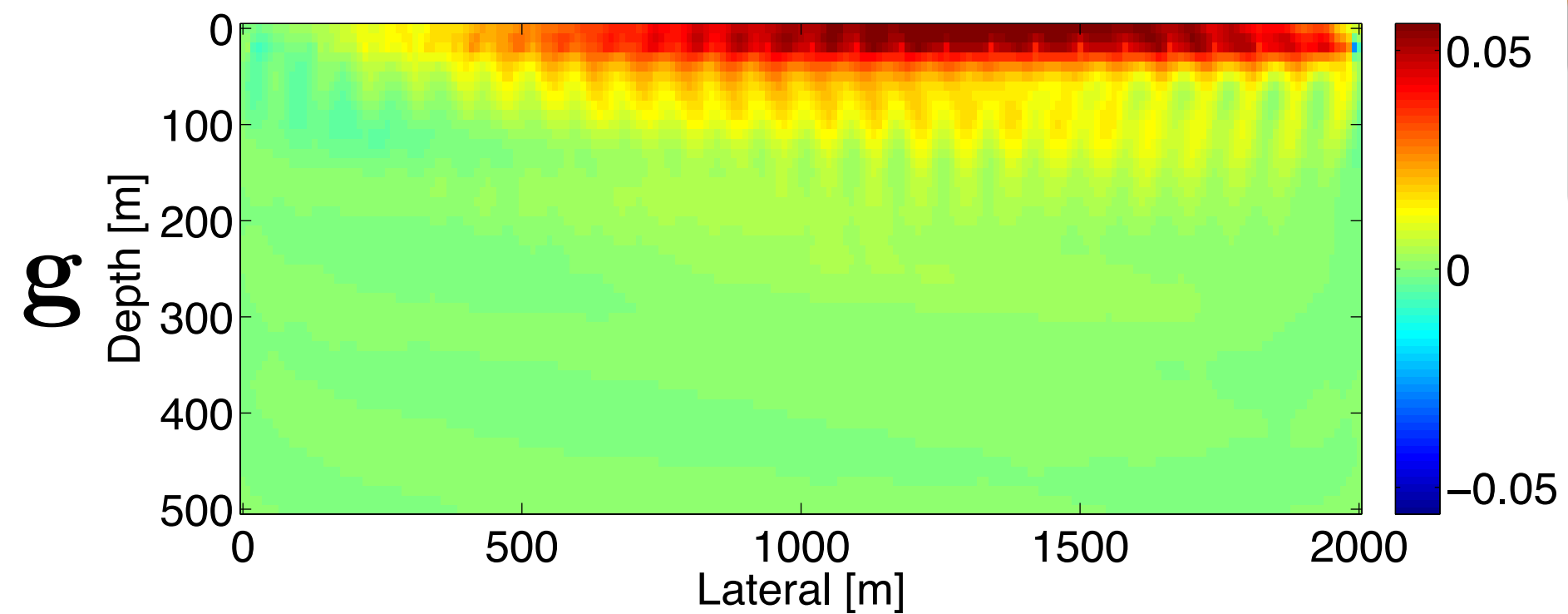
**$\bar{\mathbf{u}}$**



# WRI with source estimation



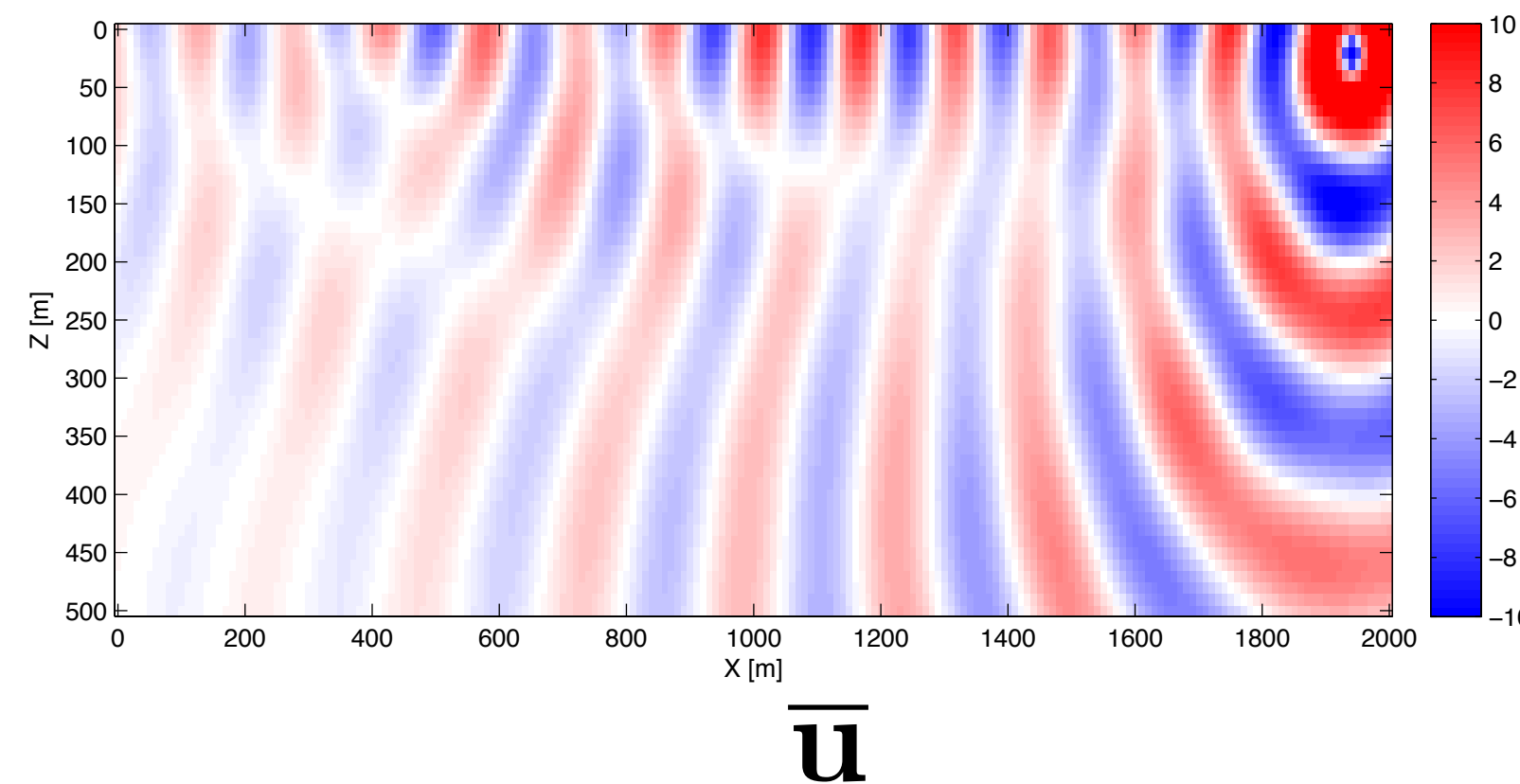
$m$



$d_{obs}$

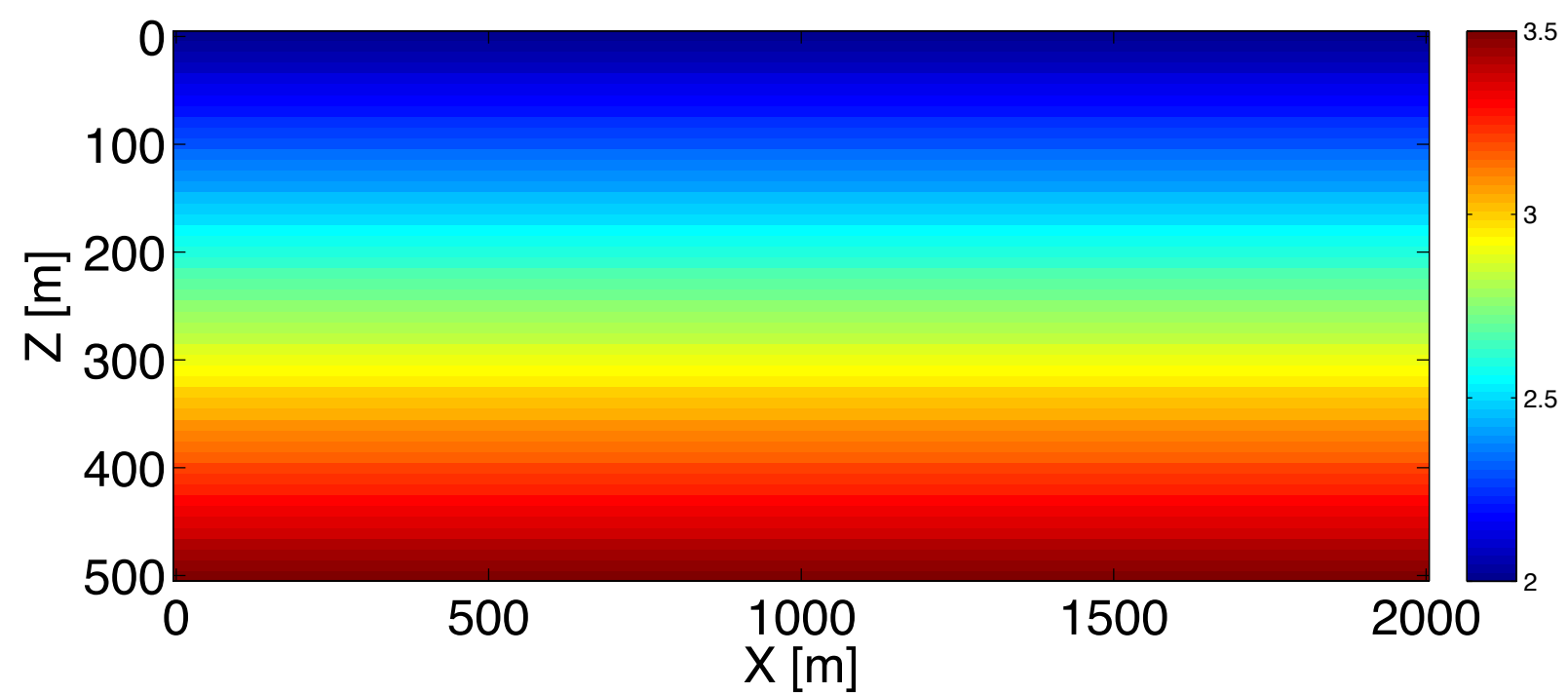
$$\begin{pmatrix} \lambda A & -\lambda e \\ P & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ d_{obs} \end{pmatrix}$$

$\bar{\alpha}$  and

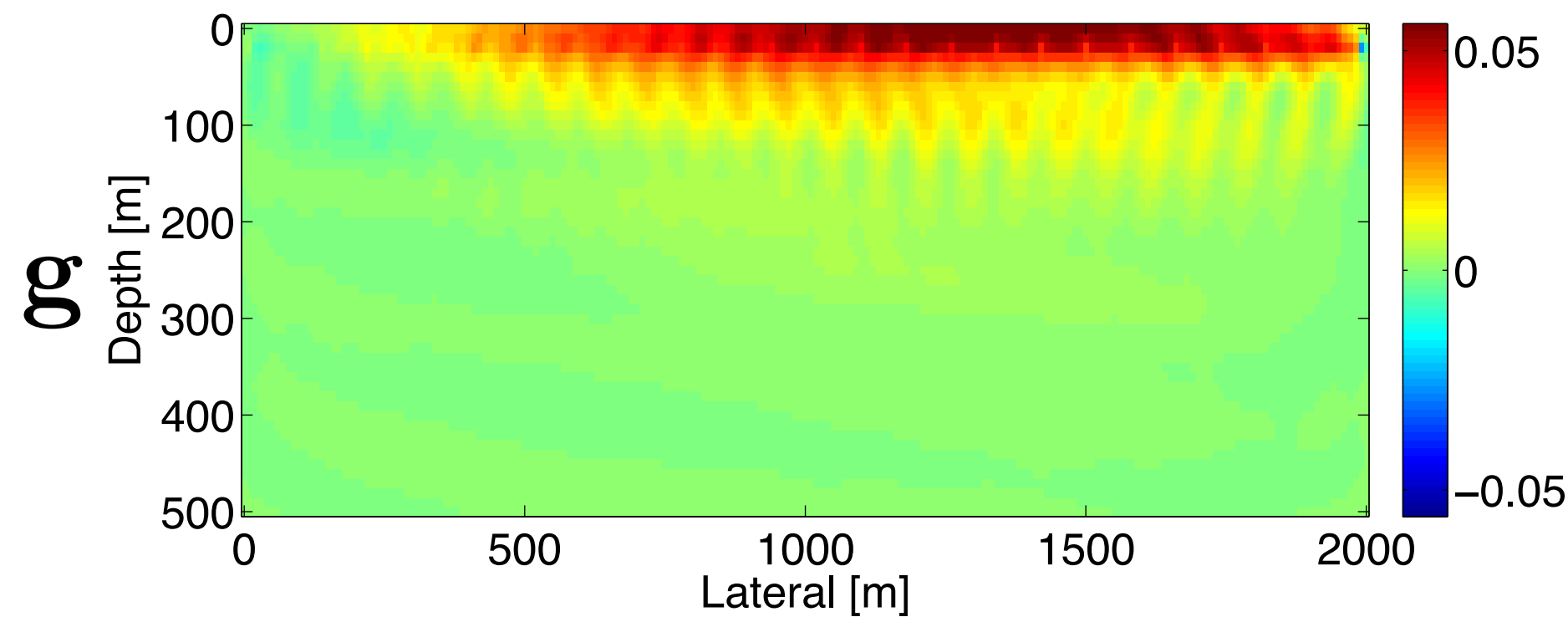


$\bar{u}$

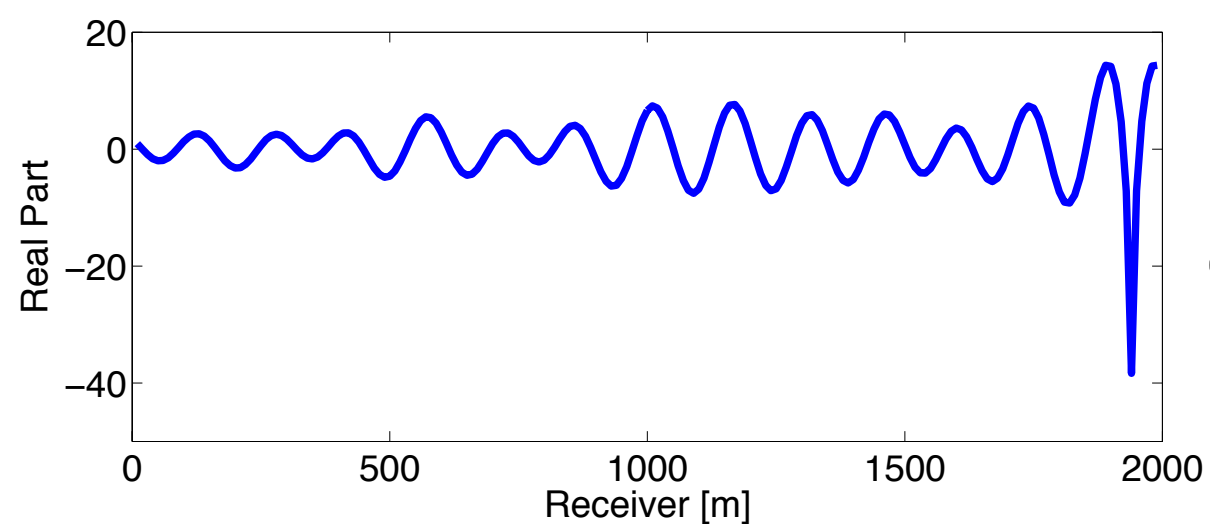
# WRI with source estimation



$m$



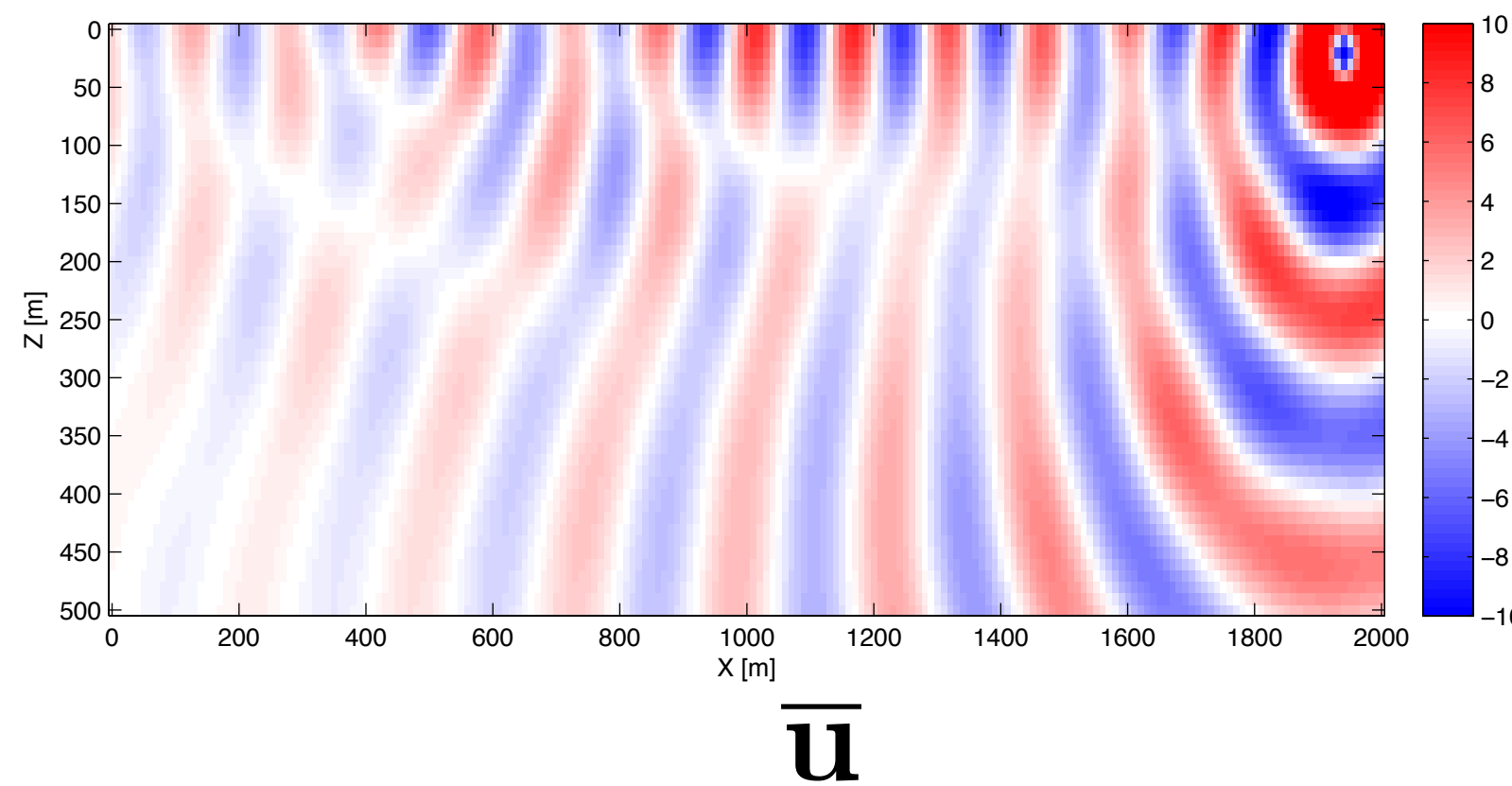
$\sigma_0$



$d_{obs}$

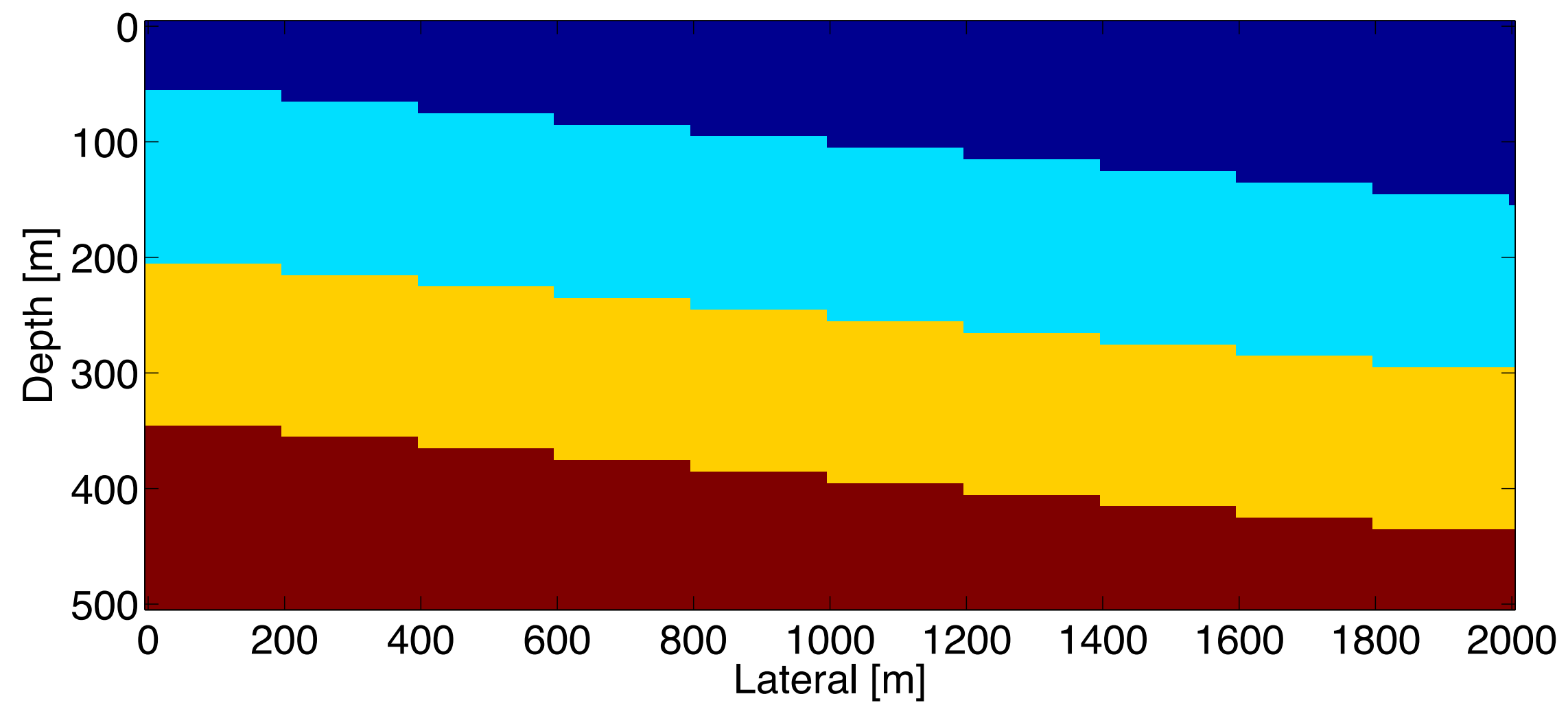
$$\begin{pmatrix} \lambda A & -\lambda e \\ \mathbf{P} & 0 \end{pmatrix} \begin{pmatrix} \bar{\mathbf{u}} \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{d}_{obs} \end{pmatrix}$$

$\bar{\alpha}$  and

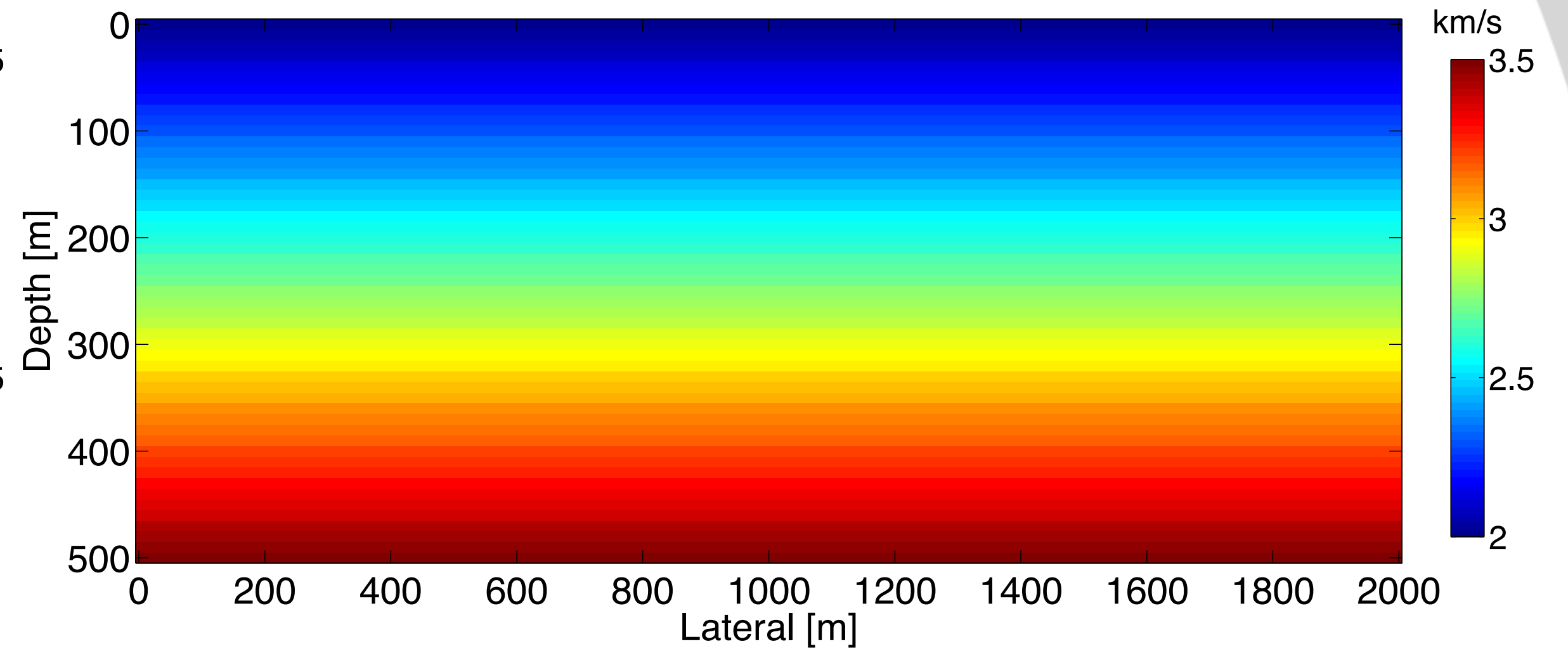


$\bar{\mathbf{u}}$

# Synthetic example

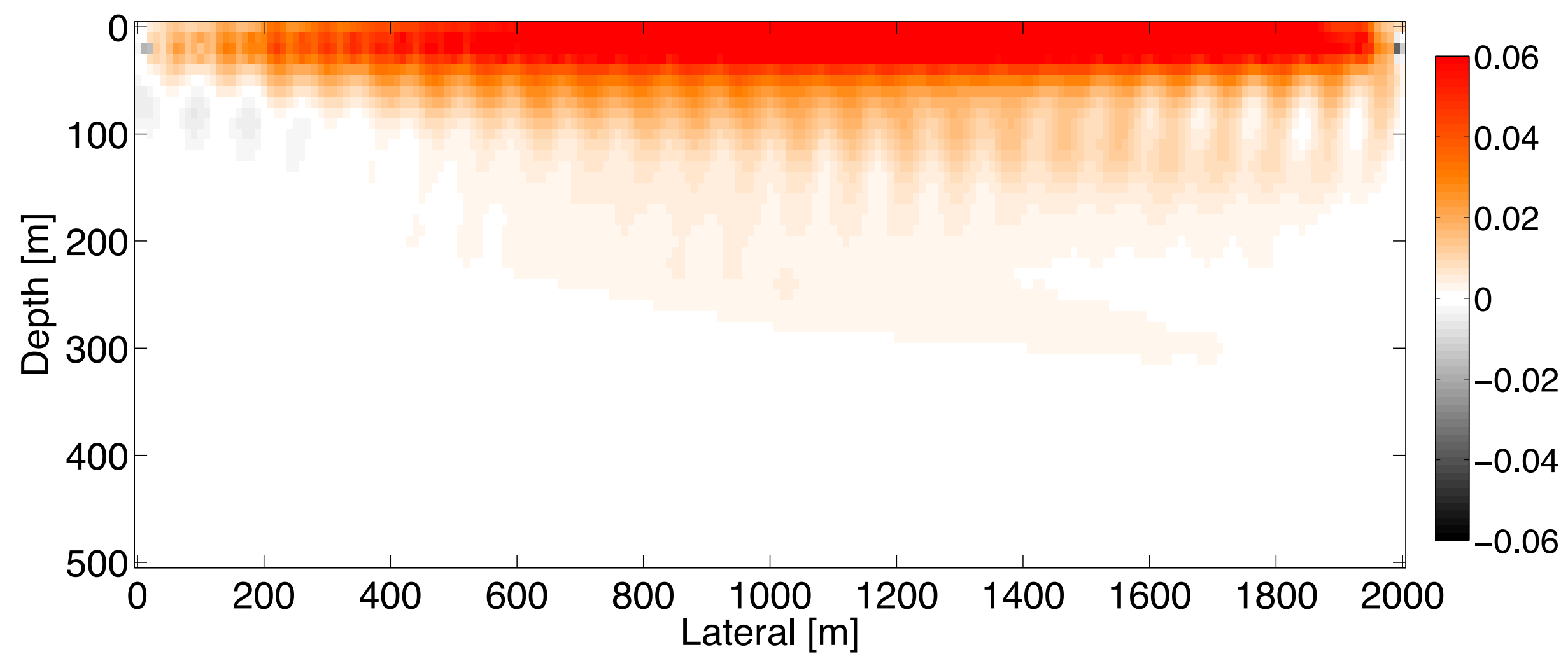


**True Model**

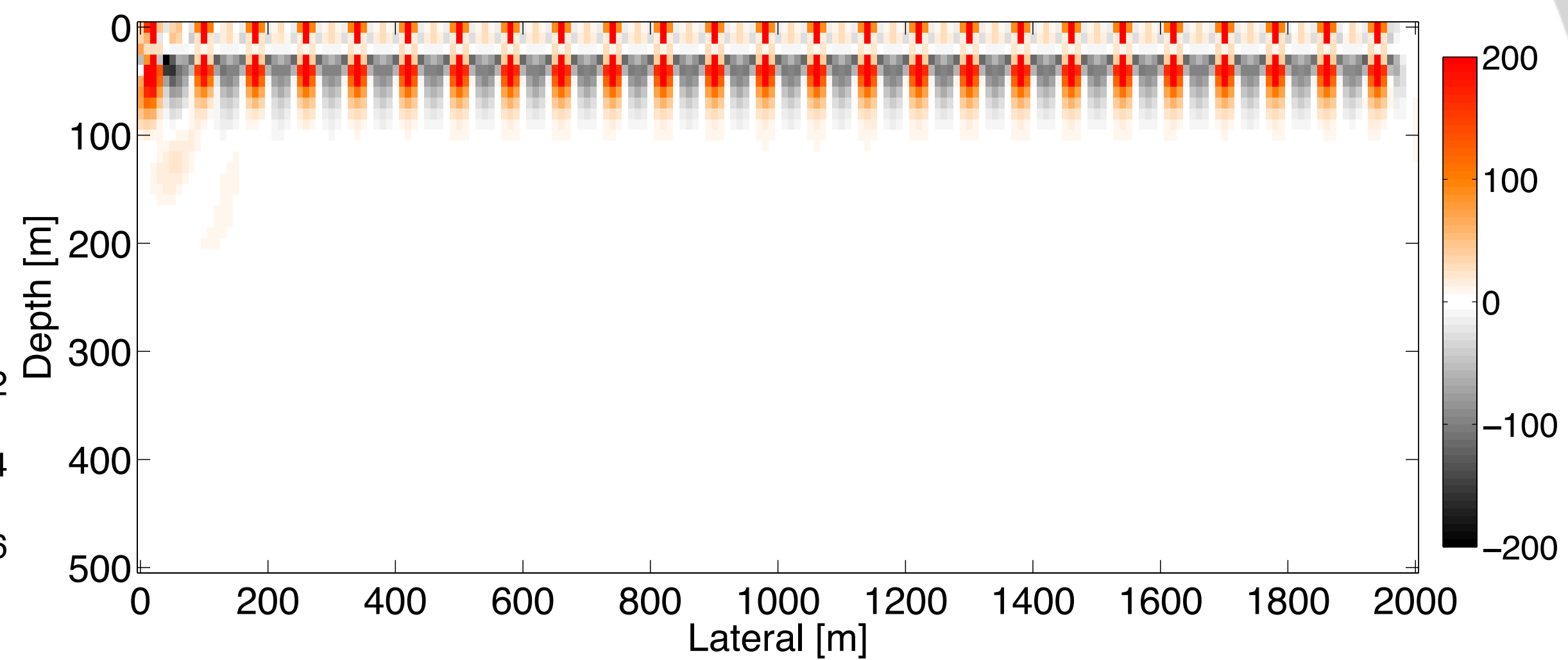


**Initial Model**

# Gradient comparison



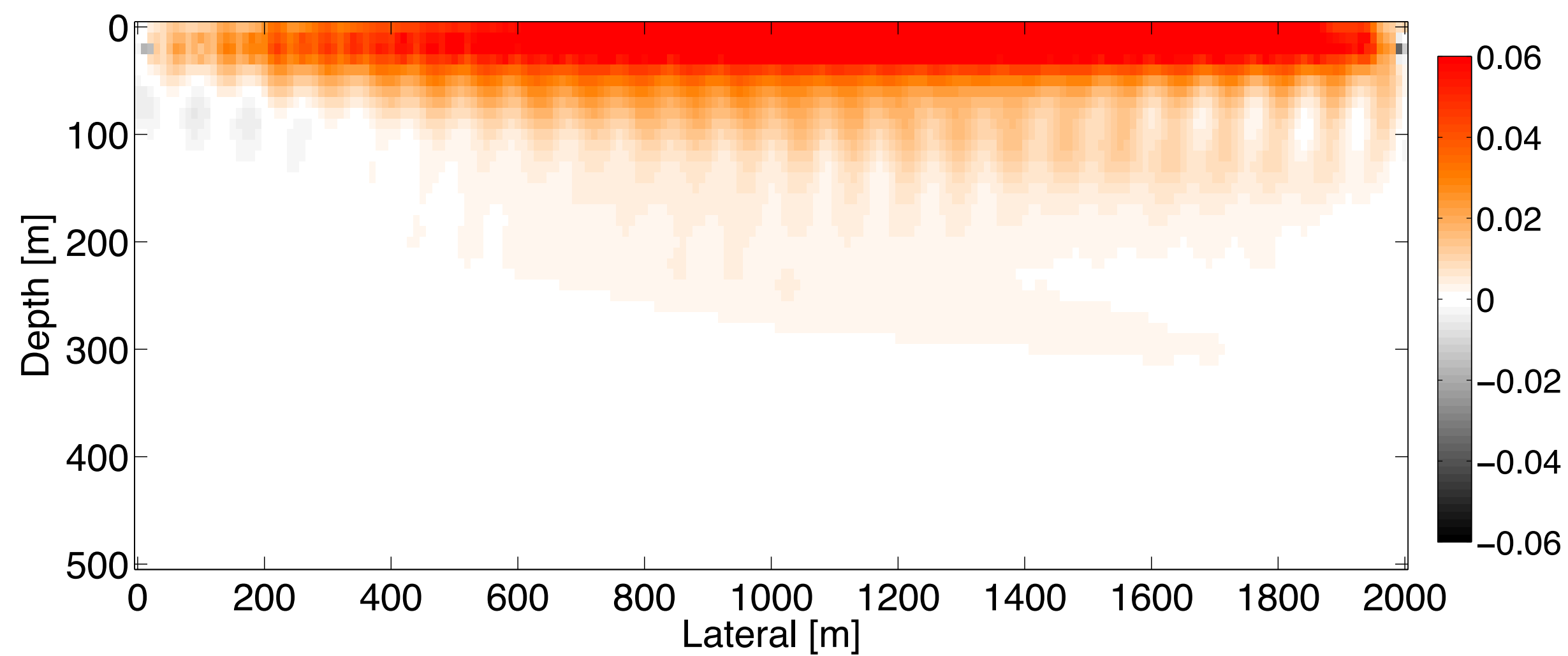
**Gradient with true source wavelet**



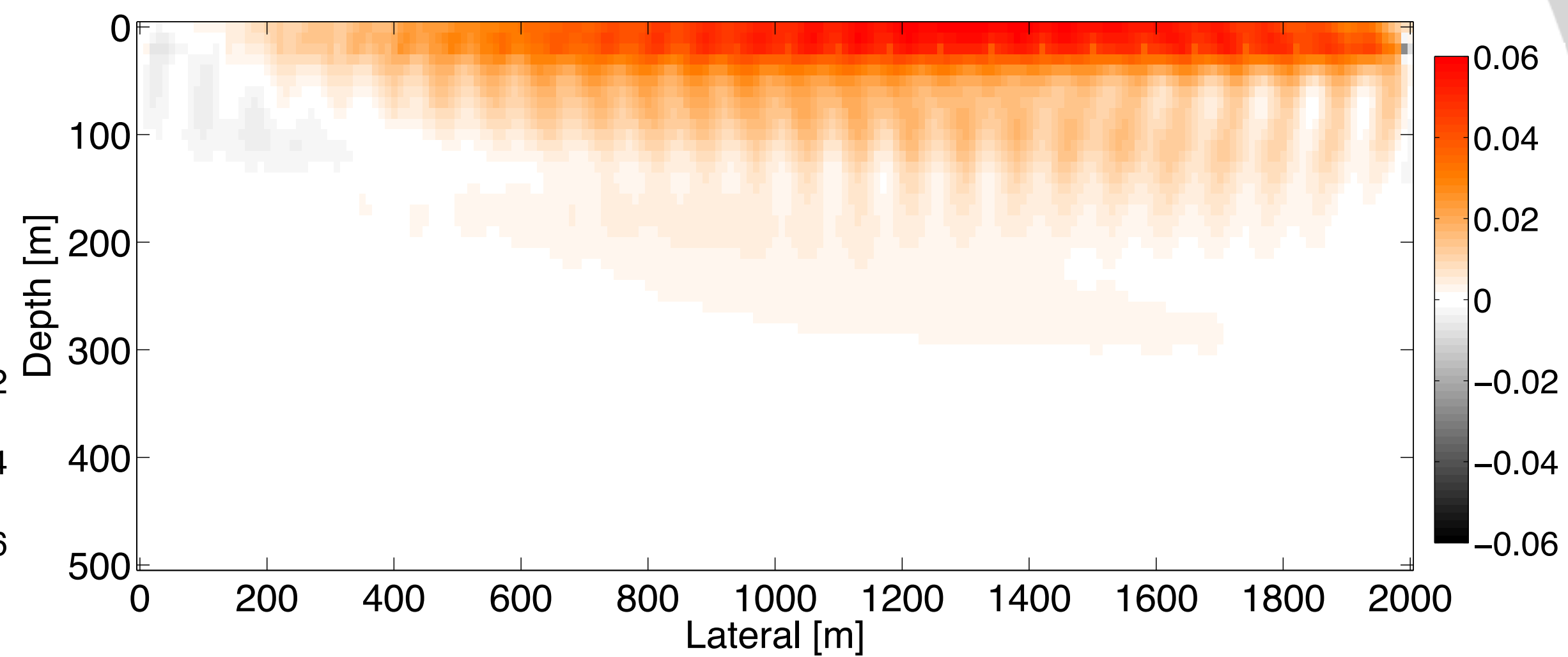
**Gradient with wrong source wavelet**



# Gradient comparison

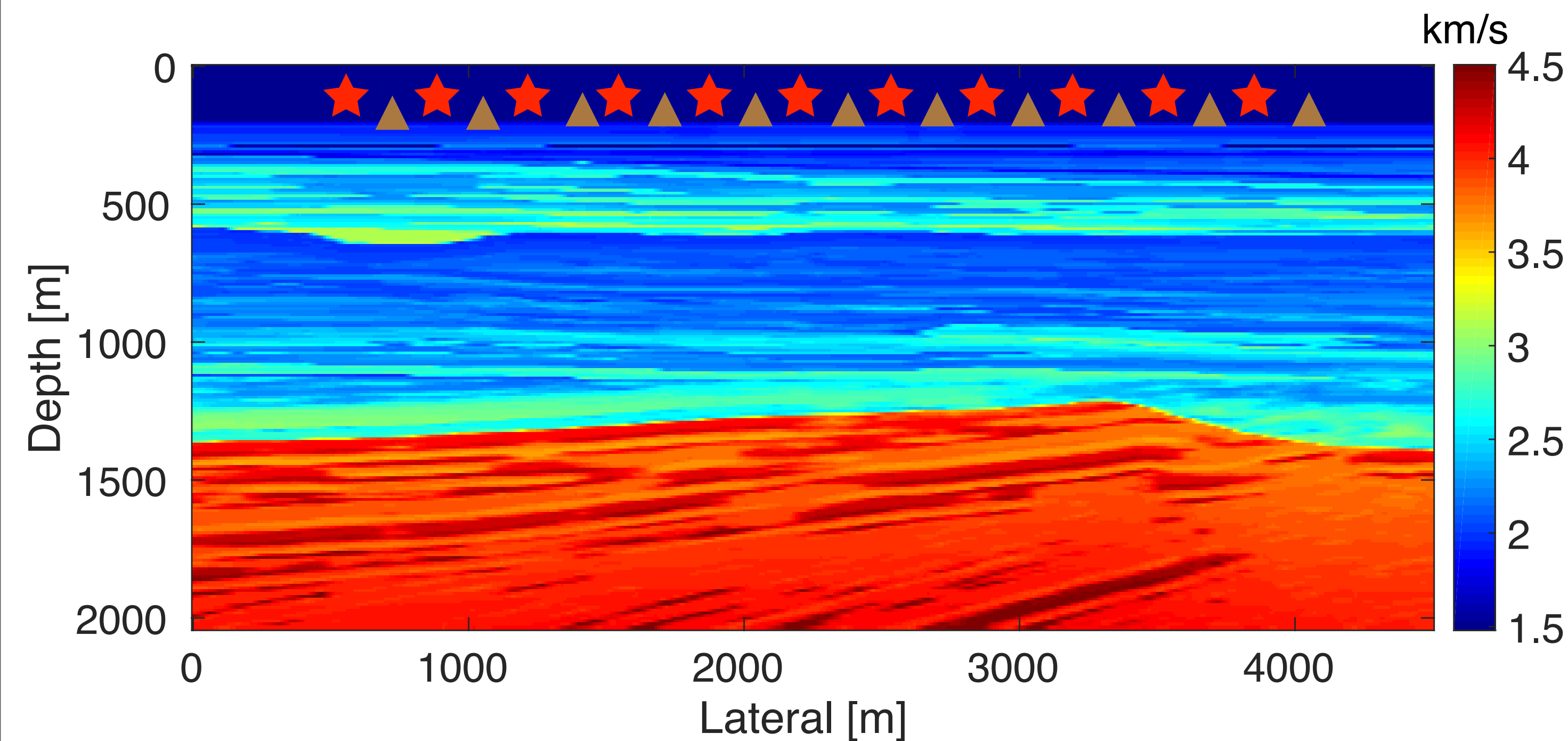


**Gradient with true source wavelet**



**Gradient with estimated source wavelet**

# BG model



## Modeling information:

**Model size:** 2000m x 4500m

**Source spacing:** 50m

**Receiver spacing:** 10m

**Fixed spread 4.5km**

**Frequency :** 2~31 Hz

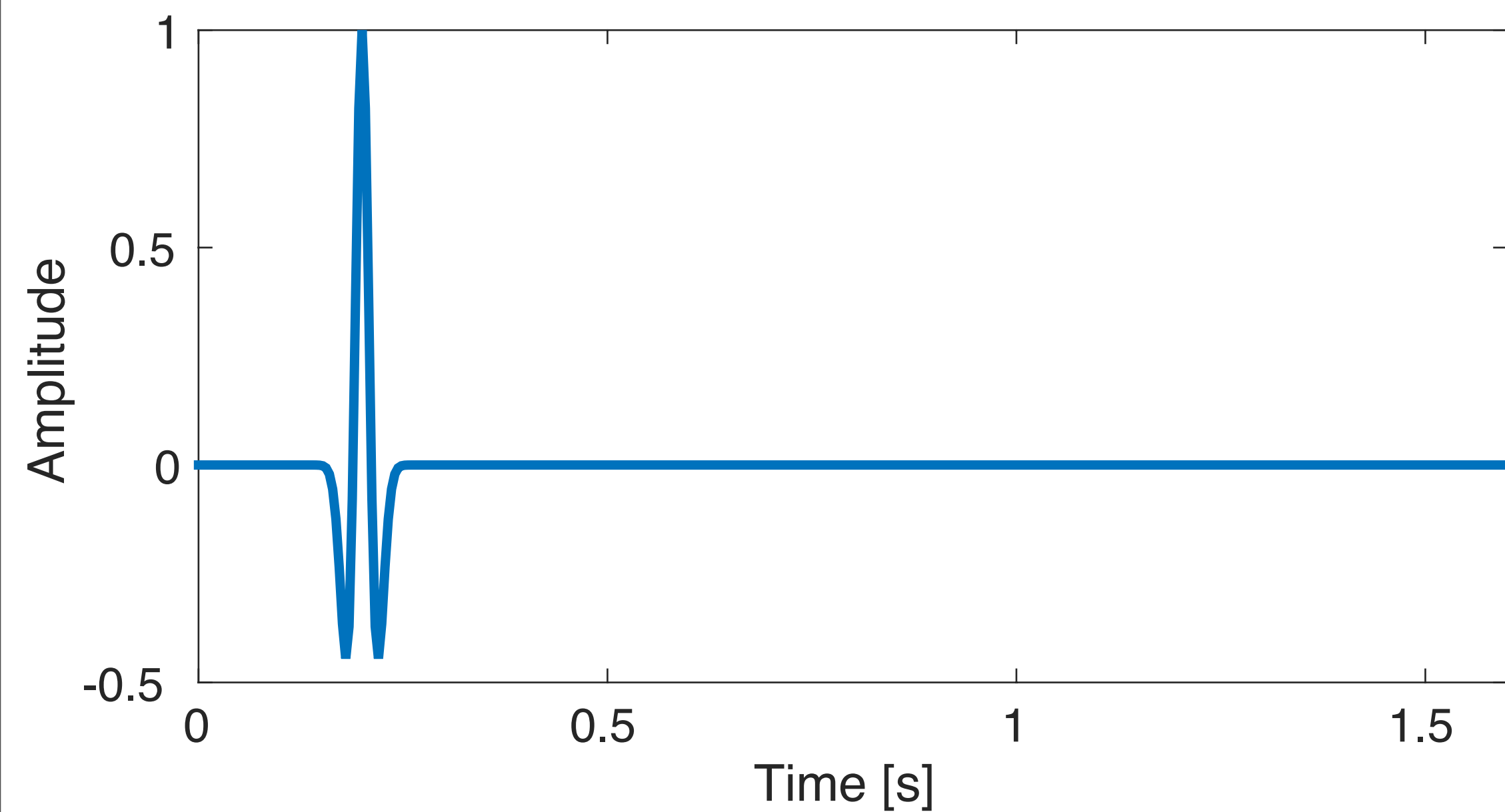
## Inversion information:

**Optimization Solver:** Gauss-Newton

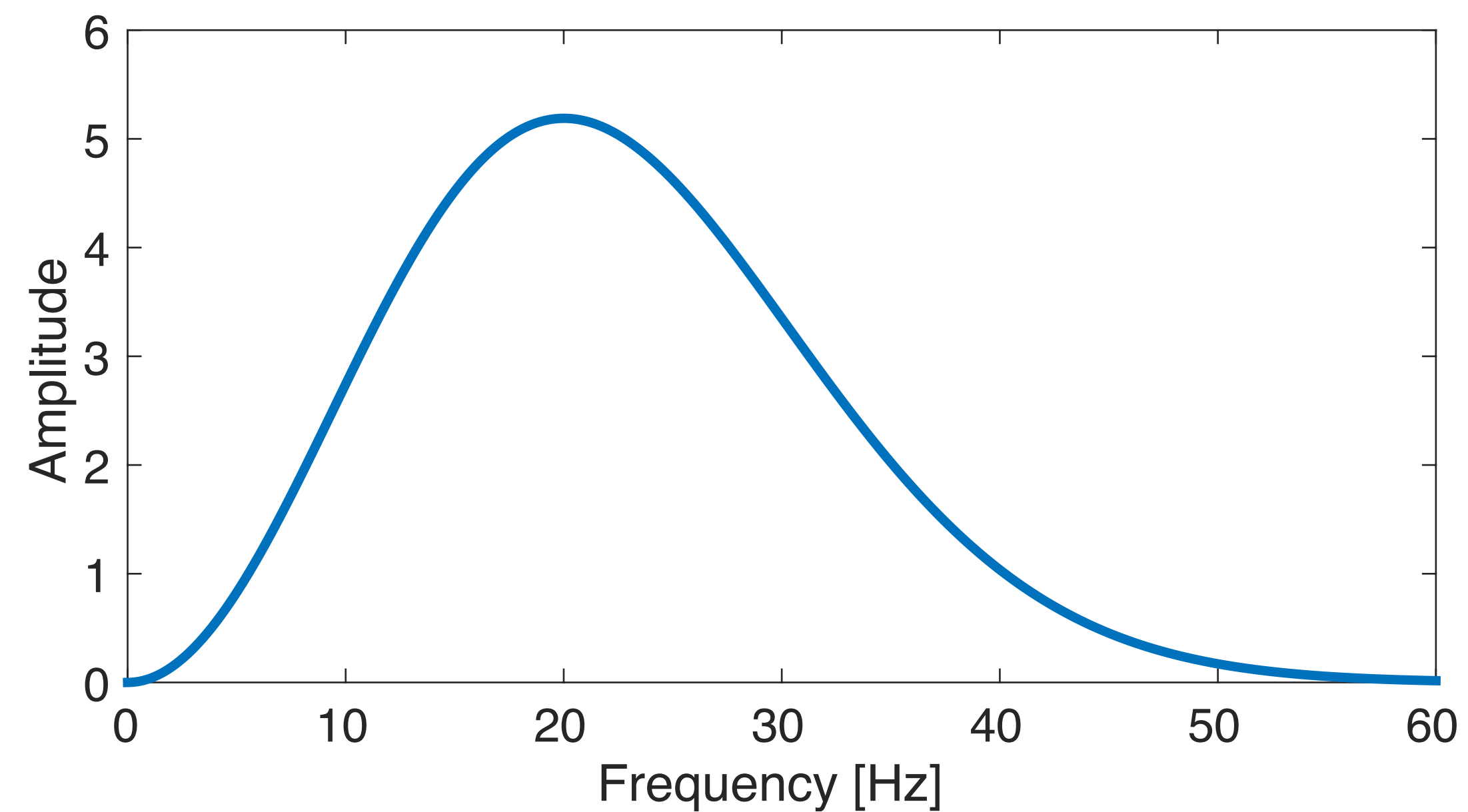
**Iterations per frequency band:** 21

**Batch size:** 15

# Source wavelet

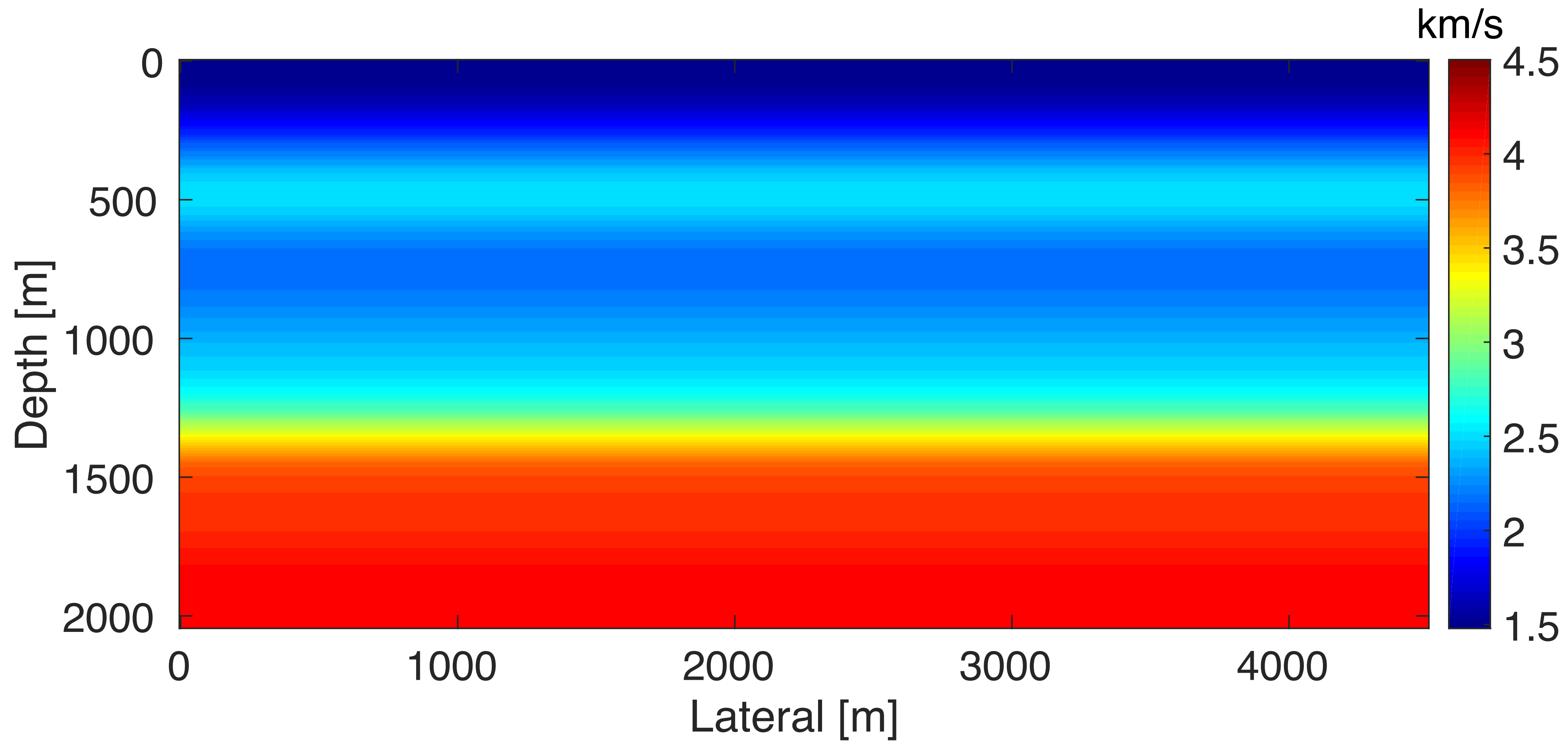


**Source Wavelet**



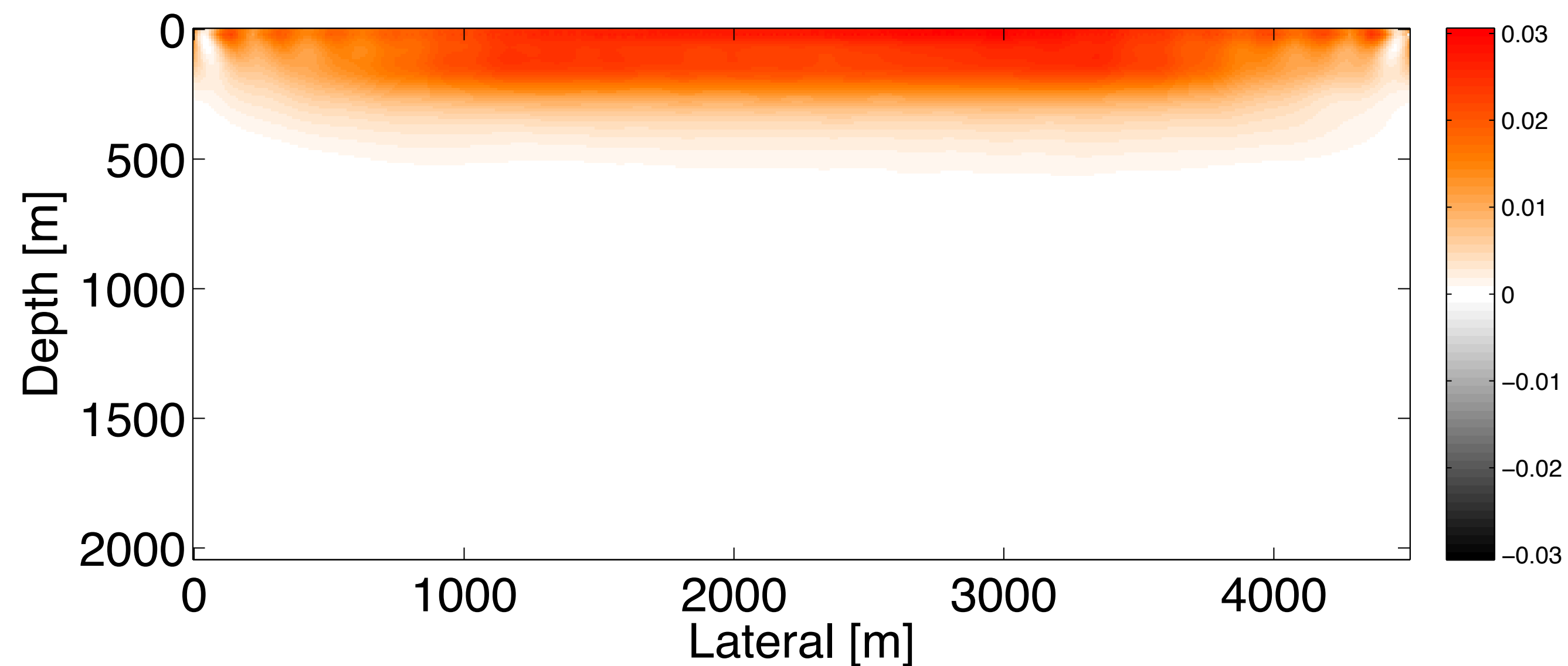
**Spectrum**

# Initial model

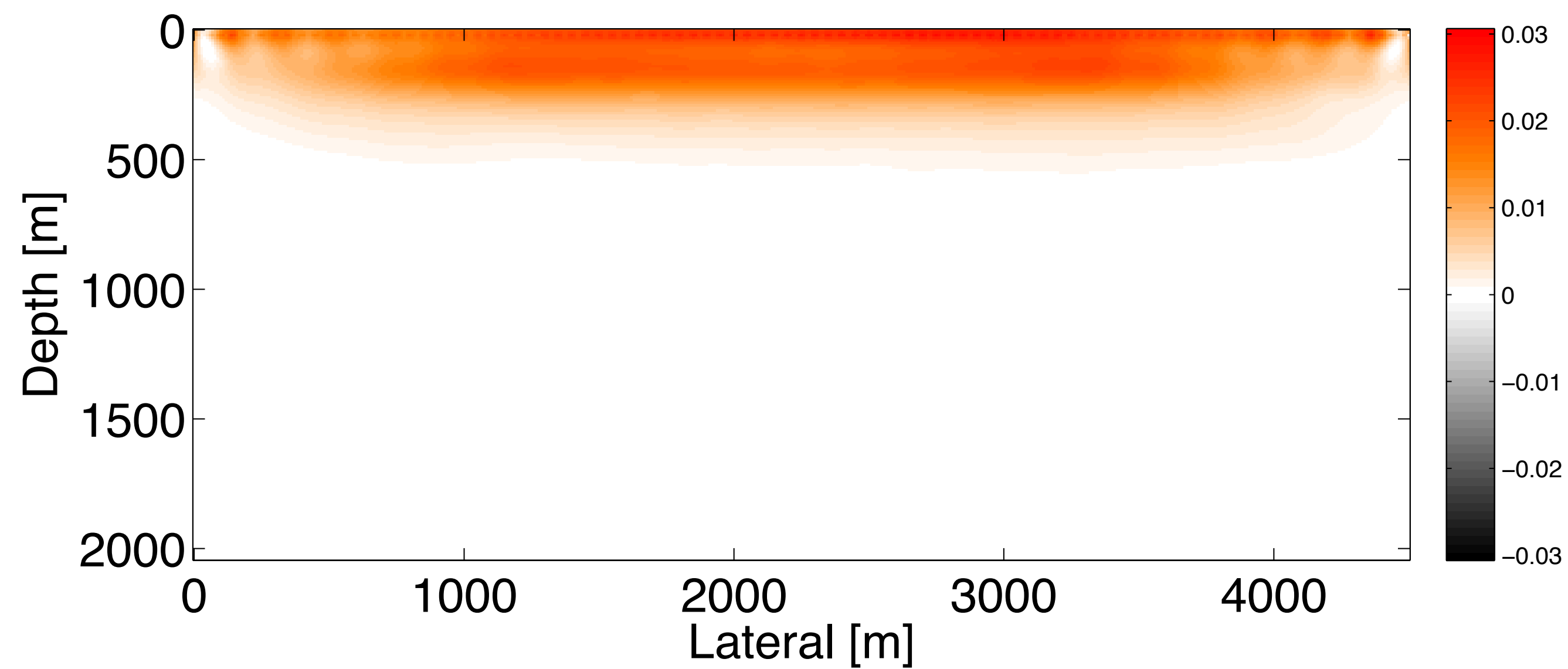




# First gradient comparison

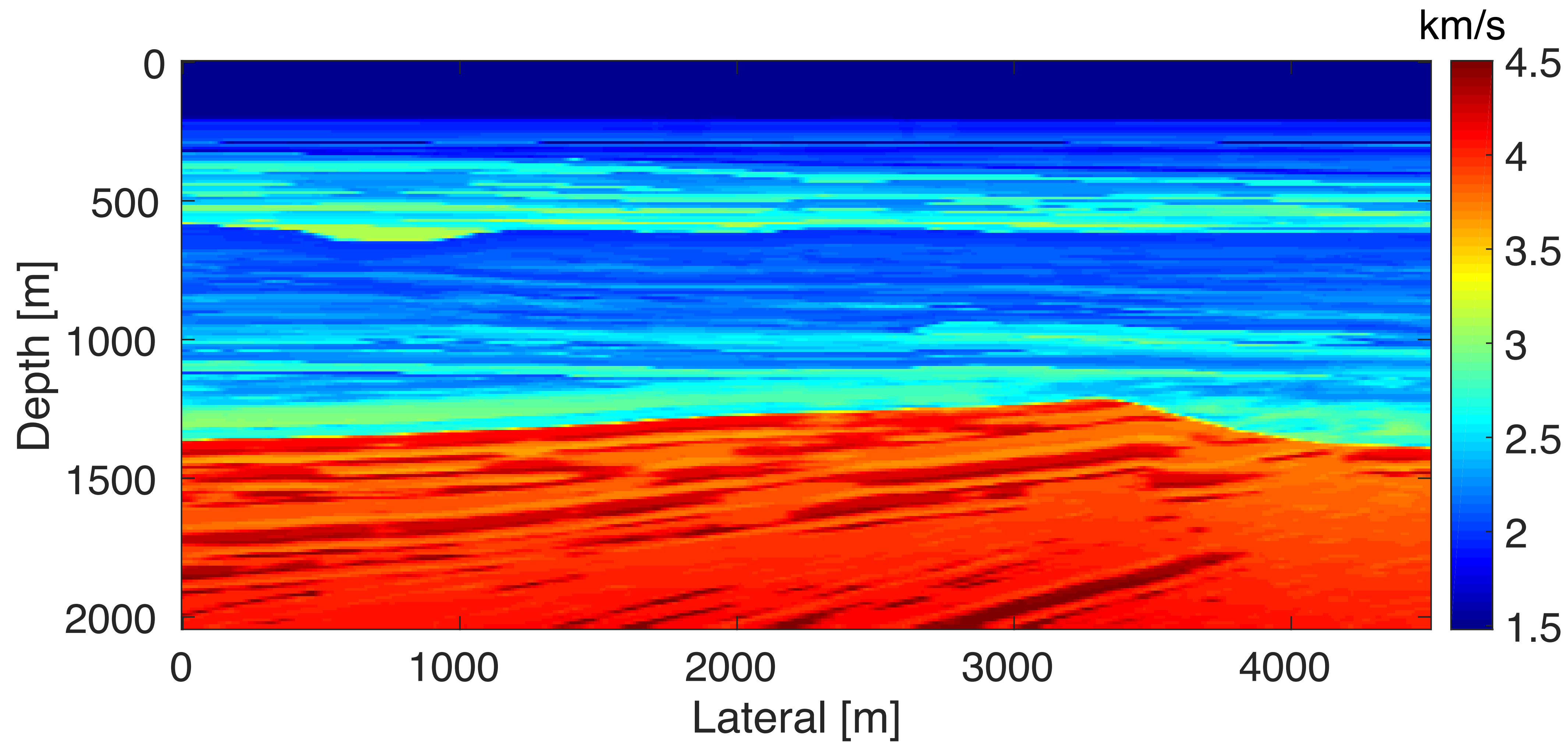


**Gradient with true source wavelet**

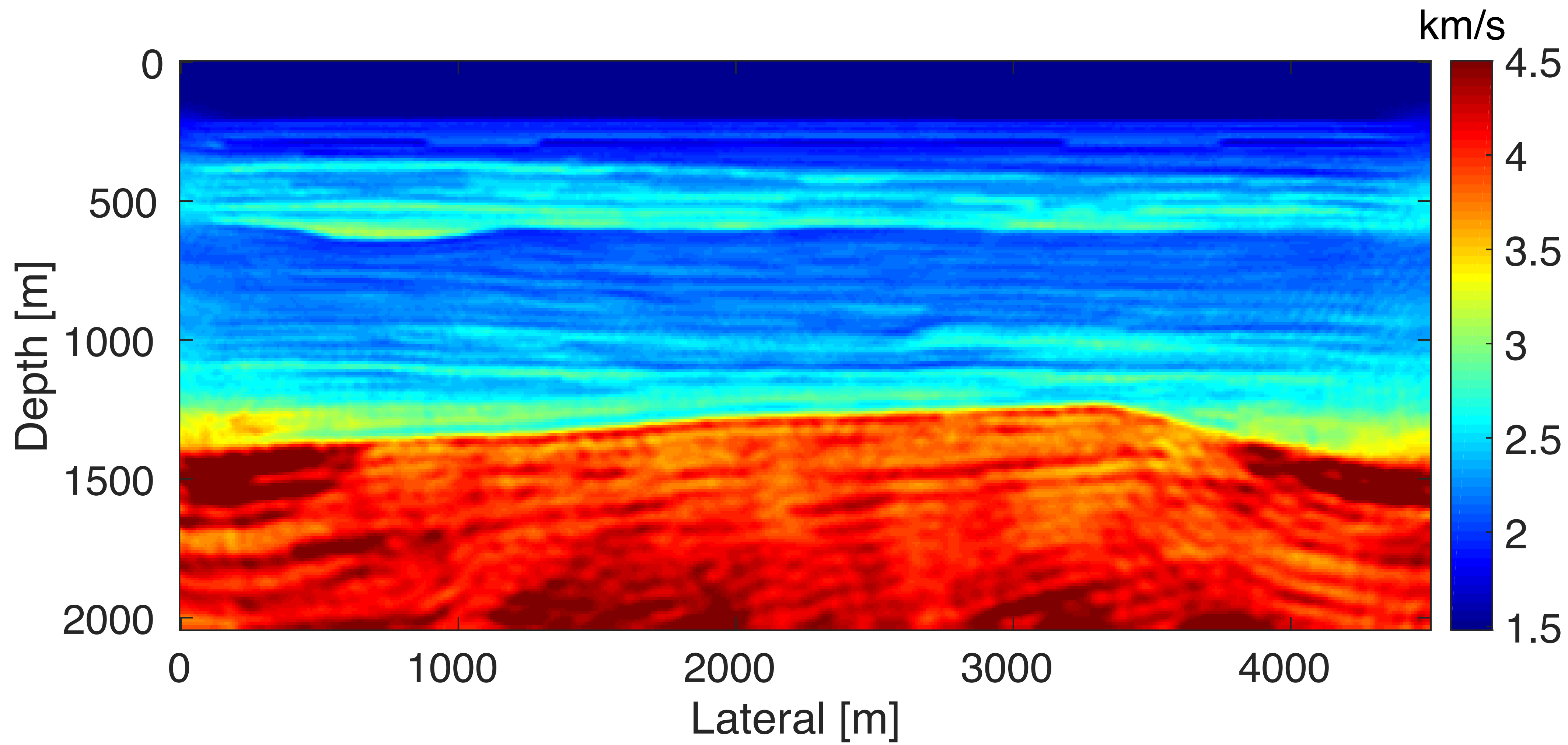


**Gradient with estimated source wavelet**

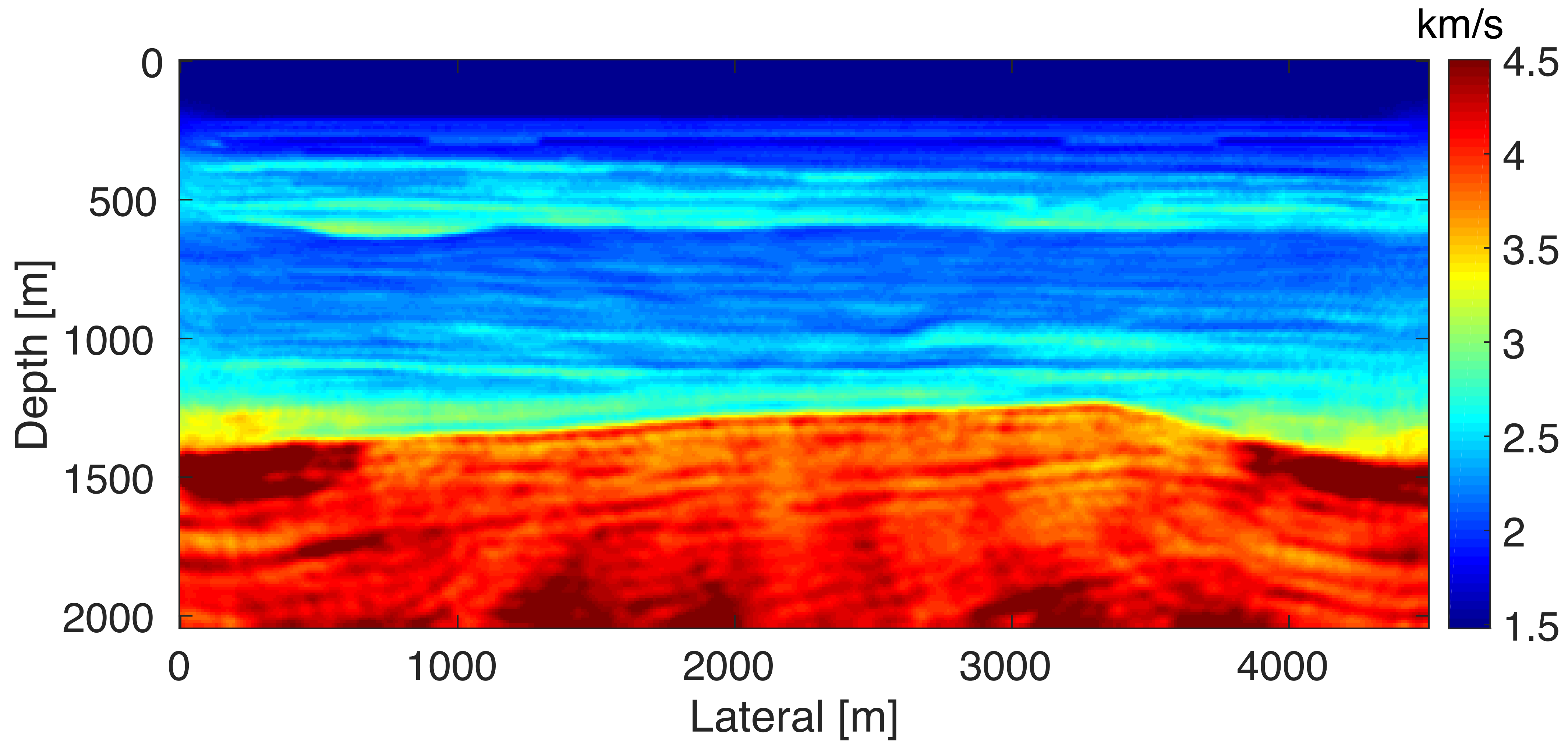
# True model



# Result with true source wavelet

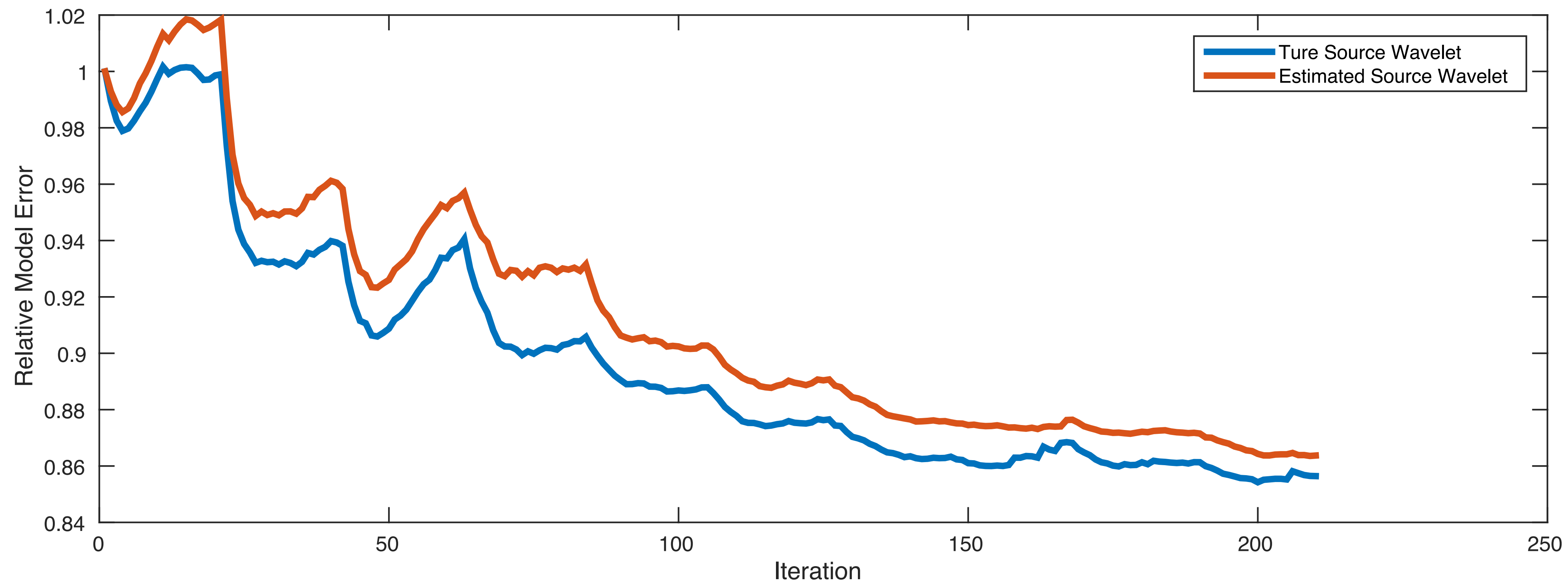


# Result with estimated source wavelet

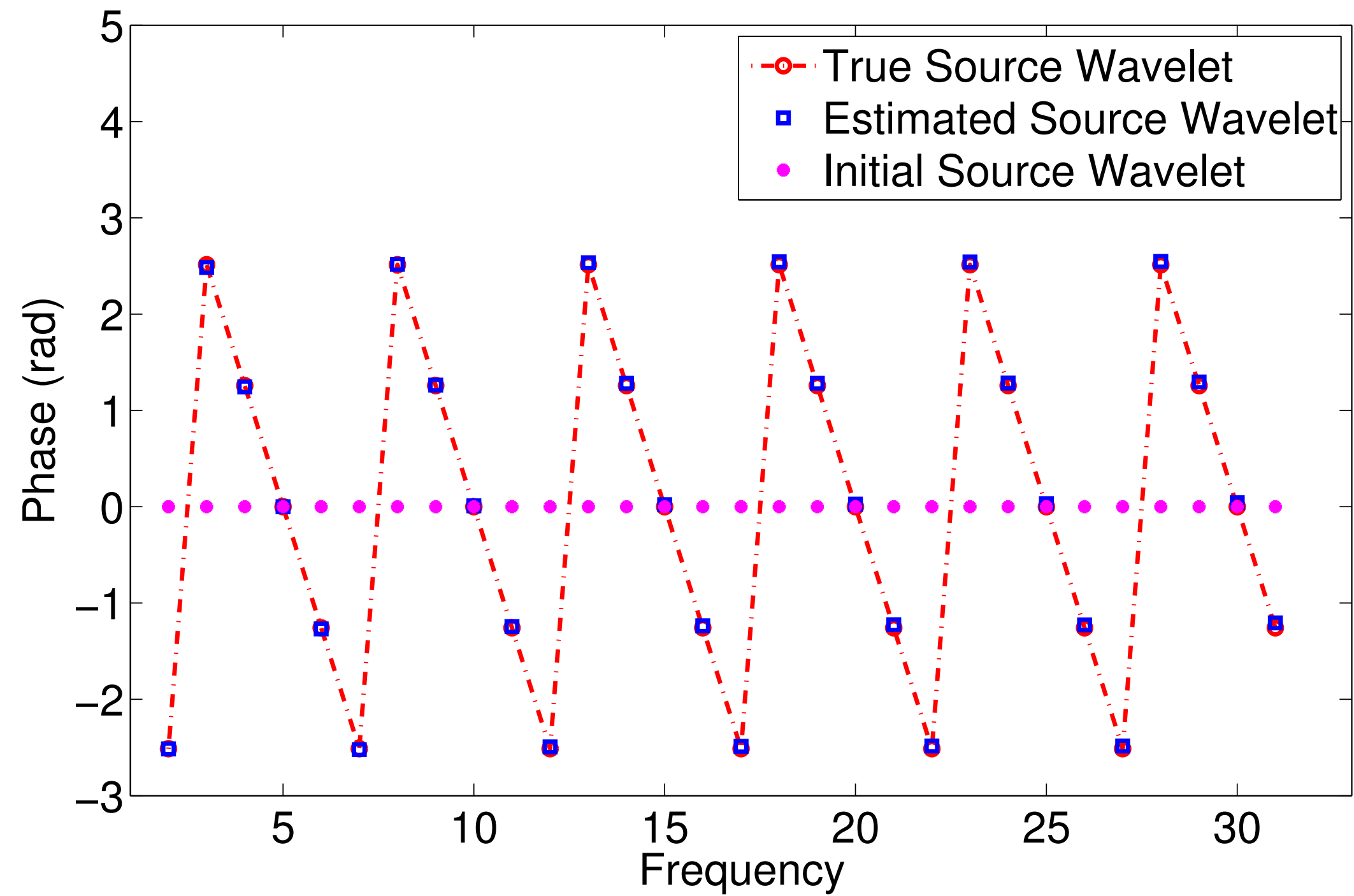




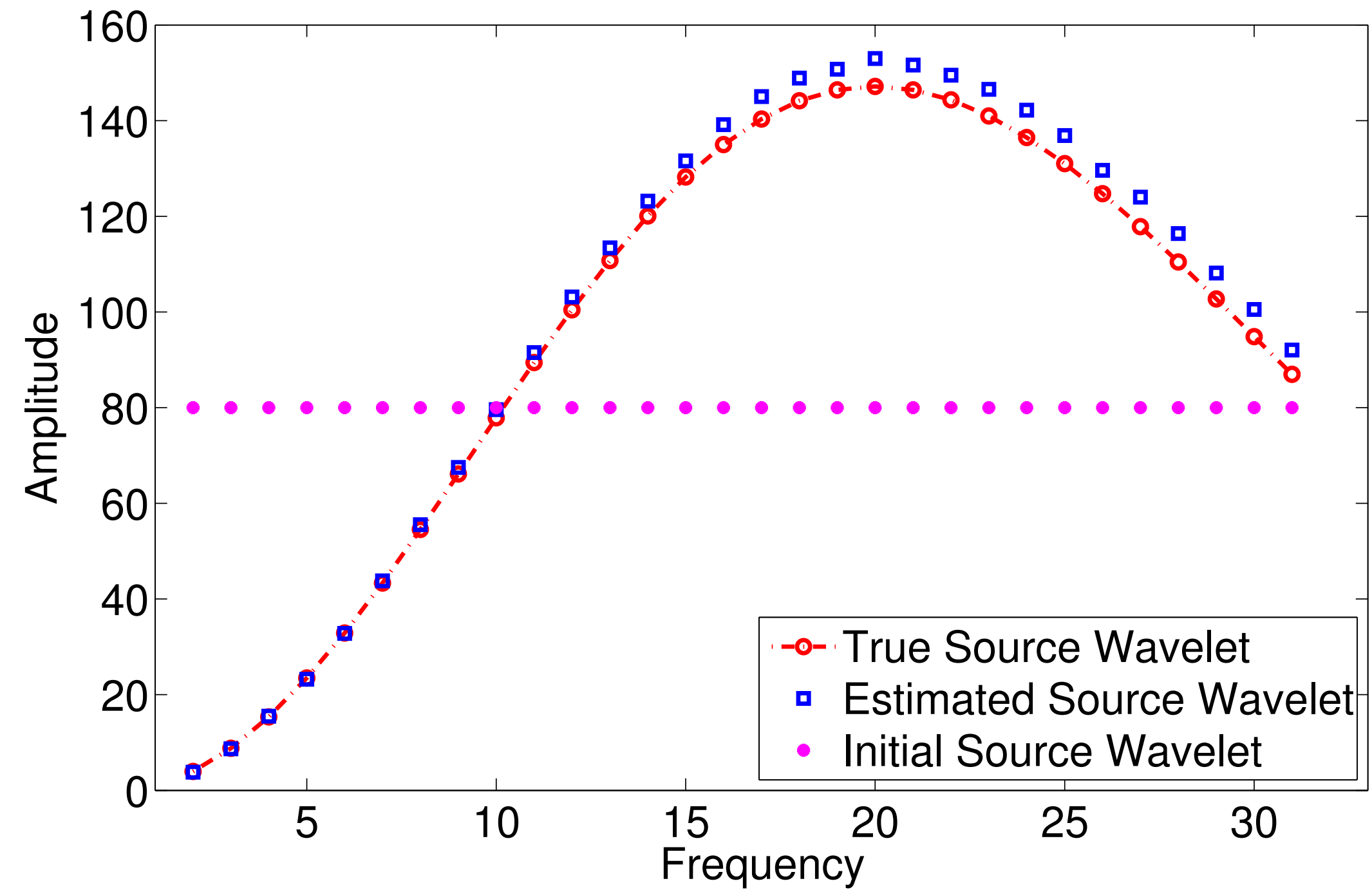
# Relative model-error comparison



# Source wavelet comparison

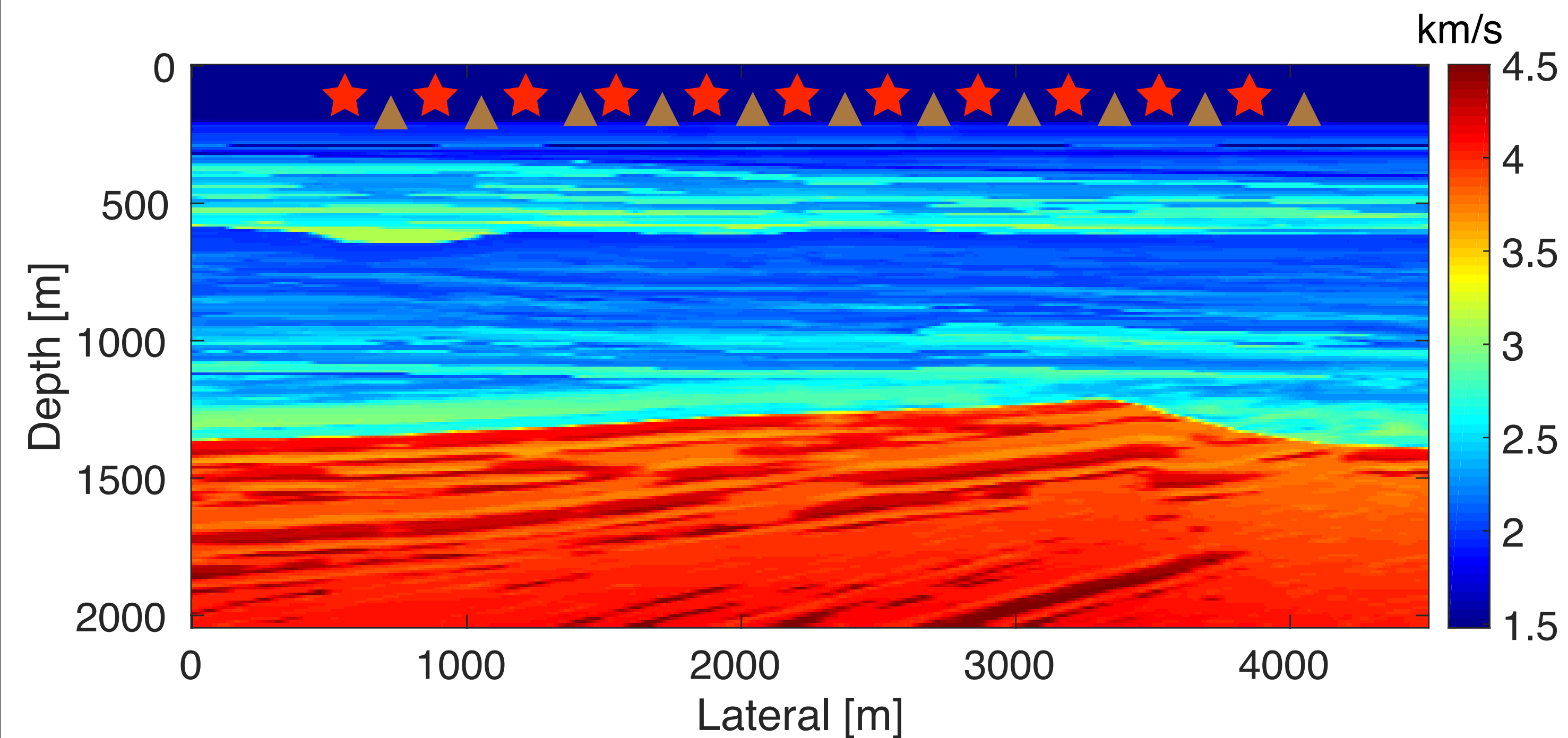


Phase



Amplitude

# BG model



**Modeling information:**  
**Model size:** 2000m x 4500m  
**Source spacing:** 50m  
**Receiver spacing:** 10m  
**Fixed spread 4.5km**

## Non-inversion crime example

### Observed Data information:

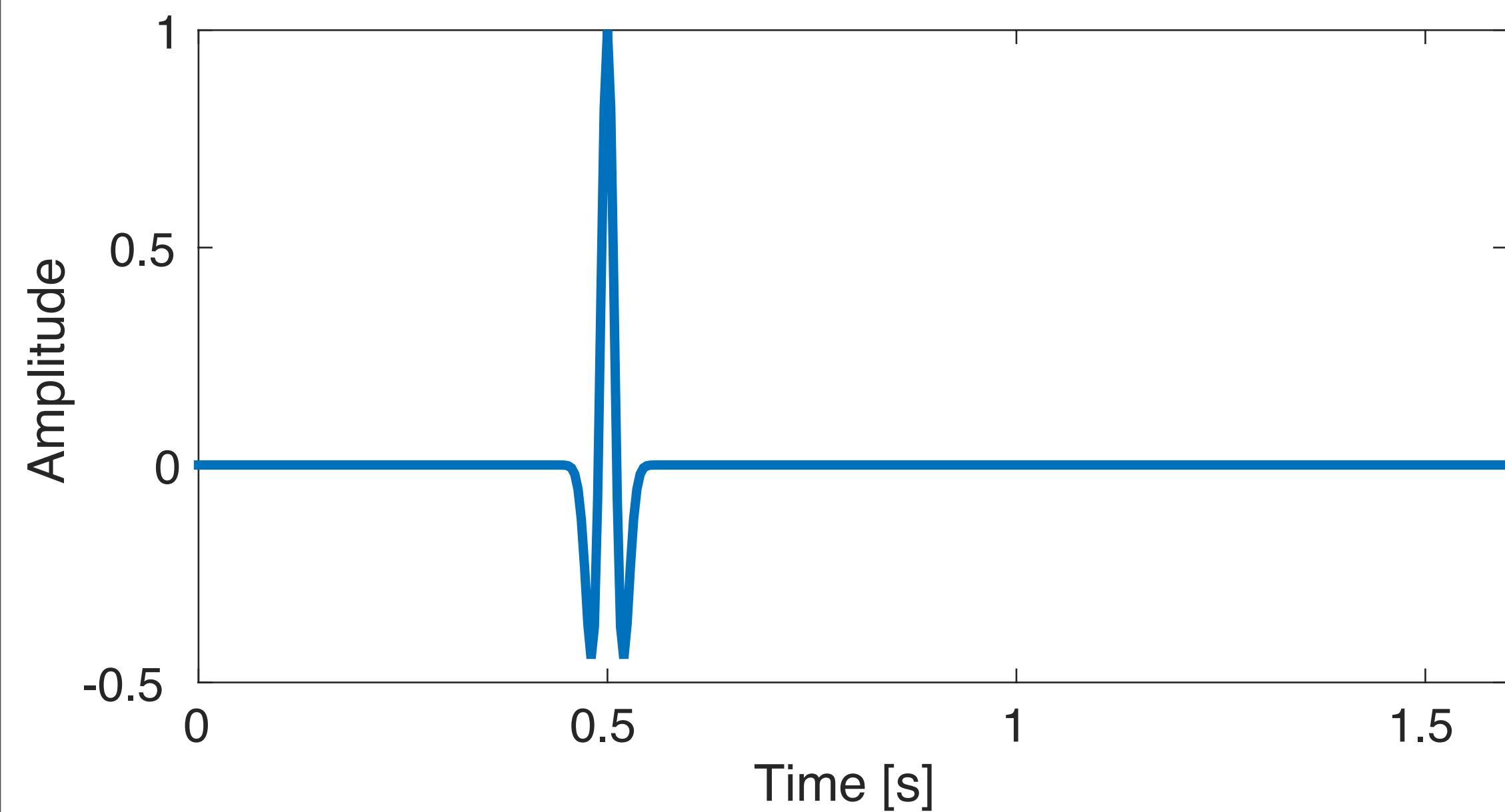
- Time domain data by iWAVE
- Grid size: 5m
- Source wavelet: 20Hz Ricker wavelet

### Inversion strategy:

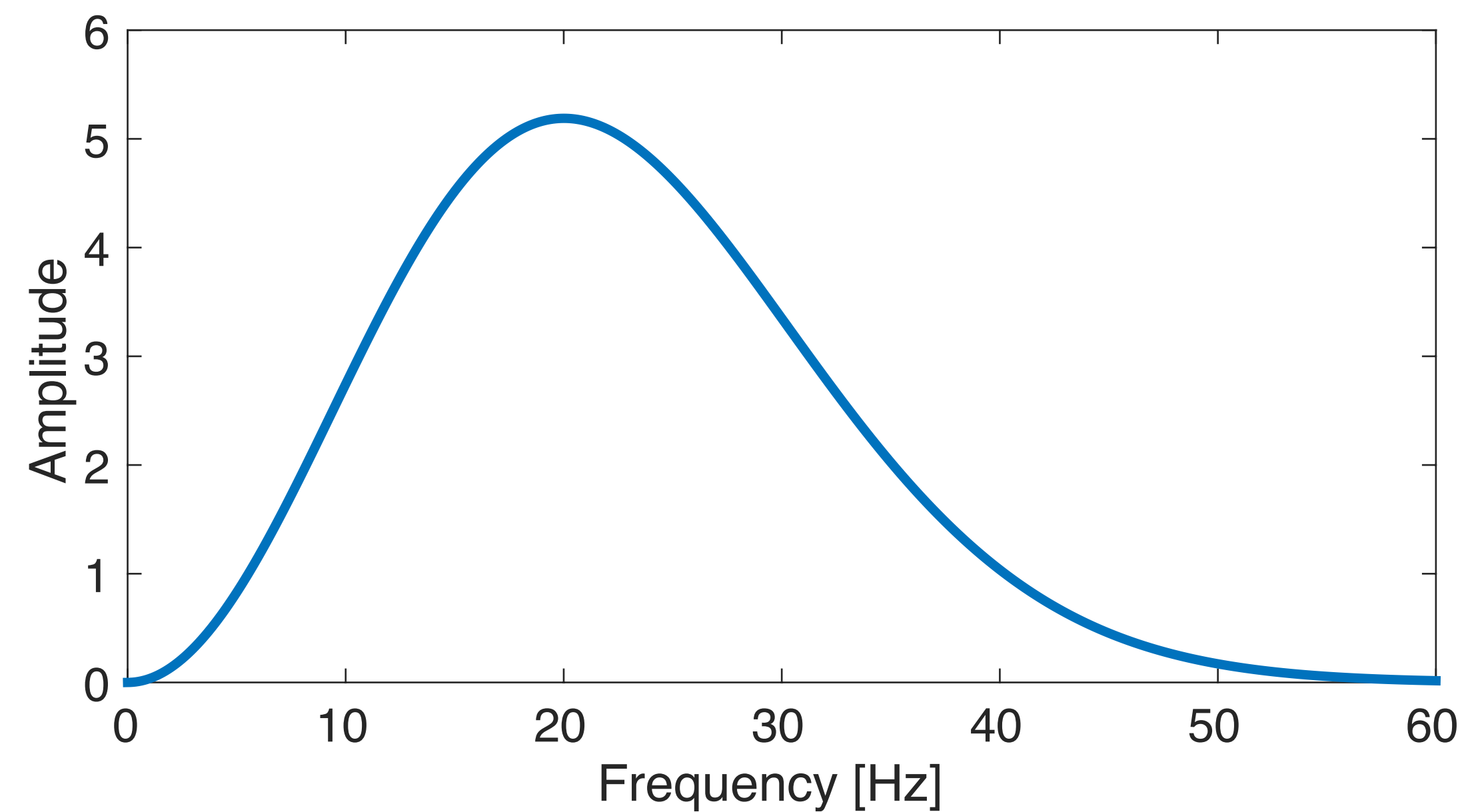
- Frequency: 2-29Hz
- Grid size: 10m (2-20 Hz), and 6m (20-29 Hz)
- Gauss-Newton method with 20 iterations per frequency band



# Source wavelet

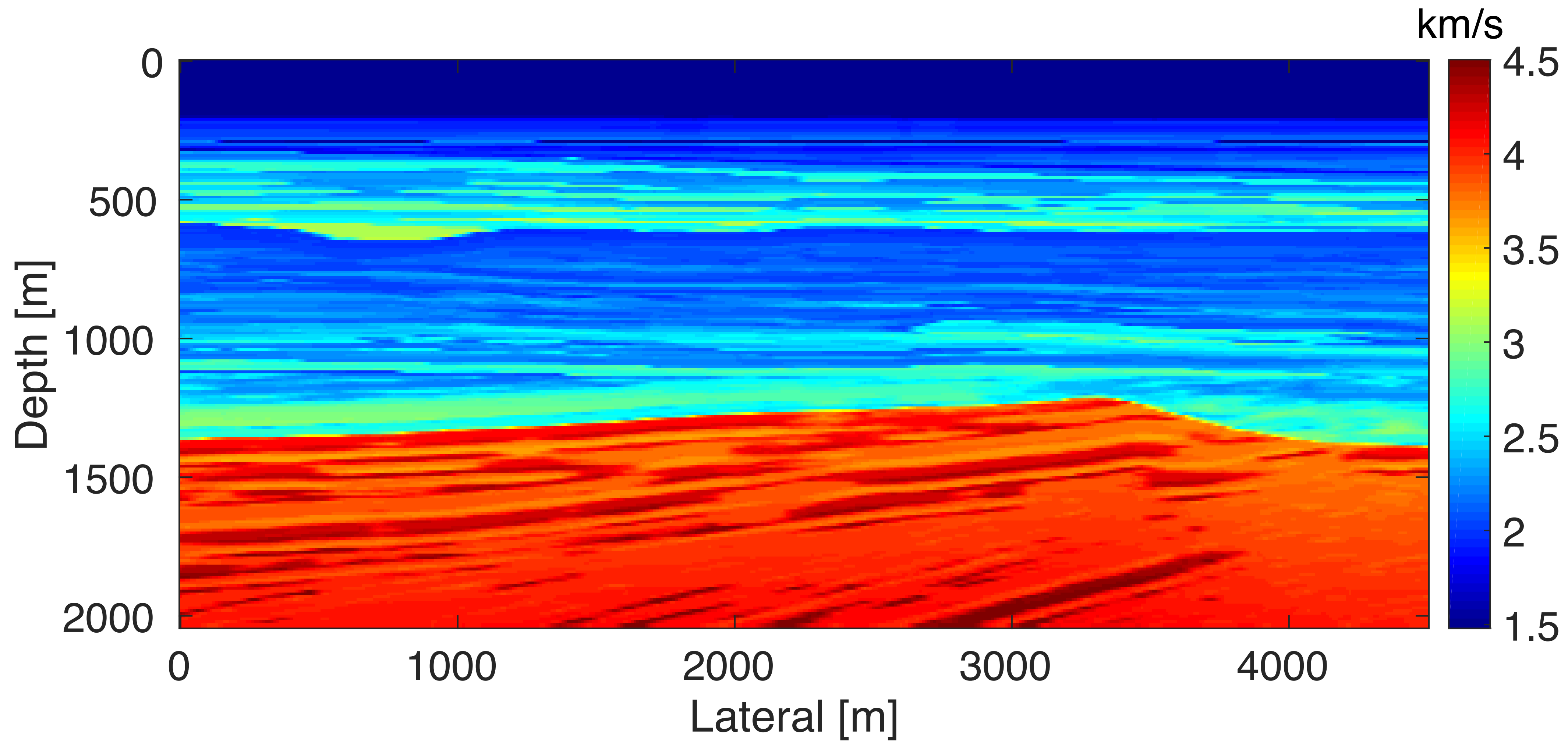


## Source wavelet

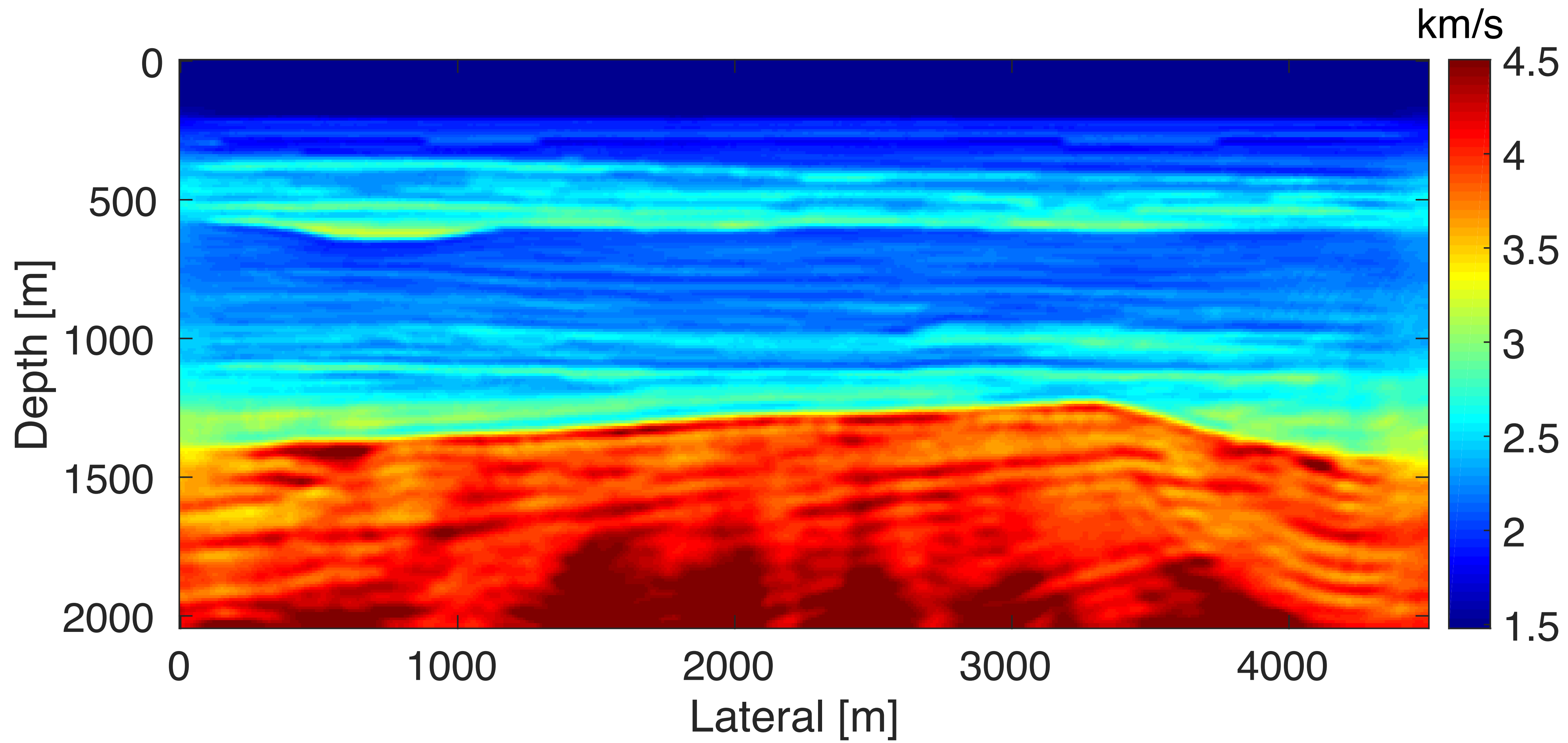


## Spectrum

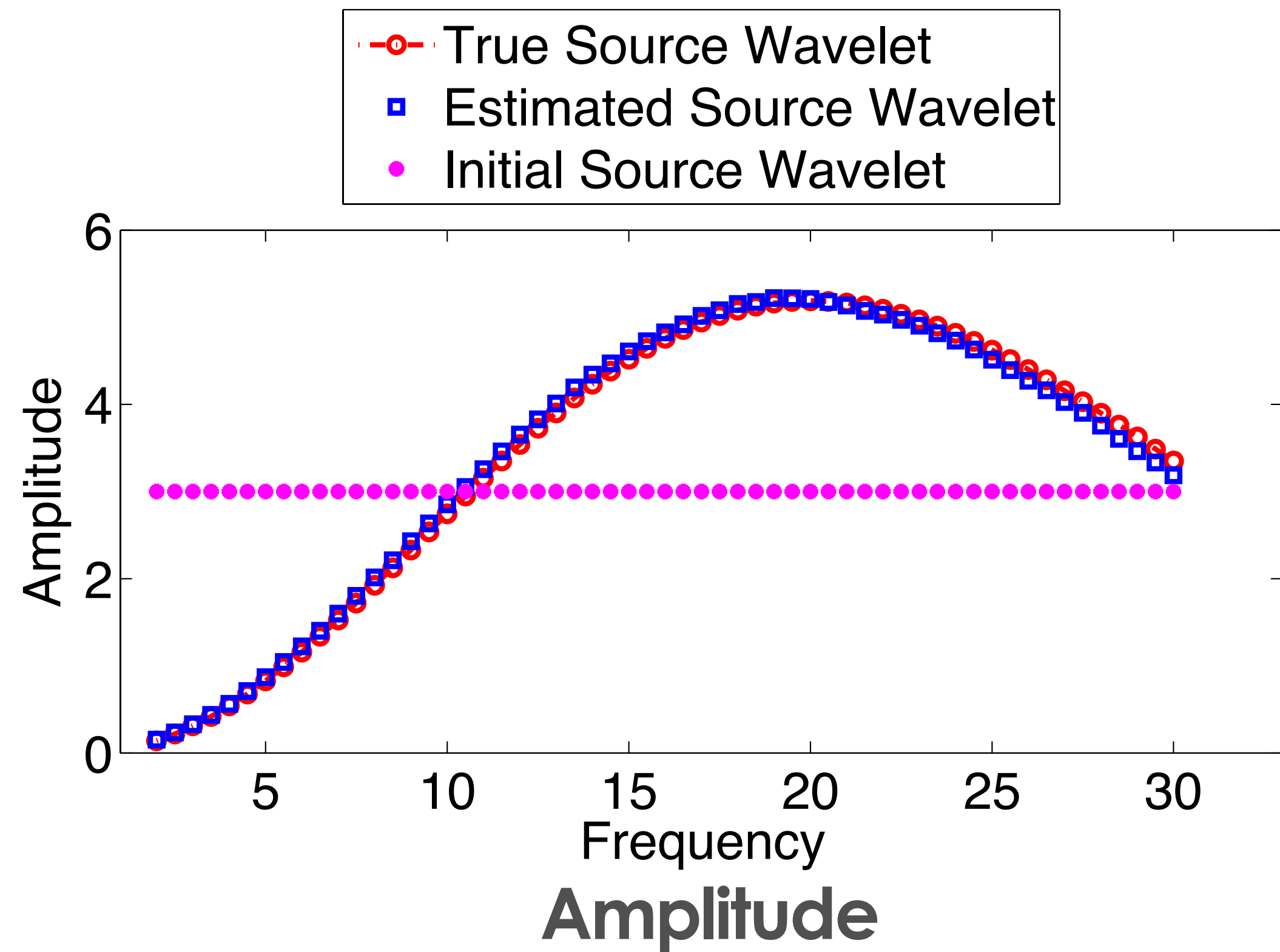
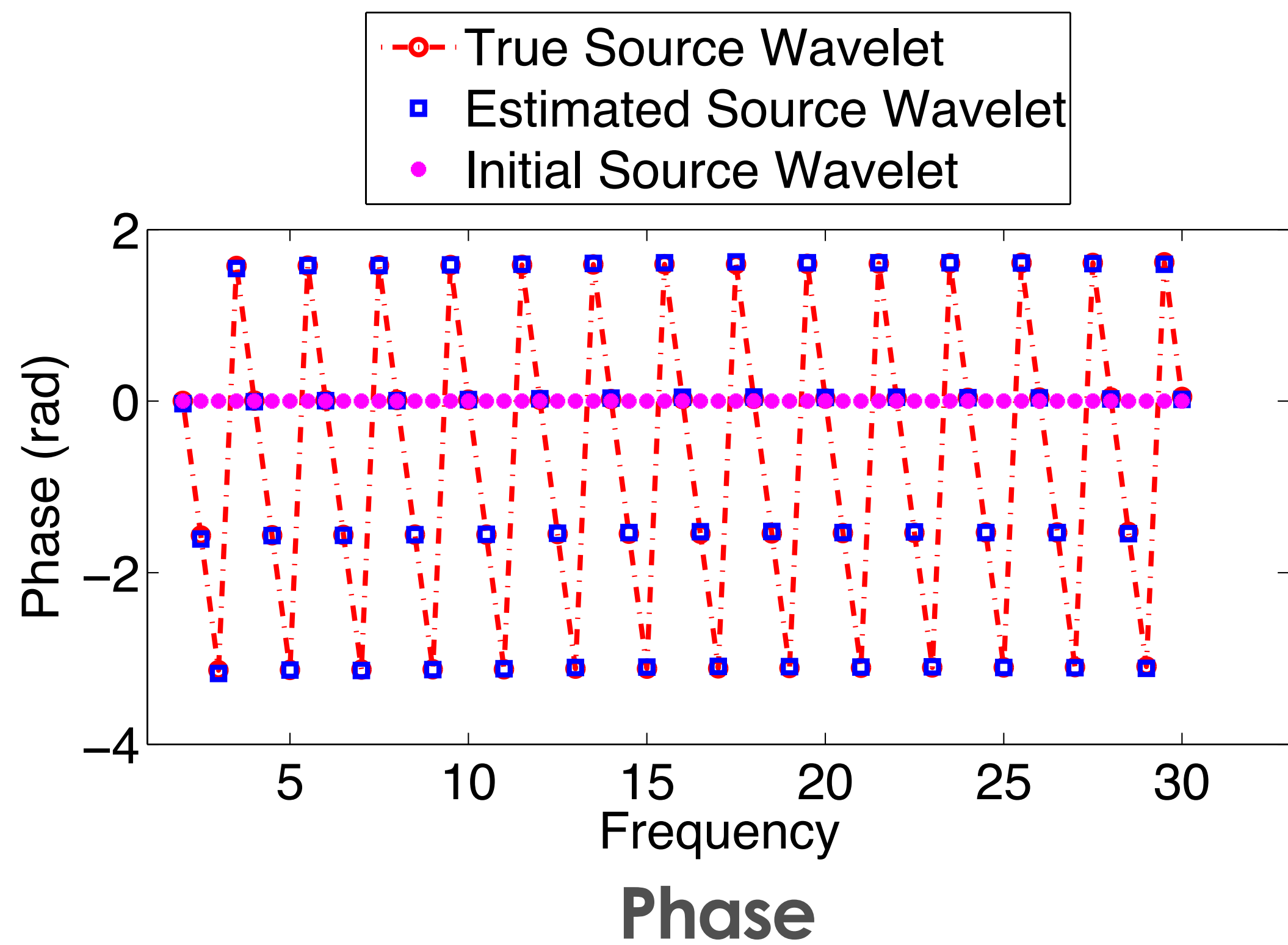
# True model



# Inversion result



# Source wavelet comparison





# Chevron blind test data



Zhilong Fang



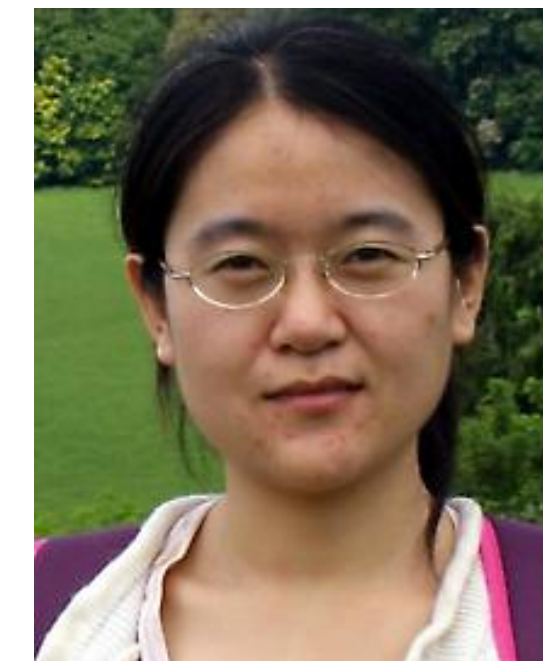
Xiang Li



Bas Peters



Brendan Smithyman

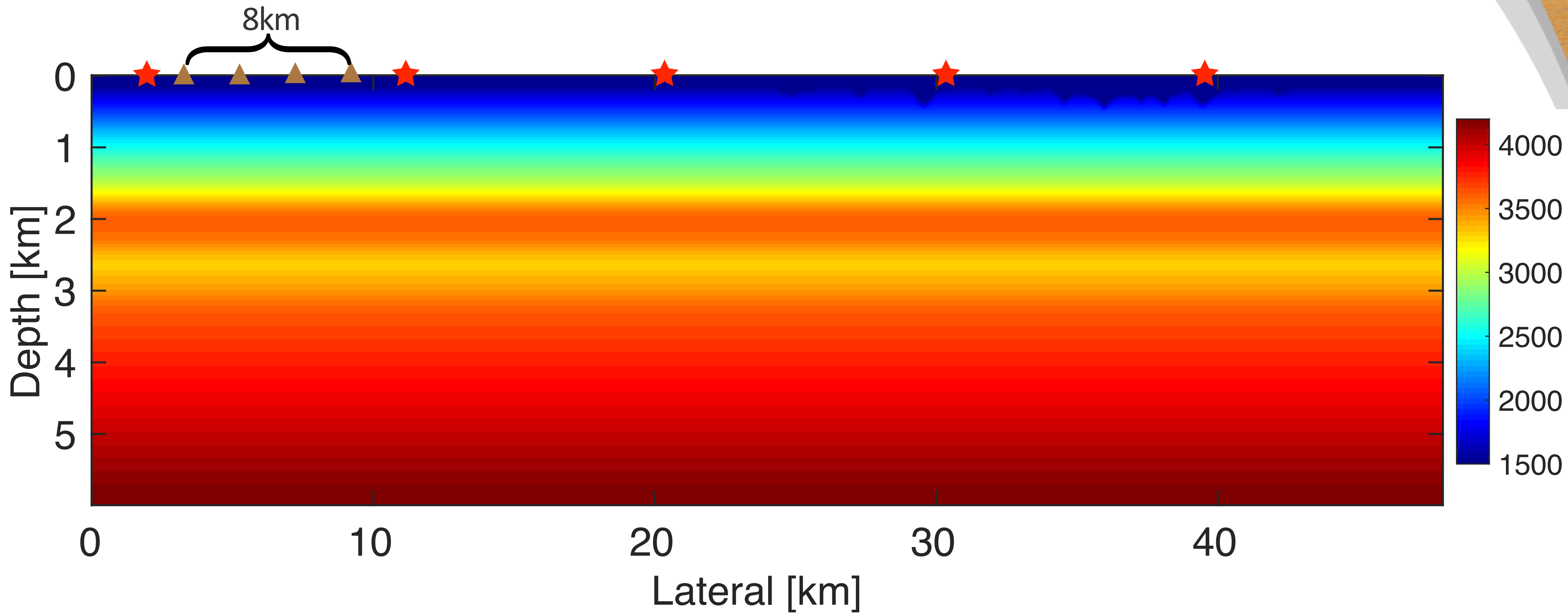


Mengmeng Yang



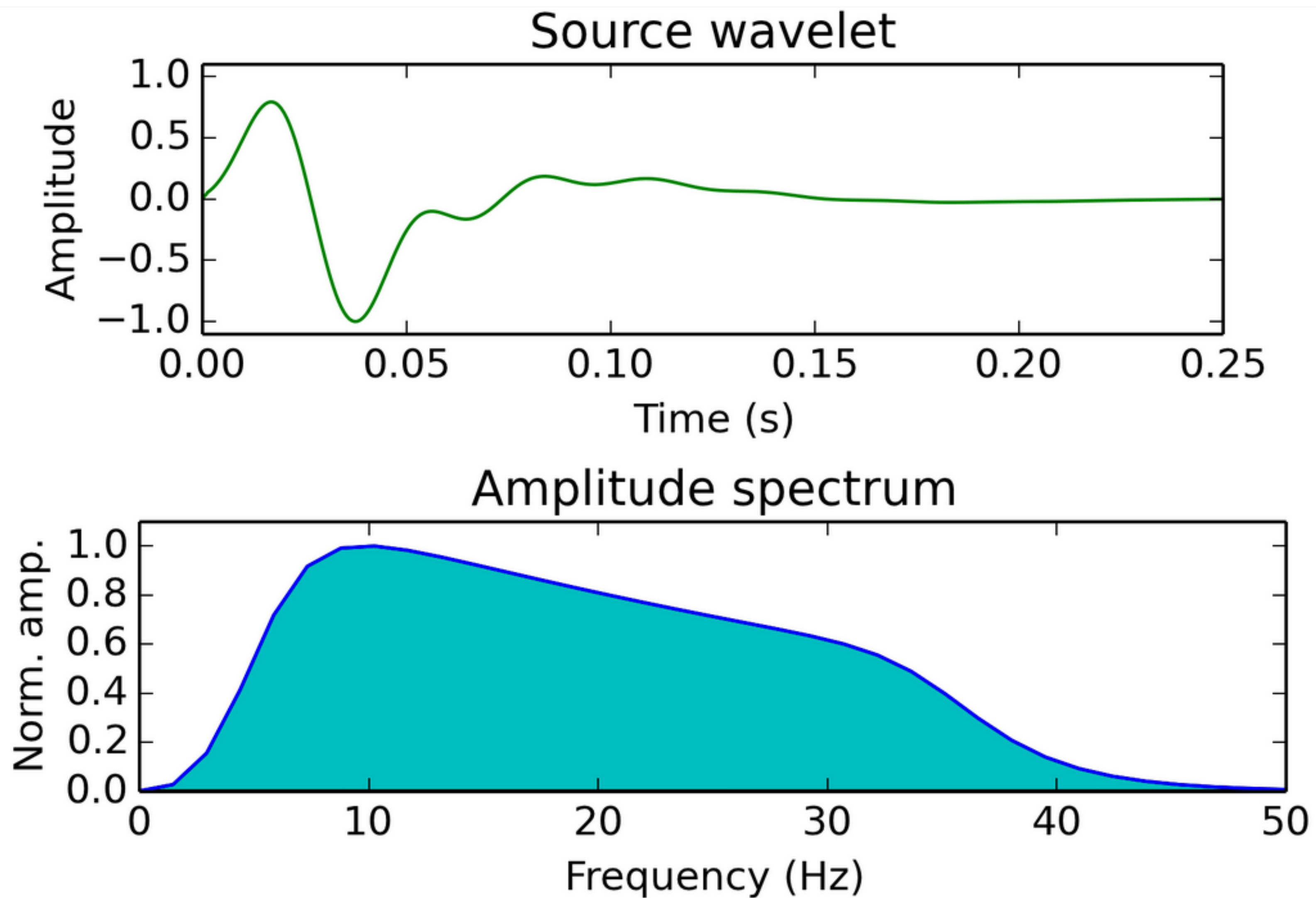
Felix J. Herrmann

# Chevron blind test data





# Chevron blind test data



# Chevron blind test data

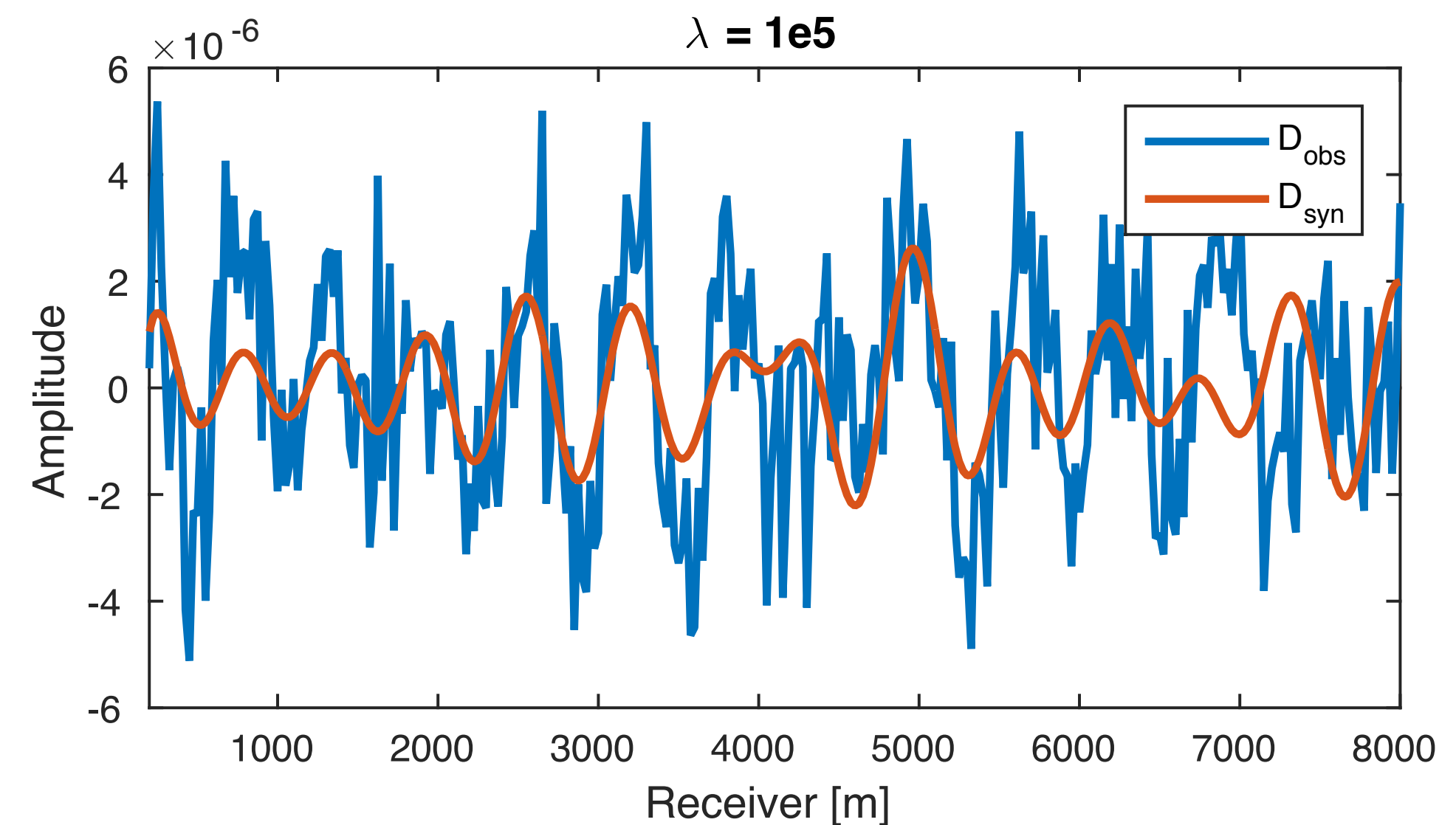
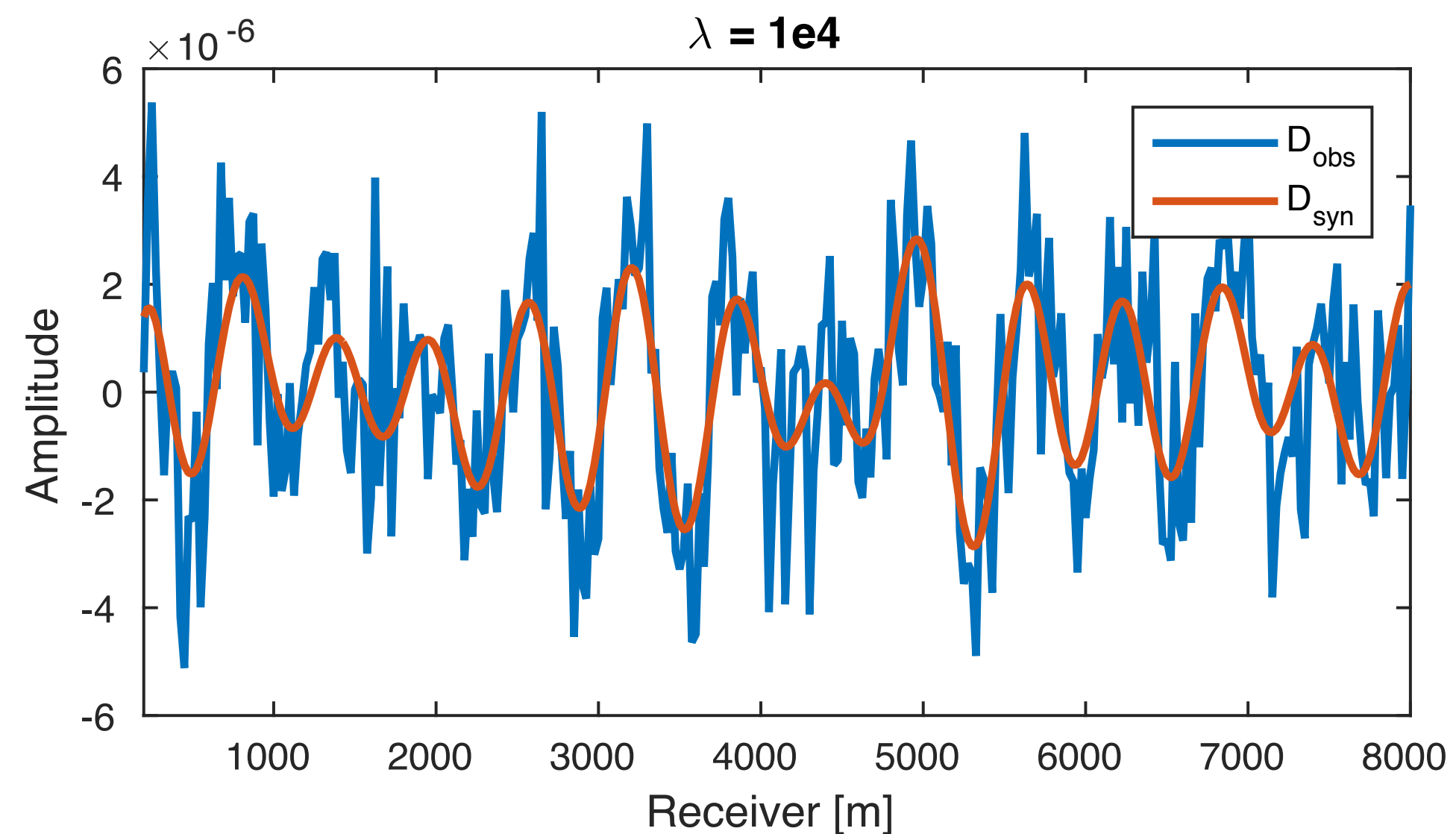
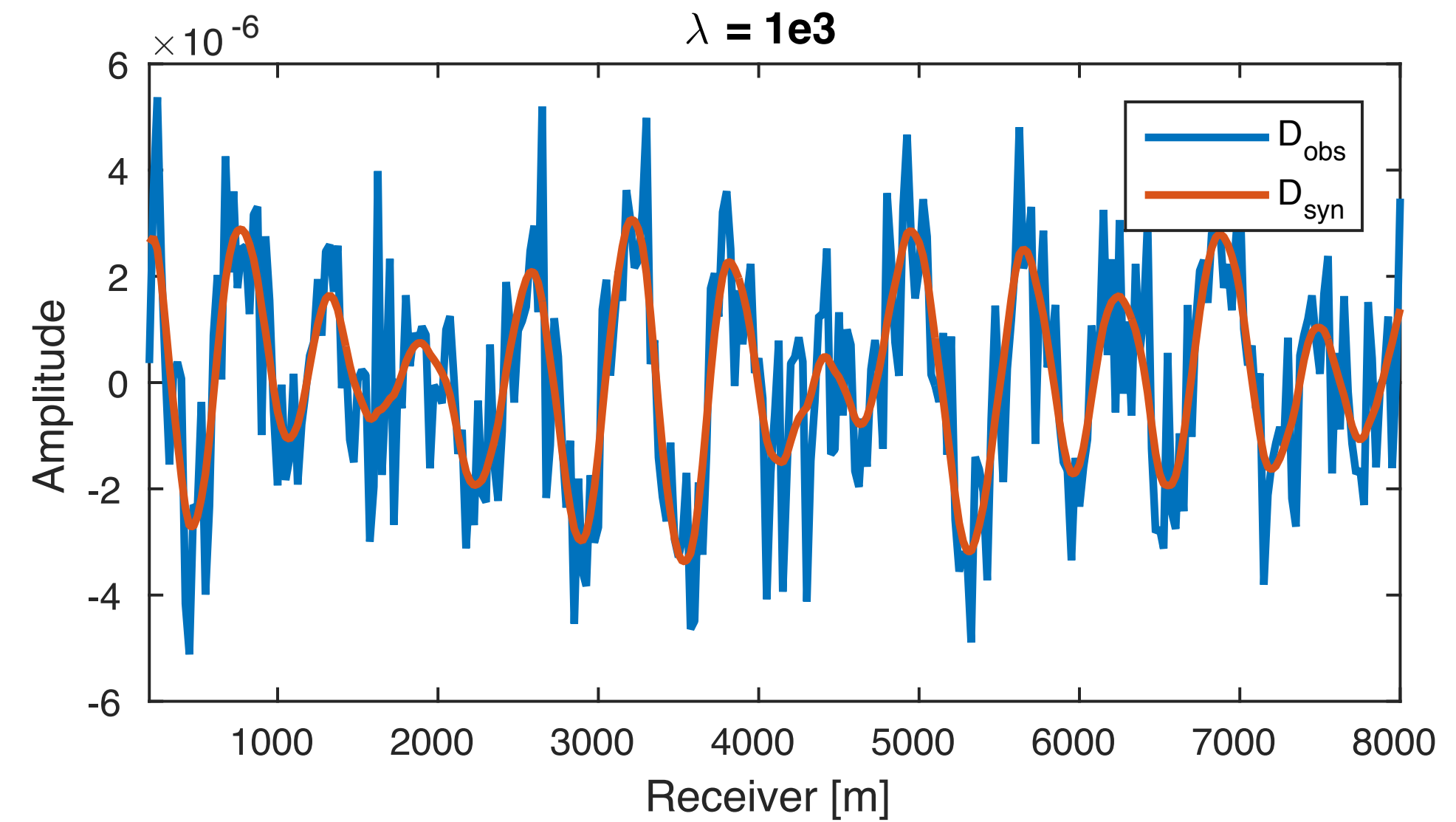
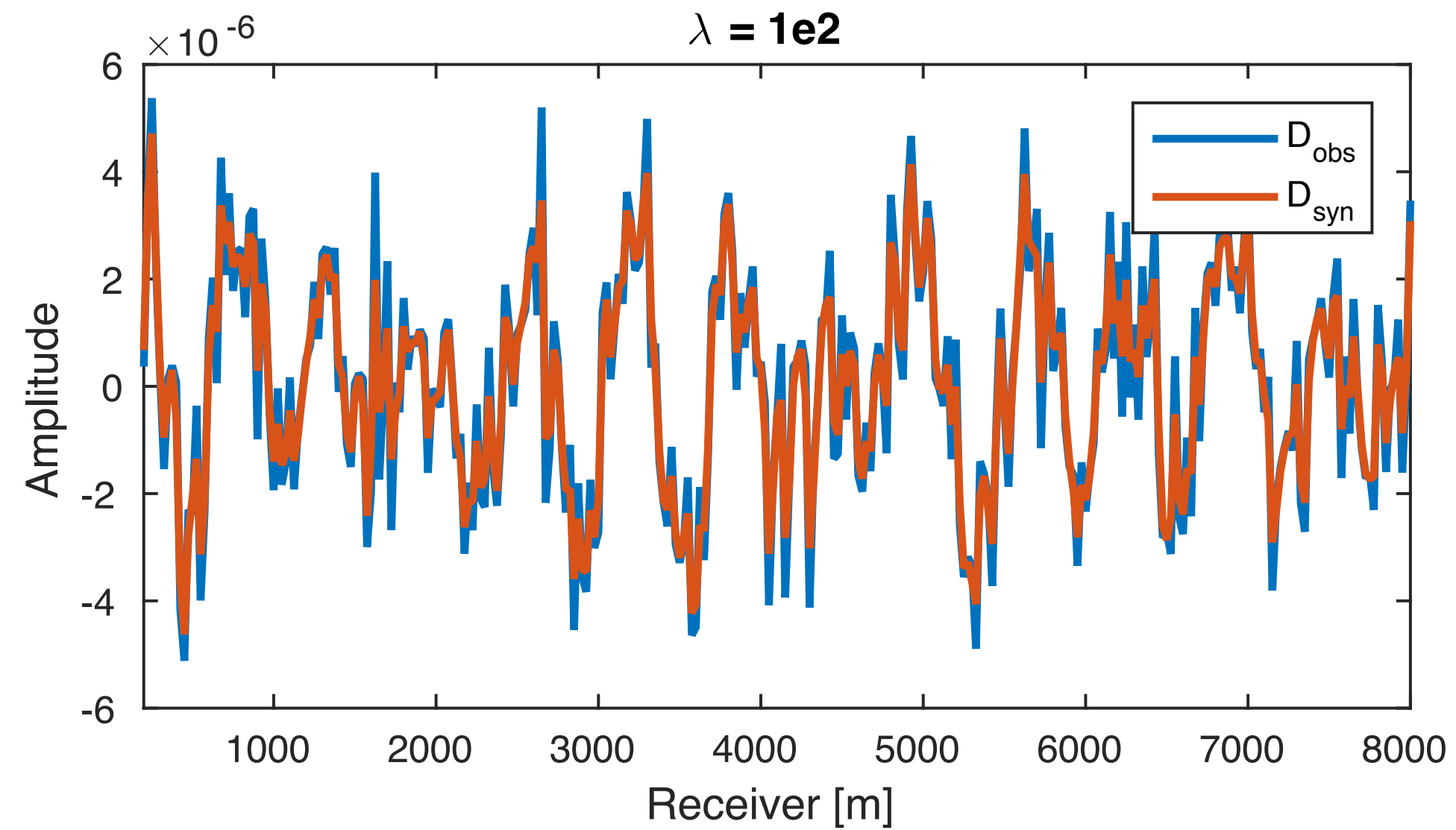
## Data-set information:

1. 1600 shots:  $d_s = 25$  m, Source depth = 15 m;
2. 321 hydrophone recs/shot:  $d_r = 25$  m, Receiver depth = 15 m;
3. Maximum offset = 8000 m;
4. Record time = 8.0 s, sample rate 4 ms;
5.  $V_p$  water = constant = 1510 m/s;
6. With free surface multiples present in the data;
7. Isotropic Elastic.

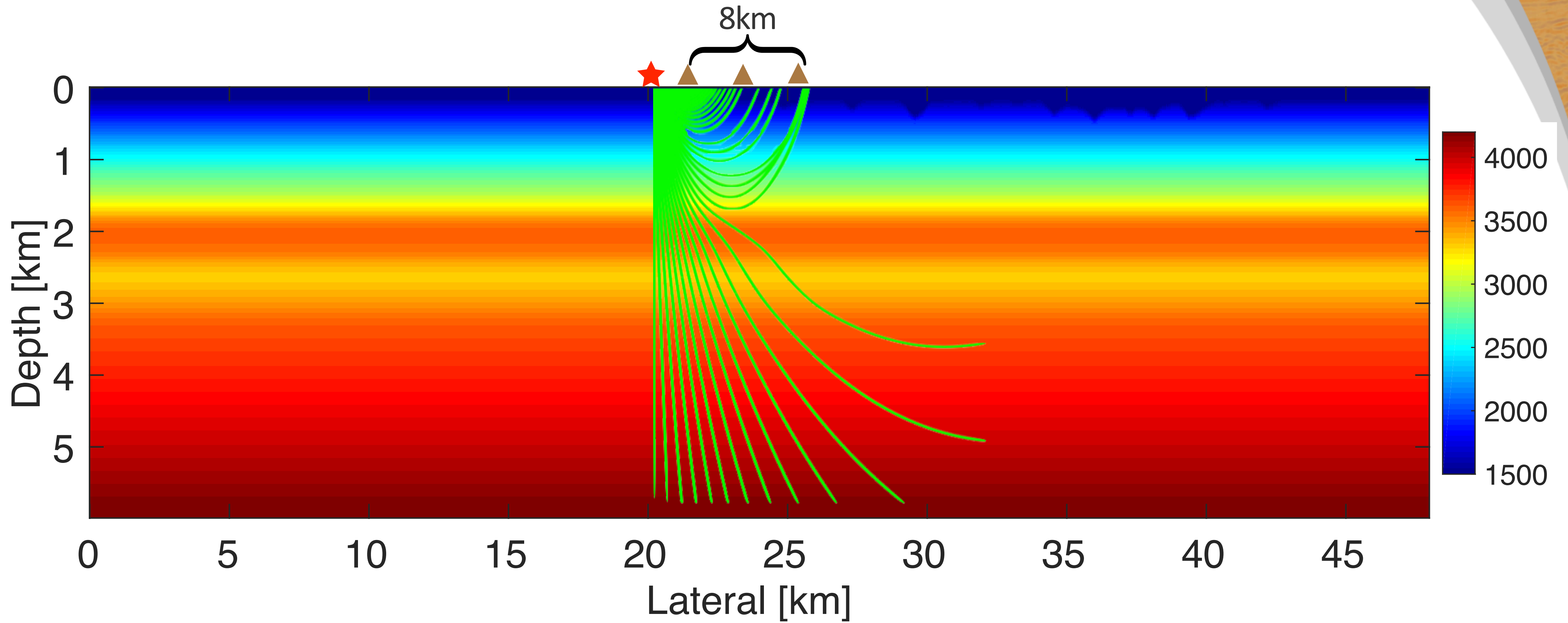


# Data comparison

## — 3 Hz Data of 800th shot



# Initial model



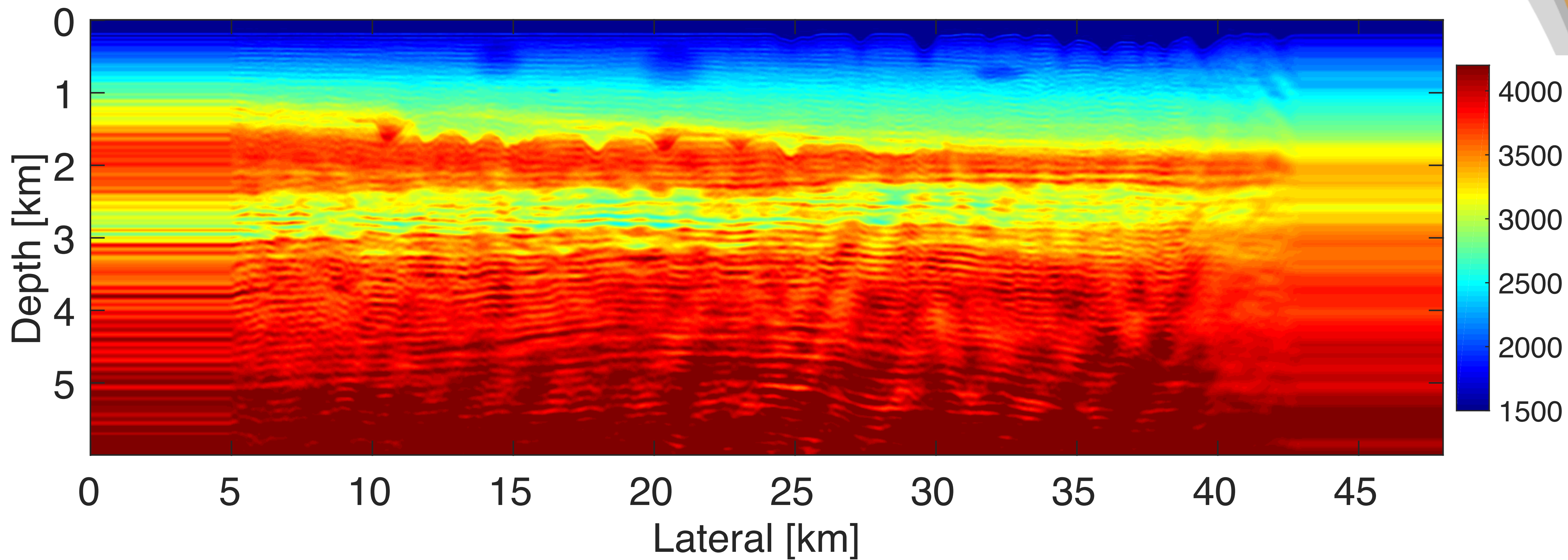
# Chevron blind test data

## Inversion strategy:

1. Frequency domain WRI with Source estimation;
2. Frequency bands: [3:0.2:5]Hz, [3:0.2:7]Hz, [3:0.2:9]Hz, [3:0.2:11]Hz, [3:0.2:15]Hz;
3. Batch sizes of random frequency subsets: 3, 6, 10, 10, 15;
4. Batch size of random source subsets: 300;
5. Optimization solver: l-BFGS with 20 iterations per frequency band;
6. 4 passes of WRI at frequency 3-11 Hz and 1 pass to 15 Hz;
7. Grid size: 20m for 3-11Hz and 12m for 3-15Hz;
8. No pre-processing !!!



# Inversion result





## WRI with minimum smoothness constraint

$$\underset{\mathbf{u}(\mathbf{m}), \mathbf{m}, \alpha(\mathbf{m})}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

## WRI with minimum smoothness constraint

$$\underset{\mathbf{u}(\mathbf{m}), \mathbf{m}, \alpha(\mathbf{m})}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

$$\text{subject to } \mathbf{m} \in \mathcal{C}_1 \cap \mathcal{C}_2$$

$$\mathcal{C}_1 \equiv \{\mathbf{m} \mid \mathbf{b}_l \leq \mathbf{m} \leq \mathbf{b}_u\}$$

$$\mathcal{C}_2 \equiv \{\mathbf{m} \mid E^* F^* (I - S) F E \mathbf{m} = 0\}$$

## WRI with minimum smoothness constraint

$$\underset{\mathbf{u}(\mathbf{m}), \mathbf{m}, \alpha(\mathbf{m})}{\text{minimize}} \sum_{k,l} \|\mathbf{P}_k \mathbf{u}_{k,l} - \mathbf{d}_{k,l}\|_2^2 + \lambda^2 \|\mathbf{A}_{k,l}(\mathbf{m}) \mathbf{u}_{k,l} - \alpha_{k,l} \mathbf{e}_{k,l}\|_2^2$$

$$\text{subject to } \mathbf{m} \in \mathcal{C}_1 \cap \mathcal{C}_2$$

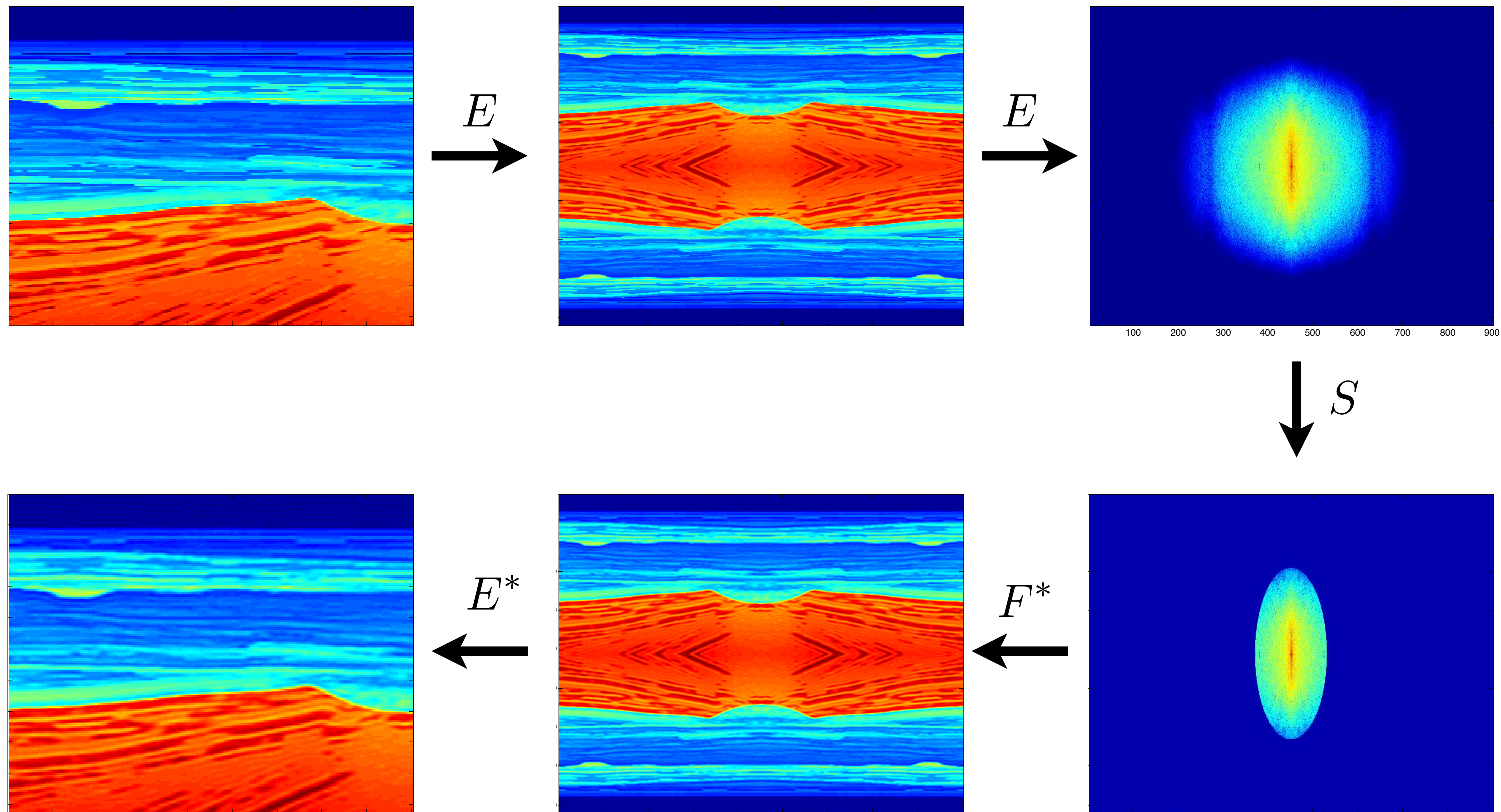
$$\mathcal{C}_1 \equiv \{\mathbf{m} \mid \mathbf{b}_l \leq \mathbf{m} \leq \mathbf{b}_u\}$$

$$\mathcal{C}_2 \equiv \{\mathbf{m} \mid E^* F^* (I - S) F E \mathbf{m} = 0\}$$

1. 2D mirror extension of the model (to avoid periodic boundaries)
2. 2D DFT
3. Remove coefficients outside ellipse (highest spatial frequencies)
4. 2D inverse DFT



# WRI with minimum smoothness constraint



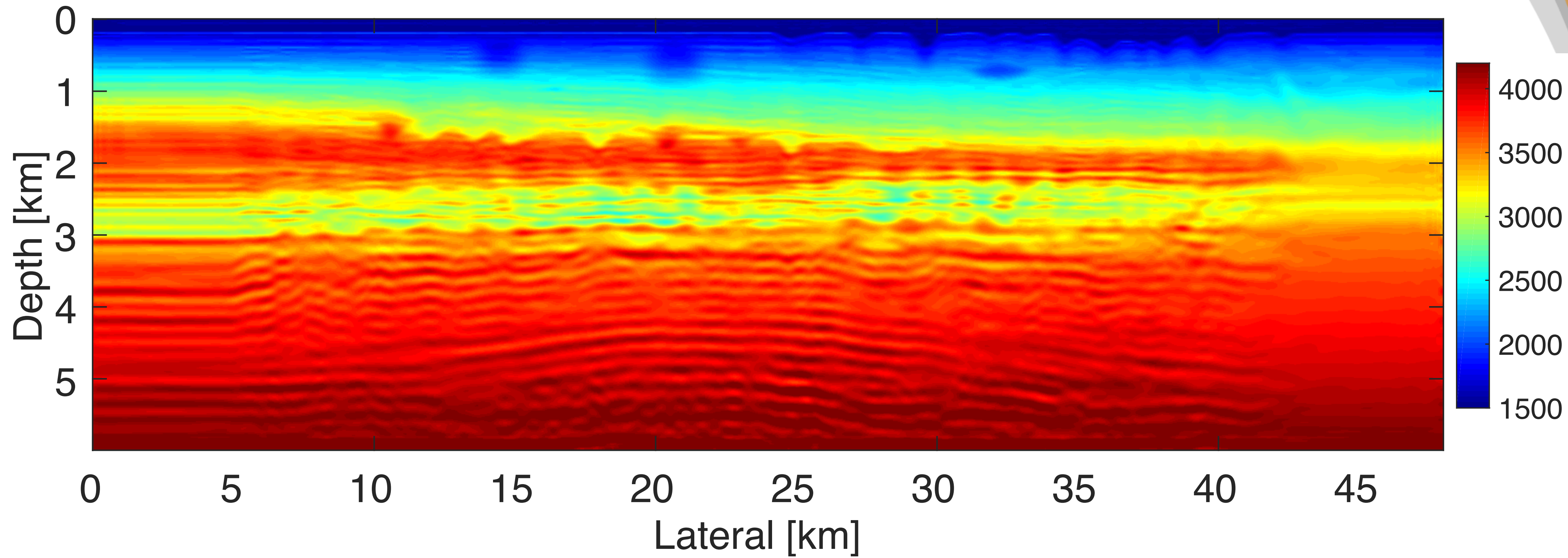


# Chevron blind test data

## Inversion strategy:

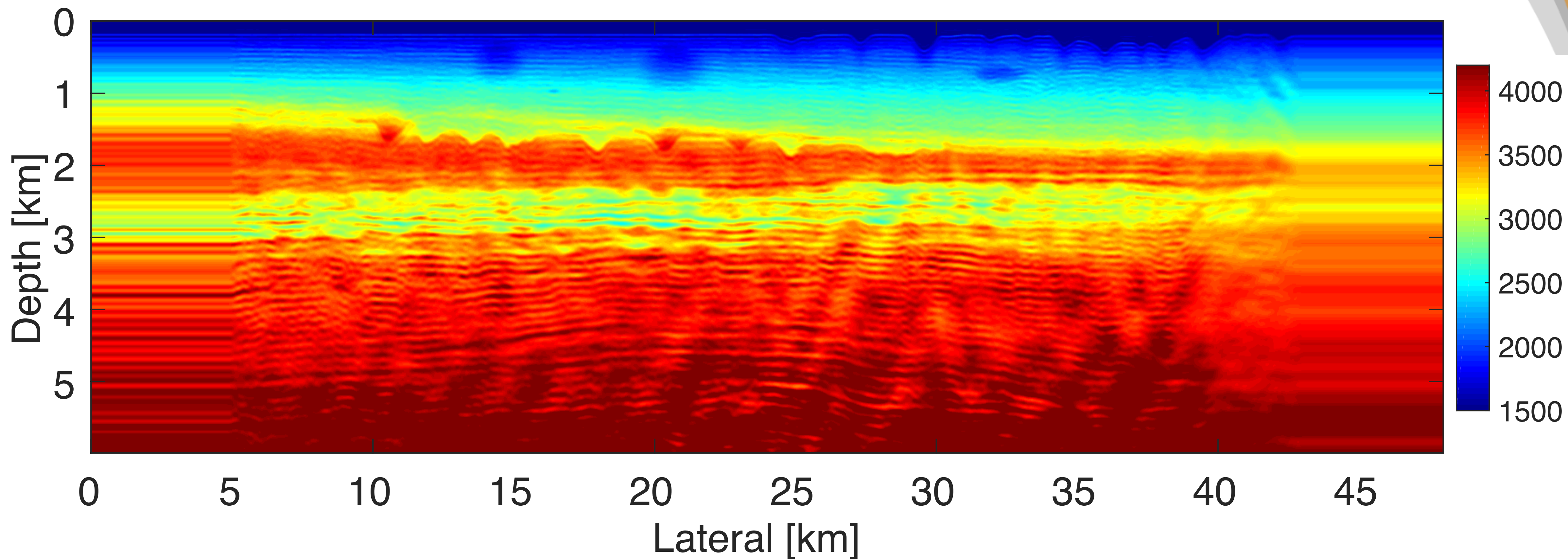
1. Frequency domain WRI with Source estimation and smoothness constraint;
2. Frequency bands: [3:0.2:5]Hz, [3:0.2:7]Hz, [3:0.2:9]Hz, [3:0.2:11]Hz;
3. Batch sizes of random frequency subsets: 3, 6, 10, 10;
4. Batch size of random source subsets: 300;
5. Optimization solver: PQN with 20 iterations per frequency band;
6. 2passes of WRI from frequency 3-11 Hz;
7. Grid size: 20m;
8. No pre-processing !!!

# Inversion result

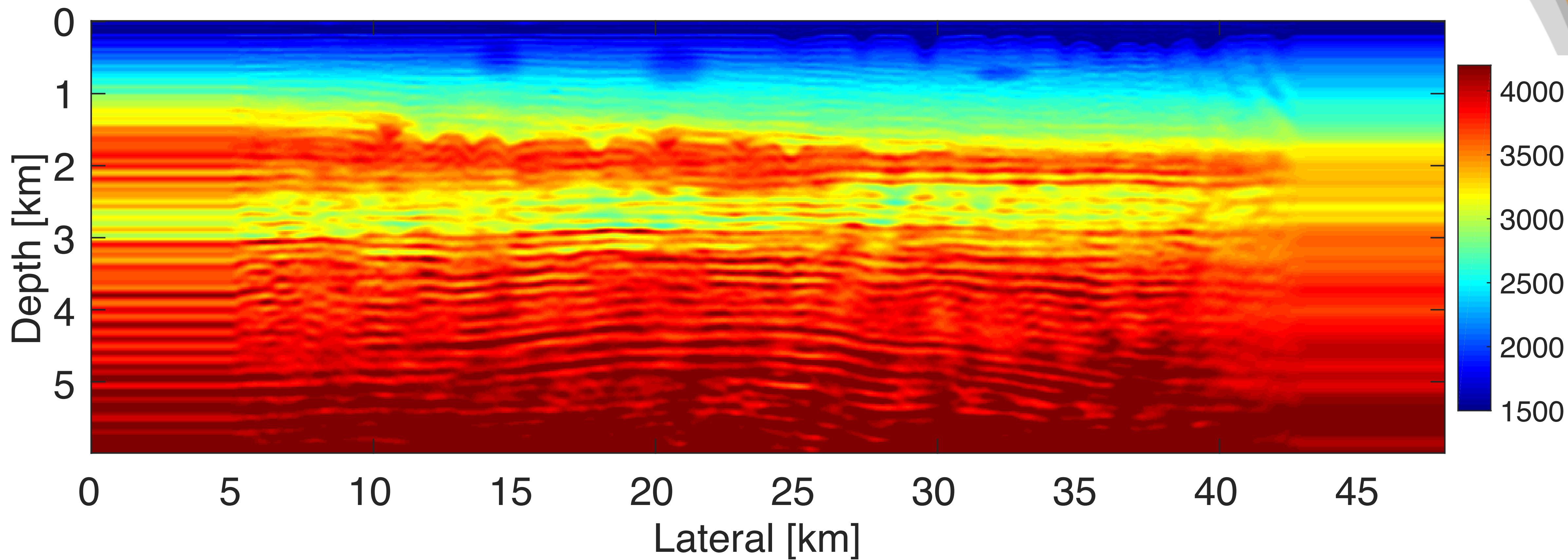




# Inversion result (presented in EAGE 2015)

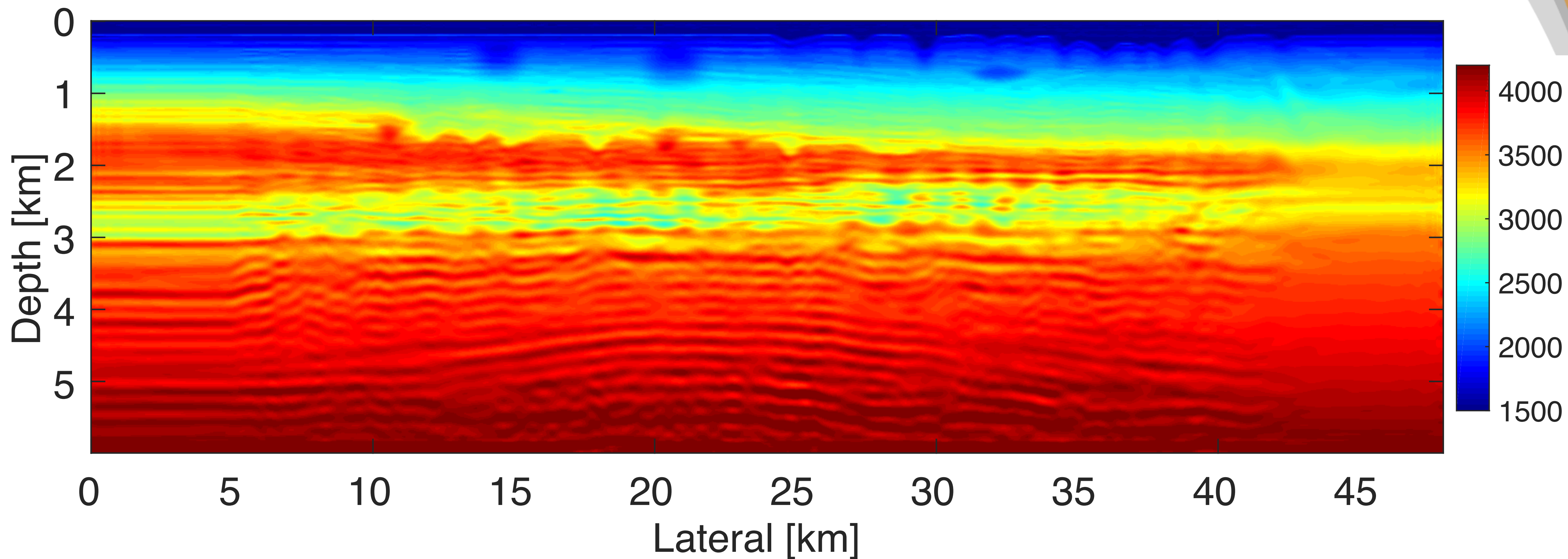


# Inversion result (without minimum smoothness constraint)

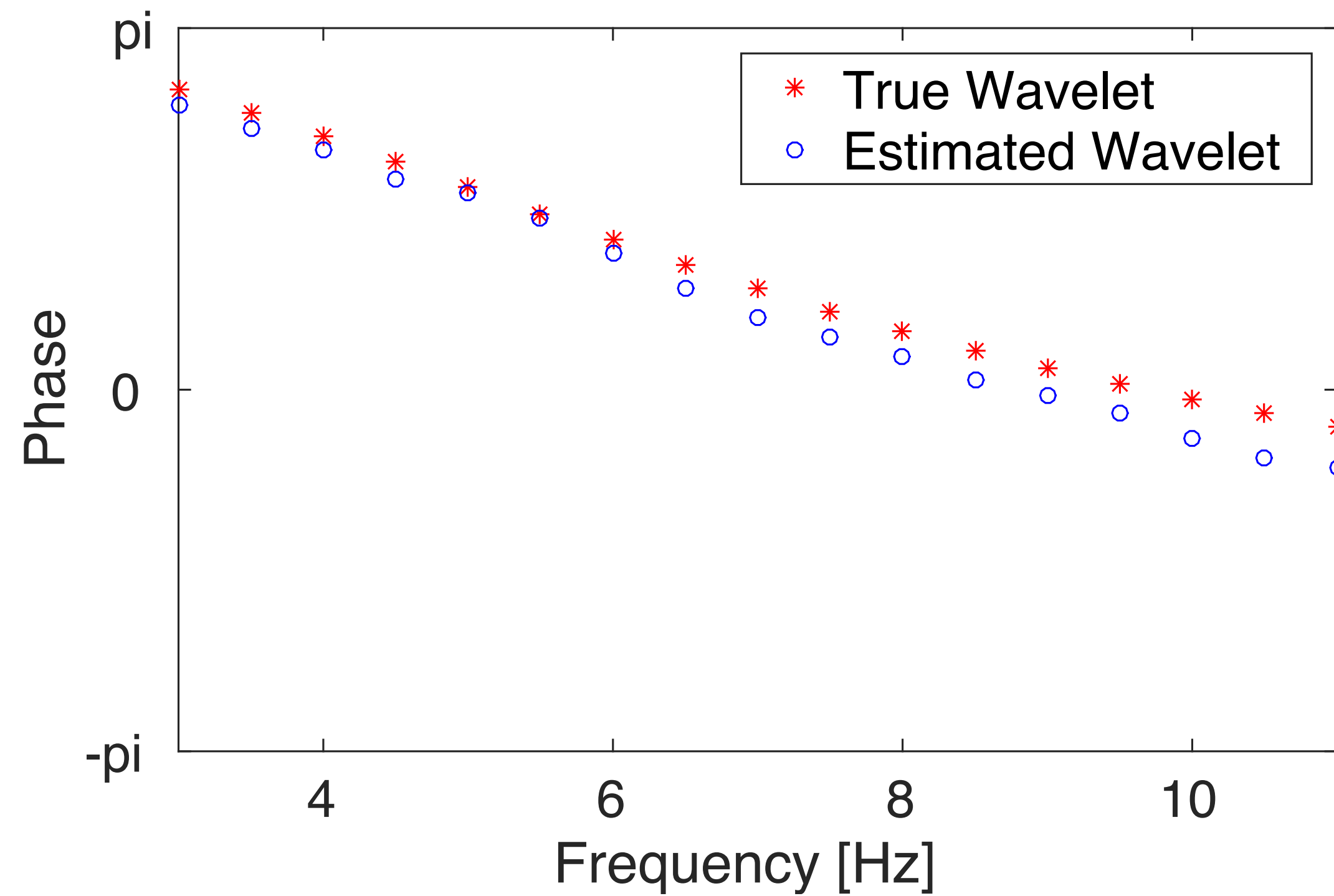




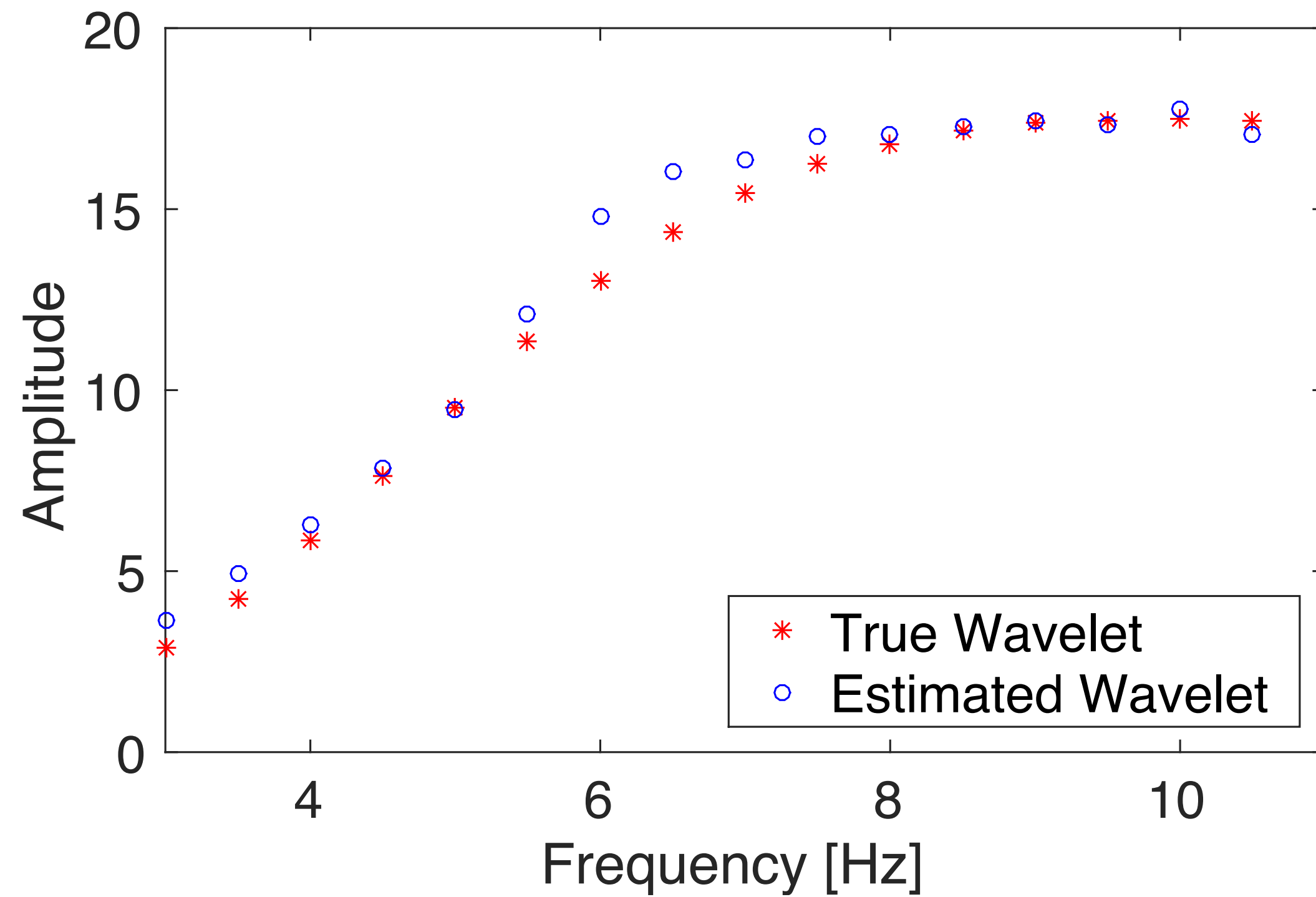
# Inversion result (with minimum smoothness constraint)



# Source wavelet comparison

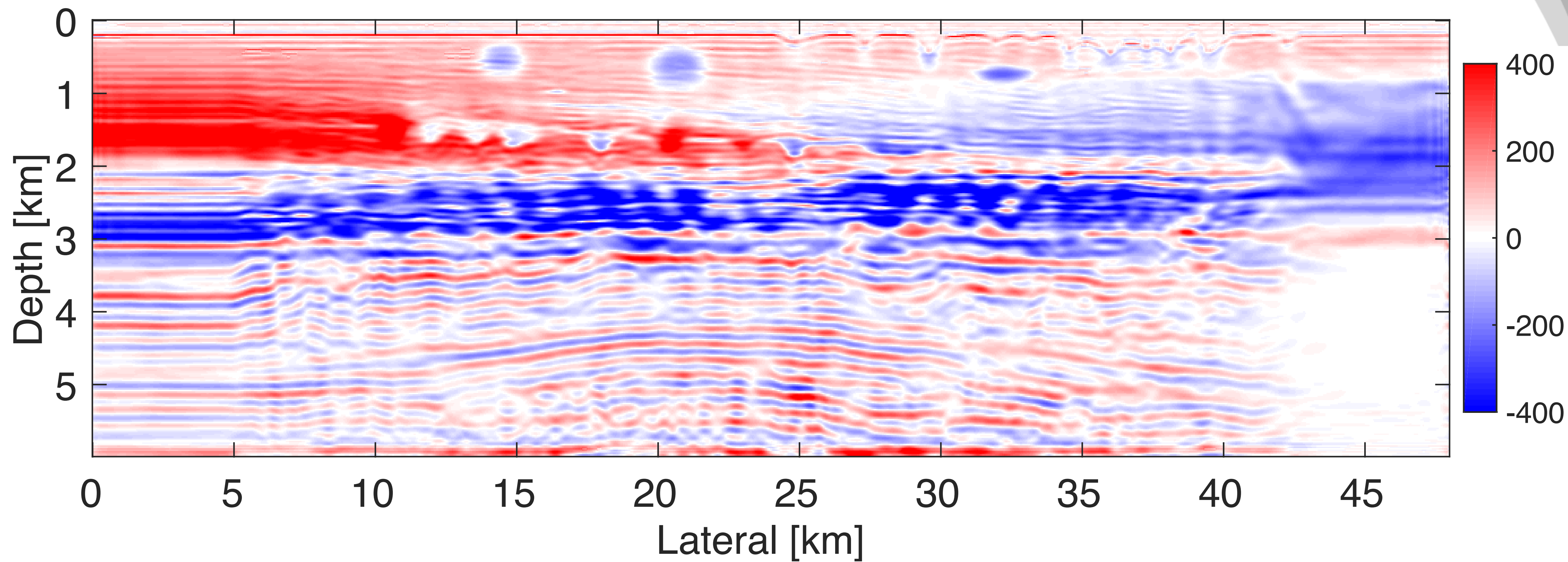


Phase



Amplitude

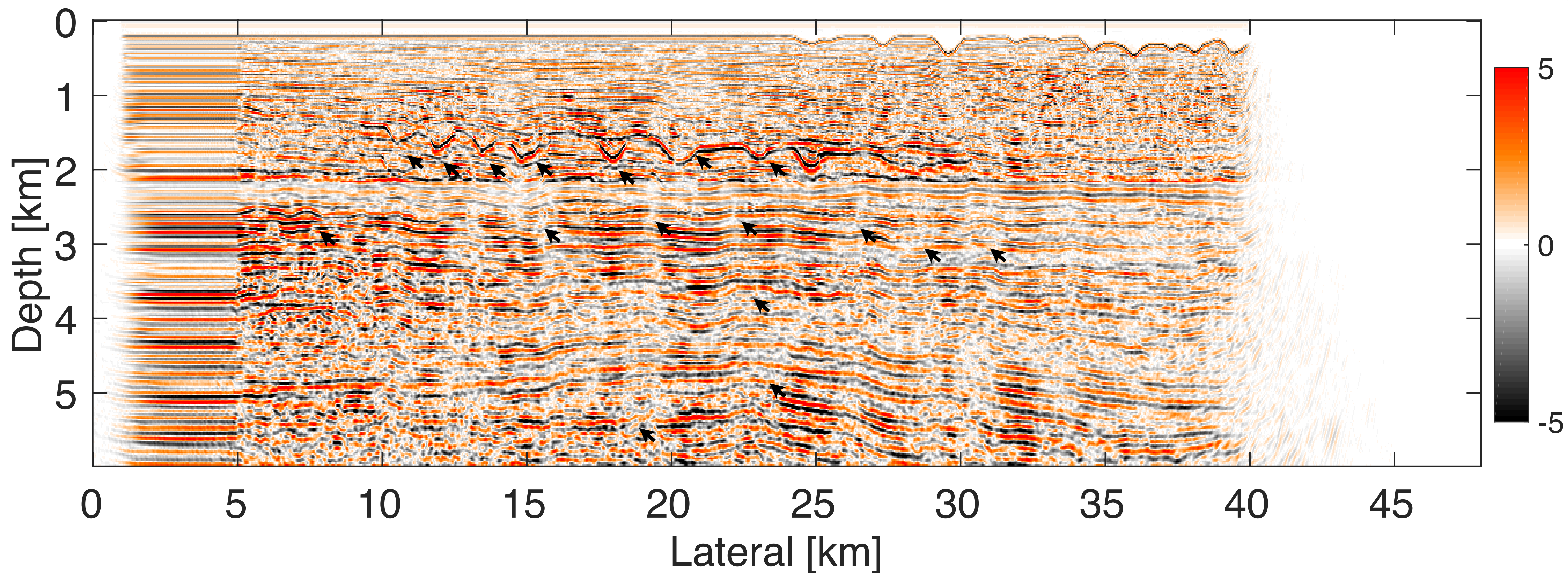
# Model update





# Kirchhoff migration

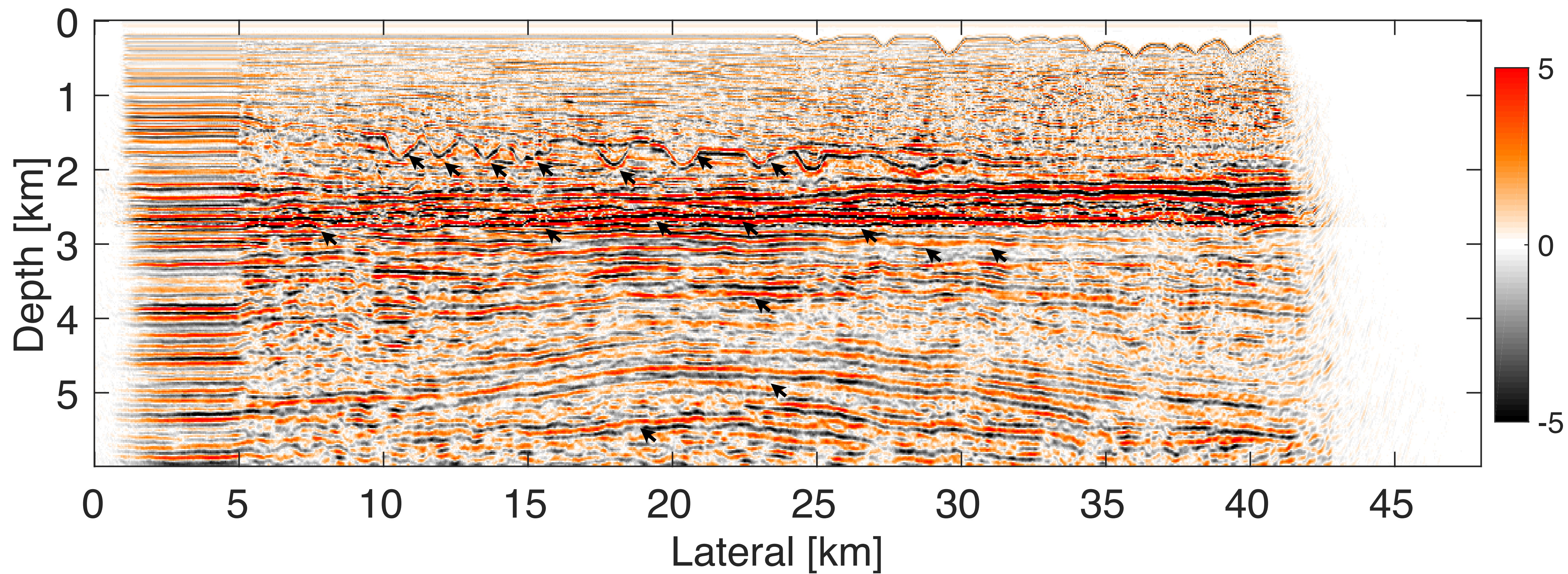
—Initial model





# Kirchhoff migration

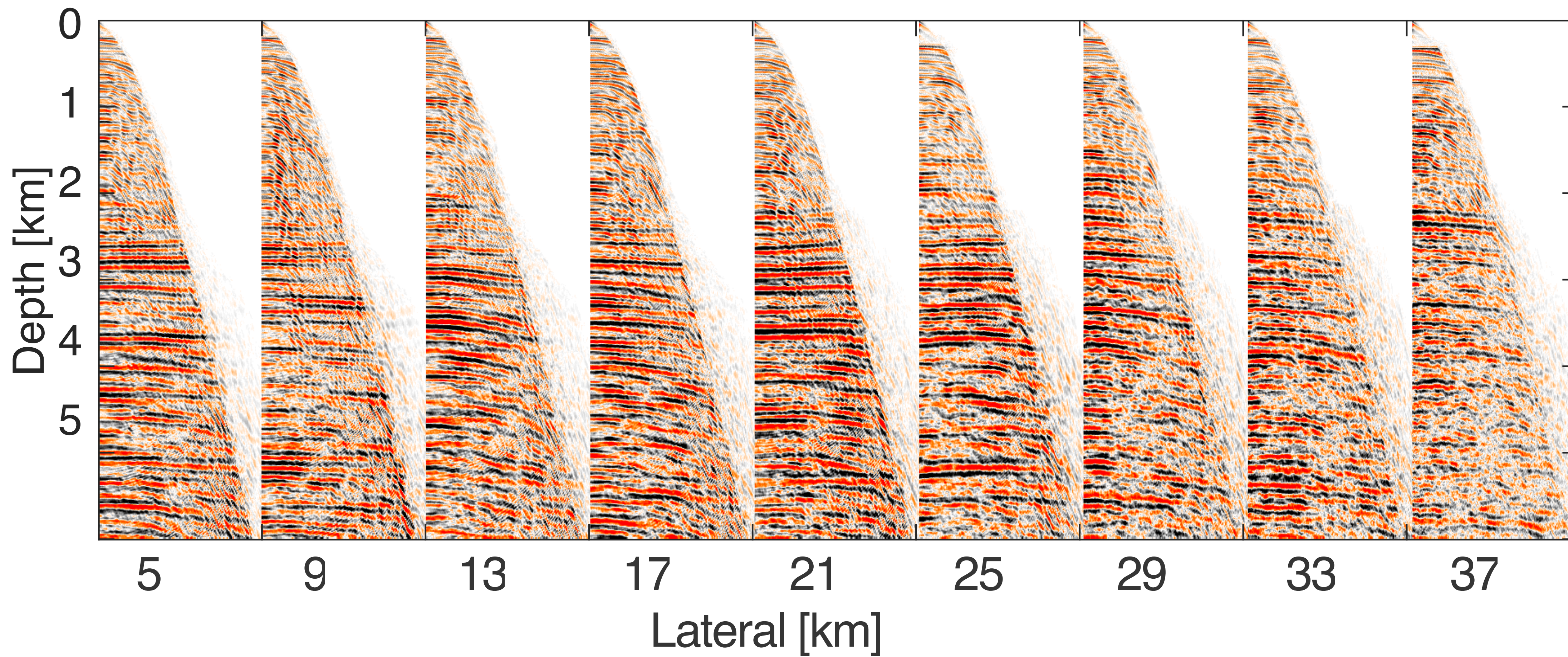
—Inversion result





# Common Image Gather

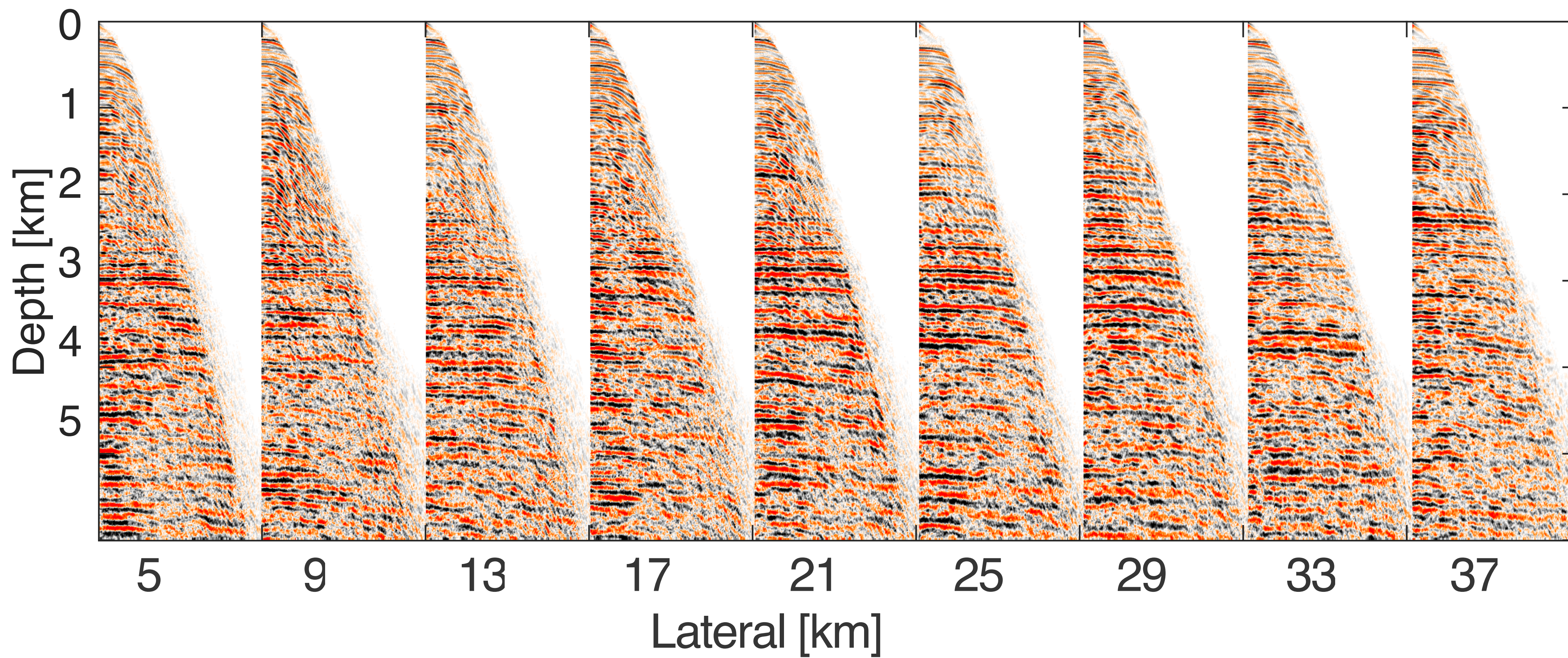
—Initial model





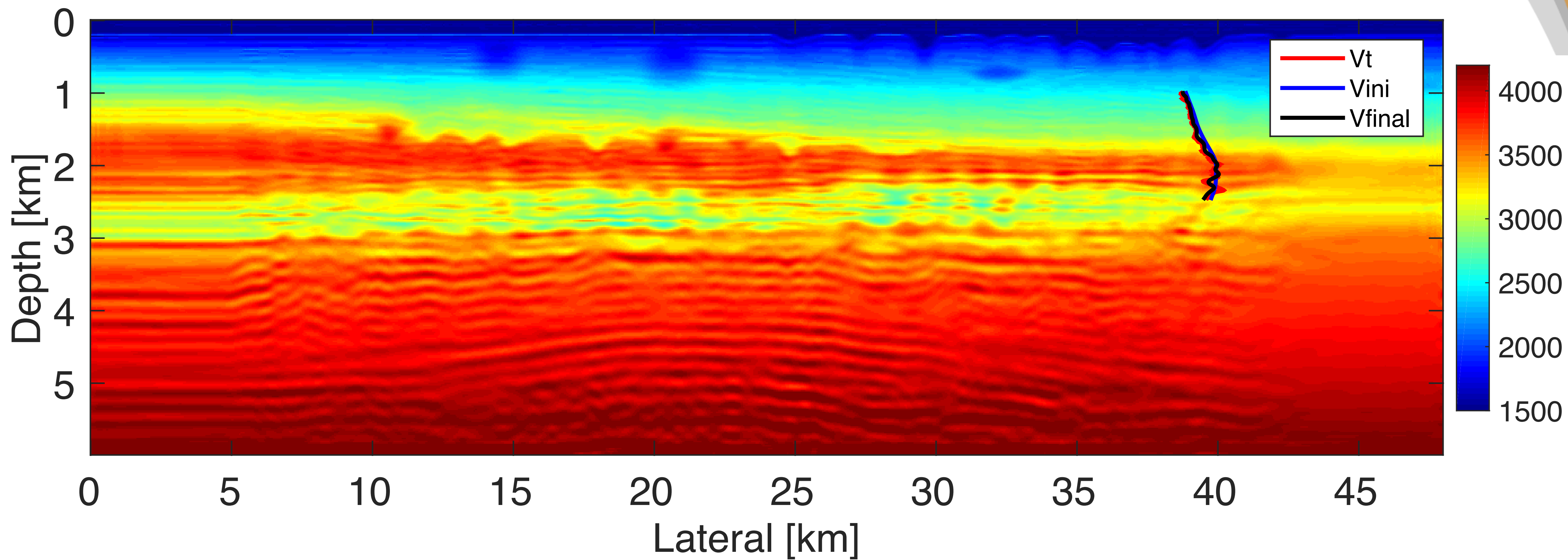
# Common Image Gather

—Inversion result



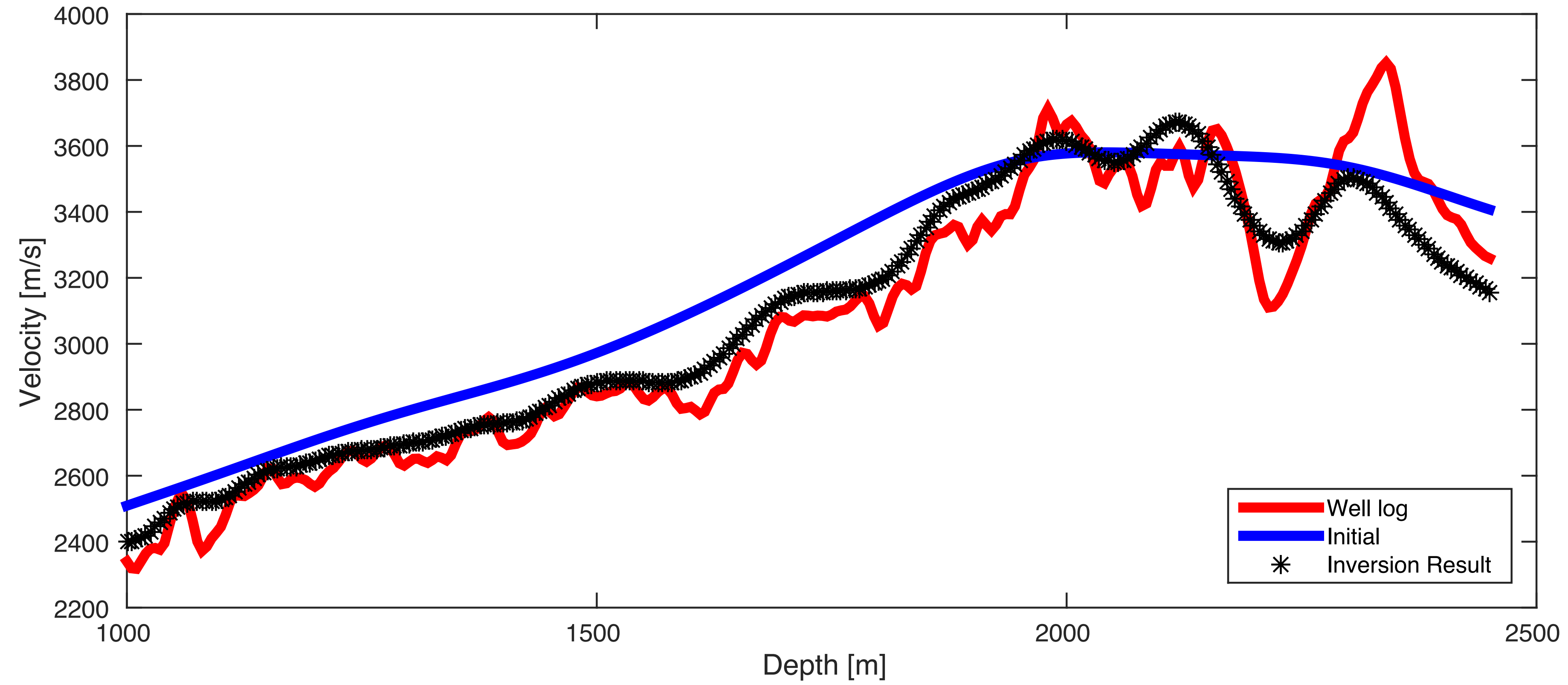


# Well-log comparison

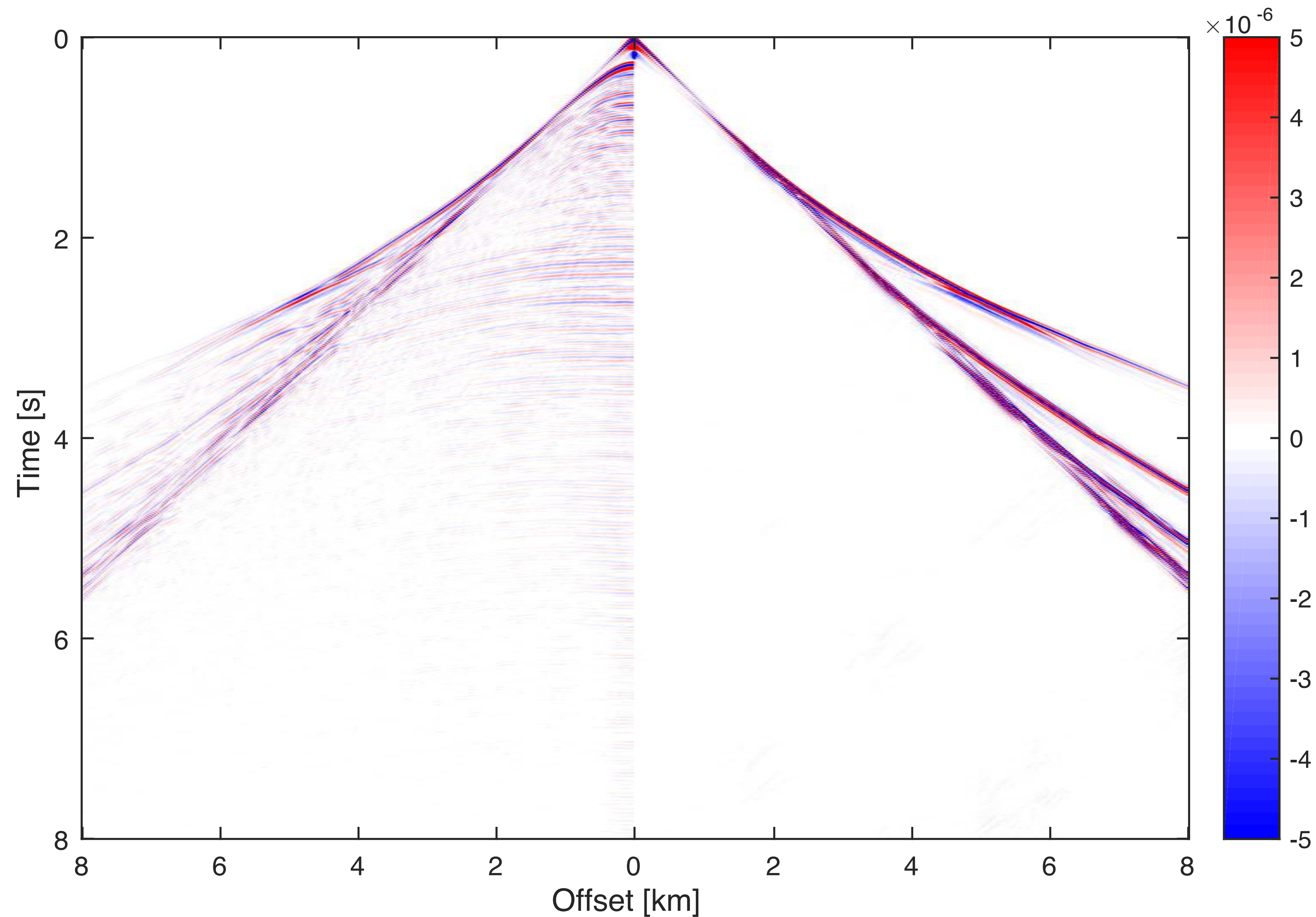




# Well-log comparison

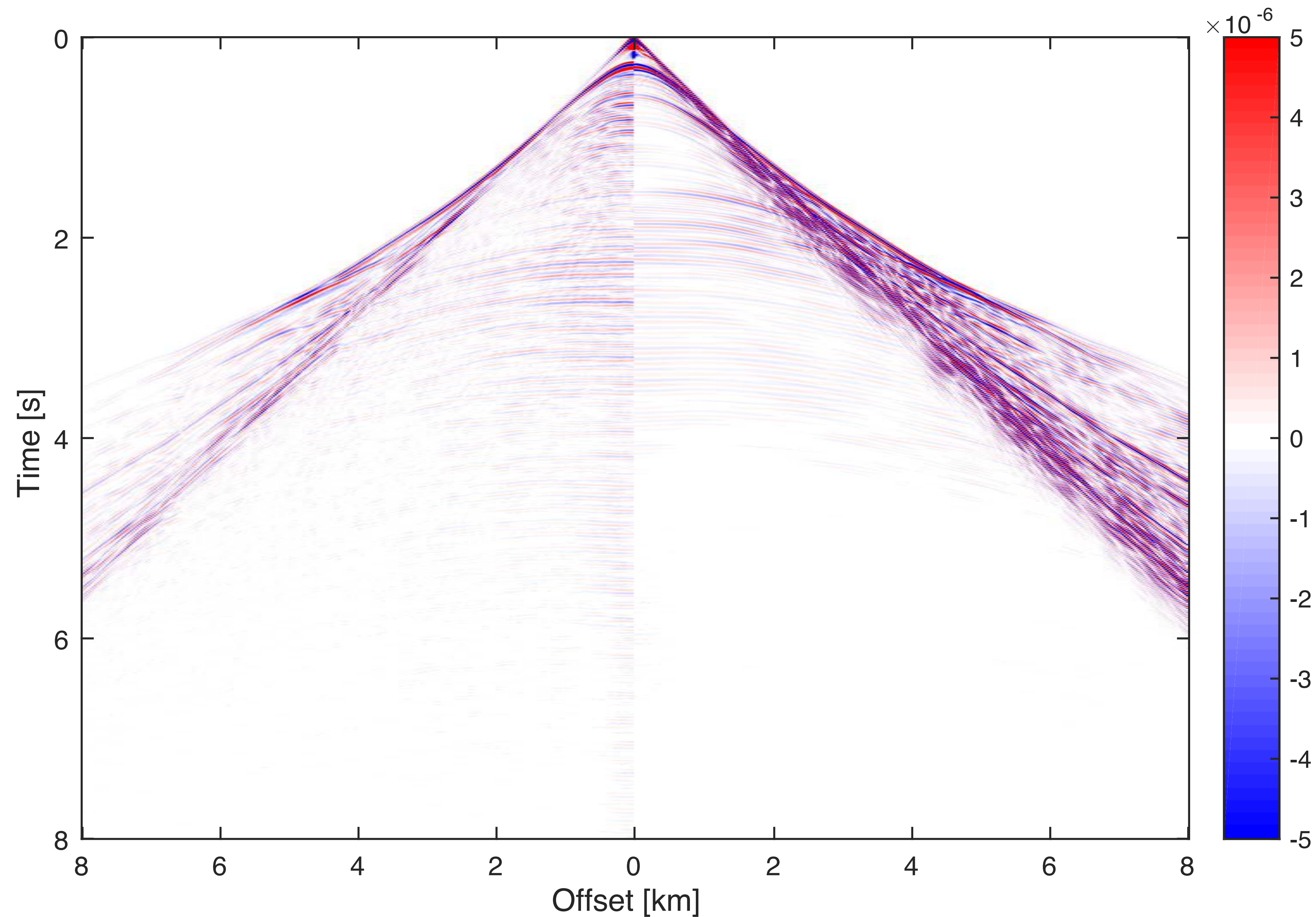


# Shot record comparison— Initial model



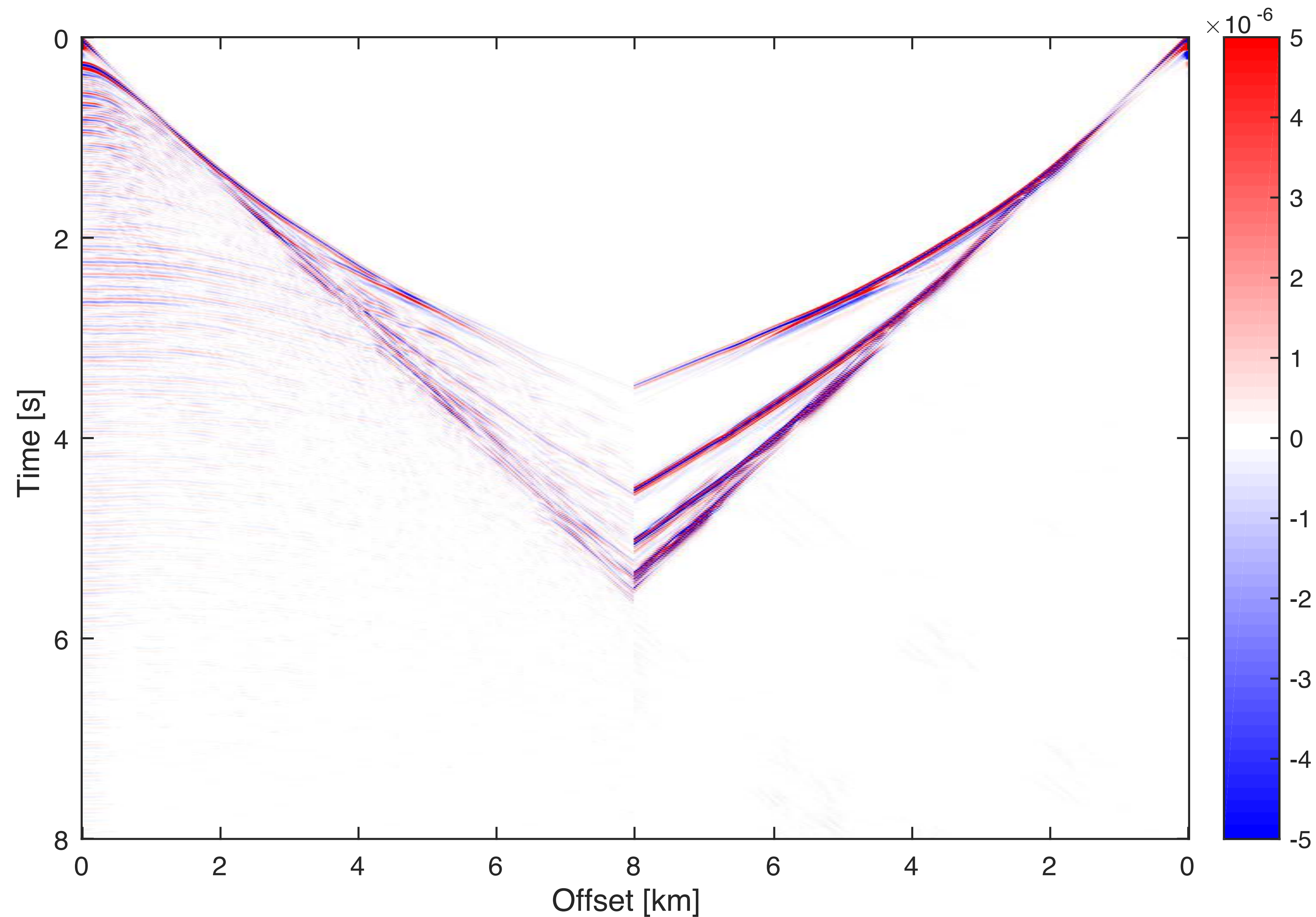


# Shot record comparison— Inversion result



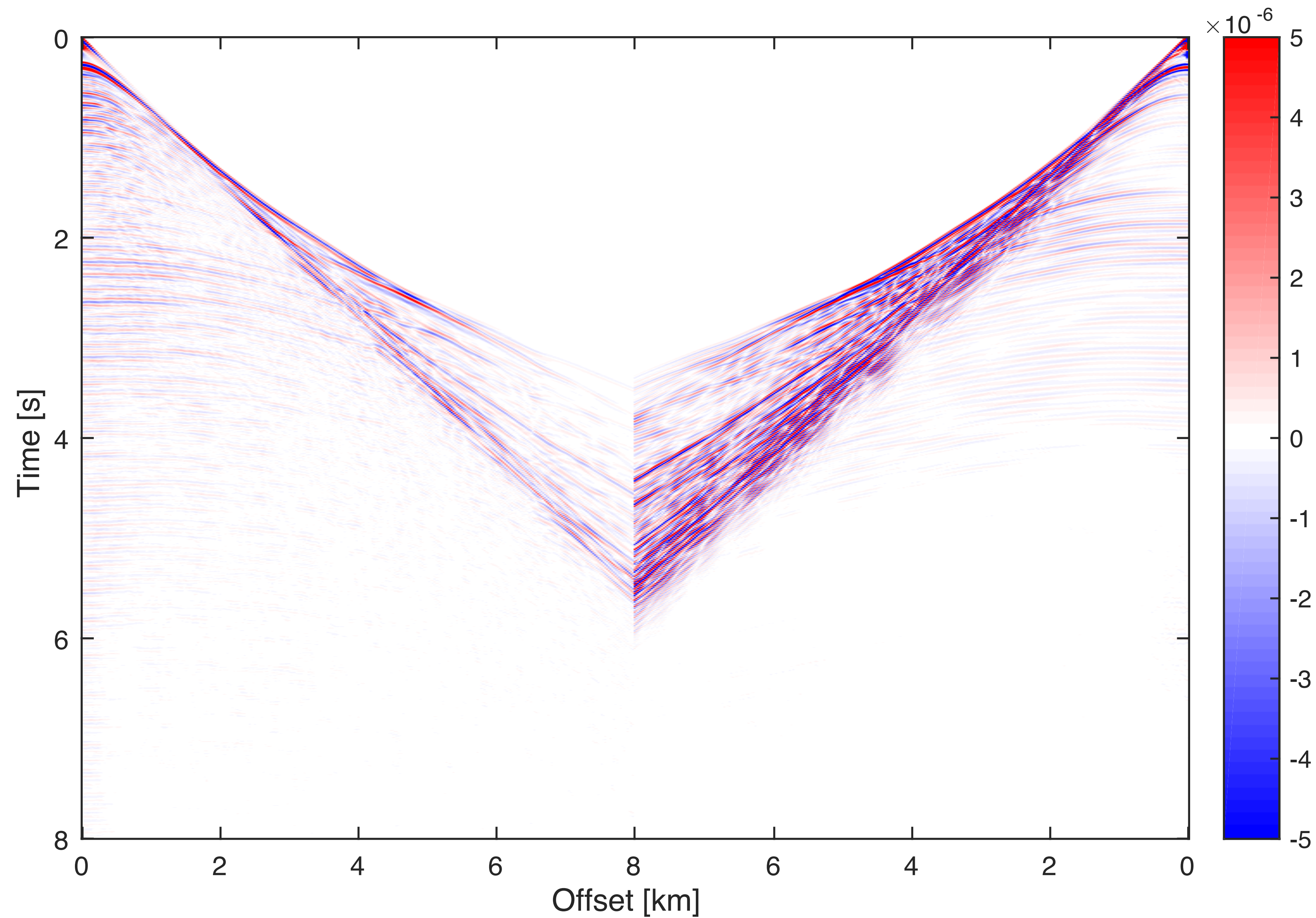


# Shot record comparison— Initial model





# Shot record comparison — Inversion result



## Conclusions

1. Using the variable projection method, we can estimate the source wavelet for the WRI.
  - Synthetic BG model
2. Source estimation enhances the robustness of WRI for field seismic data.
  - Chevron blind test data
3. Minimum smoothness constraint can remove artifacts in the inversion result and improve its quality.



## Future work

1. Use other constraint such as minimum L1 norm and TV norm constraint.
2. Study the effect of different  $\lambda$  to the final result.

# Acknowledgements

**Thank you for your attention !!**



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