SPOT, distributed data, and you

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Model problem

Suppose that you want to solve a sparsity-promoting interpolation problem

$$\min_{x} \|x\|_1$$
$$\text{such that } RMx = b$$

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Sampling operator - restricts vector to sampled locations

Model problem

Suppose that you want to solve a sparsity-promoting interpolation problem

$$\min_x \|x\|_1$$

$$\text{such that } RMx = b$$

Sparsity basis - maps (Curvelet, Fourier) coefficients to physical domain

Model problem

Suppose that you want to solve a sparsity-promoting interpolation problem

Coefficient vector

$$\min_x \|\overset{\swarrow}{x}\|_1$$

$$\text{such that } RMx = b$$

Model problem

Suppose that you want to solve a sparsity-promoting interpolation problem

$$\min_x \|x\|_1$$

$$\text{such that } RMx = b$$

Acquired data

Algorithm - linearized Bregman

$$z_{k+1} = z_k - t_k A^T (Ax_k - b)$$
$$x_{k+1} = S_\lambda(z_{k+1})$$

$A = RM$ - sampling + measurement operator

t_k - step size

$S_\lambda(x)$ - soft thresholding operator

$$S_\lambda(x) = \mathrm{sign}(x) \cdot \max(|x| - \lambda, 0)$$

Operations we need to perform

We need to repeatedly apply the *forward transform*

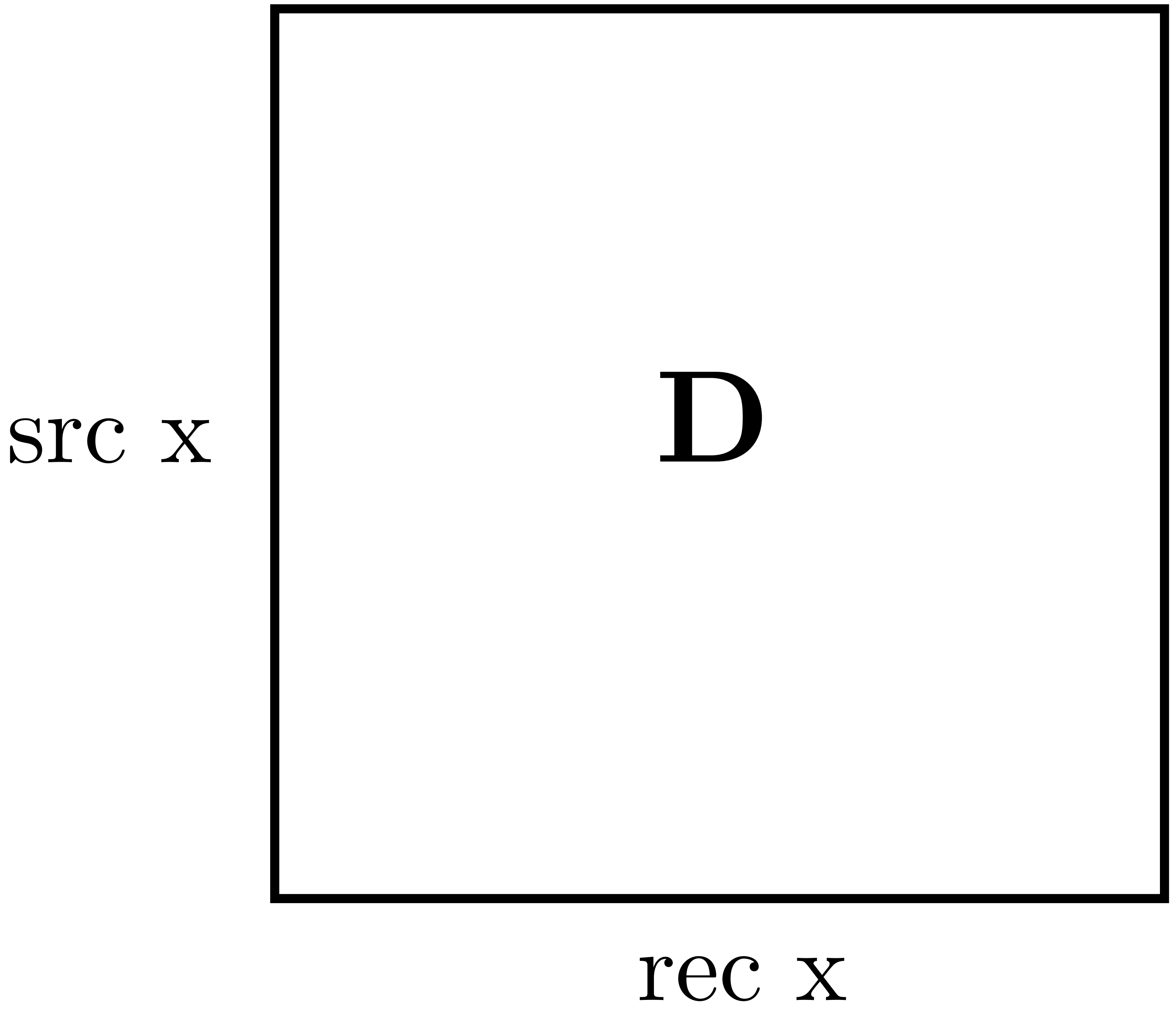
$$\text{physical space} \mapsto \text{coefficient space}$$

and apply the *adjoint transform*

$$\text{coefficient space} \mapsto \text{physical space}$$

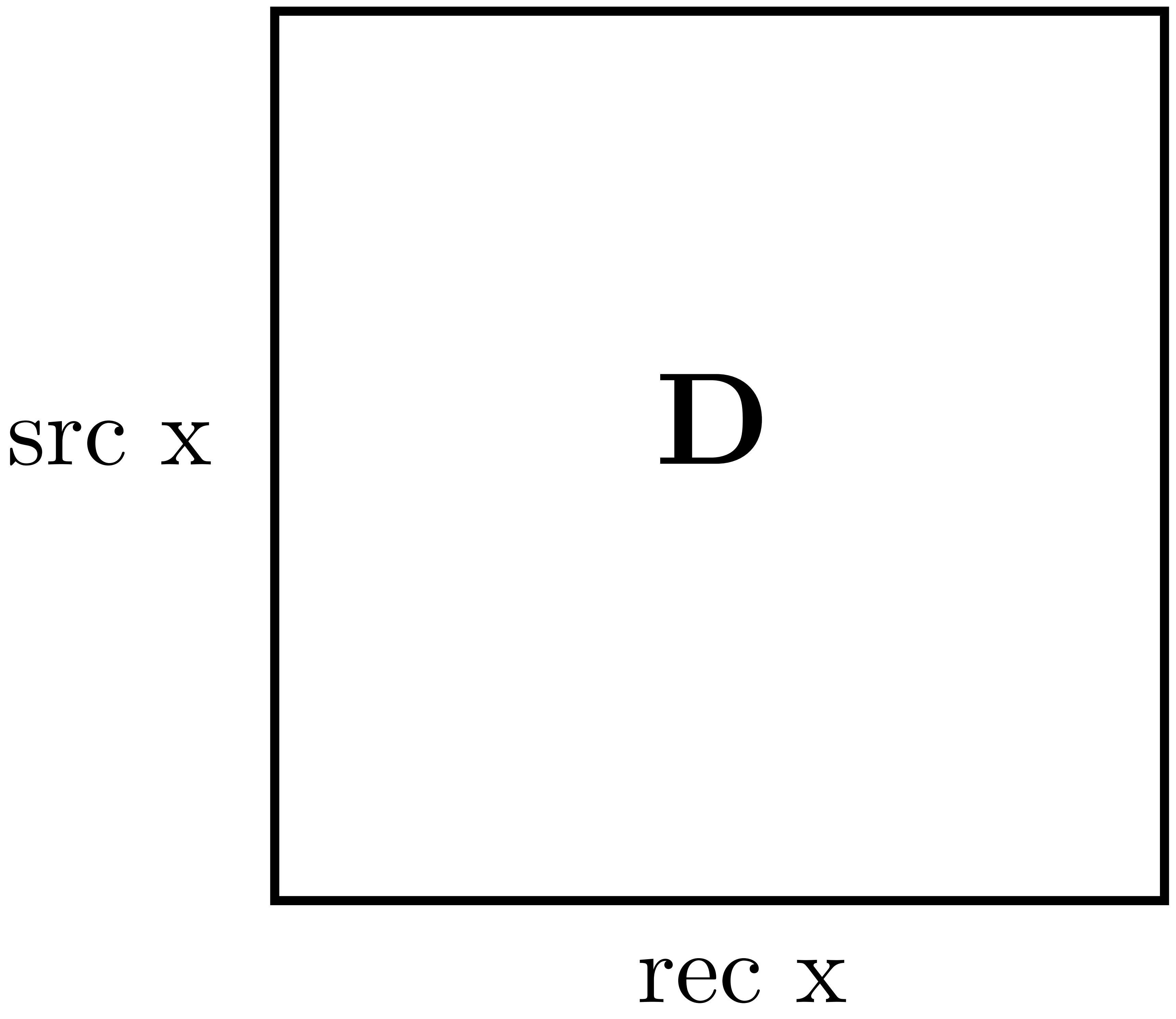
Details

Suppose we are working on two dimensional frequency slices



Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension



Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

$$\mathbf{D}$$

src x

-1.0 + 0.0i	0.5 + 1.4i	0.5 - 1.4i
1.0 + 1.2i	0.5 - 0.9i	-1.5 - 0.3i
1.0 - 1.2i	-1.5 + 0.3i	0.5 + 0.9i

rec x

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

Applying the 1D Fourier transform to each column of D

$$\mathbf{FD}$$

$k_{\mathrm{src\ x}}$

0.6 + 0.0i	-0.3 + 0.5i	-0.3 + 0.5i
0.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
2.3 + 0.0i	1.2 + 2.0i	1.2 - 2.0i

$\mathrm{rec\ x}$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

Transposing the source and receiver dimensions

$$(\mathbf{FD})^T$$

rec x

0.6 + 0.0i	0.0 + 0.0i	2.3 + 0.0i
-0.3 + 0.5i	0.0 + 0.0i	1.2 + 2.0i
-0.3 + 0.5i	0.0 + 0.0i	1.2 - 2.0i

$k_{\text{src x}}$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

Applying the 1D Fourier transform to the columns of this new array

$$\mathbf{F}(\mathbf{F}\mathbf{D})^T$$

$k_{\text{rec x}}$

0.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
1.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0+0.0i	4.0 + 0.0i

$k_{\text{src x}}$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

Transpose the resulting array

This is our final result

$$(\mathbf{F}(\mathbf{F}\mathbf{D})^T)^T$$

$k_{\text{src x}}$

0.0 + 0.0i	1.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0+0.0i	4.0 + 0.0i

$k_{\text{rec x}}$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

We can also write this as

$$\mathbf{F}\mathbf{D}\mathbf{F}^T$$

$$\begin{array}{c|c|c|c|}
\cline{2-4}
 & 0.0 + 0.0i & 1.0 + 0.0i & 0.0 + 0.0i \\
\cline{2-4}
k_{\text{src x}} & 0.0 + 0.0i & 0.0 + 0.0i & 0.0 + 0.0i \\
\cline{2-4}
 & 0.0 + 0.0i & 0.0{+}0.0i & 4.0 + 0.0i \\
\cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{k_{\text{rec x}}} & \multicolumn{1}{c}{}
\end{array}$$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

We can also write this as

$$\mathbf{FDF}^T$$

Apply $\mathbf{F}$ to the columns of $\mathbf{D}$

$k_{\text{src x}}$

0.0 + 0.0i	1.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0+0.0i	4.0 + 0.0i

$k_{\text{rec x}}$

Details

If $\mathbf{D}$ our sparsity basis of choice is the 1D Fourier basis along each dimension

We can also write this as

$$\mathbf{FDF}^T$$

Apply $\mathbf{F}$ to the rows of $\mathbf{D}$

$k_{\text{src x}}$

0.0 + 0.0i	1.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0 + 0.0i	0.0 + 0.0i
0.0 + 0.0i	0.0+0.0i	4.0 + 0.0i

$k_{\text{rec x}}$

Standard Matlab

```
op_fftsrc = @(x) fft(x)/sqrt(nsrc);
op_fftrec = @(x) fft(x)/sqrt(nrec);
op_transp = @(x) x.’;
op_m = @(x) op_transp(op_fftrec(op_transp(opfftsrc(x))));
% transformed data
op_m(D);
```

Standard Matlab

That doesn’t look too bad

- it’s not intuitive to look at - hard to tell what’s going on

What if our sparsity basis changes in one dimension?

- hard to experiment

What if our data is distributed?

- not clear what to do here

Standard Matlab

How do we get adjoints/inverses?

How can I deal with more than two dimensions?

Kronecker Product

Mathematically, we can express $\mathbf{FDF}^T$ as

$$(\mathbf{F} \otimes \mathbf{F})\mathrm{vec}(\mathbf{D})$$

Kronecker Product

Mathematically, we can express $\mathbf{F}\mathbf{D}\mathbf{F}^T$ as

$$(\mathbf{F} \otimes \mathbf{F})\mathrm{vec}(\mathbf{D})$$

$\mathbf{F} \otimes \mathbf{F}$ - kronecker product of $\mathbf{F}$ and $\mathbf{F}$

$\mathrm{vec}(\mathbf{D})$ - reshape $\mathbf{D}$ in to a vector

Kronecker Product

How you read this

$$(\mathbf{F} \otimes \mathbf{F})\mathrm{vec}(\mathbf{D})$$

Kronecker Product

How you read this

$$(\mathbf{F} \otimes \underset{\uparrow}{\mathbf{F}})\mathrm{vec}(\mathbf{D})$$

Apply $\mathbf{F}$ to the first dimension of $\mathbf{D}$

Kronecker Product

How you read this

$$(\mathbf{F} \otimes \mathbf{F})\mathrm{vec}(\mathbf{D})$$

Apply $\mathbf{F}$ to the second dimension of $\mathbf{D}$

Kronecker Product

More generally

$$(\mathbf{B} \otimes \mathbf{A})\mathrm{vec}(\mathbf{D})$$

Apply $\mathbf{A}$ to the first dimension of $\mathbf{D}$

Kronecker Product

More generally

$$(\mathbf{B} \otimes \mathbf{A})\mathrm{vec}(\mathbf{D})$$

Apply $\mathbf{B}$ to the second dimension of $\mathbf{D}$

Using the SPOT toolbox

```
A = opDFT(nsrc);
B = opDFT(nrec);
F = opKron(B,A);
% transformed data
F*vec(D);
```

Advantages

Code now looks like the math

- if you understand the underlying math, you understand what’s happening
- adjoints, inverses automatically

Easy to change the operators in both dimensions

- easier to experiment with different transforms

Handling distributed data is nearly identical to serial data

Using the SPOT toolbox - serial version

```
% D resides on the current node
A = opDFT(nsrc);
B = opDFT(nrec);
F = opKron(B,A);
% transformed data
F*vec(D);
```

Using the SPOT toolbox - serial version

```
% D is distributed along columns
A = opDFT(nsrc);
B = opDFT(nrec);
F = oppKron2Lo(B,A);
% transformed data - distributed
F*vec(D);
```

Actual Matlab code

```
% Construct sampling + measurement operators
Rsrc = opRestriction(nsrc,sampled_indices);
Rrec = opDirac(nrec);
R = opKron(Rrec,Rsrc);
Msrc = opDFT(nsrc); Mrec = opDFT(nrec);
M = opKron(Mrec,Msrc);

% Construct composite operators, subsampled data
A = R*M; b = R*vec(D);

threshold = @(x) sign(x) .* max(abs(x)-lambda,0);
```

Actual Matlab code

```
x = zeros(nsrc*nrec,1); z = zeros(nsrc*nrec,1);
for itr=1:nitr
    z = z - t*A’*(A*x-b);
    x = threshold(x);
end
```

Actual Matlab code

```
x = zeros(nsrc*nrec,1); z = zeros(nsrc*nrec,1);
for itr=1:nitr
    z = z - t*A’*(A*x-b);
    x = threshold(x);
end
```

Previous algorithm

$$
\begin{aligned}
z_{k+1} &= z_k - t_k A^T (A x_k - b) \\
x_{k+1} &= S_\lambda(z_{k+1})
\end{aligned}
$$

SPOT toolbox

Allows us to implement multidimensional operations easily and consistently

- don’t need to worry about data shuffling, parallelization, etc

Code matches the math

- easier to understand, debug

All *matrix-free* - explicit matrices are never constructed, only matrix-vector products

SPOT Toolbox

Operations such as

A*B

A\B

A+B

c*A

are wrappers to functions you implement

- matrices never formed explicitly, but Matlab treats them as regular matrices

SPOT Toolbox

Lots of existing functionality

- Sums, products, inverses, diagonal operators, random matrices
- Fourier, Curvelet transform
- Parallel multilinear (Kronecker) products
- Demigration, migration, GN Hessian, Full Hessian operators in FWI
- Jacobian, GN Hessian for Hierarchical Tucker

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https://www.slim.eos.ubc.ca/consortiumsoftware

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