Accelerating an Iterative Helmholtz Solver with FPGAs

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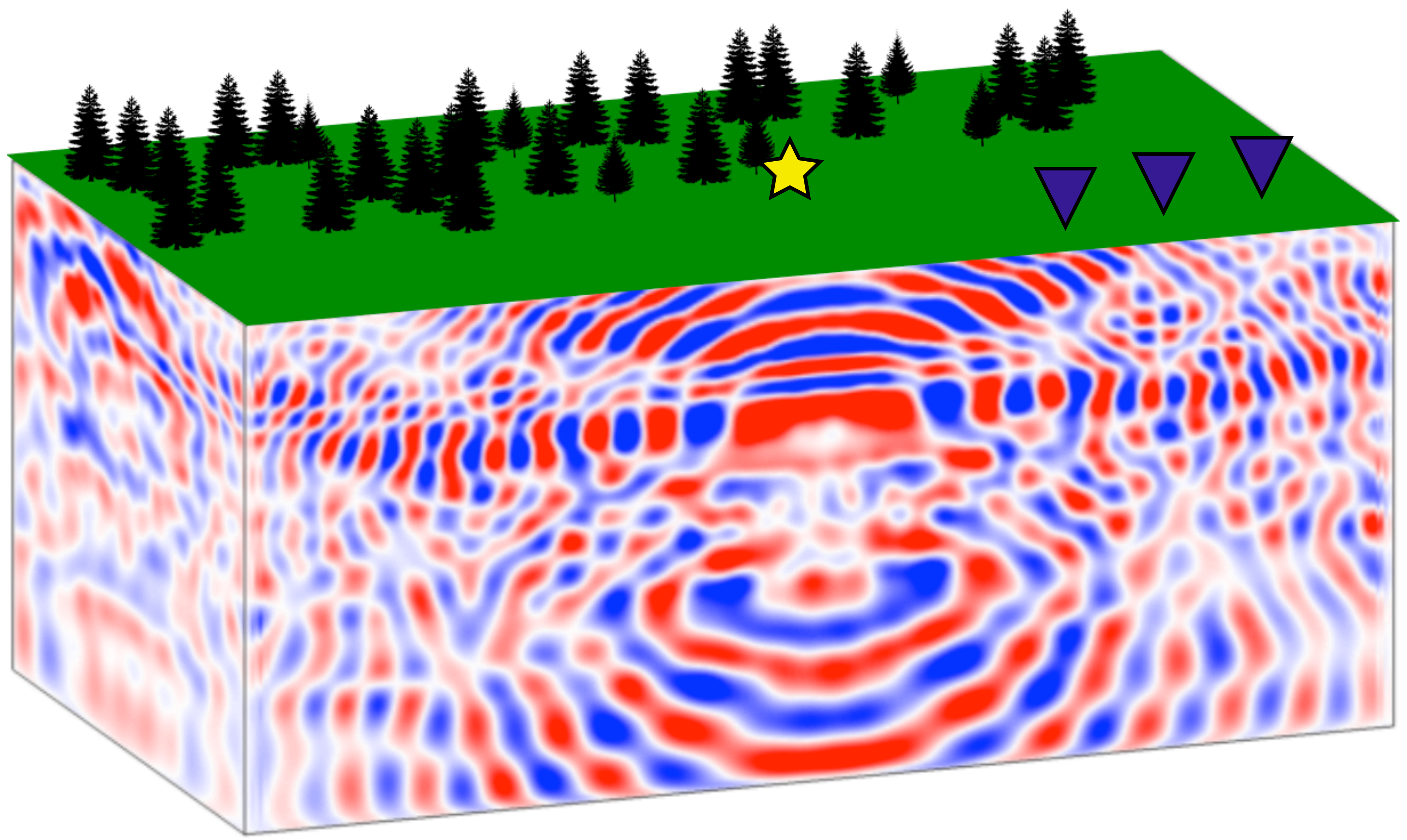
SLIM
University of British Columbia

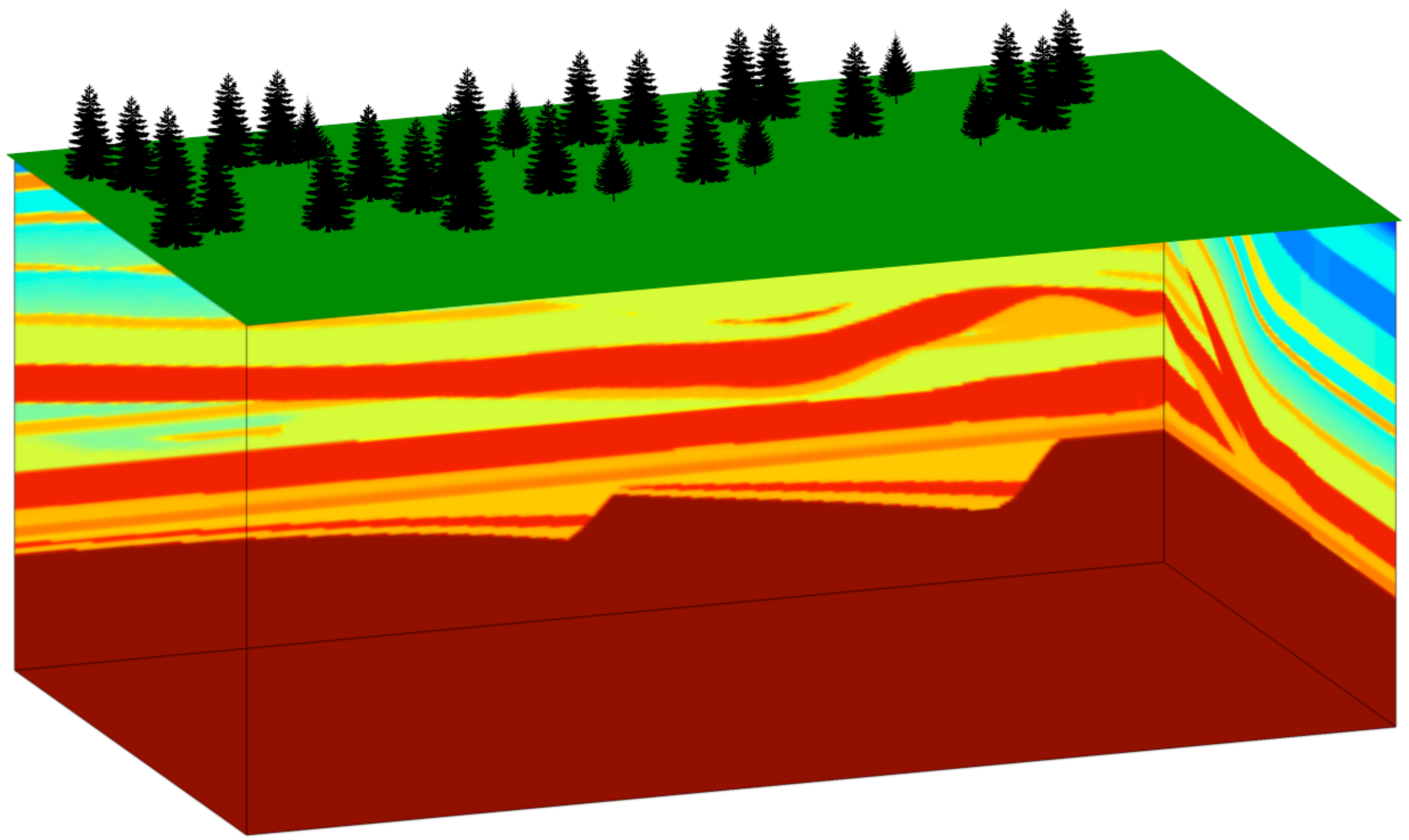
Oh by the way: I have a stutter.

Seismic Wave Simulation

Full-waveform Inversion

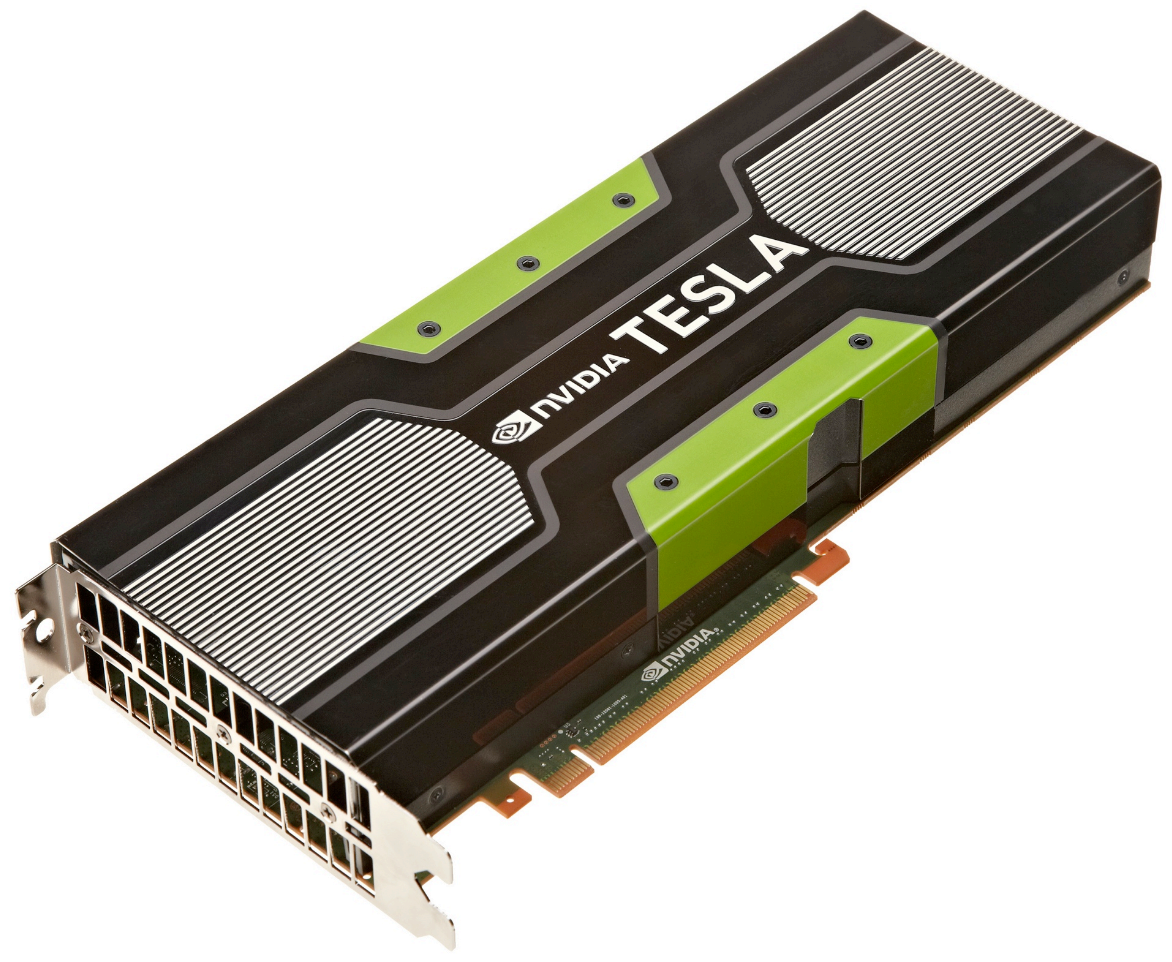


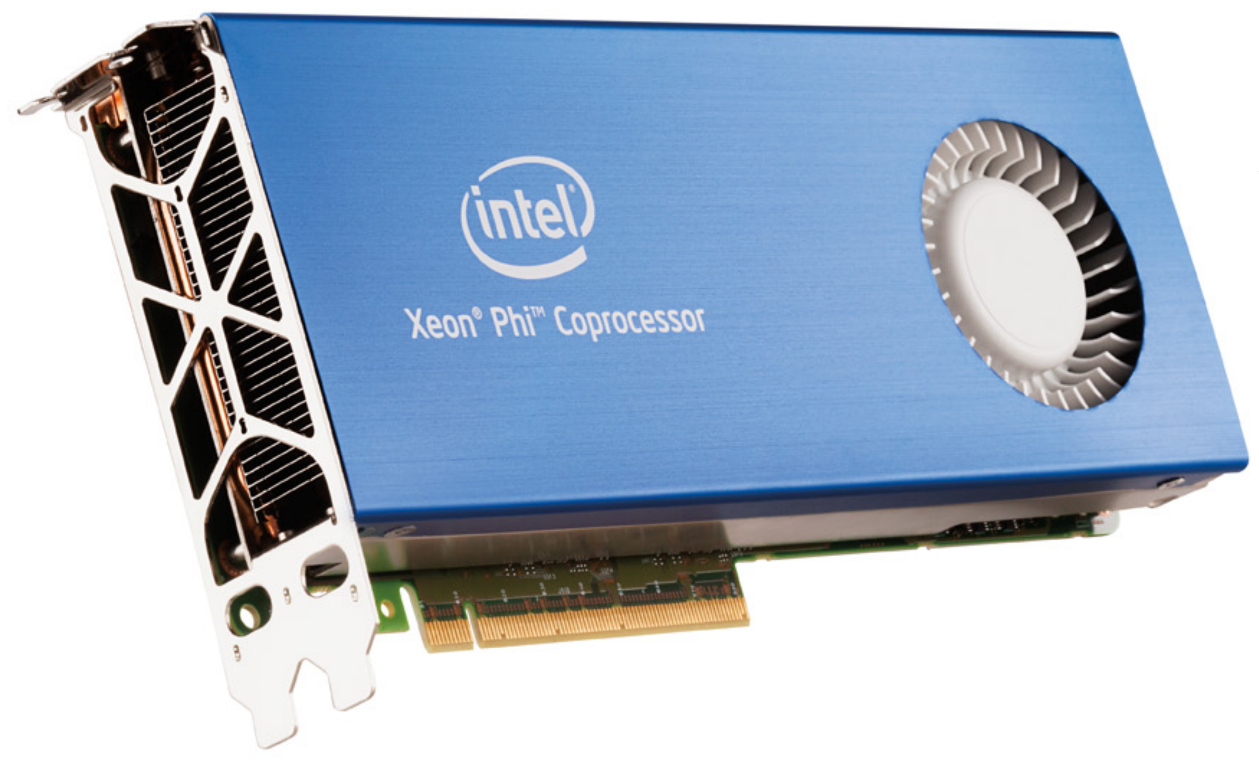
Seismic Wavefield (**u**)



Earth model (**m**)

The Accelerators Have Arrived





Top 10 of “Top 500” Supercomputers

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	National Super Computer Center in Guangzhou China	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P NUDT	3,120,000	33,862.7	54,902.4	17,808
2	DOE/SC/Oak Ridge National Laboratory United States	**Titan** - Cray XK7 , Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
3	DOE/NNSA/LLNL United States	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM	1,572,864	17,173.2	20,132.7	7,890
4	RIKEN Advanced Institute for Computational Science (AICS) Japan	K computer, SPARC64 VIIIfx 2.0GHz, Tofu interconnect Fujitsu	705,024	10,510.0	11,280.4	12,660
5	DOE/SC/Argonne National Laboratory United States	**Mira** - BlueGene/Q, Power BQC 16C 1.60GHz, Custom IBM	786,432	8,586.6	10,066.3	3,945
6	Swiss National Supercomputing Centre (CSCS) Switzerland	**Piz Daint** - Cray XC30, Xeon E5-2670 8C 2.600GHz, Aries interconnect , NVIDIA K20x Cray Inc.	115,984	6,271.0	7,788.9	2,325
7	Texas Advanced Computing Center/Univ. of Texas United States	**Stampede** - PowerEdge C8220, Xeon E5-2680 8C 2.700GHz, Infiniband FDR, Intel Xeon Phi SE10P Dell	462,462	5,168.1	8,520.1	4,510
8	Forschungszentrum Juelich (FZJ) Germany	**JUQUEEN** - BlueGene/Q, Power BQC 16C 1.600GHz, Custom Interconnect IBM	458,752	5,008.9	5,872.0	2,301
9	DOE/NNSA/LLNL United States	**Vulcan** - BlueGene/Q, Power BQC 16C 1.600GHz, Custom Interconnect IBM	393,216	4,293.3	5,033.2	1,972
10	Leibniz Rechenzentrum Germany	**SuperMUC** - iDataPlex DX360M4, Xeon E5-2680 8C 2.70GHz, Infiniband FDR IBM	147,456	2,897.0	3,185.1	3,423

FPGAs: Reconfigurable Hardware Accelerators

The Punchline

Modelling Seismic Waves

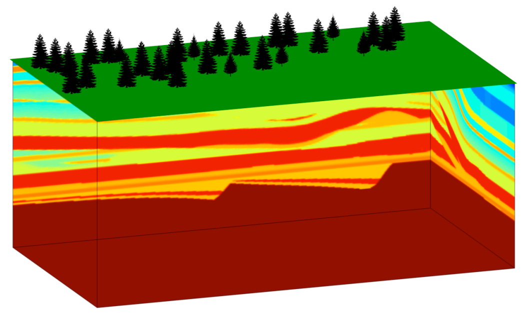
Mathematical Formulation

Modelling Seismic Waves: The Wave Equation

$$\left(\omega^2 \mathbf{m} + \Delta\right) \mathbf{u} = \mathbf{q}$$

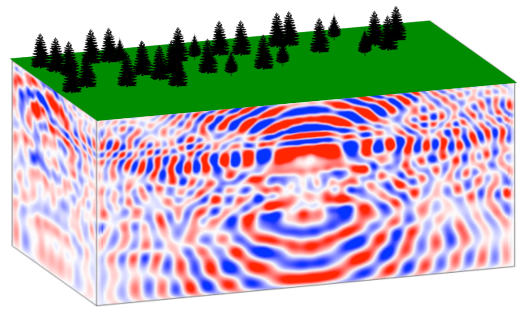
Frequency

Earth Model



Laplacian

Wavefield
(iterate)



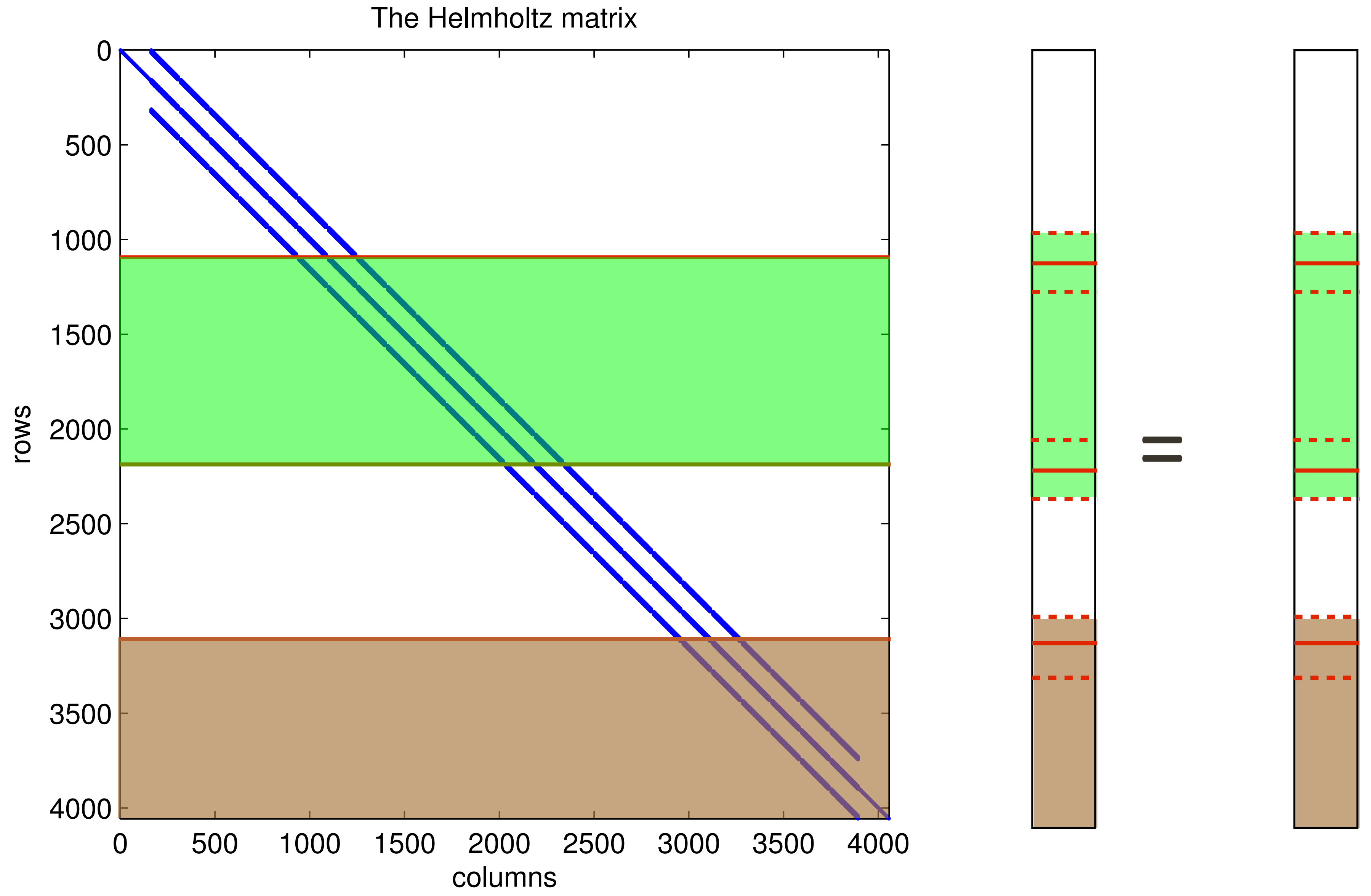
Seismic Source
(right-hand side)

☆

Modelling Seismic Waves: Discretization

[Operto, 2007]

$$A(\mathbf{m},\omega)\mathbf{u} = \mathbf{q}$$

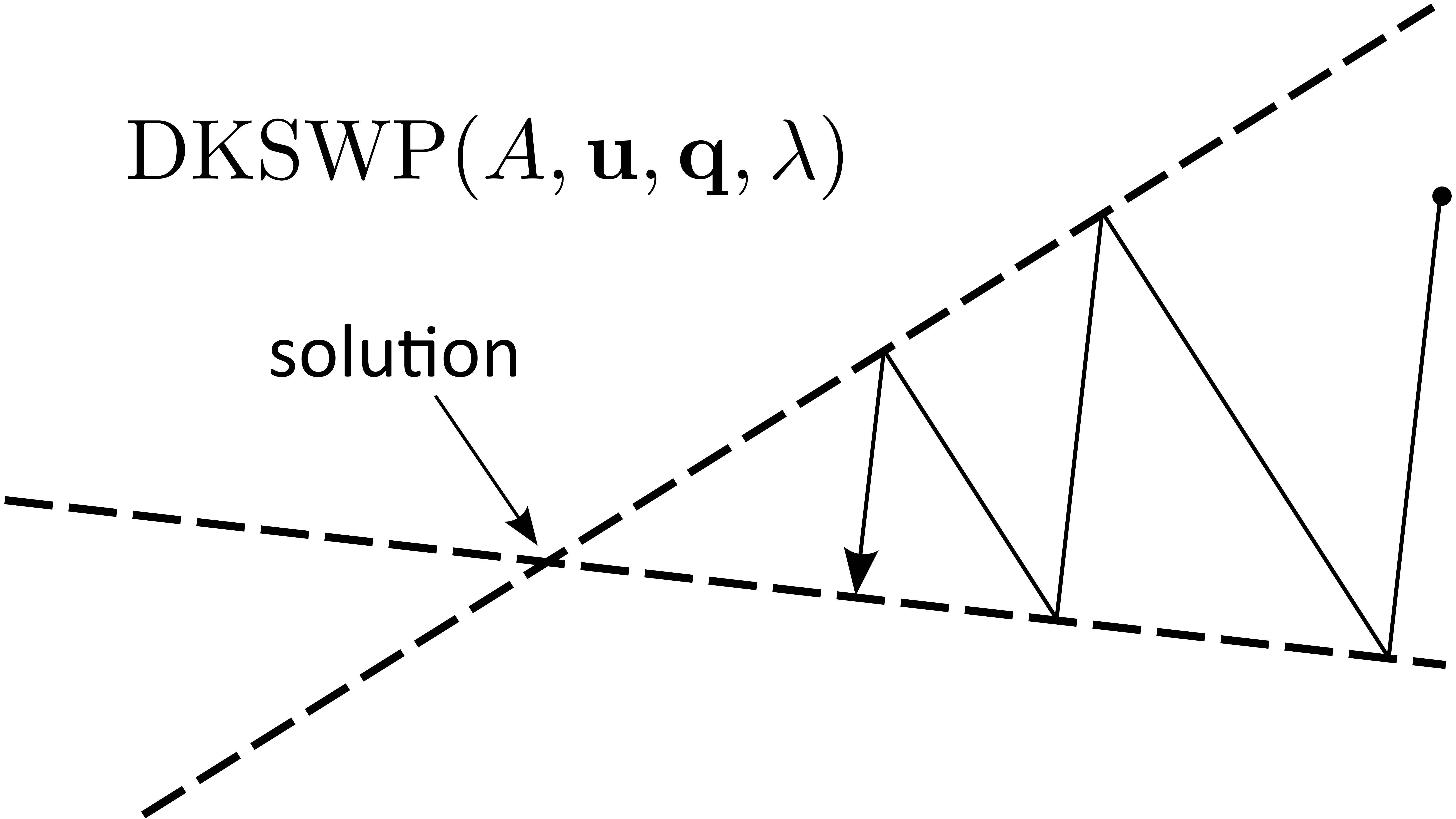


Solving the Helmholtz System

The Kaczmarz Algorithm

[Kaczmarz, 1937]

$$\mathrm{DKSWP}(A, \mathbf{u}, \mathbf{q}, \lambda)$$



Adapted from [van Leeuwen, 2012]

The Kaczmarz Algorithm: Equivalent to SSOR-NE

[Björck and Elfving, 1979]

Double Kaczmarz sweep on the original system:

$$A\mathbf{u} = \mathbf{q}$$

↔

One iteration of SSOR on the normal equations:

$$AA^*\mathbf{y} = \mathbf{q}$$

$$A^*\mathbf{y} = \mathbf{u}$$

Both are computed as:

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \lambda(b_i - \langle \mathbf{a}_i, \mathbf{u}_k \rangle)\frac{\mathbf{a}_i^*}{\|\mathbf{a}_i\|^2}$$

$$k : 1 \rightarrow 2N$$

$$i : 1 \rightarrow N, N \rightarrow 1$$

Kaczmarz + CG = CGMN

[Björck & Elfving 1979]

CGMN: Solves for Fixed Point of Kaczmarz Row Projections

$$\begin{aligned}\mathrm{DKSWP}(A, \mathbf{u}, \mathbf{q}, \lambda) &= Q_1 \cdots Q_N Q_N \cdots Q_1 \mathbf{u} + R\mathbf{q} \\ &= Q\mathbf{u} + R\mathbf{q}.\end{aligned}$$

Assume **u** is a solution and re-arrange:

$$(I - Q)\mathbf{u} = R\mathbf{q}$$

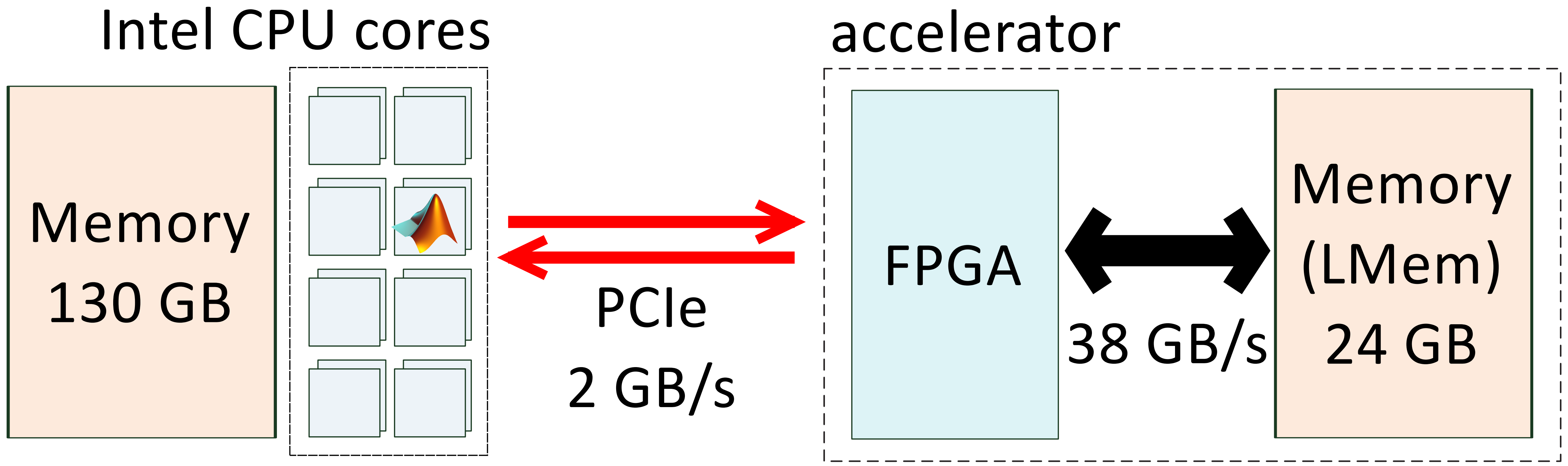
Block parallelization of Kaczmarz + CG = CARP-CG

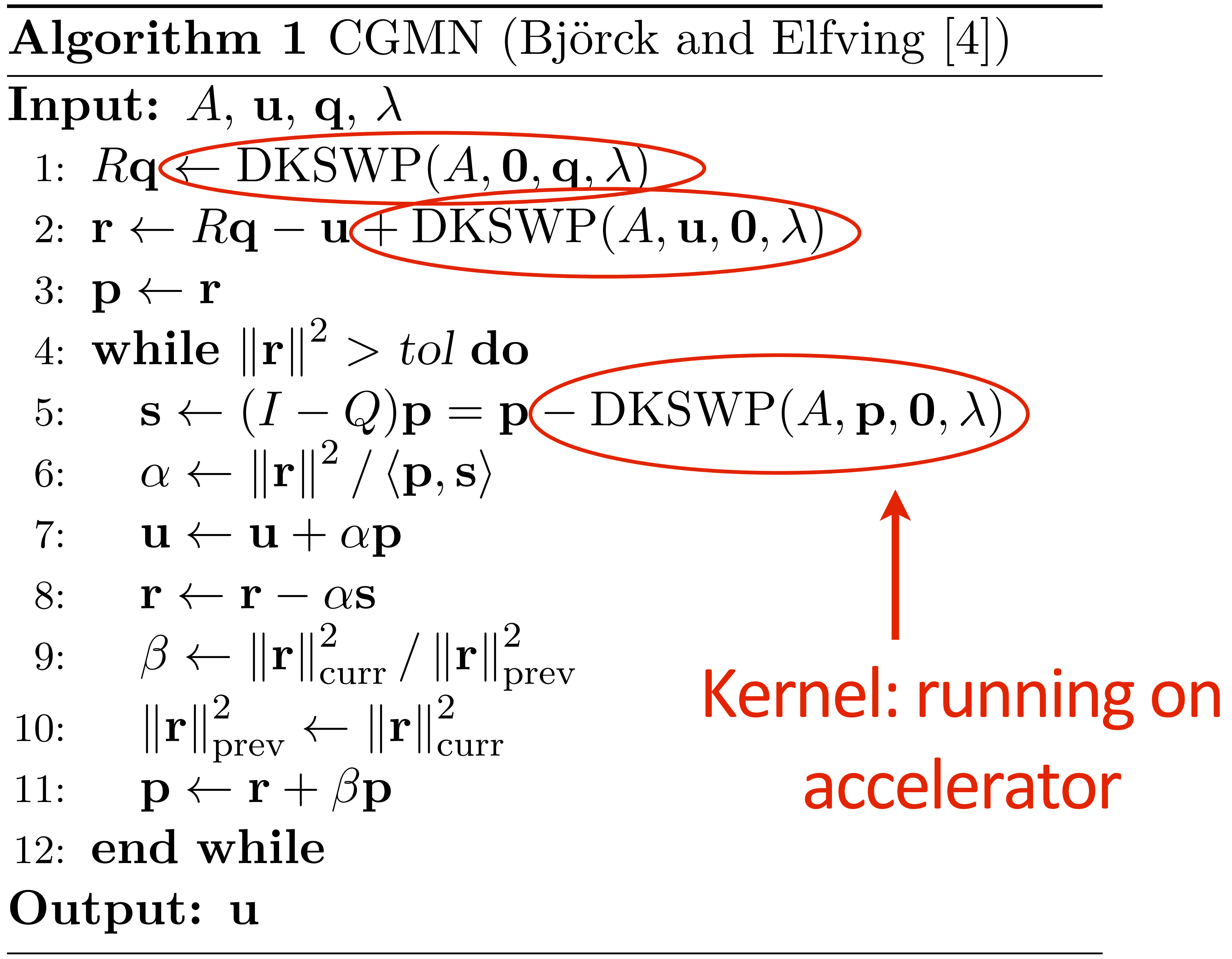
[Gordon & Gordon, 2010]

Contribution of This Work

Compute Node Overview

[Maxeler Technologies, 2011]

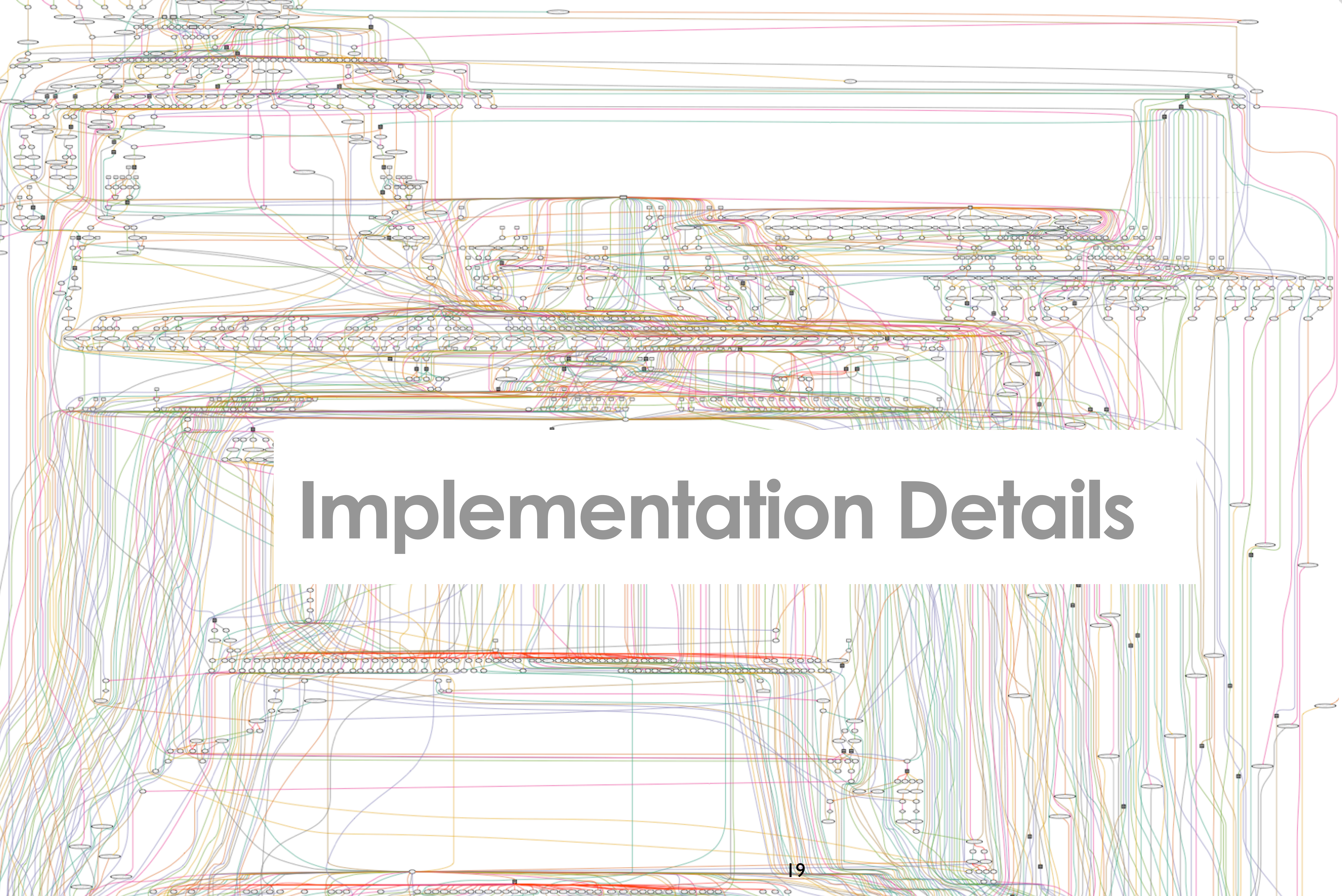




Algorithm 1 CGMN (Björck and Elfving [4])

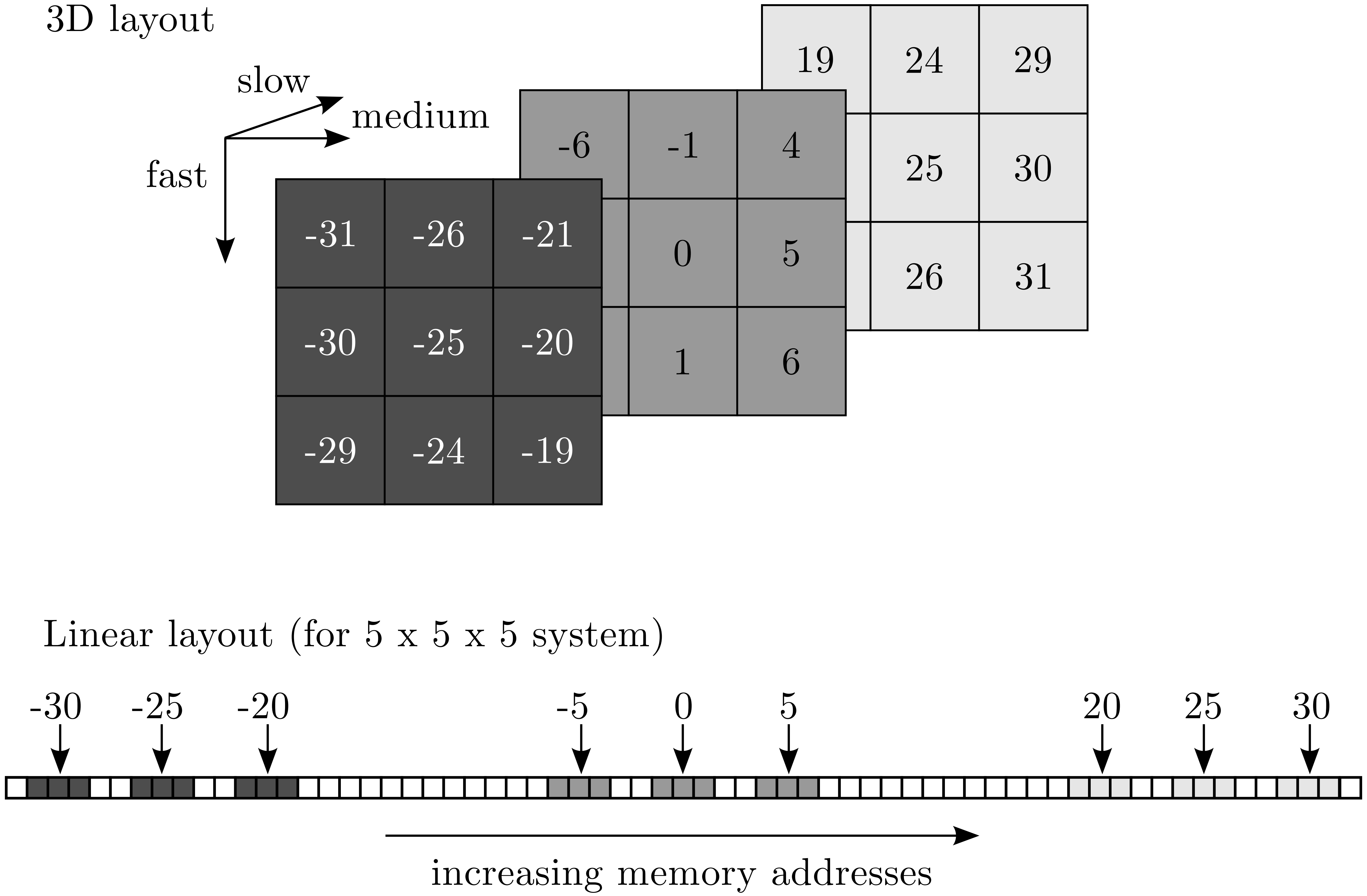
```
Input: $A$, $\mathbf{u}$, $\mathbf{q}$, $\lambda$
$R\mathbf{q} \leftarrow \mathrm{DKSWP}(A, \mathbf{0}, \mathbf{q}, \lambda)$
$\mathbf{r} \leftarrow R\mathbf{q} - \mathbf{u} + \mathrm{DKSWP}(A, \mathbf{u}, \mathbf{0}, \lambda)$
$\mathbf{p} \leftarrow \mathbf{r}$
while $\|\mathbf{r}\|^2 > tol$ do
    $\mathbf{s} \leftarrow (I - Q)\mathbf{p} = \mathbf{p} - \mathrm{DKSWP}(A, \mathbf{p}, \mathbf{0}, \lambda)$
    $\alpha \leftarrow \|\mathbf{r}\|^2 \,/\, \langle \mathbf{p}, \mathbf{s} \rangle$
    $\mathbf{u} \leftarrow \mathbf{u} + \alpha\mathbf{p}$
    $\mathbf{r} \leftarrow \mathbf{r} - \alpha\mathbf{s}$
    $\beta \leftarrow \|\mathbf{r}\|^2_{\mathrm{curr}} \,/\, \|\mathbf{r}\|^2_{\mathrm{prev}}$
    $\|\mathbf{r}\|^2_{\mathrm{prev}} \leftarrow \|\mathbf{r}\|^2_{\mathrm{curr}}$
    $\mathbf{p} \leftarrow \mathbf{r} + \beta\mathbf{p}$
end while
Output: $\mathbf{u}$
```

Kernel: running on accelerator

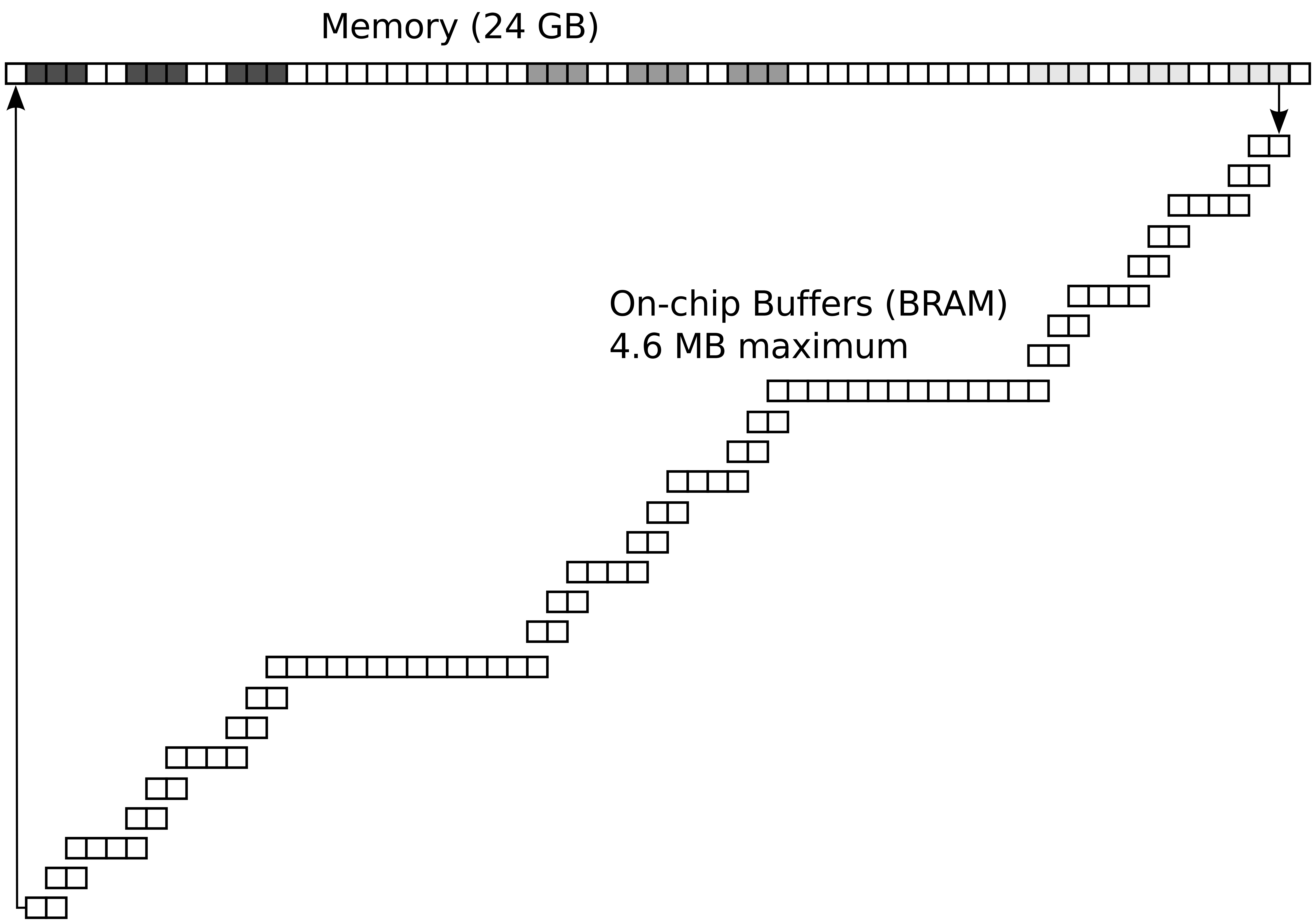


Implementation Details

Layout of 3D Wavefields in 1D Memory

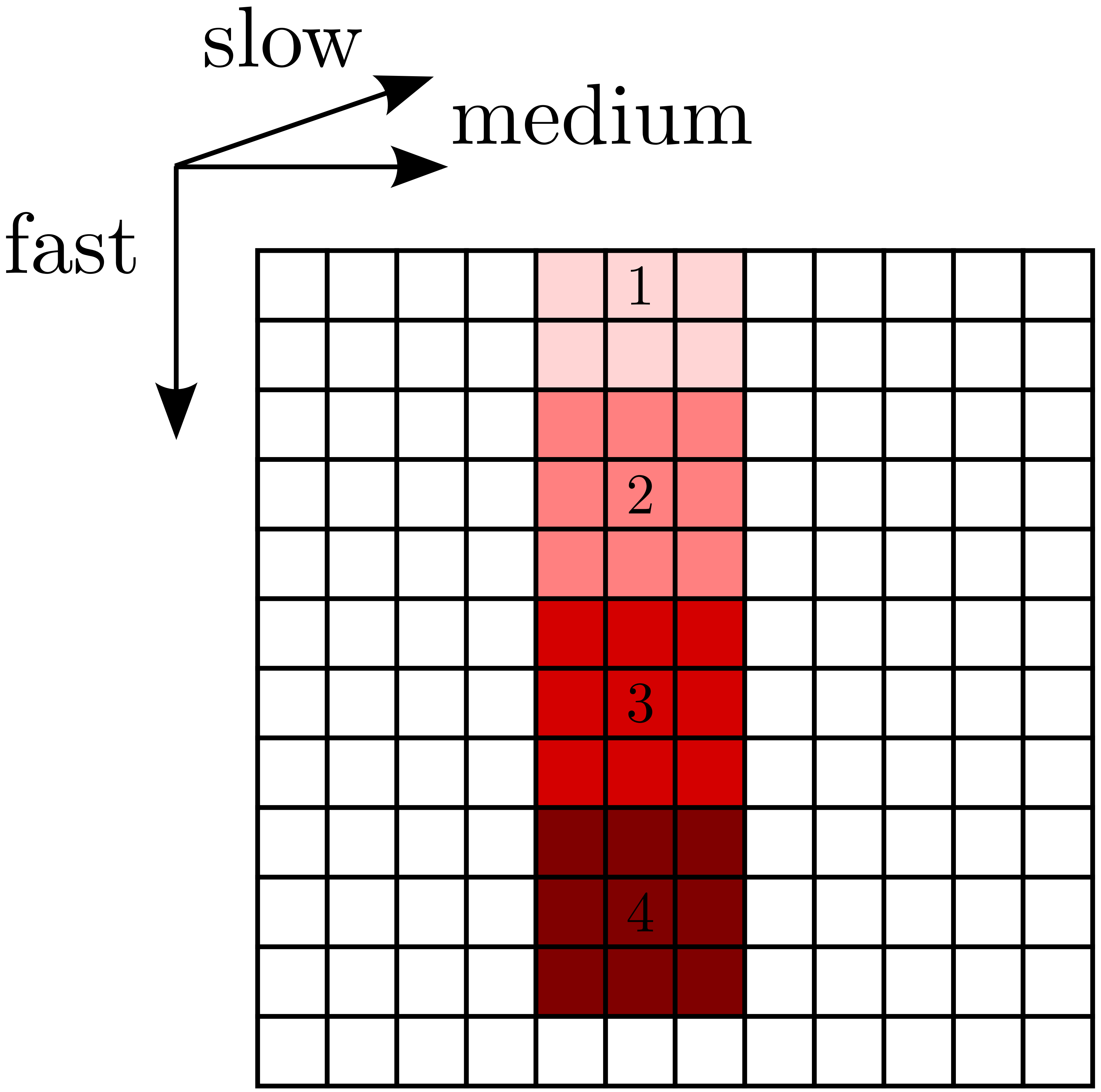


Buffering: Overcoming Latency of Memory Access

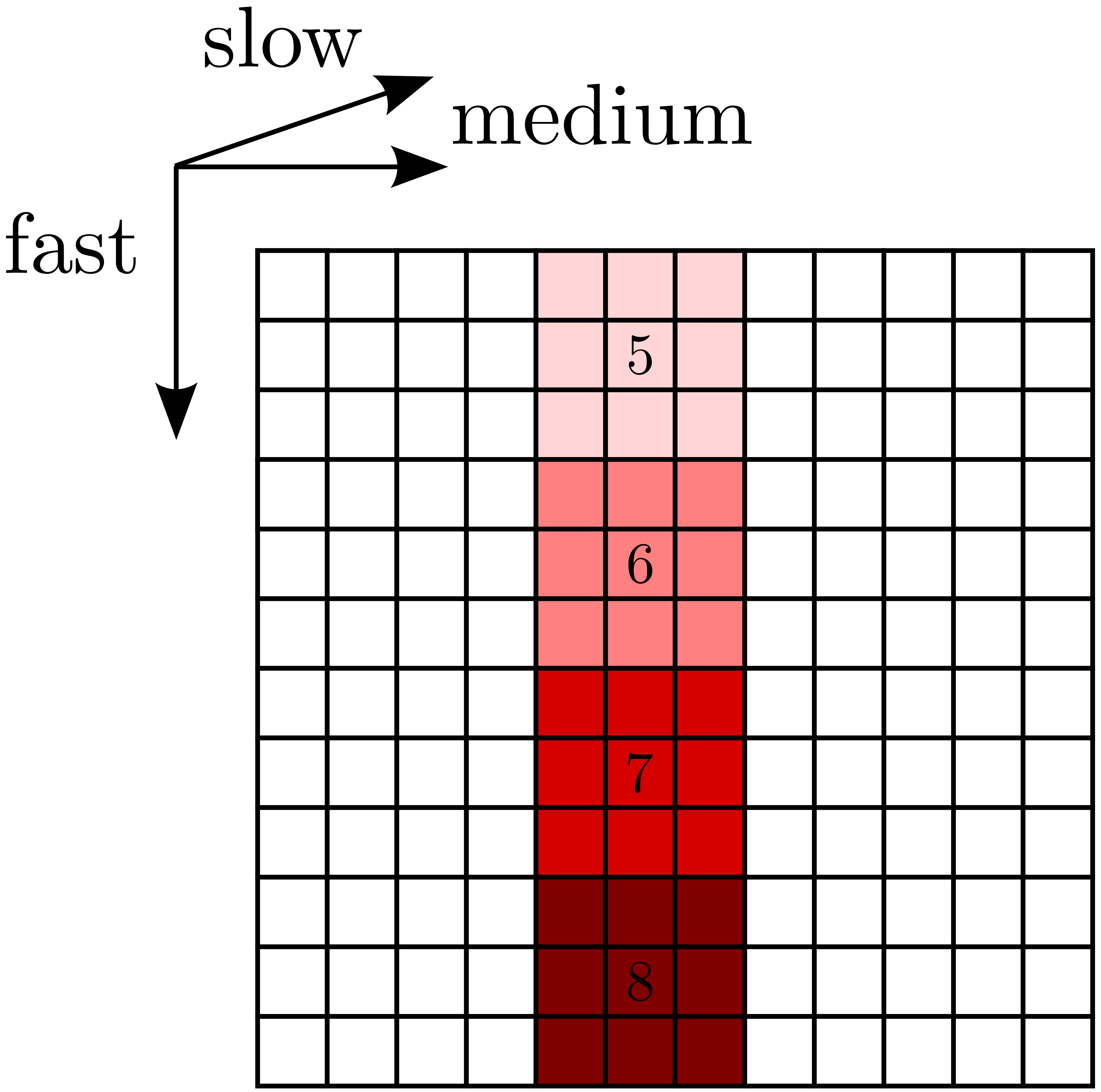

Memory (24 GB)

On-chip Buffers (BRAM)
4.6 MB maximum

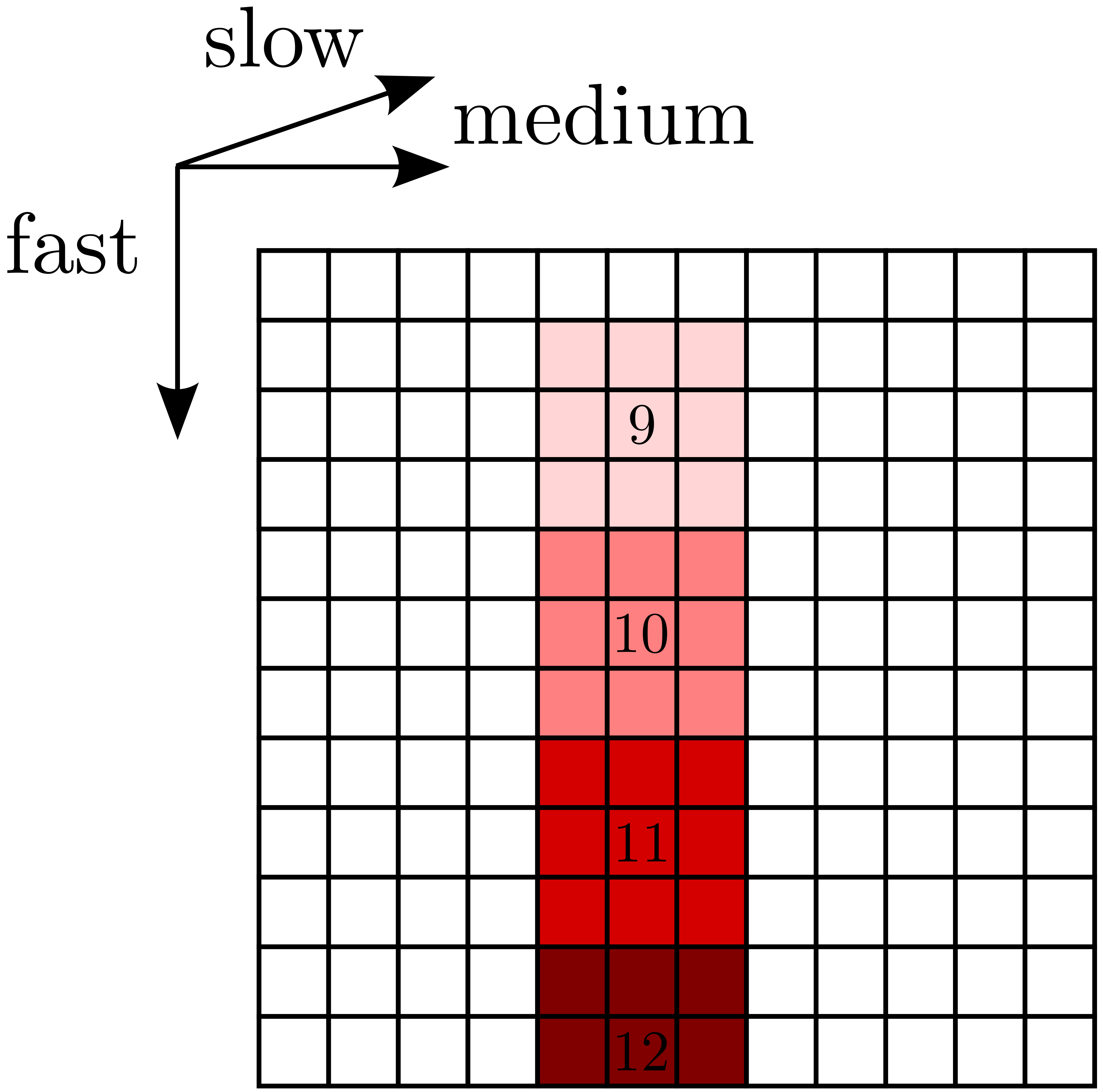
Pipelining: Overcoming Latency of Computation



Pipelining: Overcoming Latency of Computation



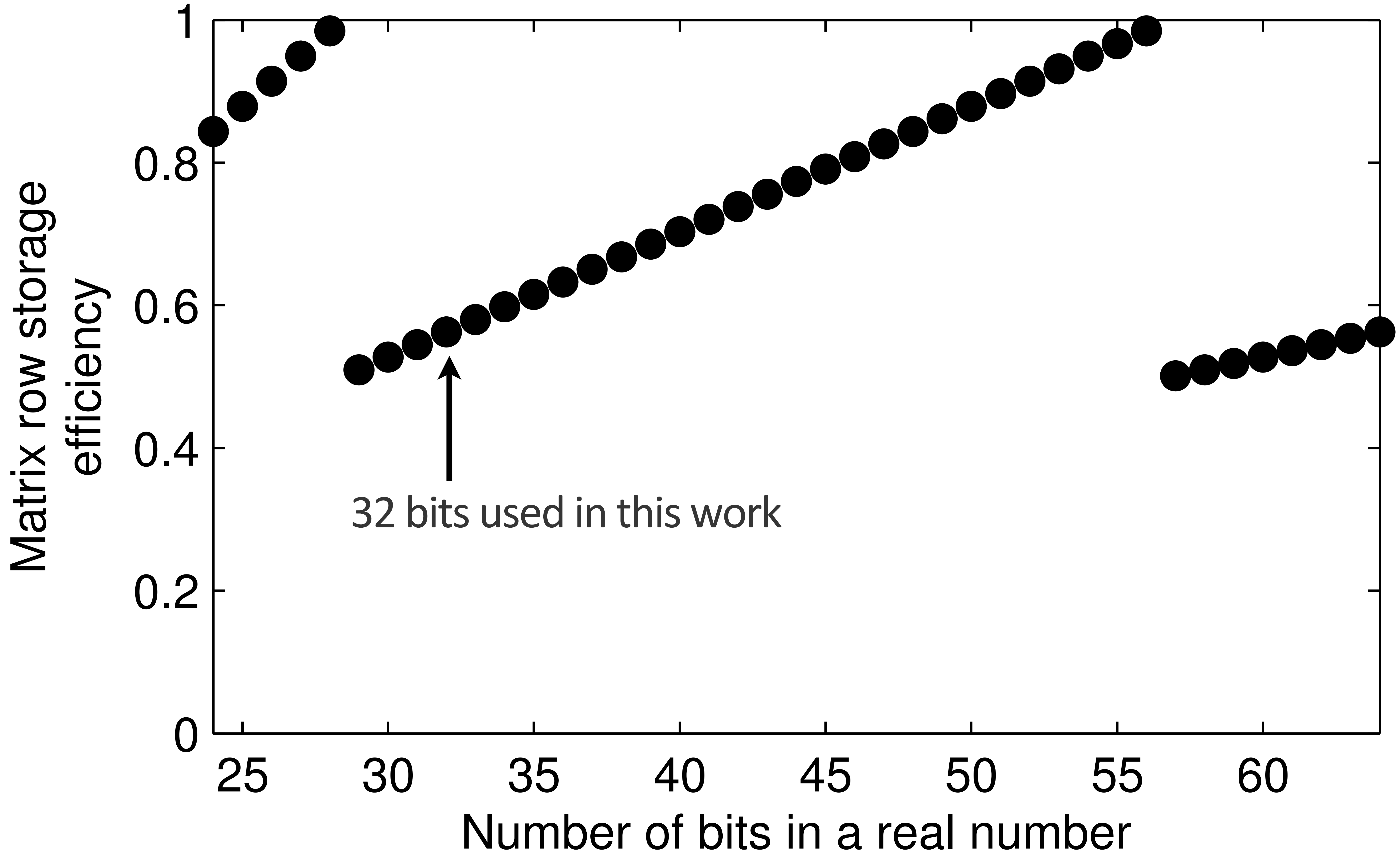
Pipelining: Overcoming Latency of Computation



Granular Memory Access: 384 bytes / burst

Number of bits in a real number	Number of bits in a complex number	Complex numbers per burst
24	48	64
32 (single precision)	64	48
48	96	32
64 (double precision)	128	24

Matrix Coefficient Representation



32 bits used in this work

Results & Discussion

What is being compared?

Reference implementation:

- Solution Algorithm
  - CARP-CG
  - written in MATLAB
  - single-threaded
  - double precision
  - running on Intel Xeon E5-2670
- Computational Kernel
  - CARP sweeps
  - written in C
  - 32 threads
  - double precision
  - running on Intel Xeon E5-2670

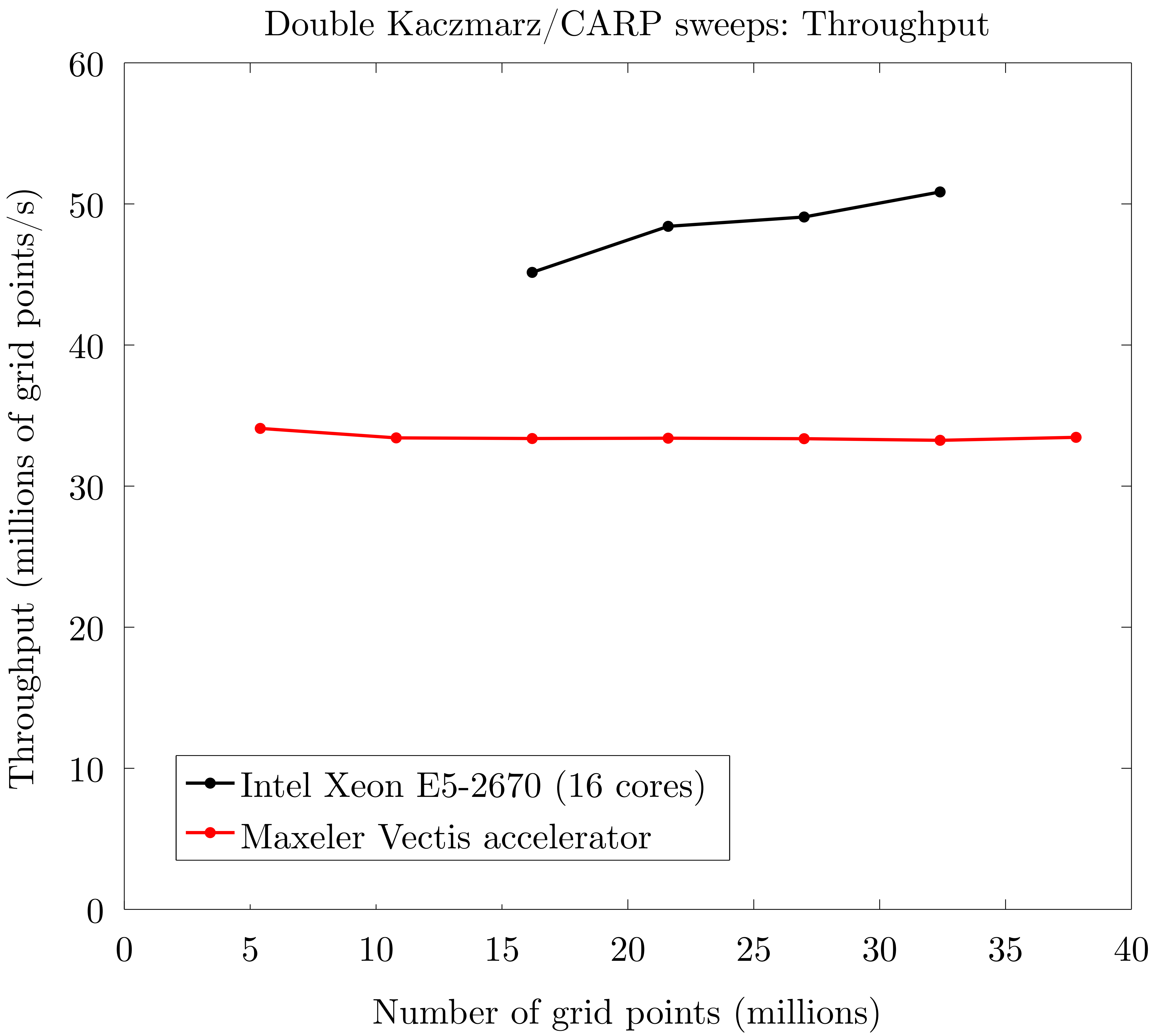
Accelerator implementation:

- Solution Algorithm
  - CGMN
  - written in MATLAB
  - single-threaded
  - single precision
  - running on Intel Xeon E5-2670
- Computational Kernel
  - Kaczmarz sweeps
  - single precision
  - running on Maxeler Vectis accelerator at 100 MHz
  - Memory clock at 303 MHz

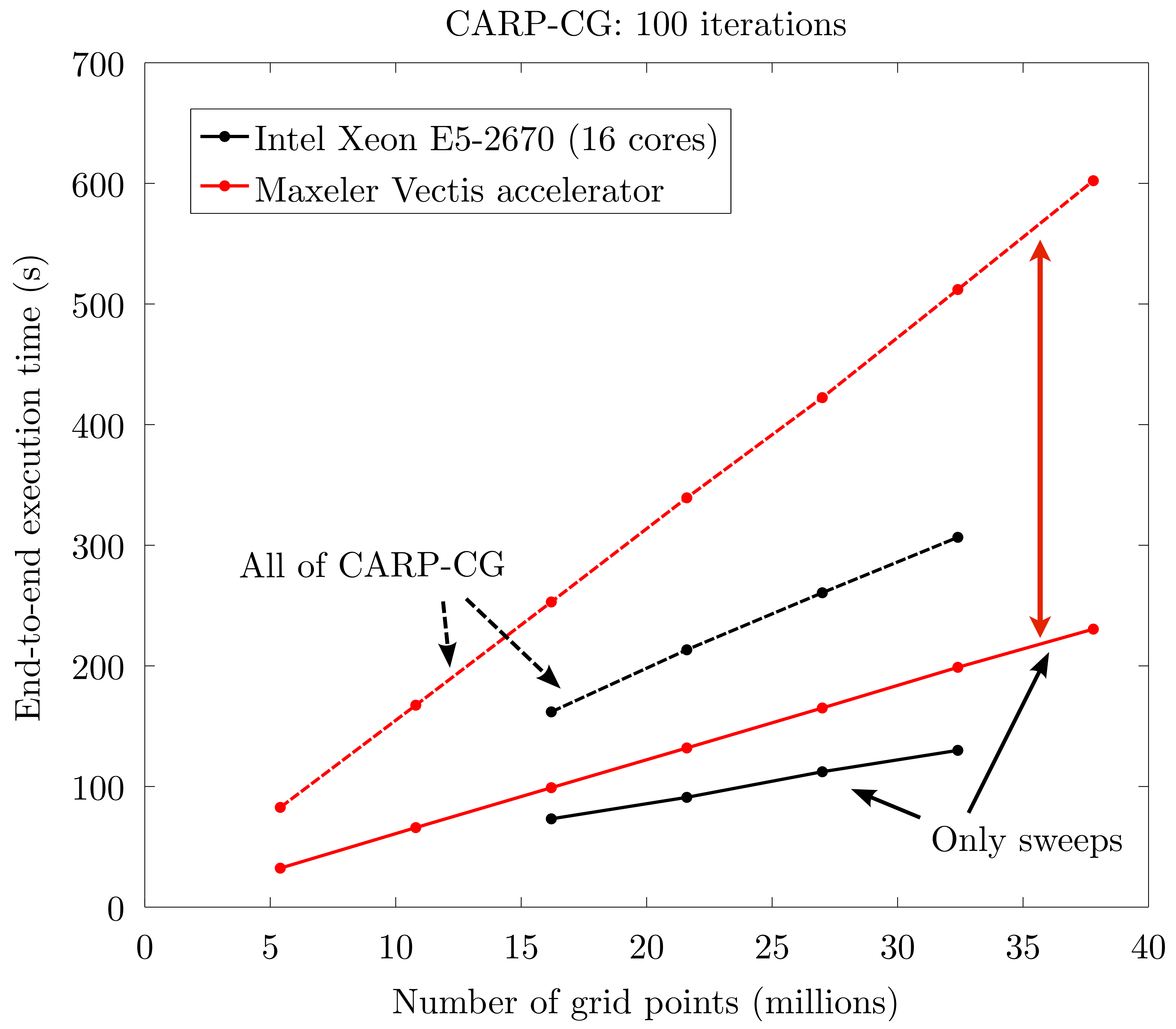
Experimental Set-up

- Solve part of the SEG/EAGE Overthrust System: 432 x 500 x Z.
- Point source.
- Zero initial guess.
- Run 100 CARP-CG/CGMN iterations.

Throughput of Kaczmarz/CARP sweeps



CARP-CG: End-to-end execution time

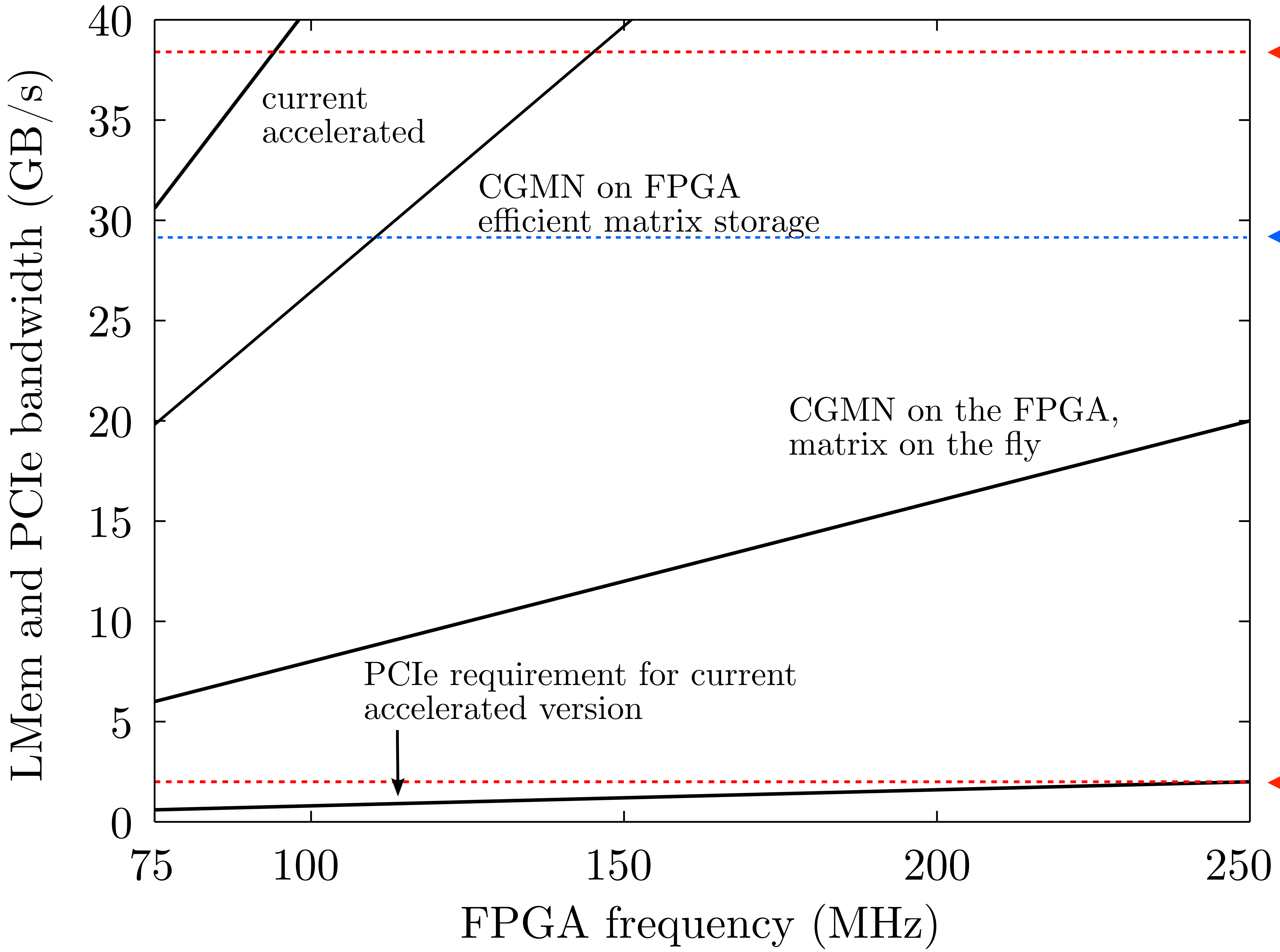


Lots of overhead!

Kaczmarz sweeps are 39% of run time.

Future Solution: Port all of CGMN to accelerator.

Avoiding Communication Bottlenecks

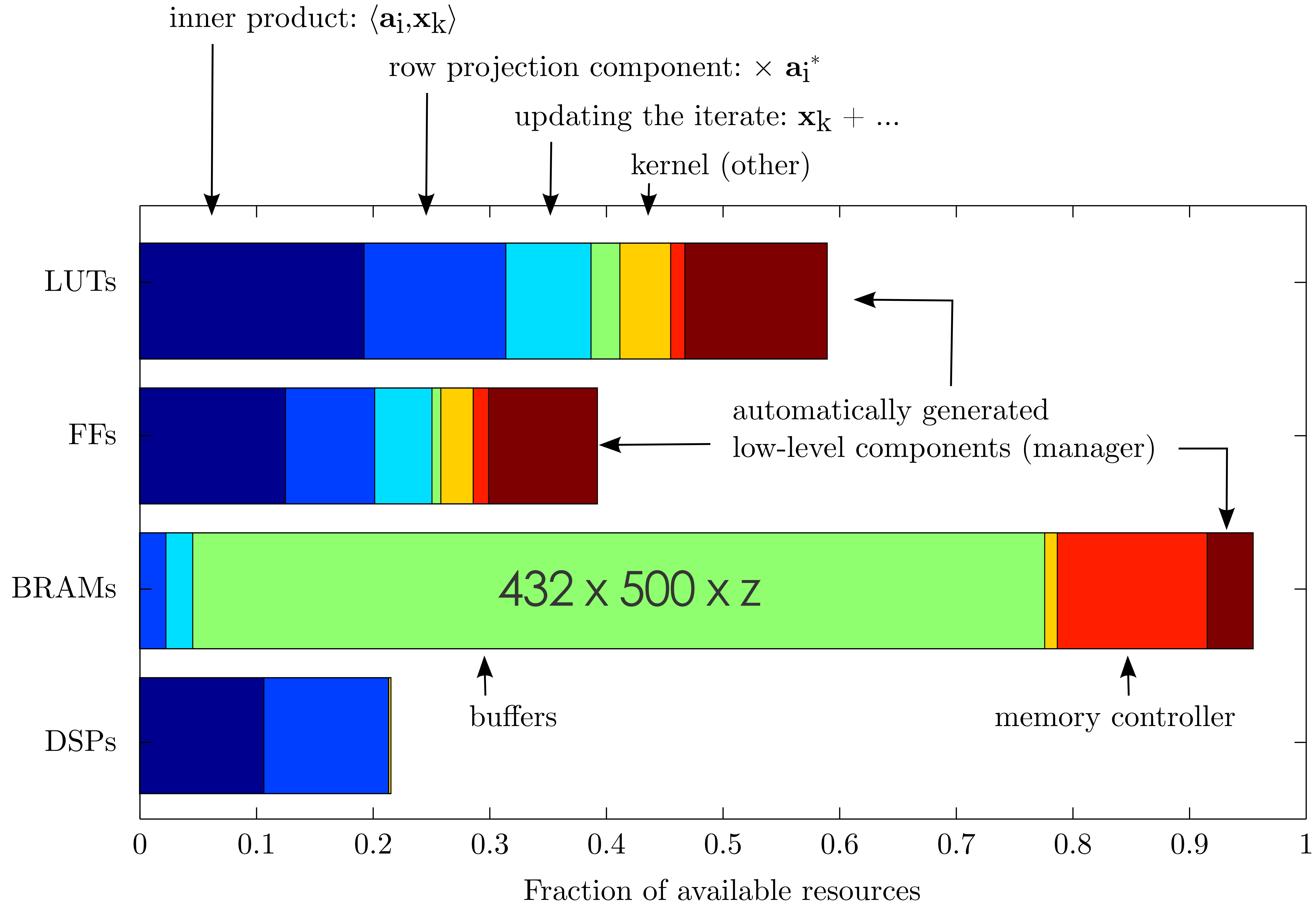


38.4 GB/s: memory bandwidth limit

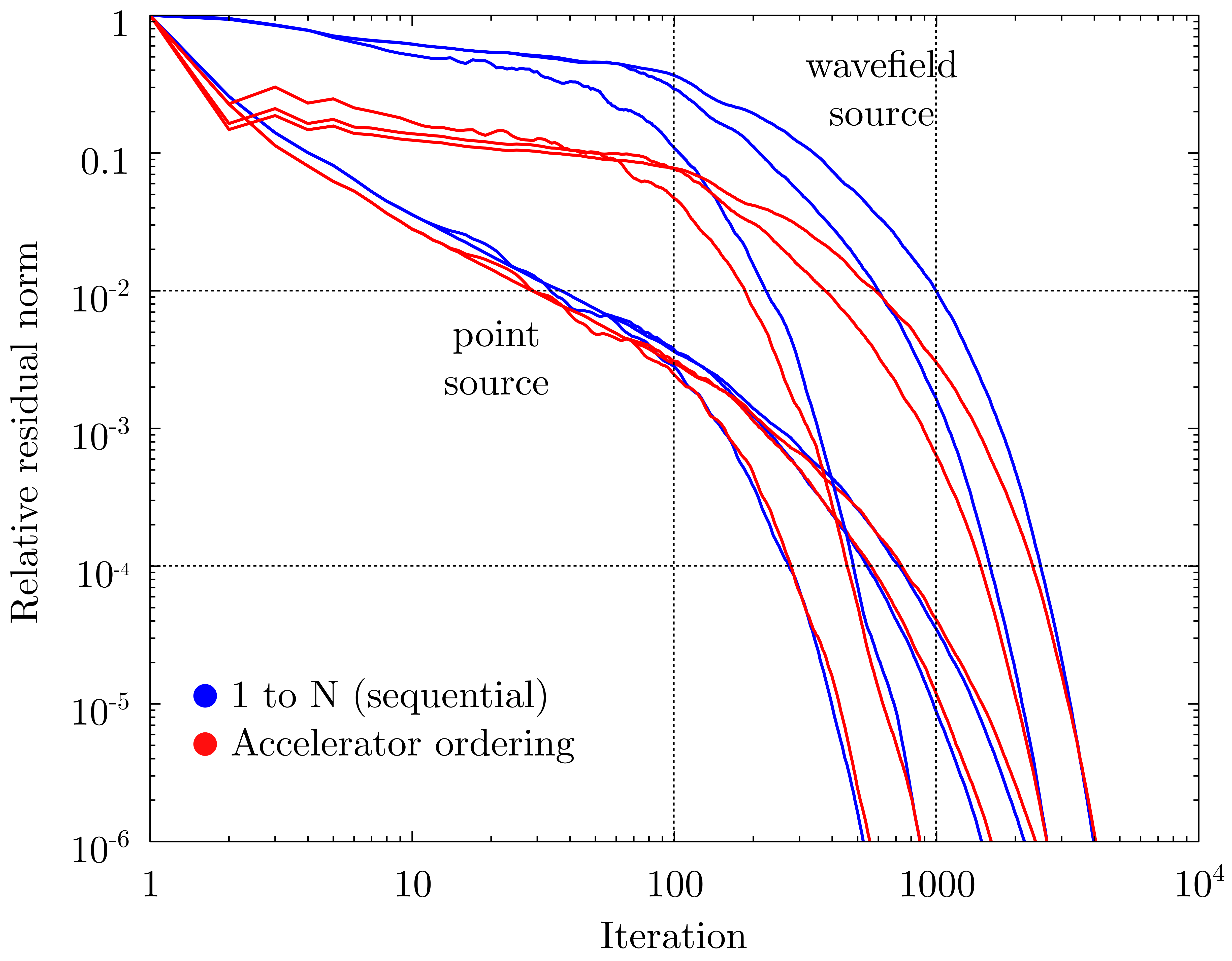
29 GB/s: bandwidth limit
in current work

2 GB/s: PCIe bandwidth limit

FPGA Resource Usage: Room for parallelism?



Effect of matrix row ordering on CGMN convergence



Conclusion

Have **implemented** frequency-domain wave simulation using reconfigurable hardware.

More work needed to realize full potential of accelerator system.

Acknowledgements

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