

Multilevel Acceleration Strategy for REPSI

Tim T.Y. Lin

Robust EPSI

L1-minimization approach to the EPSI problem

[Lin and Herrmann, 2013 *Geophysics*]

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg \min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k} \mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg \min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}} \mathbf{q}\|_2$$

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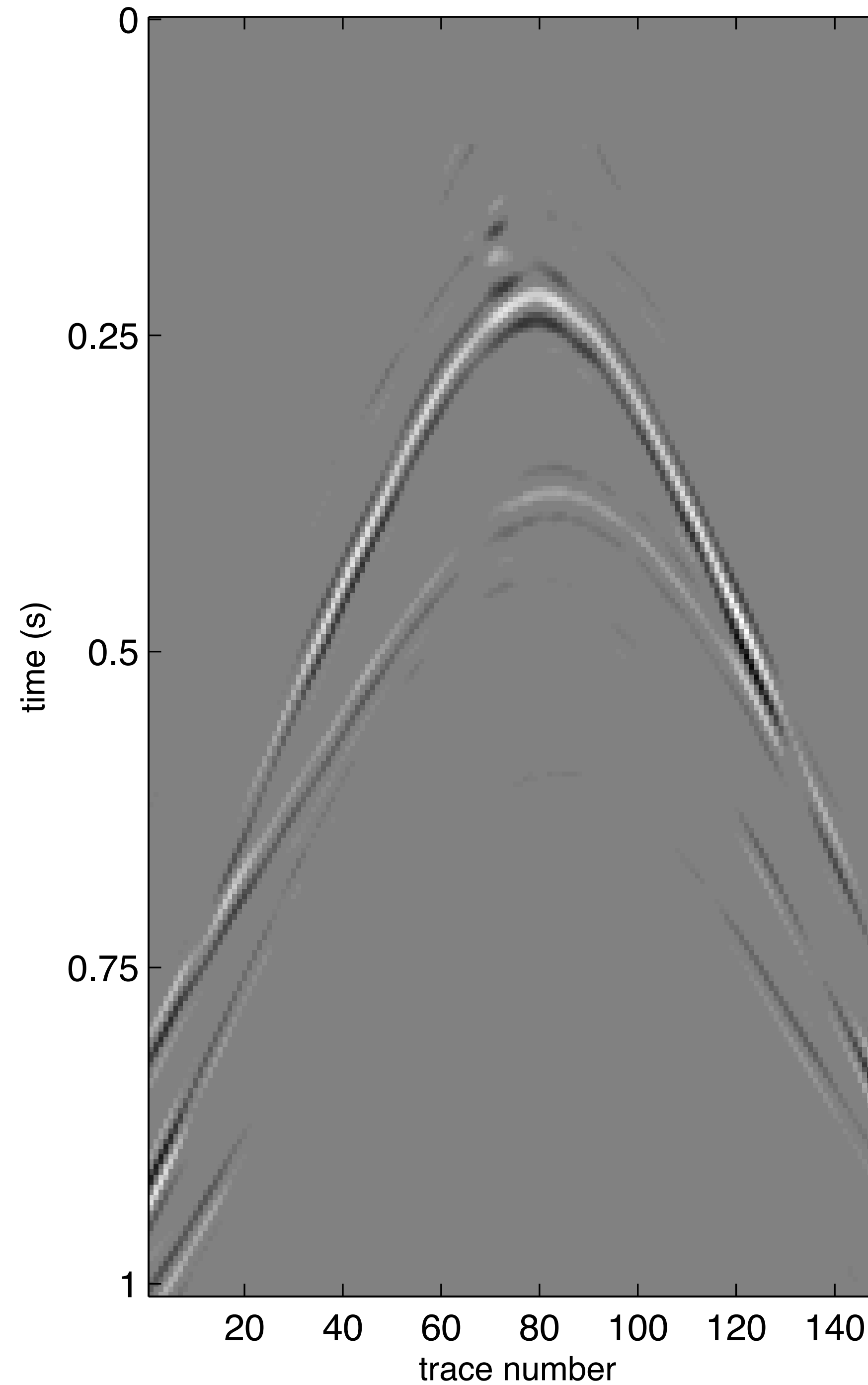
$$\mathbf{q}_{k+1} = \arg \min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}} \mathbf{q}\|_2$$

Emits sparse, or
“deconvolved” solution

L1 projection and sparsity

variable \mathbf{g} at beginning of LASSO

$$\mathbf{g}_{k+1} = \arg \min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k} \mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

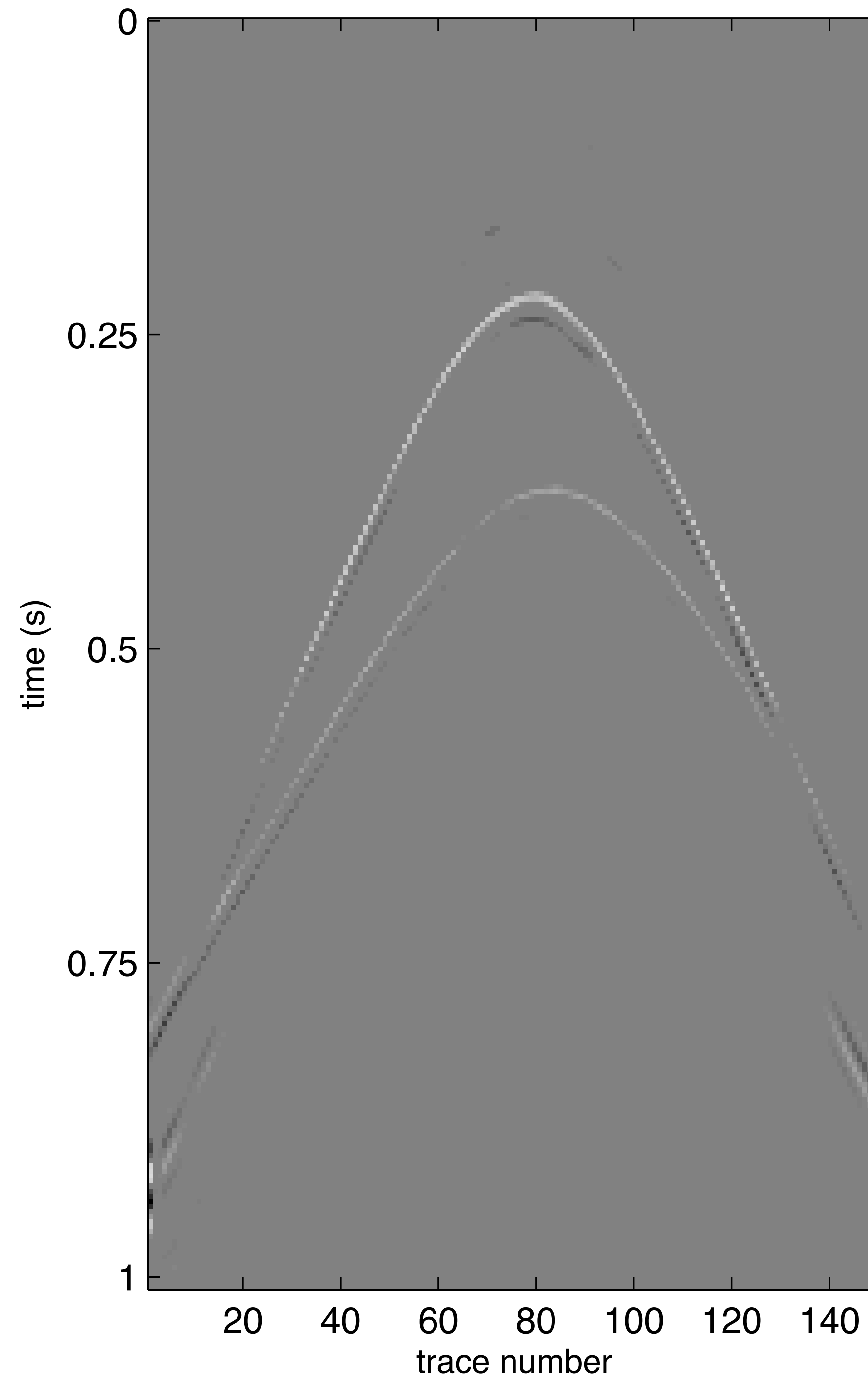


L1 projection and sparsity

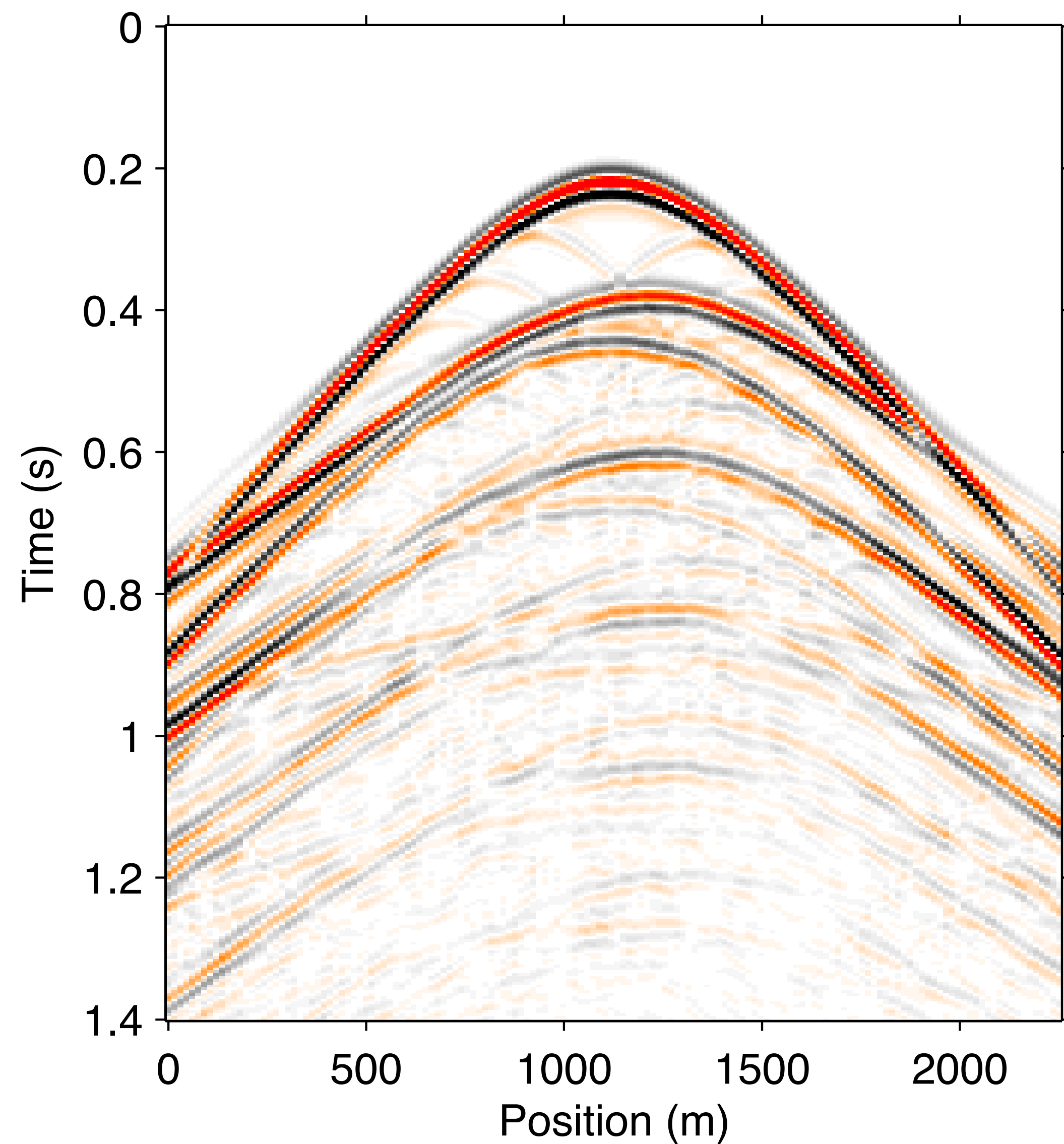
variable \mathbf{g} at **end** of LASSO

$$\mathbf{g}_{k+1} = \arg \min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k} \mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

Emits “deconvolved”
solution



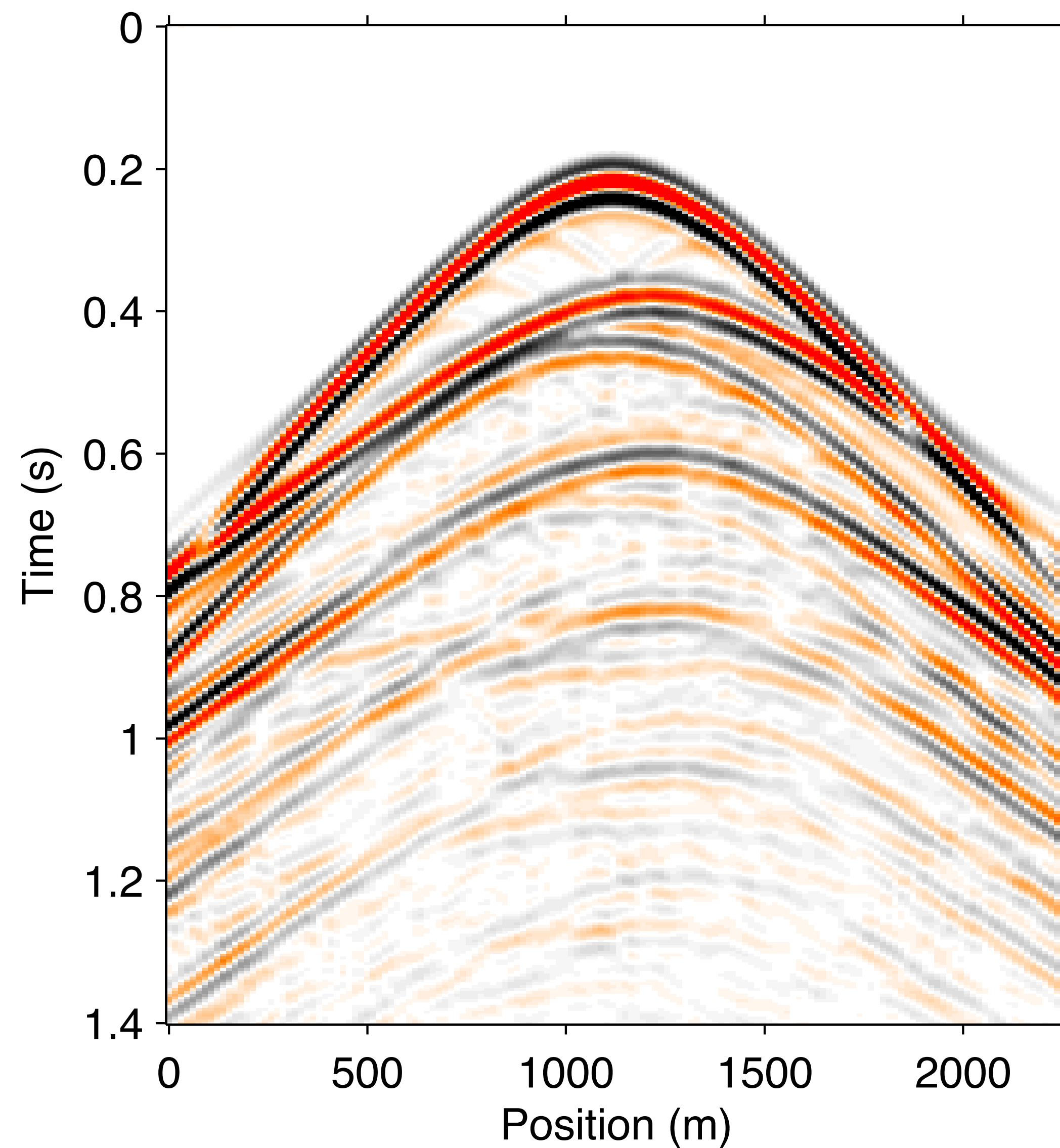
Motivation: **G** tolerates lowpass filtering



Data

modeled with Ricker 30Hz

Motivation: G tolerates lowpass filtering



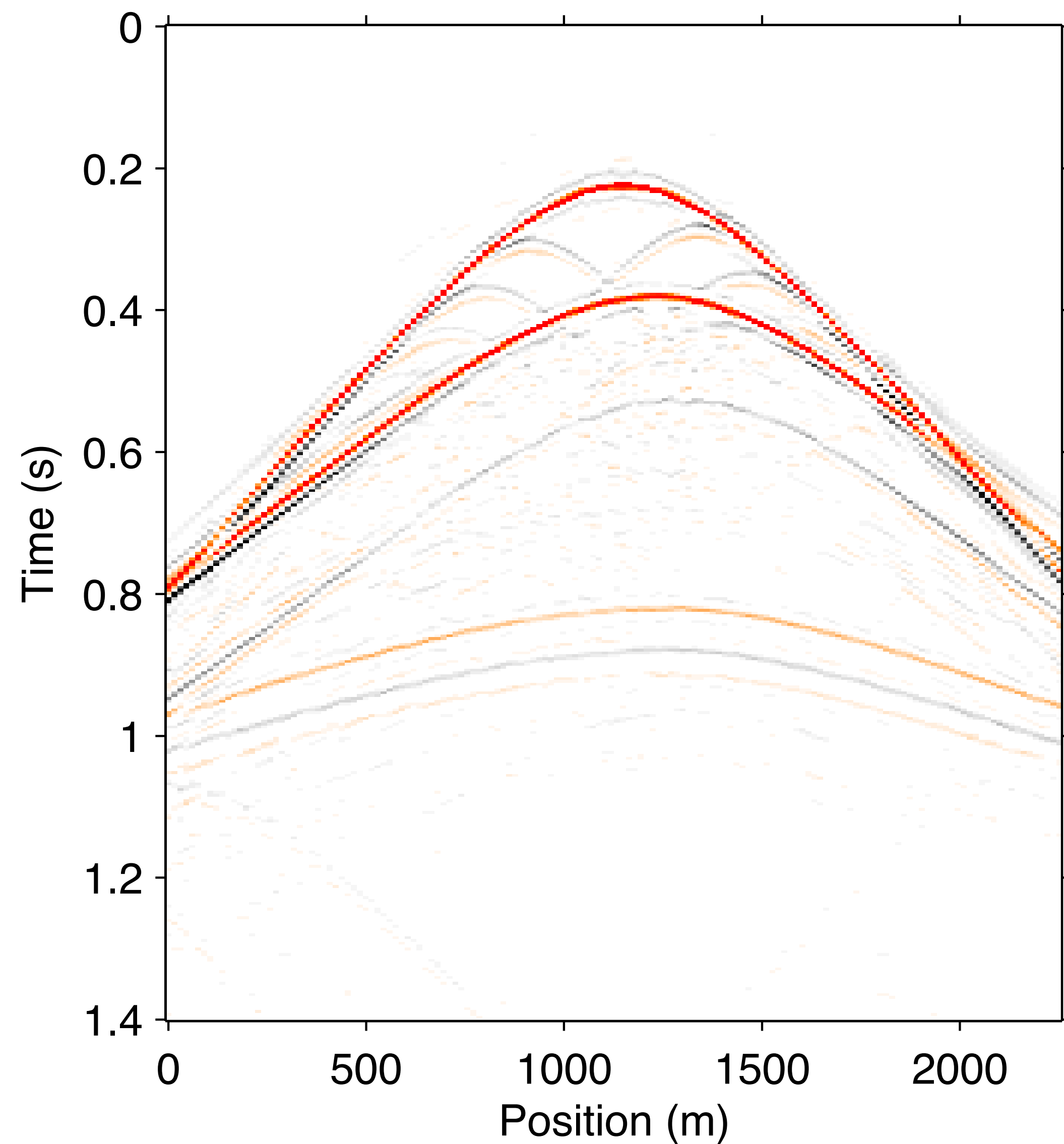
Lowpassed Data

modeled with Ricker 30Hz

lowpass at 40Hz

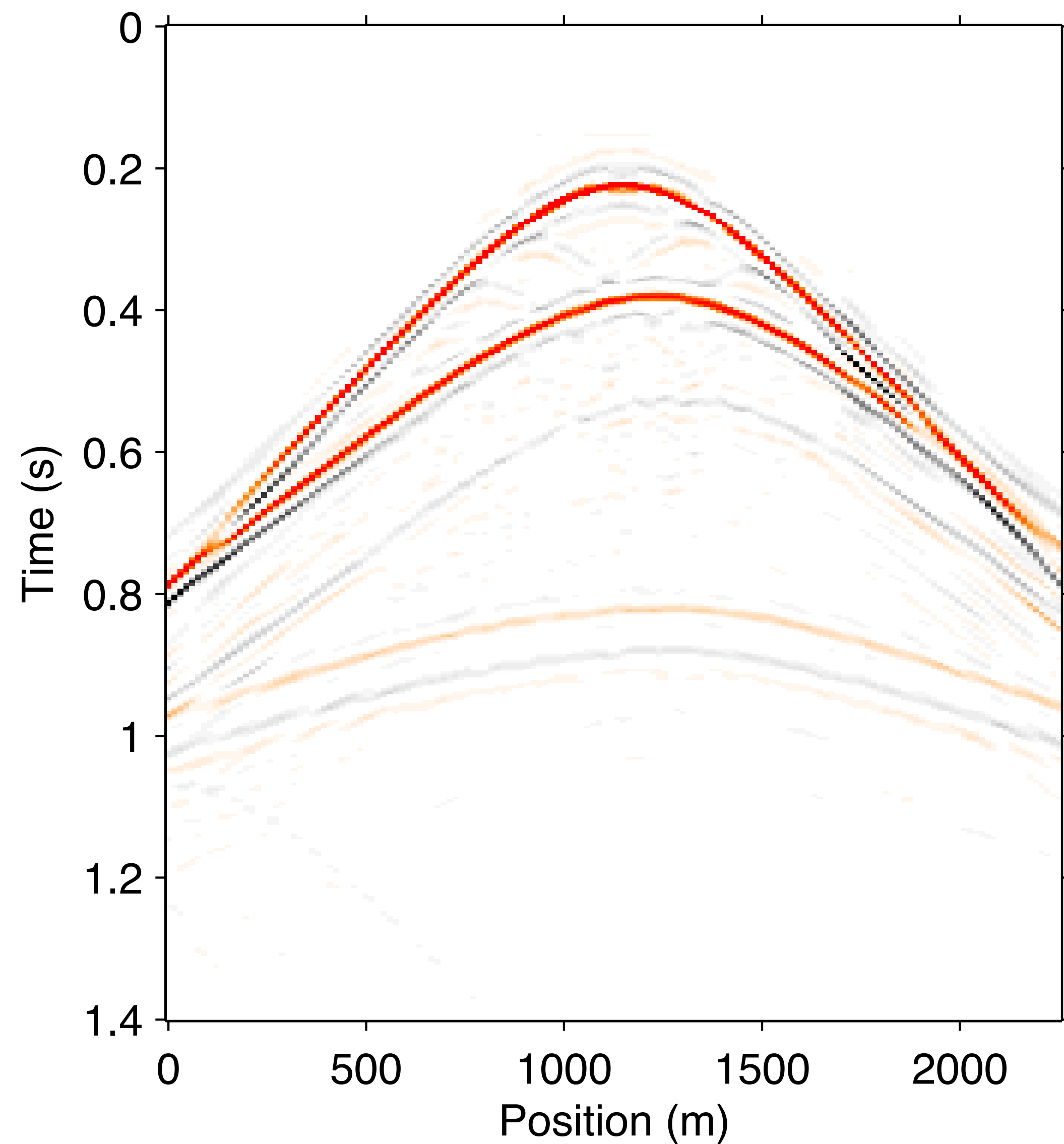
(25-order, zero-phase, Hann window)

Motivation: **G** tolerates lowpass filtering



Reference REPSI primary IR
from original data

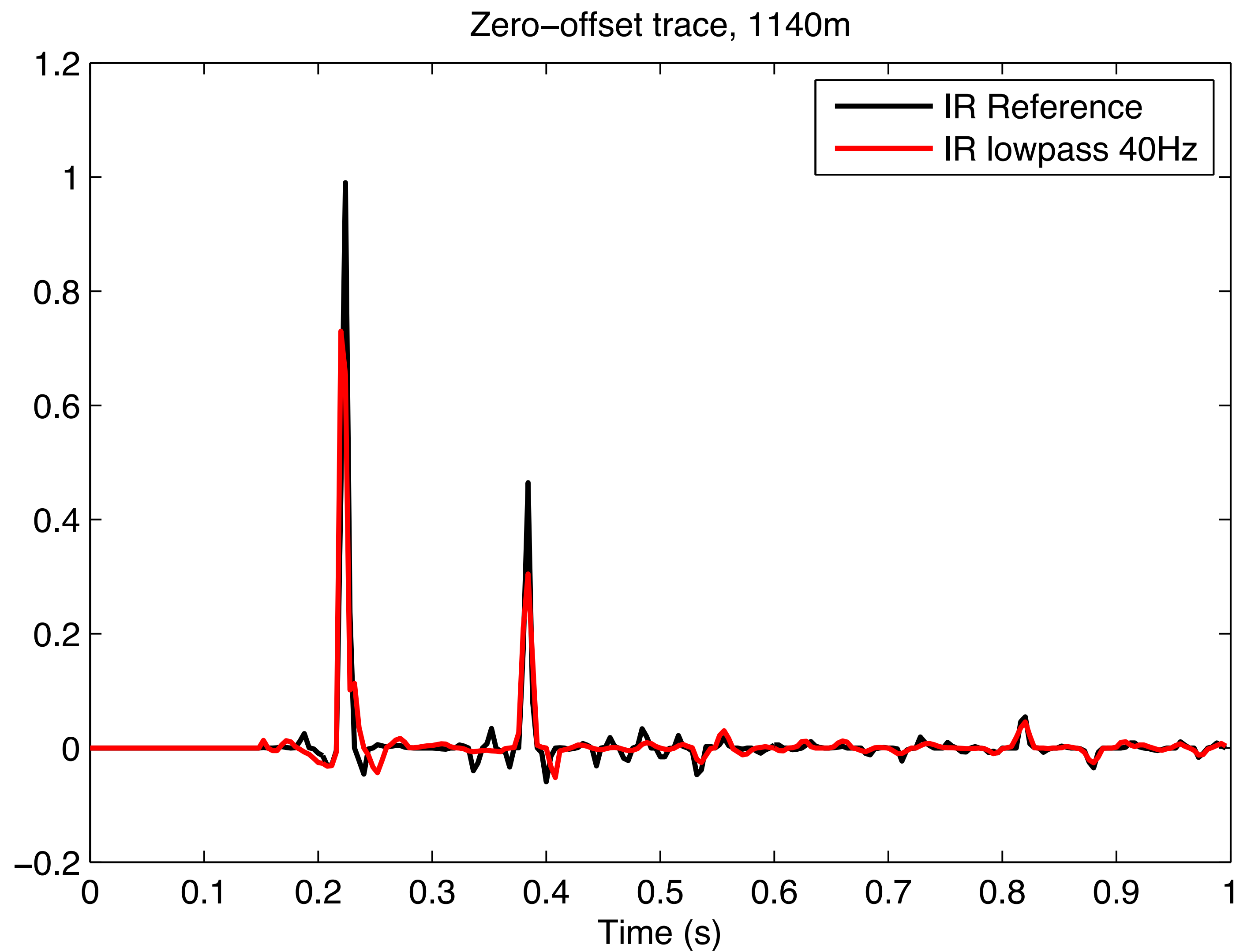
Motivation: G tolerates lowpass filtering



REPSI primary IR

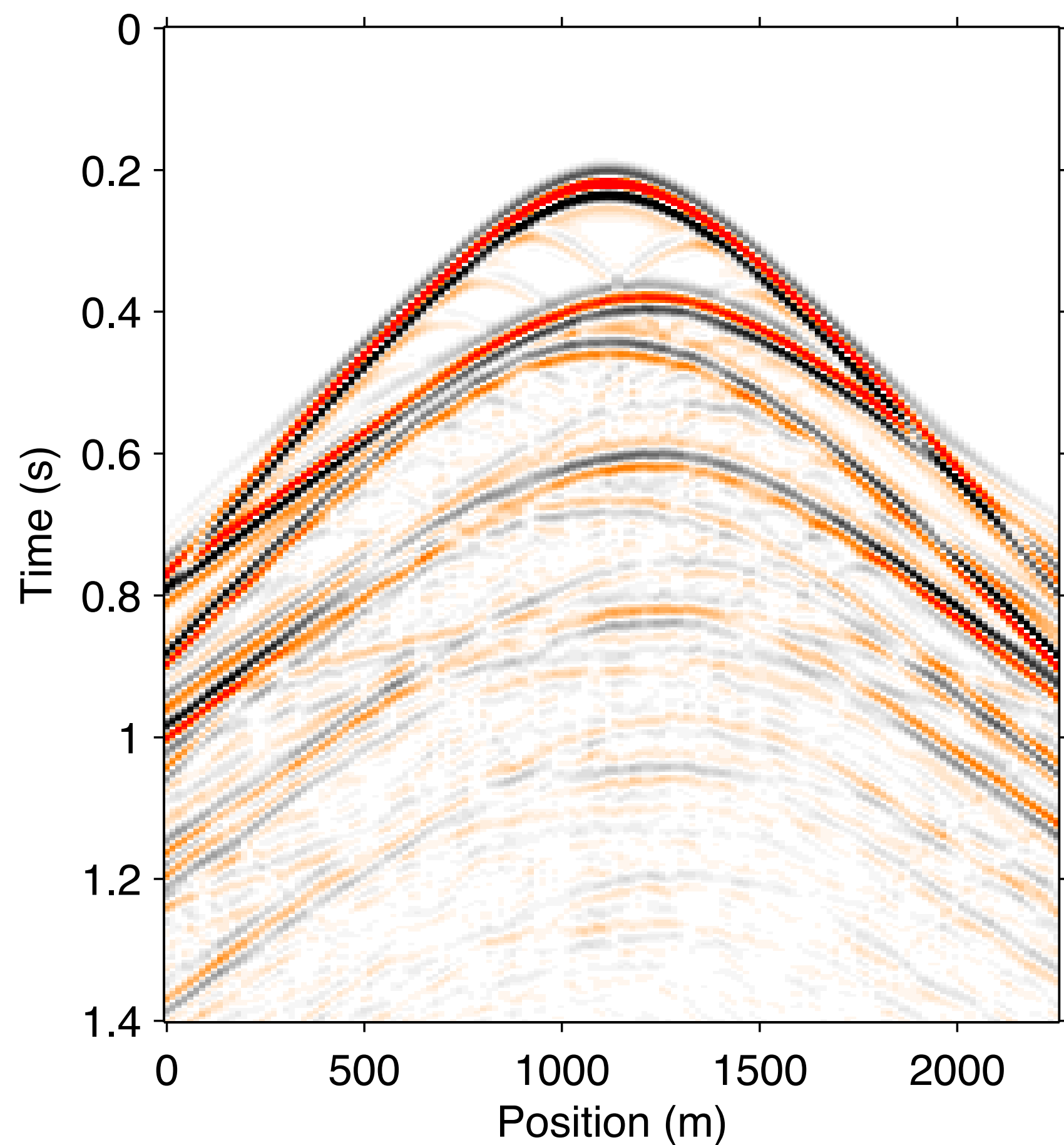
from low-passed data @ 40Hz

Motivation: G tolerates lowpass filtering

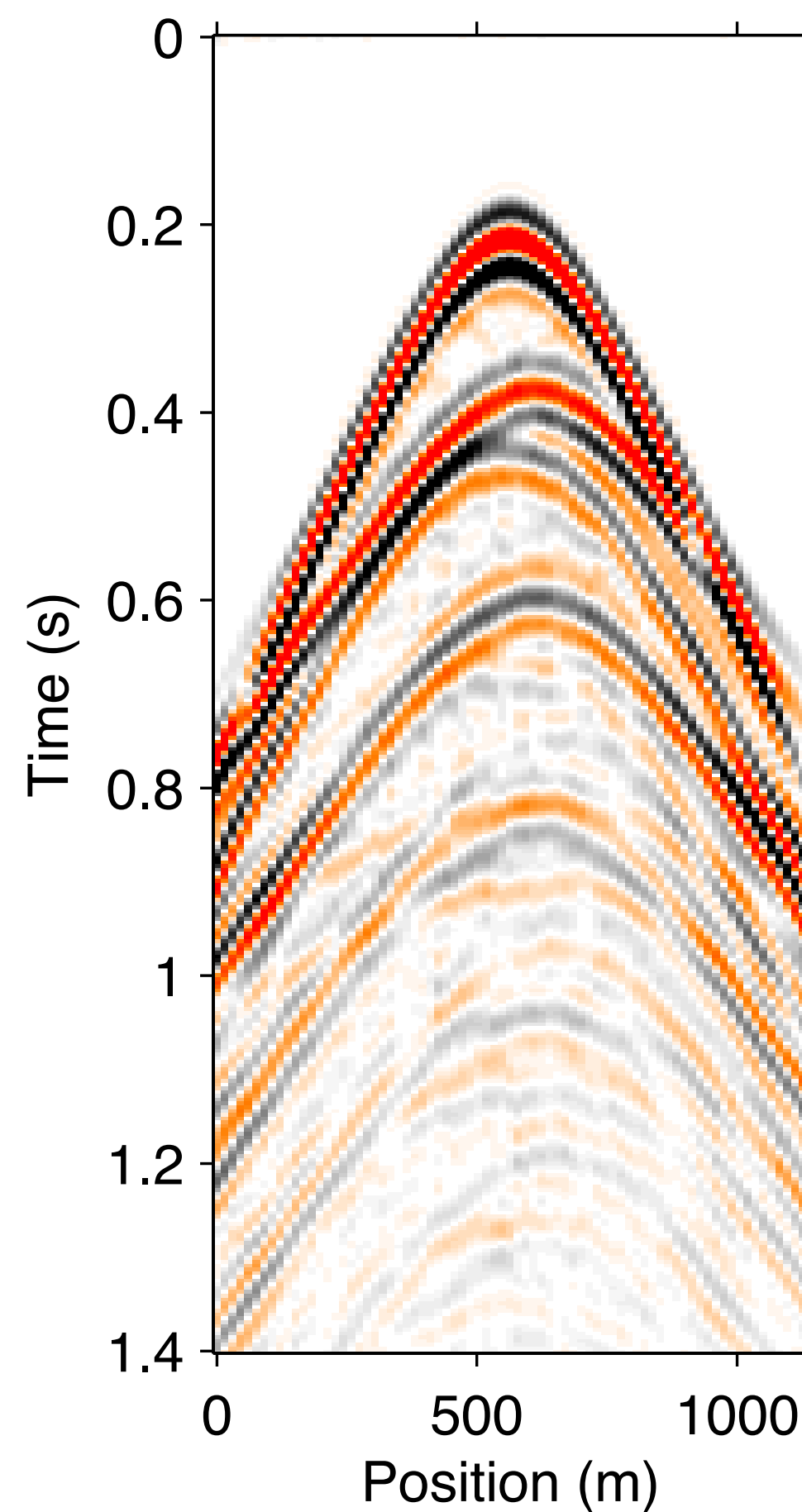


Lowpass data permits coarser sampling w/o aliasing

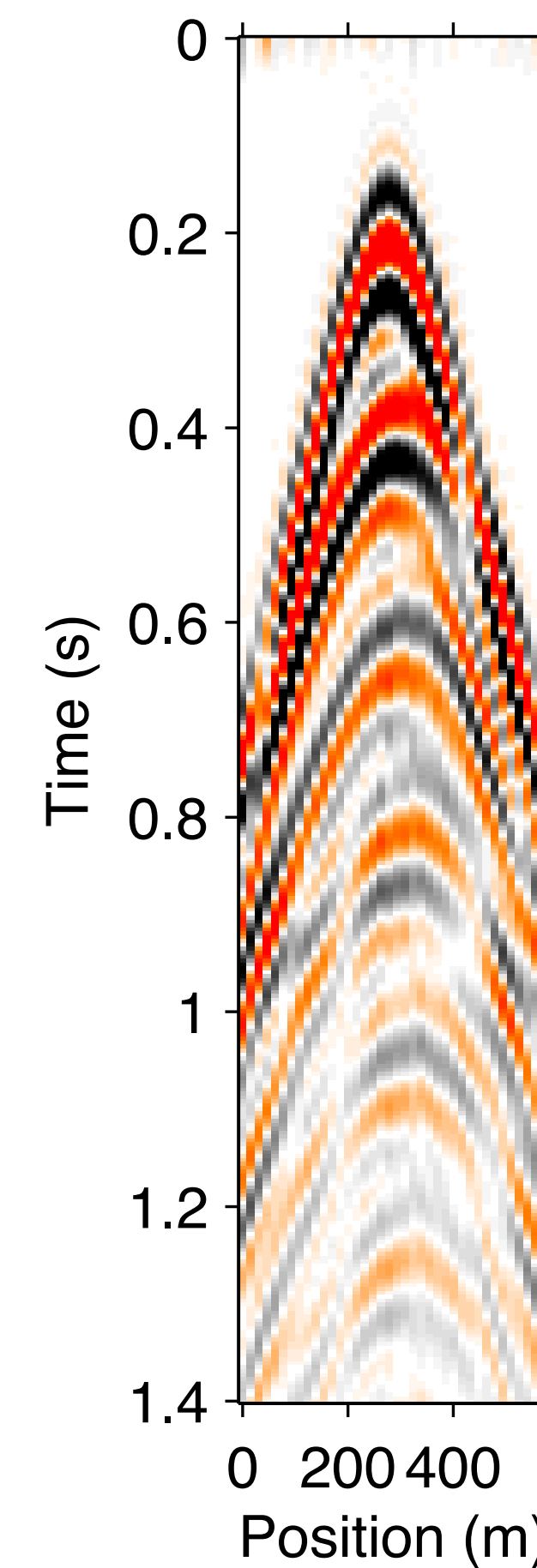
Original (dx = 15m)



2x decimated
lowpass 30Hz

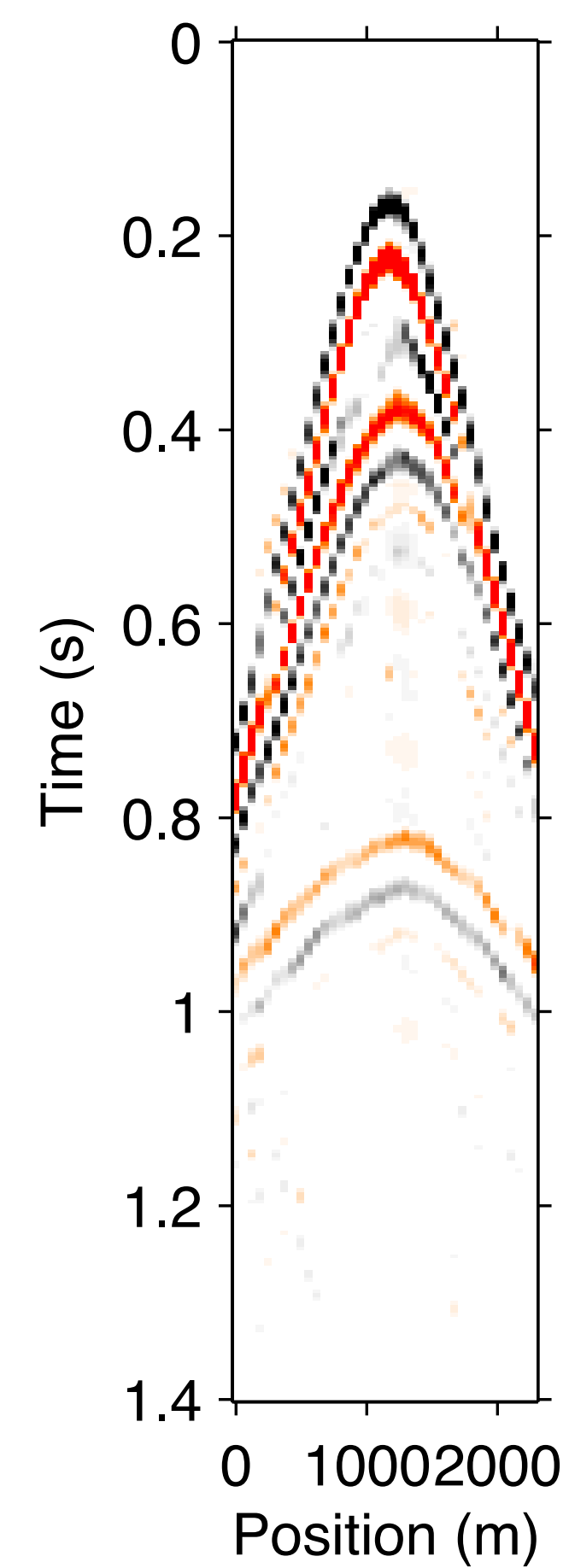
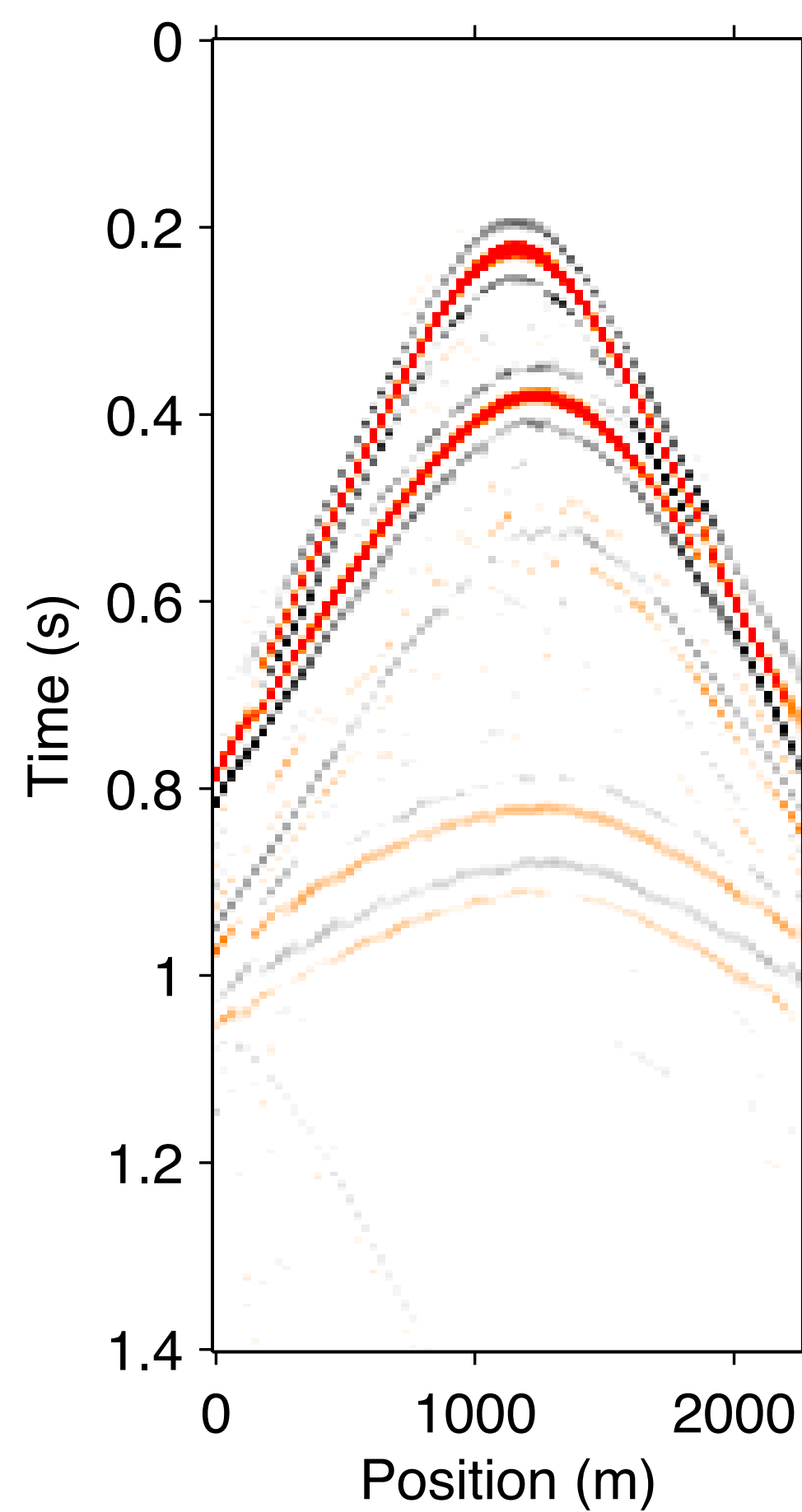
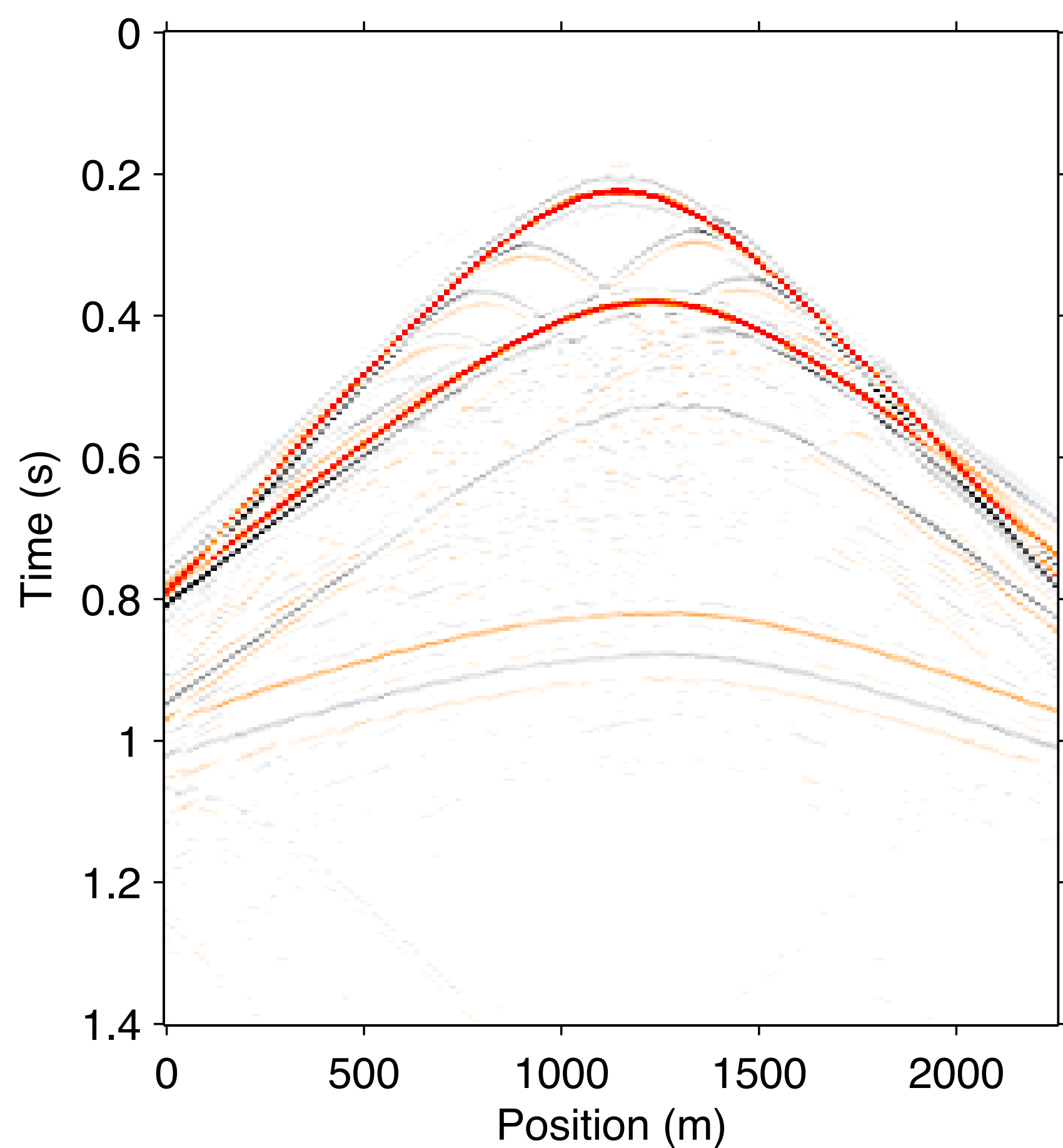


4x decimated
lowpass 15Hz

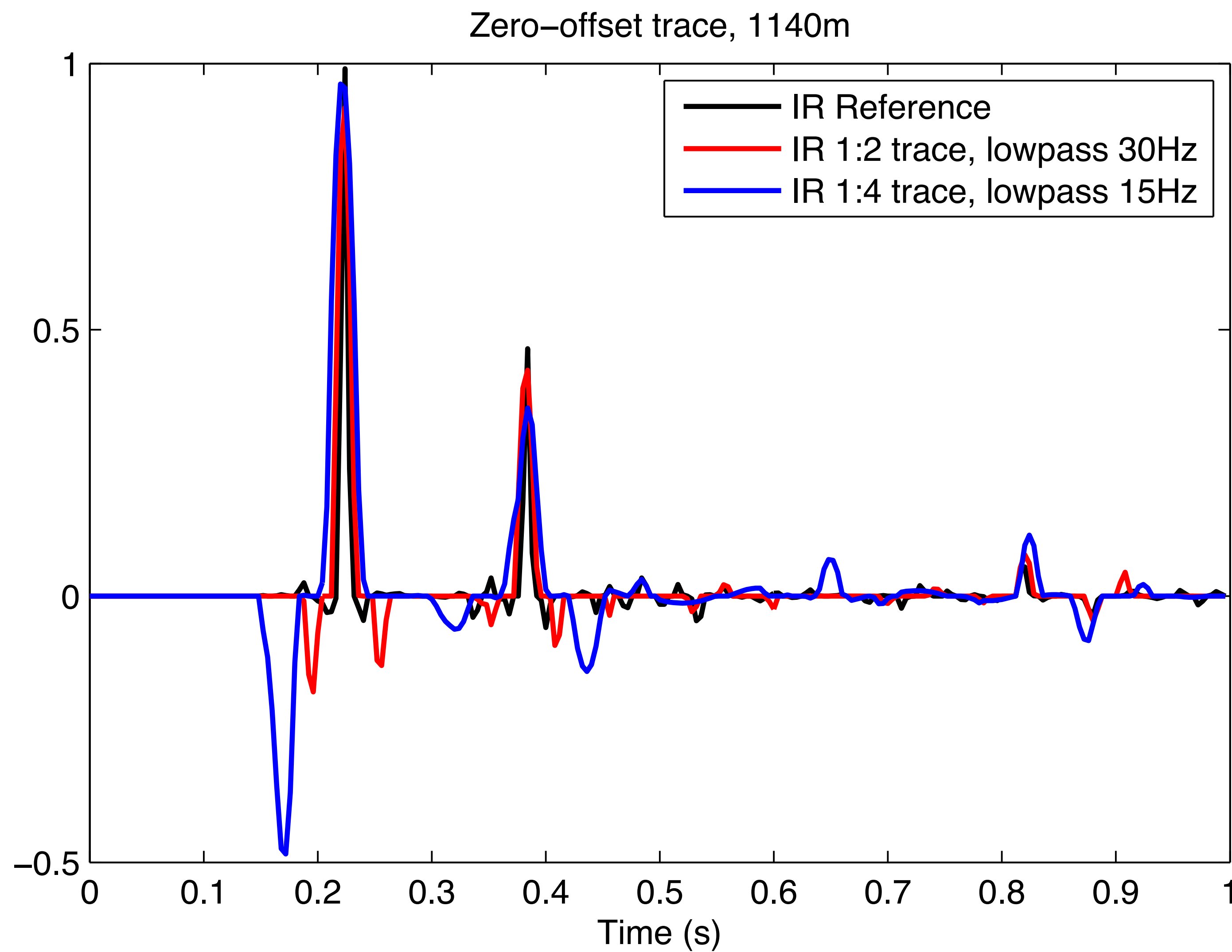


Lowpass data permits coarser sampling w/o aliasing

Impulse response solutions

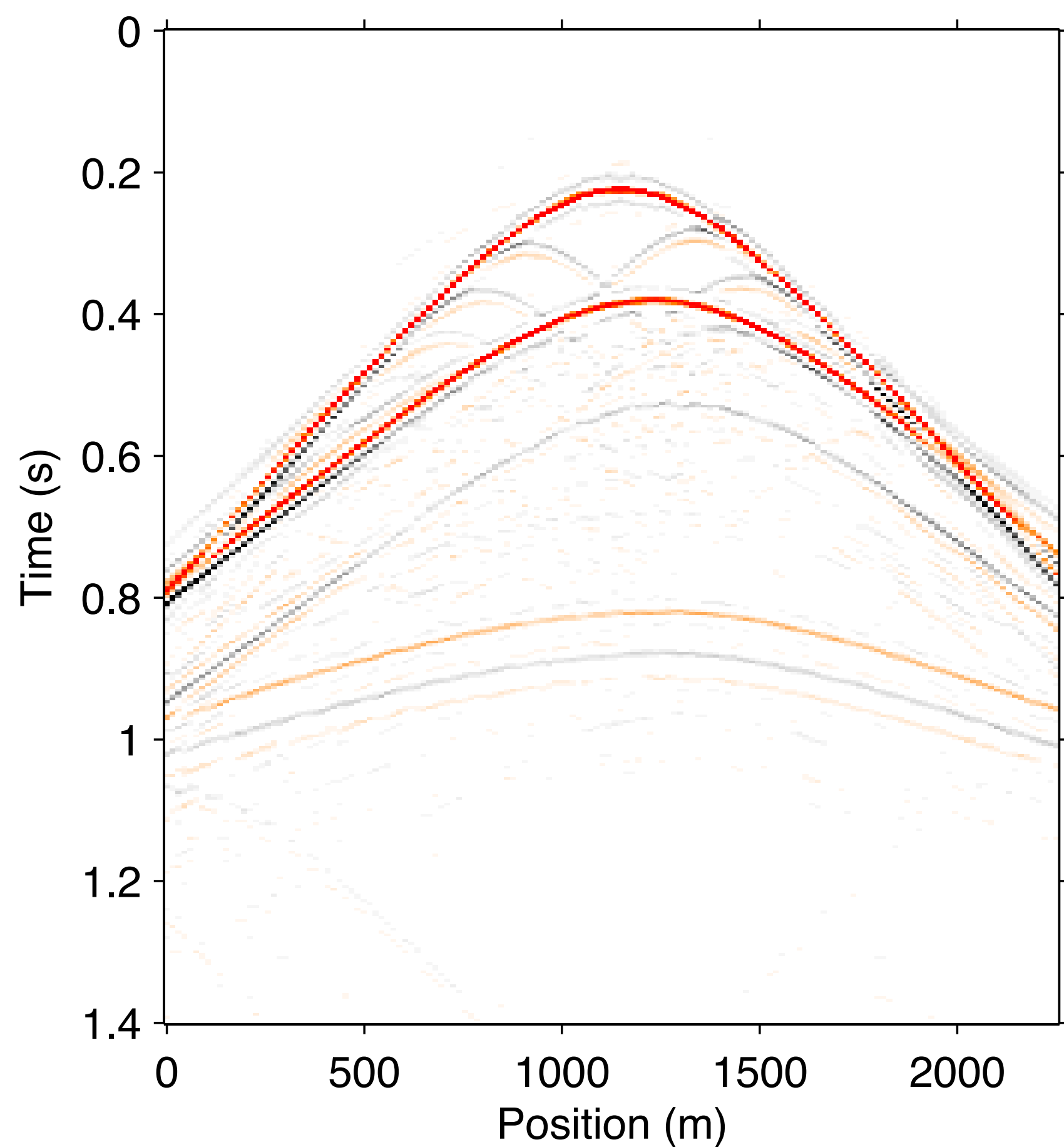


Lowpass data permits coarser sampling w/o aliasing

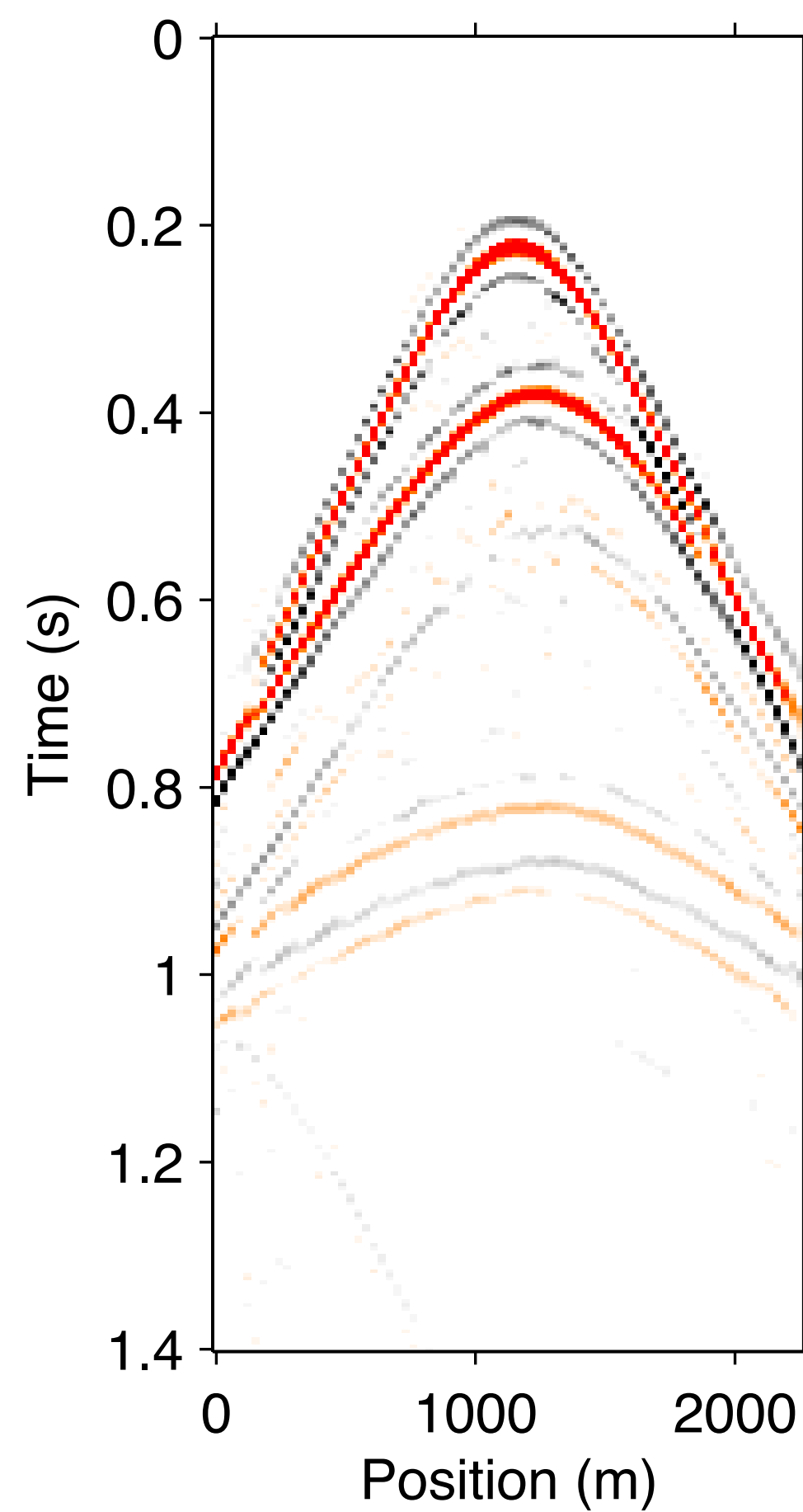


Lowpass data permits coarser sampling w/o aliasing *(much faster!)*

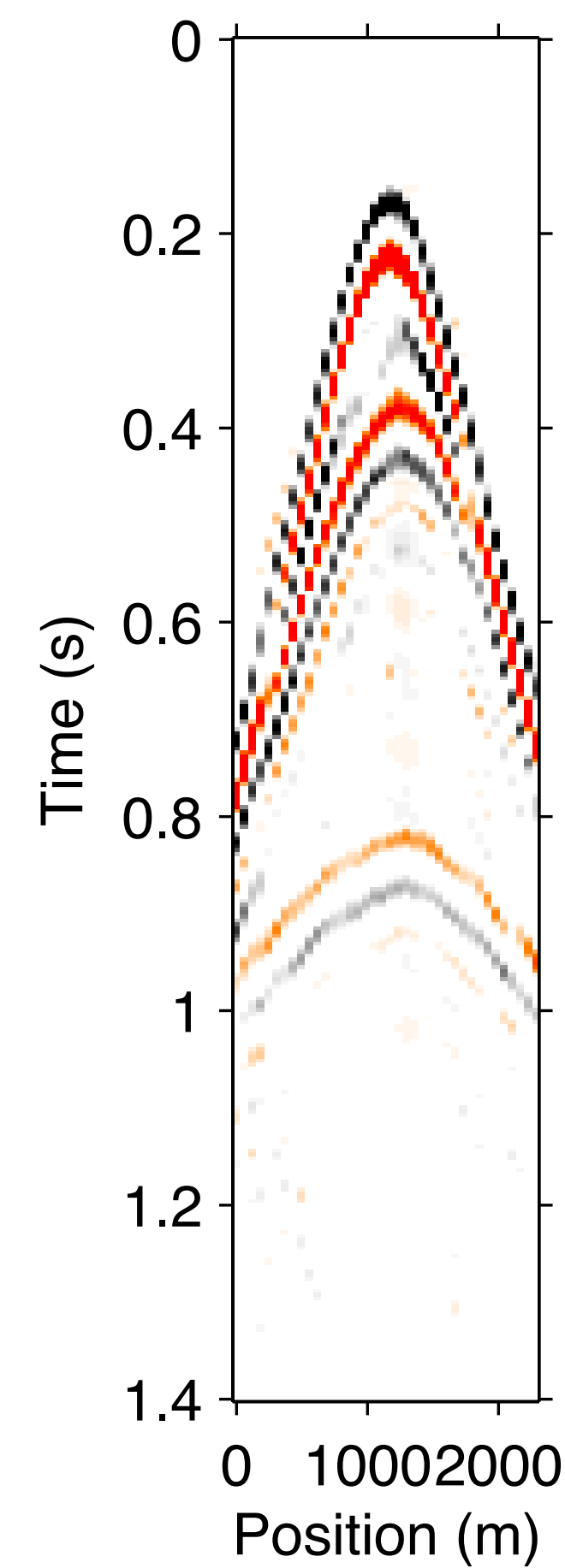
40 min



6 min



1.5 min



Multilevel strategy for EPSI

warm-start fine-scale problem with
coarse-scale solutions

Idea: Warm-start with coarse-scale solutions

EPSI takes **70-100 iterations** to converge (each iteration is doing 2 SRME multiple prediction), can we make it **FASTER**?

Since decimated datasets solve much faster, we interpolate its (slightly inaccurate) G for the initial estimate to full problem

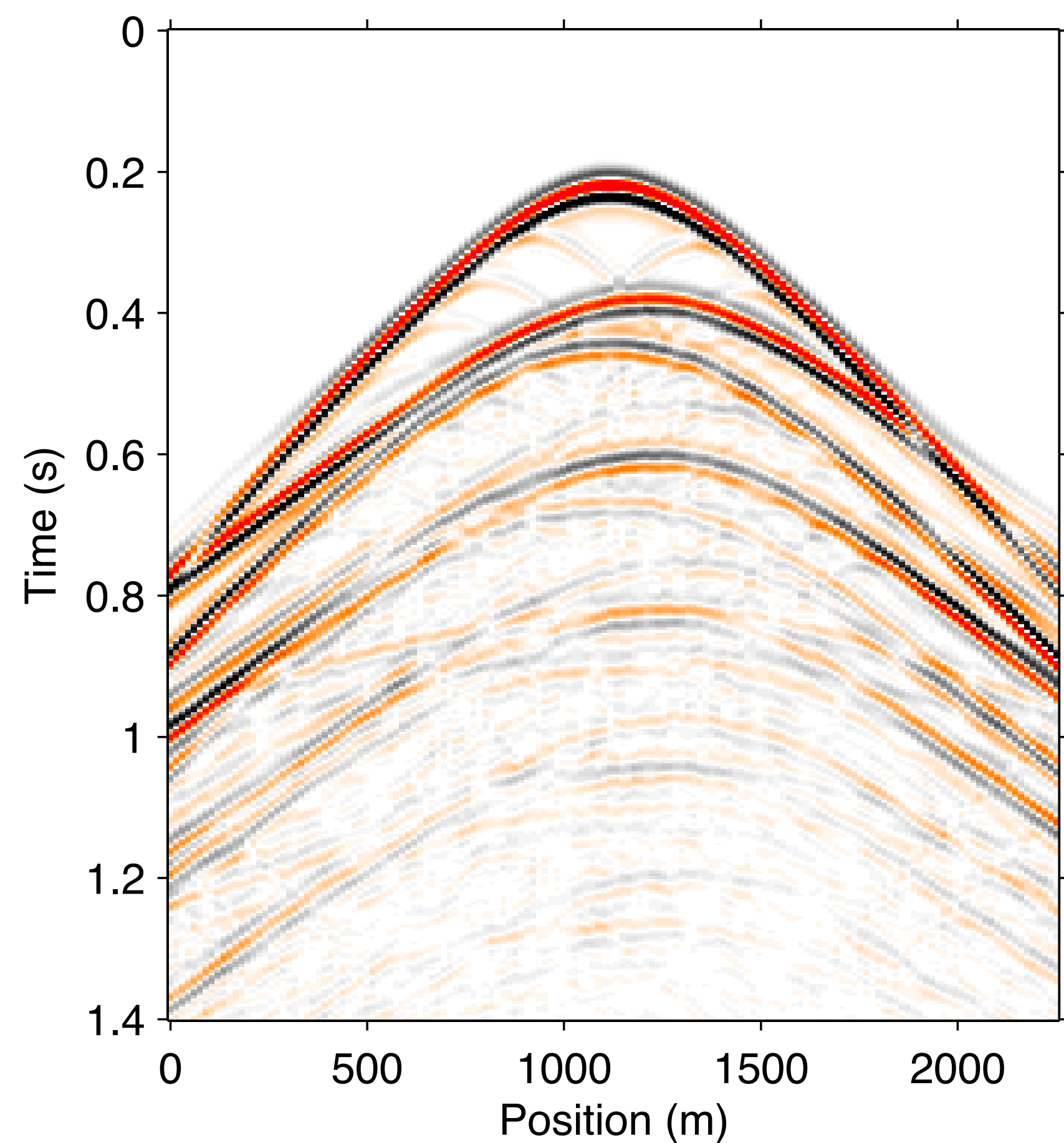
Previous Q is *discarded*

Interpolation method of G not important, just can't alias. Simple constant NMO (i.e., at water velocity) + linear interpolation works fine

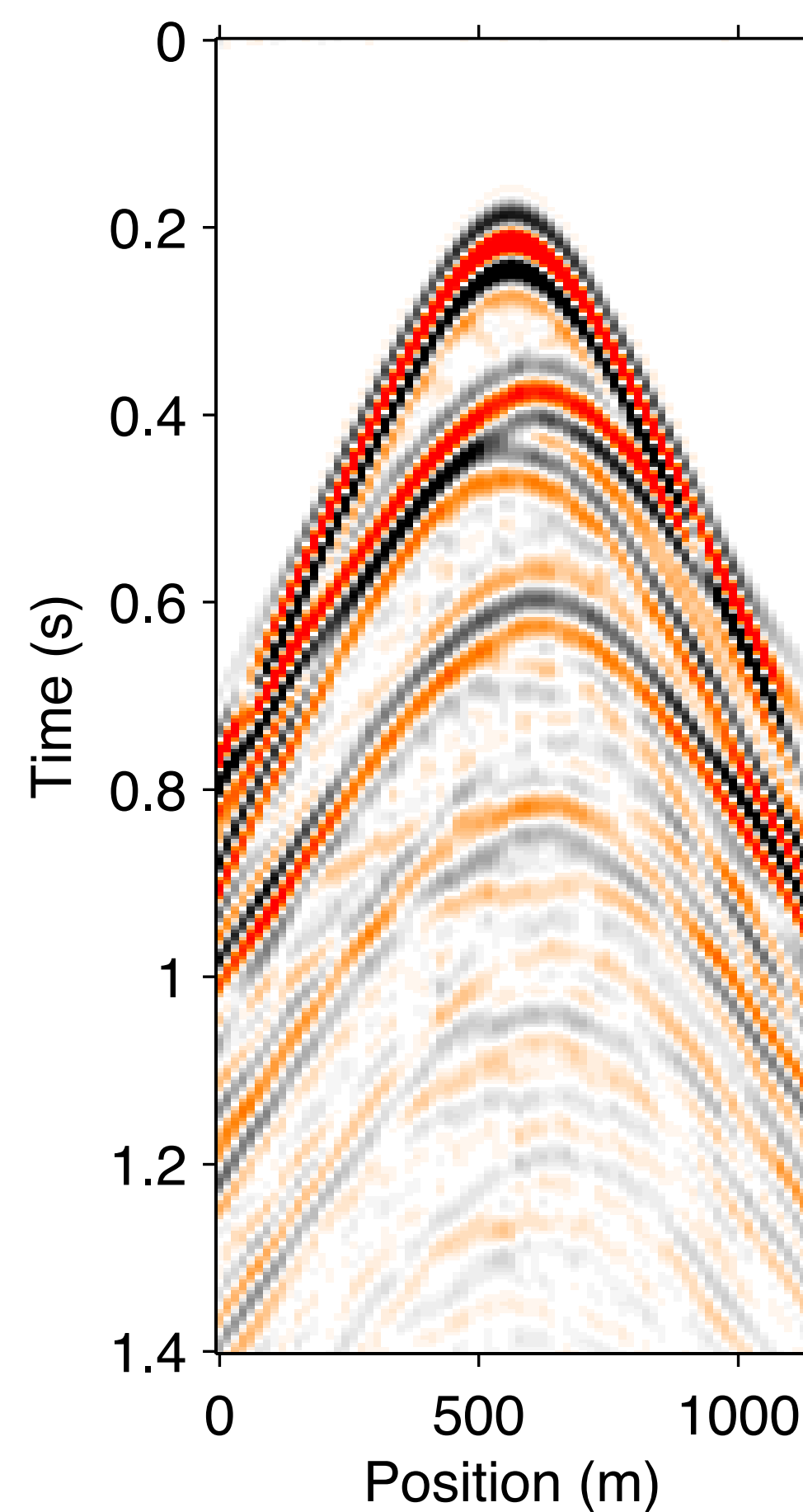
Warm-starting/continuation from coarse solution

Example

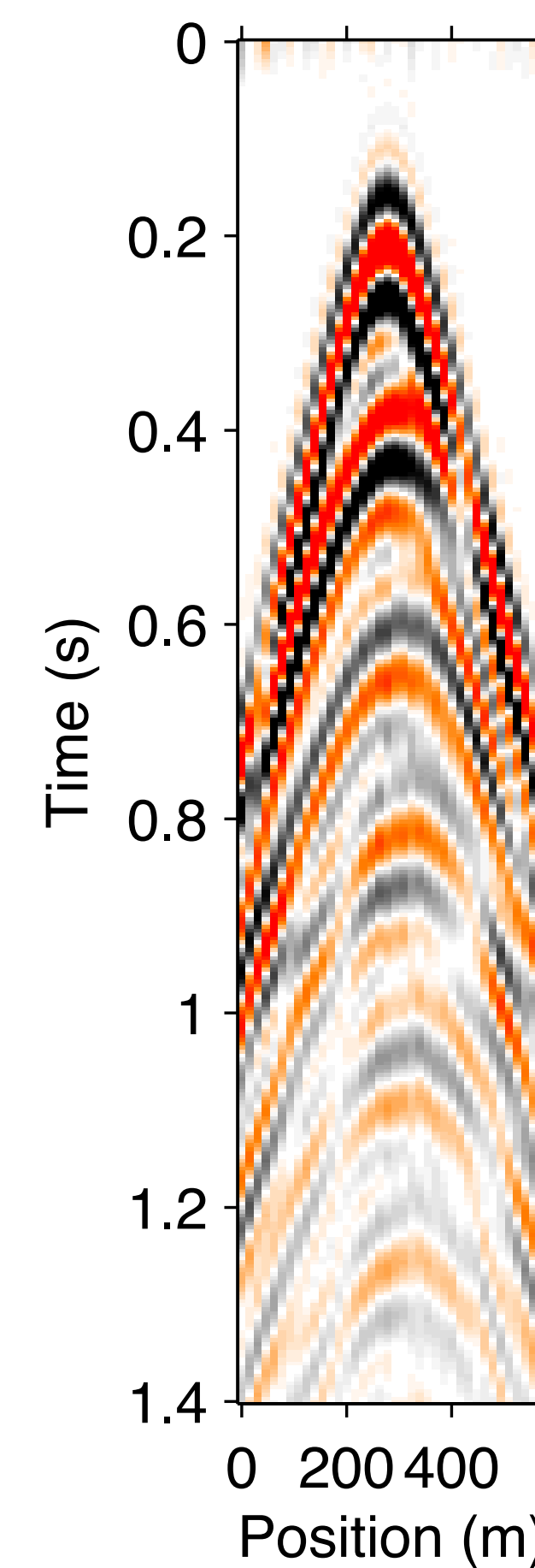
Original (dx = 15m)



2x decimated
lowpass 30Hz



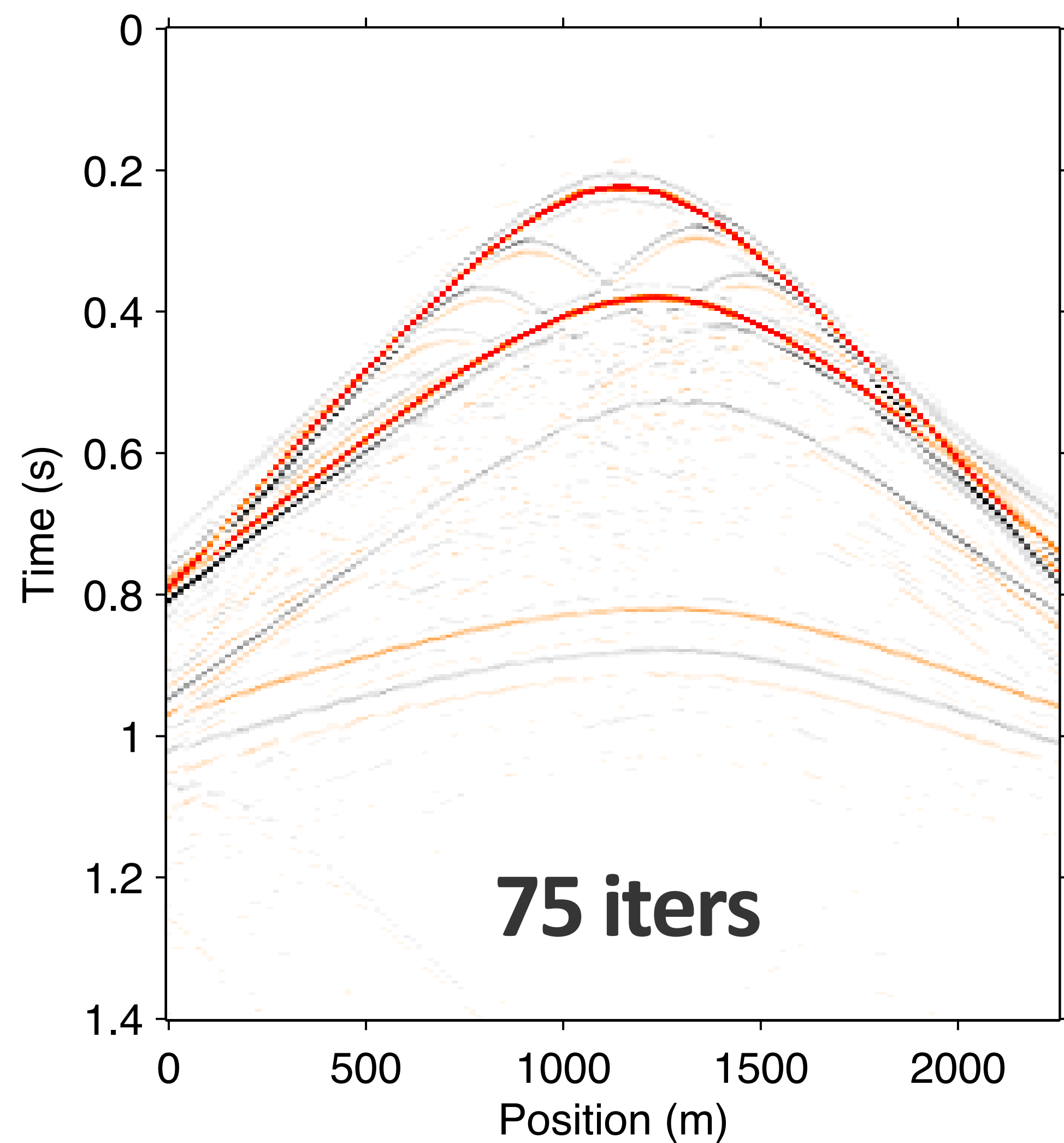
4x decimated
lowpass 15Hz



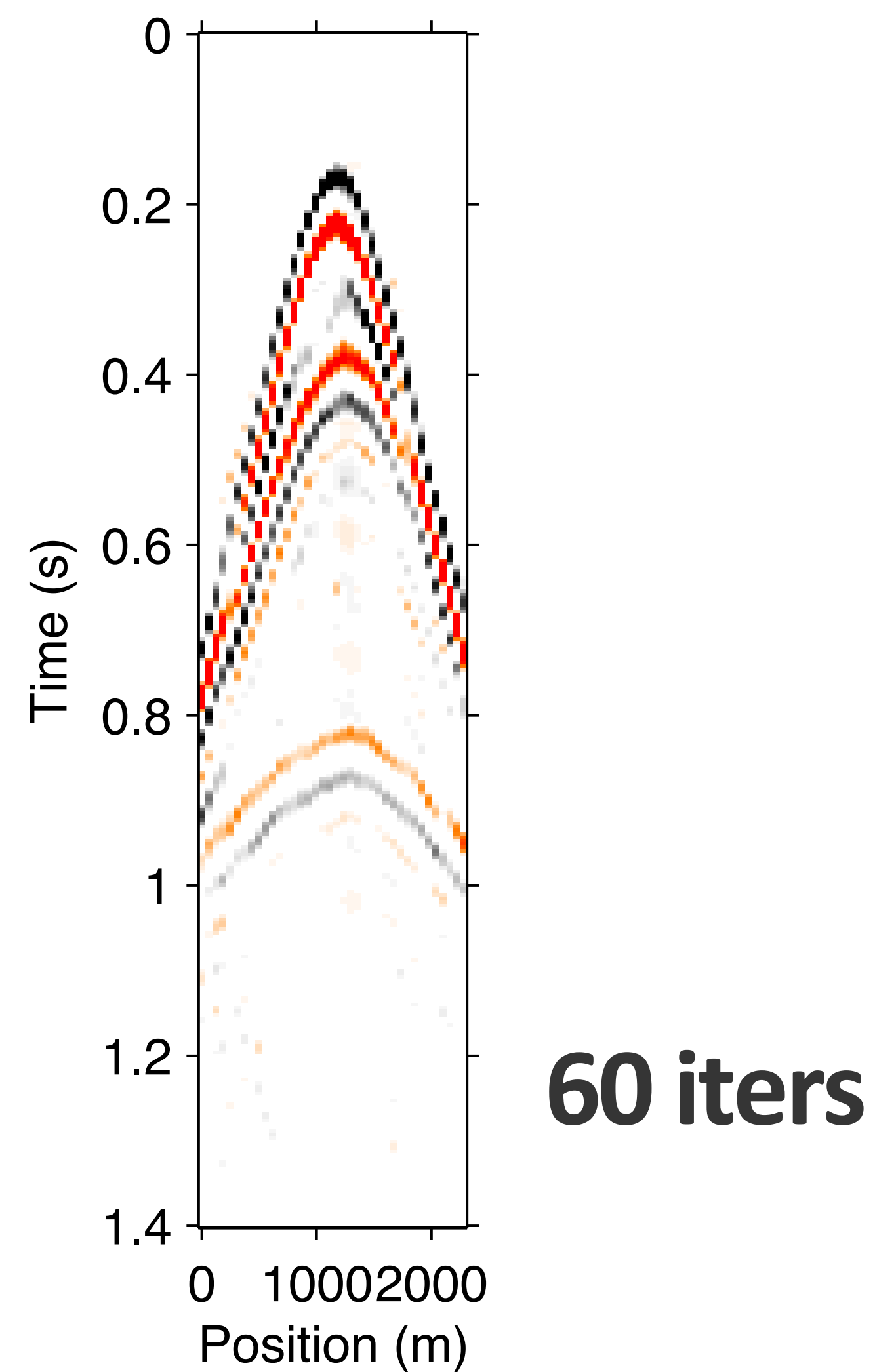
Warm-starting/continuation from coarse solution

Example

Solution of full data



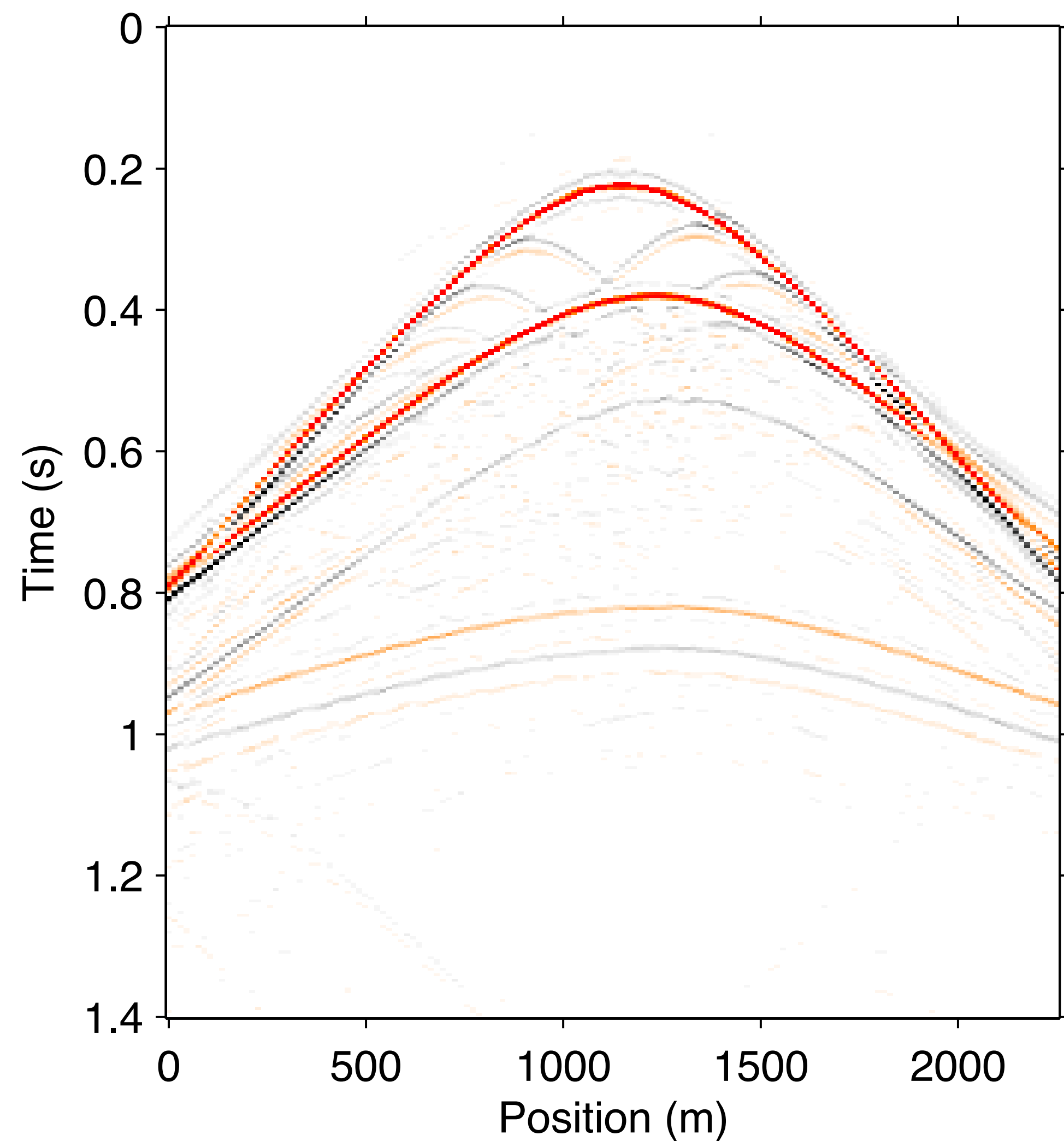
Solution of 4x decimated data



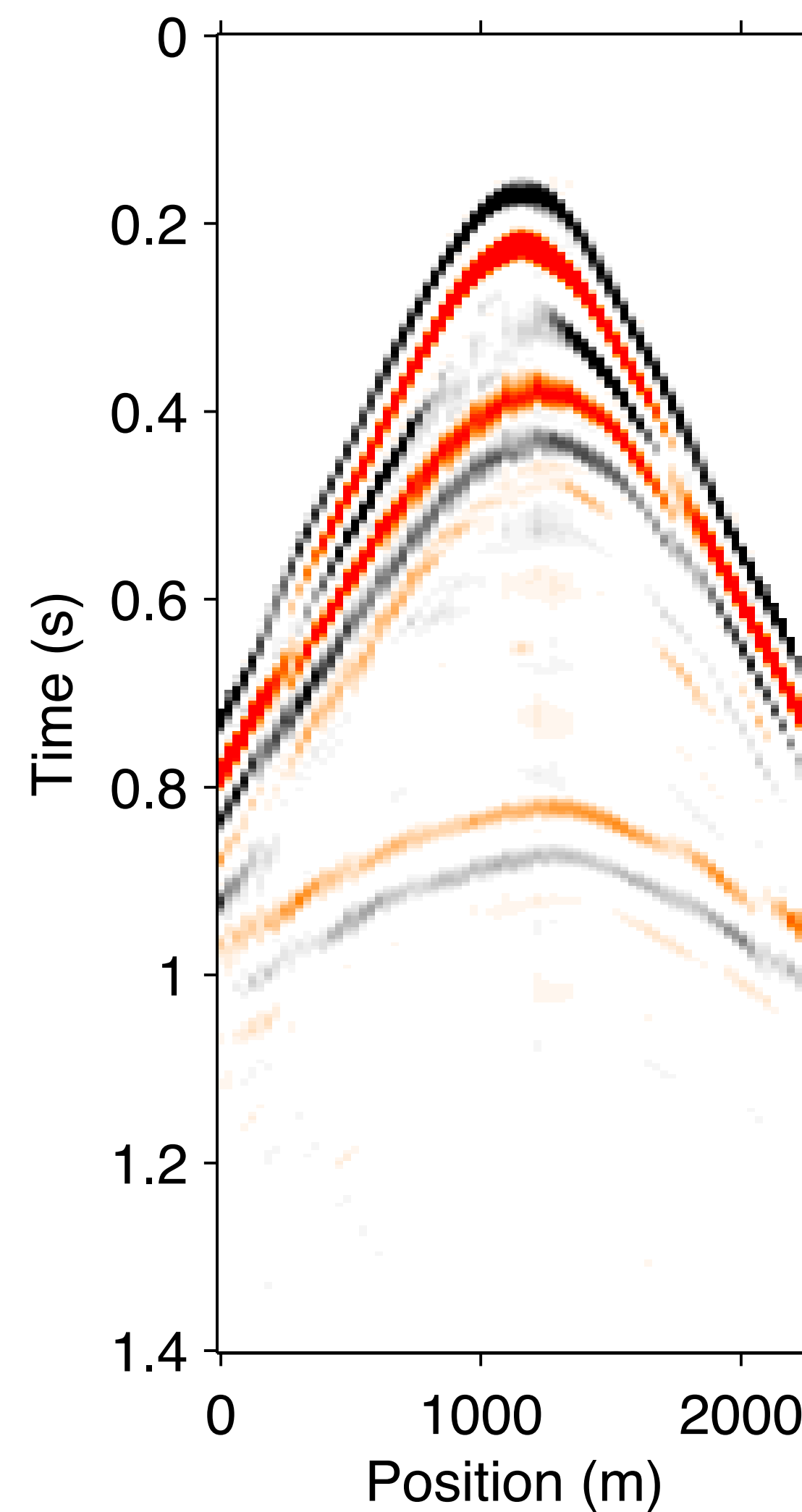
Warm-starting/continuation from coarse solution

Example

Solution of full data



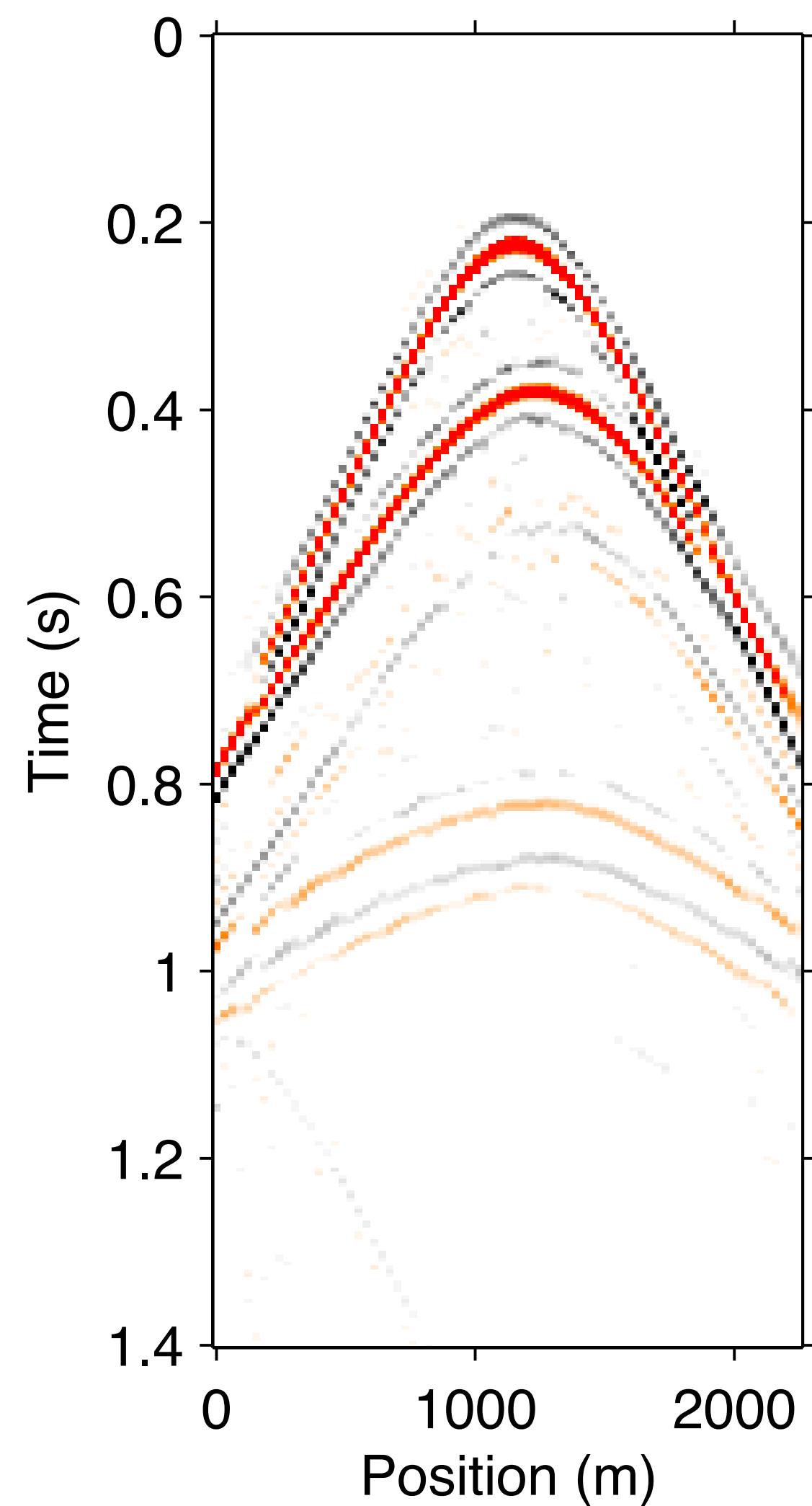
Solution of 4x decimated data
1600m/s NMO, linear interp 2x



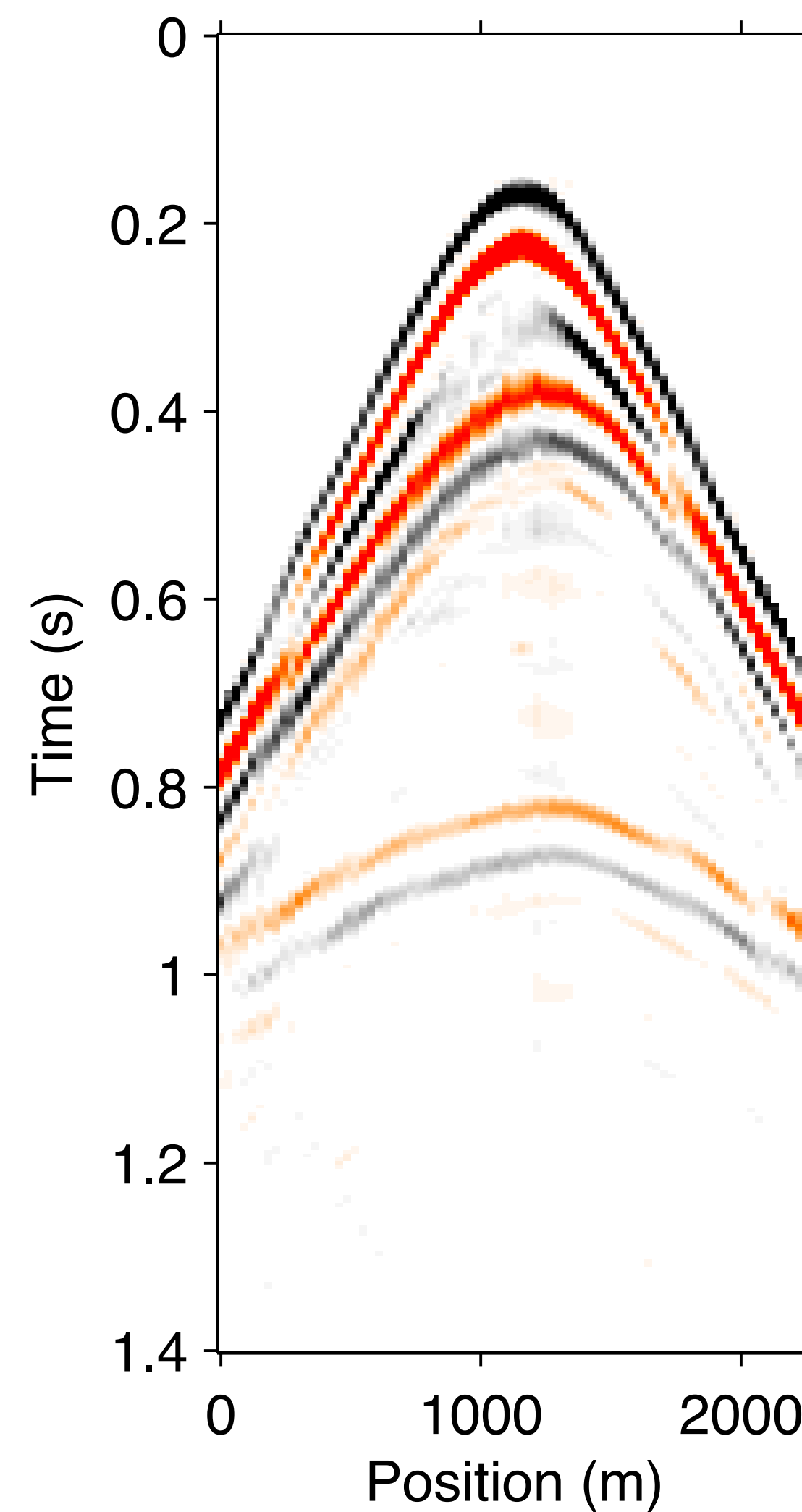
Warm-starting/continuation from coarse solution

Example

Solution of 2x decimated data



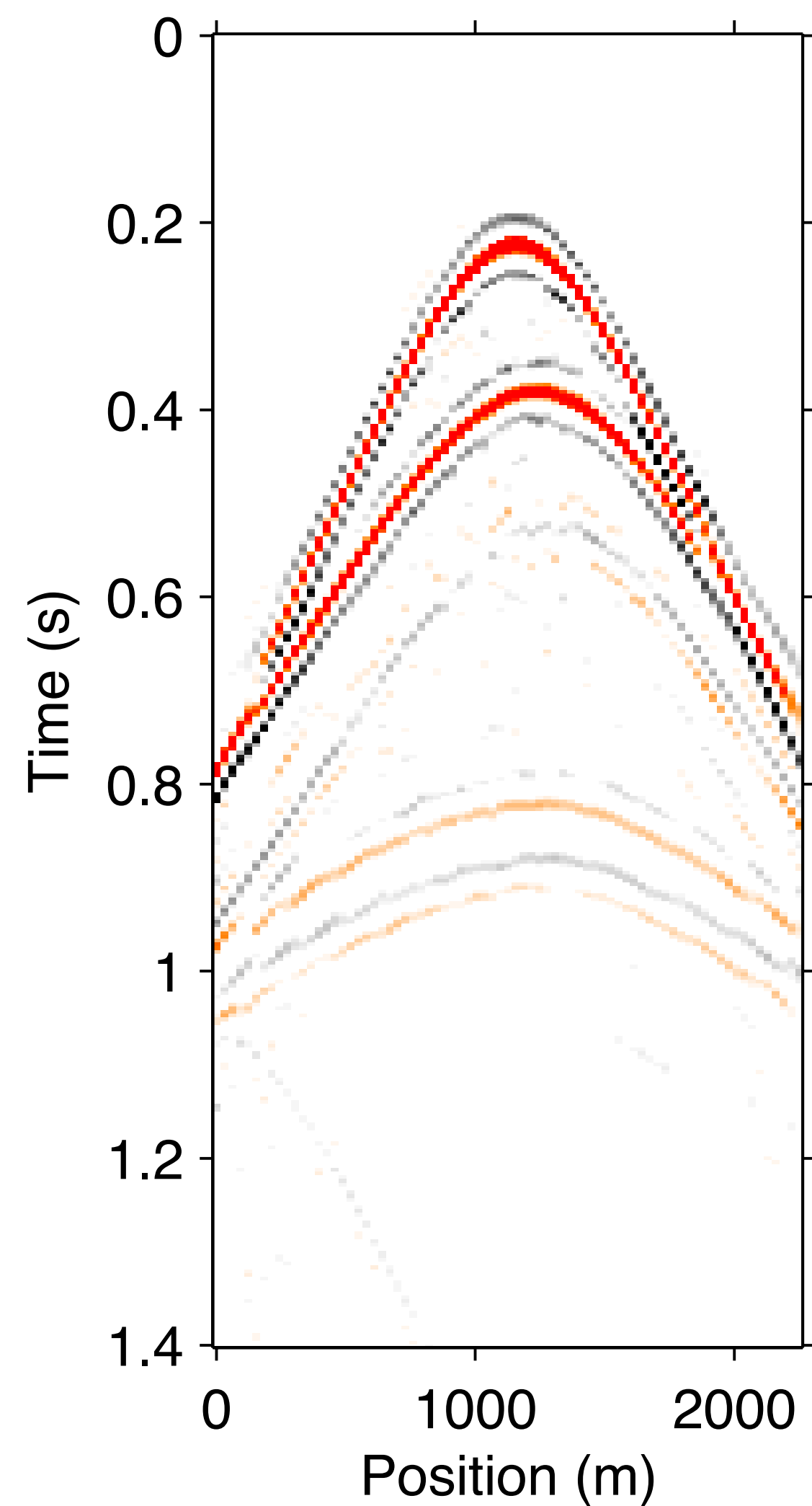
Solution of 4x decimated data
1600m/s NMO, linear interp 2x



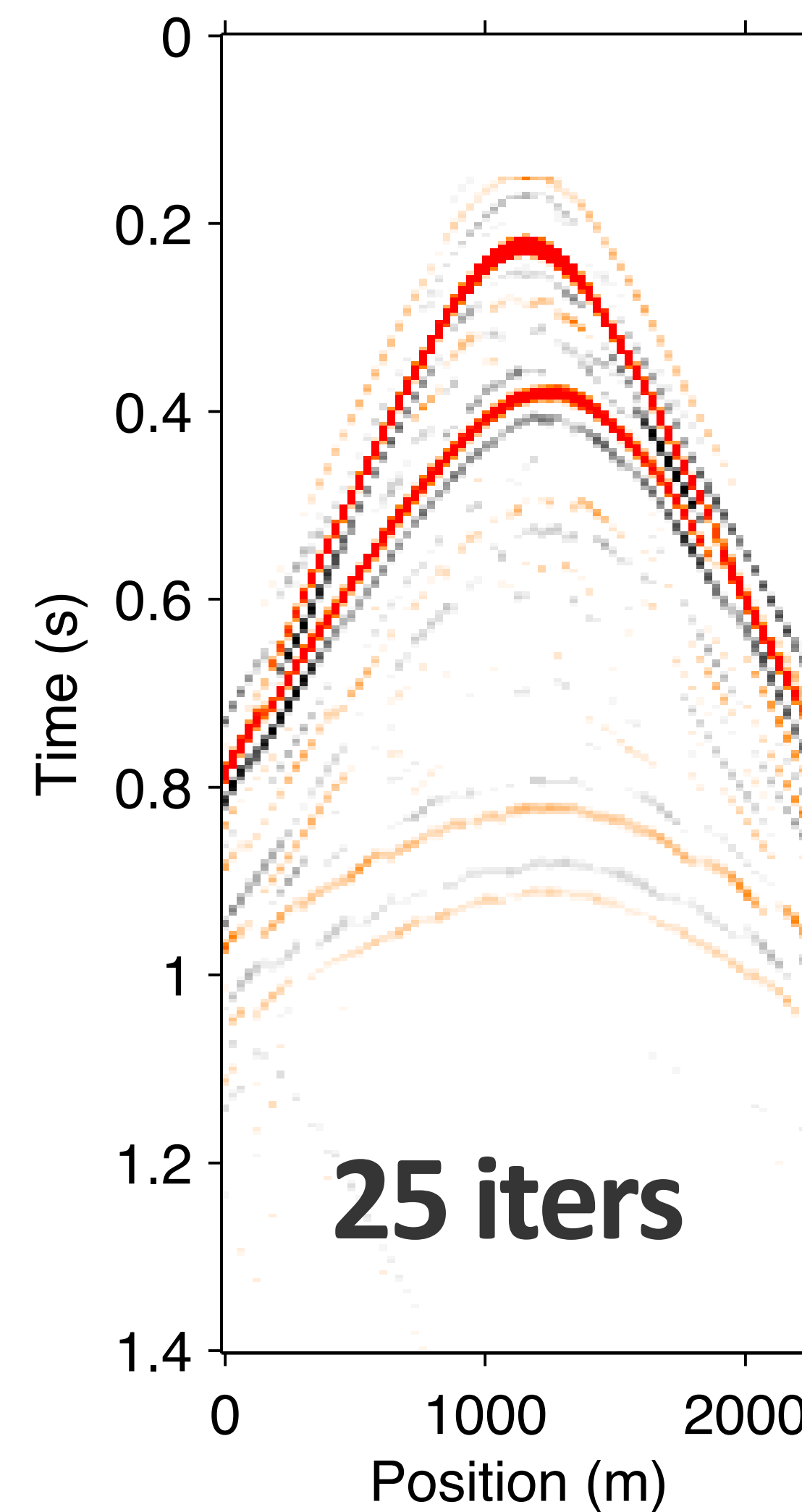
Warm-starting/continuation from coarse solution

Example

Solution of 2x decimated data



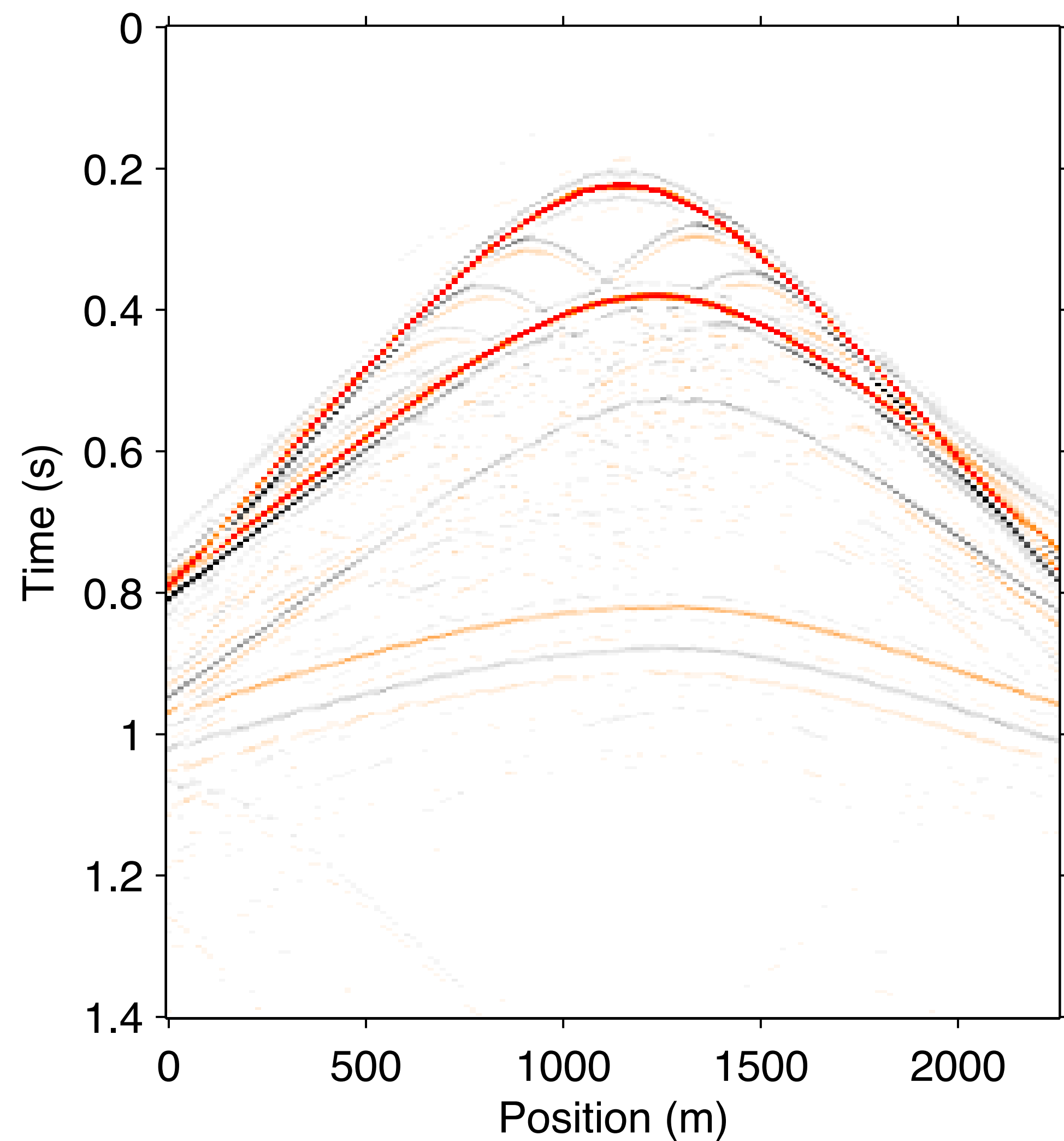
Solution on 2x dec data
continuation from 4x dec solution



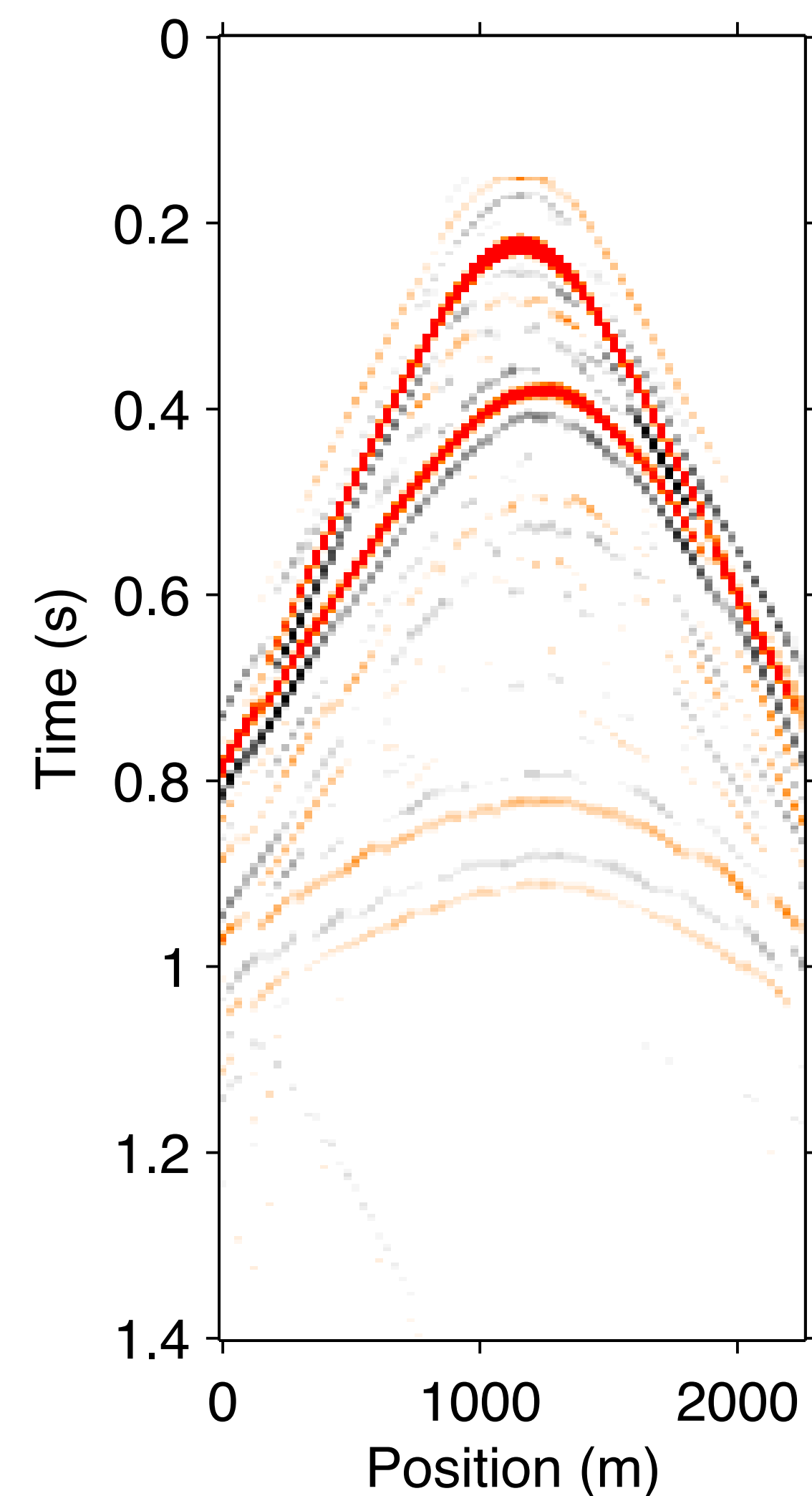
Warm-starting/continuation from coarse solution

Example

Solution of full data



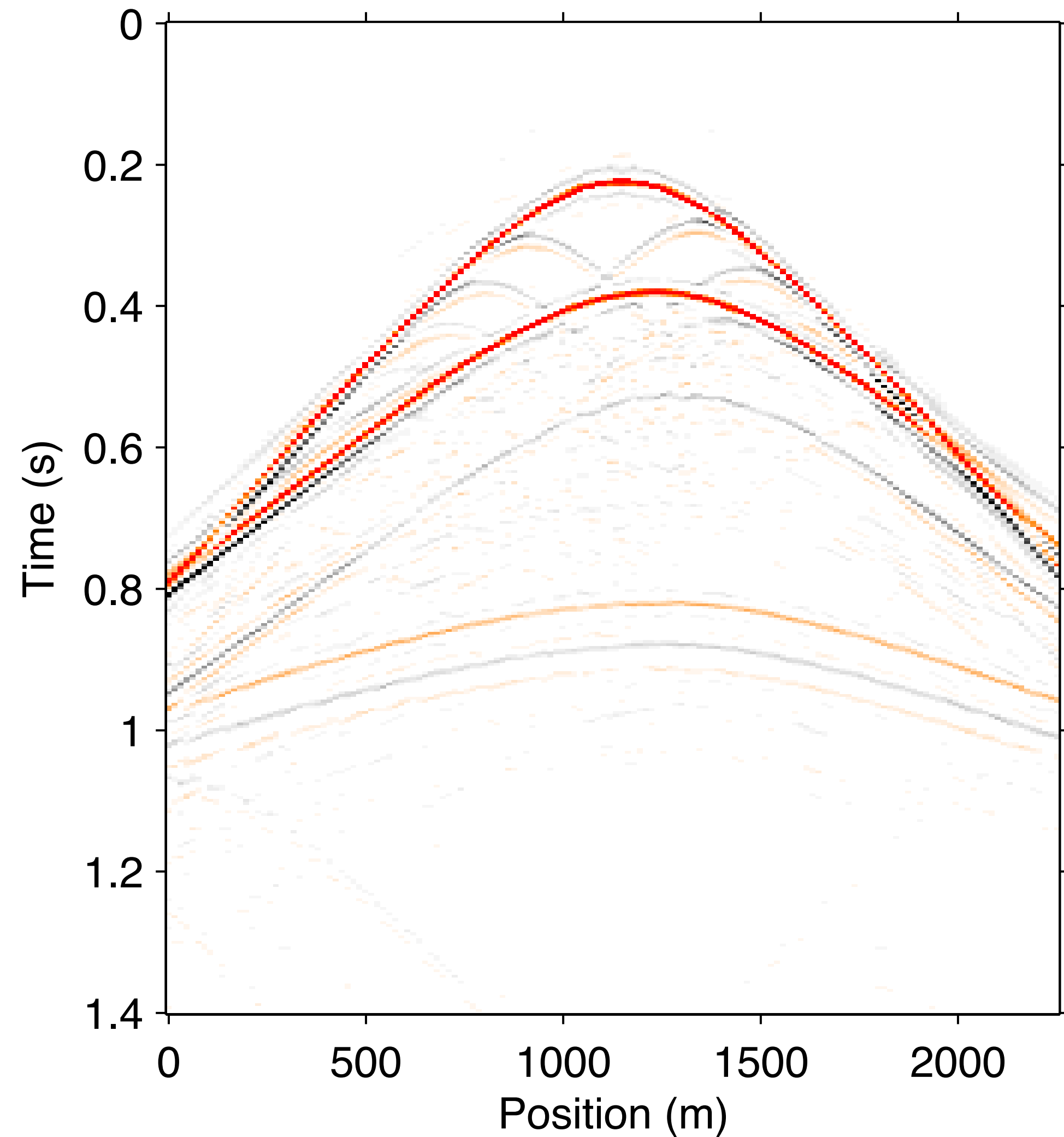
Solution on 2x dec data
continuation from 4x dec solution



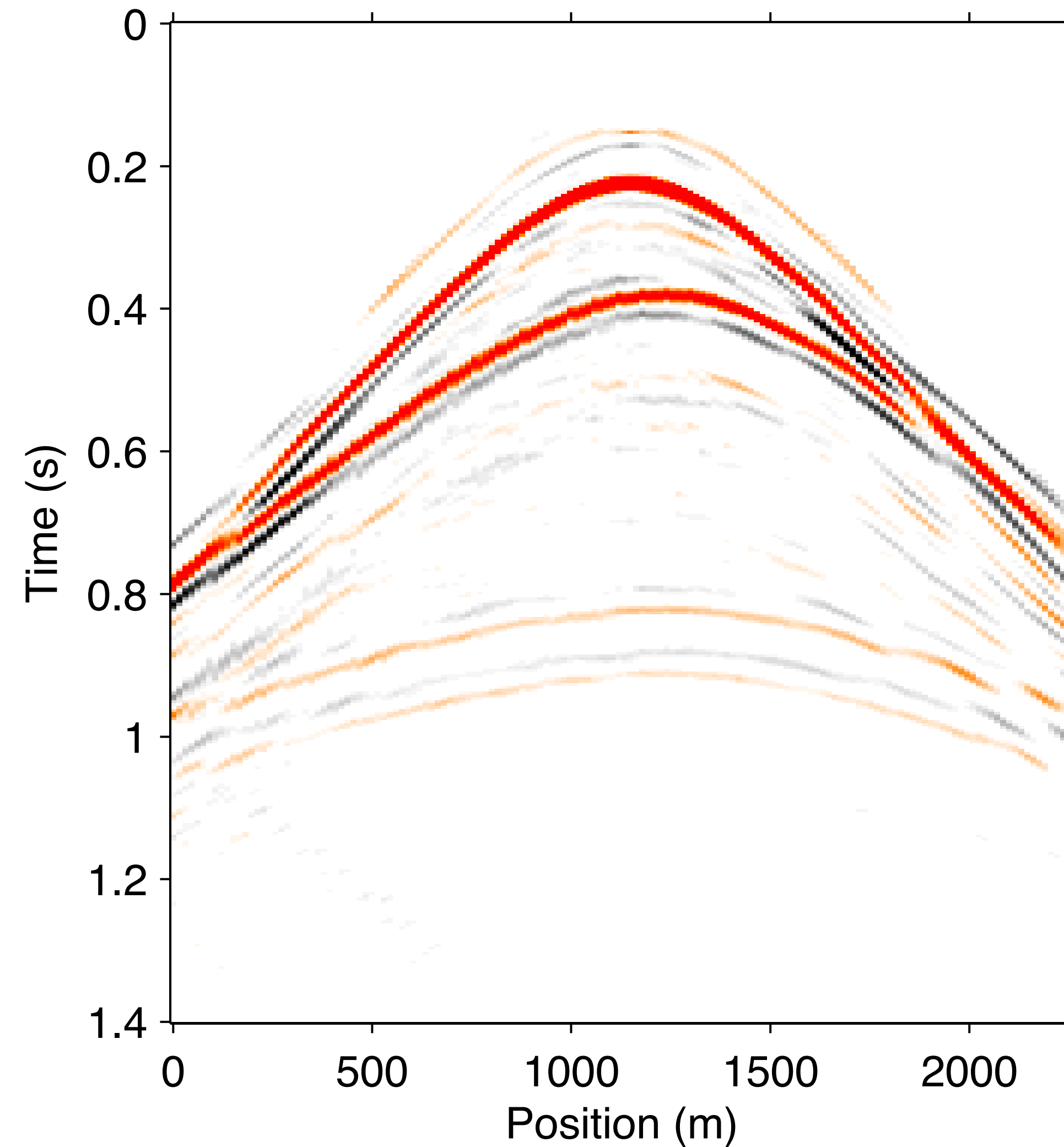
Warm-starting/continuation from coarse solution

Example

Solution of full data



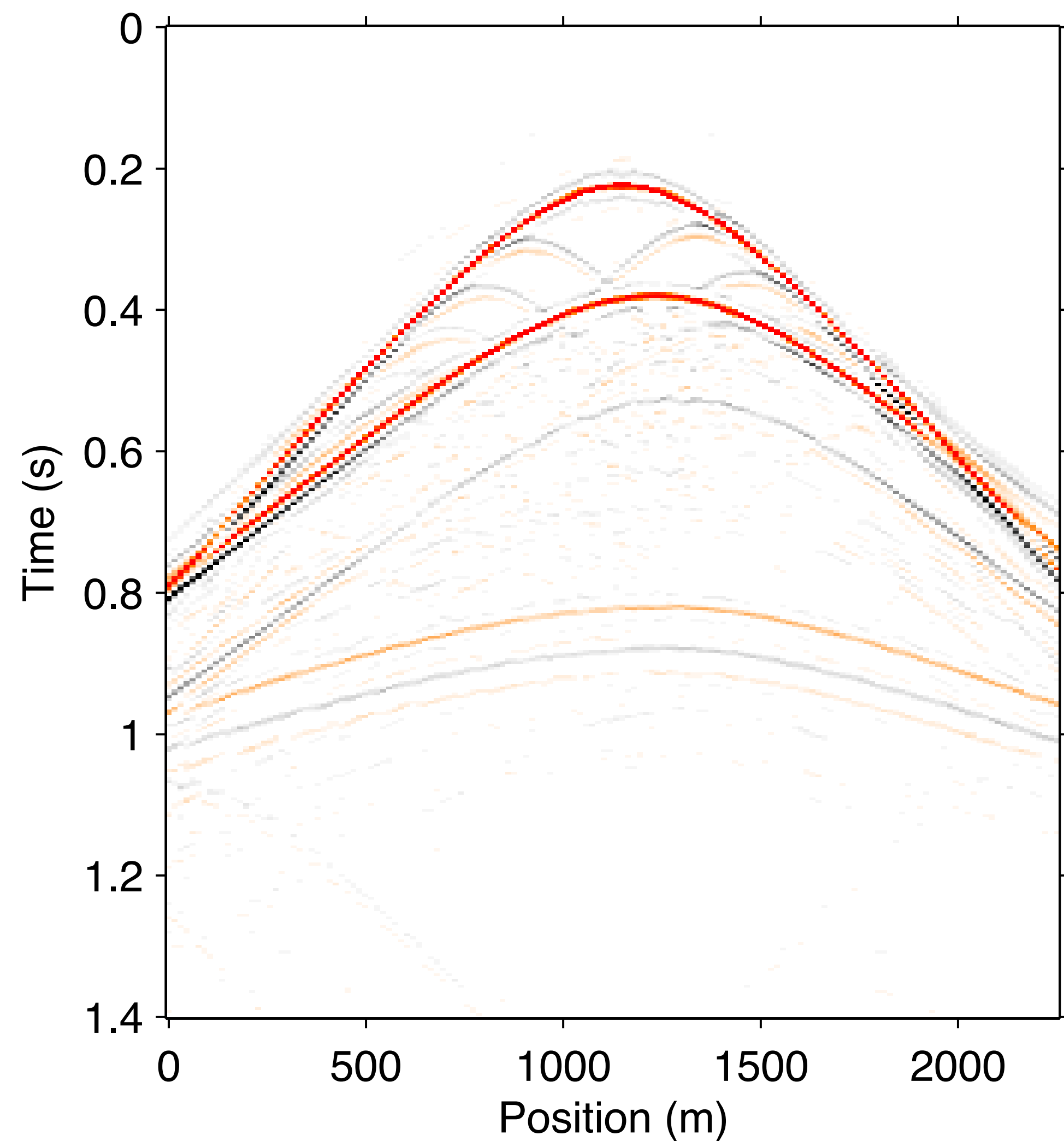
Solution on 2x dec data > interp 2x
continuation from 4x dec solution



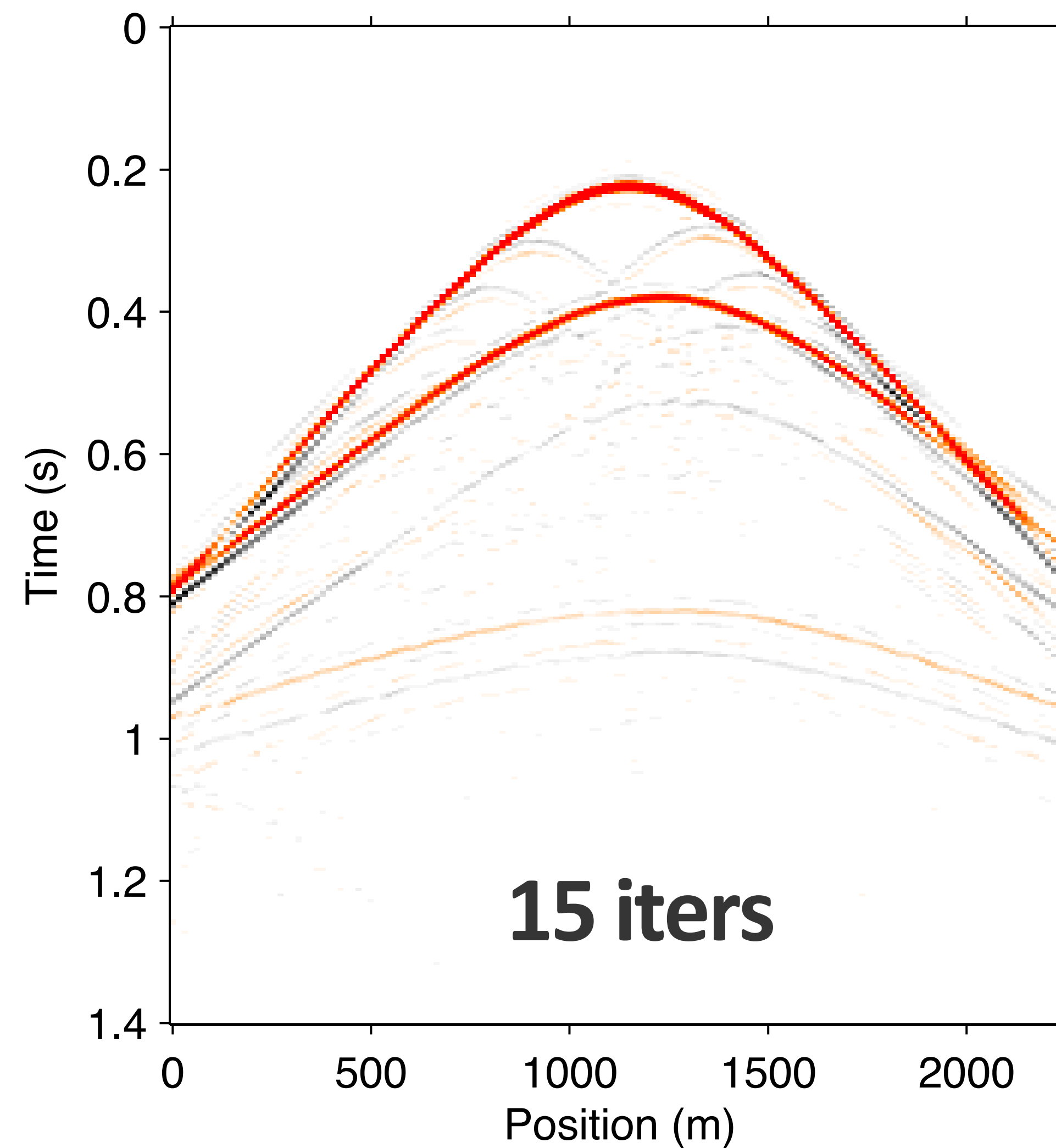
Warm-starting/continuation from coarse solution

Example

Solution of full data

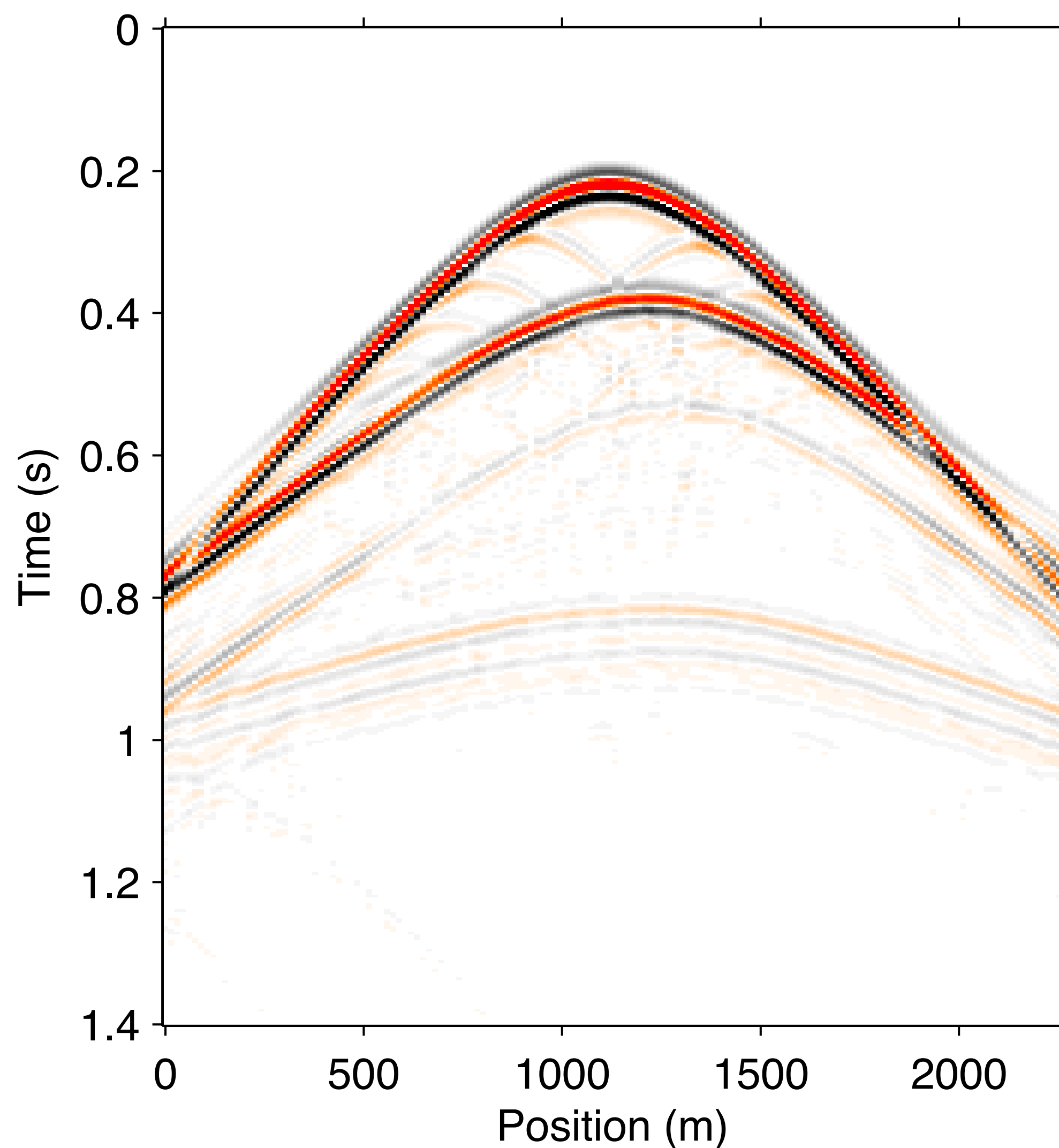


Solution on 2x dec data > interp 2x
continuation from 4x thru 2x solution



Warm-starting/continuation from coarse solution

Example

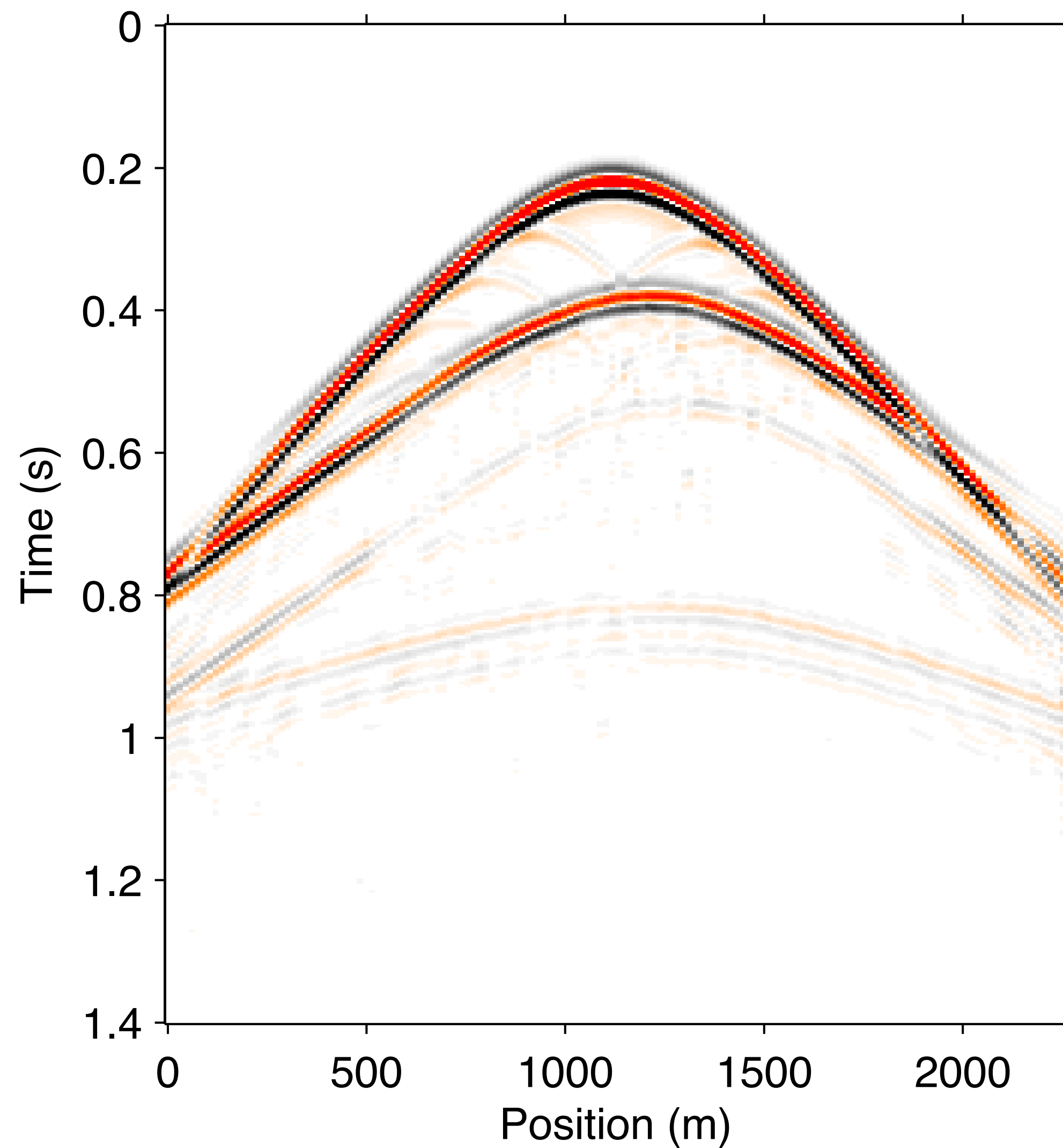


Direct Primary

Solved with plain algorithm from finest scale data

Warm-starting/continuation from coarse solution

Example



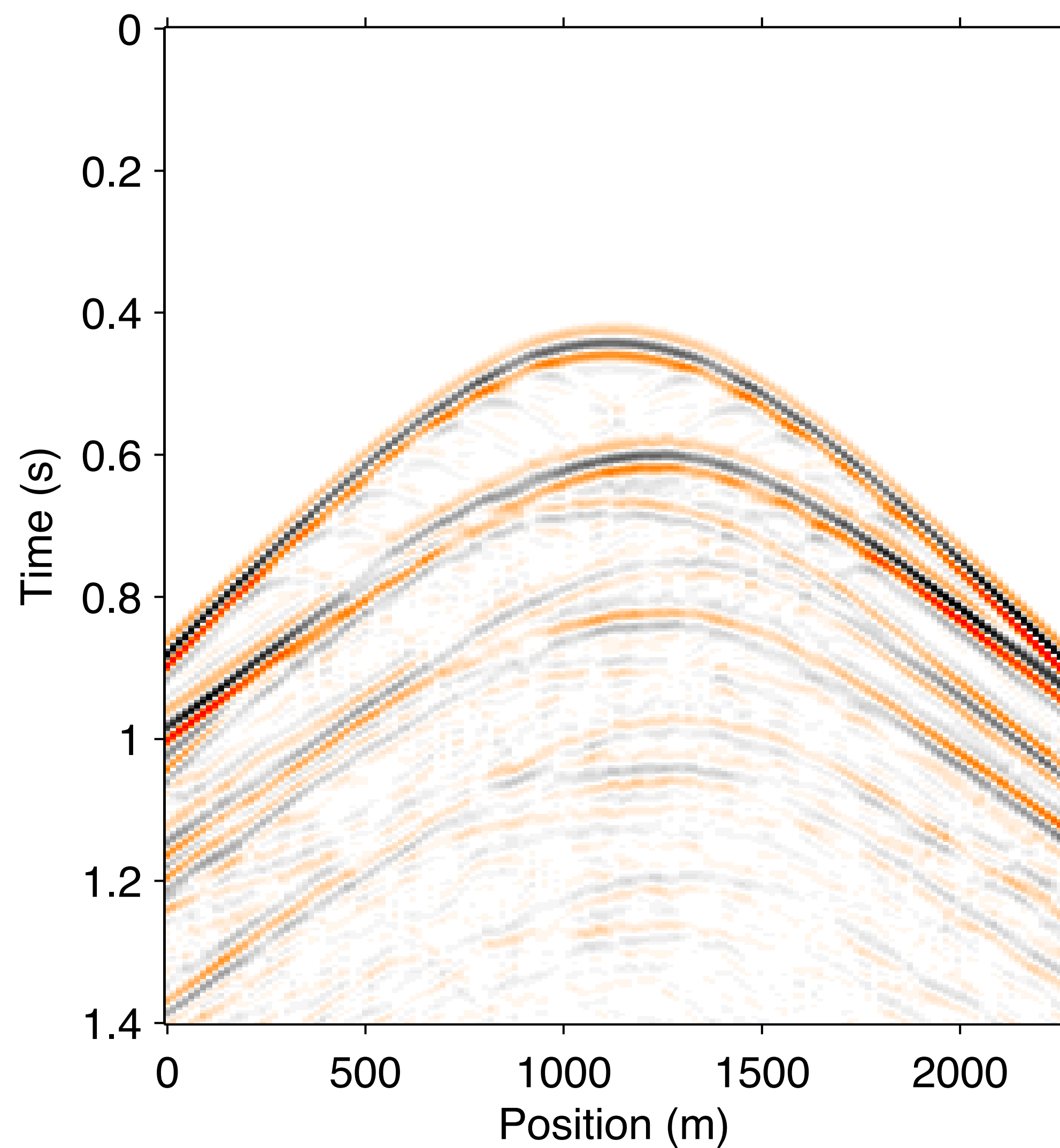
Direct Primary

Solved with spatial sampling continuation

$$dx = 60\text{m} > 30\text{m} > 15\text{m}$$

Warm-starting/continuation from coarse solution

Example

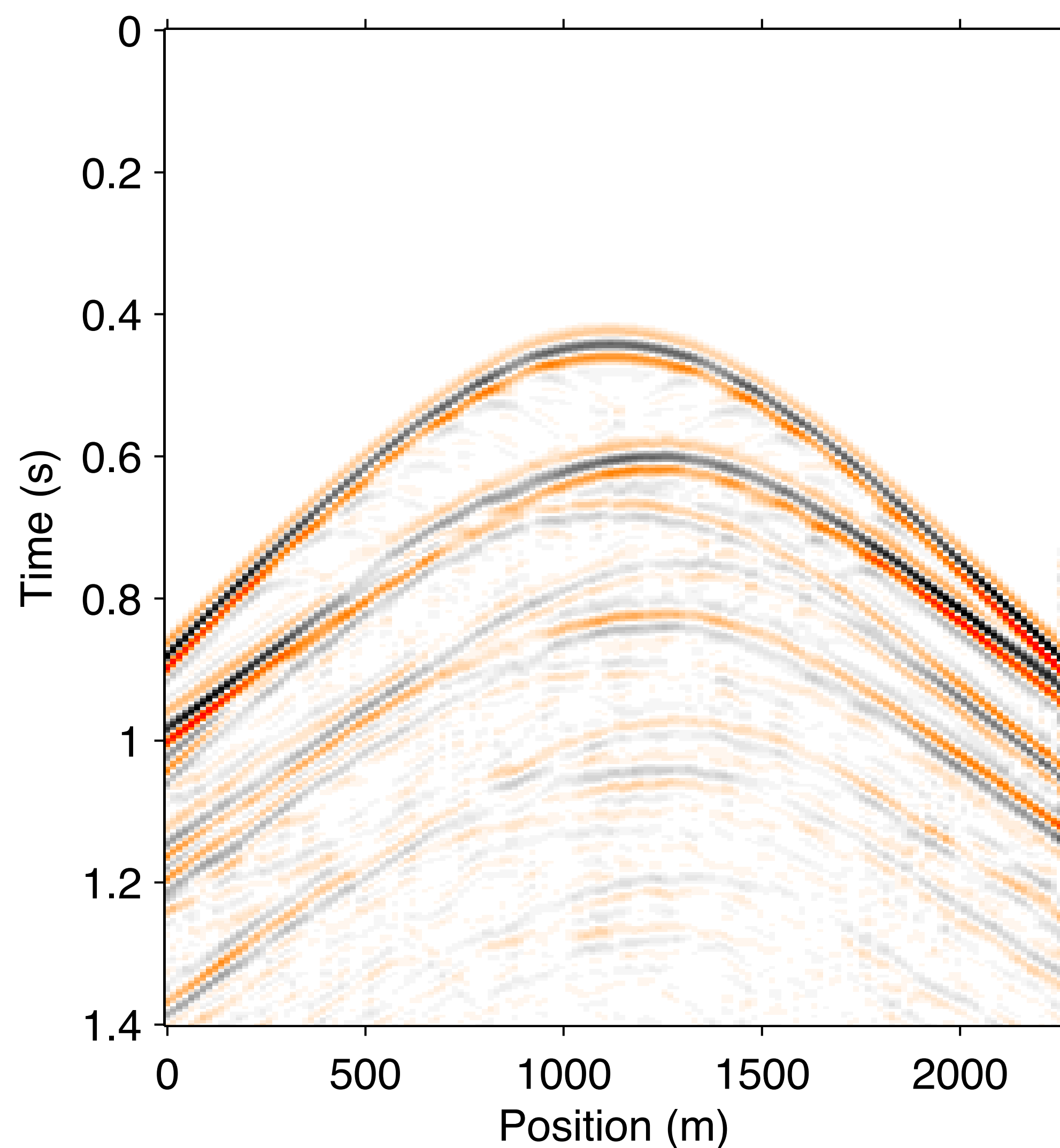


Predicted Surface Multiple

Solved with plain algorithm from finest scale data

Warm-starting/continuation from coarse solution

Example



Predicted Surface Multiple

Solved with spatial sampling continuation

$dx = 60\text{m} > 30\text{m} > 15\text{m}$

Significant speedup from bootstrapping (in 2D)

Per-iteration FLOPs cost (one forward/adjoint): $n = n_{rcv} = n_{src}$

$$\text{Cost}(n) = \underbrace{\mathcal{O}(2n_t n^2 \log n_t)}_{\text{2 times FFT}} + \underbrace{\mathcal{O}(n_f n^3)}_{\text{computing MCG \& sum in FX}}$$

$$\text{Cost}\left(\frac{1}{2}n\right) = \frac{1}{4}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{8}\mathcal{O}(n_f n^3)$$

$$\text{Cost}\left(\frac{1}{4}n\right) = \frac{1}{16}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{64}\mathcal{O}(n_f n^3)$$

Significant speedup from bootstrapping (in 2D)

Per-iteration FLOPs cost (one forward/adjoint): $n = n_{rcv} = n_{src}$

$$\text{Cost}(n) = \underbrace{\mathcal{O}(2n_t n^2 \log n_t)}_{\text{2 times FFT}} + \underbrace{\mathcal{O}(n_f n^3)}_{\text{computing MCG \& sum in FX}}$$

$$\text{Cost} \left(\frac{1}{2}n, \frac{1}{2}n_f \right) = \frac{1}{4} \mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{16} \mathcal{O}(n_f n^3)$$

$$\text{Cost} \left(\frac{1}{4}n, \frac{1}{4}n_f \right) = \frac{1}{16} \mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{128} \mathcal{O}(n_f n^3)$$

Significant speedup from bootstrapping (in 3D)

Per-iteration FLOPs cost (one forward/adjoint): $n = nx_{rcv} = ny_{rcv} = nx_{src} = ny_{src}$

$$\text{Cost}(n) = \mathcal{O}(2n_t n^4 \log n_t) + \mathcal{O}(n_f n^6)$$

2 times FFT

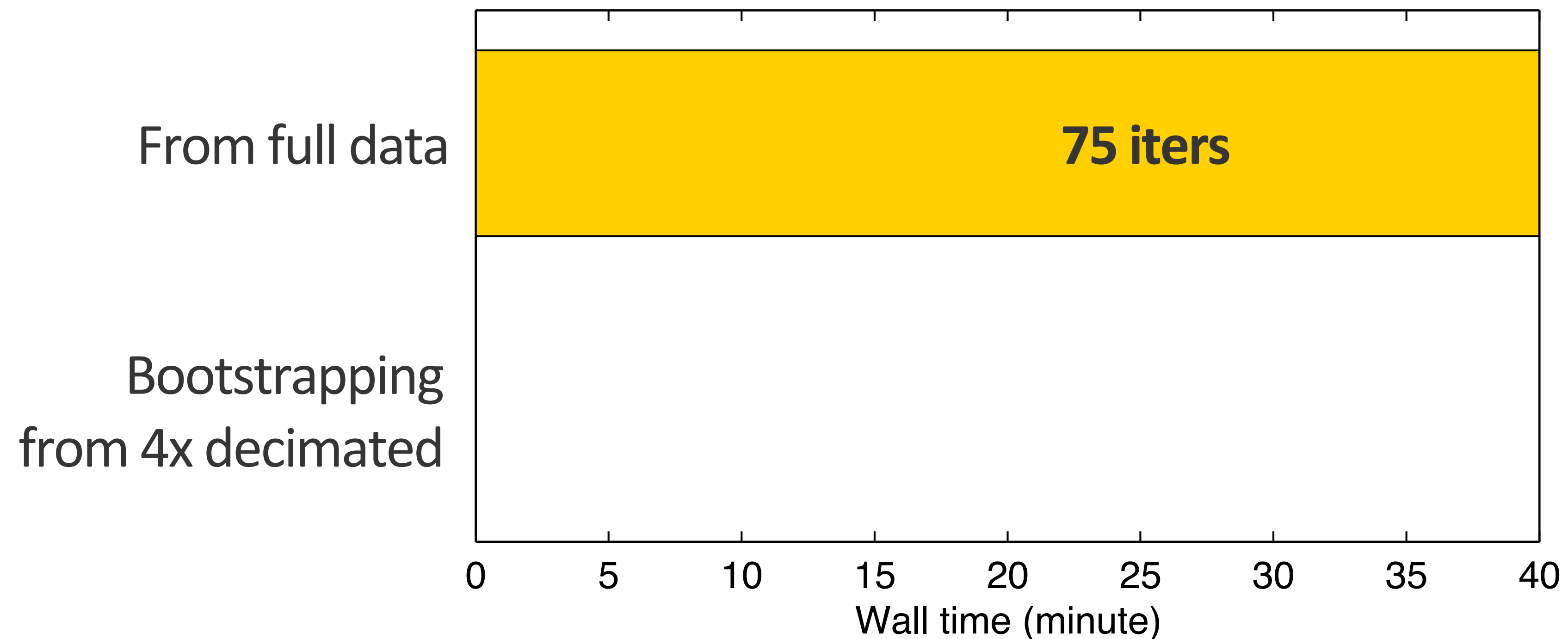
computing MCG & sum in FX

$$\text{Cost} \left(\frac{1}{2}n, \frac{1}{2}n_f \right) = \frac{1}{16} \mathcal{O}(2n_t n^4 \log n_t) + \frac{1}{128} \mathcal{O}(n_f n^6)$$

$$\text{Cost} \left(\frac{1}{4}n, \frac{1}{4}n_f \right) = \frac{1}{256} \mathcal{O}(2n_t n^4 \log n_t) + \frac{1}{8192} \mathcal{O}(n_f n^6)$$

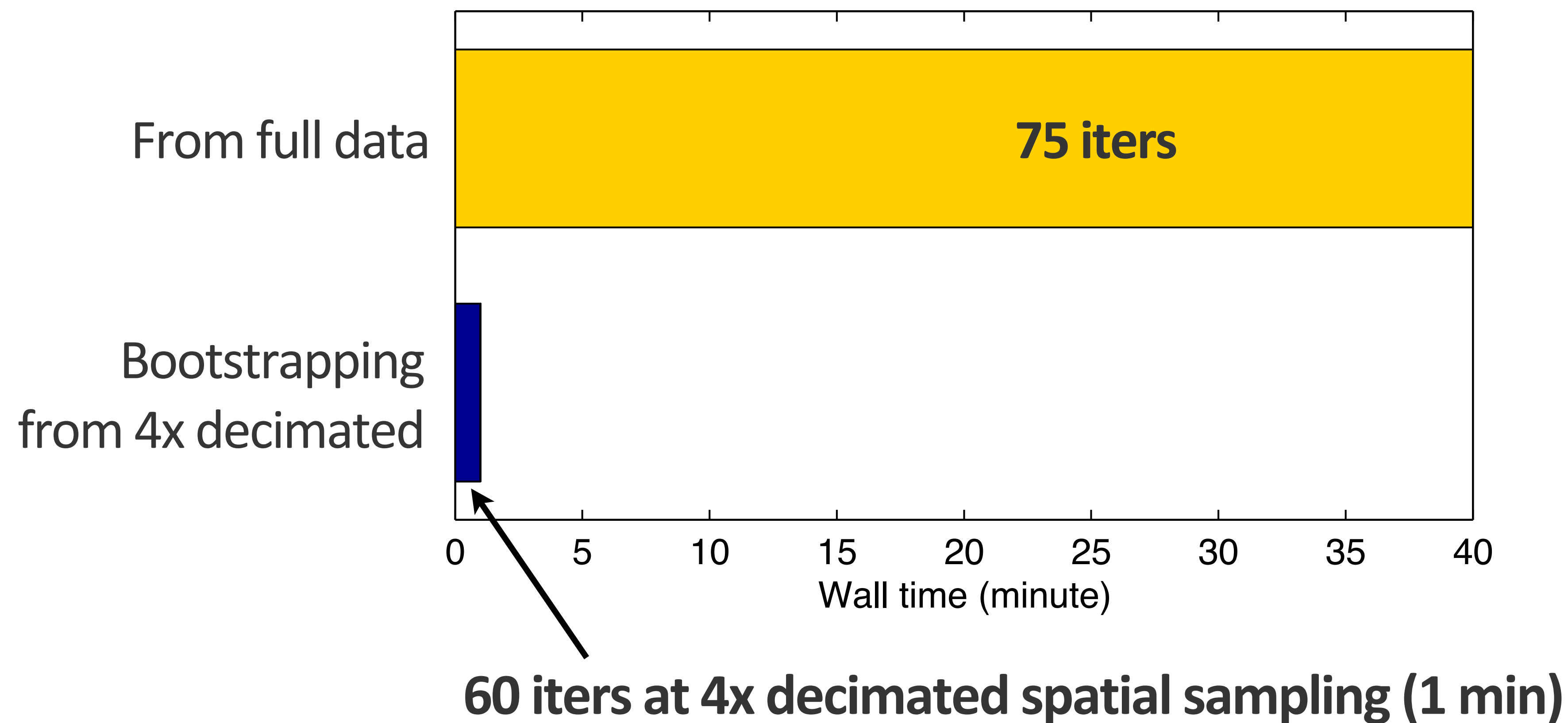
Significant speedup from bootstrapping

Wall times



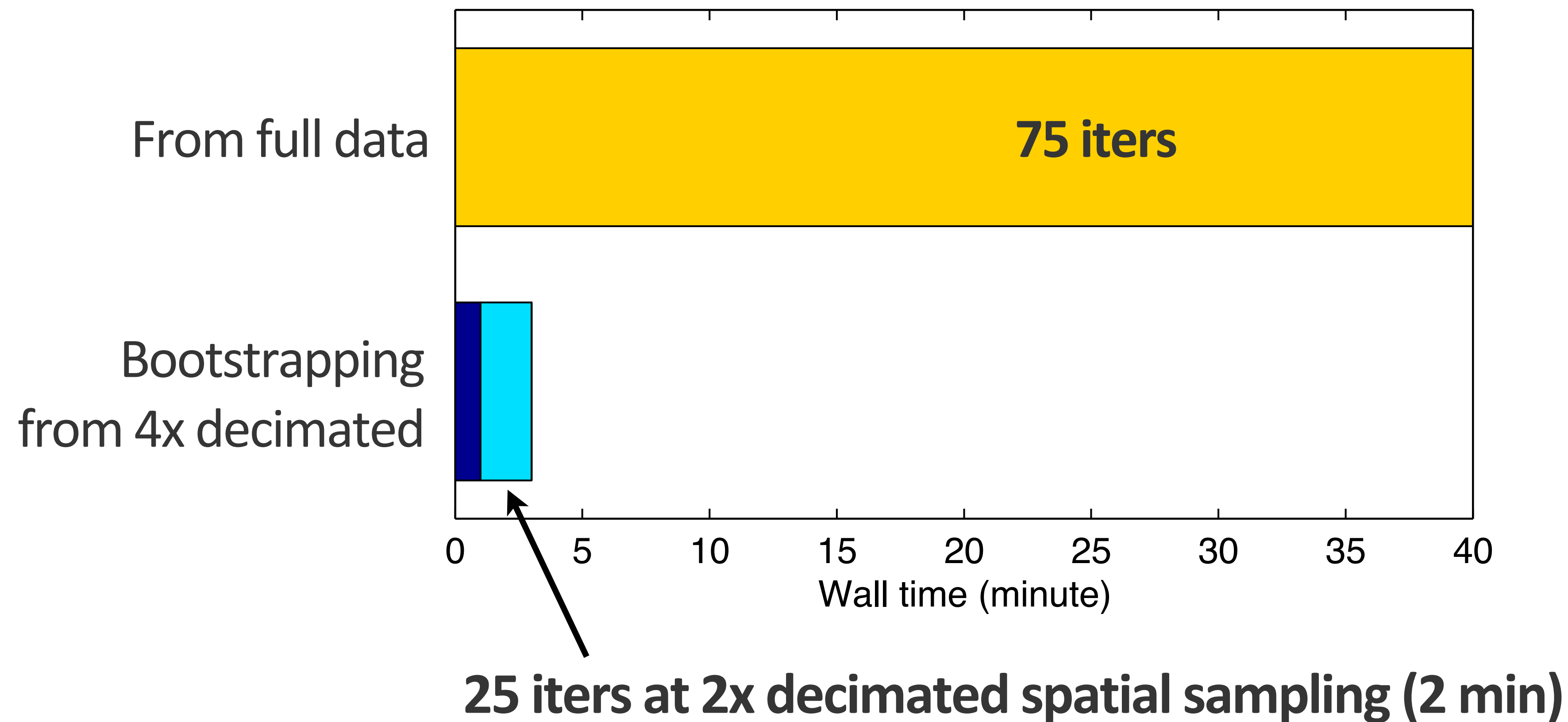
Significant speedup from bootstrapping

Wall times



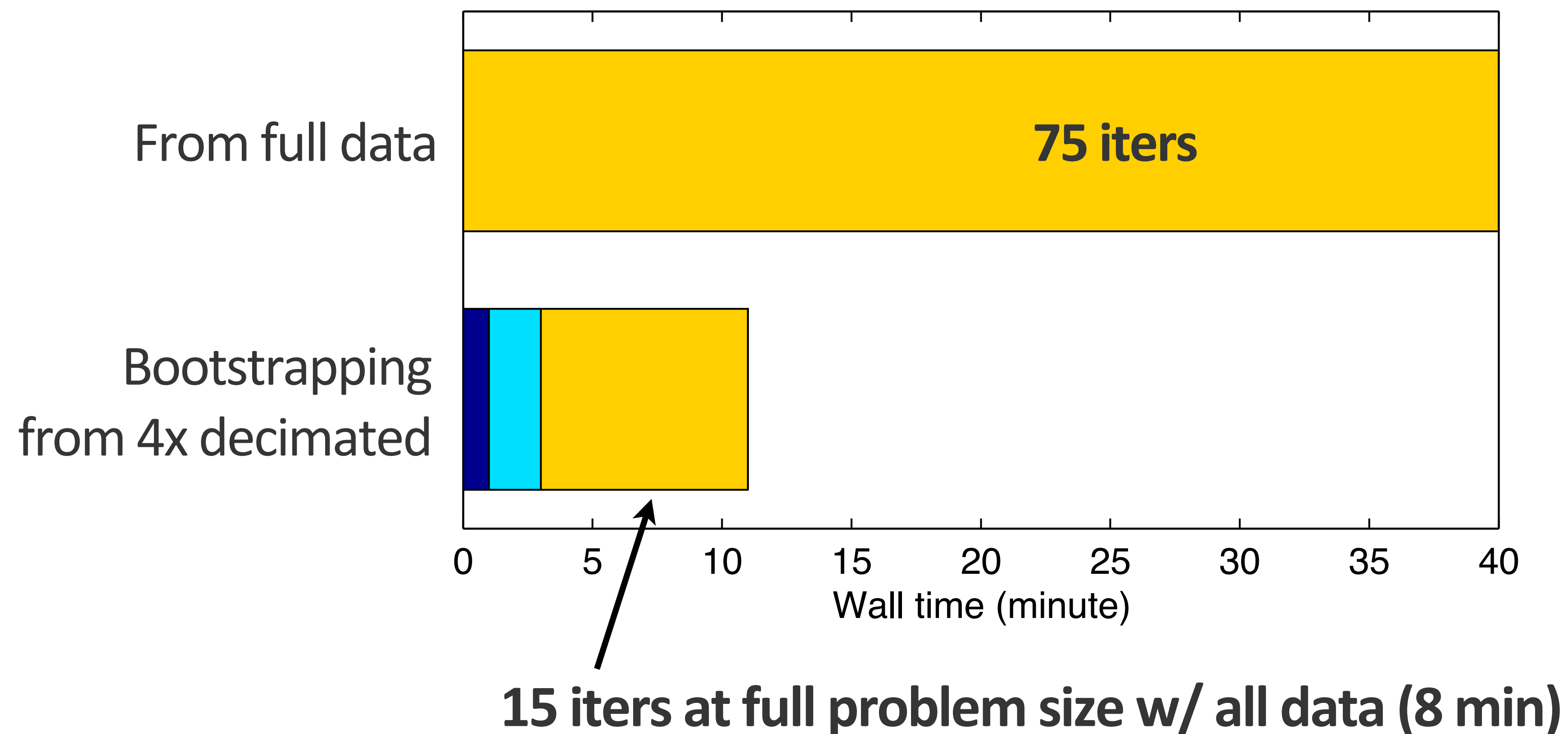
Significant speedup from bootstrapping

Wall times



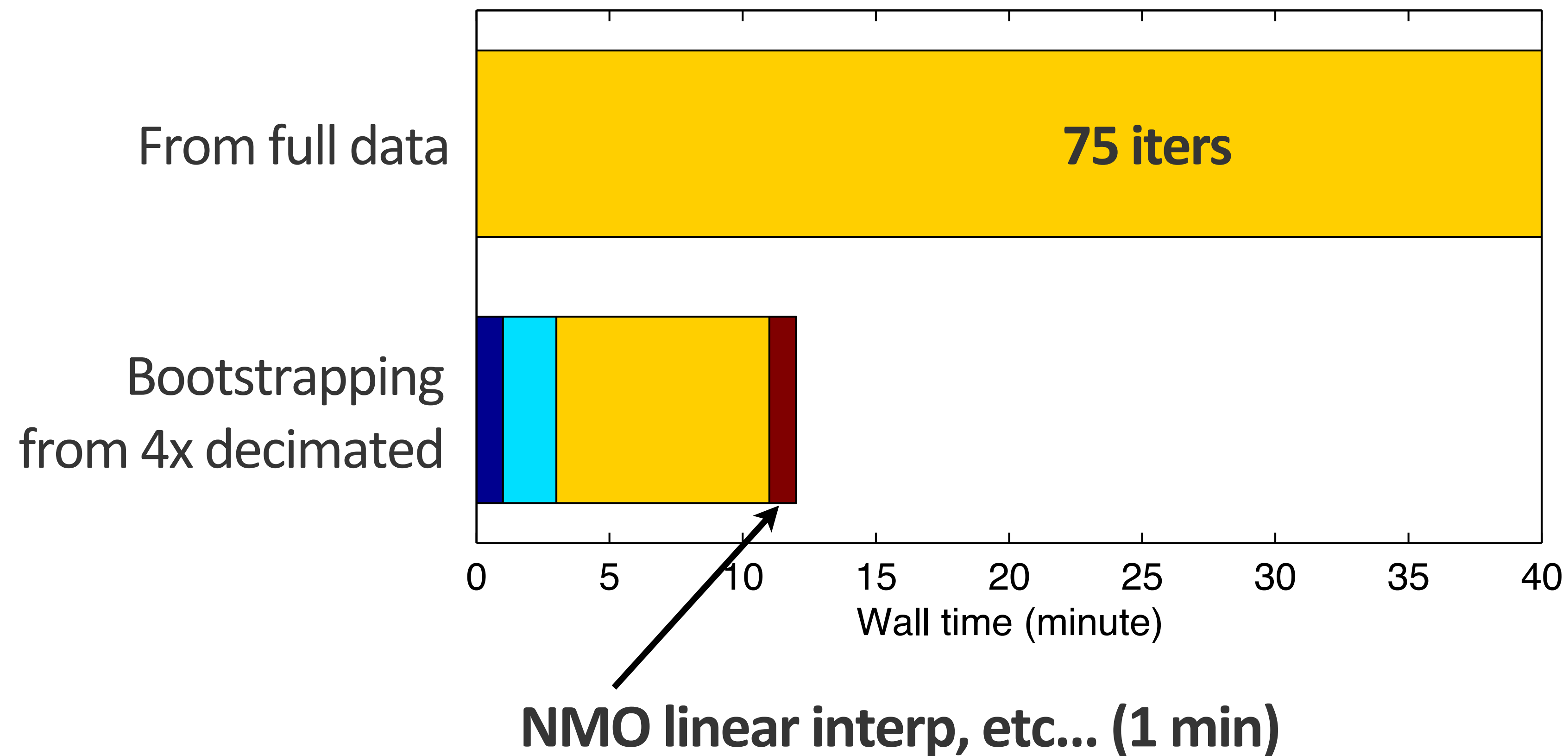
Significant speedup from bootstrapping

Wall times



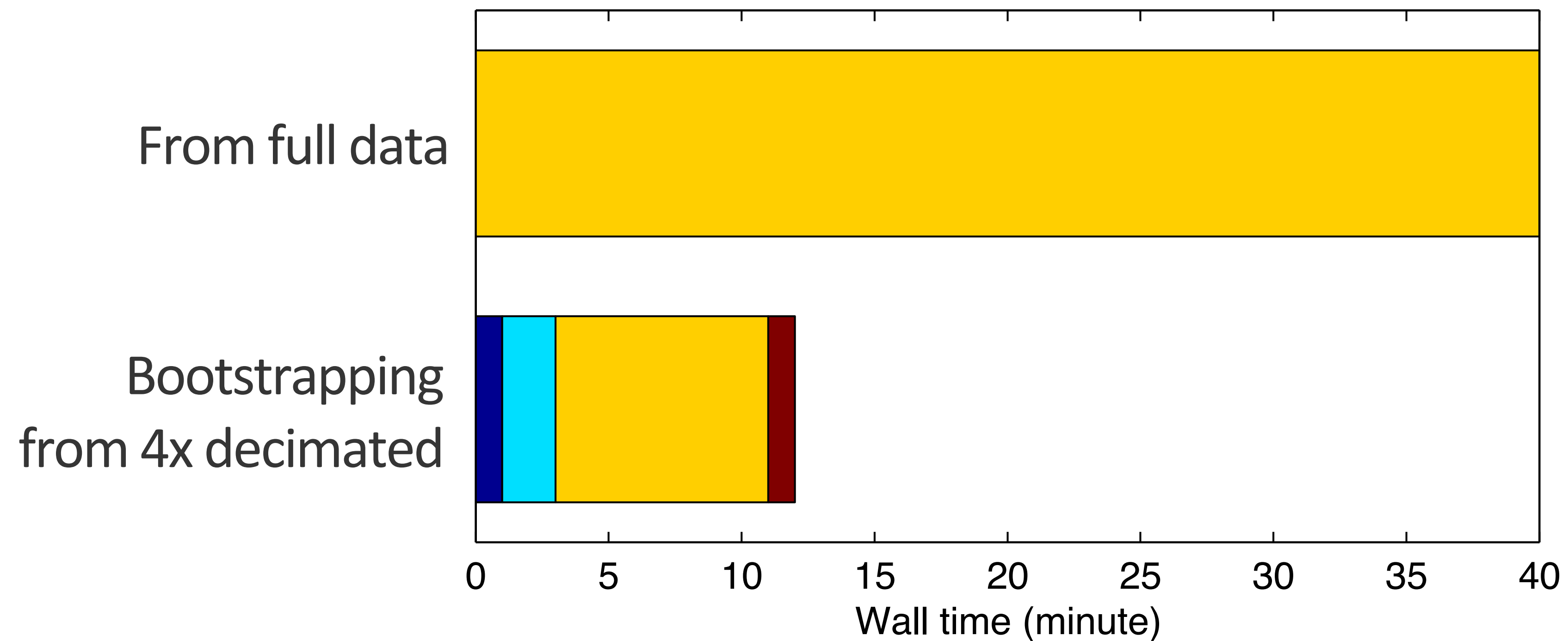
Significant speedup from bootstrapping

Wall times



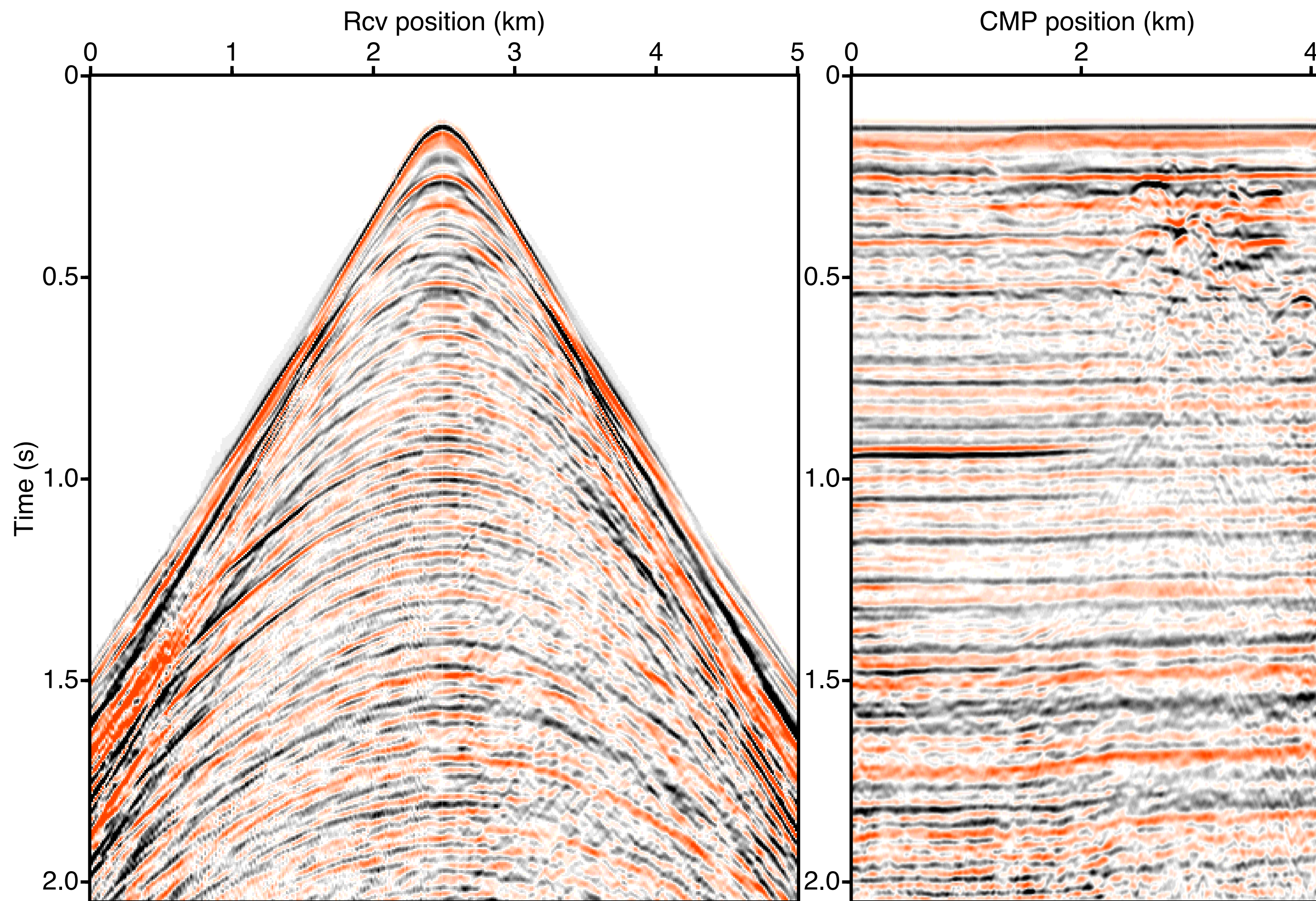
Significant speedup from bootstrapping

Wall times



Field data example

North Sea dataset



Shot gather

NMO-corrected stack

North sea data

Shot gather and stack

Streamer data
(regularized to fixed-
spread data)

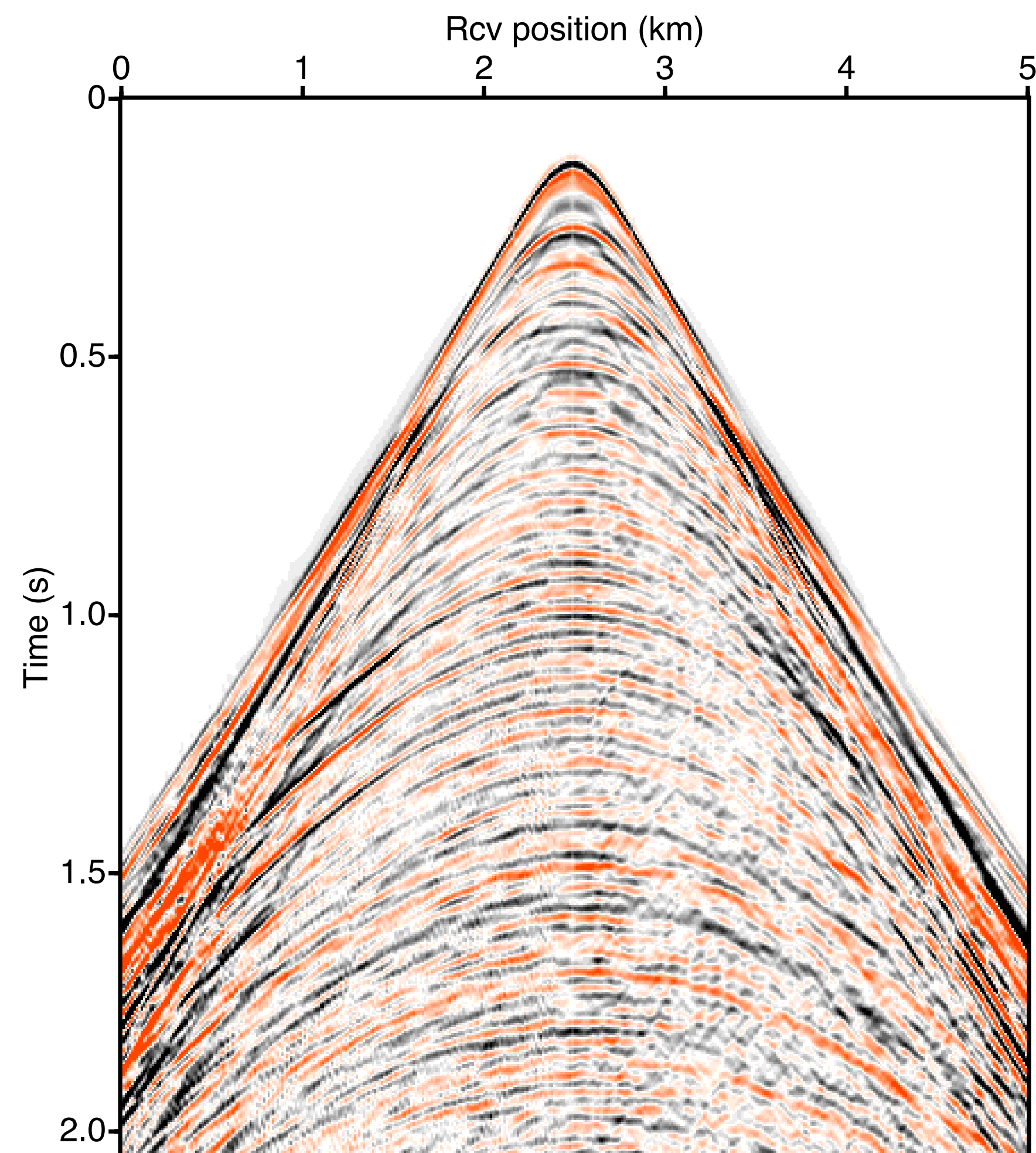
401 source and receiver

12.5 m spatial grid

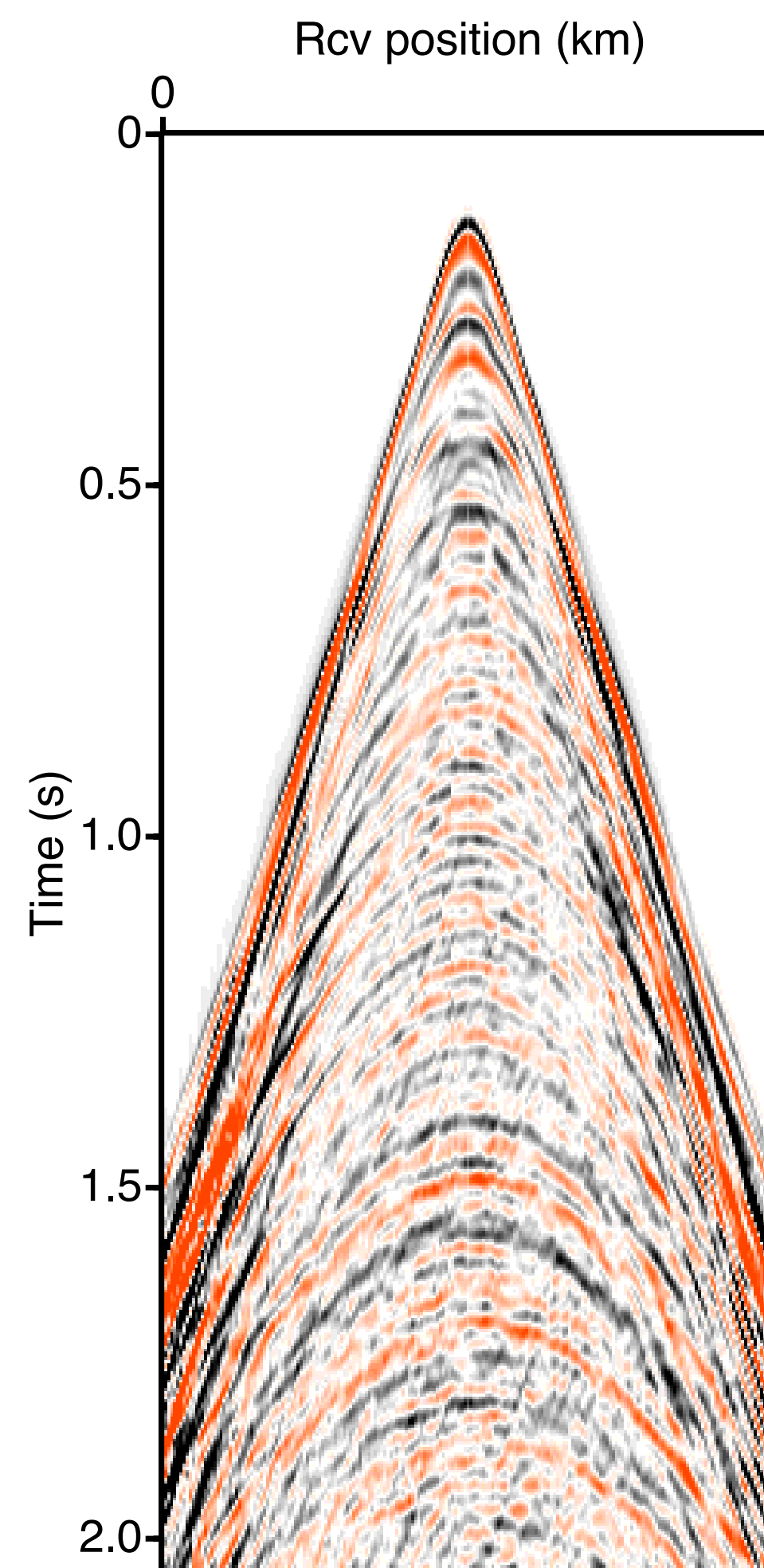
4 ms time sampling

Decimated wavefields

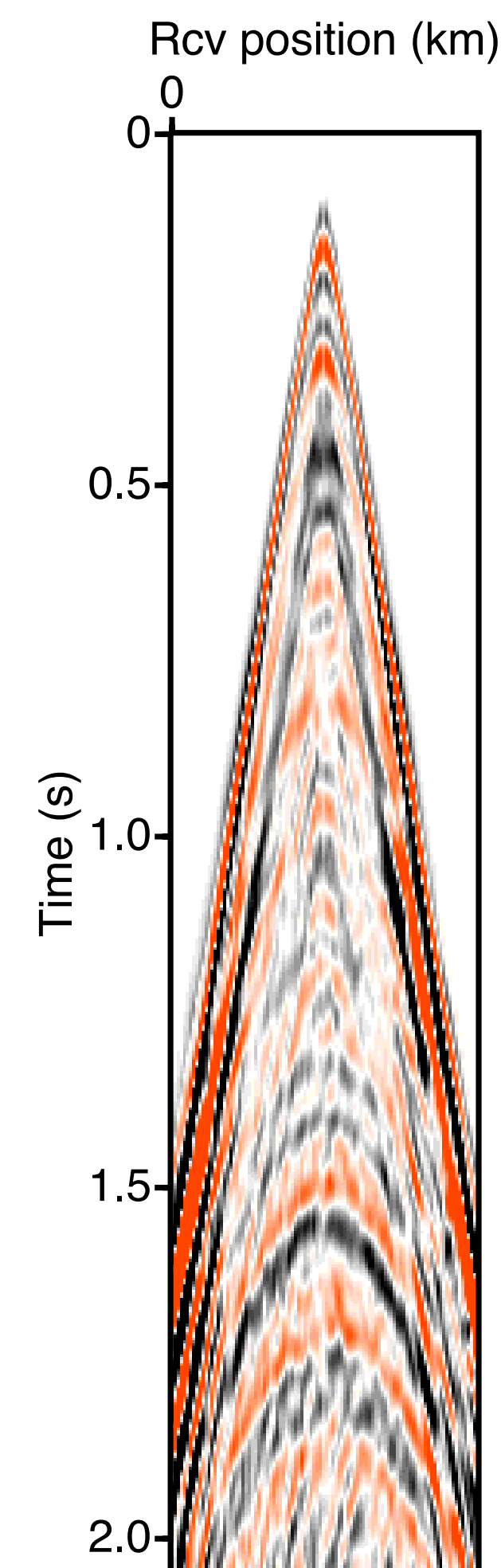
Original (dx = 12.5m)



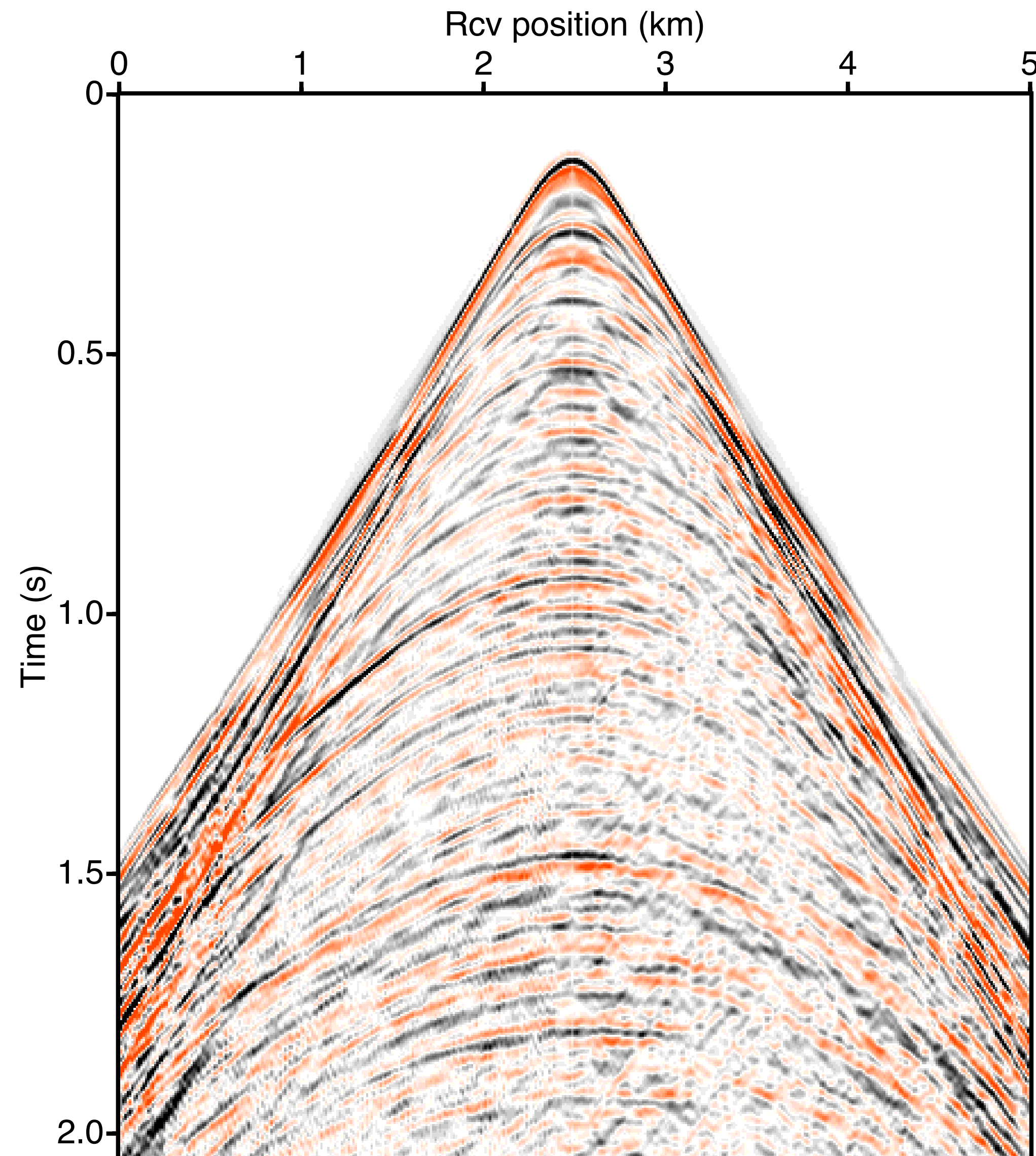
2x decimated
lowpass 40Hz



4x decimated
lowpass 20Hz



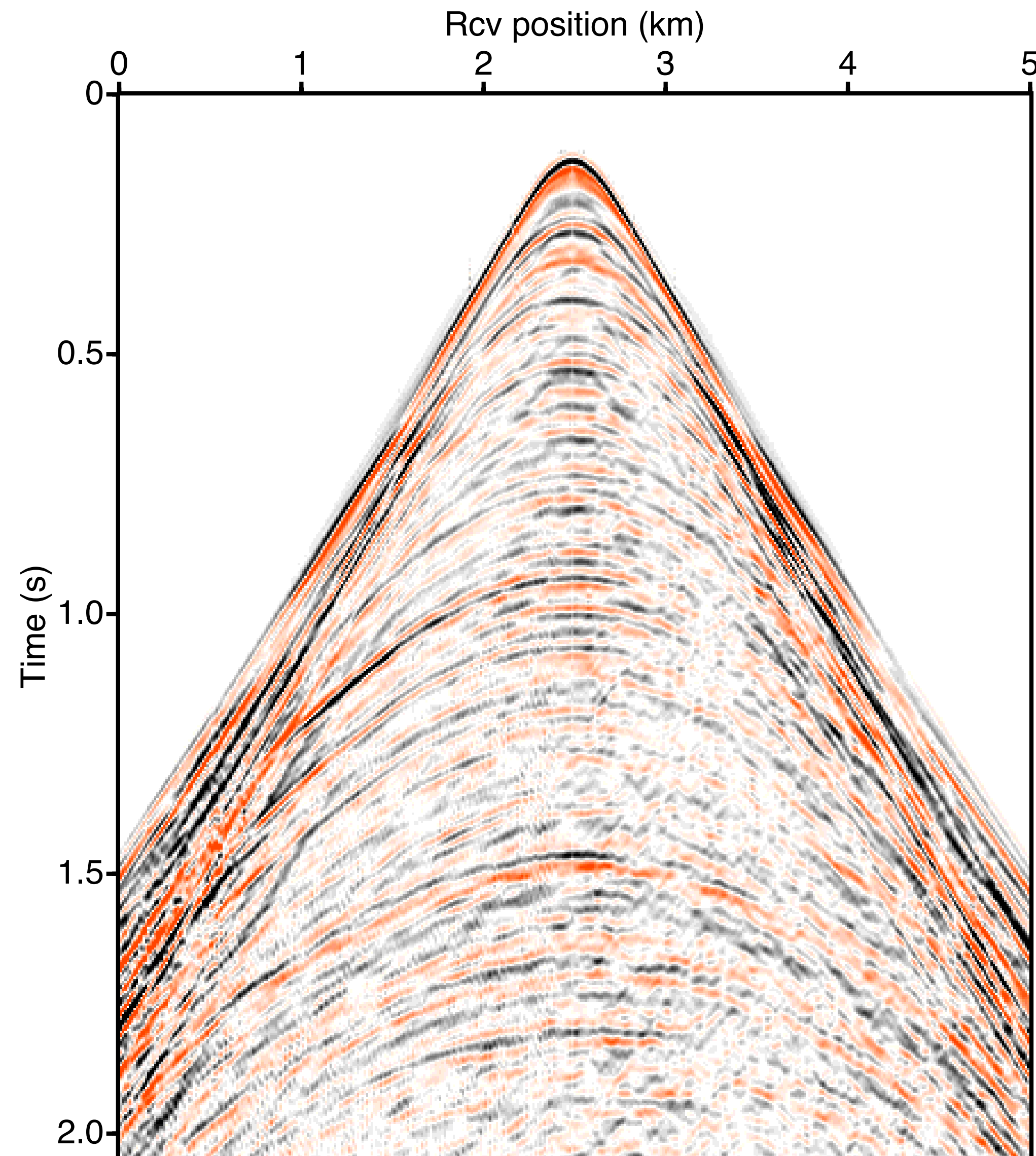
Solution wavefield comparison



Direct Primary

Solved with plain algorithm from finest scale data

Solution wavefield comparison

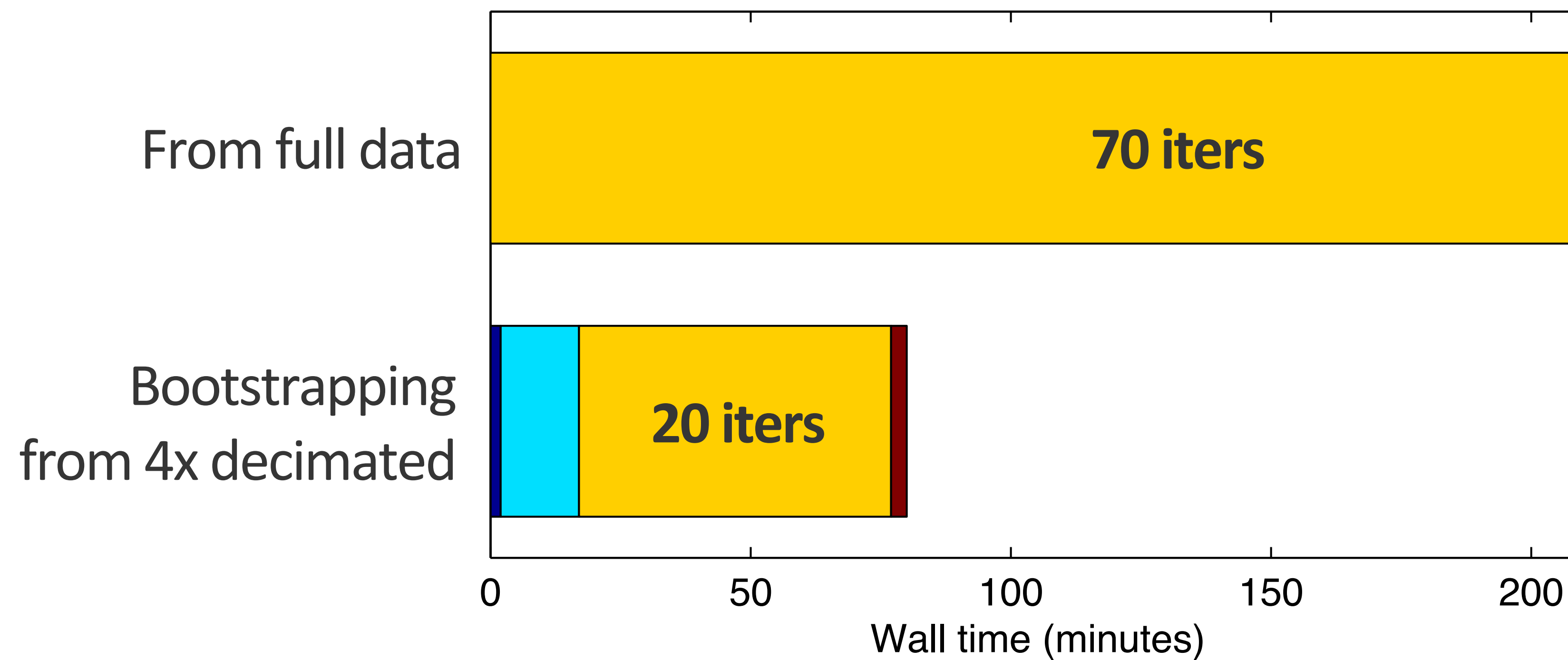


Direct Primary

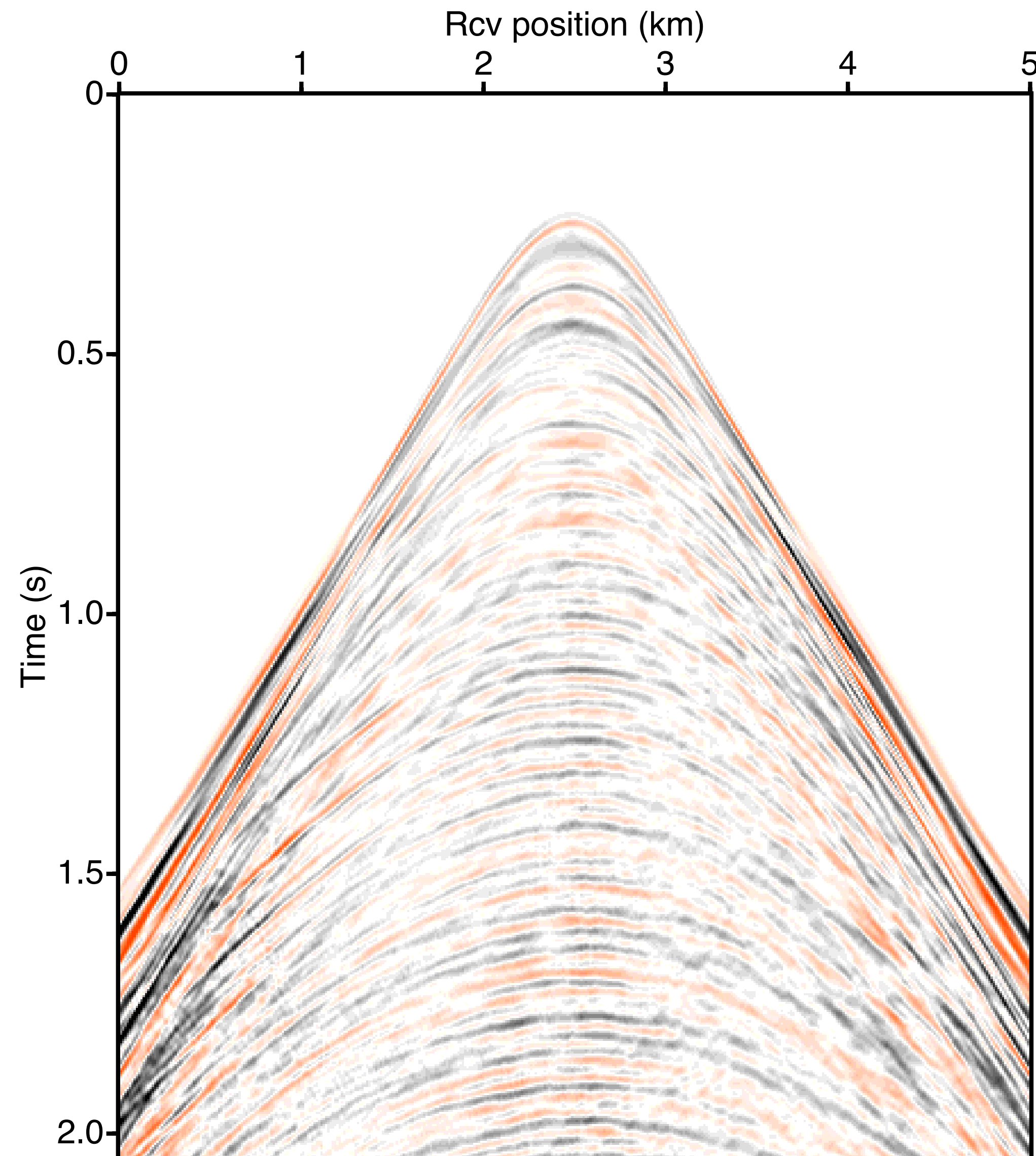
Solved with spatial sampling continuation

$dx = 50\text{m} > 25\text{m} > 12.5\text{m}$

Runtime breakdown (wall time)



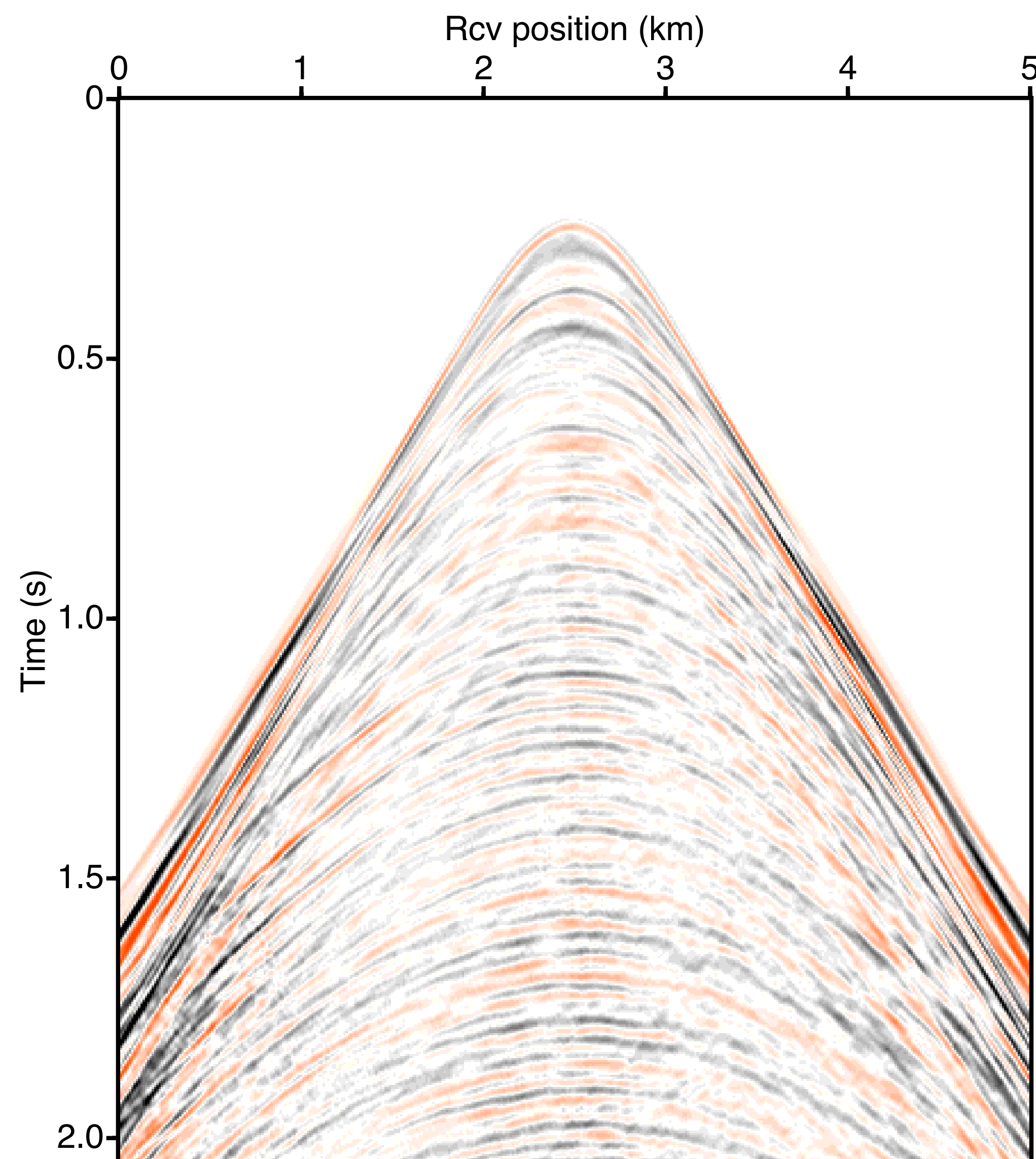
Solution multiple comparison



Predicted Surface Multiple

Solved with plain algorithm from finest scale data

Solution multiple comparison



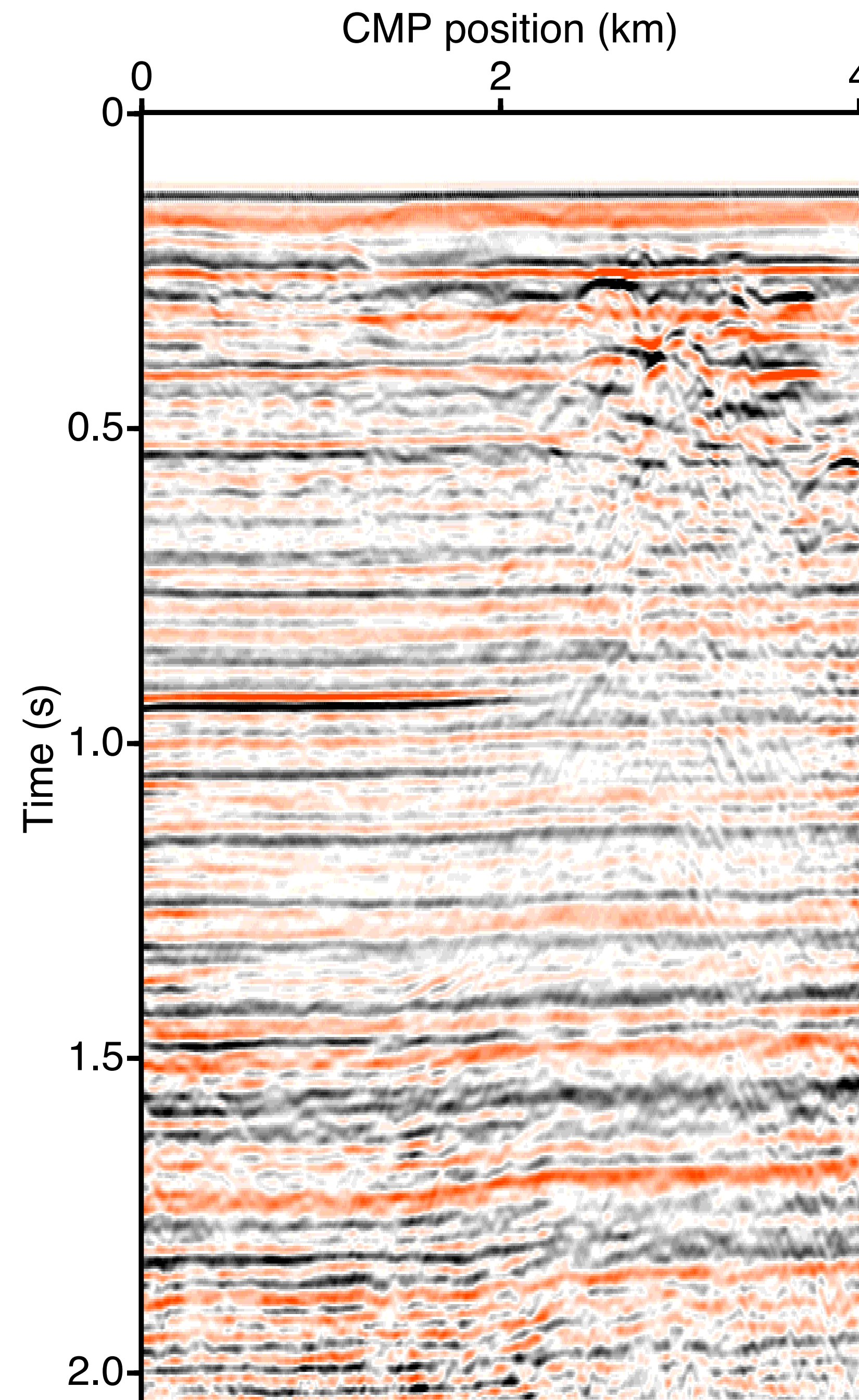
Predicted Surface Multiple

Solved with spatial sampling continuation

$dx = 50\text{m} > 25\text{m} > 12.5\text{m}$

Solution stack comparison

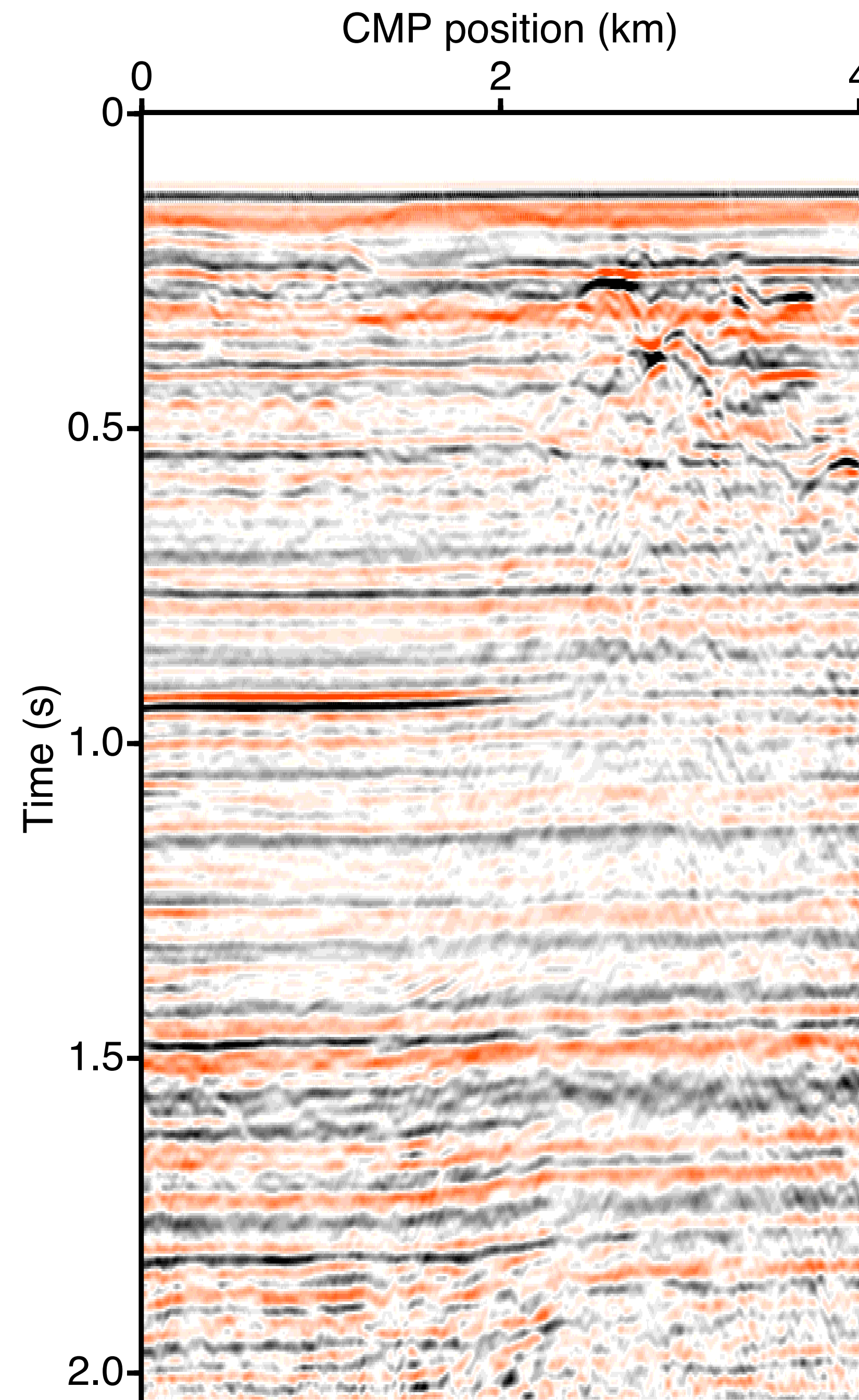
NMO Stack
original data



Solution stack comparison

REPSI Primaries NMO Stack

Solved with plain algorithm from finest scale data

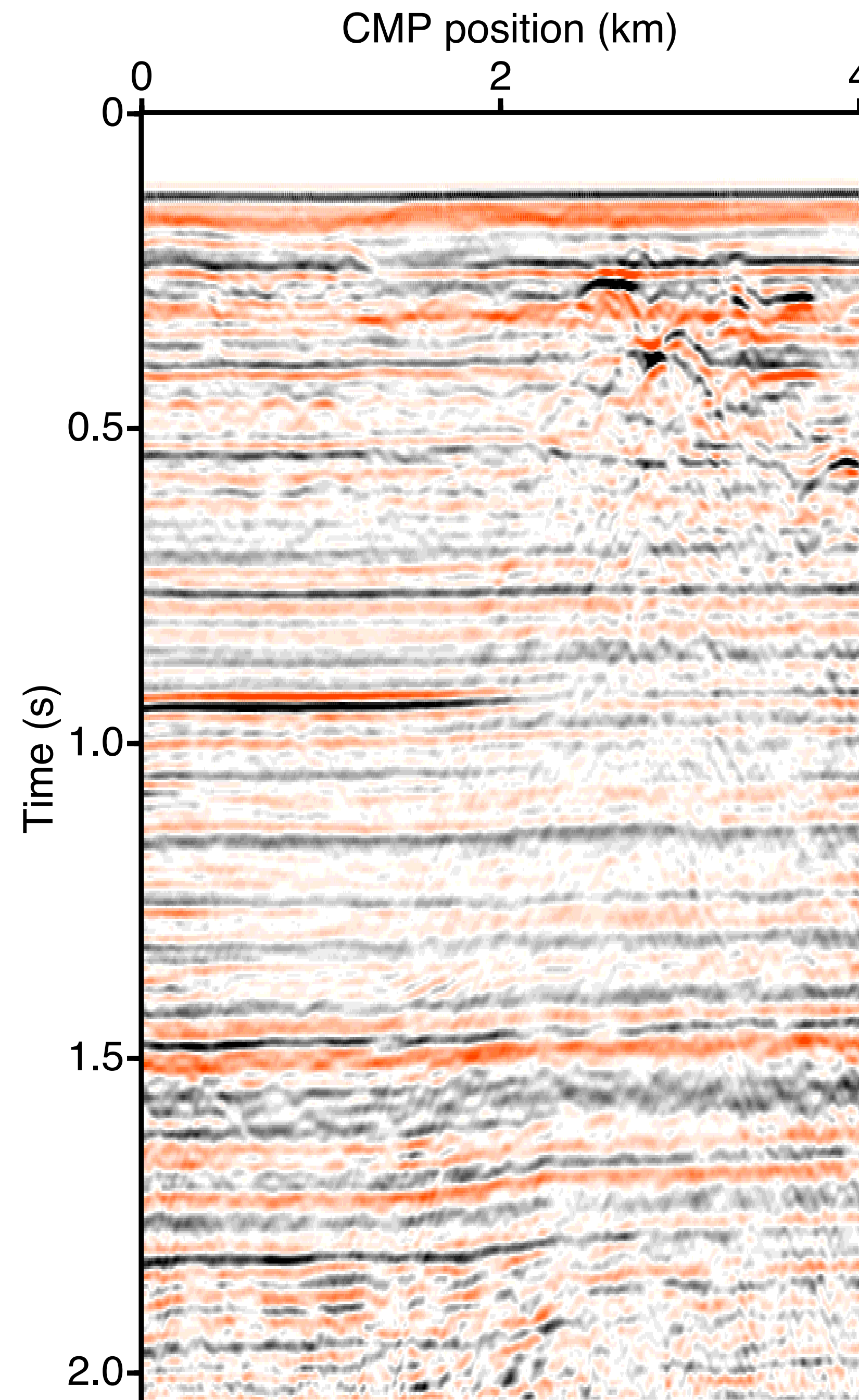


Solution stack comparison

REPSI Primaries NMO Stack

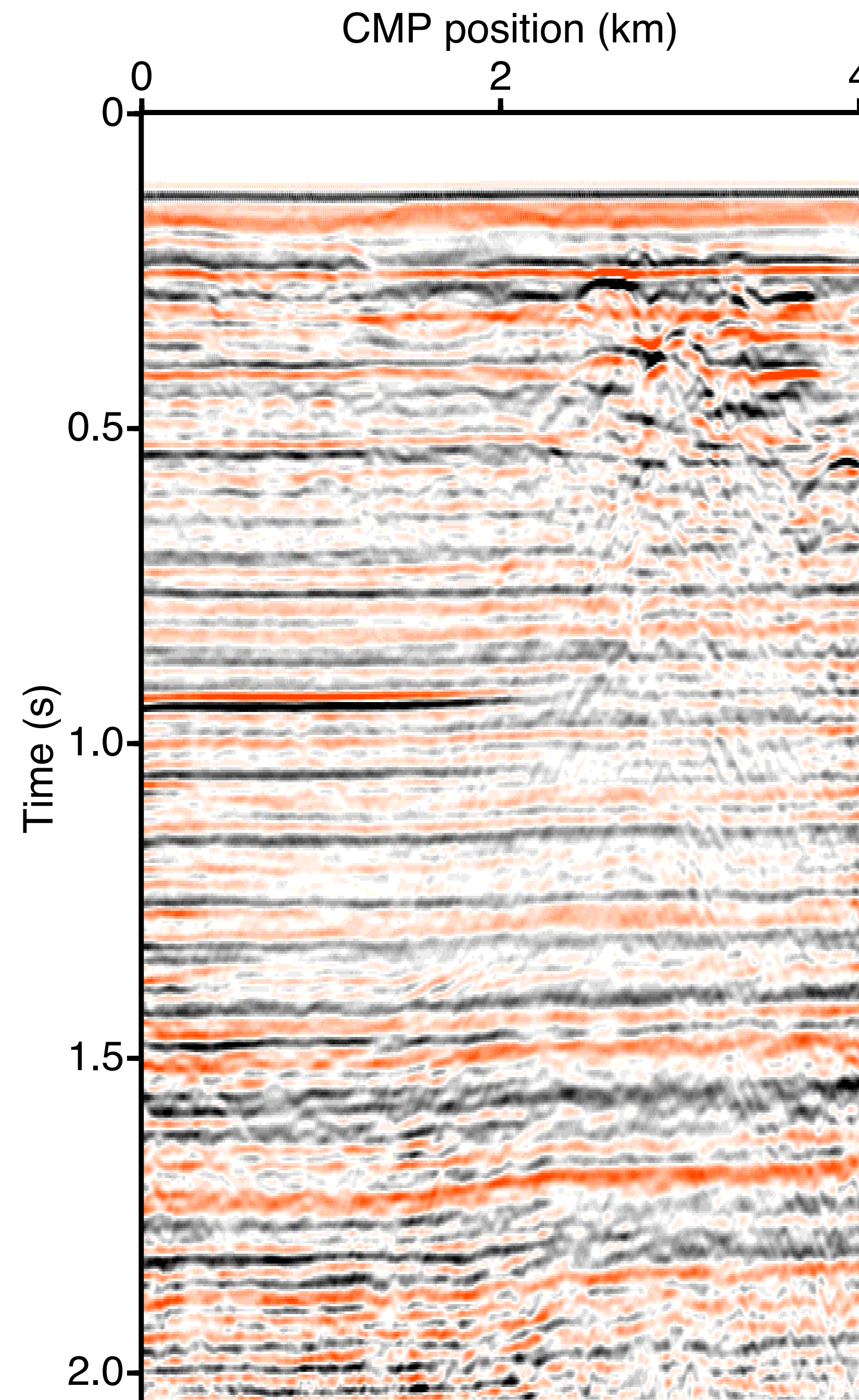
Solved with spatial sampling continuation

$dx = 50\text{m} > 25\text{m} > 12.5\text{m}$



Solution stack comparison

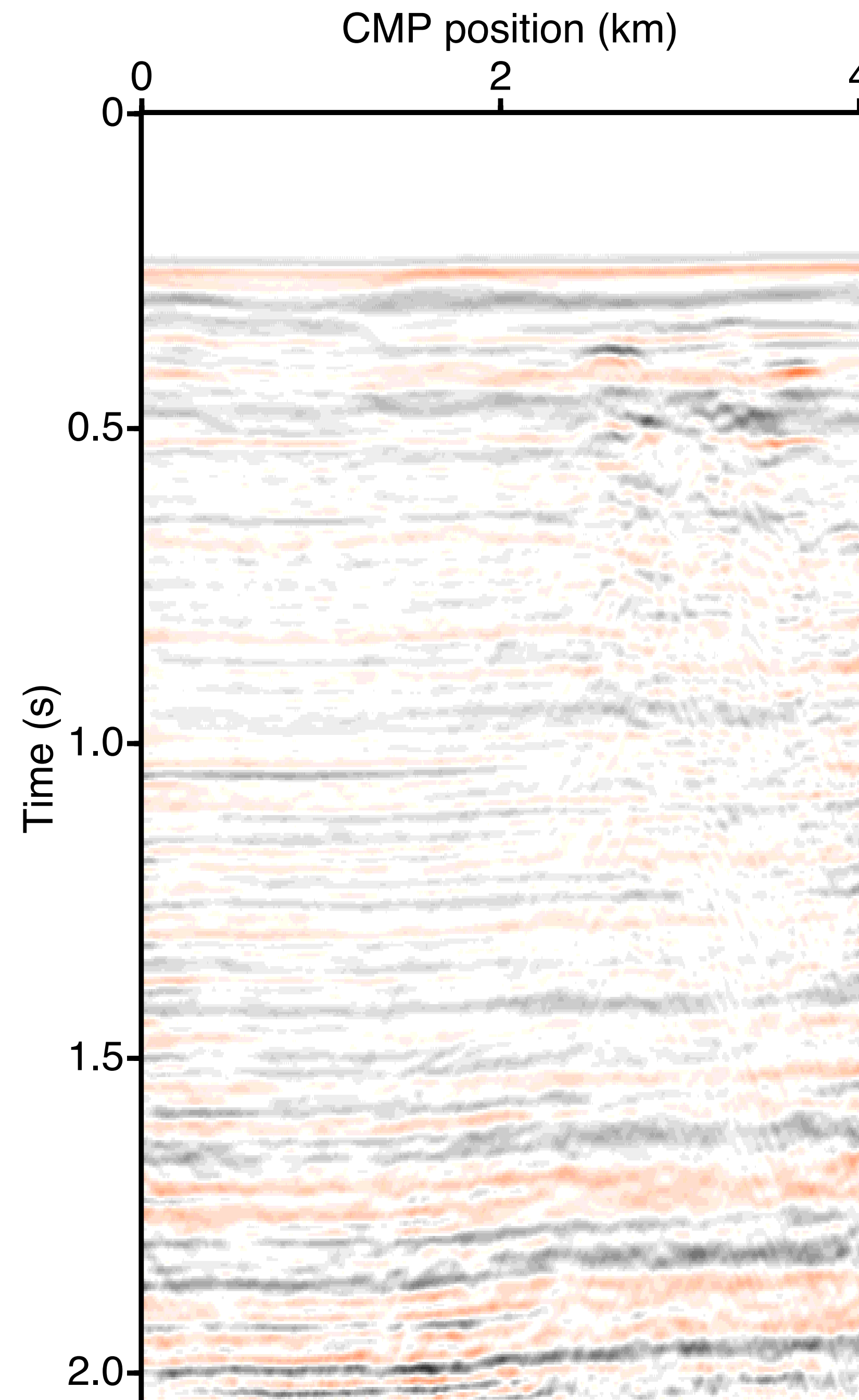
NMO Stack
original data



Solution stack comparison

REPSI Multiples NMO Stack

Solved with plain algorithm from finest scale data

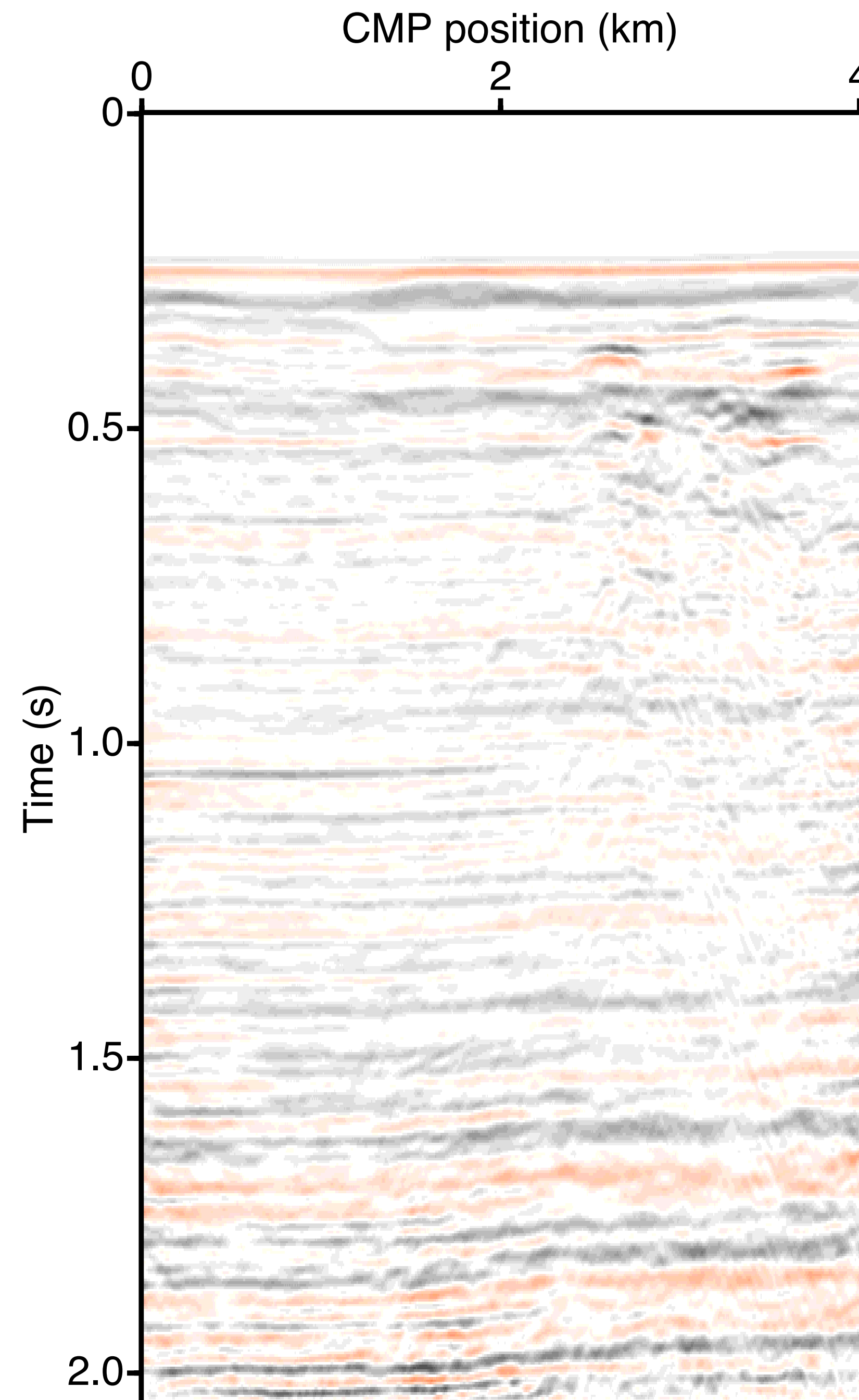


Solution stack comparison

REPSI Multiples NMO Stack

Solved with spatial sampling continuation

$dx = 50\text{m} > 25\text{m} > 12.5\text{m}$



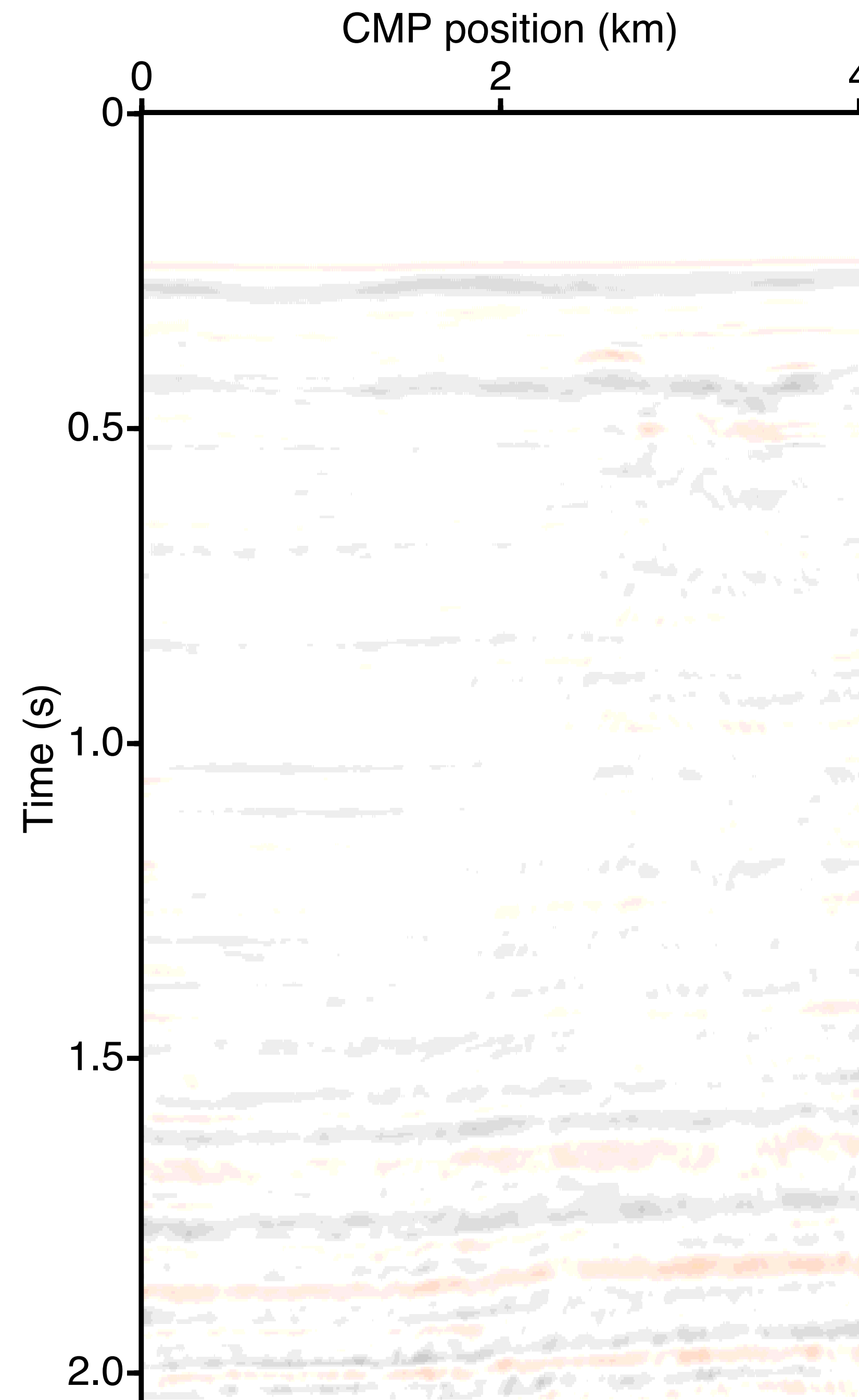
Solution stack comparison

REPSI Multiples NMO Stack

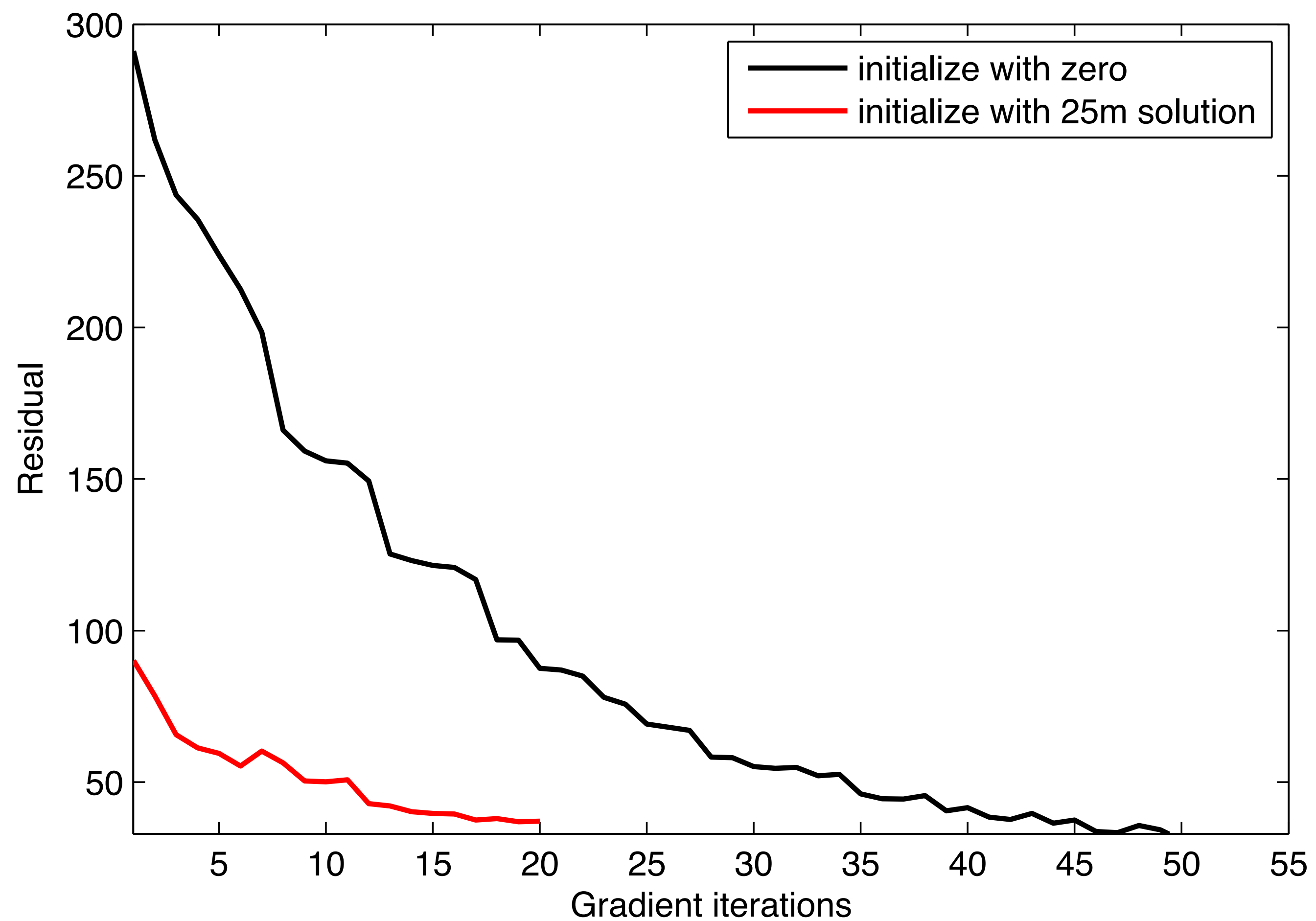
Difference:

plain algorithm

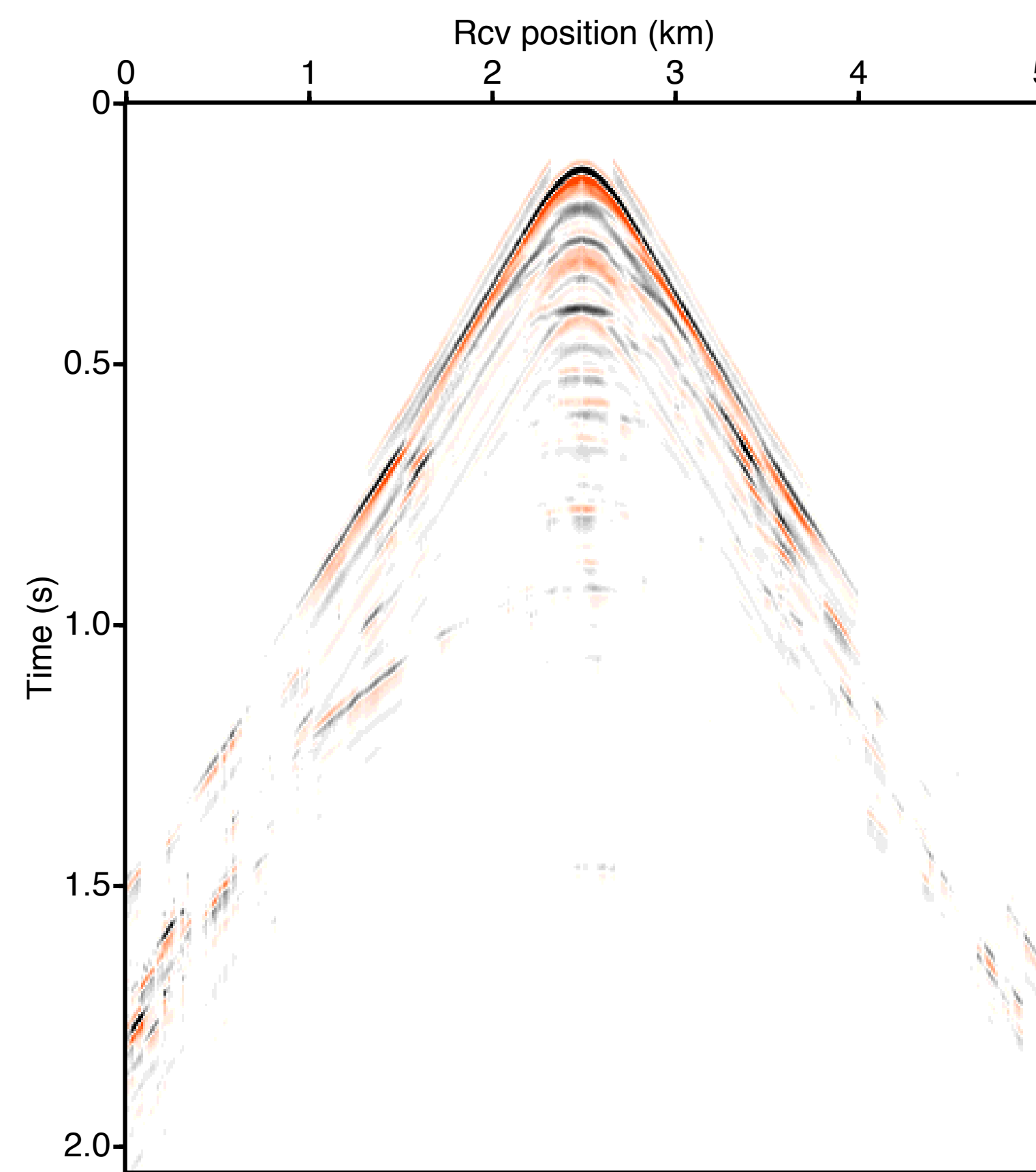
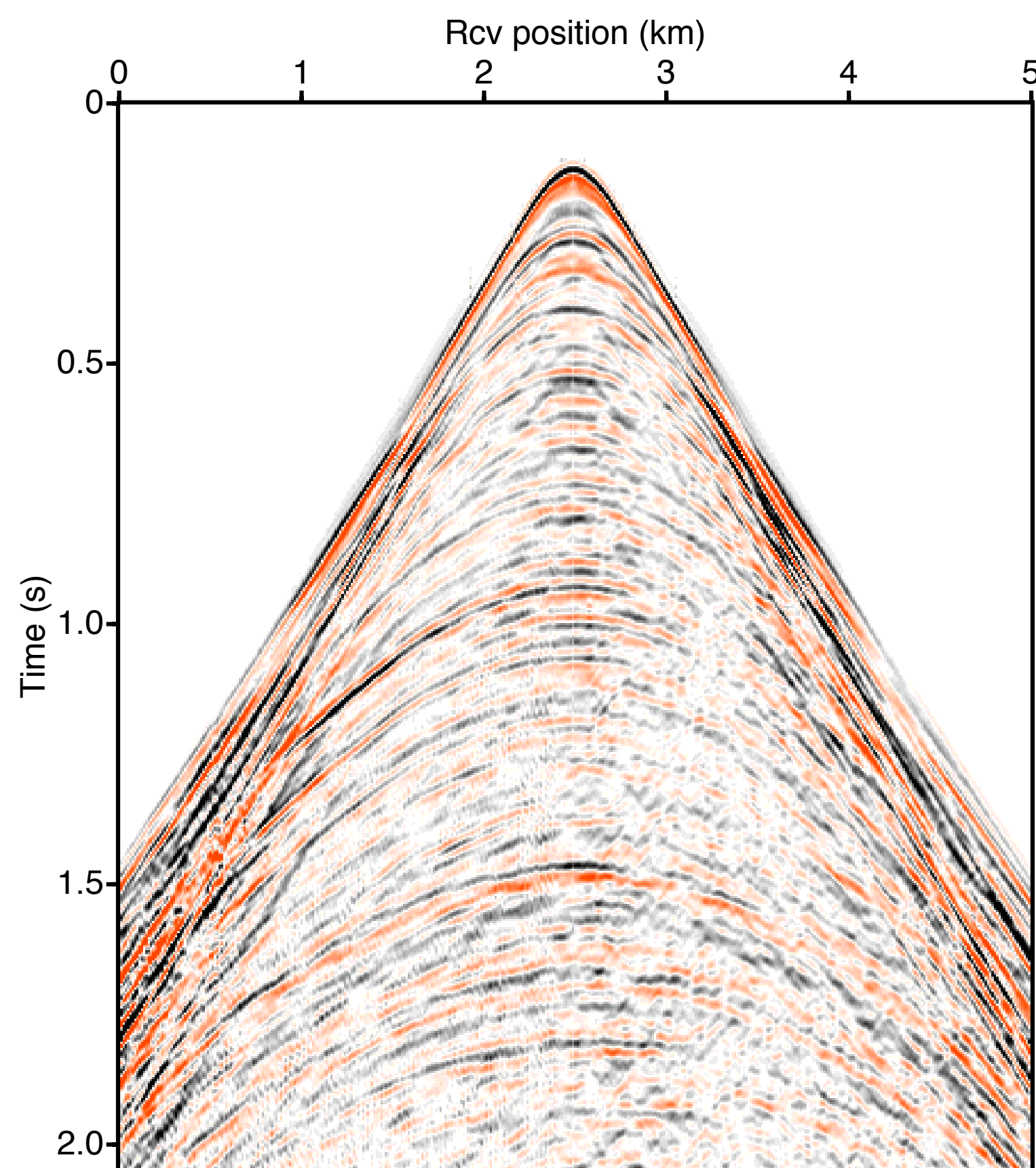
accelerated algorithm



Warm-start vs from zero residual graph (for full scale problem)



Warm-start vs from zero 'G' shot gathers



Acceleration strategy summary

Start REPSI with *decimated* data, *lowpass* to avoid spatial aliasing

Once “enough” progress is made, continue with fine-scale data

Significant savings in computation cost, 100x to 200x SRMP becomes more like 20x to 30x

How low can we go? Depends on the ability of sparsity-regularized inversion to resolve wavefronts under reduced bandwidth.

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