Implicit interpolation of trace gaps in **REPSI** using **auto-convolution terms**

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Talk outline

Brief review of REPSI

Data reconstruction in EPSI

Eliminate explicit expression for missing data

Field data example

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

true primary wavefield SRME–produced primary

$$\mathbf{P_o} = \mathbf{P} - A(f)\mathbf{P_o}\mathbf{P}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{P_o}$ primary wavefield

$A(f)$ “matching” operator

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

true primary wavefield — SRME-produced primary

$$\mathbf{P_o} \approx \mathbf{P} - A(f)\mathbf{P}\mathbf{P}$$

SRMP

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	"matching" operator

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

adaptive subtraction

$$\min_{A} \sum_{f} \|\mathbf{P} - A(f)\mathbf{P}\mathbf{P}\|$$

SRMP

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
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recorded data predicted data from SRME

$$\mathbf{P} = \mathbf{P_o} + A(f)\mathbf{P_o}\mathbf{P}$$

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From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from SRME

$$\mathbf{P} = \mathbf{P_o} + A(f)\mathbf{P_o}\mathbf{P}$$

$$\mathbf{P_o} = \mathbf{Q}\mathbf{G}$$
$$A(f) = -\mathbf{Q}^{-1}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

recorded data predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

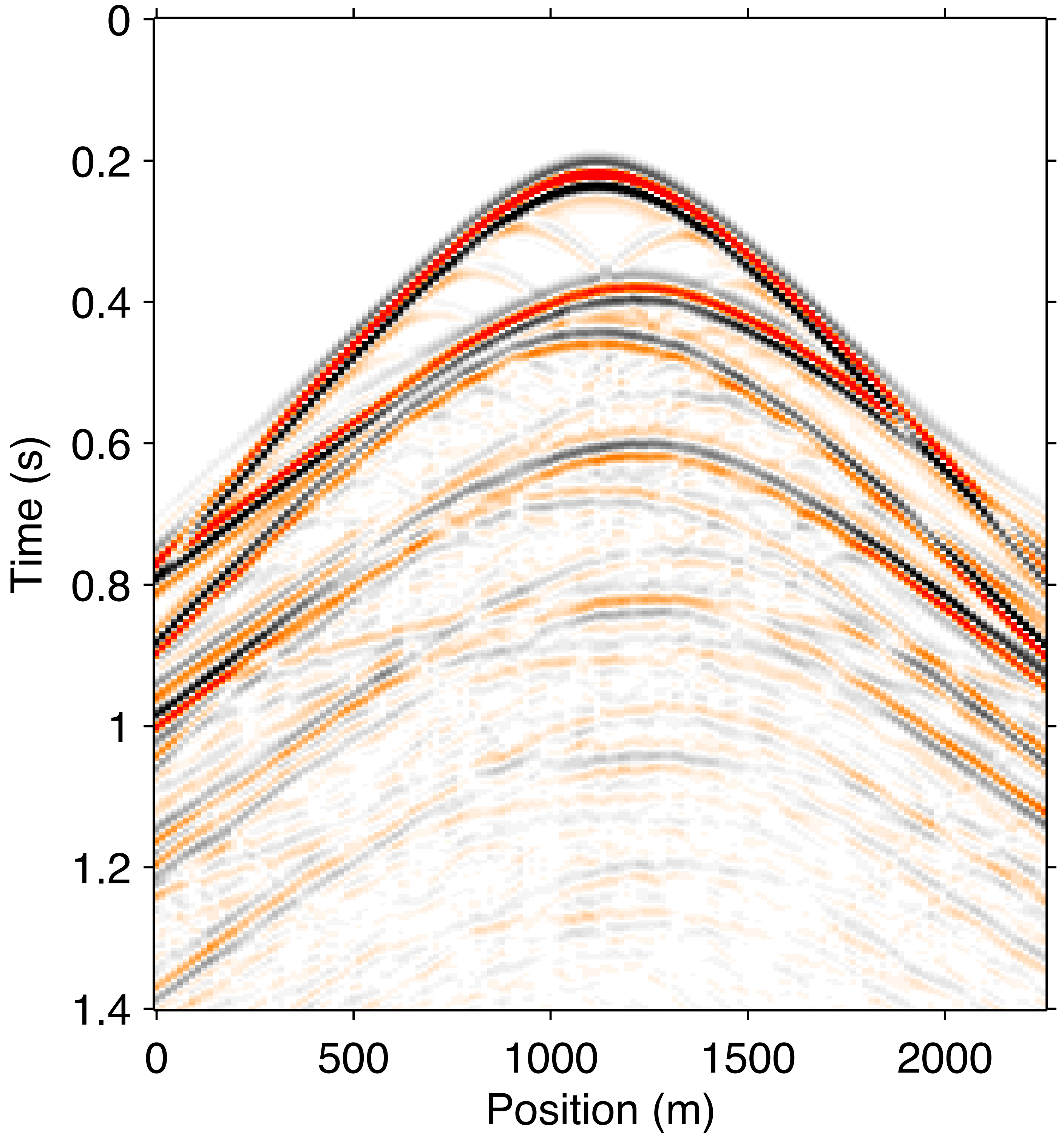
recorded data   predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI



Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

'ded data predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va … 09)

recorded data predicted data from SRME

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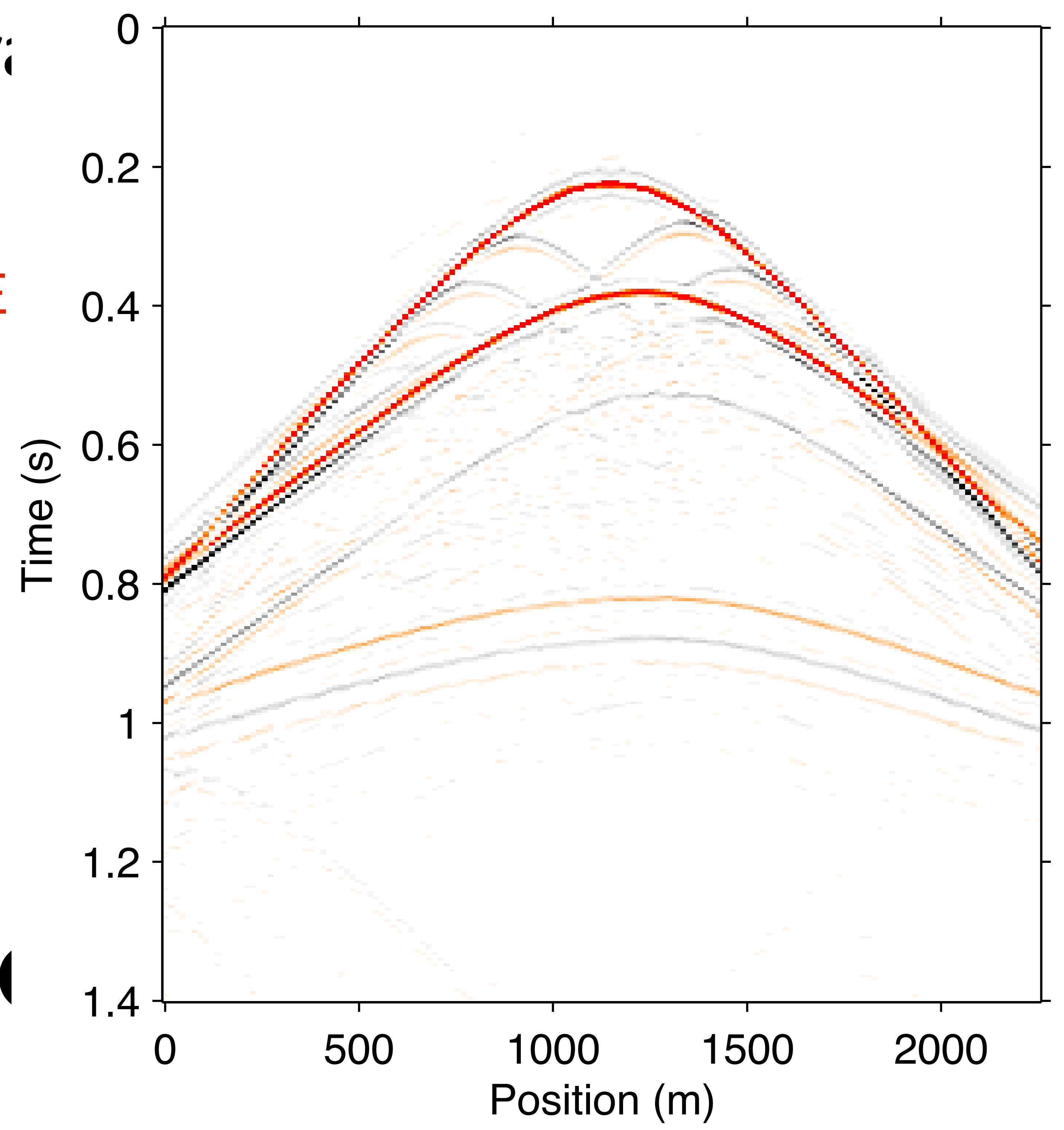

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va ... 09)

recorded data	predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

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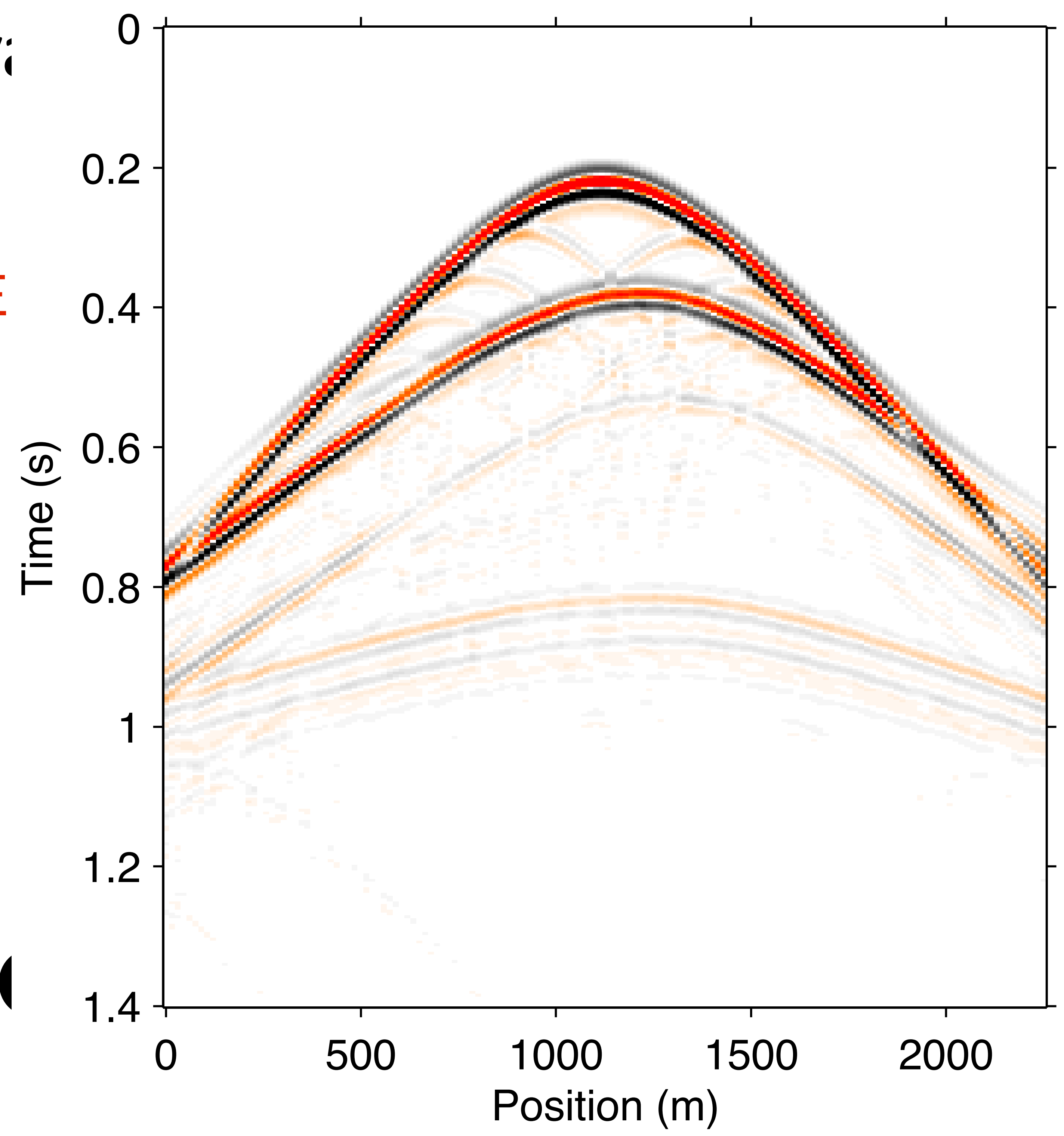

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va 09)

recorded data predicted data from SRME

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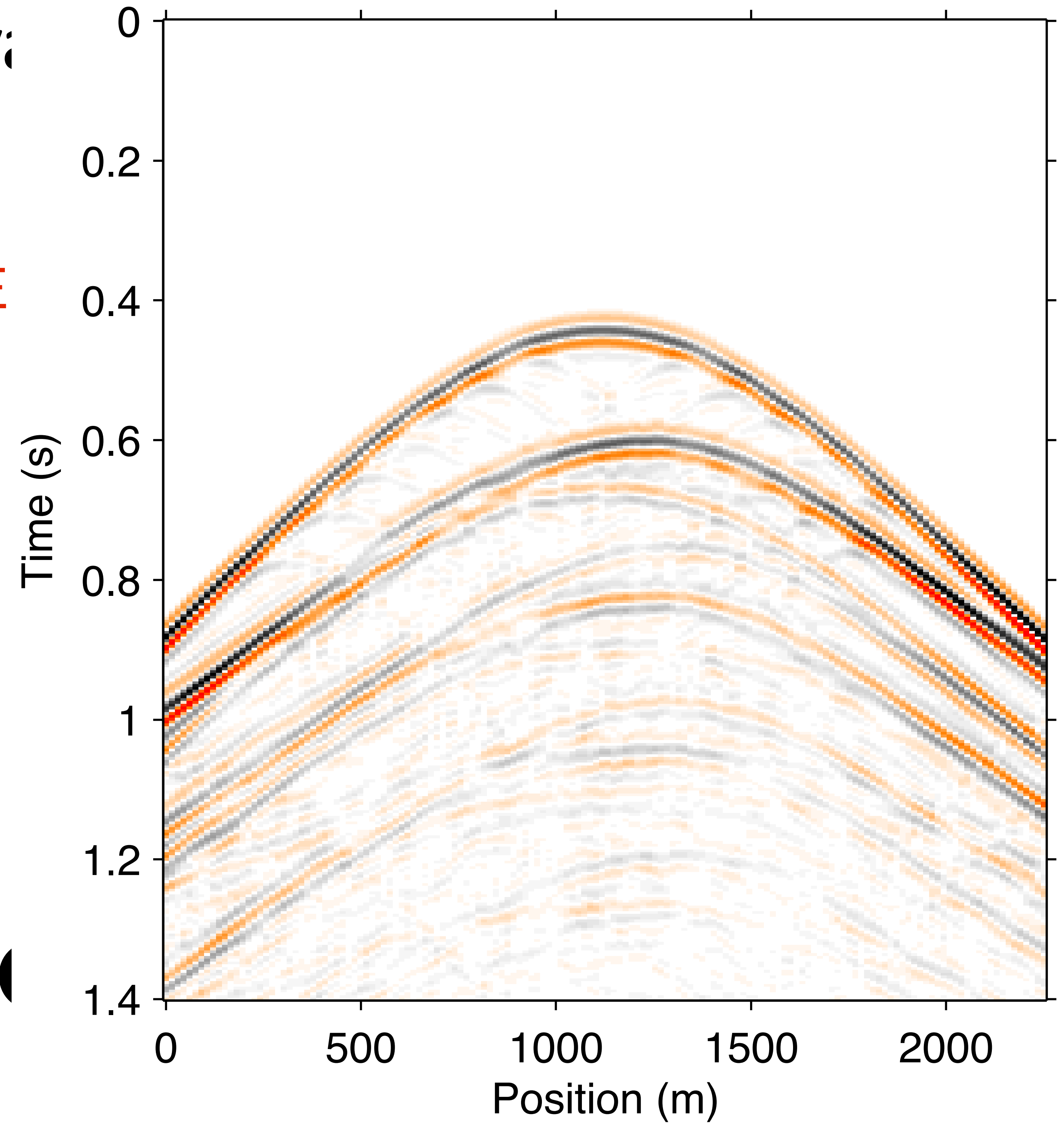

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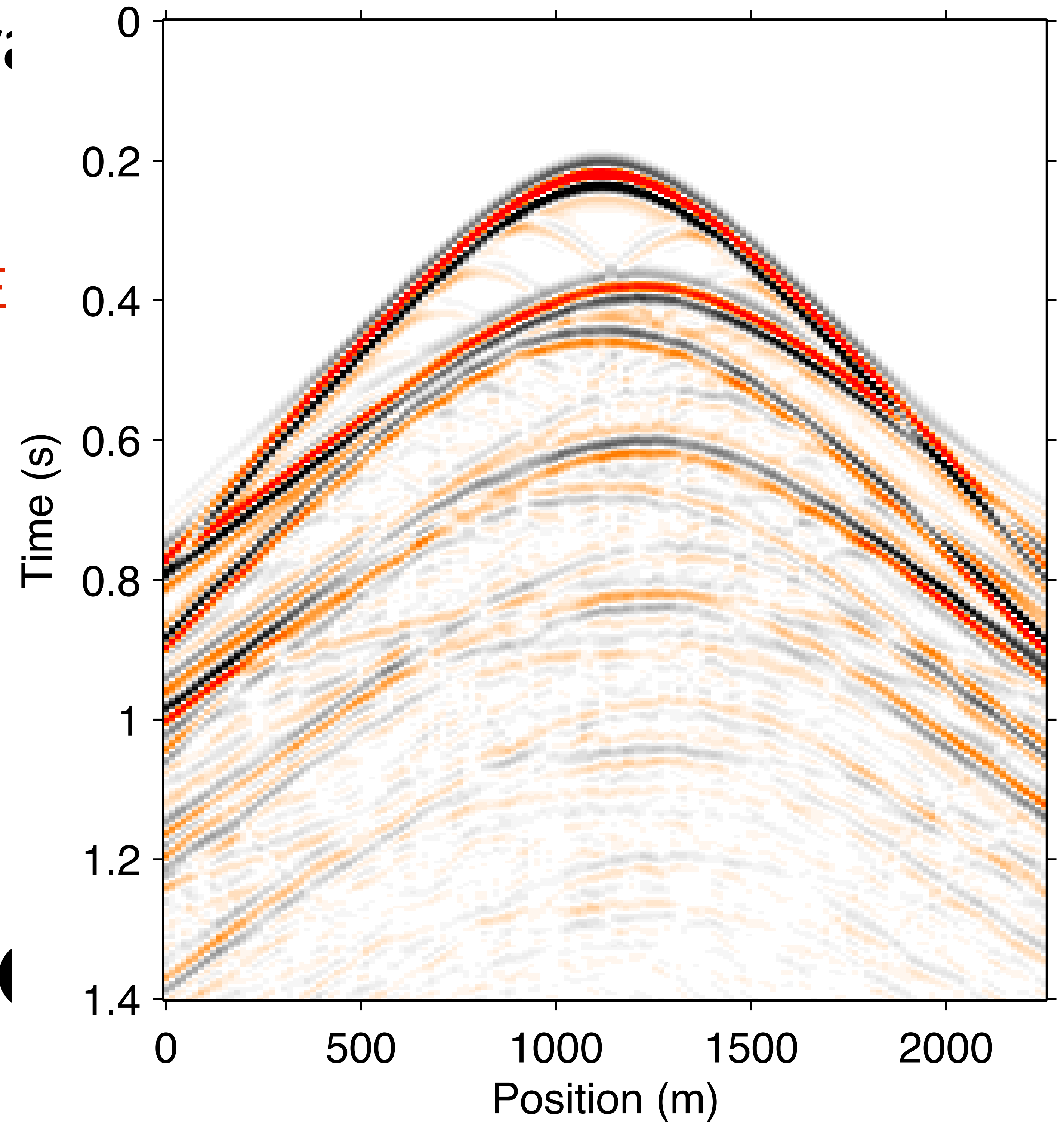

From SRME to Robust EPSI

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recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI

Two ways to obtain the final primary wavefield

"Direct" Primary "Conservative" Primary

$$\mathbf{QG} = \mathbf{P} + \mathbf{GP}$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI

In time domain (lower-case: whole dataset in time domain)

recorded data — predicted data from SRME

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

Solving the EPSI problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

In fact it is bilinear: $\mathbf{QG} = \mathbf{P} + \mathbf{GP}$

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

Solving the EPSI problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2^2$$

Solving the EPSI problem

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha \mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta \nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace

Robust EPSI
L1-minimization approach to the EPSI problem

[Lin and Herrmann, 2013 *Geophysics*]

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

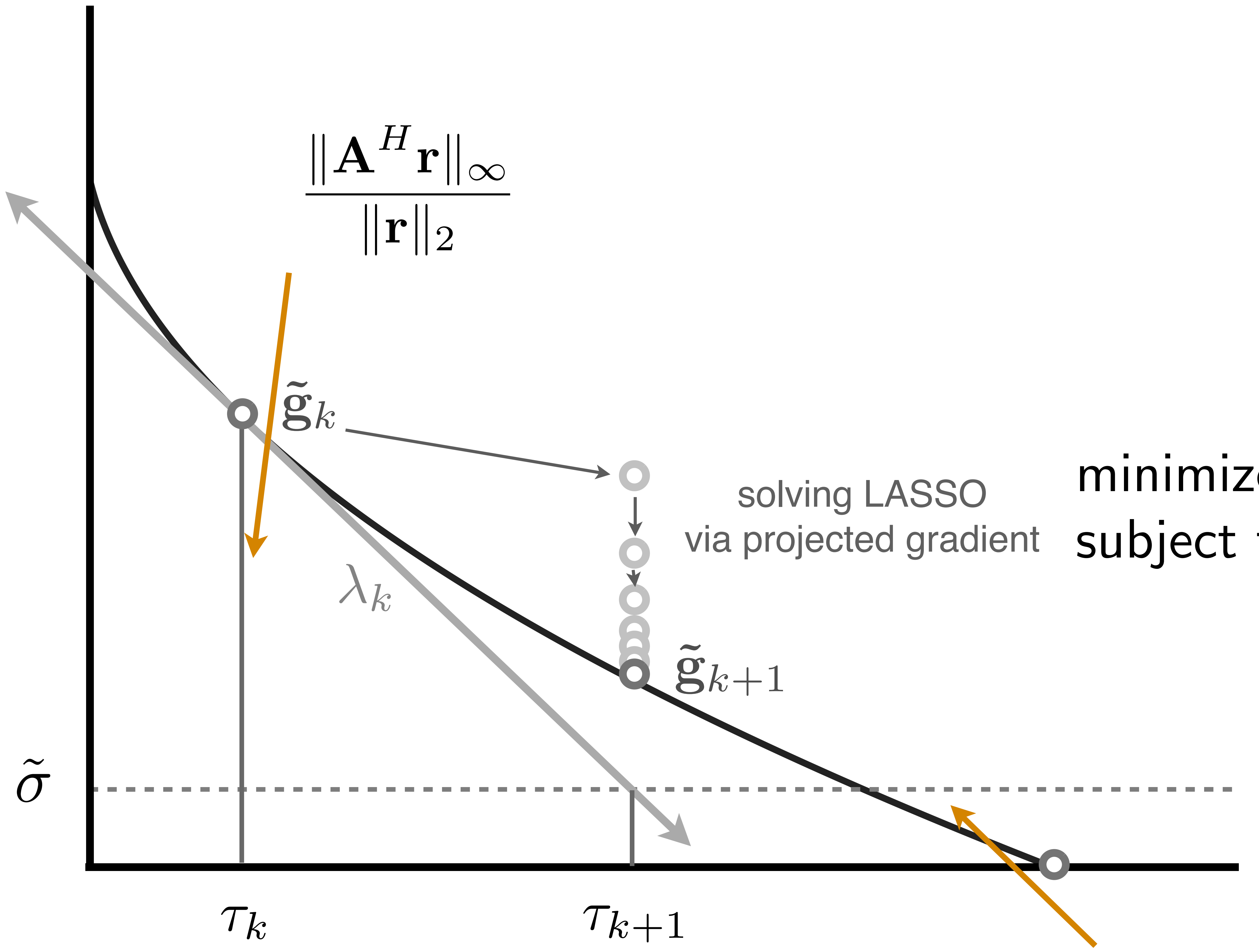
$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

Choosing Tau from the Pareto curve

$$\text{minimize} \quad \|x\|_1 \qquad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma$$

Look at the solution space and the line of optimal solutions (Pareto curve)



$$\text{minimize} \quad \|Ax - b\|_2 \qquad \text{subject to} \quad \|x\|_1 \leq \tau$$

feasible solution with smallest $\|x\|_1$

Inverting for unknown data

Robust EPSI
Inverting for unknown data

$$\Delta\mathbf{P}(\mathbf{G}_{k+1},\mathbf{R}_{k+1}) := -(\mathbf{I}+\mathbf{G}_{k+1})^H(\mathbf{R}_{k+1})$$

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k,\mathbf{q}_k)\|_2 > \sigma$

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$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha\Delta\mathbf{p}(\mathbf{g}_{k+1},\mathbf{q}_{k+1},\mathbf{p}_k)$$

Robust EPSI
Inverting for unknown data

Data changes every iteration!

$$\Delta\mathbf{P}(\mathbf{G}_{k+1},\mathbf{R}_{k+1}) := -(\mathbf{I}+\mathbf{G}_{k+1})^H(\mathbf{R}_{k+1})$$

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k,\mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

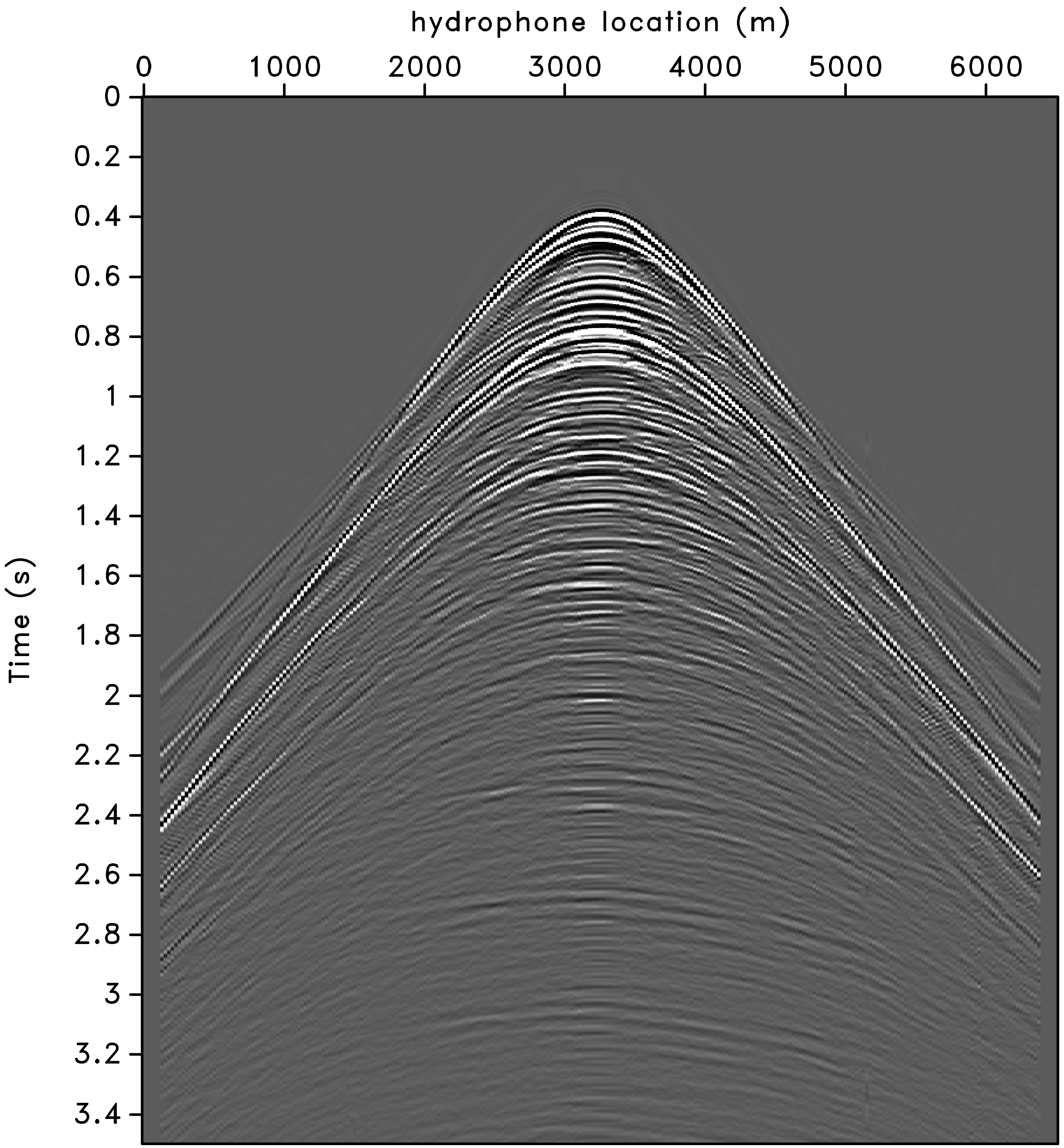
$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p}_k - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha\Delta\mathbf{p}(\mathbf{g}_{k+1},\mathbf{q}_{k+1},\mathbf{p}_k)$$

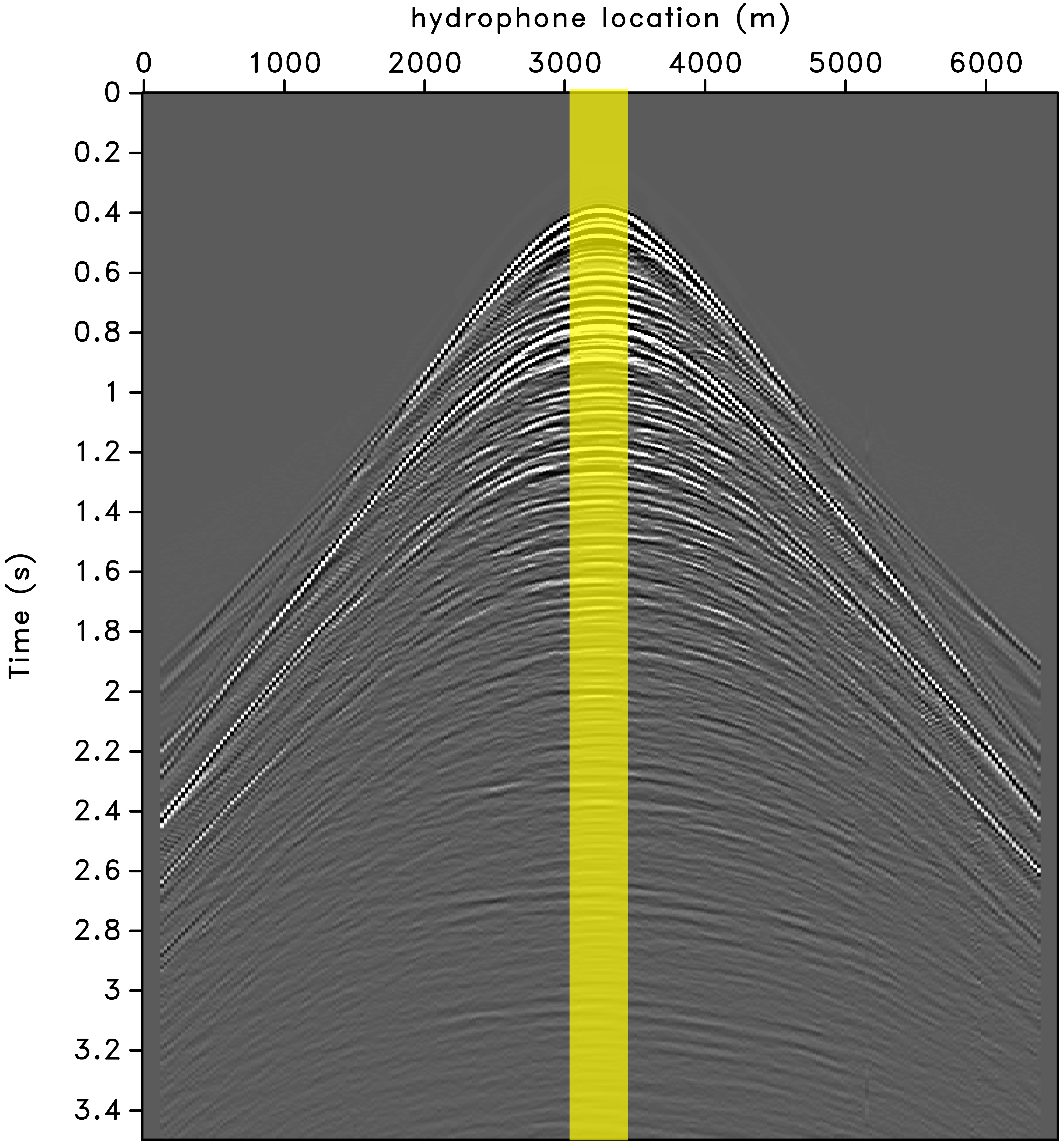
Trace Mask

Masking operator $\mathbf{K}$



Trace Mask

Bisects wavefield data to unknown/ uncertain traces
(i.e., near offset)

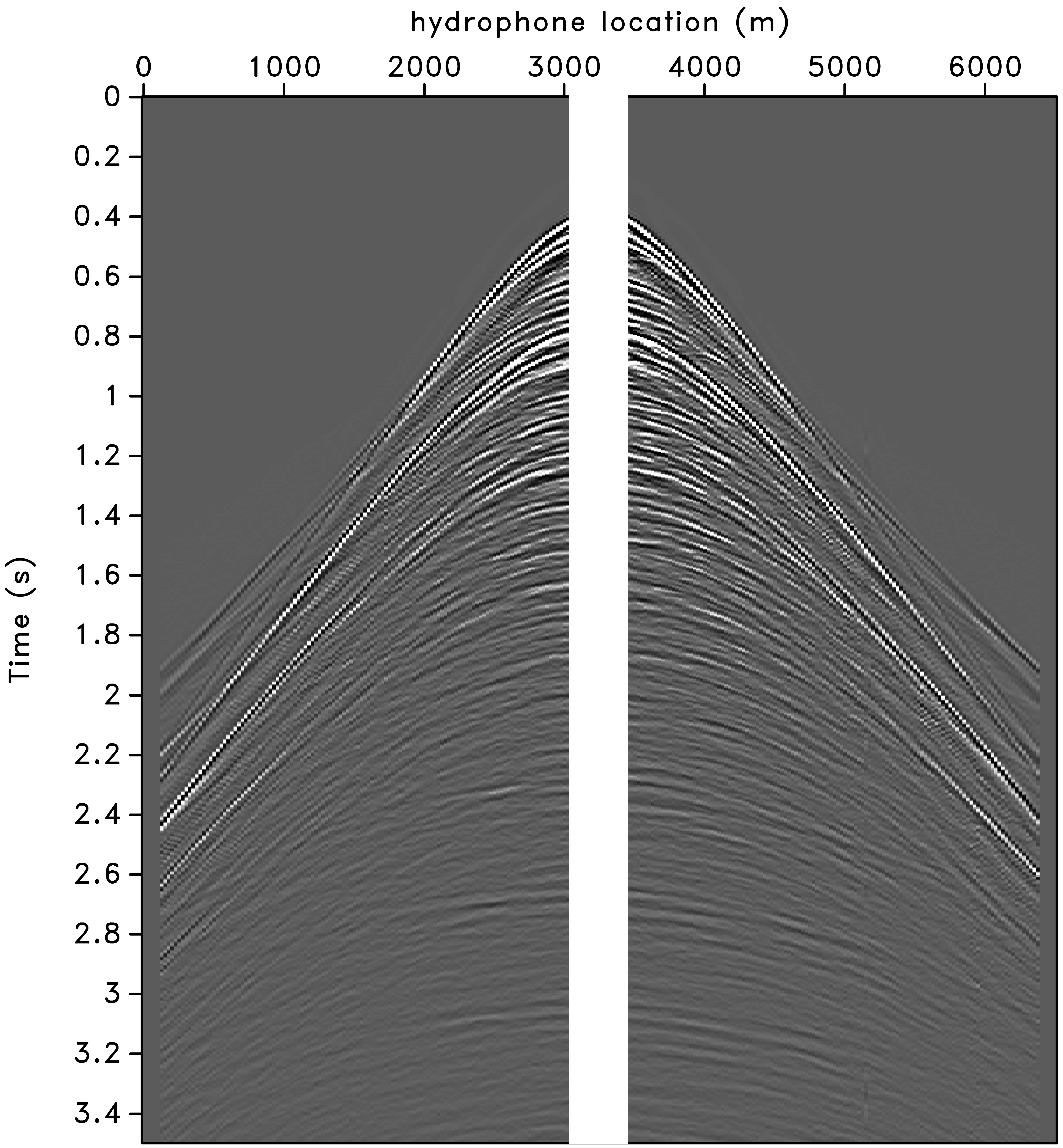


Trace Mask

Masking operator $\mathbf{K}$

Time domain: $\mathbf{K}\mathbf{p}$

Frequency slices: $\mathbf{K} \circ \mathbf{P}$

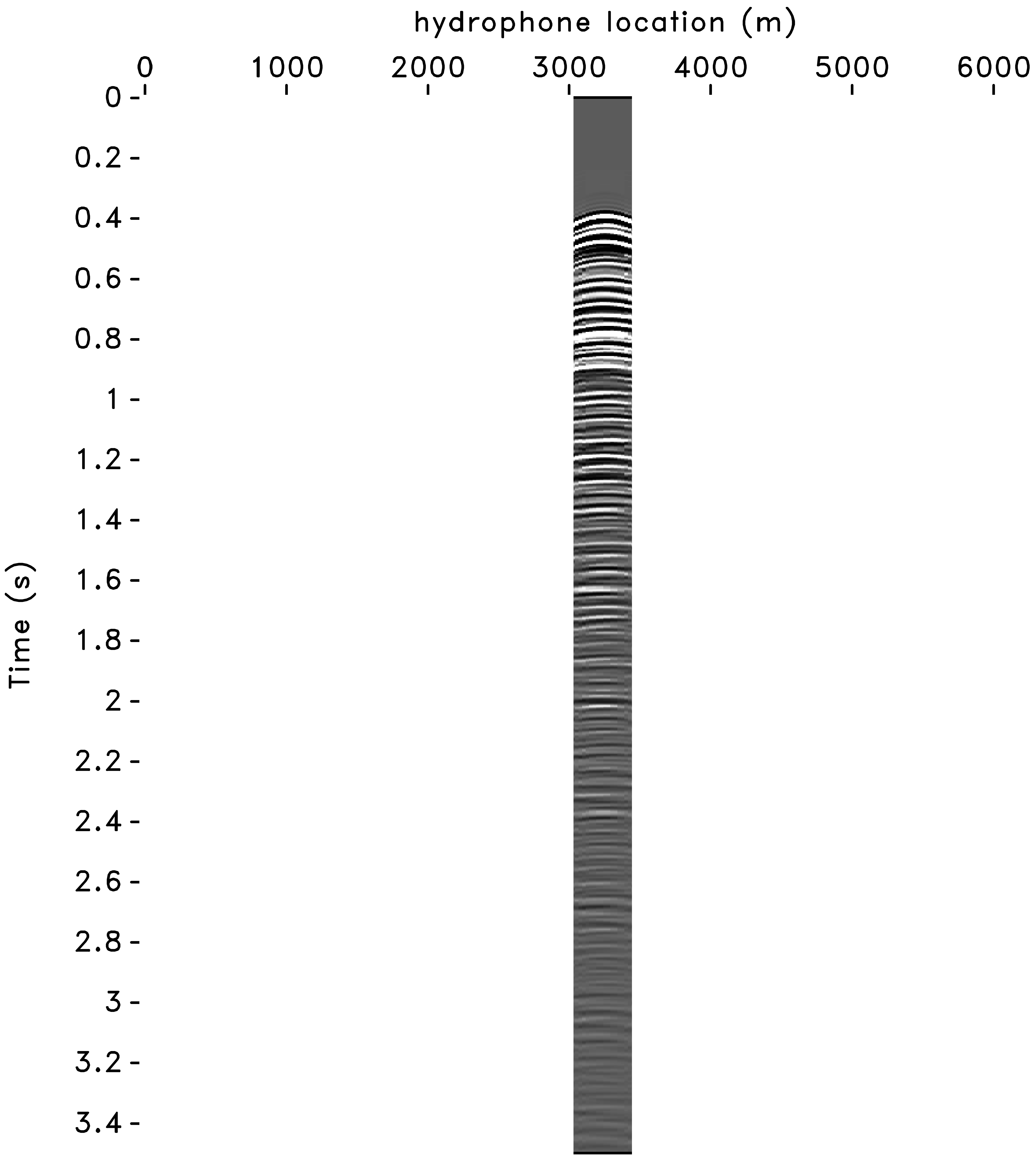


Trace Mask

Complement of Masking operator $\mathbf{K}_c$

Time domain: $\mathbf{K}_c\mathbf{p}$

Frequency slices: $\mathbf{K}_c \circ \mathbf{P}$



SLIM

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Unknown data traces: $\mathbf{P}'' := \mathbf{K_c} \circ \mathbf{P}$

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Unknown data traces:
$$
\begin{aligned}
\mathbf{P}'' &:= \mathbf{K_c} \circ \mathbf{P} \\
&= \mathbf{K_c} \circ \left(\mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''\right)
\end{aligned}
$$

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

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Unknown data traces:
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Highly dependent on G

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Unknown data traces:
$$\begin{aligned}\mathbf{P}'' &:= \mathbf{K_c} \circ \mathbf{P} \\ &= \mathbf{K_c} \circ \left(\mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''\right)\end{aligned}$$

Recursively defined

Modify the modeling operator

$$\mathcal{M}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}' + \mathbf{R}\mathbf{G}\mathbf{P}''$$

Modify the modeling operator

$$
\begin{aligned}
\widetilde{\mathcal{M}}(\mathbf{G}, \mathbf{Q}; \mathbf{P}') = &\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}' \\
&+ \mathbf{K} \circ \mathbf{R}\mathbf{G}\mathbf{K}_{\mathbf{c}} \circ (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}') \\
&+ \mathcal{O}(\mathbf{G}^3)
\end{aligned}
$$

SLIM

Modify the modeling operator

$$\widetilde{\mathcal{M}}(\mathbf{G}, \mathbf{Q}; \mathbf{P}') = \mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}'$$

2nd Order autoconvolution term $$+ \; \mathbf{K} \circ \mathbf{R}\mathbf{G}\mathbf{K}_{\mathbf{c}} \circ (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

$$+ \; \mathcal{O}(\mathbf{G}^3)$$

Modify the modeling operator

$$\begin{aligned}\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') =&\mathbf{K}\circ\big[\mathbf{GQ}+\mathbf{RGP}' \\ &+\ \mathbf{RGK_c}\circ(\mathbf{GQ}+\mathbf{RGP}') \\ &+\ \mathcal{O}(\mathbf{G}^3)\big]\end{aligned}$$

Modify the modeling operator

$$\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{K}\circ\Big[\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}'$$

2nd Order autoconvolution term $$+\ \mathbf{R}\mathbf{G}\mathbf{K}_\mathbf{c}\circ(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}')$$

3rd Order autoconvolution term $$+\ \mathbf{R}\mathbf{G}\mathbf{K}_\mathbf{c}\circ(\mathbf{R}\mathbf{G}\mathbf{K}_\mathbf{c}\circ(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}'))$$

$$+\ \mathcal{O}(\mathbf{G}^4)\Big]$$

$$:= \mathbf{K}\circ\sum_{n=0}^{\infty}(\mathbf{R}\mathbf{G}\mathbf{K}_\mathbf{c}\circ)^n(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}')\,.$$

Convergent sum

$$\mathbf{K} \circ \mathbf{P} = \mathbf{K} \circ \sum_{n=0}^{\infty} (\mathbf{R}\mathbf{G}\mathbf{K_c}\circ)^{n} (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

$$= \mathbf{K} \circ (\mathbf{I} - \mathbf{R}\mathbf{G}\mathbf{K_c}\circ)^{-1} (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

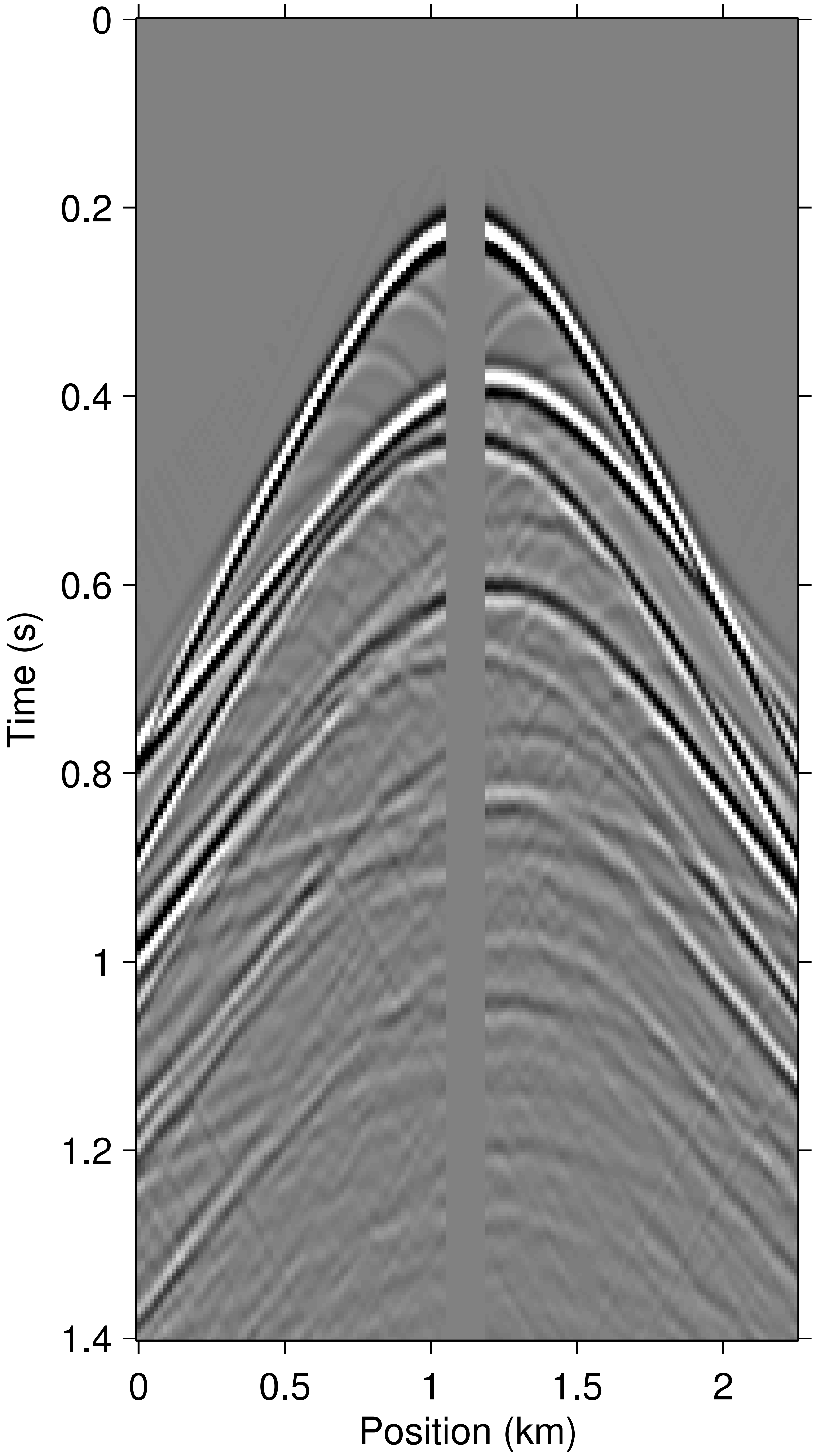
Convergent sum

$$\mathbf{K} \circ \mathbf{P} = \mathbf{K} \circ \sum_{n=0}^{\infty} (\mathbf{R}\mathbf{G}\mathbf{K}_{\mathbf{c}} \circ)^{n} (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

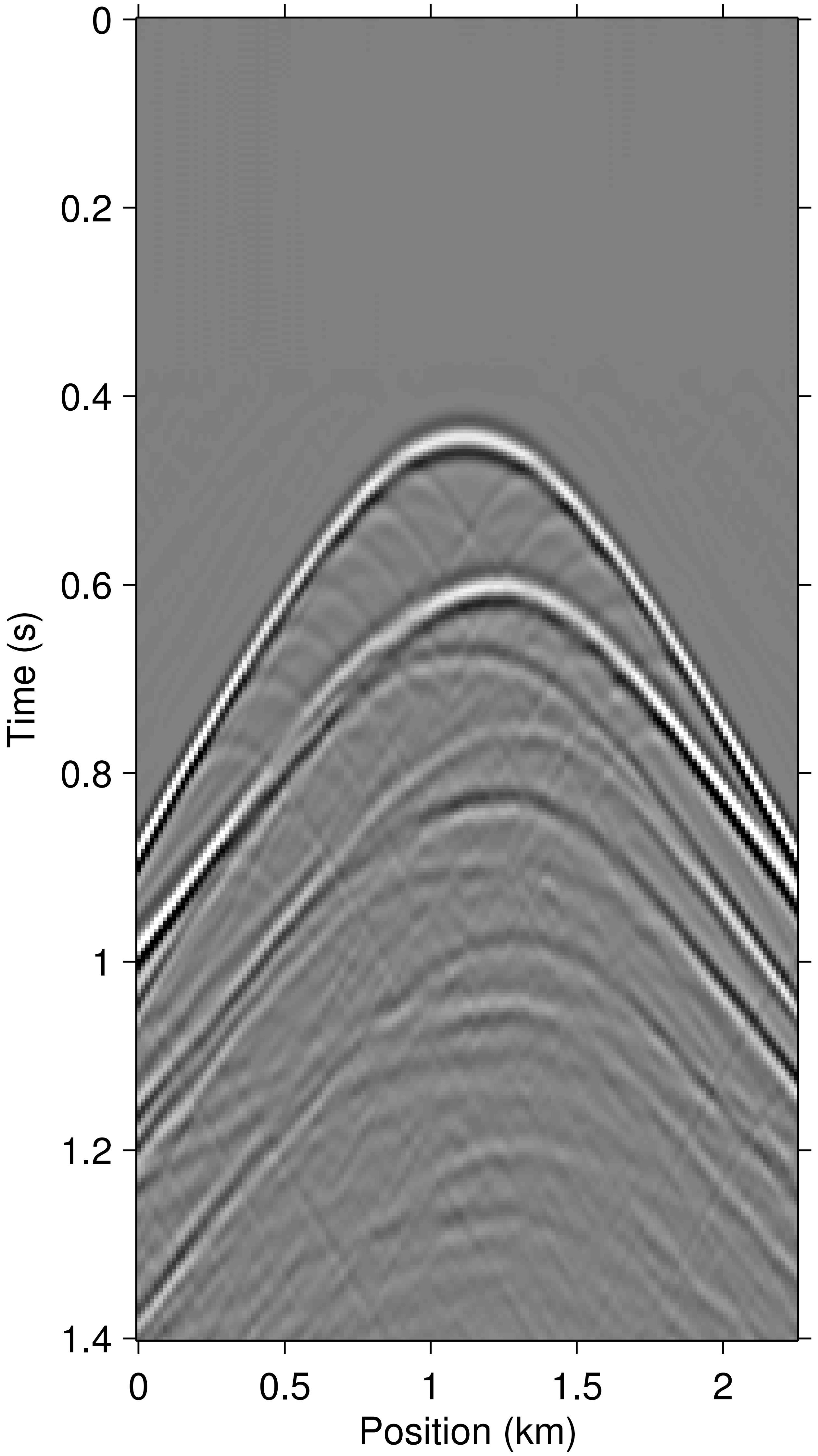
$$= \mathbf{K} \circ (\mathbf{I} - \mathbf{R}\mathbf{G}\mathbf{K}_{\mathbf{c}} \circ)^{-1} (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

Verifies the validity of the expression

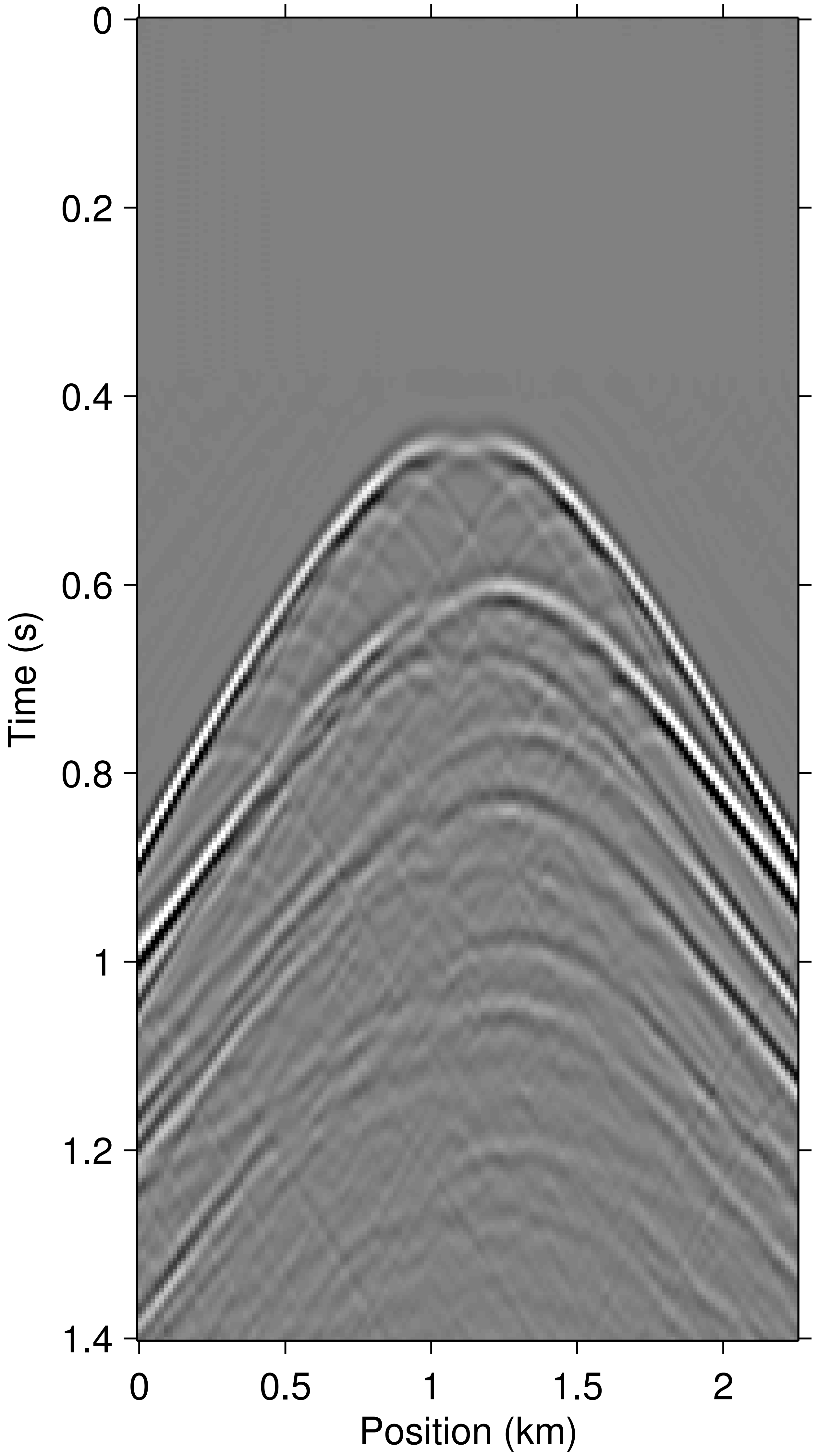
What these terms look like



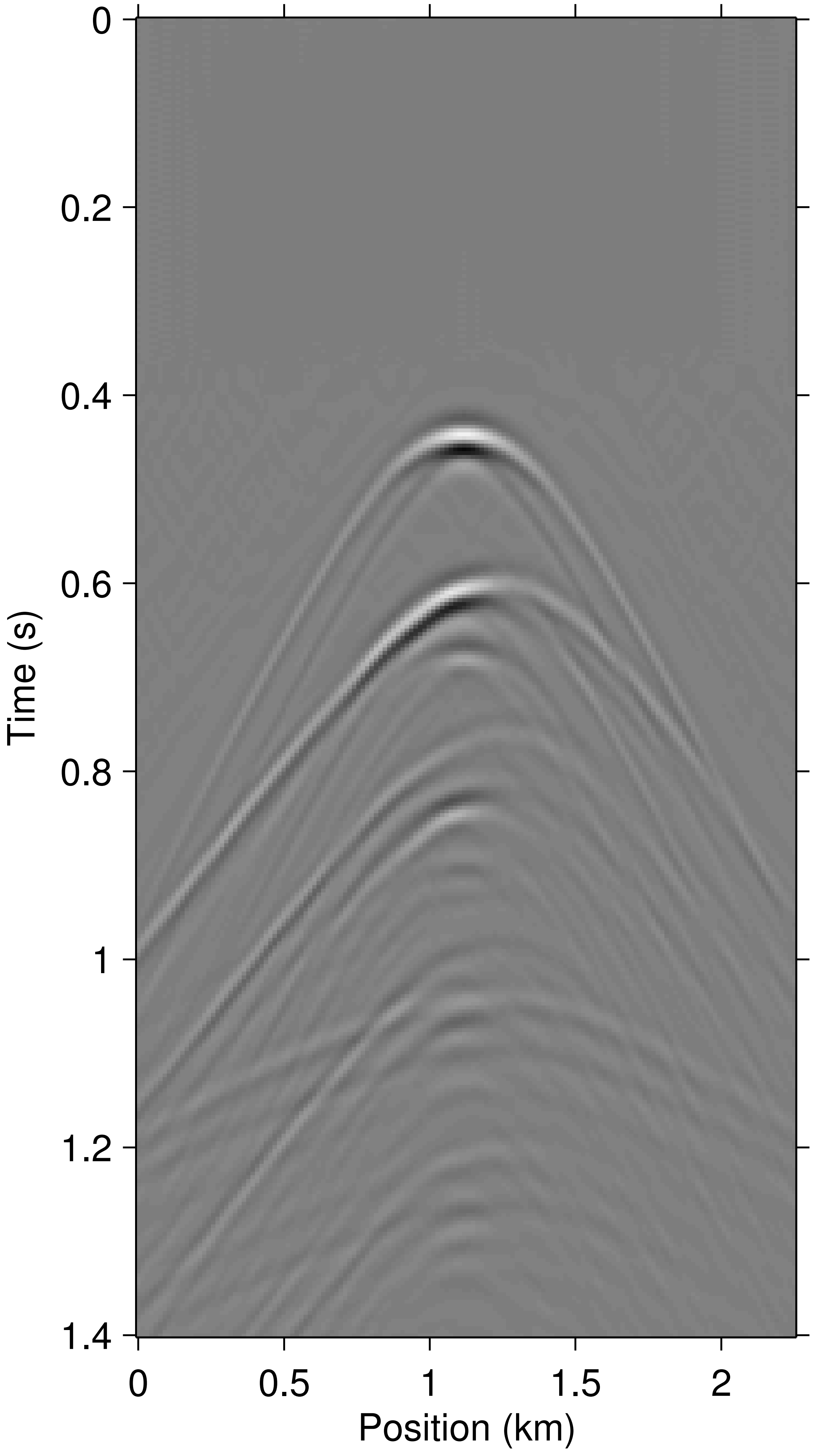
Data with missing traces P’



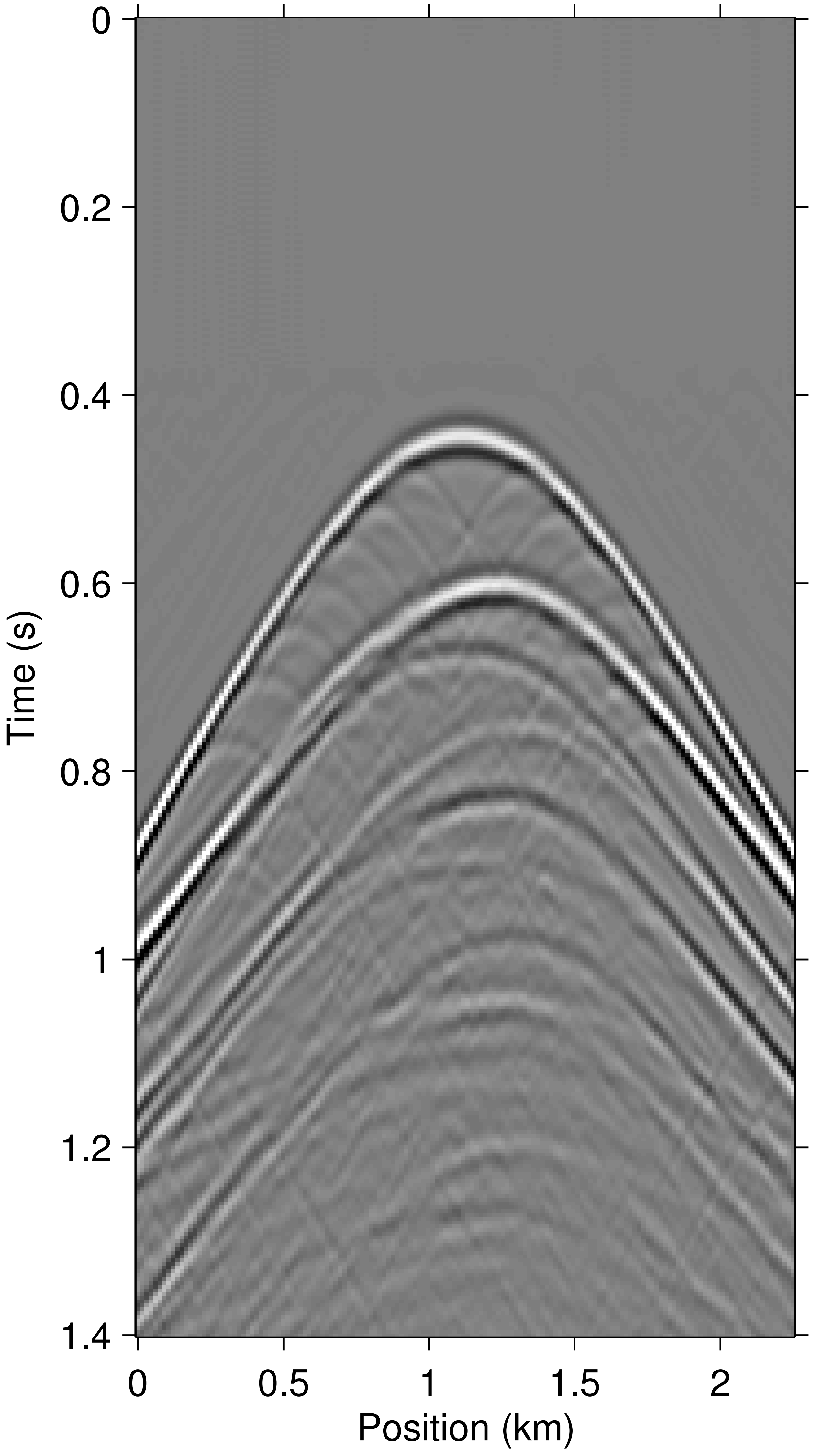
Exact multiples



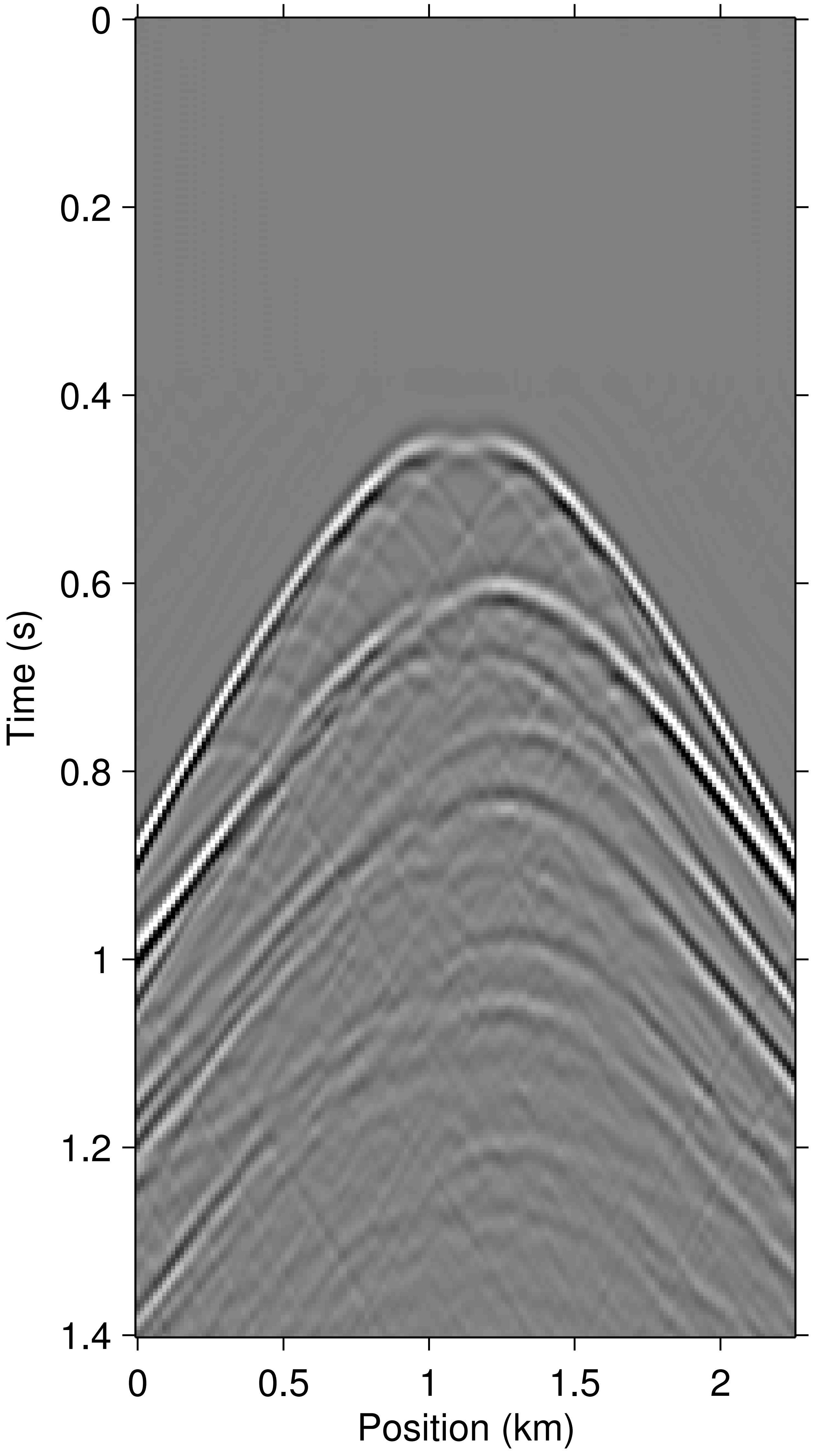
-GP’



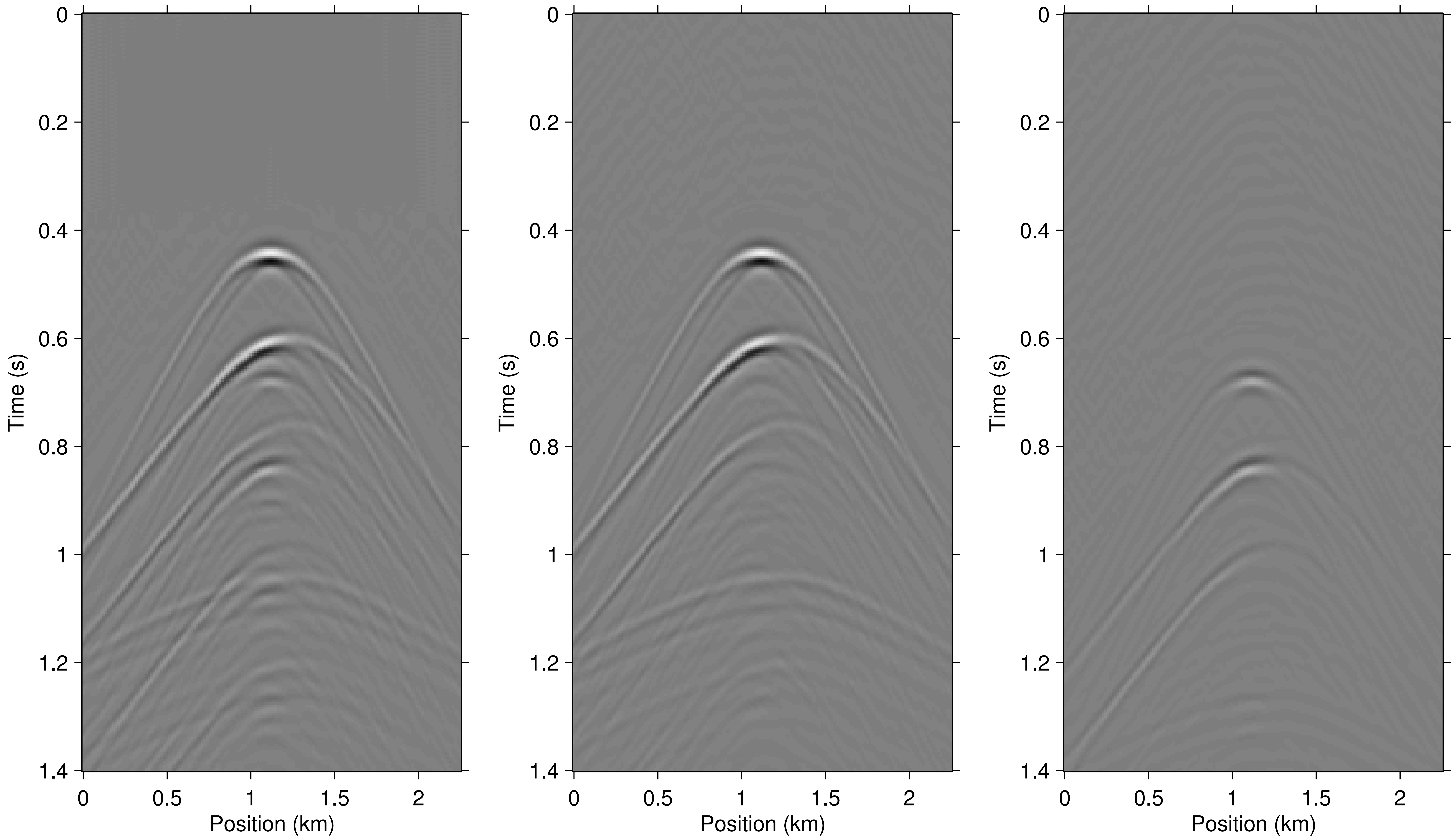
Missing contributions



Exact multiples



-GP’



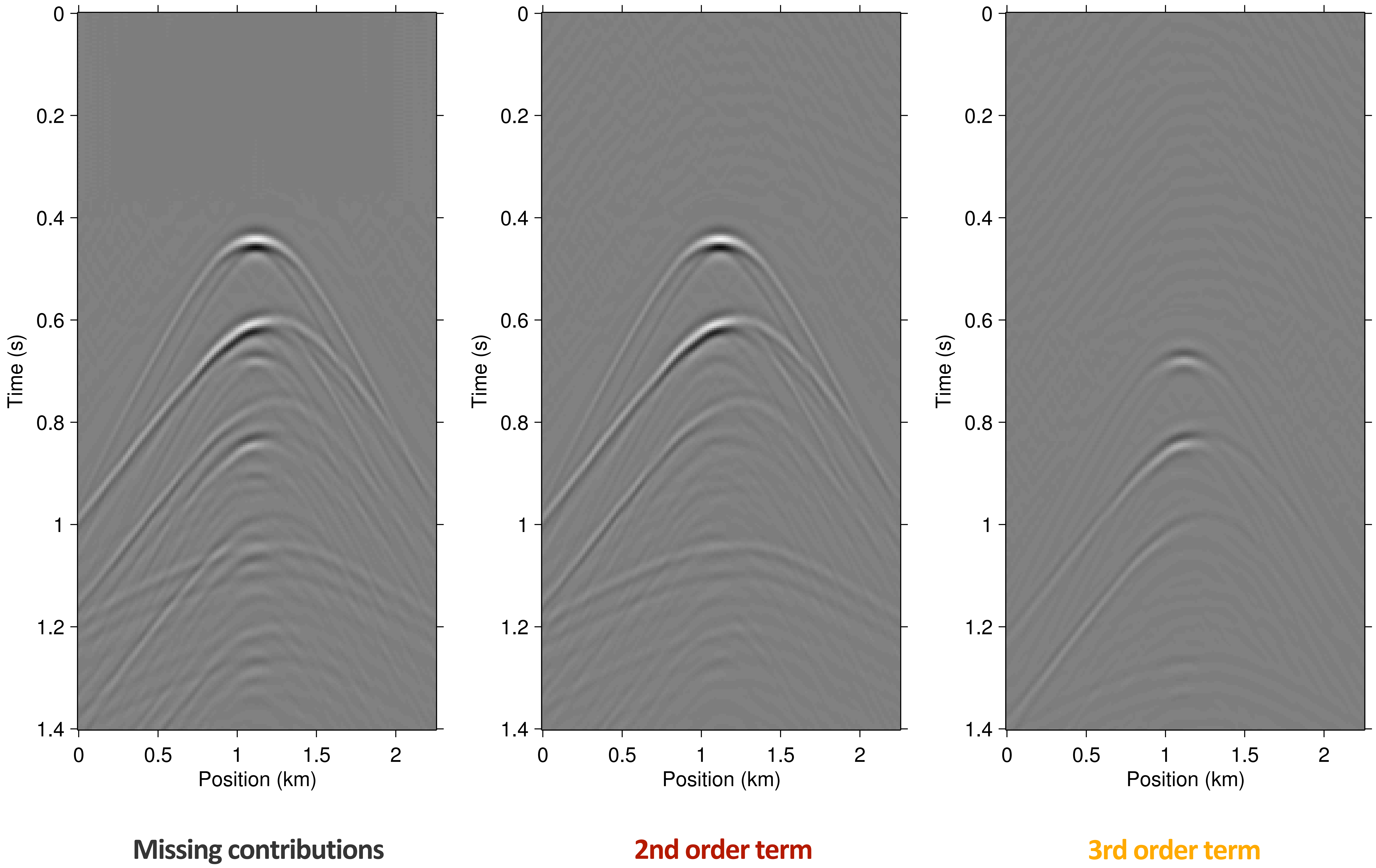
Missing contributions

2nd order term

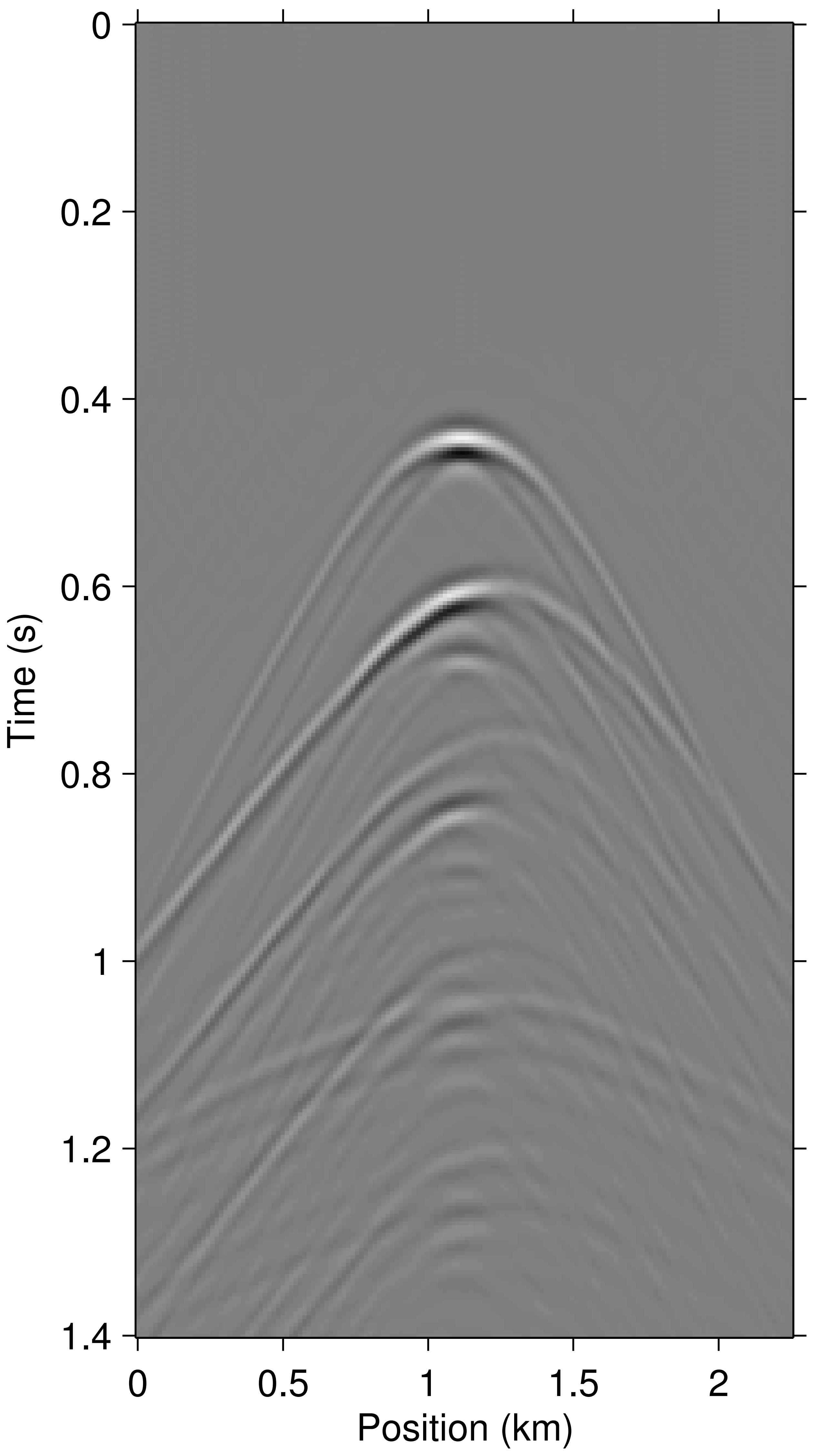
3rd order term

Modify the modeling operator

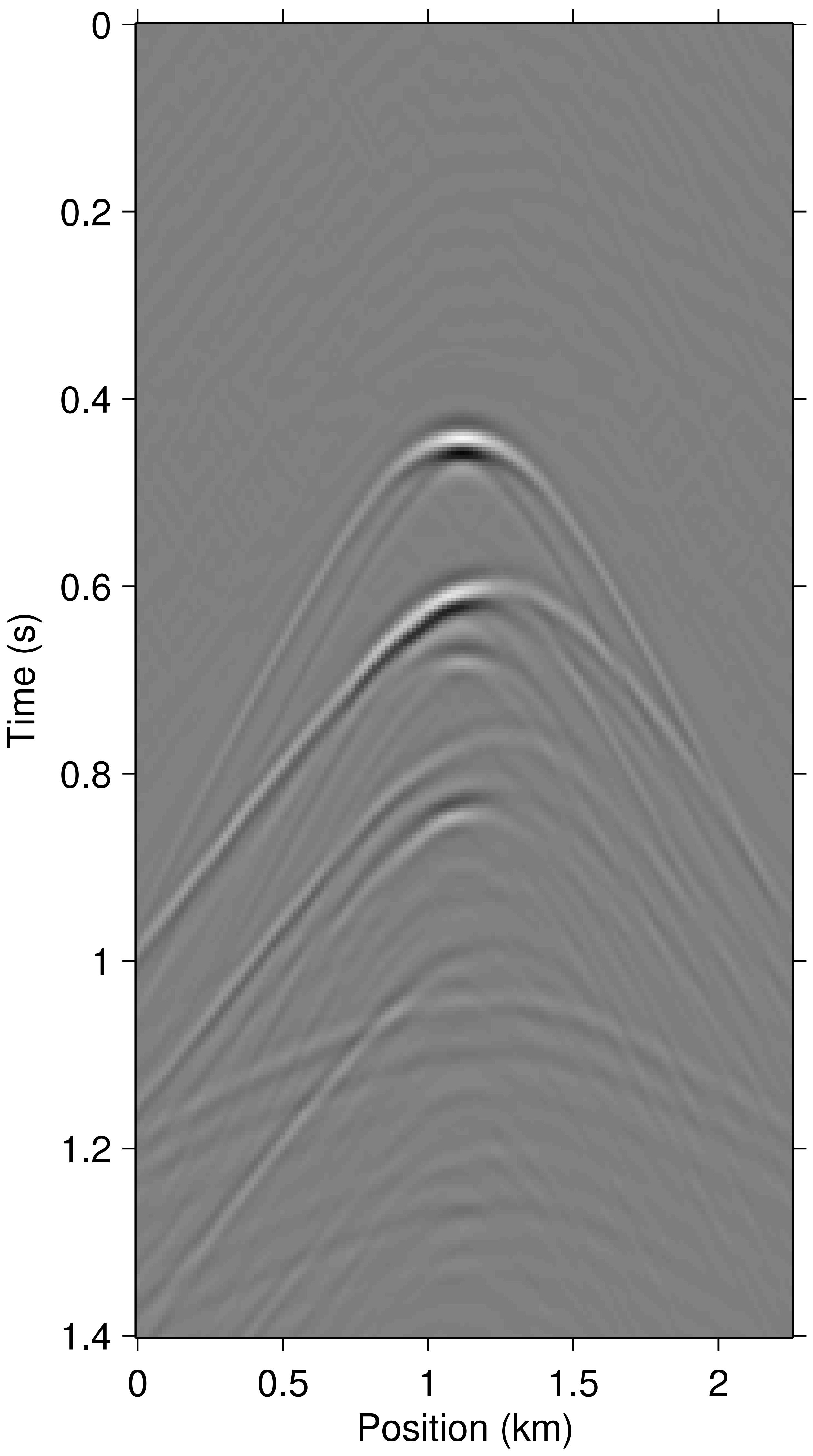
$$
\begin{aligned}
\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') =&\mathbf{K}\circ\Big[\mathbf{GQ}+\mathbf{RGP}' \\
\text{2nd Order autoconvolution term} \quad &+\ \mathbf{RGK_c}\circ(\mathbf{GQ}+\mathbf{RGP}') \\
\text{3rd Order autoconvolution term} \quad &+\ \mathbf{RGK_c}\circ(\mathbf{RGK_c}\circ(\mathbf{GQ}+\mathbf{RGP}')) \\
&+\ \mathcal{O}(\mathbf{G}^4)\Big] \\
:=&\mathbf{K}\circ\sum_{n=0}^{\infty}(\mathbf{RGK_c}\circ)^n\,(\mathbf{GQ}+\mathbf{RGP}')\,.
\end{aligned}
$$



Time (s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.5
1
1.5
2
Position (km)
Missing contributions
2nd order term
3rd order term

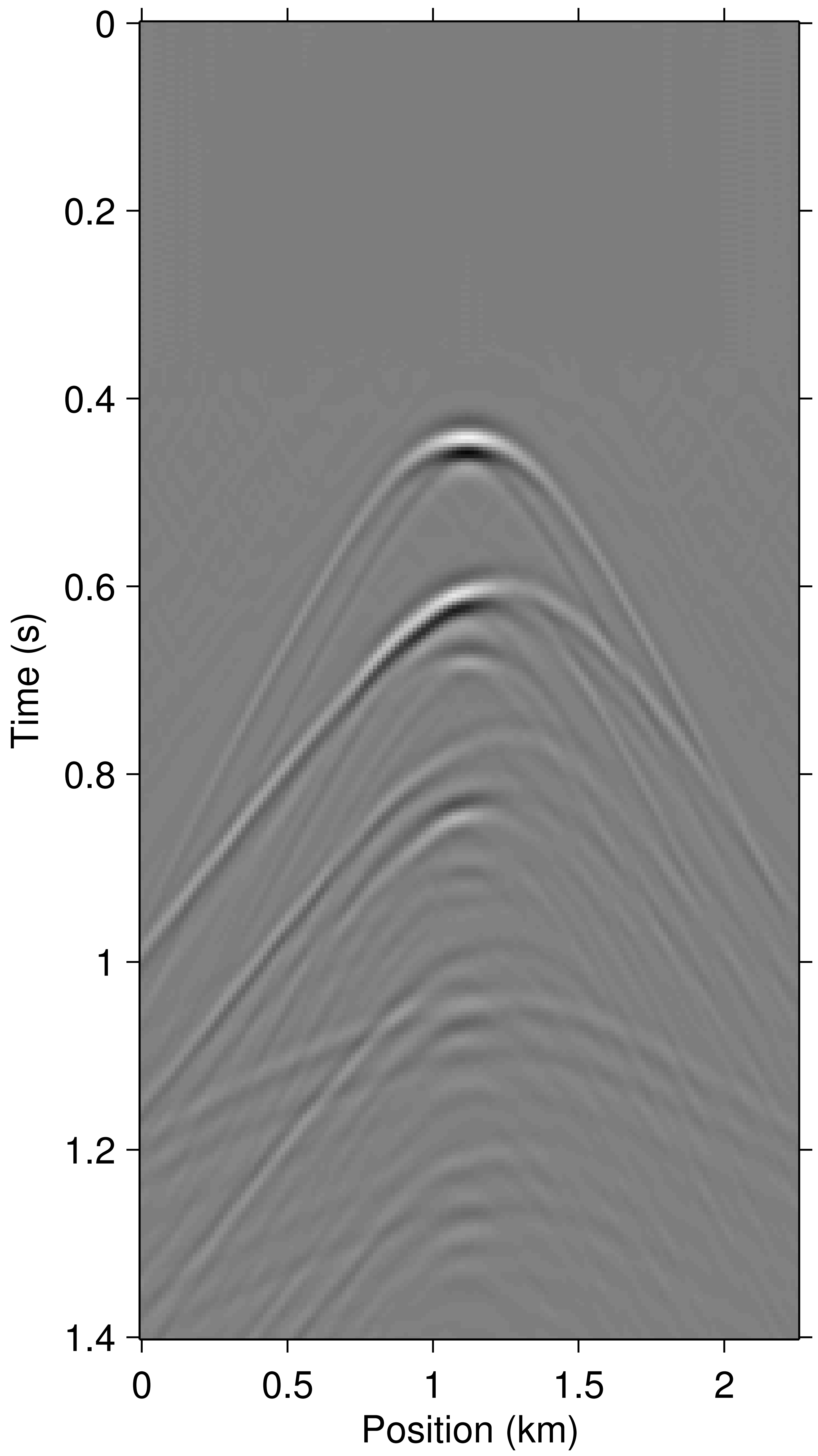


Missing contributions

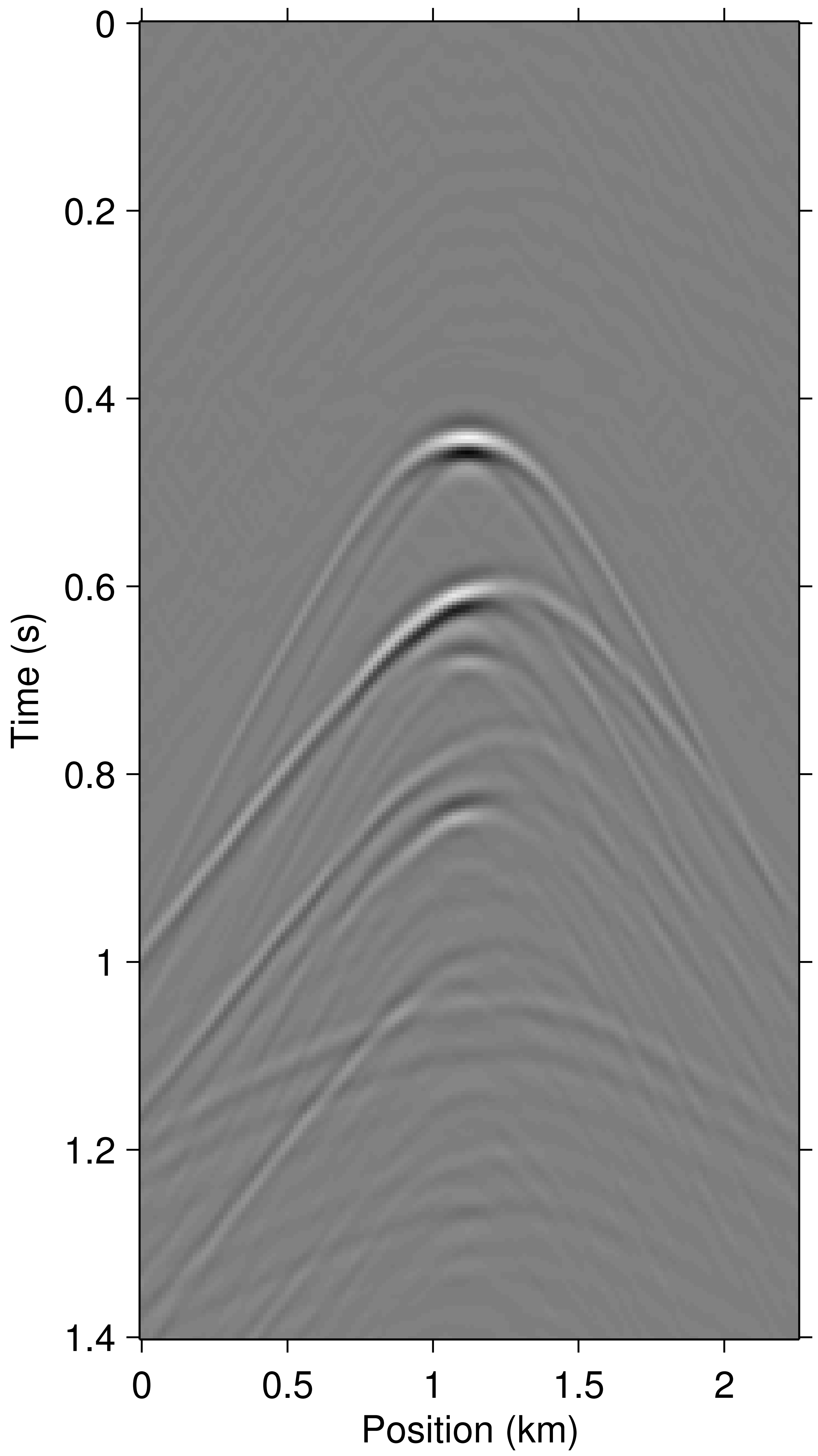


2nd order term + 3rd order

Total missing contrib.



Modeled with two terms



Solution strategy

Robust EPSI
Inverting for unknown data

$$\Delta \mathbf{P}(\mathbf{G}_{k+1}, \mathbf{R}_{k+1}) := -(\mathbf{I} + \mathbf{G}_{k+1})^H (\mathbf{R}_{k+1})$$

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p}_k - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha \Delta \mathbf{p}(\mathbf{g}_{k+1}, \mathbf{q}_{k+1}, \mathbf{p}_k)$$

Robust EPSI
Accounting for unknown data with G

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

Strategy 1: Re-linearization
Use G from previous iter in higher-order terms

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

use $\mathbf{g_k}$ in these operators

Strategy 2: Modified Gauss-Newton
Obtain Jacobian using G from previous iter

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

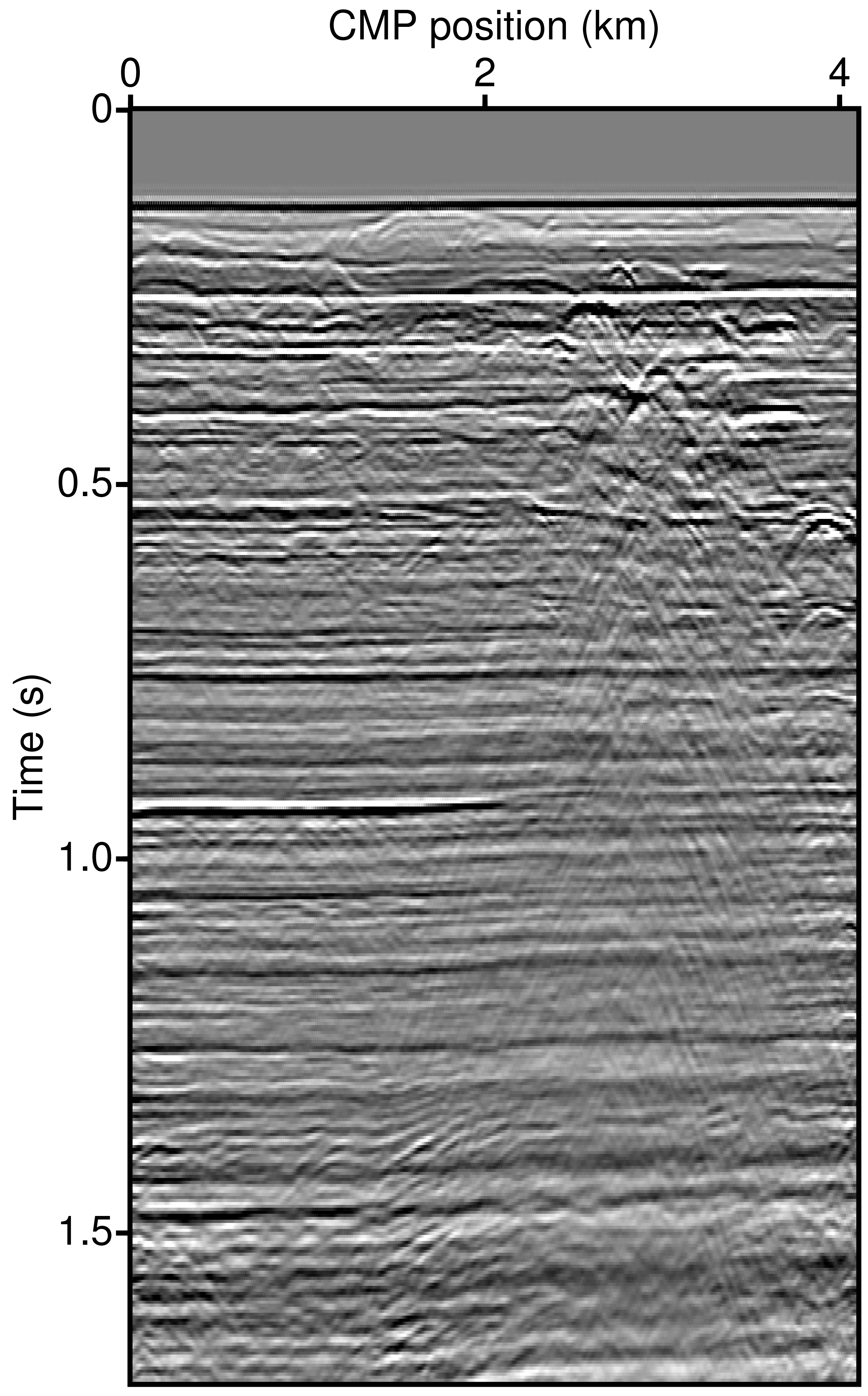
determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \arg\min_{\Delta\mathbf{g}} \|\mathbf{r}_k - \partial_{(g_k, q_k)}\widetilde{\mathcal{M}}\Delta\mathbf{g}\|_2 \text{ s.t. } \|\Delta\mathbf{g}\|_1 \leq \tau_k$$

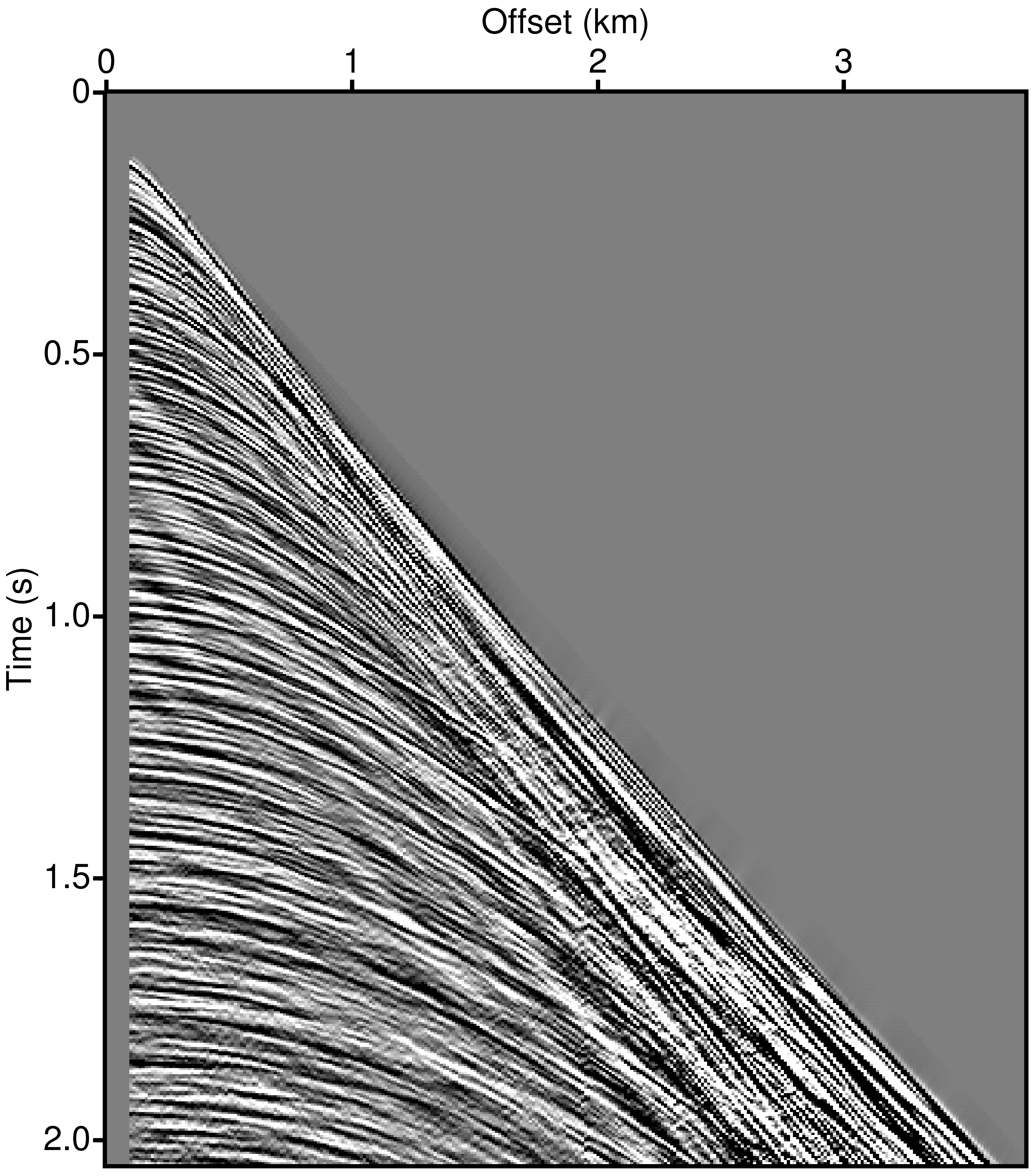
$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

Field data example

North Sea dataset



Data



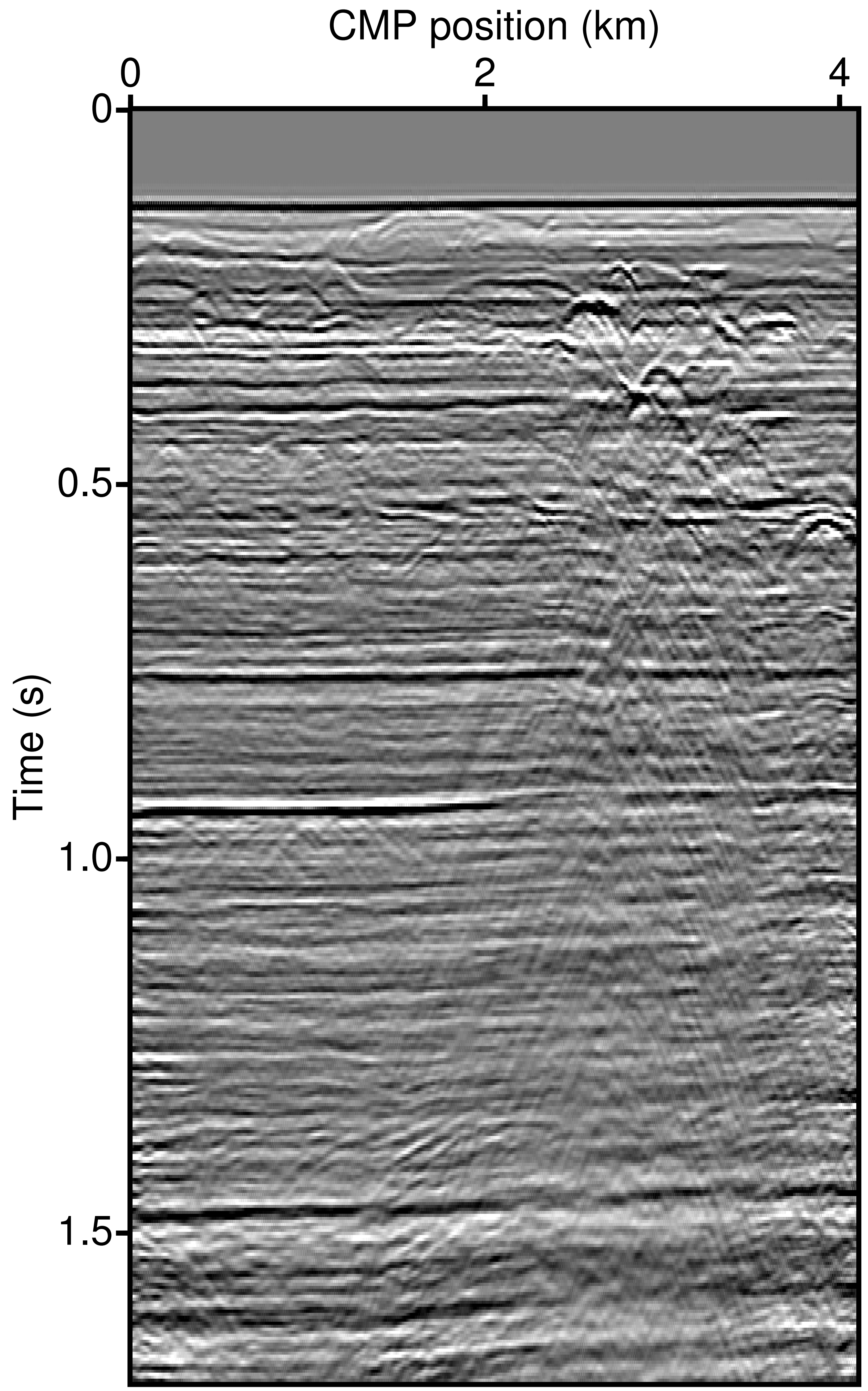
Shot

North Sea dataset

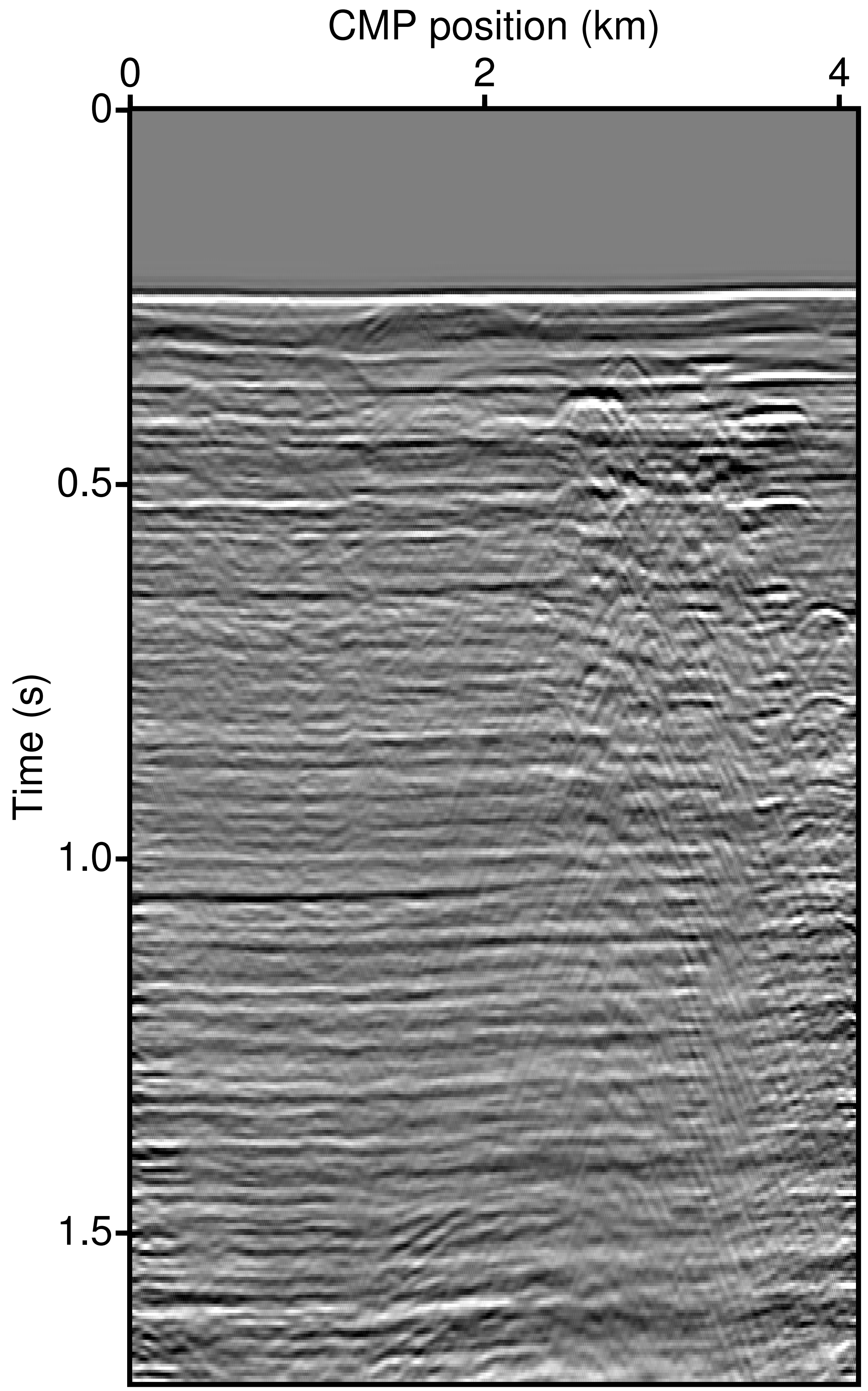
100m near-offset

regularized to 12.5m dx and 4km fixed-spread from streamer

4ms sampling



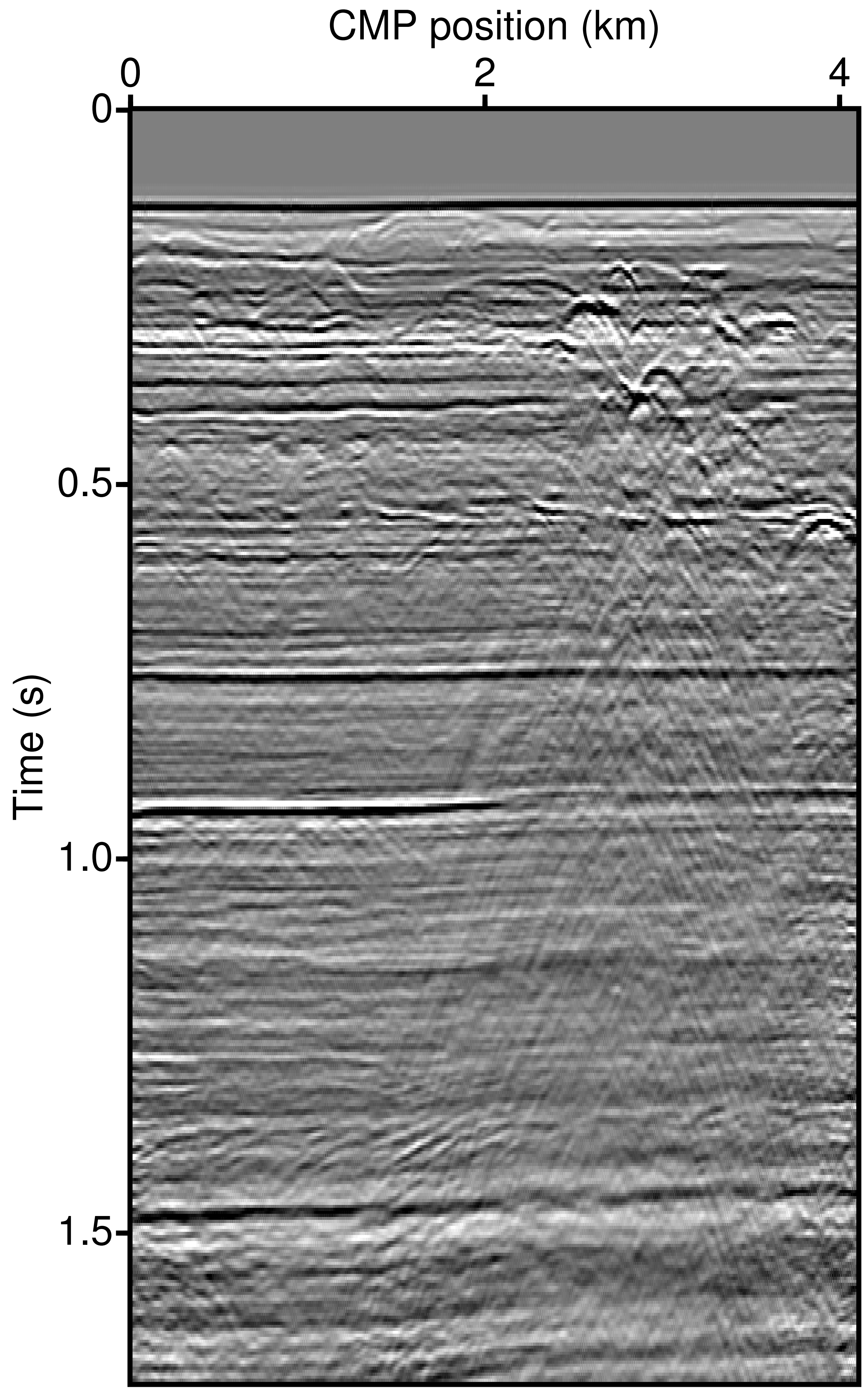
Conservative primary



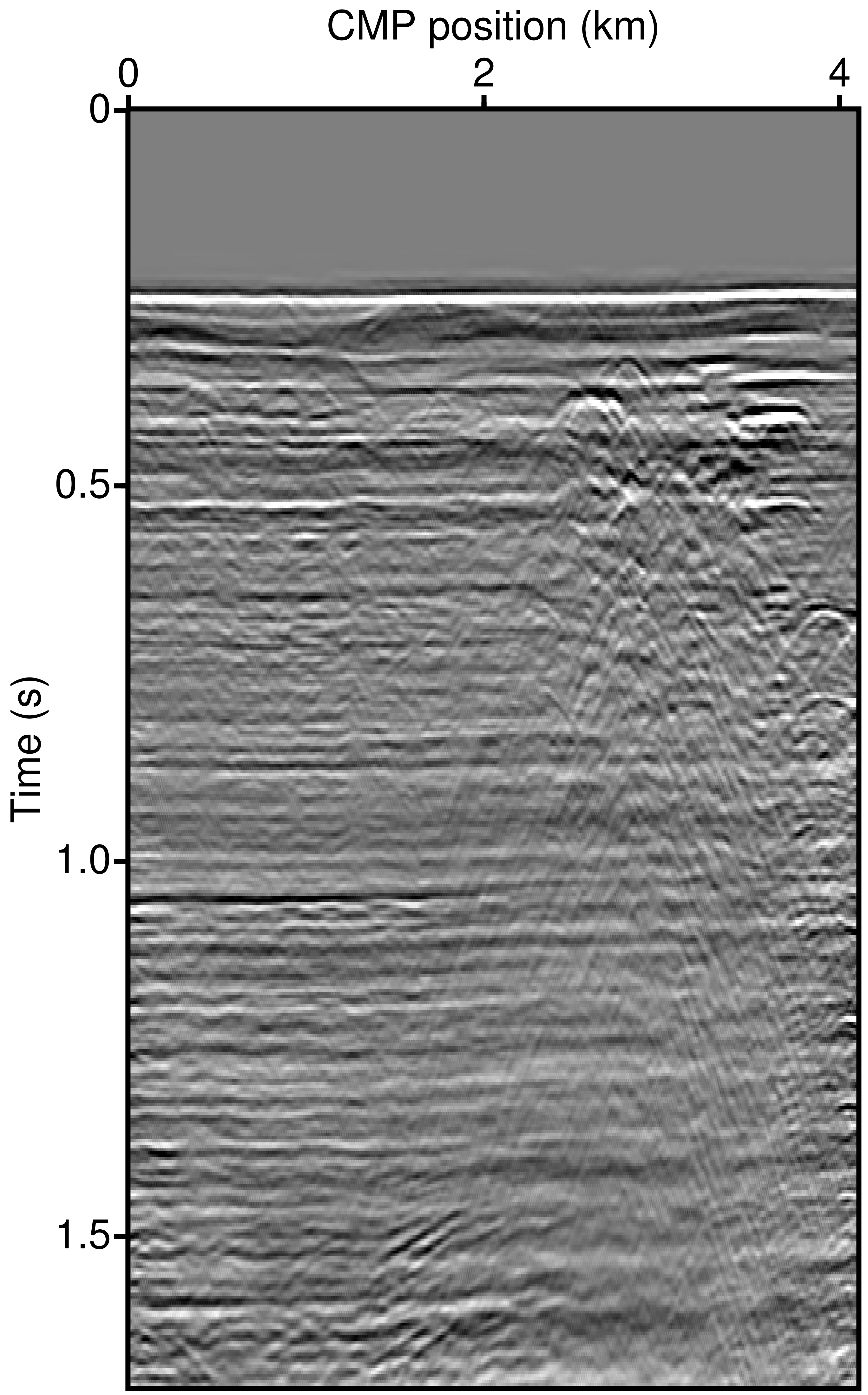
Multiple

NMO stack

Parabolic Radon Interp

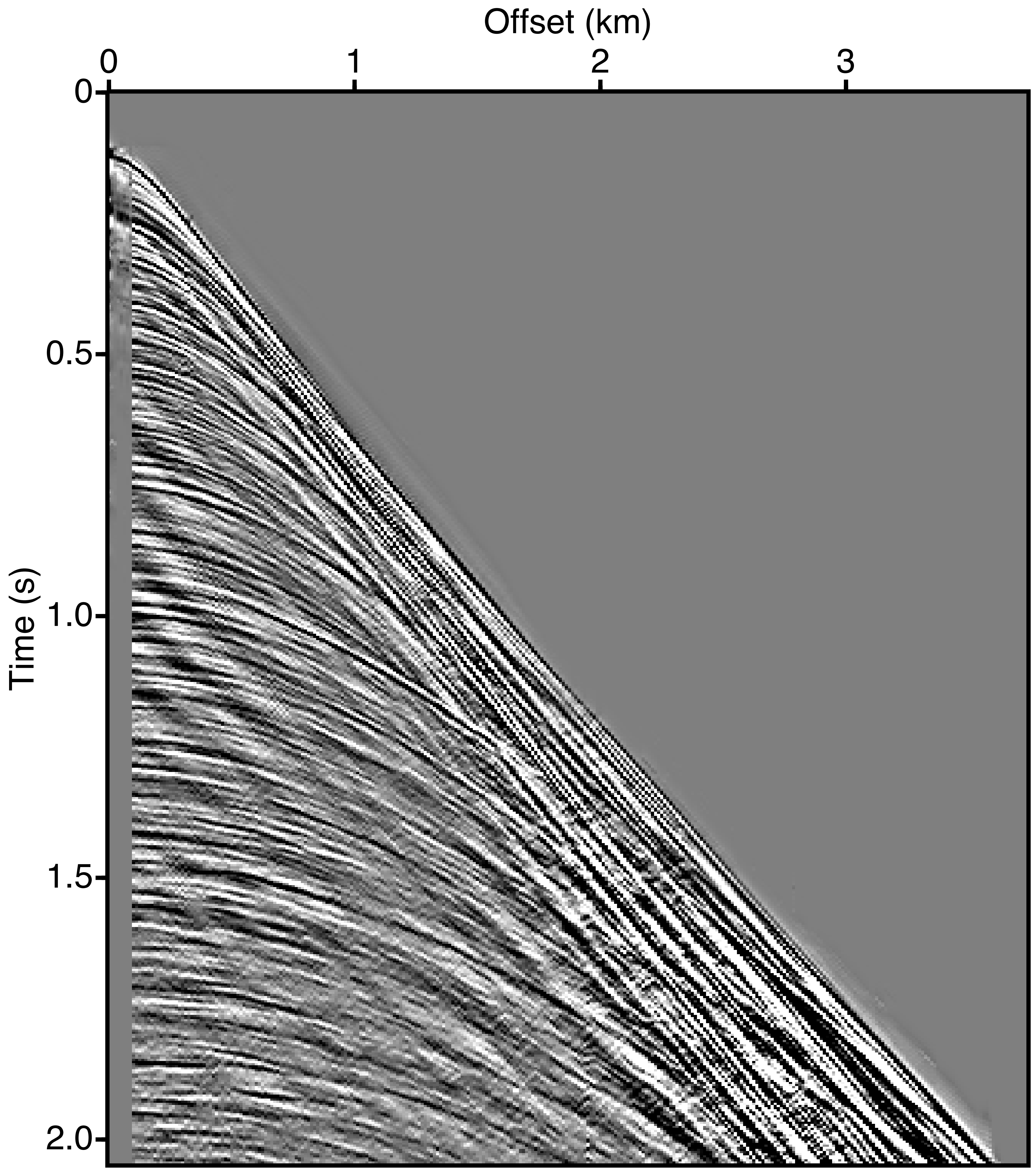


Conservative primary

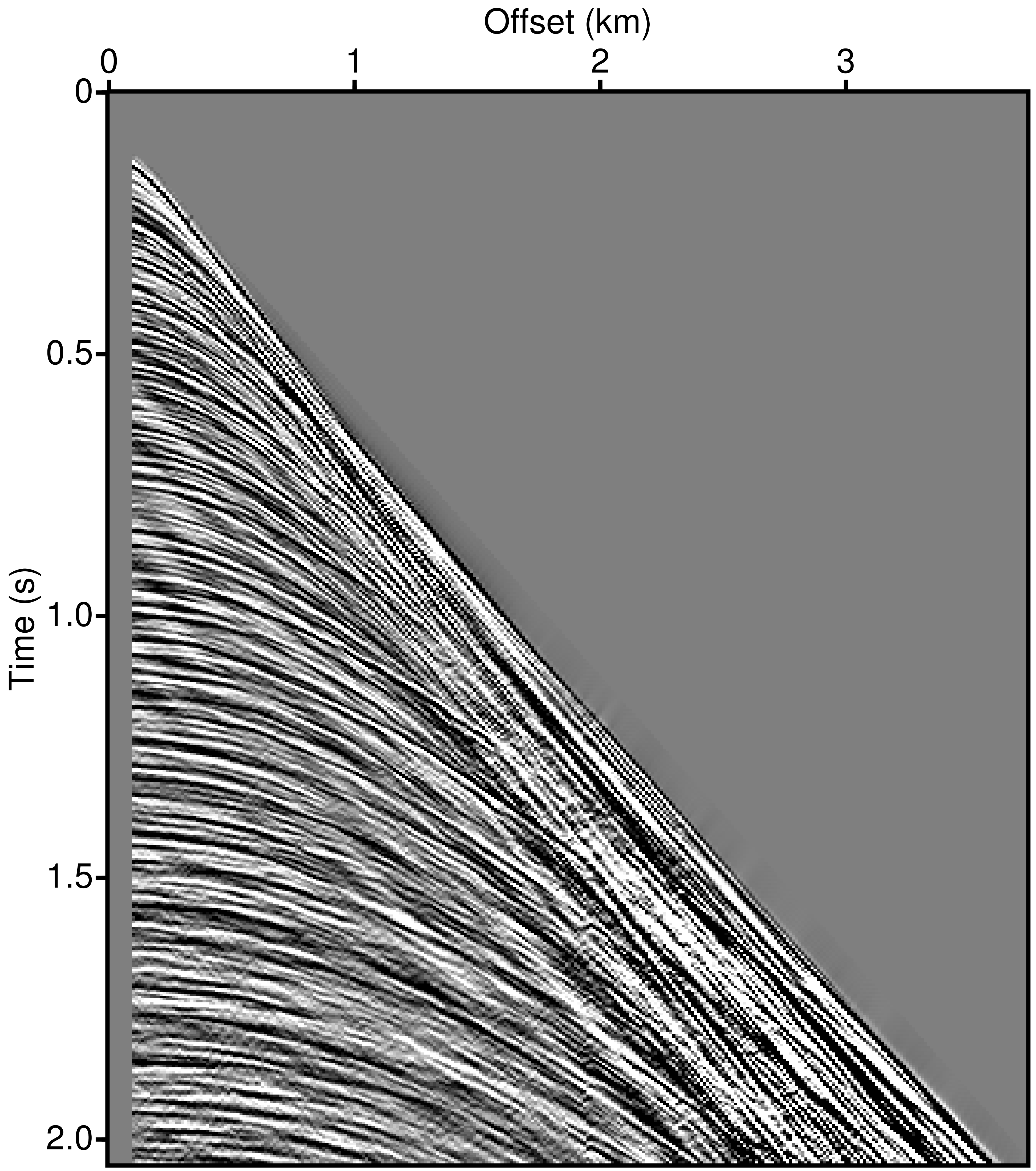


Multiple

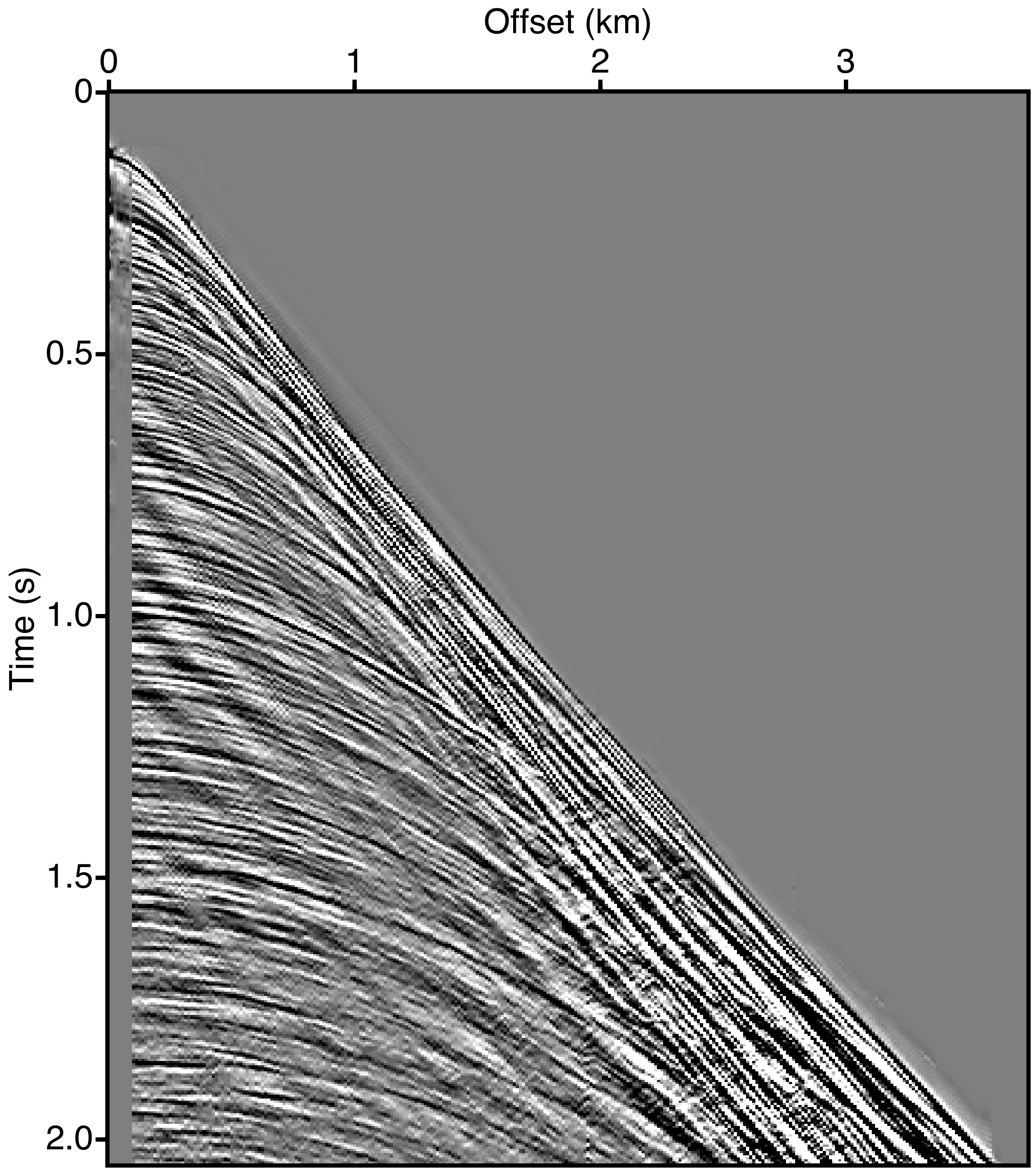
NMO stack
Re-linearization
Using 3rd Order terms



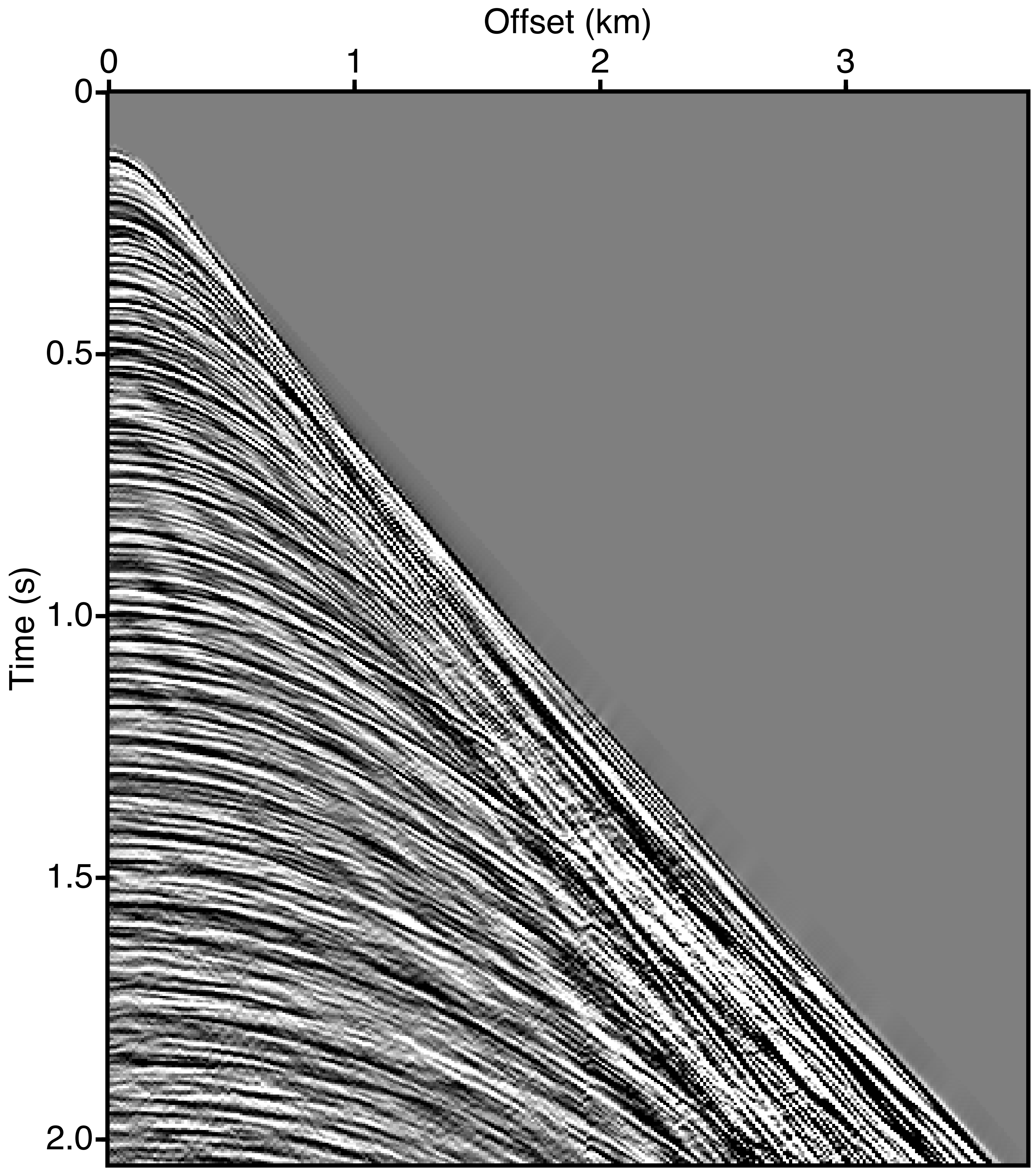
Direct primary



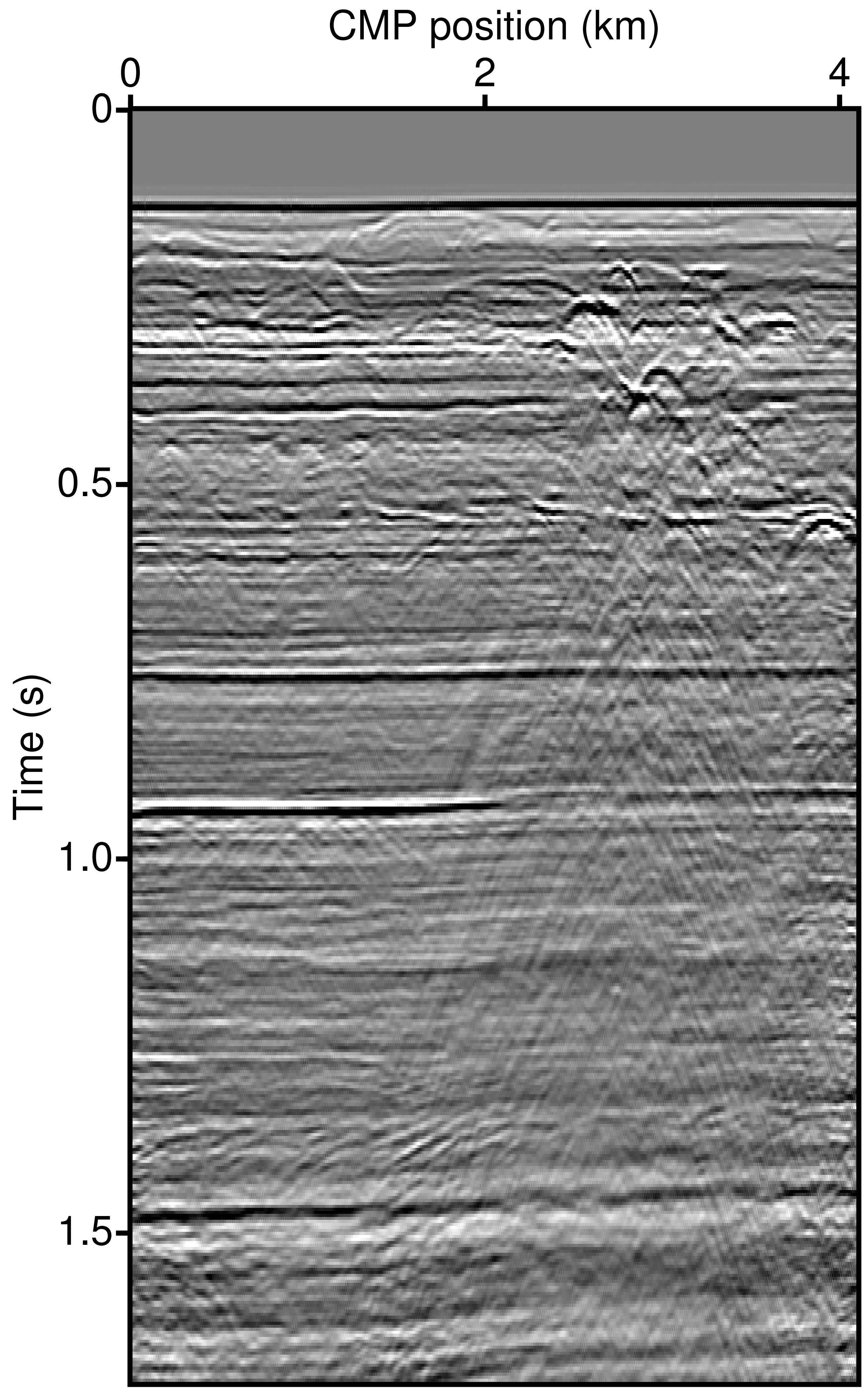
Original shots



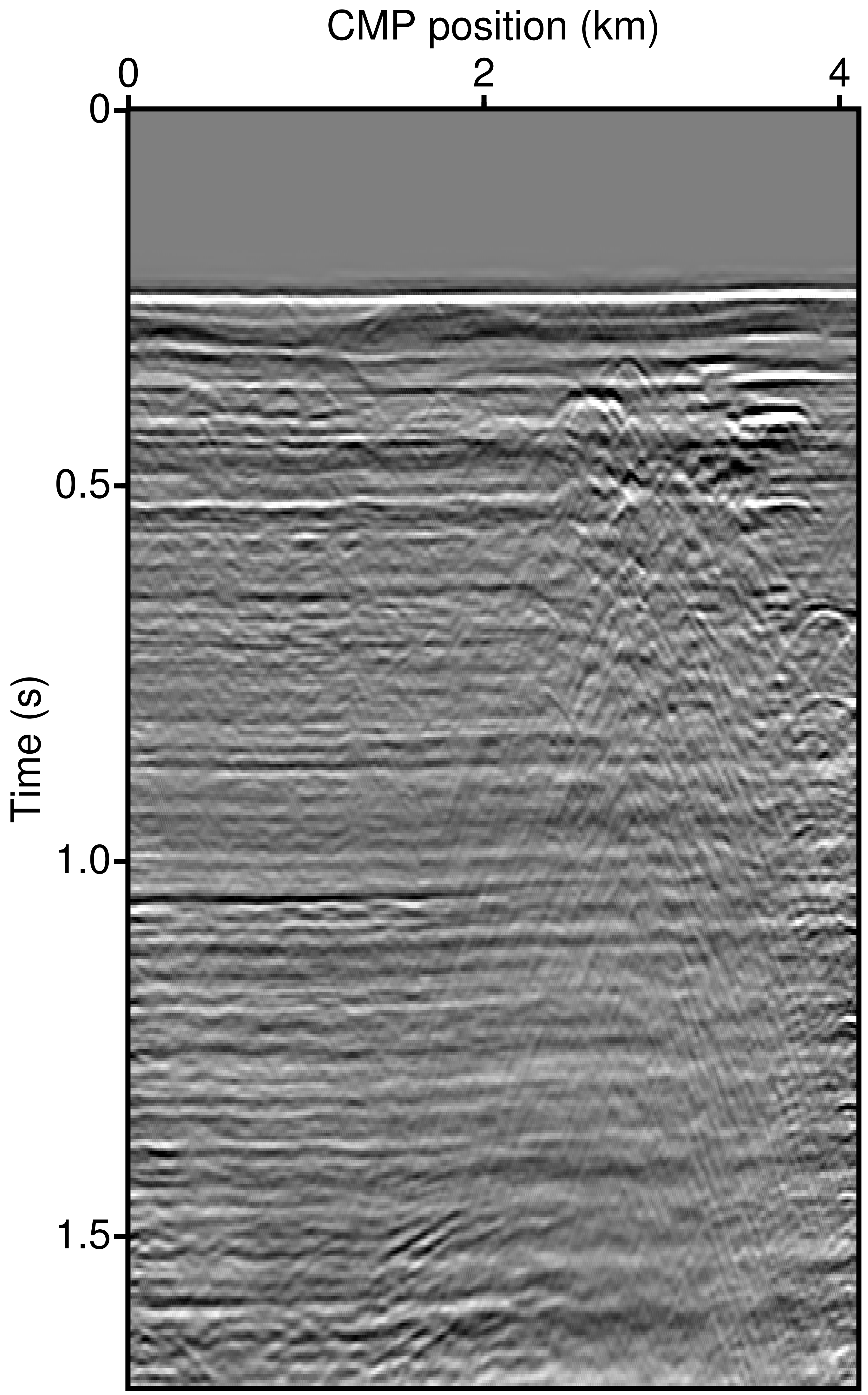
Direct primary shot



Interpolated shot



Conservative primary

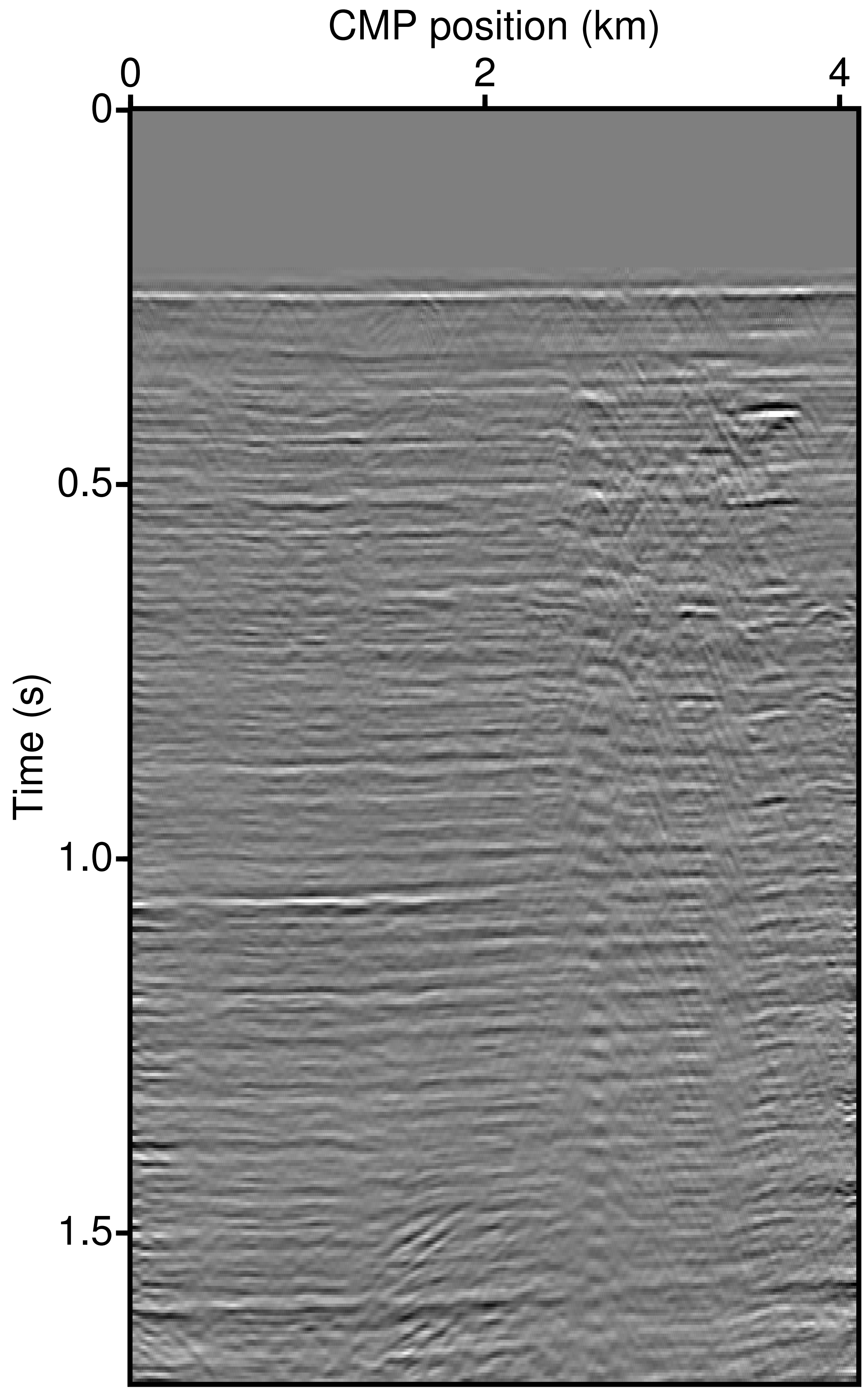


Multiple

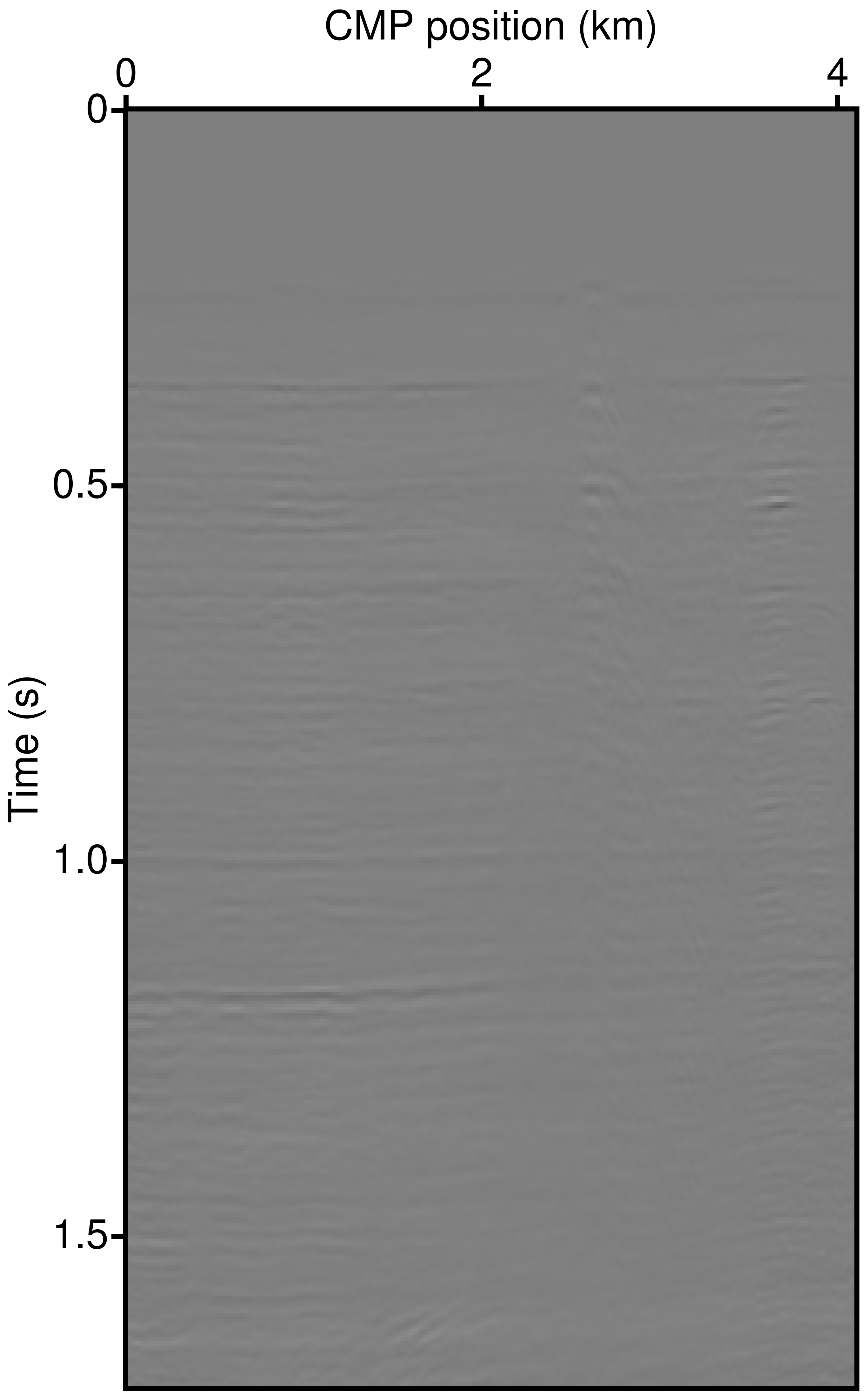
NMO stack

Re-linearization

Using 2nd Order terms

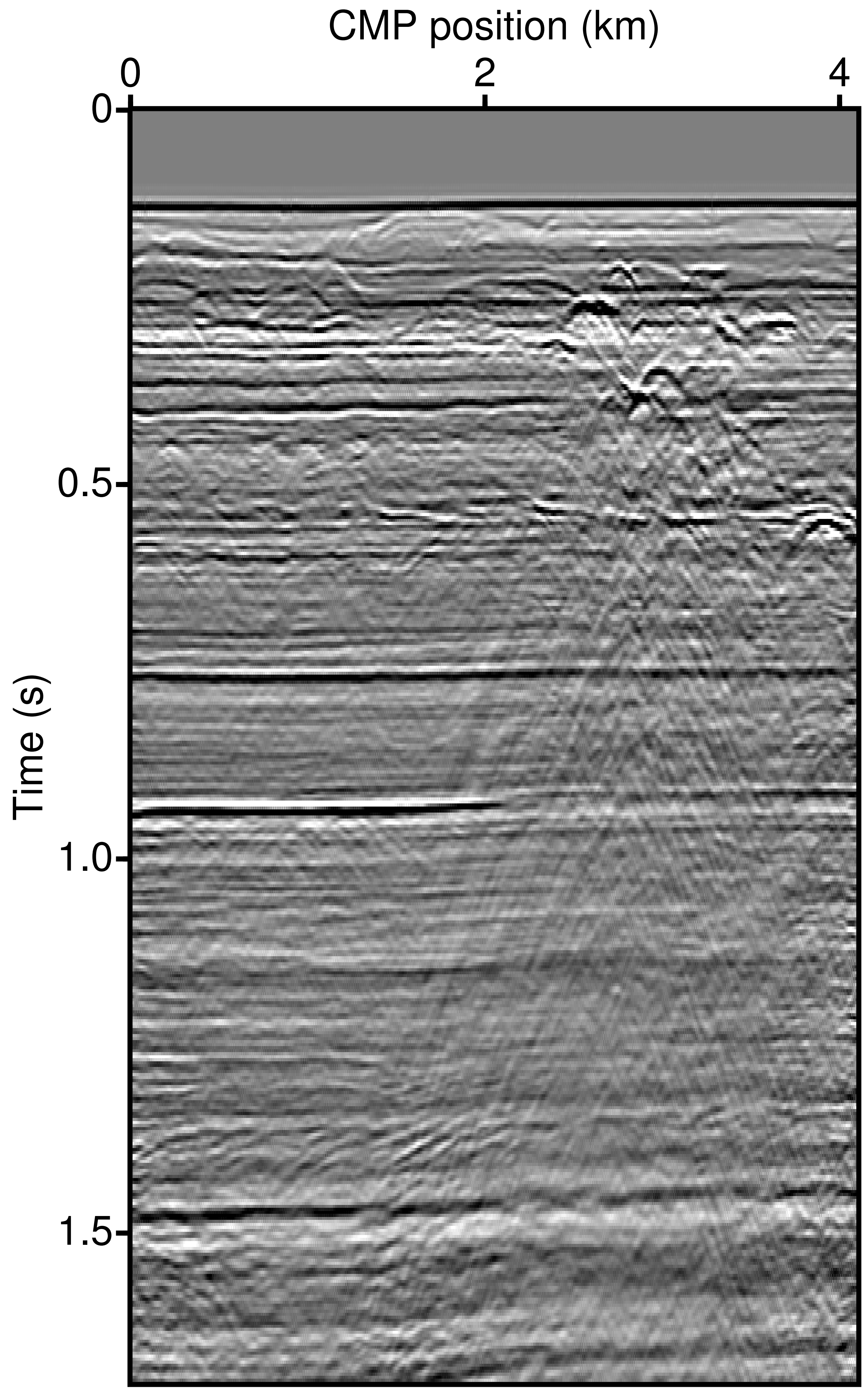


Radon interp - Re-linearization 2nd order

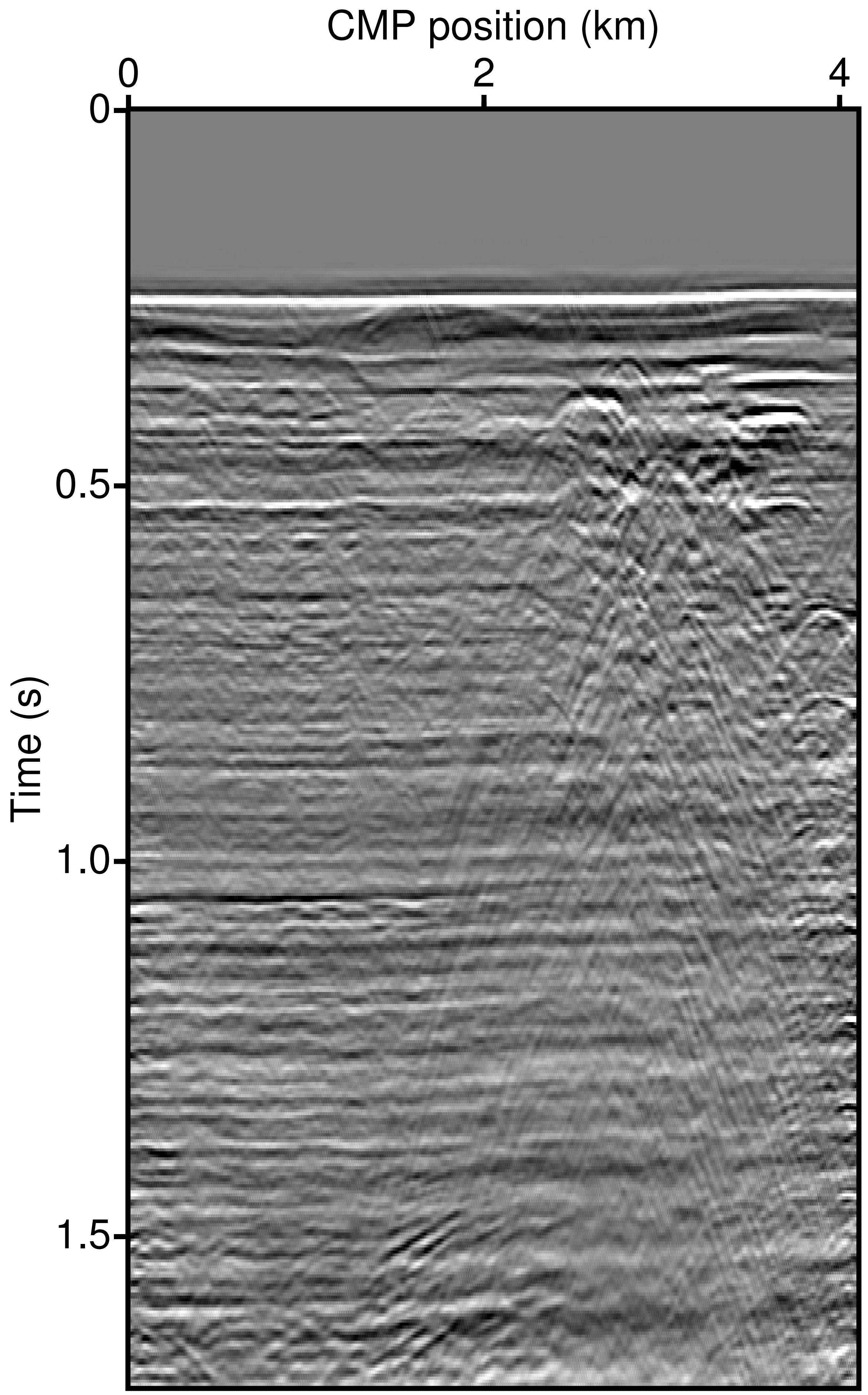


Re-linearization 3rd - 2nd order

NMO stack
Difference plots



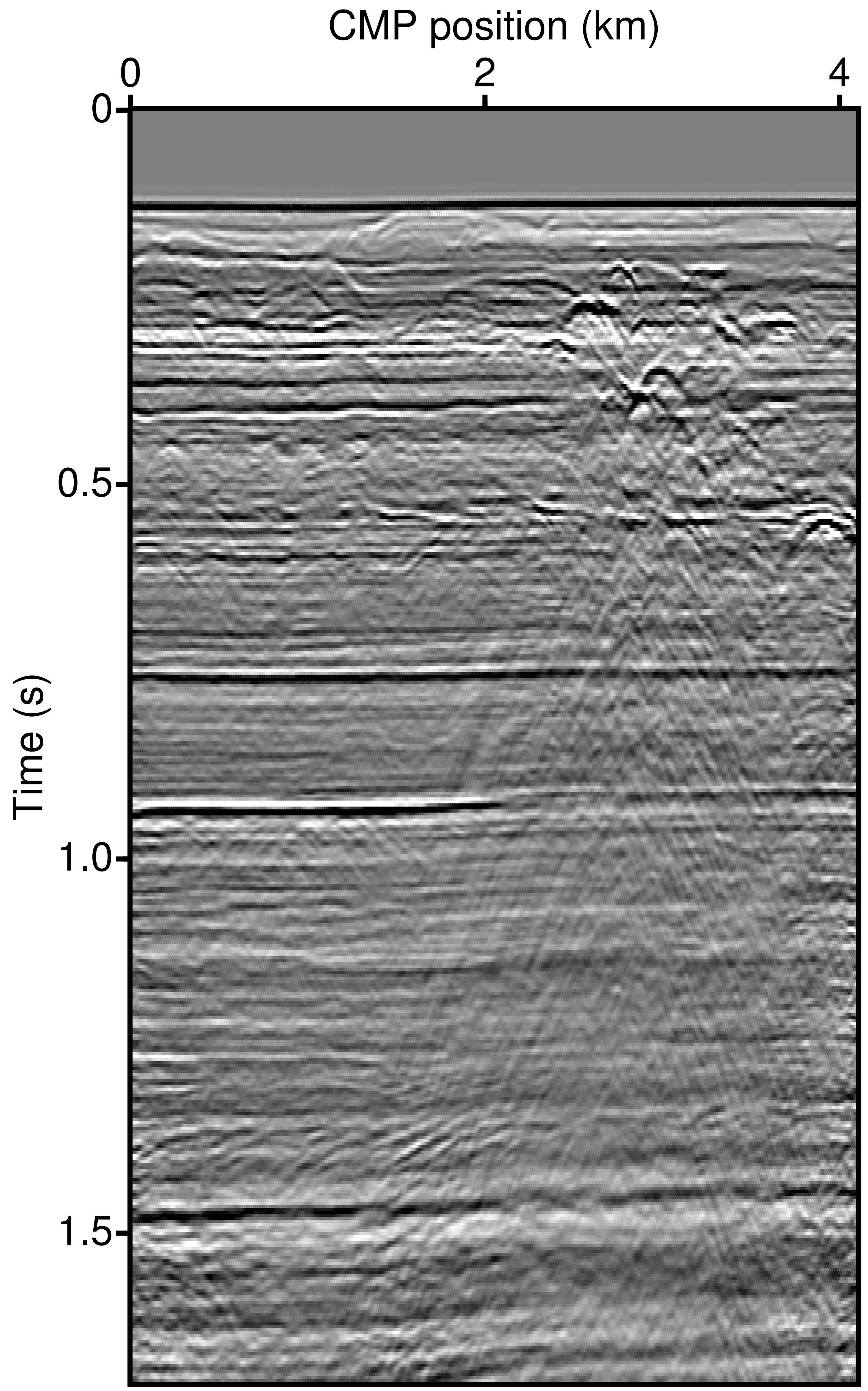
Conservative primary



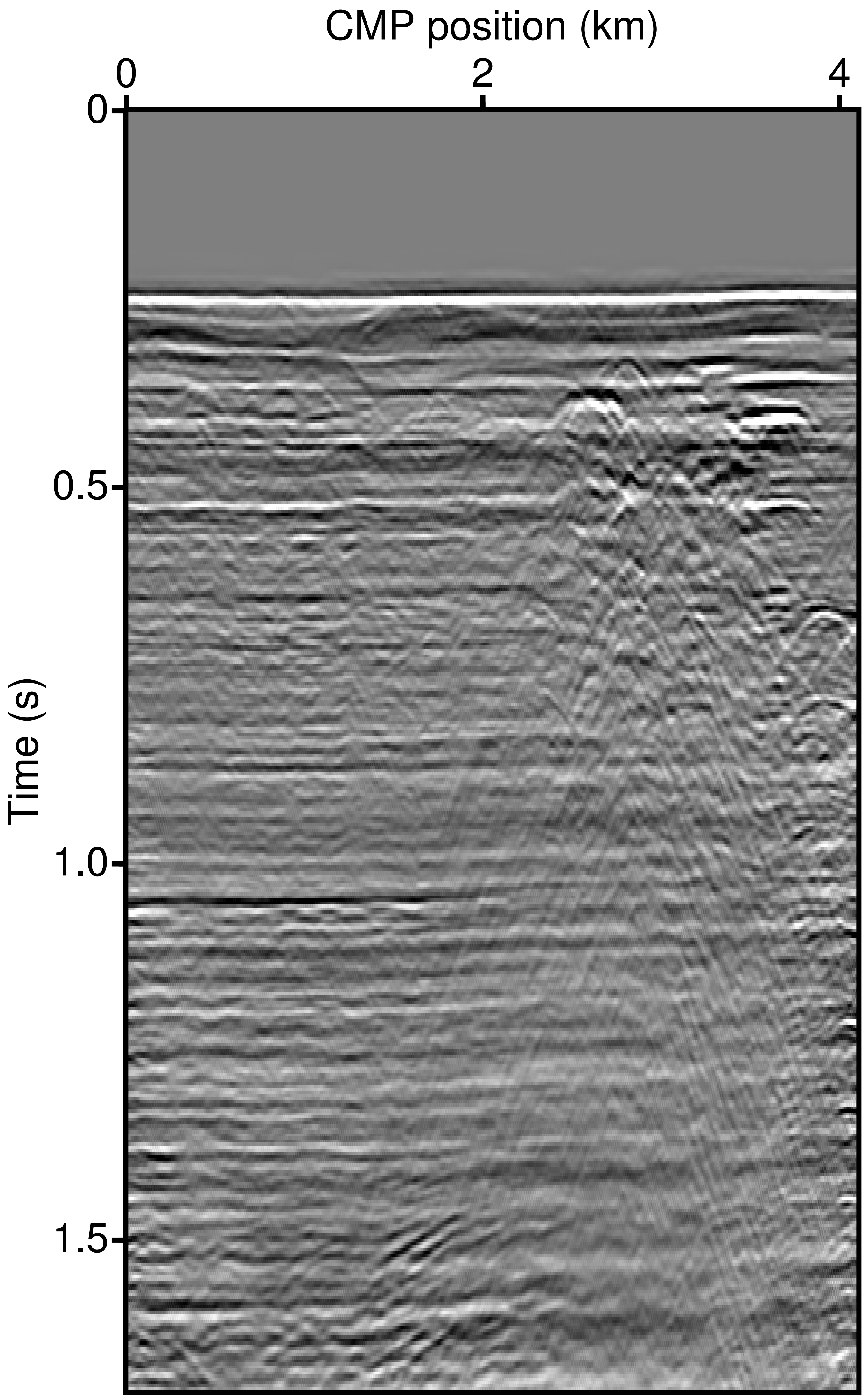
Multiple

NMO stack

Modified Gauss-Newton Using 3rd Order terms



Conservative primary

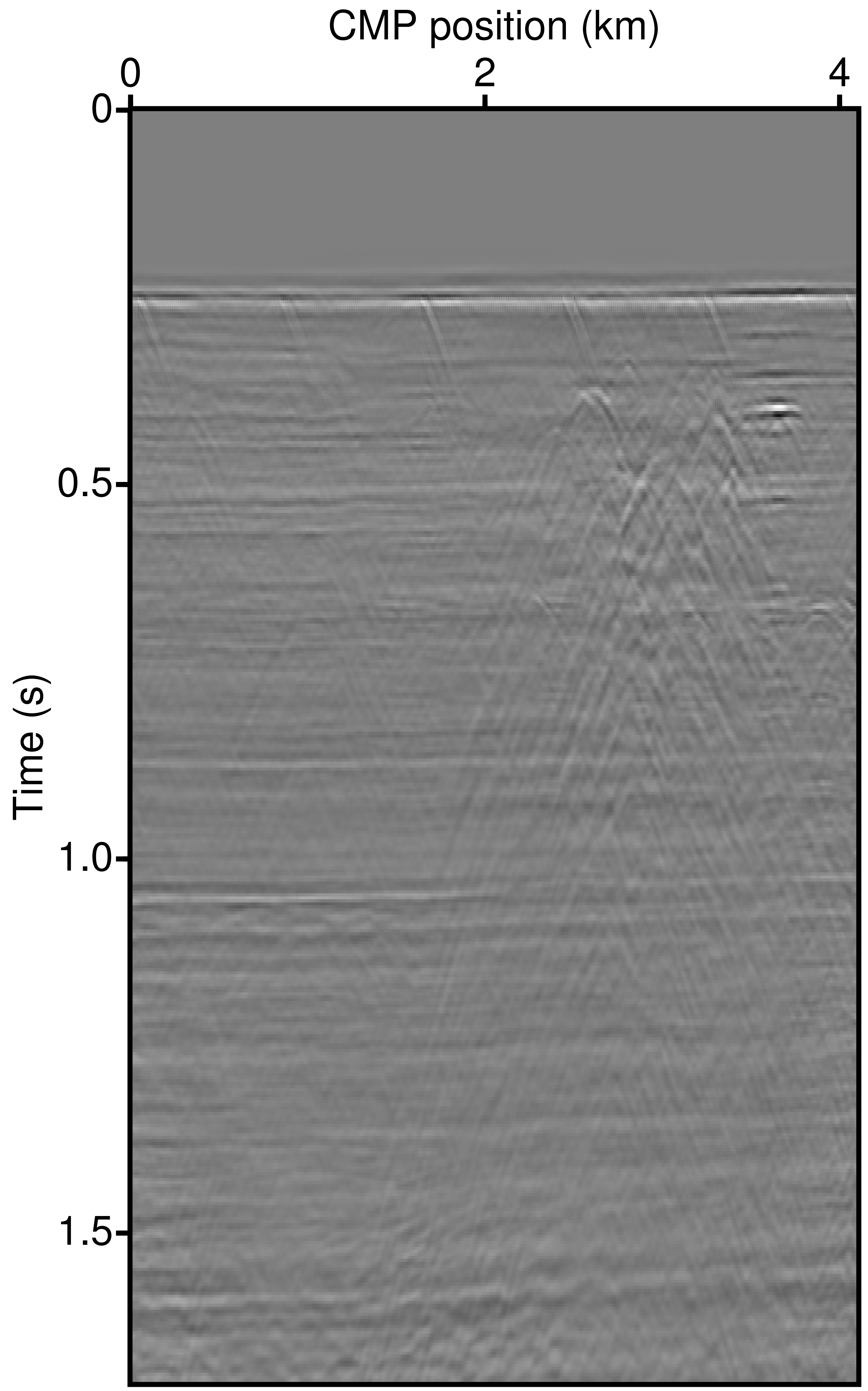


Multiple

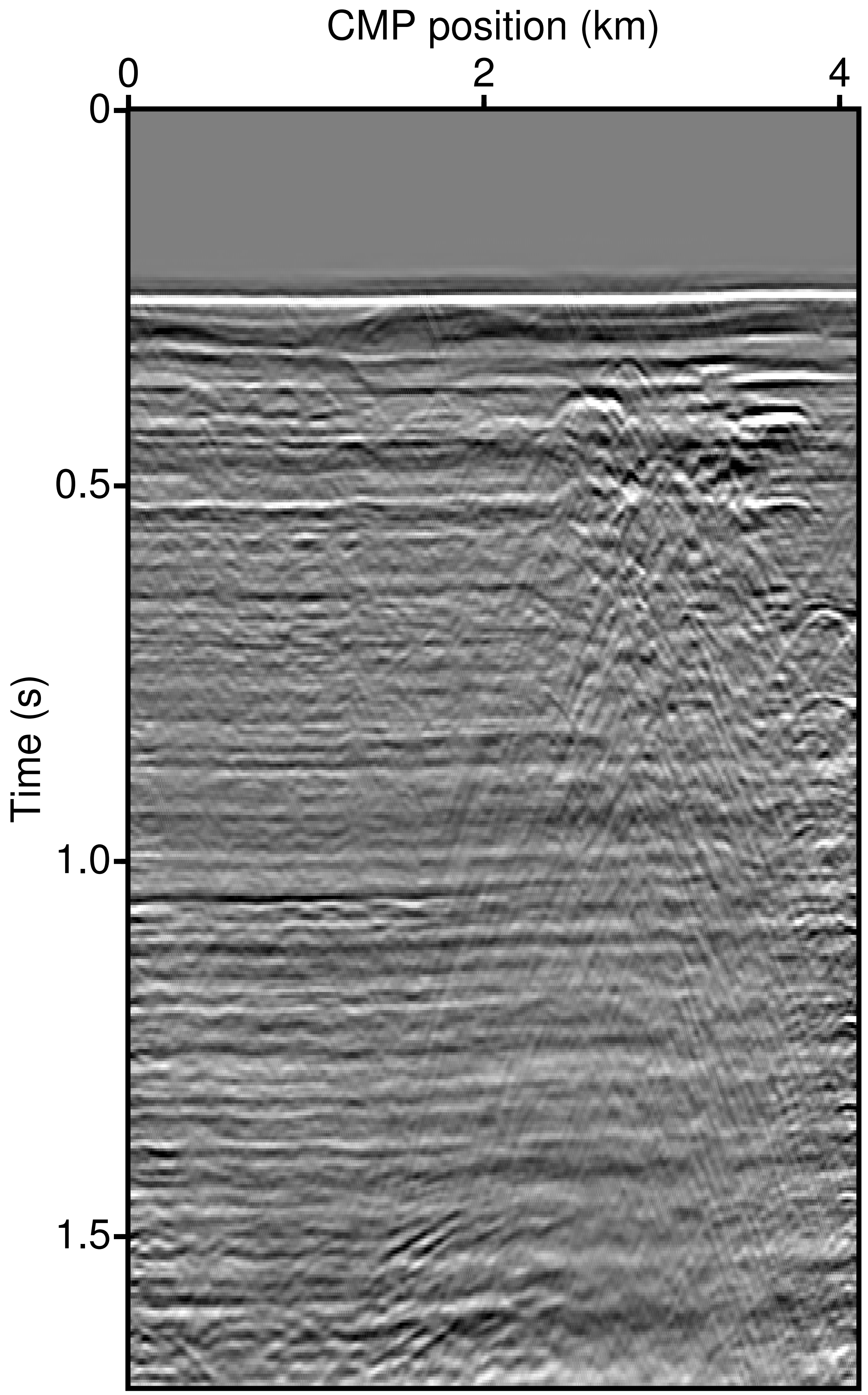
NMO stack

Re-linearization

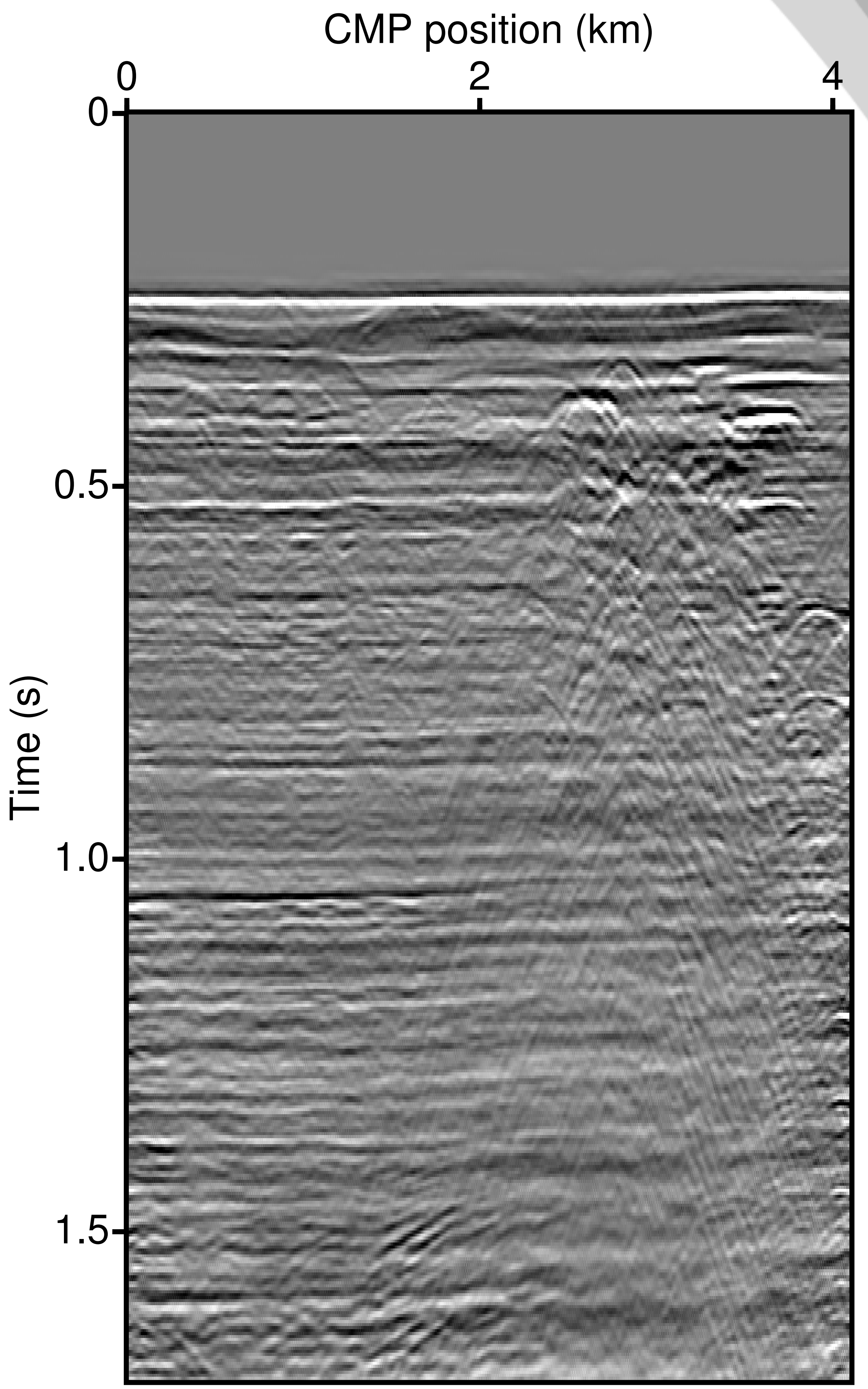
Using 3rd Order terms



Difference



GN 3rd Order Multiple



Re-lin. 3rd Order Multiple

Discussion and summary

Able to obtain an approximate forward operator with incomplete data without dependence on the missing traces

Inversion should be more stable by not changing the data at each iteration

Further work:

- More careful study needed in comparison to explicit data inversion
- More detailed comparison of algorithms
- What does the infilled G have to look like? Is it always the "physical" Green's function?

Acknowledgements

PGS for field dataset

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, CGG, Chevron, ConocoPhillips, ION, Petrobras, PGS, Total SA, WesternGeco, and Woodside.