SVD-free matrix completion for seismic data reconstruction

Rajiv Kumar, Oscar Lopez, Ernie Esser and Felix J. Herrmann

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Outline

- interpolation
  - comparison with curvelet-based reconstruction methods

- regularization
  - is binning the right approach?

Motivation

- acquisition challenges
  - missing data
  - irregular acquisition grid
- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME
- regularization
  - imaging and inversion algorithm require equi-spaced grid
- exploit *low-rank* structure of seismic data
  - *SVD-free* matrix factorization

Outline

- interpolation
  - comparison with curvelet-based reconstruction methods
- regularization
  - is binning the right approach?

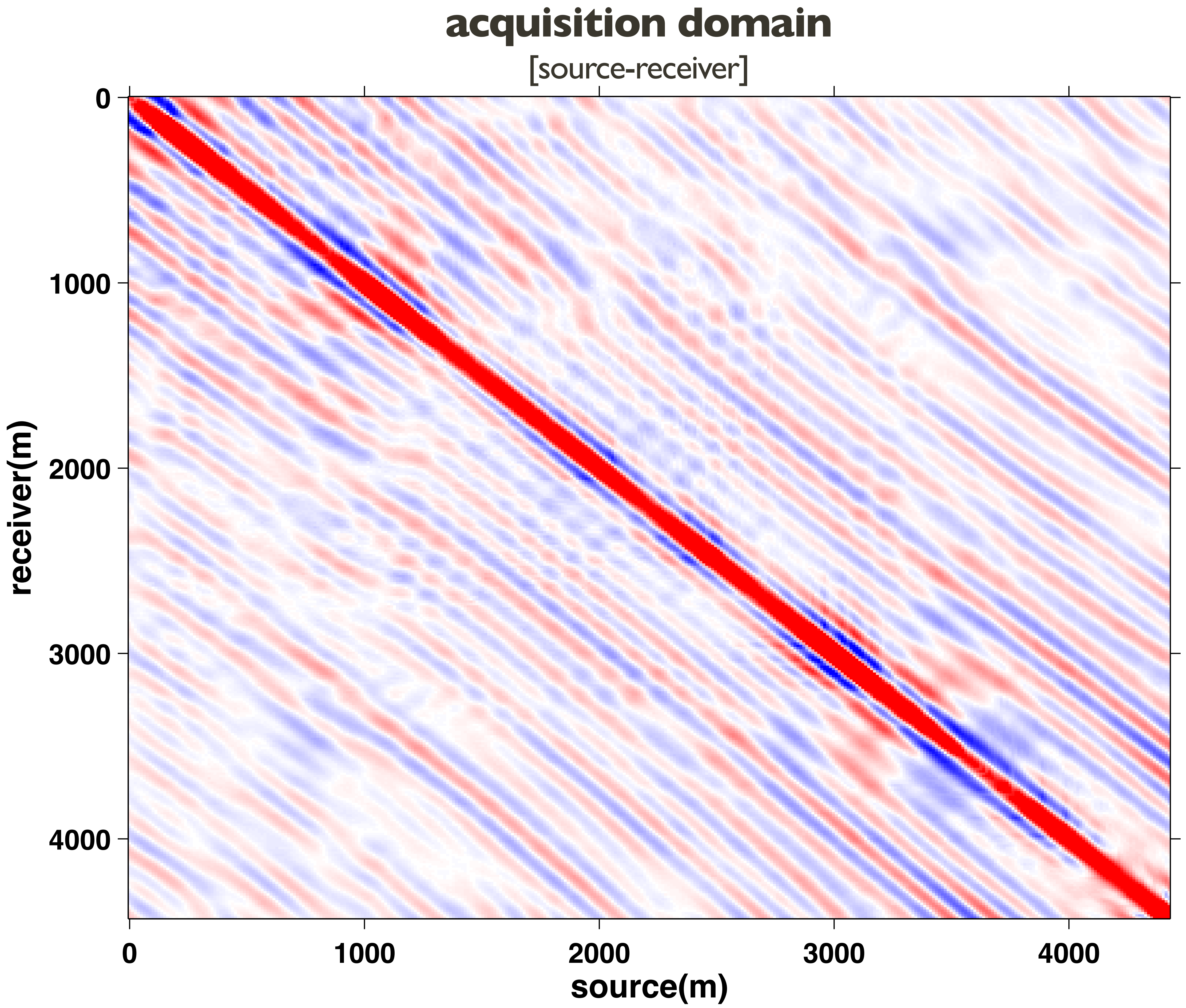
[Candes and Plan 2010, Oropeza and Sacchi 2011]

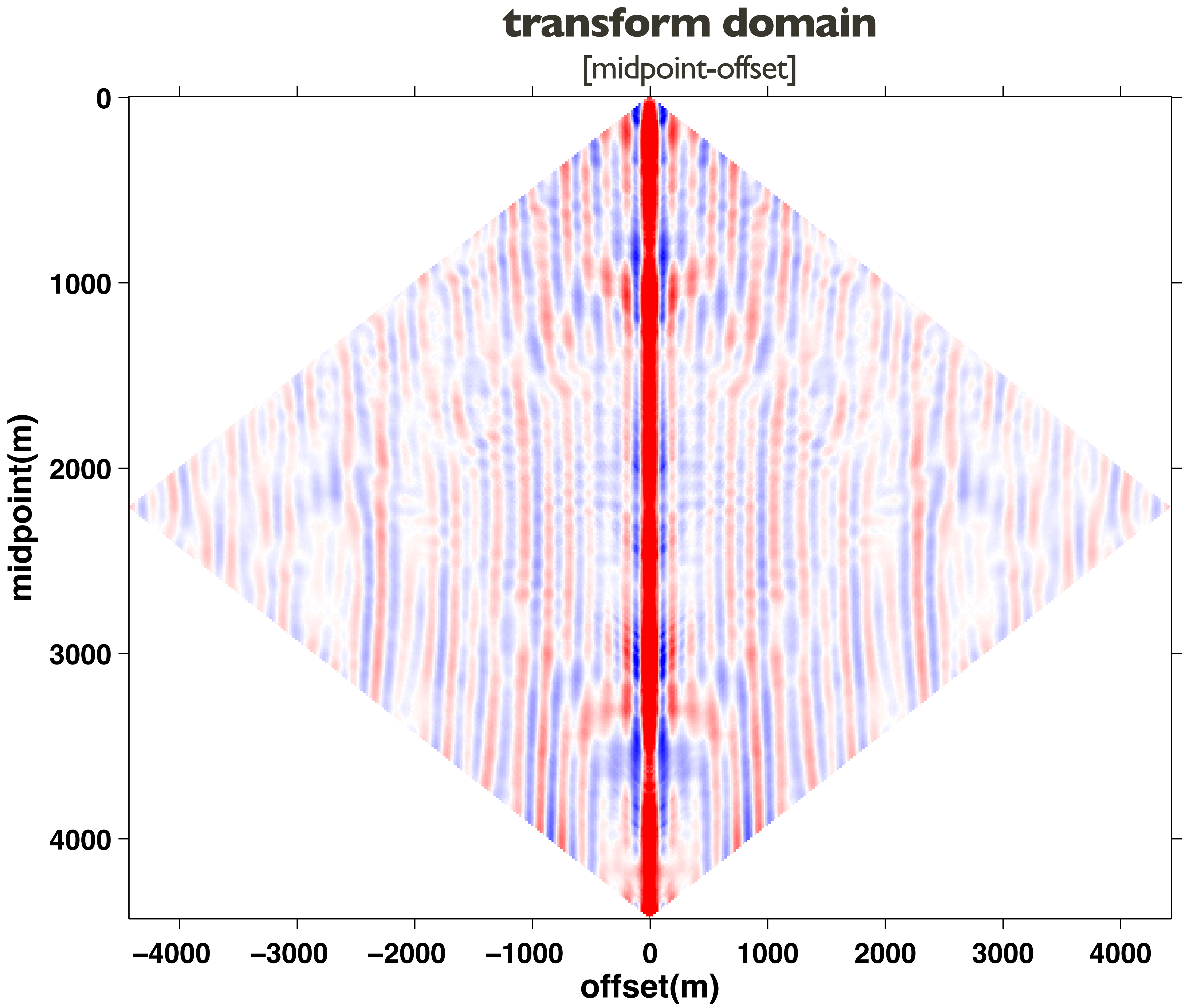
Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Low-rank structure

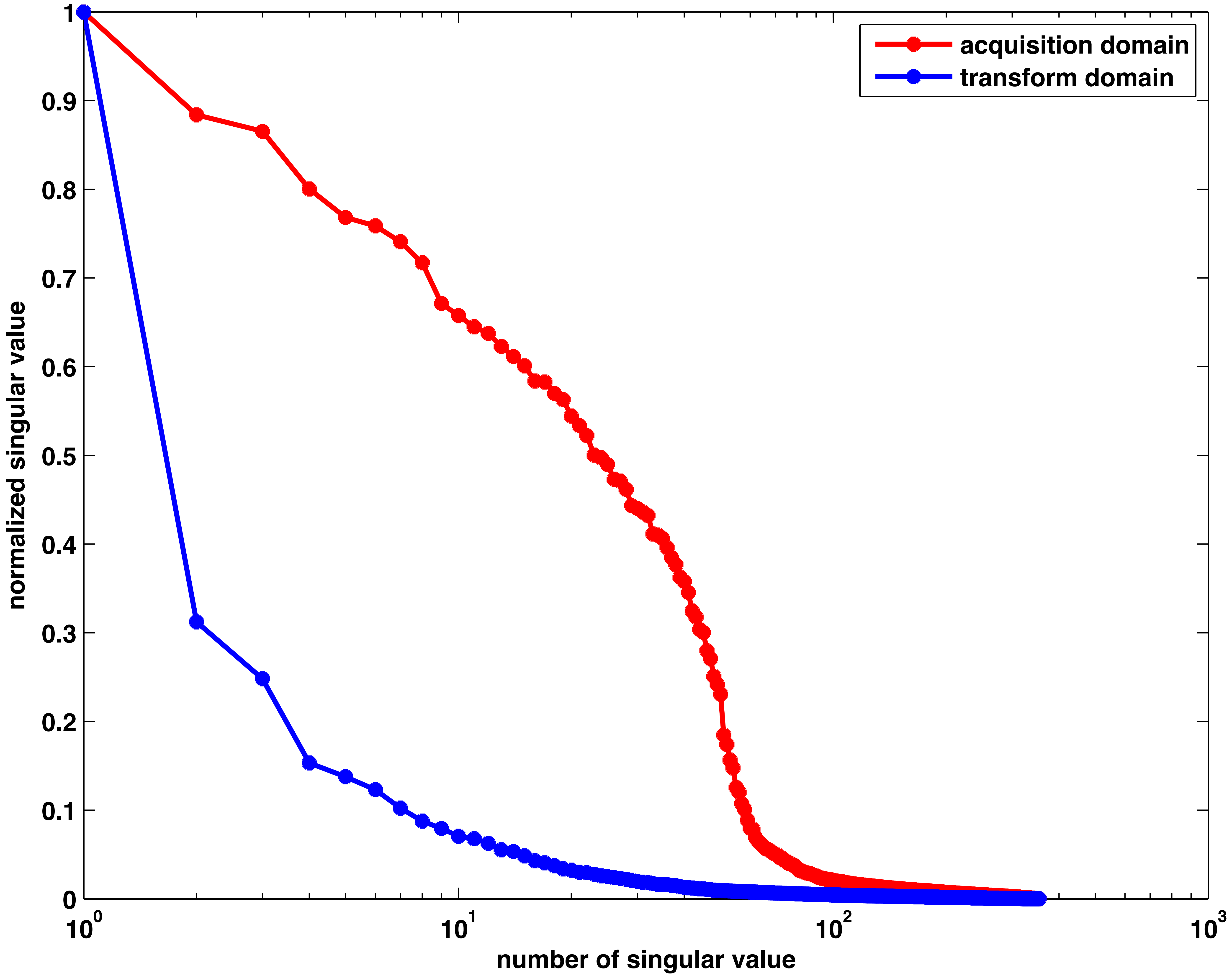
2-D acquisition





Singular value decay

2-D acquisition



Matrix completion

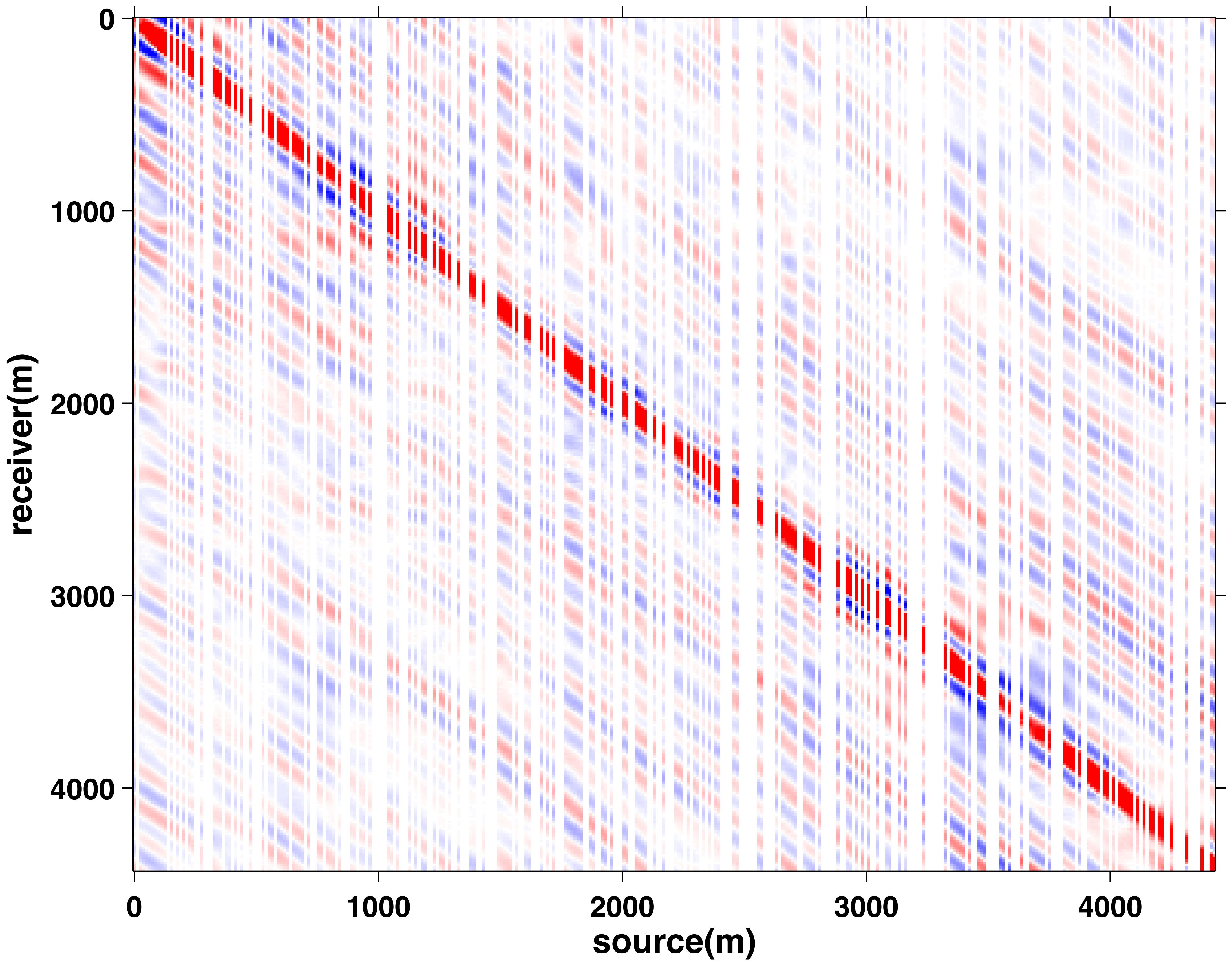
- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

2-D acquisition

uniform-random sampling

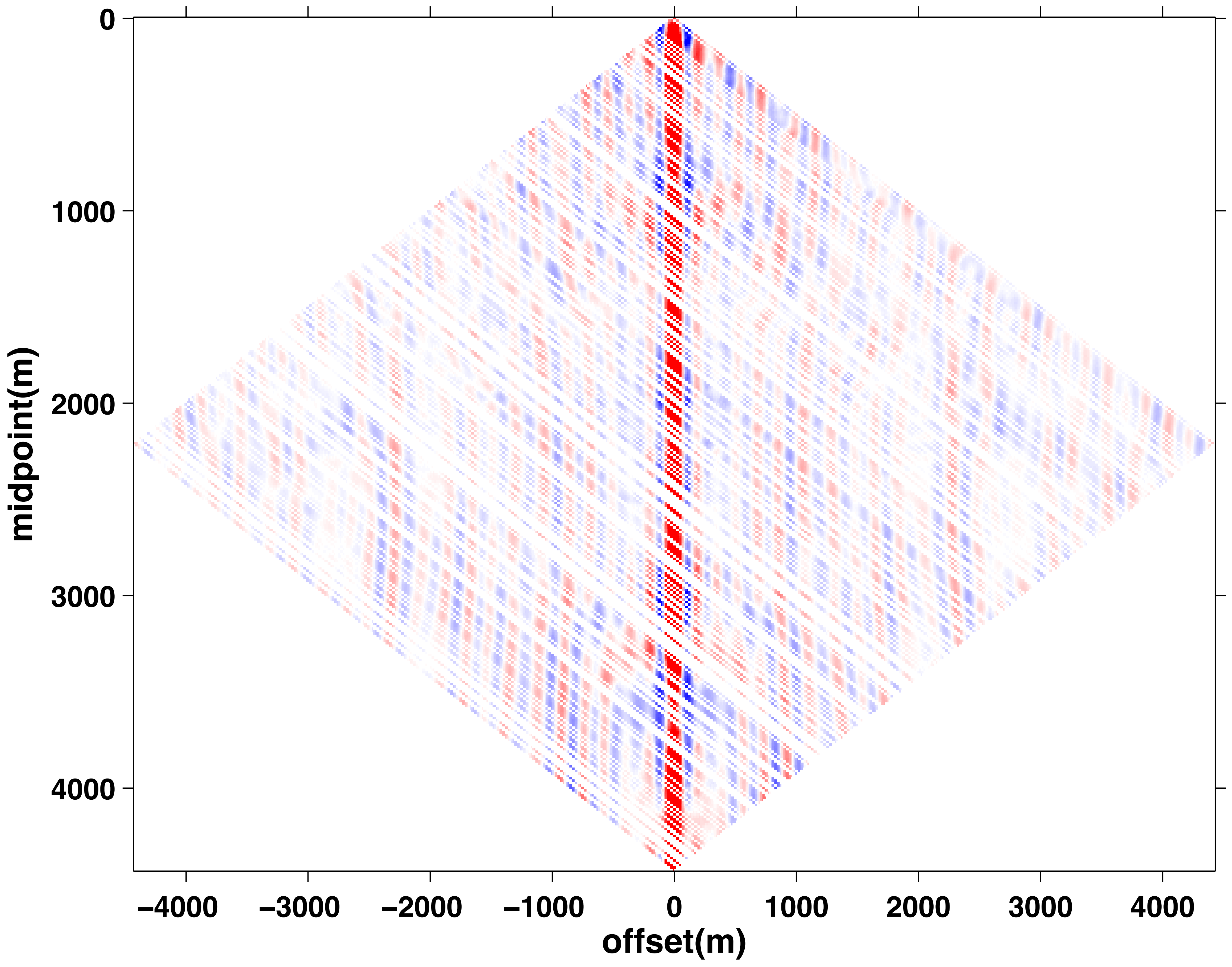
acquisition domain

missing columns *do not* increase rank

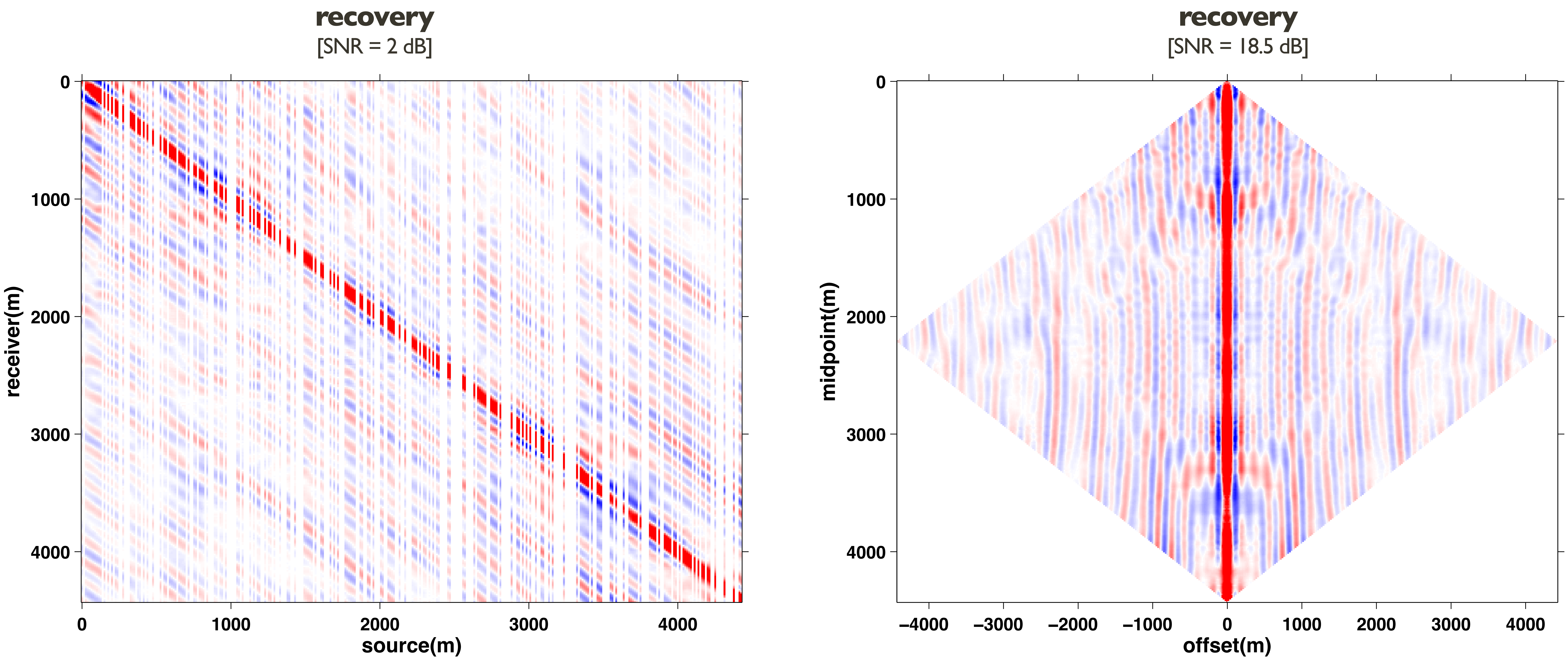


transform domain

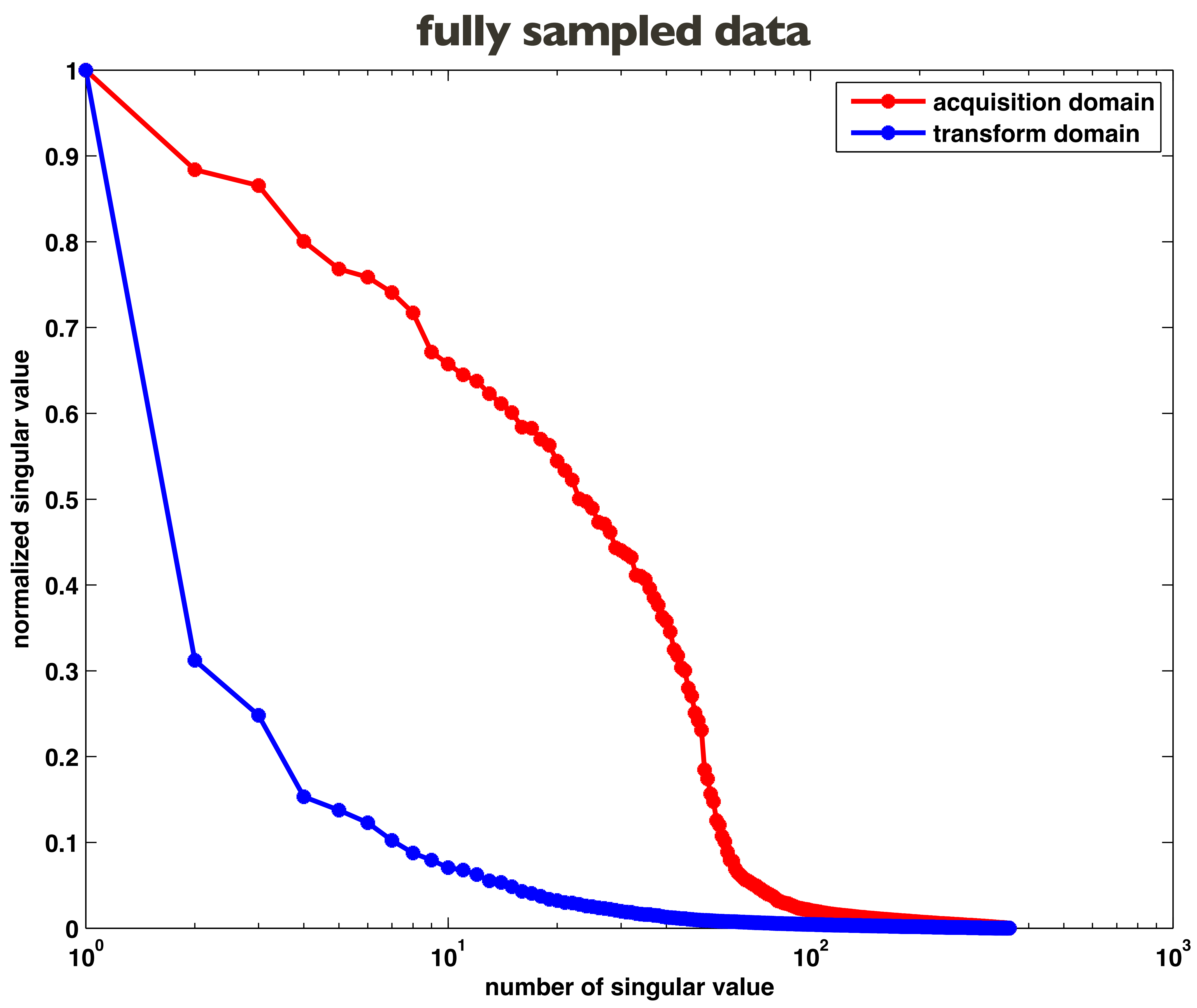
missing columns *do* increase rank

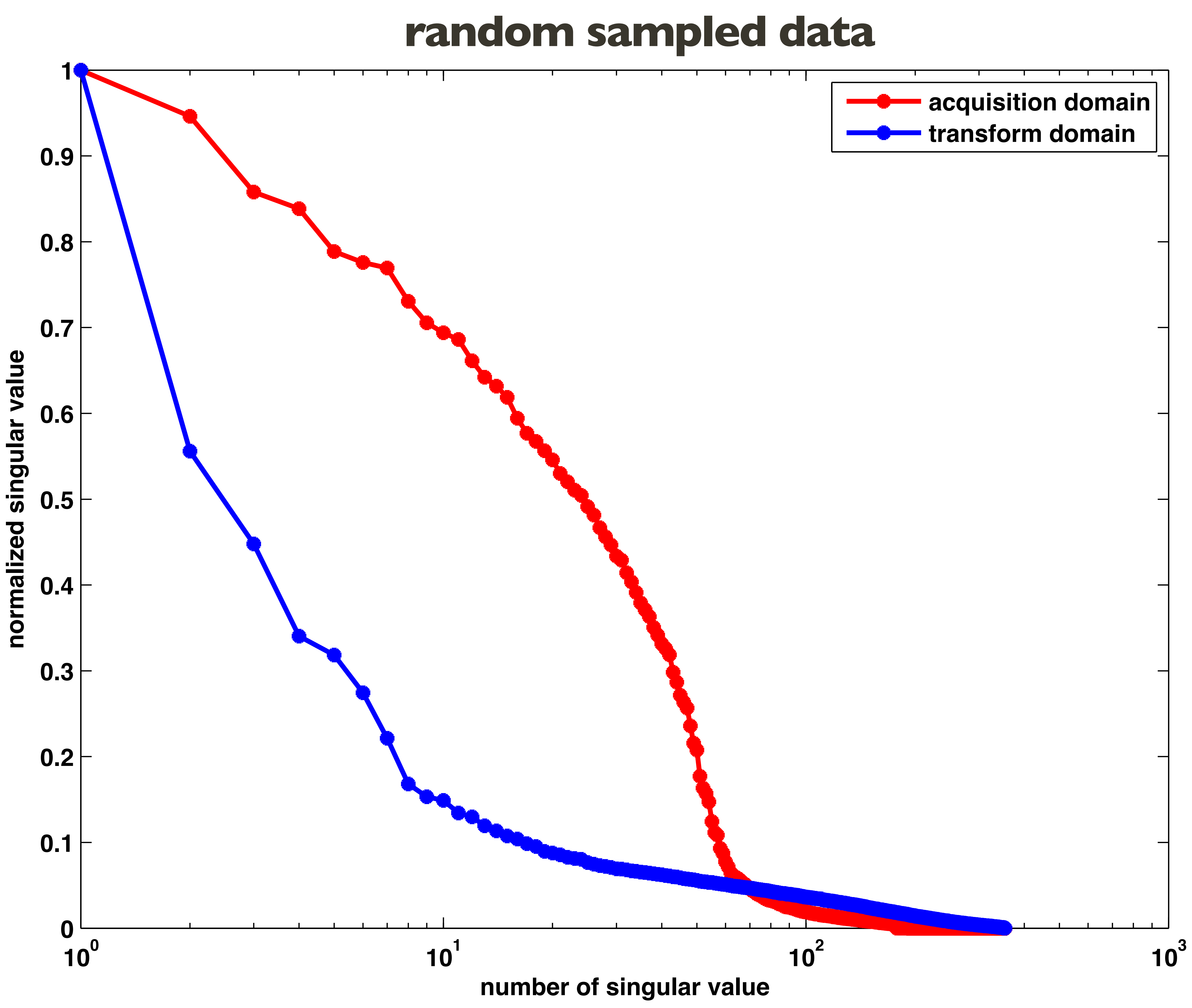


Low-rank interpolation



Randomized sampling
singular value decay





Observations

- sampling become *incoherent* in "transform" domain
- *slow decay* of singular values in "transform" domain

Matrix completion

- signal structure
  - *low rank/fast decay* of singular values

- sampling scheme
  - missing data *increase* rank in "transform domain"

- recovery using *rank penalization* scheme

Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad \text{rank}(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where $\text{rank}(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathcal{S}^H$: transform operator

Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank
- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization **[Recht et. al. 2010]**

[Recht et. al. 2010]

Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Challenges

- requires repeated application of *SVD* for projections

- expensive to compute for large system
  - curse of dimensionality

- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, Lee et. al. 2010, Recht and Re 2011]

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n\times m} \quad = \quad \mathbf{L} \in \mathbb{R}^{n\times k} \quad \mathbf{R}^H \in \mathbb{R}^{k\times m}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

Factorized formulation

- reformulate $(BPDN_\sigma)$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \;\; ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \;\; \|\mathbf{L}\mathbf{R}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

Factorized formulation

- Upper-bound on nuclear norm is defined as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

- choose k *explicitly* & avoid costly *SVD's*

Computational cost

with and without SVD

		50%		75%	
	σ	0.1	0.08	0.1	0.08
Matrix completion w/ SVD	SNR (dB)	17.3	18.3	11.6	11.5
	time (sec)	812	937	790	765
Matrix completion w/o SVD	SNR (dB)	17.6	18.4	12.6	13.3
	time (sec)	8	10	8	7

Upcoming paper

Check https://www.slim.eos.ubc.ca soon!

Computational cost

matrix completion v/s curvelet-based methods

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Matrix completion w/o SVD	SNR (dB)	17.6	18.4	12.6	13.1
	time (sec)	8	10	8	7
Curvelet-based sparsity promotion	SNR (dB)	17.4	18.6	12.5	12.8
	time (sec)	879	989	817	1010

Observation

matrix completion v/s curvelet-based methods

	Low-rank	Curvelet
computational time	O(*minutes*)	O(*hours*)
storage	$k \times (n+m)$	$8 \times nm$

Take-away message

- can avoid *"SVD"*
- faster compare to curvelet-based sparsity promotion techniques
- memory efficient compare to curvelet-based techniques

Outline

- interpolation
  - comparison with curvelet-based reconstruction methods

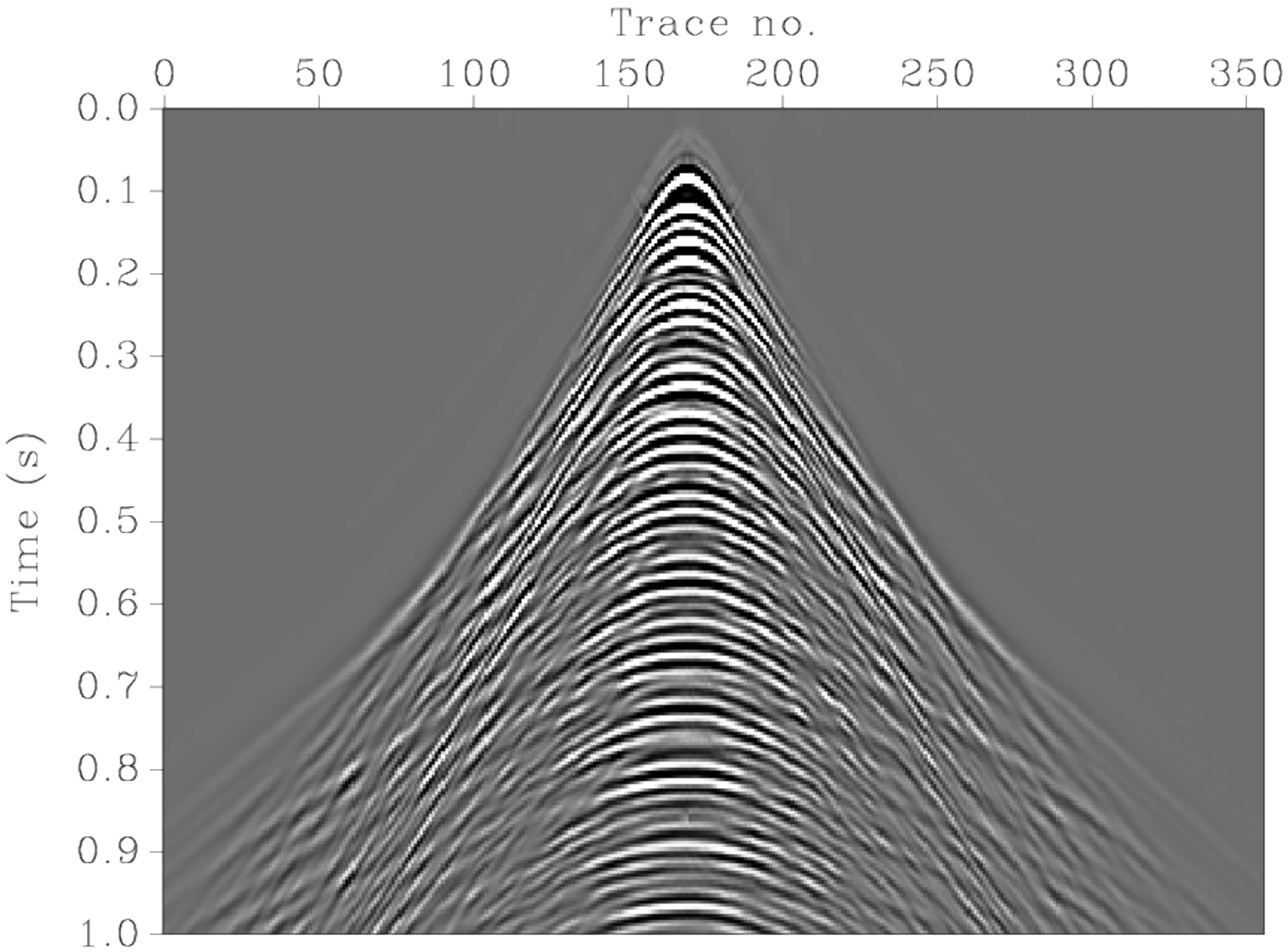
- regularization
  - is binning the right approach?

Regularization

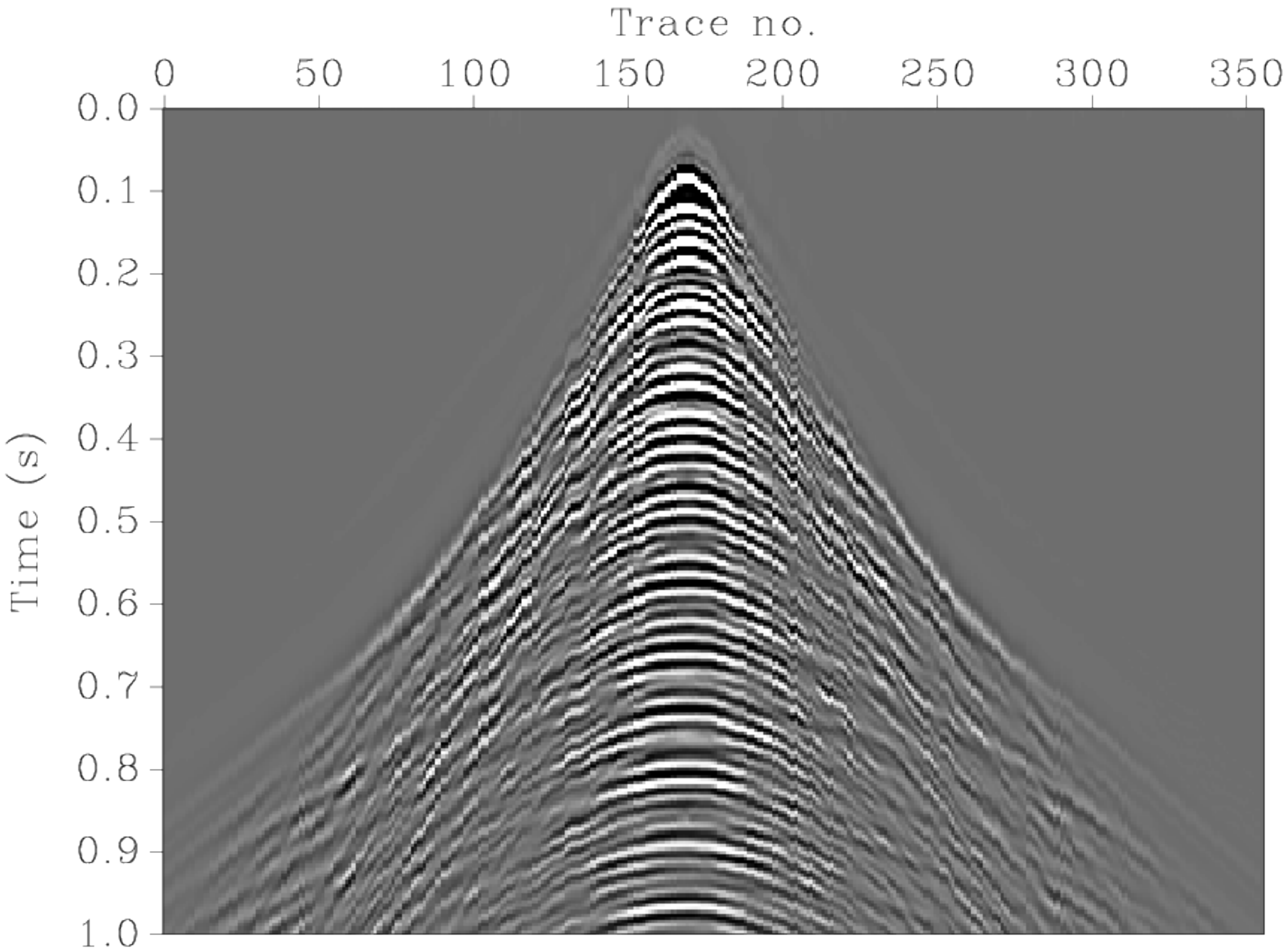
- unstructured acquisition grid
- imaging and inversion algorithm
  - regularly sampled data
- binning
  - does not preserve the data-structure

Regularization

Ground truth



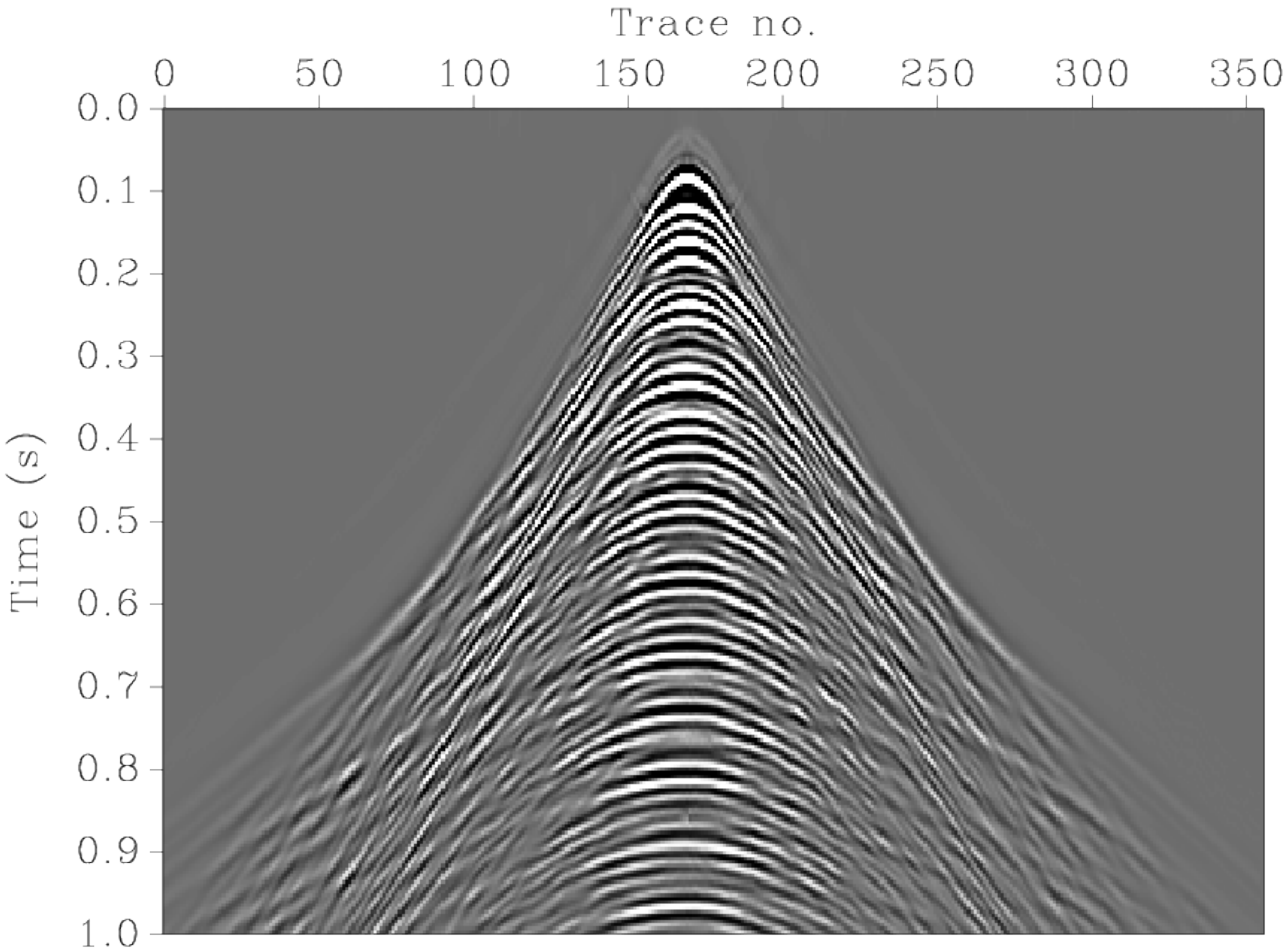
Observed data



Regularization

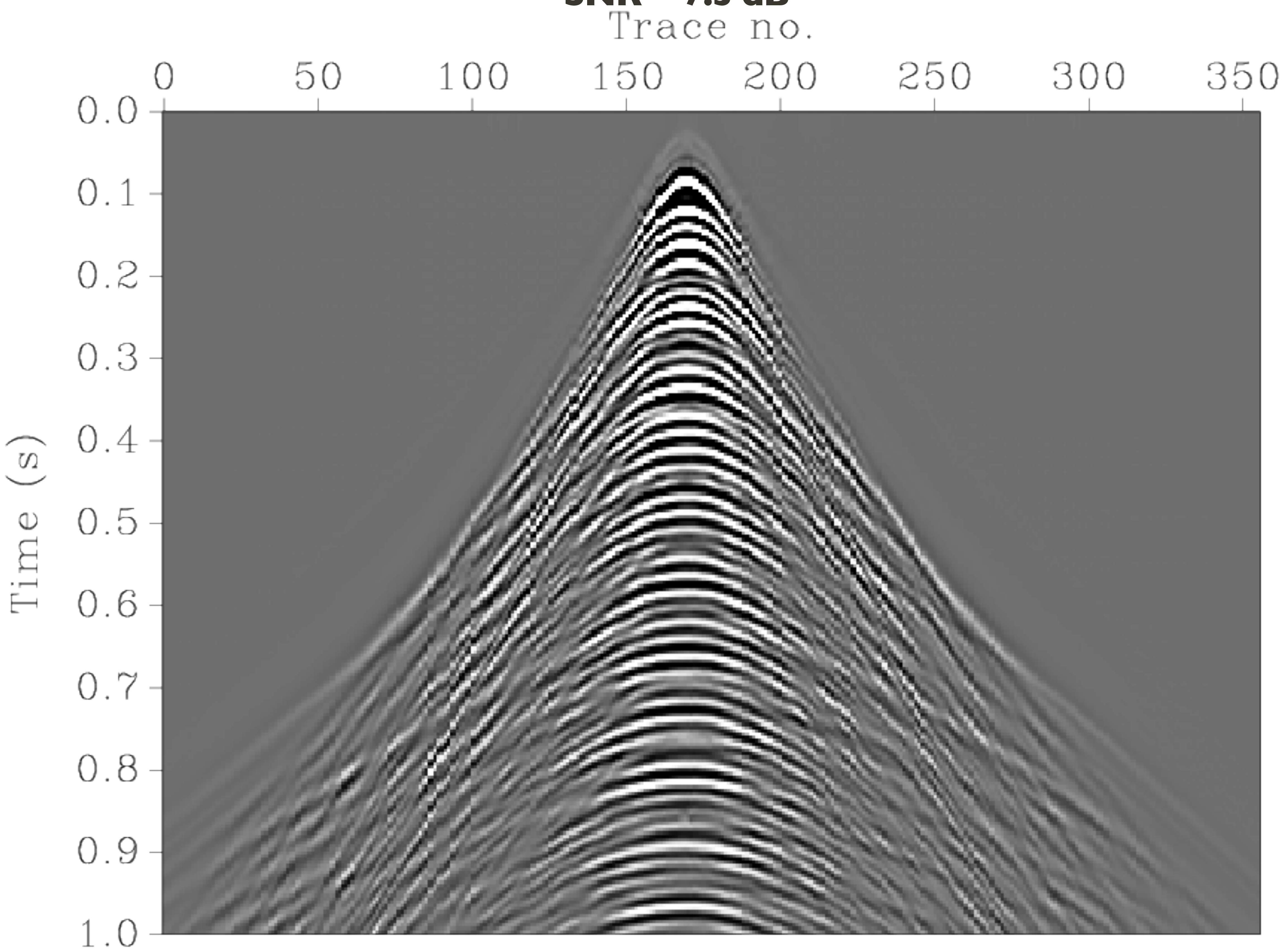
binning

Ground truth



Recovery

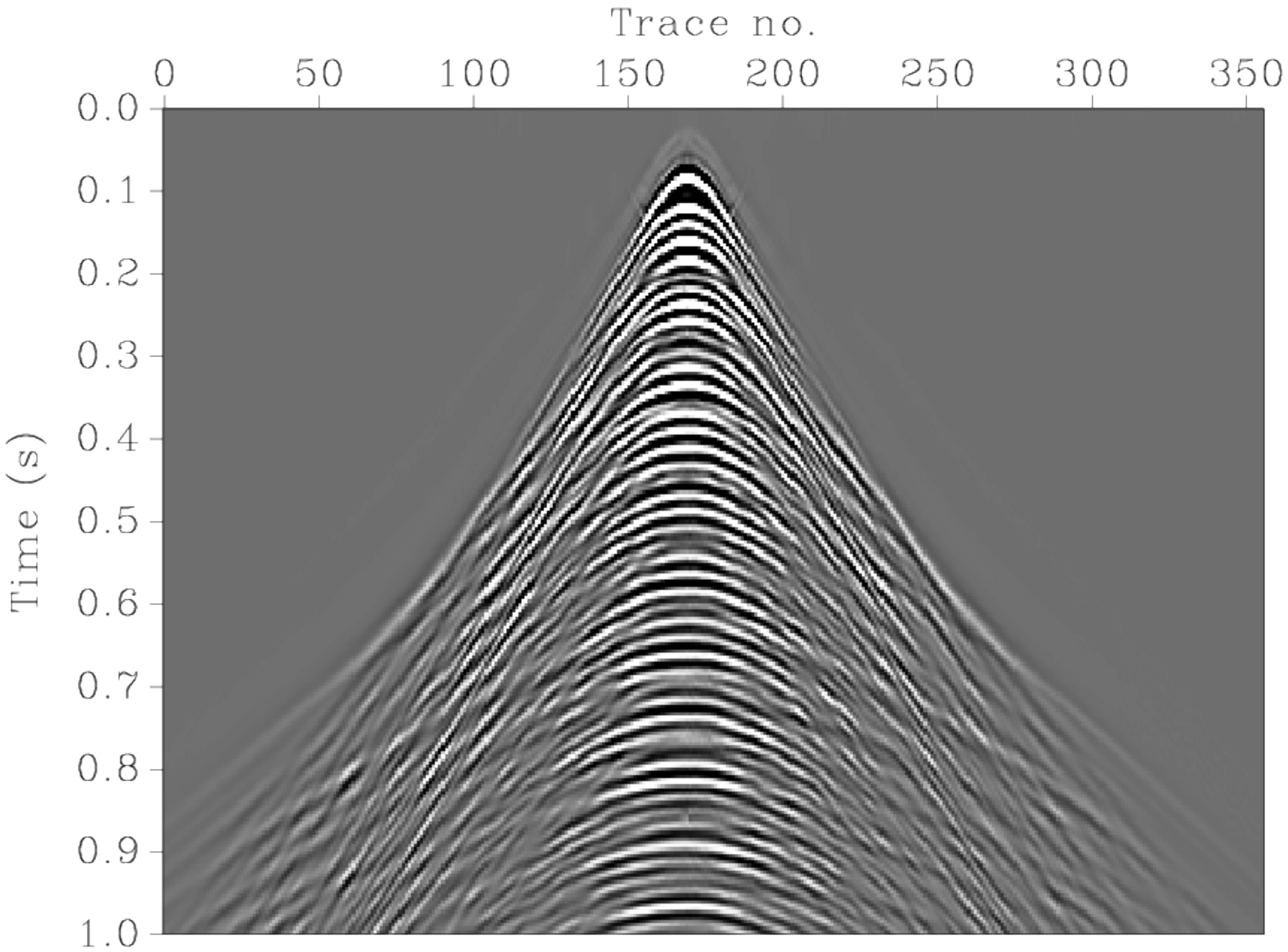
SNR = 7.3 dB



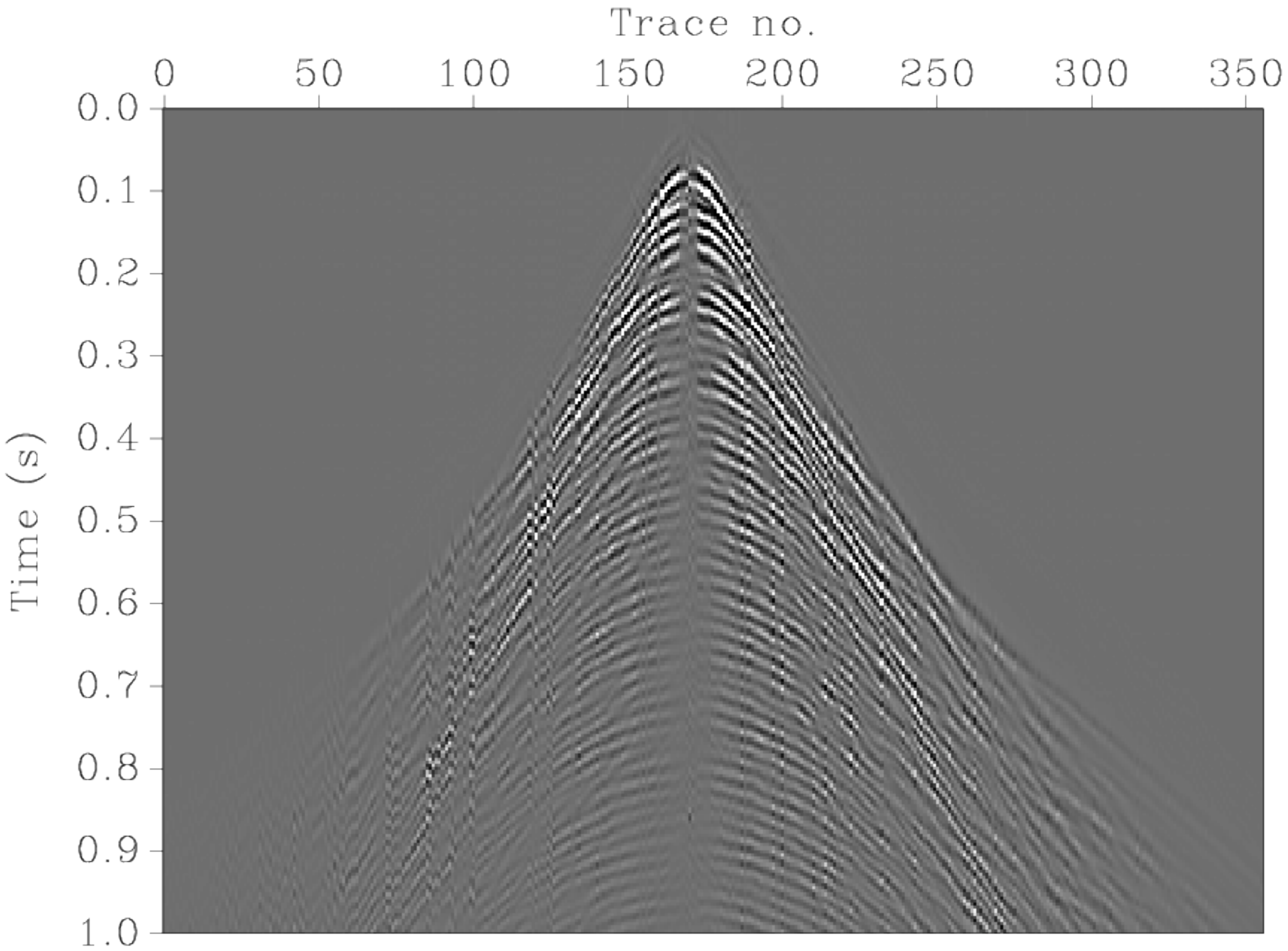
Regularization

binning

Ground truth

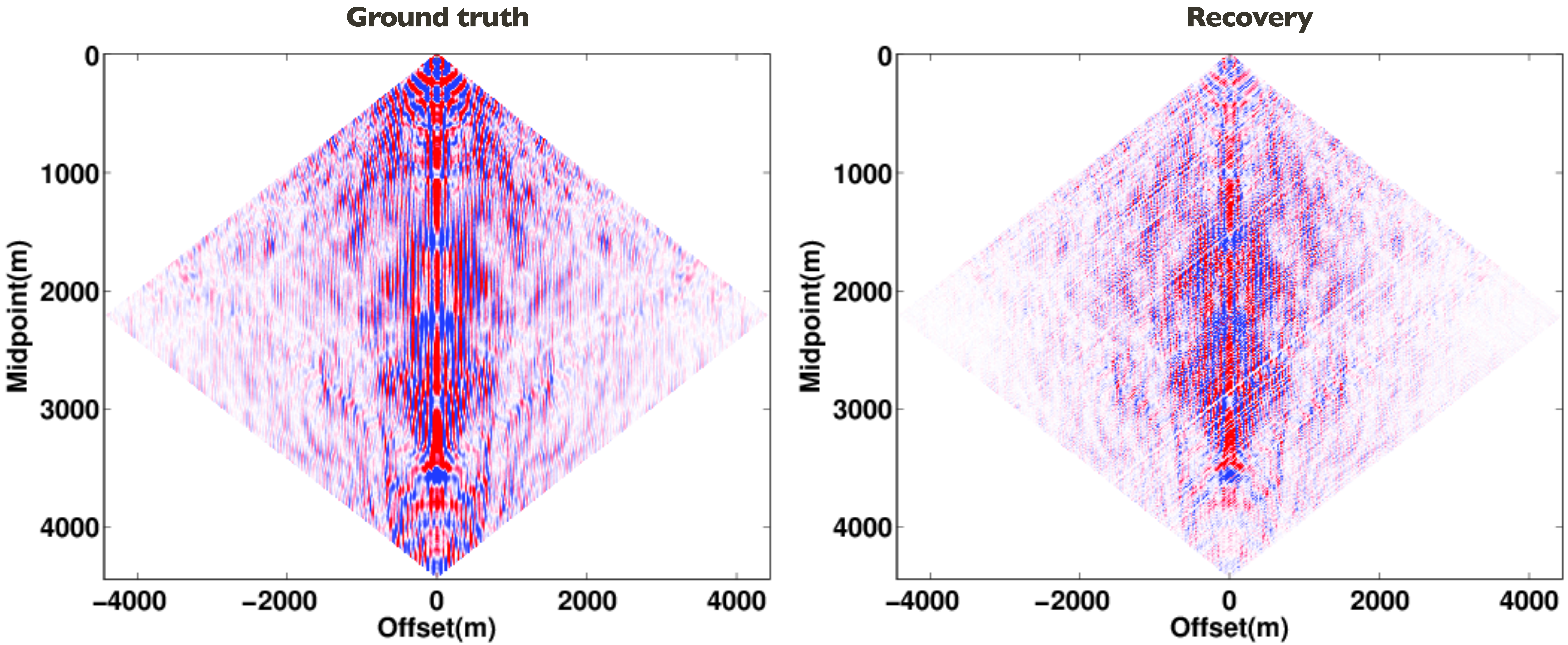


Residual



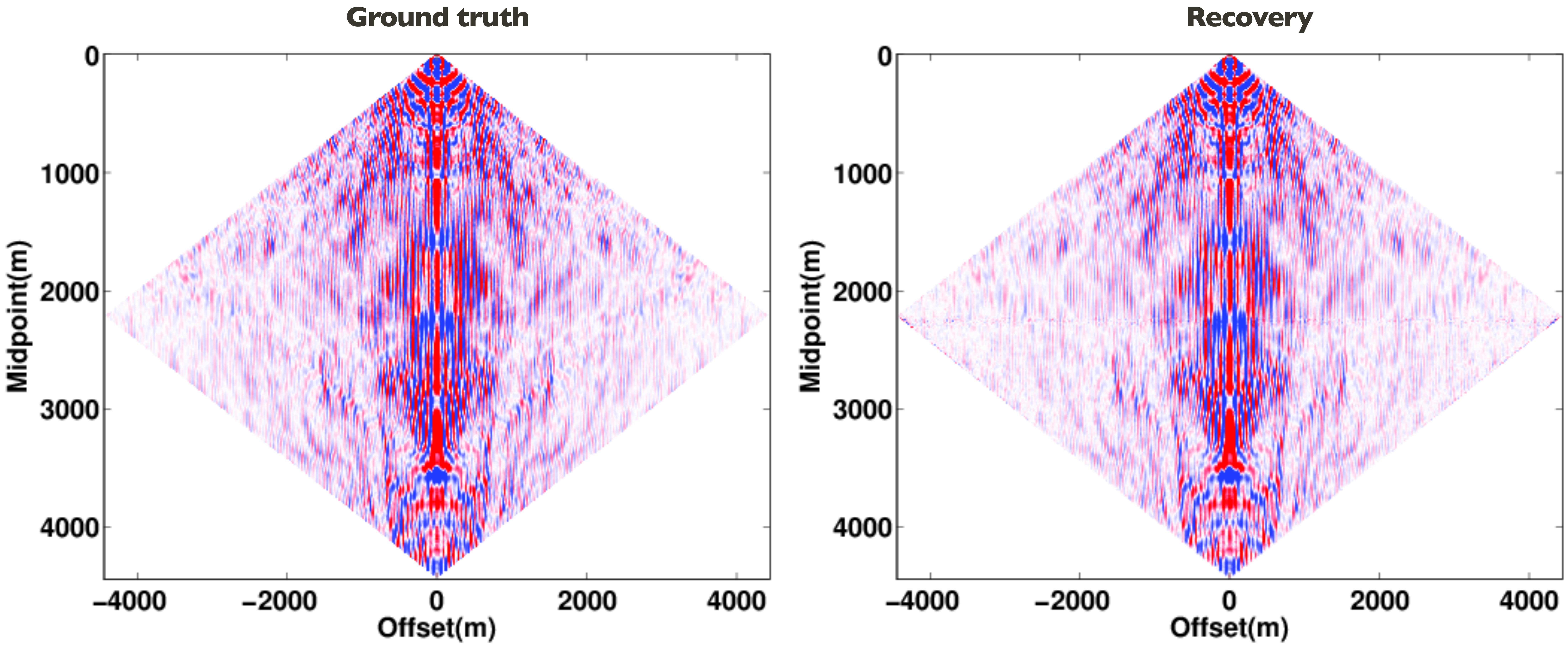
Low-rank structure

binning, midpoint-offset domain



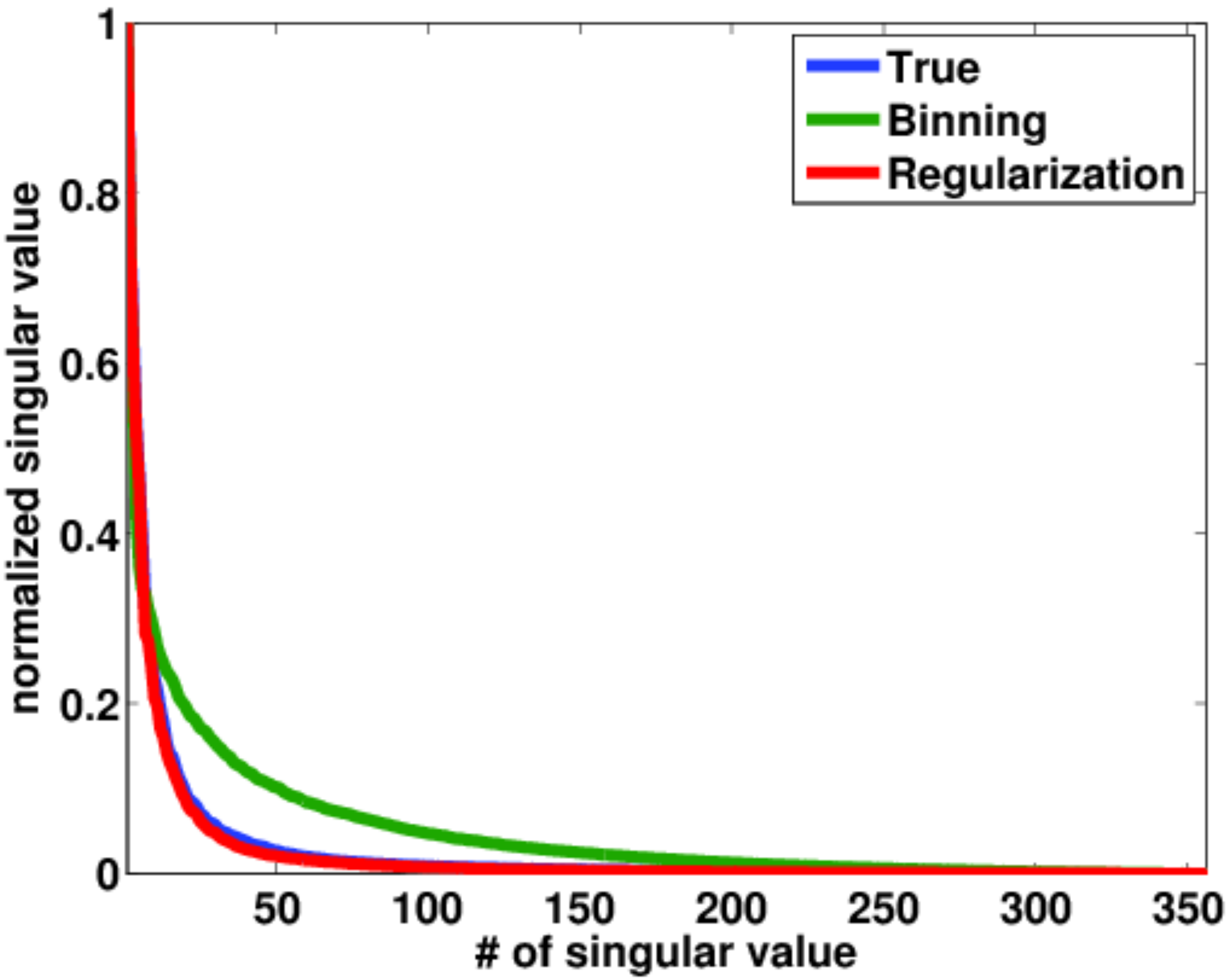
Regularization

matrix completion, midpoint-offset domain



Singular value decay

regularization v/s binning



Methodology
matrix completion

- given a regularization operator $\mathbf{N} : \mathbb{C}^{n\times m} \to \mathbb{C}^{n\times m}$ so that $\mathbf{N}(\mathbf{X}_r) = (\mathbf{X}_{ir})$, transform-sampling operator is redefine as

$$\mathcal{A} = \mathbf{RMN}^H \mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathbf{N}^H$: regularization operator
$\mathcal{S}^H$: transform operator

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Theorem

matrix completion

Let $\mathbf{X_r} \in \mathbb{C}^{n\times m}, \hat{\mathbf{X}}_r \in \mathcal{S}$ and $\mathbf{b} = \mathbf{RM}(\mathbf{X}_{ir}) + e$ with $\|e\| \leq \eta$. Let $\tilde{\mathbf{X}}$ be the solution of BPDN_σ, then

$$\|\mathcal{S}(\mathbf{X}_r - \tilde{\mathbf{X}})\| \leq \underbrace{\frac{C_1}{\sqrt{k}} \sum_{j=k+1}^{l} \sigma_j(\hat{\mathbf{X}}_r)}_{\textbf{interpolation error}} + \underbrace{(\frac{C_1}{\sqrt{k}} B_2 + 1)\|P\|_F}_{\textbf{regularization error}} + \underbrace{C_2\eta}_{\textbf{noise}}$$

where

$$P = \mathbf{N}^{-1}(\mathbf{X}_{ir}) - \mathbf{X}_r$$

$$l = min\{n, m\}$$

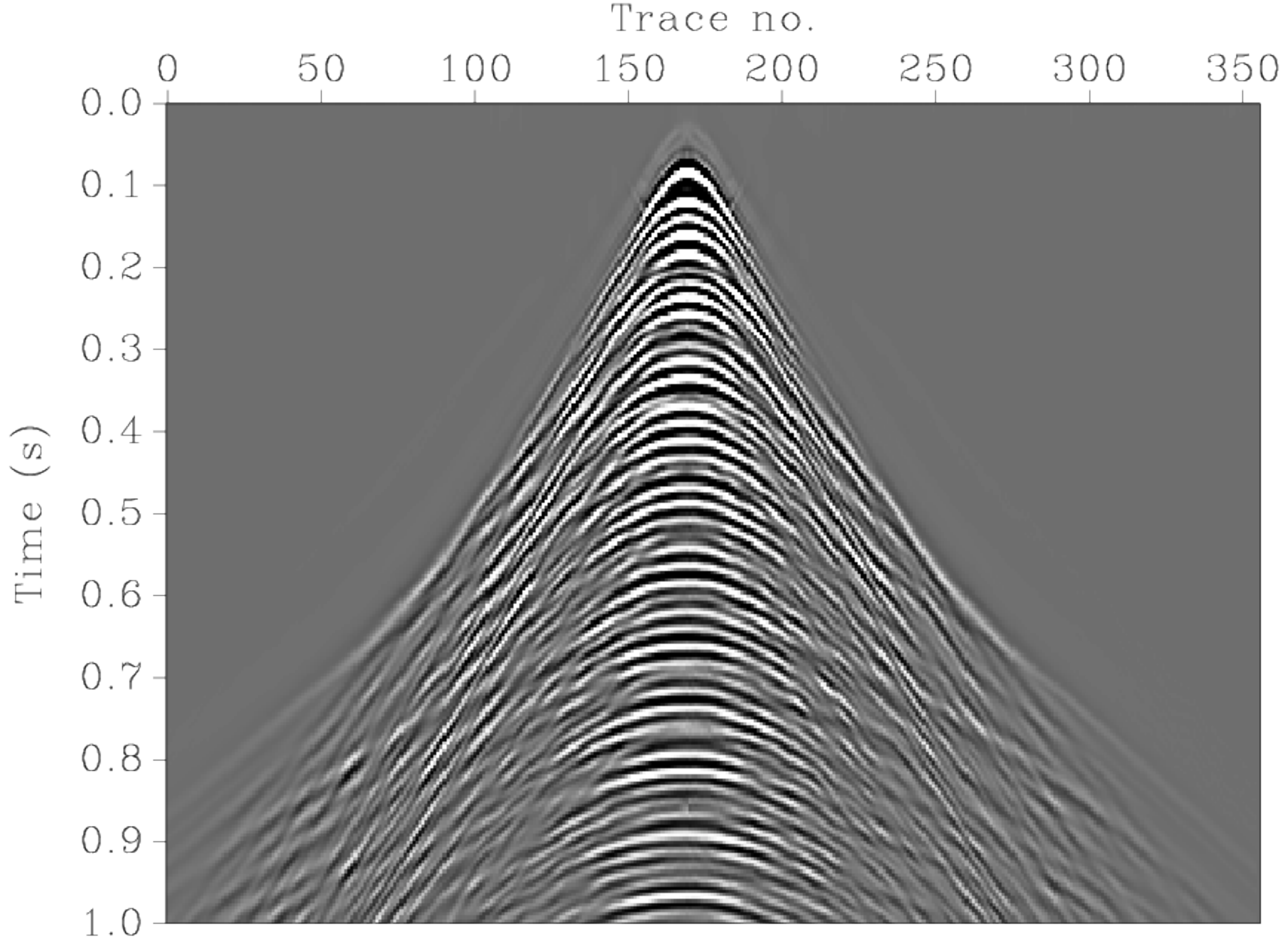
$$B_2 = (1 - \frac{k}{l})\sqrt{l}$$

$$C_1 \text{ and } C_2 > 0$$

Regularization

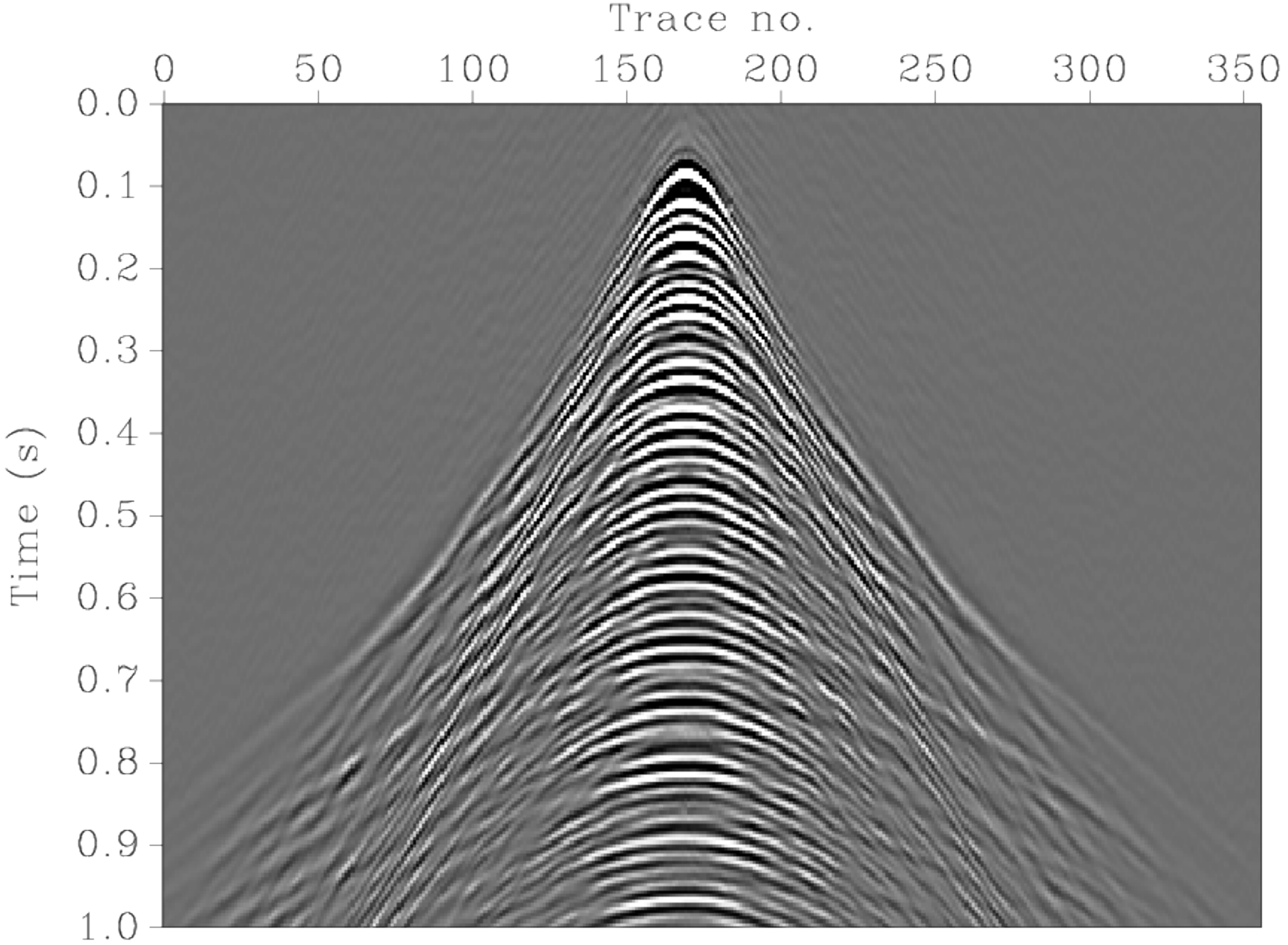
matrix completion

Ground truth



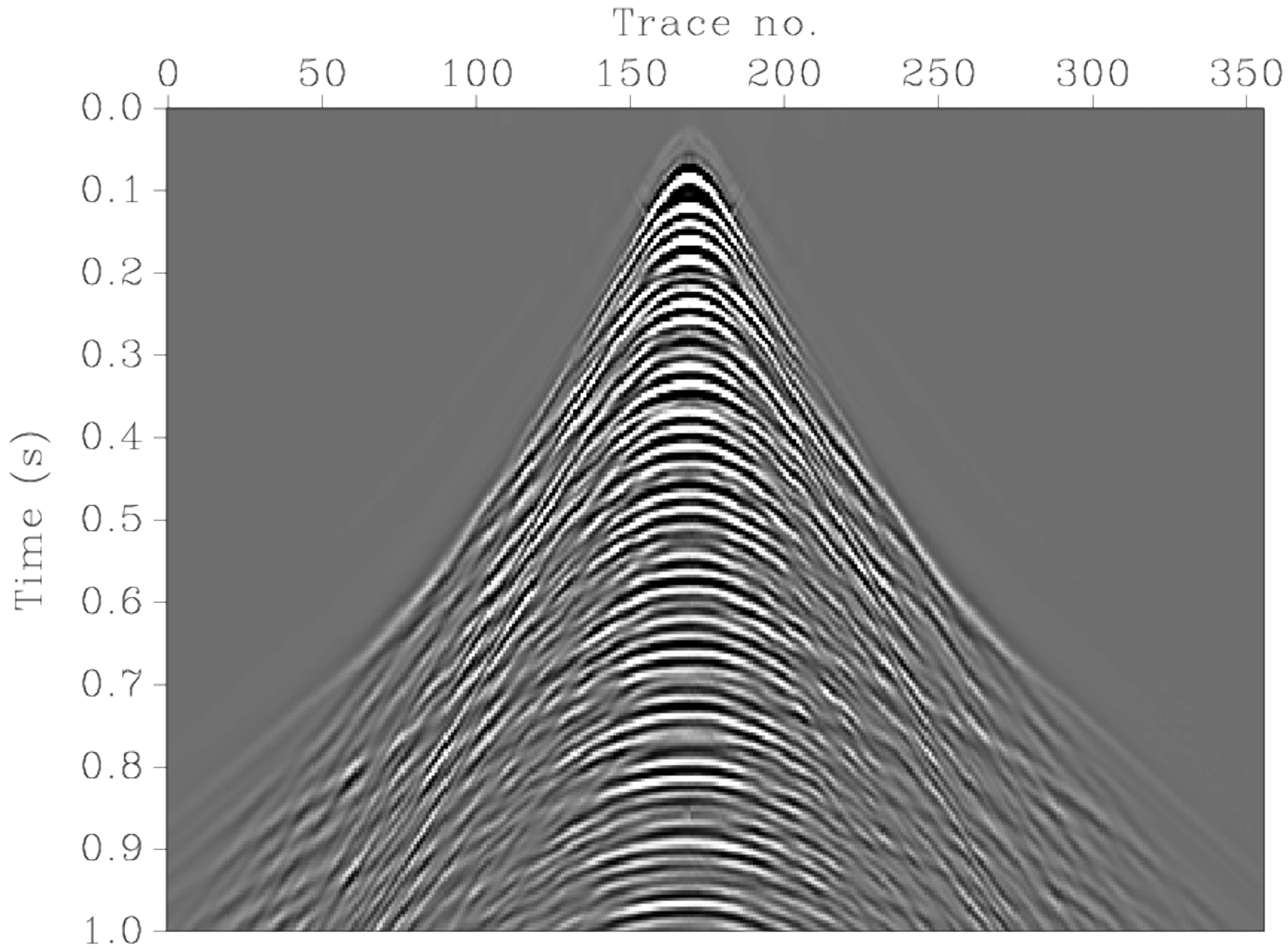
Recovery

SNR = 18.6 dB

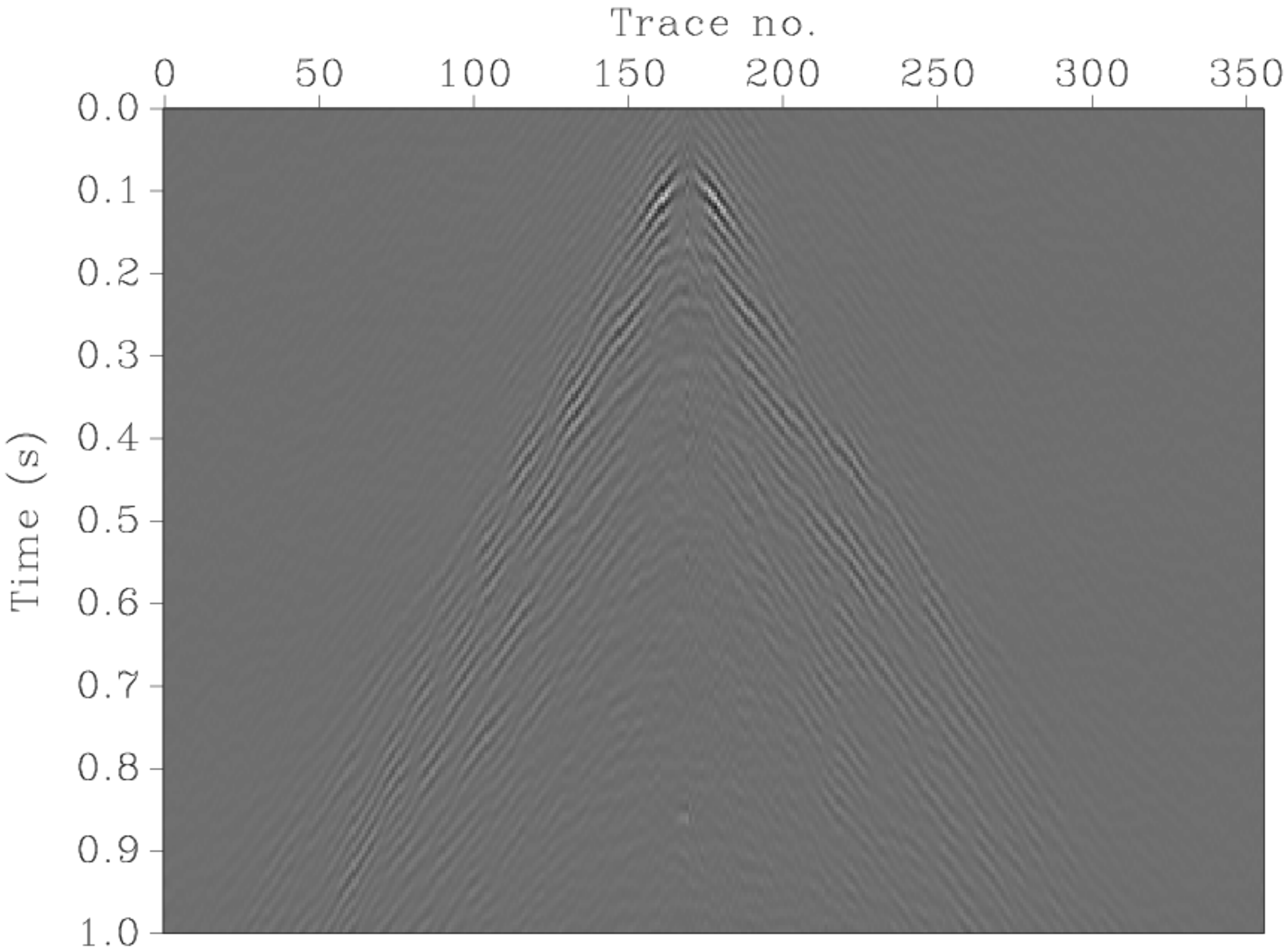


Regularization
matrix completion

Ground truth

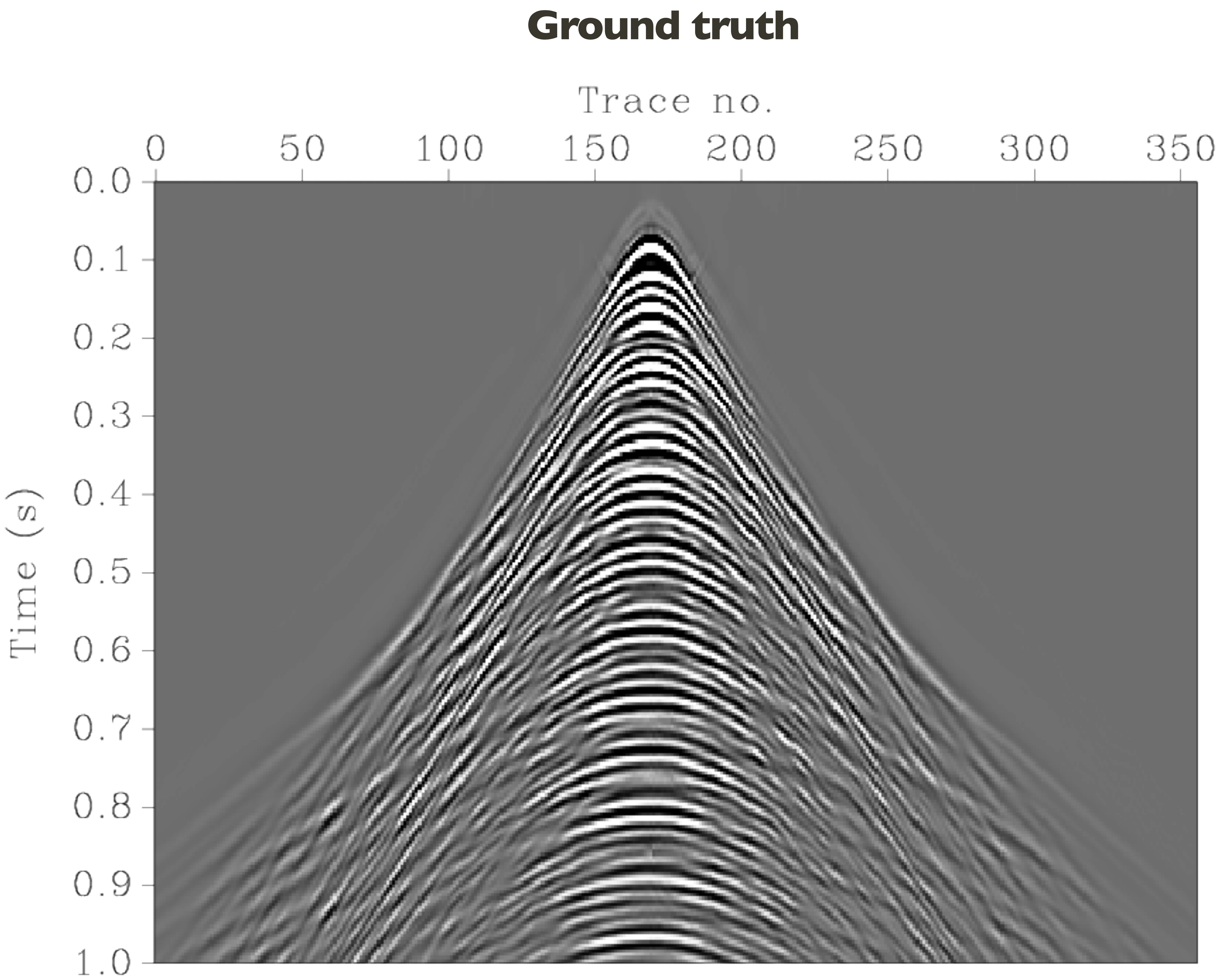


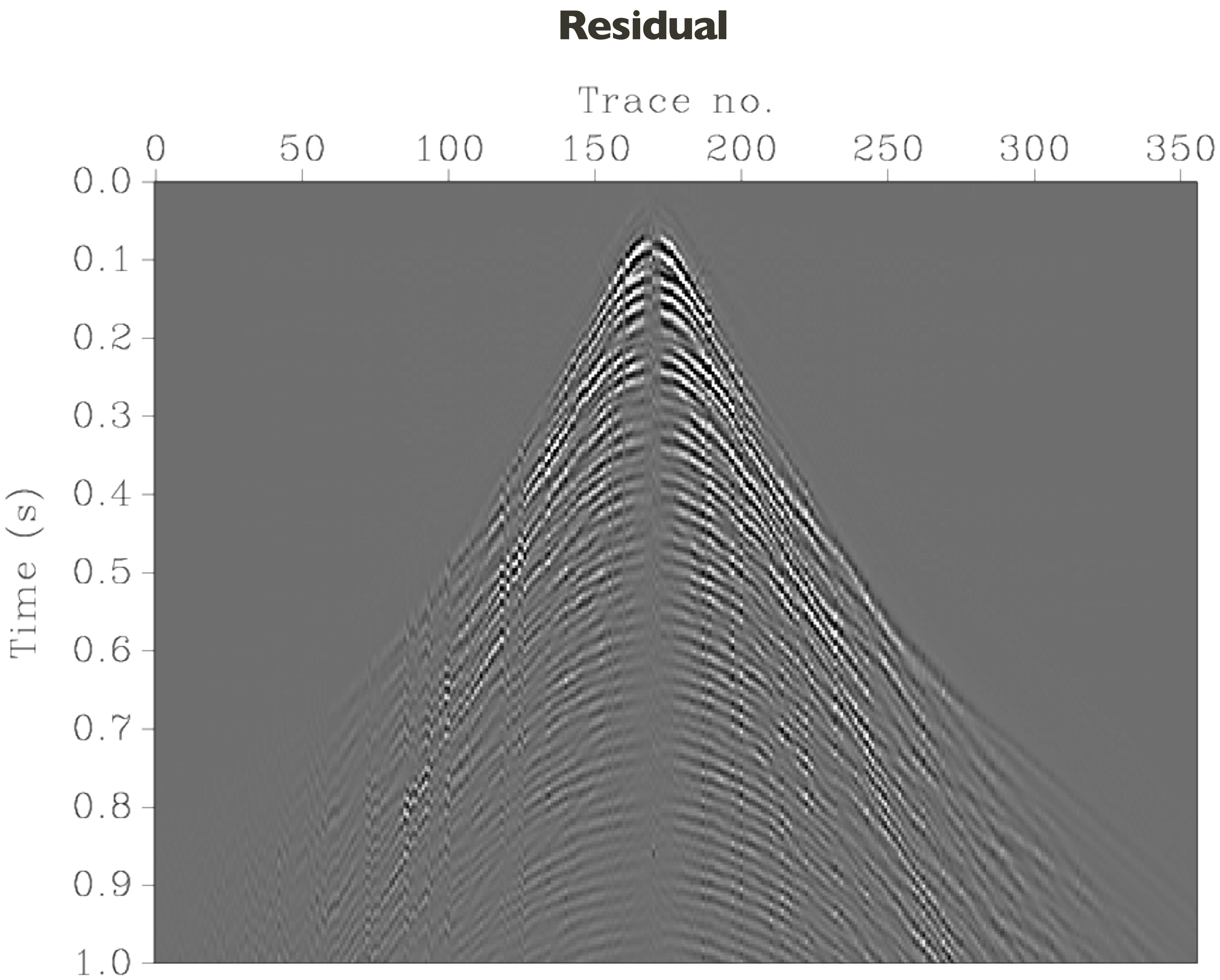
Residual



Regularization

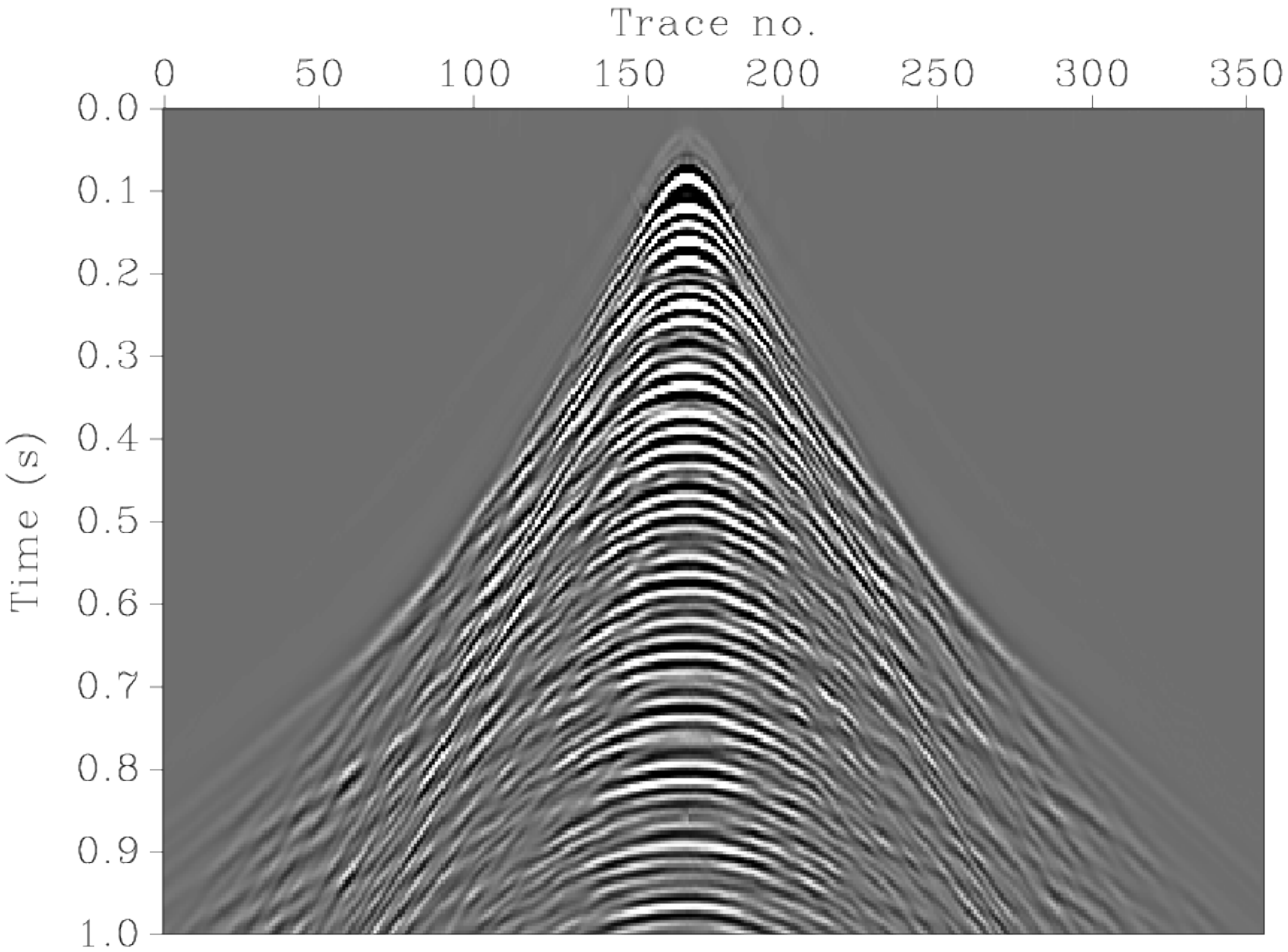
binning



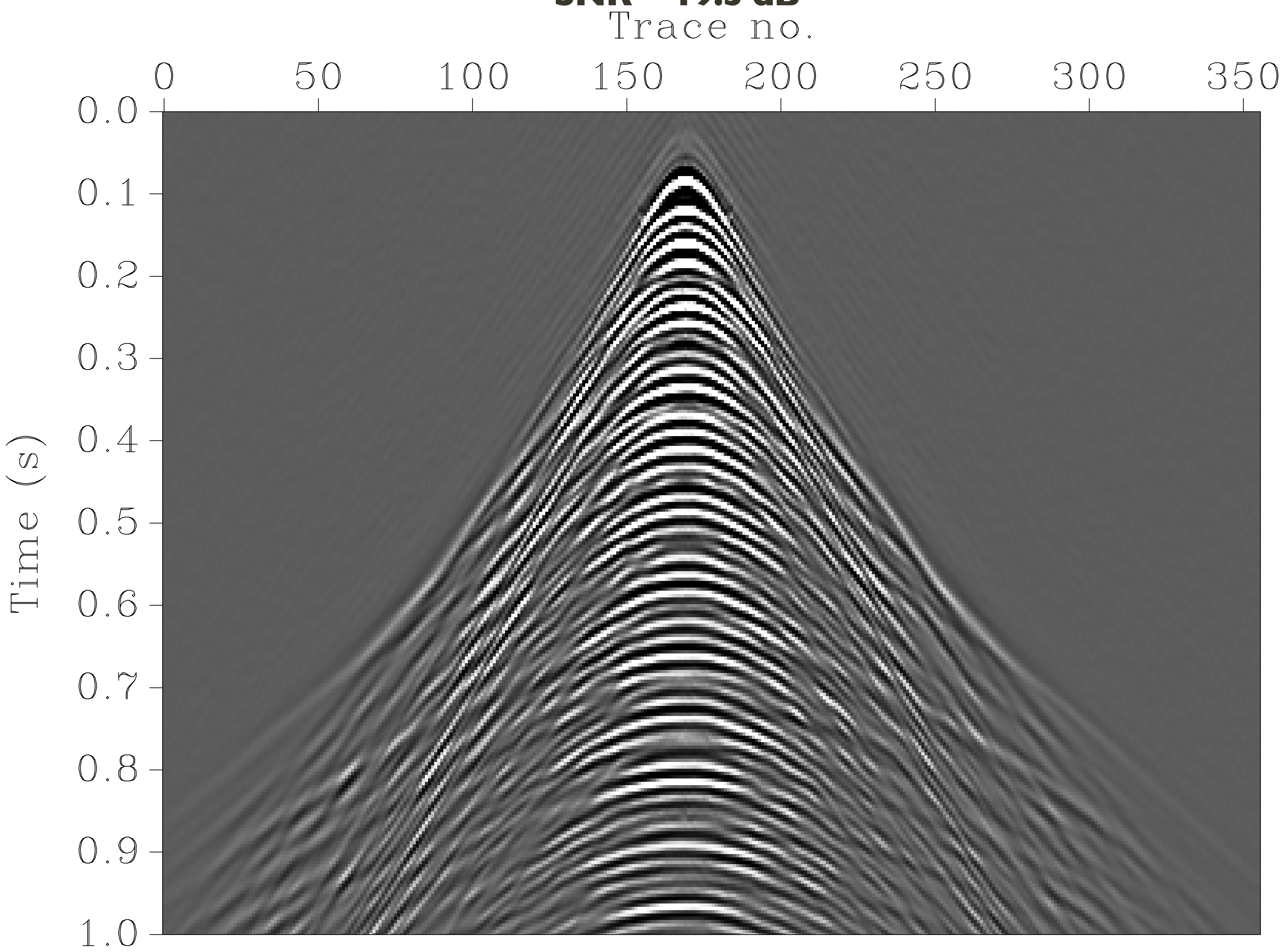


Regularization & Interpolation
matrix completion

Ground truth



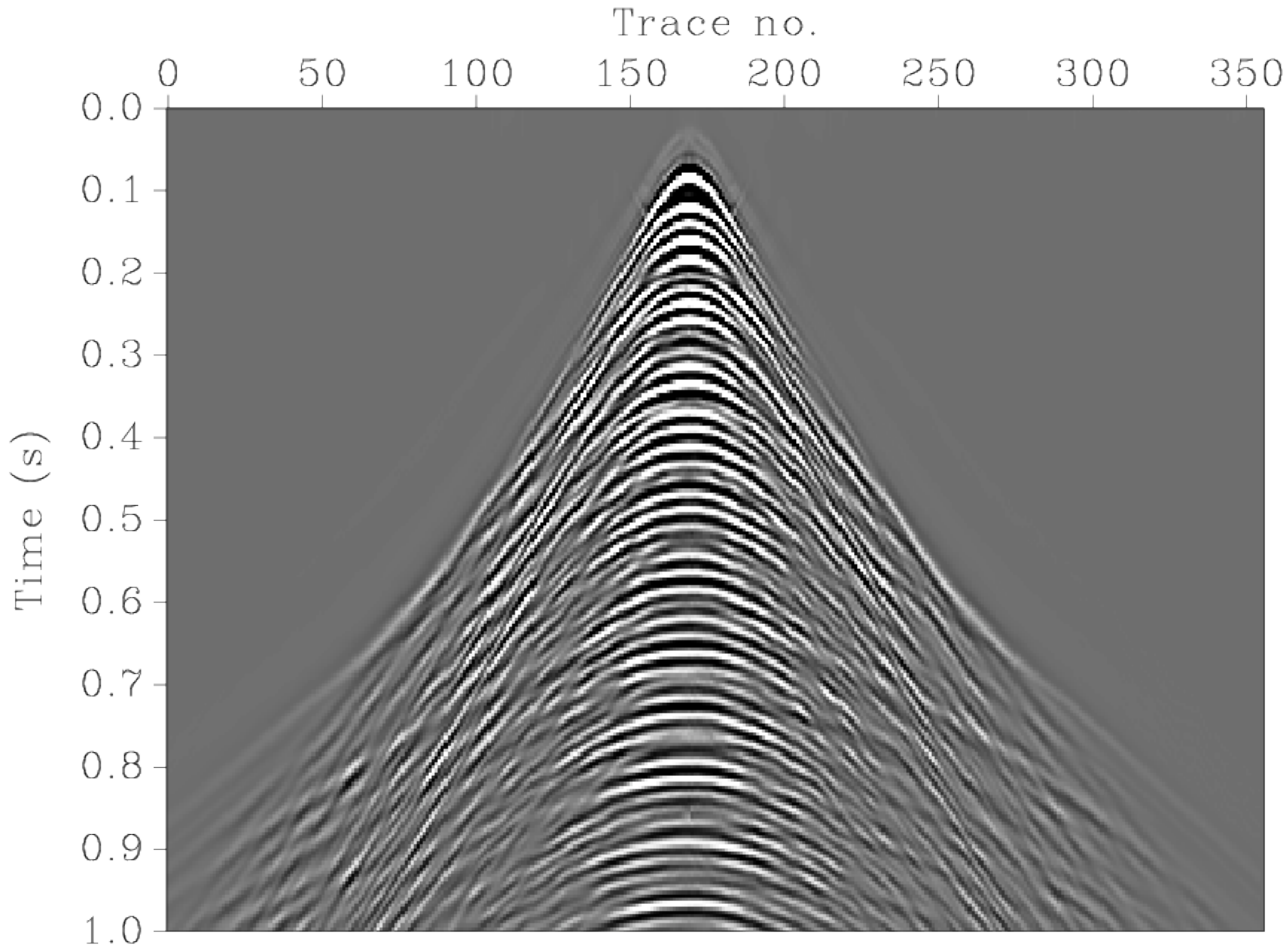
Recovery
SNR = 19.3 dB



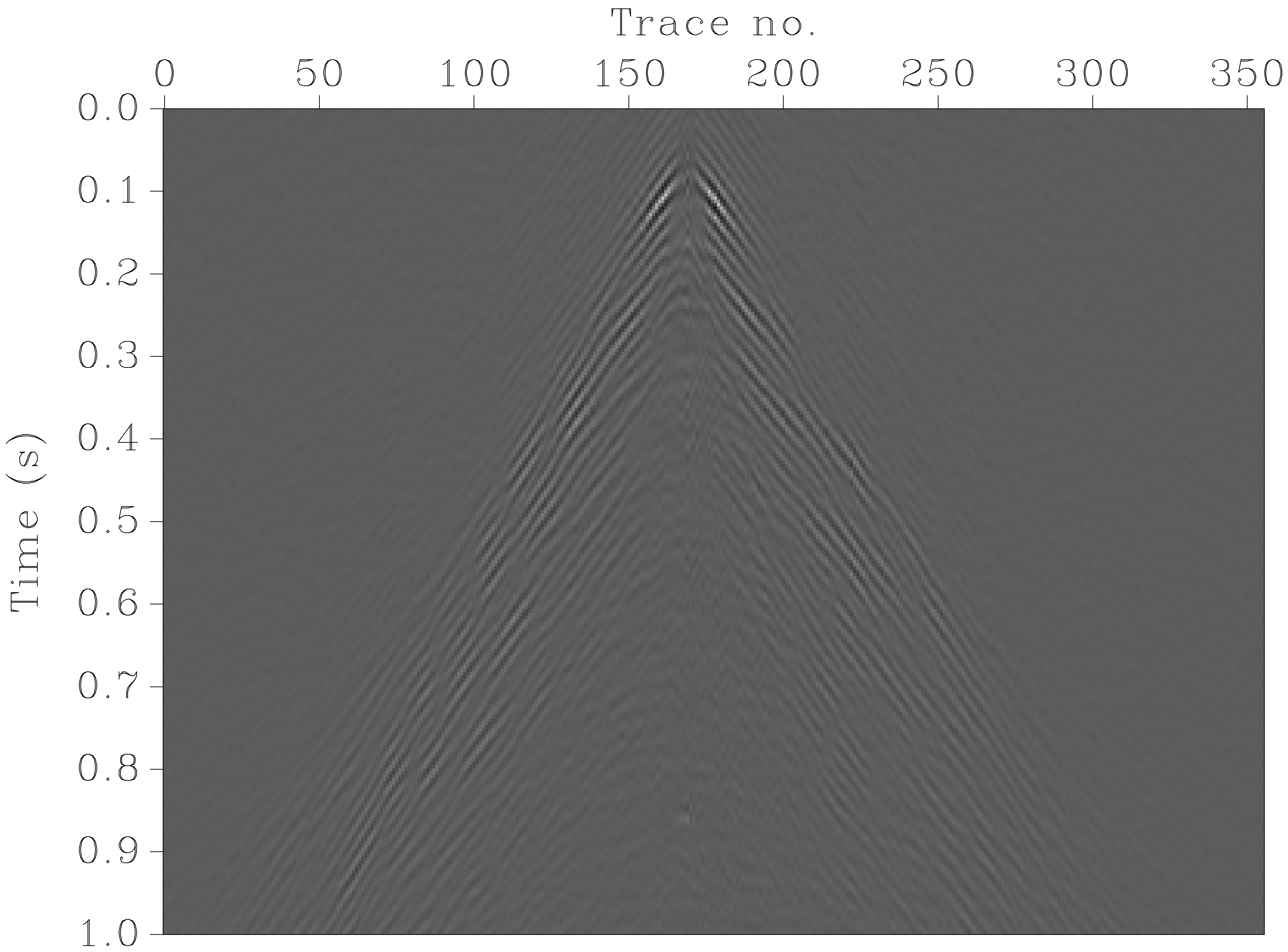
Regularization & Interpolation

matrix completion

Ground truth



Residual



Conclusion

- matrix factorization allows SVD-free low-rank methods that work fast on large data
- reconstruction quality is as good as curvelet-based techniques but computationally more feasible then curvelet
- matrix-factorization promise more compact representation
- able to handle data at unstructured grids

Future work

- incorporate irregularity along both sources & receivers coordinates
- extension to 5D seismic data volumes
- testing of matrix-factorization based methods on real-data
- comparison with tensor-based interpolation methods

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Acknowledgements

We need Real data set

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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