# Denoising in wave inversion with source blending

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SINBAD Consortium Meeting, December 9 2014

# Outline

1. An alternative solver (DC-WRI) for WRI when $\lambda$ is small.
2. Accelerating and Denoising the (DC)-WRI using principal source encoding.
3. Conclusion and future direction.

# Wavefield Reconstruction Inversion

WRI in matrix form

$$\hat{m} = \arg\min_{m,\ U} \underbrace{\|P_\Omega U - D_{obs}\|_F^2 + \lambda^2 \|A(m)U - Q\|_F^2}_{J(m,U)}.$$

We assume

- $\Omega$: locations of receivers.
- each source corresponds to the same set of receivers.

# Solving WRI by Alternative Projection

We alternatively update $m$ and $U$

$$U_{k+1} = \arg\min_U \underbrace{J(m_k, U)}_{\text{quadratic in } U} = \begin{bmatrix} \lambda A \\ P_\Omega \end{bmatrix}^\dagger \begin{bmatrix} Q \\ D_{obs} \end{bmatrix},$$

$$m_{k+1} = m_k - \gamma \frac{\partial J(m, U_{k+1})}{\partial m}\Big|_{m=m_k}.$$

- As $\lambda \to \infty$ WRI converge to FWI.
- As $\lambda \to 0$, $\begin{bmatrix} \lambda A \\ P_\Omega \end{bmatrix}$ becomes ill conditioned.

# Taking the limit

Consider the following two penalties

$$\hat{m}_\lambda = \arg\min_{m,\ U} \|P_\Omega U - D_{obs}\|_F^2 + \lambda^2 \|A(m)U - Q\|_F^2.$$

$$\hat{m}_\alpha = \arg\min_{m,\ U} \alpha^2 \|P_\Omega U - D_{obs}\|_F^2 + \|A(m)U - Q\|_F^2.$$

We assert

$$\lim_{\lambda \to 0} \hat{m}_\lambda = \lim_{\alpha \to \infty} \hat{m}_\alpha.$$

# An Alternative Method

As $\alpha \to \infty$, WRI reduces to

$$\hat{m} = \arg\min_{m,\ U} \|A(m)U - Q\|_F \equiv \widetilde{J}(m, U)$$
$$s.t. \quad P_\Omega U = D_{obs},$$

We expect

$$U_\lambda \to U.$$

$$\widehat{m}_\lambda \to \widehat{m}.$$

# The Alternative Minimization for the New Formulation

When applying the alternative minimization, we found an explicit form of $U_{k+1}$ as a minimizer of $\widetilde{J}(m_k, U)$.

$$U_{k+1} = P_{\Omega^c}^*(A(m_k)P_{\Omega^c}^*)^\dagger(Q - A(m_k)P_\Omega^* D_{obs}) + P_\Omega^* D_{obs}$$
$$m_{k+1} = m_k - \gamma \frac{\partial \widetilde{J}(m, U_{k+1})}{\partial m}\Big|_{m=m_k}.$$

$P_{\Omega^c}$ restricts the wavefield to non-receiver locations.

# DC-WRI

In WRI we need to invert

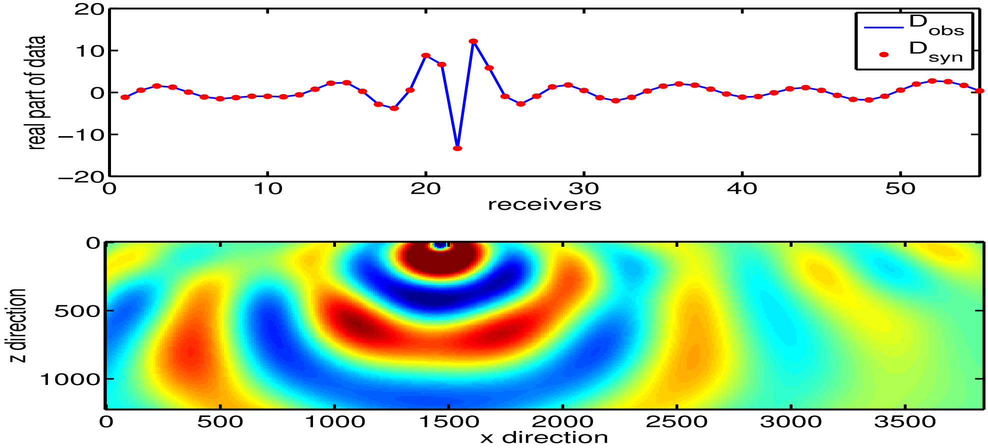
$$\begin{bmatrix} \lambda A \\ P_\Omega \end{bmatrix} \leftarrow \text{a data augmented operator}$$

and now we need to invert

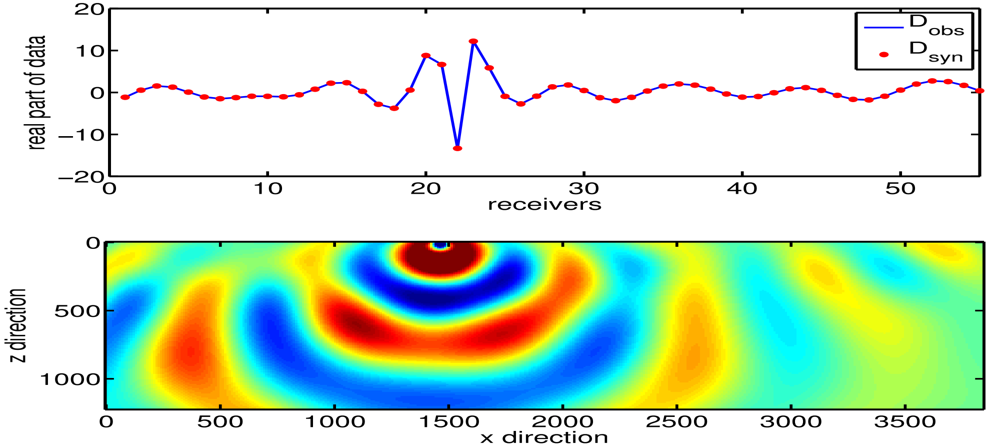
$$A(m_k)P^*_{\Omega^c} \leftarrow \text{a data constrained operator}$$

We call the new method Data-Constrained Wavefield Reconstruction Inversion (DC-WRI).

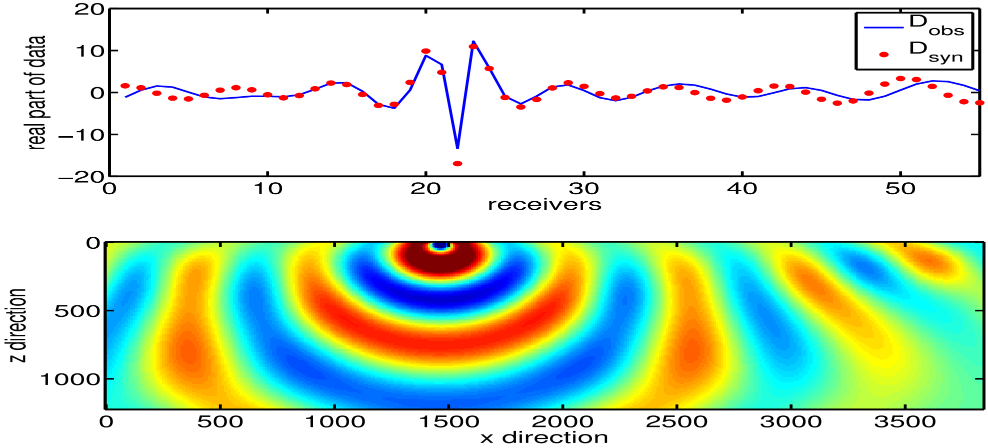
# Wavefield in DC-WRI



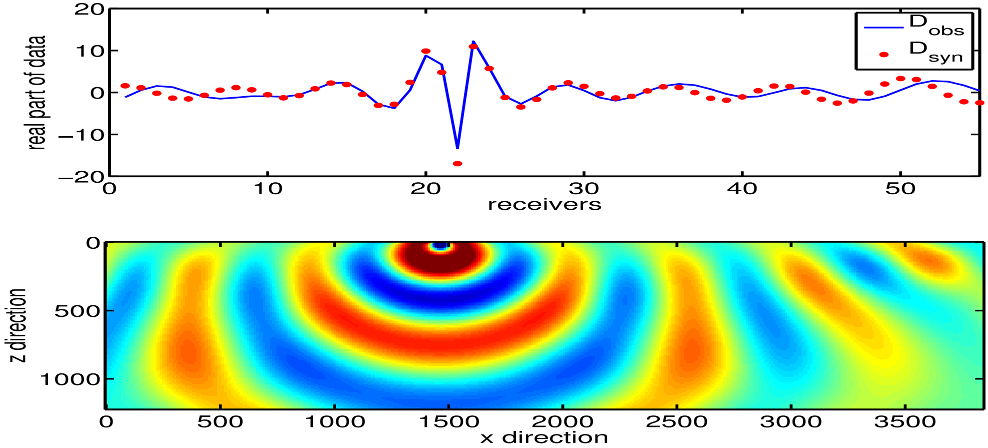
# Wavefield in WRI with $\lambda = 10^{-5}$



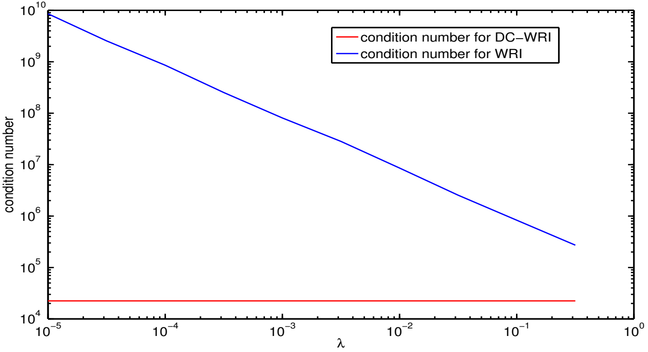
# Wavefield in WRI with $\lambda = 10^5$



# Wavefield in FWI

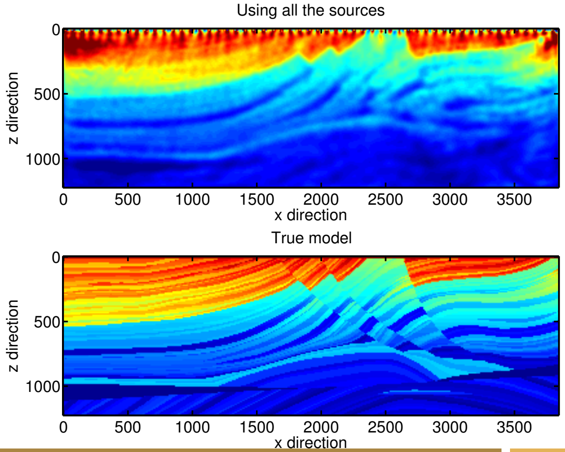


# Comparison of condition numbers



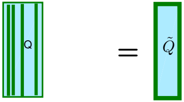
# Noisy data?

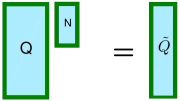
Source offset: 20m, Frequency: 5-7Hz: SNR=9.5(dB) 8-20 Hz: SNR=25 dB

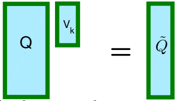


# Source encoding

Source encoding techniques:

- randomly picking the sources: $[(q_{n_1}, ...q_{n_k}] \leftarrow$ 


- Gaussian encoding: $QN \leftarrow$ 


- Encoding using singular vectors $QV_k$ 


  $V_k$ is orthogonal, deterministic, model dependent.

# Source encoding

What is a reasonable $V_k$?

- $\widehat{m}$ is closely related to the accuracy of reconstructions of $U$;
- we want to reconstruct the wavefields for only a subset of "sources";
- Low rank approximation (or PCA)

$$U = W\Sigma V^T \approx W_k \Sigma_k V_k^T$$

- $Q \leftrightarrow U$, $QV_k \leftrightarrow UV_k = W_k\Sigma_k \leftarrow$ $\boxed{\text{the principal component of } U}$.

# Source encoding

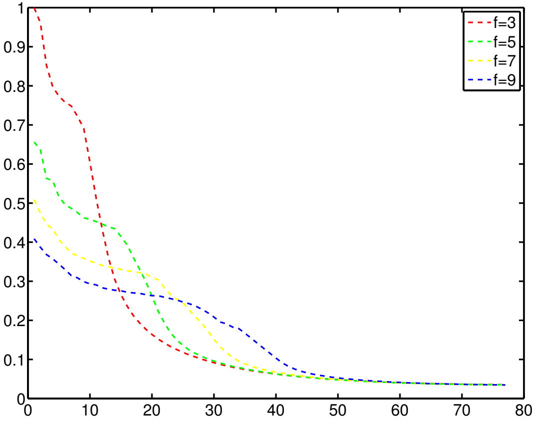
What is a reasonable $V_k$?

- $\widehat{m}$ is closely related to the accuracy of reconstructions of $U$.
- Reconstruct the wavefields for all the sources vs. those for a subset of sources.
- The best subset? Low rank approximation (or PCA)

$$U = W\Sigma V^T \approx W_k \Sigma_k V_k^T$$

- $Q \leftrightarrow U, \quad \underbrace{QV_k}_{\text{encoded source}} \quad \leftrightarrow UV_k = W_k \Sigma_k \leftarrow \boxed{\text{the principal component of } U}.$

# Normalized singular values of $U$ for $f = 3, 5, 7, 9$

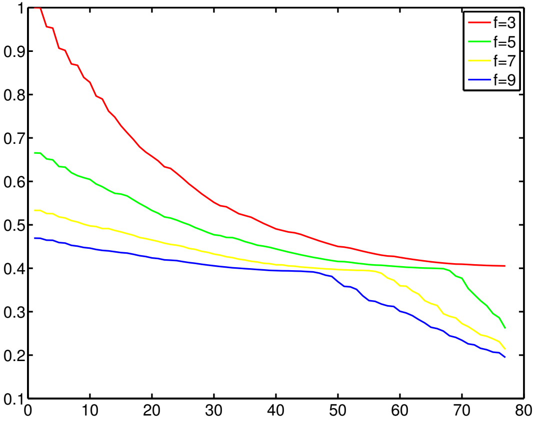


# Source encoding with noise

What if we have noise?

- the principle directions of $D_{obs}$ have the largest SNR when the noise is i.i.d.
- the principle direcitons are obtained through $D_{obs} = \widetilde{W}\widetilde{\Sigma}\widetilde{V}^T \approx \widetilde{W}_k\widetilde{\Sigma}_k\widetilde{V}_k^T$.
- $Q \leftrightarrow U$, $QP_{\widetilde{V}_k}V_k \leftrightarrow UP_{\widetilde{V}_k}V_k \leftarrow$ $\boxed{\text{projecting } V_k \text{ onto directions with high SNR}}$.

# Normalized singular values of $D_{obs}$ for $f = 3, 5, 7, 9$



# Sketched SVD

Sketched SVD: - [Gilbert, Anna C., Jae Young Park, and Michael B. Wakin].

The right singular vectors of $X$ is close to those of $GX$, where $G$ is Gaussian with rows much less than columns.

**Theorem 1.**

*$X$ is an $n \times d$ matrix with rank $k$. Let $G$ be an a random Gaussian matrix with $m < n$. If $m \geq O(k\epsilon^{-2}\log(1/\epsilon) + \log(1/\delta))$, then with probably over $1 - \delta$*

$$\|V_X - V_{GX}\|_2 \leq \epsilon \frac{C\|X\|}{d(X)},$$

*where $V_X$ and $V_{GX}$ are the first $k$ right singular vectors of $X$ and $GX$, $d(X) = \max_{i \neq j} |\sigma_i - \sigma_j|$.*

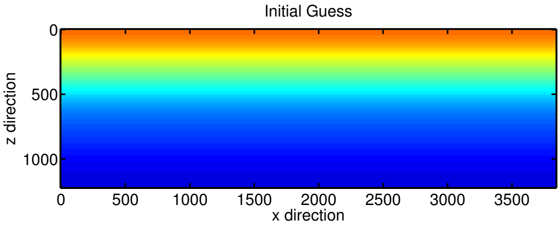
# Summary of the algorithm

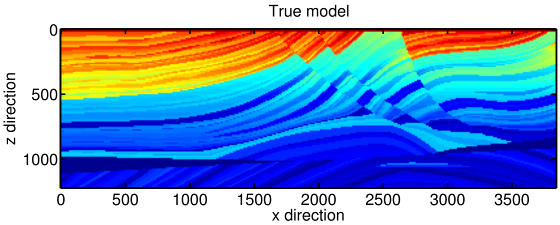
Input: $D, m_0$, some $k_1, k_2 \in \mathbb{Z}$ $k_1 < k_2$ Output: $\hat{m}$.

1. set $f = f_{\min}$;
2. find the SVD of $D_{obs}$ $(nr \times ns)$ and store $\widetilde{V}_{k_2}$;
3. solve the first iteration of DC-WRI and get $U$;
4. apply sketched SVD on $U$ and obtained $V_{k_1}$;
5. construct the mixed sources by $q \cdot P_{\widetilde{V}_{k_2}} V_{k_1}$ ;
6. solve DC-WRI using the mixed sources $q \cdot P_{\widetilde{V}_{k_2}} V_{k_1}$ and mixed data $d \cdot P_{\widetilde{V}_{k_2}} V_{k_1}$ for all frequencies.

# Examples

Receiver and source offset: 50m Frequency: 4 HZ, no noise



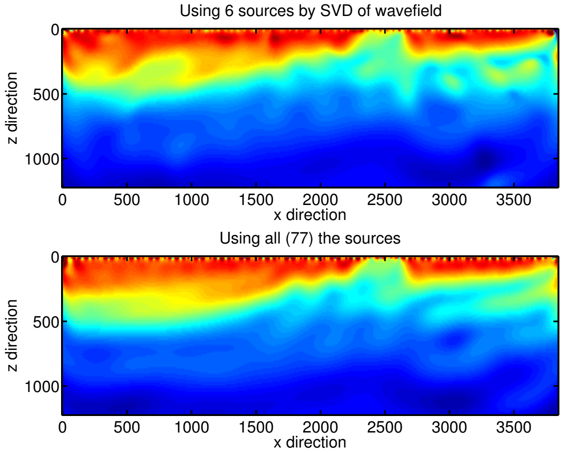



**Figure :** Up: 6 principal sources from wavefield SVD. Down: all (77) sources.

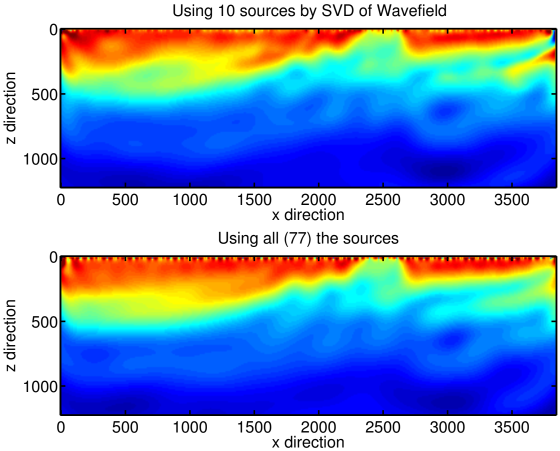



**Figure :** Up 10 principal sources from wavefield SVD. Down: all (77) sources.

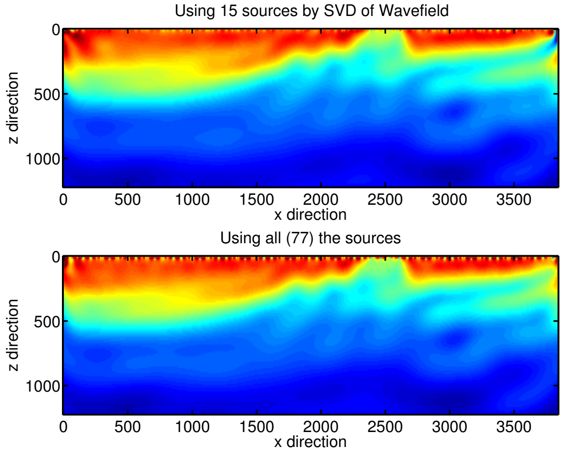



**Figure :** Up: 15 principal sources from wavefield SVD. Down: all (77) sources.

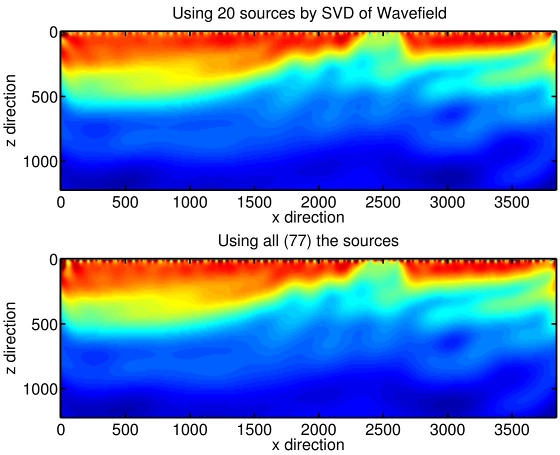



**Figure :** Up: 15 principal sources from wavefield SVD. Down: all (77) sources.

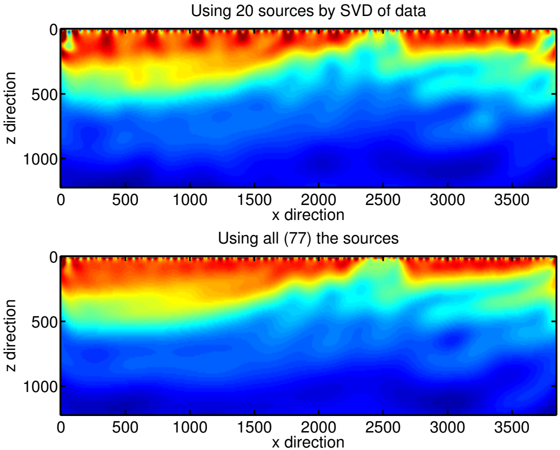
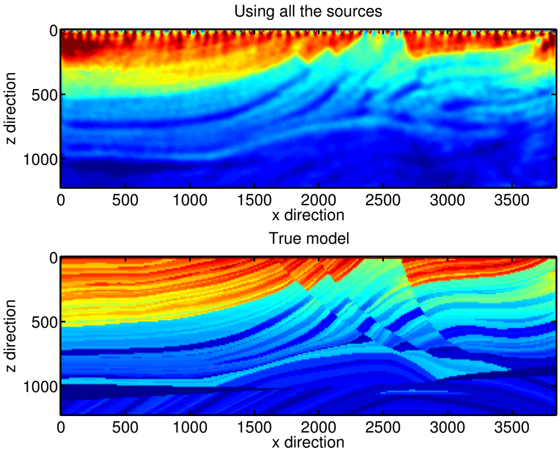



**Figure :** Up: 20 principal sources from SVD on the data. Down: all (77) sources.

# Source offset: 20m, Frequency: 5-7Hz: SNR=9.5(dB) 8-20 Hz: SNR=25 dB



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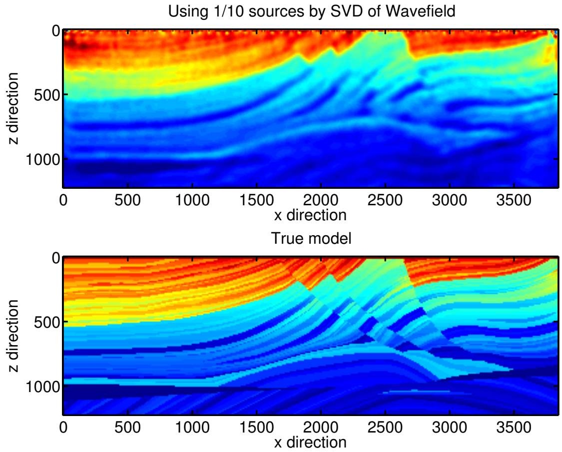


# Summary

We proposed

- a method that overcomes the ill-conditioning problem of small parameter regime of WRI;
- a new source encoding method that could accelerate and stabilize both the new approach and WRI with any $\lambda$.

Future direction:

- Testing cases with missing traces, data completion, or other non-OBS scenarios;
- Bringing the technique to time domain.

# Acknowledgement

Thank you!

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, CGG, Chevron, ConocoPhillips, ION, Petrobras, PGS, Statoil, Total SA, Sub Salt Solutions, WesternGeco, and Woodside.

# Data Constrained WRI

Misfit:

$$J(m,u) = \|P_\Omega U - D_{obs}\|_F^2 + \lambda^2 \|A(m)U - Q\|_F^2$$

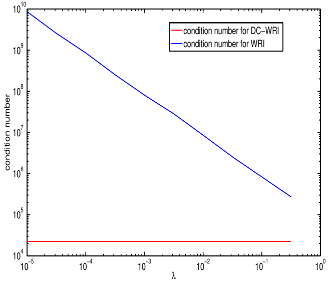
Data Augmented System (WRI)

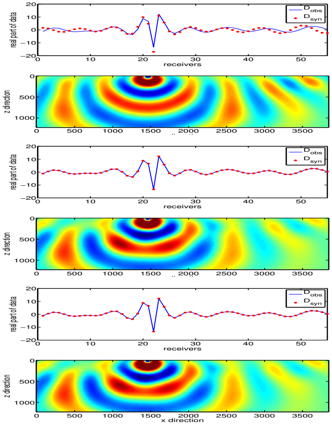
$$\arg\min_U J(m_k, U) = \begin{bmatrix} \lambda A \\ P_\Omega \end{bmatrix}^\dagger \begin{bmatrix} Q \\ D_{obs} \end{bmatrix}$$

Data Constrained System (DC-WRI)

$$\arg\min_U J(m_k, U) = (A(m_k) P_{\Omega^c}^*)^\dagger \begin{bmatrix} Q \\ D_{obs} \end{bmatrix}$$

# Comparison of condition numbers




Up: $\lambda = 10^{10}$ Middle: $\lambda = 10^{-10}$ Down: DC-WRI