Sparse inversion **simplified**

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SLIM

University of British Columbia

Motivation

- Many techniques developed in this group use our in-house *sophisticated* and *complex* l1 solver -- SPGl1.

- Implementation of SPGl1 in low-level languages, e.g., C or Fortran, can be a daunting task to many *geoscientists*.

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Solution

Recent developments in Compressive Sensing (CS) using Linearized Bregman Projection (LBP).

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|_2 \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$

For λ large enough, it converges to the solution of the Basis Pursuit (BP) problem.

Benefits

- **Easy implementation** (details in examples)
- Framework provided also for matrix completion

Preliminary applications

- Fast compressive imaging (by Ning Tu)
- Seismic data interpolation (by Rajiv Kumar)

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Fast compressive imaging using LBP

leveraging the sparse randomized block-Kaczmarz solver:

obtain submatrix $\underline{\mathbf{A}}^k$ and data $\underline{\mathbf{b}}^k$ at the k^{th} iteration

compute residual of the previous step $\mathbf{r}^k = \underline{\mathbf{A}}^k \mathbf{x}^k - \underline{\mathbf{b}}$

compute gradient $\mathbf{g}^k = \underline{\mathbf{A}}^{k'} \mathbf{r}^k$

compute steplength $t^k = \dfrac{\|\mathbf{r}^k\|_2^2}{\|\mathbf{g}^k\|_2^2}$

gradient descent $\mathbf{z}^{k+1} = \mathbf{z}^k - t^k \mathbf{g}^k$

soft thresholding $\mathbf{x}^{k+1} = \mathcal{S}_\lambda(\mathbf{z}^{k+1})$

LBP vs. SPGl1: computer codes

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LBP vs. SPGl1: computer codes

Start your stopwatch!

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LBP vs. SPGl1: computer codes

Start your stopwatch!

```
r = b_sub - A_sub*x;
g = A_sub'*r;
rnorm = norm(r,2);
gnorm = norm(g,2);
sl = (rnorm/gnorm)^2;
z = z+sl*g;
x = sign(x).*max(0,abs(x)-lambda);
```

LBP

E. van den Berg and M. Friedlander, 2008

LBP vs. SPGl1: computer codes

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r = b_sub - A_sub*x;
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gnorm = norm(g,2);
sl = (rnorm/gnorm)^2;
z = z+sl*g;
x = sign(x).*max(0,abs(x)-lambda);
```

LBP

```
function [x,r,g,info] = spgl1( A, b, tau, sigma, x, options )

m = length(b);

%----------------------------------------------------------------------
% Check arguments.
%----------------------------------------------------------------------
if ~exist('options','var'), options = []; end
if ~exist('x','var'), x = []; end
if ~exist('sigma','var'), sigma = []; end
if ~exist('tau','var'), tau = []; end

if nargin < 2 || isempty(b) || isempty(A)
   error('At least two arguments are required');
elseif isempty(tau) && isempty(sigma)
   tau = 0;
   sigma = 0;
   singleTau = false;
elseif isempty(sigma) % && ~isempty(tau)  <-- implied
   singleTau = true;
else
   if isempty(tau)
      tau = 0;
   end
   singleTau = false;
end

%----------------------------------------------------------------------
% Grab input options and set defaults where needed.
%----------------------------------------------------------------------
defaultopts = spgSetParms(...
'fid'        ,      1 , ... % File ID for output
'verbosity'  ,      2 , ... % Verbosity level
'iterations' ,   10*m , ... % Max number of iterations
'nPrevVals'  ,      3 , ... % Number previous func values for linesearch
'bpTol'      ,  1e-06 , ... % Tolerance for basis pursuit solution
'optTol'     ,  1e-04 , ... % Optimality tolerance
'decTol'     ,  1e-04 , ... % Req'd rel. change in primal obj. for Newton
'stepMin'    ,  1e-16 , ... % Minimum spectral step
'stepMax'    ,  1e+05 , ... % Maximum spectral step
'rootMethod' ,      2 , ... % Root finding method: 2=quad,1=linear (not used).
'activeSetIt',    Inf , ... % Exit with EXIT_ACTIVE_SET if nnz same for # its.
'subspaceMin',      0 , ... % Use subspace minimization
'iscomplex'  ,    NaN , ... % Flag set to indicate complex problem
'maxMatvec'  ,    Inf , ... % Maximum matrix-vector multiplies allowed
'weights'    ,      1 , ... % Weights W in ||Wx||_1
'Kaczmarz'   ,      0 , ... % Toggles whether Kaczmarz mode is on (experimental)
'KaczScale'  ,      1 , ... % Scaling factor for Tau when using Kaczmarz-type submatrices
'quitPareto' ,      0 , ... % Exits when pareto curve is reached
'minPareto'  ,      3 , ... % If quitPareto is on, the minimum number of iterations before checking for quitPareto conditions
'lineSrchIt' ,      1 , ... % Maximum number of line search iterations for spgLineCurvy, originally 10 ...
'feasSrchIt' ,  10000 , ... % Maximum number of feasible direction line search iteraitons, originally 10 ...
'ignorePErr' ,      0 , ... % Ignores projections error by issuing a warning instead of an error ...
'project'    , @NormL1_project , ...
'primal_norm', @NormL1_primal  , ...
'dual_norm'  , @NormL1_dual      ...
   );
options = spgSetParms(defaultopts, options);

fid           = options.fid;
logLevel      = options.verbosity;
maxIts        = options.iterations;
nPrevVals     = options.nPrevVals;
bpTol         = options.bpTol;
optTol        = options.optTol;
decTol        = options.decTol;
stepMin       = options.stepMin;
stepMax       = options.stepMax;
activeSetIt   = options.activeSetIt;
subspaceMin   = options.subspaceMin;
maxMatvec     = max(3,options.maxMatvec);
weights       = options.weights;
quitPareto    = options.quitPareto;
minPareto     = options.minPareto;
Kaczmarz      = options.Kaczmarz;
lineSrchIt    = options.lineSrchIt;
feasSrchIt    = options.feasSrchIt;
ignorePErr    = options.ignorePErr;

% maxLineErrors TEMPORARILY DISABLED to prevent very lar[illegible]g issue[illegible]ff a[illegible]orithm
maxLineErrors = Inf;     % Maximum number of line-search failures.
pivTol        = 1e-12;  % Threshold for significant Newton step.

%----------------------------------------------------------------------
% Initialize local variables.
%----------------------------------------------------------------------
iter          = 0;  itnTotLSQR = 0; % Total SPGL1 and LSQR iterations.
nProdA        = 0;  nProdAt    = 0;
lastFv        = -inf(nPrevVals,1);  % Last m function values.
nLineTot      = 0;                  % Total no. of linesearch steps.
printTau      = false;
nNewton       = 0;
bNorm         = norm(b,2);
stat          = false;
timeProject   = 0;
timeMatProd   = 0;
nnzIter       = 0;                  % No. of its with fixed pattern.
nnzIdx        = [];                 % Active-set indicator.
subspace      = false;              % Flag if did subspace min in current itn.
stepG         = 1;                  % Step length for projected gradient.
testUpdateTau = 0;                  % Previous step did not update tau

% Determine initial x, vector length n, and see if problem is complex
explicit = ~(isa(A,'function_handle'));
```

SPGl1

E. van den Berg and M. Friedlander, 2008

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LBP vs. SPGl1: computer codes

Start your stopwatch!

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LBP

SPGl1

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% Determine initial x, vector length n, and see if problem is complex
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```

So apparent that you do not really need a stopwatch.

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LBP vs. SPGl1: where are the complexities

SPGl1

- automatic calculates the sparsity parameter
  - determining whether the Pareto curve is reached
  - computing local slope of the Pareto curve
- non-deterministic line-search

LBP

- so far uses more "**heuristic**" threshold
  - easy implementation comes at a price
- uses deterministic line search
  - also the least costly among all gradient descent methods

Herrmann and Li, 2011; Tu and Herrmann, 2012; Tu and Herrmann, 2013

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LBP vs. SPGl1: rerandomization benefits

Rerandomization

- **redraw** source experiments/frequencies
  - speedup of convergence in terms of model errors
  - improved robustness to linearization errors
  - ***no overhead*** on simulation cost

SPGl1

- redraw **occasionally**
  - once the Pareto curve is reached

LBP

- redraw **every iteration**
  - leveraging the sparse Kaczmarz solver

Herrmann and Li, 2011; Tu and Herrmann, 2012

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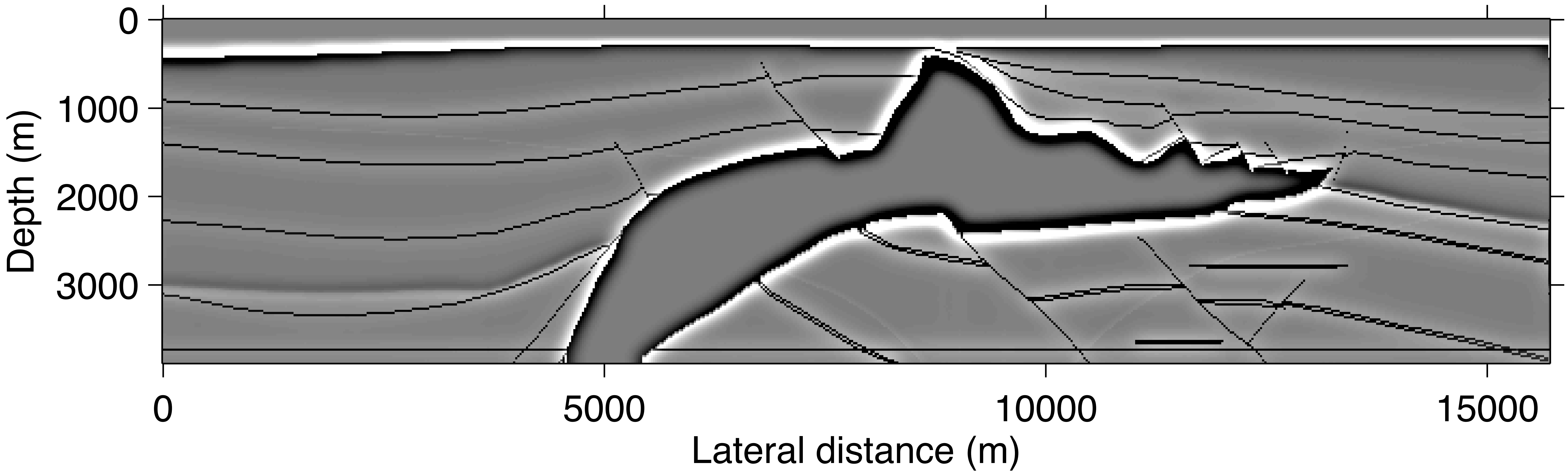
LBP vs. SPGl1: imaging examples

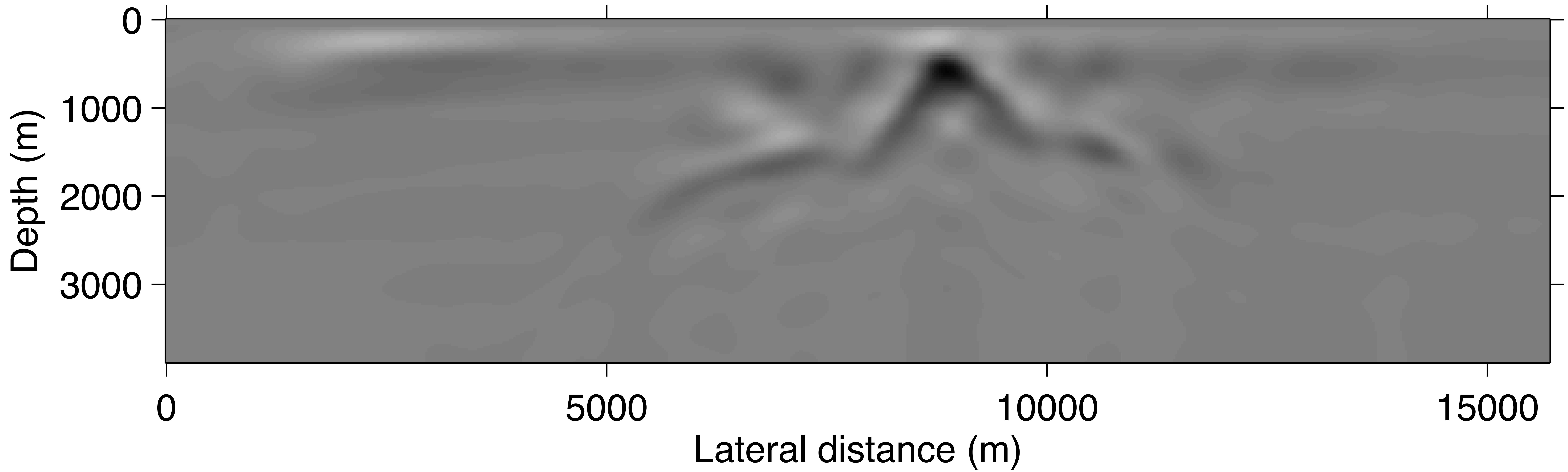
fast least-squares migrations of
- primaries
- primaries + surface multiples

- simulation cost ~1 RTM of all the data
  - ***very cheap*** (50 fold reduction) per-iteration cost by source/freq. subsampling
- input data generated by linearized modelling
- thresholding **heuristics**
  - keep 1% of curvelet coefficients at first
  - gradually relaxed to 5%

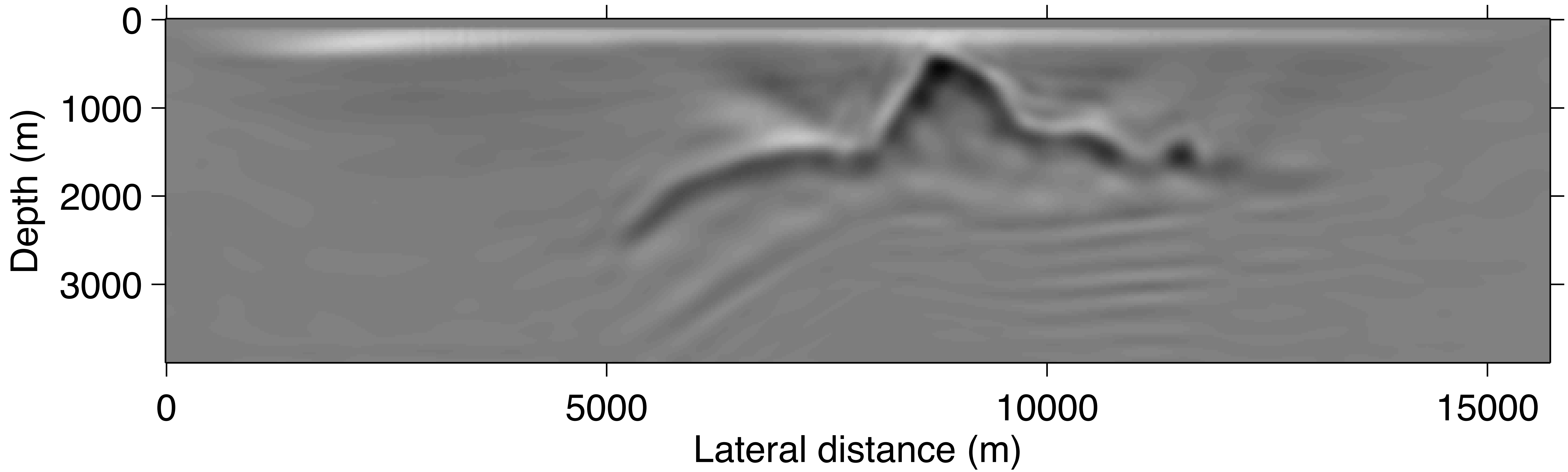
Fast imaging of primaries: true image



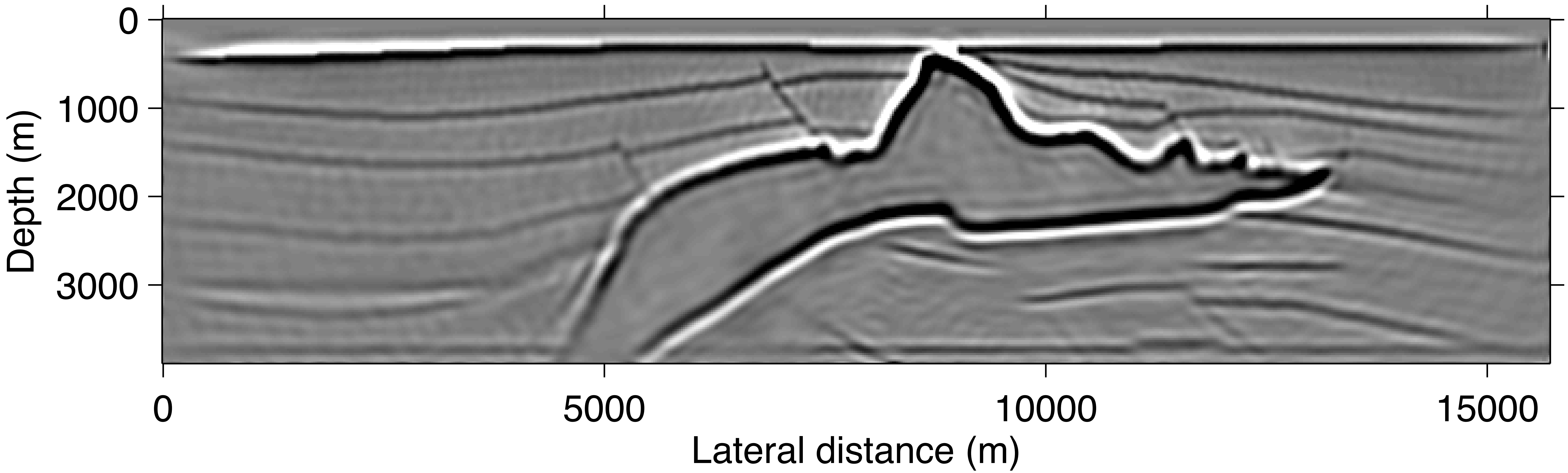
Fast imaging of primaries: by SPGl1, 1st iteration



Fast imaging of primaries: by LBP, 1st iteration

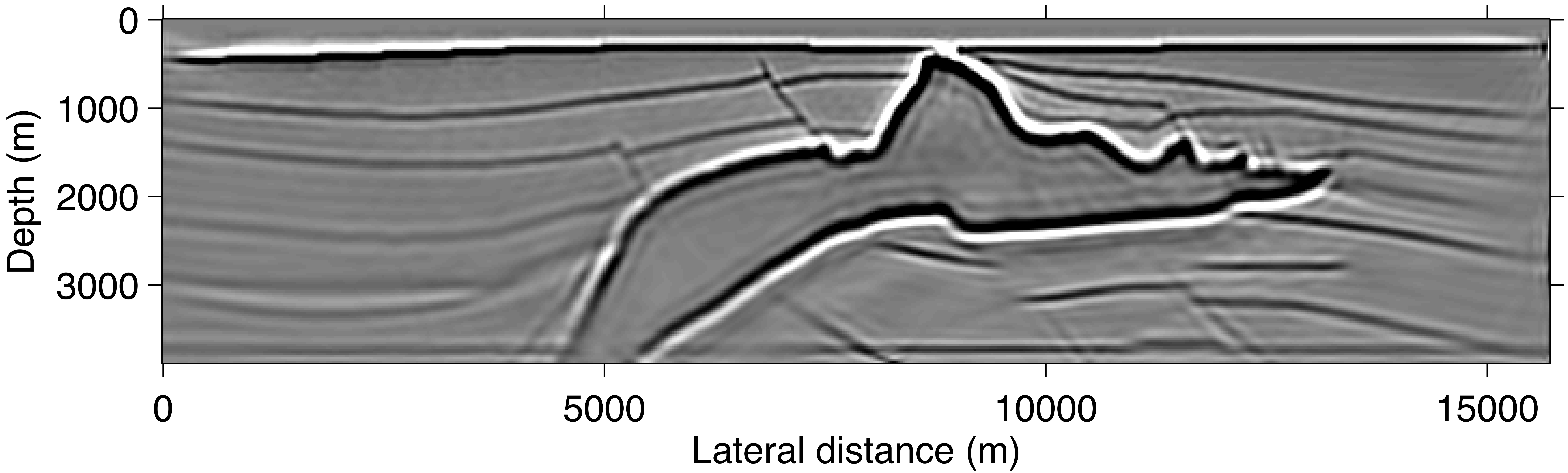


Fast imaging of primaries: by SPGl1, final image



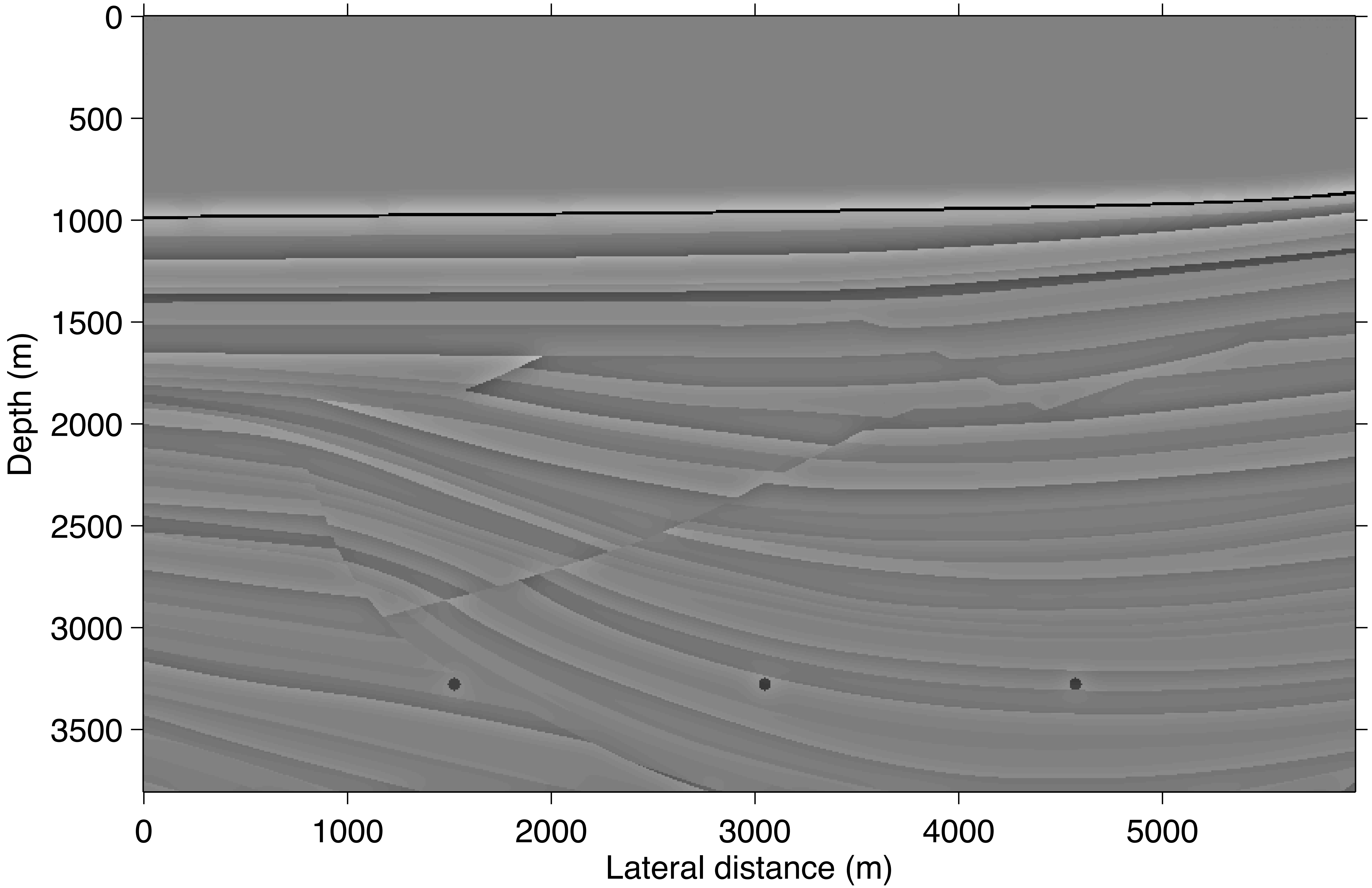
Simulation cost **~1 RTM** using all the data

Fast imaging of primaries: by **LBP**, final image

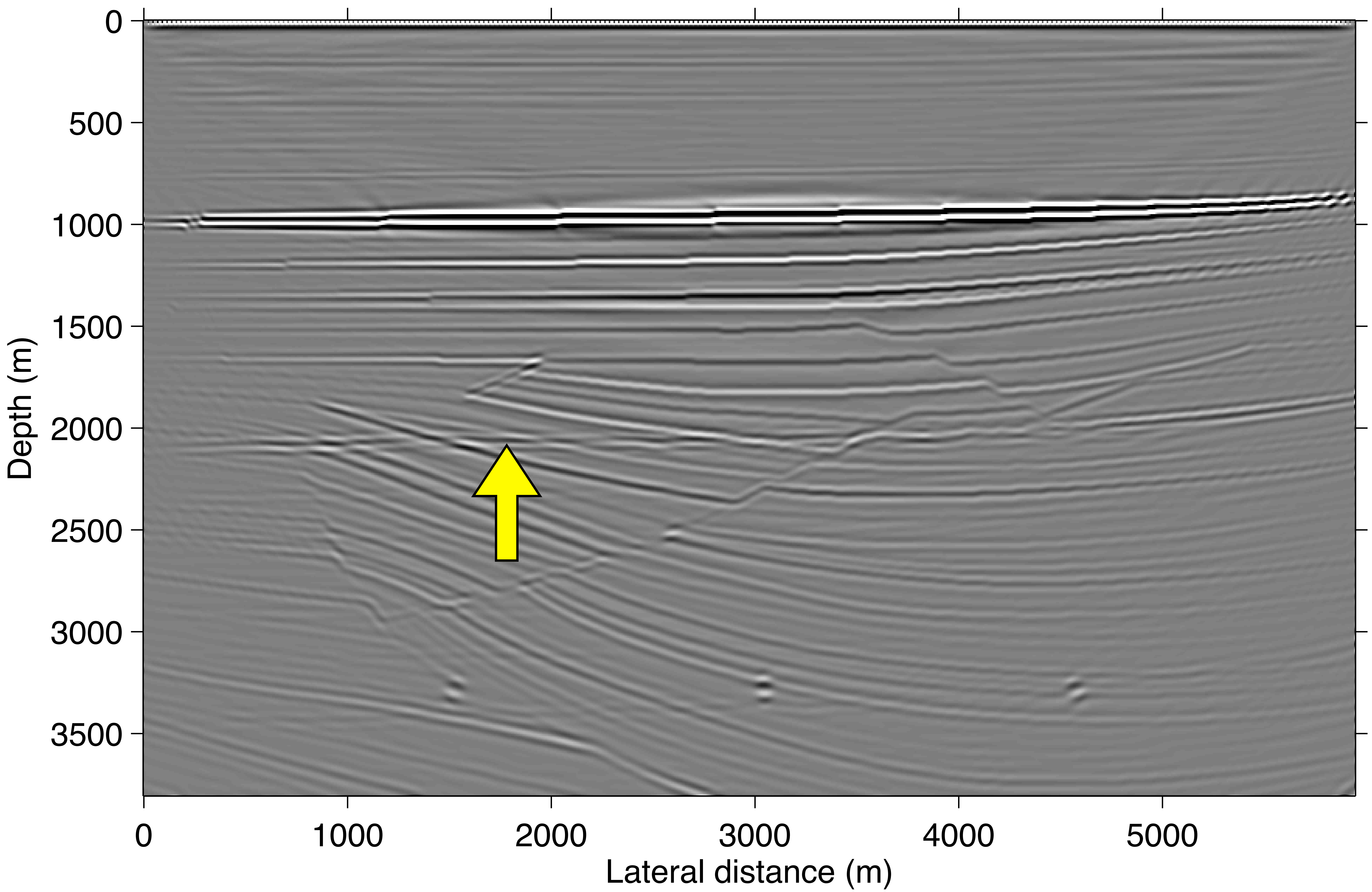


Simulation cost **~1 RTM** using all the data

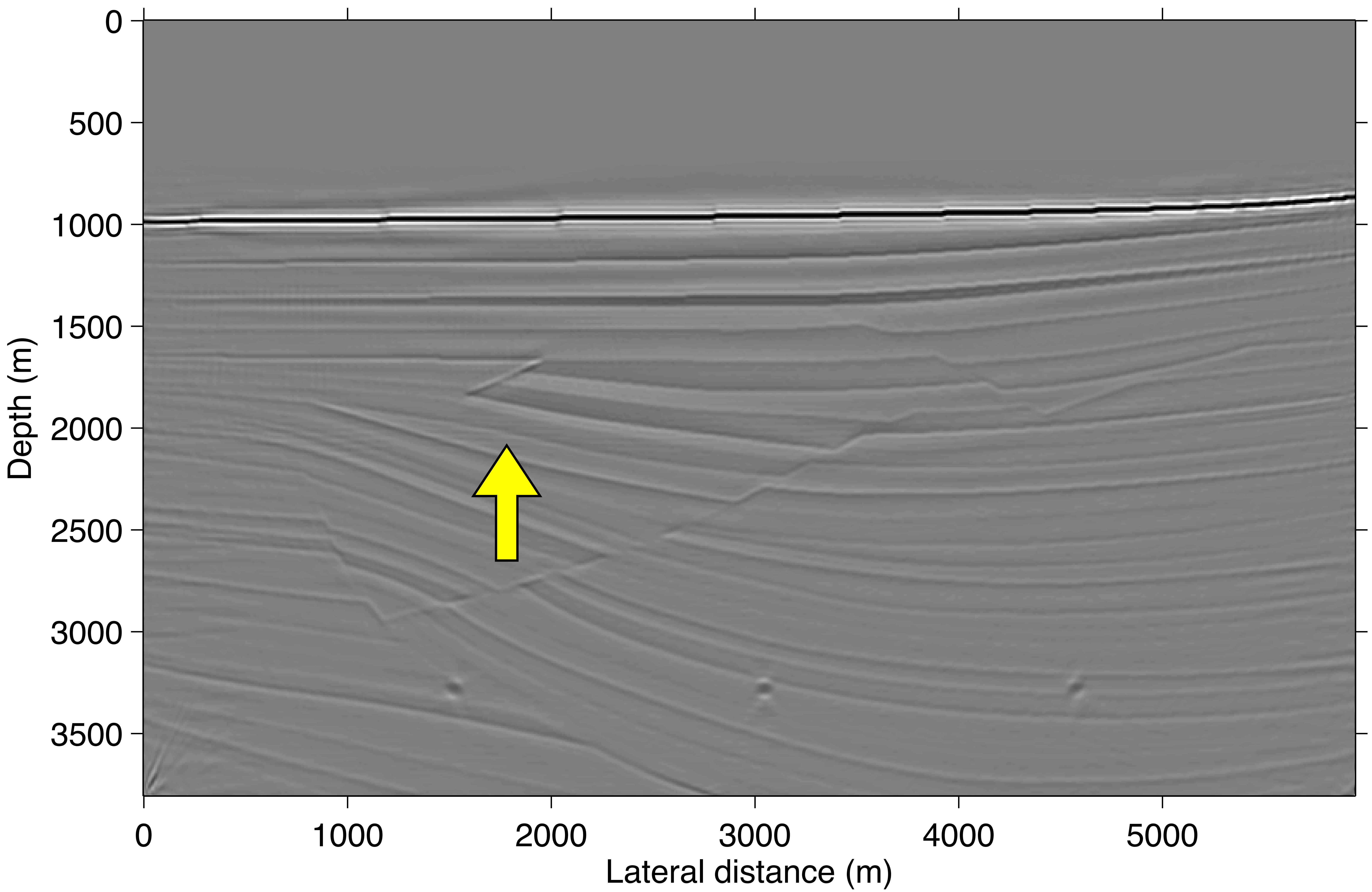
Fast imaging with multiples: true image



RTM with multiples [more details in Tue talk @ 4:30PM]

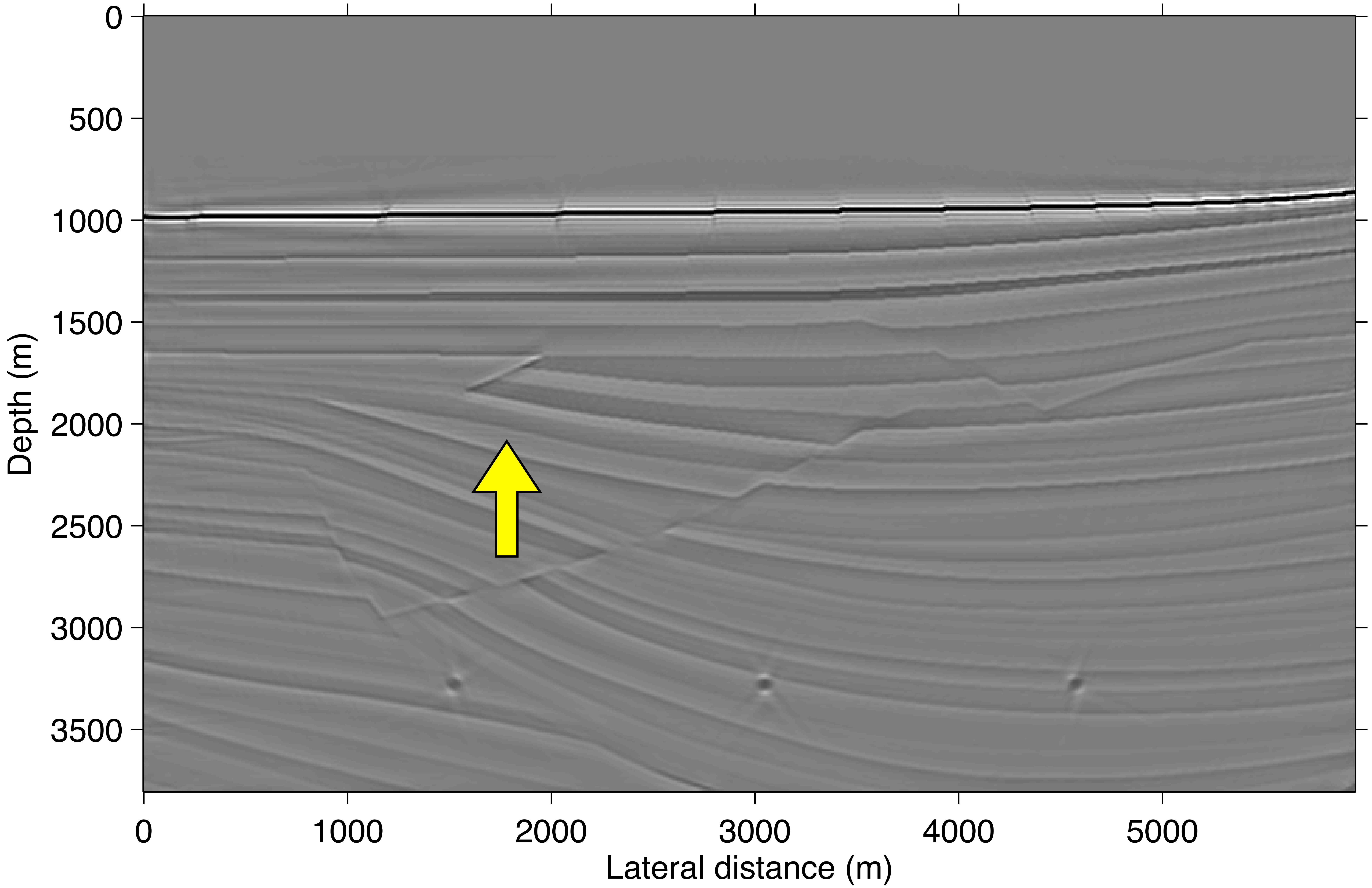


Fast imaging with multiples: by SPGl1, final image



Simulation cost **~1 RTM** using all the data

Fast imaging with multiples: by LBP, final image



Simulation cost **~1 RTM** using all the data

Data interpolation via matrix completion

$$\min_{\mathbf{X}} \quad \|\mathbf{X}\|_* \quad \text{s.t. } \|\mathcal{A}(X) - \mathbf{b}\|_2^2 \leq \sigma$$

where

$$\mathcal{A}(.) = \mathbf{M}\mathcal{S}^H(.)$$

$\mathbf{M}$ time-jittered operator

$\mathcal{S}^H$ transform operator

Algorithm

Huang et. al. '11

$$\text{Input} : \mathbf{X}^o = \hat{\mathbf{X}}^o = \mathbf{P}^o = \hat{\mathbf{P}}^o = 0, \mu = 5n, \tau = 1/\mu, \alpha = 1$$

for $k = 0, 1, \ldots$ **do**

$$\mathbf{X}^{k+1} := \text{Shrink}(\mathbf{X}^k - \mu(\tau \mathcal{A}'(\mathcal{A}(X) - \mathbf{b}) - \mathbf{P^k}), \mu)$$

$$\mathbf{P}^{k+1} := \hat{\mathbf{P}}^k - (\tau \mathcal{A}'(\mathcal{A}(\hat{\mathbf{X}}^k) - \mathbf{b}) - (\mathbf{X^{k+1}} - \hat{\mathbf{X}}^{\mathbf{k}})/\mu$$

$$\hat{\mathbf{X}}^{k+1} := \alpha_k \mathbf{X}^{k+1} + (1 - \alpha_k)\mathbf{X}^k$$

$$\hat{\mathbf{P}}^{k+1} := \alpha_k \mathbf{P}^{k+1} + (1 - \alpha_k)\mathbf{P}^k$$

$$\alpha_k := 1 + 2/(k+2) * ((k+2)/2 - 1)$$

end for

Algorithm

Huang et. al. '11

$$\mathrm{Shrink}(Y, \gamma) := U \mathrm{diag}(\max(\sigma - \gamma, 0)) V^H$$

where

$$Y = U \mathrm{diag}(\sigma) V^H$$

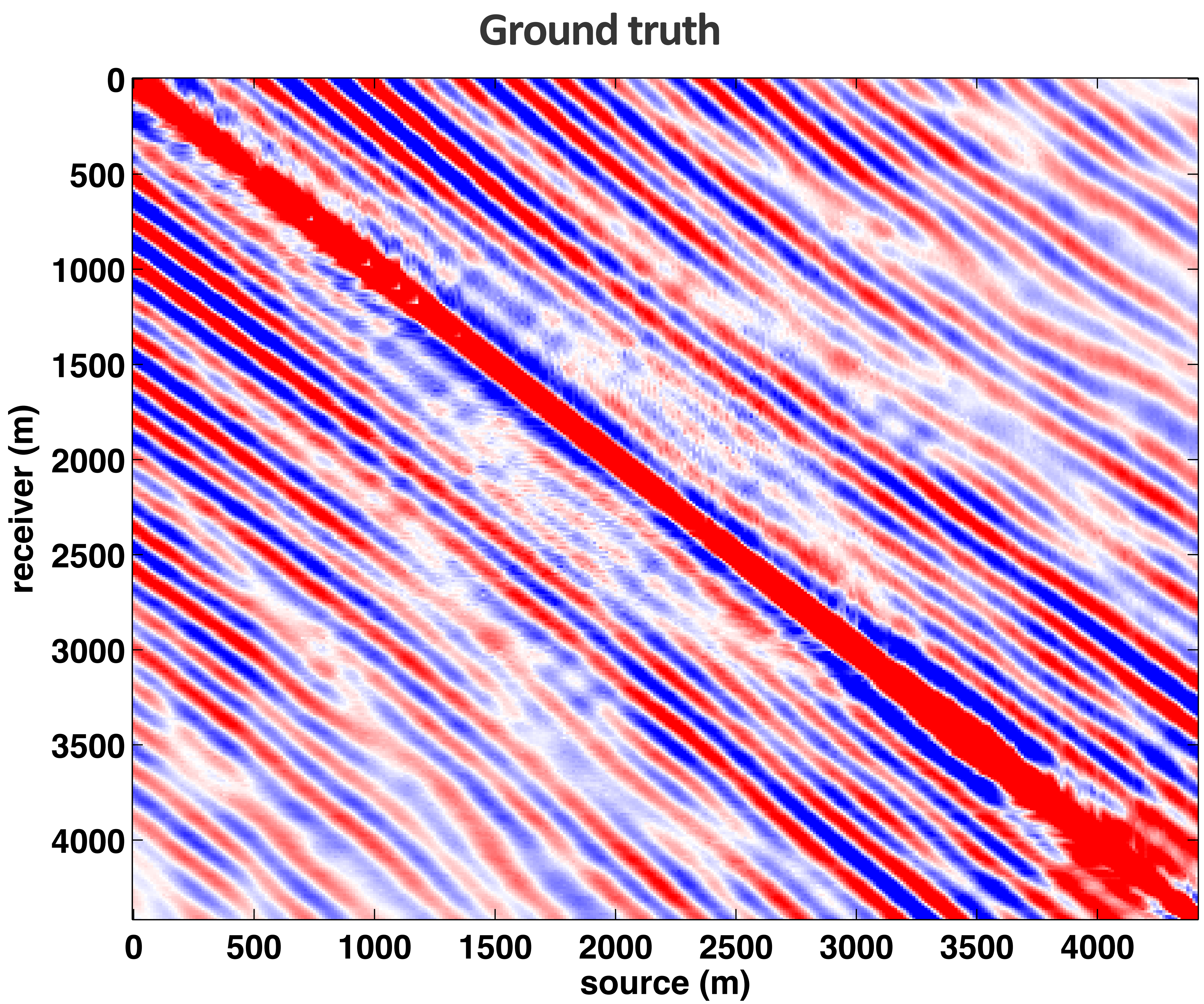
Matlab Code

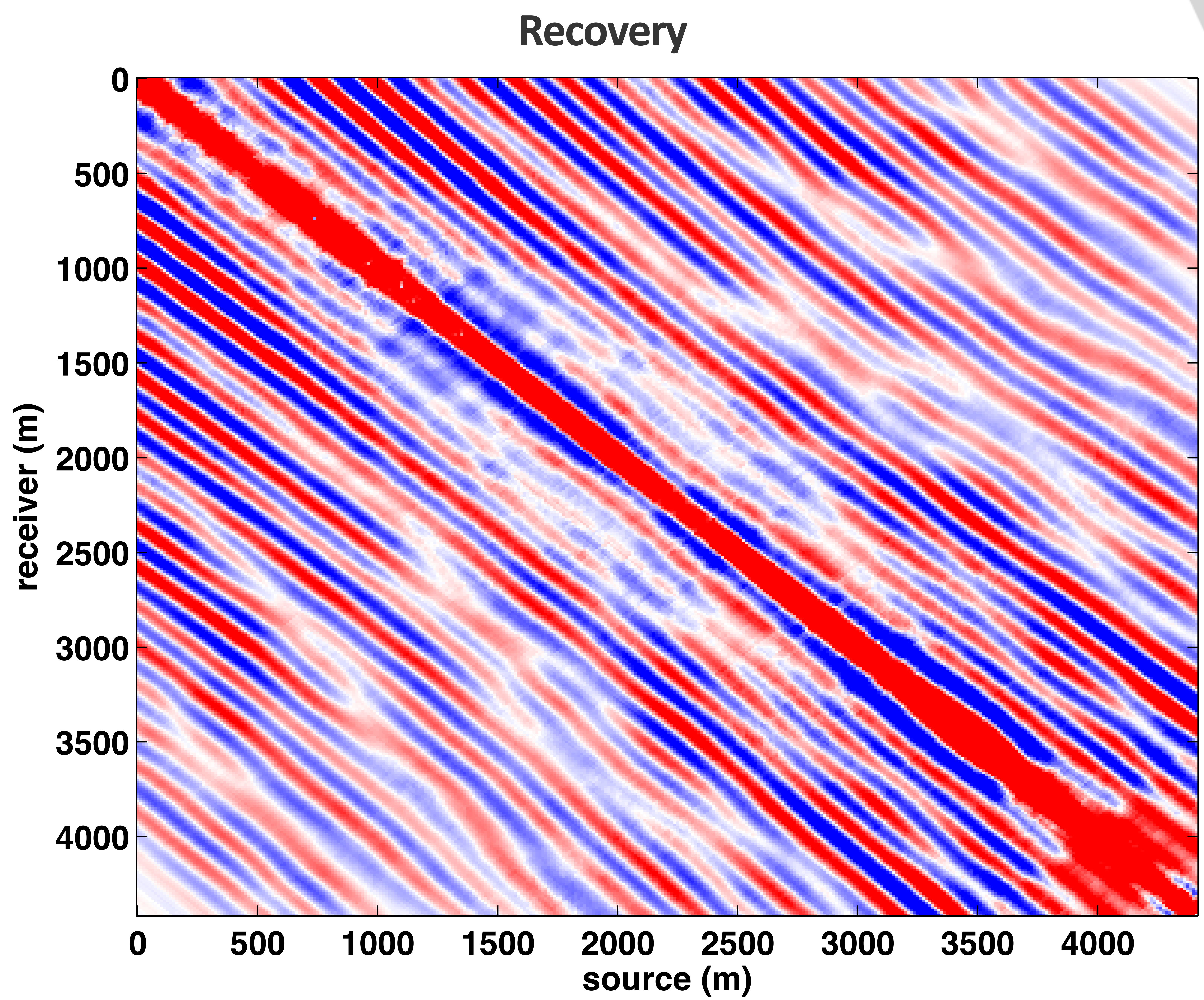
```
% as per Theorem 3.3
alpha = 1 + (2/(k+2)) * (1./theta - 1);
% As per algorithm 4 on page no 446
res   = A(X) - M;
X     = reshape(X - mu*(tau*(Adj(res) - P)),params.mhnumr, params.mhnumc);
X     = shrink( X , mu);
P     = Phat - tau*(Adj(A(Xhat) - M)) - (X - Xhat)./mu;
Xhat  = alpha*X + (1- alpha) * Xprev;
Phat  = alpha*P + (1- alpha) * Pprev;
Xprev = X;
Pprev = P;
% update theta
theta = 2/(k+2);
obj   = norm(A(X) - M,'fro')/norm(M,'fro');
k     = k+1;
```

Gulf of Suez
70% missing source

LBP

SNR = 15.9 dB





LBP

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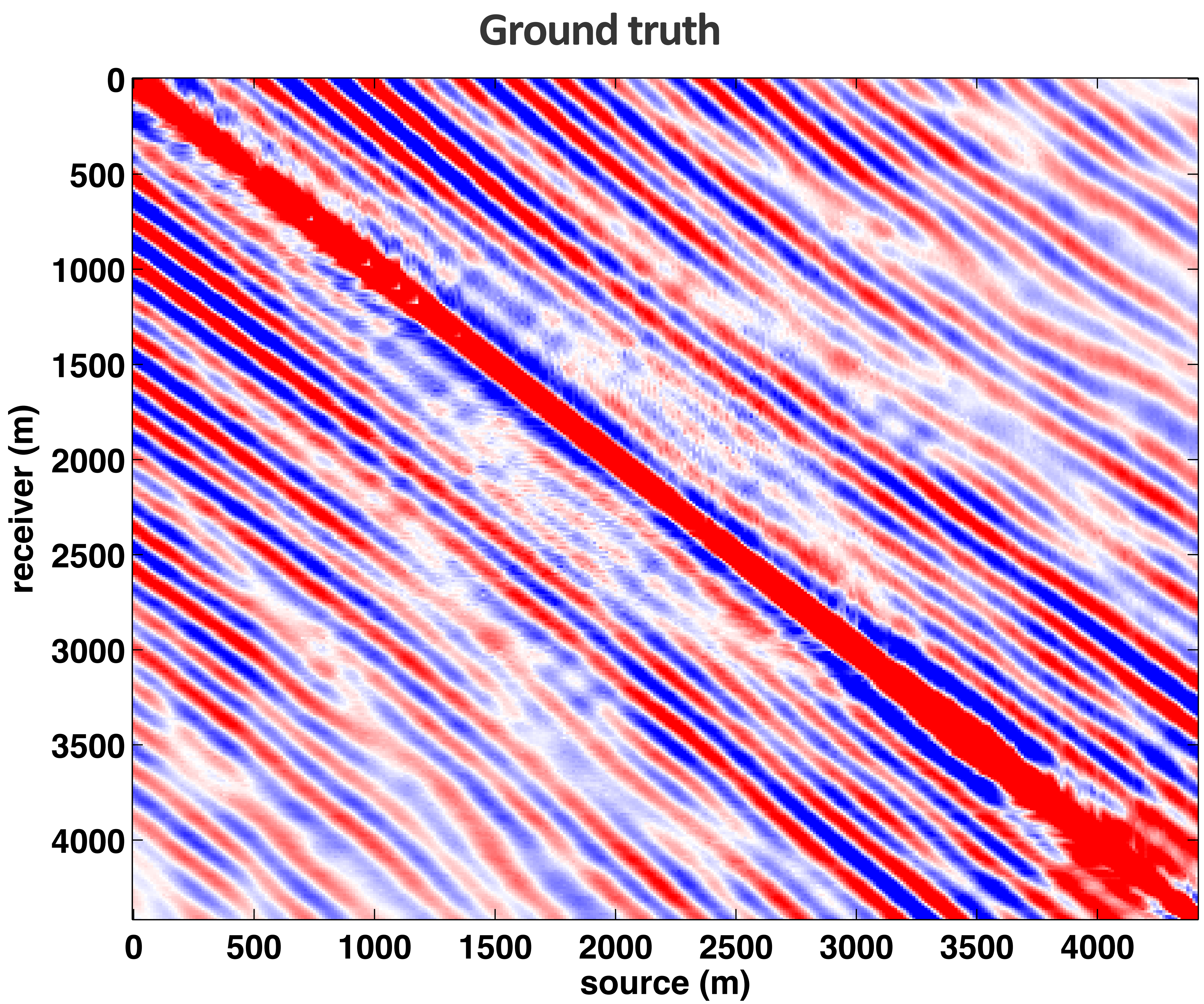
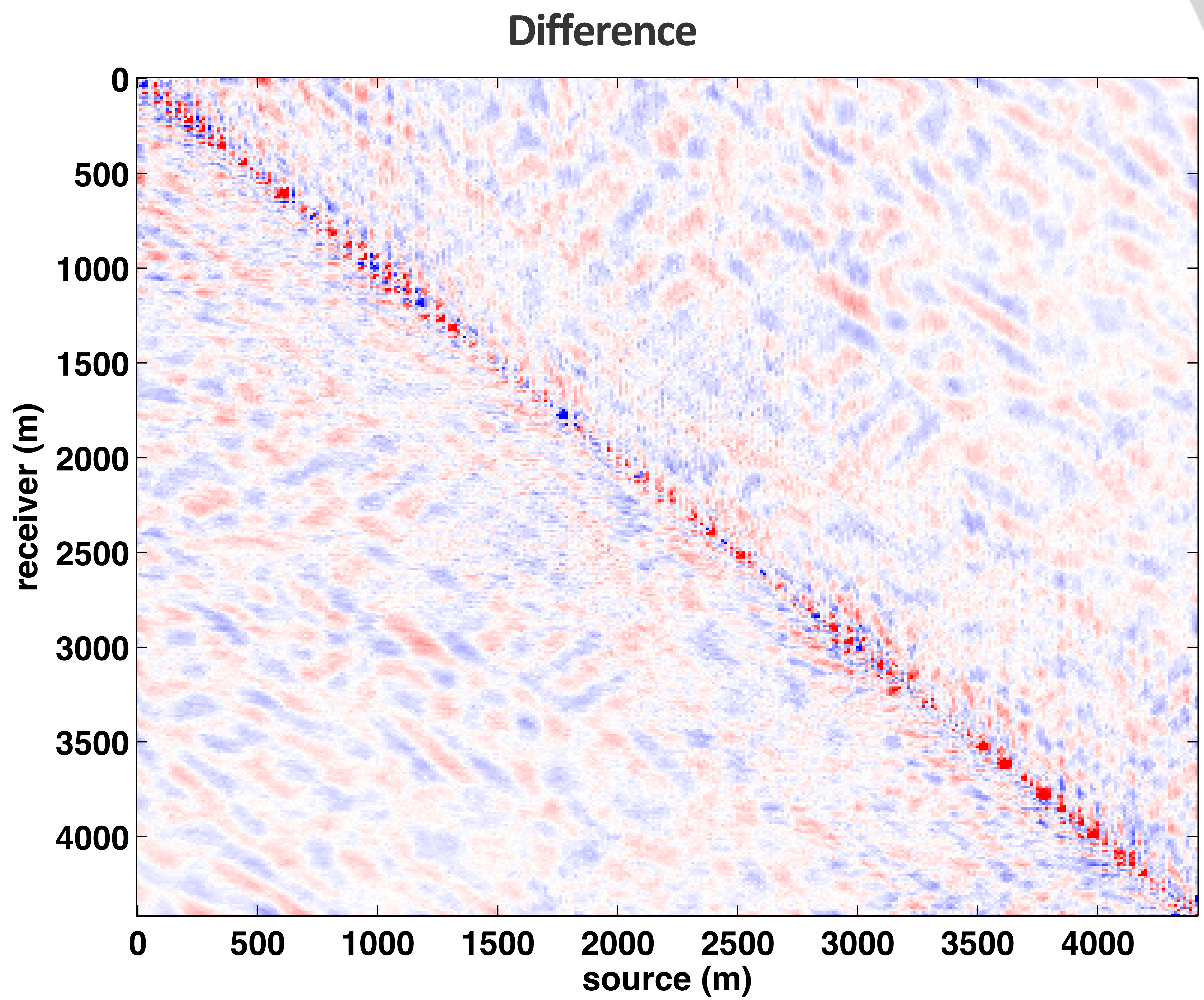
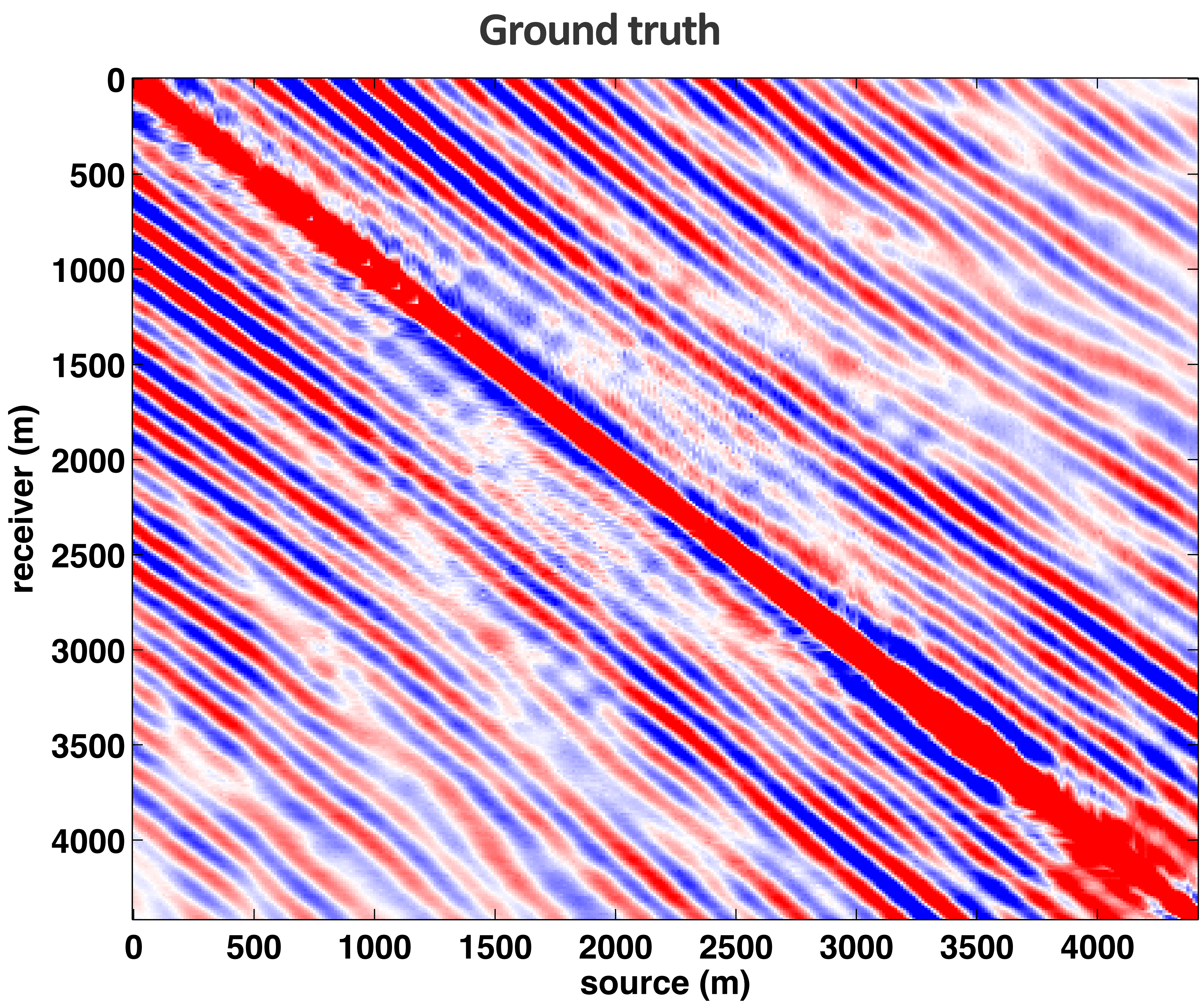
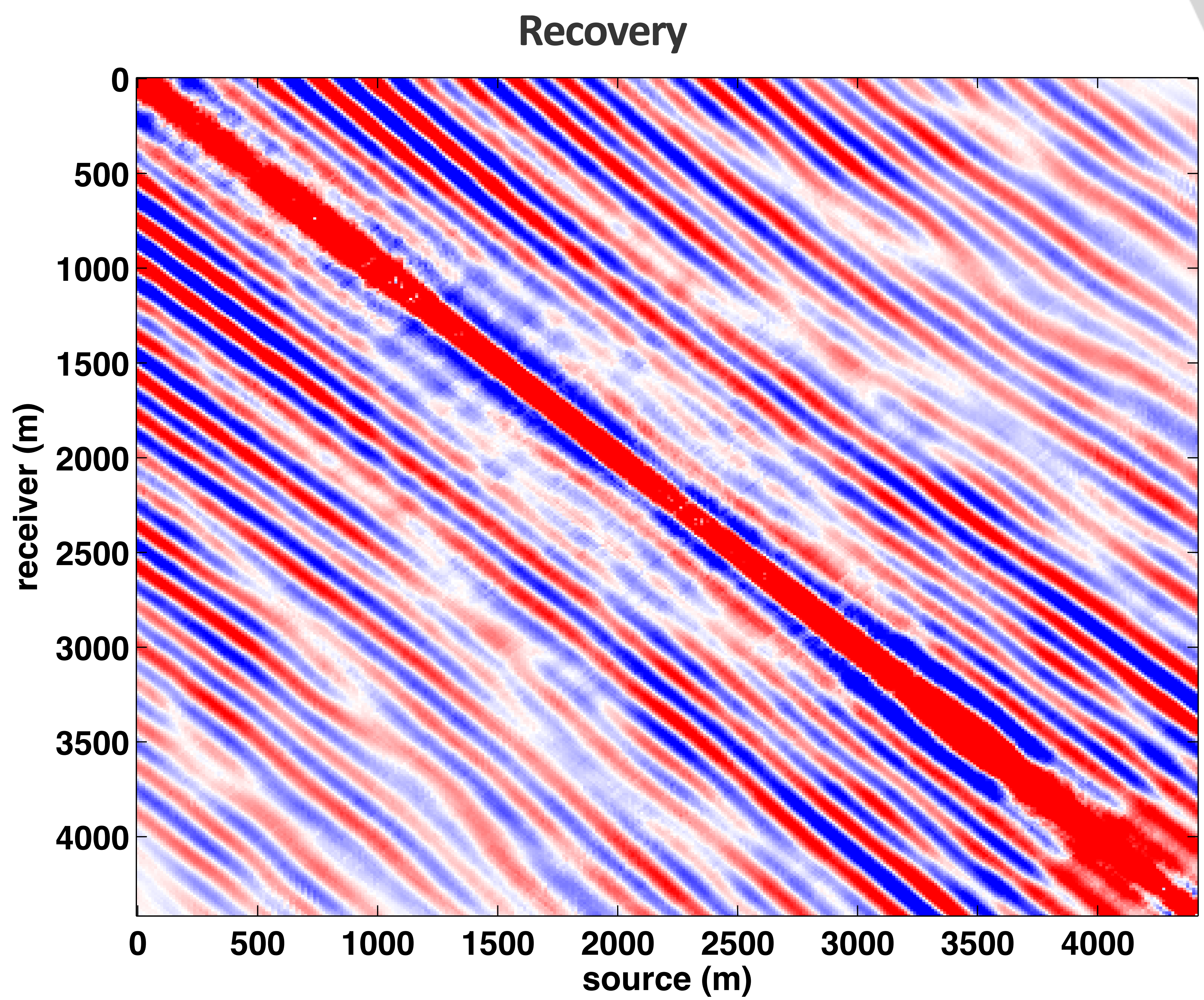




SPGL1

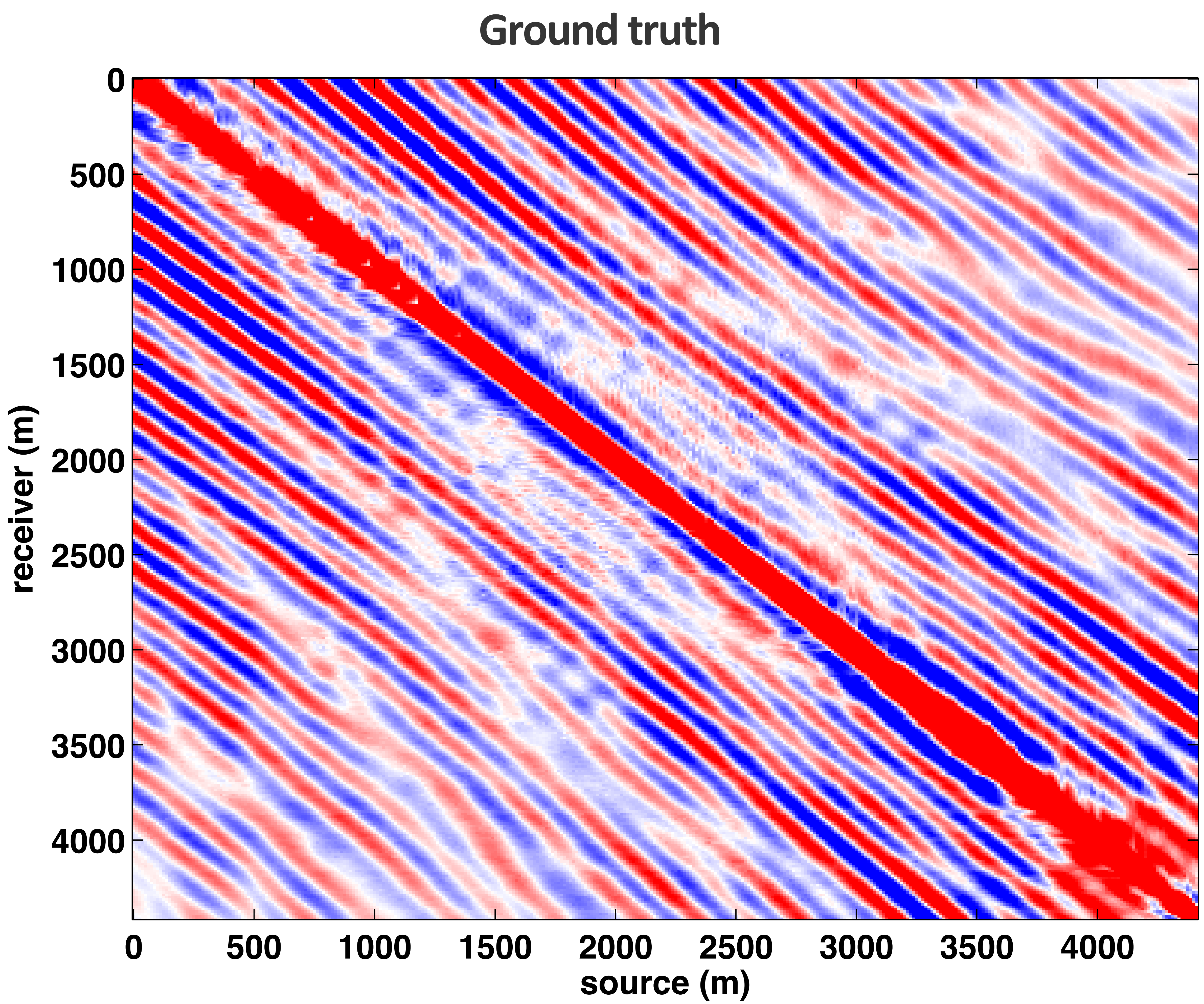
SNR = 15.3 dB

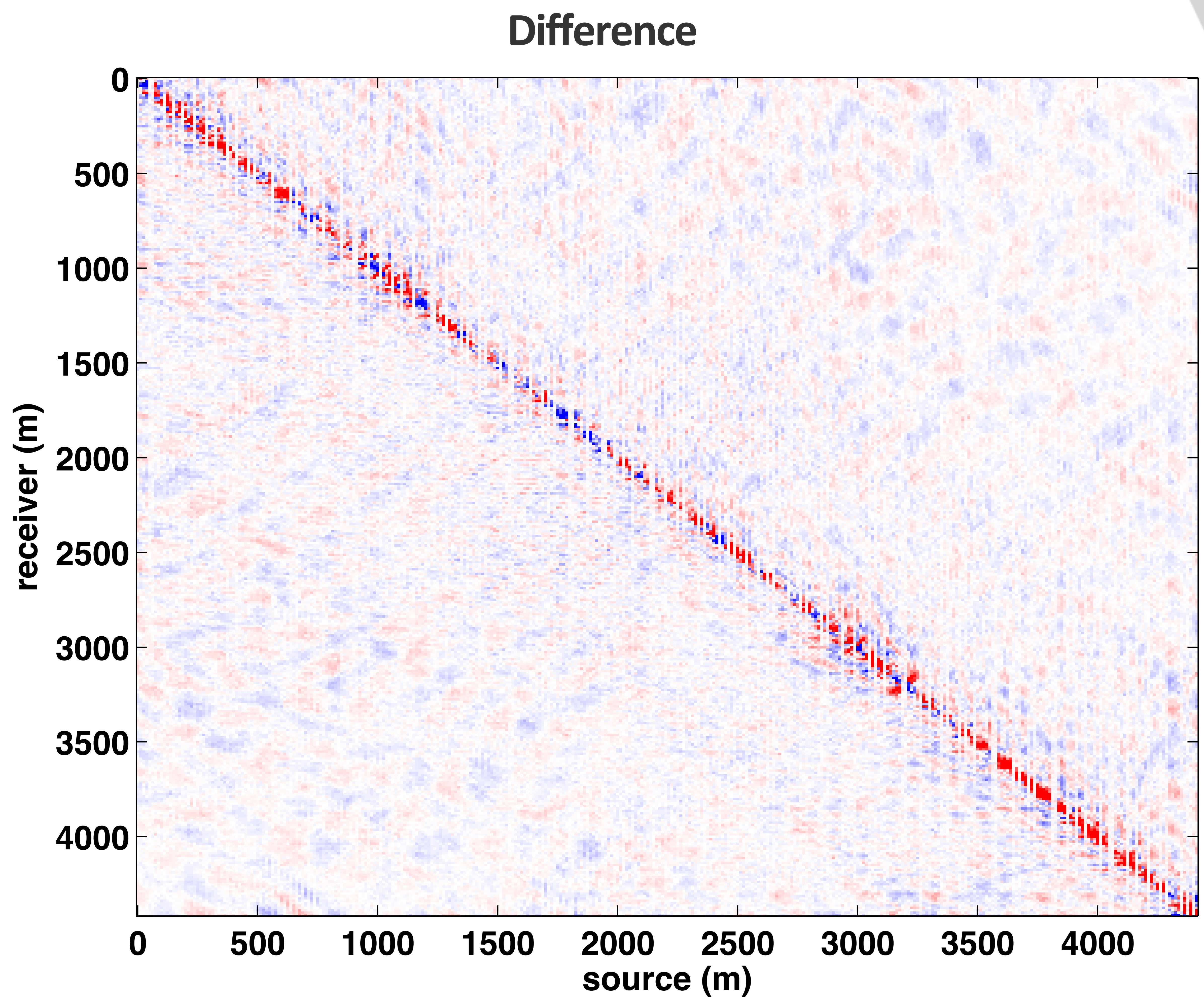




SPGL1

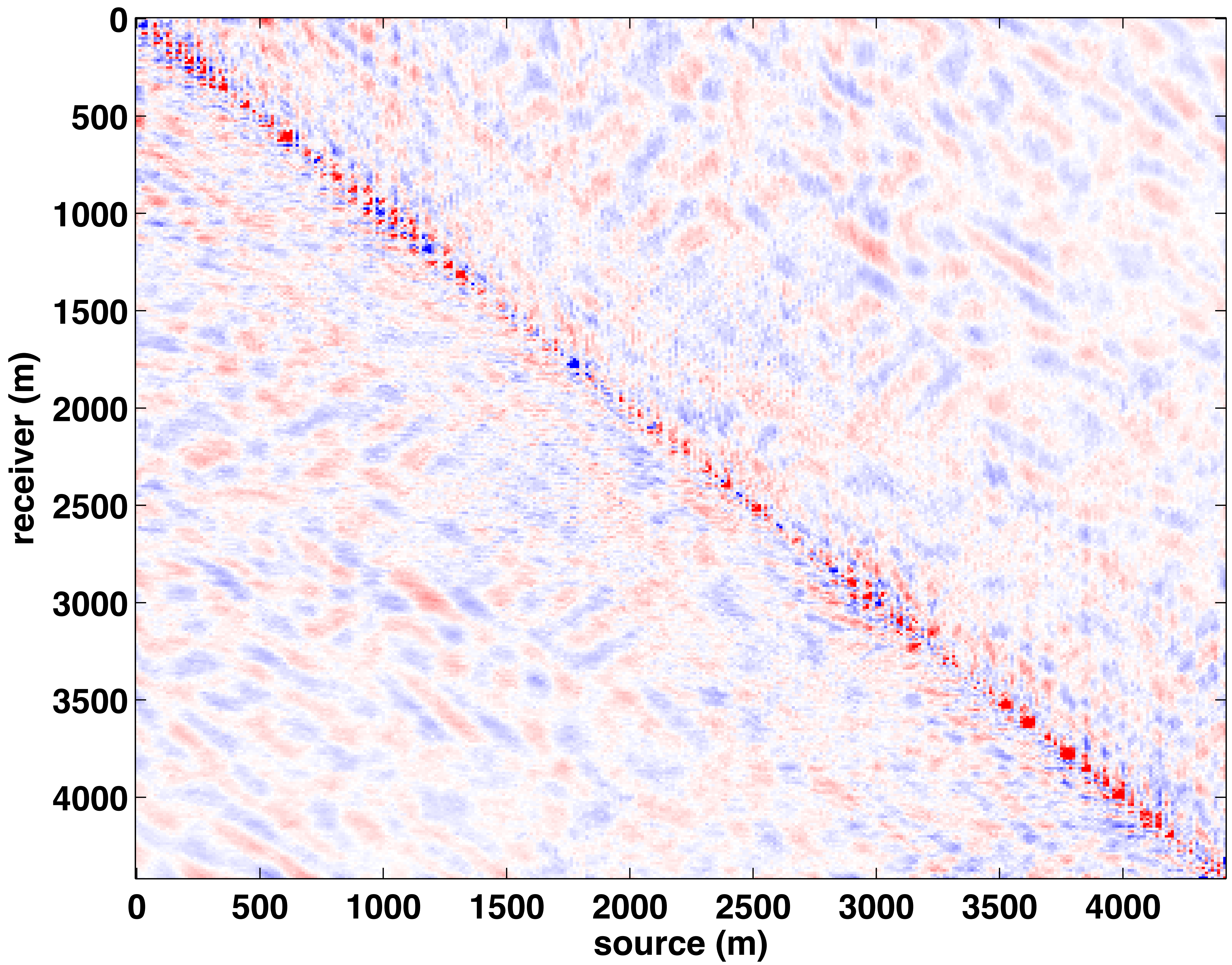
SNR = 15.3 dB



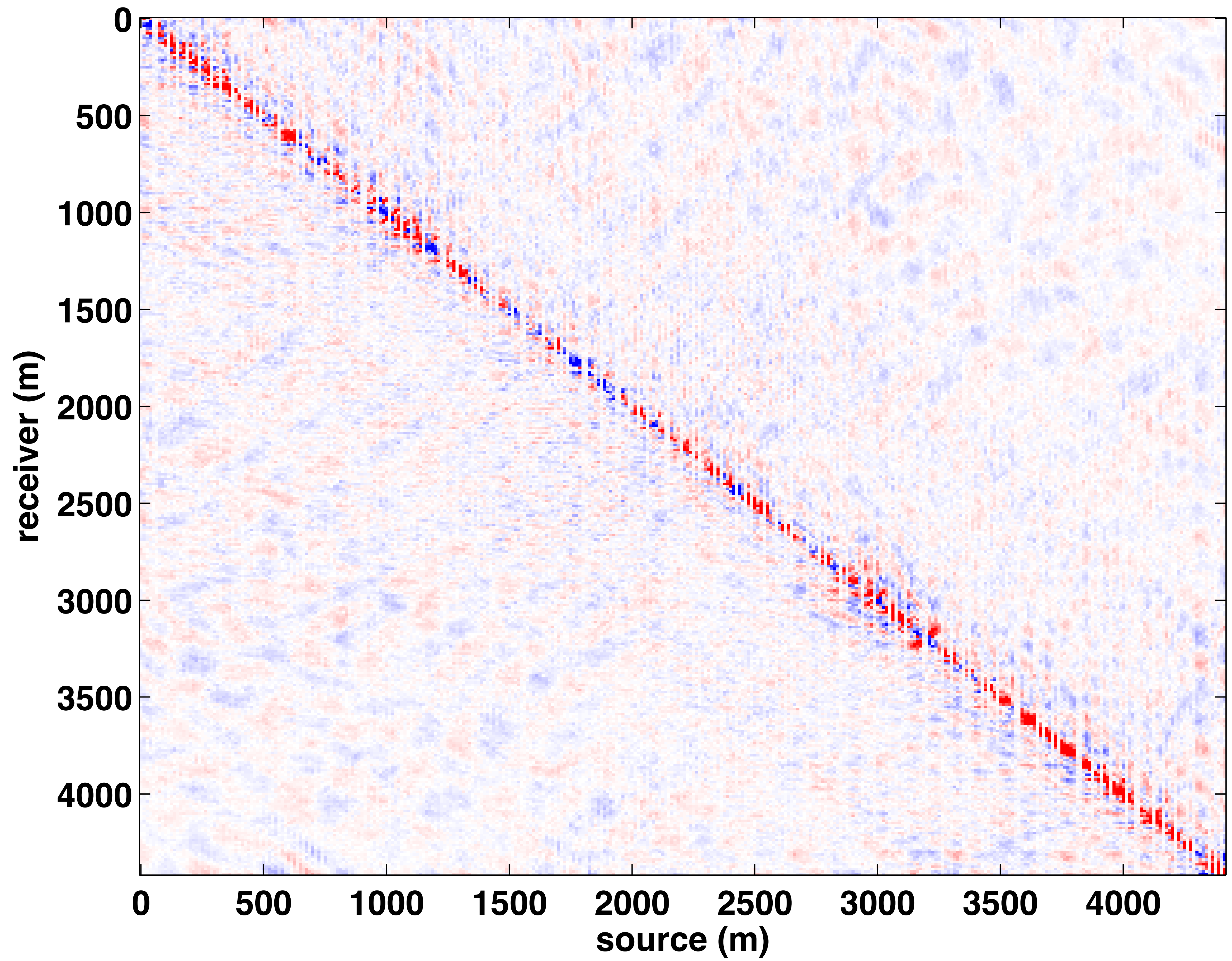


SPGL1 v/s LBP

LBP



SPGL1



Conclusion

- effectiveness of LBP-based sparse inversion approaches preliminarily verified

- **simple implementation**
  - to promote the uptake of SLIM technologies by industry

Future work

- more extensive testing
- more investigation into the heuristics
  - relating the heuristics to the solution path of SPGl1
- possible extension to other applications
- software release

Acknowledgements

Thank you all for your attention!

SINBAD

NSERC CRSNG

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