Fast imaging with **source** estimation

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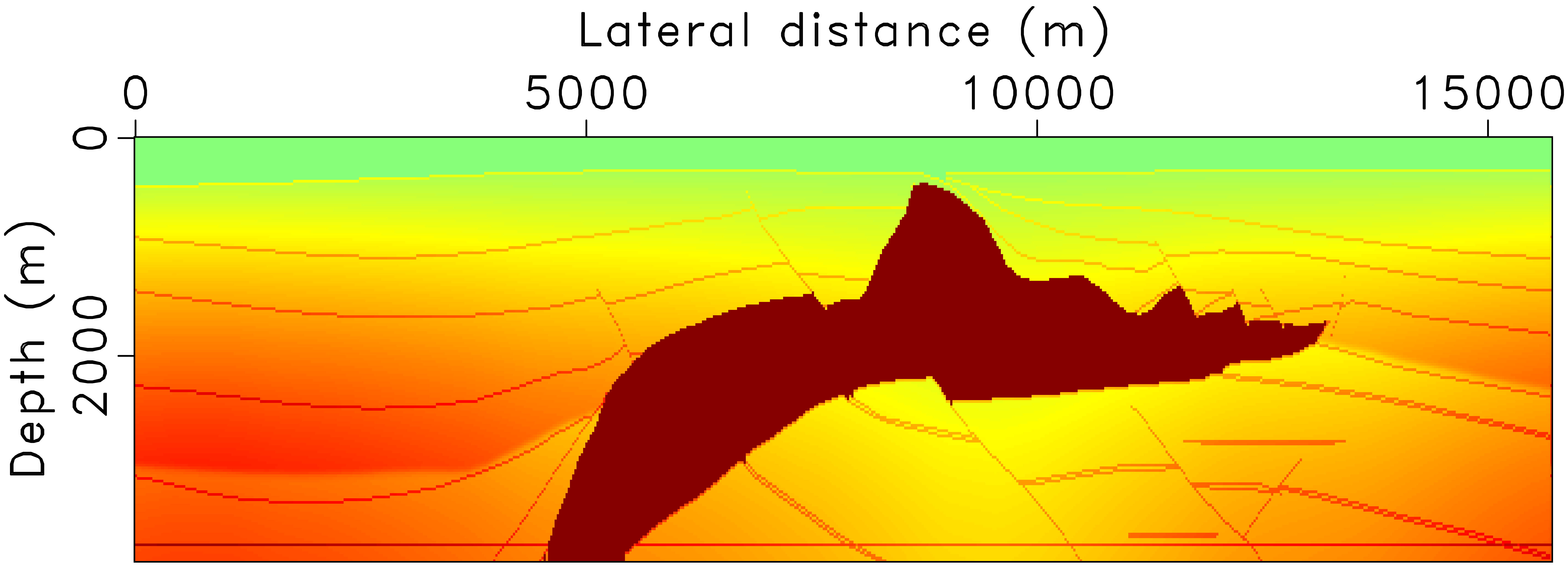
Motivation

- Conventional RTM requires knowledge of the source wavelet as prior information.
- A wrong wavelet leads to a wrong image.

Example

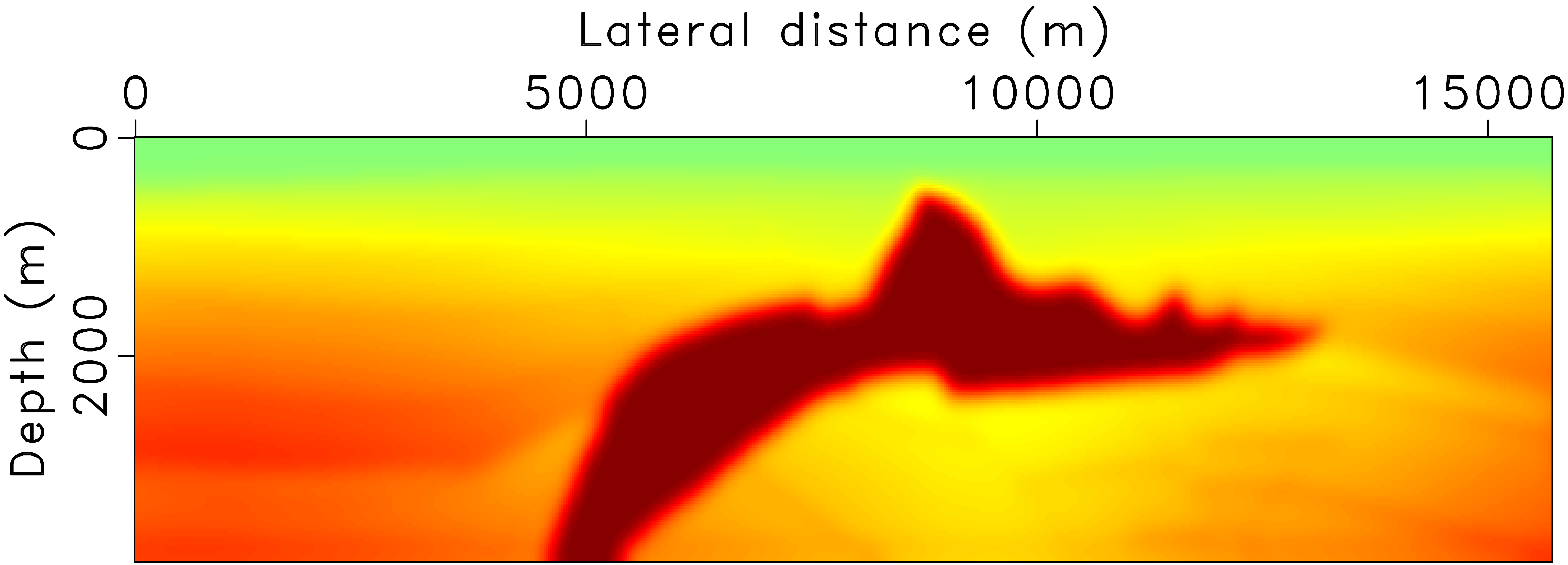
- SEG/EAGE salt model, 3.9km deep, 15.7km wide, 24.38m grid spacing
- 5Hz Ricker wavelet, 8s recording, 96 freq. samples
- 323 co-located sources/receivers with 48.768m (160 ft.)spacing at 24.384m (80 ft.) depth
- data modelled using iWave with absorbing surface, i.e., primaries only

Example



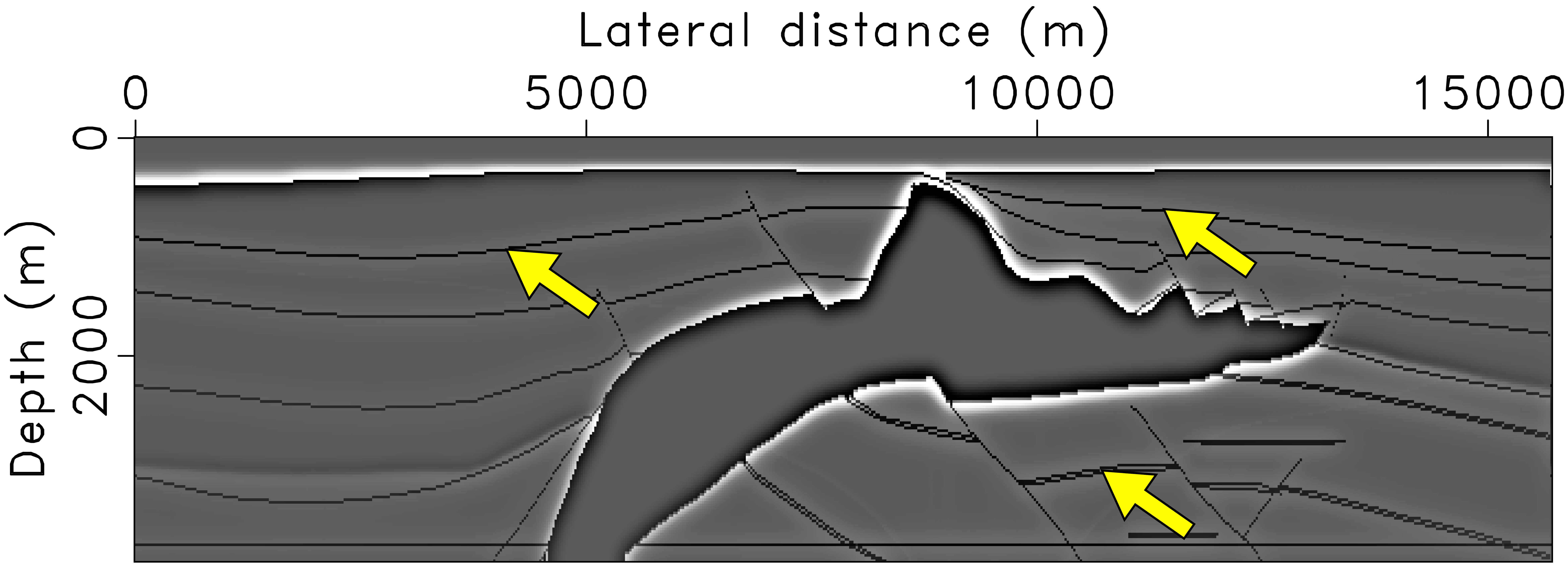
True model

Example



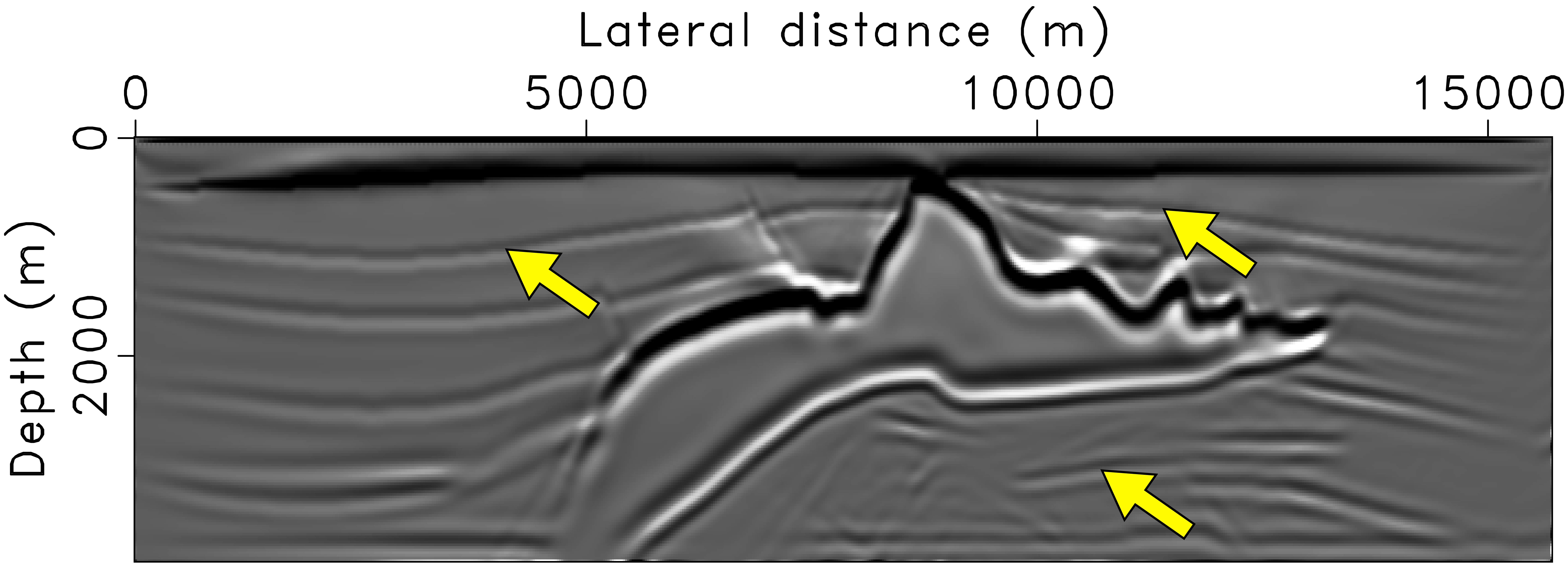
Background model

Example



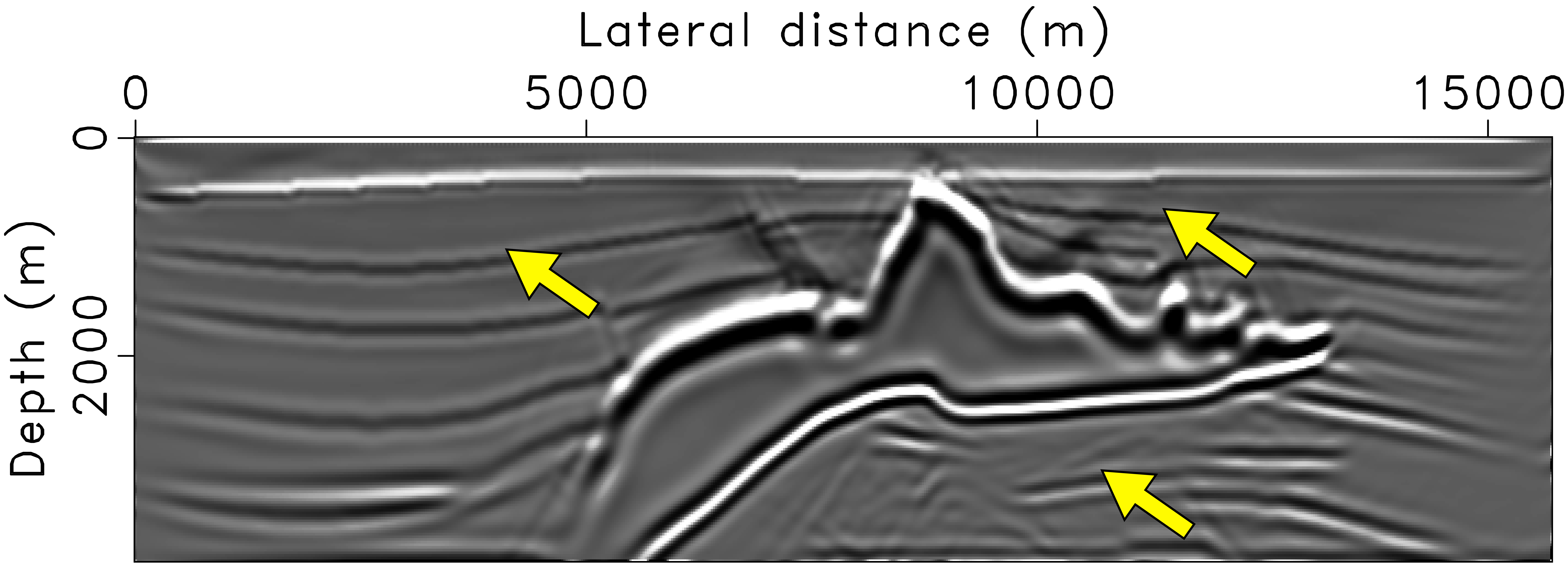
True model perturbations

Example



RTM with the **true** source wavelet

Example



RTM with a **wrong** source wavelet (0.1s phase shift)

Solution

We would like to borrow ideas from:

- source estimation by *variable projection*
  - separable *non-linear* least-squares
- *compressive* imaging by *sparse* inversion
  - least-squares with *sparse* constraint

Question:

Are these two techniques *compatible*?

Problem formulation with unknown source

$$\min_{\mathbf{x},\boldsymbol{w}} f(\mathbf{x},\boldsymbol{w}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau.$$

- $\mathbf{C}$: curvelet transform
- $\underline{\cdot}$: subsampled source / receiver wavefields
- $\nabla\mathbf{F}$: linearized modelling operator
- Σ, Ω : randomized sim. sources / frequency subset
- τ : sparsity constraint
- $\boldsymbol{w}$: unknown source wavelet spectra

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Challenges

The core gradient step becomes

$$\mathbf{x}^{k+1} = \mathcal{P}_{\mathcal{X}}[\mathbf{x}^k + \lambda \nabla_{\mathbf{x}} f(\mathbf{x}, \boldsymbol{w})|_{\mathbf{x}=\mathbf{x}^k, \boldsymbol{w}=\boldsymbol{w}^k}]$$

with

$$\mathcal{X} \doteq \{\mathbf{x} : \|\mathbf{x}\|_1 \leq \tau\}.$$

Challenges:

- evaluation of the gradient
- computing the sparsity level

Pratt R. G., 1999; Aravkin and van Leeuwen, 2012

Gradient descent using variable projection

With an estimate of the solution vector $\mathbf{x}$, the source estimates can be obtained by:

$$\widetilde{w}_i(\mathbf{x}) = \frac{\sum_{j\in\Sigma} < \underline{\mathbf{d}}_{i,j}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}{\sum_{j\in\Sigma} < \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}.$$

Then the optimization problem is reduced to:

$$\min_{\mathbf{x}} \overline{f}(\mathbf{x}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, \widetilde{w}_i(\mathbf{x})\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau,$$

with $\nabla_{\mathbf{x}}\overline{f}(\mathbf{x}) = \nabla_{\mathbf{x}} f(\mathbf{x}, \widetilde{\boldsymbol{w}}(\mathbf{x}))$.

Computing the sparsity level

nonlinear LASSO

$$\min_{\mathbf{x},\boldsymbol{w}} f(\mathbf{x},\boldsymbol{w}) \doteq \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2$$

$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau.$$

⇕

nonlinear BPDN

$$\operatorname*{argmin}_{\mathbf{x},\boldsymbol{w}} \|\mathbf{x}\|_1$$

$$\text{subject to} \quad \sum_{i\in\Omega}\sum_{j\in\Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, w_i\underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}\|_2^2 \leq \sigma^2.$$

⇓

compute τ using Newton's method with $\boldsymbol{w} = w(\mathbf{x})$ with **rerandomization**

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Examples

Examples: using ideal data

input data simulated by linearized modelling using the same modelling engine as inversion:

$$\mathbf{d}_{i,j} = \nabla\mathbf{F}[\mathbf{m}_0, w_i\mathbf{s}_j]\mathbf{dm}$$

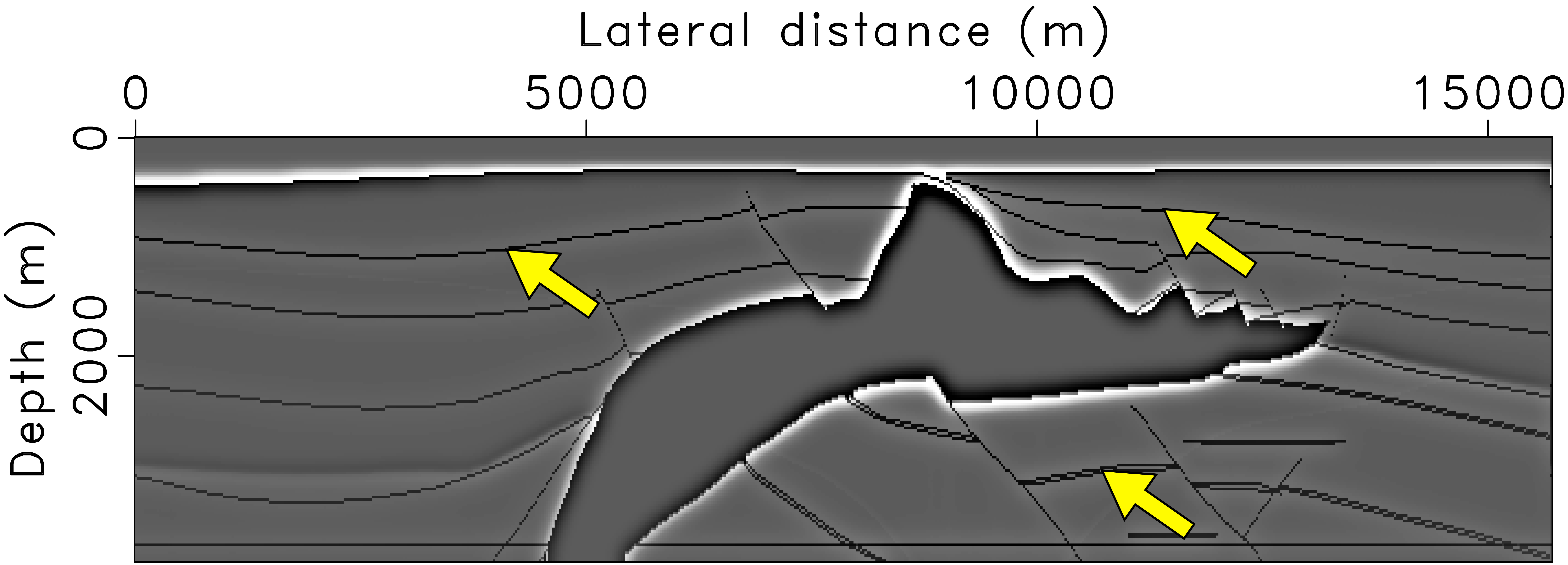
Fast inversion w/ source estimation:

-**no assumption made** about the phase of the wavelet

-initial guess simply an **impulse** with a **wrong** phase

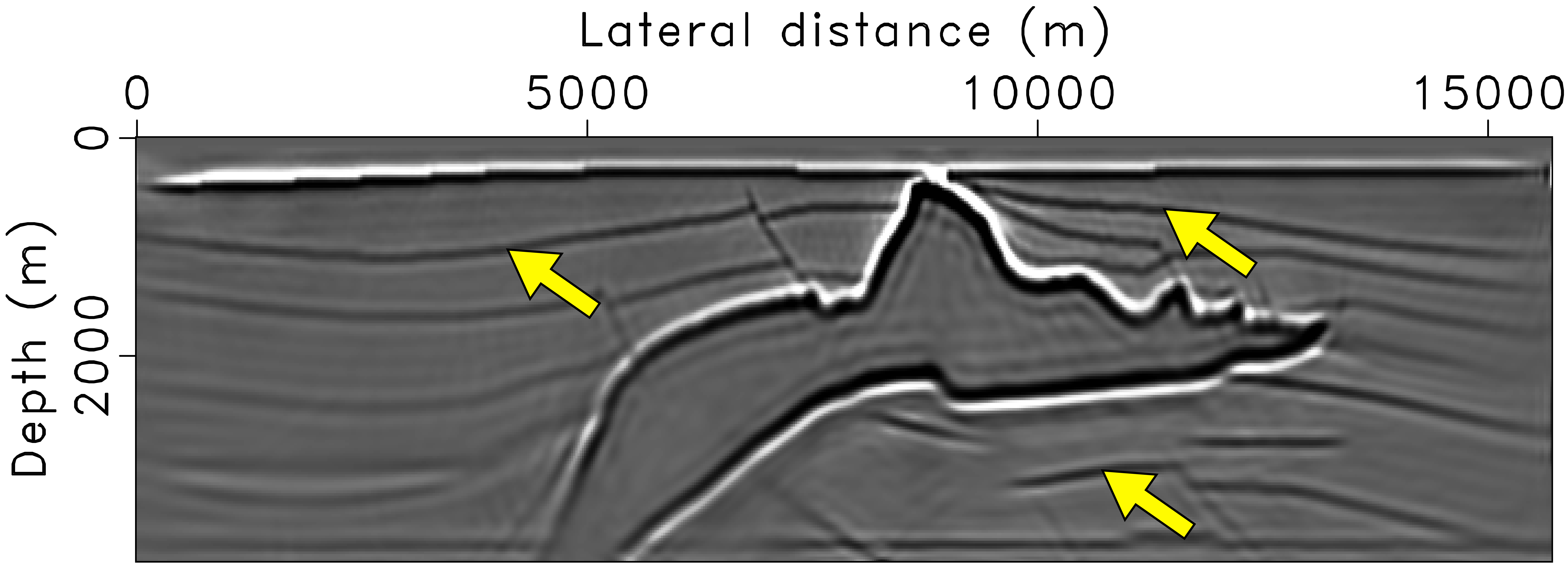
-simulation cost **~1 RTM** of all the data

Examples: using ideal data



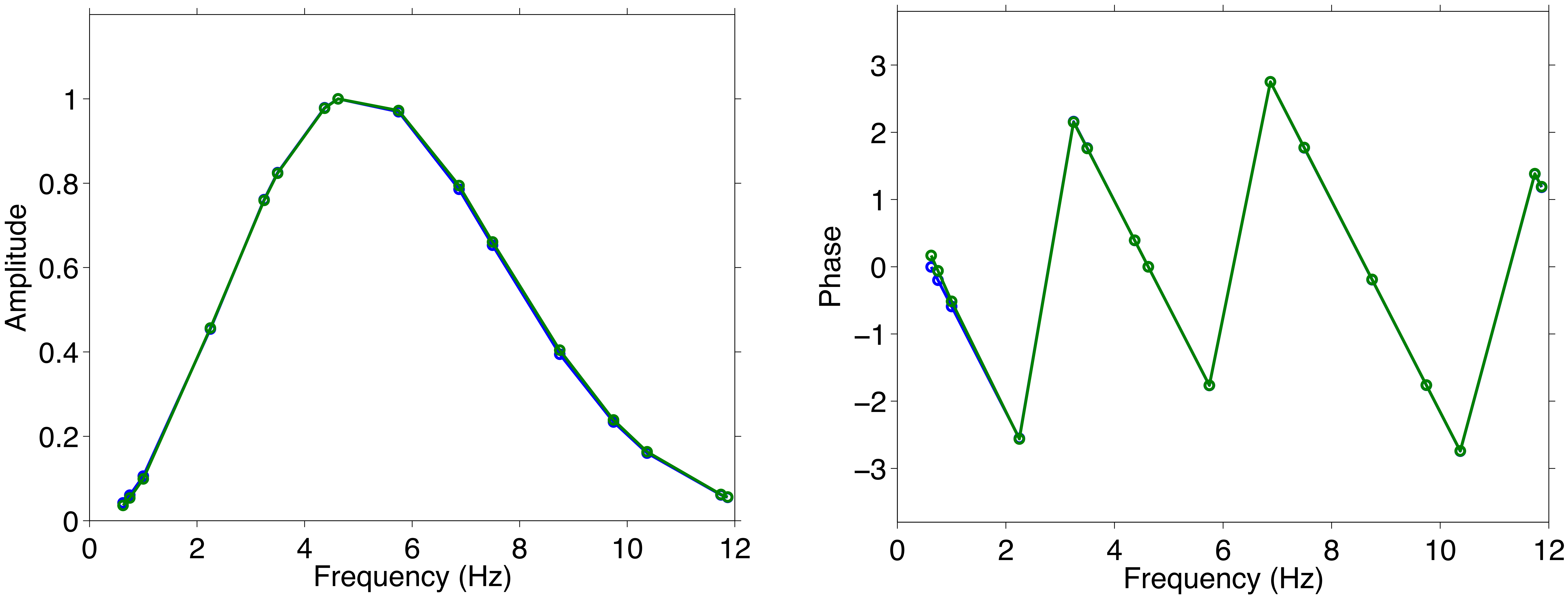
True model perturbations

Examples: using **ideal** data



Fast imaging w/ source estimation

Examples: using **ideal** data



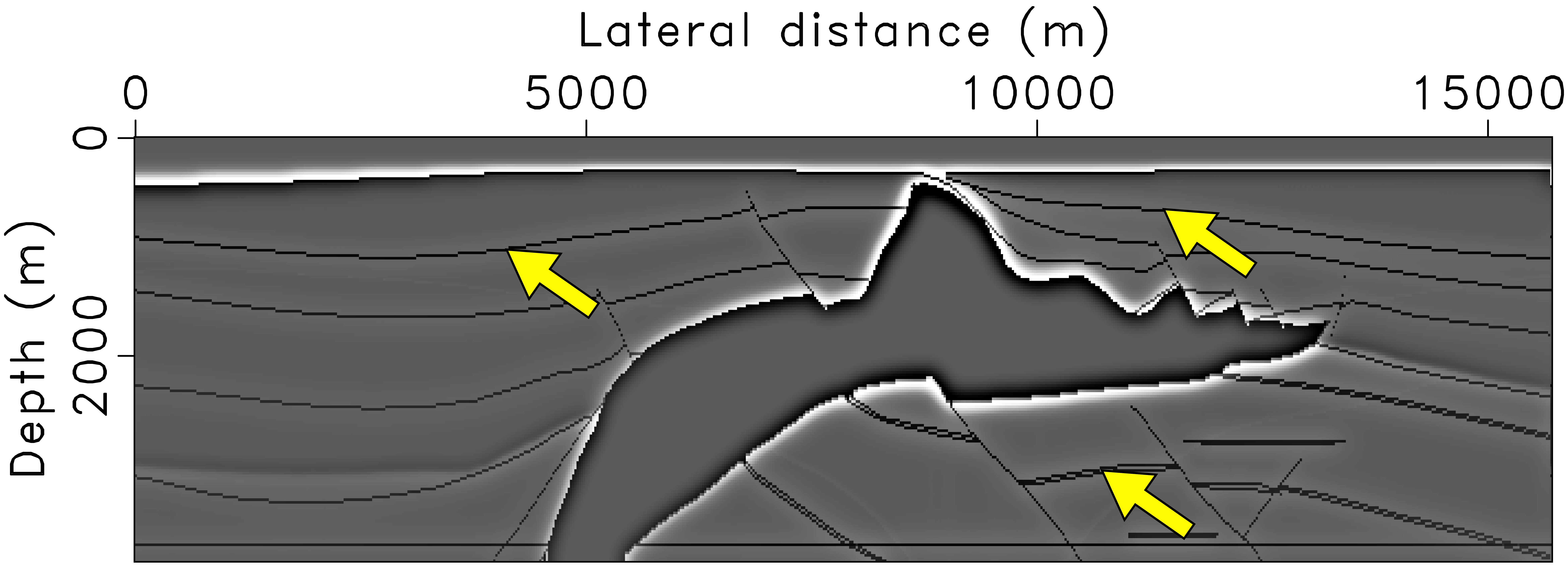
Source estimates of the last subproblem: *after normalization*

Examples: a more realistic setup

input data simulated using iWave, inverted using our in-house frequency-domain modelling engine:

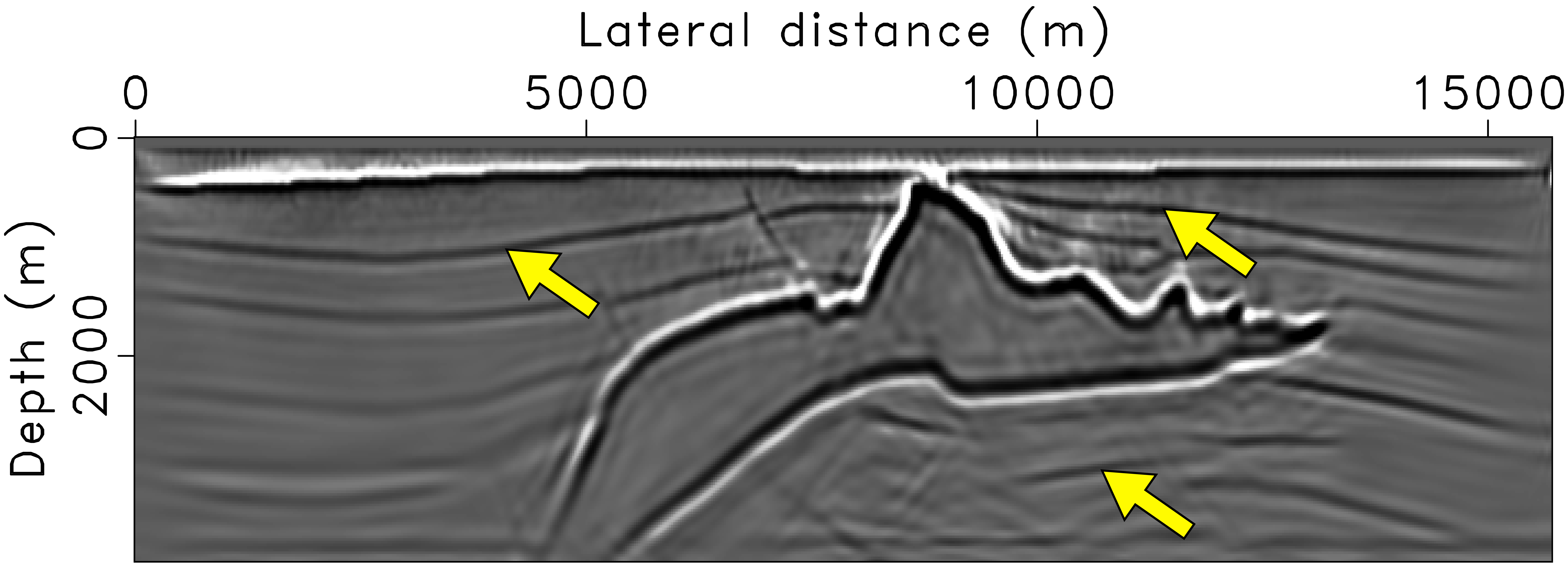
$$\mathbf{d}_{i,j} = \mathbf{F}[\mathbf{m}, w_i \mathbf{s}_j] - \mathbf{F}[\mathbf{m}_0, w_i \mathbf{s}_j]$$

Examples: a more **realistic** setup



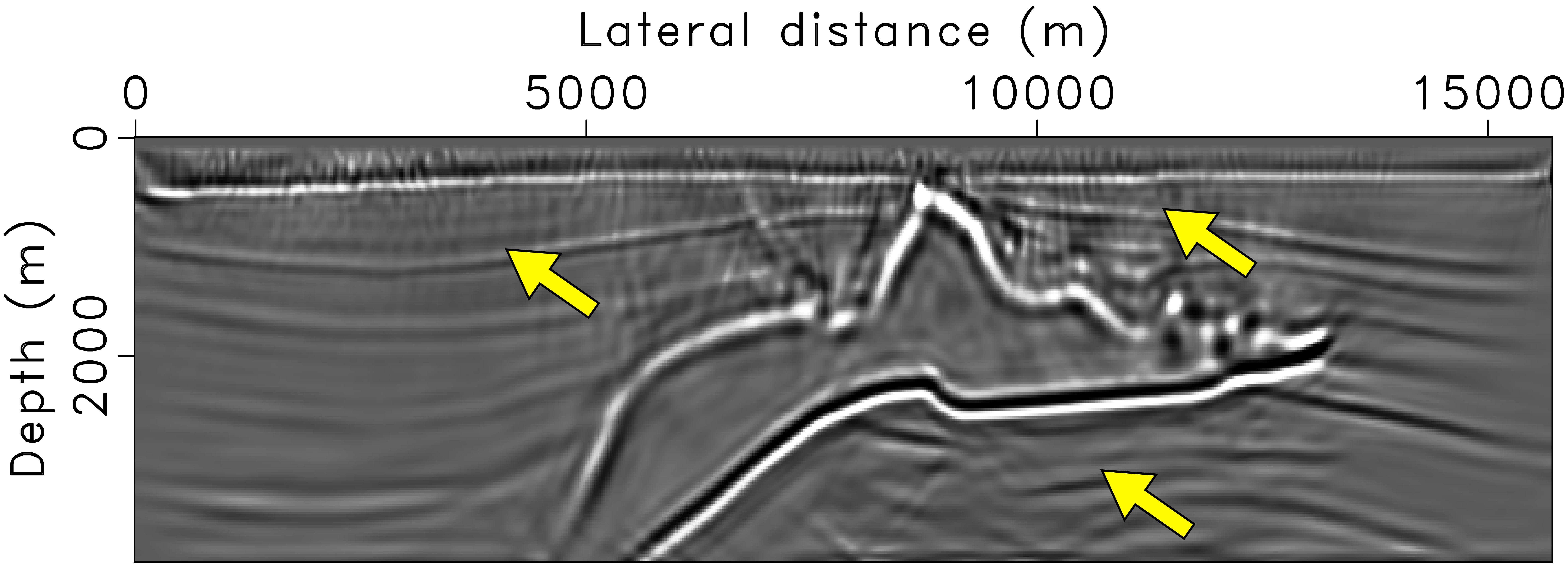
True model perturbations

Examples: a more **realistic** case



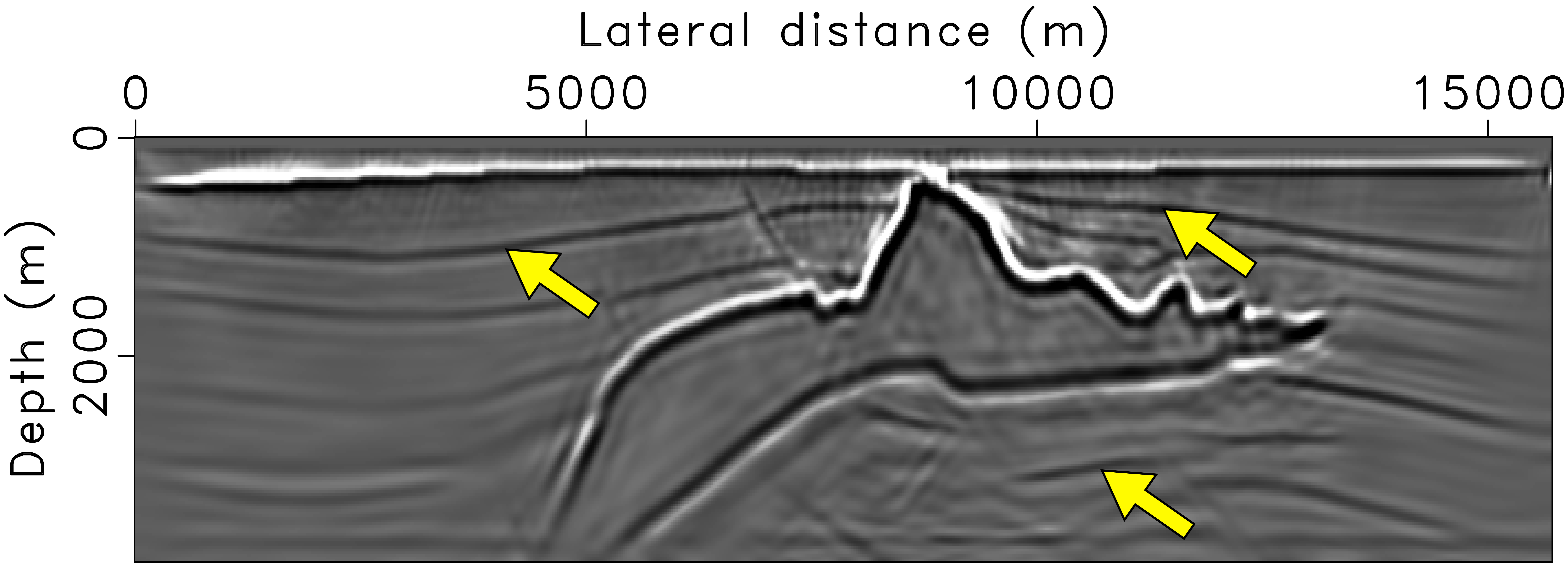
Fast imaging w/ **true** source

Examples: a more **realistic** case



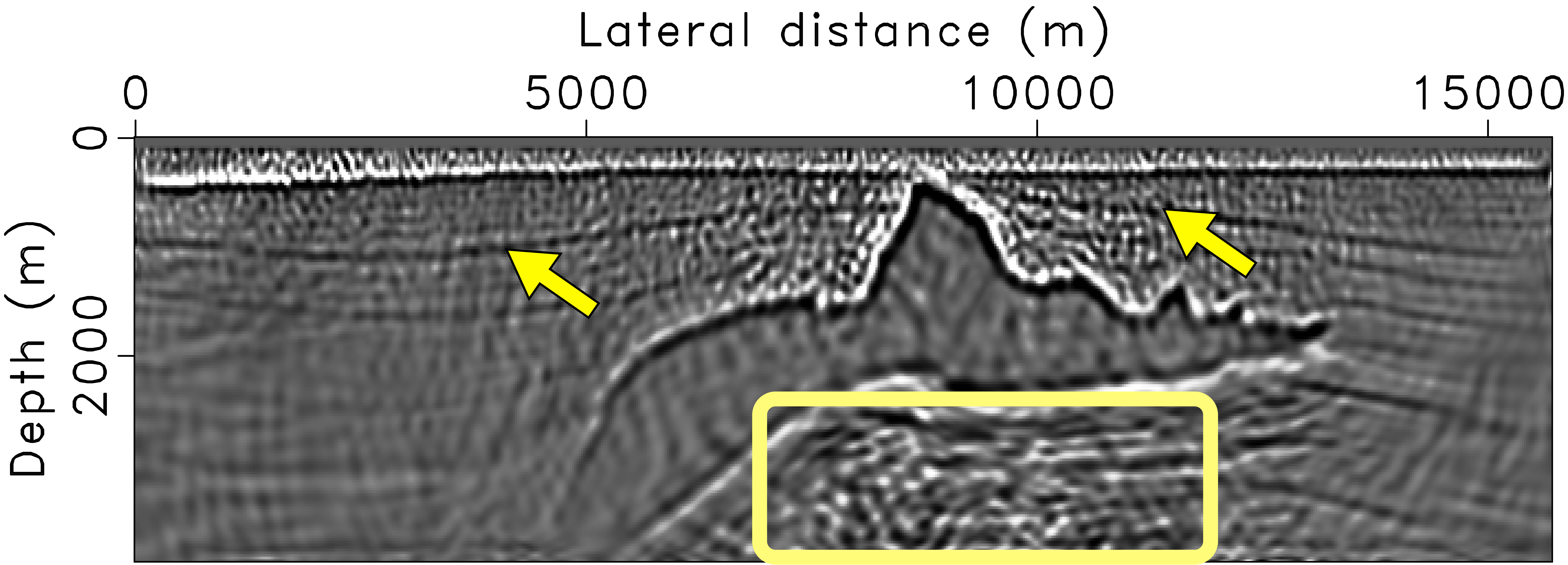
Fast imaging w/ a **wrong** source, 0.1s phase error

Examples: a more **realistic** case



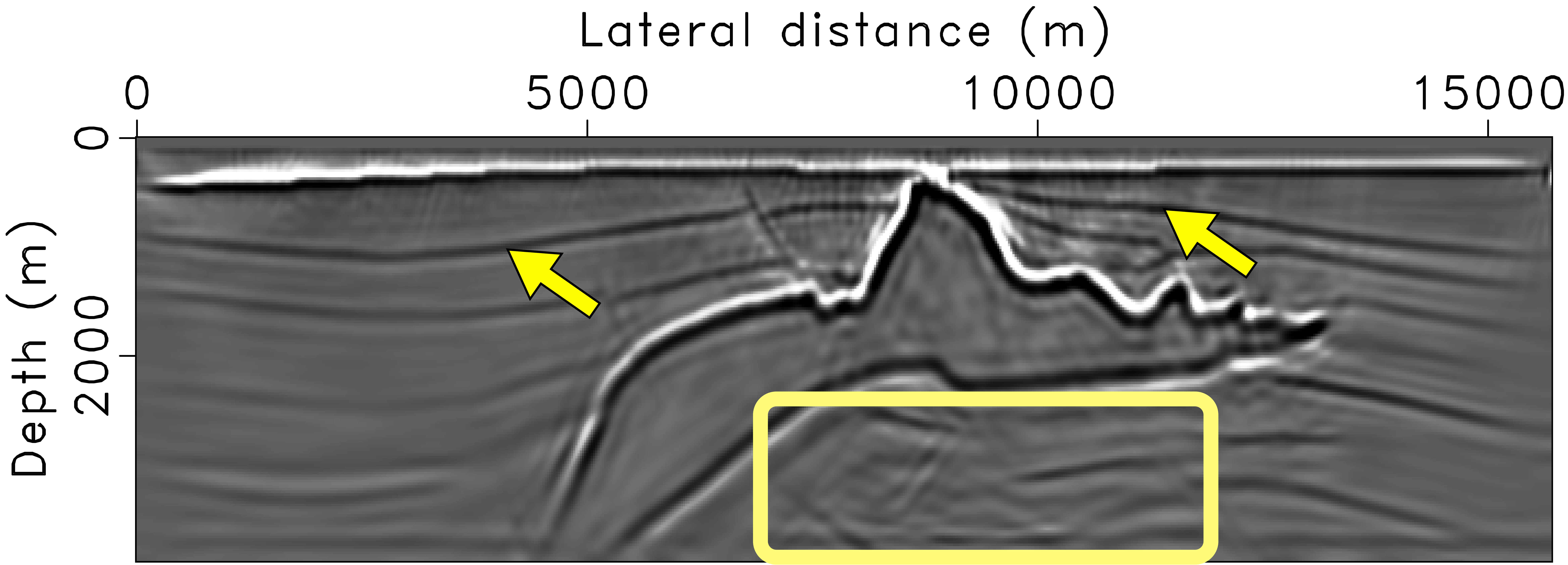
Fast imaging w/ source **estimation**

Examples: a more **realistic** case



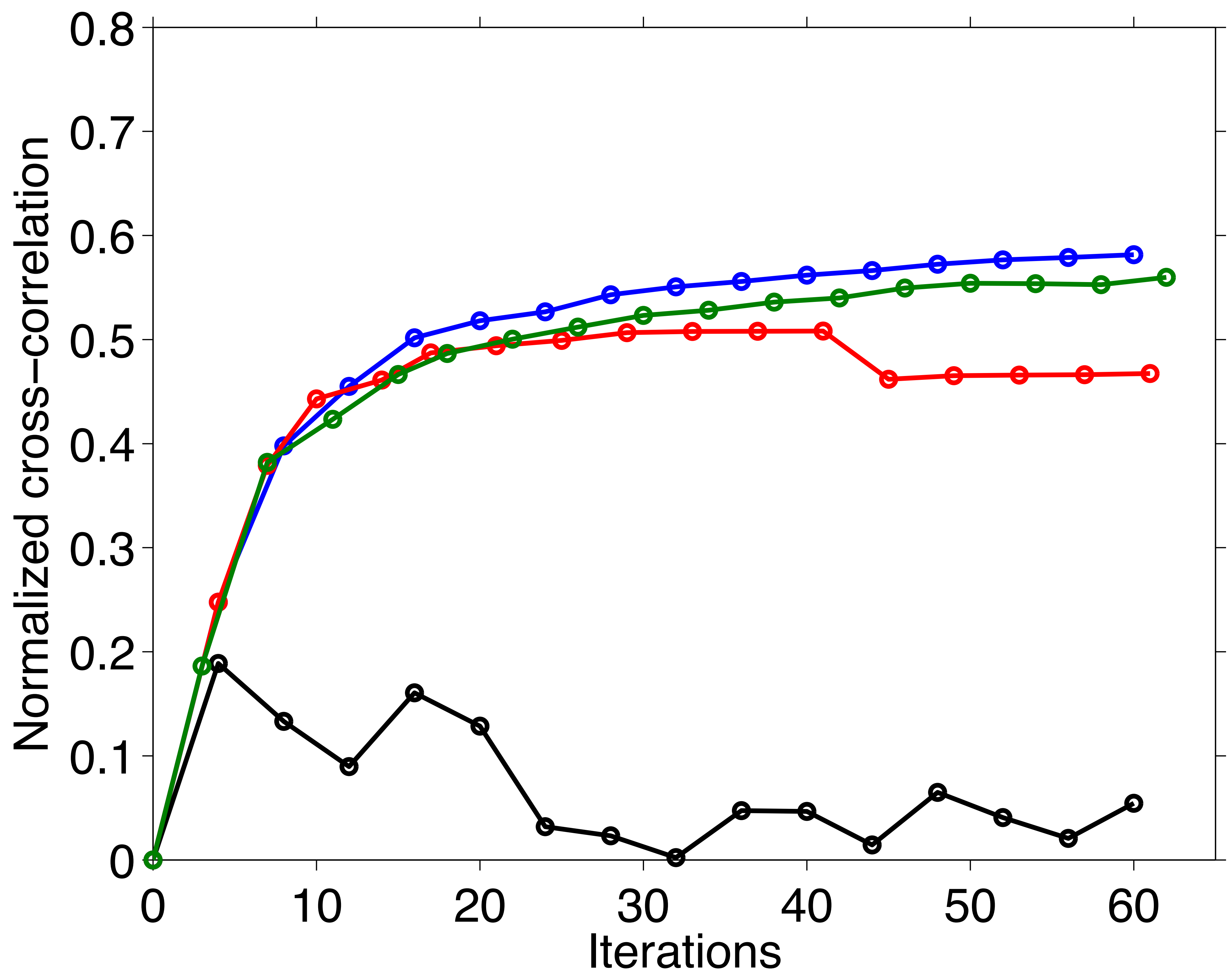
Fast imaging w/ source estimation, **w/o rerandomization**

Examples: a more **realistic** case



Fast imaging w/ source estimation, **w/ rerandomization**

Examples: a more realistic case



Normalized cross-correlation:

$$\mathrm{NCC}(\mathbf{v}_1, \mathbf{v}_2) = \frac{< \mathbf{v}_1, \mathbf{v}_2 >}{\|\mathbf{v}_1\|_2 \|\mathbf{v}_2\|_2}$$

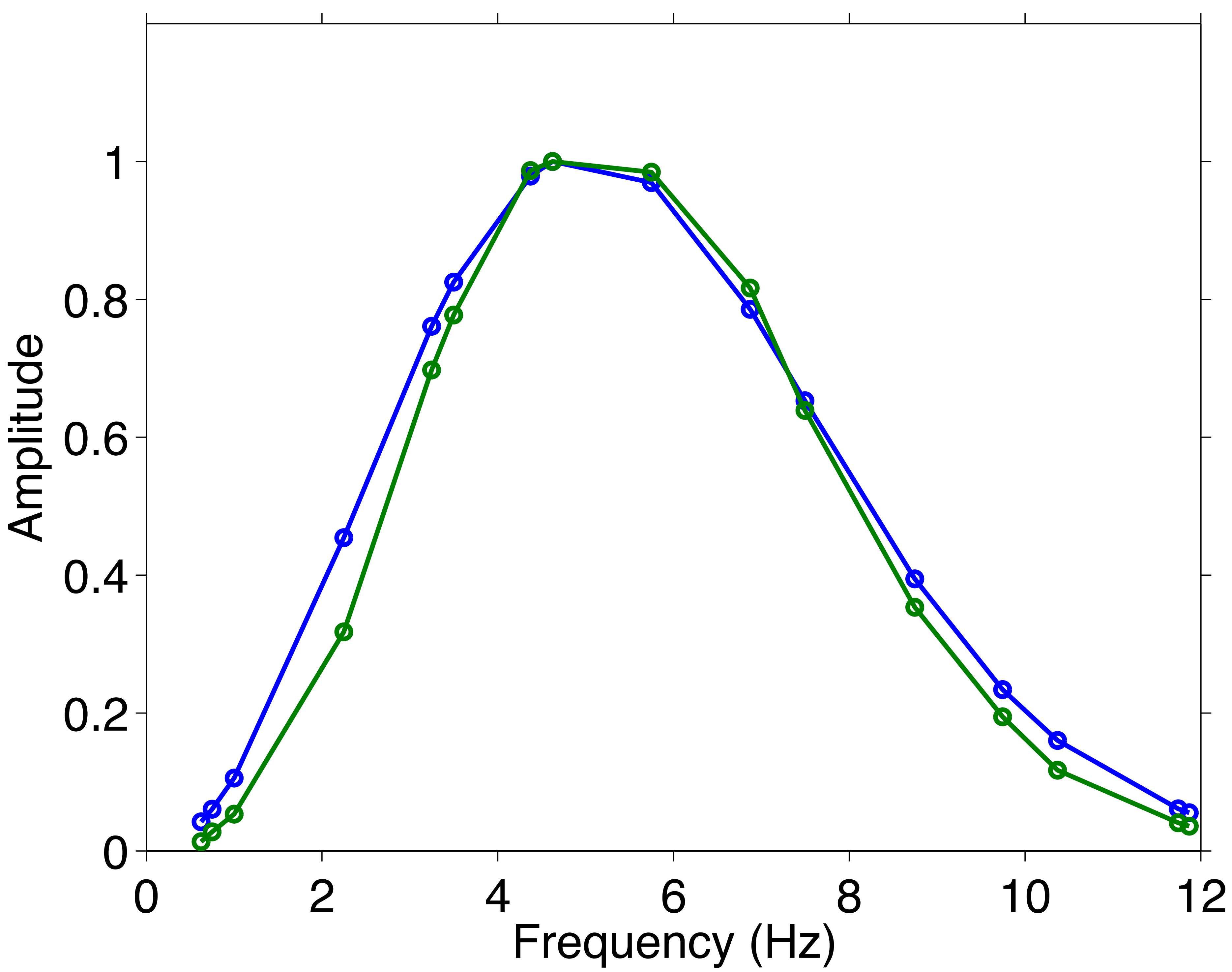
blue: ***true*** source wavelet

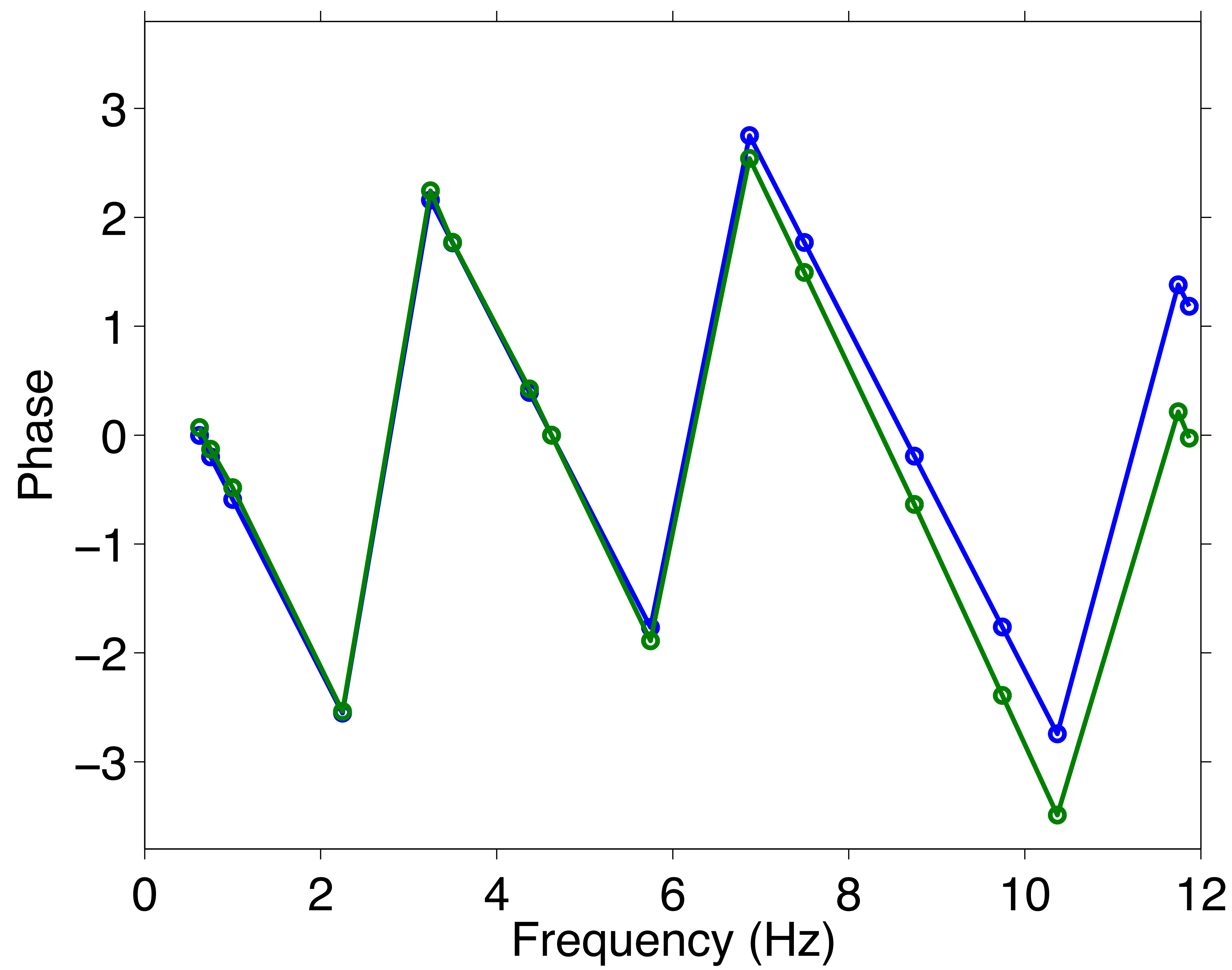
green: source ***estimation***

red: source estimation ***w/o*** rerandomization

black: ***wrong*** source wavelt

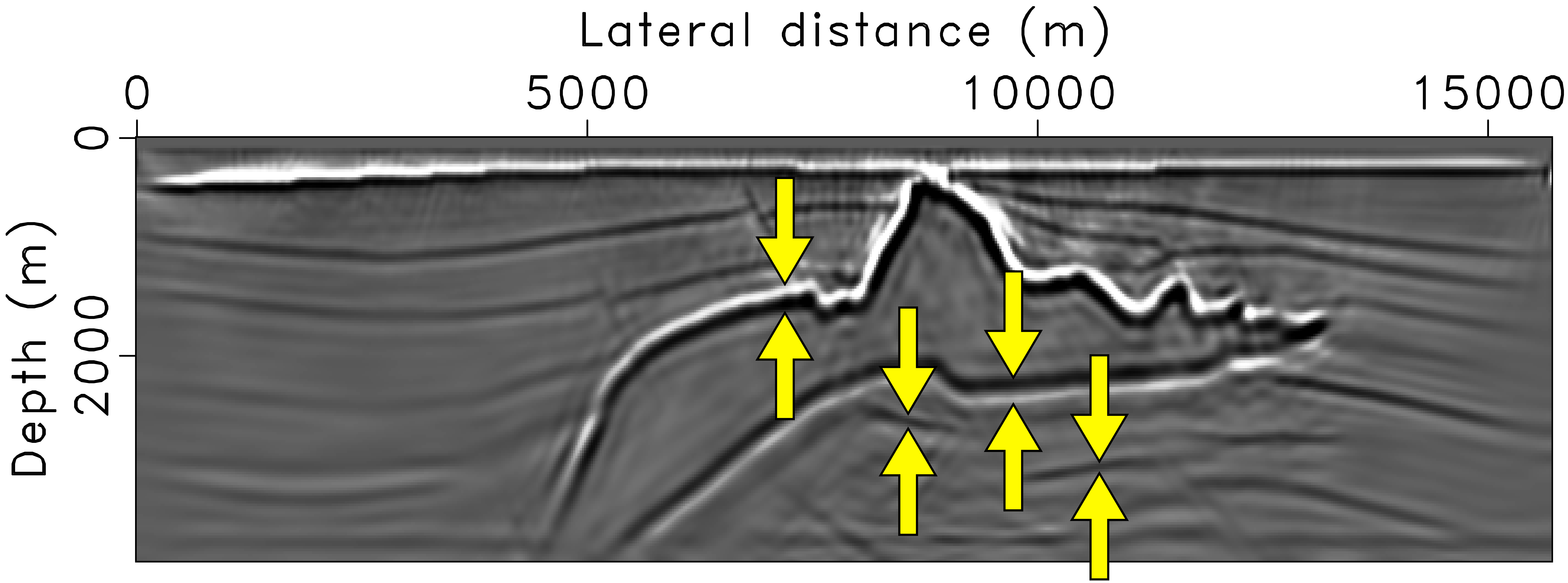
Examples: a more realistic case





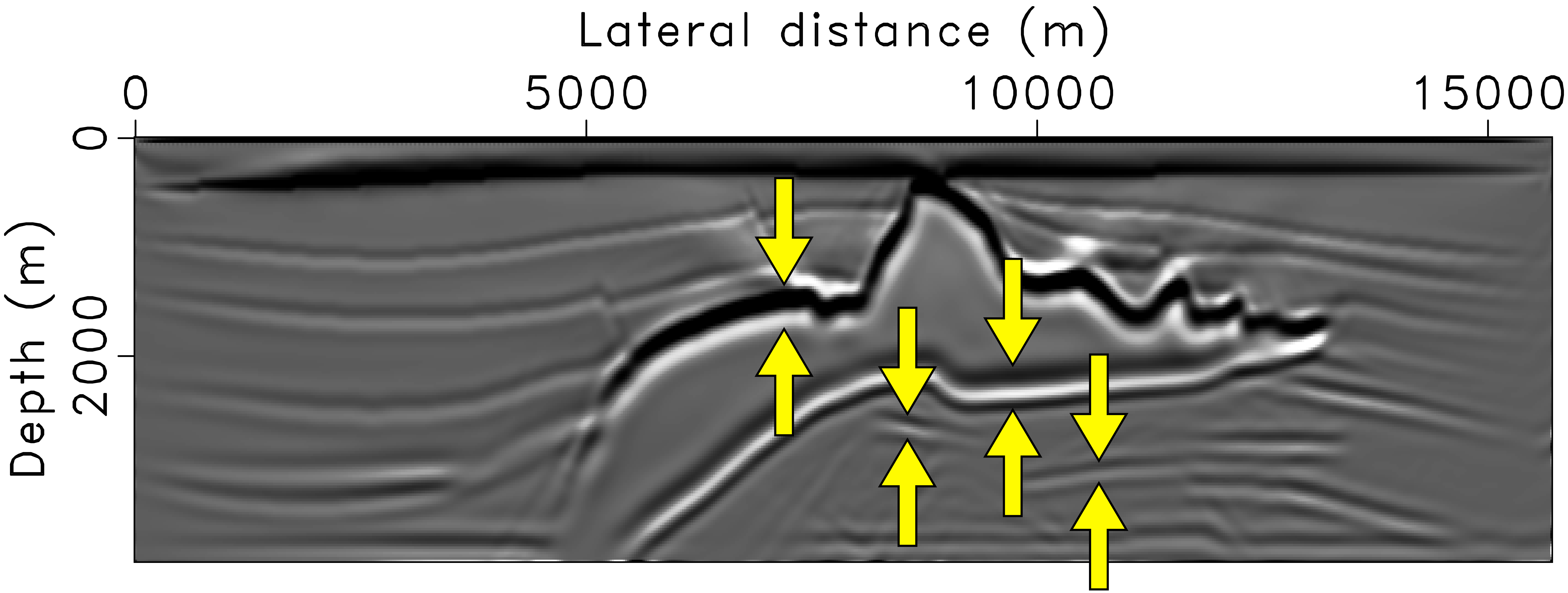
Source estimates of the last subproblem: *after normalization*

Inversion vs RTM: spatial **resolution**



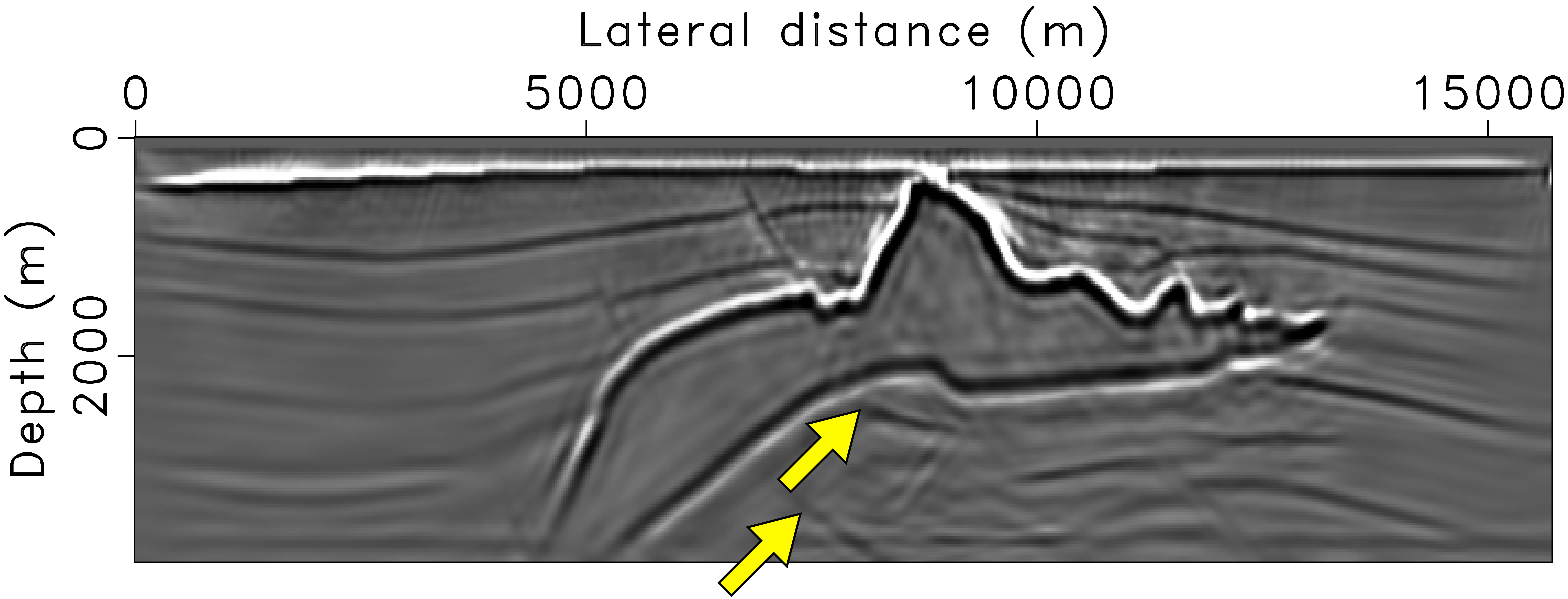
Inversion w/ source **estimation**

Inversion vs RTM: spatial **resolution**



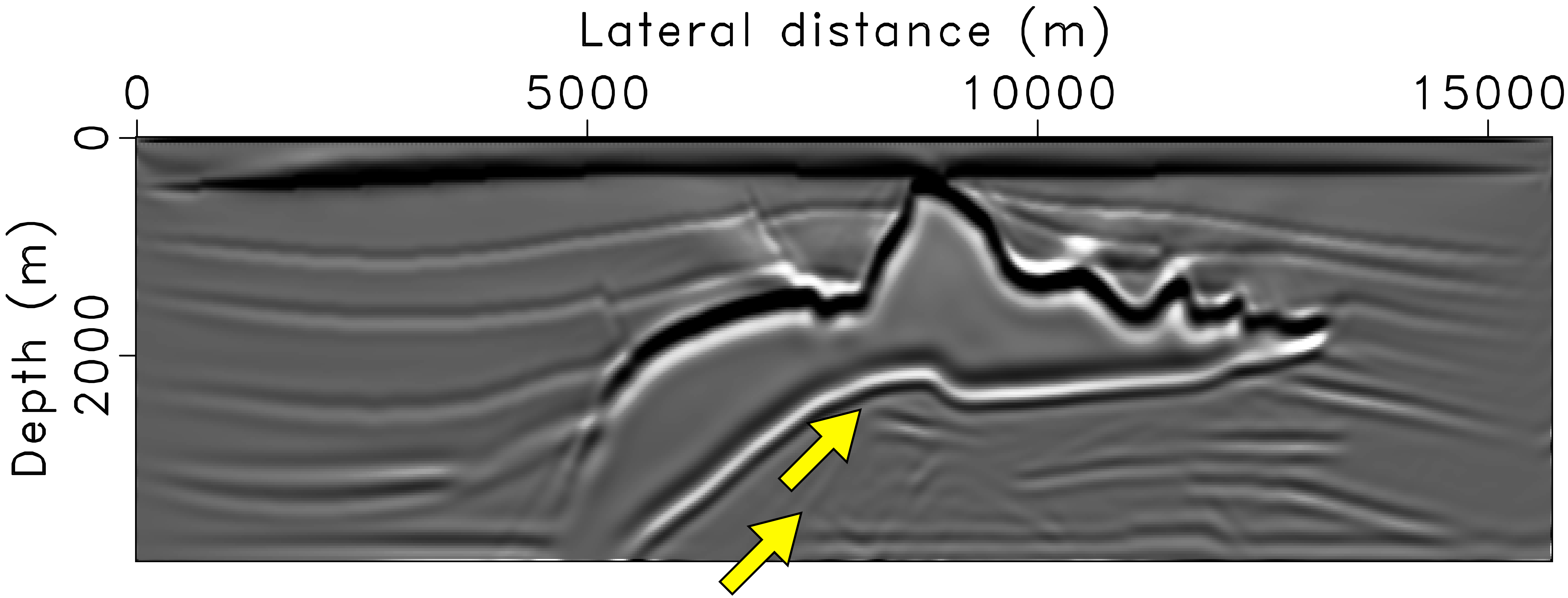
RTM with the **true** source wavelet

Inversion vs RTM: subsalt structures



Inversion w/ source **estimation**

Inversion vs RTM: subsalt structures



RTM with the **true** source wavelet

Challenge

Non-deterministic amplitude **ambiguity**:

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i \in \Omega} \sum_{j \in \Sigma} \|\underline{\mathbf{d}}_{i,j} - \nabla \mathbf{F}[\mathbf{m}_0, w_i \underline{\mathbf{s}}_j] \mathbf{C}^* \mathbf{x}\|_2^2$$

$$= f(\alpha \mathbf{x}, \frac{1}{\alpha} \boldsymbol{w})$$

Solution

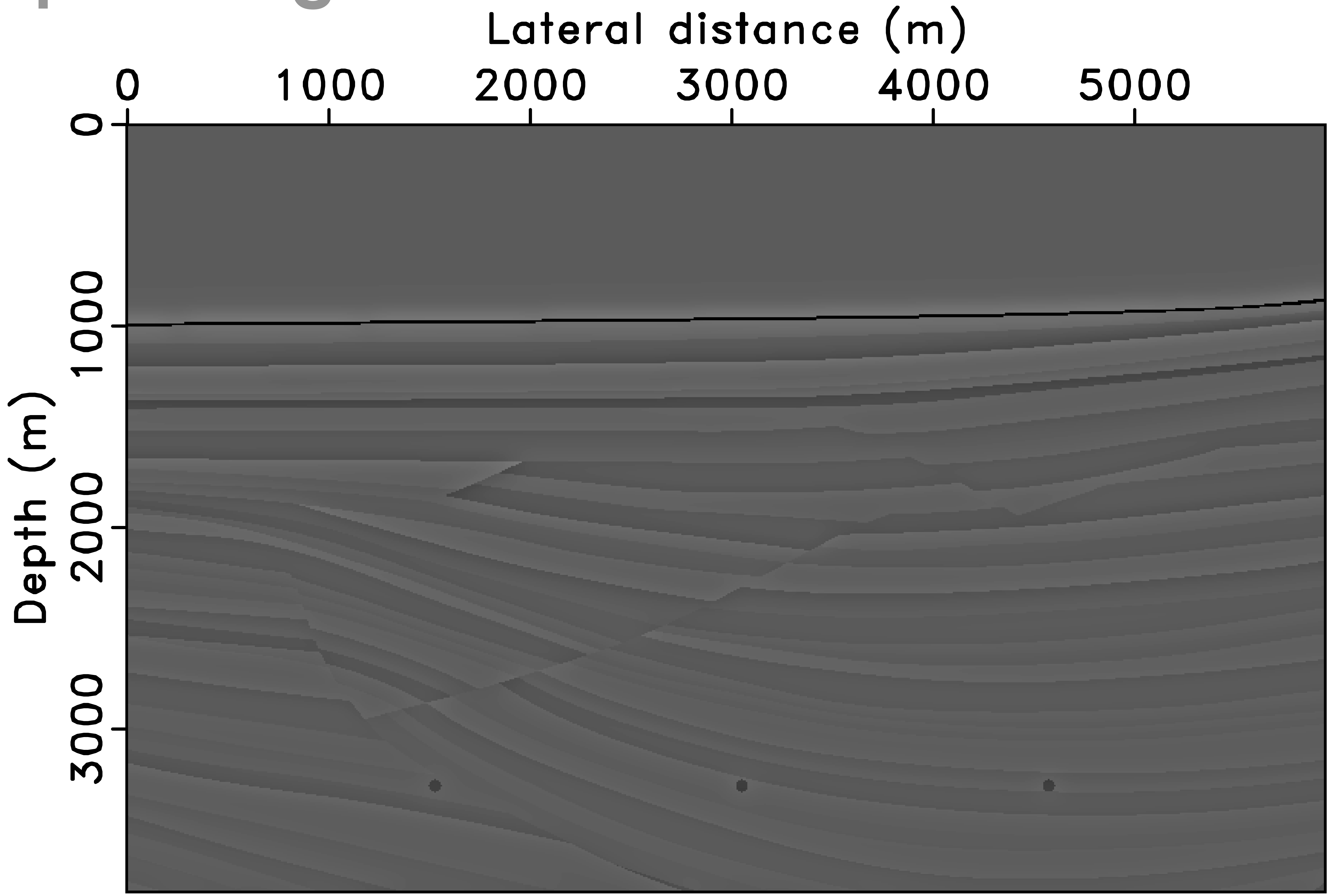
Incorporating surface-related multiples:

$$f(\mathbf{x}, \boldsymbol{w}) \doteq \sum_{i \in \Omega} \sum_{j \in \Sigma} \| \underline{\mathbf{d}}_{i,j} - \nabla \mathbf{F}[\mathbf{m}_0, w_i \underline{\mathbf{s}}_j - \underline{\mathbf{d}}_{i,j}] \mathbf{C}^* \mathbf{x} \|_2^2$$

Deterministic source estimates:

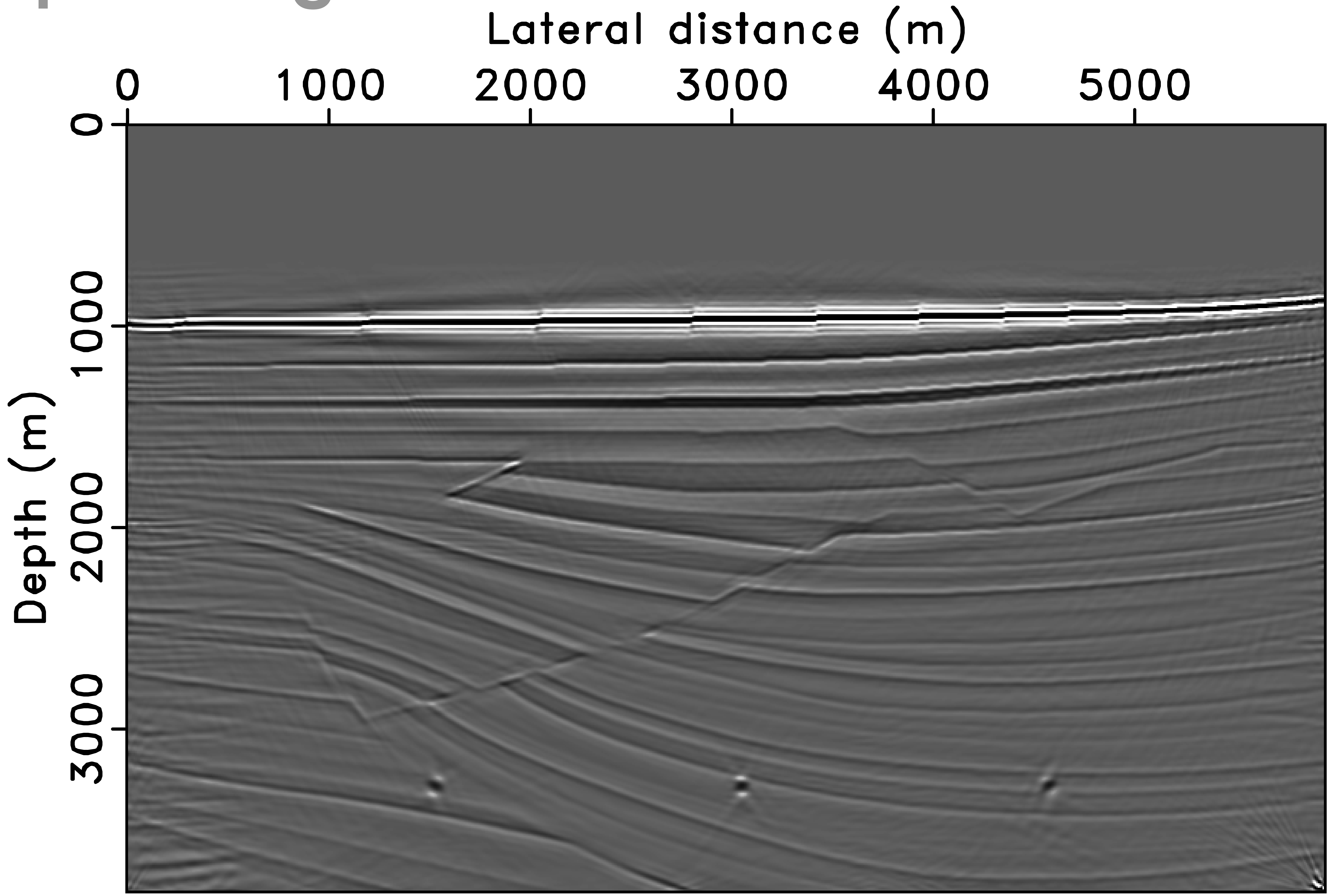
$$\tilde{w}_i(\mathbf{x}) = \frac{\sum_{j \in \Sigma} < \underline{\mathbf{d}}_{i,j} - \nabla \mathbf{F}[\mathbf{m}_0, -\underline{\mathbf{d}}_{i,j}] \mathbf{C}^* \mathbf{x}, \nabla \mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j] \mathbf{C}^* \mathbf{x} >}{\sum_{j \in \Sigma} < \nabla \mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j] \mathbf{C}^* \mathbf{x}, \nabla \mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j] \mathbf{C}^* \mathbf{x} >}$$

Example: using ideal data



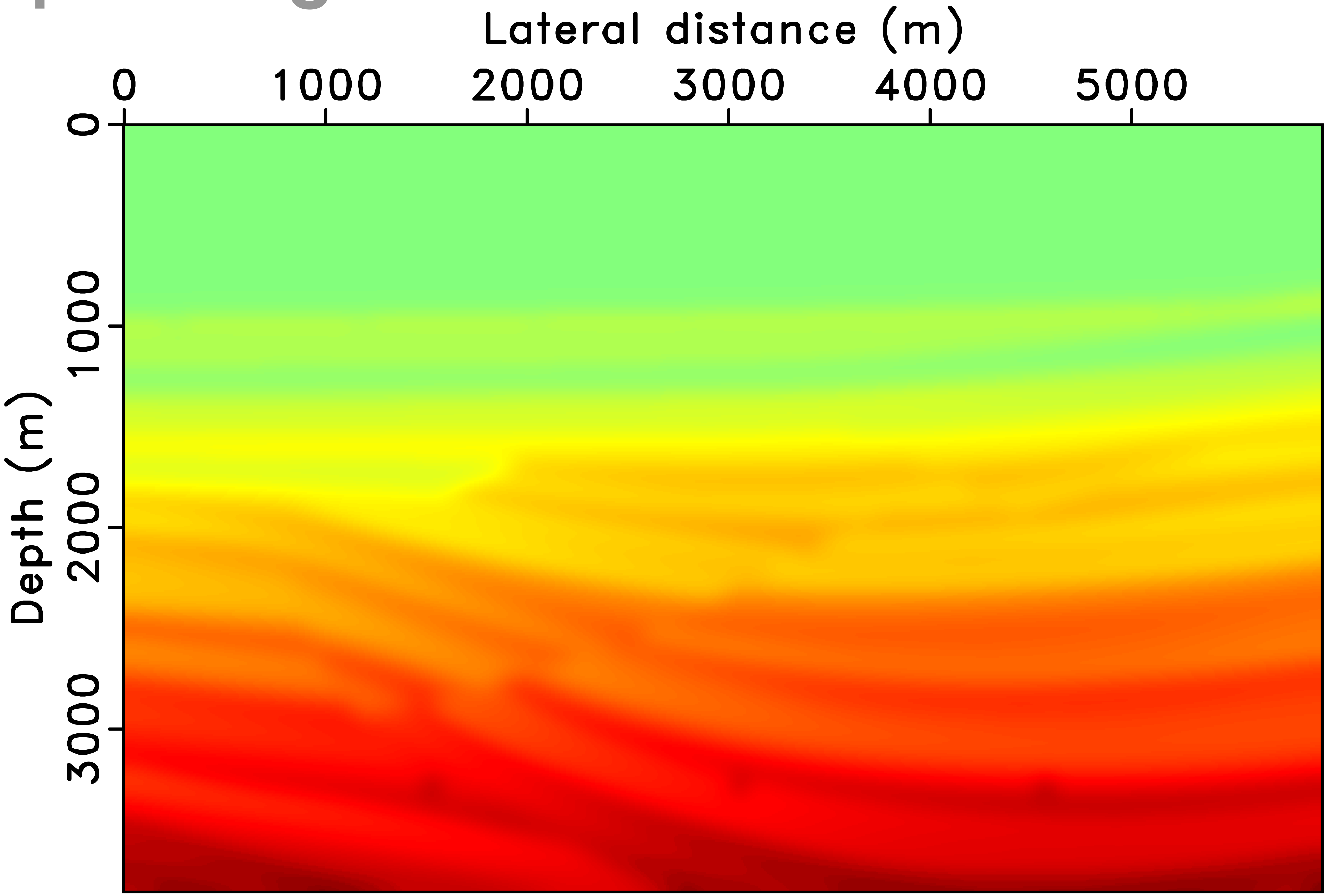
True model perturbations

Example: using **ideal** data



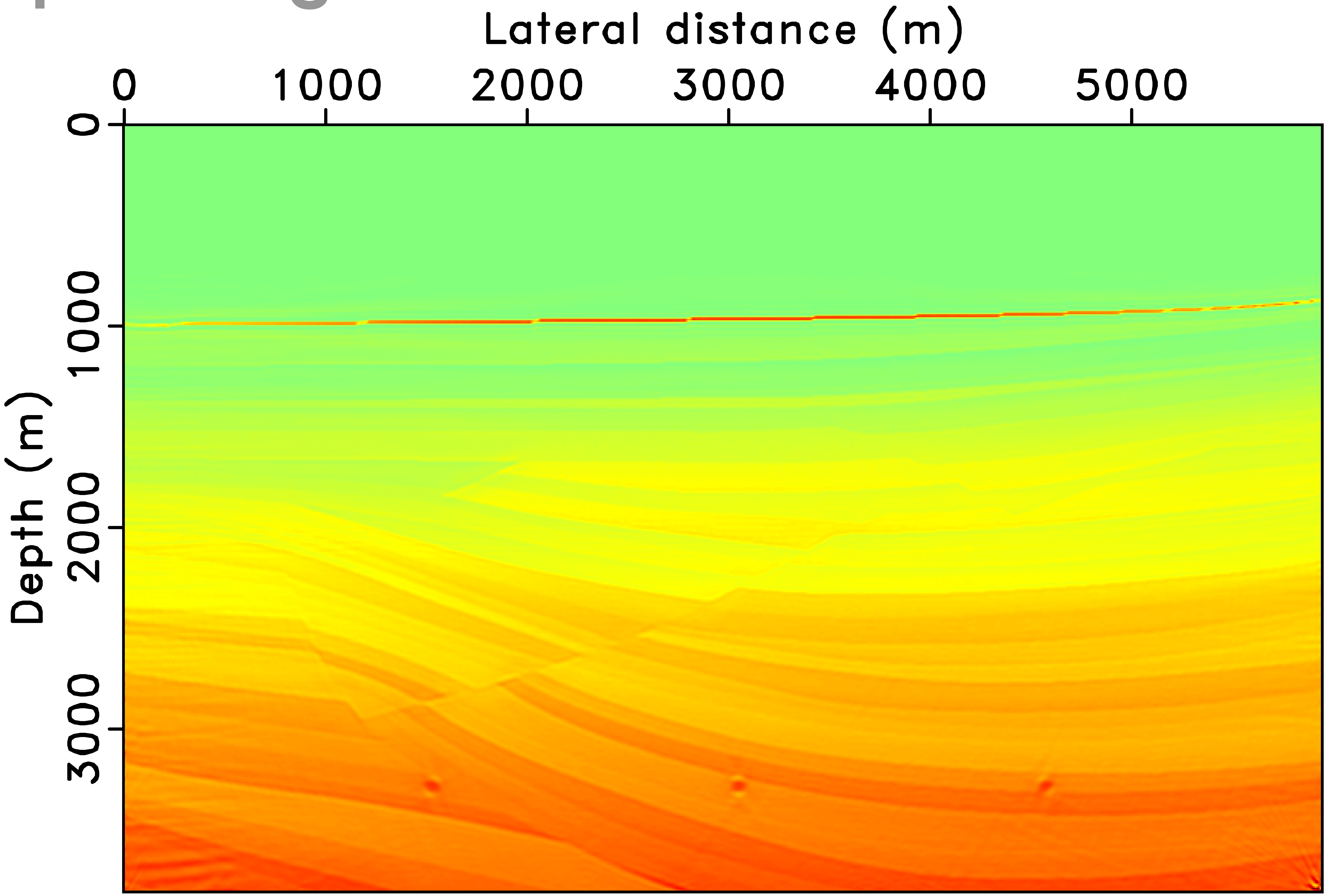
Fast imaging w/ source **estimation**

Example: using ideal data



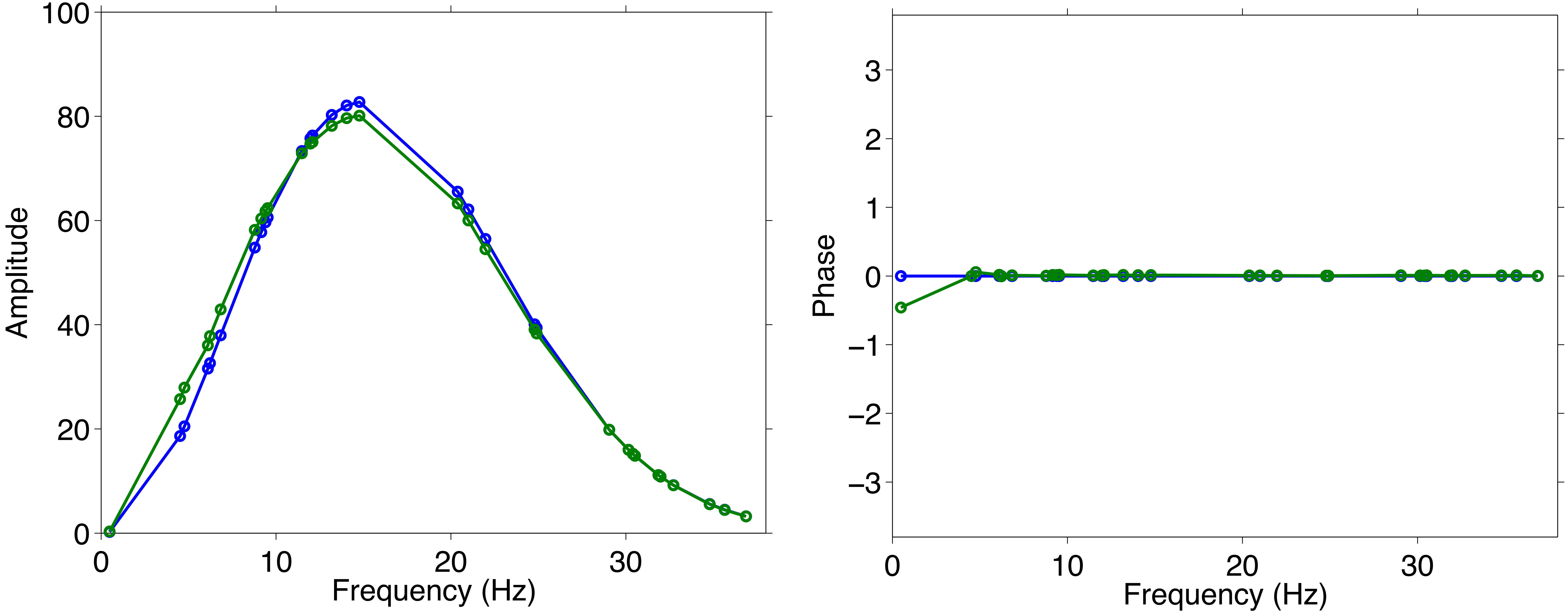
Background model

Example: using **ideal** data



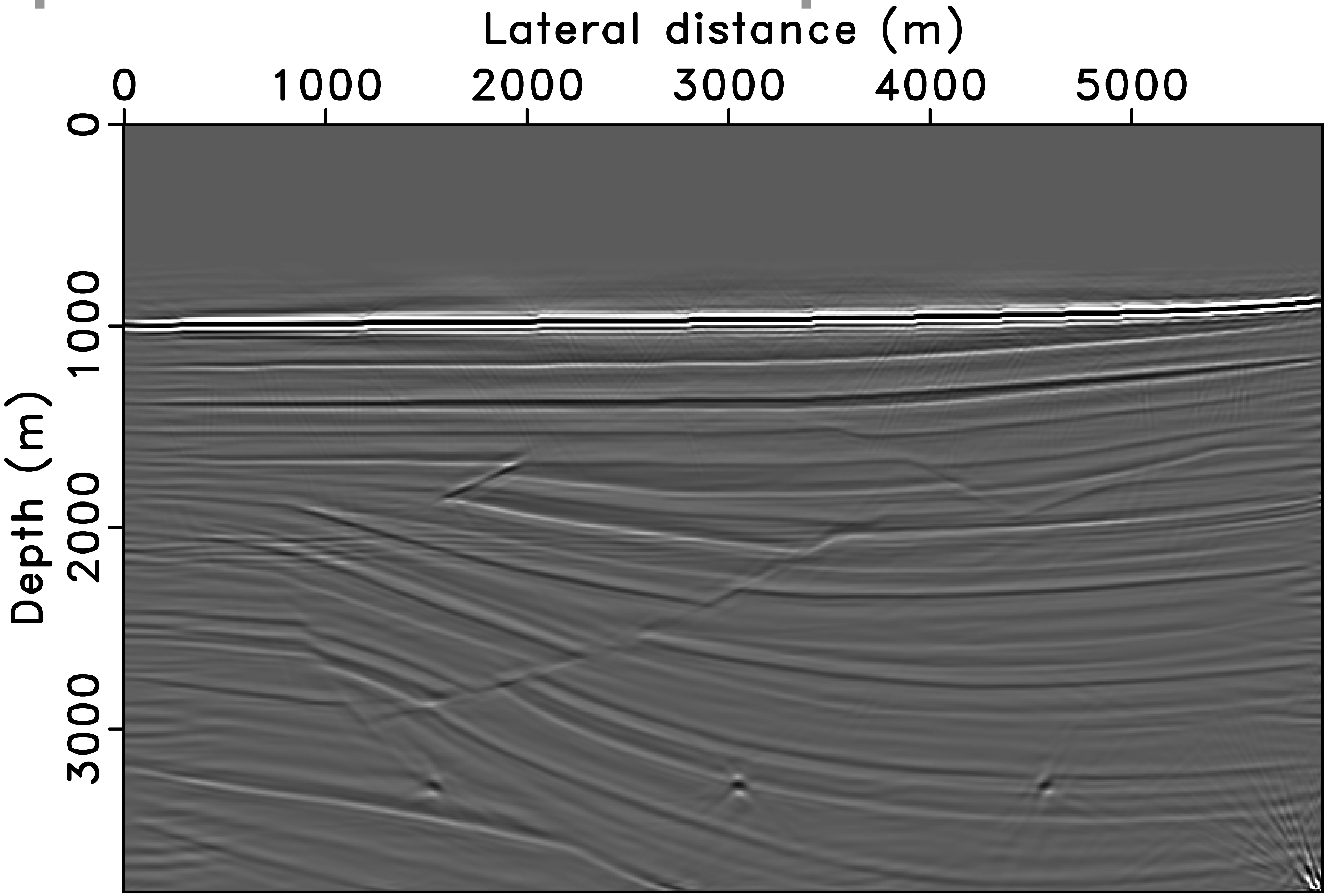
Adding inversion **result** back to background model, **no normalization** of any kind

Example: using **ideal** data



Source estimates of the last subproblem, **no normalization** used

Example: a more **realistic** setup



Simulation

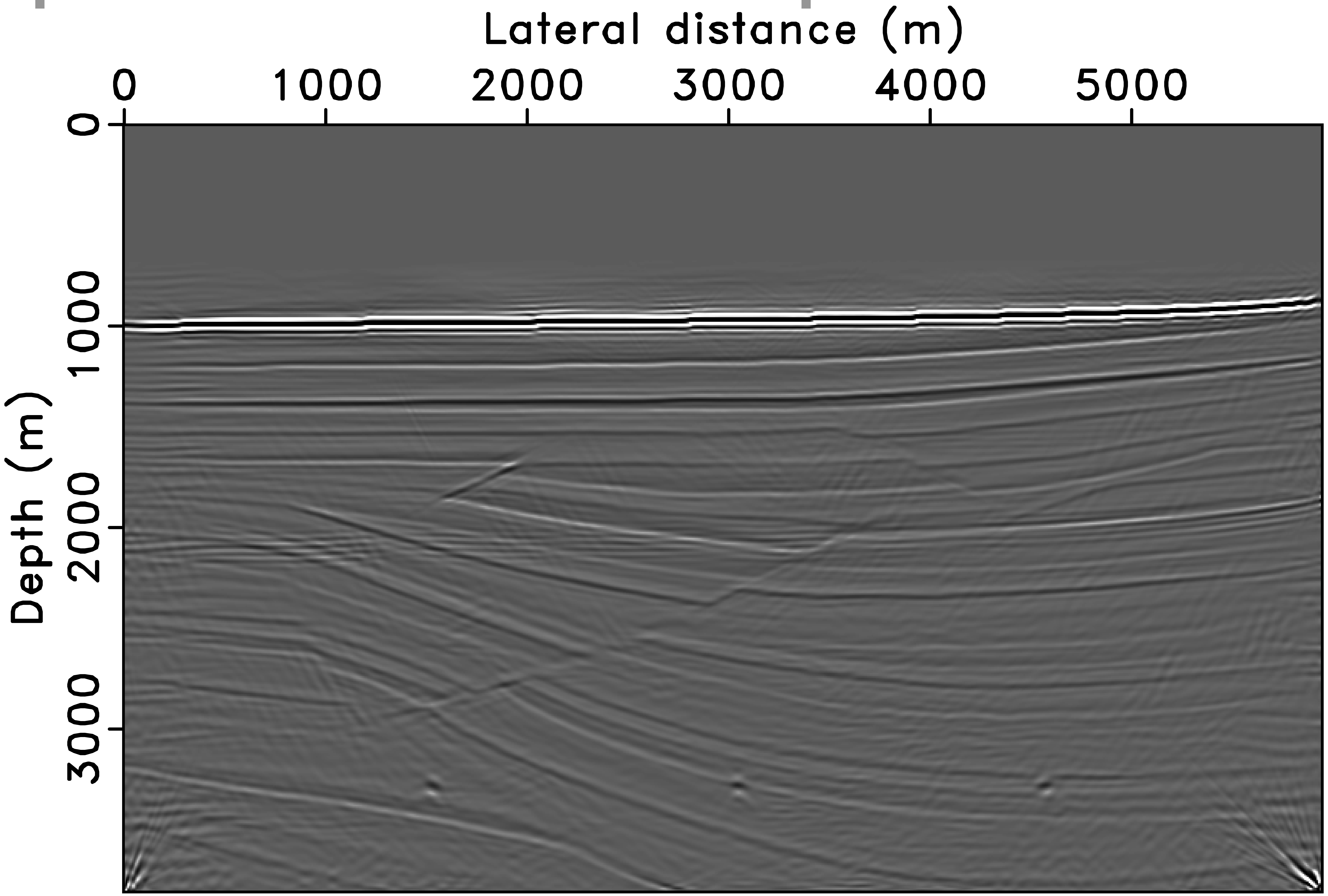
iWave

Inversion

in-house modelling engine

Fast imaging w/ **true** source

Example: a more **realistic** setup

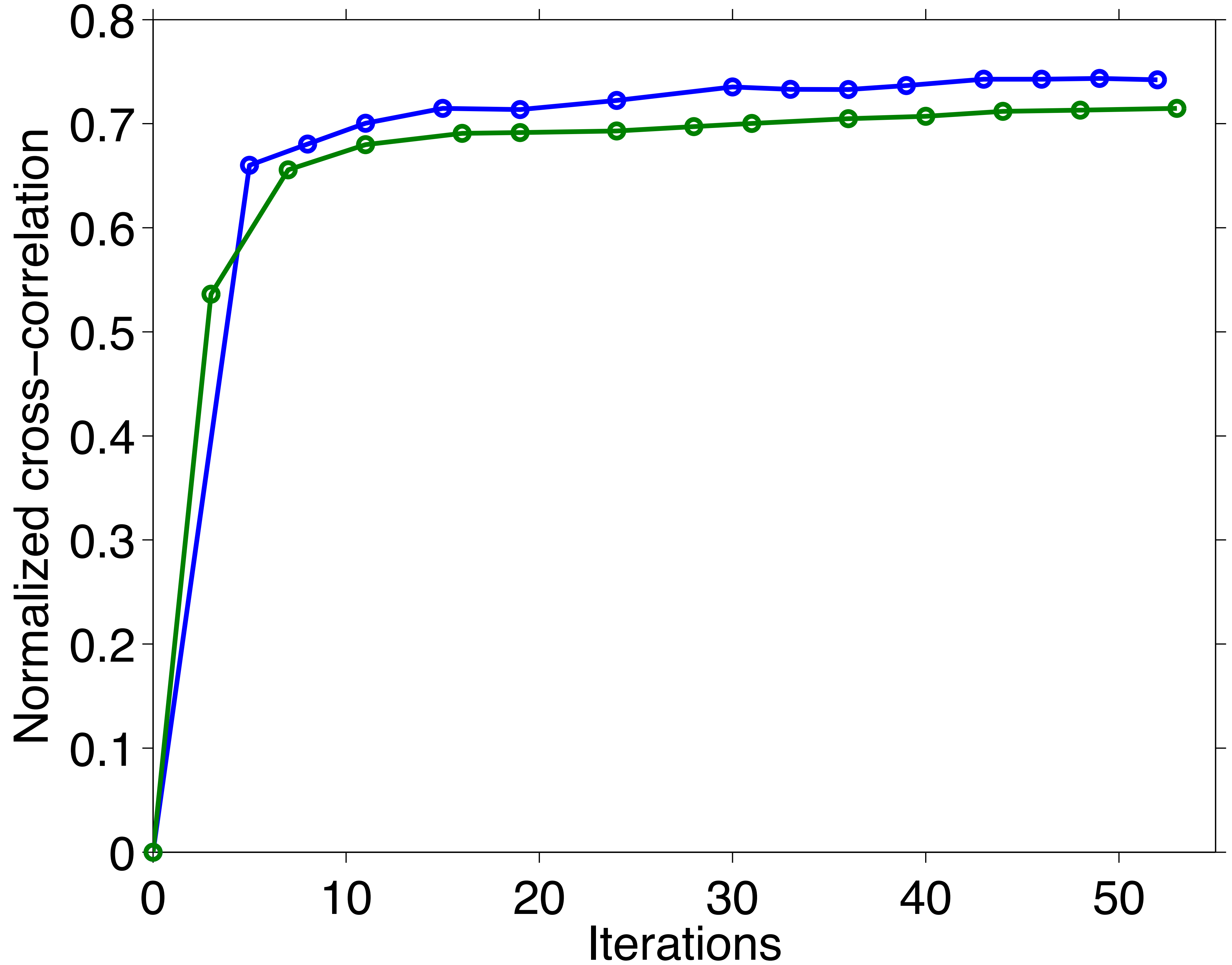


Fast imaging w/ source **estimation**

Caveat:
Deterministic amplitude difference from the true model

- iWave simulation
- in-house inversion

Example: a more **realistic** setup



blue: ***true*** source wavelet

green: source ***estimation***

Conclusions

Wrong source estimates lead to wrong images.

High-fidelity source estimation can be done in the inversion procedure in a **fast** fashion.

- by variable projection with a sparse constraint
- no assumption on the phase of the source is made

Amplitude ambiguity in the source can be mitigated by using surface-related multiple in the inversion.

To be continued...

Application to 2D field data, Wed 11:55 AM

Imaging the Nelson data set using surface-related multiples

Future work

- applications to 3D field data
- extension of the method to the time domain

Acknowledgements

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Pseudo code

Input:

total upgoing wavefield , background velocity model $\mathbf{m}_0$, tolerance $\sigma = 0$, iteration limit k_{max}

Initialization:

iteration index $k \leftarrow 0$, subproblem index $l \leftarrow 0, \mathbf{x}_l \leftarrow \mathbf{0}, w_i = 1$ for all i $\in 1, \cdots, n_f$

while $k < k_{max}$ **do**

$\Omega_l, \Sigma_l, \underline{\mathbf{d}}_{i,j}, \underline{\mathbf{s}}_j \leftarrow$ new independent draw

$\tau_l \leftarrow$ determine from τ_{l-1} and σ by root finding on the Pareto curve

$$\mathbf{x}_l \leftarrow \begin{cases} \mathrm{argmin}_{\mathbf{x}} \sum_{i\in\Omega_l, j\in\Sigma_l} \|\underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}_i[\mathbf{m}_0, w_i(\mathbf{x})\underline{\mathbf{s}}_j - \underline{\mathbf{d}}_{i,j}]\mathbf{C}^{\mathrm{H}}\mathbf{x}\|_2^2 \\ \text{subject to } \|\mathbf{x}\|_1 \leq \tau_l \end{cases}$$ //warm

start with $\mathbf{x}_{l-1}$, solved in k_l iterations, in each iteration, compute

$$w_i(\mathbf{x}) = \frac{\sum_{j\in\Sigma} < \underline{\mathbf{d}}_{i,j} - \nabla\mathbf{F}[\mathbf{m}_0, -\underline{\mathbf{d}}_{i,j}]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}{\sum_{j\in\Sigma} < \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x}, \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{s}}_j]\mathbf{C}^*\mathbf{x} >}$$

$k \leftarrow k + k_l, l \leftarrow l + 1$

end while

Output: Model perturbation estimate $\delta\mathbf{m} = \mathbf{C}^{\mathrm{H}}\mathbf{x}$

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