Fast imaging with surface-related multiples by sparse inversion

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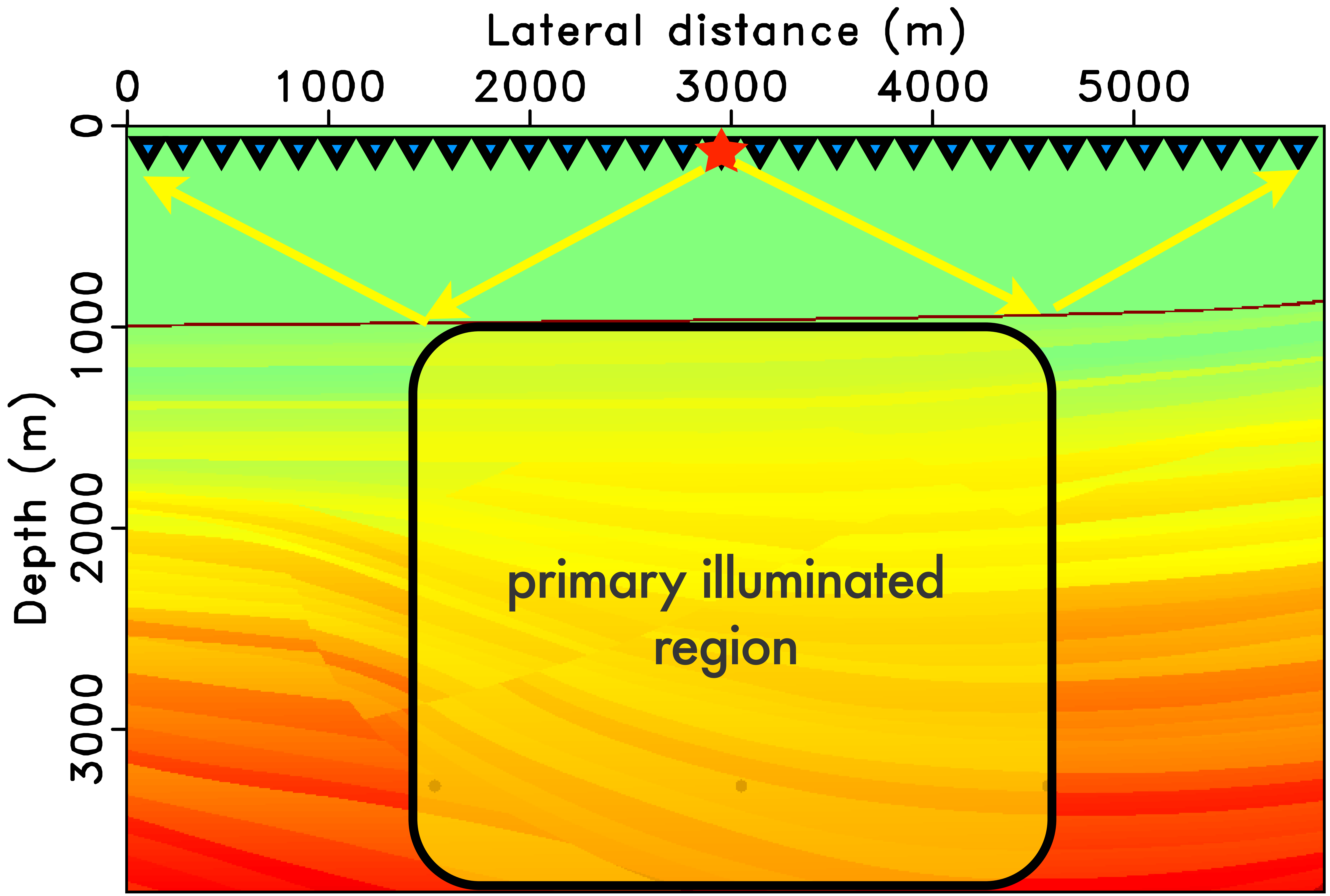
Berkhout 1993; Guitton 2002; Muijs et al., 2007; Verschuur 2011; Long et al. 2013; Wong et al., 2014

Motivation

- make use of primaries and multiples *simultaneously*
  - higher SNR from primaries and extra illumination from multiples
- eliminate *acausal* artifacts from multiples by *inversion*
  - cross-correlation of wrong orders of up-/down-going wavefields
- look for a computationally *efficient* approach
  - dense matrix products in multiple prediction by SRME or EPSI
  - expensive simulation in iterative inversion

Illustration: primary wave propagation



RTM image of primaries: using 1 source

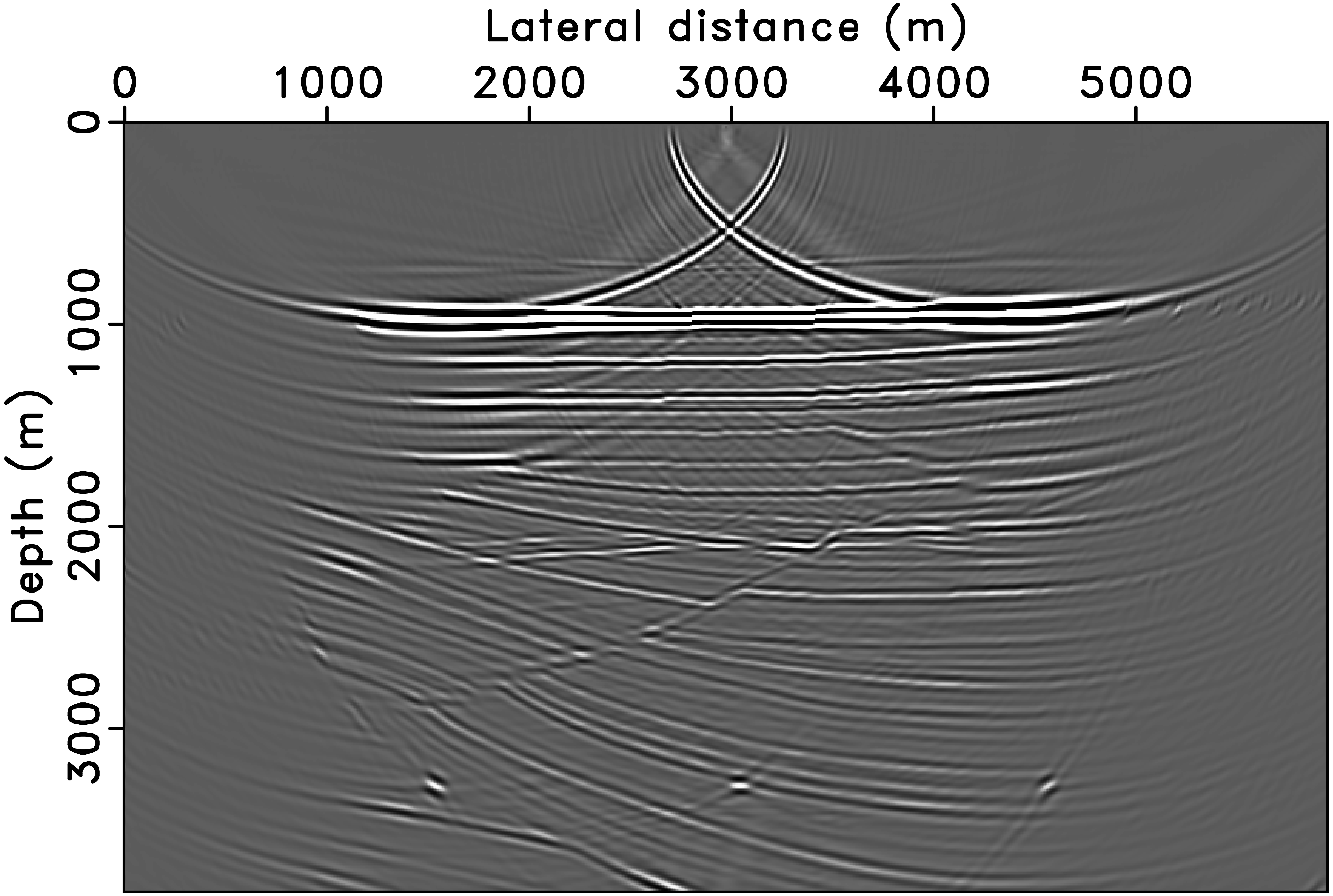
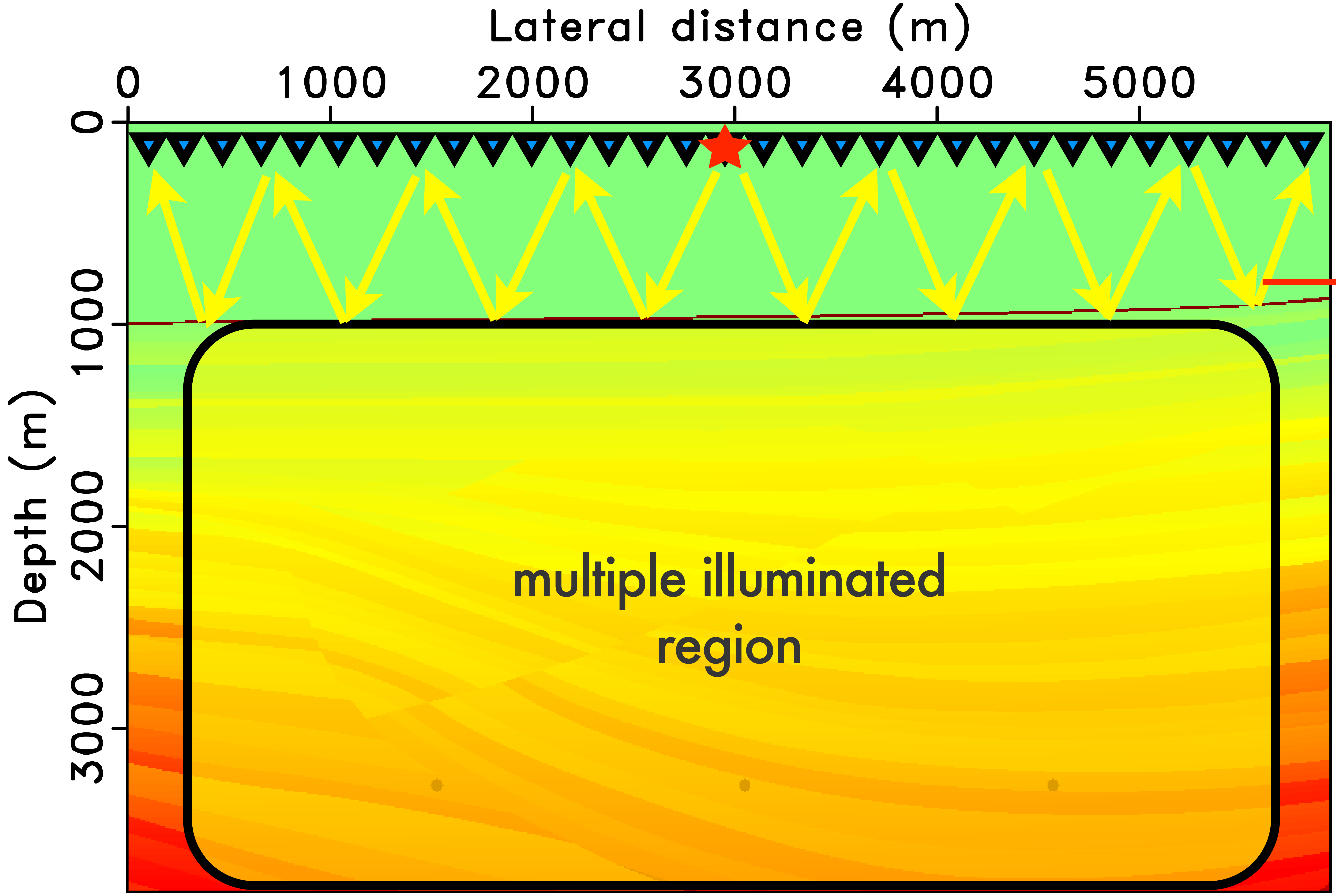


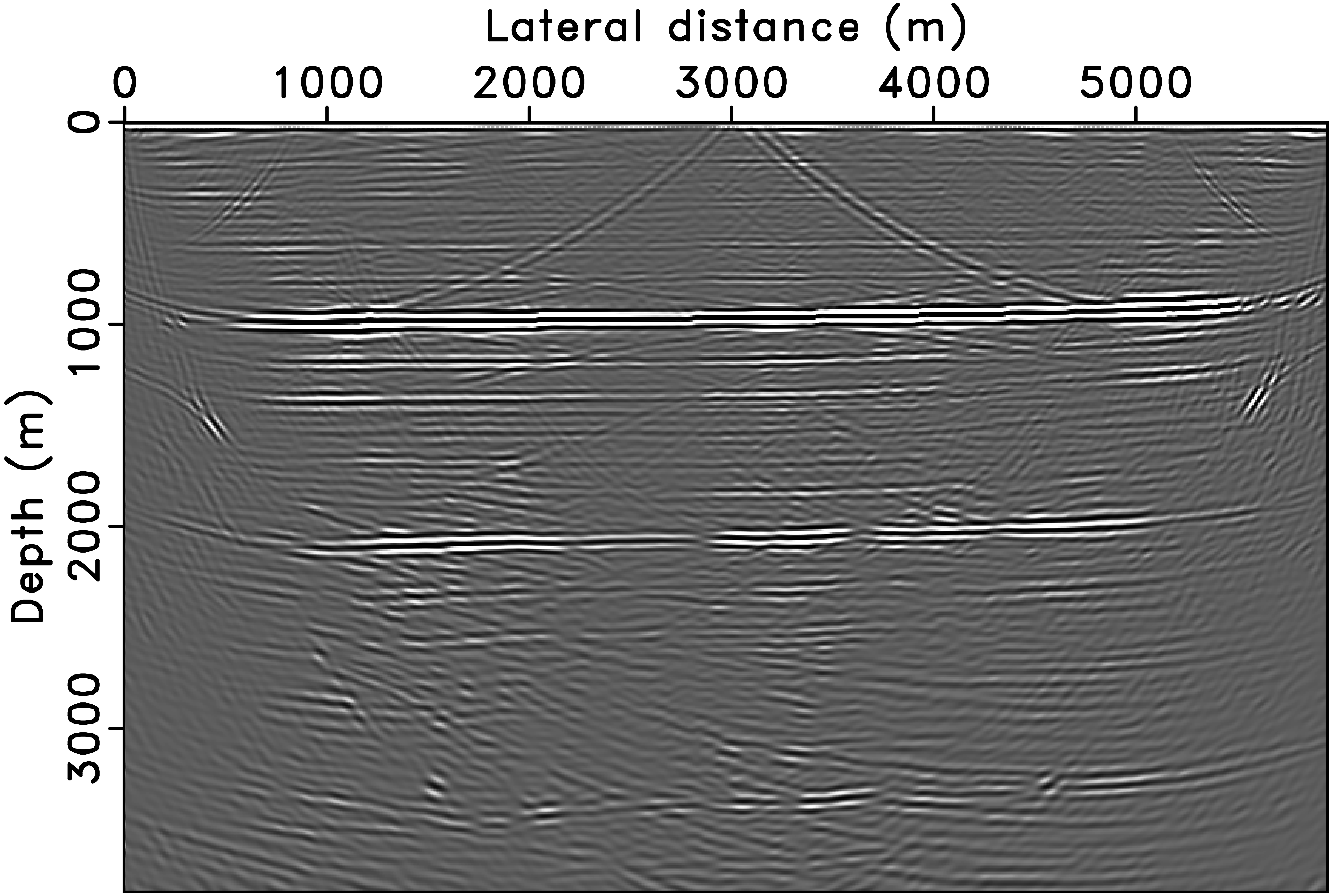
Illustration: surface-multiples propagation

[Each receiver serves as a virtual secondary source]



much smaller incident angles compared to primaries

RTM image of multiples: using 1 source



Muijs et al., 2007; Lin et al., 2010; Liu et al., 2011; Whitmore et al., 2010; Verschuur et al., 2011; Wong et al., 2012

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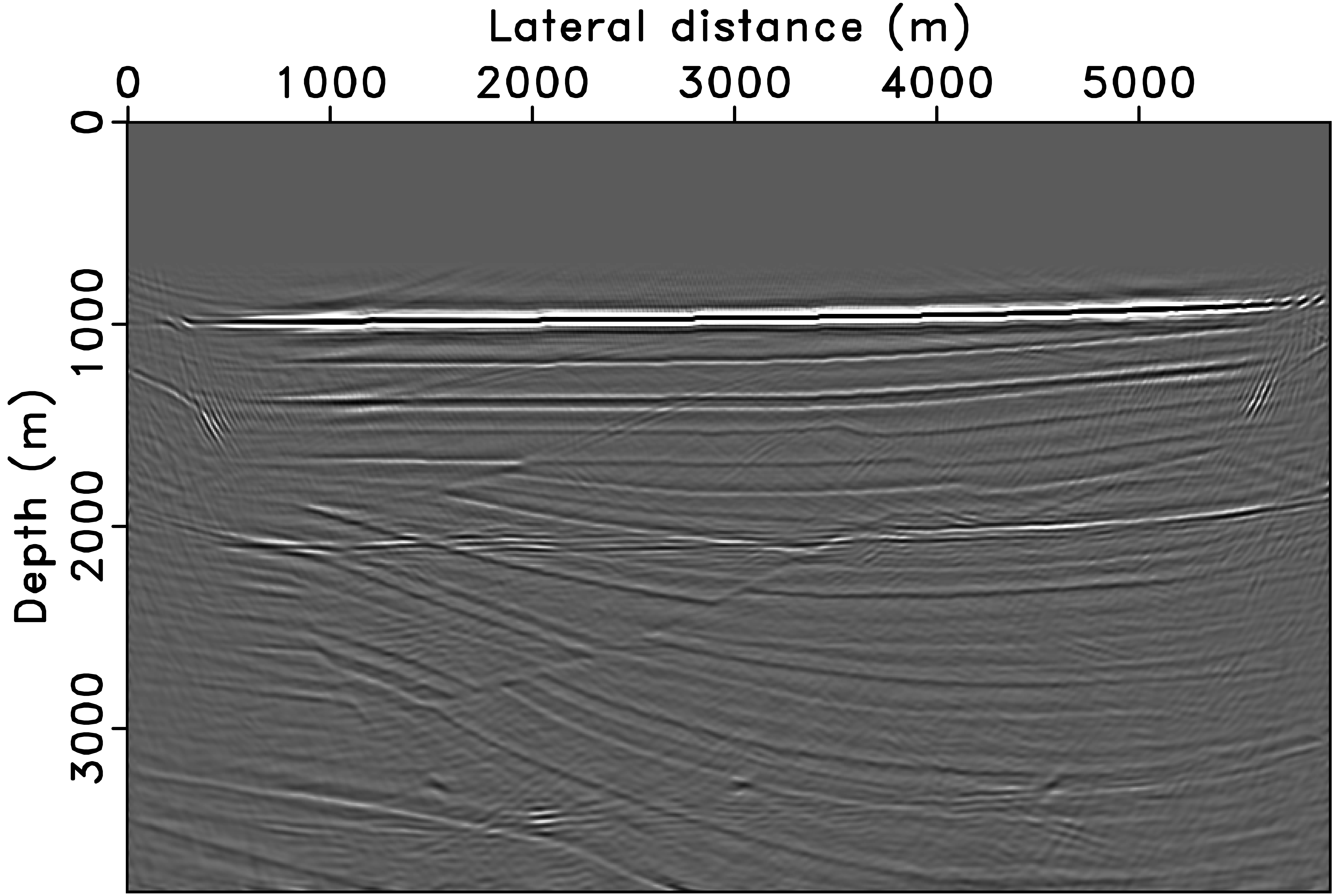
Inversion of multiples: 1 source, 60 iterations

[~120X the simulation cost of the 1 source RTM image]



Herrmann and Li, 2012; Tu and Herrmann, 2012 a,b

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Caveat: internal multiples are ***not*** modelled in multiple prediction by the SRME relation

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Solutions

- Model ***all orders*** of surface-related ***multiples*** using ***two-way*** wave-equations based on the **SRME** relation
- **Joint** inversion of primaries and multiples given true source wavelet
- ***Eliminate*** expensive dense matrix products in multiple prediction
- ***Fast*** least-squares ***inversion*** with a cost of a ***single*** **RTM** of all the data

Verschuur et al., 1992

Method: the SRME relation

$$\mathbf{P}_i = \mathbf{X}_i(\mathbf{S}_i + \mathbf{R}_i\mathbf{P}_i)$$

$\mathbf{P}$: total up-going wavefield

$\mathbf{X}$: surface-free dipole Green's function

$\mathbf{S}$: point-source wavefield $= w_i\mathbf{I}$

$\mathbf{R}$: surface reflectivity $\approx -\mathbf{I}$

i : frequency index $1 \cdots n_f$

Eliminating dense matrix-matrix products

[SRME relation & wave-equation solver]

Linearized modelling of the surface-free Green's function:

$$
\begin{aligned}
\mathbf{X}_i &\approx \nabla \mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}, \mathbf{I}] \\
&= \omega^2 \mathbf{D}_r \mathbf{H}_i[\mathbf{m}_0]^{-1} \mathbf{H}_i[\mathbf{m}_0]^{-1} \mathrm{diag}(\delta\mathbf{m}) \mathbf{D}_s^* \mathbf{I} \\
&\doteq \nabla \mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^* \mathbf{I})
\end{aligned}
$$

$\nabla \mathcal{F}_i$: linearized modelling

$\mathbf{m}_0$: background model

$\delta\mathbf{m}$: model perturbations

$\mathbf{I}$: impulsive source array

$\mathbf{D}_r$: data restriction at receivers

$\mathbf{H}_i$: time-harmonic Helmholtz operator

$\mathbf{D}_s^*$: source injection operator

Tu and Herrmann, 2012 a,b

Eliminating dense matrix-matrix products

[SRME relation & wave-equation solver]

Combined with free-surface physics:

$$\begin{aligned}\mathbf{P}_i &\approx \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}, \mathbf{I}](\mathbf{S}_i - \mathbf{P}_i)\\ &= \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*\mathbf{I})(\mathbf{S}_i - \mathbf{P}_i) \longrightarrow \text{Dense matrix products}\\ &= \nabla\mathcal{F}_i[\mathbf{m}_0, \delta\mathbf{m}](\mathbf{D}_s^*(\mathbf{S}_i - \mathbf{P}_i)) \longrightarrow \text{Wave-equation solves with total downgoing data}\\ &\doteq \nabla\mathcal{F}_i[\mathbf{m}_0, \mathbf{S}_i - \mathbf{P}_i].\end{aligned}$$

Herrmann and Li, 2012; Tu and Herrmann, 2012 a,b

Compressive imaging with sparsity promotion

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^{\mathrm{H}} \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{x}\|_1$$

$$\text{subject to } \sum_{i\in\Omega} \|\underline{\mathbf{p}}_i - \nabla\mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{S}}_i - \underline{\mathbf{P}}_i]\mathbf{C}^{\mathrm{H}}\mathbf{x}\|_2^2 \leq \sigma^2$$

$\mathbf{C}$: curvelet transform

$\underline{\cdot}$: wavefields with subsampled source experiments,

i.e., $\underline{\mathbf{P}}_i = \mathbf{P}_i\mathbf{E}$, $\underline{\mathbf{p}}_i = \mathrm{vec}(\underline{\mathbf{P}}_i)$.

Ω : randomized frequency subset

σ : misfit tolerance

Tu and Herrmann, 2012 b; Tu et al., 2013 a

Rerandomization

SPGl1 solves a series of LASSO subproblems

$$\underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i \in \Omega} \|\underline{\mathbf{p}}_i - \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{S}}_i - \underline{\mathbf{P}}_i]\mathbf{C}^{\mathrm{H}}\mathbf{x}\|_2^2$$

$$\text{subject to } \|\mathbf{x}\|_1 \leq \tau$$

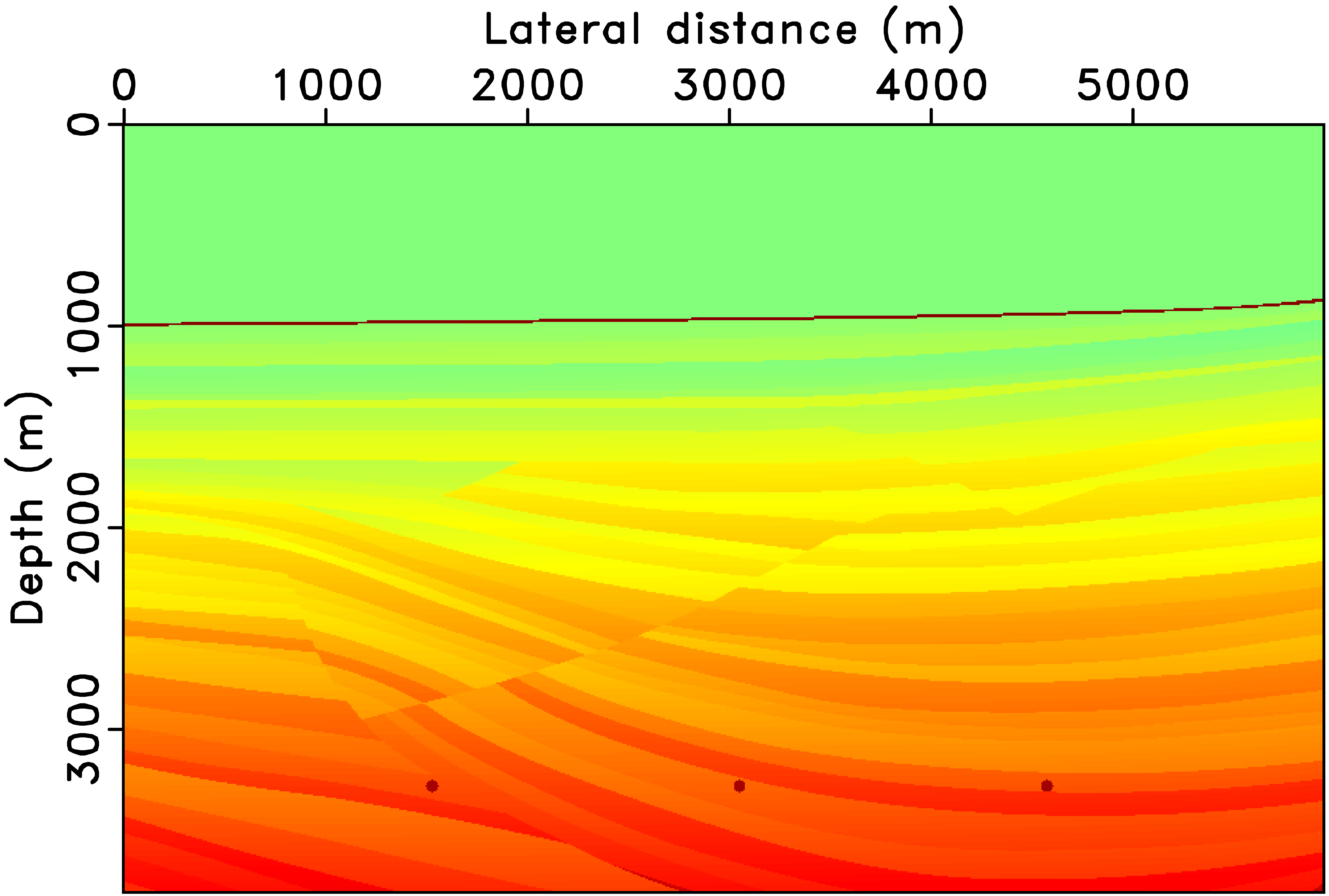
for gradually relaxed τ's computed using Newton's method.

Rerandomization:

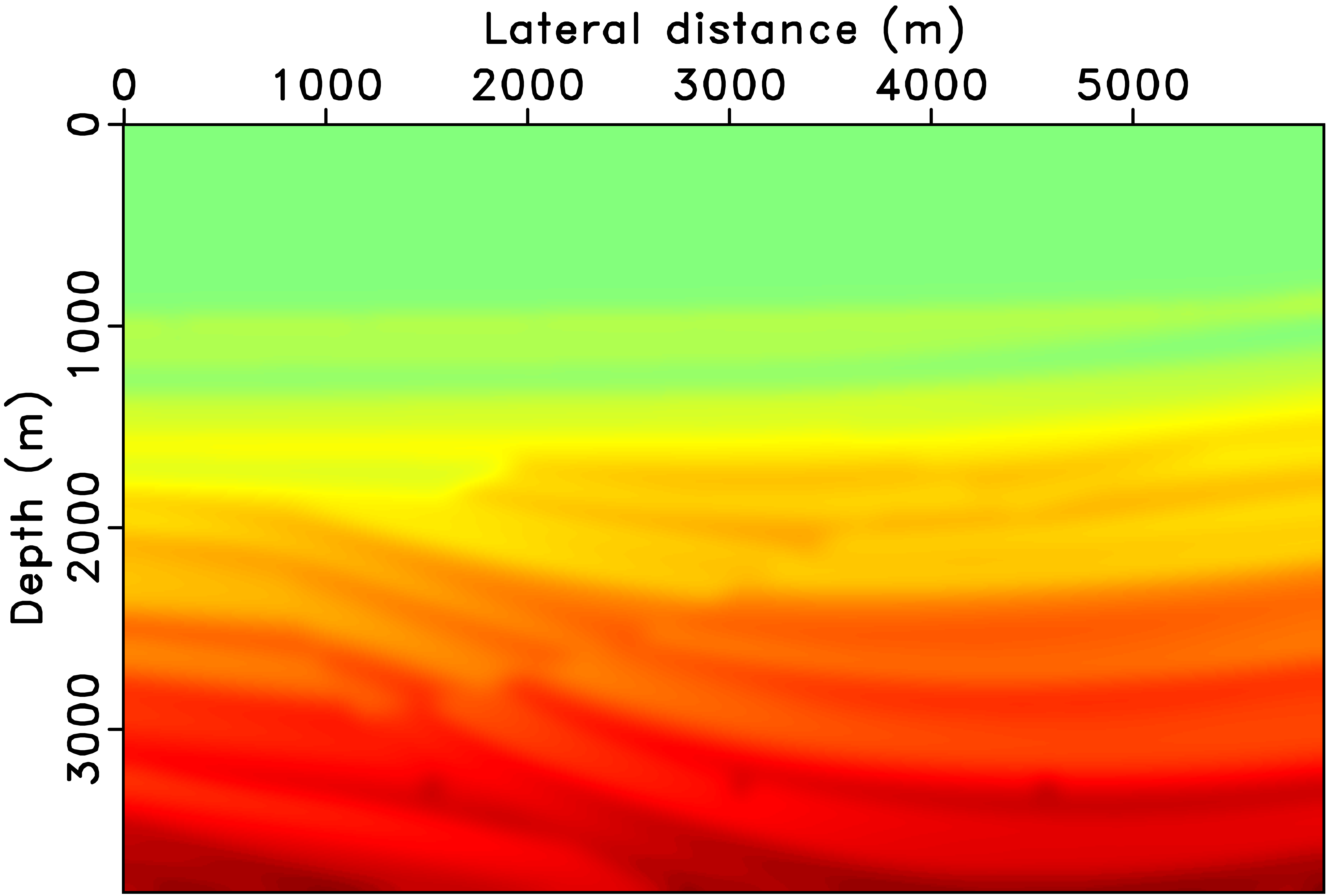
- new independent sim. sources/freq. for each subproblem.
- faster progress to solution
- higher robustness to linearization errors

Synthetic example

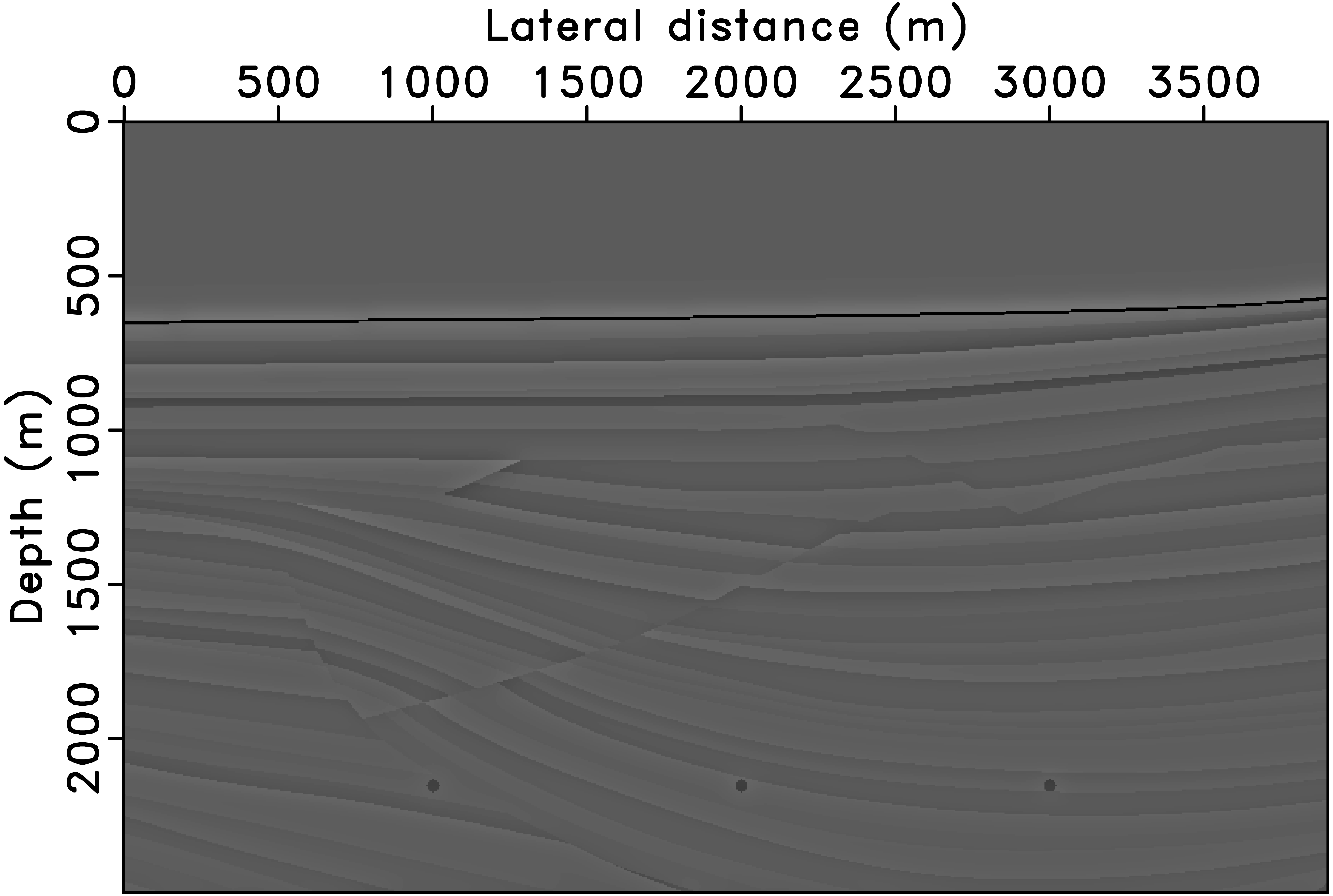
- model cropped from the Sigsbee 2B model, 3.8km deep, 6km wide, 7.62m grid spacing
- 15Hz Ricker wavelet, 8.184s recording time, 311 freq. samples
- 261 co-located sources/receivers, fixed spread, 22.86m spacing, 7.62m deep
- data modelled using iWave with free-surface BC.
- 31 (~10%) frequencies, 26 (~10%) simultaneous sources
- 50 iterations, simulation cost ~1 RTM with all the data
- comparison between **RTM** (adjoint migration) w. multiples and **fast inversion** w. multiples



True velocity model

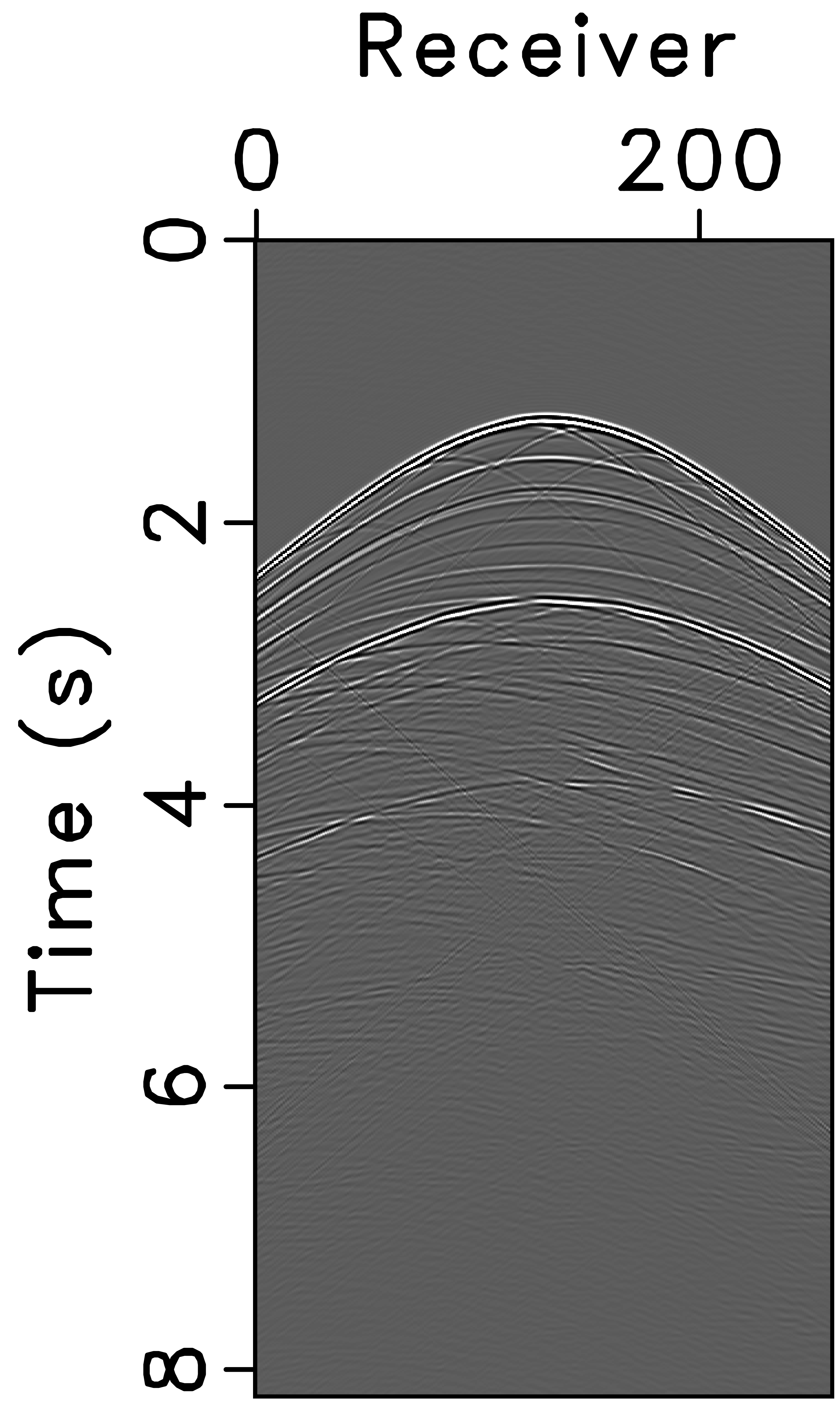


Background velocity model

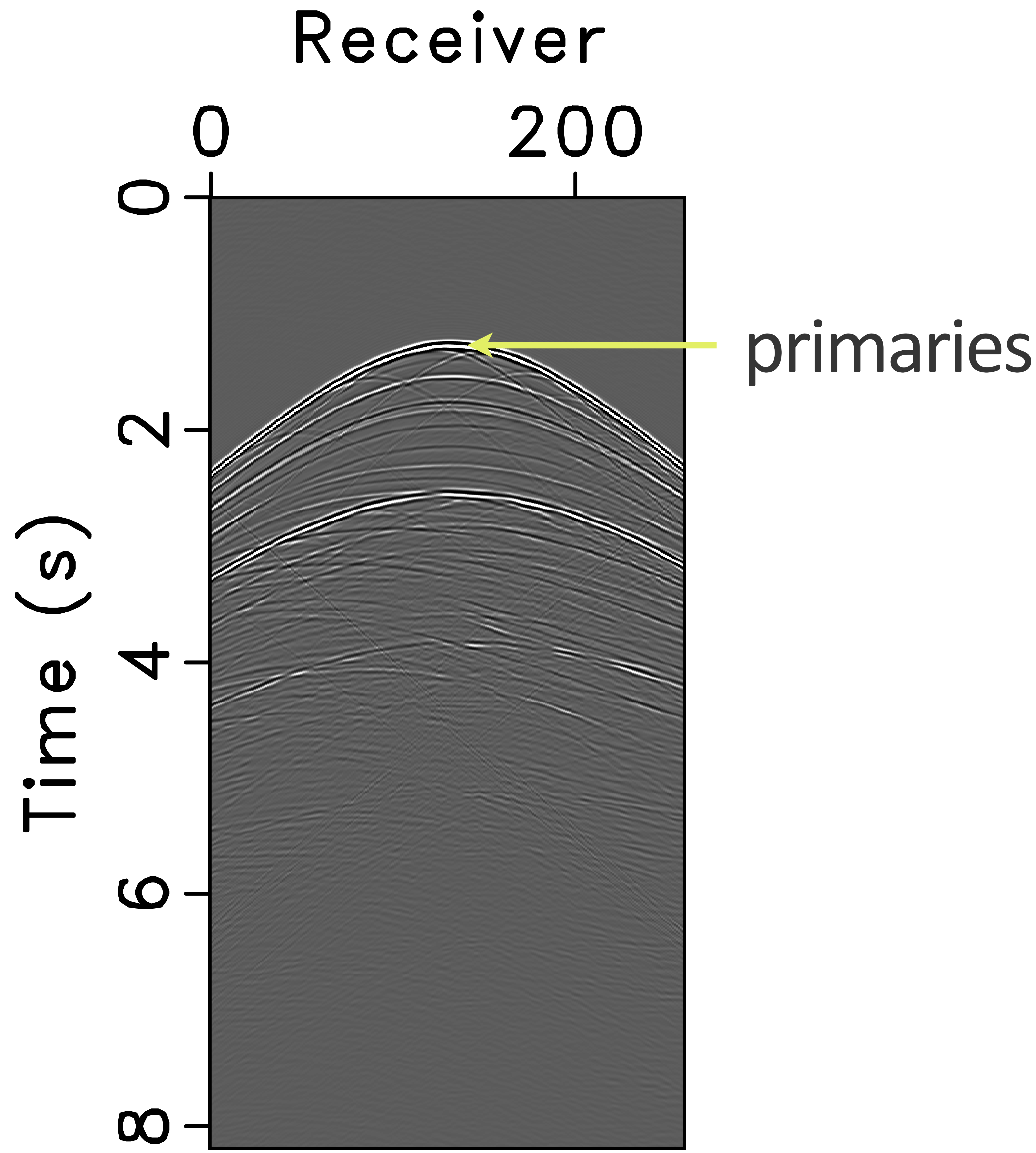


True perturbations

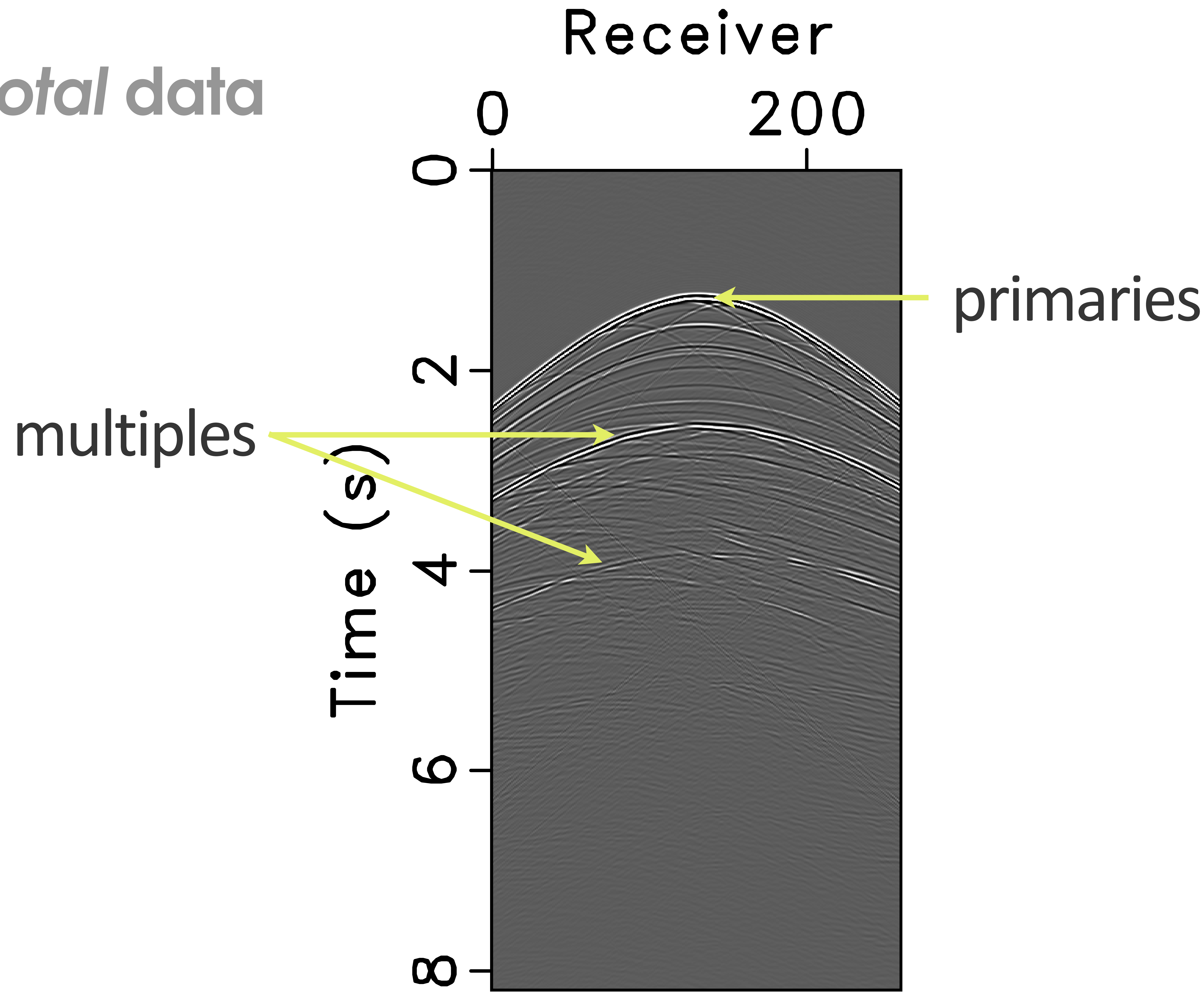
A shot-gather of *total* data



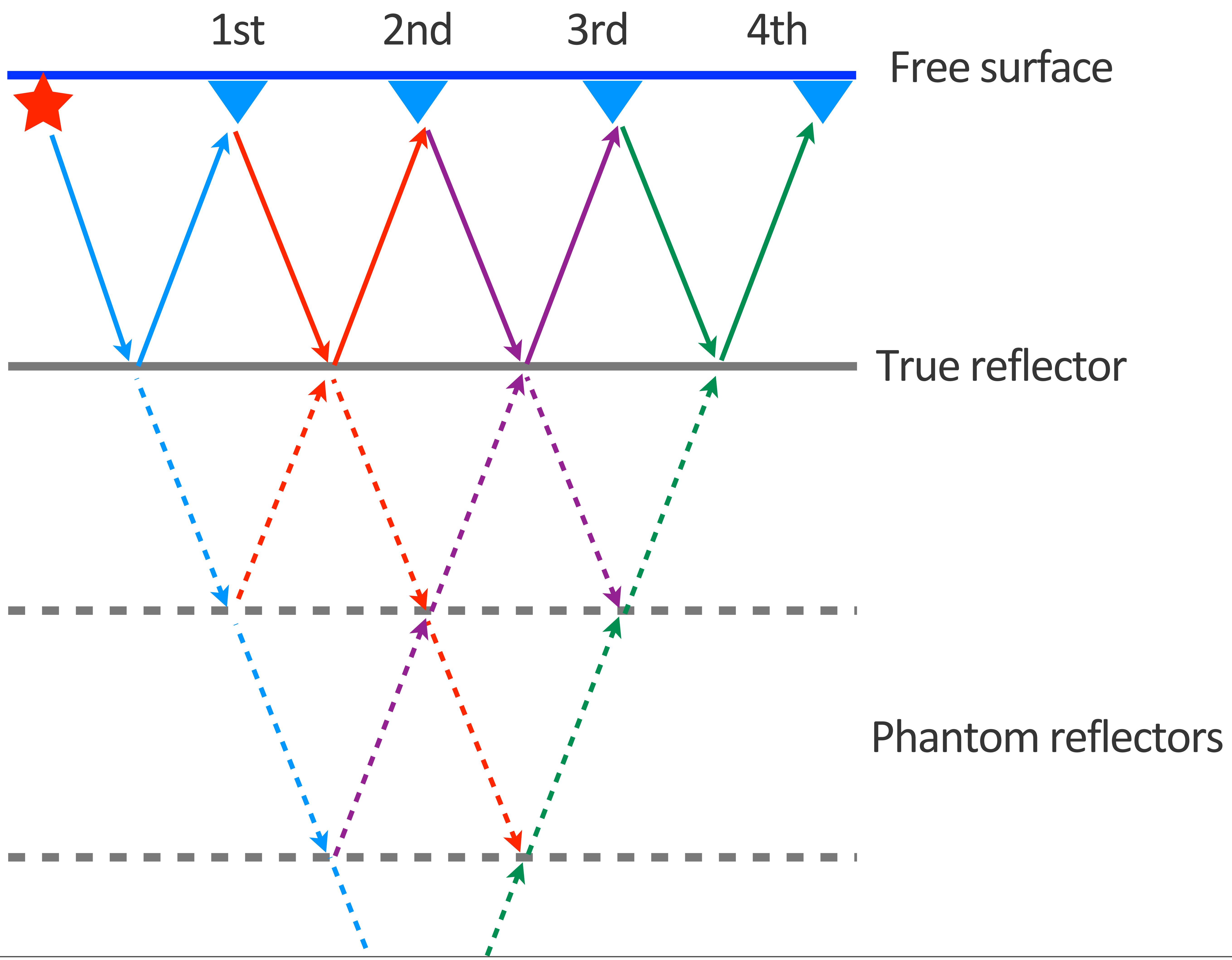
A shot-gather of *total* data

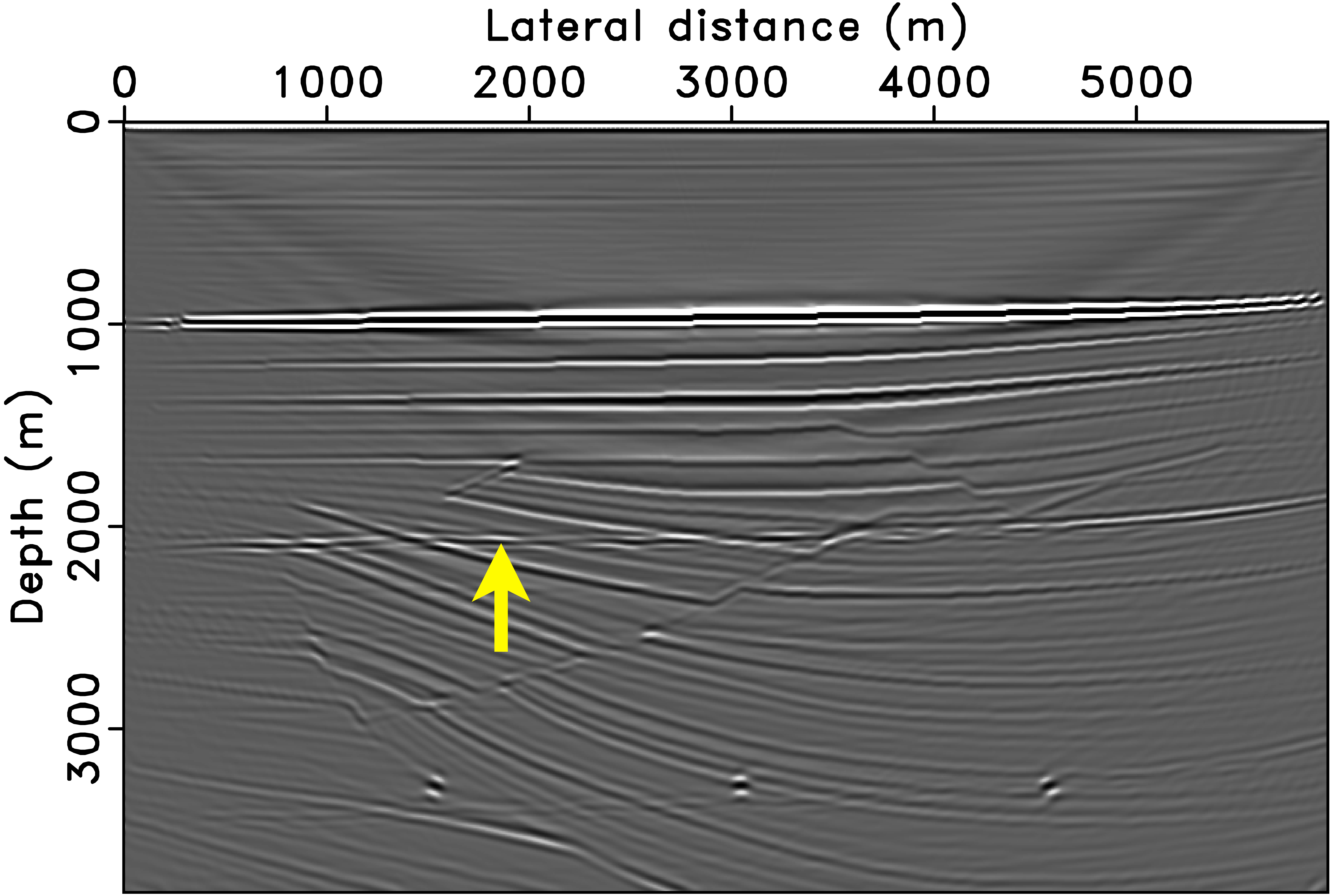


A shot-gather of *total* data



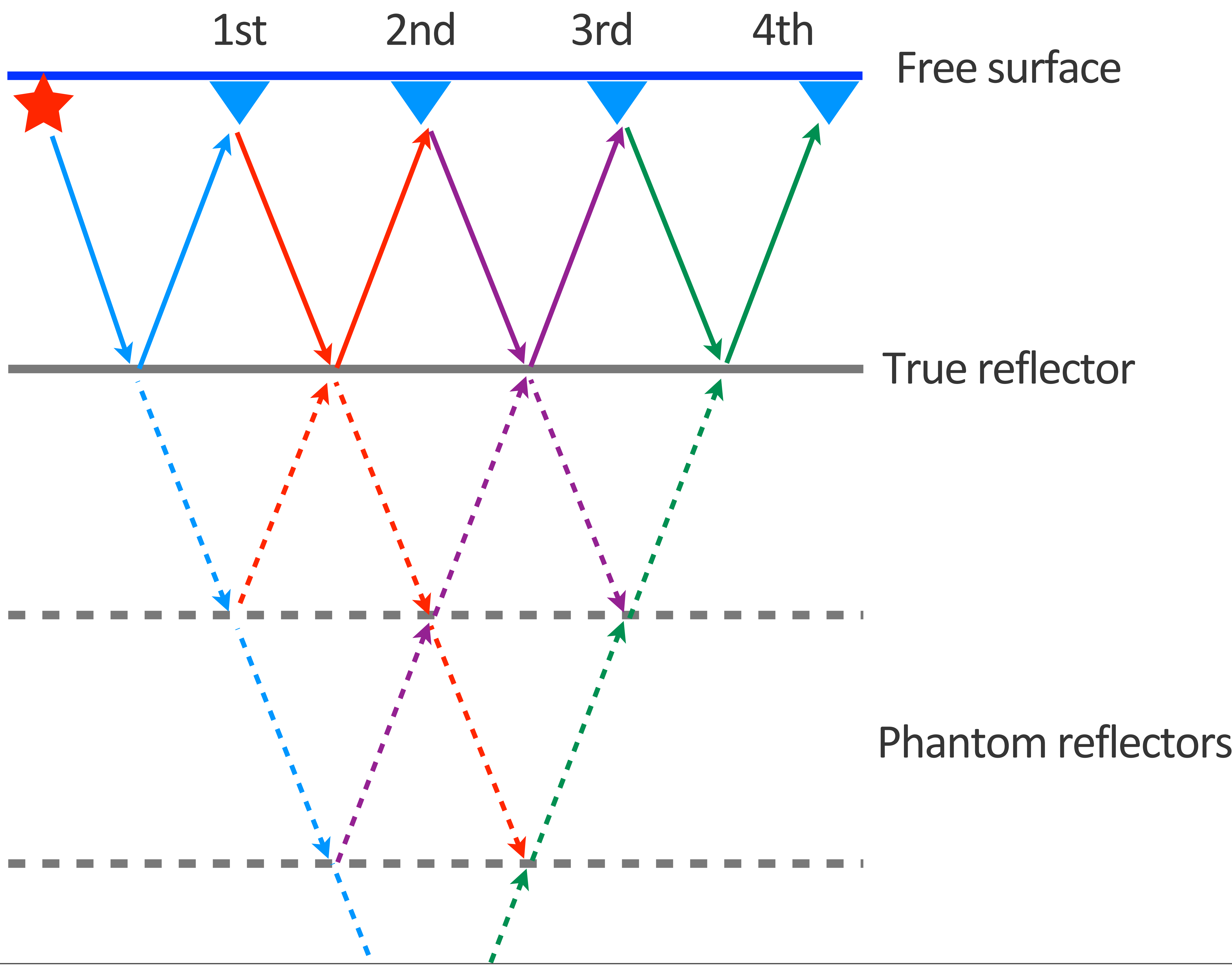
Cross-correlation imaging



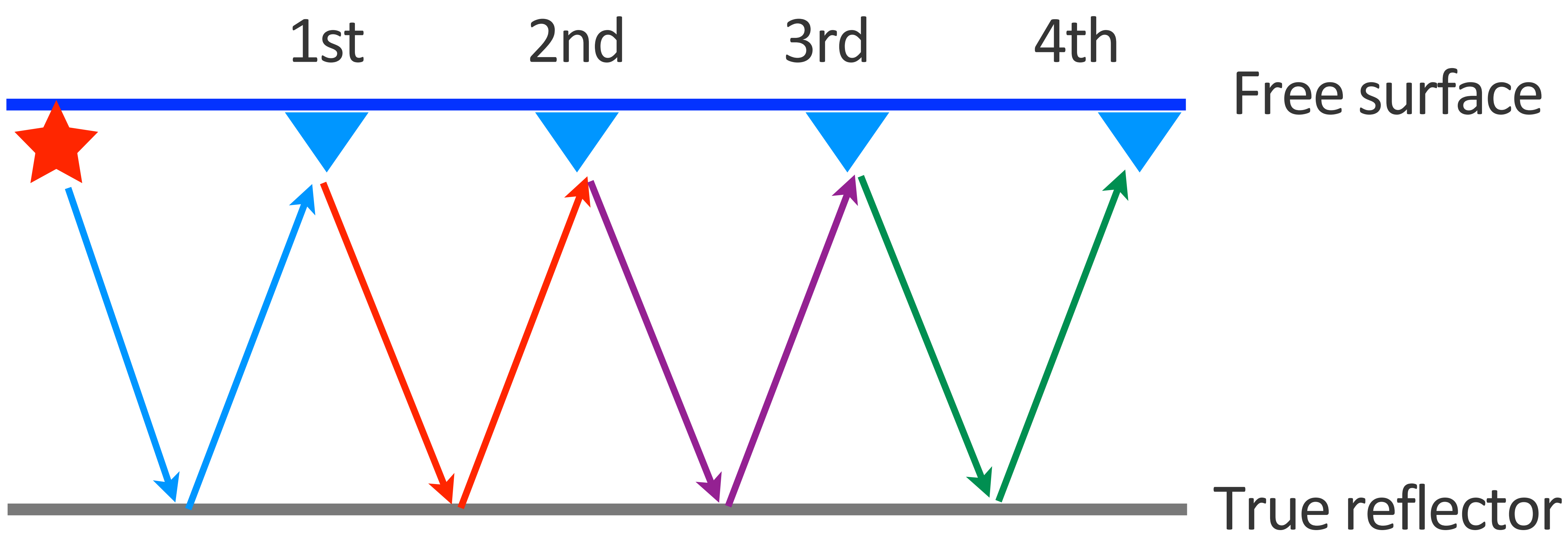


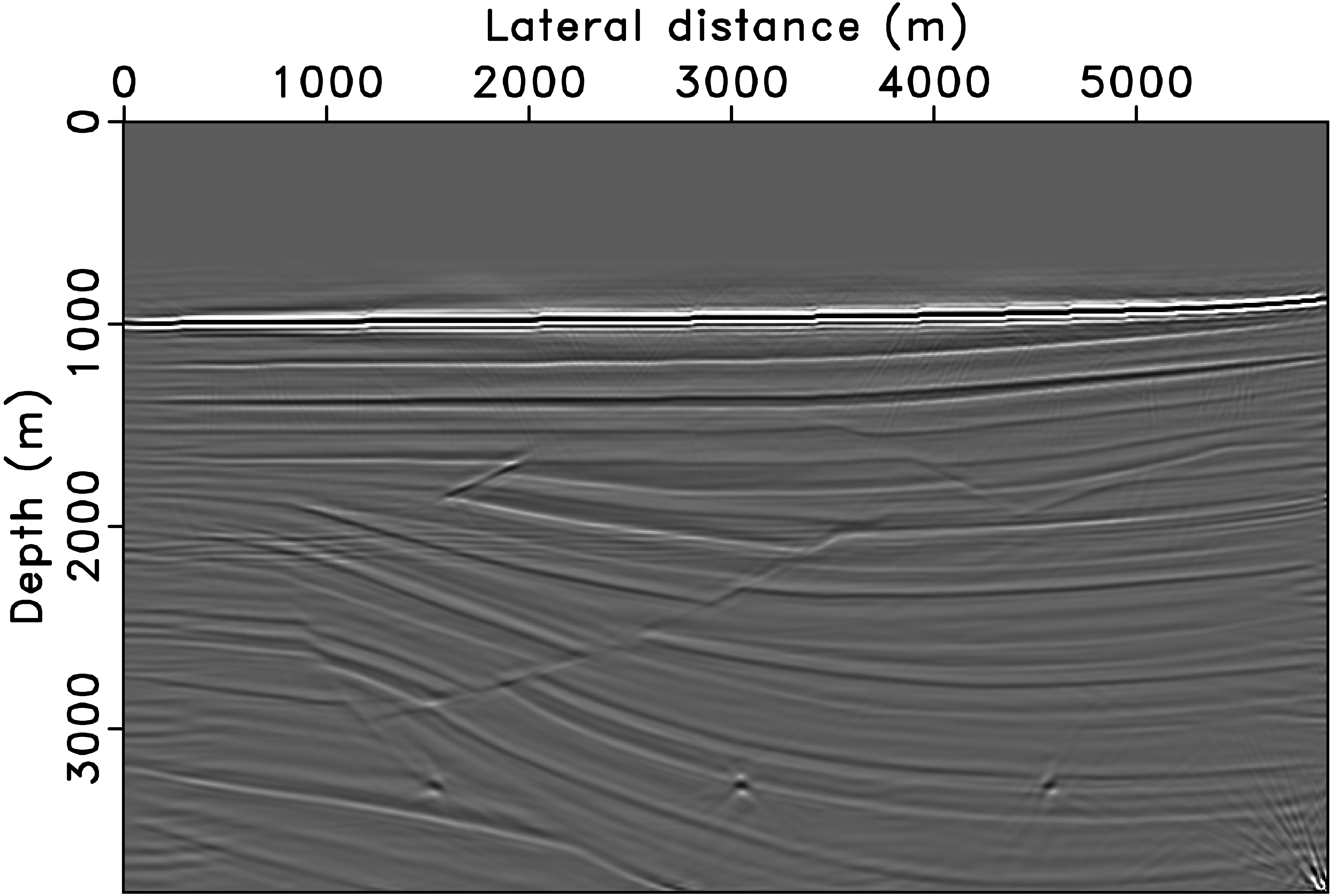
RTM image, with **total downgoing** data as areal source

Cross-correlation imaging

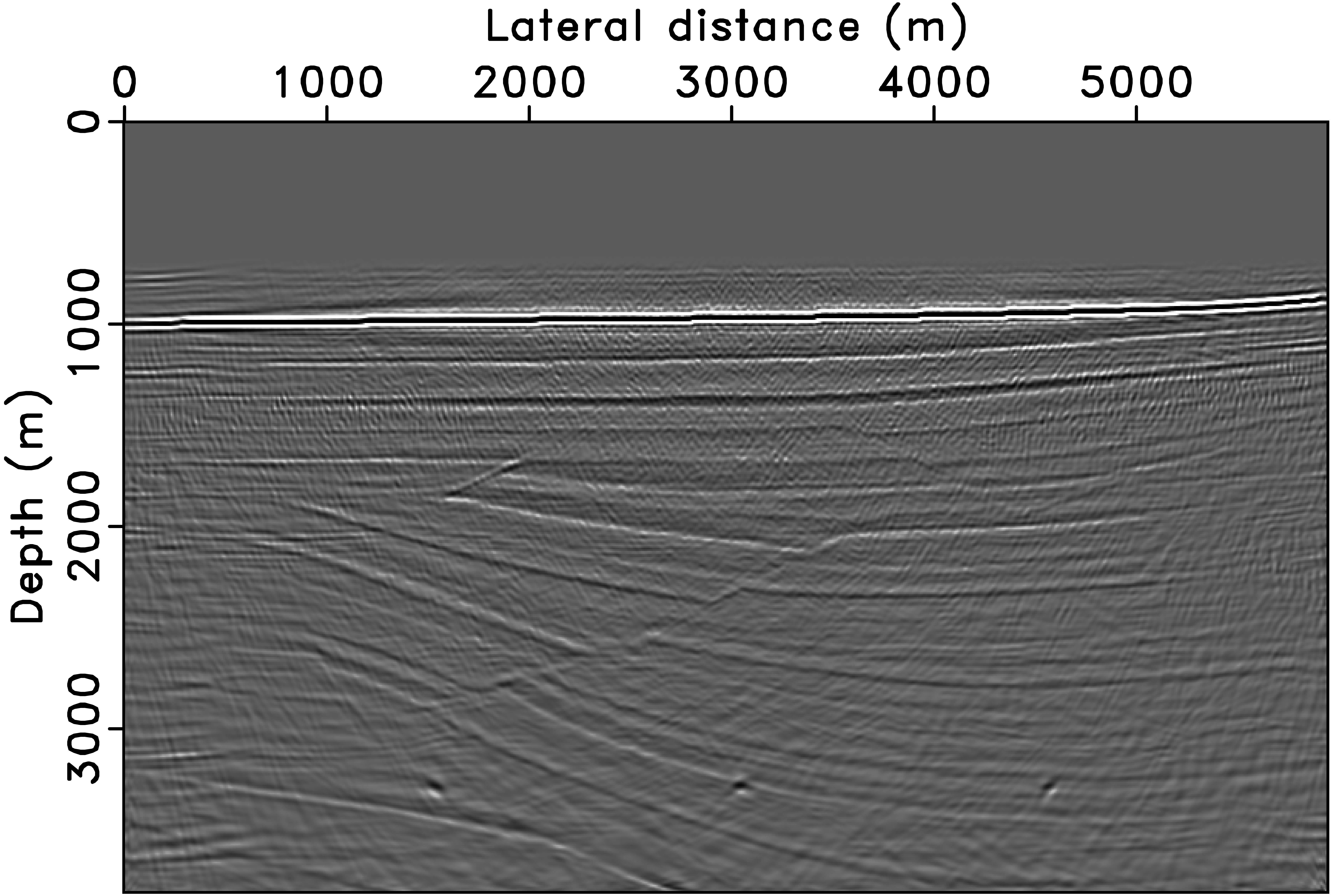


Inversion





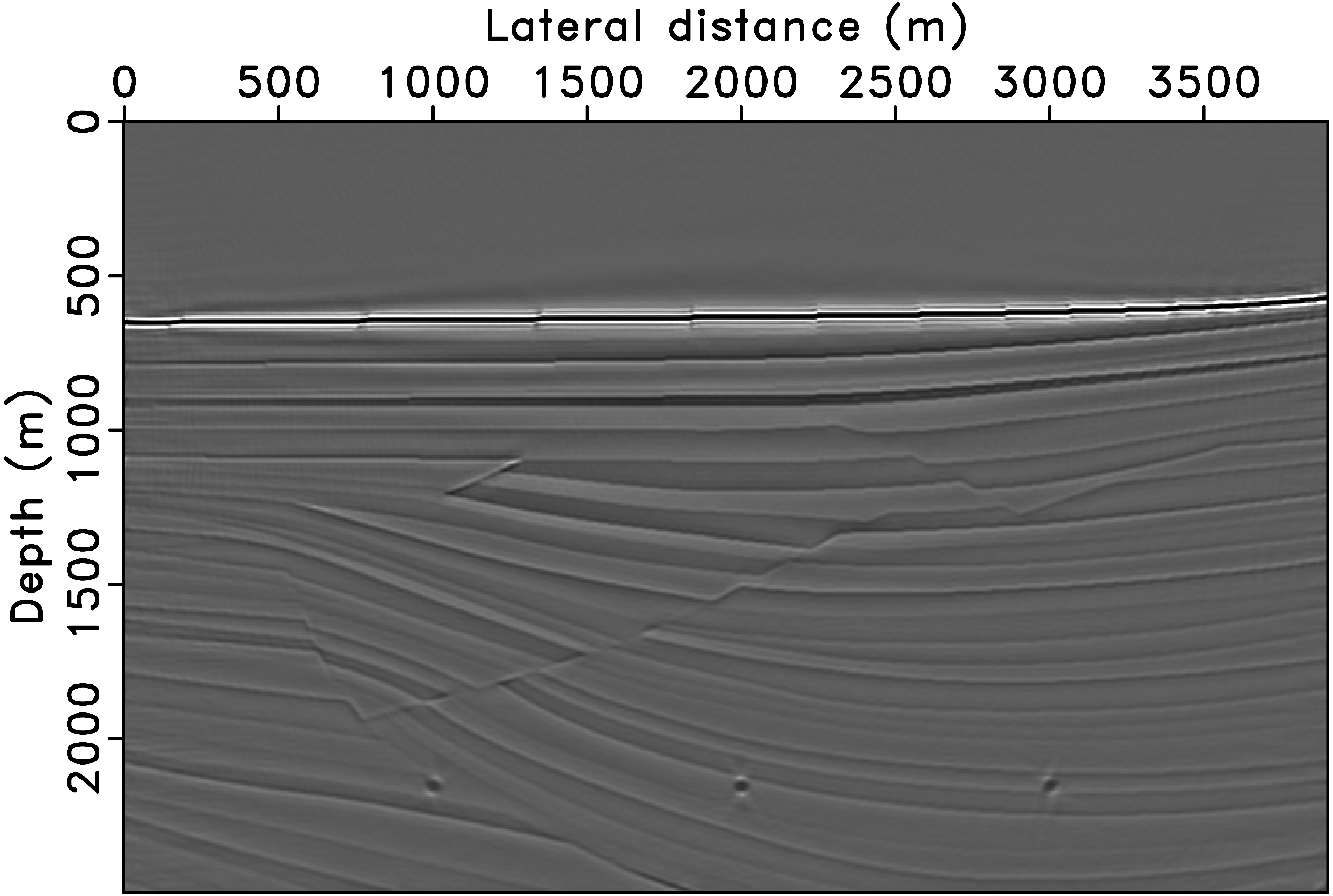
Fast inversion w. sparsity promotion, simulation cost ~**1 RTM** of all data



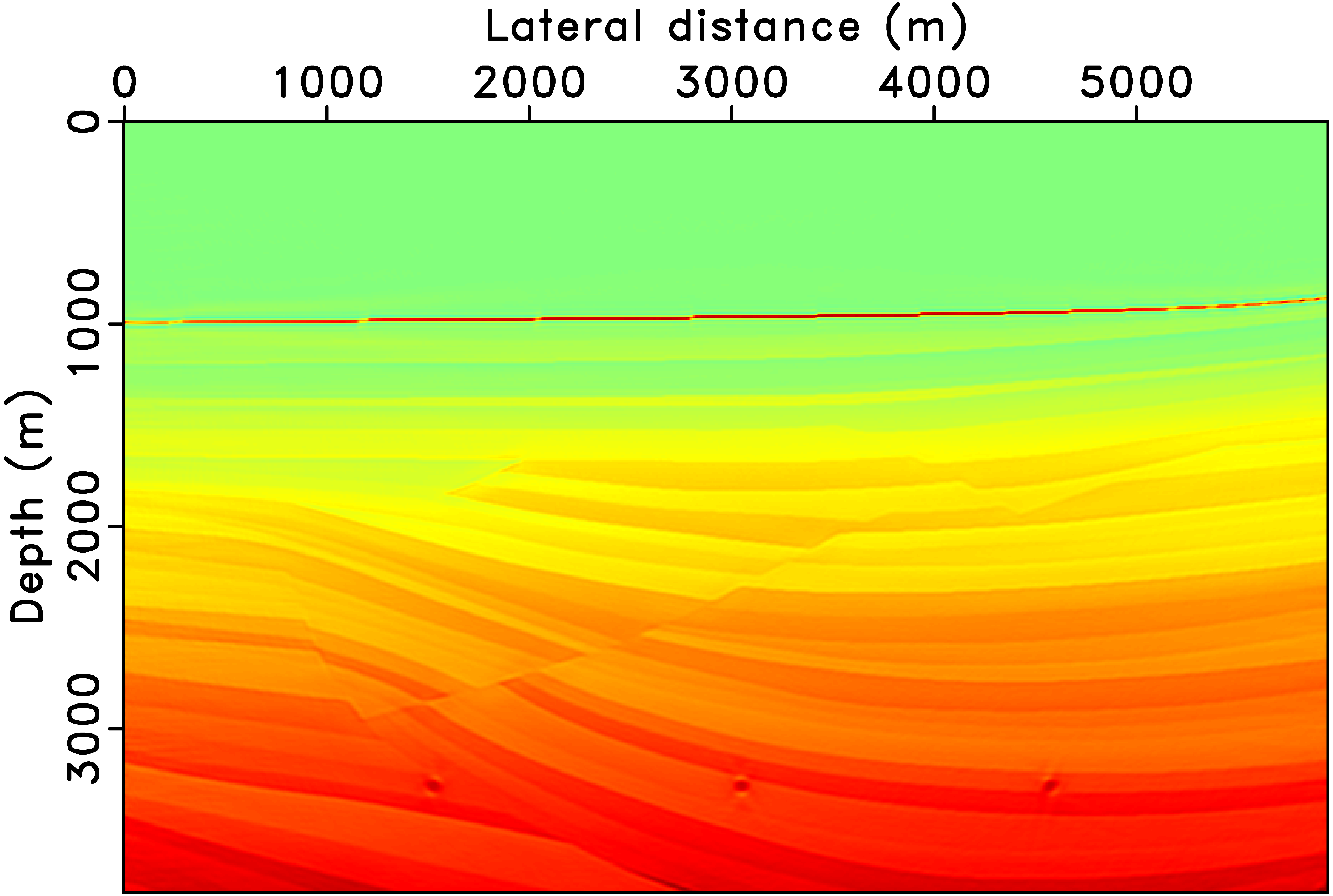
same as the previous image but **without** rerandomization

Further questions

- Where are the remnant artifacts from?
  - discrepancy between modelling and inversion engines
  - inaccurate multiple prediction because of finite aperture
- What happens if there are errors in the background velocity?
- What can we do for an unknown source wavelet?



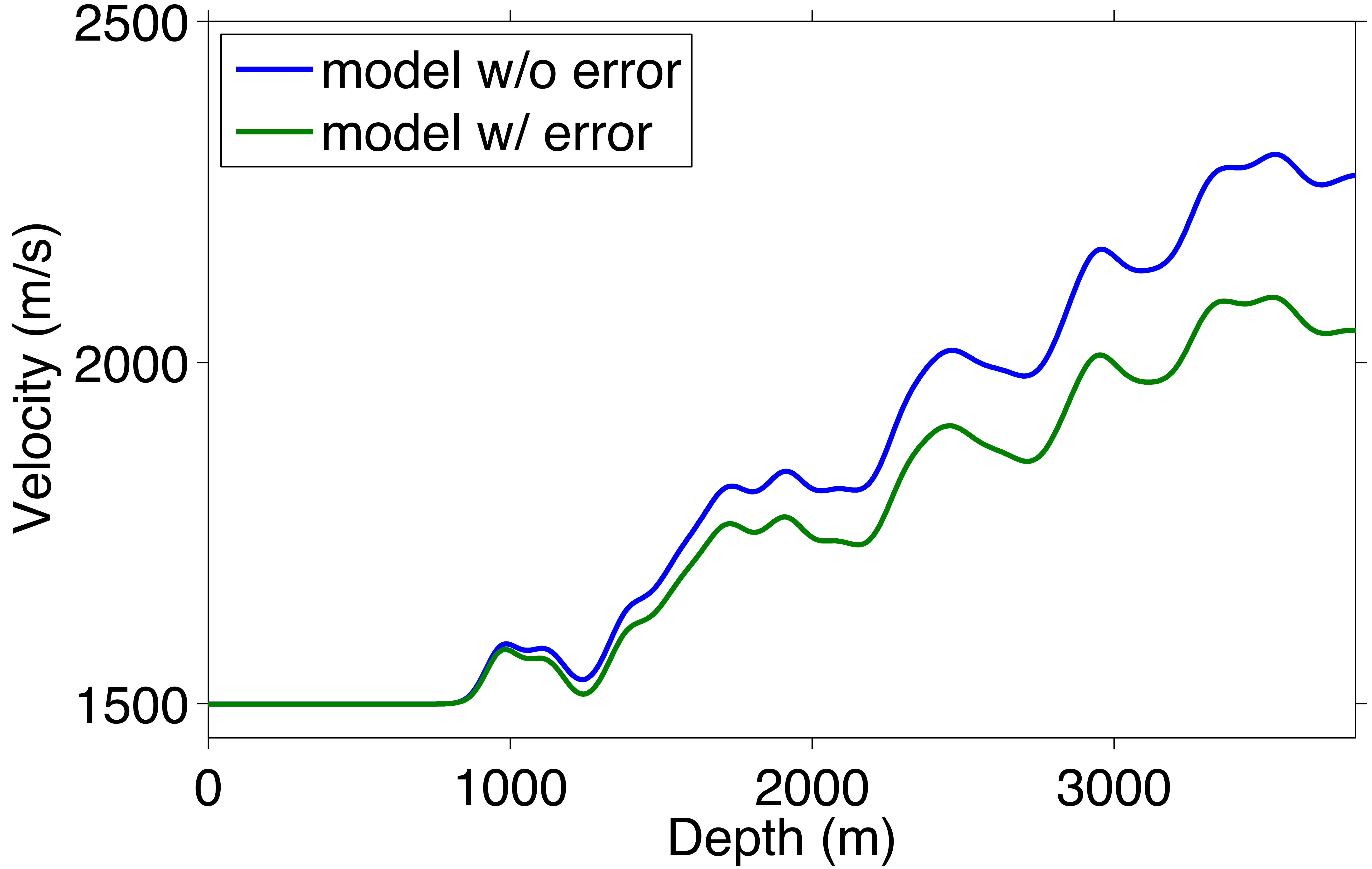
Fast inversion -- the same modelling and inversion engine

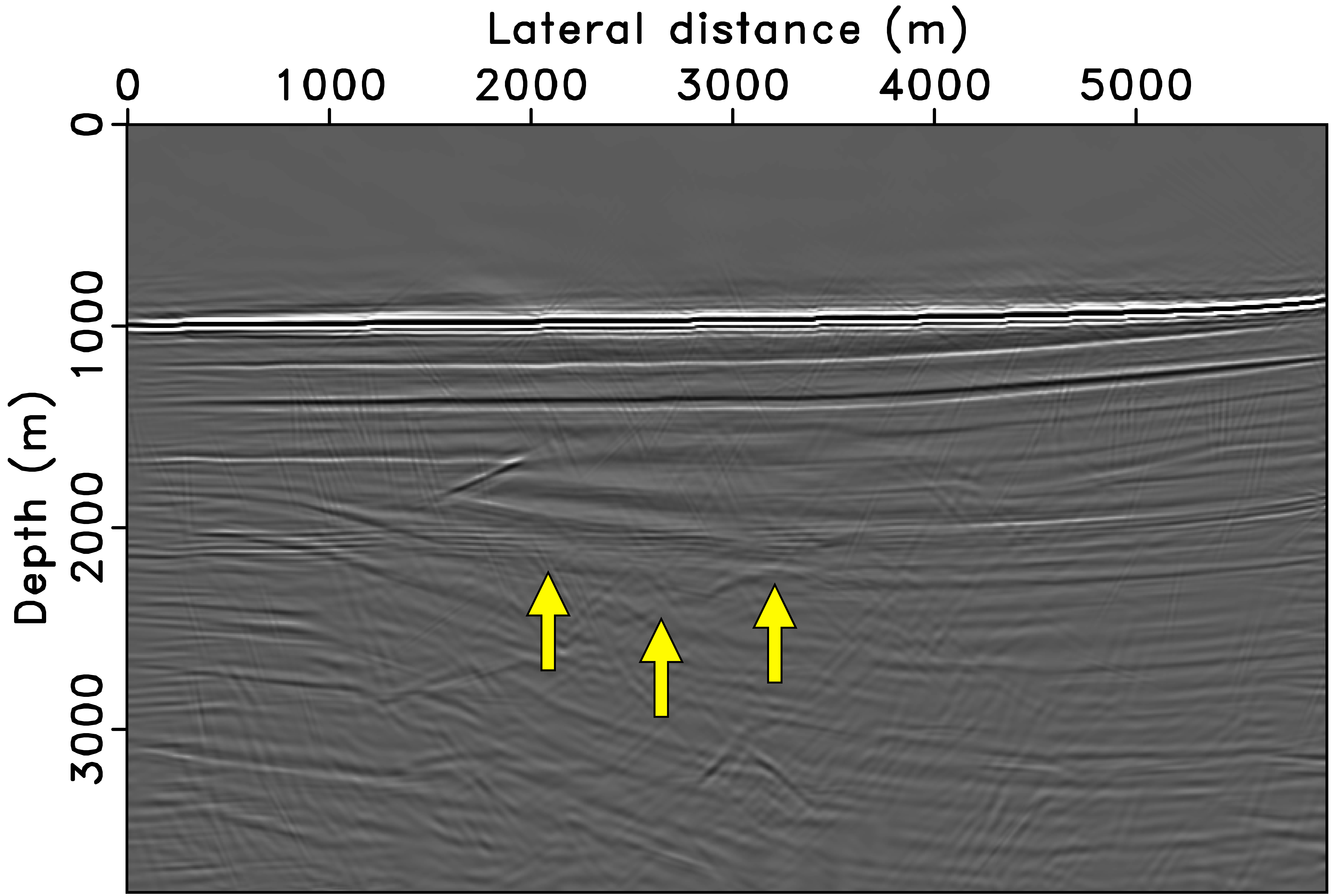


True-amplitude imaging: adding perturbations back to the background model

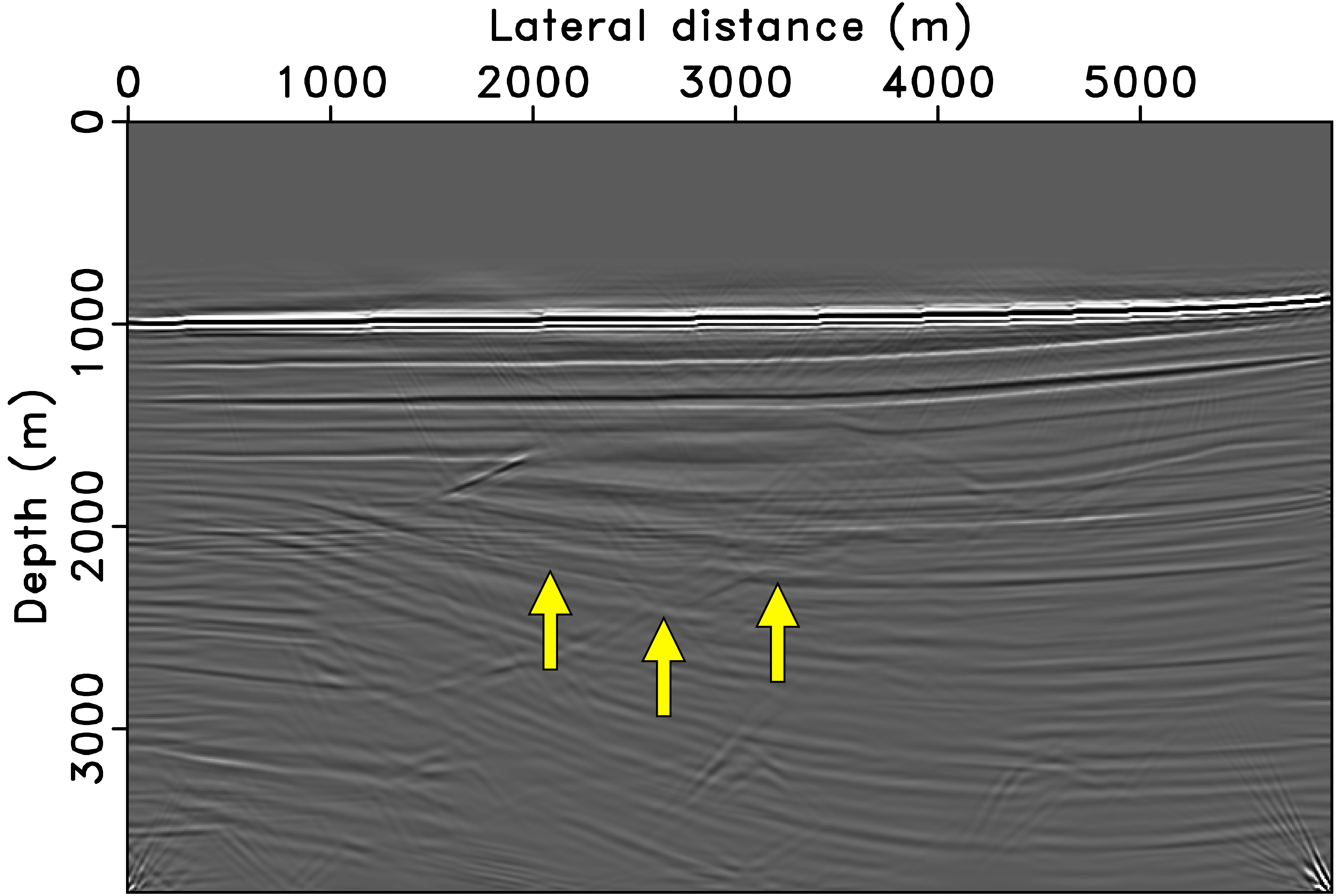
caveat: we assume we know the **true** source wavelet here

Improve robustness to velocity errors by curvelet-domain sparsity-promoting





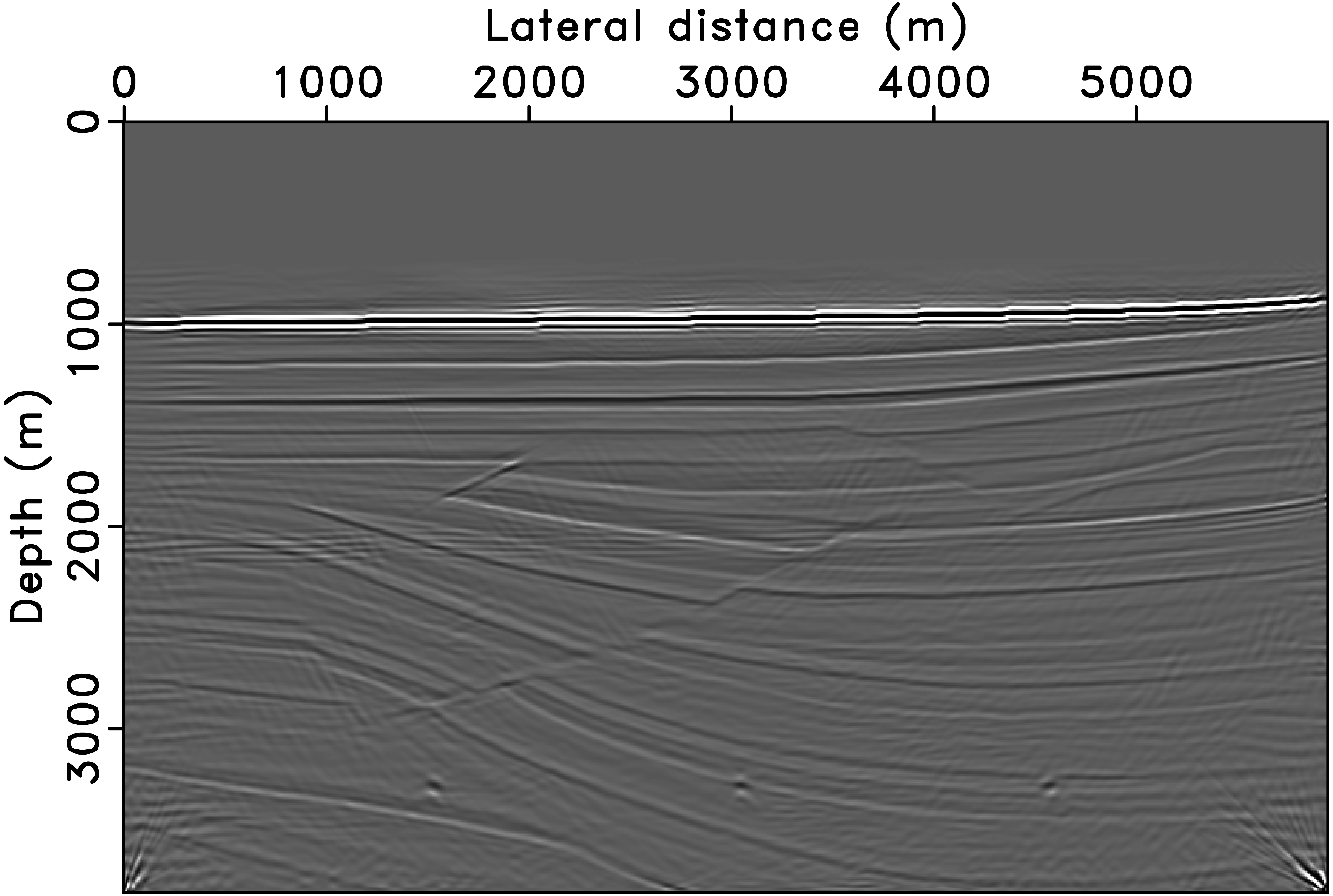
Fast inversion without curvelet-domain sparsity promotion



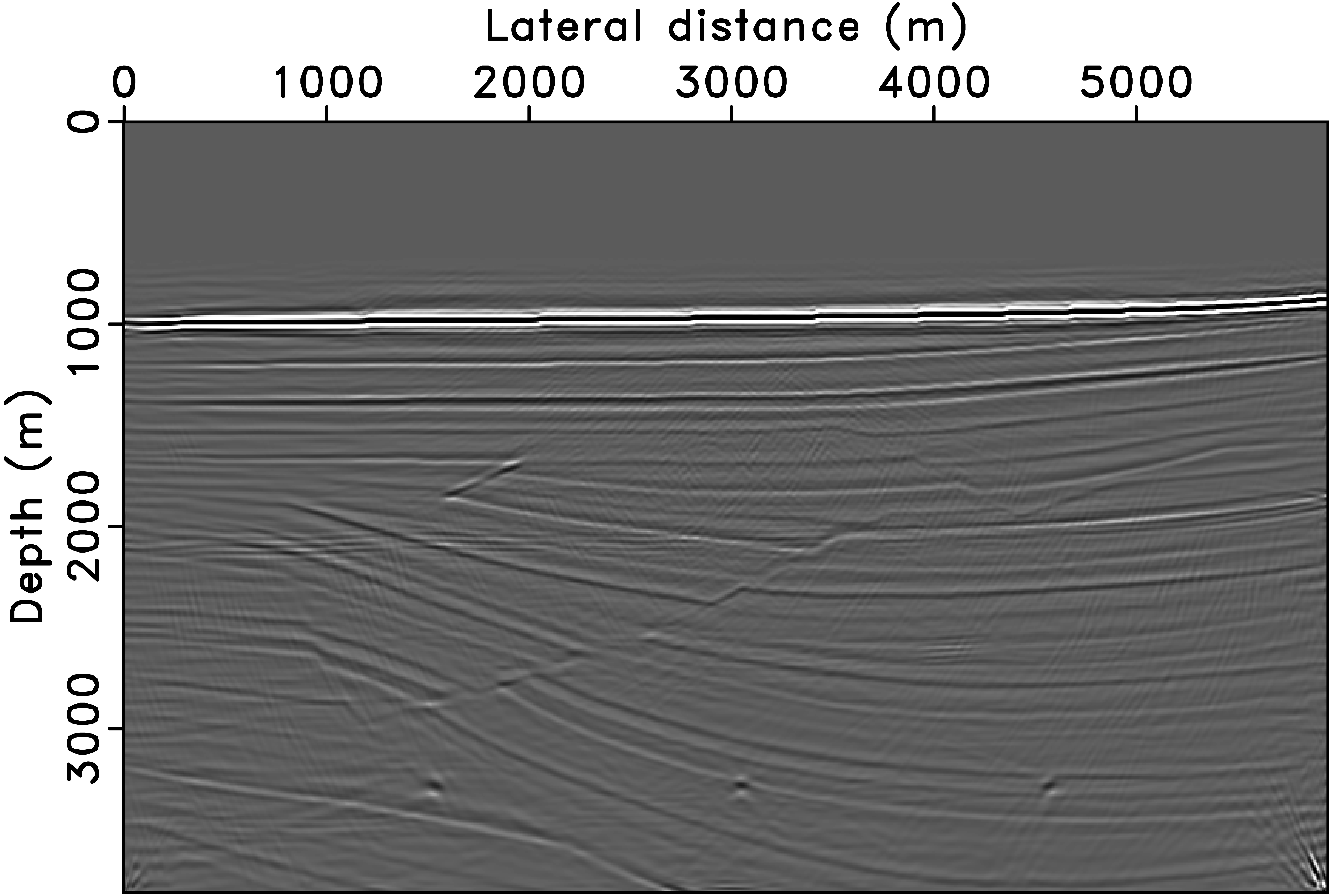
Fast inversion with curvelet-domain sparsity promotion

Unknown source wavelet

- on-the-fly source estimation by variable projection
- imaging of multiples alone
  - ***caveat***: separating primaries/multiples by for example SRME also requires knowledge of the source wavelet



Fast inversion w. source **estimation**



Fast inversion of multiples **alone**

Conclusions

Multiples can be useful signal in seismic imaging.

- increased illumination coverage
- cross-correlation (**RTM**) not enough to image multiples
- acausal artifacts can be suppressed by **inversion**

Primaries and multiples can be jointly imaged.

- based on the SRME relation
- in practice made possible by on-the-fly source estimation

Inversion can be carried out efficiently by

- eliminating dense matrix products in multiple prediction.
- reducing simulation cost using compressive imaging.

To be continued...

Application to 2D field data, Wed 11:55 AM

Imaging the Nelson data set using surface-related multiples

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Future work

extension to general interferometric imaging

- in essence, the method combines
  - multi-dimensional deconvolutions of wavefields
    - in this case upgoing w.r.t. the downgoing
  - linearized imaging procedure in an efficient way

Acknowledgements

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SINBAD

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Pseudo code

Input:

total upgoing wavefield $\mathbf{P}_i$, point-source wavefield $\mathbf{S}_i$,

background velocity model $\mathbf{m}_0$, tolerance $\sigma = 0$, iteration limit k_{max}

Initialization:

$k \leftarrow 0, l \leftarrow 0, \mathbf{x}_l \leftarrow \mathbf{0}$

while $k < k_{max}$ **do**

$\Omega_l, \mathbf{E}_l \leftarrow$ new independent draw, $\underline{\mathbf{P}}_i = \mathbf{P}_i \mathbf{E}_l,\ \underline{\mathbf{S}}_i = \mathbf{S}_i \mathbf{E}_l,\ \underline{\mathbf{p}}_i = \mathrm{vec}(\underline{\mathbf{P}}_i)$

$\tau_l \leftarrow$ determine from τ_{l-1} and σ by root finding on the Pareto curve

$$\mathbf{x}_l \leftarrow \begin{cases} \mathrm{argmin}_{\mathbf{x}} \sum_{i \in \Omega_l} \|\underline{\mathbf{p}}_i - \nabla \mathbf{F}_i[\mathbf{m}_0, \underline{\mathbf{S}}_i - \underline{\mathbf{P}}_i]\mathbf{C}^{\mathrm{H}}\mathbf{x}\|_2^2 \\ \text{subject to } \|\mathbf{x}\|_1 \leq \tau_l \end{cases}$$ //warm start with

$\mathbf{x}_{l-1}$, solved in k_l iterations

$k \leftarrow k + k_l, l \leftarrow l + 1$

end while

Output: Model perturbation estimate $\delta \mathbf{m} = \mathbf{C}^{\mathrm{H}}\mathbf{x}$

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