Single- and multi-parameter WRI — synthetic examples

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Part 1- single-parameter WRI

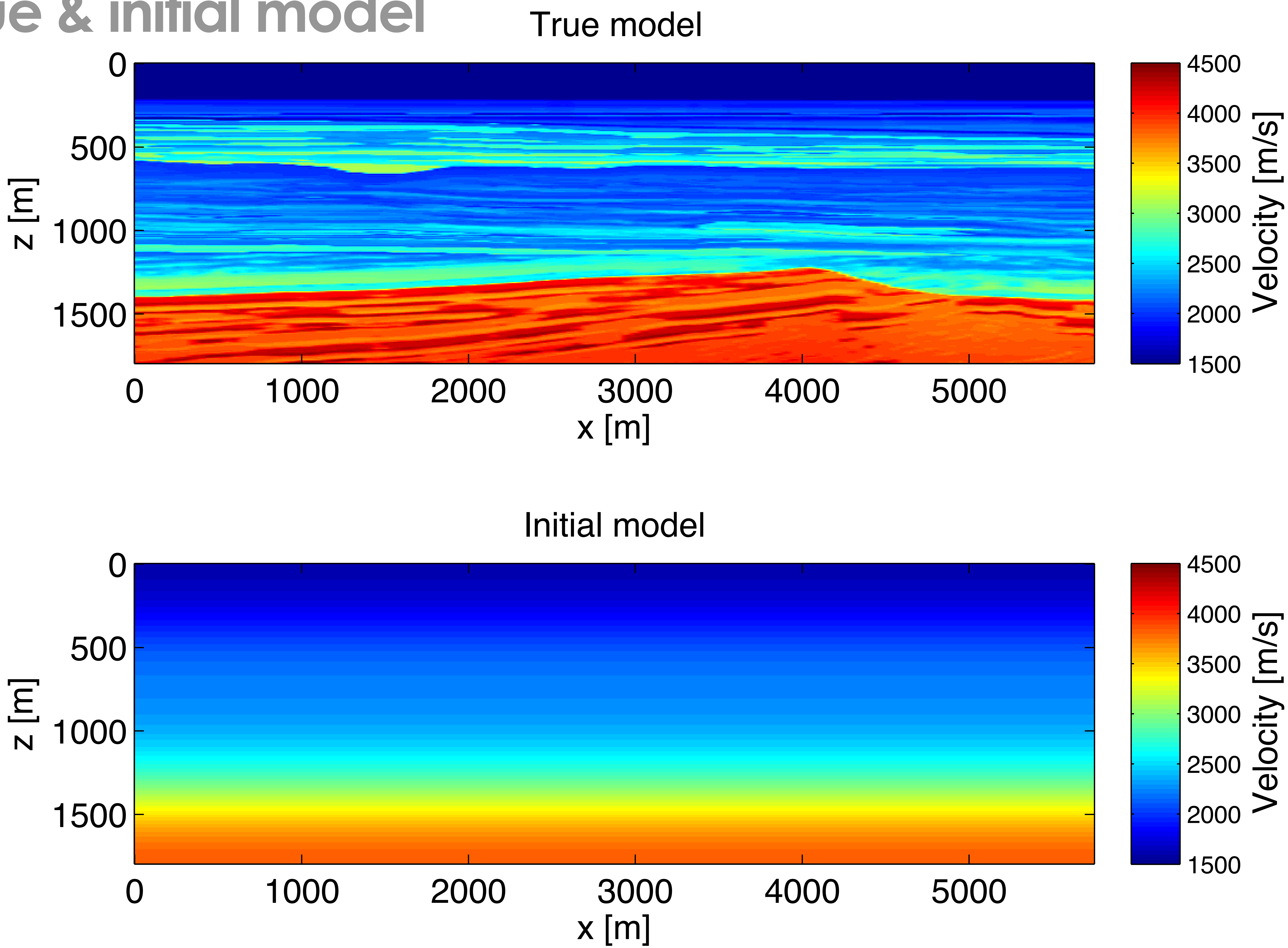
Goals

- Evaluate FWI & WRI in case of a poor start model & missing low-frequencies.
- Test whether WRI still works when (“mildly”) wrong physics is used.

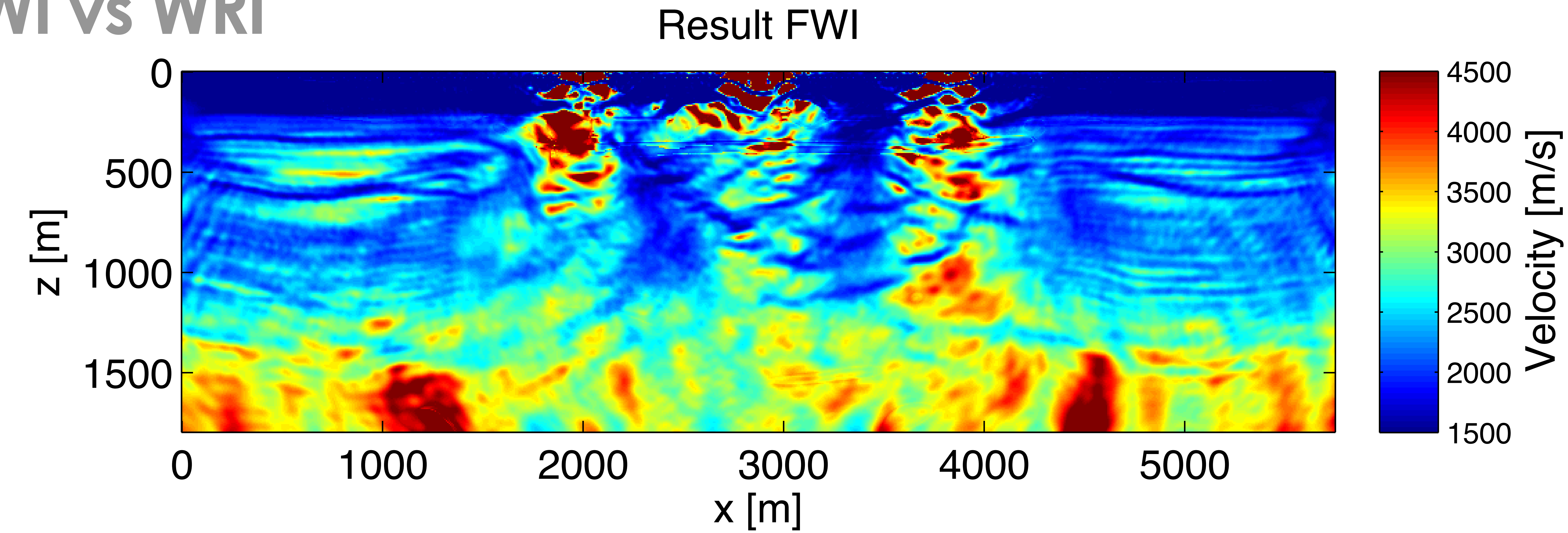
Example – BG Compass model

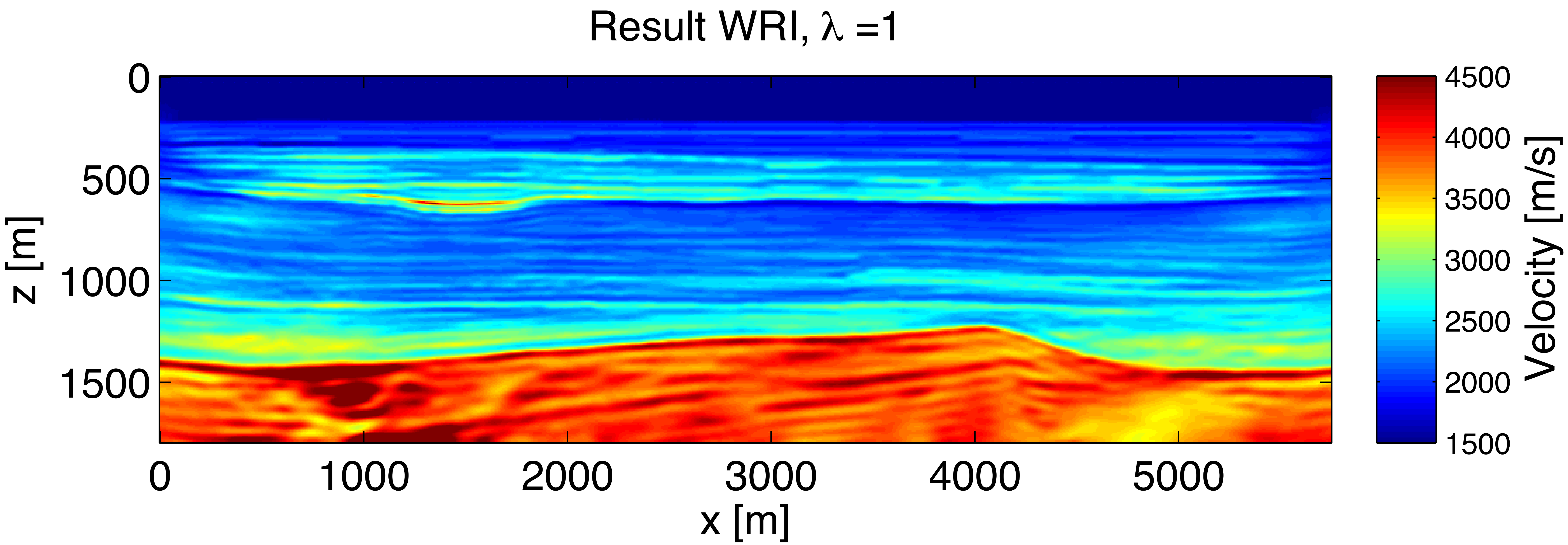
- Low frequencies missing, 24 frequency batches (15 iterations each) {5 6} ,{6 7},... ,{28 29} Hertz. Each interval contains 5 frequencies.
- 103 sources/receivers w/ 55m sample interval
- Inaccurate initial model

True & initial model

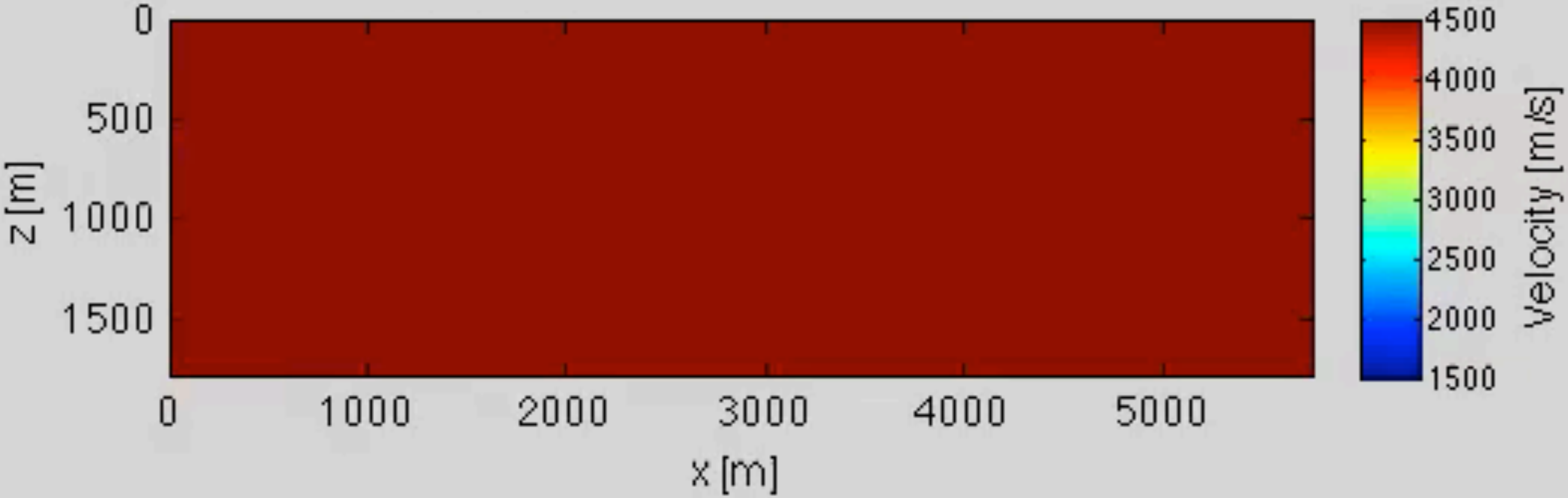


FWI vs WRI

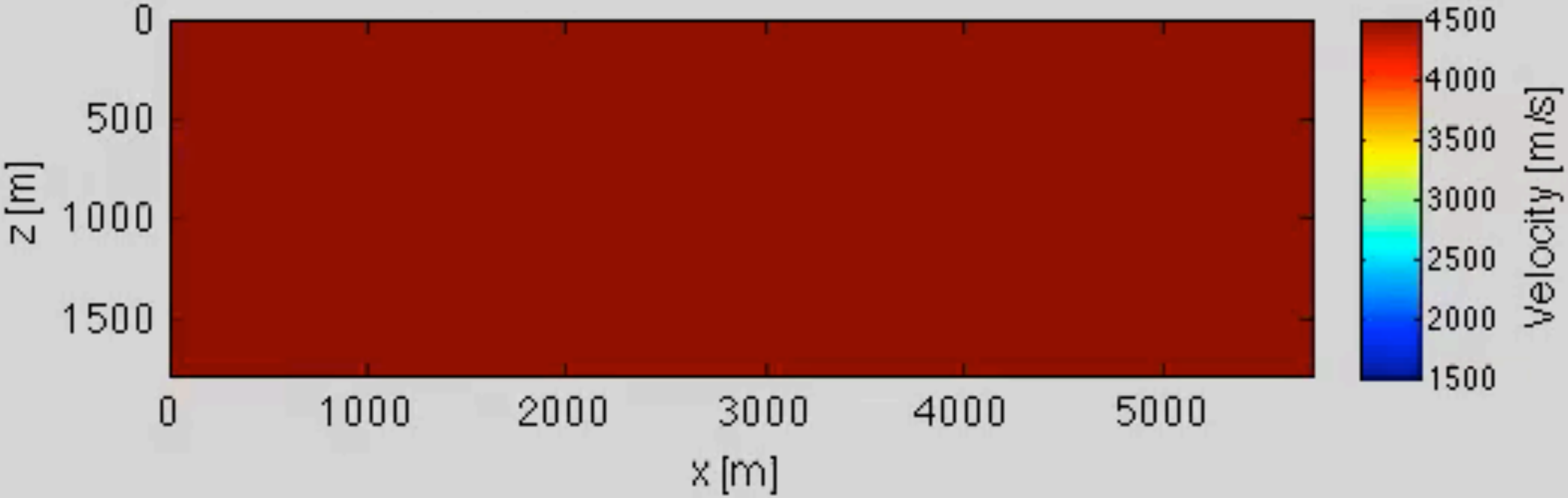

Result FWI
0
500
1000
1500
z [m]
0
1000
2000
3000
4000
5000
x [m]
4500
4000
3500
3000
2500
2000
1500
Velocity [m/s]


Result WRI, λ =1
0
500
1000
1500
z [m]
0
1000
2000
3000
4000
5000
x [m]
4500
4000
3500
3000
2500
2000
1500
Velocity [m/s]

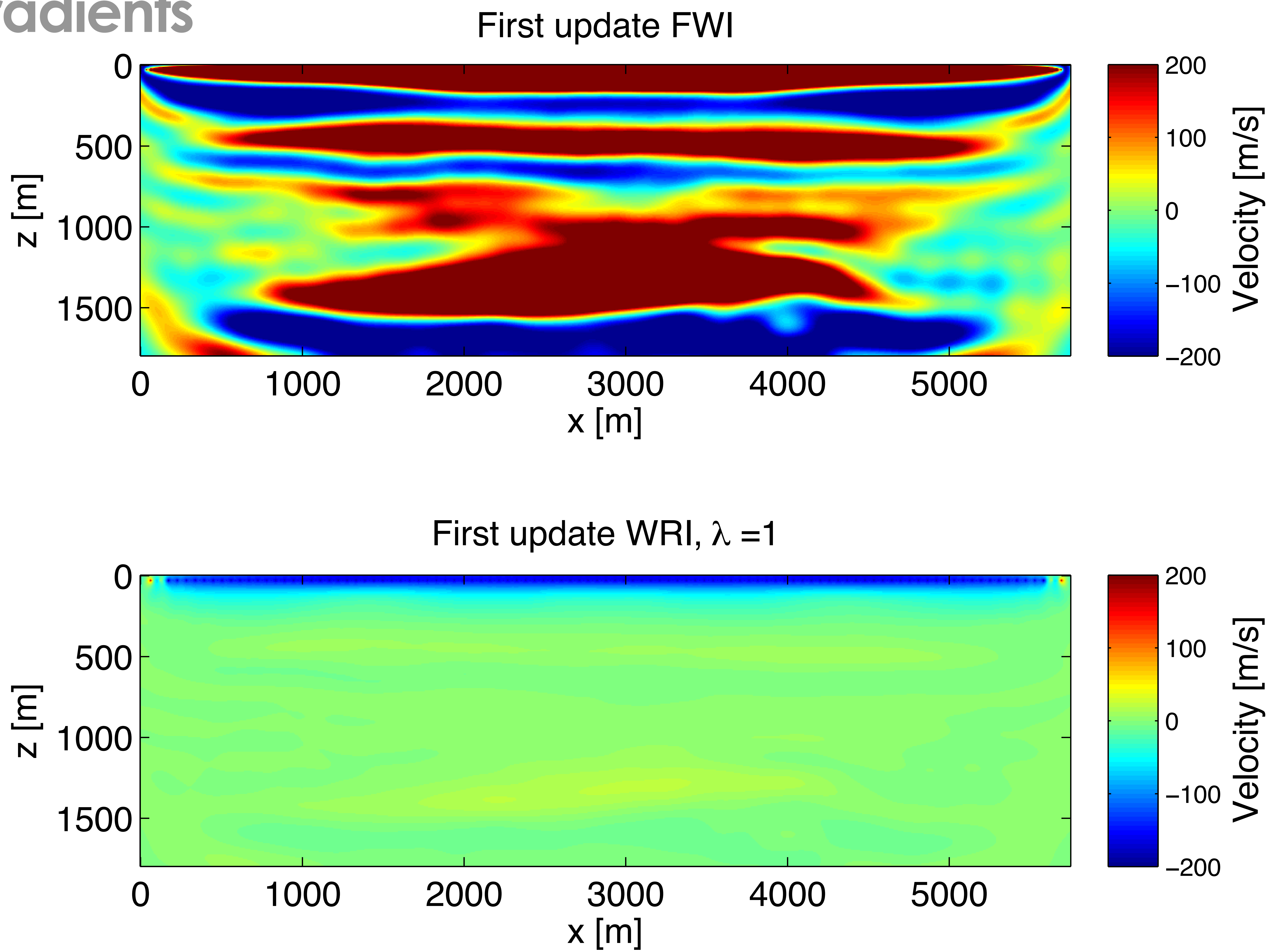
Model estimate at every iteration



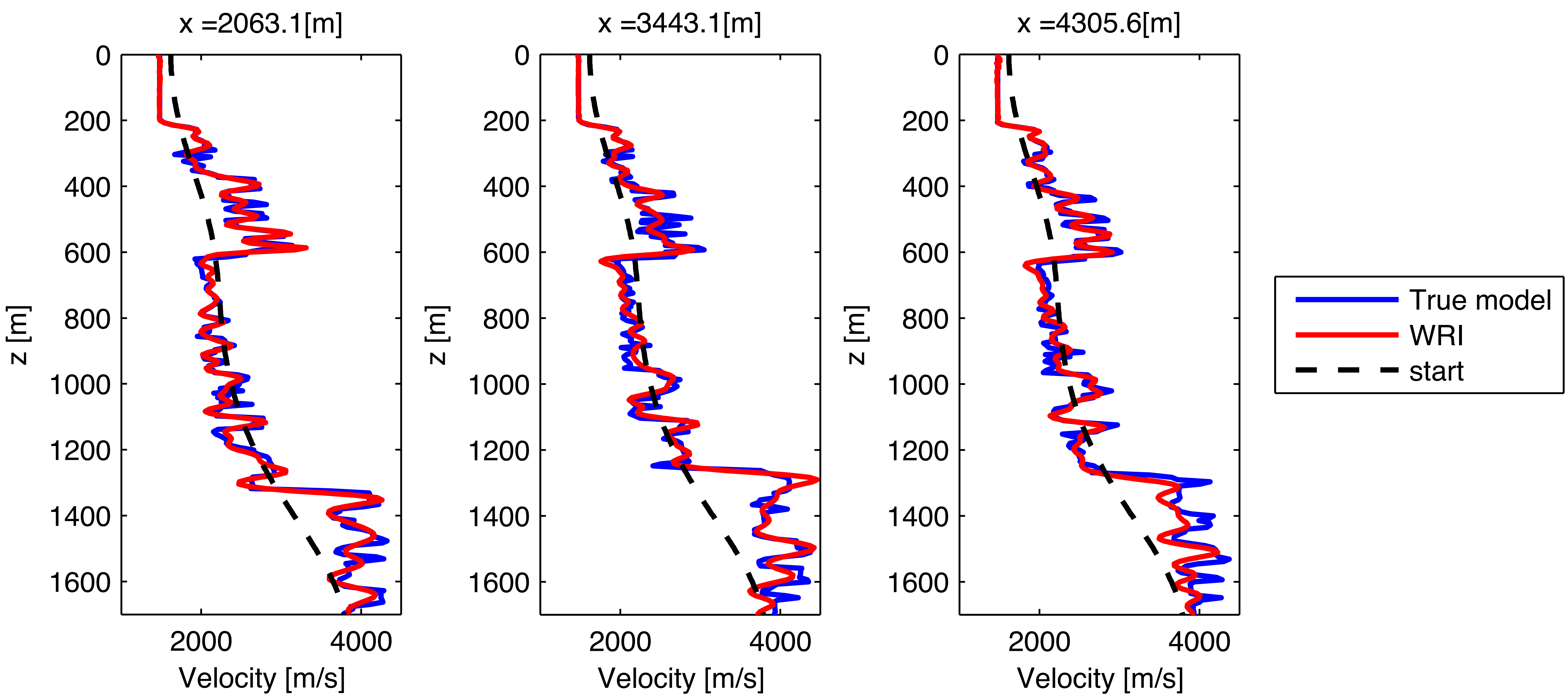
Model estimate at every iteration



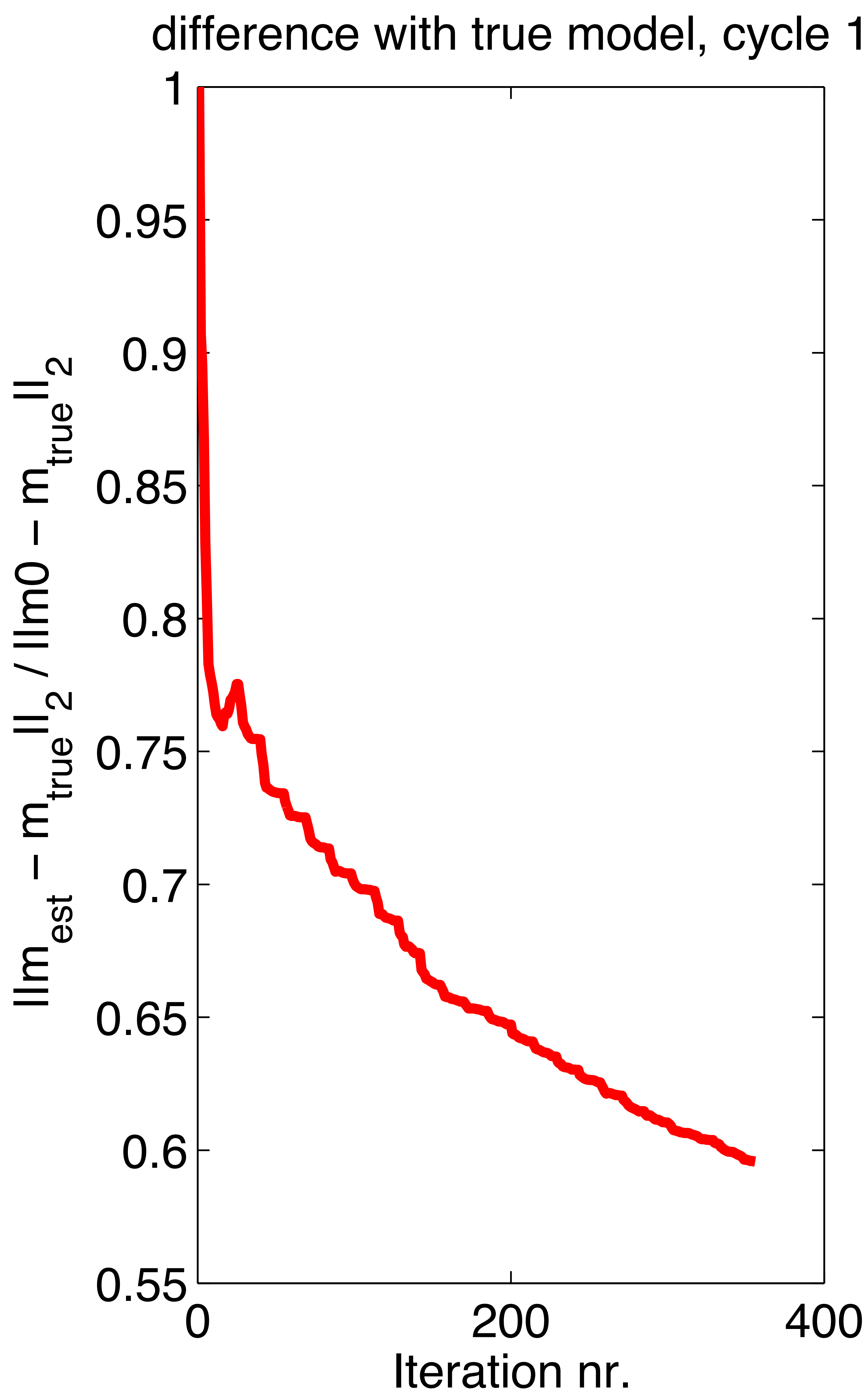
Gradients

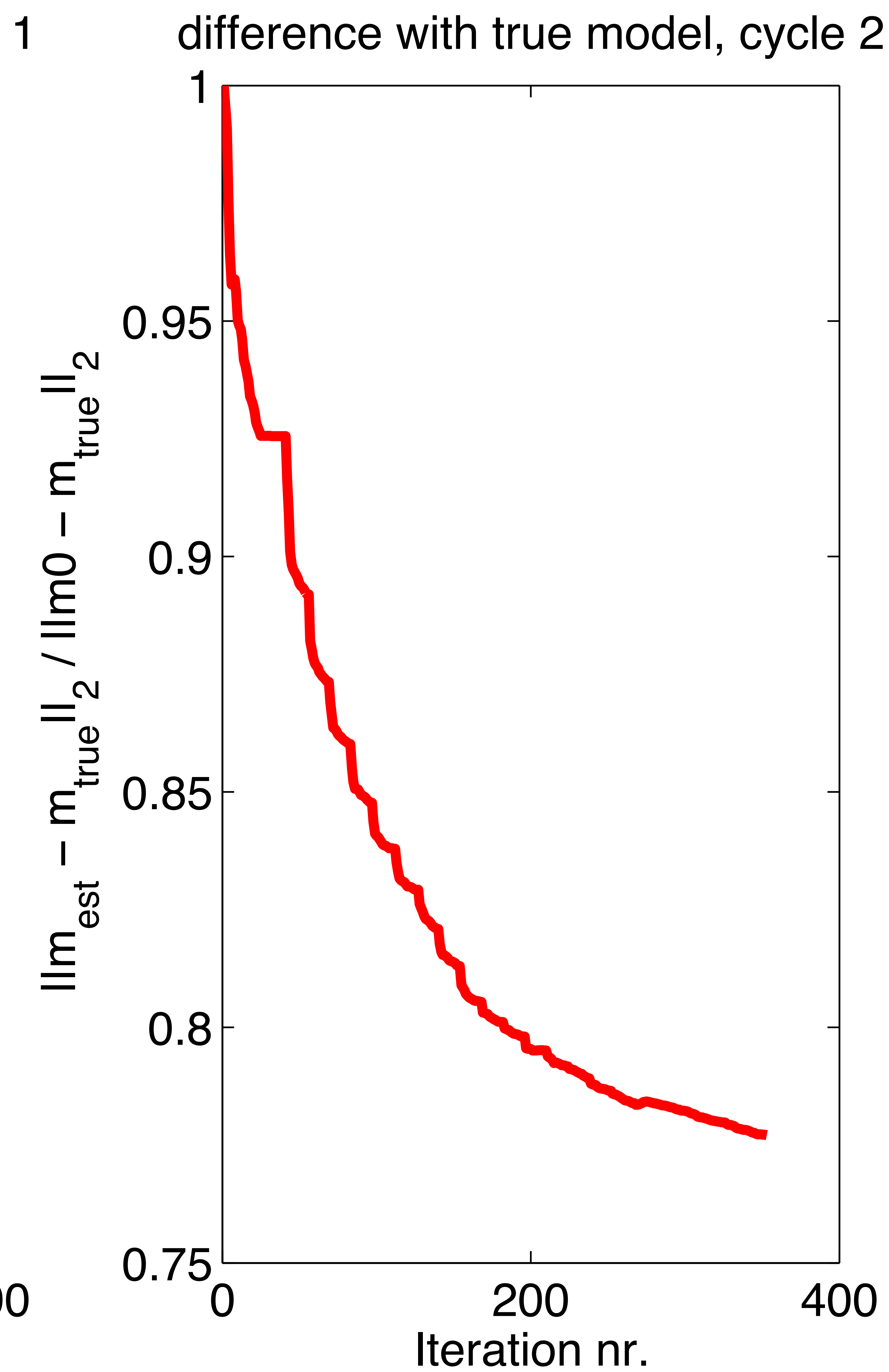

First update FWI
z [m]
x [m]
0
500
1000
1500
0
1000
2000
3000
4000
5000
Velocity [m/s]
200
100
0
−100
−200
First update WRI, λ =1
z [m]
x [m]
0
500
1000
1500
0
1000
2000
3000
4000
5000
Velocity [m/s]
200
100
0
−100
−200

Cross sections

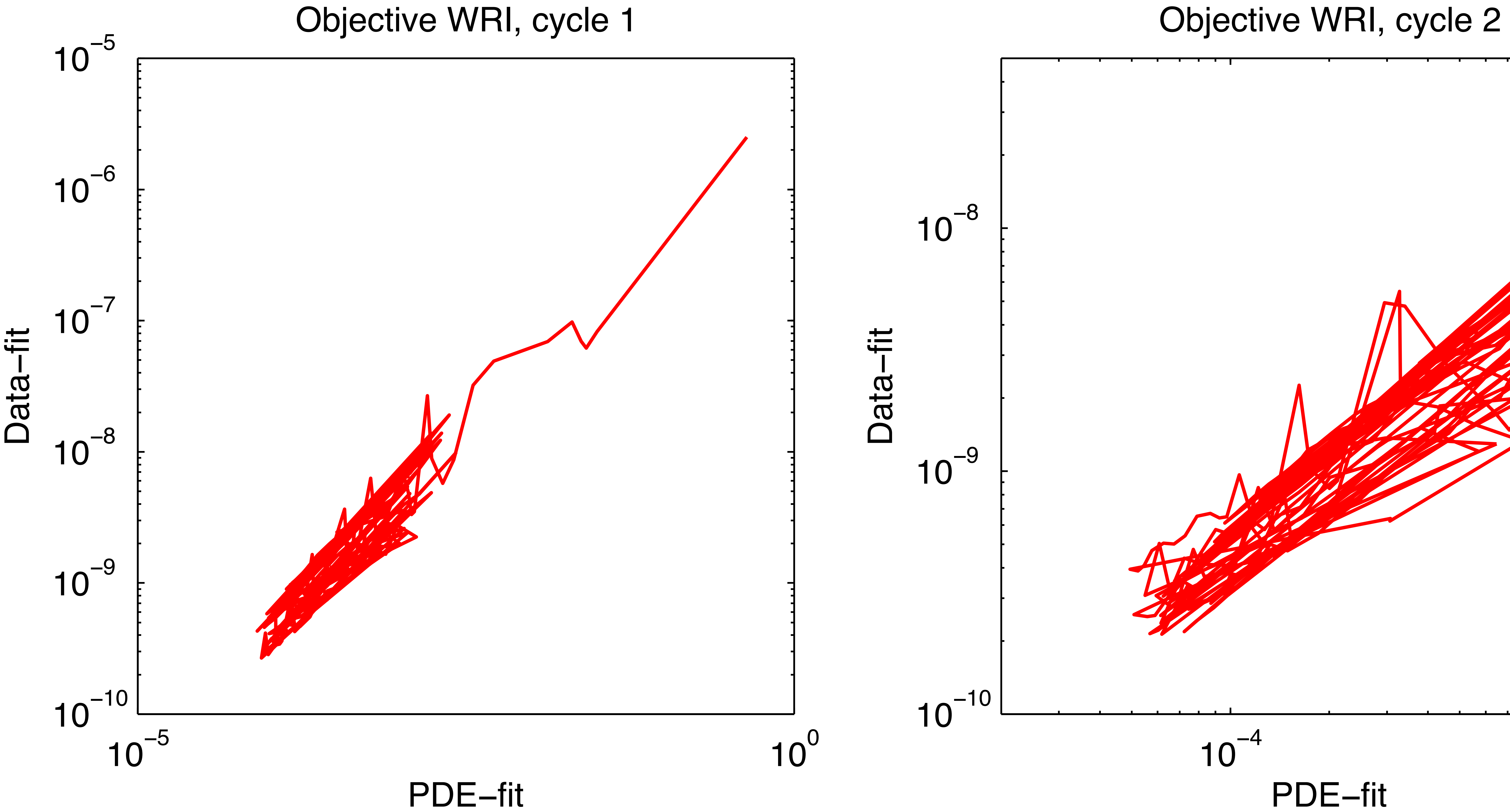


Relative model errors



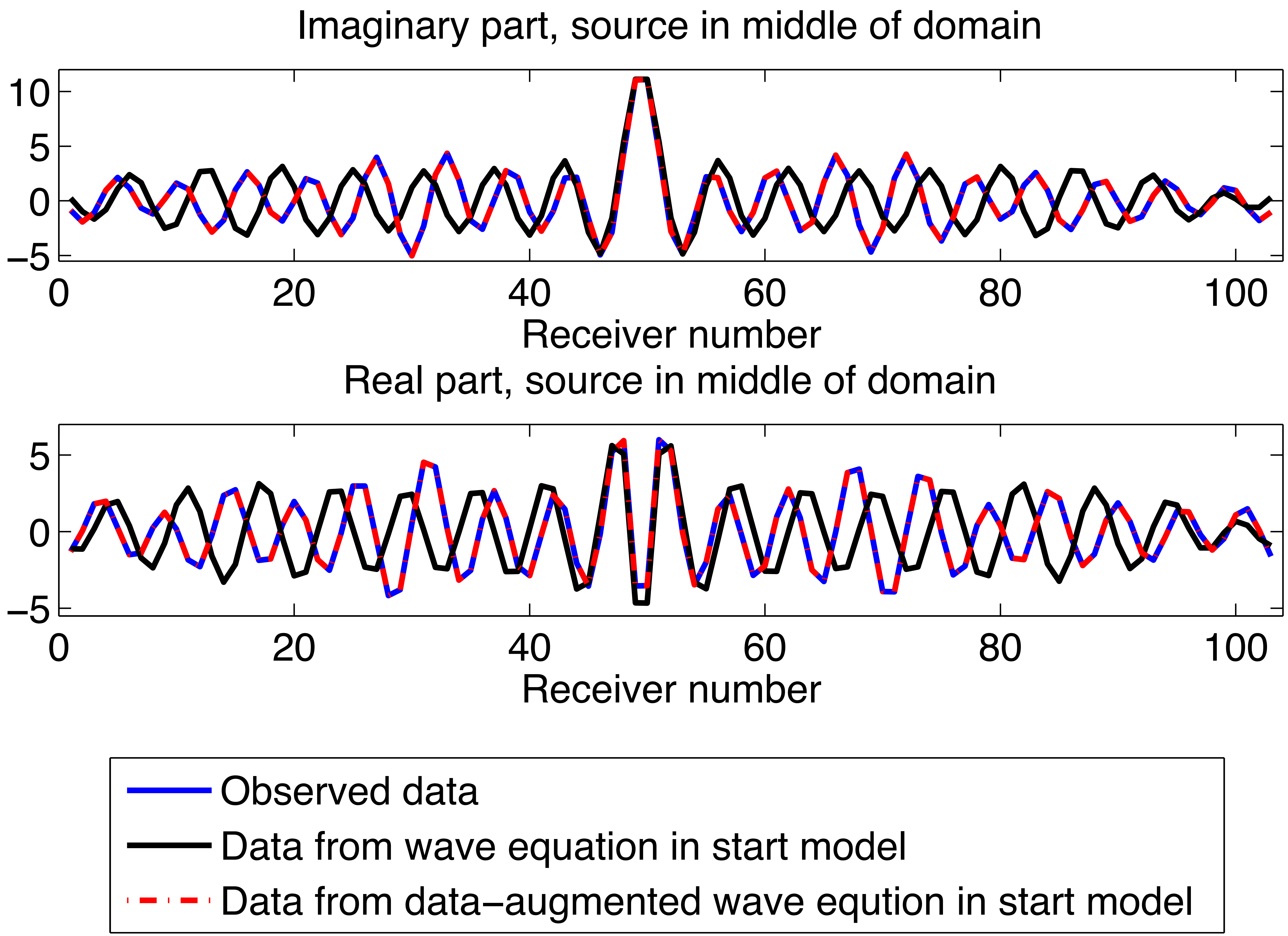


Objective function value



Data fit increases at some iterates

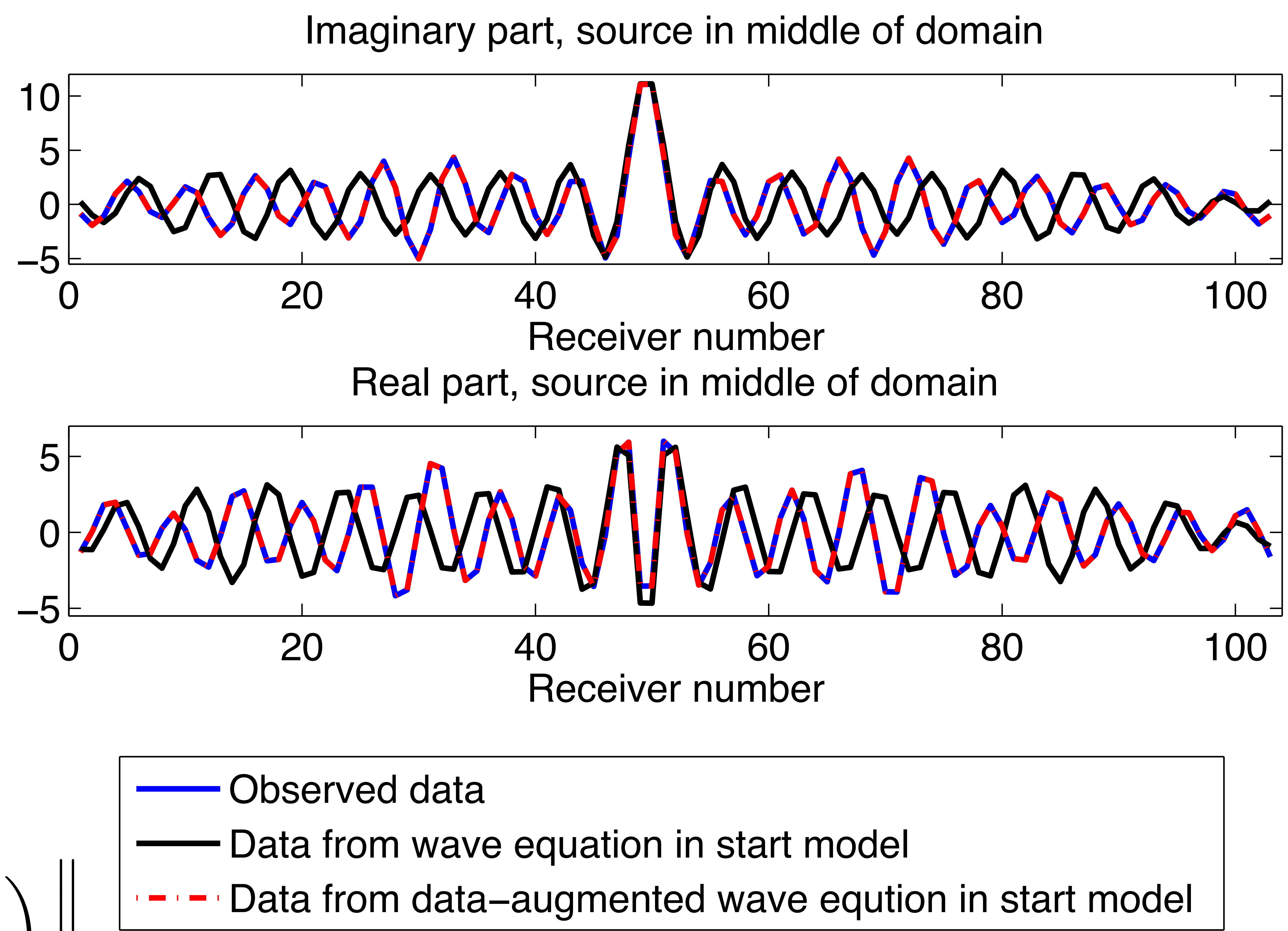
Data fit



Imaginary part, source in middle of domain
10
5
0
−5
0
20
40
60
80
100
Receiver number
Real part, source in middle of domain
5
0
−5
0
20
40
60
80
100
Receiver number
Observed data
Data from wave equation in start model
Data from data−augmented wave eqution in start model

Predicted fields in initial model, 5Hz

- WRI does not work with physical wavefields
- WRI uses the ‘data-augmented’ wavefield
- for λ small enough, the initial field will match the data closely.



$$\bar{\mathbf{u}}_{kl} = \arg\min_{\mathbf{u}_{kl}} \left\| \begin{pmatrix} \lambda H_k(\mathbf{m}) \\ P \end{pmatrix} \mathbf{u}_{kl} - \begin{pmatrix} \lambda \mathbf{q}_{kl} \\ \mathbf{d}_{kl} \end{pmatrix} \right\|_2$$

Example – BG Compass model no inverse crime

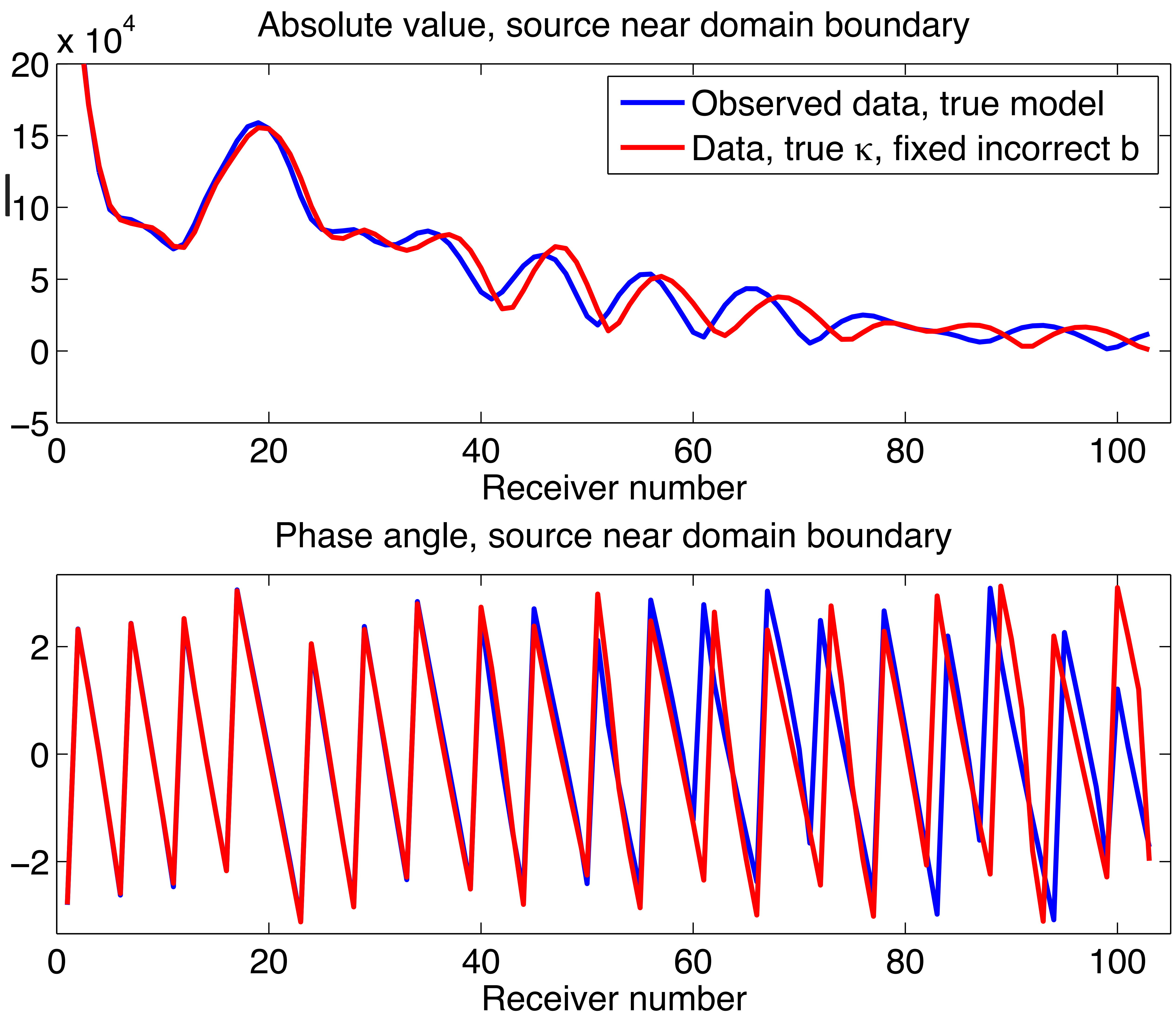
- Generate 'observed' data using a compressibility and buoyancy model.
- Invert for compressibility, fixed and inaccurate buoyancy.
- Obtain velocity model from inverted compressibility and fixed inaccurate buoyancy.

- Low frequencies missing, 15 frequency batches (15 iterations each) {5 6} ,{6 7},... ,{19 20} Hertz. Each interval contains 5 frequencies.

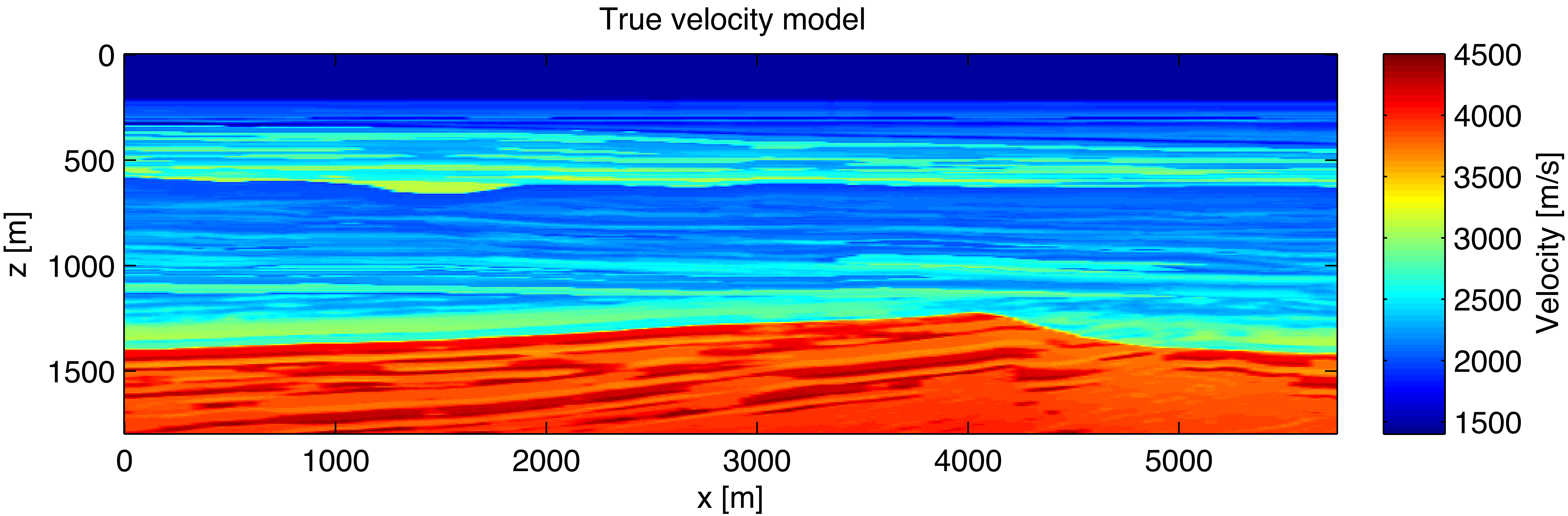
Fields in initial model, 5Hz

- Wavefield in true model (blue).
- Wavefield in true compressibility model with fixed and inaccurate buoyancy model (red).

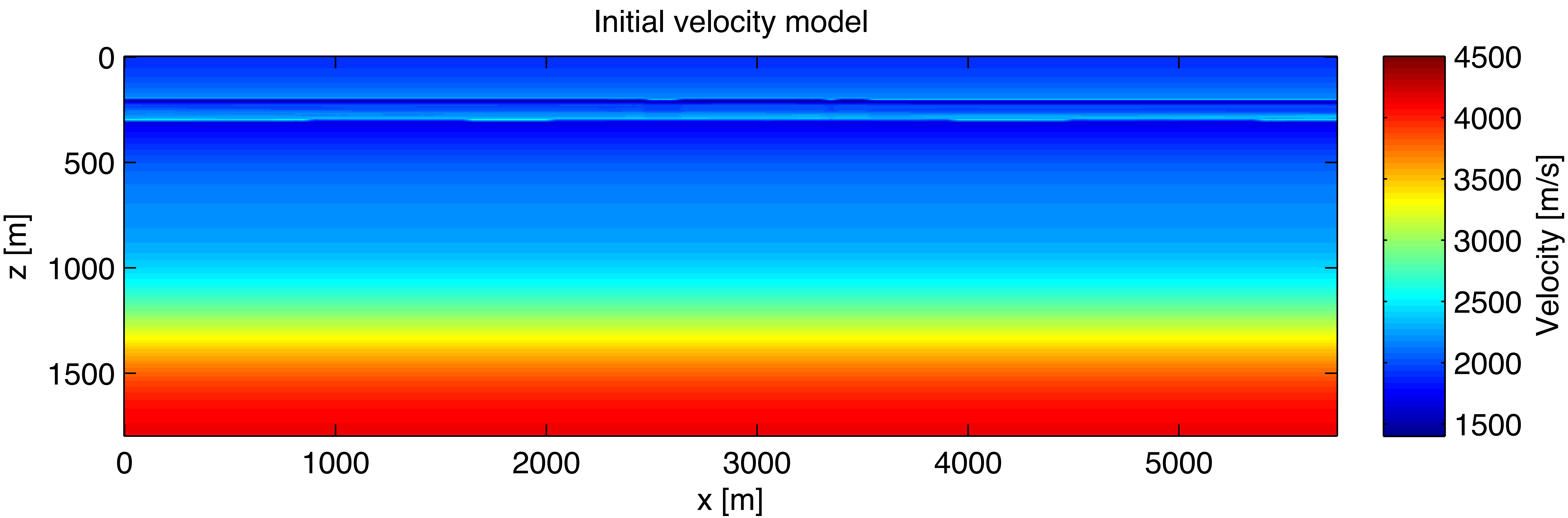
- Perfect model estimation still results in nonzero data fit.



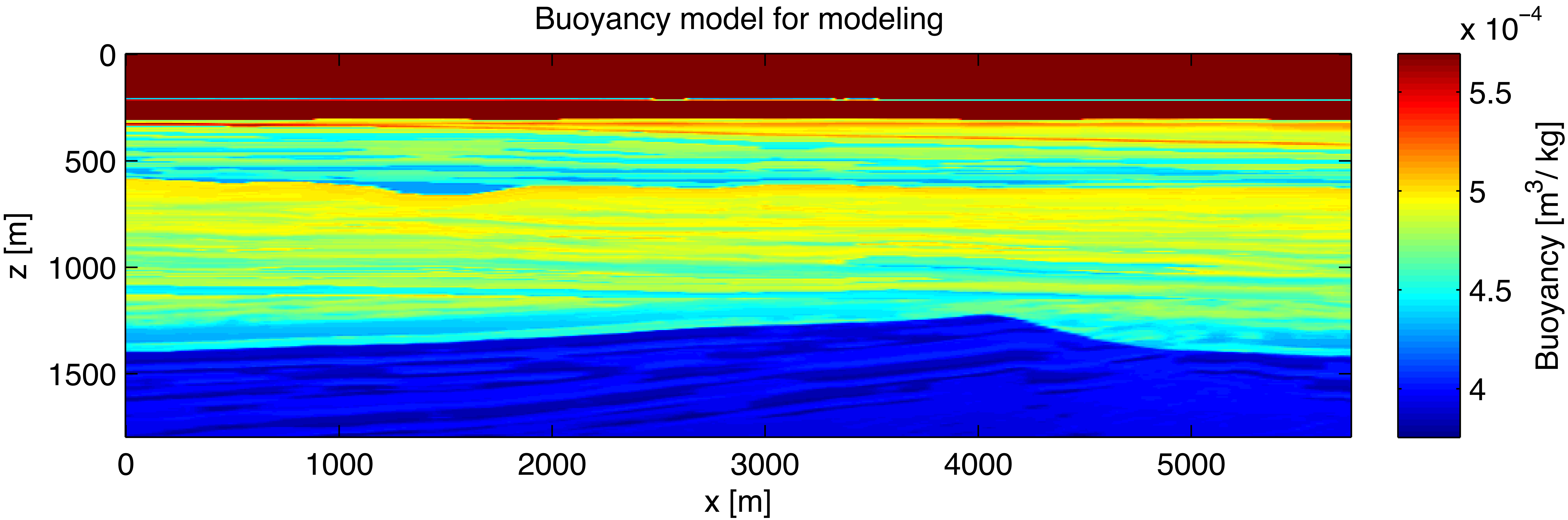
True velocity



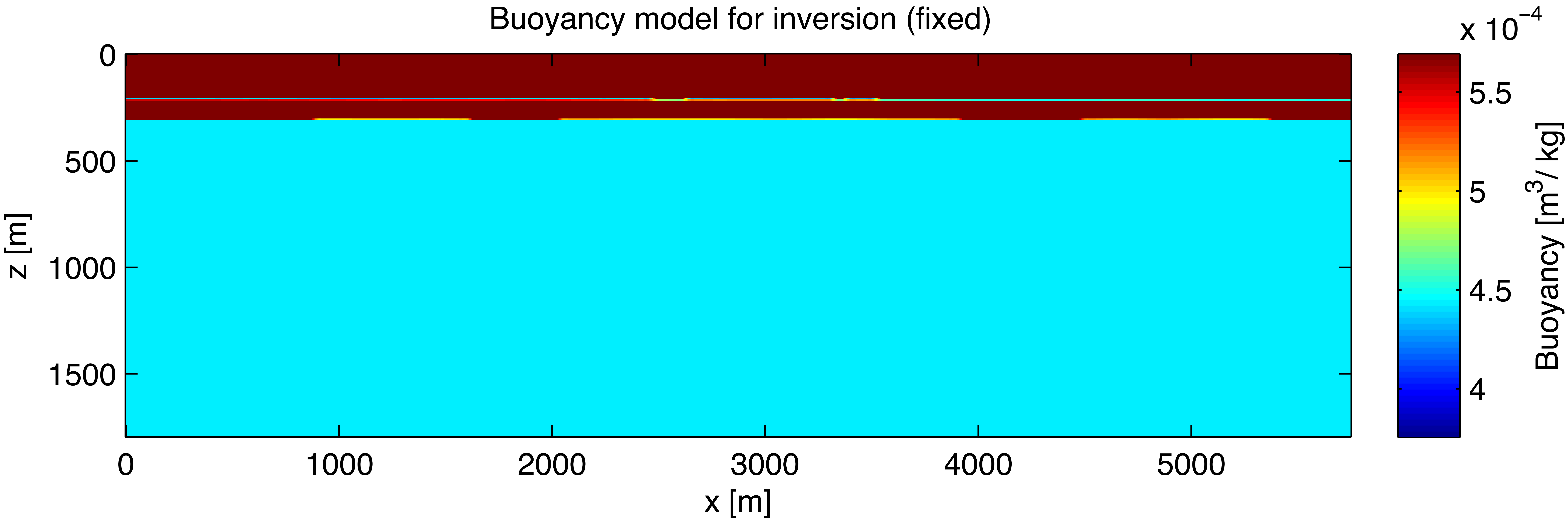
Initial velocity



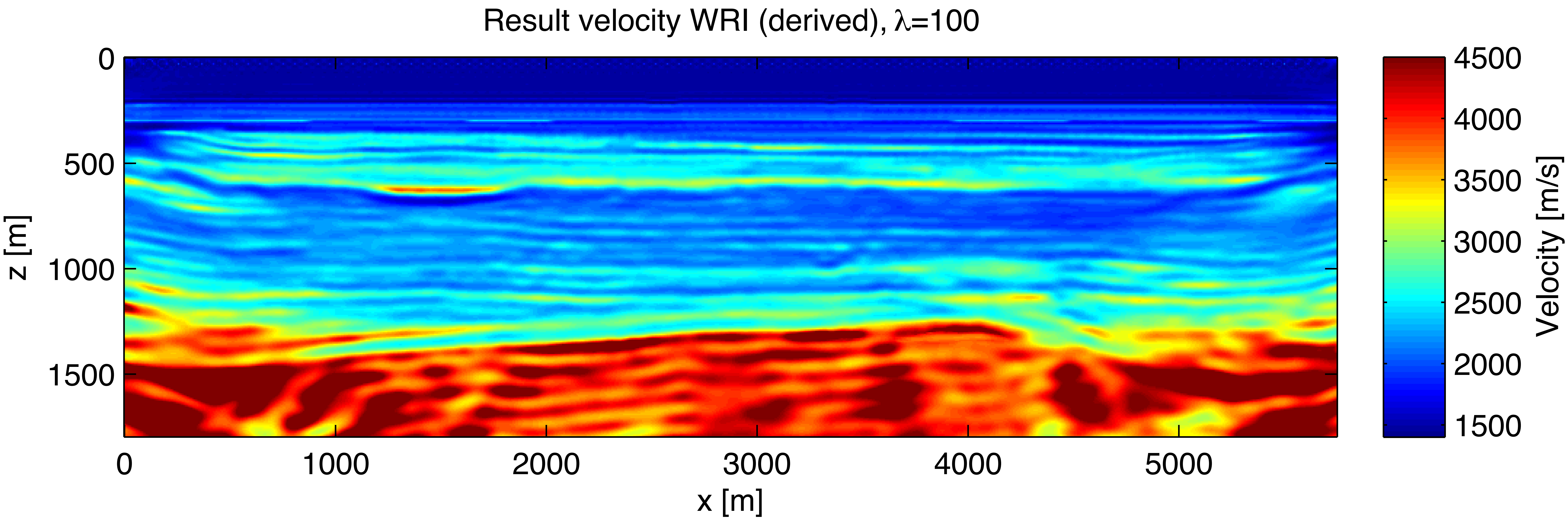
Buoyancy for modeling 'observed data'



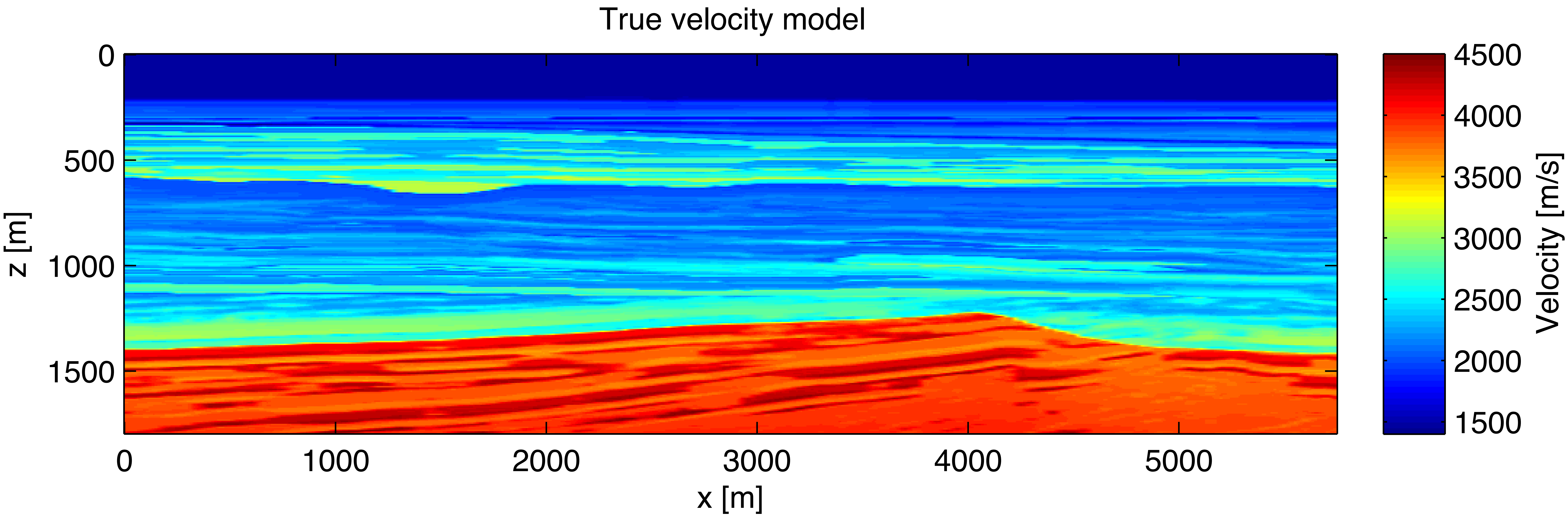
Fixed buoyancy for inversion



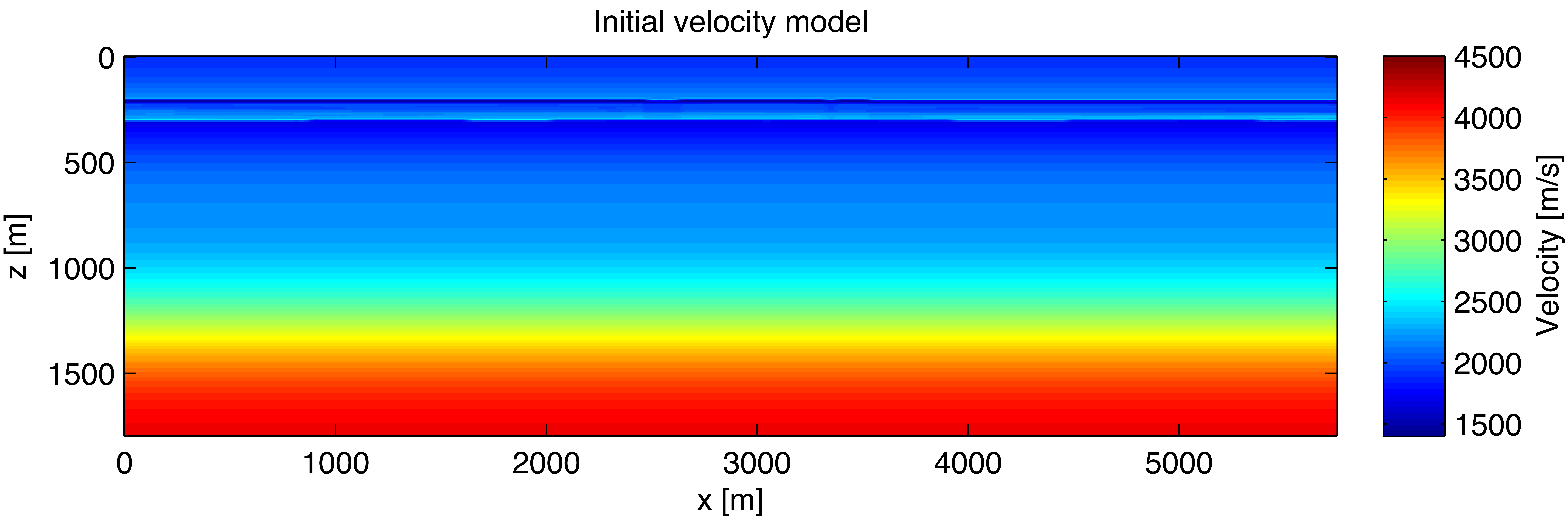
Final velocity estimate using WRI



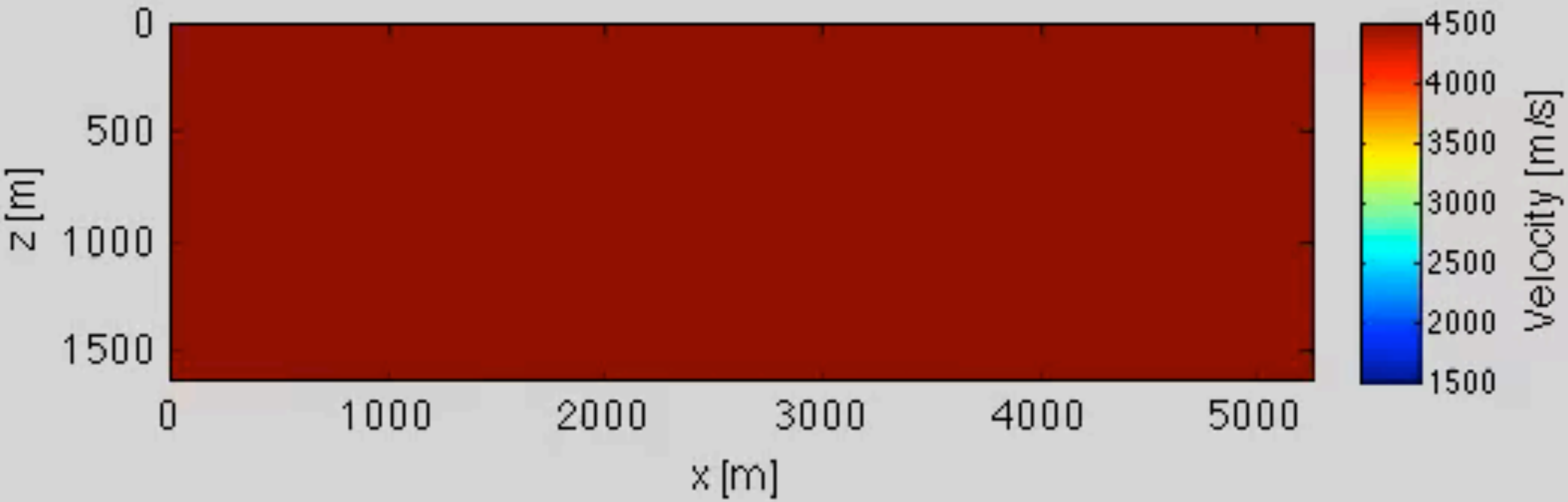
True velocity



Initial velocity

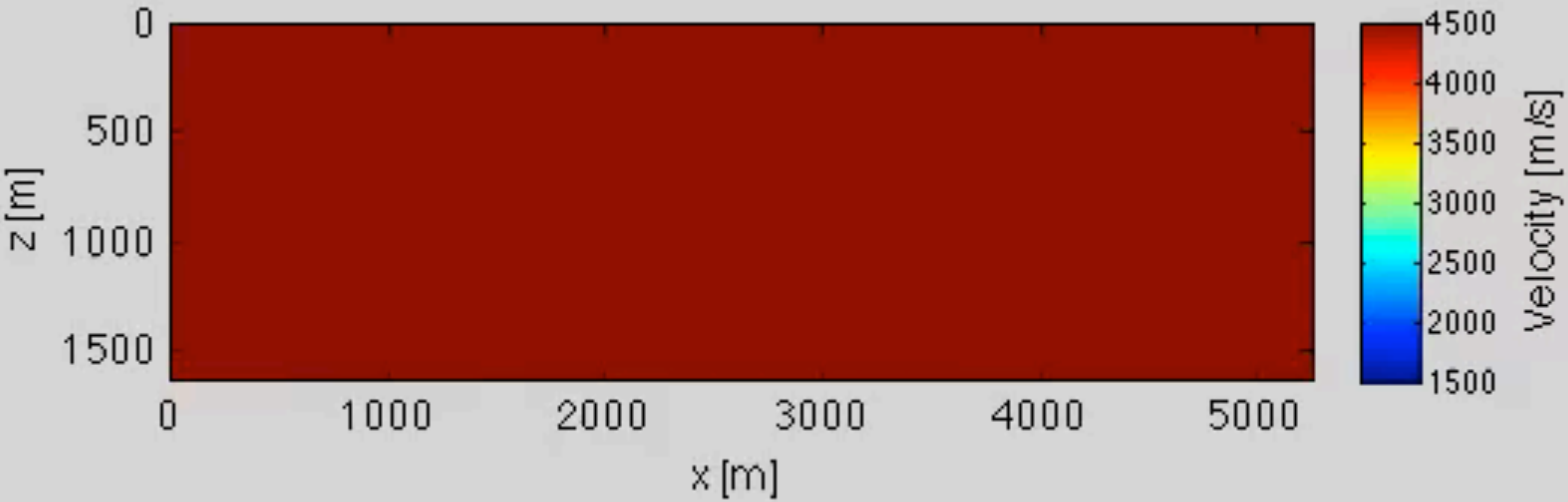


Cycle 1

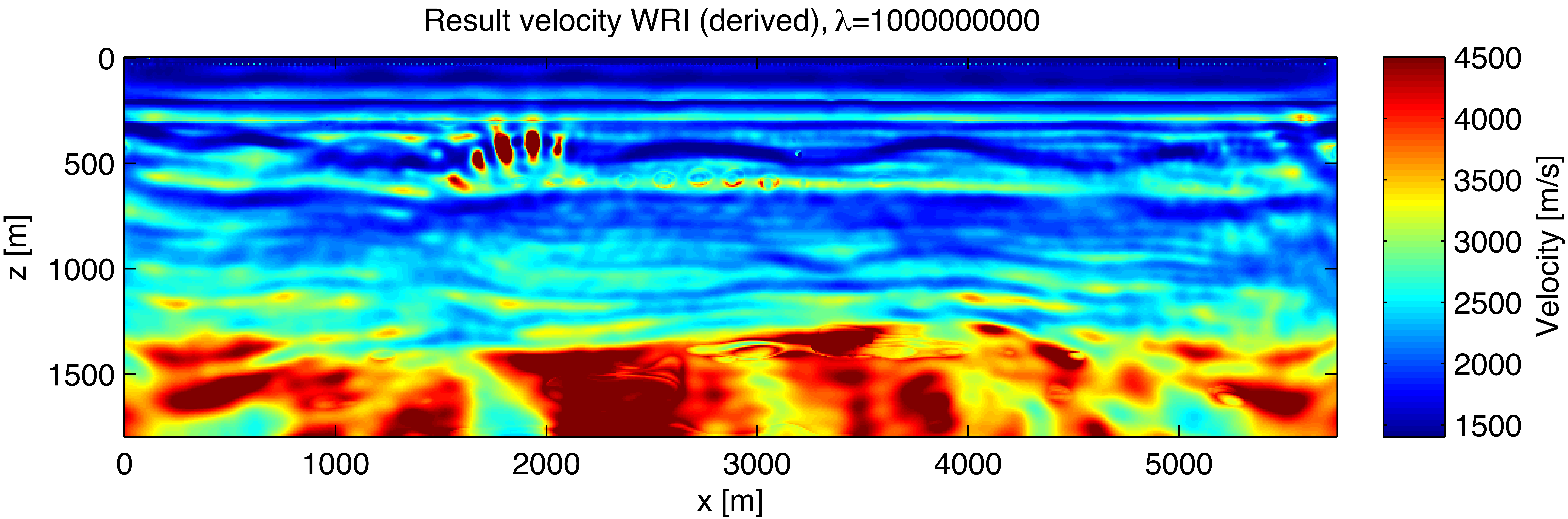


0
500
1000
1500
z [m]
0
1000
2000
3000
4000
5000
x [m]
4500
4000
3500
3000
2500
2000
1500
Velocity [m/s]

Cycle 2

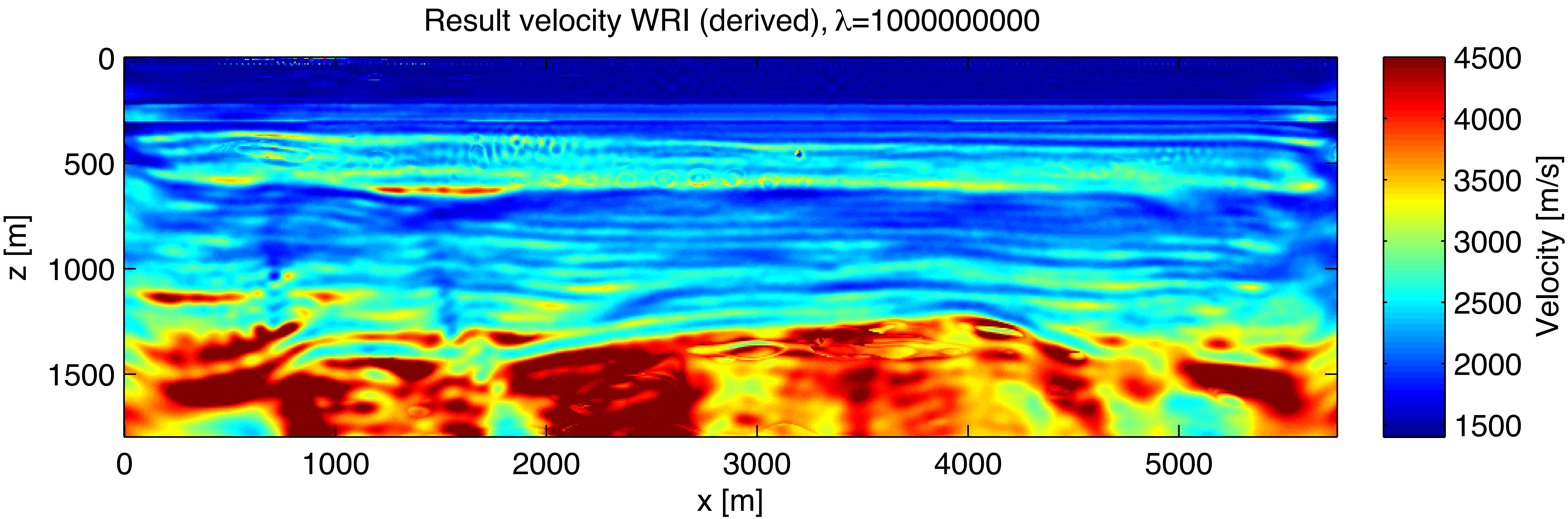


0
500
1000
1500
z [m]
0
1000
2000
3000
4000
5000
x [m]
4500
4000
3500
3000
2500
2000
1500
Velocity [m/s]



λ large -> does not fit data at the start

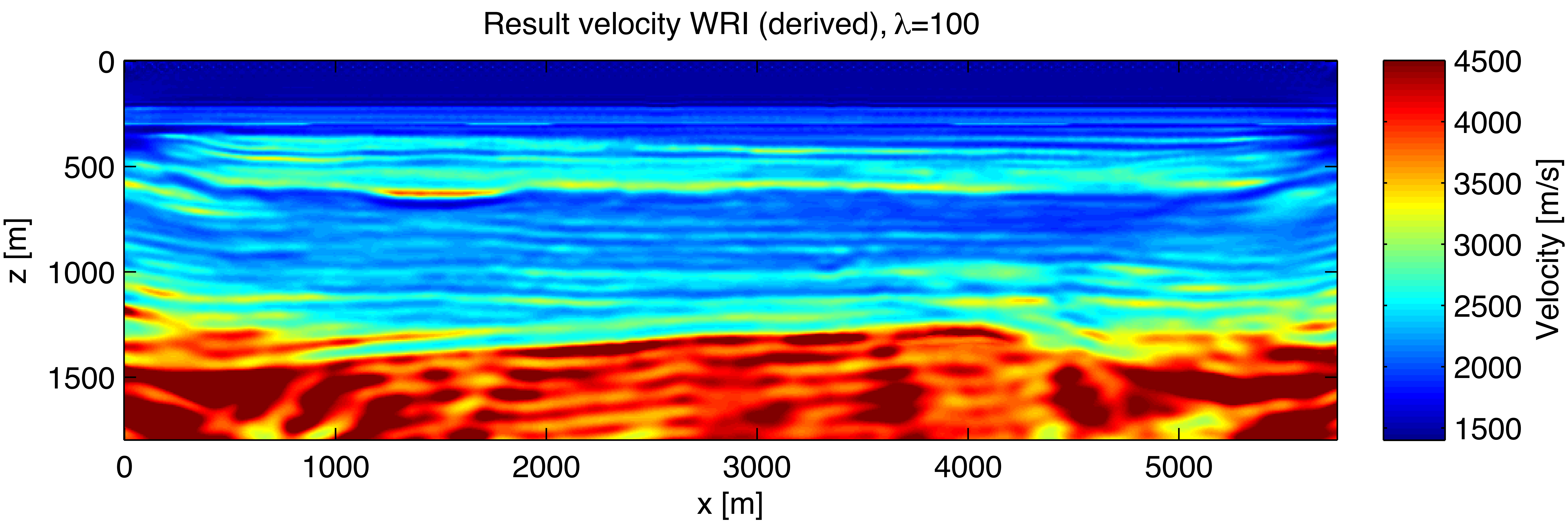
1st cycle through data



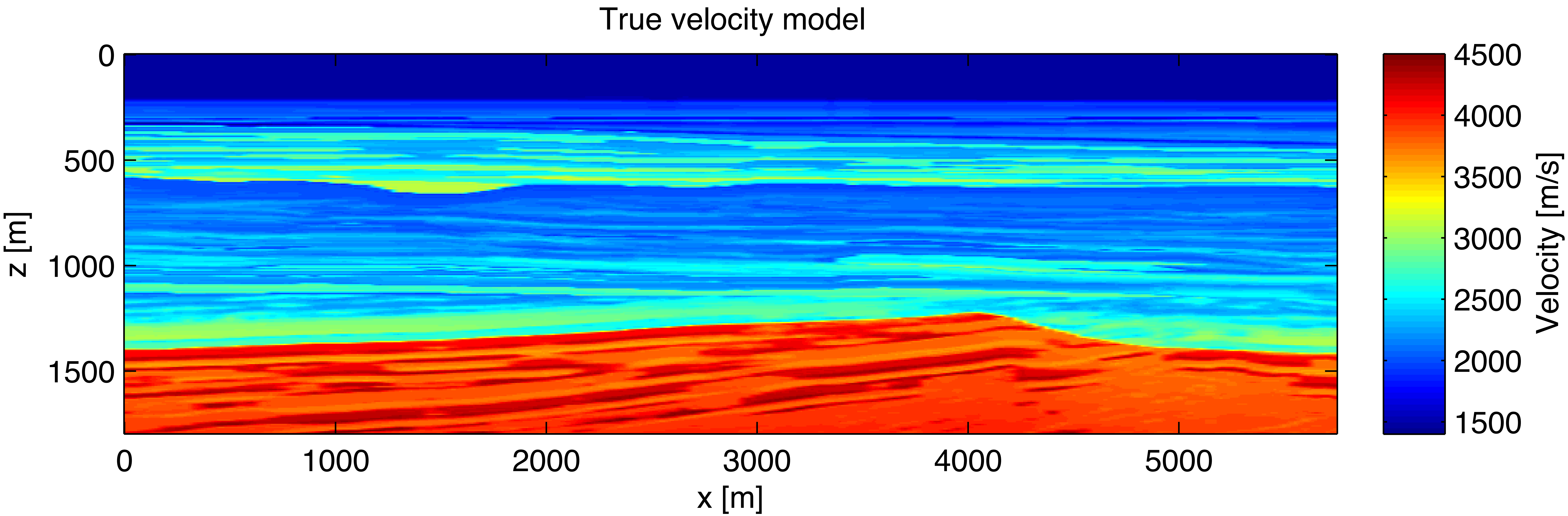
λ large -> does not fit data at the start

2nd cycle through data

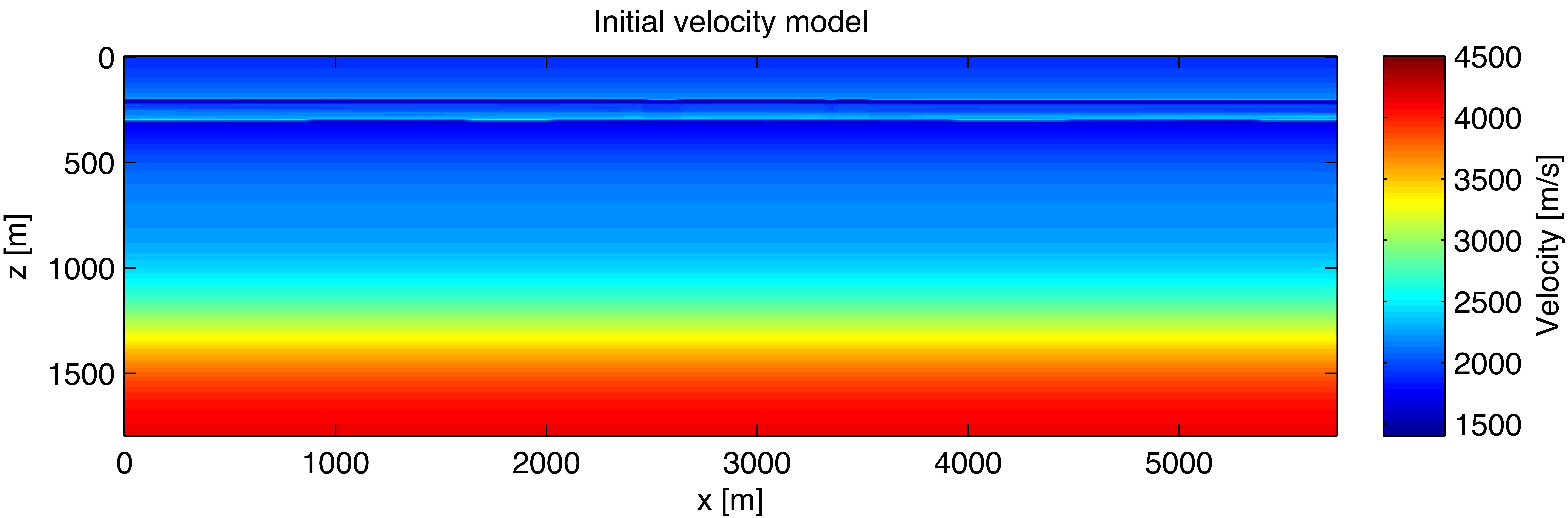
Final velocity estimate using WRI



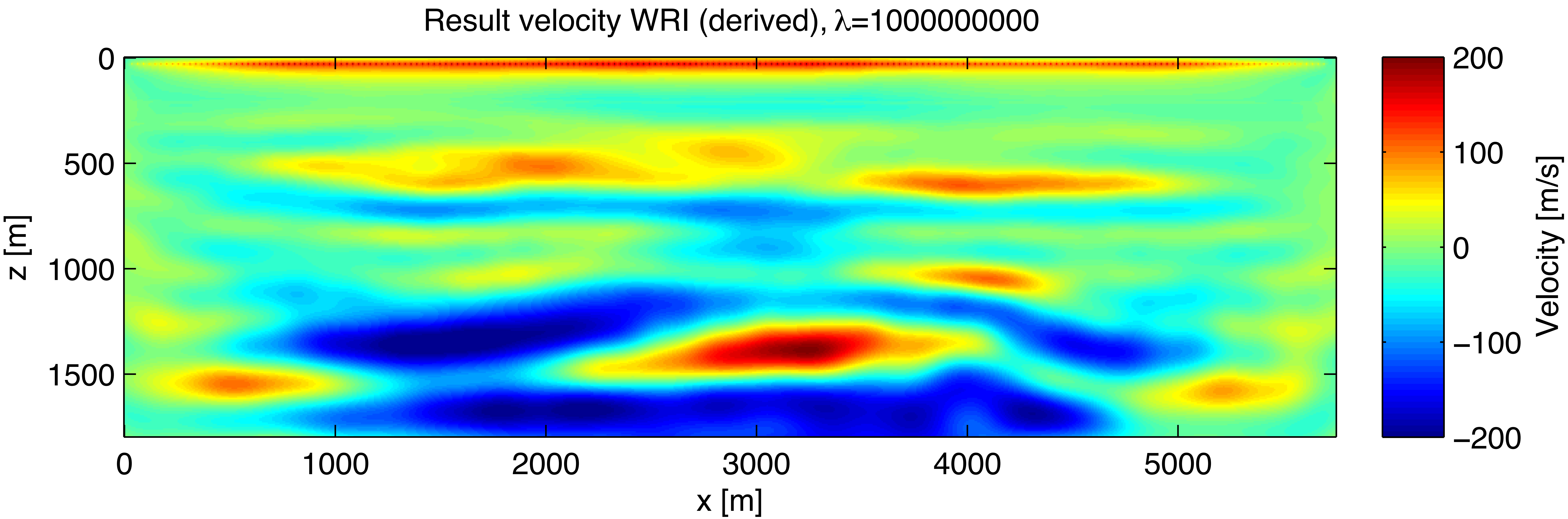
True velocity



Initial velocity

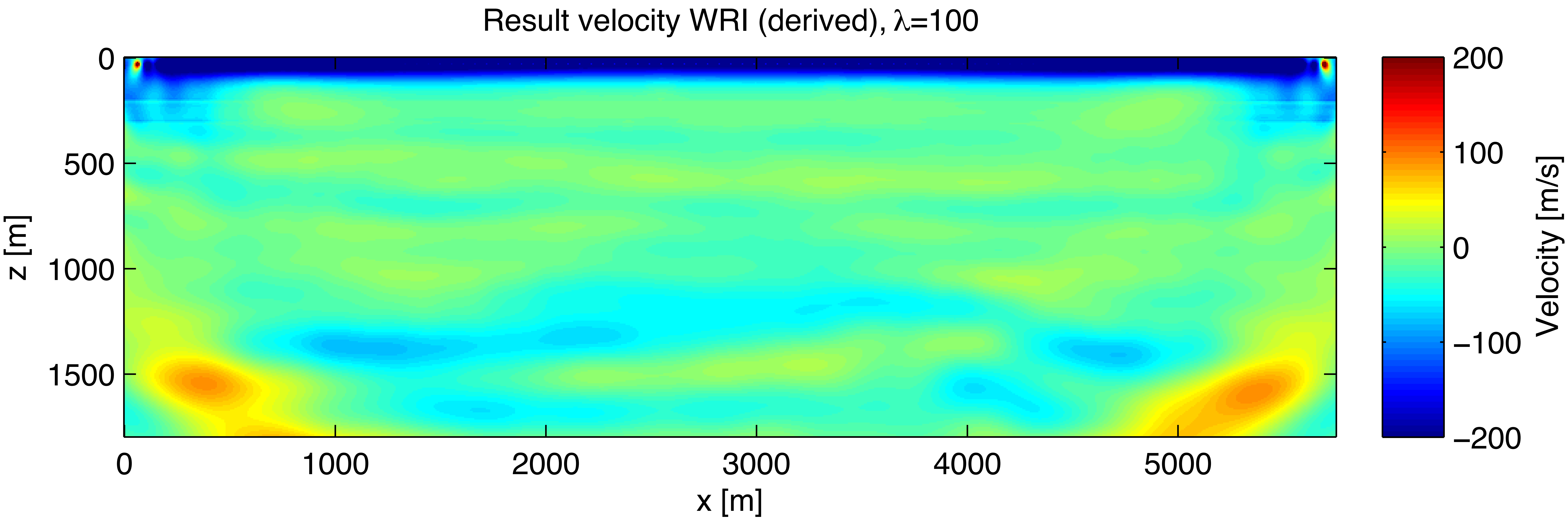


1st update



λ large -> does not fit data at the start

1st update



λ small -> does not fit data at the start

Conclusions - single-parameter WRI

WRI can outperform FWI when starting models are inaccurate.

WRI still works when mildly wrong physics is use–i.e., no inverse crime.

Fitting data at the start seems key.

Software

Single parameter WRI and the first example are in the SLIM software release.

Part 2 - Multi-parameter WRI

Multi-parameter PDE-constrained optimization

Main issues with multi-parameter inverse problems:

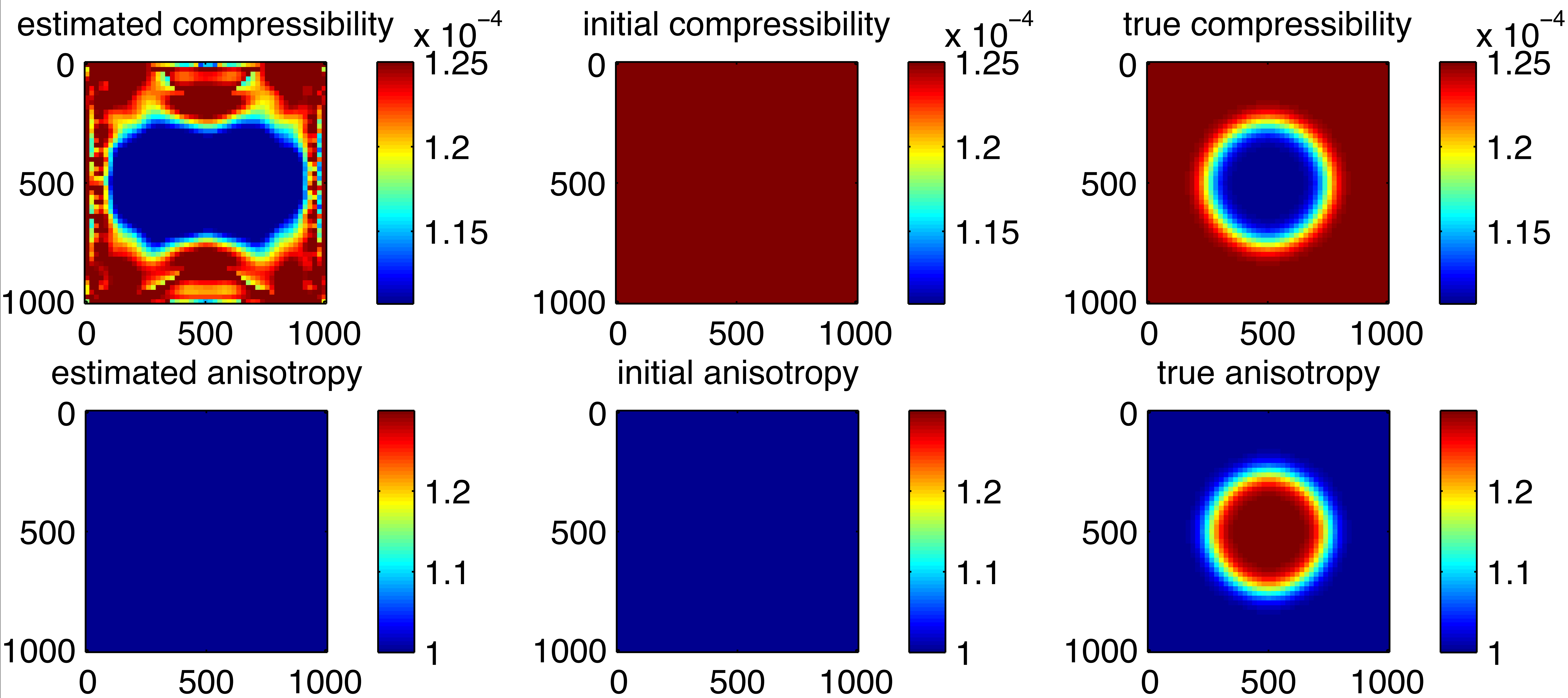
- non uniqueness
- parameter scaling

Multi-parameter PDE-constrained optimization

Cross-well toy example

Update parameters simultaneously using a quasi-Newton method.

Multi-parameter PDE-constrained optimization



Multi-parameter PDE-constrained optimization

Data fit > 99%

Proposed solutions include:

- find ‘best’ parameterization
- sequential/alternating inversion
- regularization
- manual scaling of gradients

Problems:

- sensitive to parameter choices
- manual fine-tuning

Multi-parameter PDE-constrained optimization

Observation:

- Hessians naturally provide information on 'scaling' and 'coupling'.

Using Hessians was also proposed by Lavoué et al. 2014.

Adjoint-state based FWI leads to dense Hessians.

Problems:

- cannot store dense Hessian
- matrix-vector product cost extra PDE solves

Goals

- multi-parameter WRI

- Obtain an approximation of the Hessian without solving extra PDE's.
- Approximation must be sparse and available in memory explicitly.

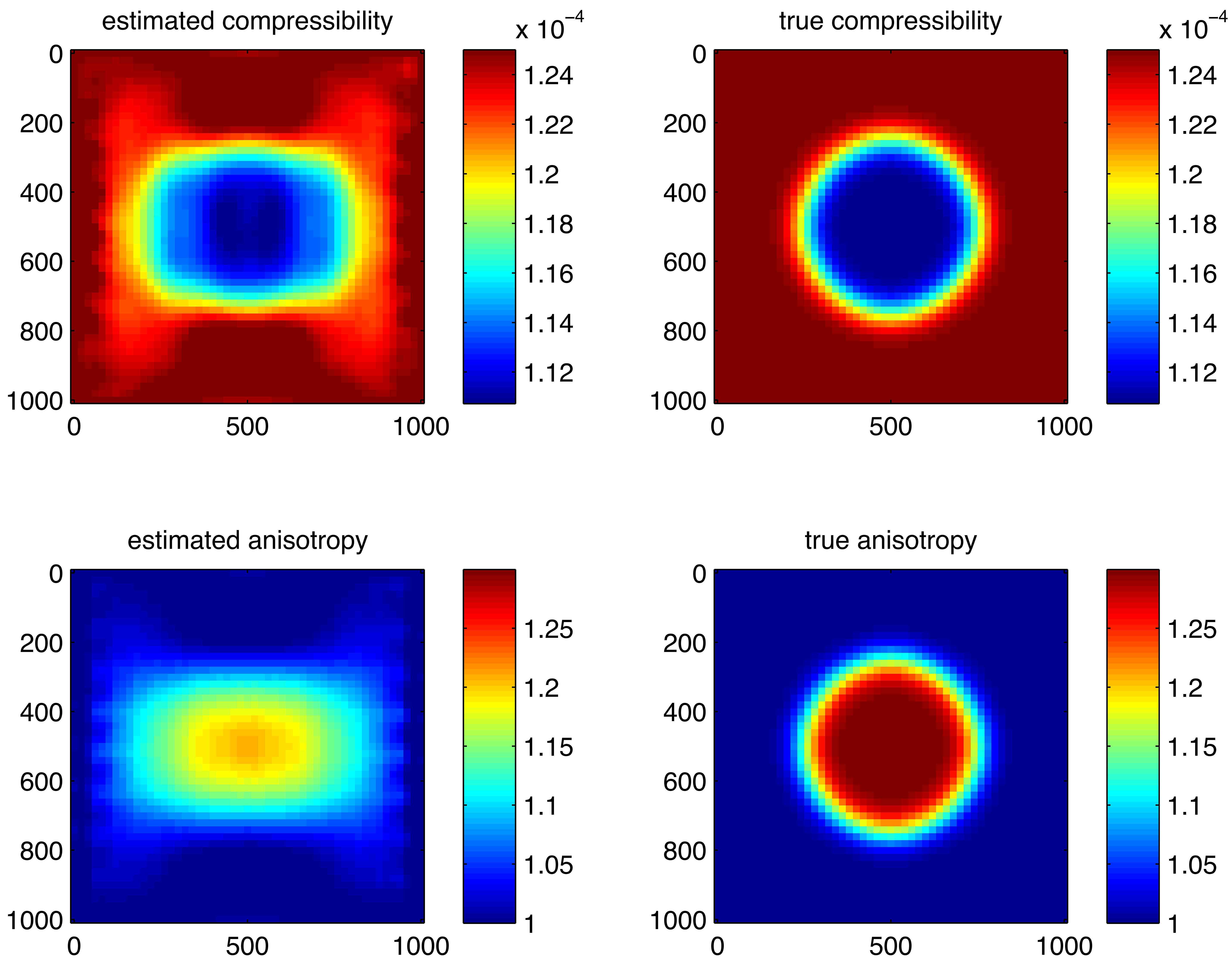
Goals

- multi-parameter WRI

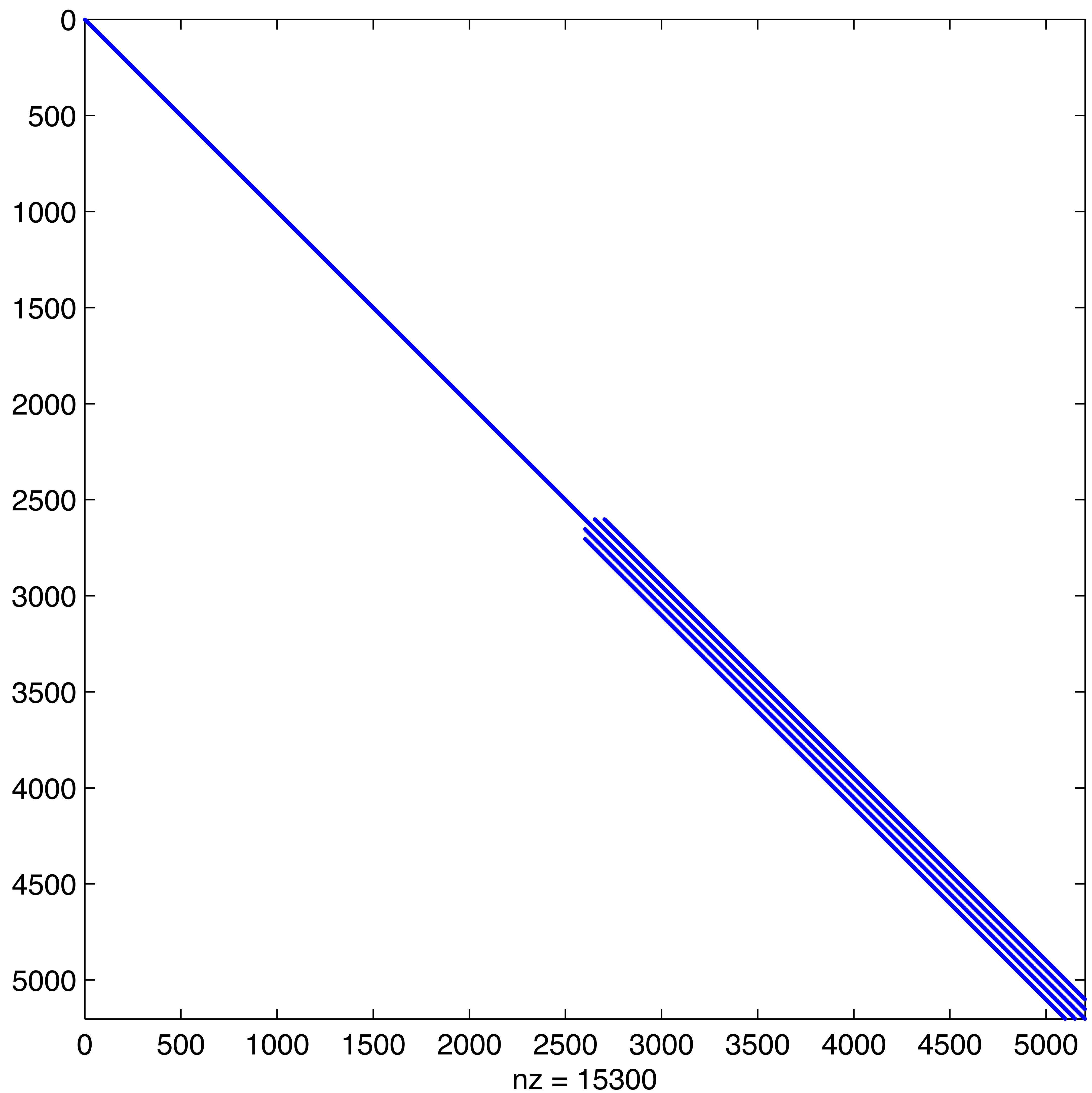
- Obtain an approximation of the Hessian without solving extra PDE's.
- Approximation must be sparse and available in memory explicitly.

This is possible w/ WRI!

Multi-parameter WRI example



A reduced-space algorithm



$$\tilde{H} = \begin{pmatrix} \nabla^2_{\boldsymbol{\kappa},\boldsymbol{\kappa}}\phi_\lambda & 0 \\ 0 & \nabla^2_{\mathbf{b},\mathbf{b}}\phi_\lambda \end{pmatrix} = \begin{pmatrix} G^*_{\boldsymbol{\kappa}} G_{\boldsymbol{\kappa}} & 0 \\ 0 & G^*_{\mathbf{b}} G_{\mathbf{b}} \end{pmatrix}$$

Approximate reduced Hessian sparsity pattern

Derivation similar to single parameter WRI Hessian approximation

A reduced-space algorithm

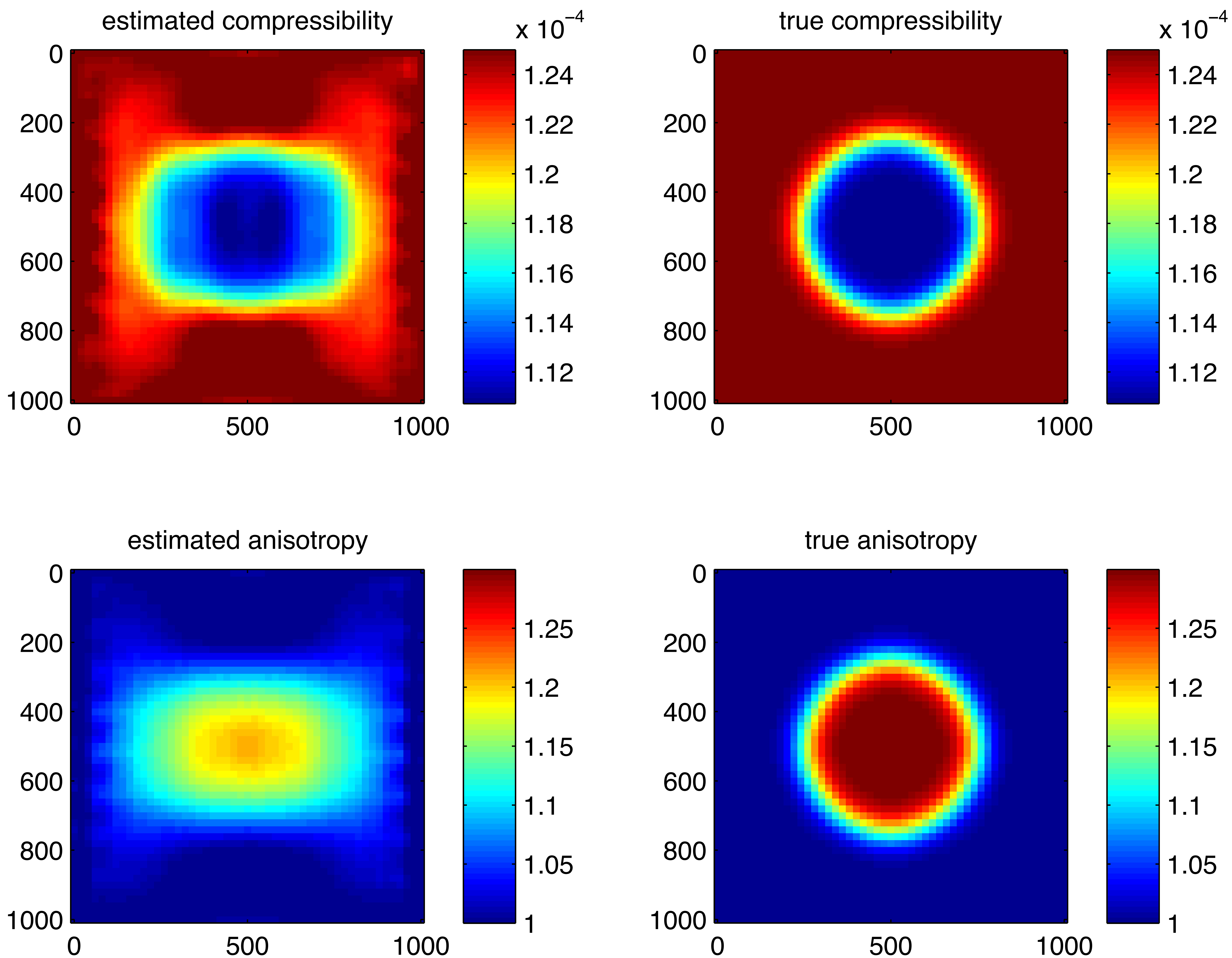
Algorithm 1 Waveform inversion with a sparse Hessian approximation.

while Not converged **do**

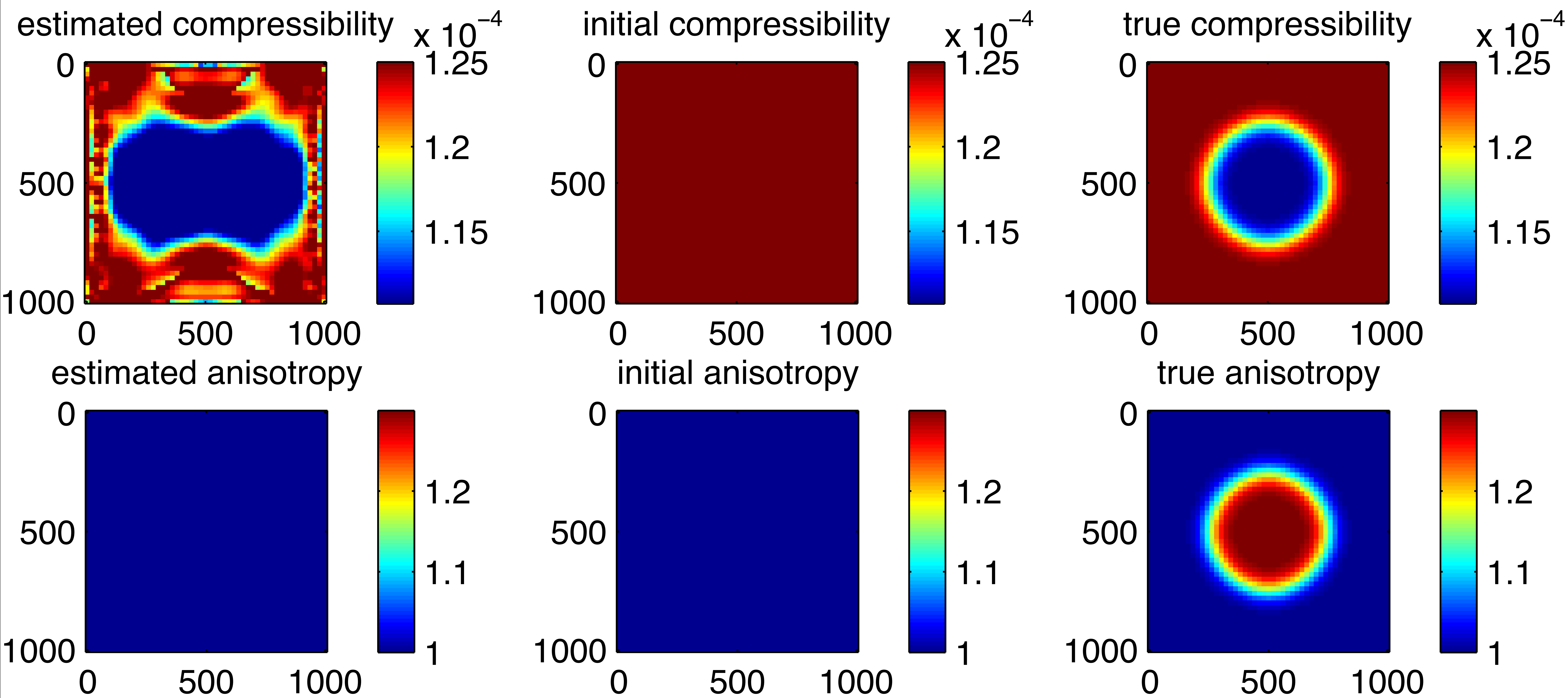
1. $\bar{\mathbf{u}} = \arg\min_{\mathbf{u}} \left\| \begin{pmatrix} \lambda \mathbf{A}(\mathbf{b},\kappa) \\ \mathbf{P} \end{pmatrix} \mathbf{u} - \begin{pmatrix} \lambda \mathbf{q} \\ \mathbf{d} \end{pmatrix} \right\|_2$ `// Solve`
2. $\mathbf{G}_\kappa, \mathbf{G}_\mathbf{b}, \nabla_\mathbf{b}\bar{\phi}_\lambda, \nabla_\kappa\bar{\phi}_\lambda$ `// Form`
3. $\mathbf{p}_{gn} = \tilde{\mathbf{H}}^{-1}\mathbf{g}$ `// Solve`
4. find steplength α `// Linesearch`
5. $\mathbf{m} = \mathbf{m} + \alpha \mathbf{p}_{gn}$ `// update model`

end

A reduced-space algorithm – example



Conventional quasi-Newton result



Summary & Conclusions - multi-parameter

WRI provides access to a reduced Hessian approximation which is:

- sparse
- easy to invert
- scales different parameter classes based on the optimization, data and PDE itself

Can extend to more than two parameters.

Different optimization strategies may not remove non uniqueness.

No theoretical guarantees yet.

Current & future work

Apply single- and multi-parameter WRI to real data.

(talk on Wednesday)

3D WRI using iterative solvers

(another talk on Wednesday)

References

Peters, Bas, and Felix J. Herrmann. 2014. “A Sparse Reduced Hessian Approximation for Multi-Parameter Wavefield Reconstruction Inversion.” EAGE 2014.

R.E. Kleinman and P.M.van den Berg . 1992. A modified gradient method for two- dimensional problems in tomography. Journal of Computational and Applied Mathematics.

Biswanath Banerjee and Timothy F. Walsh and Wilkins Aquino and Marc Bonnet. 2013. Large scale parameter estimation problems in frequency-domain elastodynamics using an error in constitutive equation functional. Computer Methods in Applied Mechanics and Engineering.

Leeuwen, Tristan van, and Felix J. Herrmann. 2013. “Mitigating Local Minima in Full-Waveform Inversion by Expanding the Search Space.” Geophysical Journal International .

Leeuwen, Tristan van, and Felix J. Herrmann. 2013. “A Penalty Method for PDE-Constrained Optimization.” UBC.

Bas Peters, Felix J. Herrmann, and Tristan van Leeuwen. 2014. “Wave-equation based inversion with the penalty method: adjoint-state versus wavefield-reconstruction inversion”. EAGE.