Quadratic-penalty based full-space methods for waveform inversion

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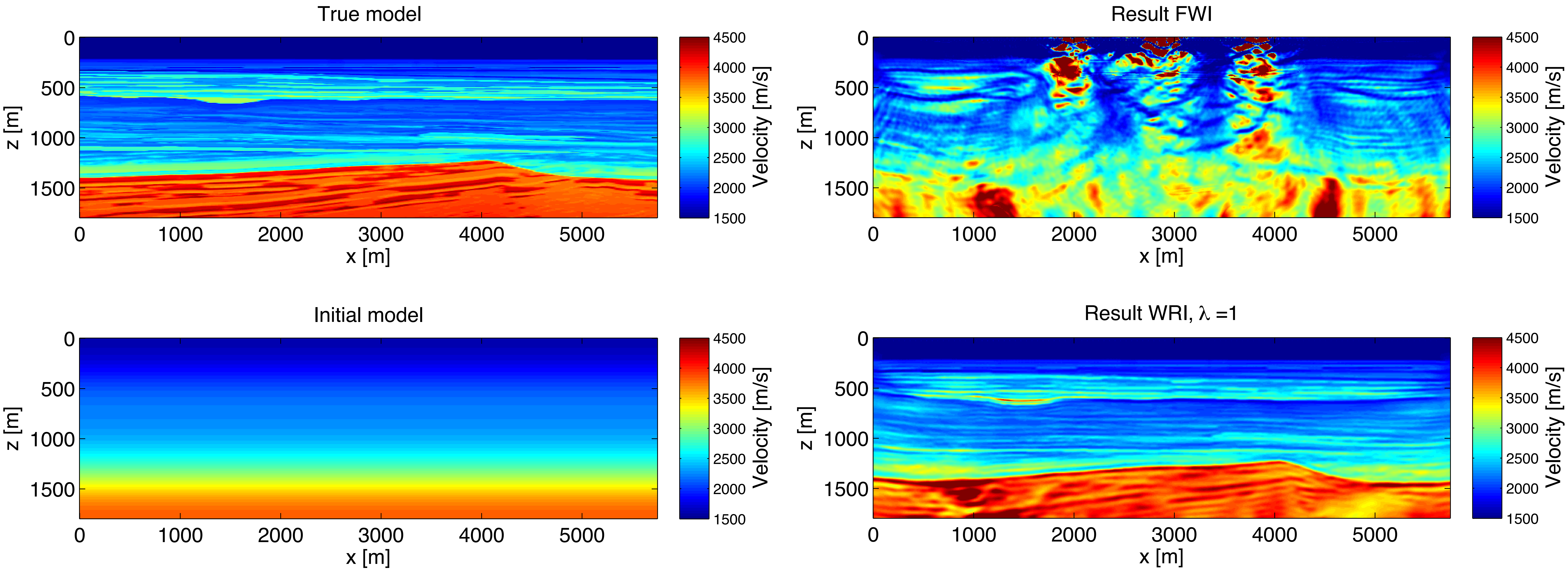
Waveform inversion

Works well if initial model is good

Waveform inversion – poor start model



Example from **[Peters et al. 2013]**

Wavefield reconstruction inversion

[T. van Leeuwen & F.J. Herrmann, 2013]

Less sensitive to

- starting models compared to FWI
- missing low-frequency data

Avoids cycle-skipping problems by virtue of extending the search space.

But, requires reasonable accurate solve of the augmented wave equation...

Wavefield reconstruction inversion

Still limitations on quality of start models & missing low-frequencies

Can we do better?

Maybe full-space methods?

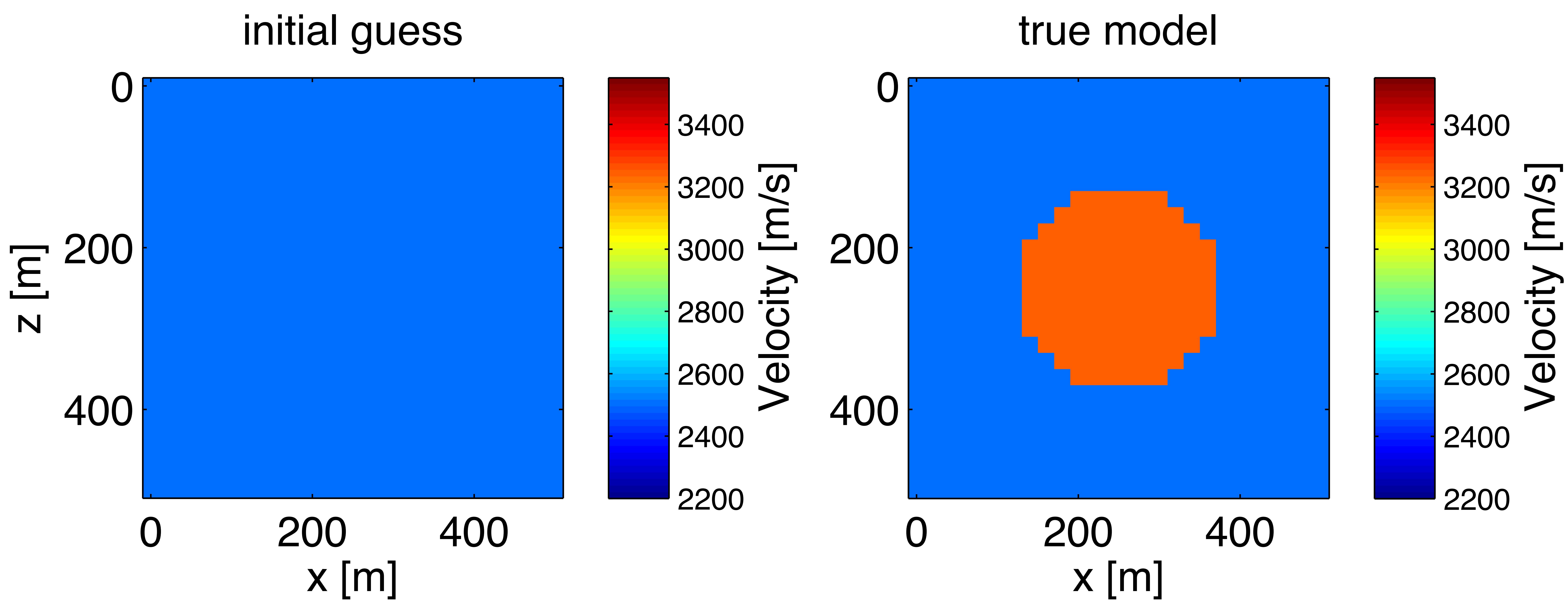
- at the "expense" of storing 2 copies of monochromatic wavefields

But, at the gain of no longer insisting on accurate solves...

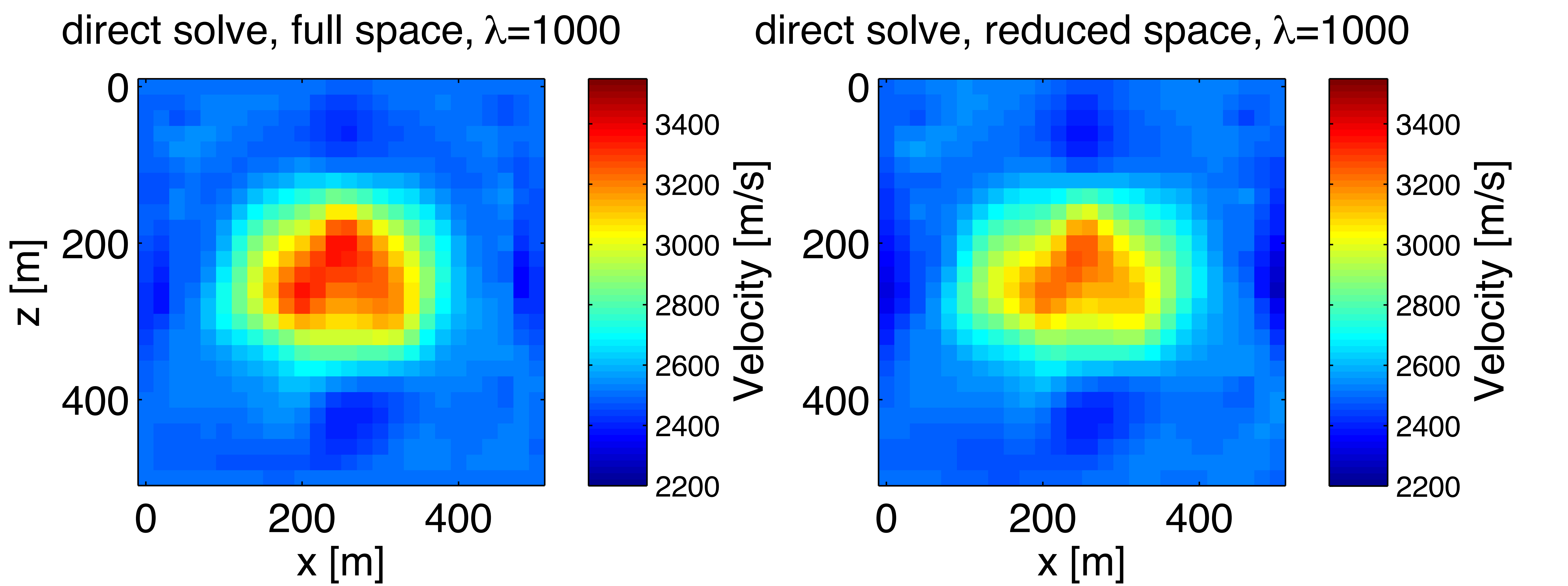
Toy problem

- cross-well setting
- 4 frequencies [6-10] Hz
- 5 simultaneous sources
- 5 receivers

Toy problem

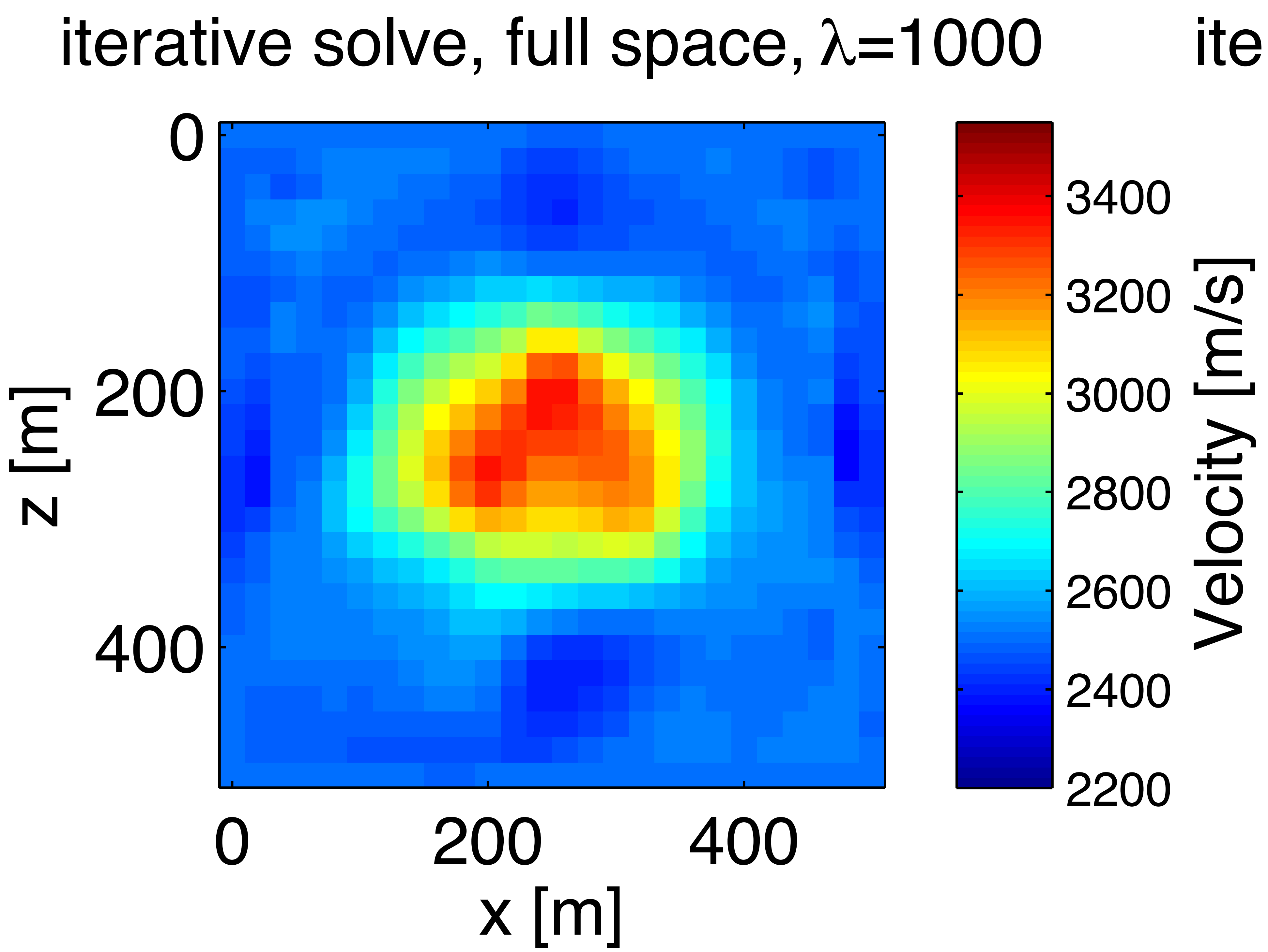

initial guess
true model
0
200
400
0
200
400
z [m]
x [m]
3400
3200
3000
2800
2600
2400
2200
Velocity [m/s]

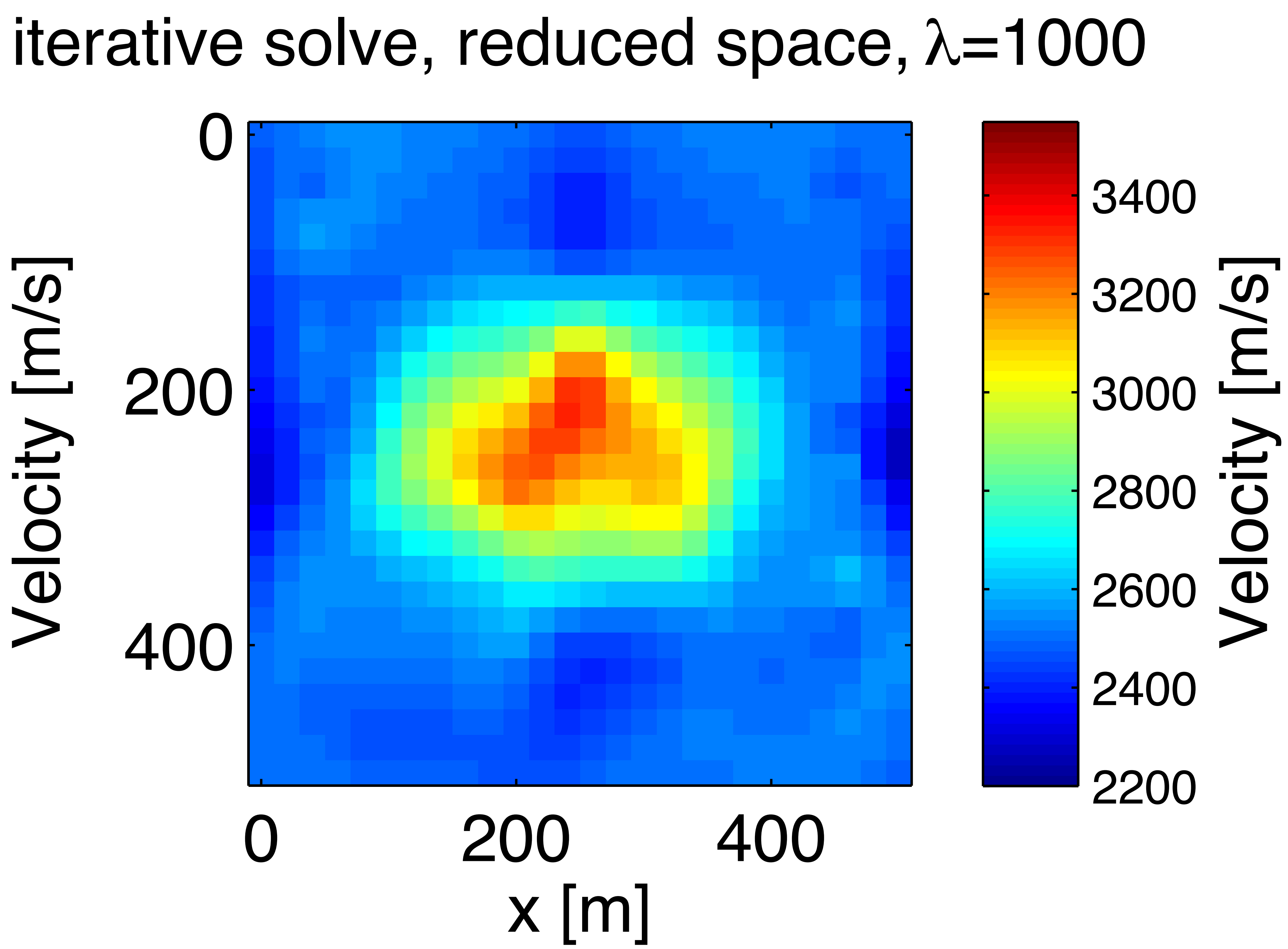
Toy problem



direct solution for least-squares problems

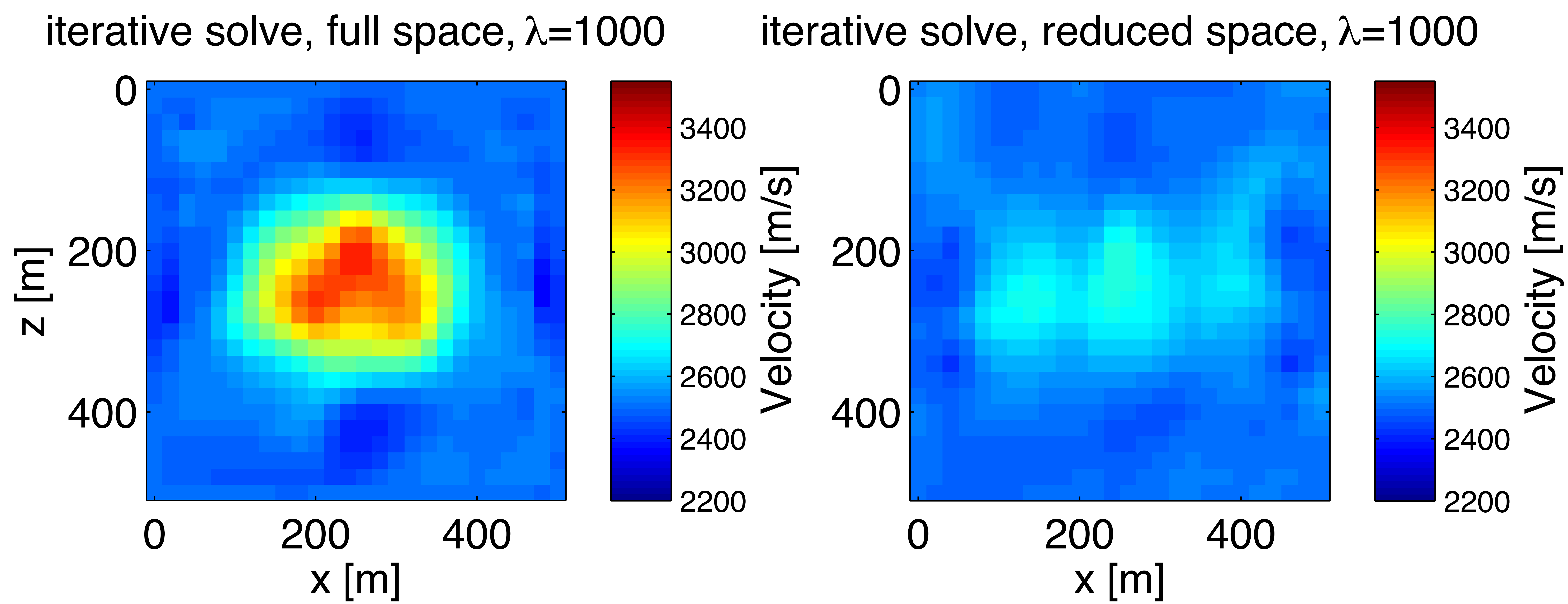
Toy problem





accurate iterative solution for least-squares problems

Toy problem

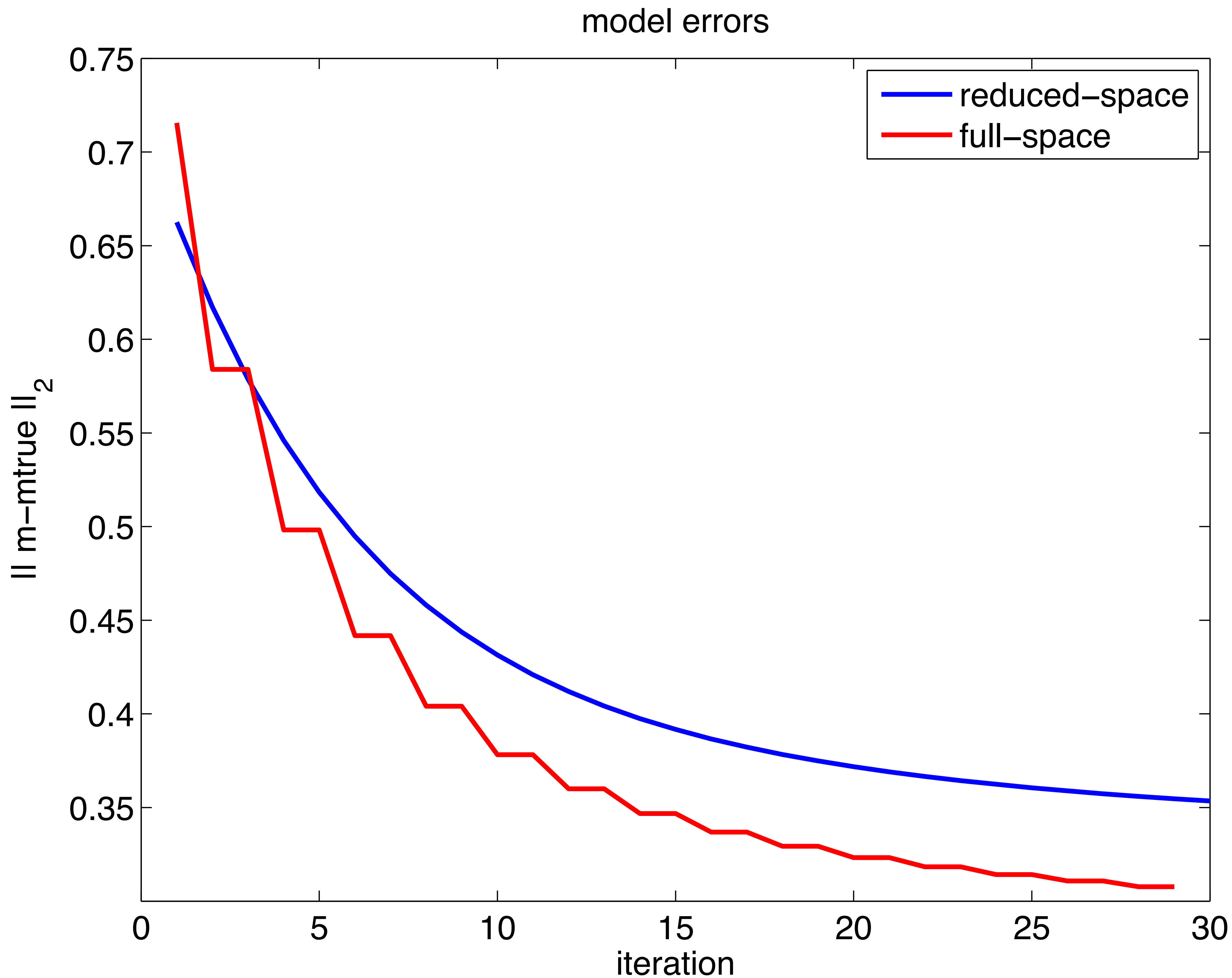


inaccurate iterative solution for least-squares problems

Toy problem

model errors,

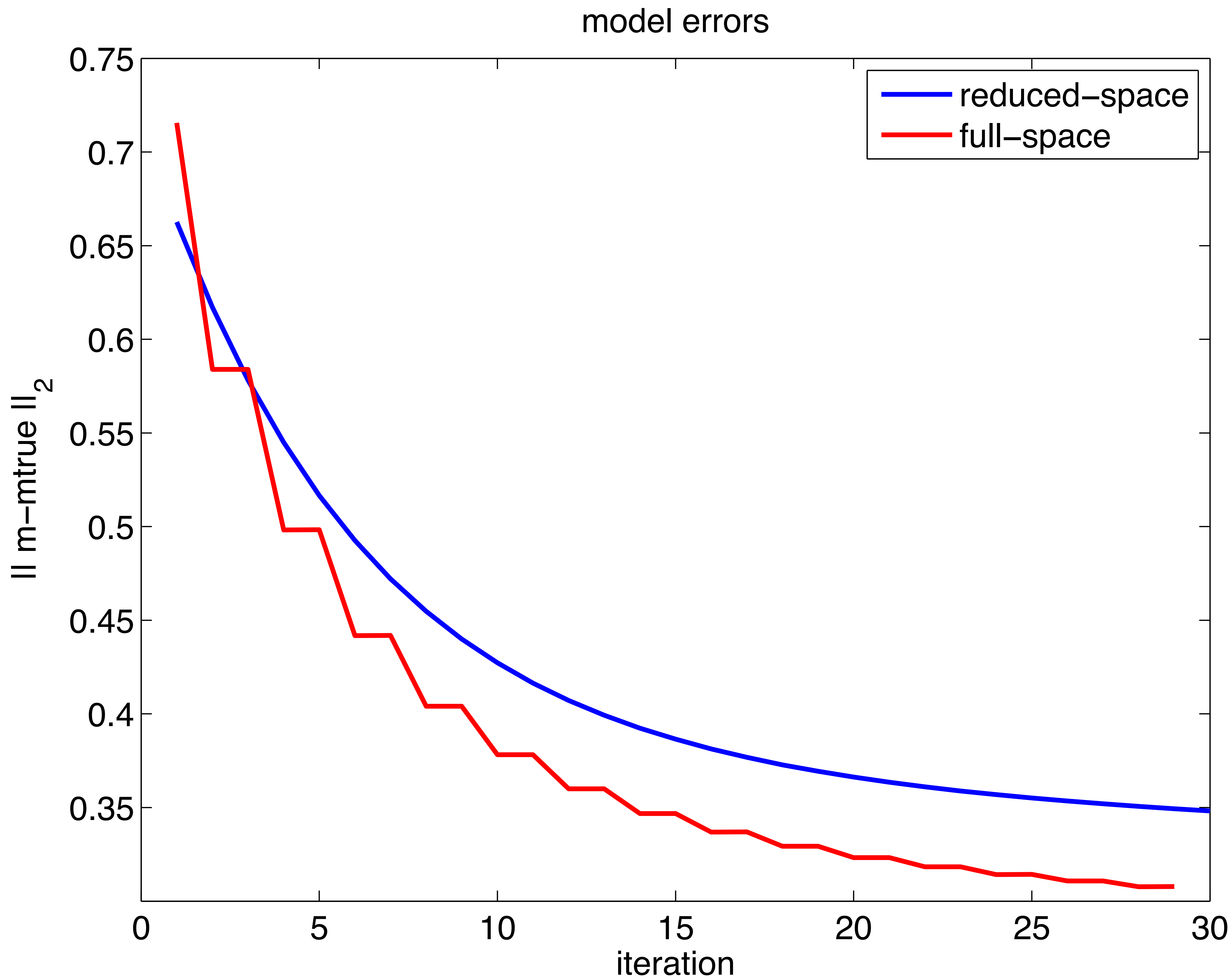
direct solver



Toy problem

model errors,

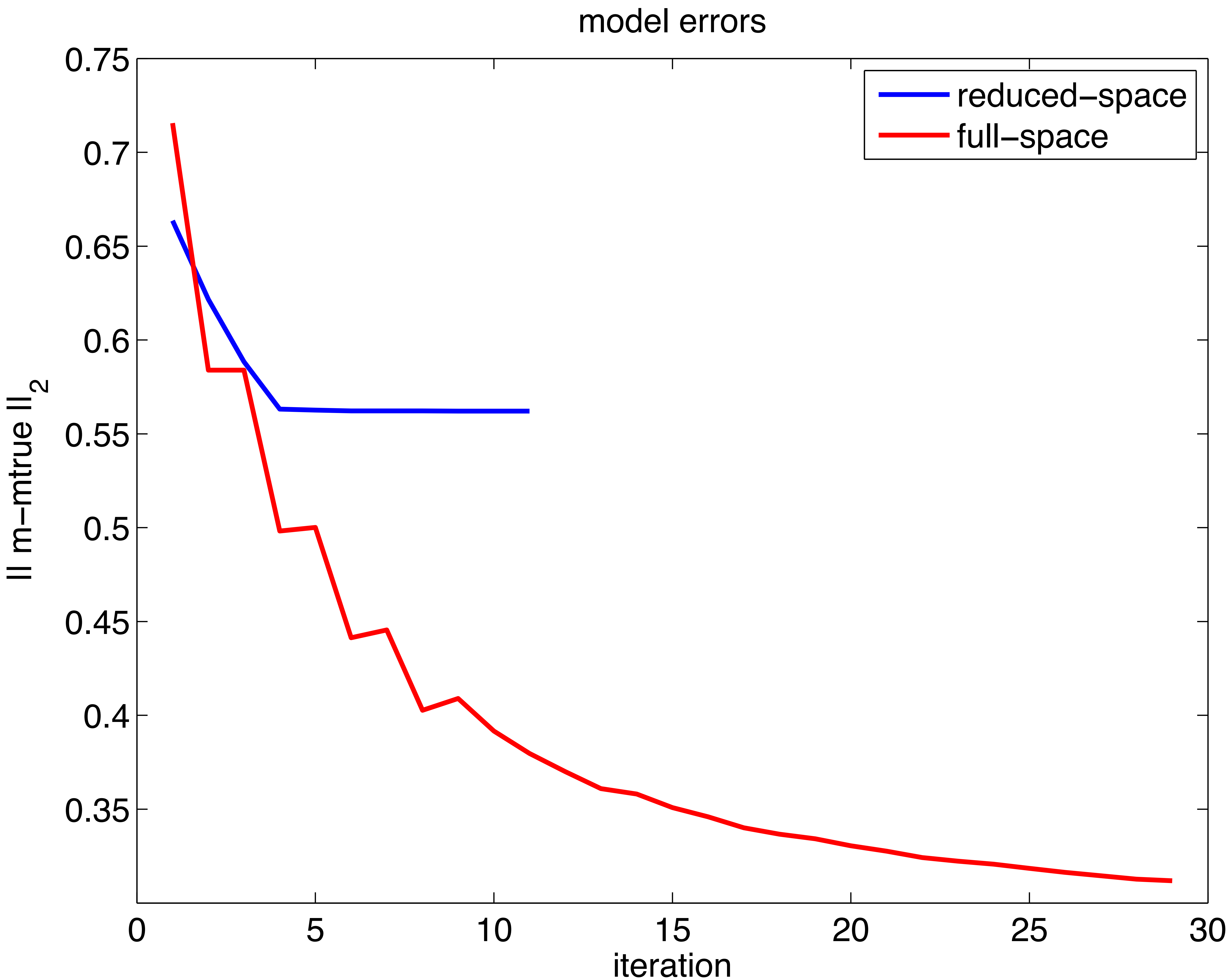
accurate iterative solutions



Toy problem

model errors,

inaccurate iterative solutions



Toy problem

Inexact iterative linear system solves:

- full-space method not very sensitive
- WRI & FWI quite sensitive

Full-space vs reduced-space methods

Bottom line

reduced-space: solve for the fields
update the medium parameters

full-space: update fields & medium parameters

Full-space vs reduced-space methods

Bottom line

reduced-space: solve for the fields
update the medium parameters ← alternating strategy

full-space: update fields & medium parameters

Full-space vs reduced-space methods

Bottom line

reduced-space: solve for the fields
update the medium parameters ← alternating strategy

full-space: update fields & medium parameters ← joint updating

Full-space vs reduced-space methods

FWI & WRI are in the reduced-space class –i.e., wave-equations are solved

Full-space is commonly used in a Lagrangian setting.

Because of memory requirements, rarely used in (academic) geophysics. **[EM: E. Haber et al., 2004 ; Seismic: M. J. Grothe et al., 2011]**

Short derivation

Problem formulation:

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}\|\mathbf{P}\mathbf{u}-\mathbf{d}\|_2^2 \quad \text{s.t.} \quad \mathbf{H}(\mathbf{m})\mathbf{u}=\mathbf{q}$$

Short derivation

Quadratic penalty form (WRI):

$$\phi(\mathbf{m},\mathbf{u},\lambda) = \frac{1}{2}\|\mathbf{P}\mathbf{u}-\mathbf{d}\|_2^2 + \frac{\lambda^2}{2}\|\mathbf{H}(\mathbf{m})\mathbf{u}-\mathbf{q}\|_2^2$$

Short derivation

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$$\begin{pmatrix}\mathbf{P}_1 & & & \\ & \mathbf{P}_2 & & \\ & & \ddots & \\ & & & \mathbf{P}_k\end{pmatrix}\begin{pmatrix}\mathbf{u}_1\\ \mathbf{u}_2\\ \vdots\\ \mathbf{u}_k\end{pmatrix} - \begin{pmatrix}\mathbf{d}_1\\ \mathbf{d}_2\\ \vdots\\ \mathbf{d}_k\end{pmatrix} \qquad \begin{pmatrix}\mathbf{H}_1 & & & \\ & \mathbf{H}_2 & & \\ & & \ddots & \\ & & & \mathbf{H}_k\end{pmatrix}\begin{pmatrix}\mathbf{u}_1\\ \mathbf{u}_2\\ \vdots\\ \mathbf{u}_k\end{pmatrix} - \begin{pmatrix}\mathbf{q}_1\\ \mathbf{q}_2\\ \vdots\\ \mathbf{q}_k\end{pmatrix}$$

Short derivation

SLIM

Newton's method

Hessian

gradient

$$\begin{pmatrix} \mathbf{P}^*\mathbf{P} + \lambda^2\mathbf{H}^*\mathbf{H} & \nabla^2_{\mathbf{u},\mathbf{m}}\phi \\ \nabla^2_{\mathbf{m},\mathbf{u}}\phi & \lambda^2\mathbf{G}^*_{\mathbf{m}}\mathbf{G}_{\mathbf{m}} \end{pmatrix} \begin{pmatrix} \delta_{\mathbf{u}} \\ \delta_{\mathbf{m}} \end{pmatrix} = - \begin{pmatrix} \mathbf{P}^*(\mathbf{Pu} - \mathbf{d}) + \lambda^2\mathbf{H}^*(\mathbf{Hu} - \mathbf{q}) \\ \lambda^2\mathbf{G}^*_{\mathbf{m}}(\mathbf{Hu} - \mathbf{q}) \end{pmatrix}$$

updates for medium parameters

updates for all fields

Short derivation

Newton's method:

$$\begin{pmatrix} \mathbf{P}^*\mathbf{P} + \lambda^2\mathbf{H}^*\mathbf{H} & \nabla^2_{\mathbf{u},\mathbf{m}}\phi \\ \nabla^2_{\mathbf{m},\mathbf{u}}\phi & \lambda^2\mathbf{G}^*_{\mathbf{m}}\mathbf{G}_{\mathbf{m}} \end{pmatrix} \begin{pmatrix} \delta_{\mathbf{u}} \\ \delta_{\mathbf{m}} \end{pmatrix} = -\begin{pmatrix} \mathbf{P}^*(\mathbf{P}\mathbf{u} - \mathbf{d}) + \lambda^2\mathbf{H}^*(\mathbf{H}\mathbf{u} - \mathbf{q}) \\ \lambda^2\mathbf{G}^*_{\mathbf{m}}(\mathbf{H}\mathbf{u} - \mathbf{q}) \end{pmatrix}$$

Short derivation

Approximate Hessian:

$$\begin{pmatrix} \mathbf{P}^*\mathbf{P} + \lambda^2\mathbf{H}^*\mathbf{H} & 0 \\ 0 & \lambda^2\mathbf{G}_{\mathbf{m}}^*\mathbf{G}_{\mathbf{m}} \end{pmatrix} \begin{pmatrix} \delta_{\mathbf{u}} \\ \delta_{\mathbf{m}} \end{pmatrix} = -\begin{pmatrix} \mathbf{P}^*(\mathbf{P}\mathbf{u} - \mathbf{d}) + \lambda^2\mathbf{H}^*(\mathbf{H}\mathbf{u} - \mathbf{q}) \\ \lambda^2\mathbf{G}_{\mathbf{m}}^*(\mathbf{H}\mathbf{u} - \mathbf{q}) \end{pmatrix}$$

Short derivation

Approximate Hessian:

$$\begin{pmatrix} \mathbf{P}^*\mathbf{P} + \lambda^2\mathbf{H}^*\mathbf{H} & 0 \\ 0 & \lambda^2\mathbf{G}_{\mathbf{m}}^*\mathbf{G}_{\mathbf{m}} \end{pmatrix} \begin{pmatrix} \delta_{\mathbf{u}} \\ \delta_{\mathbf{m}} \end{pmatrix} = - \begin{pmatrix} \mathbf{P}^*(\mathbf{Pu} - \mathbf{d}) + \lambda^2\mathbf{H}^*\big(\mathbf{Hu} - \mathbf{q}\big) \\ \lambda^2\mathbf{G}_{\mathbf{m}}^*\big(\mathbf{Hu} - \mathbf{q}\big) \end{pmatrix}$$

Can be solved inexactly (cheap)!

Algorithm

0. construct initial guess $\mathbf{m}$ for medium and $\mathbf{u}_i$ for each field

while not converged **do**

1. form Hessian and gradient `// form (~free)`
2. ignore the $\nabla^2_{\mathbf{u},\mathbf{m}}\phi, \nabla^2_{\mathbf{m},\mathbf{u}}\phi$ blocks `// approximate`
3. find $\delta\mathbf{m}$ & each $\delta\mathbf{u}_i$ in parallel `// solve`
4. find steplength α using linesearch `// evaluate (~free)`
5. $\mathbf{m} = \mathbf{m} + \alpha\delta\mathbf{m}$ & $\mathbf{u} = \mathbf{u} + \alpha\delta\mathbf{u}$ `// update model and fields`

end

Algorithm

0. construct initial guess $\mathbf{m}$ for medium and $\mathbf{u}_i$ for each field

while not converged **do**

1. form Hessian and gradient `// form (~free)`
2. ignore the $\nabla^2_{\mathbf{u},\mathbf{m}}\phi, \nabla^2_{\mathbf{m},\mathbf{u}}\phi$ blocks `// approximate`
3. find $\delta\mathbf{m}$ & each $\delta\mathbf{u}_i$ in parallel `// solve` ← medium and field updates are independent
4. find steplength α using linesearch `// evaluate (~free)`
5. $\mathbf{m} = \mathbf{m} + \alpha\delta\mathbf{m}$ & $\mathbf{u} = \mathbf{u} + \alpha\delta\mathbf{u}$ `// update model and fields`

end

Algorithm

0. construct initial guess $\mathbf{m}$ for medium and $\mathbf{u}_i$ for each field

while not converged **do**

depend on the updated model and updated fields

1. form Hessian and gradient `// form (~free)`
2. ignore the $\nabla^2_{\mathbf{u},\mathbf{m}}\phi, \nabla^2_{\mathbf{m},\mathbf{u}}\phi$ blocks `// approximate`
3. find $\delta\mathbf{m}$ & each $\delta\mathbf{u}_i$ in parallel `// solve`
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end

Full-space vs reduced-space methods

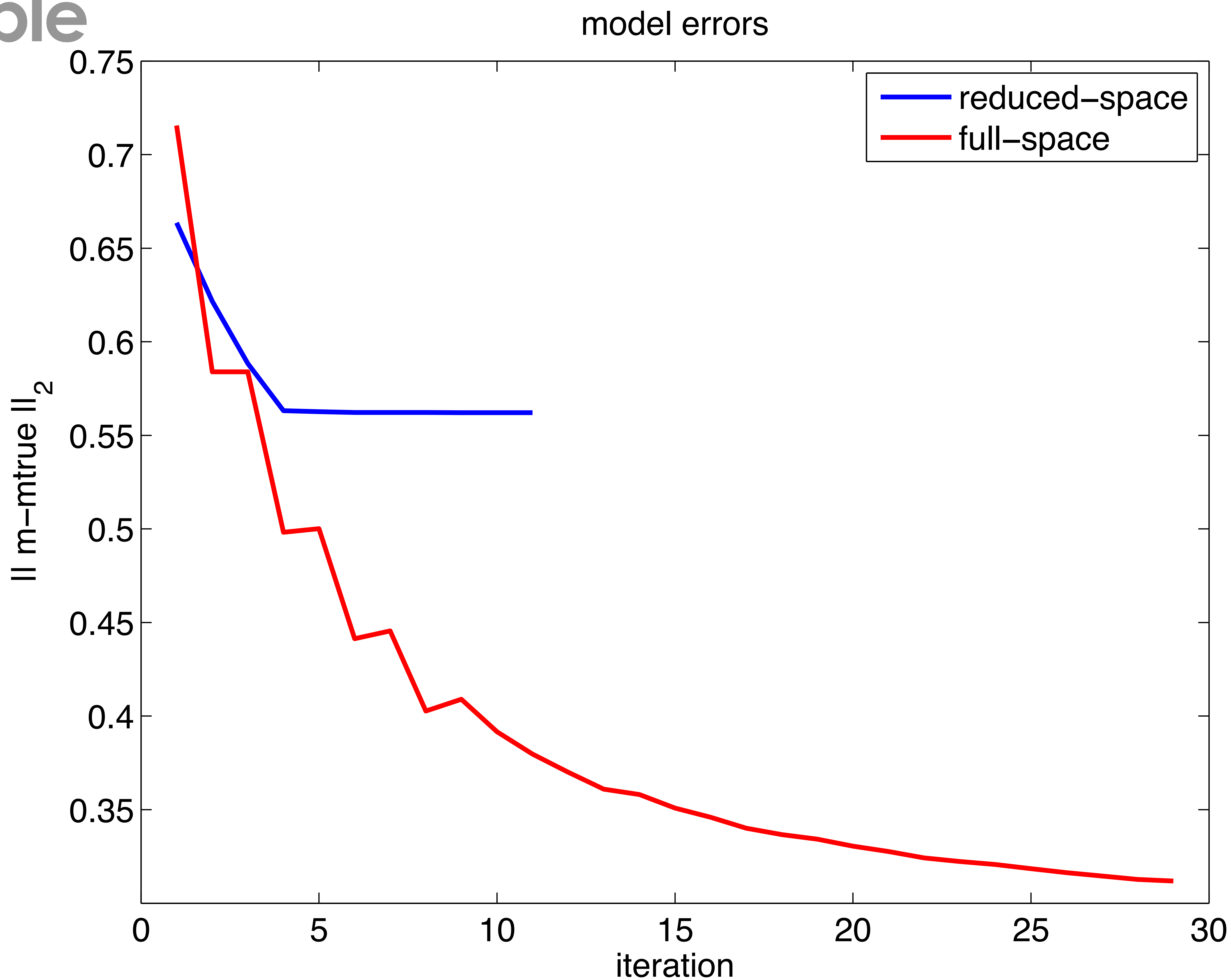
	FWI & WRI	full
Hessian	dense	sparse
Hessian	solve “PDE’s”	~free
gradient	solve “PDE’s”	~free
memory	2 fields per parallel process	all fields in memory (can be distributed over nodes)
function evaluation	solve “PDE’s”	~free

~free = sparse matrix-vector products

Previous example

model errors,

inaccurate iterative solutions



Inexact full-space vs inexact reduced-space

FWI & WRI:

- error in objective function value
- error in gradient
- error in Hessian

(error in gradient, error in Hessian) → error in medium parameter update

Inexact full-space vs inexact reduced-space

FWI & WRI:

- error in objective function value
- error in gradient
- error in Hessian

→ error in medium parameter update

Full-space from WRI:

- objective function value always exact
- gradient always exact
- Hessian always exact

→ 0 iterations → gradient descent
many iterations → Newton's method

Toy examples

Using a direct solver:

- similar reconstruction quality compared to WRI+diagonal Hessian approximation
- need to test on more realistic models.

Memory requirements

save all fields for all frequencies & sources

can be distributed over multiple nodes

Feasible? Need

- parallel computing
- simultaneous sources
- small frequency batches

Computational cost

Independent update computation

No communication between compute nodes to compute updates

1 iteration of WRI ≈ 1 iteration of full-space Newton type quadratic penalty

Conclusions

Constructed a full-space method which:

- updates fields & medium parameters simultaneously
- computational cost ≈ reduced-space methods
- similar parallelism as in FWI & WRI
- many properties are different from FWI & WRI
- promising results with iterative solvers

- con: need to store all fields
- but, less storage needed compared to Lagrangian full-space methods

Current & future work

Test on more realistic examples.
Evaluate reconstruction quality compared to WRI.
Maximize benefit from inexact update computation.

Acknowledgements

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