Use what's in common: time-lapse FWI with distributed Compressive Sensing

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Preamble

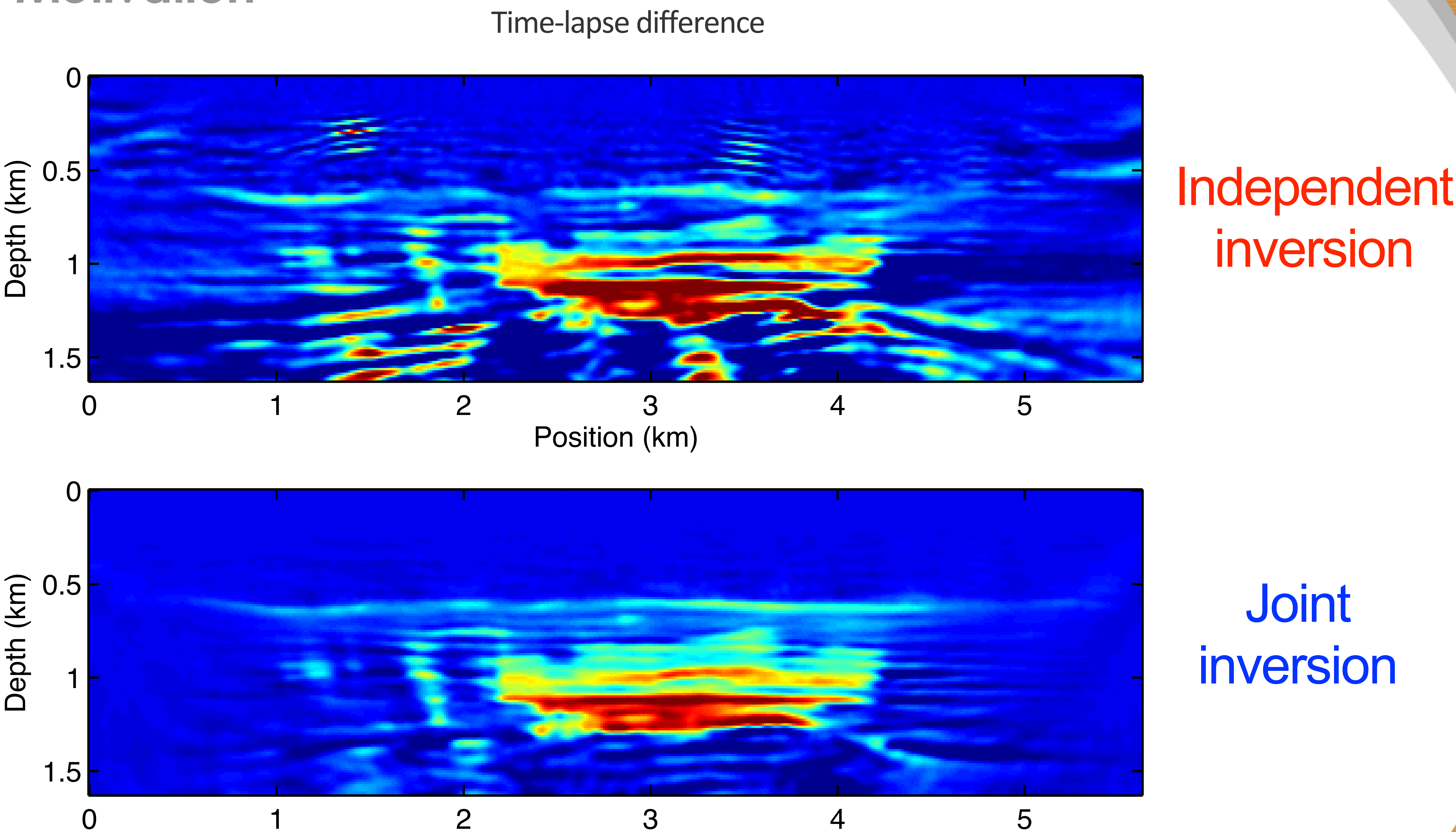
Our first attempt at FWI for time-lapse seismic

Using ideas from distributed Compressive Sensing

Fast inversion formulation

Improved time-lapse inversion results

Motivation


Time-lapse difference
Depth (km)
0
0.5
1
1.5
0
1
2
3
4
5
Position (km)
Independent inversion
Joint inversion

Formulation

Full-waveform inversion

$$\underset{\mathbf{m},\alpha}{\text{minimize}}\ \frac{1}{2}\|\mathbf{D} - \alpha\boldsymbol{\mathcal{F}}[\mathbf{m}]\|_F^2$$

$\mathbf{D}$: observed data

$\boldsymbol{\mathcal{F}}$: forward modelling kernel

α : source wavelet

$\mathbf{m}$: model parameters

Xiang Li, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann,
"Fast randomized full-waveform inversion with compressive sensing",
Geophysics, vol. 77, p. A13-A17, 2012.

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Formulation

Modified Gauss-Newton

$$\tilde{\mathbf{x}}^k = \arg\min_{\mathbf{x}} \frac{1}{2} \| \underbrace{\mathbf{D}^k - \boldsymbol{\mathcal{F}}(\mathbf{m}^k)}_{\mathbf{b}} - \nabla \underbrace{\boldsymbol{\mathcal{F}}(\mathbf{m}^k)\mathbf{C}^T}_{\mathbf{A}} \underset{\mathbf{x}}{\mathbf{x}} \|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}\|_1 < \tau^k$$

$$\mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{C}^T \tilde{\mathbf{x}}$$

Full-waveform inversion w/ time-lapse

Independent inversion:

$$\text{for} \quad i = 1, 2$$

$$\tilde{\mathbf{x}}_i^k = \arg\min_{\mathbf{x}_i} \frac{1}{2} \| \underbrace{\mathbf{D}_i^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)}_{\mathbf{b}_i} - \nabla \underbrace{\boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)\mathbf{C}^T}_{\mathbf{A}_i} \underbrace{\mathbf{x}_i}_{\mathbf{x}_i} \|_2^2 \quad \text{s.t.} \quad \|\mathbf{x}_i\|_1 < \tau_i^k$$

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T \tilde{\mathbf{x}}_i$$

Objective: Invert for baseline, monitor and difference

....but time-lapse data/models/images share information.

Dror Baron , Marco F. Duarte , Shriram Sarvotham , Michael B. Wakin , Richard G. Baraniuk. An Information-Theoretic Approach to Distributed Compressed Sensing (2005)

Distributed compressive sensing
– *joint* recovery model (JRM)

vintages

$$\mathbf{x}_1 = \mathbf{z}_0 + \mathbf{z}_1$$
$$\mathbf{x}_2 = \mathbf{z}_0 + \mathbf{z}_2$$

differences

common component

$$\overbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{A}_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}^{\mathbf{z}} = \overbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}}^{\mathbf{b}}$$

baseline

monitor

Key idea:

1. use the fact that *different* vintages *share* common information
2. invert for *common* components & *differences* w.r.t. the *common* components with *sparse* recovery

Joint full-waveform inversion w/ time-lapse

Joint inversion:

$$\tilde{\mathbf{z}}_k = \arg\min_{\mathbf{z}_k} \frac{1}{2}\|\underbrace{\mathbf{D}_i^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)}_{\mathbf{b}_k} - \nabla\underbrace{\boldsymbol{\mathcal{F}}(\mathbf{m}_i^k)\mathbf{C}^T}_{\mathbf{A}_k}\mathbf{z}_k\|_2^2 \quad \text{s.t.} \quad \|\mathbf{z}_k\|_1 < \tau^k$$

$$\mathbf{z}_k$$

Model update:

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T(\tilde{\mathbf{z}}_0^k + \tilde{\mathbf{z}}_i^k)$$

Joint full-waveform inversion w/ time-lapse

Joint inversion:

$$\mathbf{b}_k = \begin{bmatrix} \mathbf{D}_1^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_1^k) \\ \mathbf{D}_2^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_2^k) \end{bmatrix}$$

$$\mathbf{A}_k = \begin{bmatrix} \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_1^k)\mathbf{C}^T & \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_1^k)\mathbf{C}^T & \mathbf{0} \\ \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_2^k)\mathbf{C}^T & \mathbf{0} & \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_2^k)\mathbf{C}^T \end{bmatrix}$$

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{z}_0^k \\ \mathbf{z}_1^k \\ \mathbf{z}_2^k \end{bmatrix}$$

$$\mathbf{m}_i^{k+1} = \mathbf{m}_i^k + \mathbf{C}^T(\tilde{\mathbf{z}}_0^k + \tilde{\mathbf{z}}_i^k)$$

How is this different from recently published methods ?

Other joint inversion methods

Robust joint full-waveform inversion of time-lapse seismic data sets with total-variation regularization

Musa Maharramov, Biondo Biondi

$$\alpha\|\mathbf{D}_b^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_b^k) - \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_b^k)\delta\mathbf{m}_b^k\|_2^2$$

$$+\beta\|\mathbf{D}_m^k - \boldsymbol{\mathcal{F}}(\mathbf{m}_m^k) - \nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_m^k)\delta\mathbf{m}_m^k\|_2^2$$

$$+\cdots+\delta\|\mathbf{WR}(\mathbf{m}_b^k - \mathbf{m}_m^k) - \boldsymbol{\Delta}\mathbf{m}^{\mathrm{PRIOR}}\|_1 \longrightarrow \text{min}$$

Model update:

$$\mathbf{m}_b^{k+1} = \mathbf{m}_b^k + \delta\mathbf{m}_b^k$$

$$\mathbf{m}_m^{k+1} = \mathbf{m}_m^k + \delta\mathbf{m}_m^k$$

Our approach:

$$\mathbf{m}_b^{k+1} = \mathbf{m}_b^k + \boxed{\delta\mathbf{m}_0^k} + \delta\mathbf{m}_b^k$$

$$\mathbf{m}_m^{k+1} = \mathbf{m}_m^k + \boxed{\delta\mathbf{m}_0^k} + \delta\mathbf{m}_m^k$$

Other joint inversion methods

Robust joint full-waveform inversion of time-lapse seismic data sets with total-variation regularization

Musa Maharramov, Biondo Biondi

$$\alpha\|\mathbf{D}_b^k - \mathcal{F}(\mathbf{m}_b^k) - \nabla\mathcal{F}(\mathbf{m}_b^k)\delta\mathbf{m}_b^k\|_2^2$$

$$+\beta\|\mathbf{D}_m^k - \mathcal{F}(\mathbf{m}_m^k) - \nabla\mathcal{F}(\mathbf{m}_m^k)\delta\mathbf{m}_m^k\|_2^2$$

$$+\cdots+\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_b^k - \mathbf{m}_m^k) - \Delta\mathbf{m}^{\text{PRIOR}}\|_1 \longrightarrow \text{min}$$

Model update:

$$\mathbf{m}_b^{k+1} = \mathbf{m}_b^k + \delta\mathbf{m}_b^k$$

$$\mathbf{m}_m^{k+1} = \mathbf{m}_m^k + \delta\mathbf{m}_m^k$$

No guaranteee of improved vintages

Our approach:

$$\mathbf{m}_b^{k+1} = \mathbf{m}_b^k + \boxed{\delta\mathbf{m}_0^k} + \delta\mathbf{m}_b^k$$

$$\mathbf{m}_m^{k+1} = \mathbf{m}_m^k + \boxed{\delta\mathbf{m}_0^k} + \delta\mathbf{m}_m^k$$

Good vintage recovery assured

Other joint inversion methods

Time-lapse image-domain tomography using adjoint-state methods

Jeffrey Shragge, Tongning Yang and Paul Sava

Minimize image imperfections

$$\mathcal{H}_1 = \frac{1}{2}\|P_1(\mathbf{x},\lambda)\mathbf{r}_1(\mathbf{x},\lambda)\|^2_{\mathbf{x},\lambda}$$

$\mathbf{r}_1(\mathbf{x},\lambda)$ Extended image gather volume

$P_1(\mathbf{x},\lambda) = |\lambda|$ Differential semblance operator

$$\mathcal{H}_2 = \frac{1}{2}\|P_2(\mathbf{x},\lambda)\mathbf{r}_2(\mathbf{x},\lambda)\|^2_{\mathbf{x},\lambda}$$

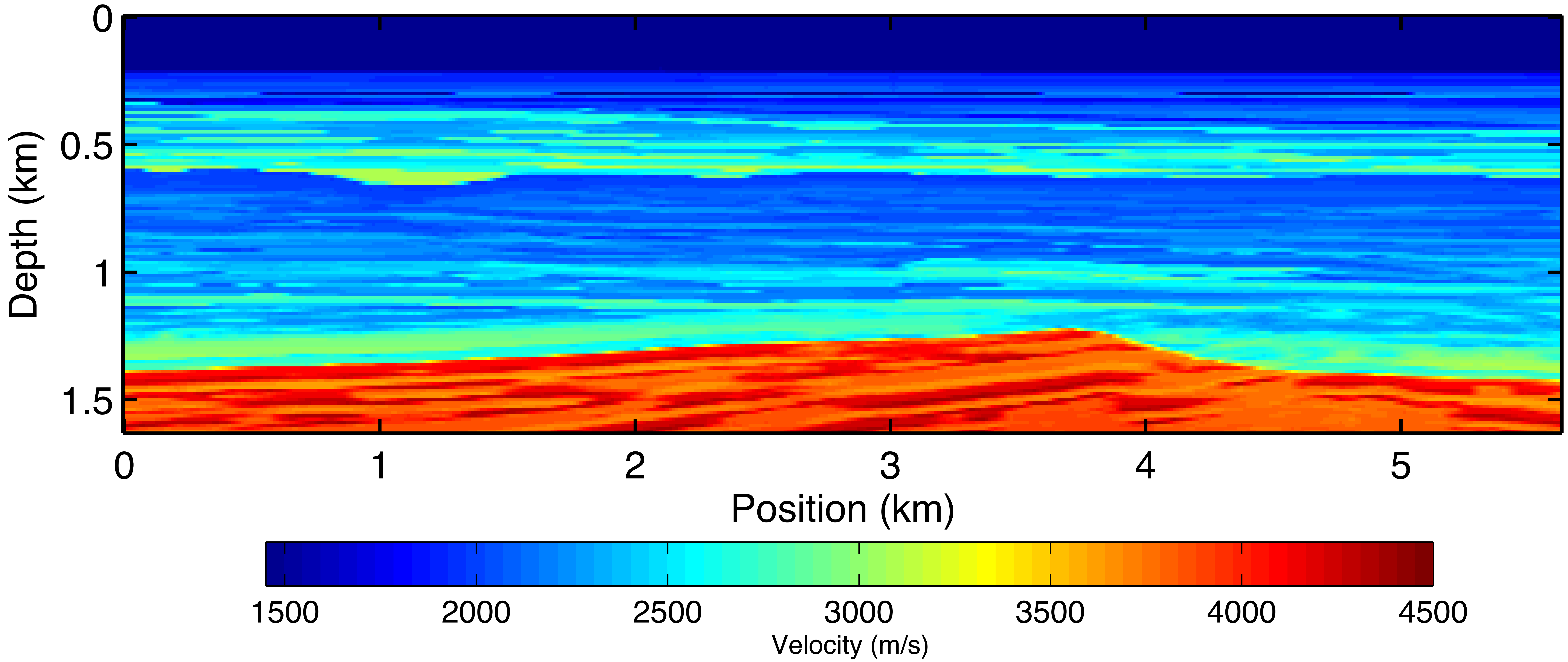
$$P_2 = P_{4D}(\mathbf{r}_1)$$

$$P_{4D} = \mathrm{sech}^2(< \mathbf{r}_1^2 > / \max(< \mathbf{r}_1^2 >))$$

Application

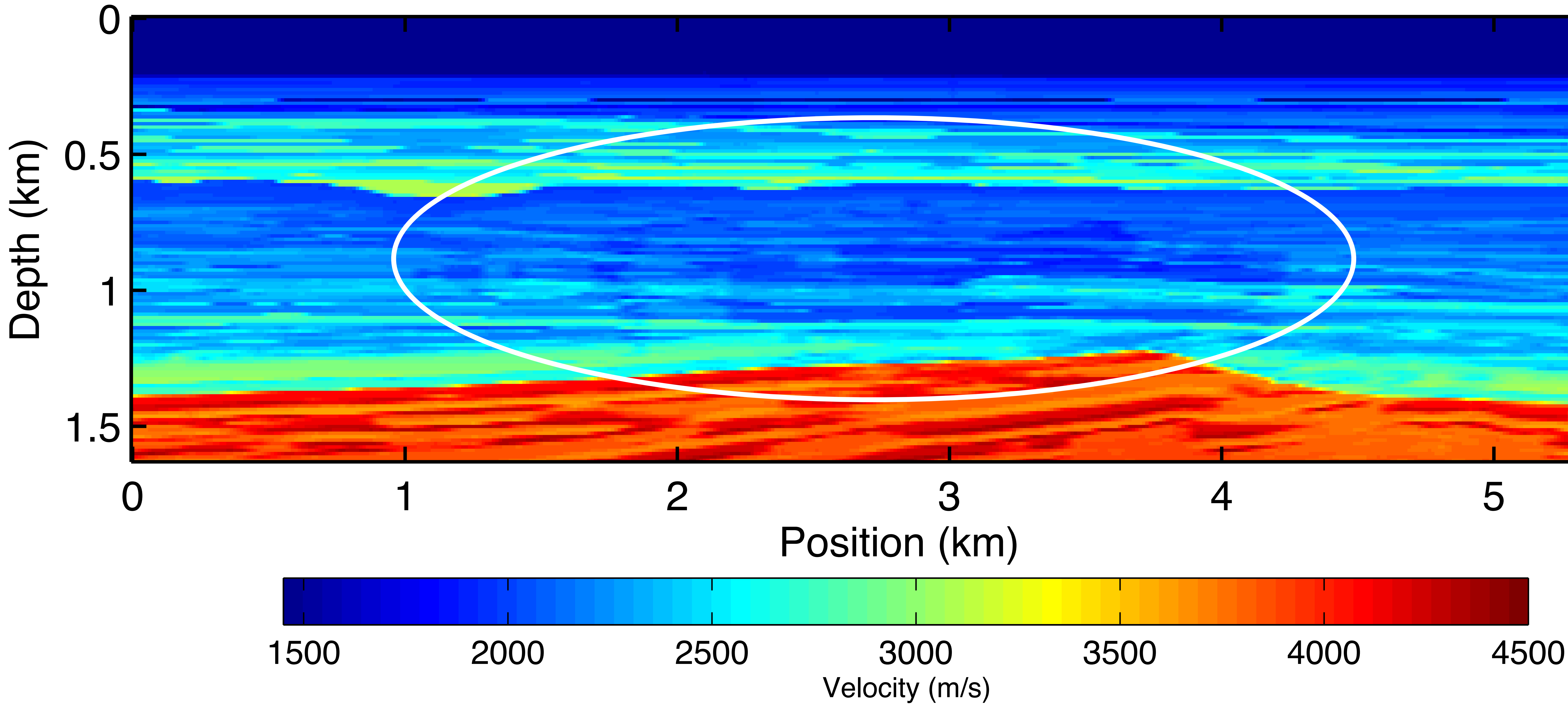
Baseline

BG Compass model

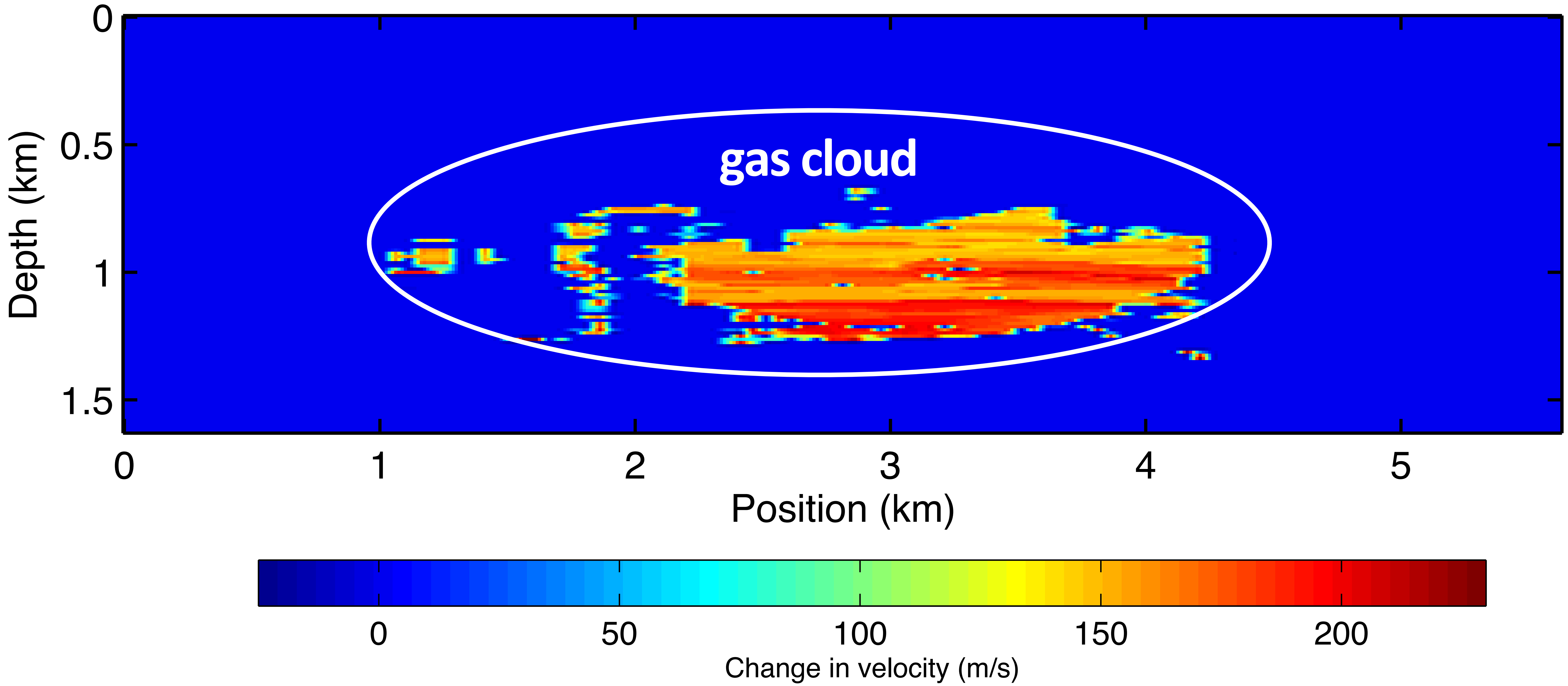


Monitor

BG Compass model

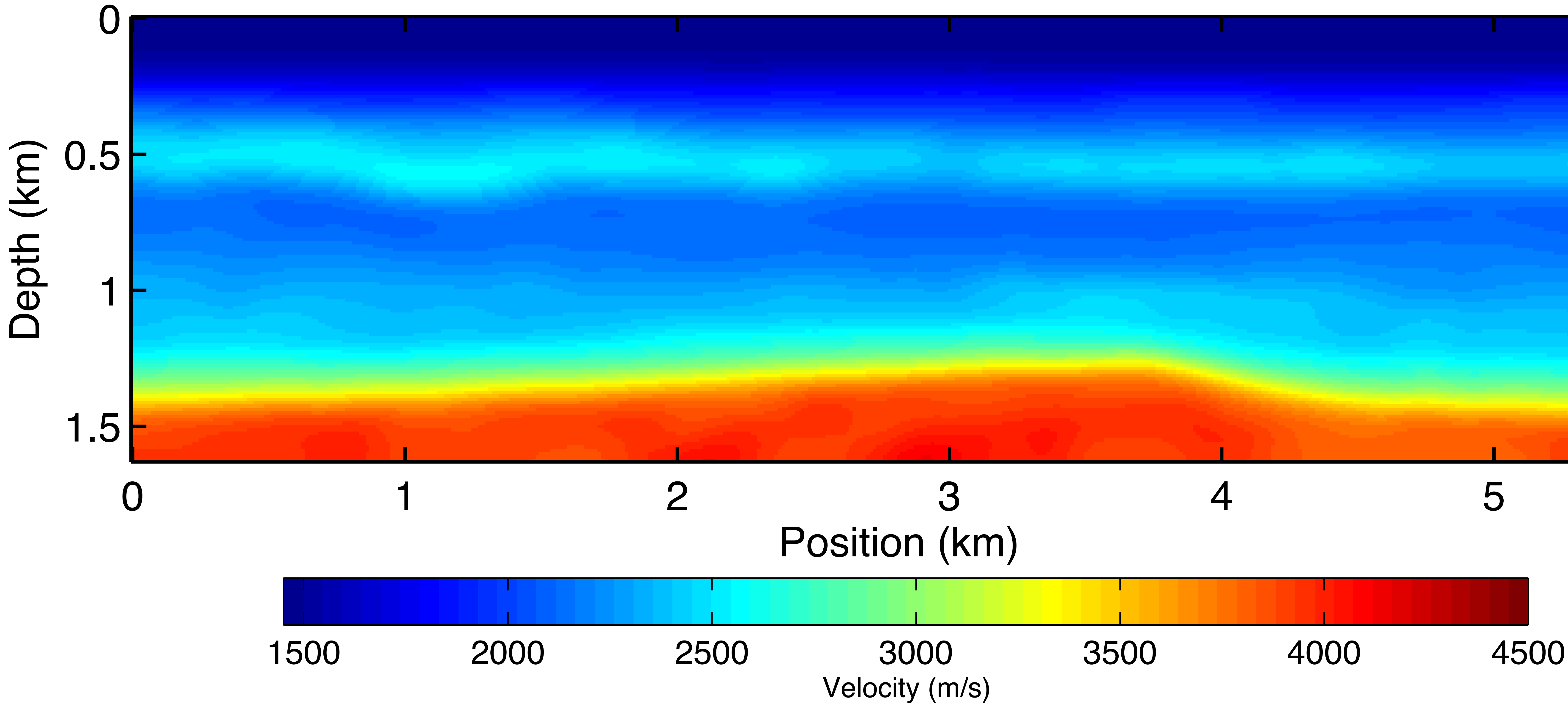


Time-lapse



gas cloud
Depth (km)
0
0.5
1
1.5
0
1
2
3
4
5
Position (km)
0
50
100
150
200
Change in velocity (m/s)

Starting model



Xiang Li, Aleksandr Y. Aravkin, Tristan van Leeuwen, and Felix J. Herrmann, "Fast randomized full-waveform inversion with compressive sensing", *Geophysics*, vol. 77, p. A13-A17, 2012.

Example

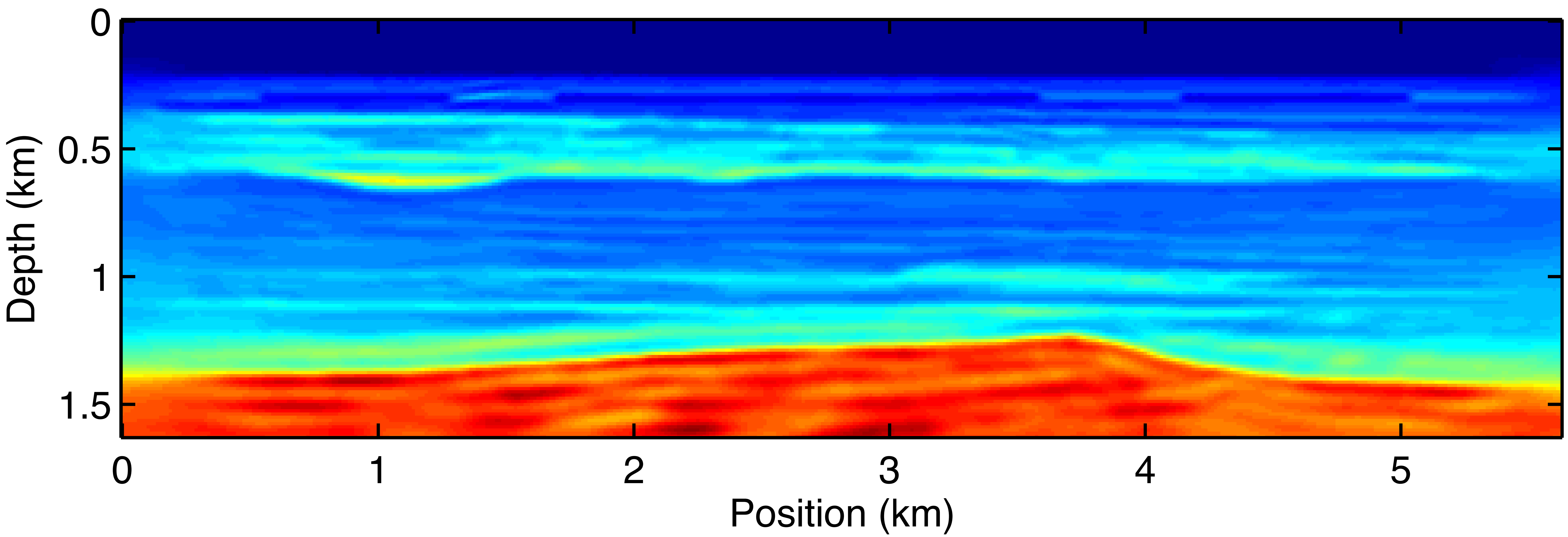
Modeling parameters

- 150 shots randomly sampled @ minimum 12.5m, maximum 137.5m interval
- 226 receivers @ 25m interval
- 80 frequencies from 3 to 22.5Hz
- Ricker wavelet @ 12Hz
- Maximum offset @ 5.6km

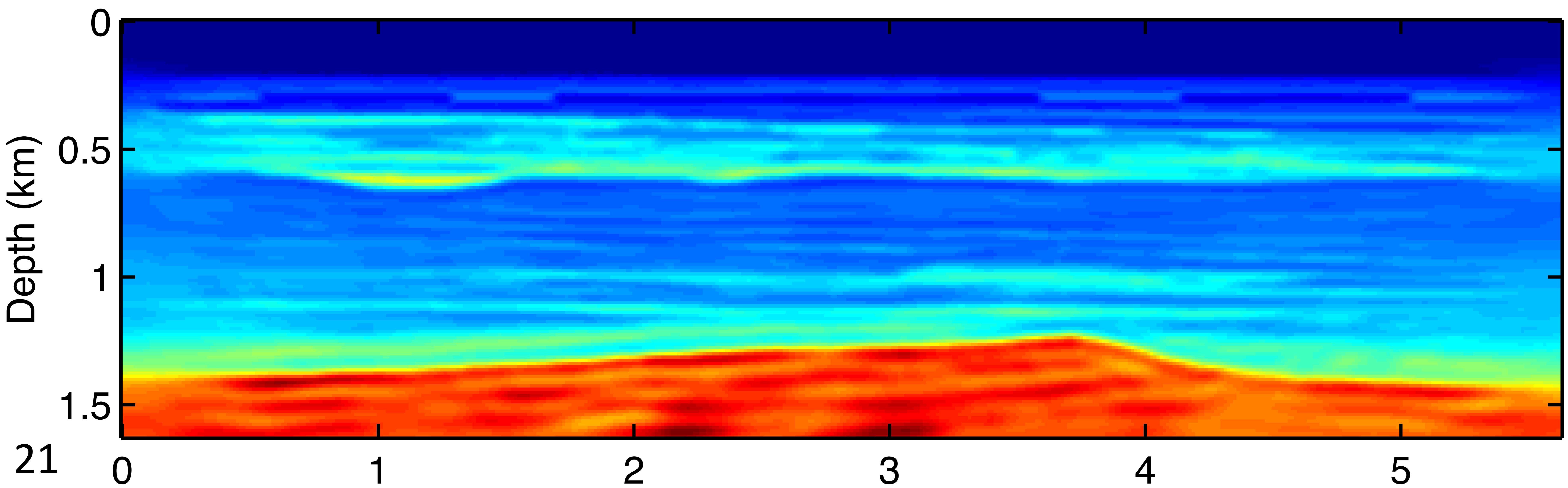
Modified Gauss-Newton

- Assume *good* initial model
- *Baseline* : use few simultaneous shots, with renewal
- *Monitor* : repeat similar encoding as baseline
- Started inversion at 3Hz
- 8 frequencies per band
- 10 Gauss-Newton subproblems per band
- Approximately 10 iterations per subproblem

Baseline inversion

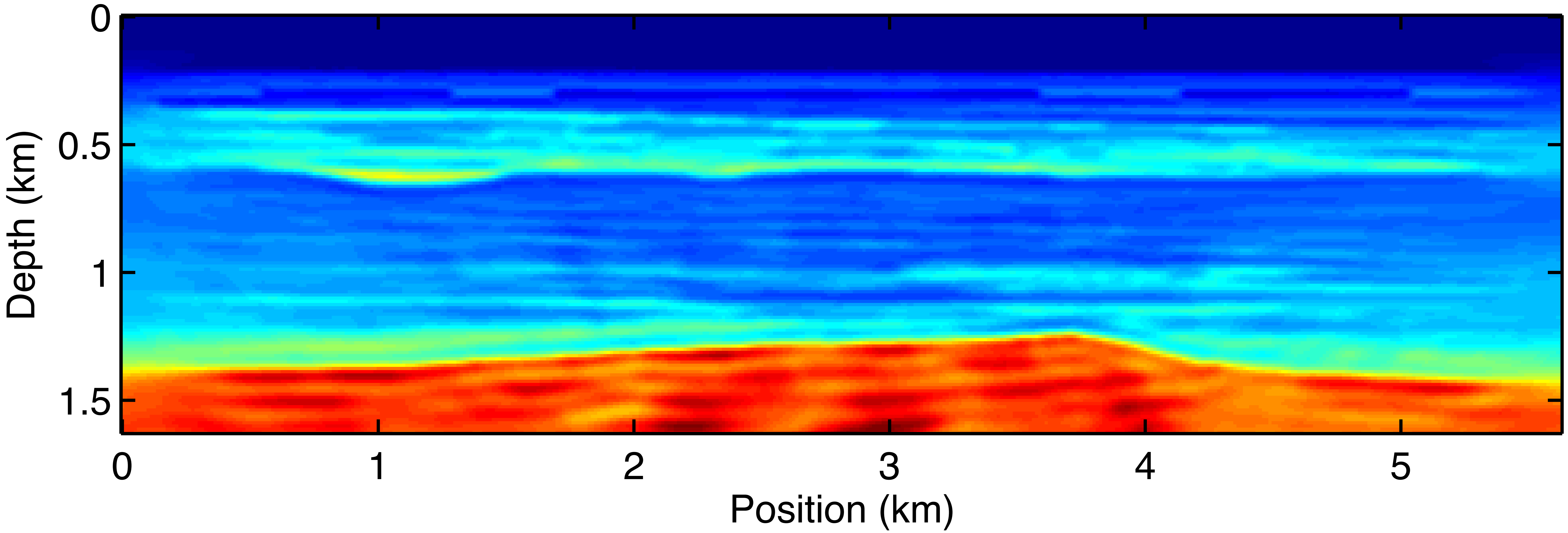


Independent

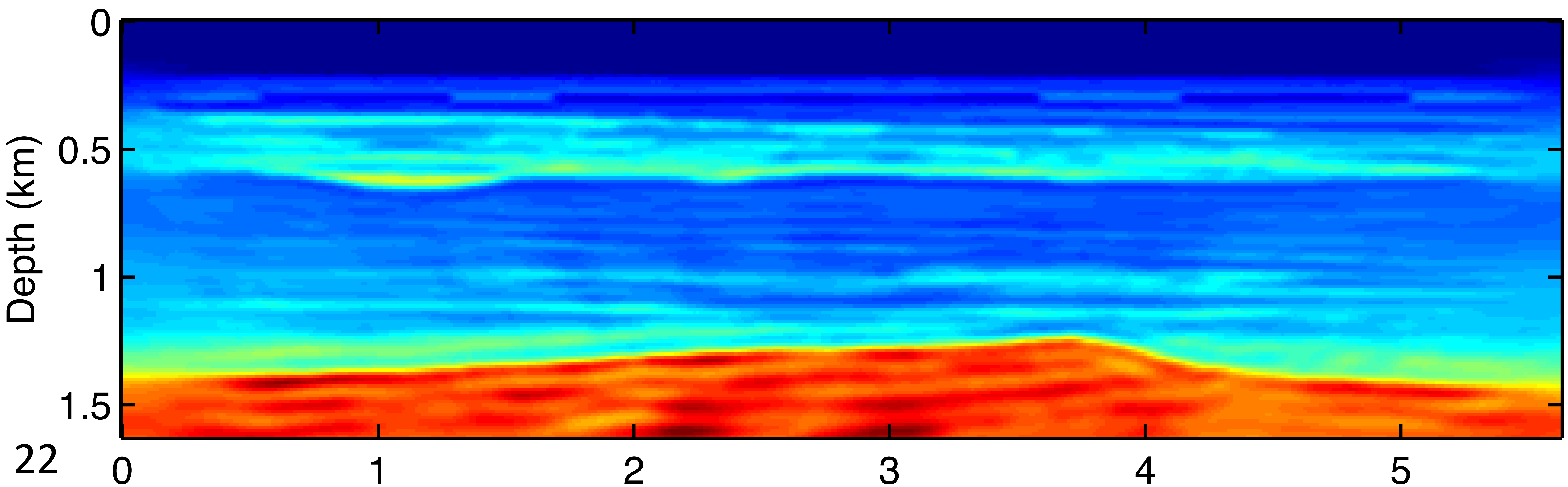


Joint

Monitor inversion

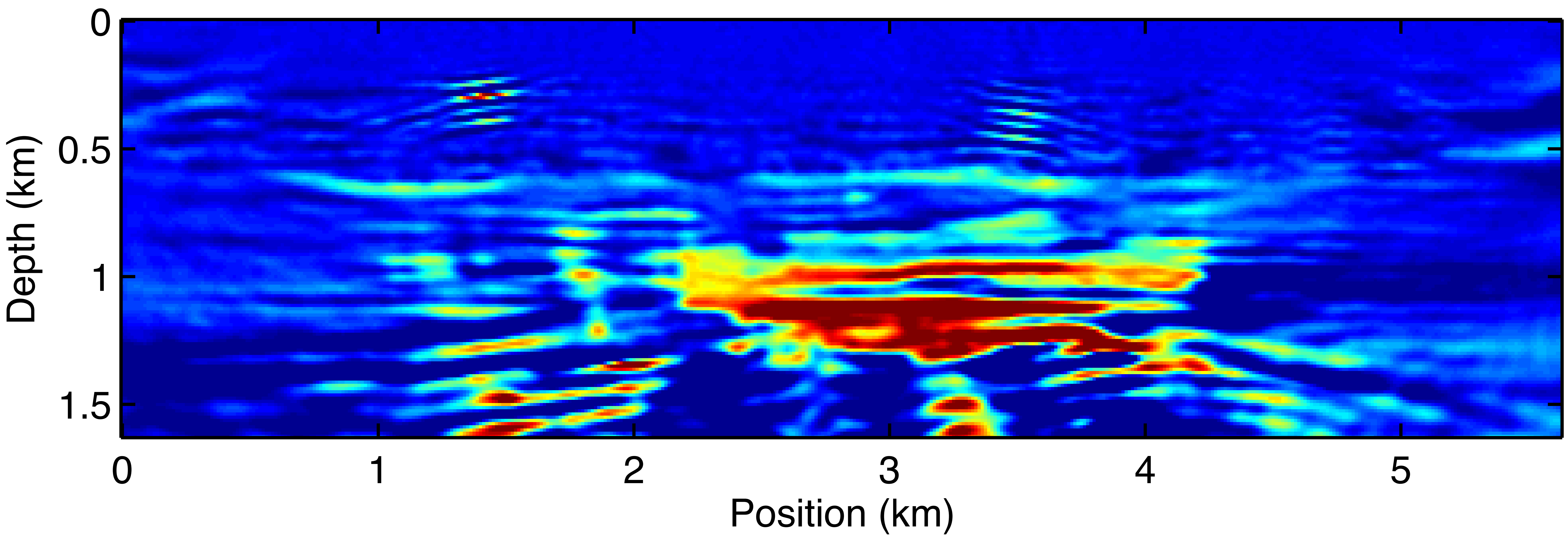


Independent

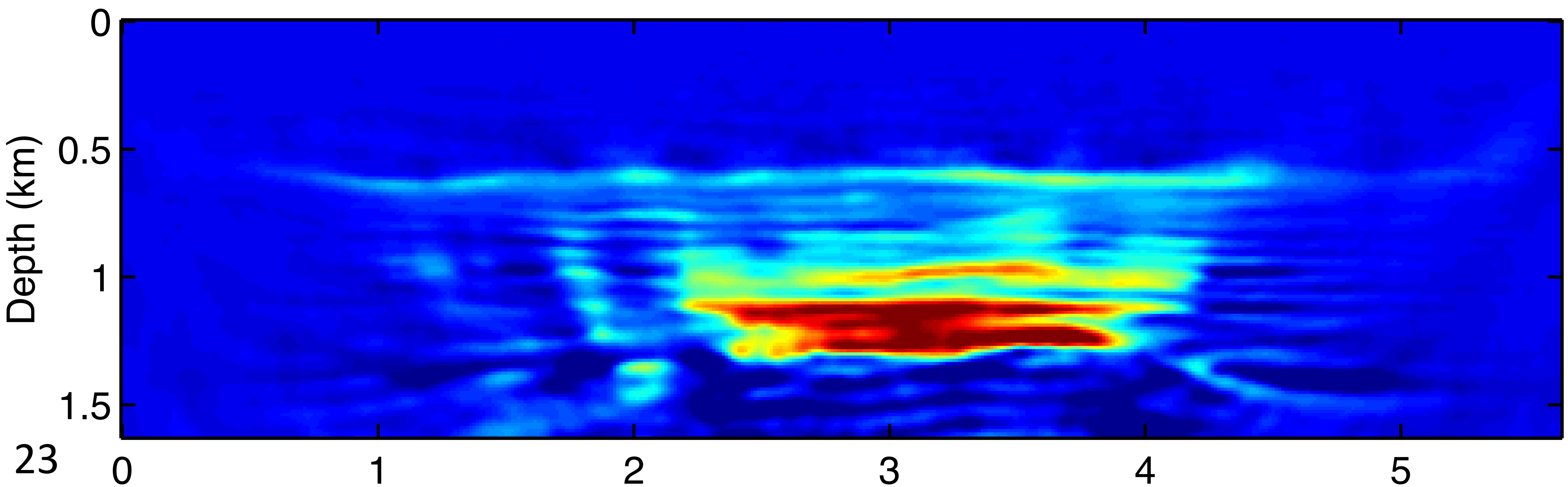


Joint

Time-lapse difference

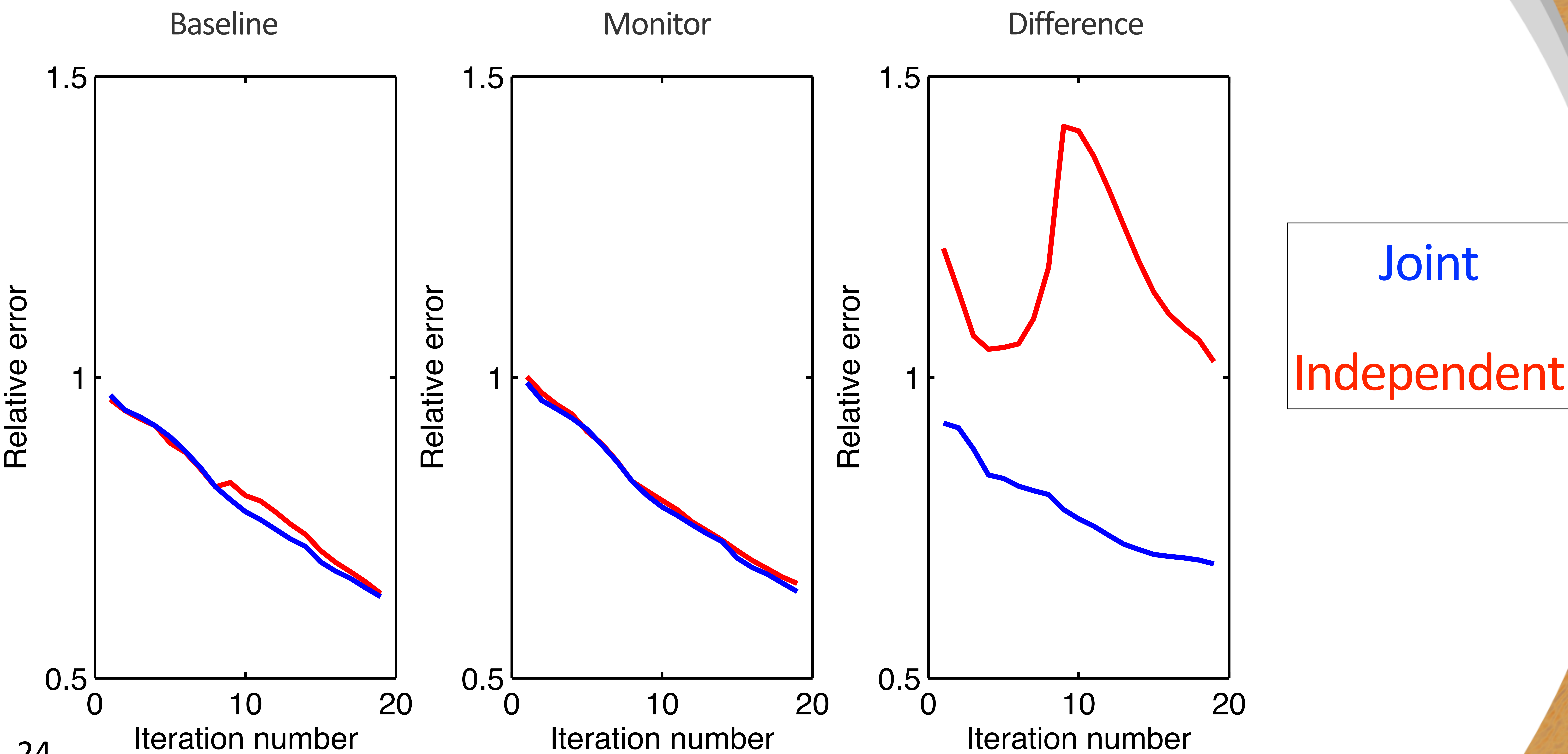


Independent



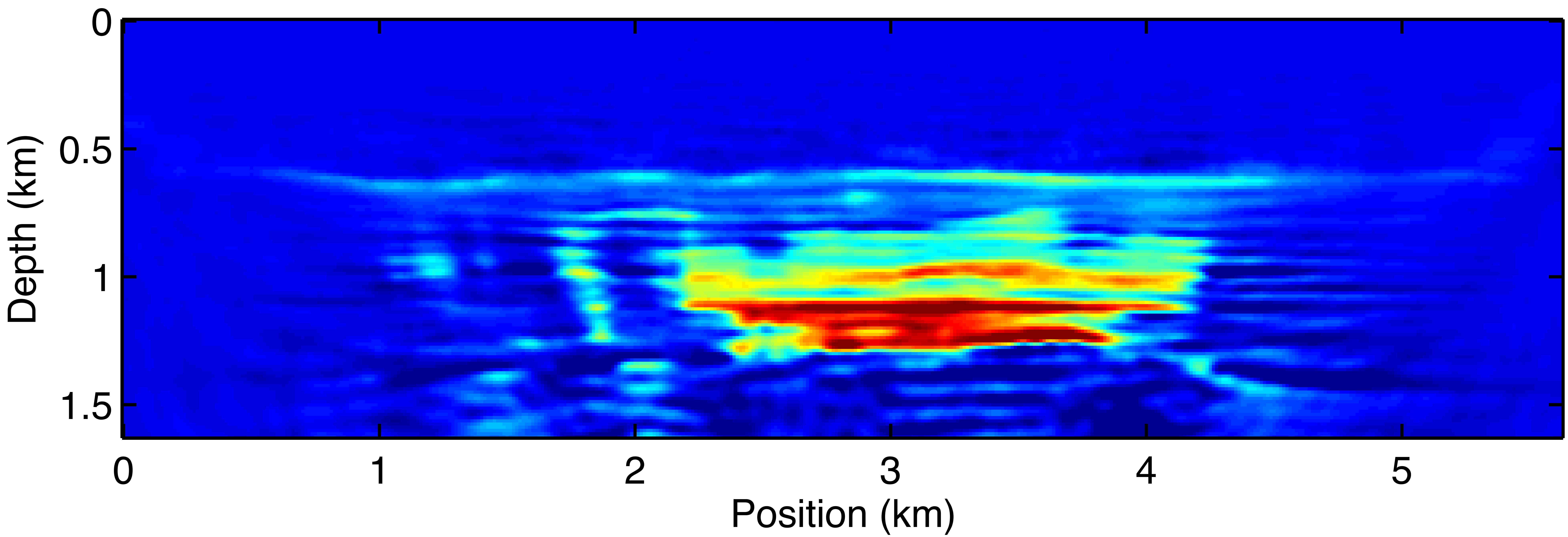
Joint

Model error

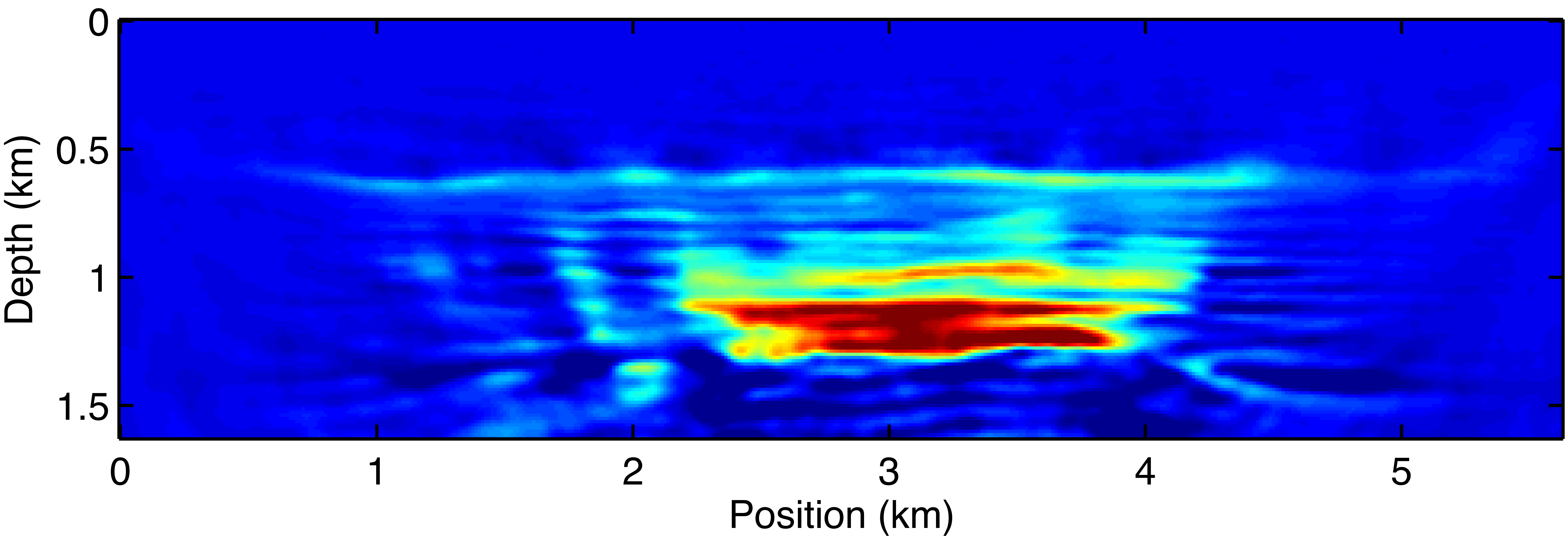

Baseline
Monitor
Difference
1.5
1
0.5
0
10
20
Relative error
Iteration number
Joint
Independent

After the second pass with the *joint recovery model*

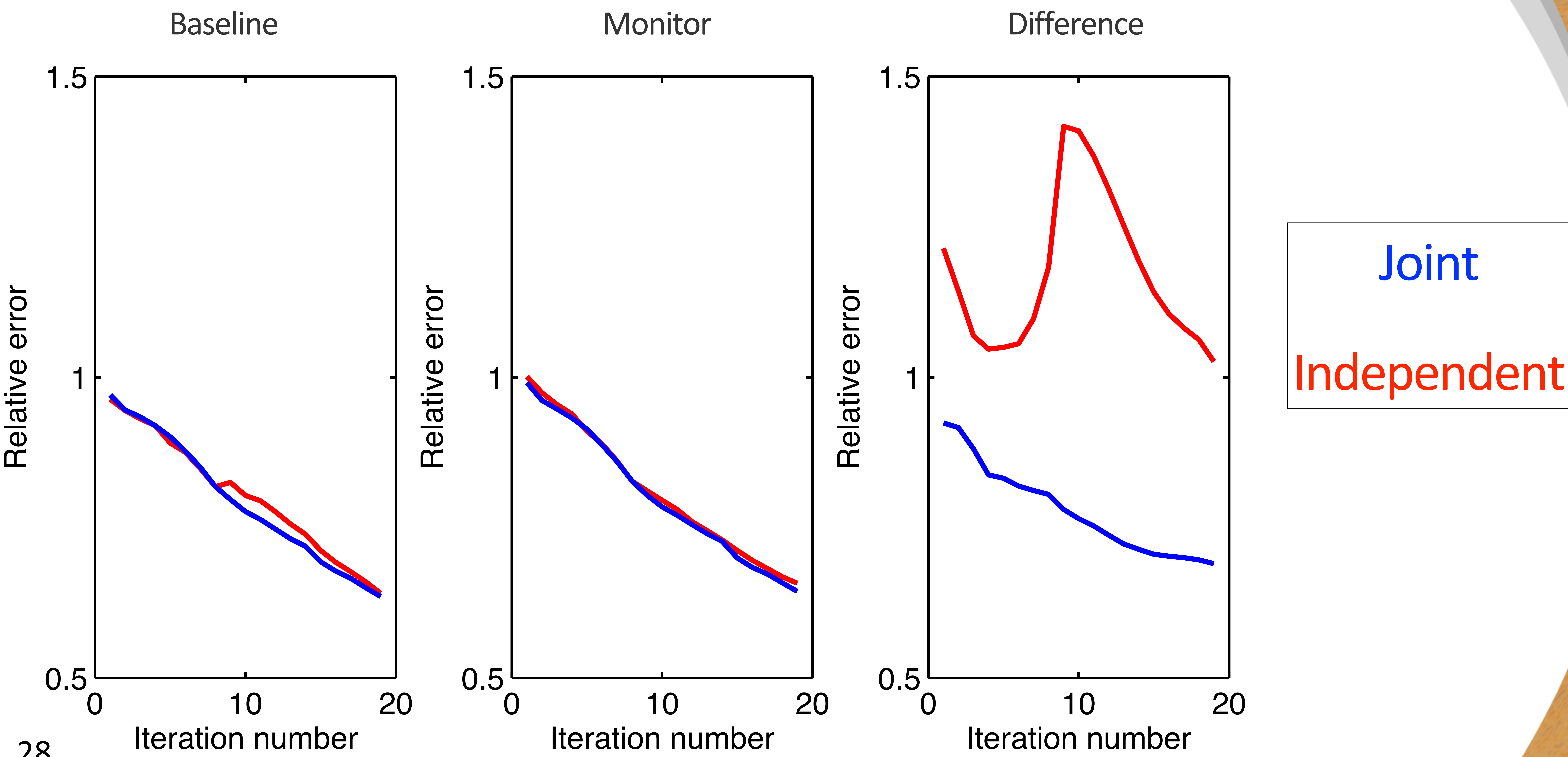
Second pass



First pass

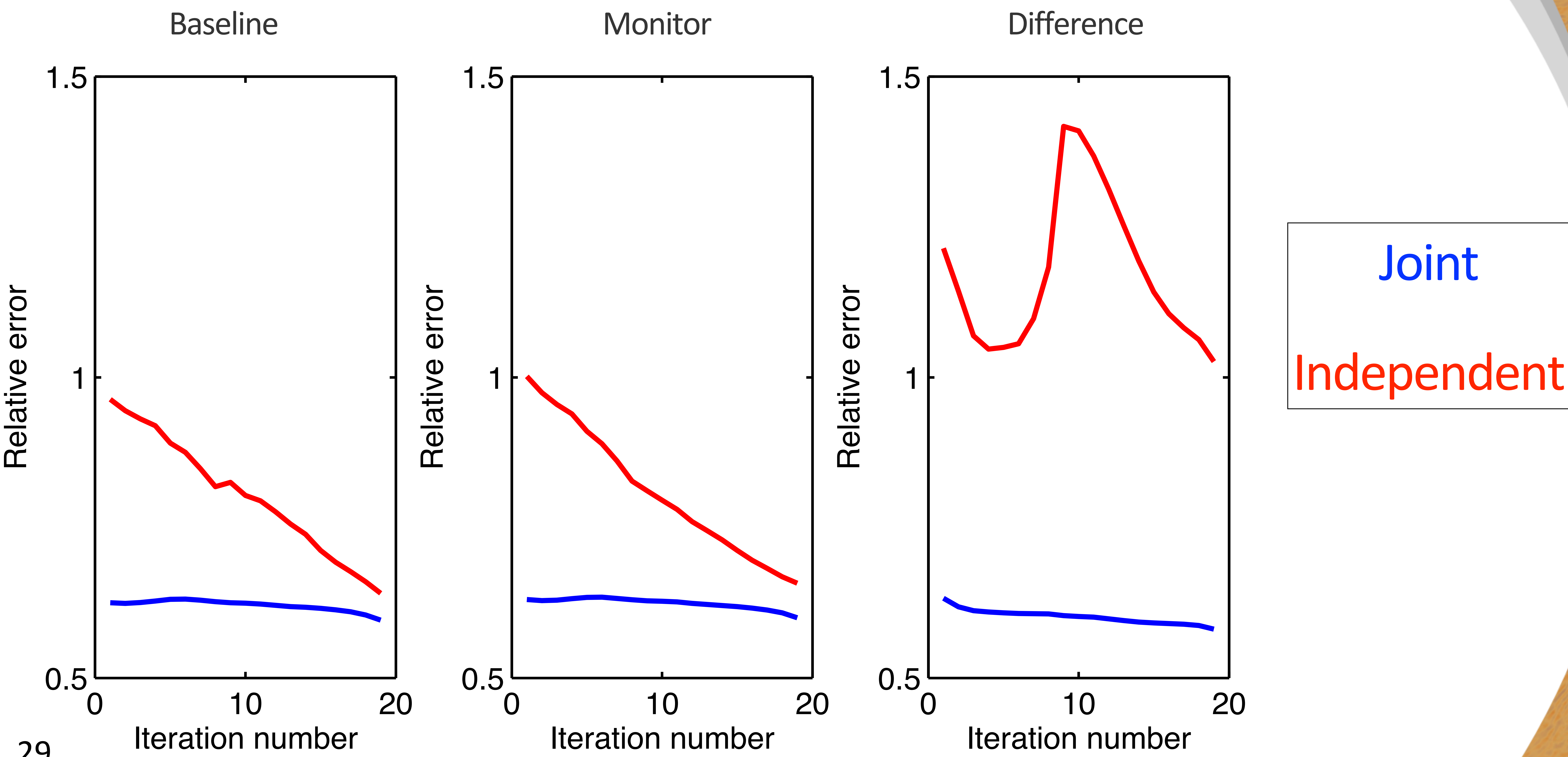


Model error – pass 1

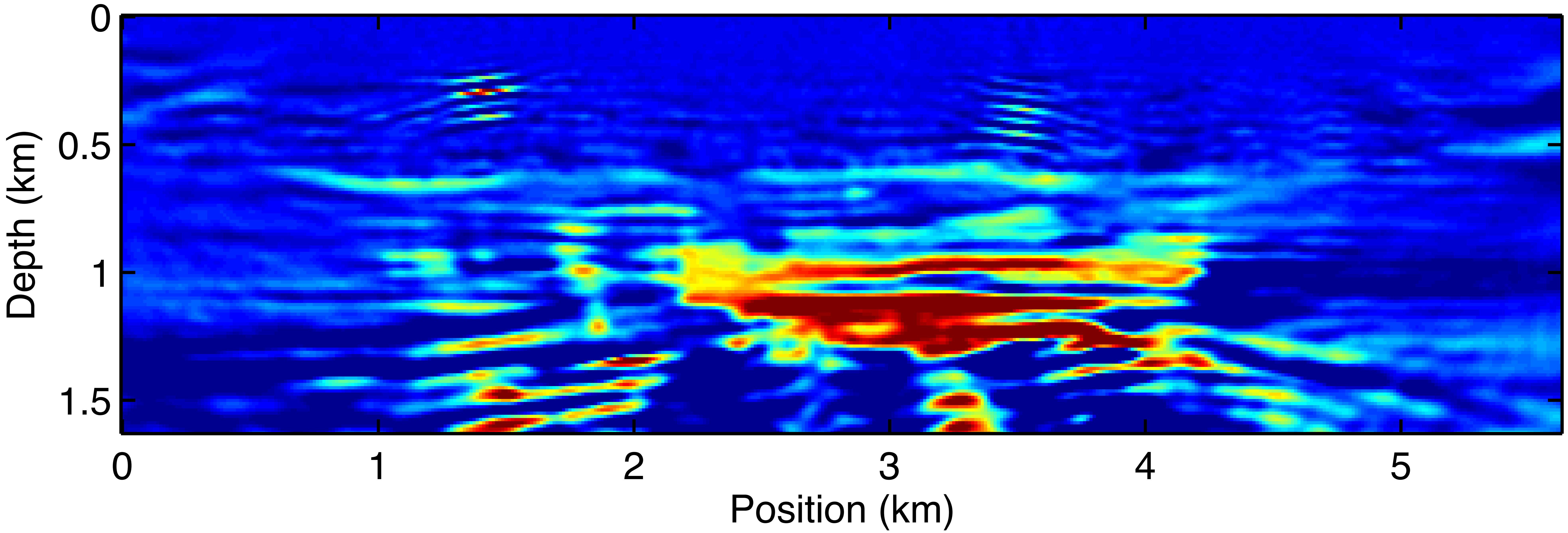


Tuesday, December 9, 14

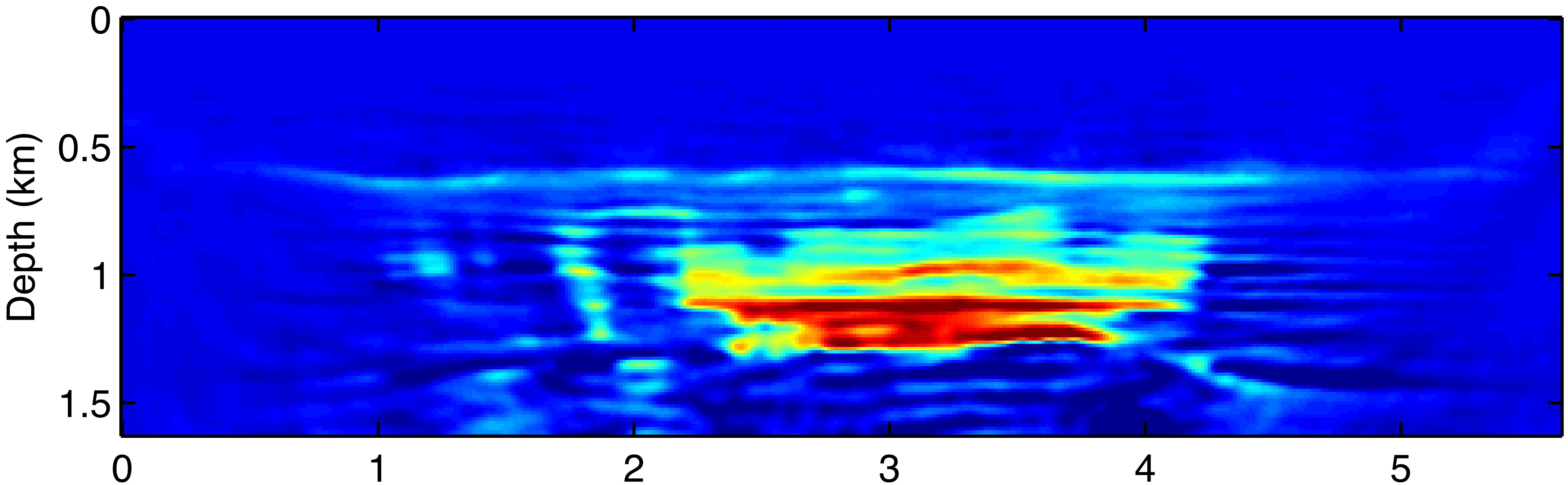
Model error – pass 2



Joint vs Independent



Independent inversion



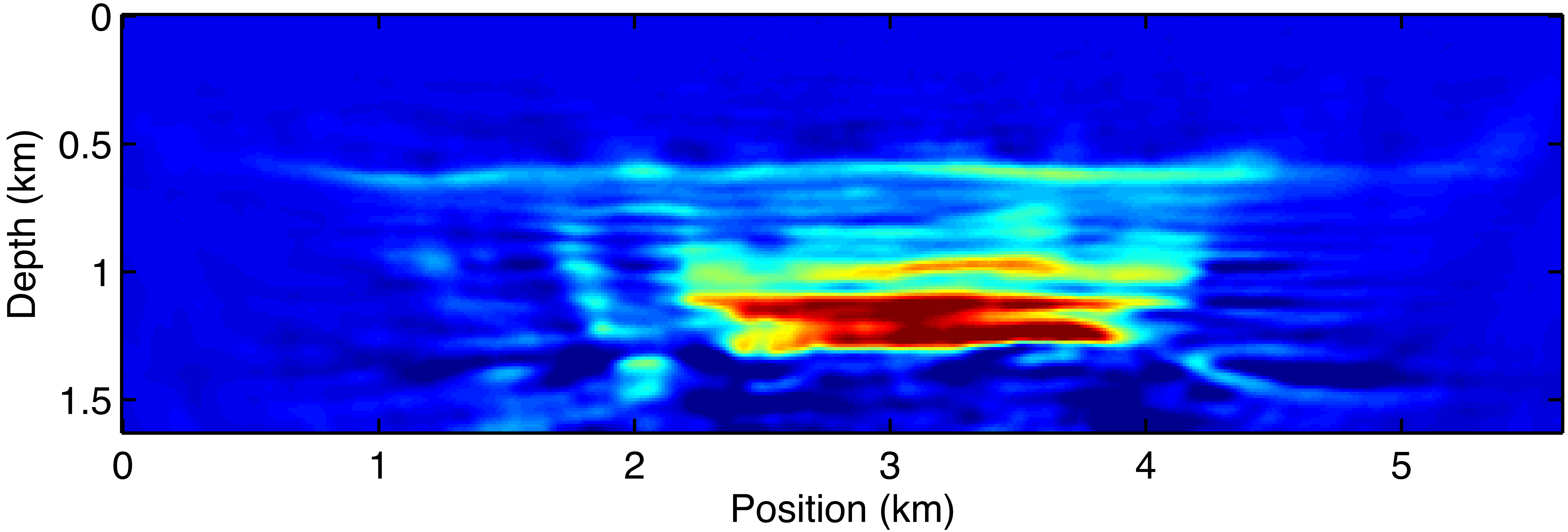
Joint inversion

SLIM

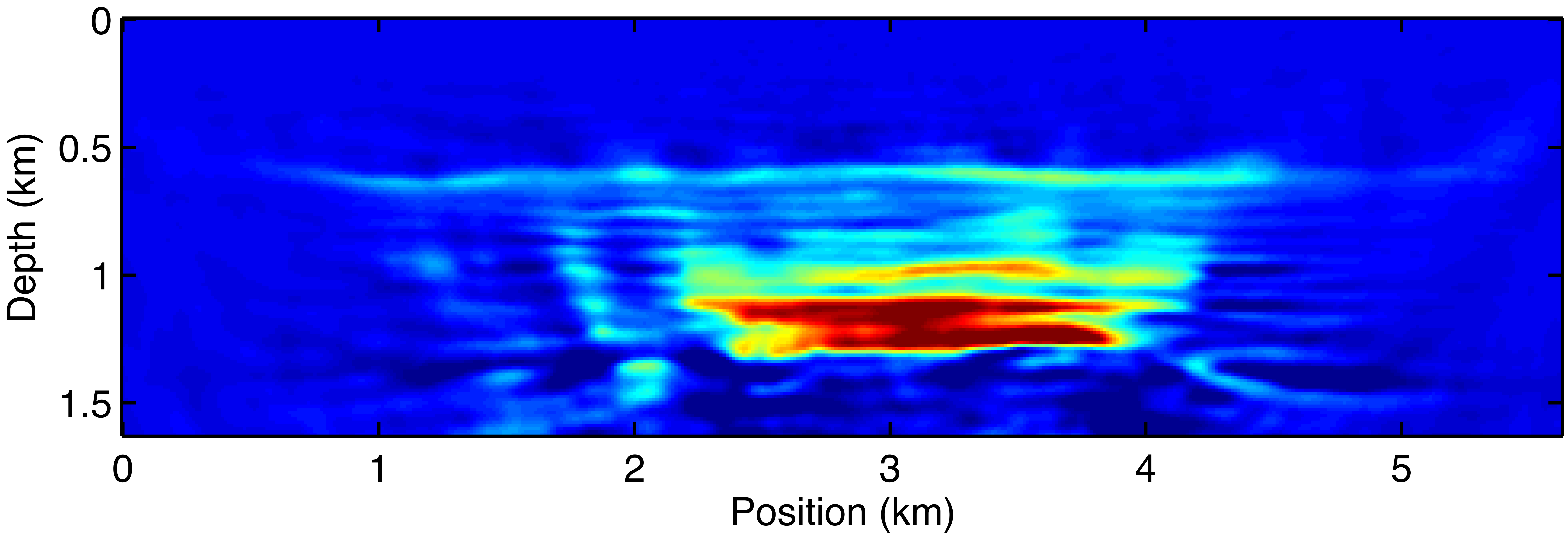
What happens when the geometry is different?

-results as at 11:00p.m. Monday 08 Dec.

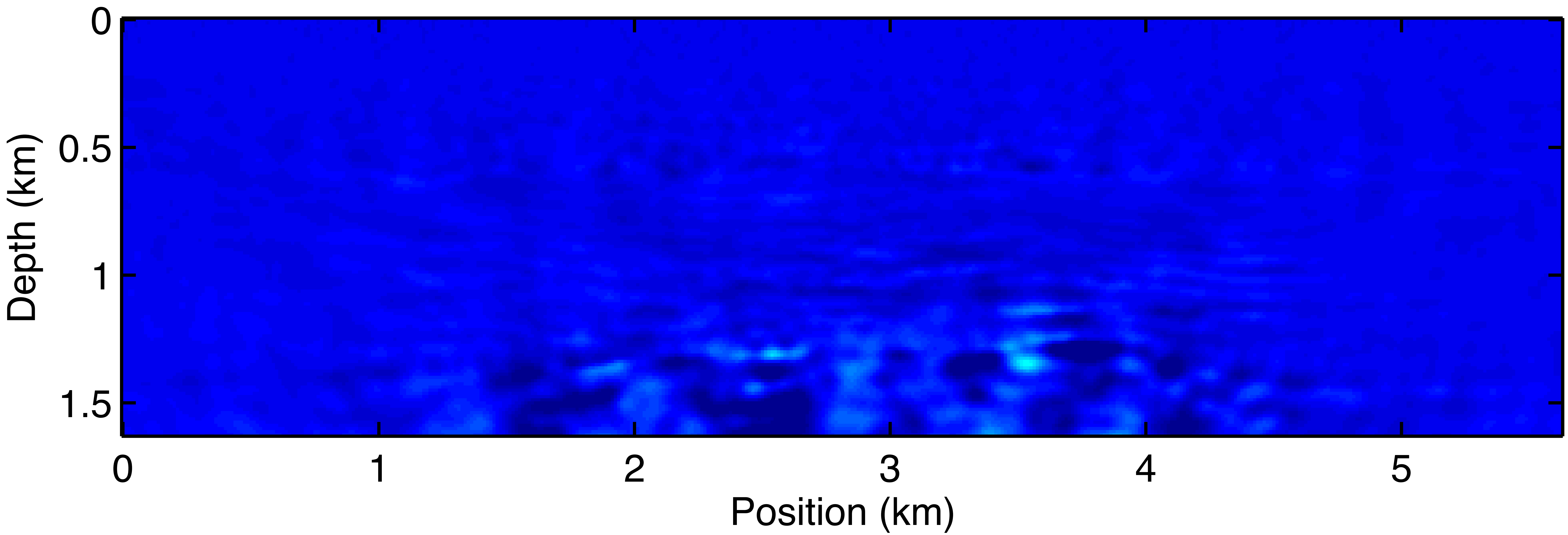
Different



Same



Error



No significant difference !!!

Conclusions

We can do FWI on time-lapse data and obtain excellent inversion results.

Randomization speeds-up computation using ideas from Compressive Sensing

Significant attenuation of artifacts in time-lapse difference model with the *joint recovery model*

“The key is in exploiting the shared information”.

Future work

Asymmetric acquisition geometry

- w/ & w/o repetition or “controlled” repetition

Multiple surveys

Software release

Uncertainty quantification

Other SLIM inversion algorithms

3-D FWI on time-lapse data set

Acknowledgements

BG Group for the velocity model.

Thank you for your attention!

SINBAD

NSERC CRSNG

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, CGG, Chevron, ConocoPhillips, ION, Petrobras, PGS, Statoil, Total SA, Sub Salt Solutions, WesternGeco, and Woodside.