Tuesday afternoon

Surface multiples, source wavelets, and imaging

SLIM

University of British Columbia

First half

Extreme SRME -> EPSI

Surface-related multiple removal as a deconvolution problem

- Get explicitly the *source wavelet* **Q**
- and a (discrete) *Green's function for the primary wavefield* **G**

Tim Improving model for **G**: deal with data gaps

Ernie Improving model for **Q**: better deconvolution

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Second half

EPSI's model (separating **G** and **Q**) is very helpful for imaging

- multiples naturally translate to injecting data into source term
- helps with scaling ambiguity in inversion imaging
- improves azimuthal range of (extended) image gathers

Ning Incorporating into a fast inversion imaging scheme

Kumar Computing extended image gathers with this model

Ning How the EPSI model helps with source scaling

Dealing with **acquisition gaps** in Robust EPSI **without interpolation**

Tim T.Y. Lin

SLIM
University of British Columbia

Talk outline

Brief review of EPSI and Robust EPSI

1. Modifying EPSI prediction model to account for missing data *

2. Algorithmic implications for Robust EPSI

Bonus: Multi-scale EPSI for acceleration and deconvolution (Ernie)

*missing data in aperture gaps, *not* undersampling

EPSI model of surface multiples

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

true primary wavefield   SRME-produced primary

$$\mathbf{P_o} = \mathbf{P} - A(f)\mathbf{P_o}\mathbf{P}$$

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	“matching” operator

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

true primary wavefield

SRME-produced primary

$$\mathbf{P_o} \approx \mathbf{P} - A(f)\mathbf{P}\mathbf{P}$$

SRMP

$\mathbf{P}$	total up-going wavefield
$\mathbf{P_o}$	primary wavefield
$A(f)$	"matching" operator

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

adaptive subtraction

$$\min_{A} \sum_{f} \|\mathbf{P} - A(f)\boxed{\mathbf{PP}}\|$$

SRMP

$\mathbf{P}$ total up-going wavefield

$\mathbf{P_o}$ primary wavefield

$A(f)$ “matching” operator

From SRME to Robust EPSI

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From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

recorded data   predicted data from SRME

$$\mathbf{P} = \mathbf{P_o} + A(f)\mathbf{P_o}\mathbf{P}$$

$$\mathbf{P_o} = \mathbf{QG}$$
$$A(f) = -\mathbf{Q}^{-1}$$

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

From SRME to Robust EPSI

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$\mathbf{G}$	primary impulse response

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (van Groenestijn and Verschuur, 2009)

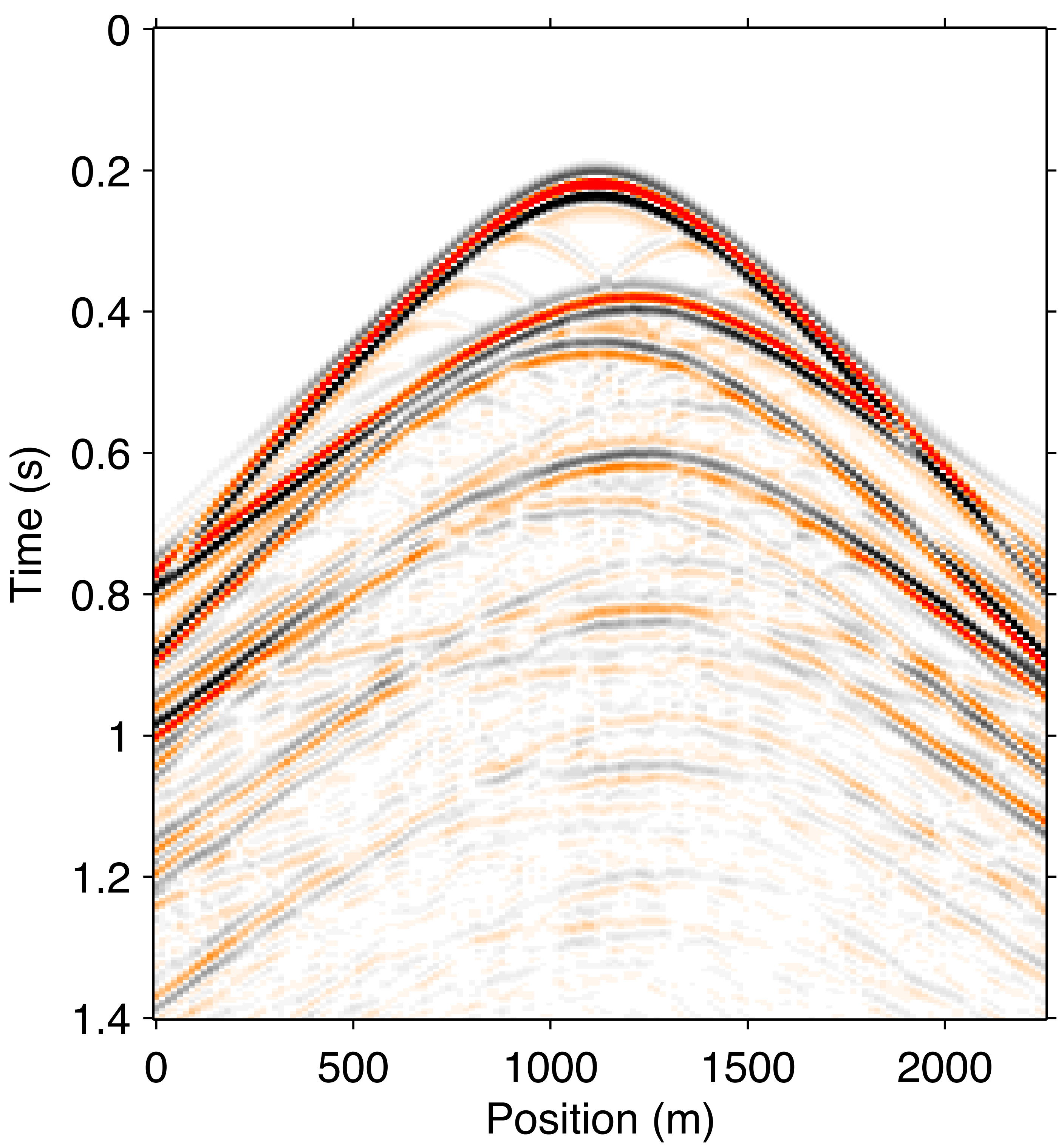
recorded data predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Do Inversion for G and Q by minimizing:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI



Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

rded data   predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{QG} - \mathbf{GP})\|_2^2$$

From SRME to Robust EPSI

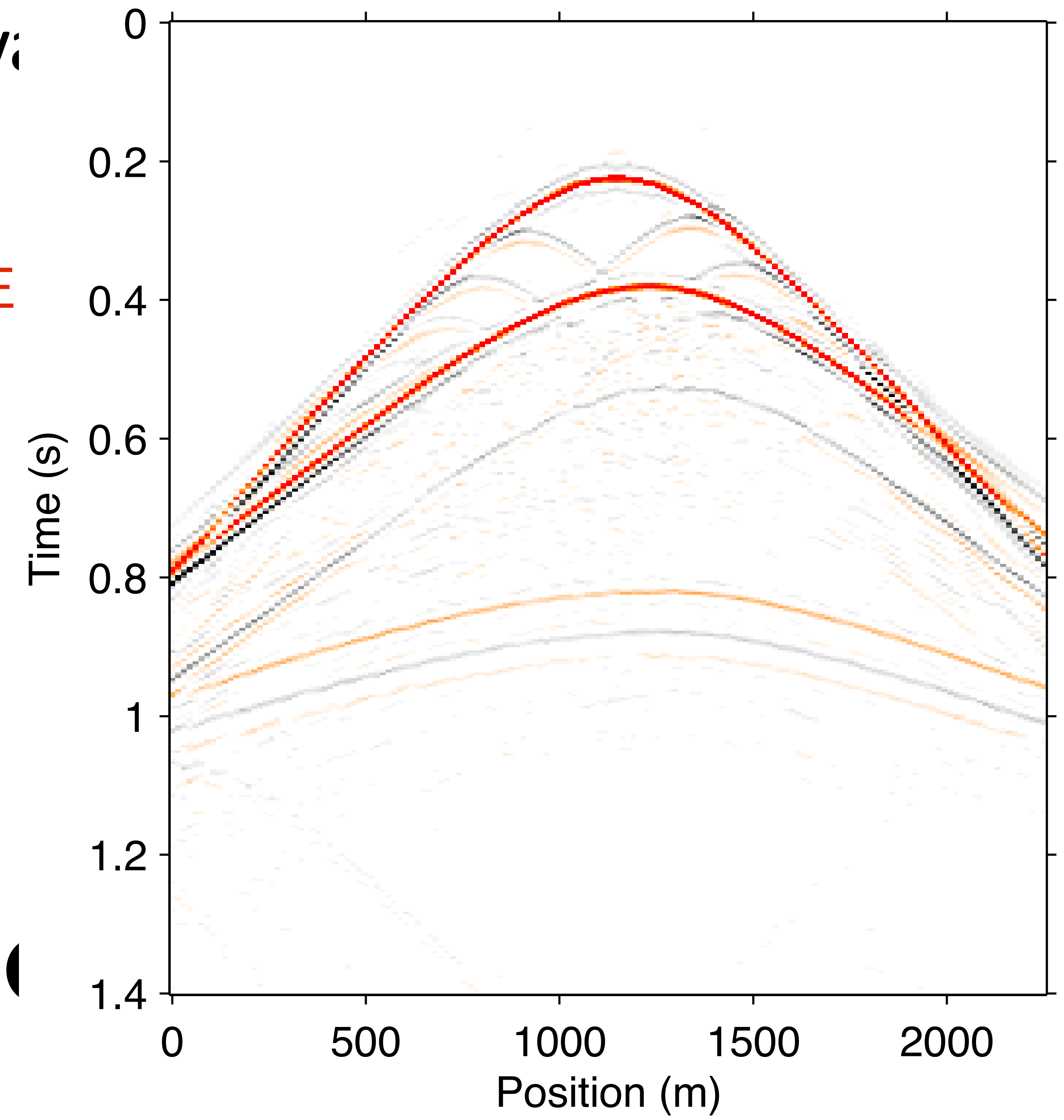
Based on **Estimation of Primaries by Sparse Inversion** (va 09)

recorded data predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Inversion objective:

$$f(\mathbf{G},\mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{Q}$$

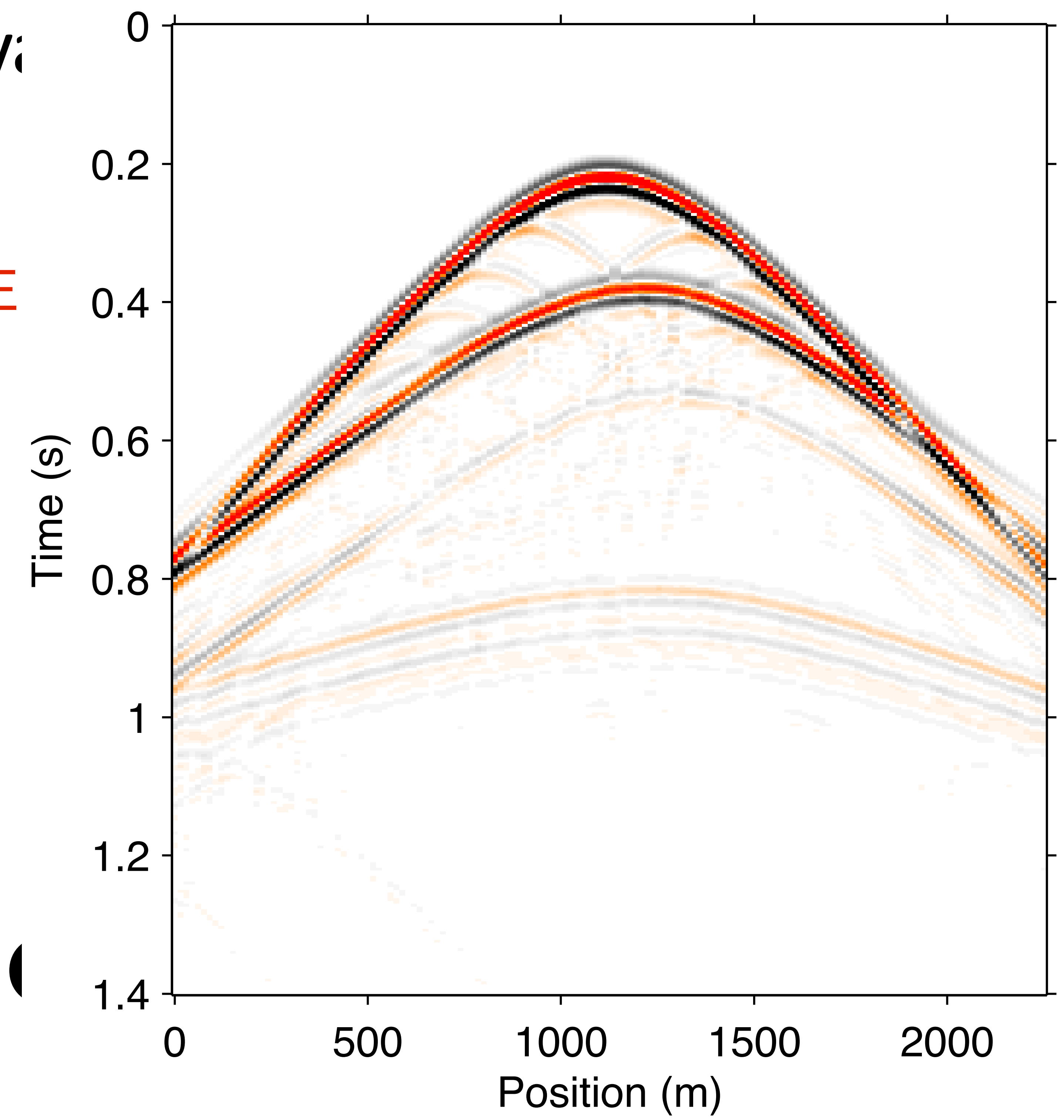


From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va 09)

recorded data predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$



Inversion objective:

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From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va ... 09)

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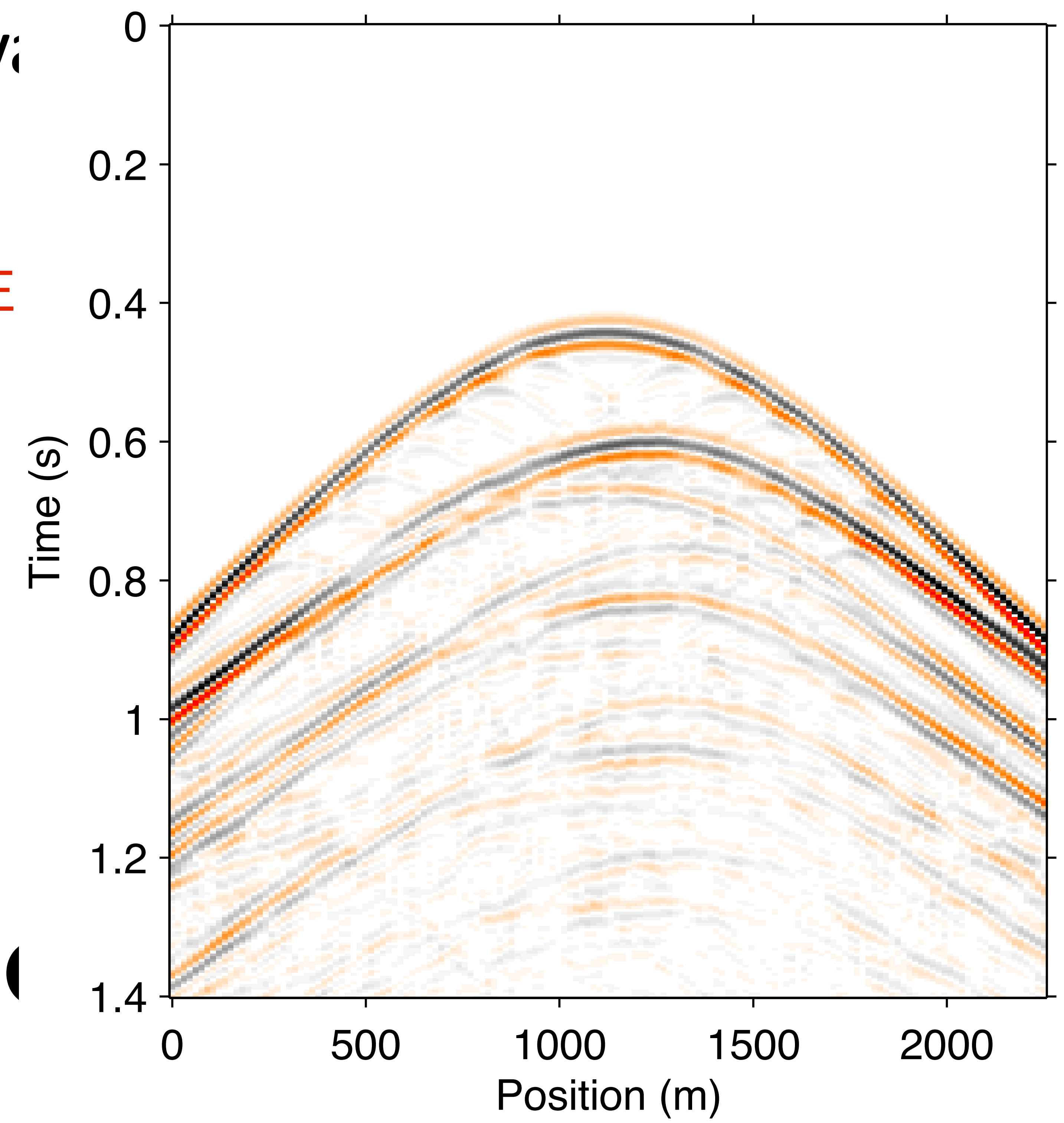

From SRME to Robust EPSI

Based on **Estimation of Primaries by Sparse Inversion** (va)9)

recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{QG} - \mathbf{GP}$$

Inversion objective:

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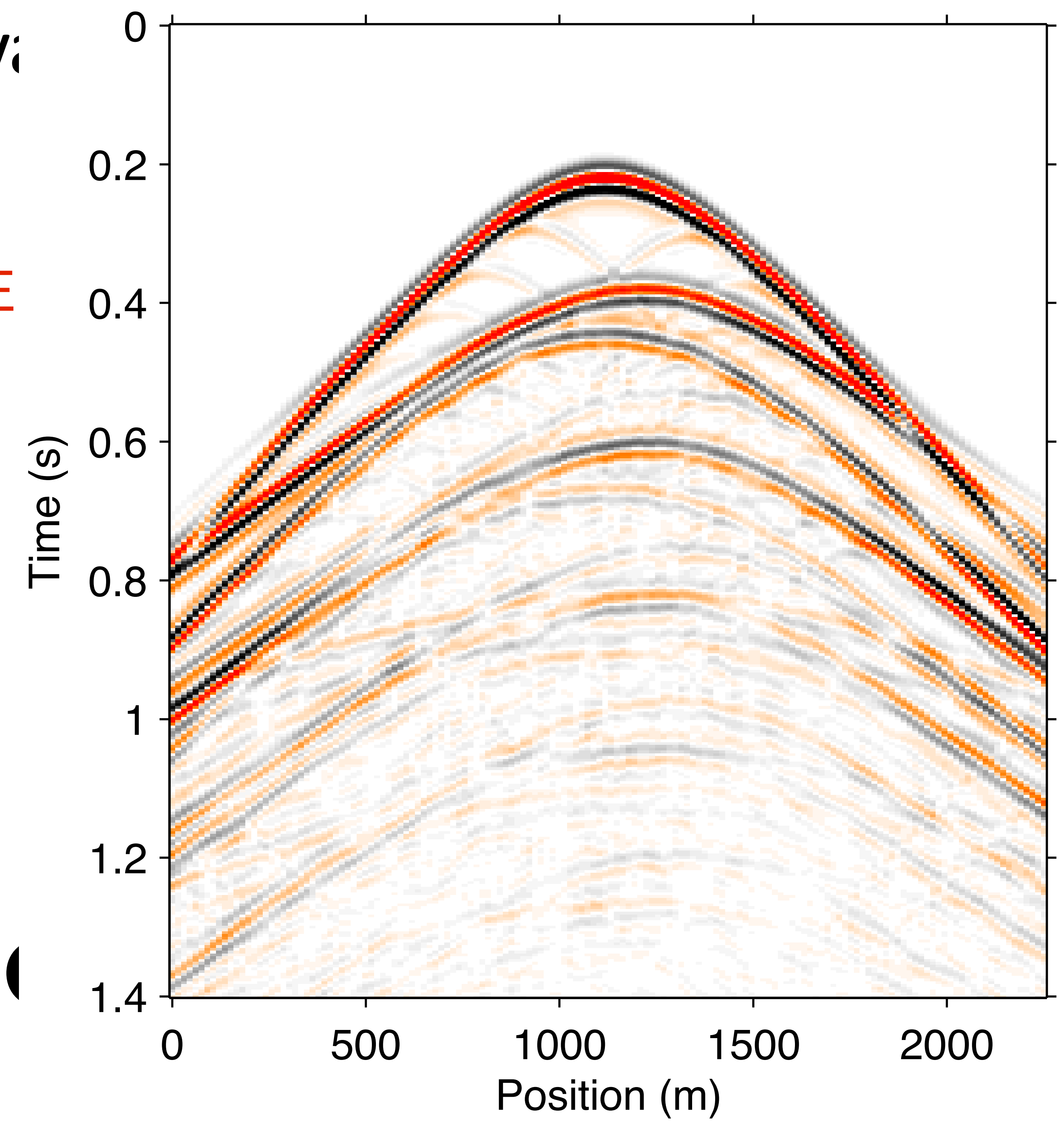


So what happens...

when there are gaps in your aperture?

From SRME to Robust EPSI

Even assuming we already have the perfect G...

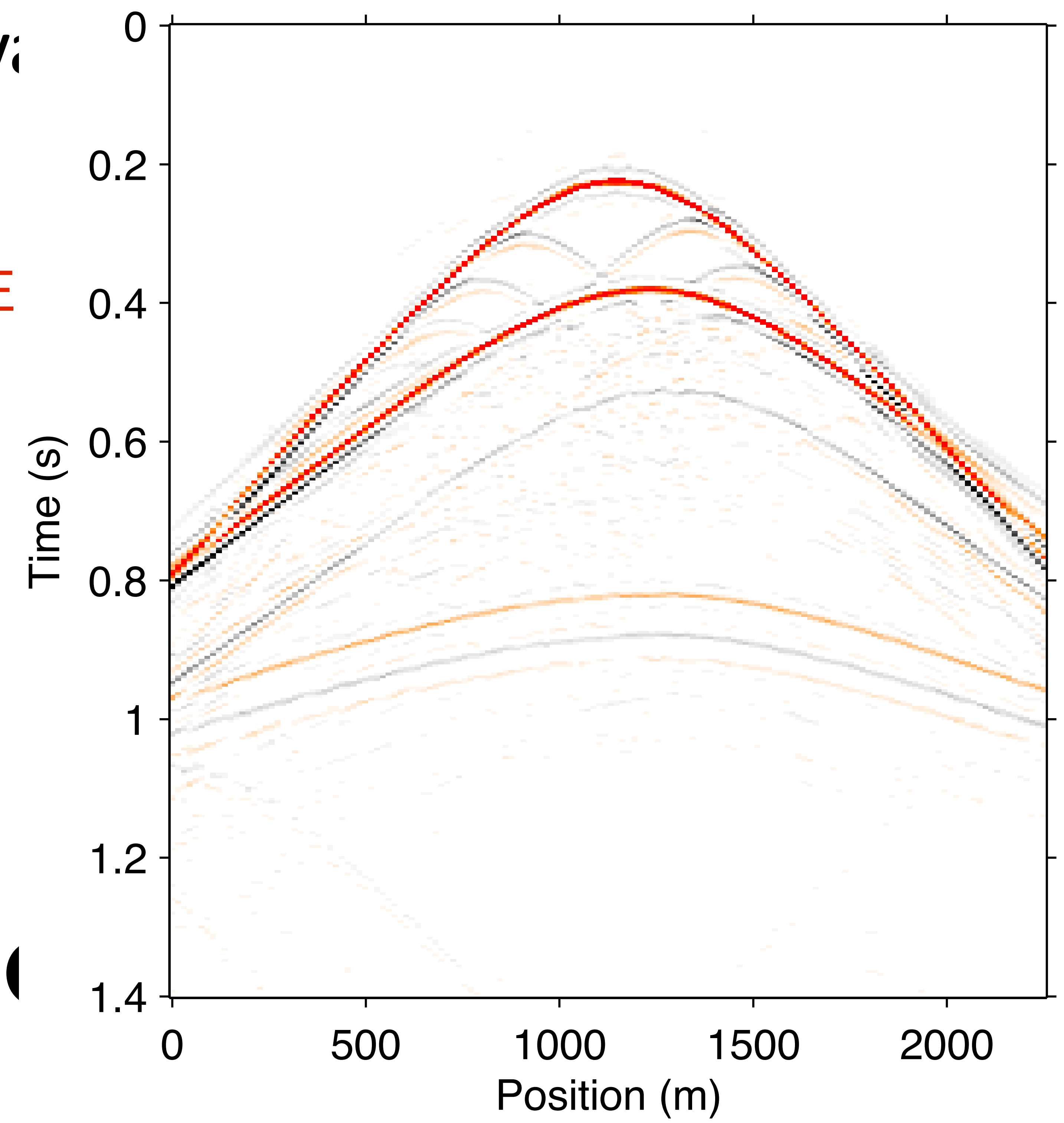
Based on **Estimation of Primaries by Sparse Inversion** (va... 09)

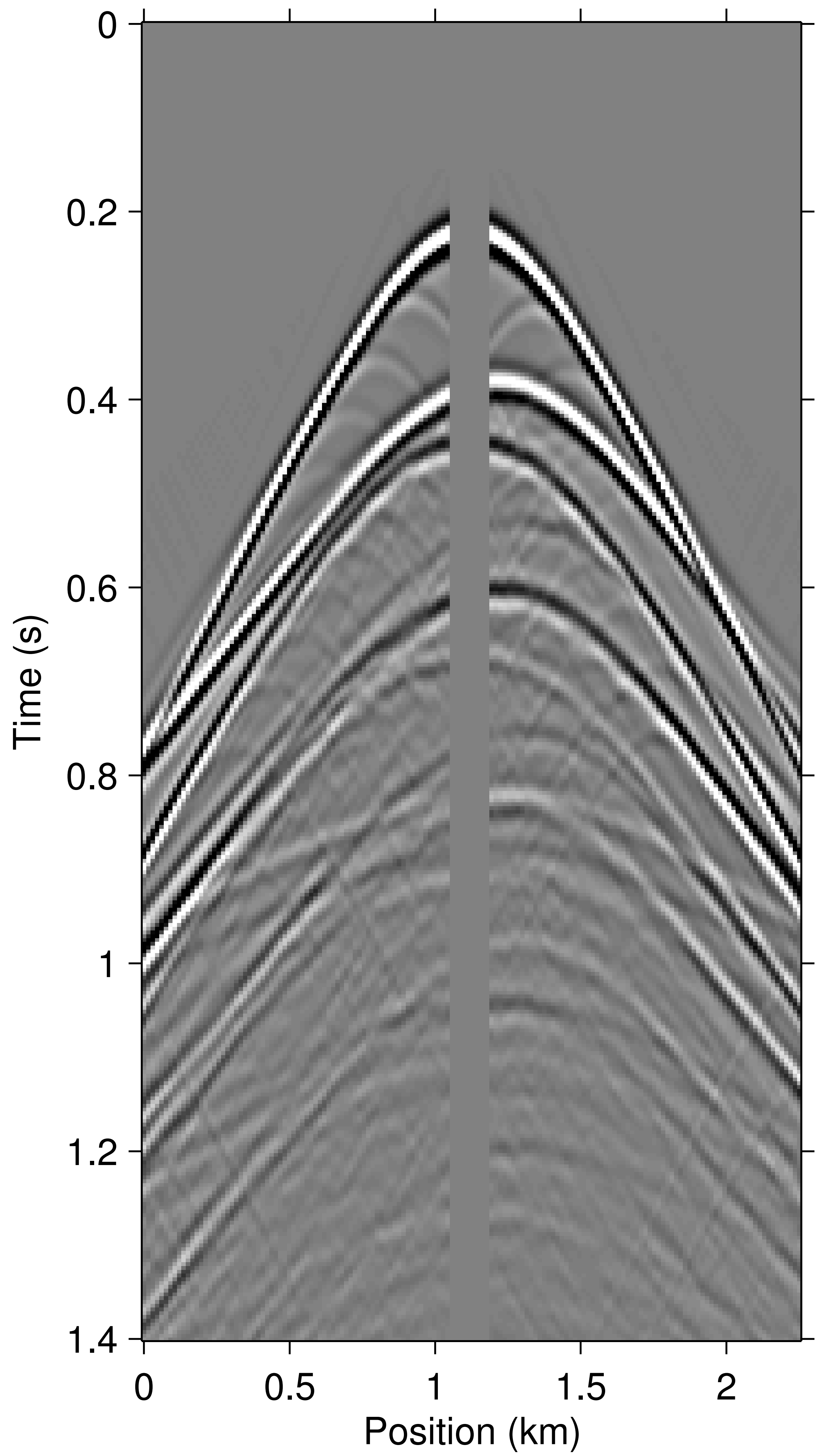
recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

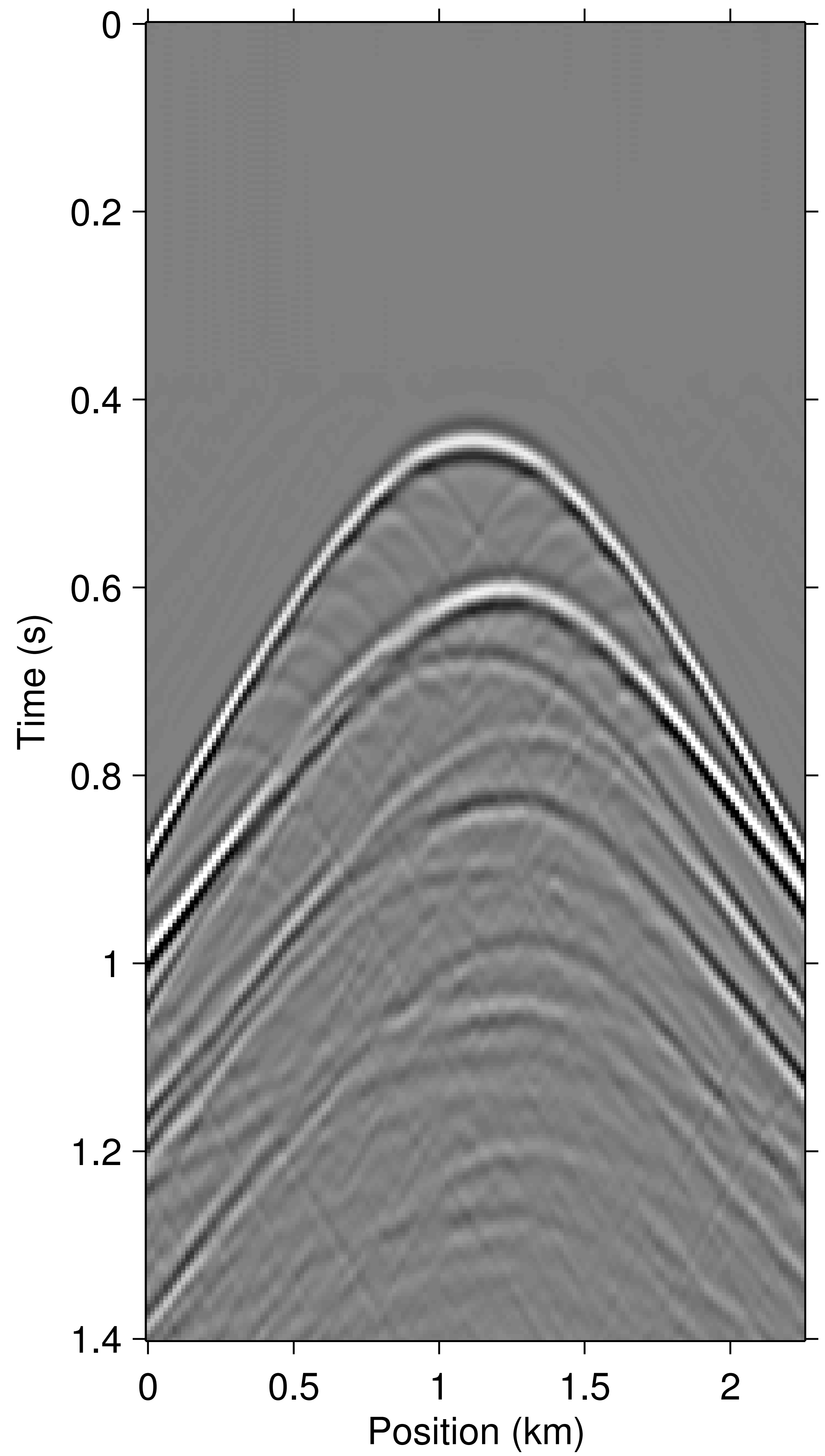
Inversion objective:

$$f(\mathbf{G},\mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{Q}$$

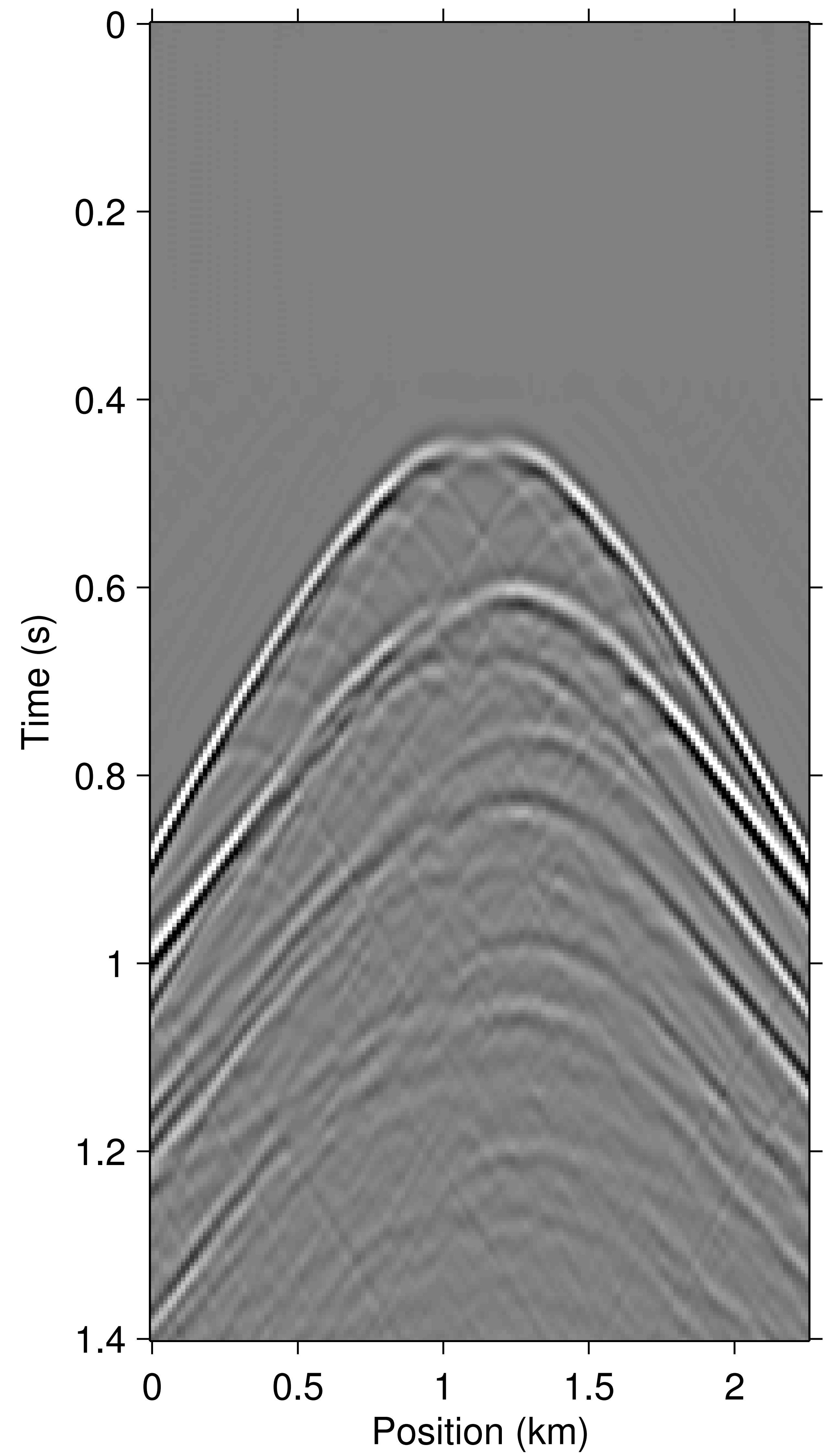




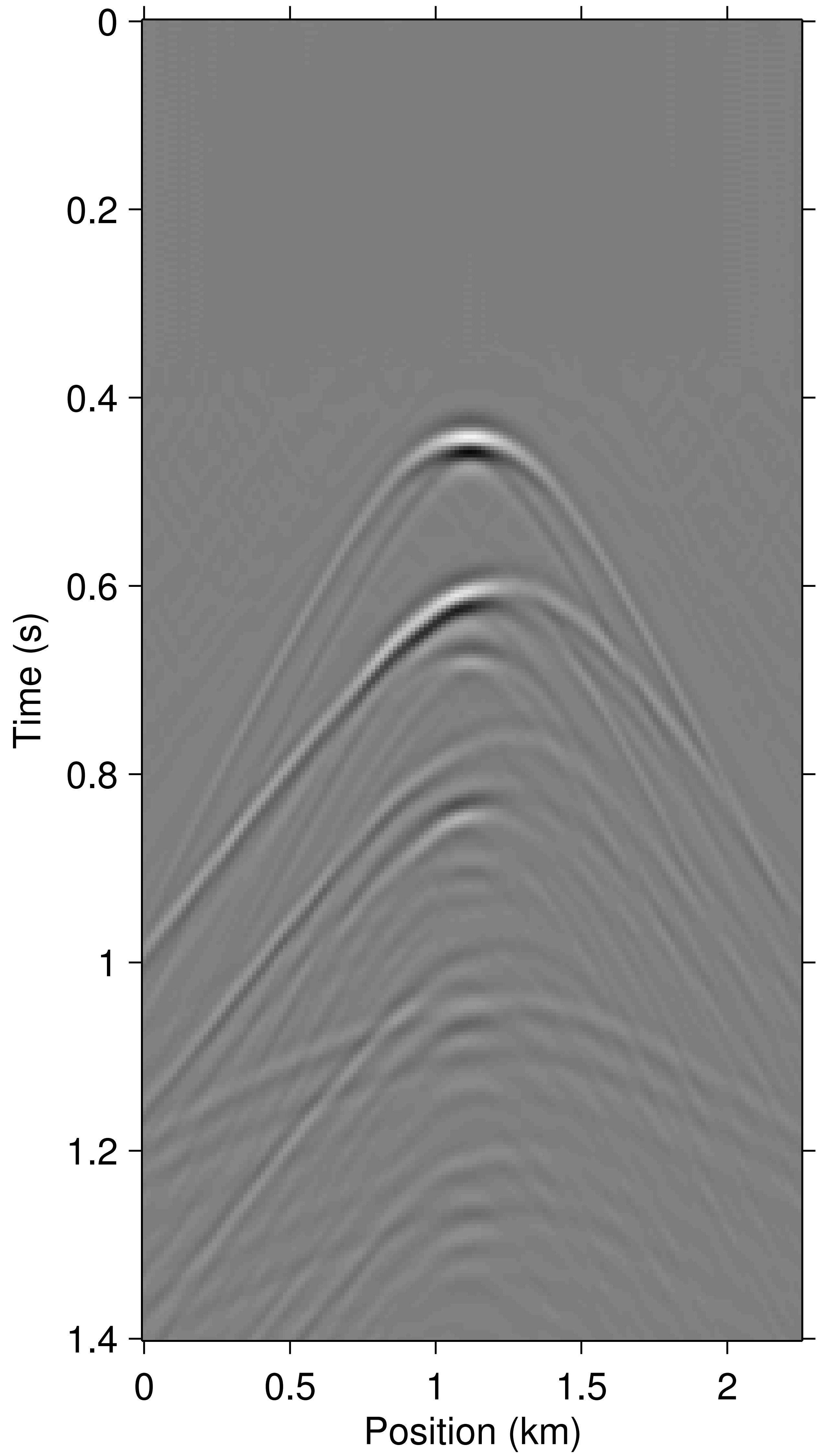
Data with missing traces P’



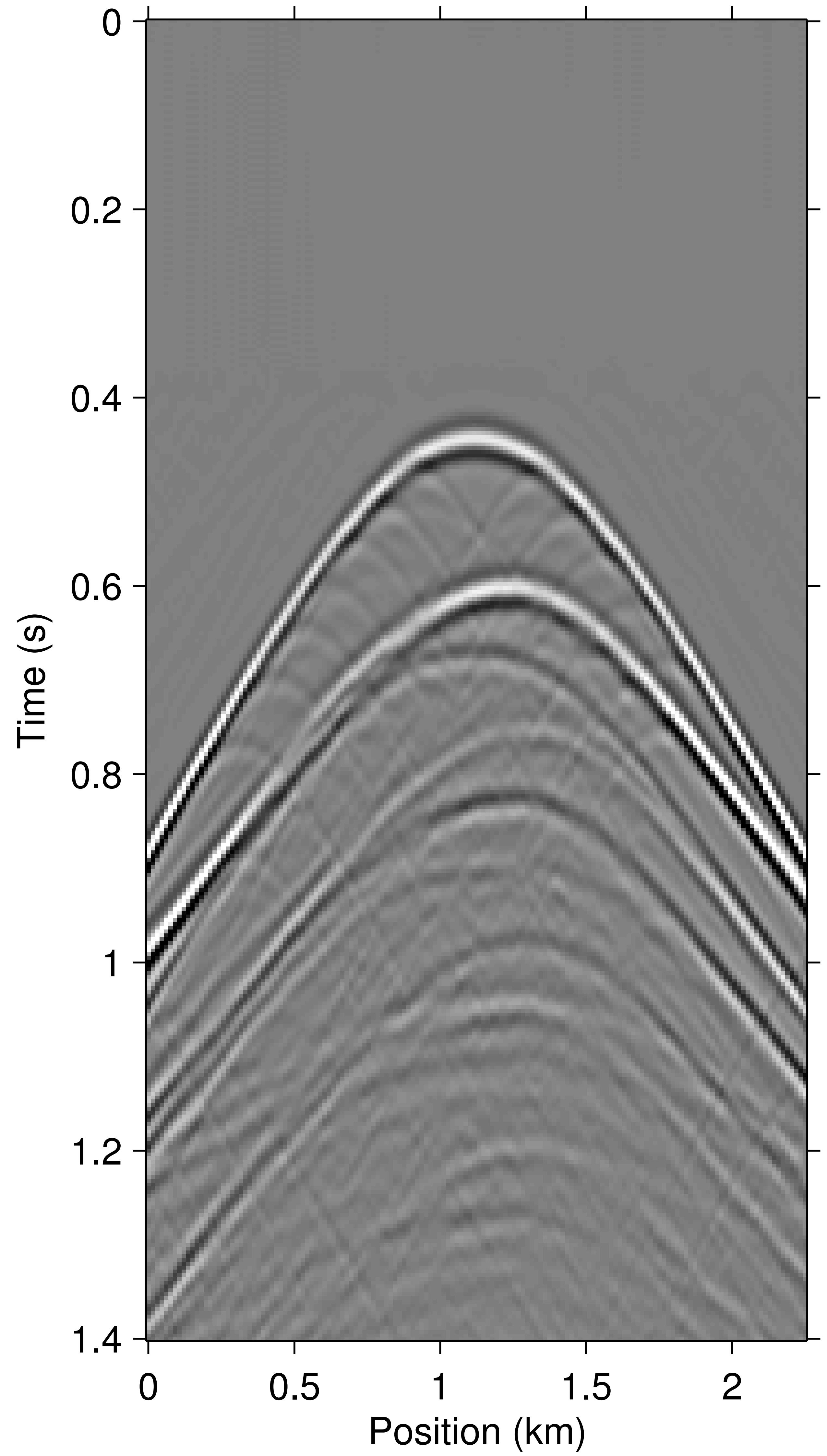
Exact multiples



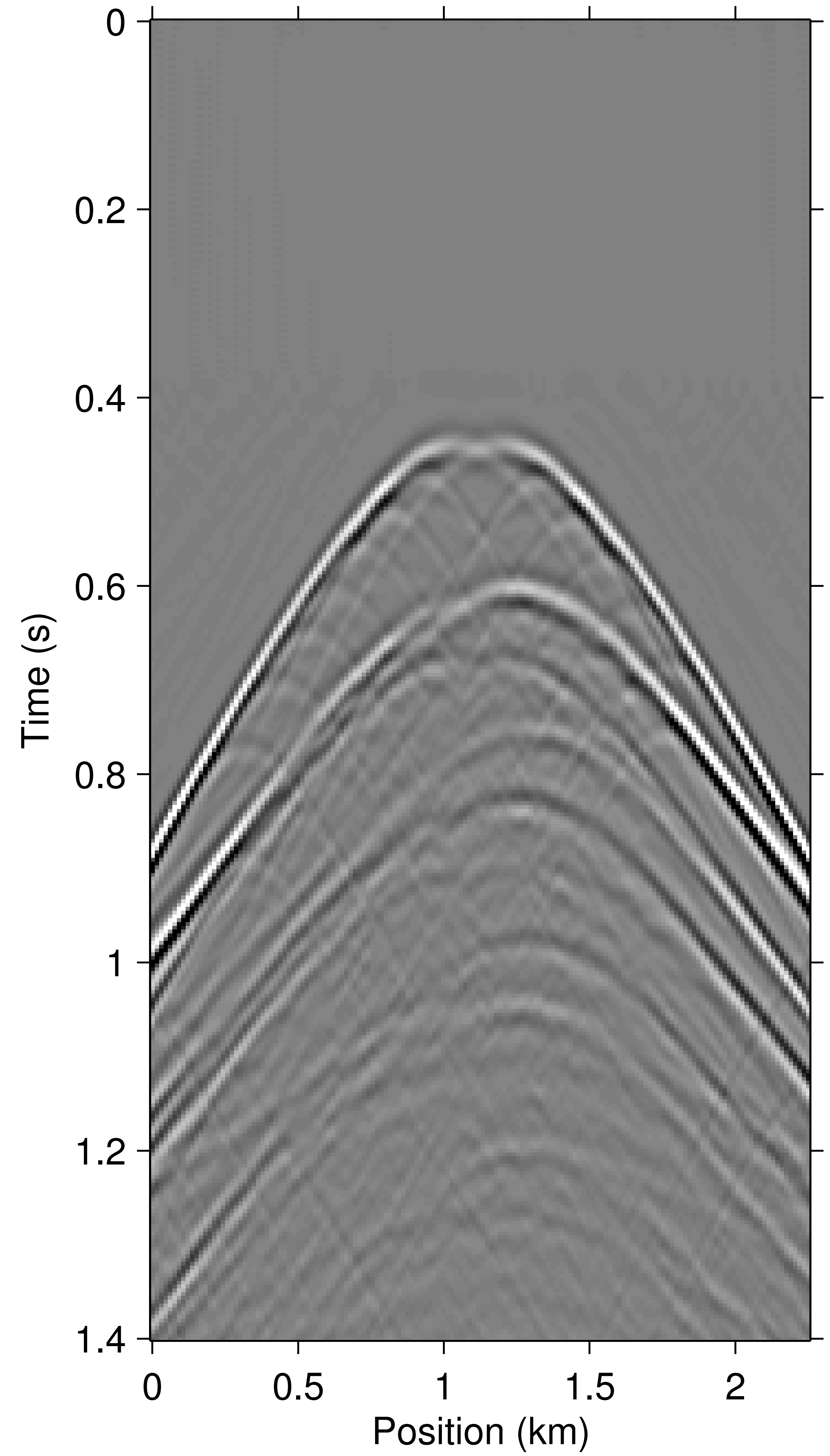
EPSI Predicted -GP’
(with perfect G)



Missing contributions



Exact multiples



EPSI Predicted -GP’
(with perfect G)

Main idea

Modify the relationship

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

to account for the missing contribution

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A brief discussion of the inversion

recorded data — predicted data from SRME

$$\mathbf{P} = \mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P}$$

Inversion objective:

$$f(\mathbf{G}, \mathbf{Q}) = \frac{1}{2}\|\mathbf{P} - (\mathbf{Q}\mathbf{G} - \mathbf{G}\mathbf{P})\|_2^2$$

Write the EPSI data prediction as an operator M

In time domain (lower-case: whole dataset in time domain)

recorded data predicted data from SRME

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathcal{M}(\mathbf{g}, \mathbf{q}) := \mathcal{F}_{\mathrm{t}}^{\dagger} \mathrm{BlockDiag}_{\omega_1 \cdots \omega_{nf}} [(q(\omega)\mathbf{I} - \mathbf{P})^{\dagger} \otimes \mathbf{I}] \mathcal{F}_{\mathrm{t}} \mathbf{g}$$

Inversion objective:

$$f(\mathbf{g}, \mathbf{q}) = \frac{1}{2} \|\mathbf{p} - \mathcal{M}(\mathbf{g}, \mathbf{q})\|_2^2$$

Solving the EPSI problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

In fact it is bilinear: $\mathbf{QG} = \mathbf{P} + \mathbf{GP}$

$$\mathbf{M}_{\tilde{q}}\mathbf{g} = \mathcal{M}(\mathbf{g}, \tilde{\mathbf{q}}) \qquad \mathbf{M}_{\tilde{g}}\mathbf{q} = \mathcal{M}(\mathbf{q}, \tilde{\mathbf{g}})$$

Solving the EPSI problem

Linearizations

$$\mathbf{p} = \mathcal{M}(\mathbf{g}, \mathbf{q})$$

$$\mathbf{M}_{\tilde{q}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{g}}\right)_{\tilde{q}}$$

$$\mathbf{M}_{\tilde{g}} = \left(\frac{\partial \mathcal{M}}{\partial \mathbf{q}}\right)_{\tilde{g}}$$

Associated objectives:

$$f_{\tilde{q}}(\mathbf{g}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{q}}\mathbf{g}\|_2^2 \qquad f_{\tilde{g}}(\mathbf{q}) = \frac{1}{2}\|\mathbf{p} - \mathbf{M}_{\tilde{g}}\mathbf{q}\|_2^2$$

Solving the EPSI problem

Do:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \alpha\mathcal{S}(\nabla f_{q_k}(\mathbf{g}_k))$$
$$\mathbf{q}_{k+1} = \mathbf{q}_k + \beta\nabla f_{g_{k+1}}(\mathbf{q}_k)$$

Gradient sparsity

$\mathcal{S}$: pick largest ρ elements per trace

Robust EPSI
L1-minimization approach to the EPSI problem

[Lin and Herrmann, 2013 *Geophysics*]

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

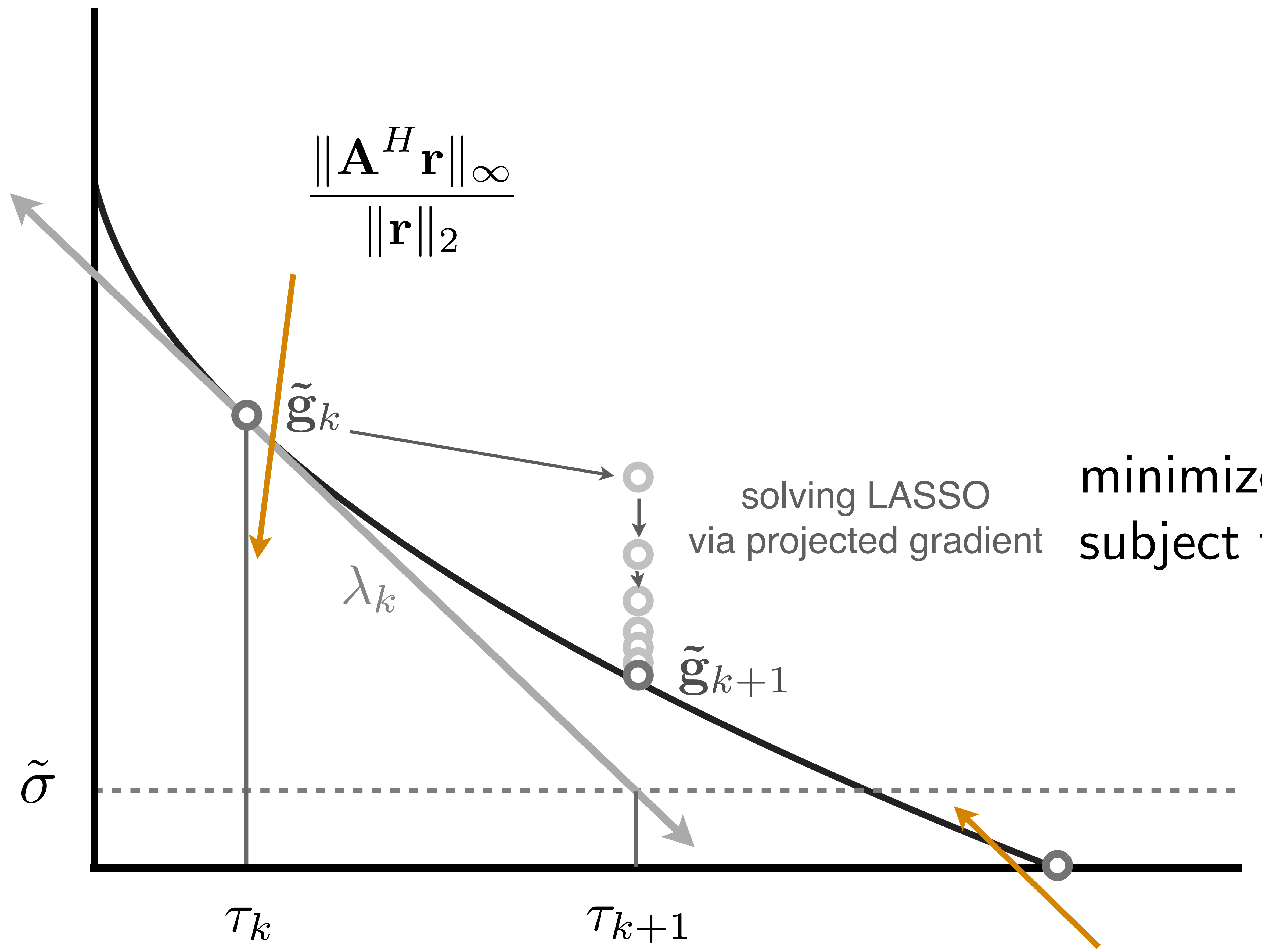
$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

Choosing Tau from the Pareto curve

$$\begin{array}{ll} \text{minimize} & \|x\|_1 \\ \text{subject to} & \|Ax-b\|_2 \le \sigma \end{array}$$

Look at the solution space and the line of optimal solutions (Pareto curve)



$$\begin{array}{ll} \text{minimize} & \|Ax-b\|_2 \\ \text{subject to} & \|x\|_1 \le \tau \end{array}$$

feasible solution with smallest $\|x\|_1$

Can we do better?

Inverting for unknown data **1.0**
(naive method: explicit reconstruction)

Robust EPSI
Inverting for unknown data

$$\Delta\mathbf{P}(\mathbf{G}_{k+1},\mathbf{R}_{k+1}) := -(\mathbf{I}+\mathbf{G}_{k+1})^H(\mathbf{R}_{k+1})$$

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k,\mathbf{q}_k)\|_2 > \sigma$

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$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha\Delta\mathbf{p}(\mathbf{g}_{k+1},\mathbf{q}_{k+1},\mathbf{p}_k)$$

Robust EPSI
Inverting for unknown data 1.0

Data changes every iteration!

$$\Delta \mathbf{P}(\mathbf{G}_{k+1}, \mathbf{R}_{k+1}) := -(\mathbf{I} + \mathbf{G}_{k+1})^H (\mathbf{R}_{k+1})$$

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Robust EPSI
Inverting for unknown data 1.0

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Data changes every iteration!

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$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p}_k - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

G and P updated independently

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha \Delta \mathbf{p}(\mathbf{g}_{k+1}, \mathbf{q}_{k+1}, \mathbf{p}_k)$$

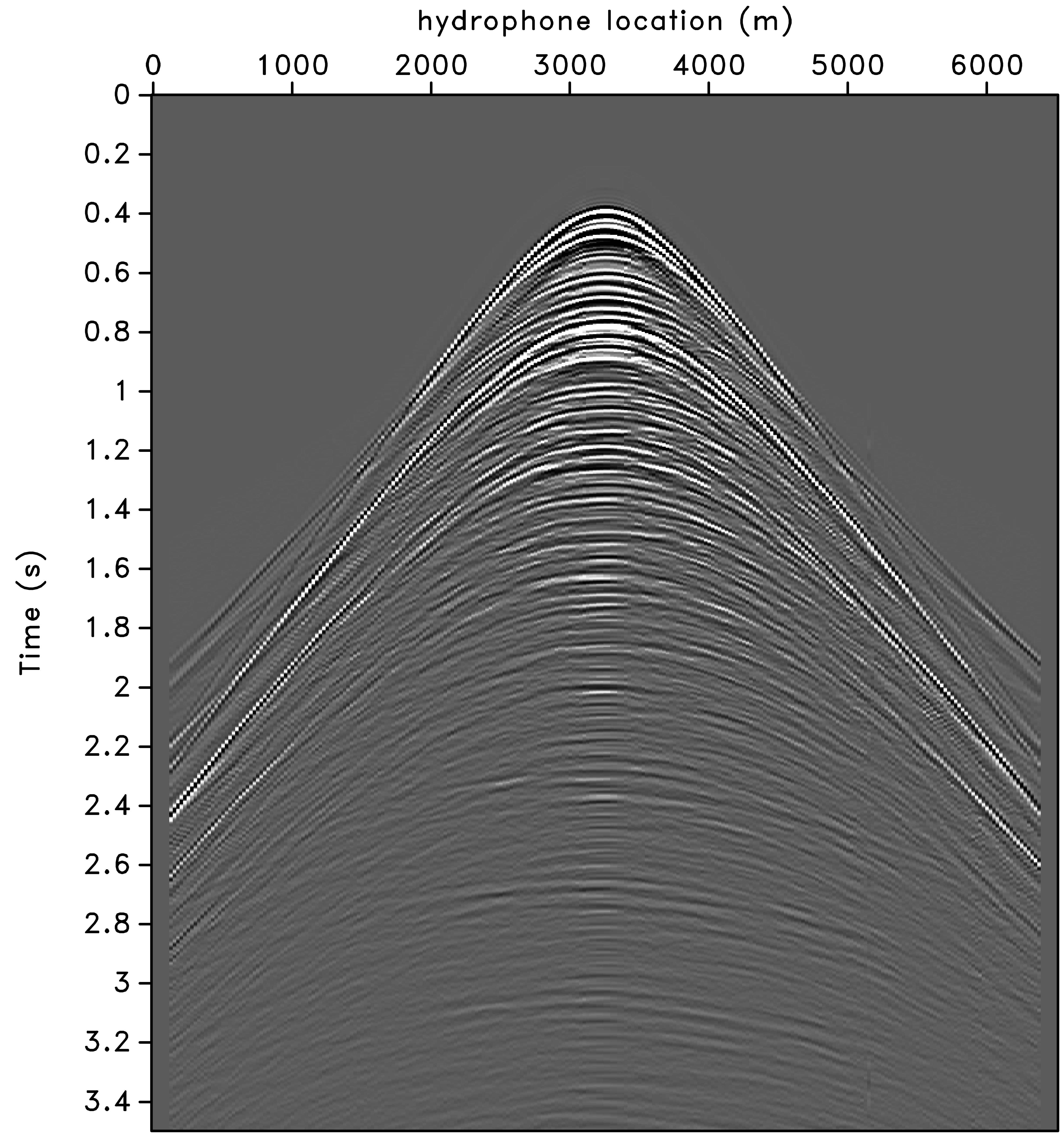
Inverting for unknown data **2.0**

1. No changing observations

2. Relate changes in P to changes in G

Trace Mask

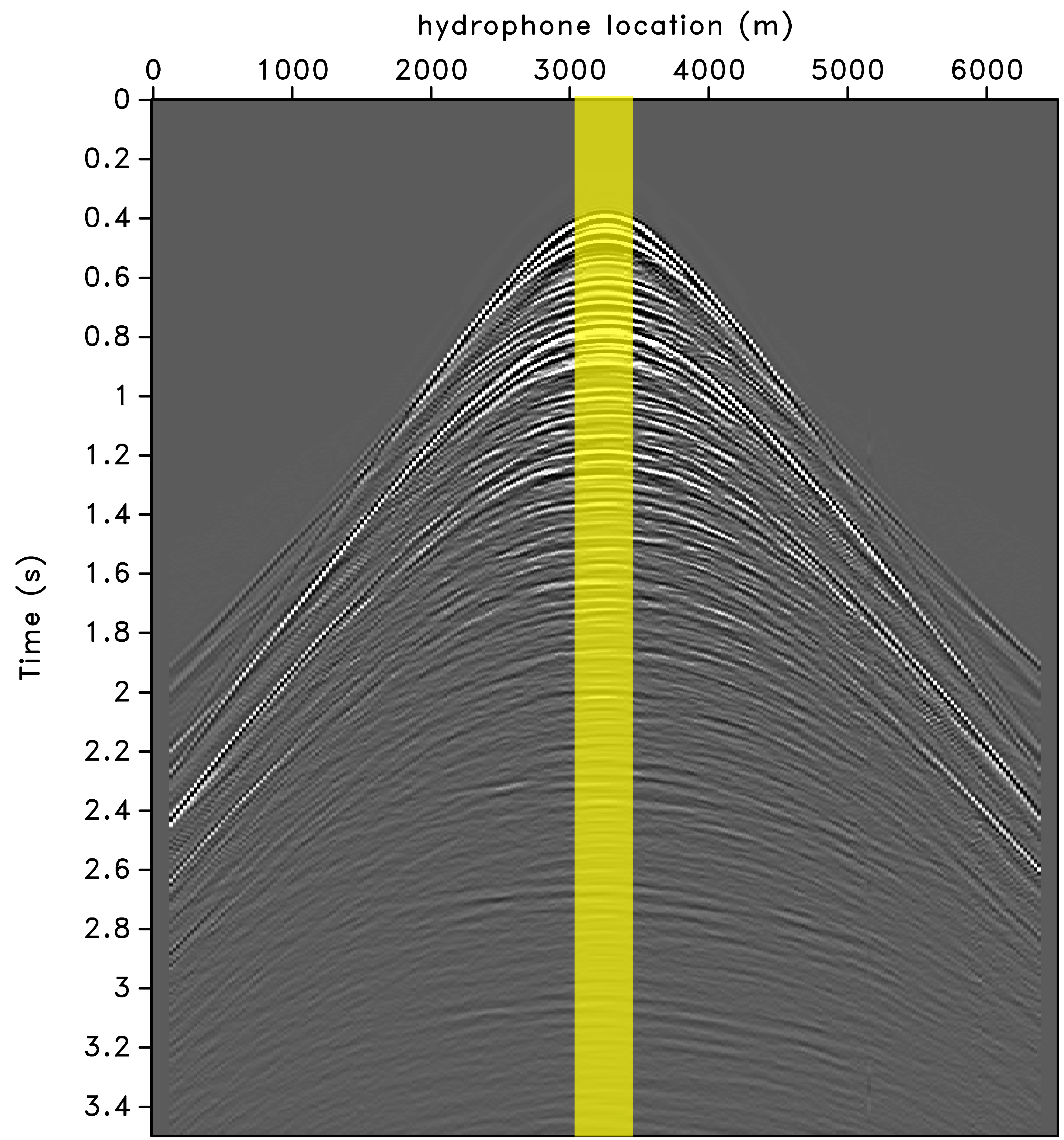
Masking operator $\mathbf{K}$



Trace Mask

Bisects wavefield data to unknown/ uncertain traces

(e.g., near-offset)

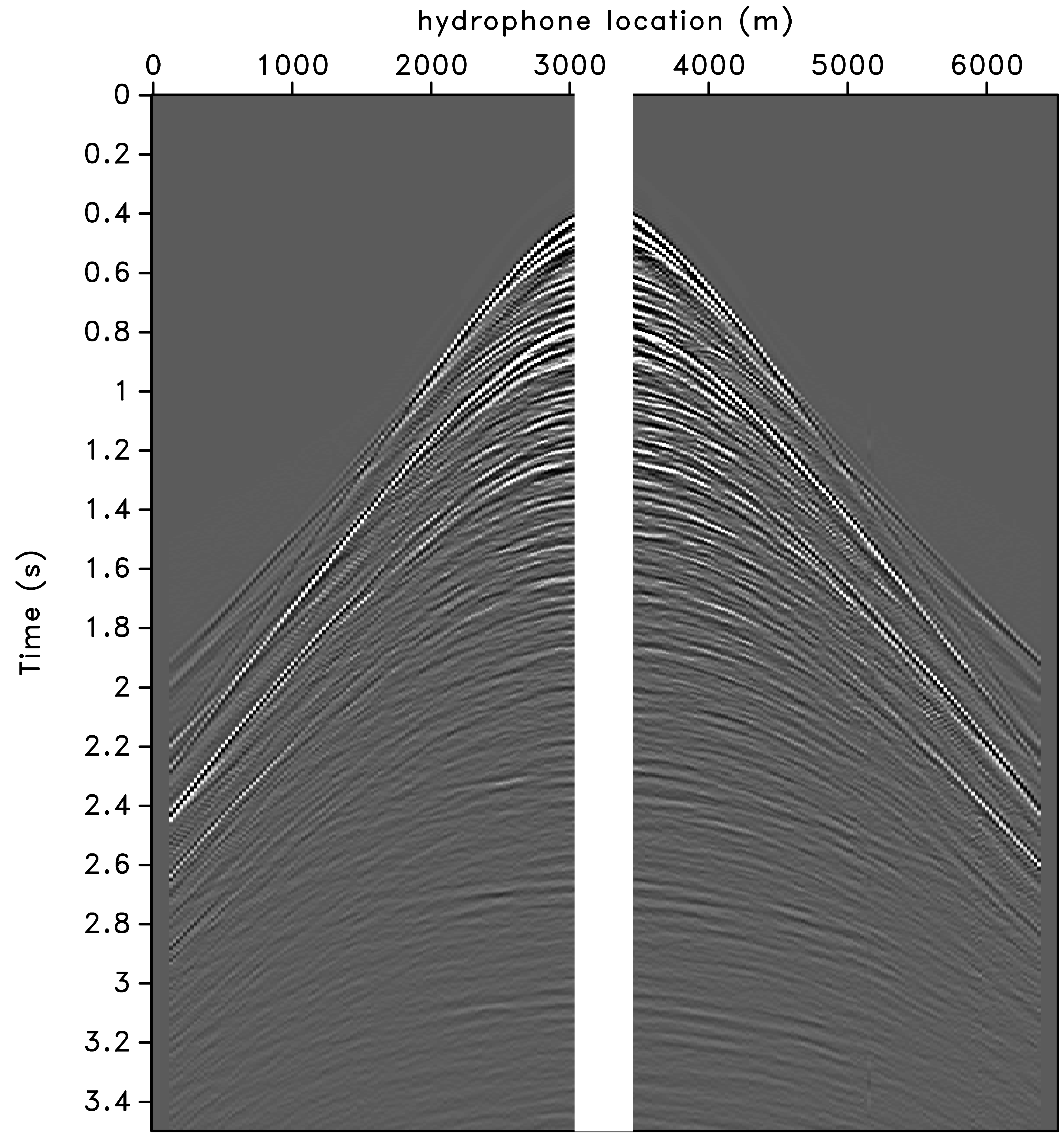


Trace Mask

Masking operator $\mathbf{K}$

Time domain: $\mathbf{Kp}$

Frequency slices: $\mathbf{K} \circ \mathbf{P}$

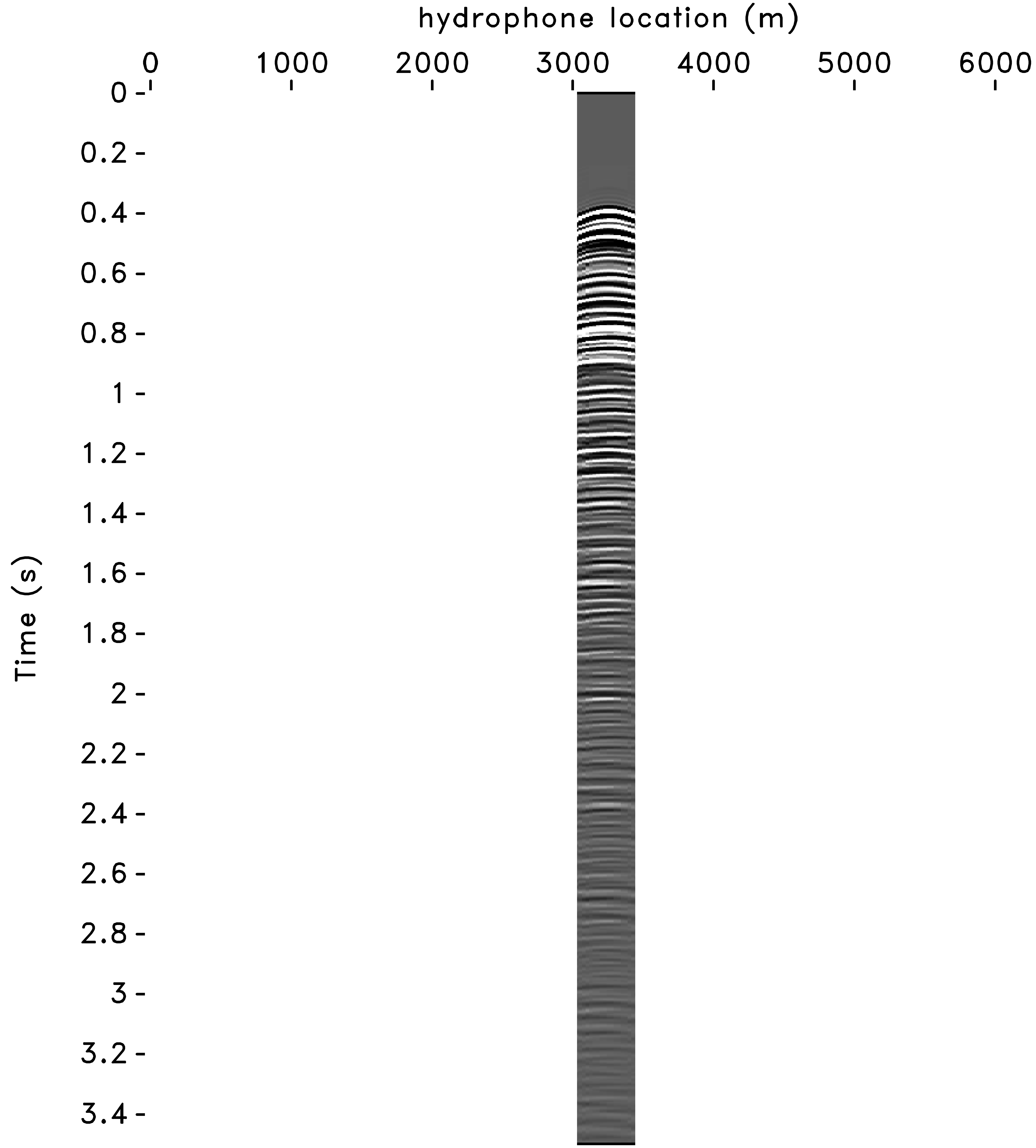


Trace Mask

Complement of Masking operator $\mathbf{K}_c$

Time domain: $\mathbf{K}_c\mathbf{p}$

Frequency slices: $\mathbf{K}_c \circ \mathbf{P}$



Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Unknown data traces: $\mathbf{P}'' := \mathbf{K_c} \circ \mathbf{P}$

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Unknown data traces: $\mathbf{P}'' := \mathbf{K_c} \circ \mathbf{P}$

$$= \mathbf{K_c} \circ \left(\mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''\right)$$

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

Known data traces: $\mathbf{P}' := \mathbf{K} \circ \mathbf{P}$

Trace stencil

Unknown data traces: $\mathbf{P}'' := \mathbf{K_c} \circ \mathbf{P}$

$$= \mathbf{K_c} \circ \left(\mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''\right)$$

Bisected data variables $\mathbf{P}' + \mathbf{P}'' = \mathbf{P}$

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Trace stencil

Unknown data traces: $\mathbf{P}'' := \mathbf{K_c} \circ \mathbf{P}$

$$= \mathbf{K_c} \circ \left(\mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''\right)$$

Recursively defined

Modify the modeling operator

$$\mathcal{M}(\mathbf{G}, \mathbf{Q}; \mathbf{P}') = \mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}' + \mathbf{R}\mathbf{G}\mathbf{P}''$$

Modify the modeling operator

$$\mathcal{M}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGP}''$$

Tilde: modified with higher-order terms

$$\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{GQ} + \mathbf{RGP}' + \mathbf{RGK_c} \circ (\mathbf{GQ} + \mathbf{RGP}') + \mathcal{O}(\mathbf{G}^3)$$

Modify the modeling operator

$$\begin{aligned}\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') =& \mathbf{GQ} + \mathbf{RGP}' \\ &+ \mathbf{RGK_c} \circ (\mathbf{GQ} + \mathbf{RGP}') \\ &+ \mathcal{O}(\mathbf{G}^3)\end{aligned}$$

Modify the modeling operator

$$\widetilde{\mathcal{M}}(\mathbf{G}, \mathbf{Q}; \mathbf{P}') = \mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}' + \mathbf{R}\mathbf{G}\mathbf{K_c} \circ (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}') + \mathcal{O}(\mathbf{G}^3)$$

Modify the modeling operator

$$\widetilde{\mathcal{M}}(\mathbf{G}, \mathbf{Q}; \mathbf{P}') = \mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}'$$

2nd Order autoconvolution term (of G) →

$$+ \mathbf{R}\mathbf{G}\mathbf{K_c} \circ (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

$$+ \mathcal{O}(\mathbf{G}^3)$$

Modify the modeling operator

Trace mask over all modeled wavefield

$$\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{K} \circ \big[\mathbf{GQ} + \mathbf{RGP}'$$

2nd Order autoconvolution term

$$+ \ \mathbf{RGK_c} \circ (\mathbf{GQ} + \mathbf{RGP}')$$

$$+ \ \mathcal{O}(\mathbf{G}^3)\big]$$

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Modify the modeling operator

$$\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') = \mathbf{K}\circ\Big[\mathbf{GQ}+\mathbf{RGP}'$$

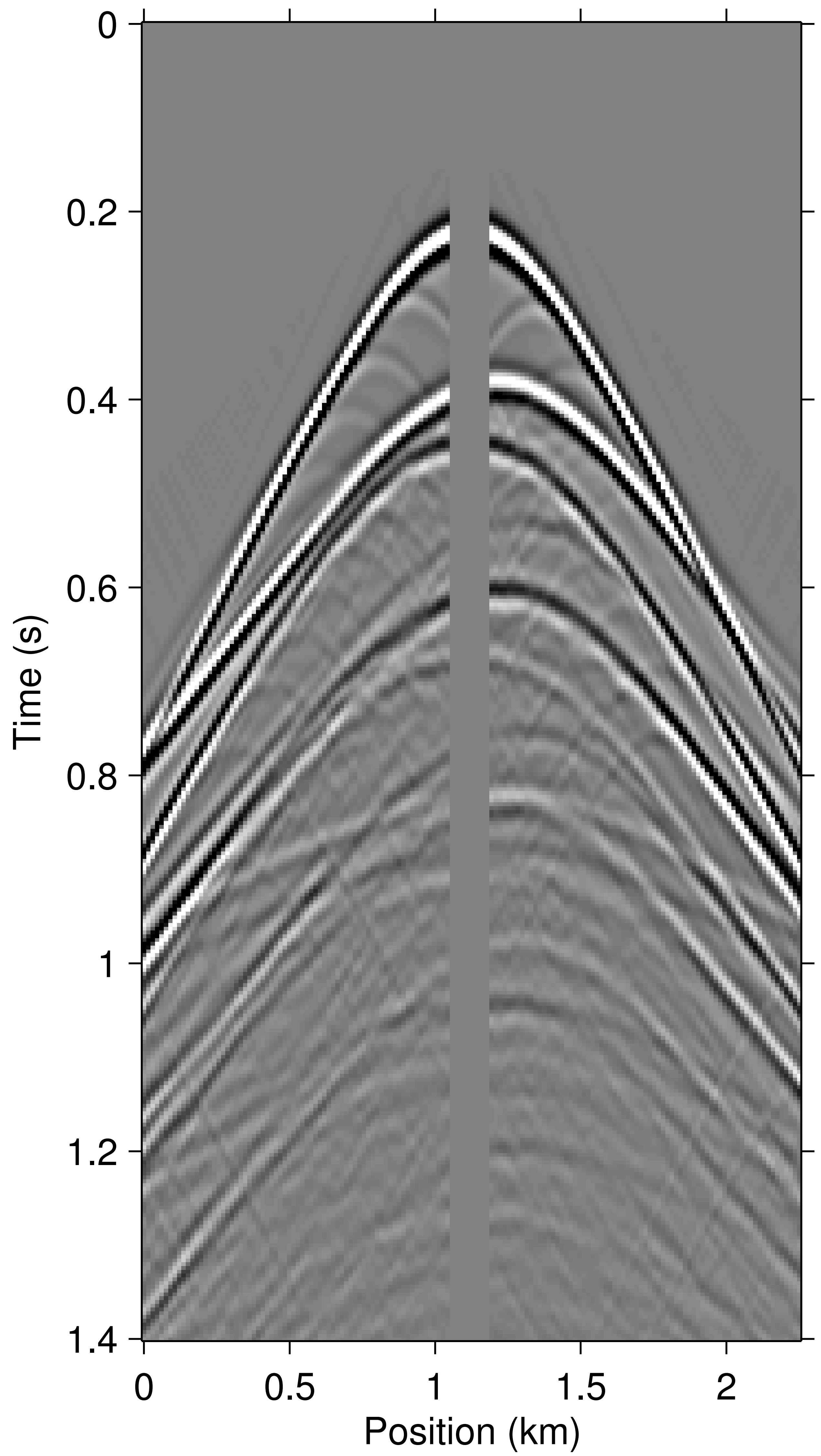
2nd Order autoconvolution term $$+\ \mathbf{RGK_c}\circ(\mathbf{GQ}+\mathbf{RGP}')$$

3rd Order autoconvolution term $$+\ \mathbf{RGK_c}\circ(\mathbf{RGK_c}\circ(\mathbf{GQ}+\mathbf{RGP}'))$$

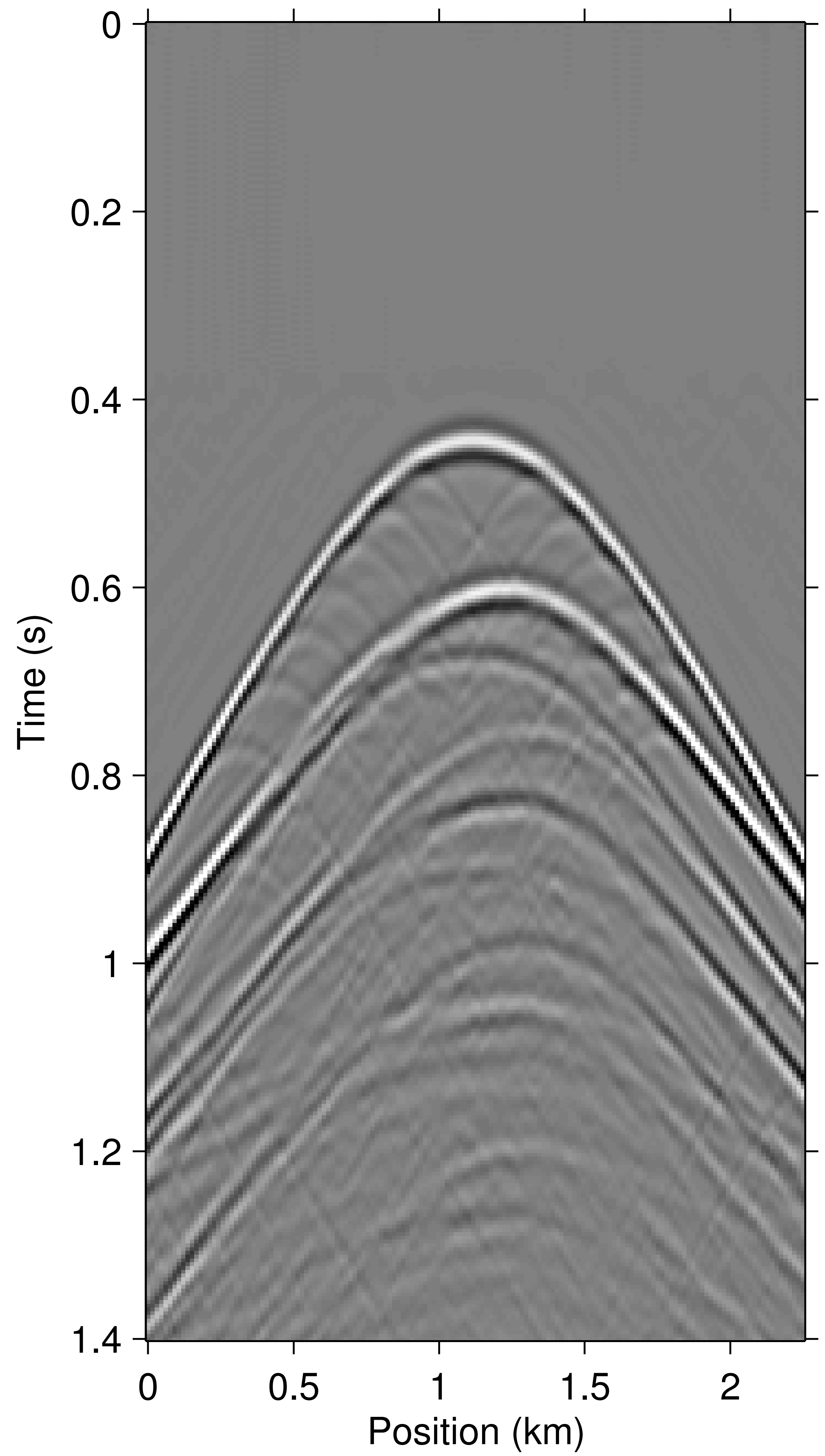
$$+\ \mathcal{O}(\mathbf{G}^4)\Big]$$

$$:= \mathbf{K}\circ\sum_{n=0}^{\infty}(\mathbf{RGK_c}\circ)^n\,(\mathbf{GQ}+\mathbf{RGP}')\,.$$

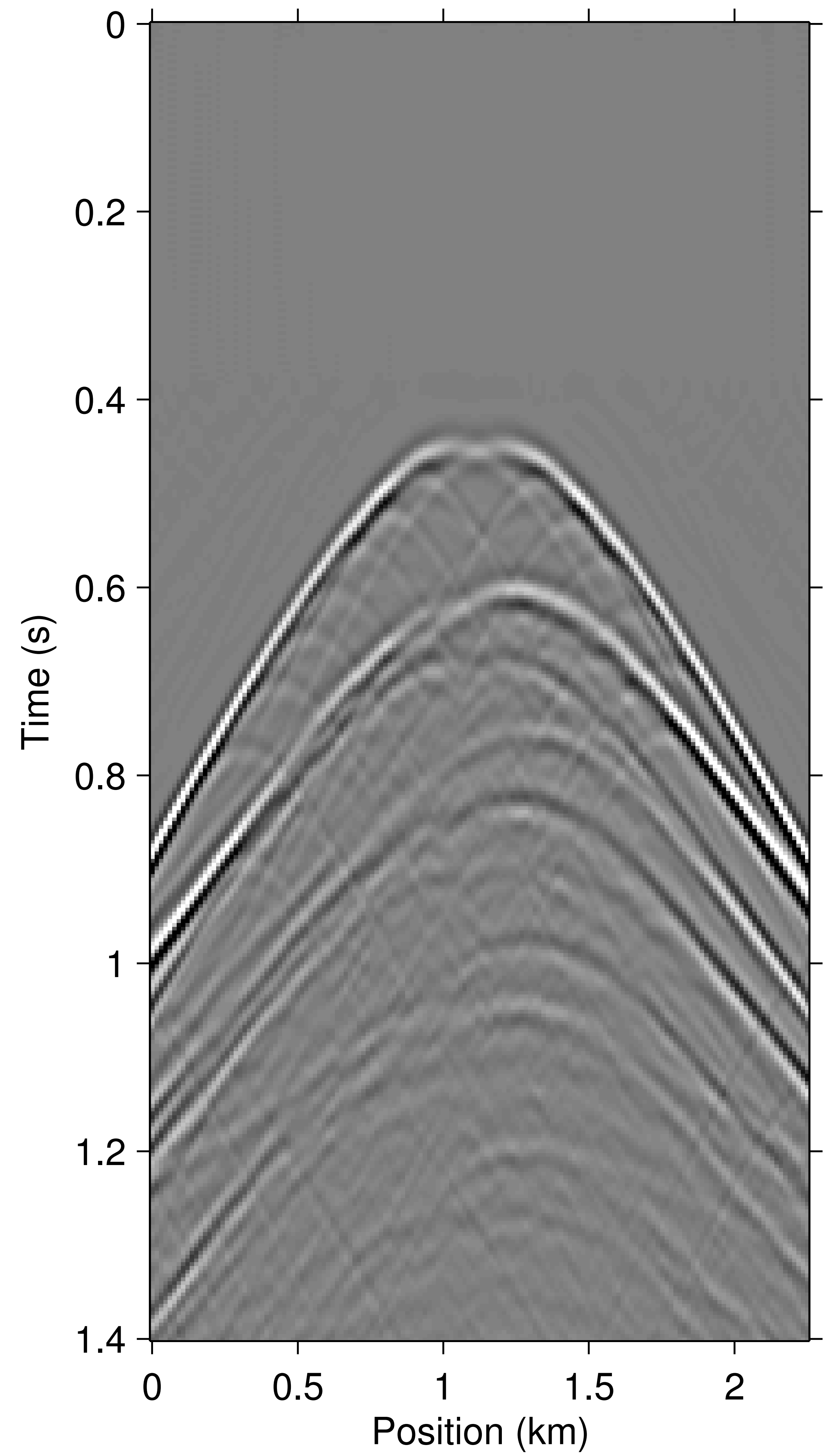
What these terms look like



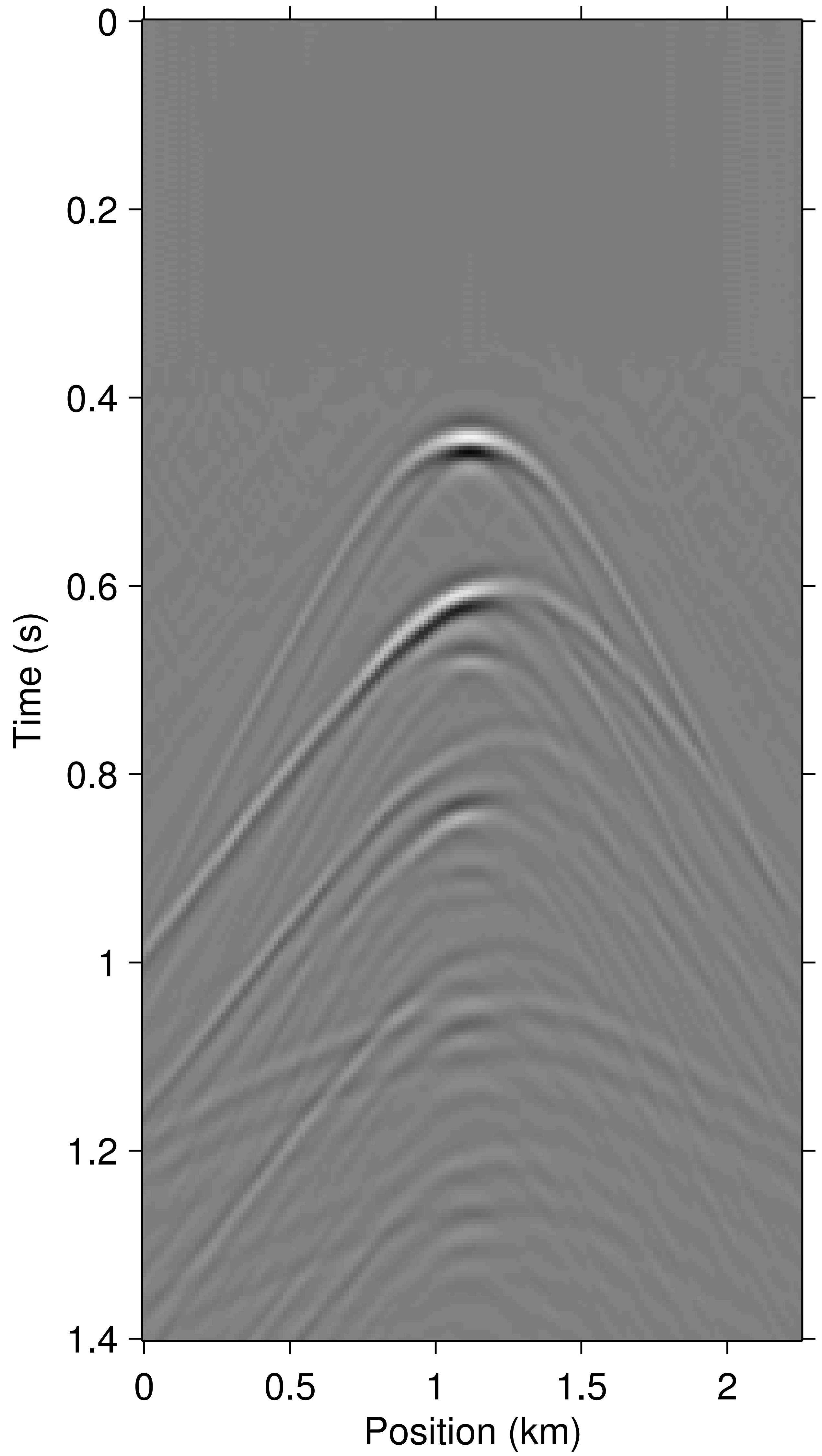
Data with missing traces P'



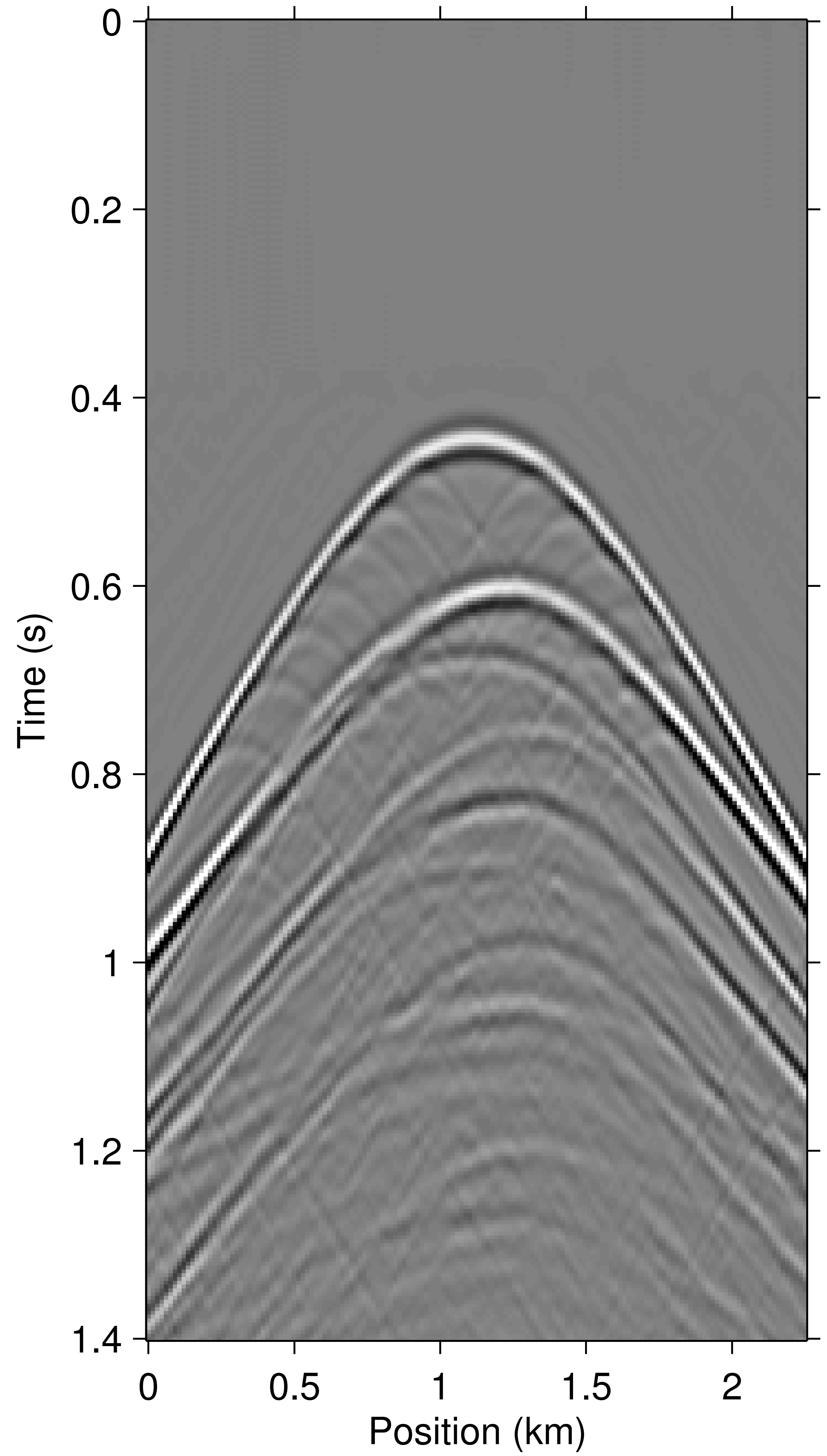
Exact multiples



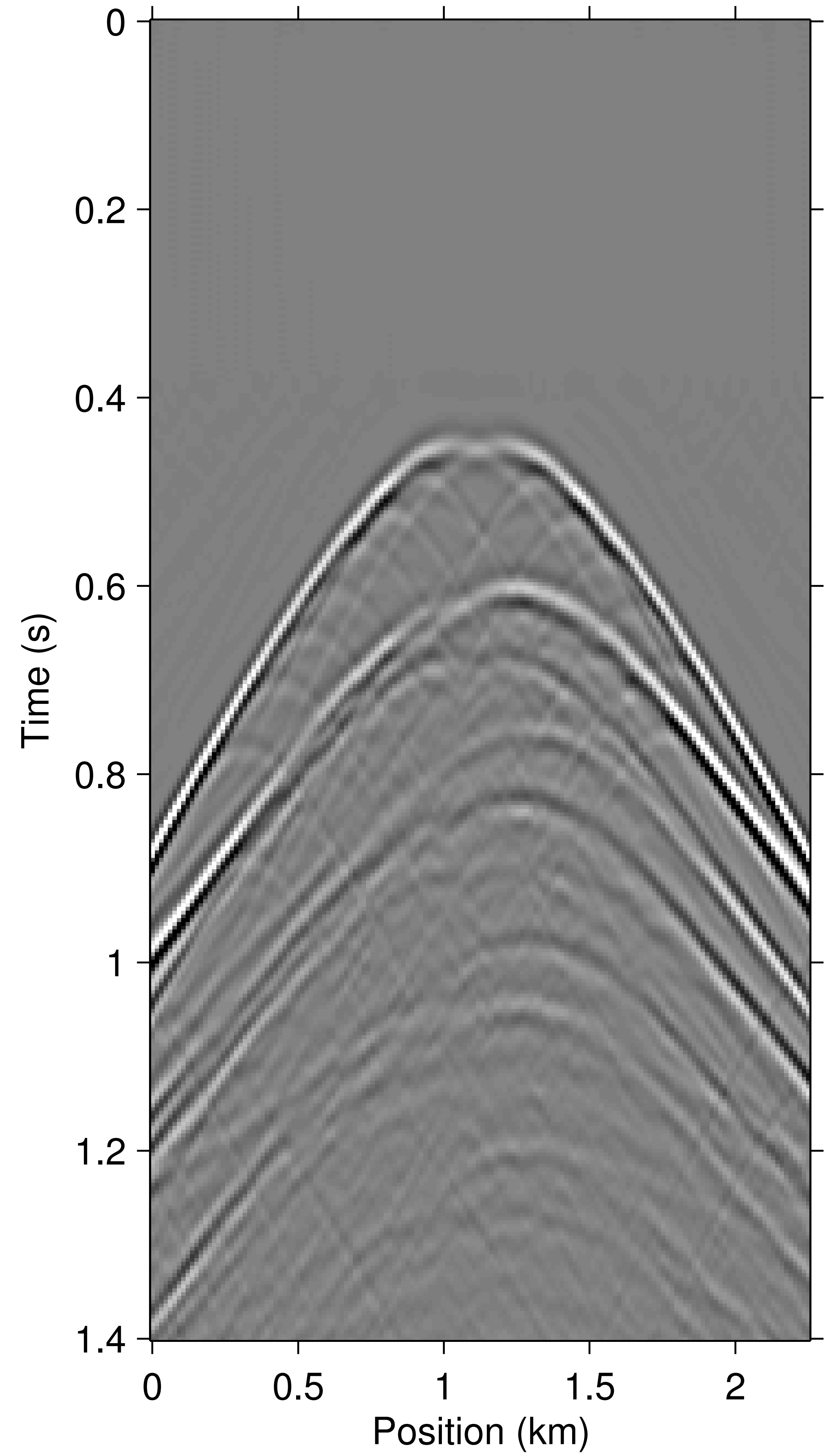
EPSI Predicted -GP'
(with perfect G)



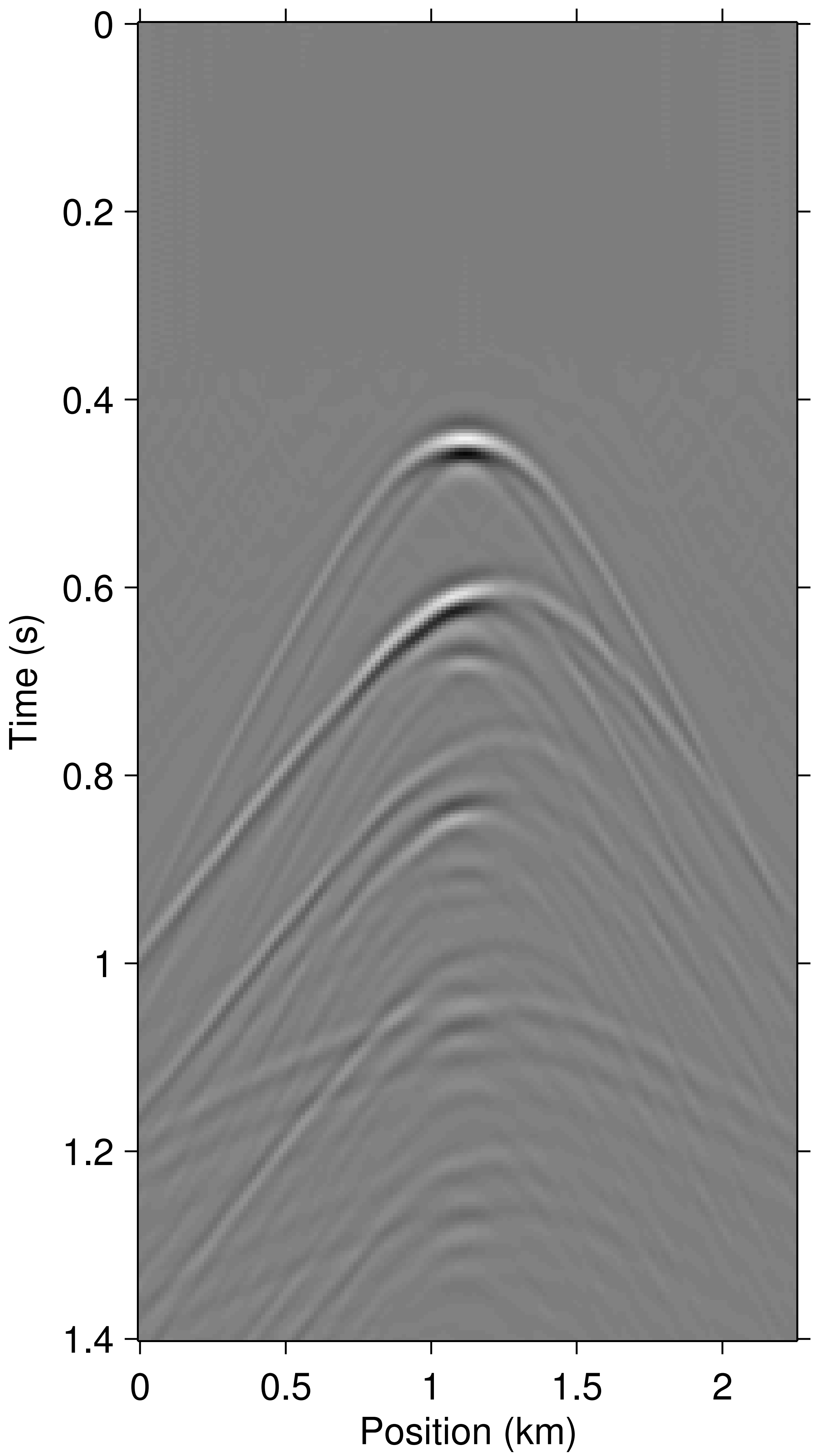
Missing contributions



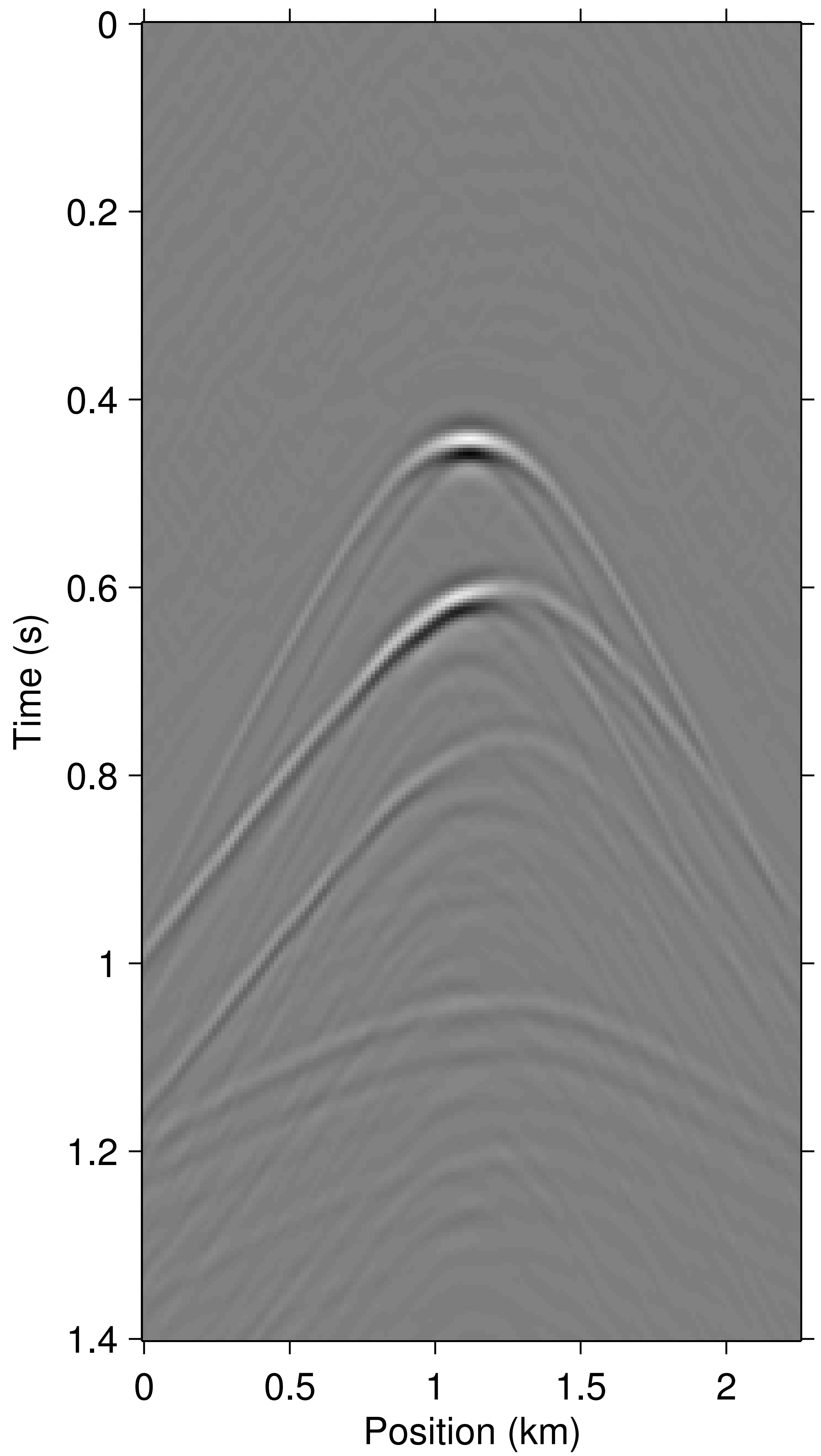
Exact multiples



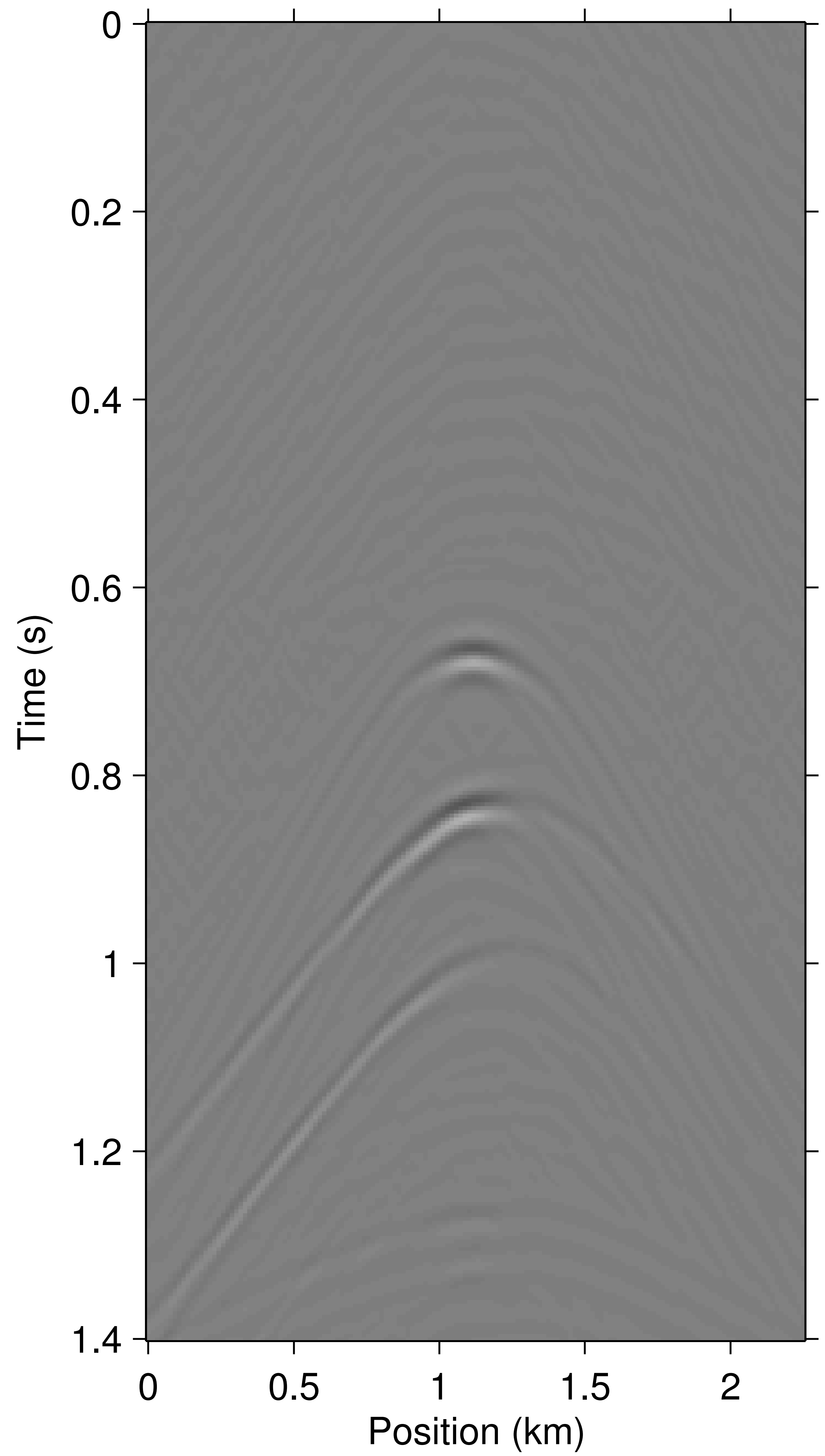
EPSI Predicted -GP’
(with perfect G)



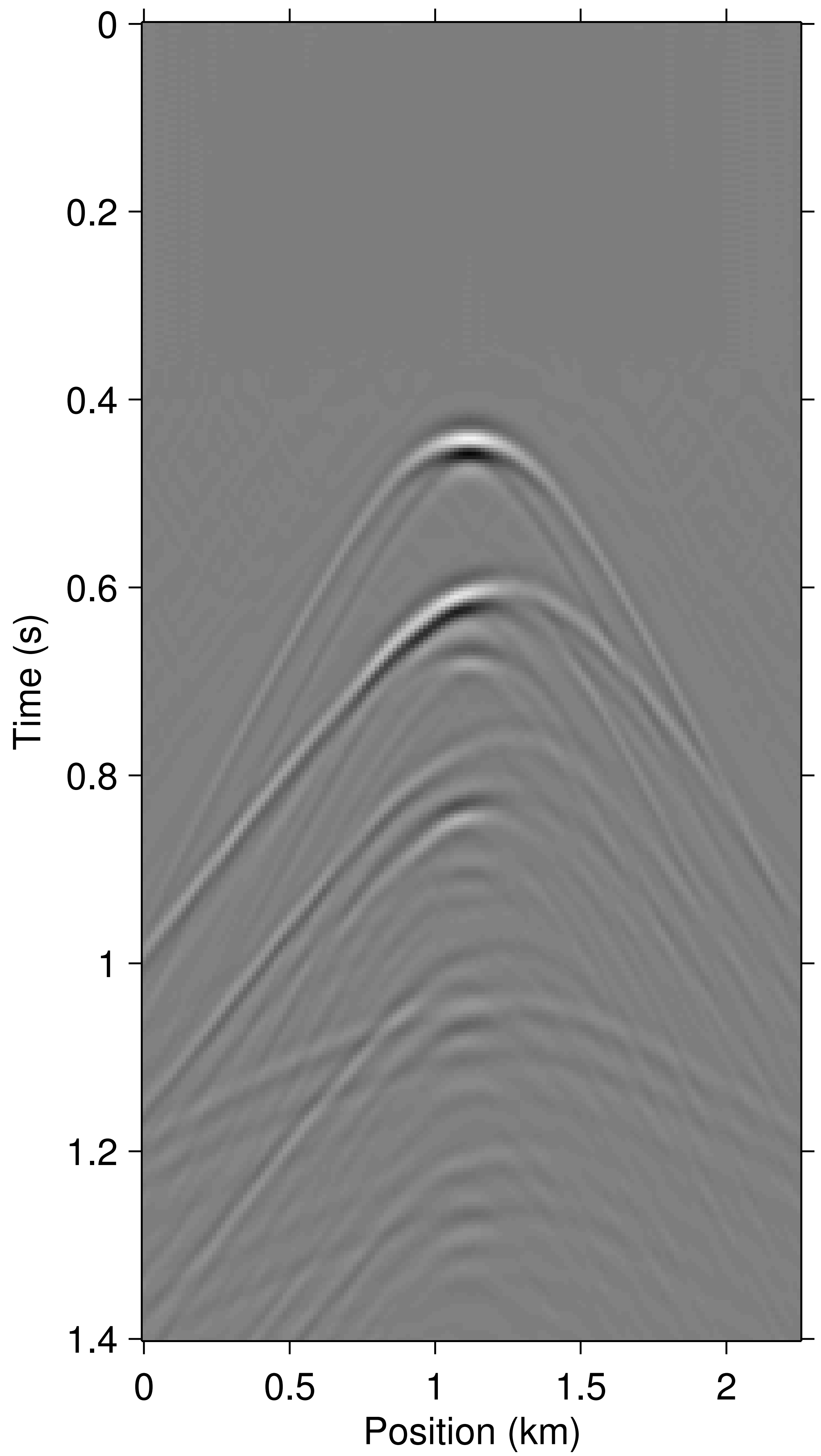
Missing contributions



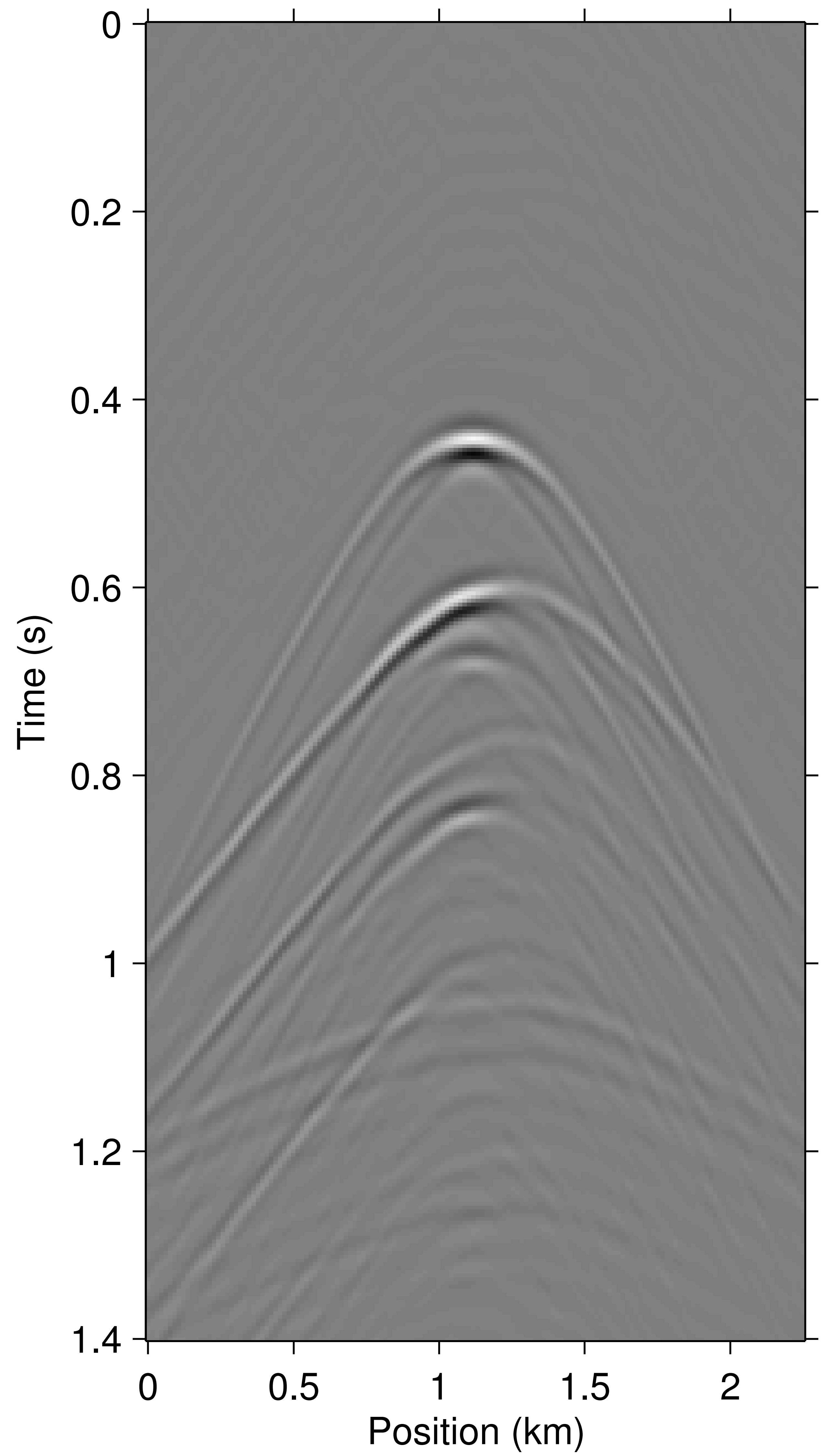
2nd order term



3rd order term

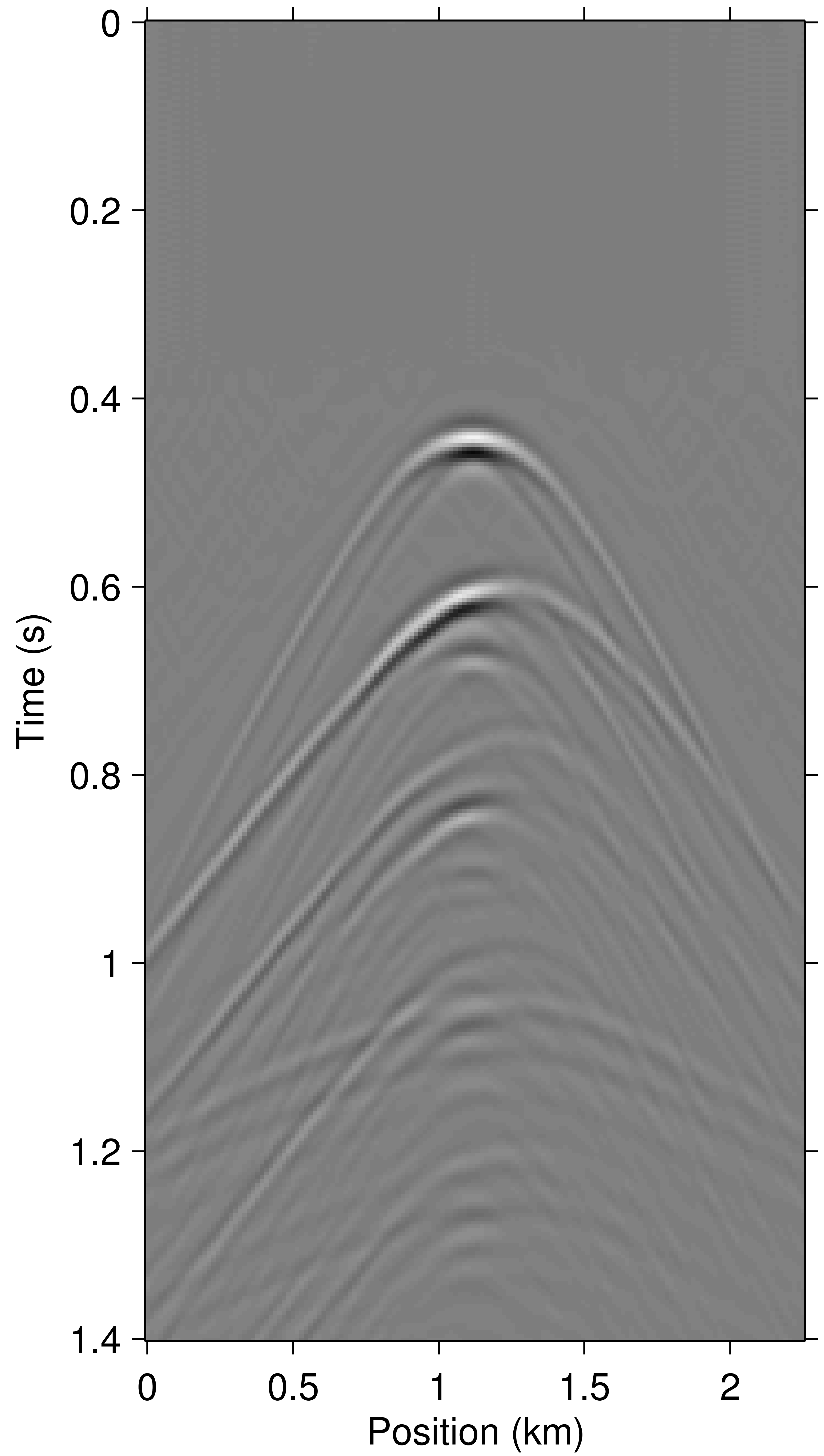


Missing contributions

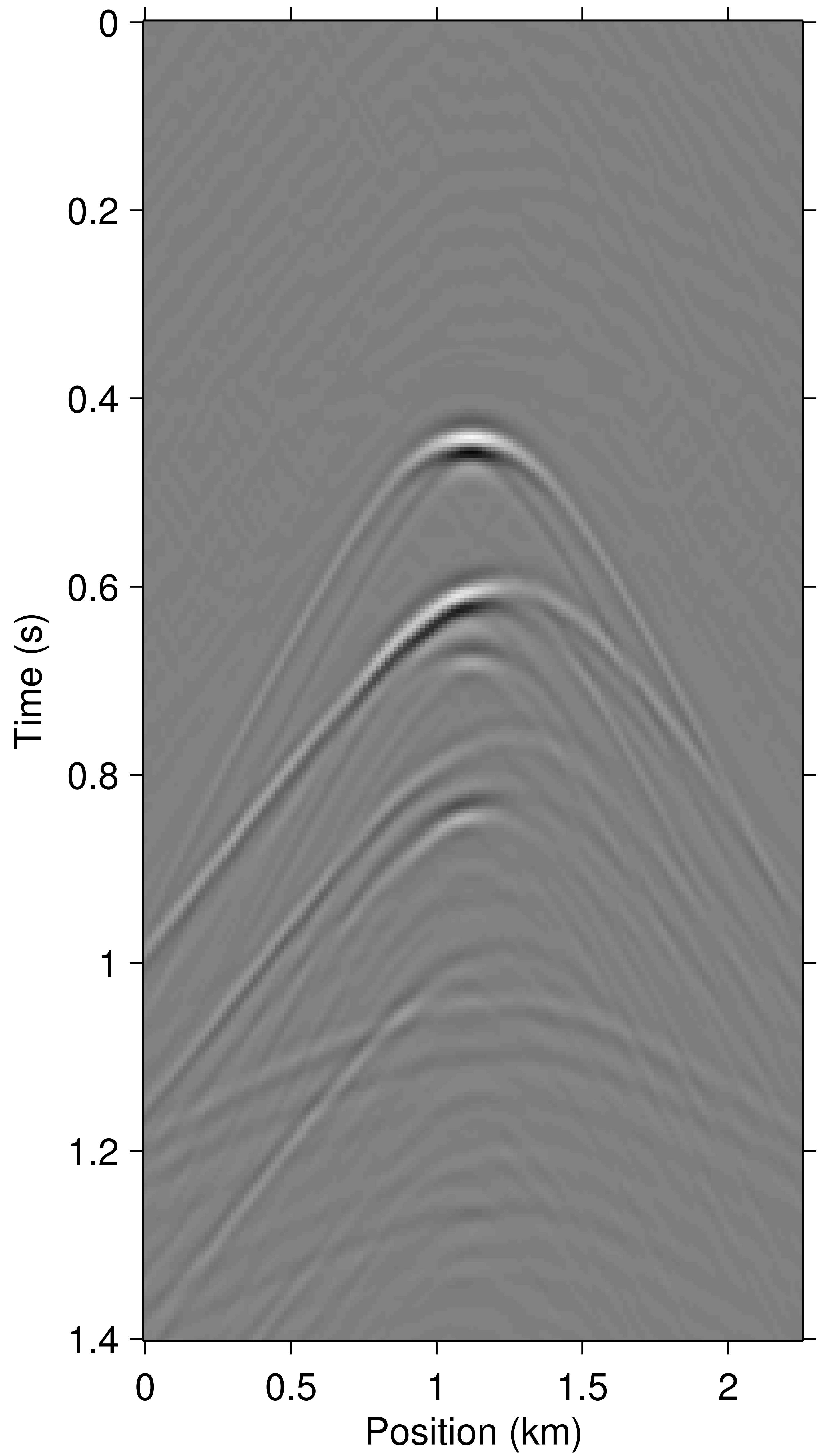


2nd order term + 3rd order

Total missing contrib.



Modeled with two terms



Main Result

Just one or two of these terms is enough to account for missing traces

Modify the modeling operator

$$
\begin{aligned}
\widetilde{\mathcal{M}}(\mathbf{G},\mathbf{Q};\mathbf{P}') =& \mathbf{K}\circ\big[\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}' \\
\text{2nd Order autoconvolution term} \quad & + \ \mathbf{R}\mathbf{G}\mathbf{K_c}\circ(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}') \\
\text{3rd Order autoconvolution term} \quad & + \ \mathbf{R}\mathbf{G}\mathbf{K_c}\circ(\mathbf{R}\mathbf{G}\mathbf{K_c}\circ(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}')) \\
& + \ \mathcal{O}(\mathbf{G}^4)\big] \\
:=& \mathbf{K}\circ\sum_{n=0}^{\infty}(\mathbf{R}\mathbf{G}\mathbf{K_c}\circ)^n(\mathbf{G}\mathbf{Q}+\mathbf{R}\mathbf{G}\mathbf{P}').
\end{aligned}
$$

Convergent sum

$$\mathbf{K} \circ \mathbf{P} = \mathbf{K} \circ \sum_{n=0}^{\infty} (\mathbf{R}\mathbf{G}\mathbf{K_c} \circ)^n (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

$$= \mathbf{K} \circ (\mathbf{I} - \mathbf{R}\mathbf{G}\mathbf{K_c} \circ)^{-1} (\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}')$$

SLIM

Convergent sum

$$\mathbf{K} \circ \mathbf{P} = \mathbf{K} \circ \sum_{n=0}^{\infty} \left(\mathbf{R}\mathbf{G}\mathbf{K_c}\circ\right)^n \left(\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}'\right)$$

$$= \mathbf{K} \circ \left(\mathbf{I} - \mathbf{R}\mathbf{G}\mathbf{K_c}\circ\right)^{-1} \left(\mathbf{G}\mathbf{Q} + \mathbf{R}\mathbf{G}\mathbf{P}'\right)$$

Verifies the validity of the expression

Solution strategy

Robust EPSI
Inverting for unknown data 1.0

$$\Delta \mathbf{P}(\mathbf{G}_{k+1}, \mathbf{R}_{k+1}) := -(\mathbf{I} + \mathbf{G}_{k+1})^H (\mathbf{R}_{k+1})$$

While $\|\mathbf{p}_k - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p}_k - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p}_k - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \alpha \Delta \mathbf{p}(\mathbf{g}_{k+1}, \mathbf{q}_{k+1}, \mathbf{p}_k)$$

Autoconvolving Robust EPSI
Accounting for unknown data with G

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}, \mathbf{q_k})\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

Strategy 1: Re-linearization
Using G from previous iter in higher-order terms

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

fix at $\mathbf{g_k}$ for autoconv terms

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

Strategy 2: Modified Gauss-Newton
Obtain Jacobian using G from previous iter

While $\|\mathbf{p} - \widetilde{\mathcal{M}}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

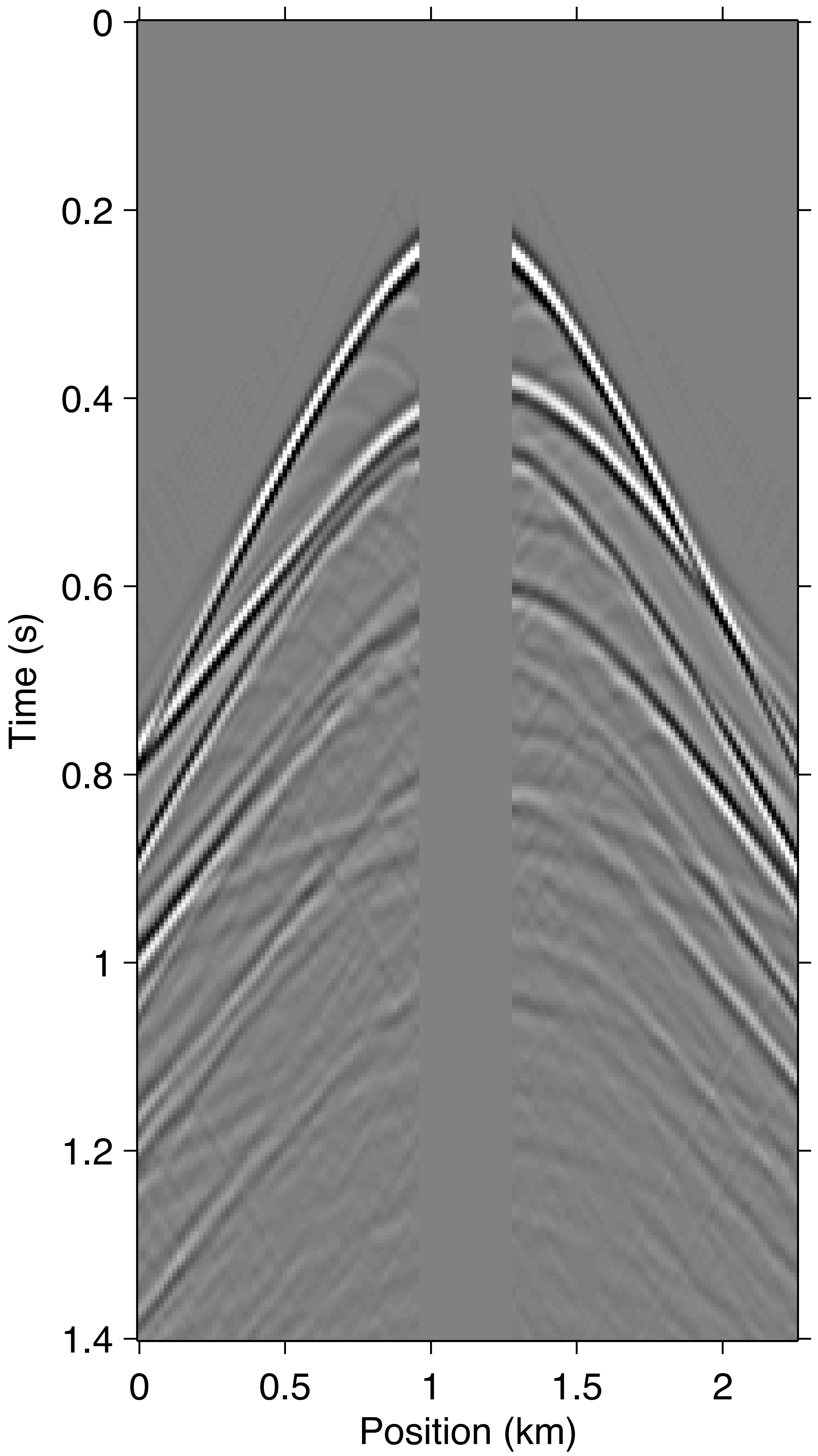
determine new τ_k from the Pareto curve

$$\mathbf{g}_{k+1} = \mathbf{g}_k + \arg\min_{\Delta\mathbf{g}} \|\mathbf{r}_k - \partial_{(g_k,q_k)}\widetilde{\mathcal{M}}\Delta\mathbf{g}\|_2 \text{ s.t. } \|\Delta\mathbf{g}\|_1 \leq \tau_k$$

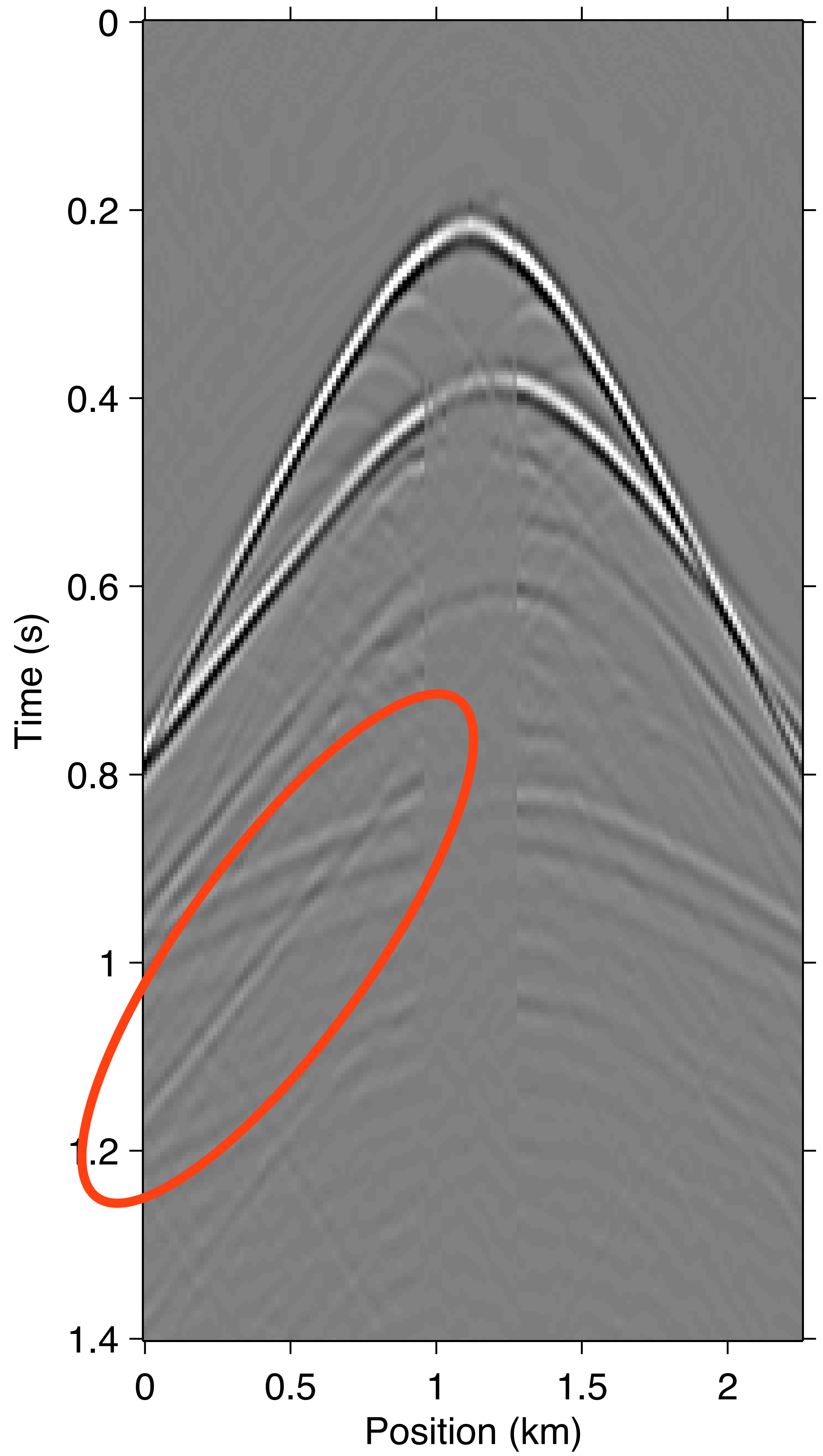
$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \widetilde{\mathbf{M}}_{g_{k+1}}\mathbf{q}\|_2$$

Compare with explicit updating of P

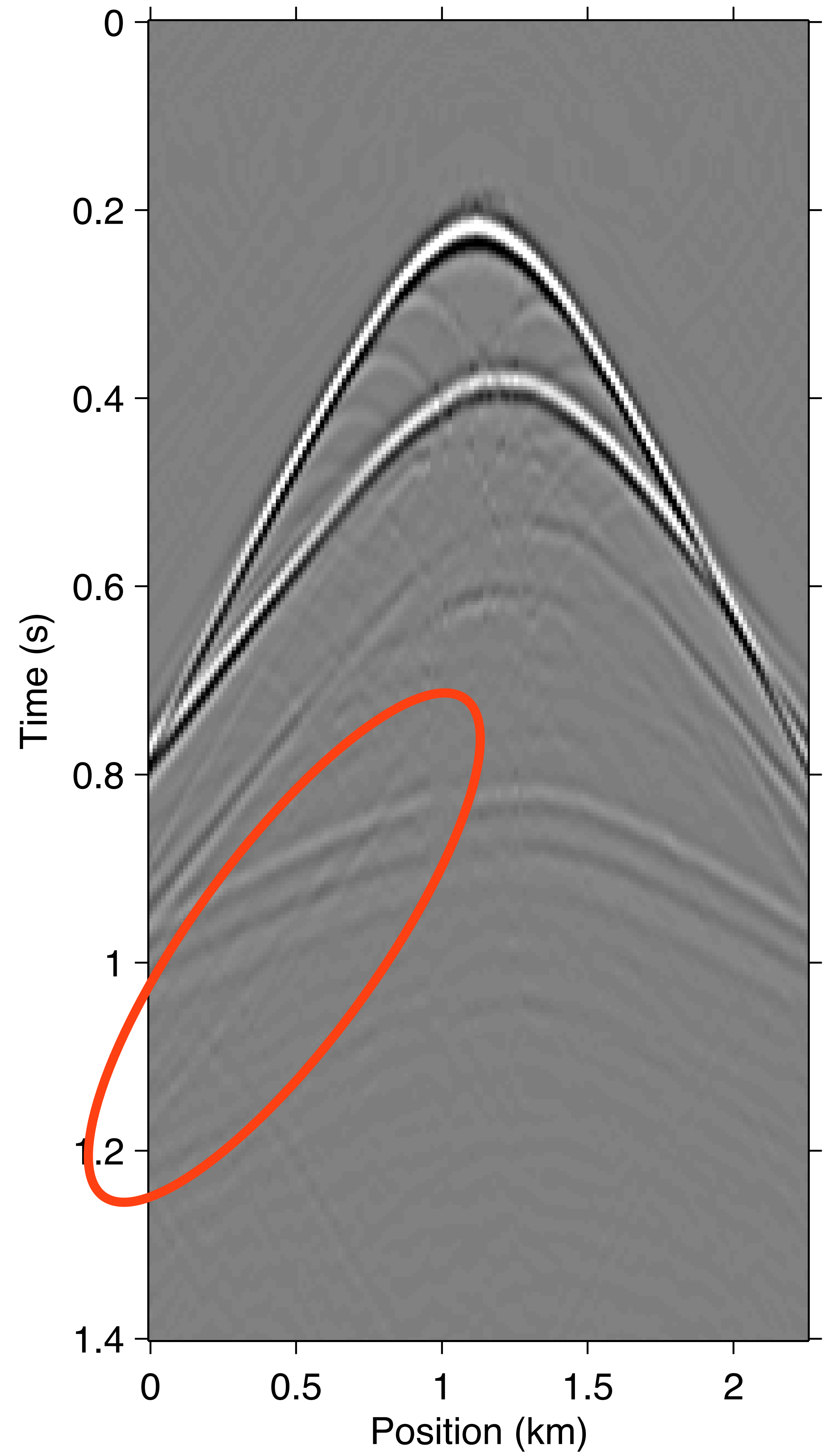
150m near-offset
2km max offset



Data with missing near-offset



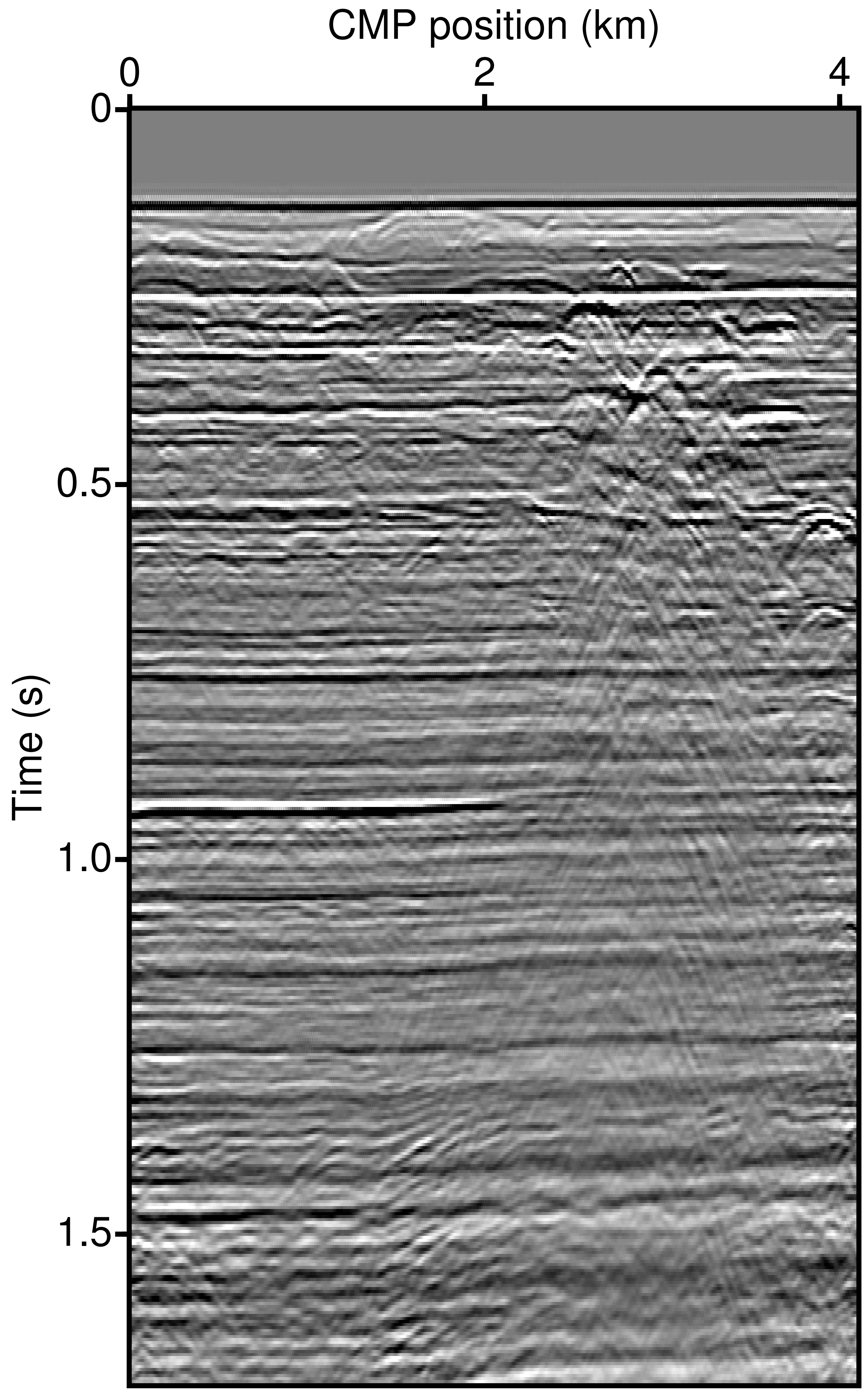
Explicit data updates



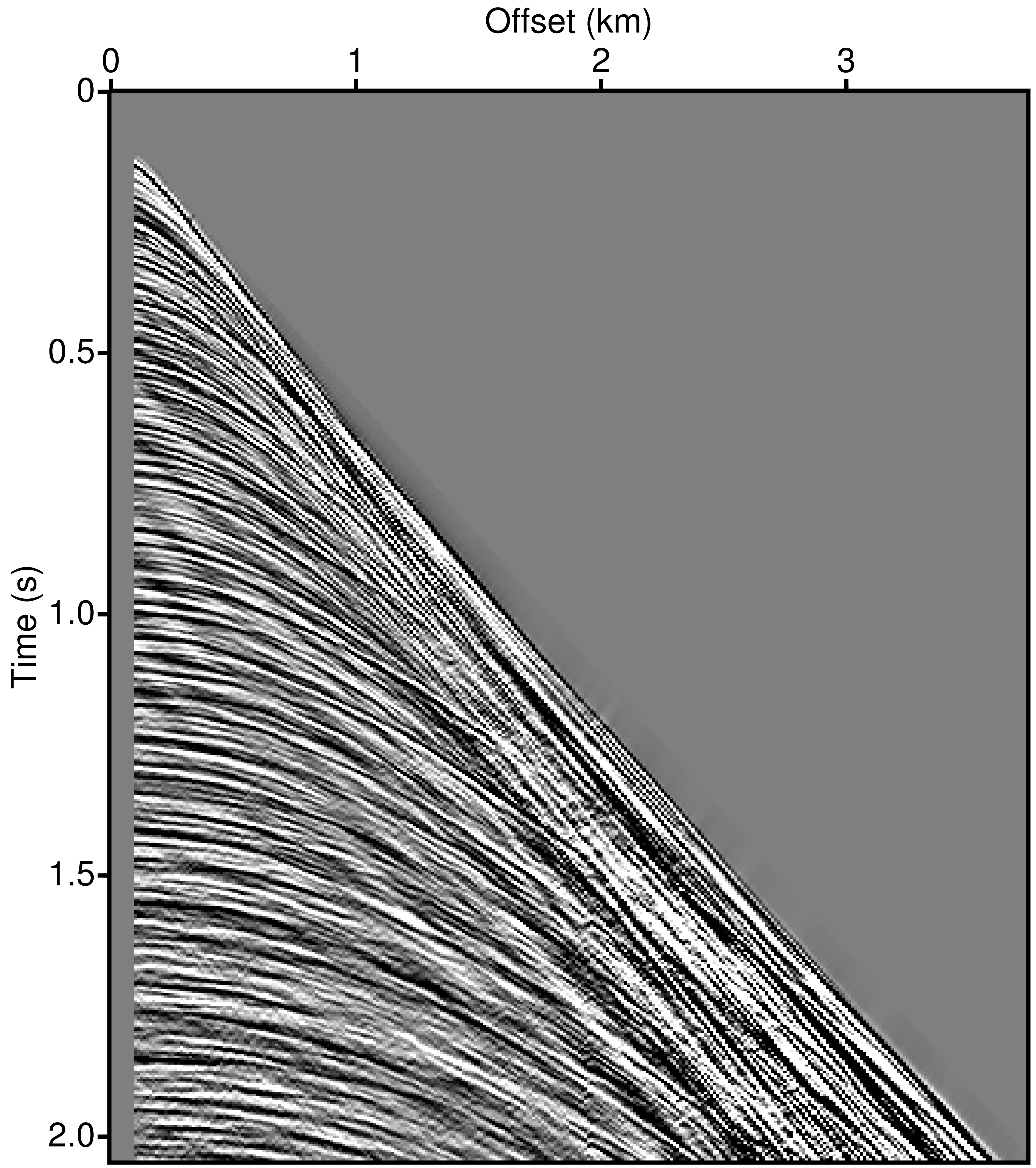
Autoconvolving upto 3rd-order (GN)

Field data example

North Sea dataset



Data



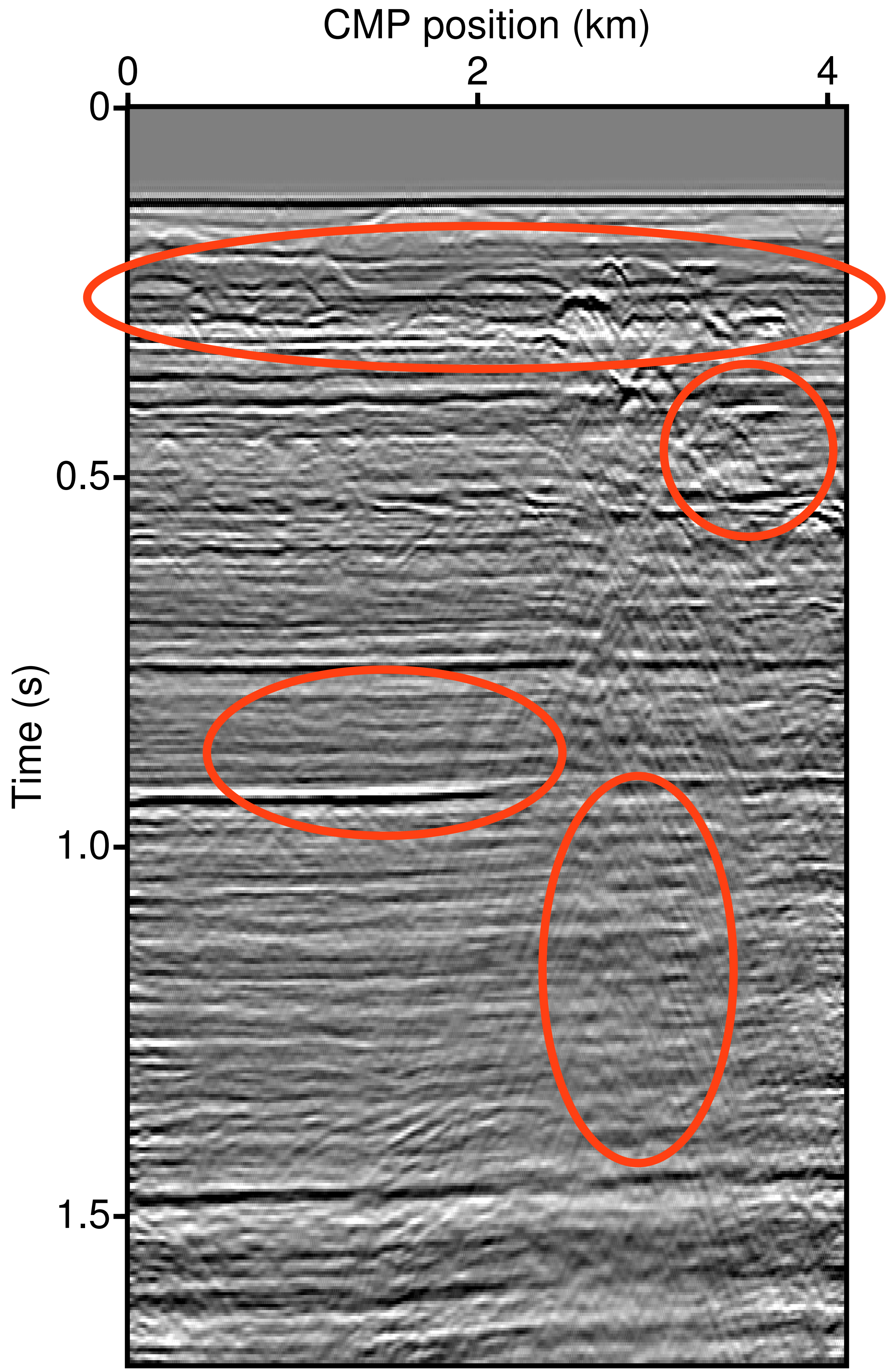
Shot

North Sea dataset

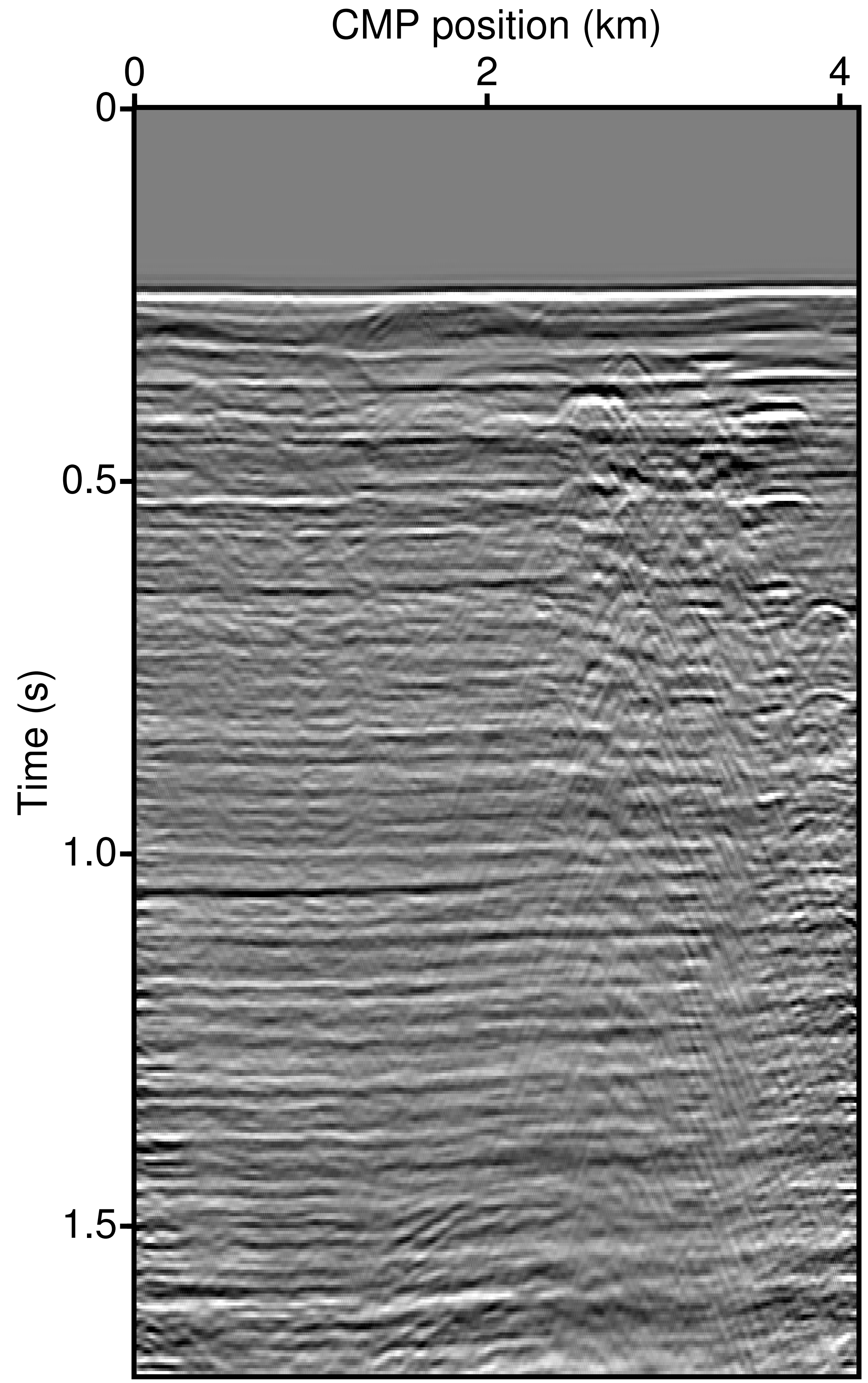
100m near-offset

regularized to 12.5m dx and 4km fixed-spread from streamer

4ms sampling

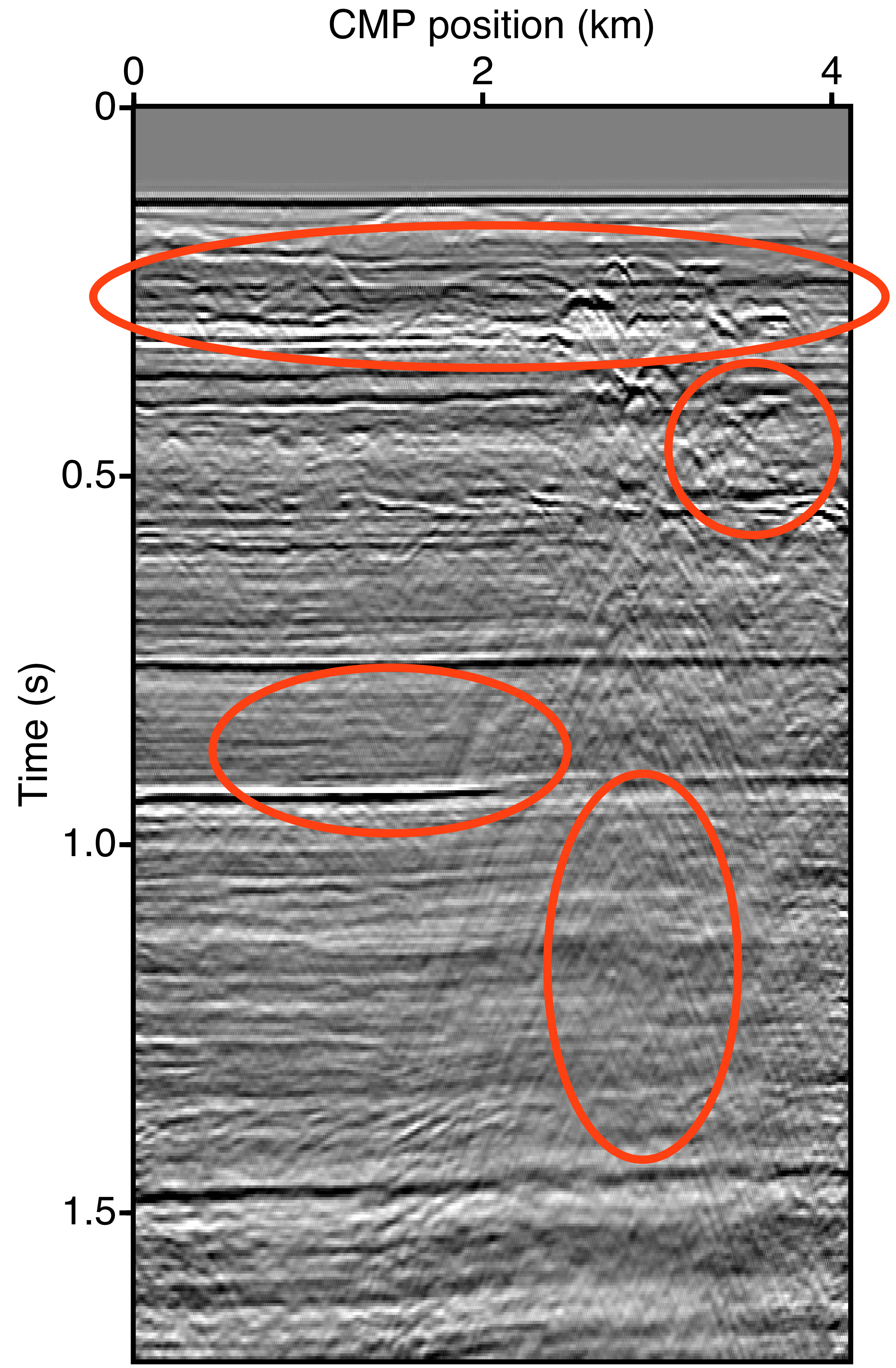


Conservative primary

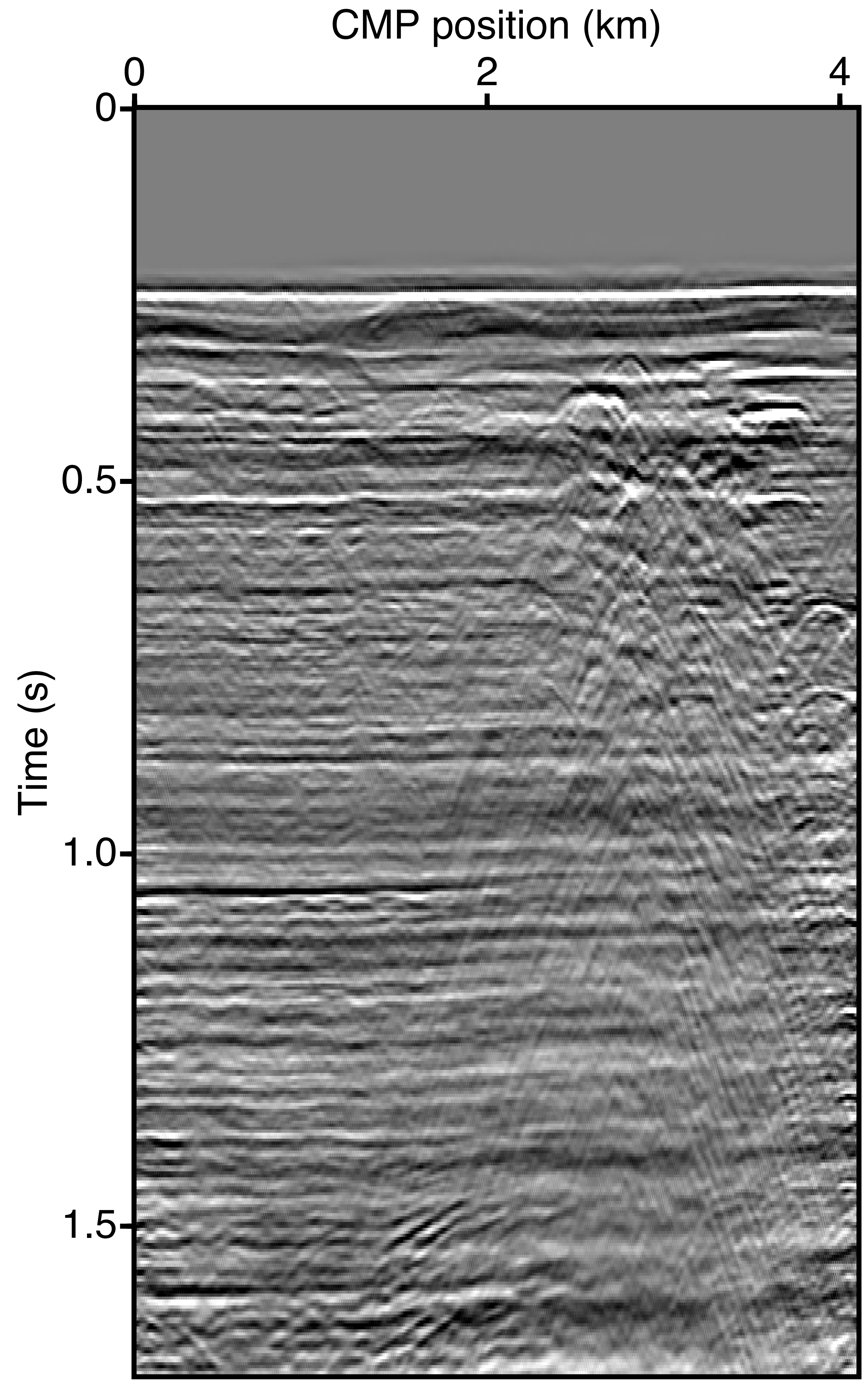


Multiple

NMO stack
Parabolic Radon Interp

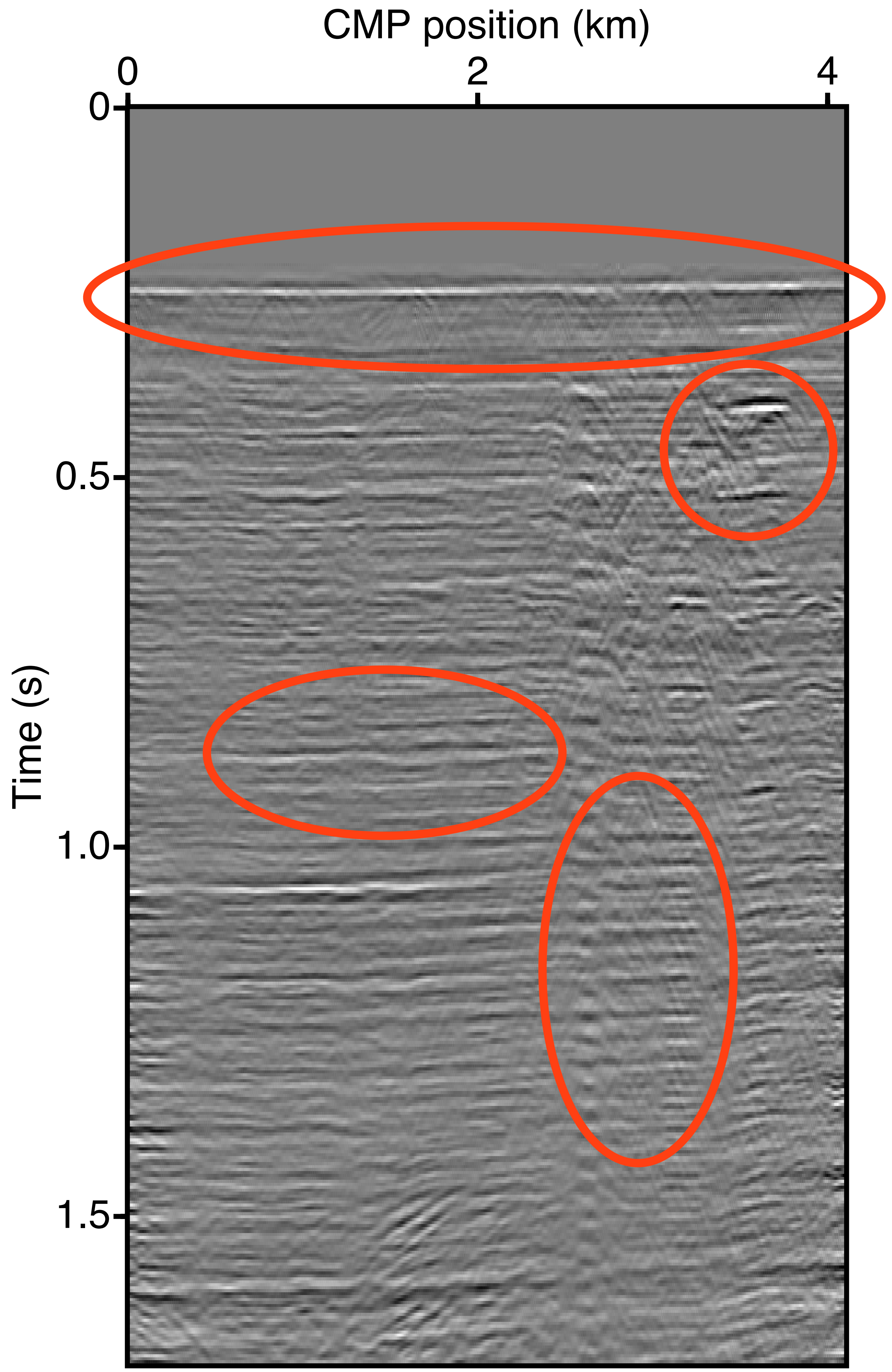


Conservative primary

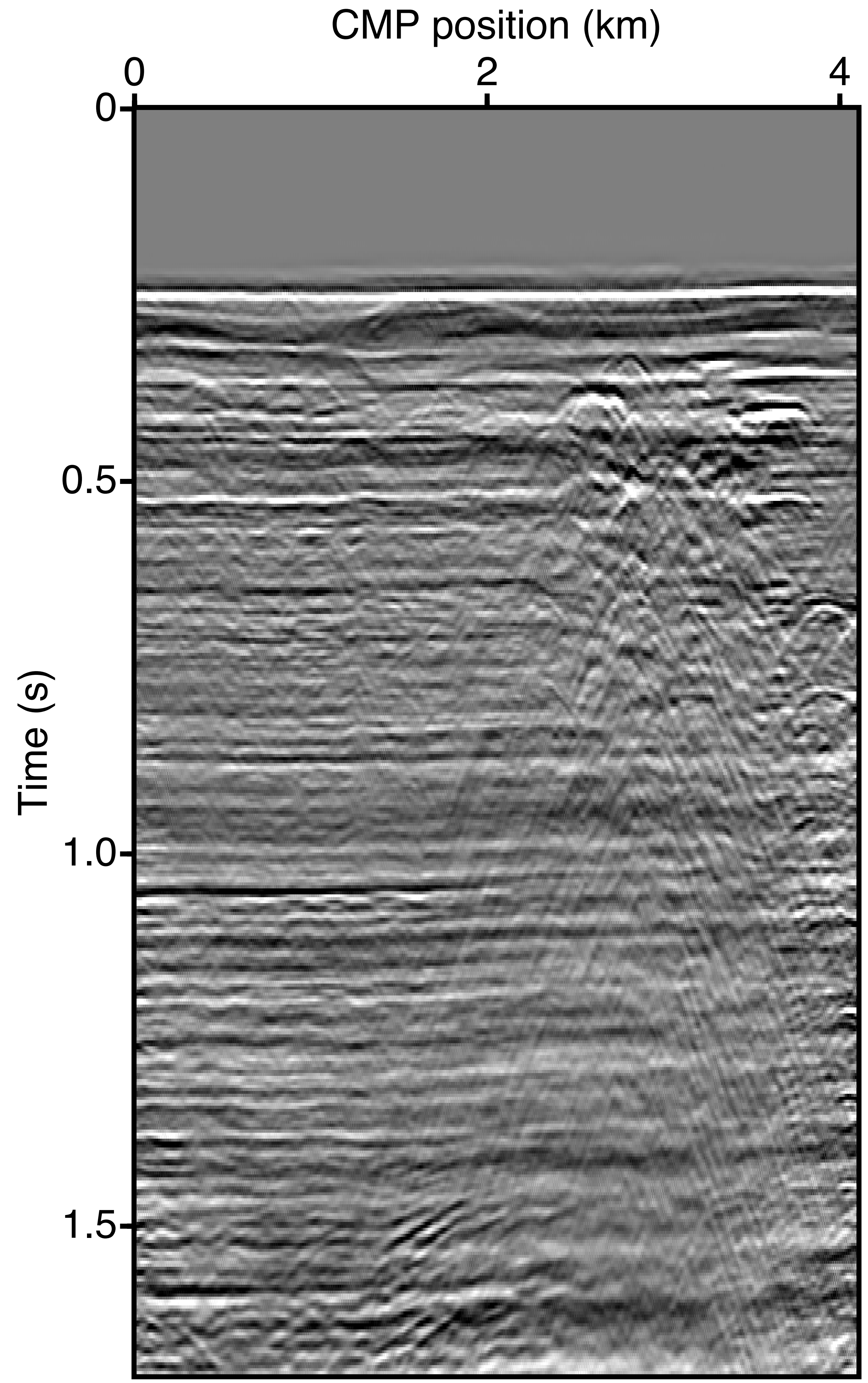


Multiple

NMO stack
Re-linearization
Using 3rd Order terms



Difference from Radon interp.

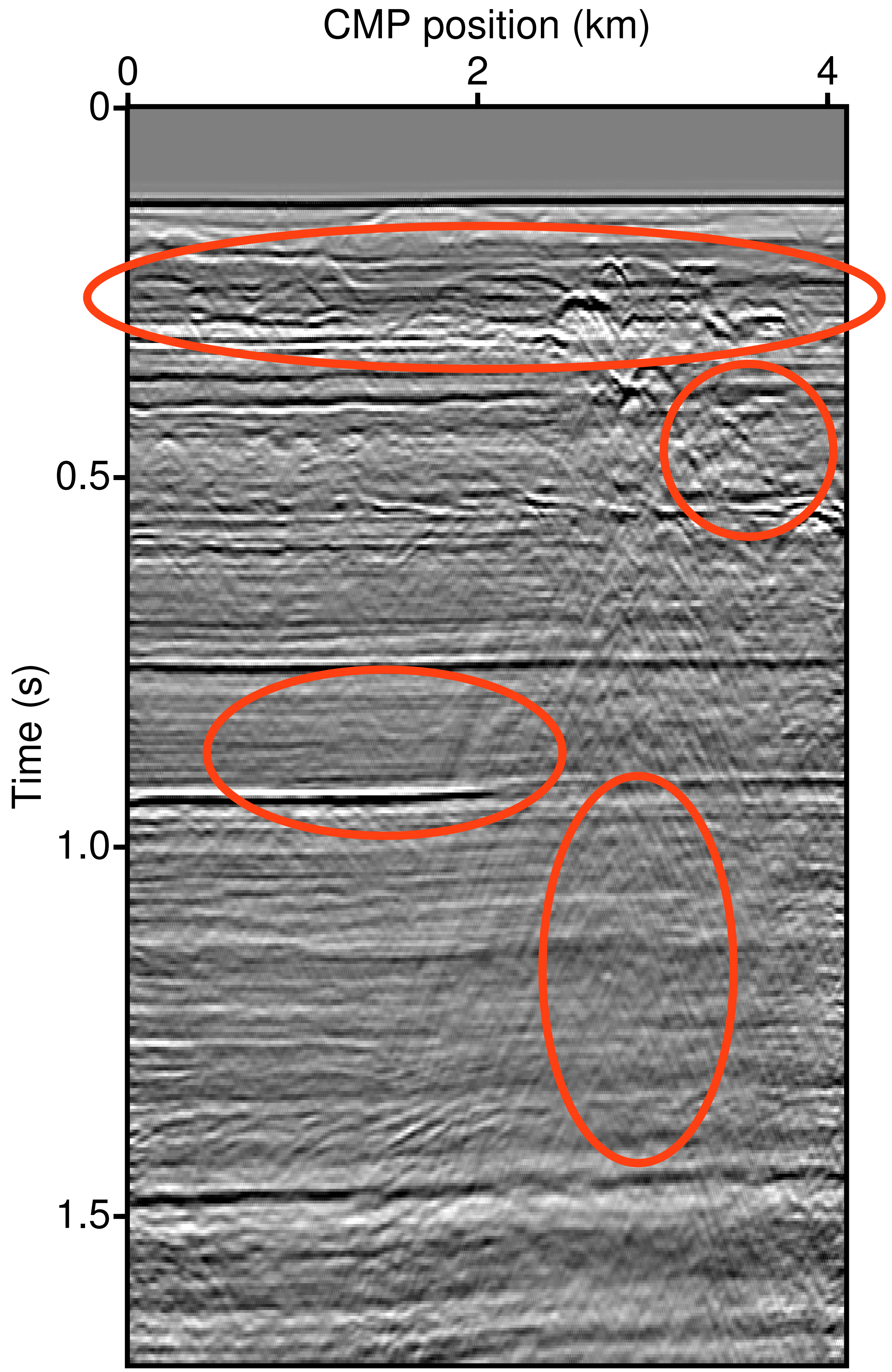


Multiple

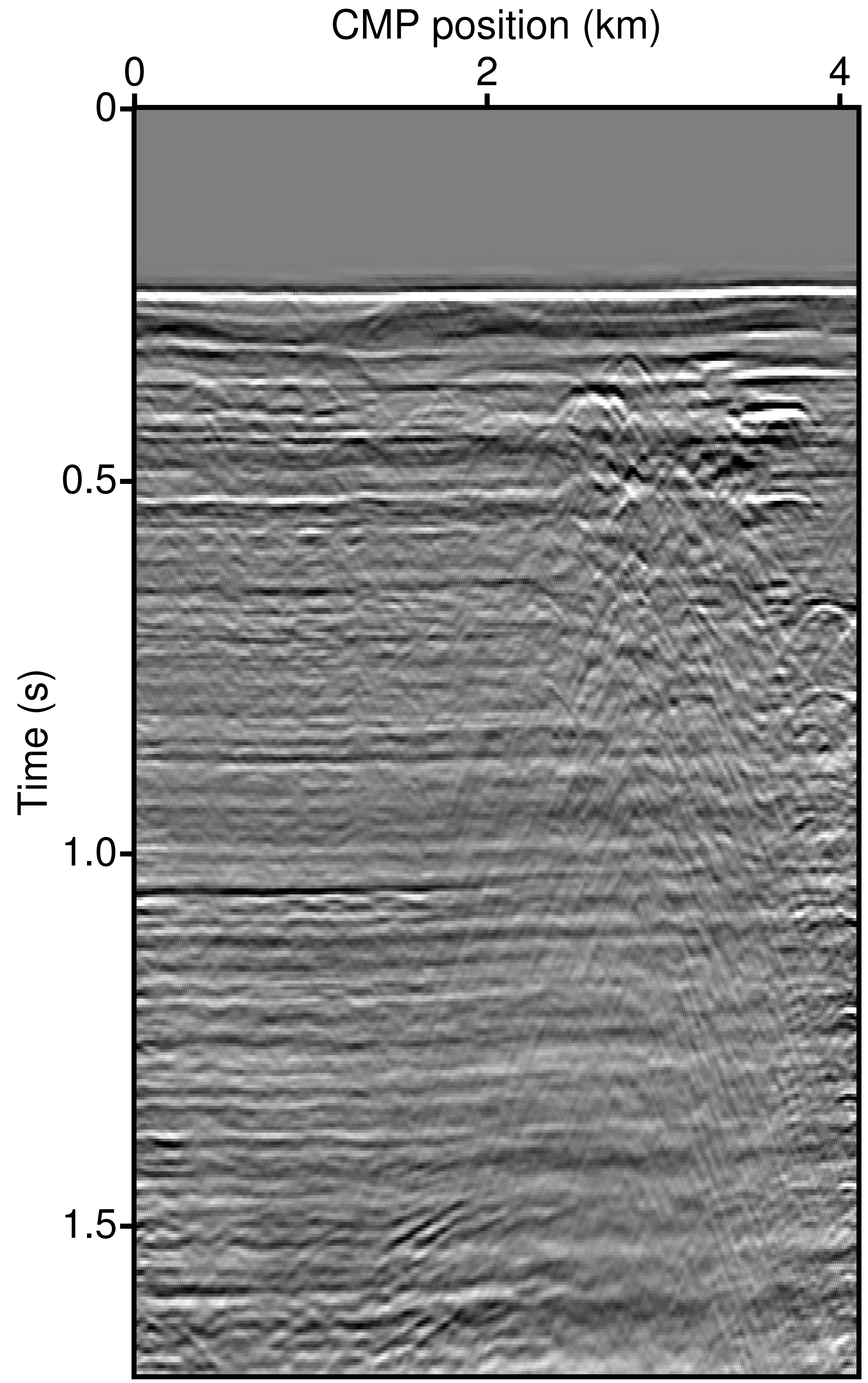
NMO stack

Re-linearization

Using 3rd Order terms



Conservative primary

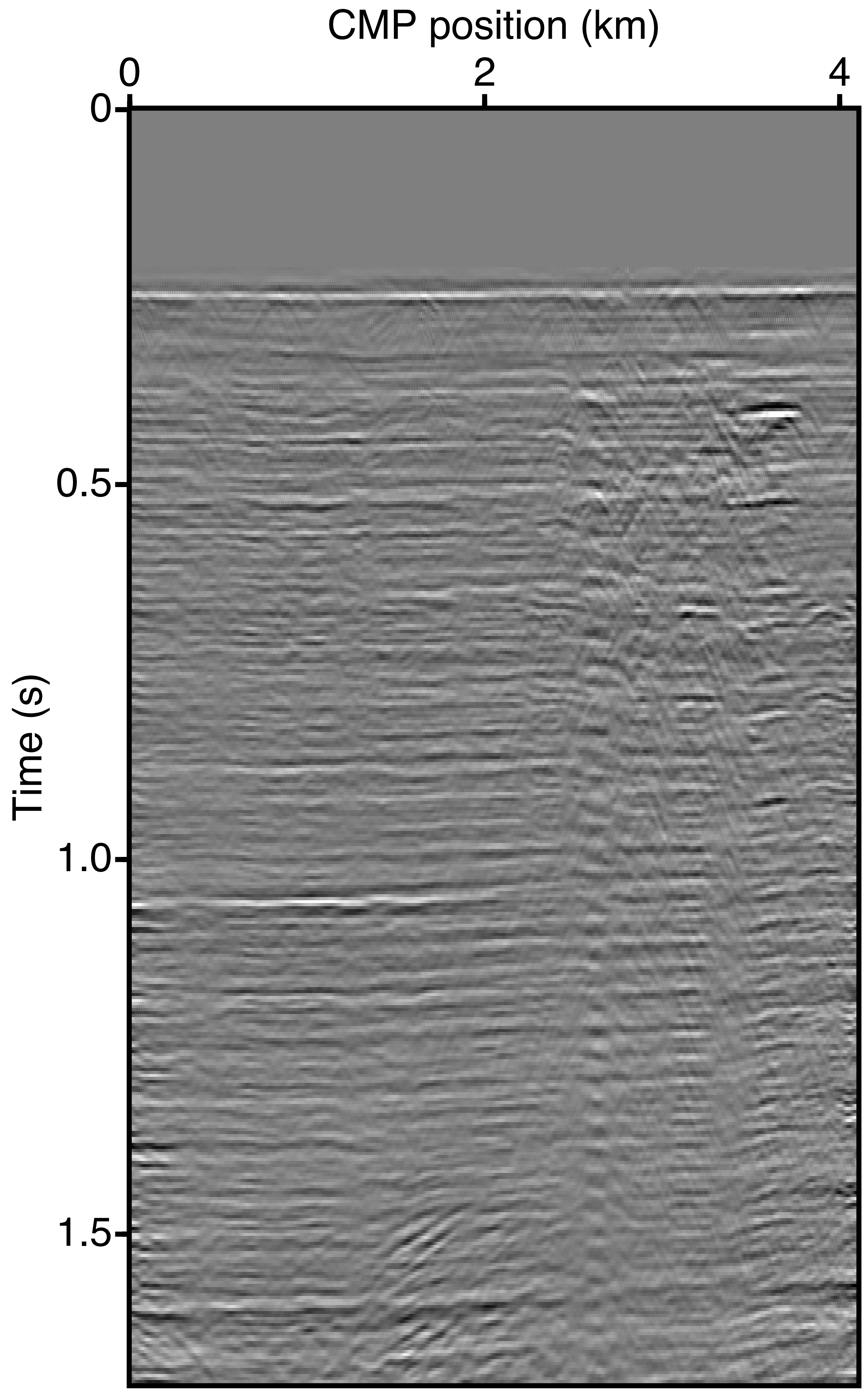


Multiple

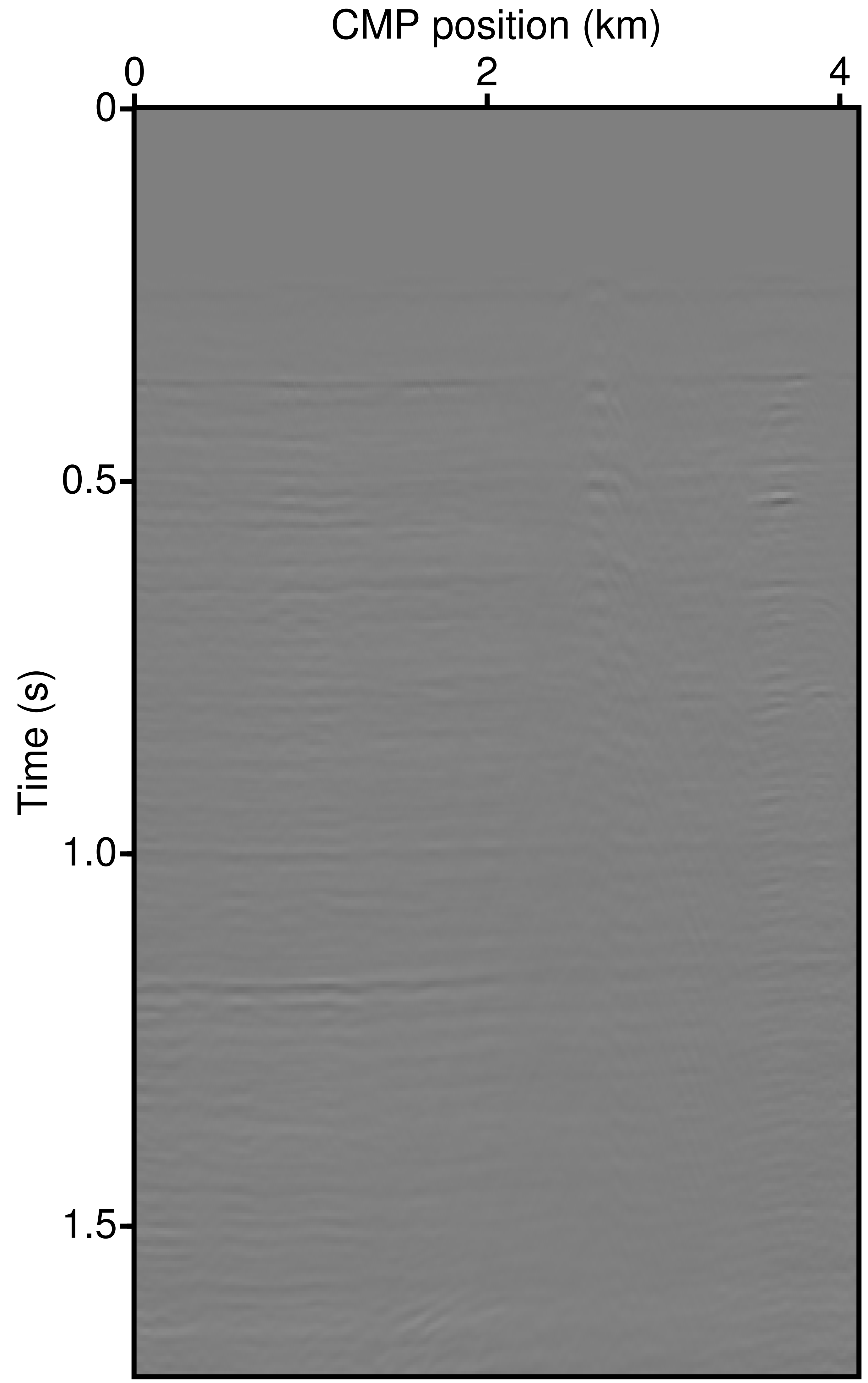
NMO stack

Re-linearization

Using 2nd Order terms

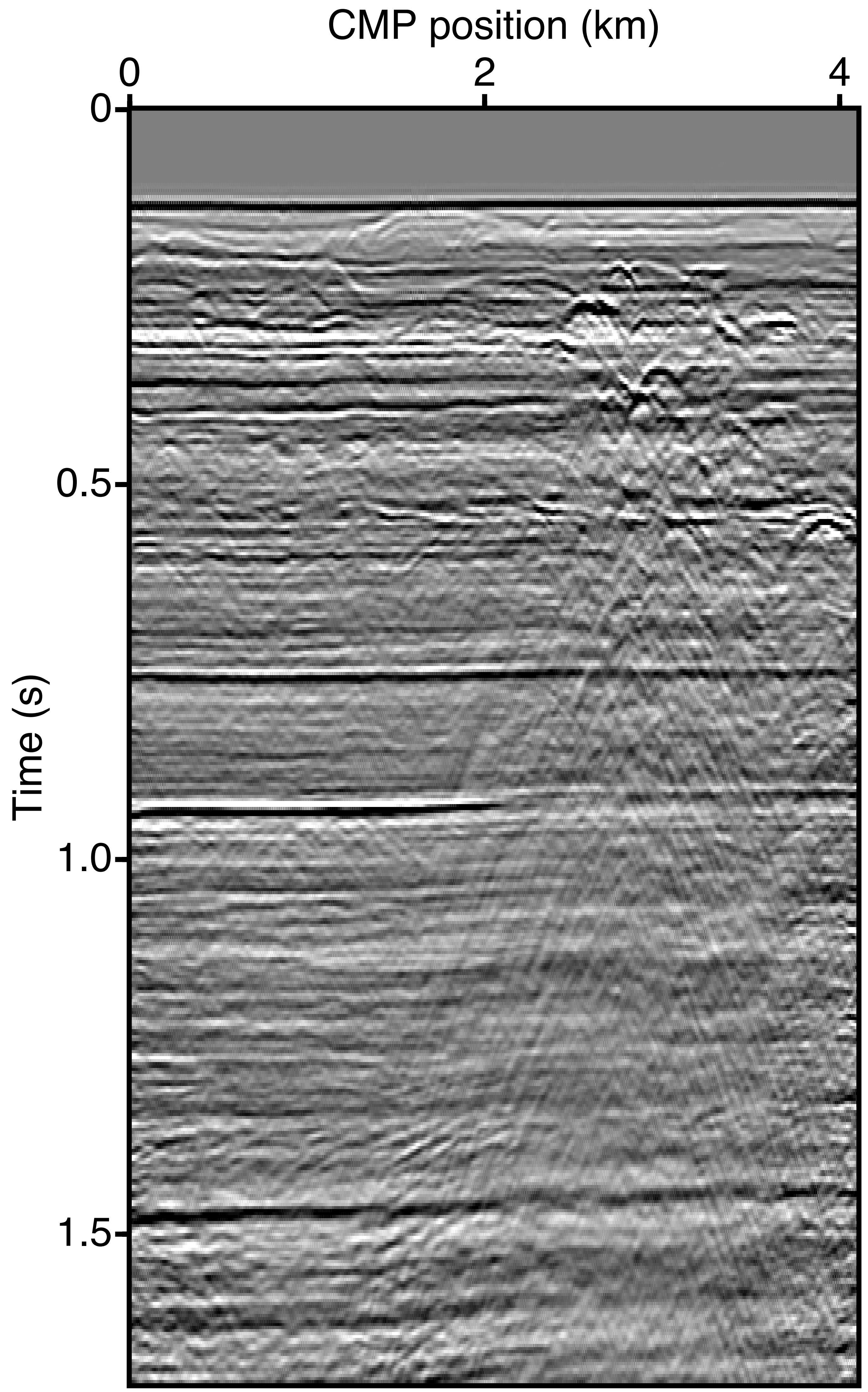


Radon interp - Re-linearization 2nd order

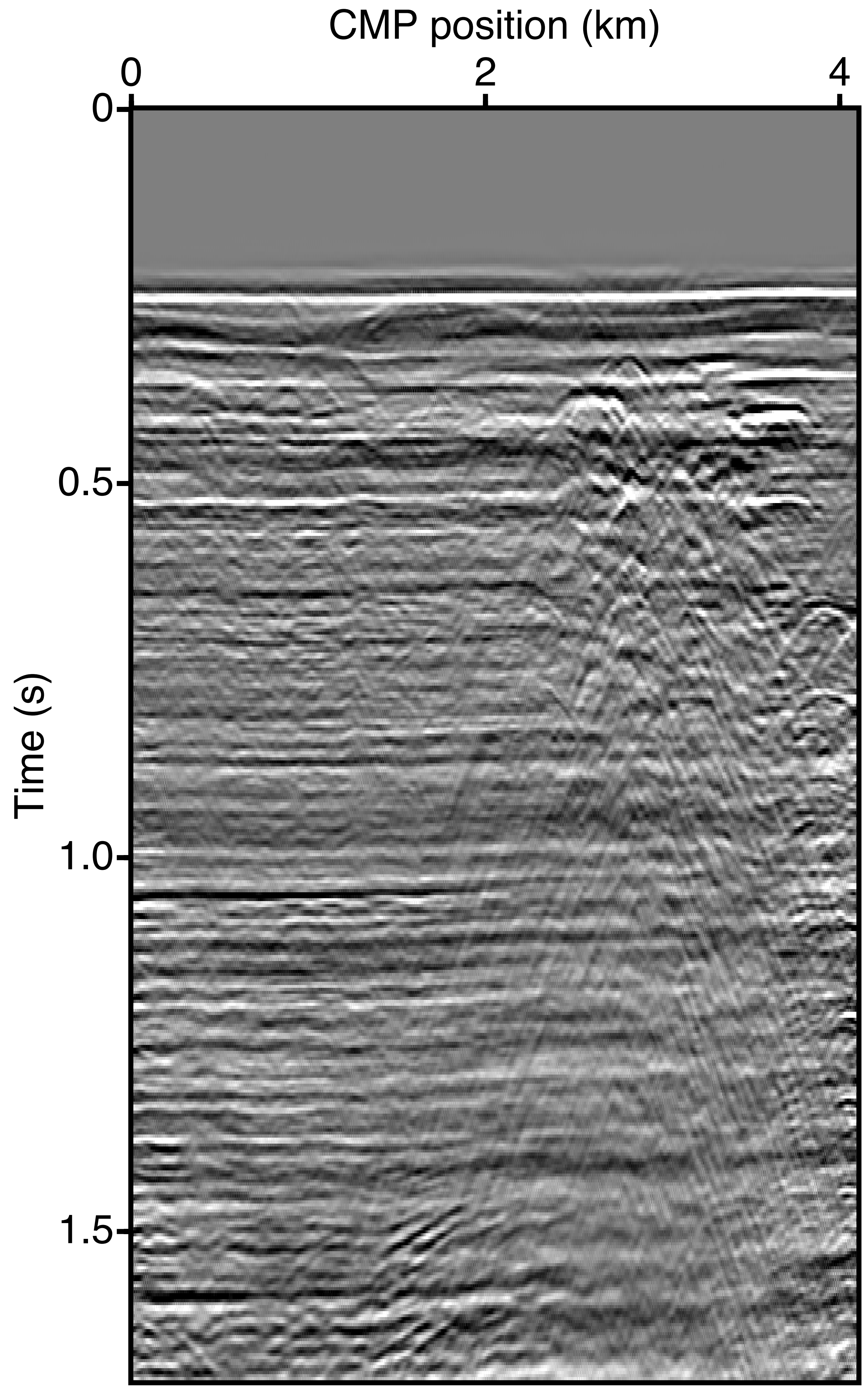


Re-linearization 3rd - 2nd order

NMO stack
Difference plots

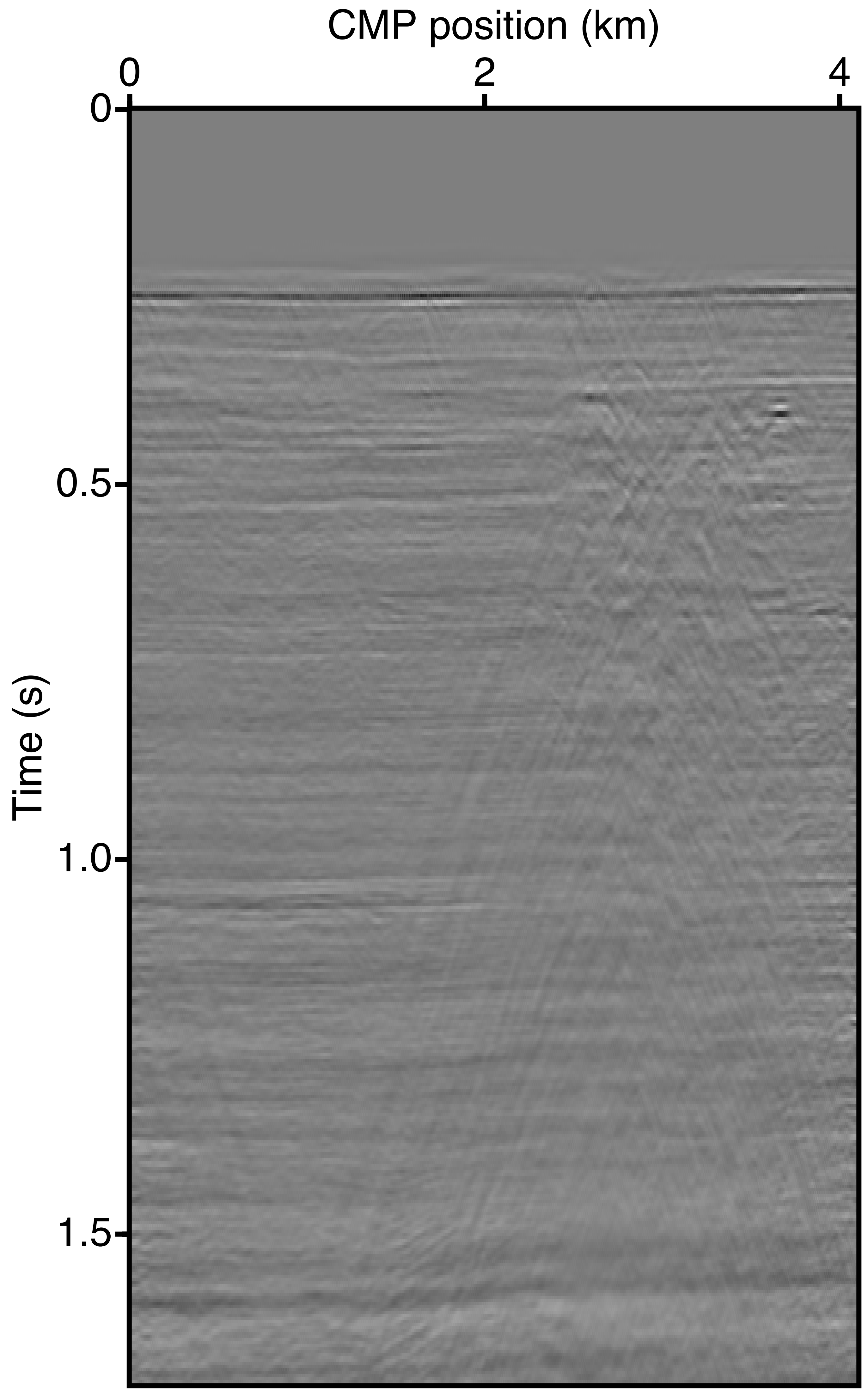


Conservative primary

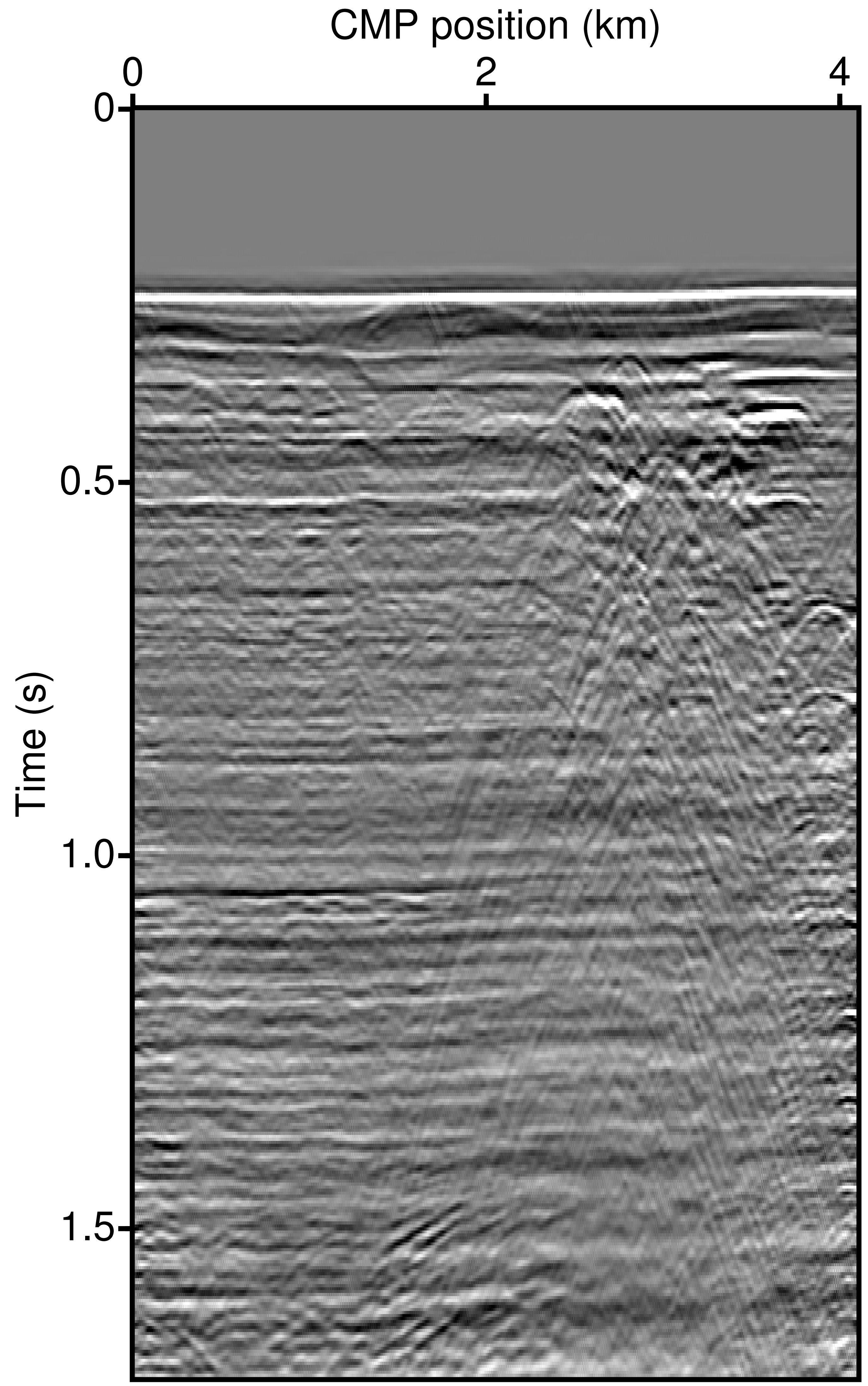


Multiple

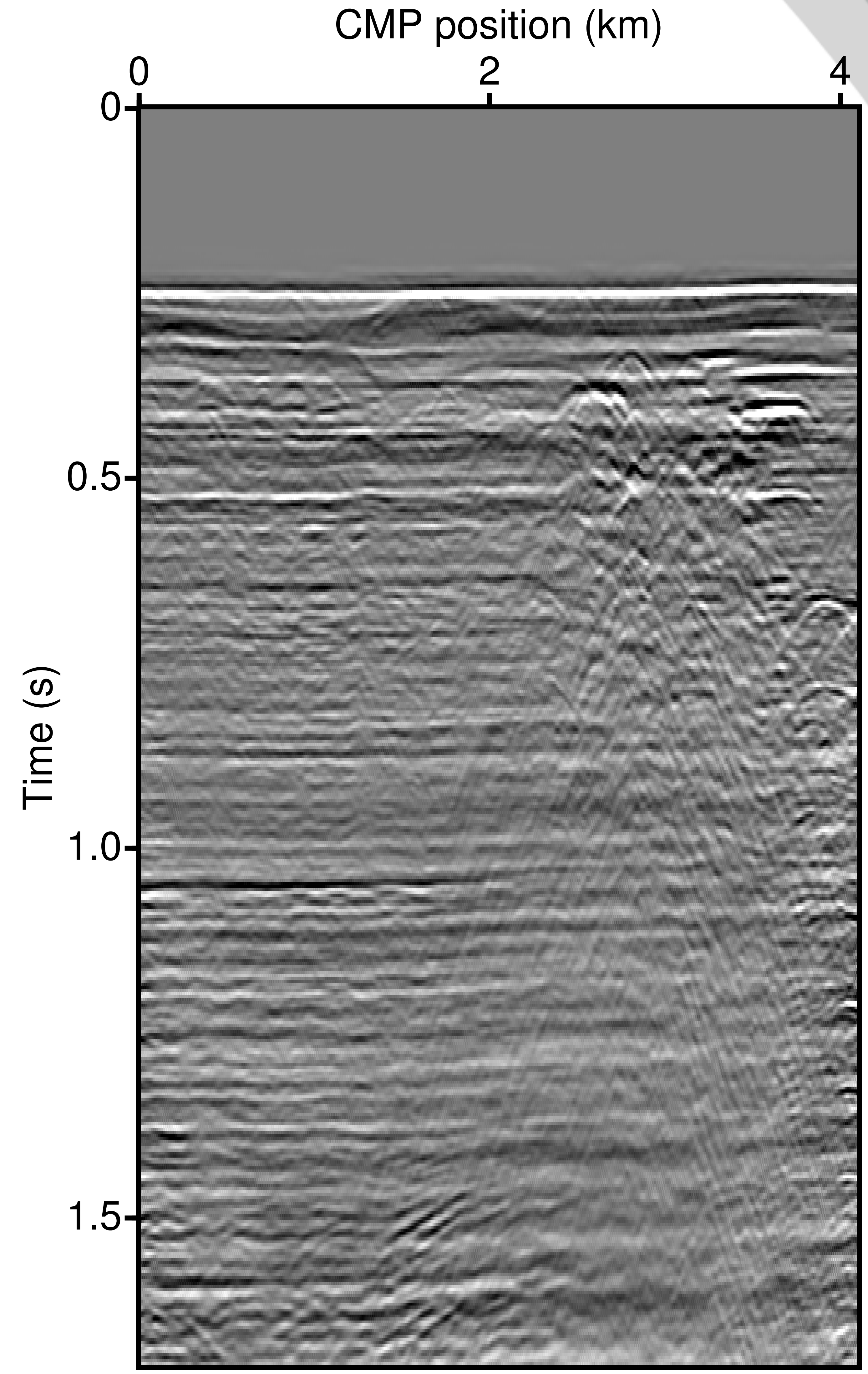
NMO stack
Modified Gauss-Newton
Using 3rd Order terms



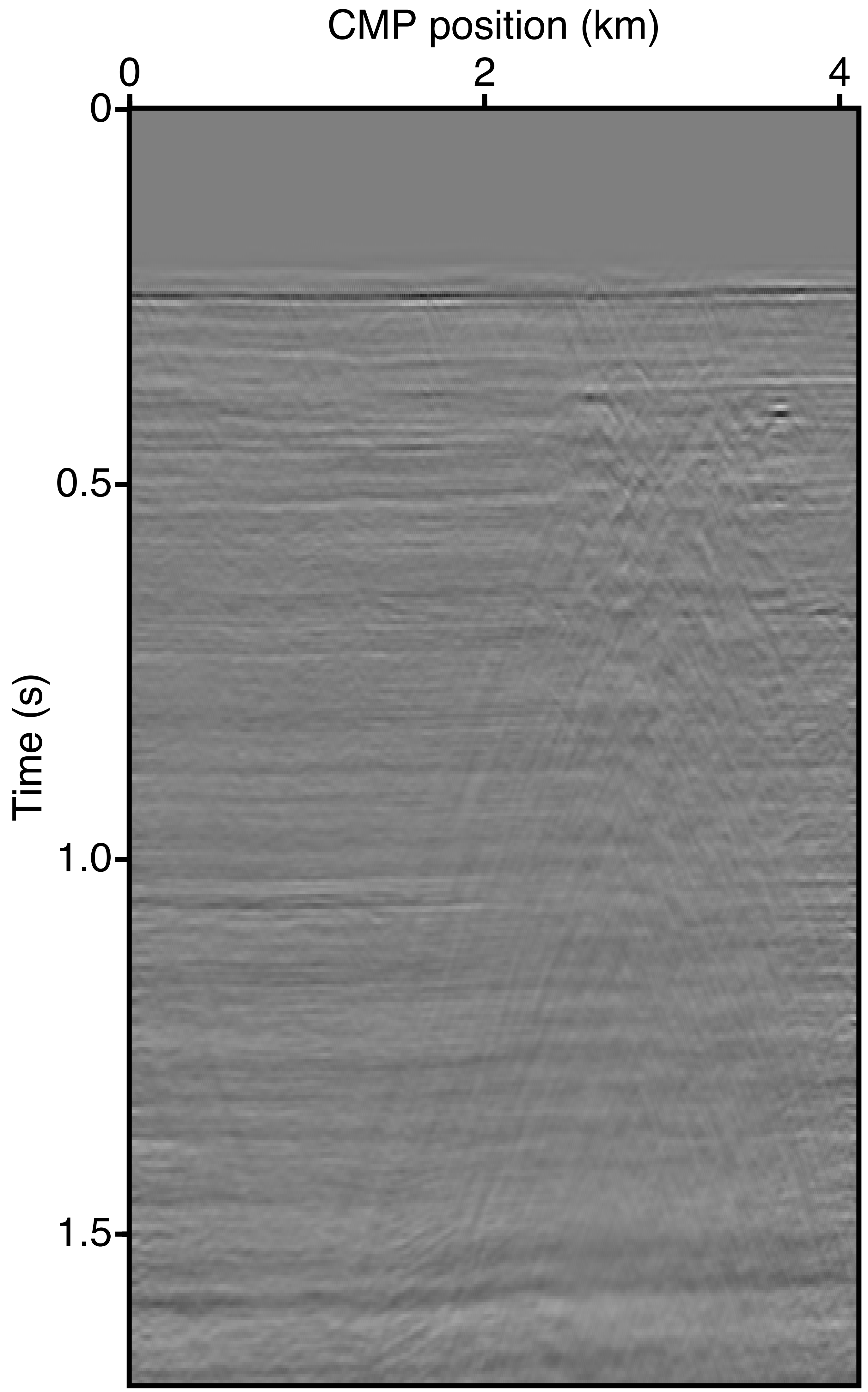
Diff: GN vs Relinearize



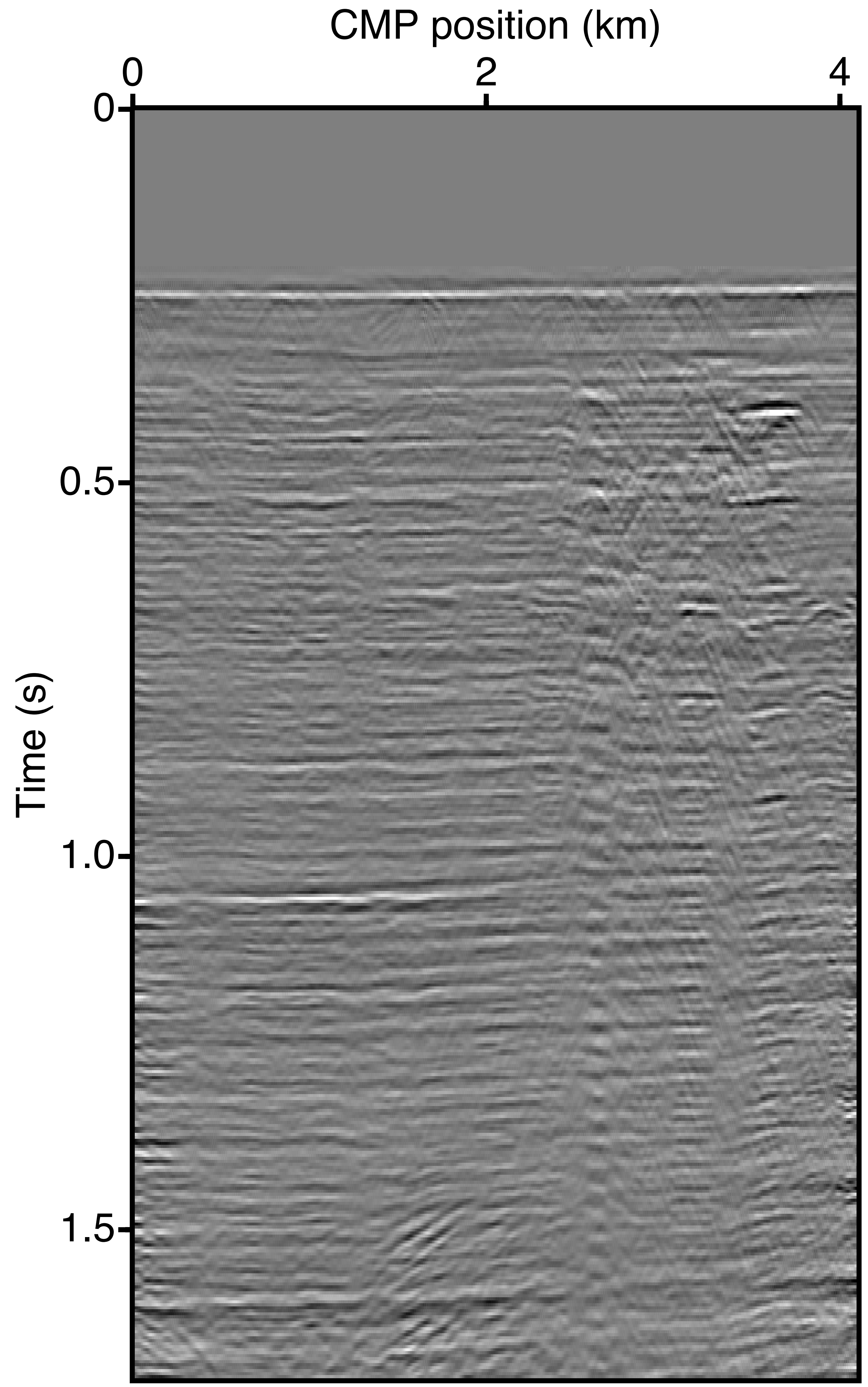
GN 3rd Order Multiple



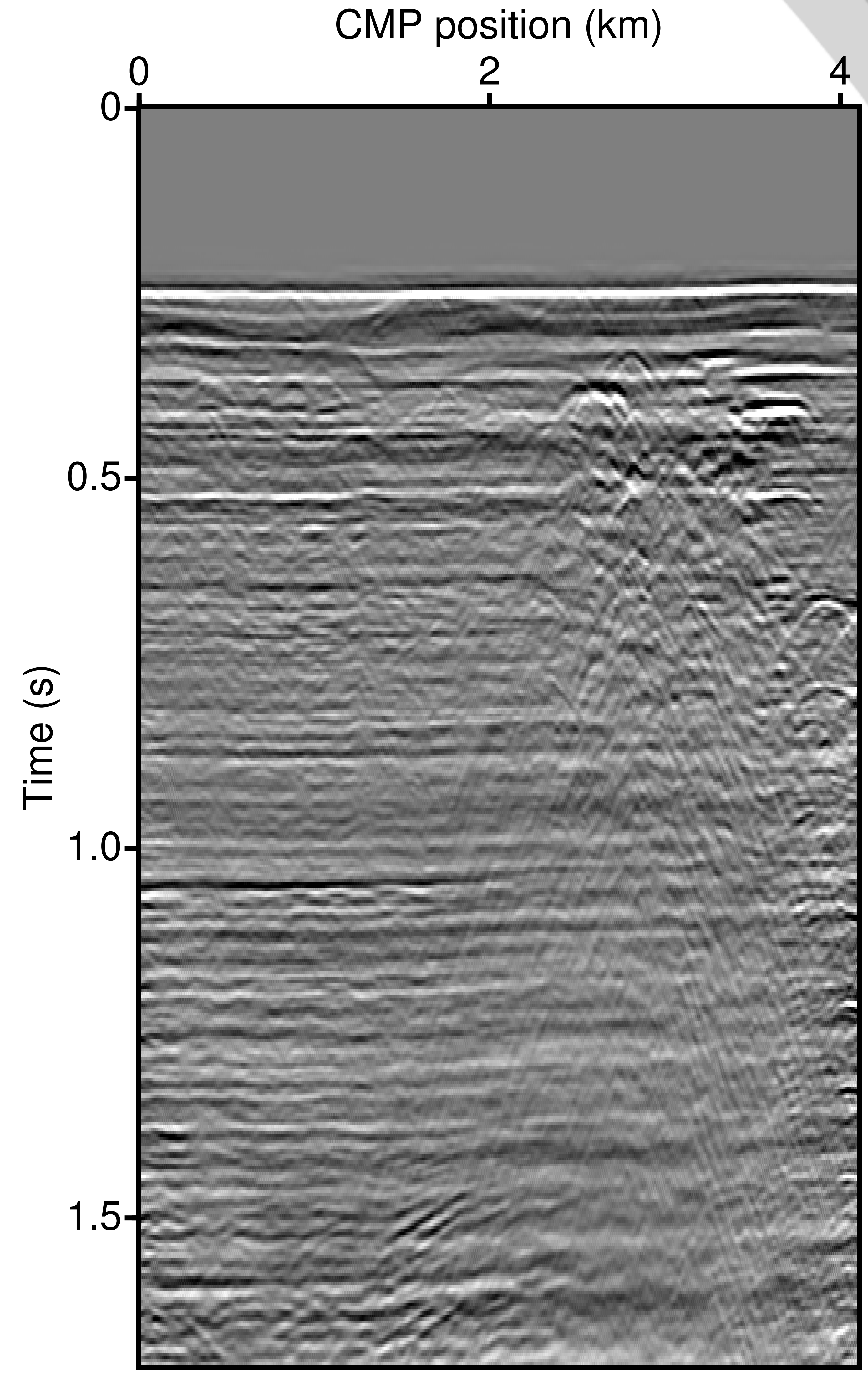
Re-lin. 3rd Order Multiple



Diff: GN vs Relinearize



Diff: Radon interp vs Relinearize



Re-lin. 3rd Order Multiple

Summary for Autoconvolving REPSI

Able to obtain an approximate forward operator with incomplete data without dependence on the missing traces

Inversion should be more stable by not changing the data at each iteration

Regularization on **G** is automatically reflected in the model for the missing traces

Bonus: Multi-scale EPSI for acceleration, and its relationship to deconvolution

EPSI as a convolutional model

Traditional convolution model

$$\text{Up-going Primary} = \mathbf{GQ}$$

EPSI Model

$$\text{Up-going Primary} + \text{Multiples} = \mathbf{GQ} - \mathbf{GP}$$

additional info on G

$\mathbf{P}$ total up-going wavefield

$\mathbf{Q}$ down-going source signature

$\mathbf{G}$ primary impulse response

Robust EPSI
L1-minimization approach to the EPSI problem

[Lin and Herrmann, 2013 *Geophysics*]

While $\|\mathbf{p} - \mathcal{M}(\mathbf{g}_k, \mathbf{q}_k)\|_2 > \sigma$

determine new τ_k from the Pareto curve

Emits sparse, or "deconvolved" solution

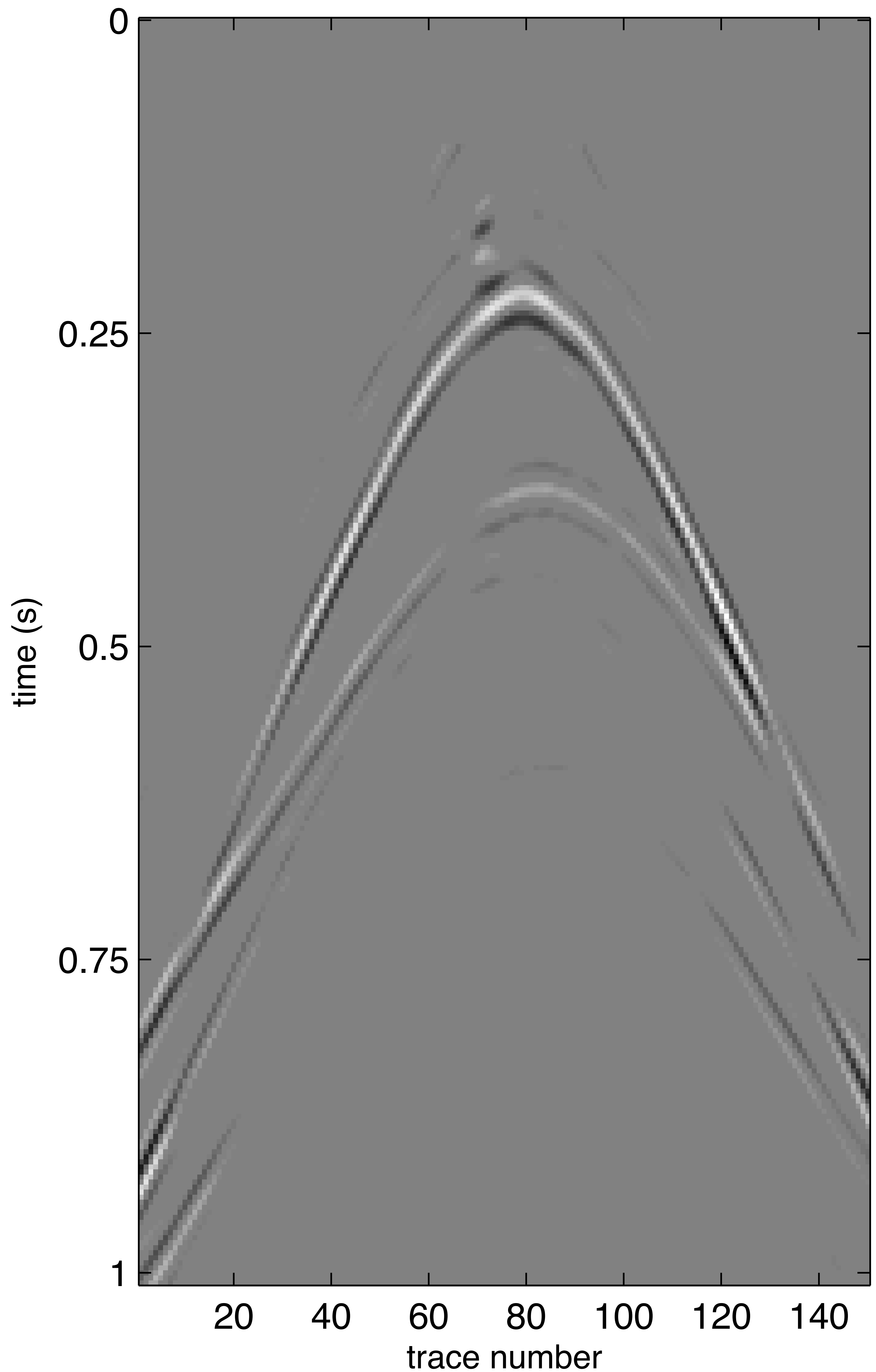
$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

$$\mathbf{q}_{k+1} = \arg\min_{\mathbf{q}} \|\mathbf{p} - \mathbf{M}_{g_{k+1}}\mathbf{q}\|_2$$

L1 projection and sparsity

variable g at beginning of LASSO

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

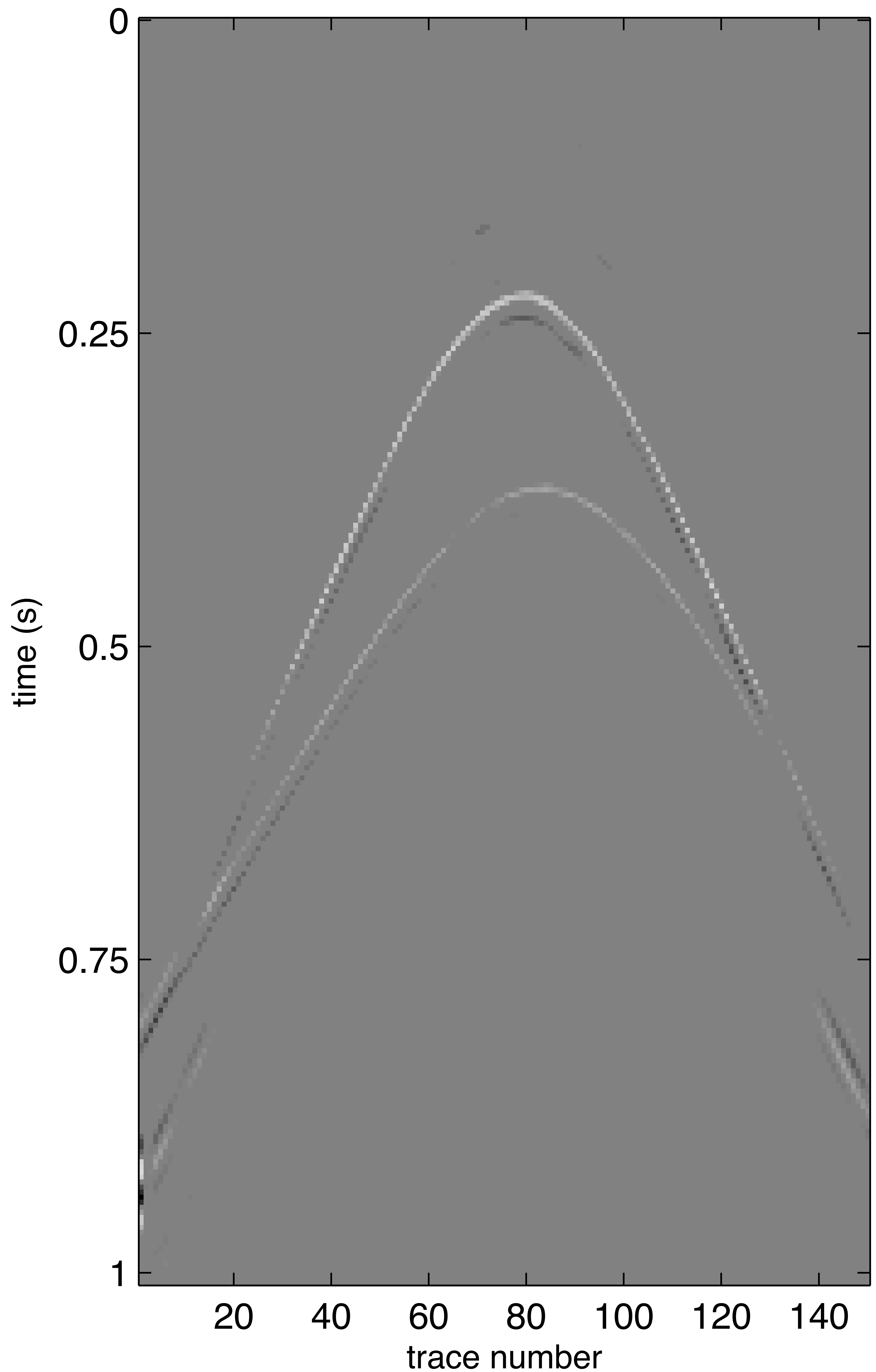


L1 projection and sparsity

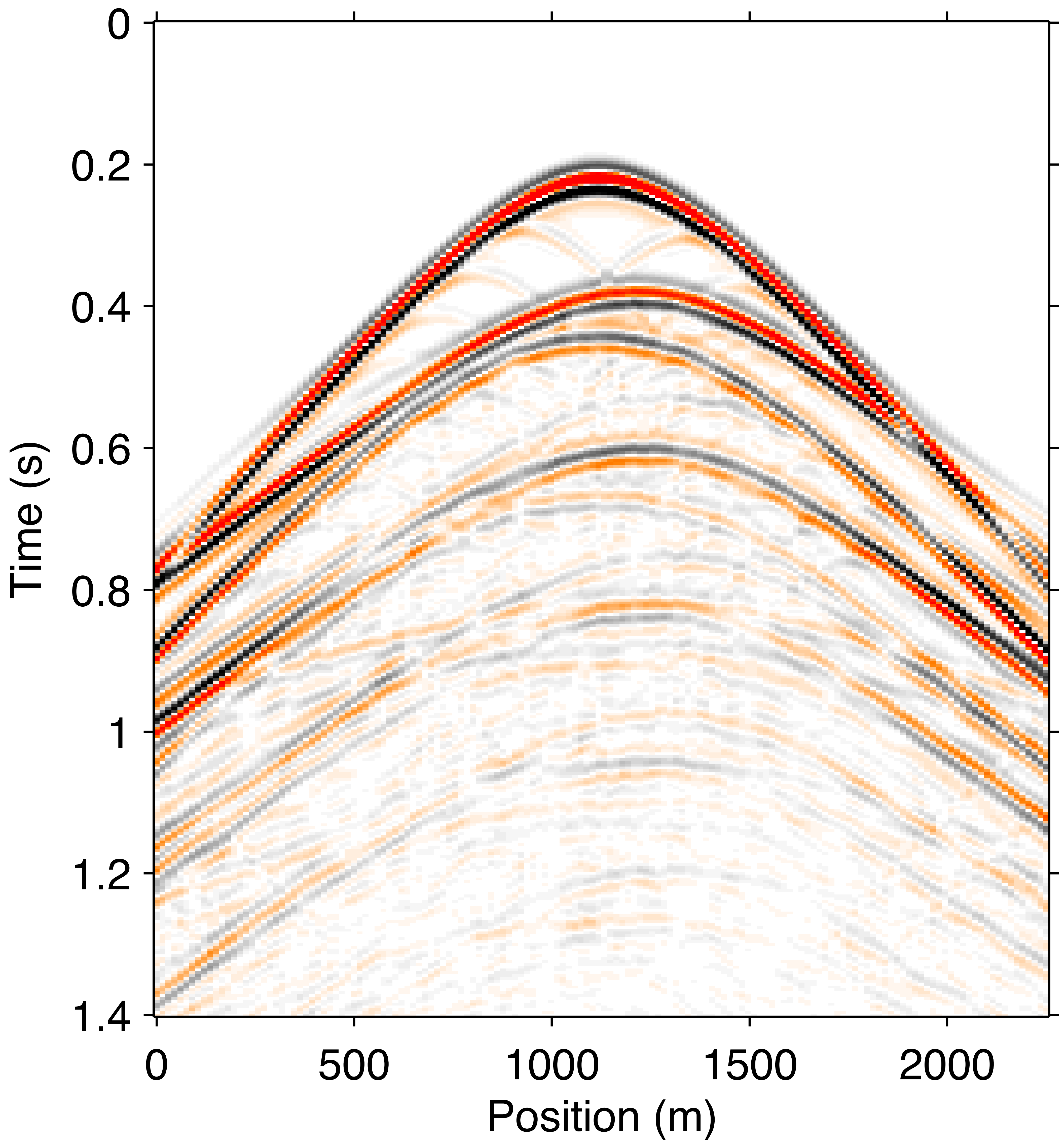
variable g at end of LASSO

$$\mathbf{g}_{k+1} = \arg\min_{\mathbf{g}} \|\mathbf{p} - \mathbf{M}_{q_k}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_1 \leq \tau_k$$

Emits “deconvolved” solution



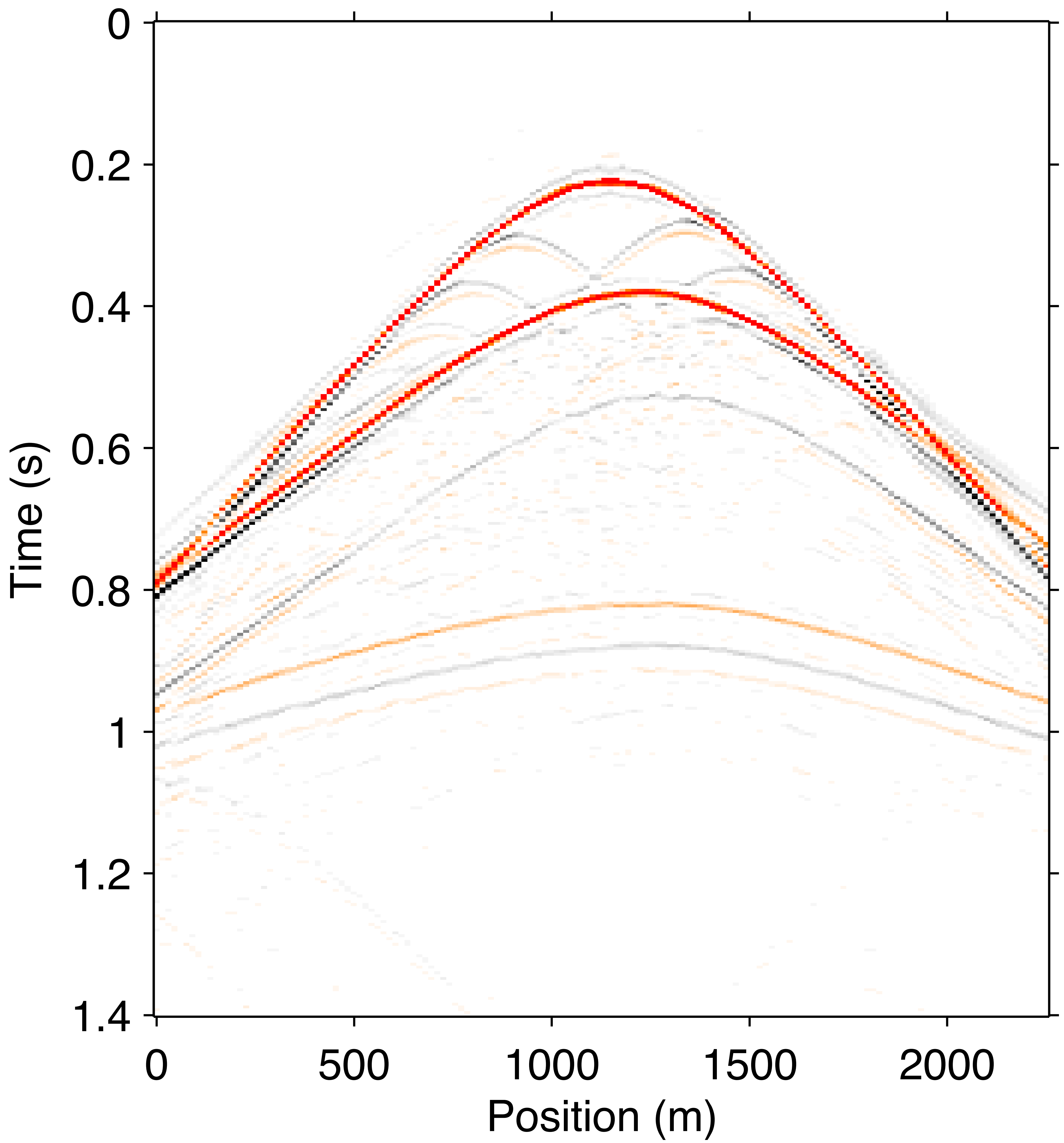
Motivation: G tolerates lowpass filtering



Data

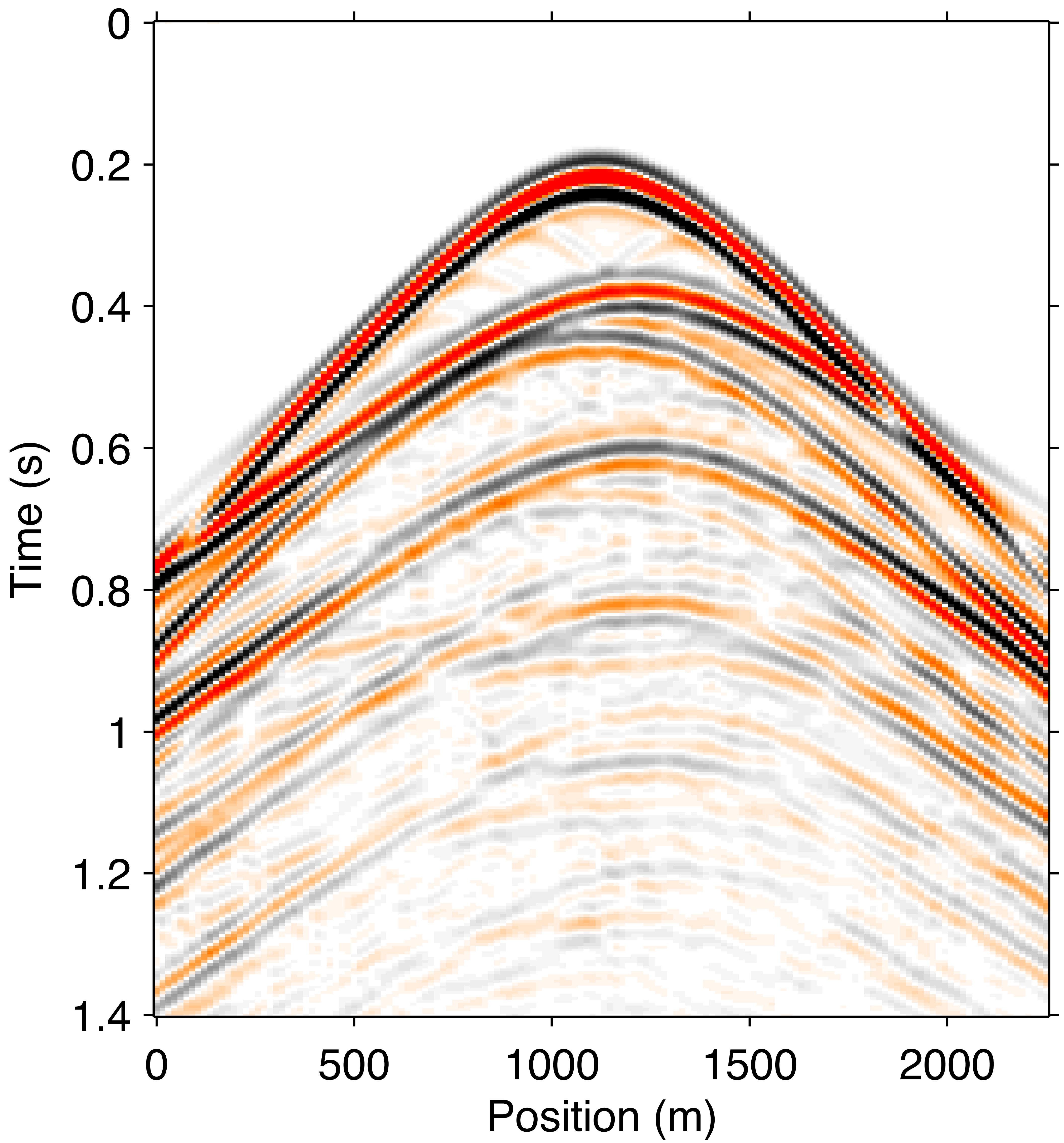
modeled with Ricker 30Hz

Motivation: G tolerates lowpass filtering



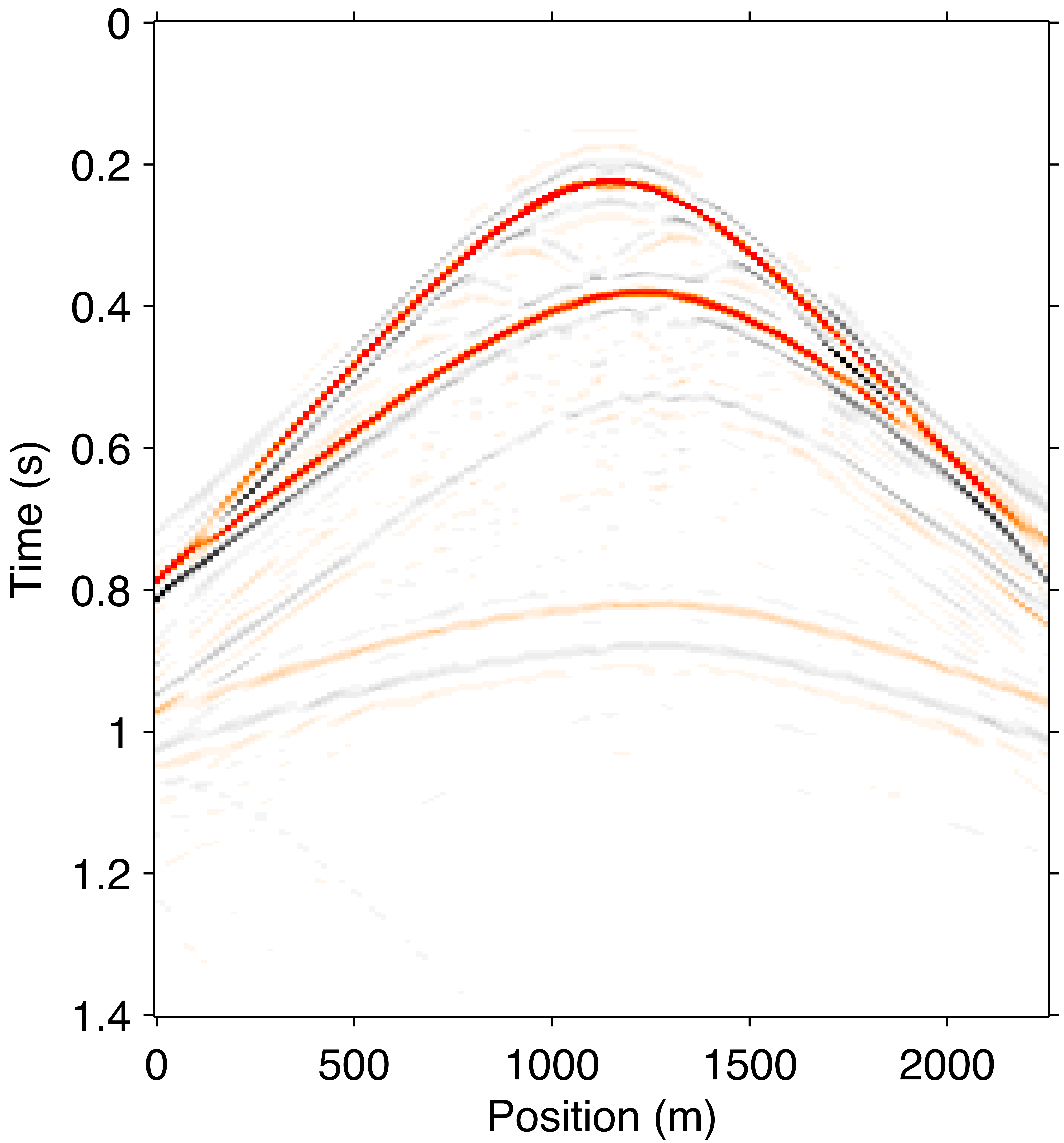
Reference REPSI primary IR
from original data

Motivation: G tolerates lowpass filtering



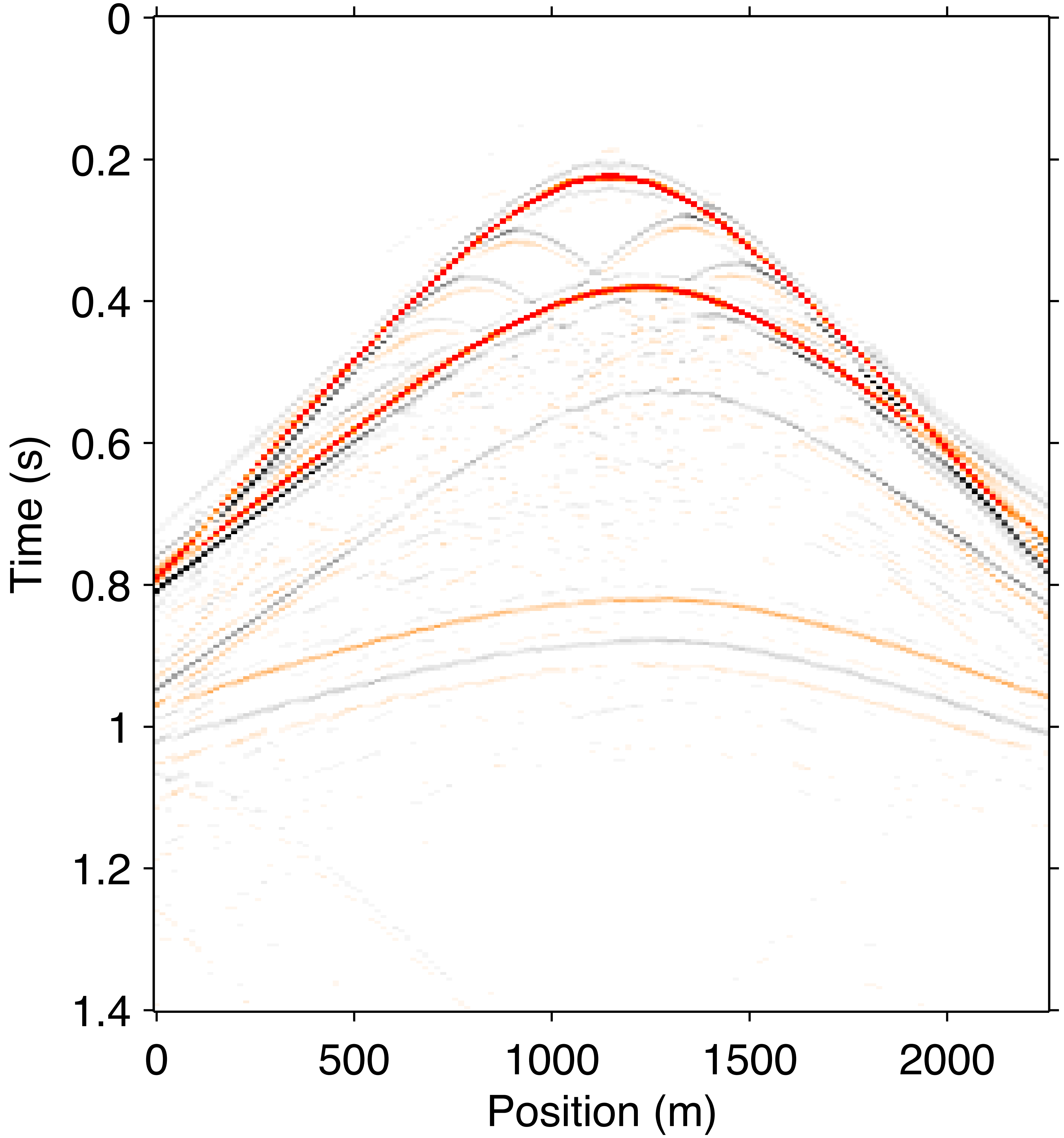
Lowpassed Data
modeled with Ricker 30Hz
lowpass at 40Hz
(25-order, zero-phase, Hann window)

Motivation: G tolerates lowpass filtering



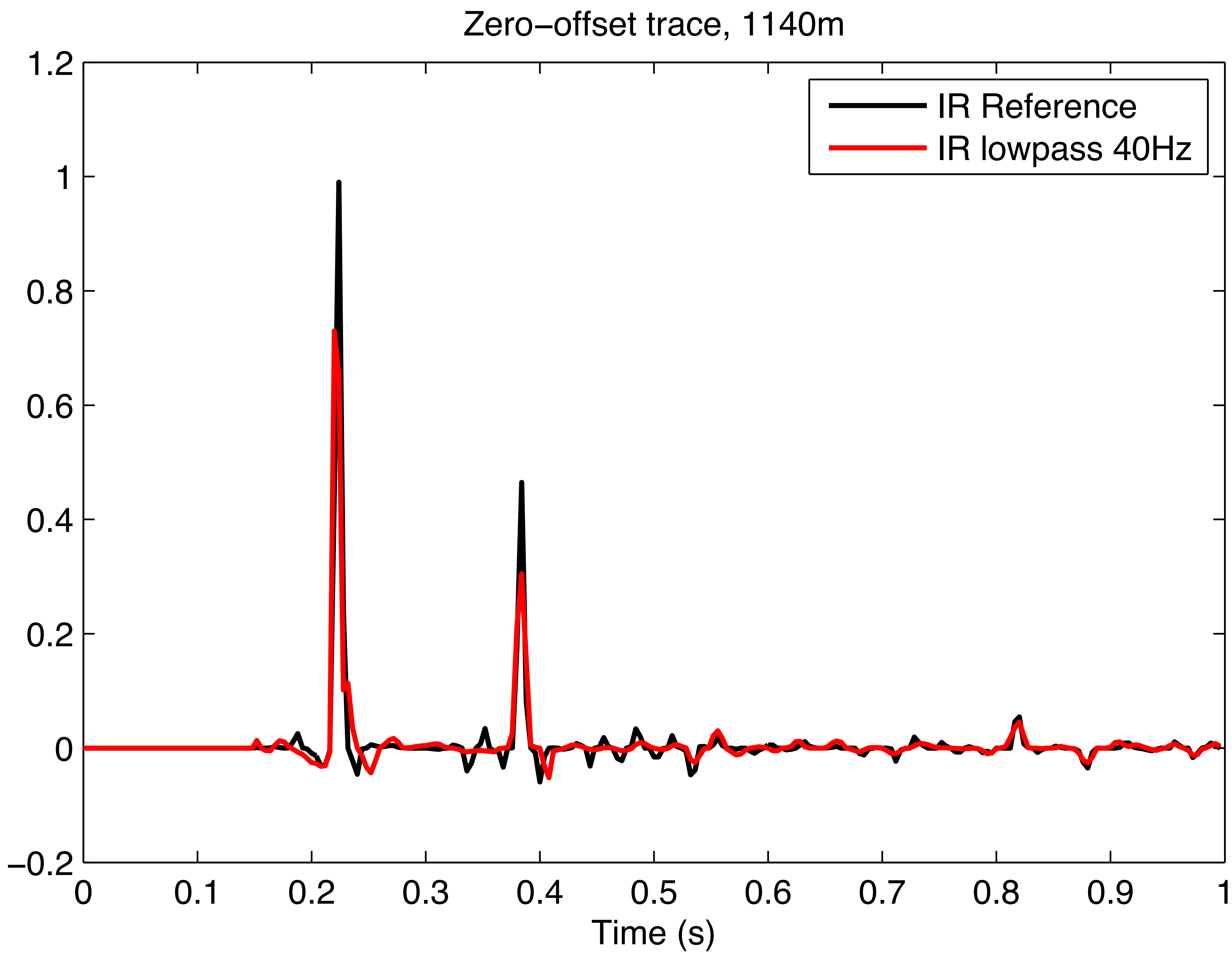
REPSI primary IR
from low-passed data @ 40Hz

Motivation: G tolerates lowpass filtering

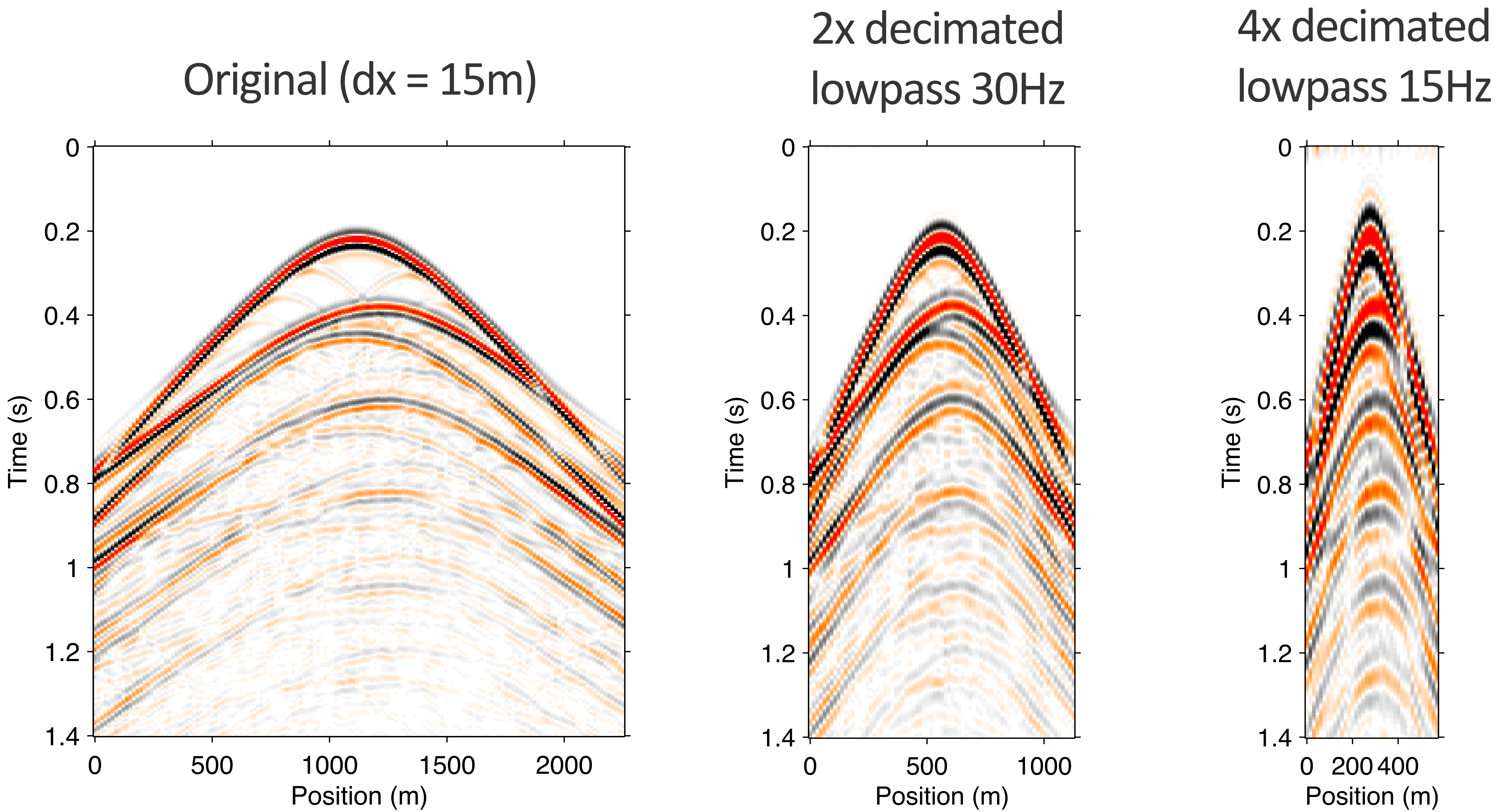


Reference REPSI primary IR
from original data

Motivation: G tolerates lowpass filtering

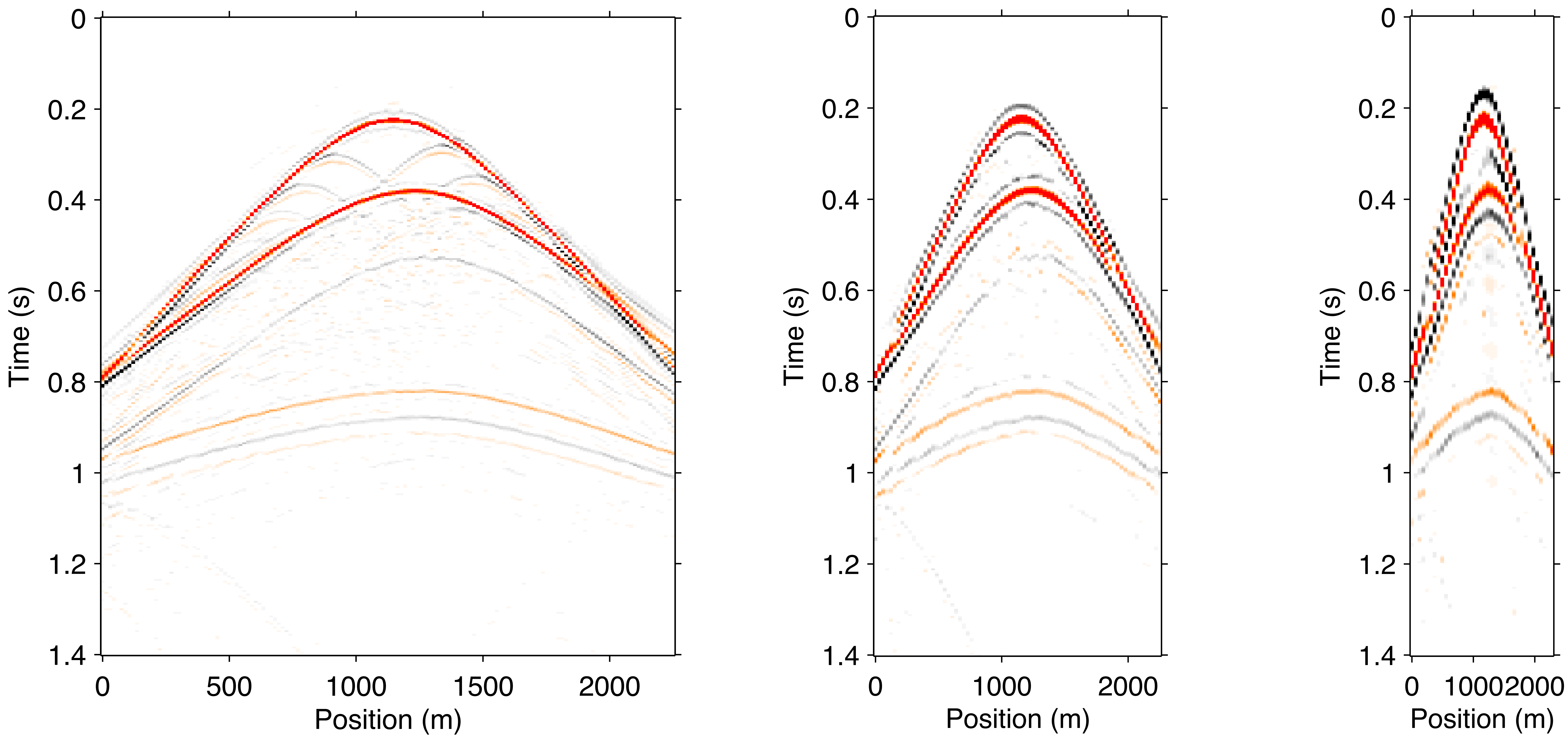


Lowpass data permits coarser sampling w/o aliasing

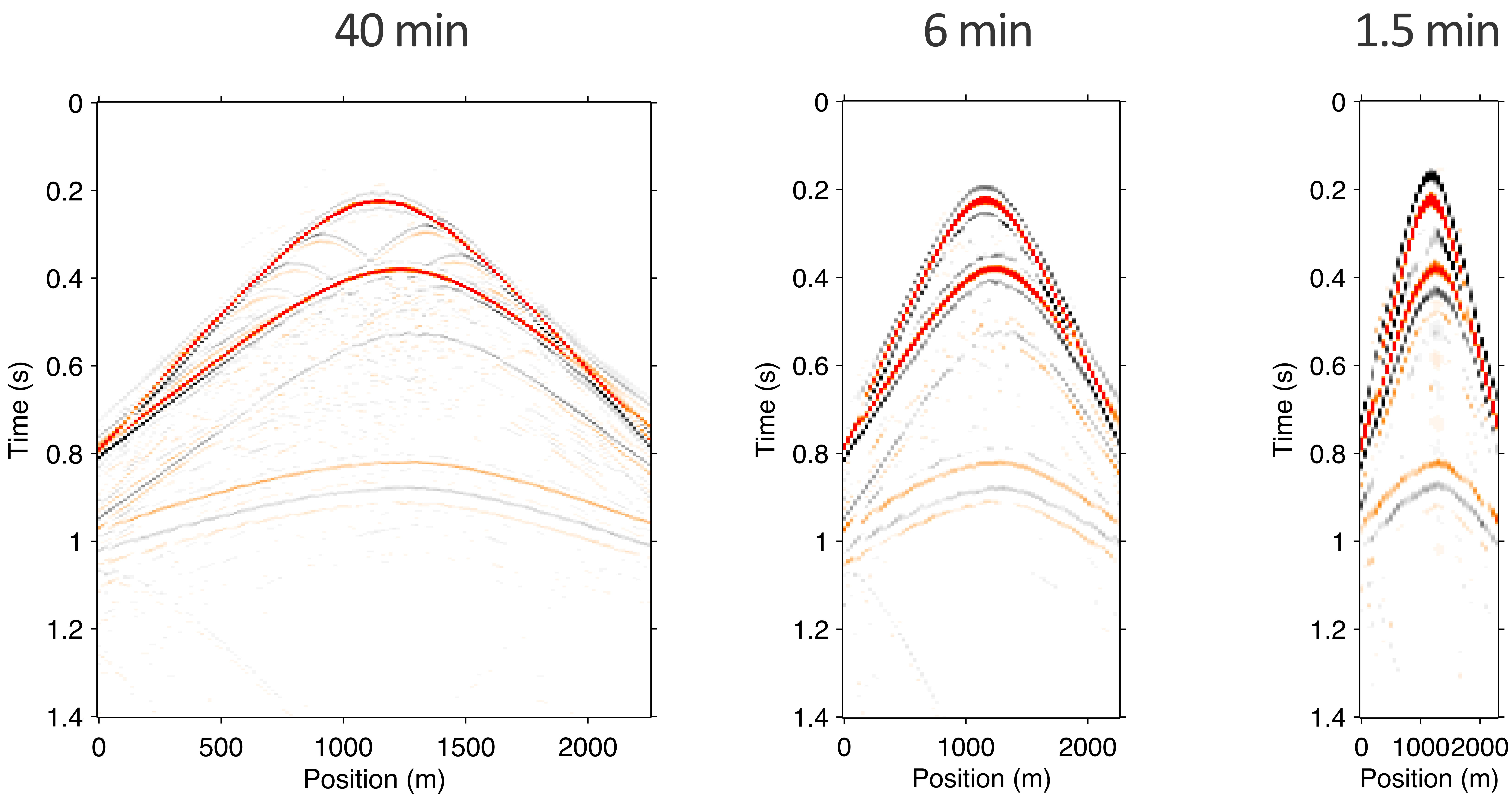


Lowpass data permits coarser sampling w/o aliasing

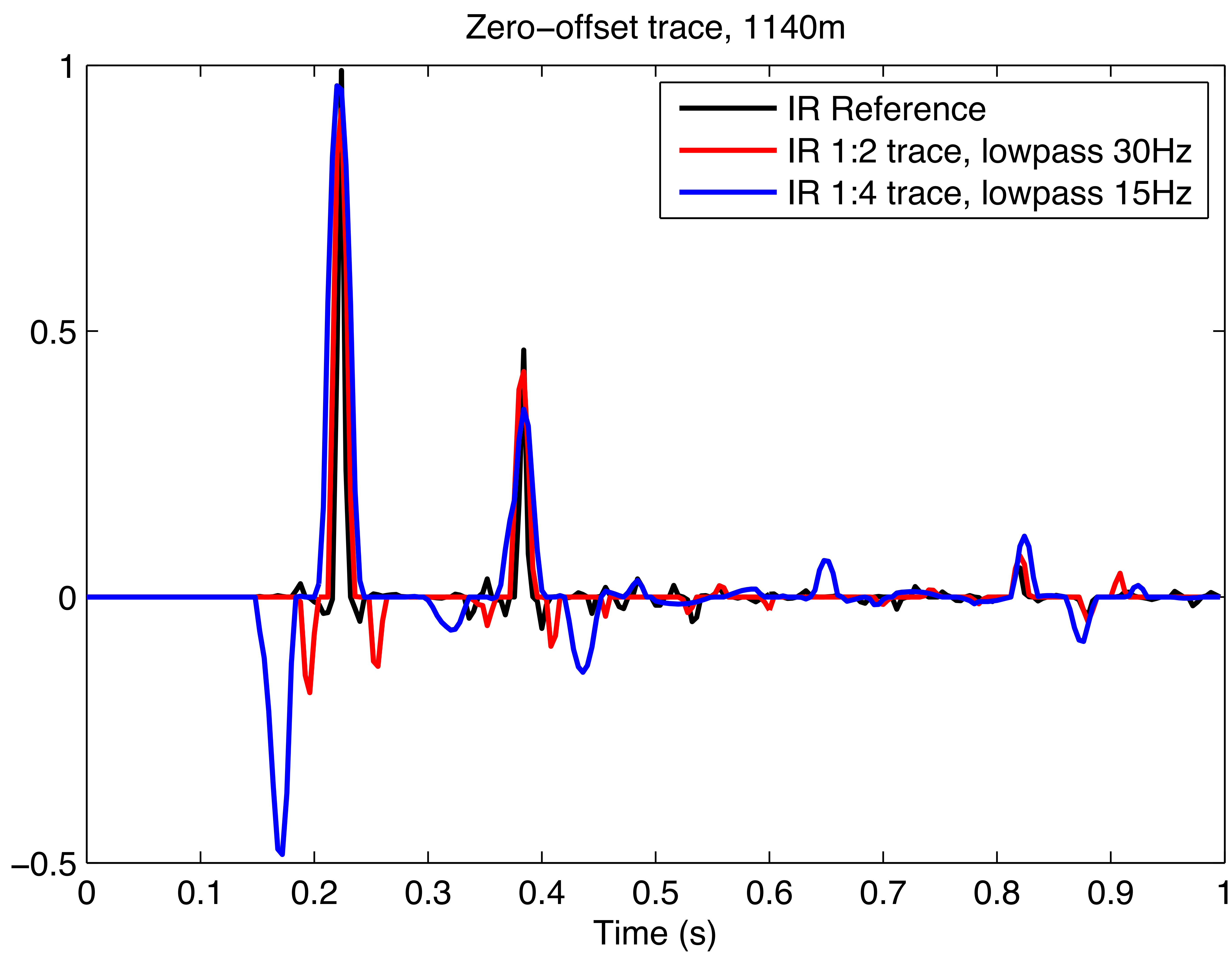
Impulse response solutions



Lowpass data permits coarser sampling w/o aliasing
(much faster!)



Lowpass data permits coarser sampling w/o aliasing



Multilevel strategy for EPSI

warm-start fine-scale problem *(slow)*
with coarse-scale solutions *(fast)*

Significant speedup from bootstrapping (in 2D)

Per-iteration FLOPs cost (one forward/adjoint): $n = n_{\text{rcv}} = n_{\text{src}}$

$$\text{Cost}(n) = \mathcal{O}(2n_t n^2 \log n_t) + \mathcal{O}(n_f n^3)$$

2 times FFT   computing MCG & sum in FX

$$\text{Cost}\left(\frac{1}{2}n\right) = \frac{1}{4}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{8}\mathcal{O}(n_f n^3)$$

$$\text{Cost}\left(\frac{1}{4}n\right) = \frac{1}{16}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{64}\mathcal{O}(n_f n^3)$$

Significant speedup from bootstrapping (in 2D)

Per-iteration FLOPs cost (one forward/adjoint): $n = n_{\mathrm{rcv}} = n_{\mathrm{src}}$

$$\text{Cost}(n) = \mathcal{O}(2n_t n^2 \log n_t) + \mathcal{O}(n_f n^3)$$

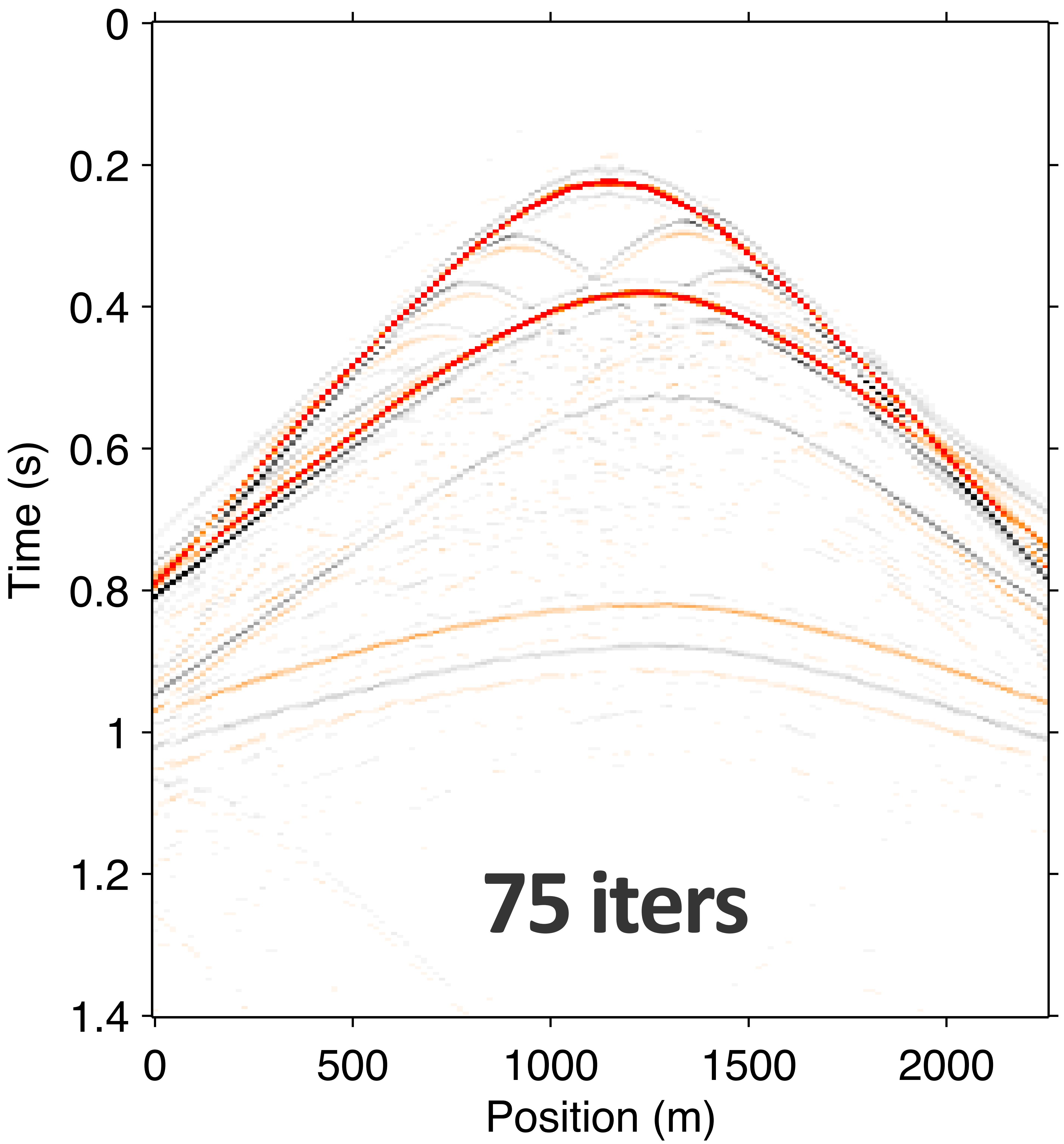
2 times FFT   computing MCG & sum in FX

$$\text{Cost}\left(\frac{1}{2}n, \frac{1}{2}n_f\right) = \frac{1}{4}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{16}\mathcal{O}(n_f n^3)$$

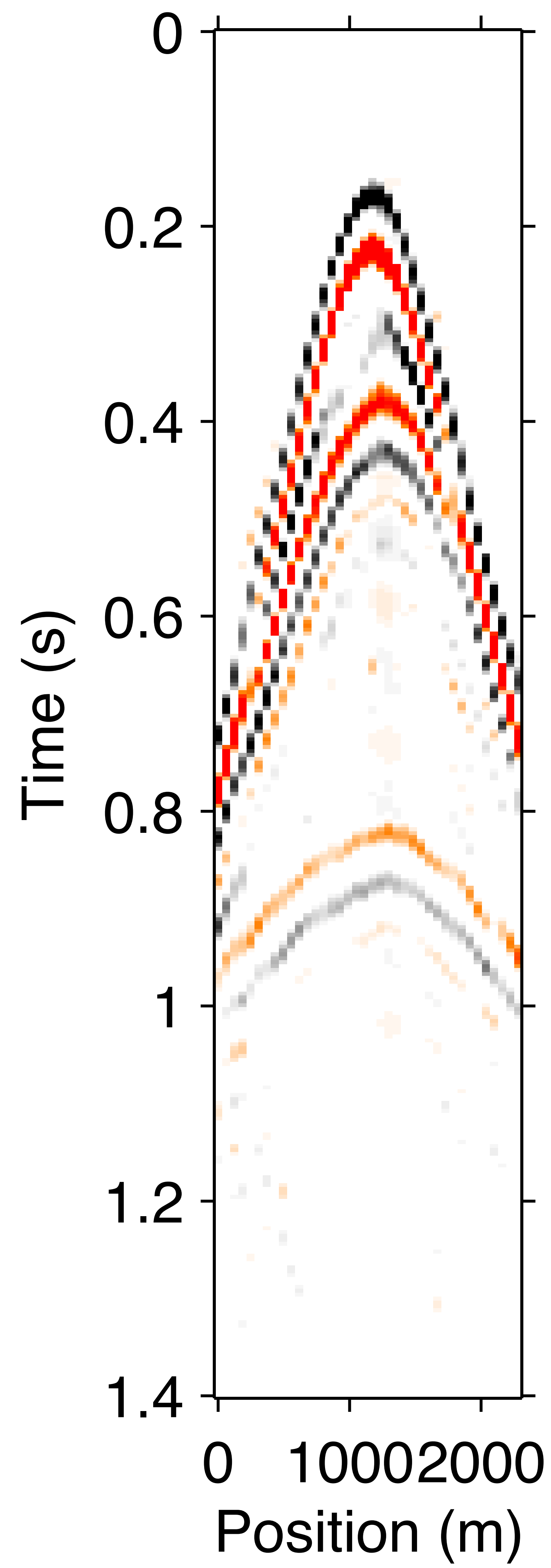
$$\text{Cost}\left(\frac{1}{4}n, \frac{1}{4}n_f\right) = \frac{1}{16}\mathcal{O}(2n_t n^2 \log n_t) + \frac{1}{128}\mathcal{O}(n_f n^3)$$

Warm-starting/continuation from coarse solution
Example

Solution of full data

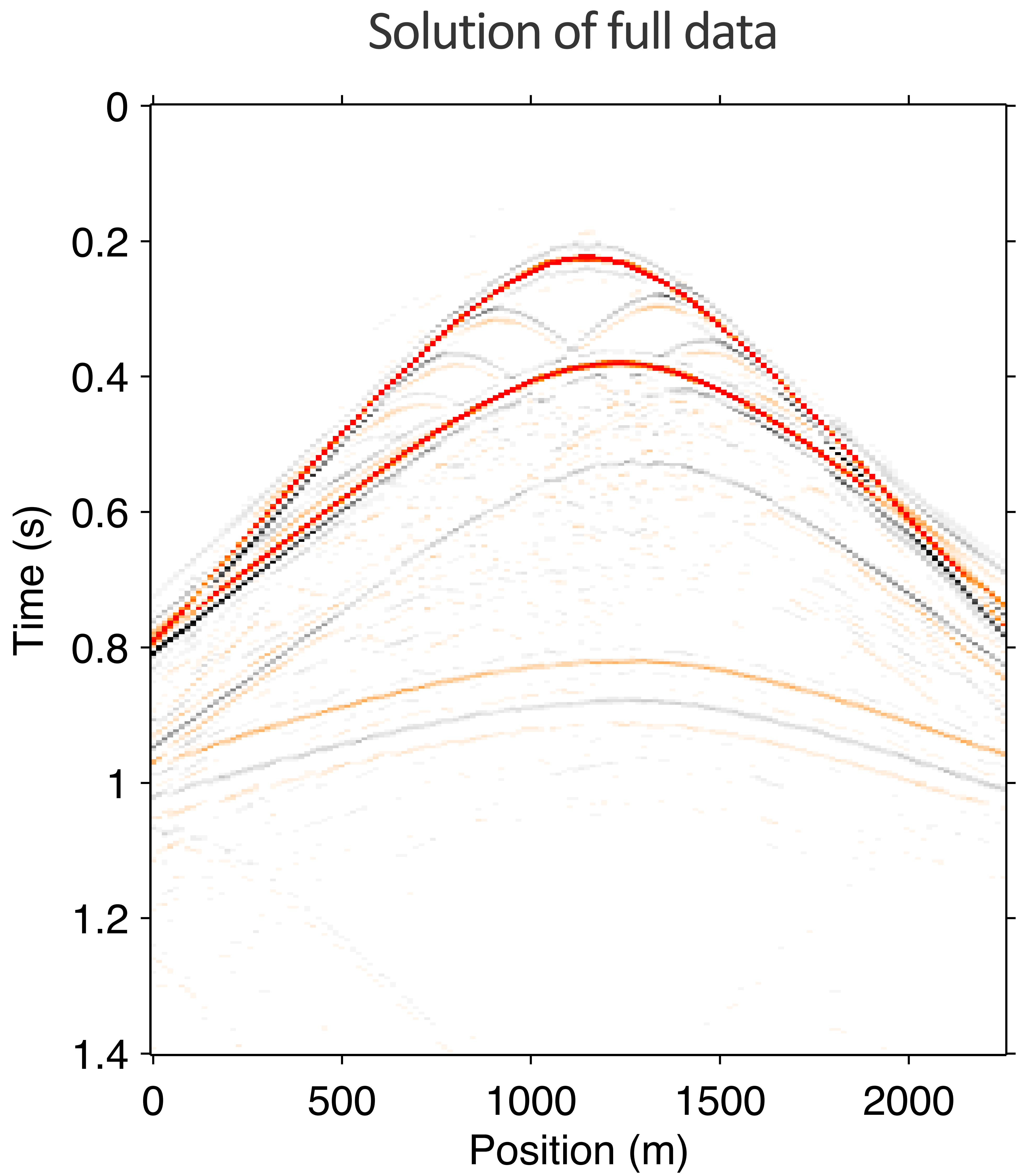


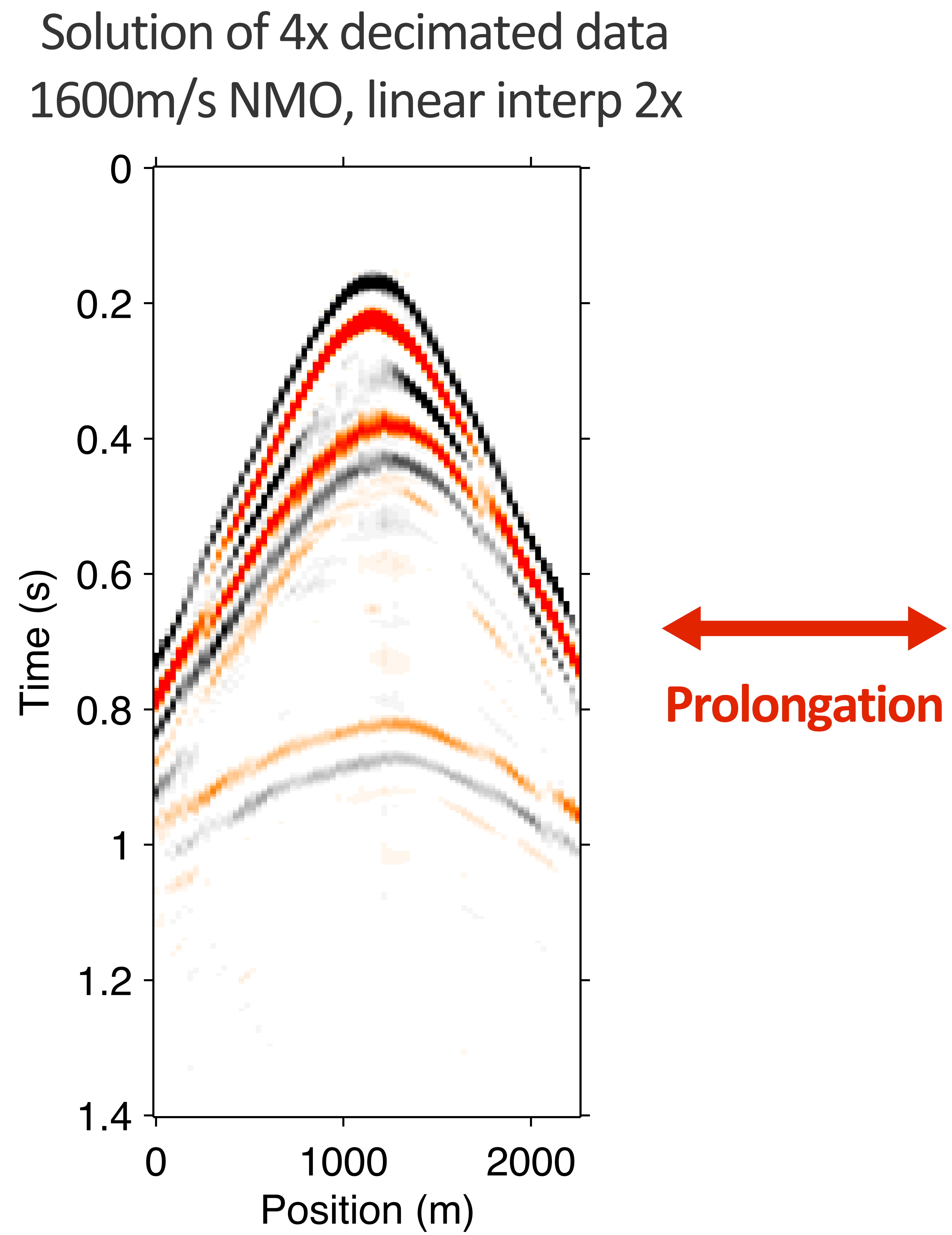
Solution of 4x decimated data



60 iters

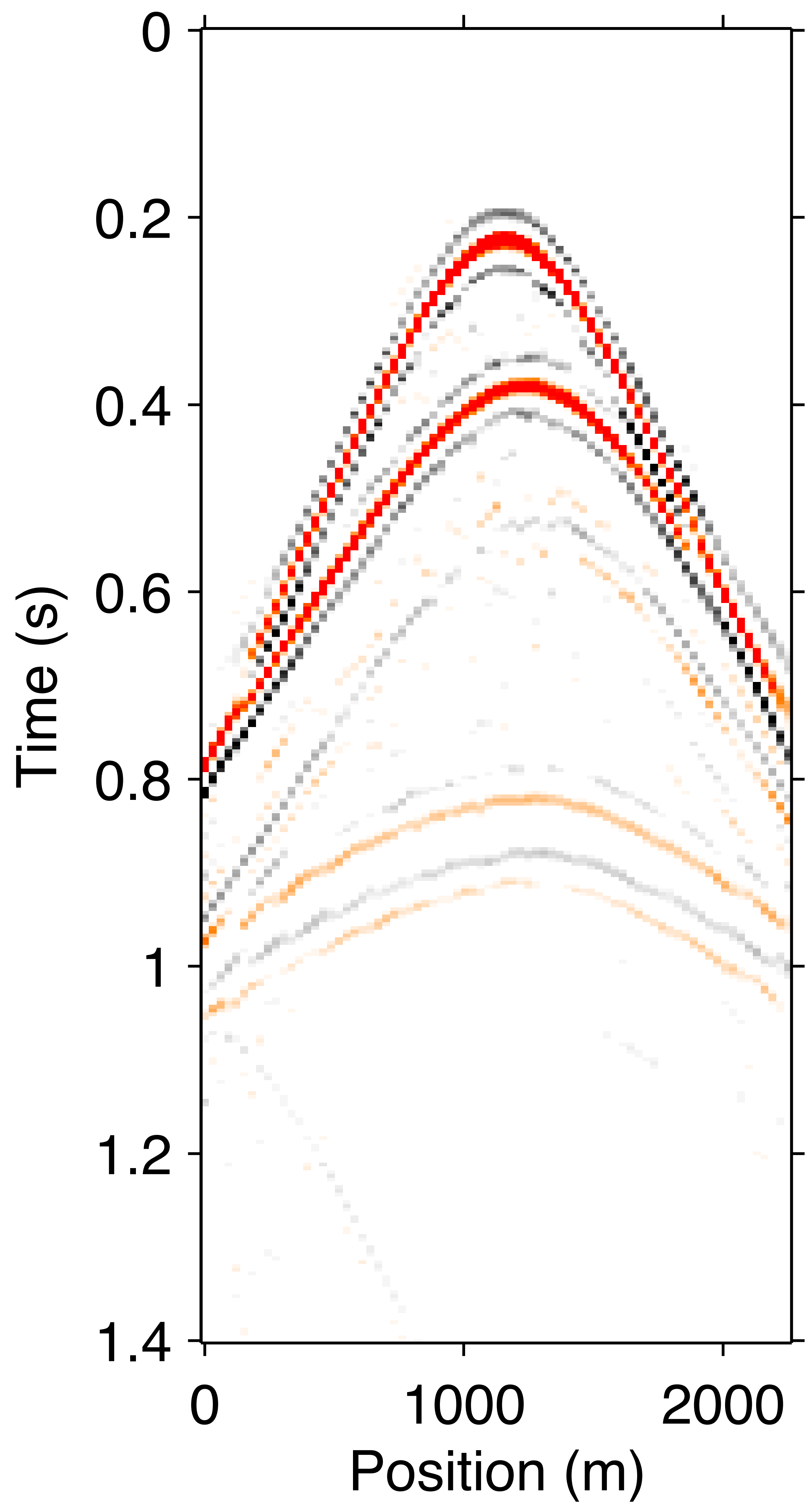
Warm-starting/continuation from coarse solution
Example



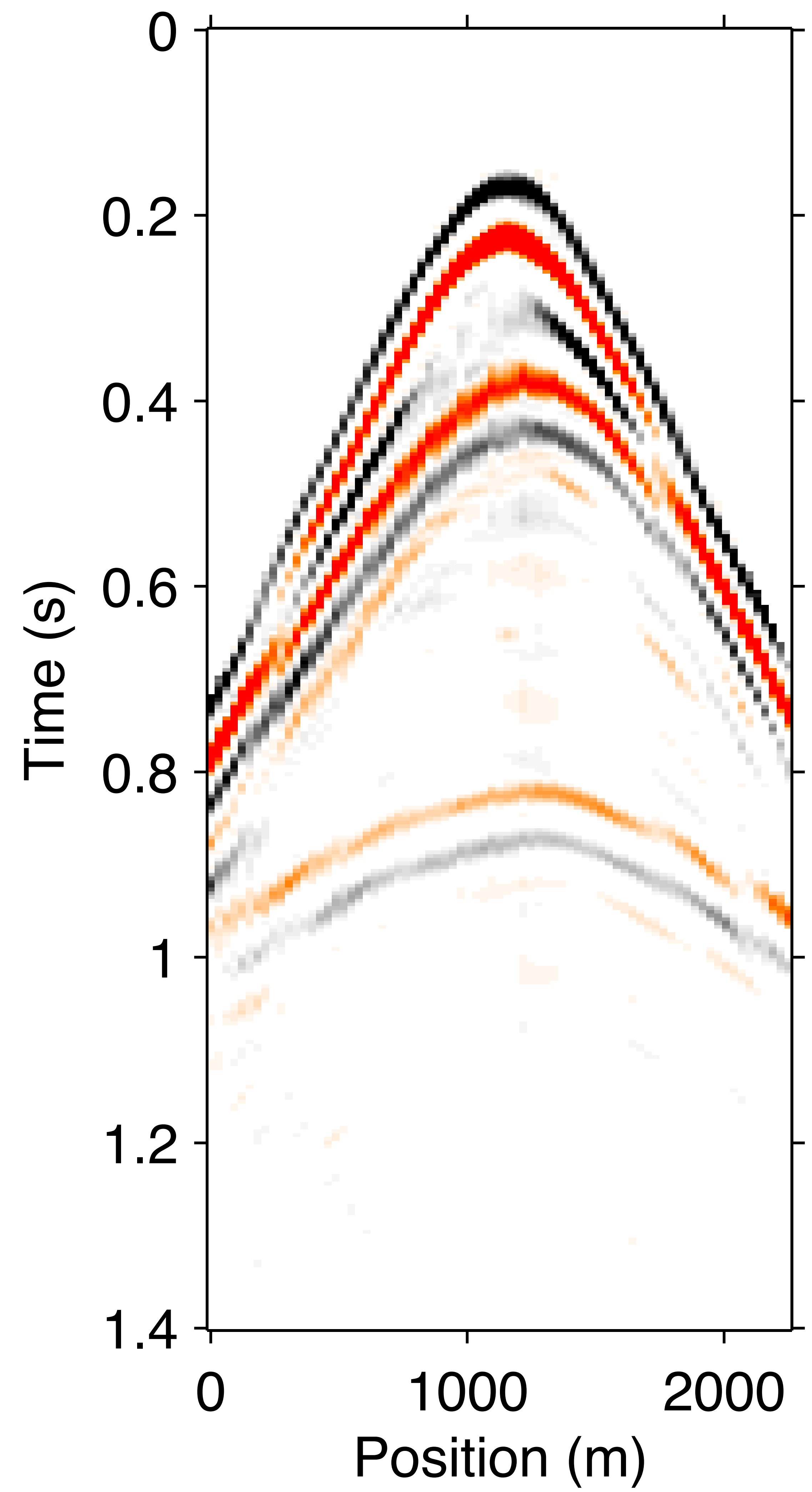


Warm-starting/continuation from coarse solution
Example

Solution of 2x decimated data



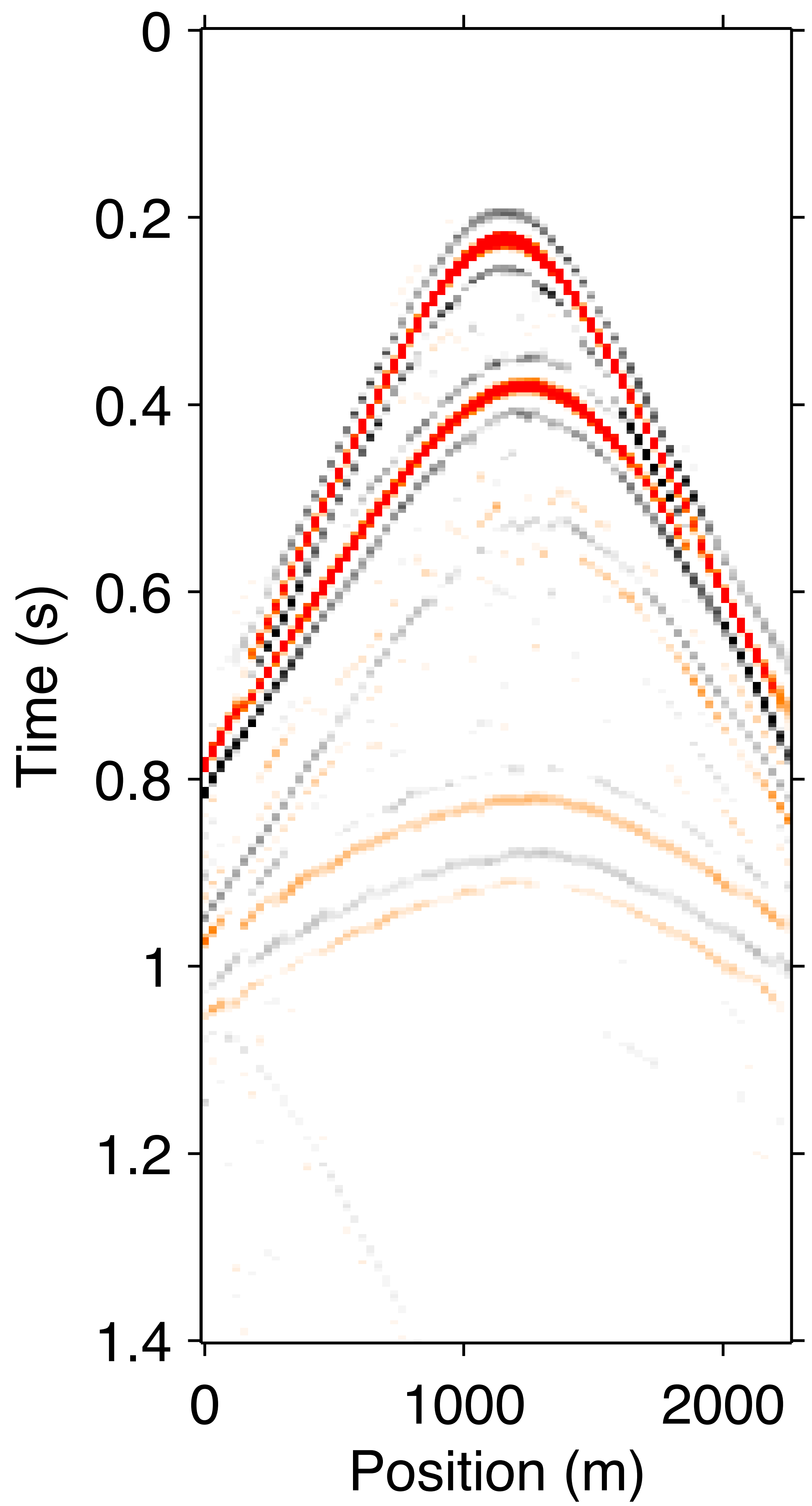
Solution of 4x decimated data
1600m/s NMO, linear interp 2x



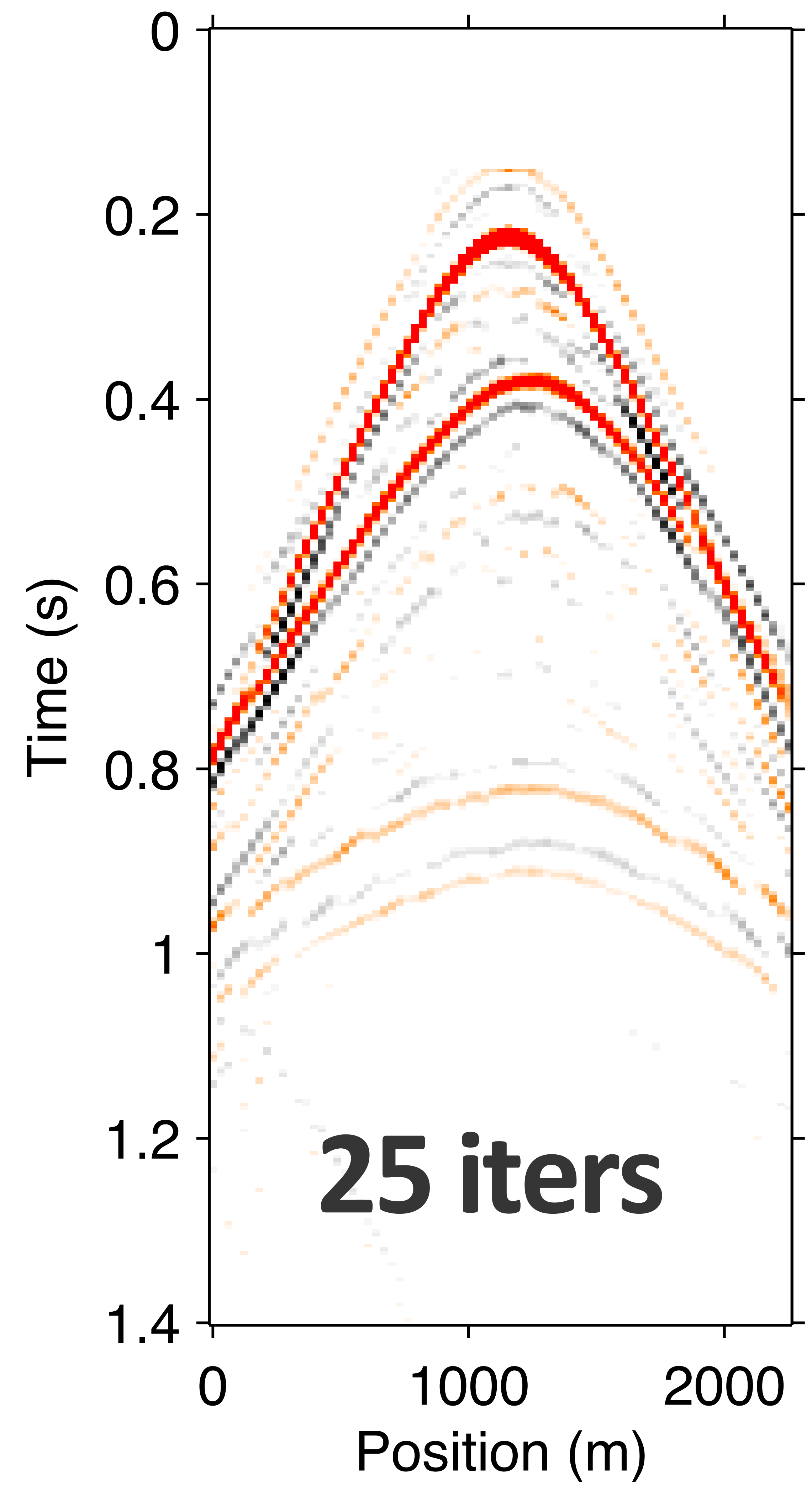
Solve

Warm-starting/continuation from coarse solution
Example

Solution of 2x decimated data

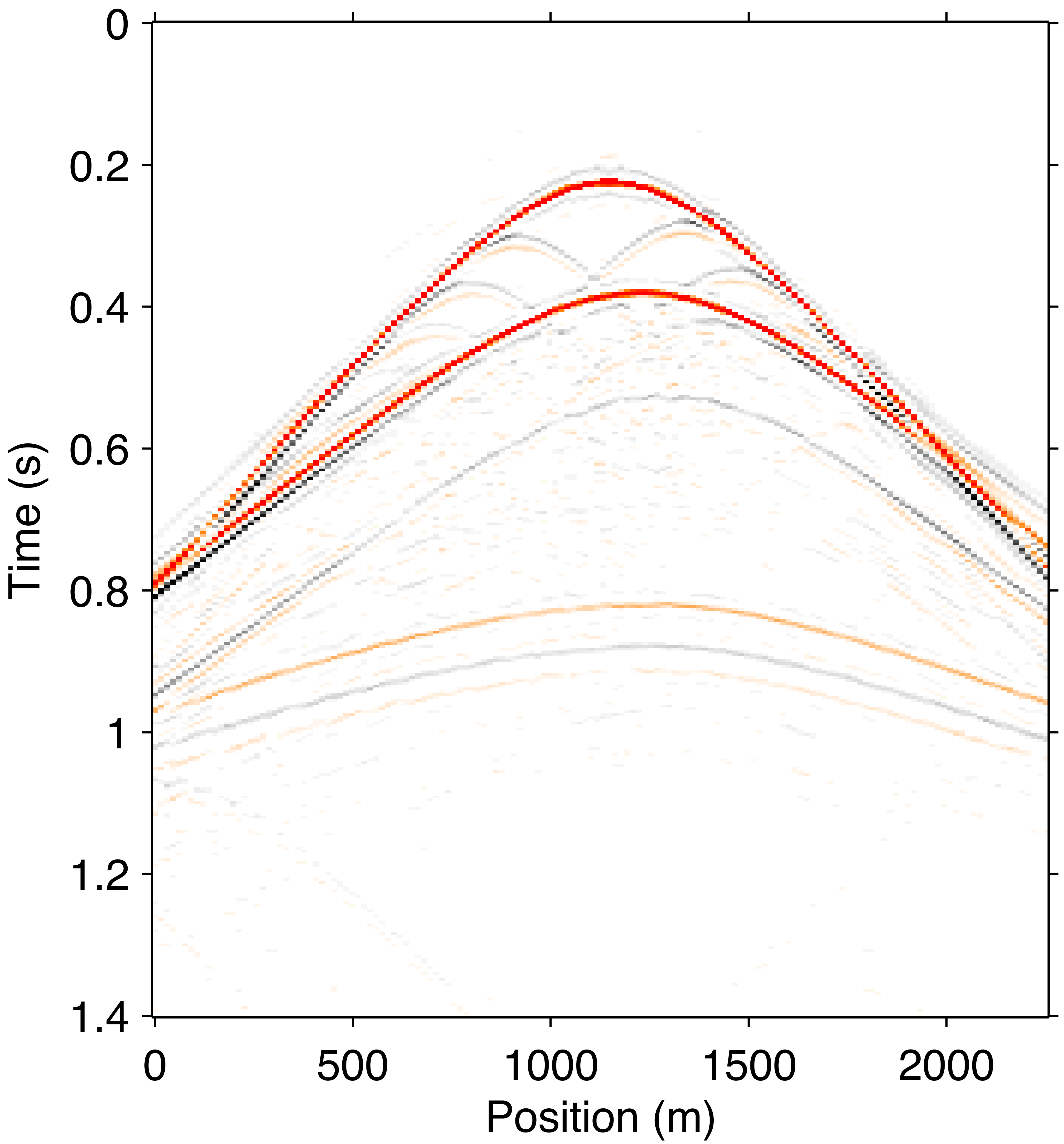


Solution on 2x dec data
continuation from 4x dec solution

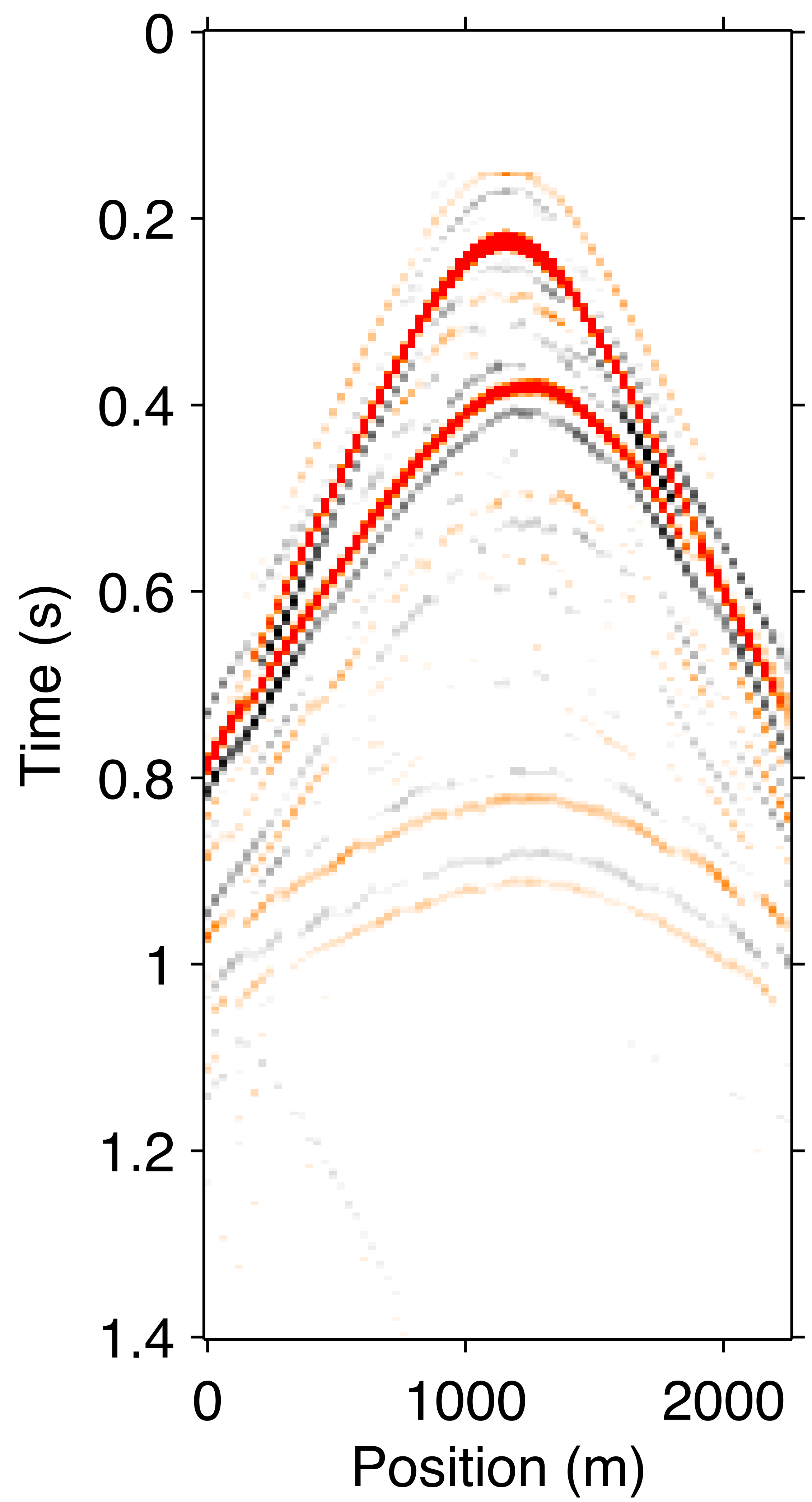


Warm-starting/continuation from coarse solution
Example

Solution of full data

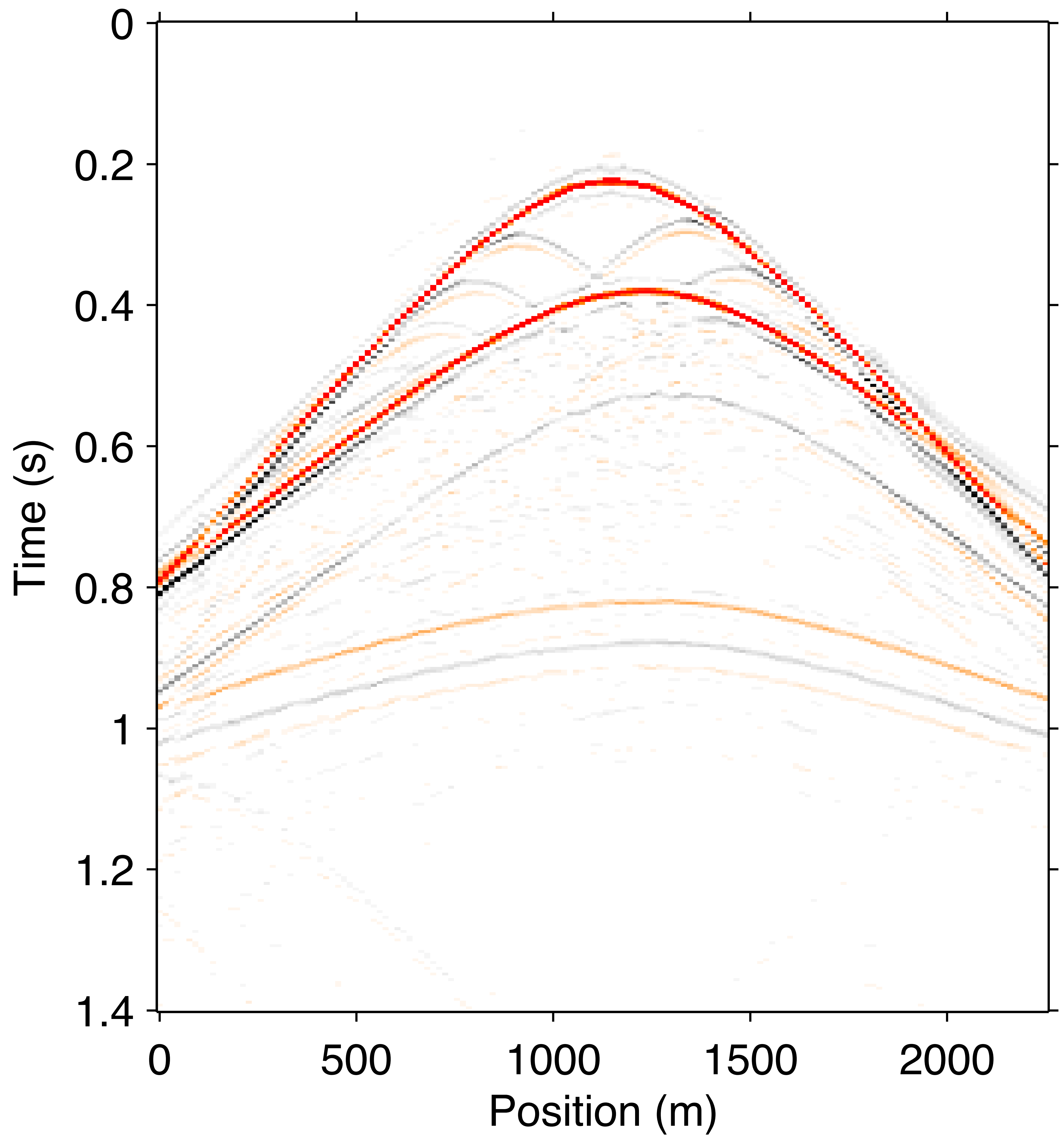


Solution on 2x dec data
continuation from 4x dec solution

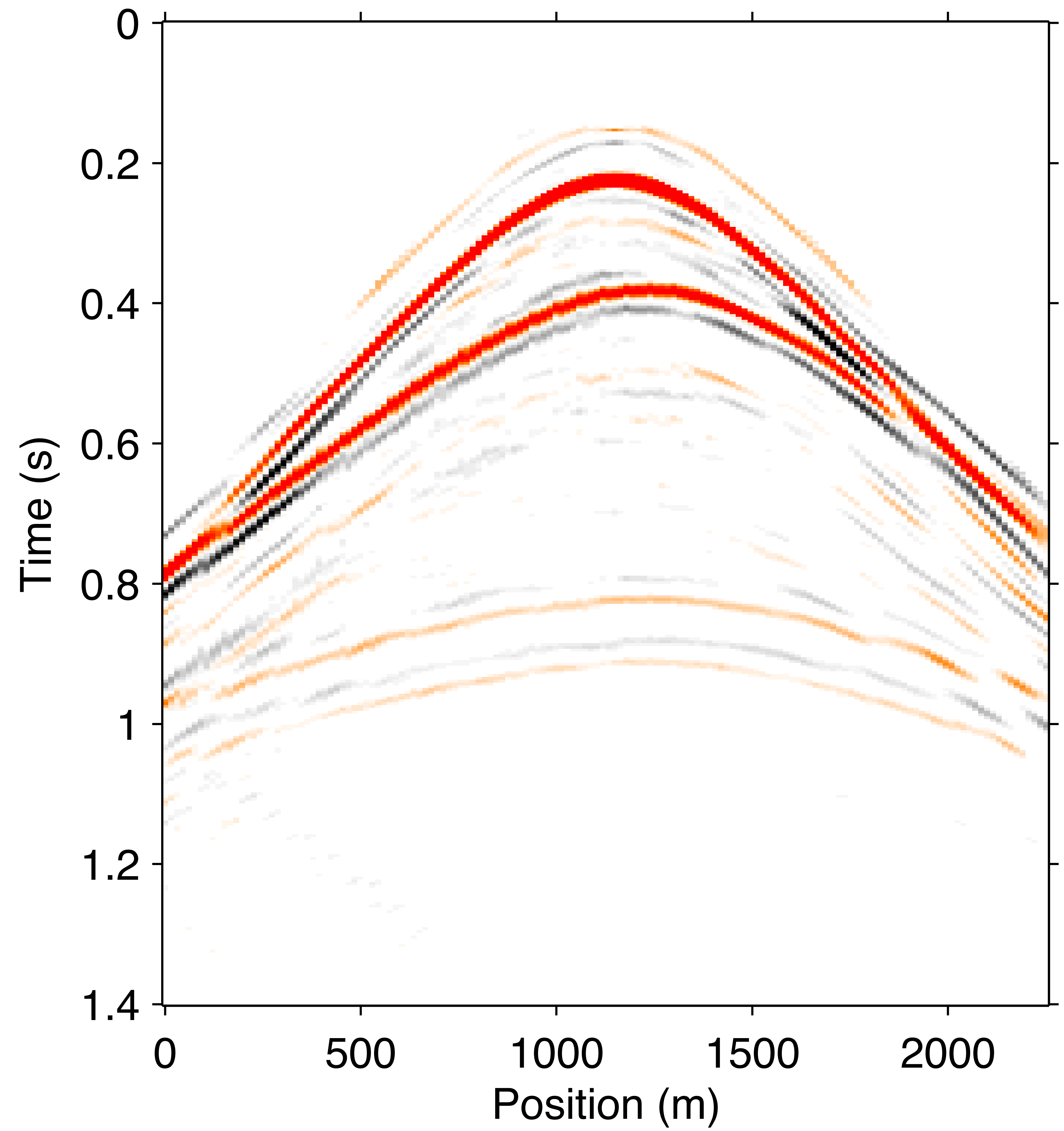


Warm-starting/continuation from coarse solution
Example

Solution of full data



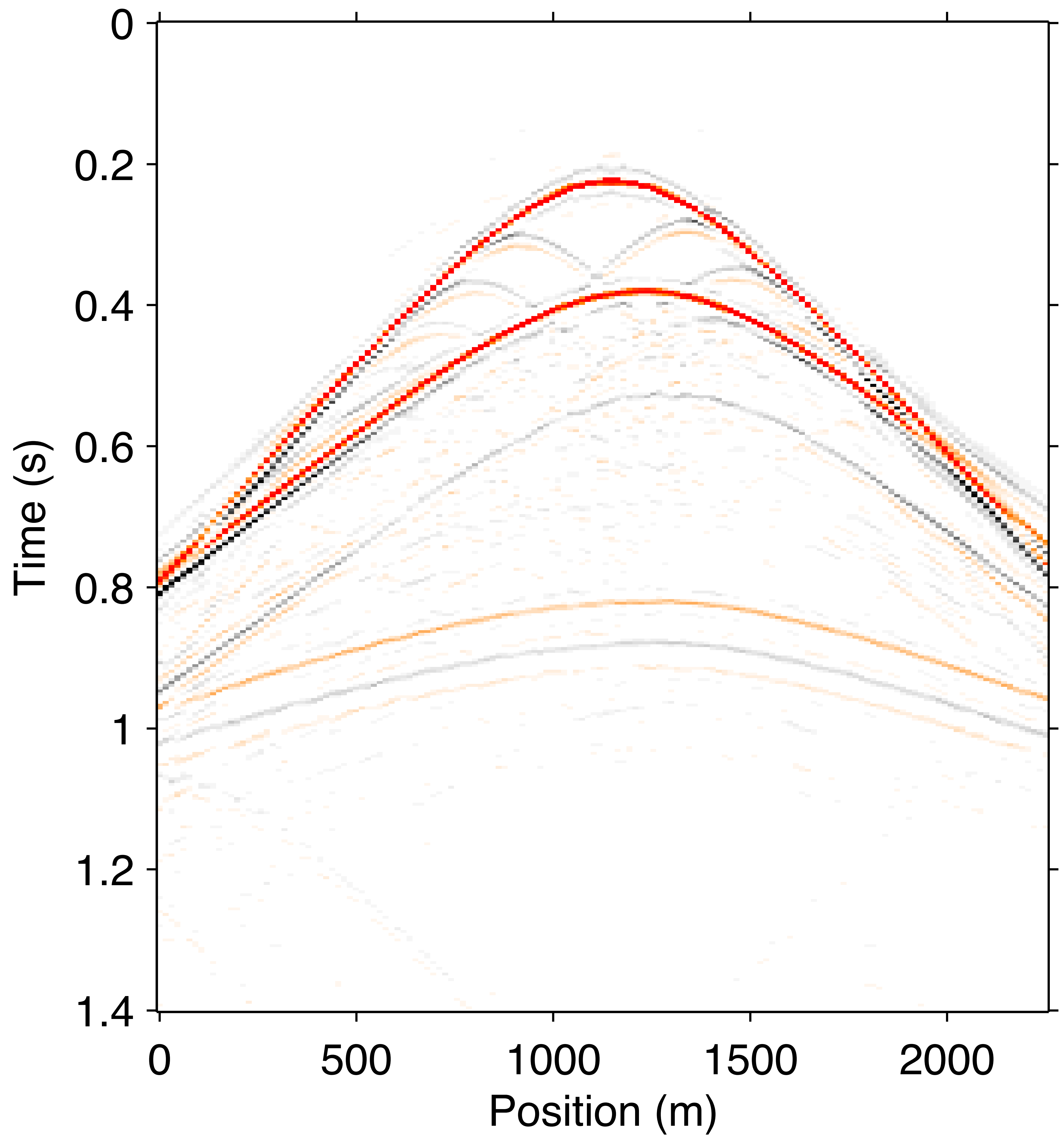
Solution on 2x dec data > interp 2x
continuation from 4x dec solution



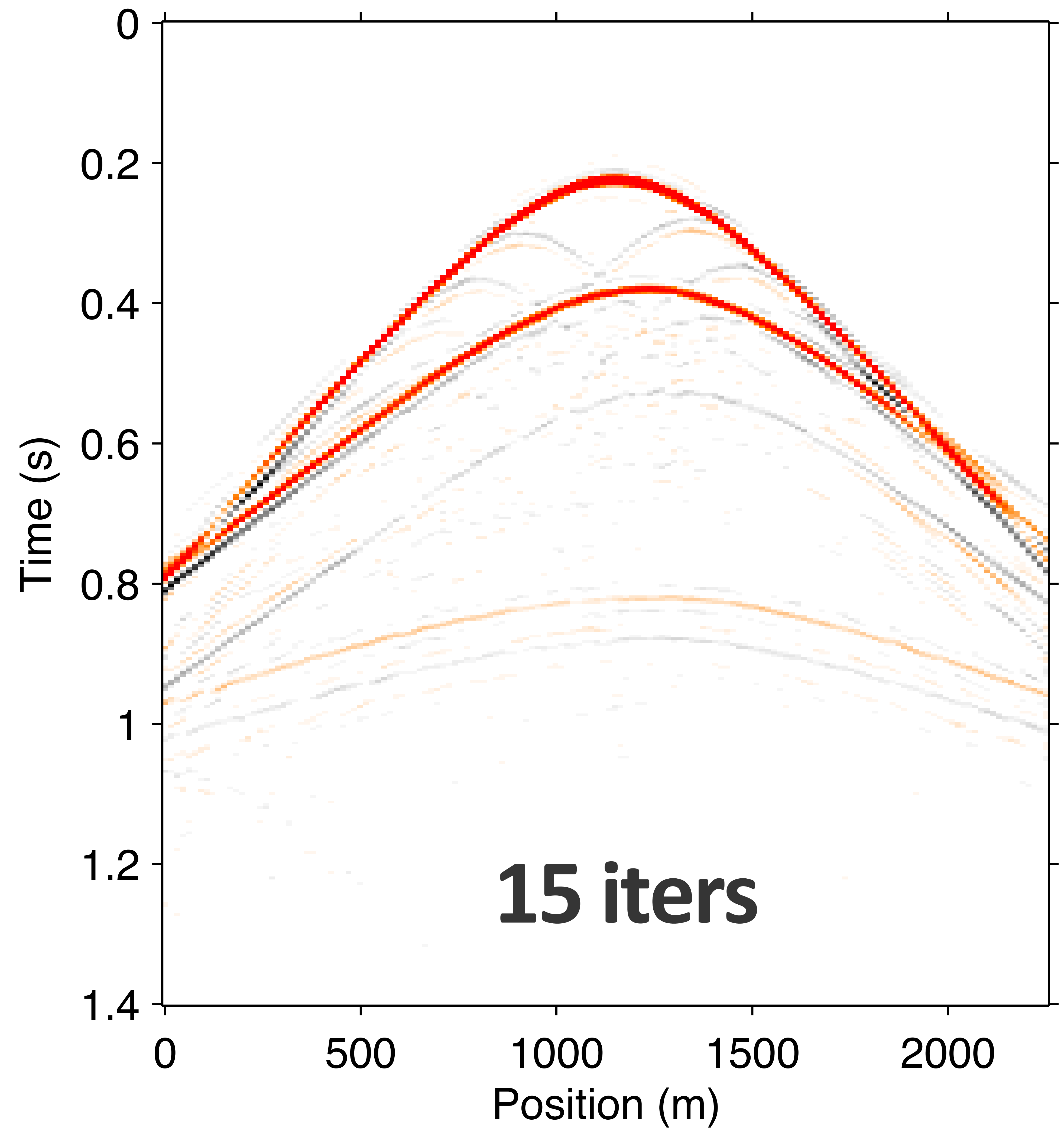
Prolongation

Warm-starting/continuation from coarse solution
Example

Solution of full data

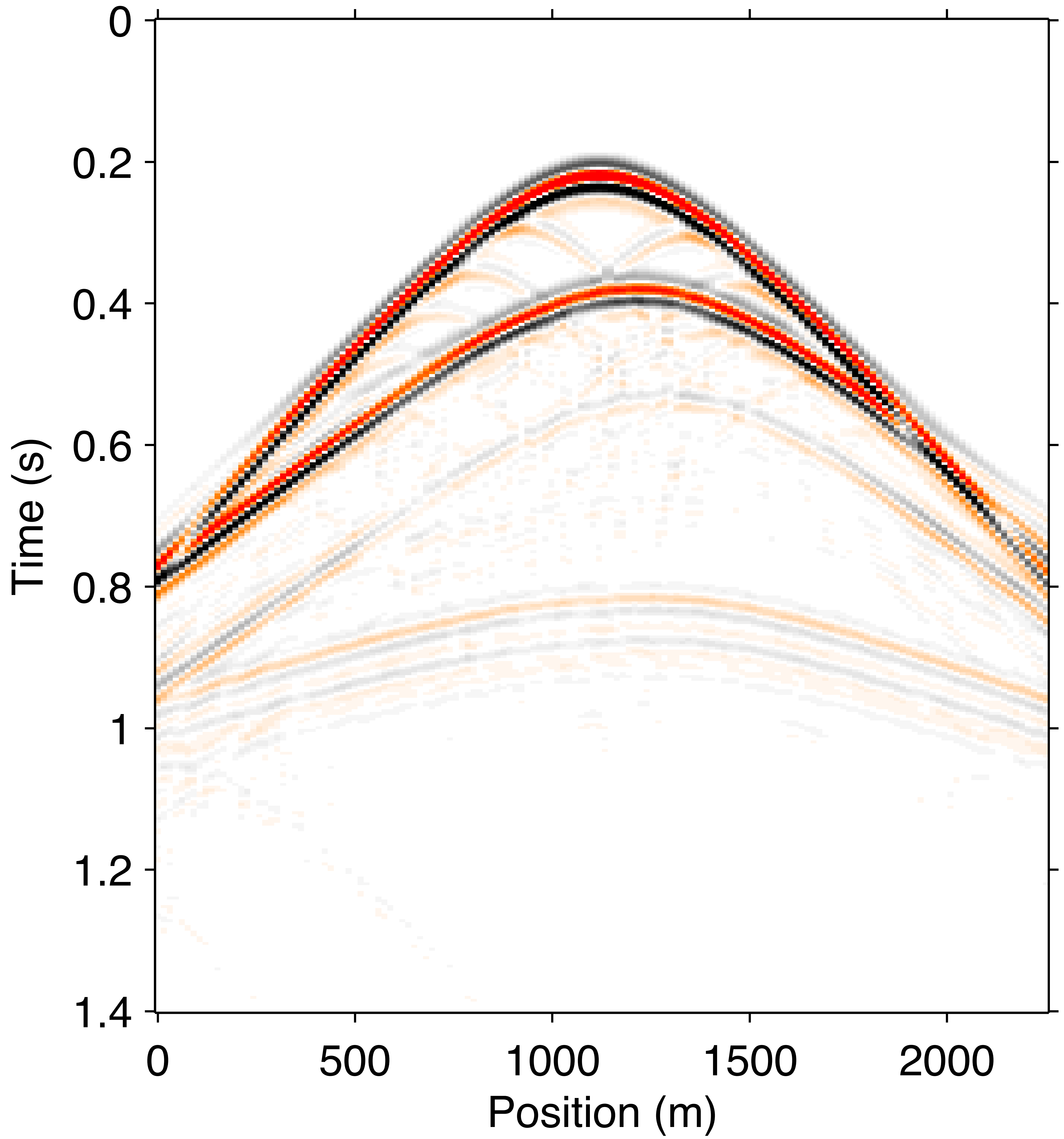


Solution on 2x dec data > interp 2x
continuation from 4x thru 2x solution



Solve

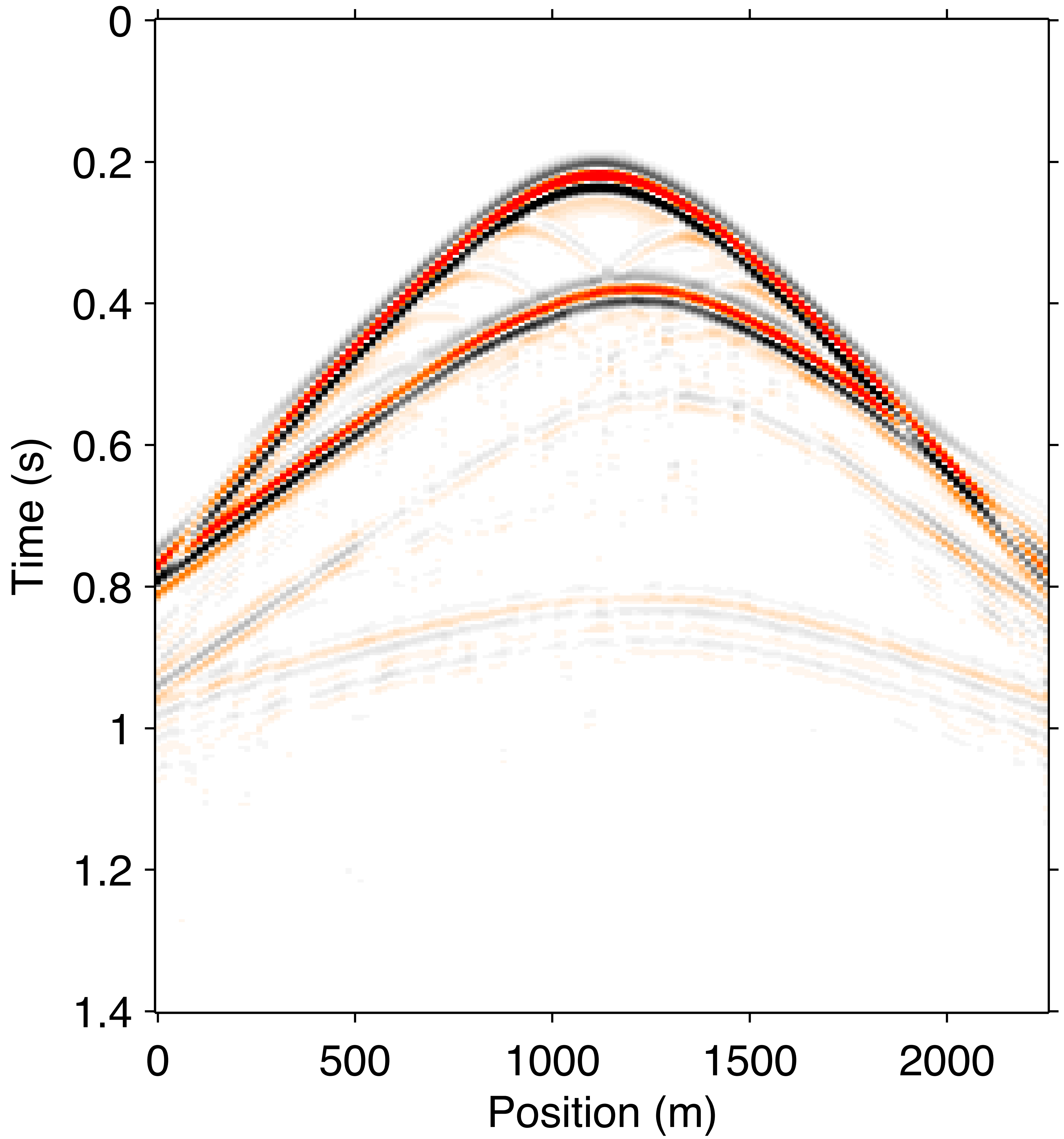
Warm-starting/continuation from coarse solution
Example



Direct Primary

Solved with plain algorithm from finest scale data

Warm-starting/continuation from coarse solution
Example

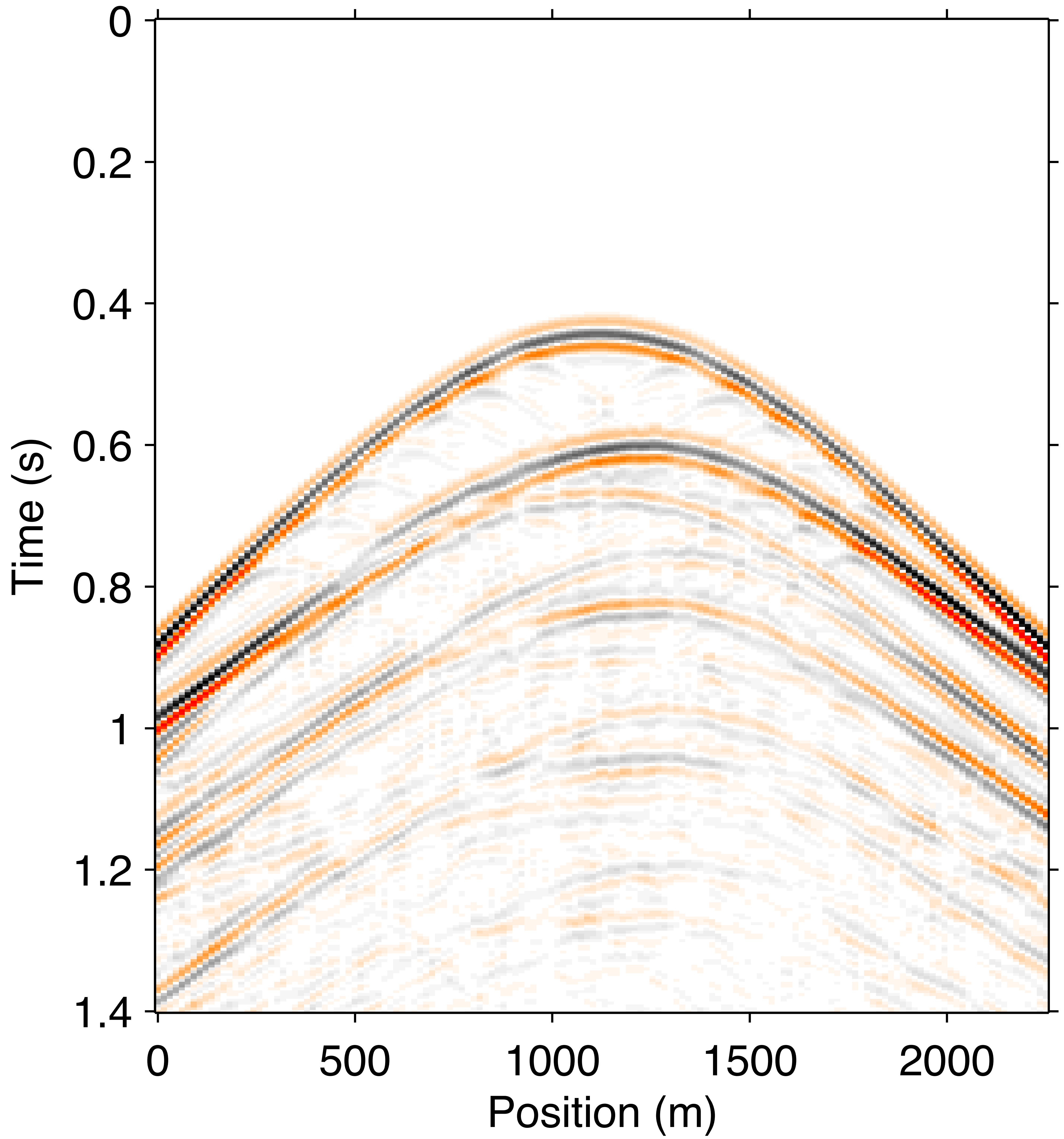


Direct Primary

Solved with spatial sampling continuation

dx = 60m > 30m > 15m

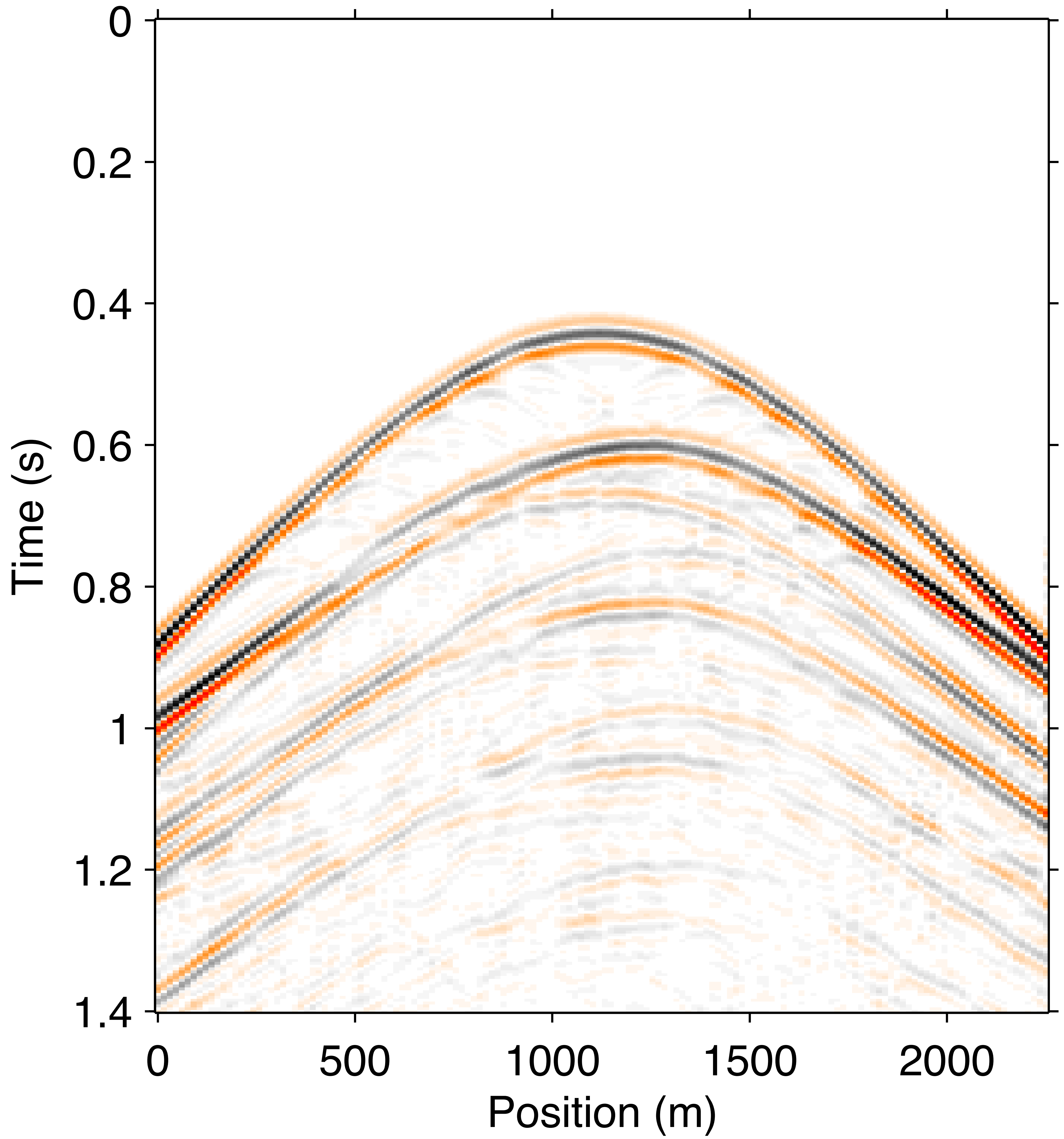
Warm-starting/continuation from coarse solution
Example



Predicted Surface Multiple

Solved with plain algorithm from finest scale data

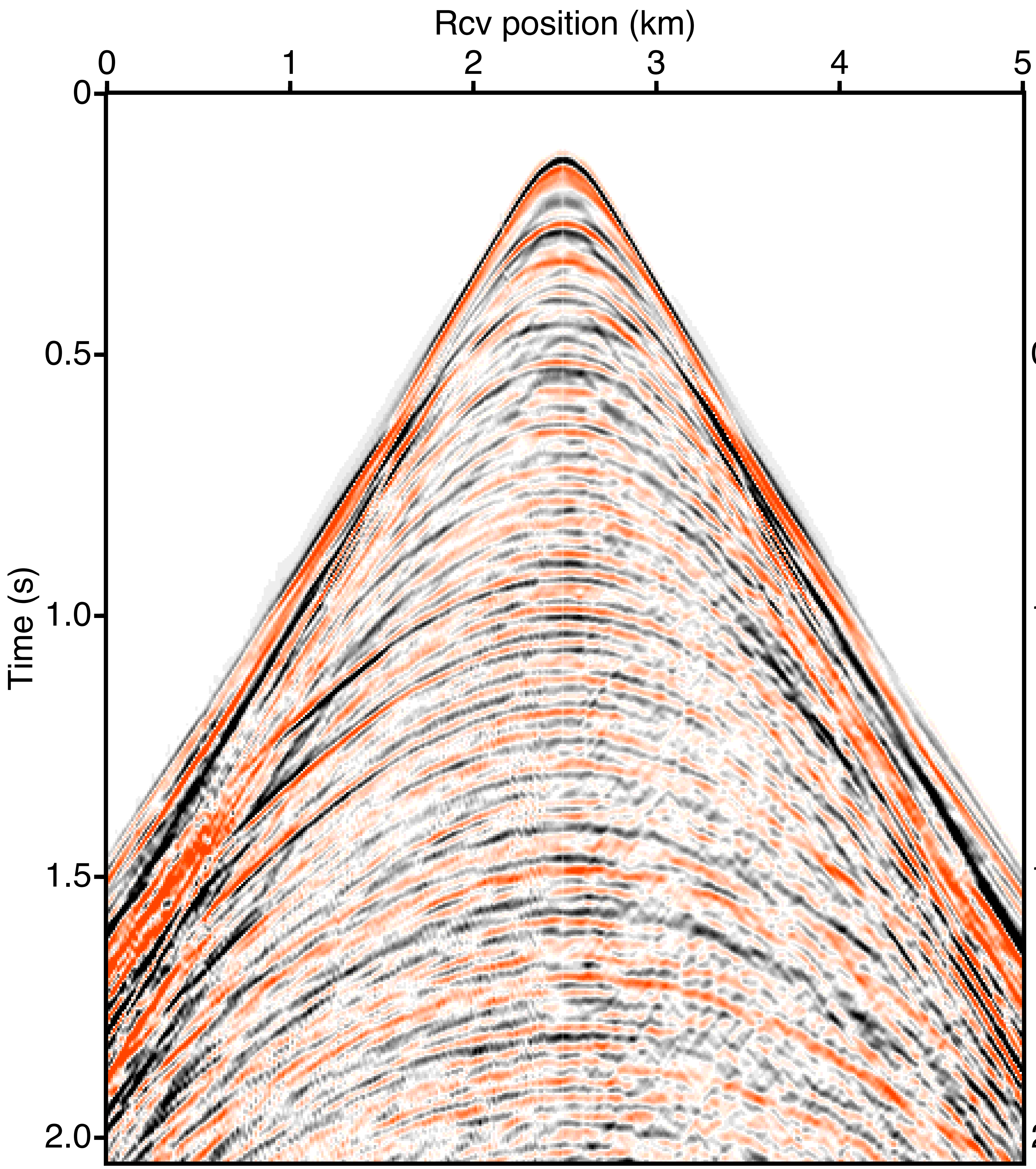
Warm-starting/continuation from coarse solution
Example



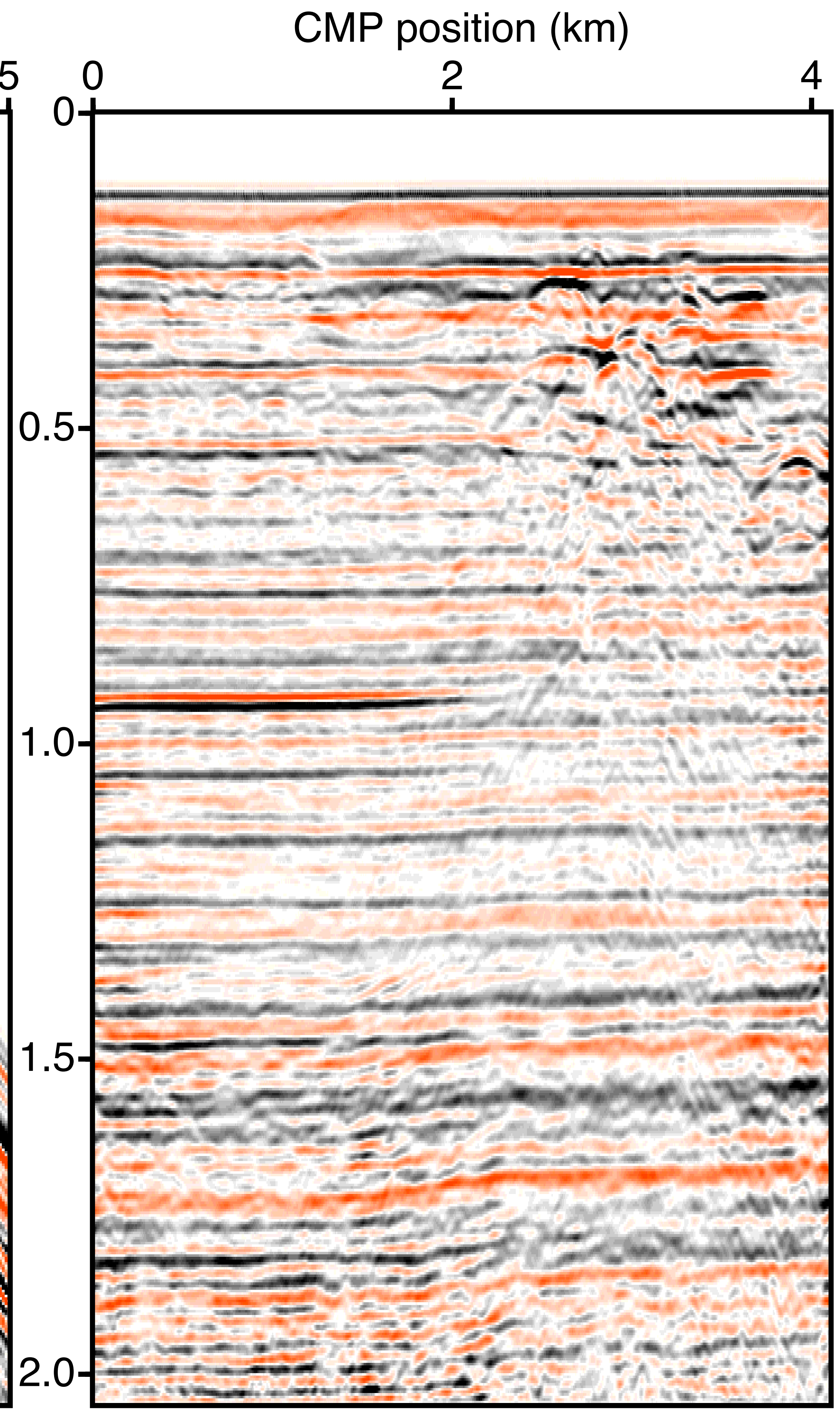
Predicted Surface Multiple

Solved with spatial sampling continuation

dx = 60m > 30m > 15m



Shot gather



NMO-corrected stack

North sea data

Shot gather and stack

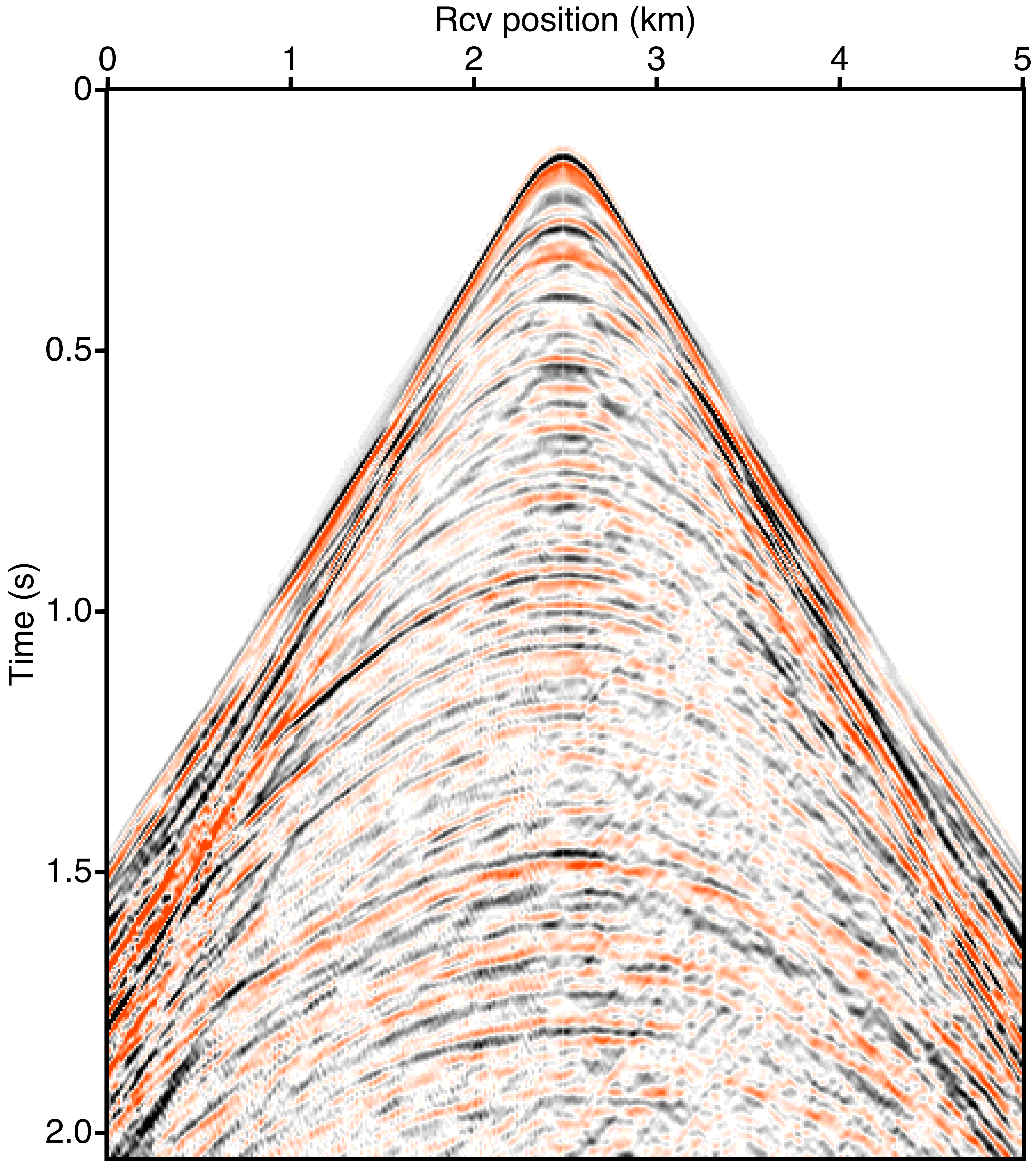
Streamer data
(regularized to fixed-spread data)

401 source and reciever

12.5 m spatial grid

4 ms time sampling

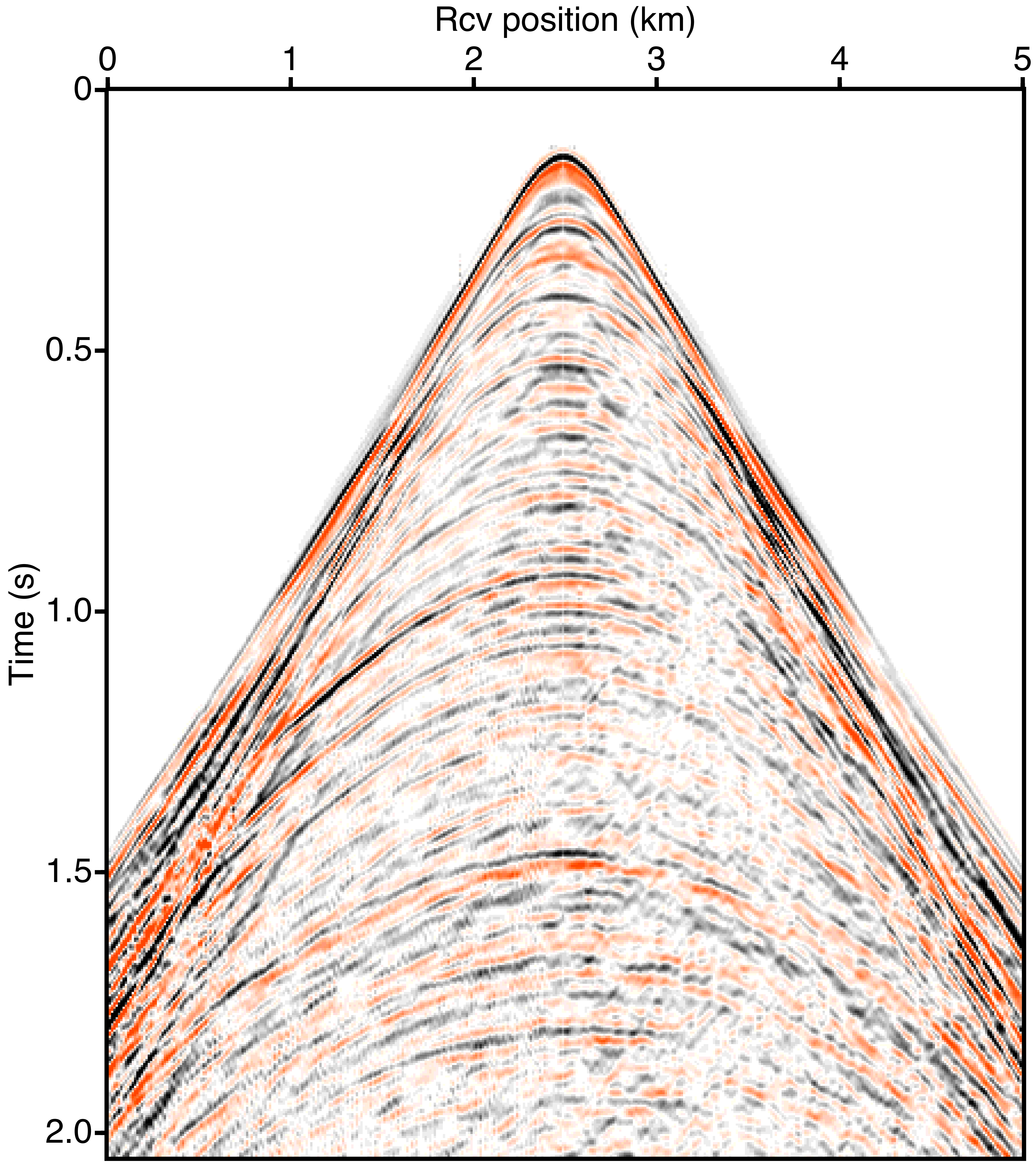
Solution wavefield comparison



Direct Primary

Solved with plain algorithm from finest scale data

Solution wavefield comparison

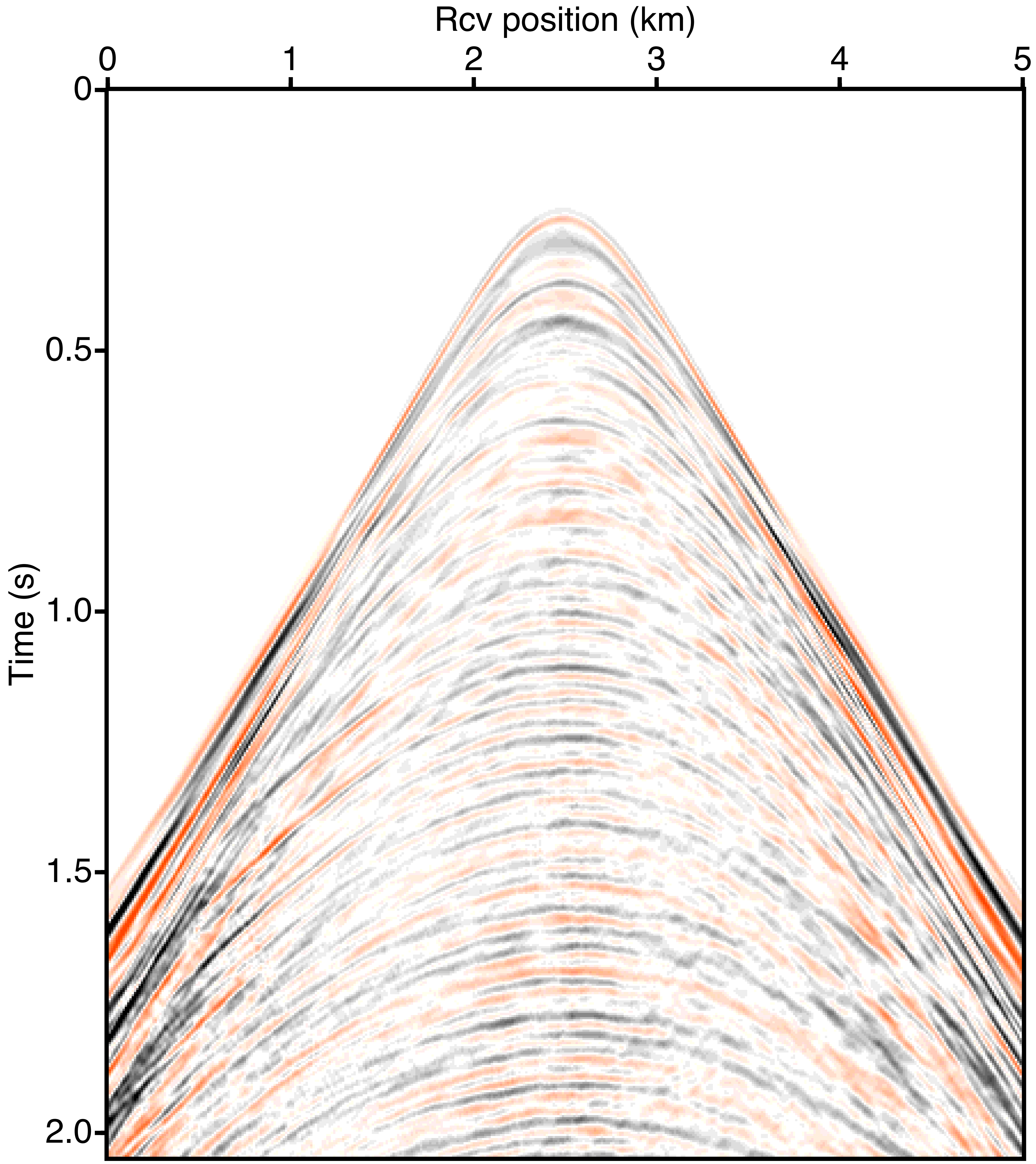


Direct Primary

Solved with spatial sampling continuation

dx = 50m > 25m > 12.5m

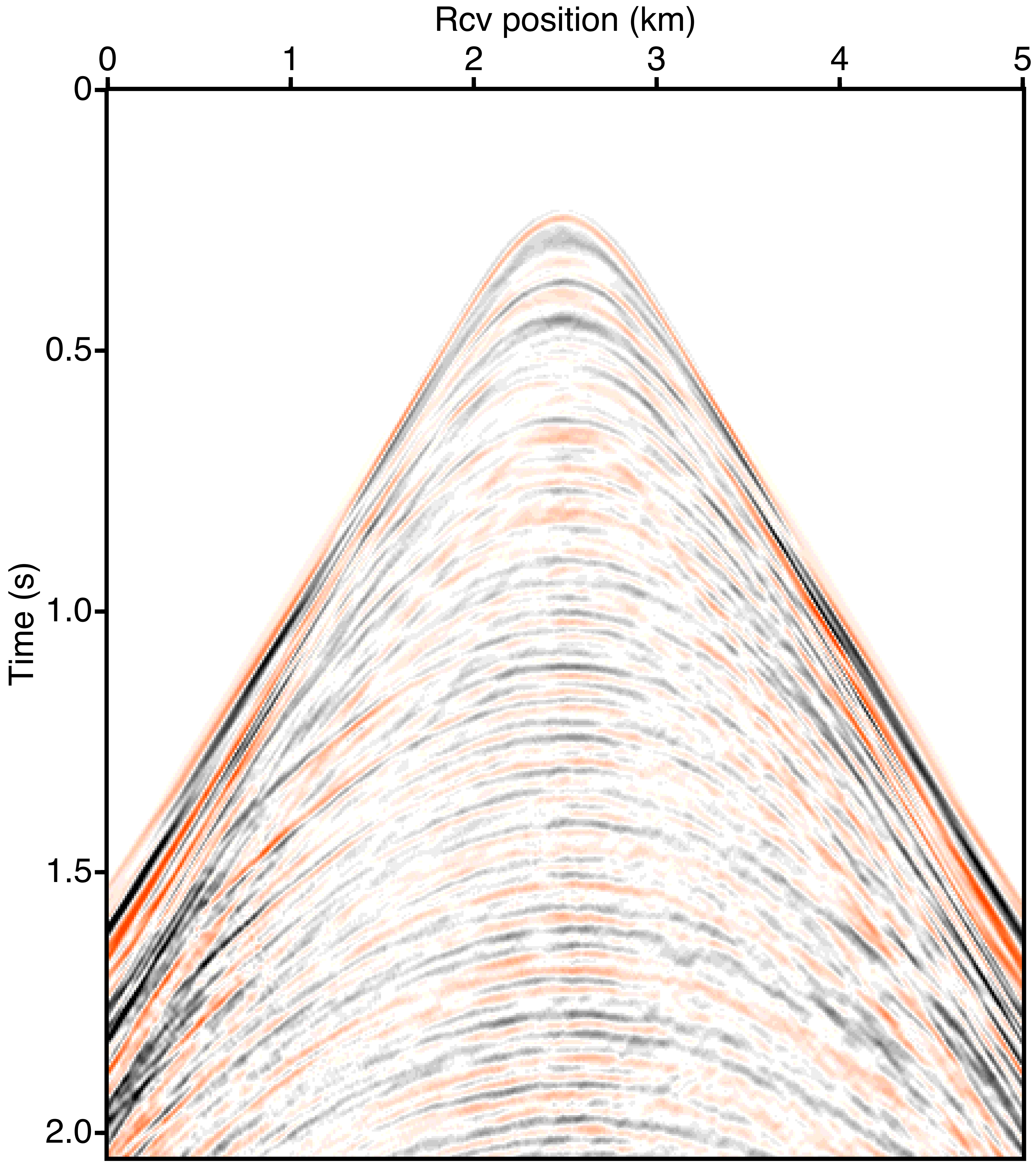
Solution multiple comparison



Predicted Surface Multiple

Solved with plain algorithm from finest scale data

Solution multiple comparison



Predicted Surface Multiple

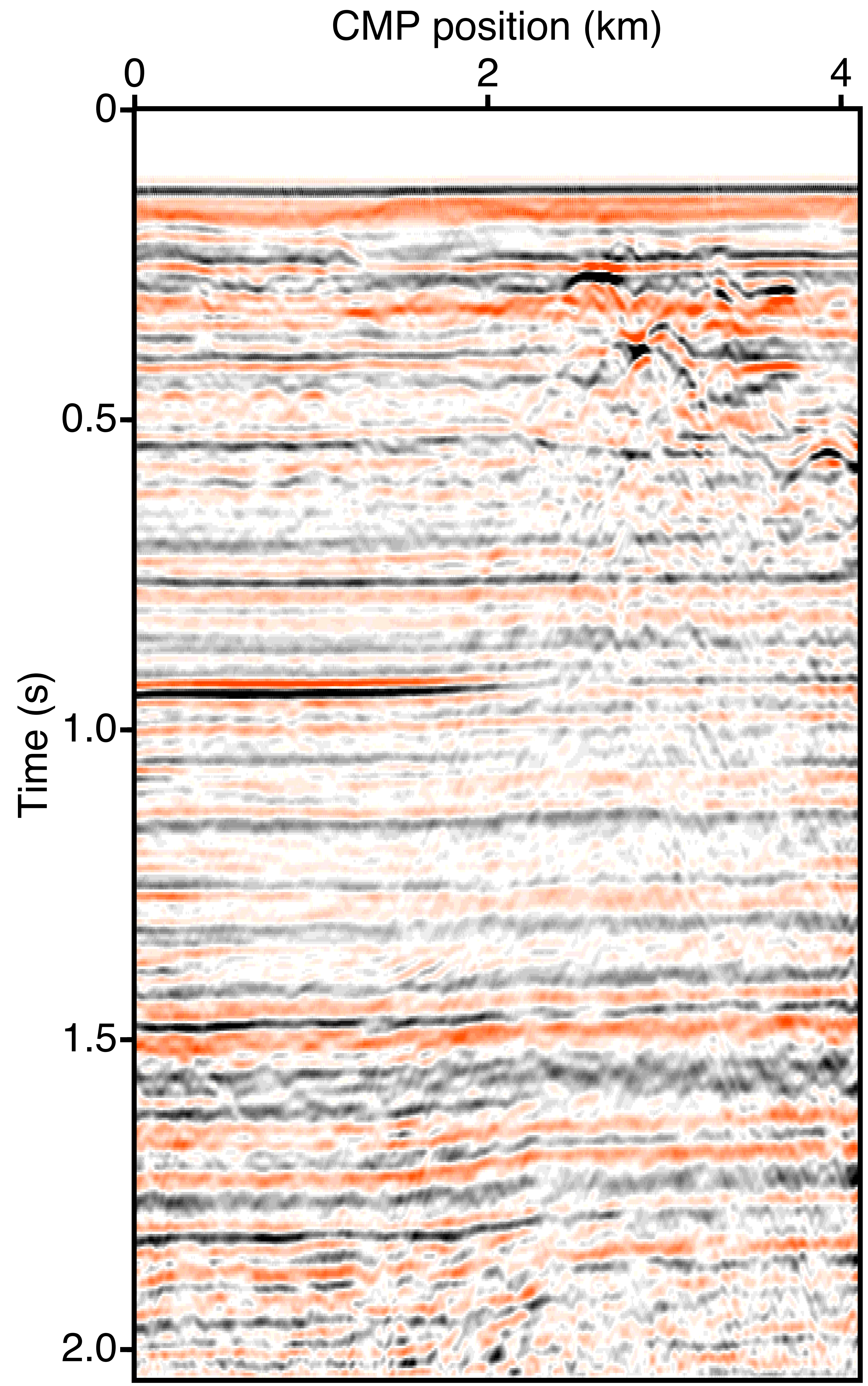
Solved with spatial sampling continuation

dx = 50m > 25m > 12.5m

Solution stack comparison

REPSI Primaries NMO Stack

Solved with plain algorithm from finest scale data

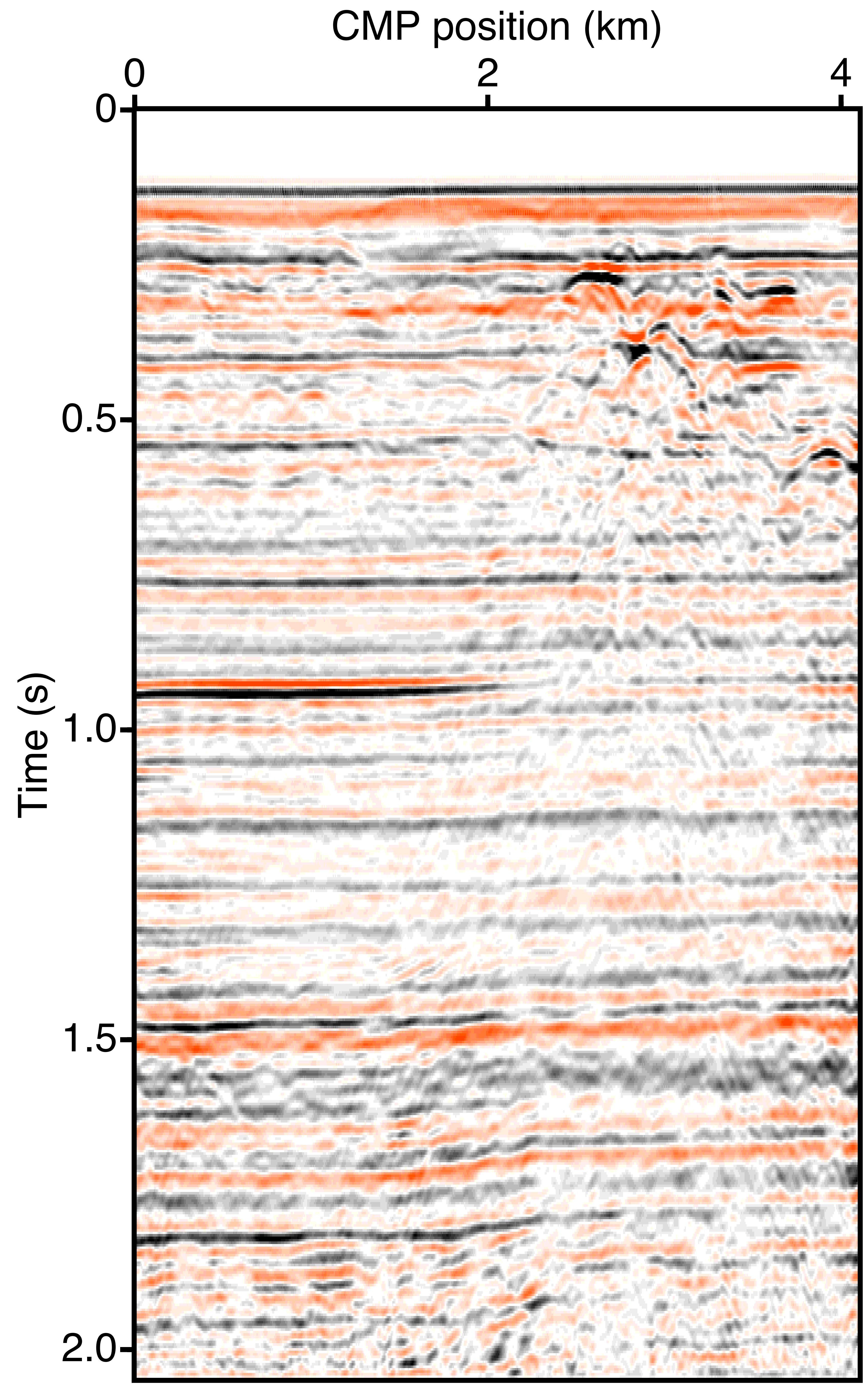


Solution stack comparison

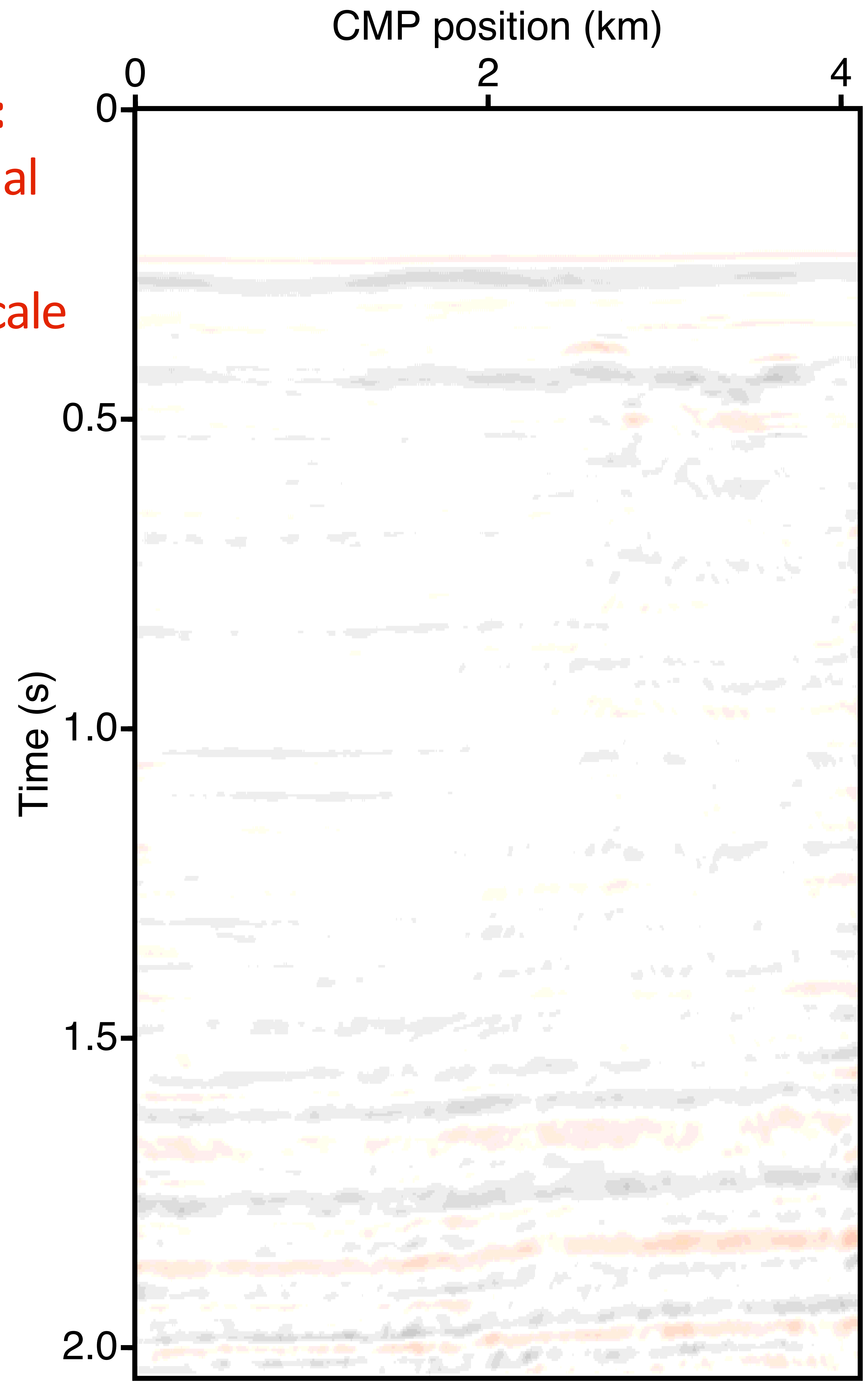
REPSI Primaries NMO Stack

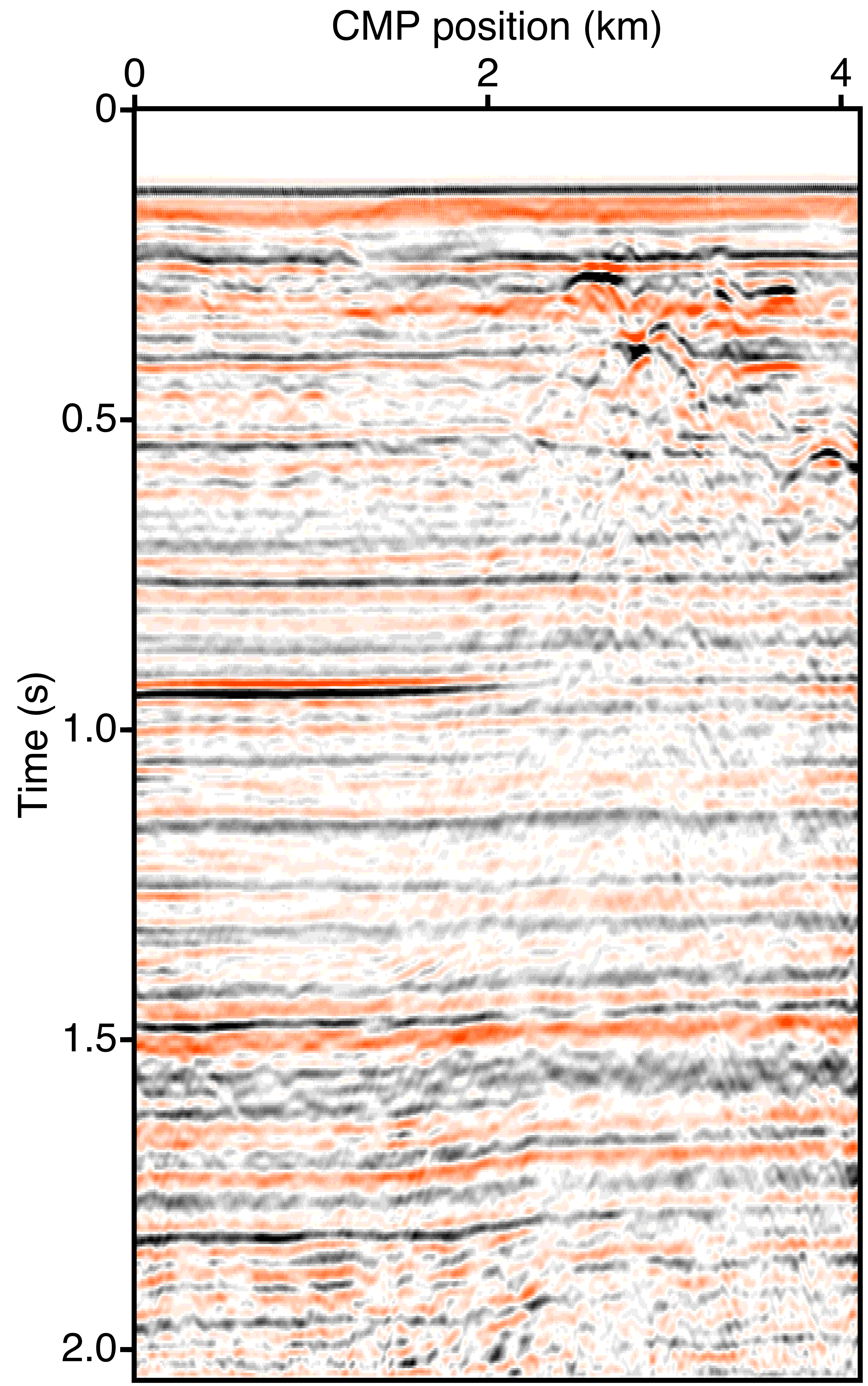
Solved with spatial sampling continuation

dx = 50m > 25m > 12.5m



Diff:
Normal
vs
Multiscale

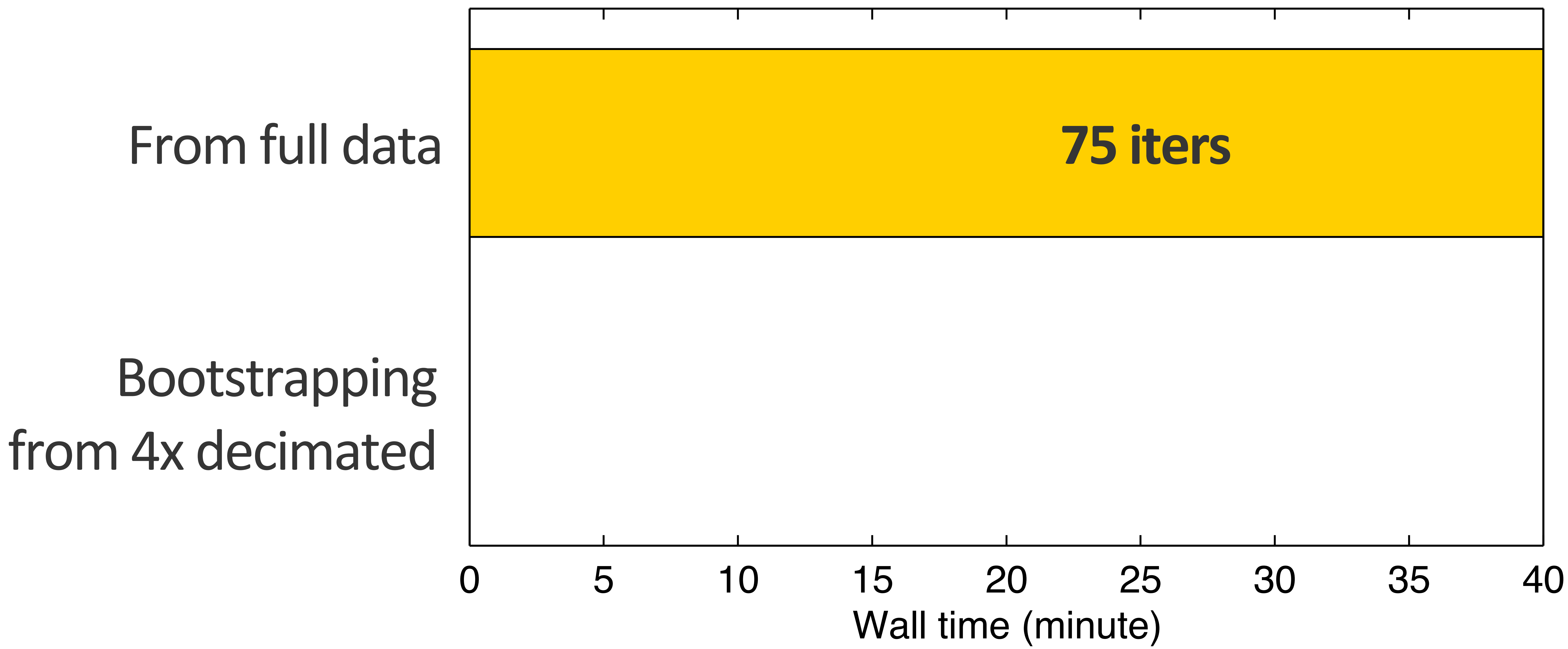




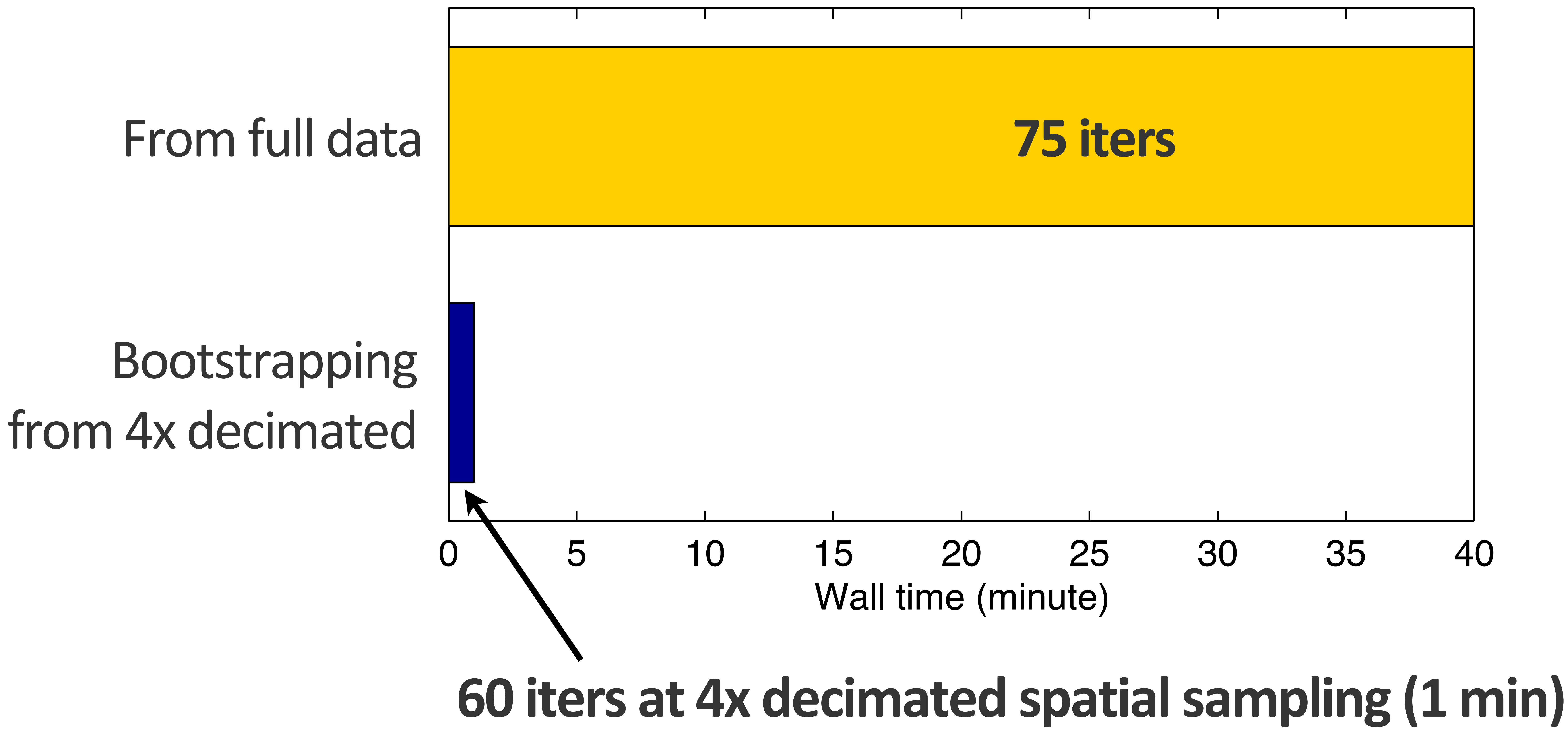
SLIM

Tuesday, December 9, 14

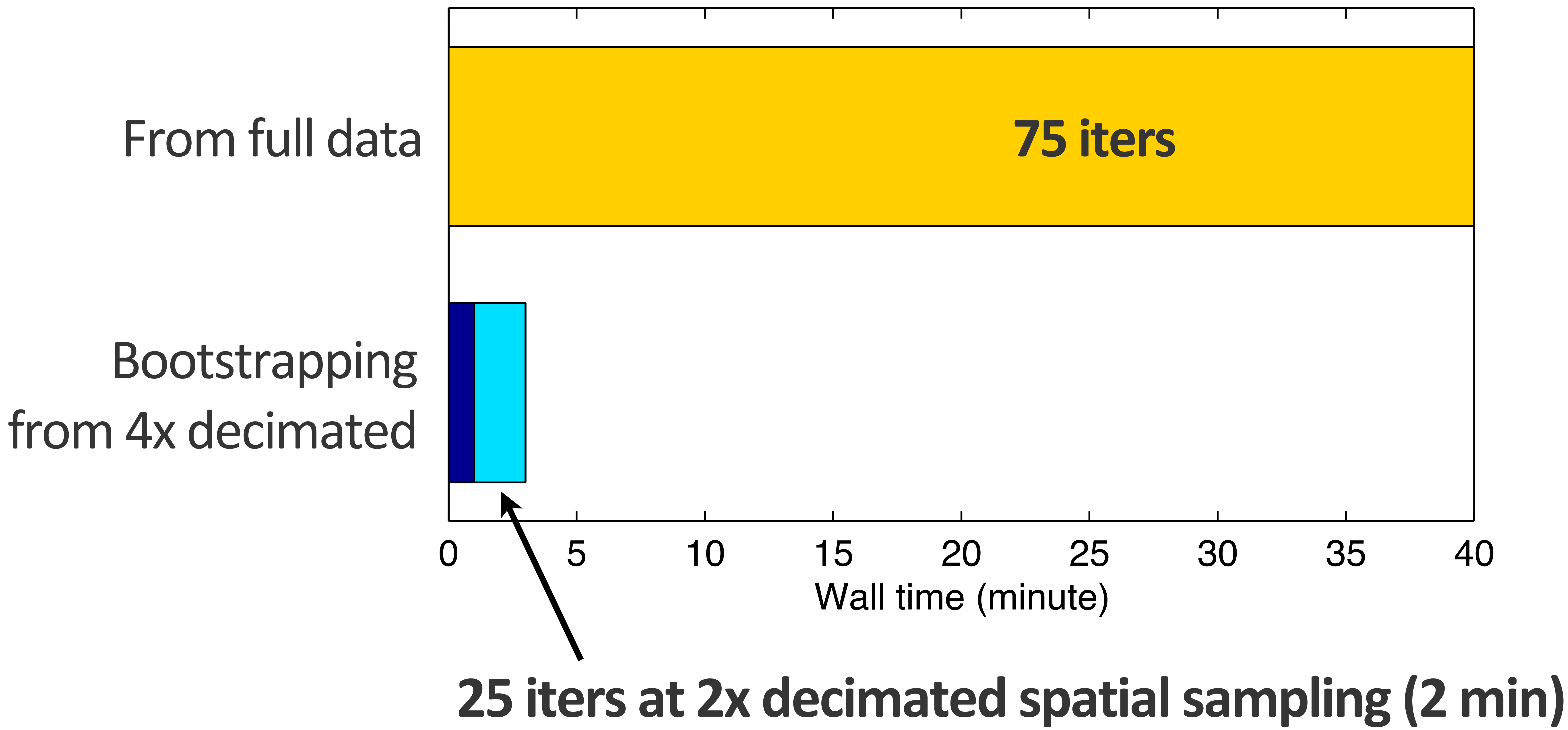
Significant speedup from bootstrapping
Wall times



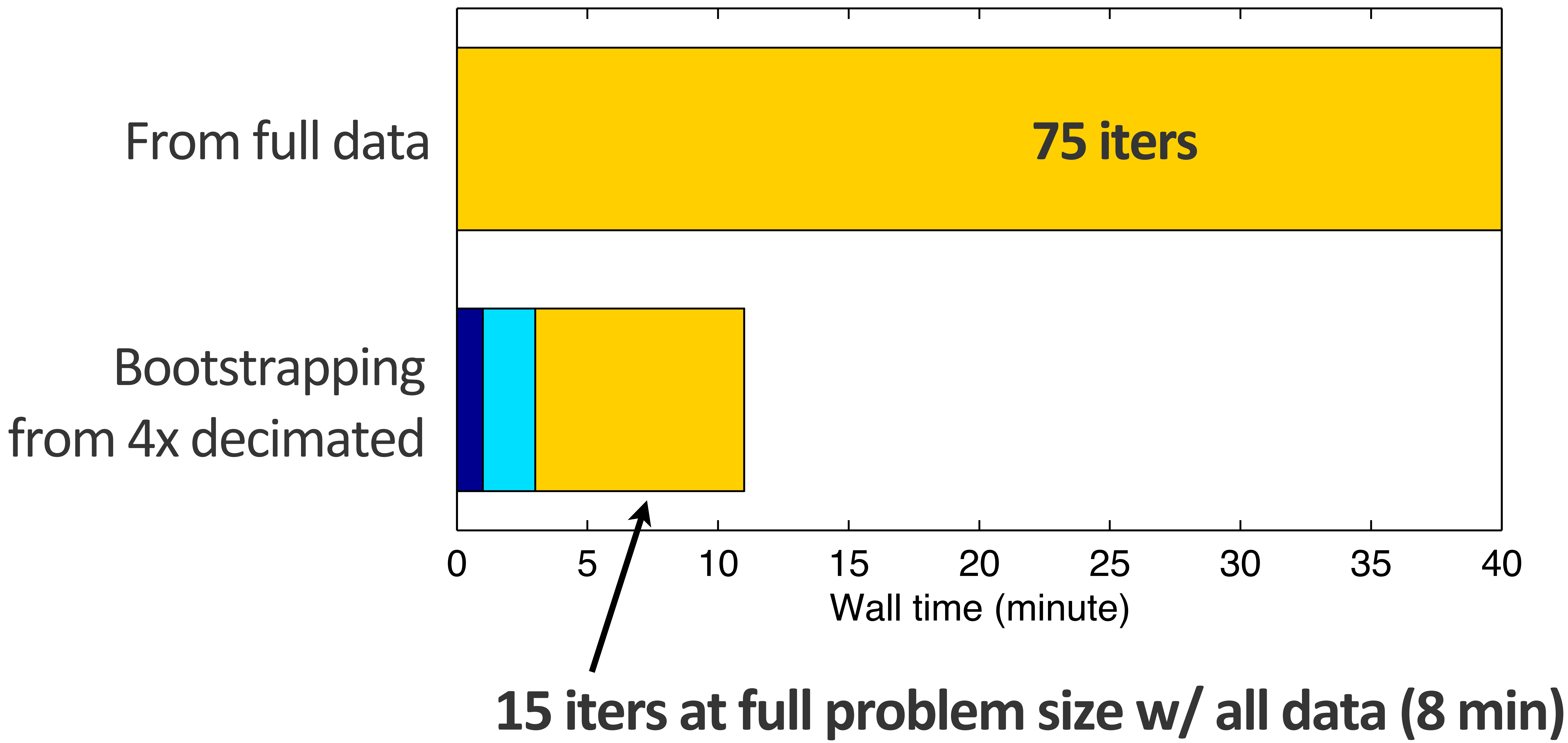
Significant speedup from bootstrapping
Wall times



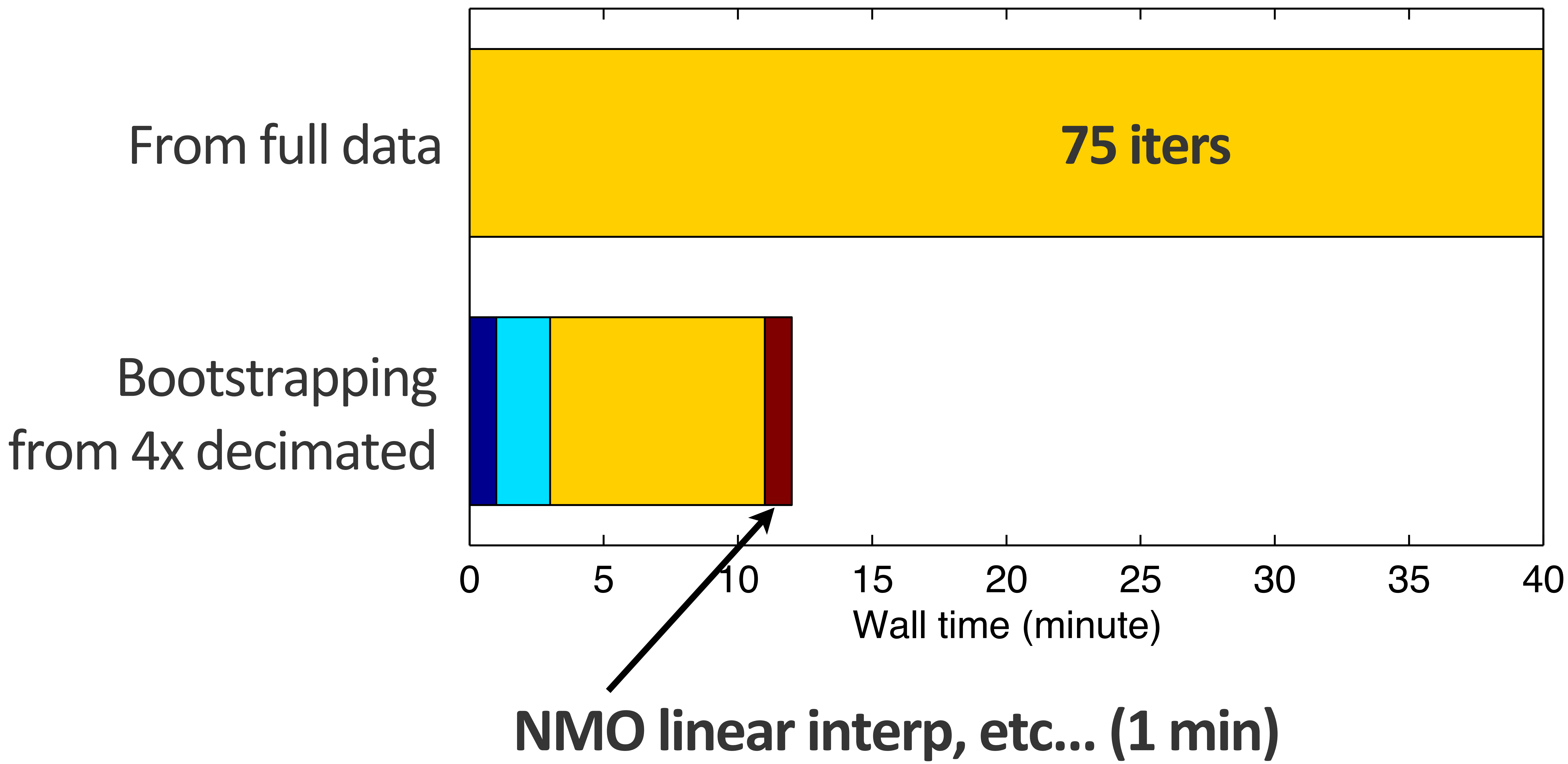
Significant speedup from bootstrapping
Wall times



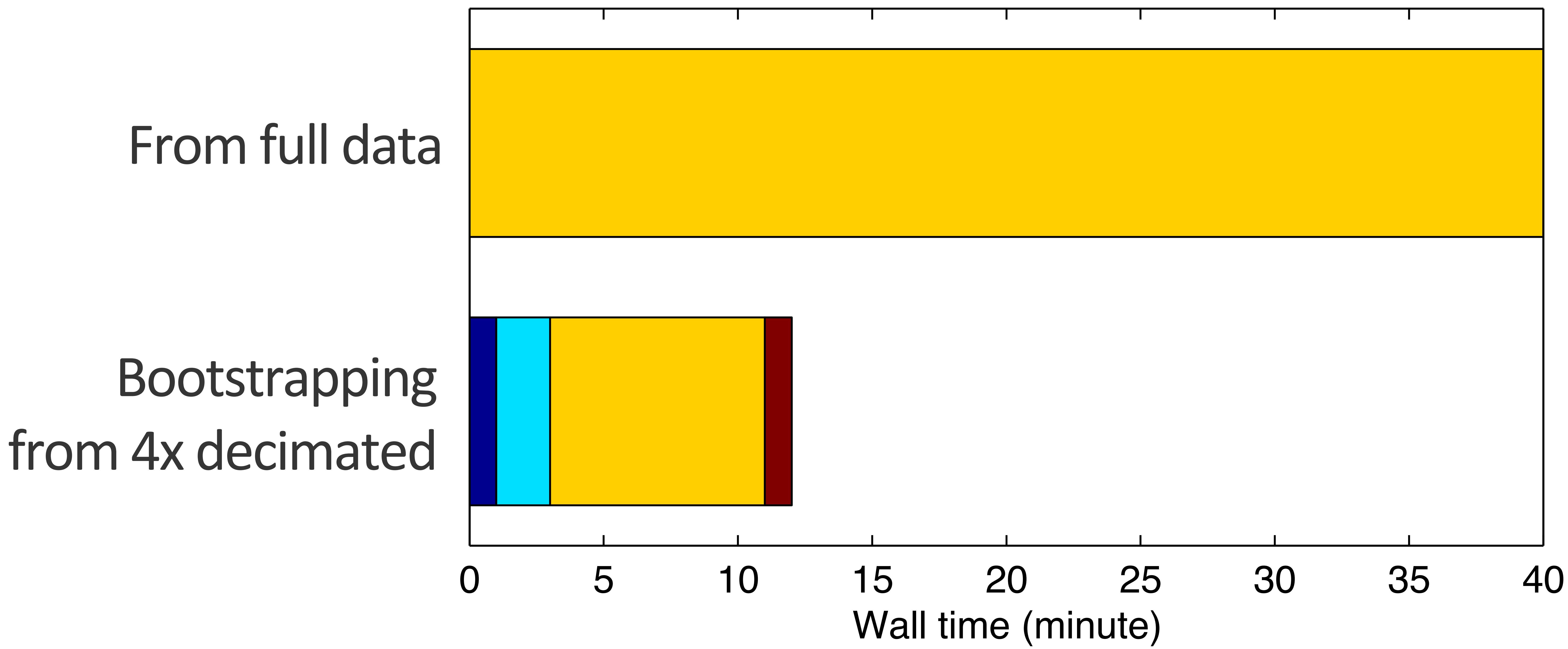
Significant speedup from bootstrapping
Wall times



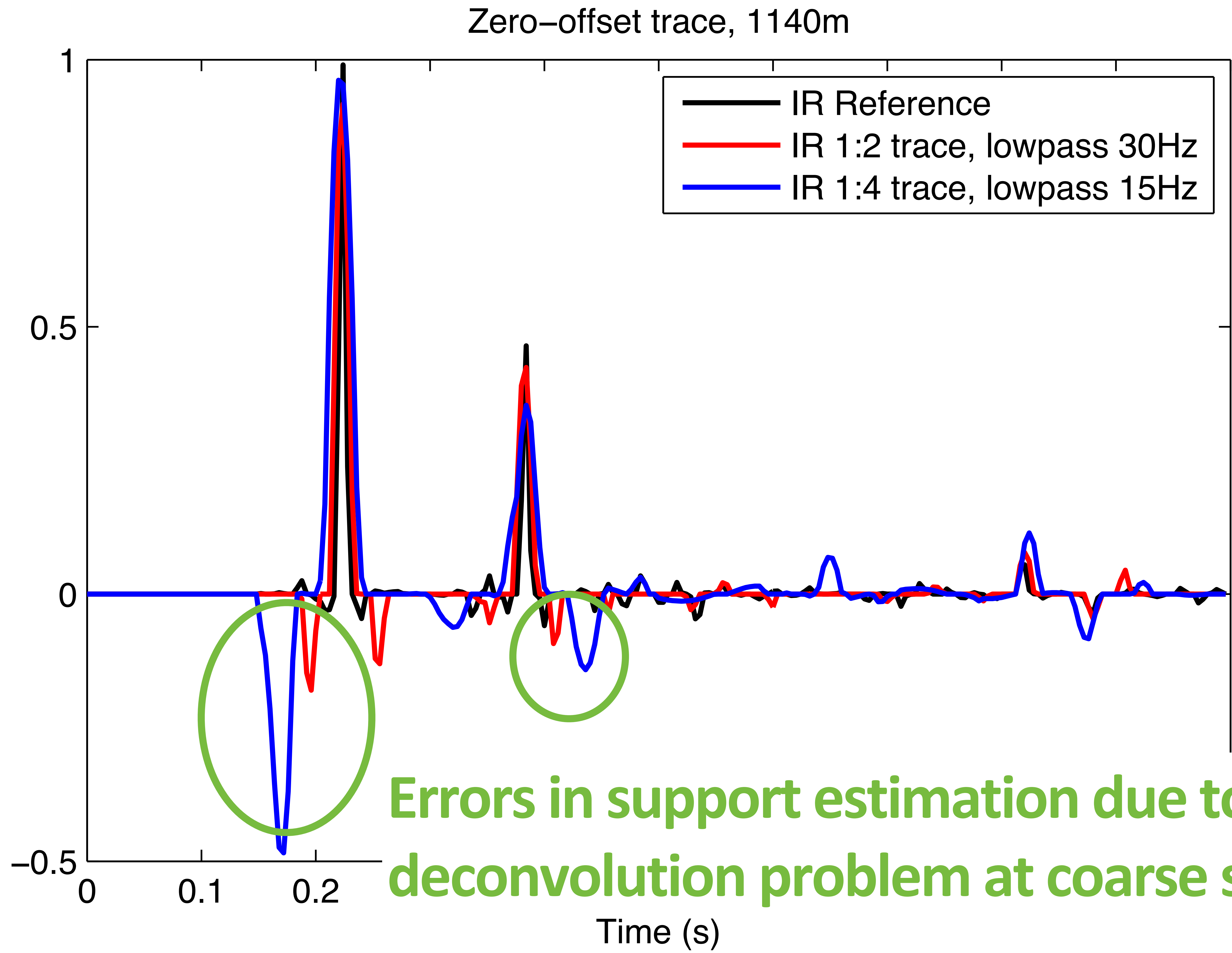
Significant speedup from bootstrapping
Wall times



Significant speedup from bootstrapping
Wall times



Lowpass data permits coarser sampling w/o aliasing



Errors in support estimation due to more difficult deconvolution problem at coarse scales

EPSI as a convolutional model

Traditional convolution model

$$\text{Up-going Primary} = \mathbf{GQ}$$

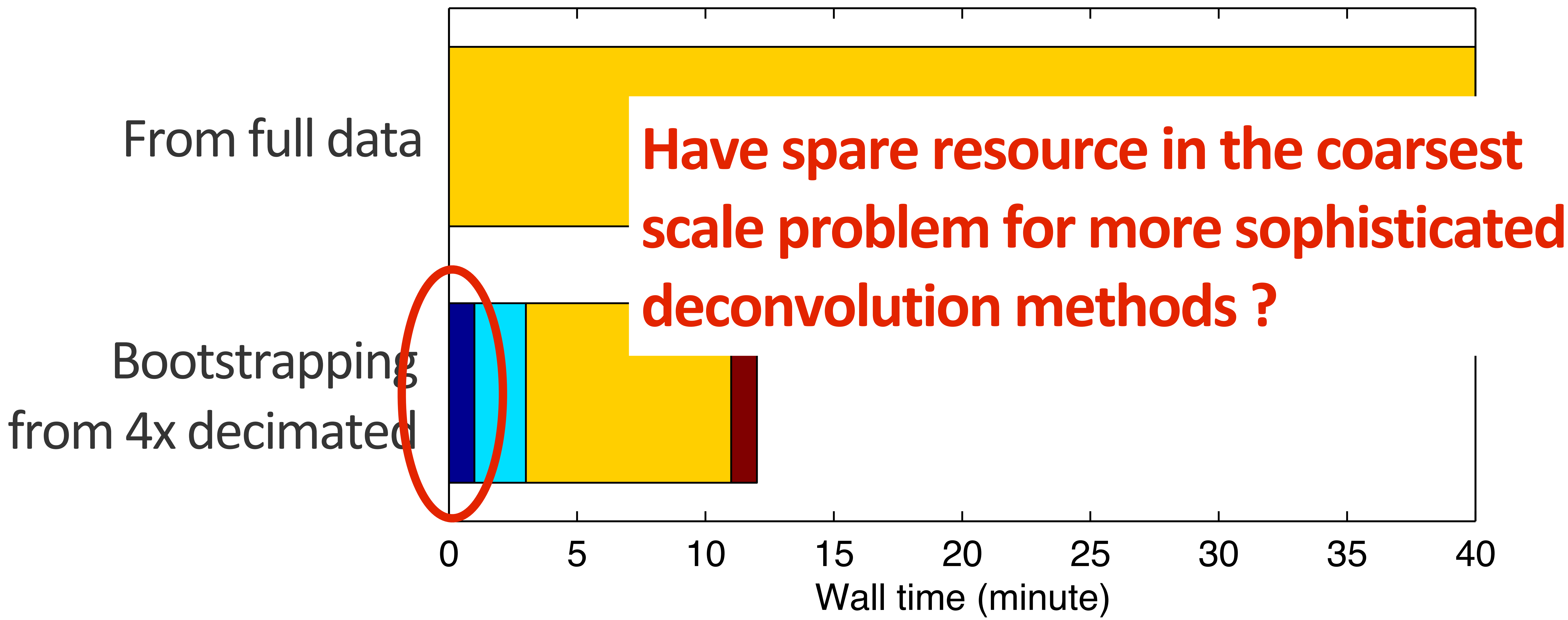
EPSI Model

$$\text{Up-going Primary} + \text{Multiples} = \mathbf{GQ} - \mathbf{GP}$$

additional info on G

$\mathbf{P}$	total up-going wavefield
$\mathbf{Q}$	down-going source signature
$\mathbf{G}$	primary impulse response

Significant speedup from bootstrapping
Wall times



Yes, we are working on that...
(Ernie's talk, next)