Minimal Residual Iterative Methods for Time-Harmonic Wave-Equation

Rafael Lago, Felix Herrmann

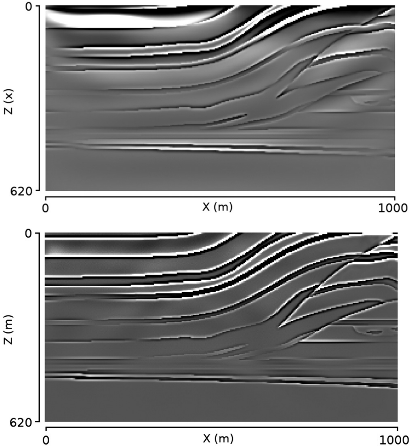
SINBAD Consortium Meeting, Whistler, December 10th 2014

Frequency Domain - Why?

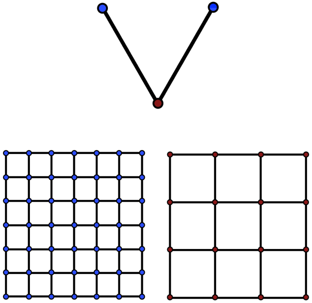
"Migration in the frequency domain on multicore CPU ***is faster*** *than RTM in the time domain accelerated by GPU, given enough computer nodes to calculate all frequencies in parallel."*

Knibbe, Mulder, Oosterlee, Vuik
Geophysics 2013

Figures: Time (38.7 min) vs. frequency (9.5min)



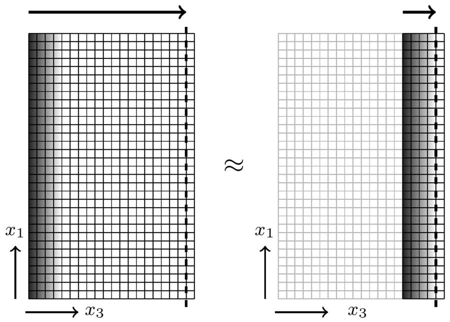
Complex Shifted Laplacian Multigrid



Erlangga 2005, Calandra et al. 2013

Complex Shifted Laplacian Multigrid

Parallel Sweeping Preconditioner



Erlangga 2005, Calandra et al. 2013

Poulson et al. 2013

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Parallel Sweeping Preconditioner

Multifrontal HSS

Erlangga 2005, Calandra et al. 2013

Poulson et al. 2013

Wang et al. 2010, 2011, 2012

Complex Shifted Laplacian Multigrid

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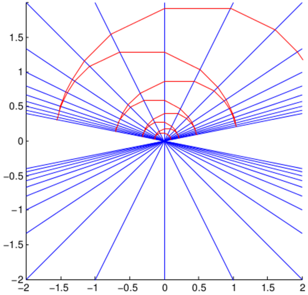
Kaczmarz Sweeps



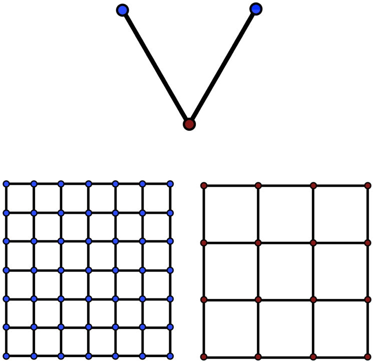
Erlangga 2005, Calandra et al. 2013

Poulson et al. 2013

Wang et al. 2010, 2011, 2012

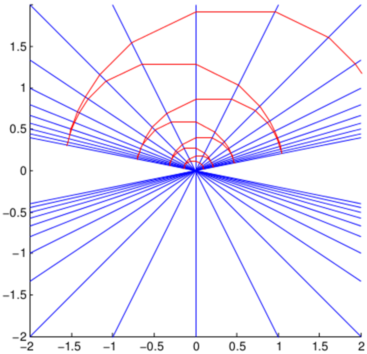
Gordon and Gordon 2012, Lago et al. 2014

Complex Shifted Laplacian Multigrid



VS

Kaczmarz Sweeps



Complex Shifted Laplacian Multigrid

- $\mathcal{T}_{2V}$ for FGMRES

[Calandra et al. 2012]

Kaczmarz Sweeps

- CGMN and CRMN*

[Lago et al. 2014]

* *parallel version: CARP-CG and CARP-CR*

Complex Shifted Laplacian Multigrid

- $\mathcal{T}_{2V}$ for FGMRES
- iterative

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Complex Shifted Laplacian Multigrid

- $\mathcal{T}_{2V}$ for FGMRES
- iterative
- controllable memory use†

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† *more detail on memory use later*

Complex Shifted Laplacian Multigrid

- $\mathcal{T}_{2V}$ for FGMRES
- iterative
- controllable memory use†
- controllable accuracy

[Calandra et al. 2012]

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Complex Shifted Laplacian Multigrid

- $\mathcal{T}_{2V}$ for FGMRES
- iterative
- controllable memory use†
- controllable accuracy
- few (expensive) iterations

[Calandra et al. 2012]

Kaczmarz Sweeps

- CGMN and CRMN*
- iterative
- controllable memory use†
- controllable accuracy
- many (cheap) iterations

[Lago et al. 2014]

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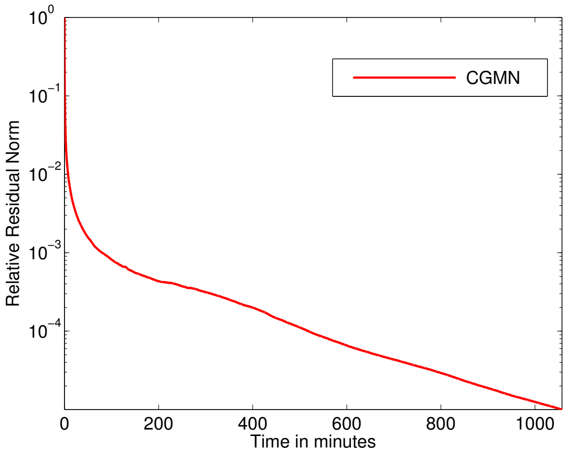
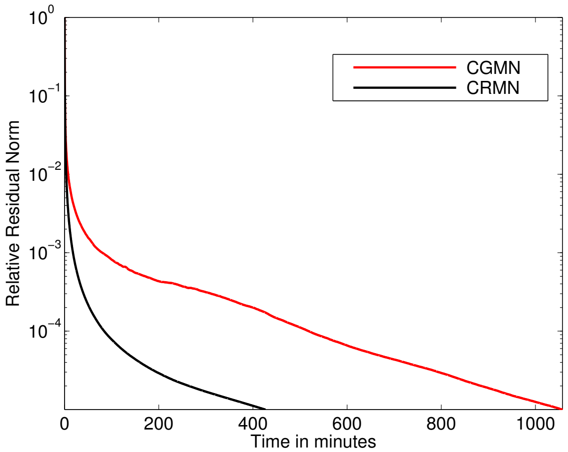
† *more detail on memory use later*

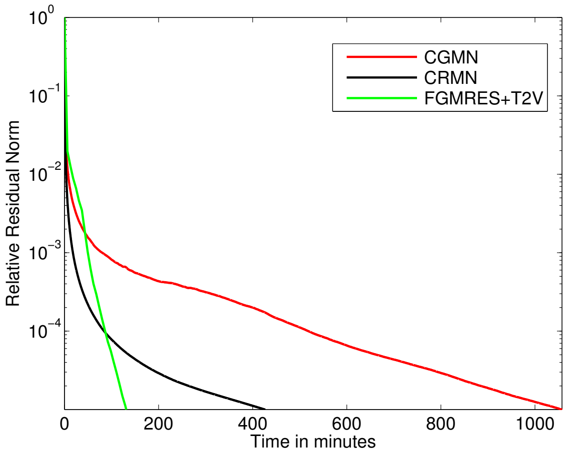
The Comparison



The Comparison



The Comparison



Why so fast?

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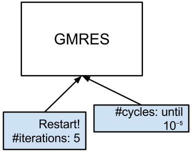
Why so fast? Calandra et al. 2012



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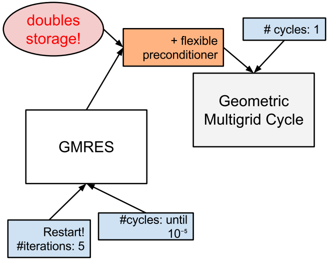

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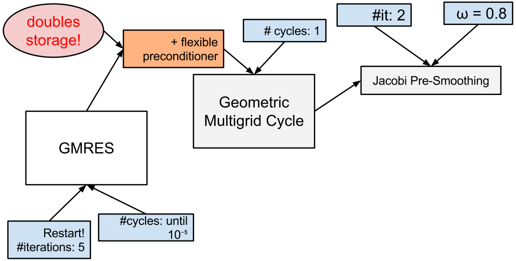

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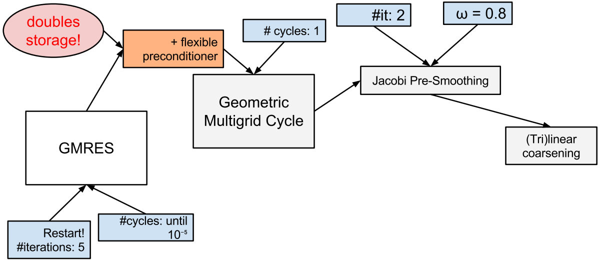

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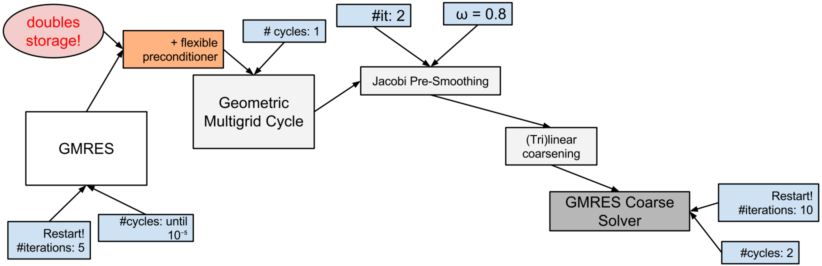

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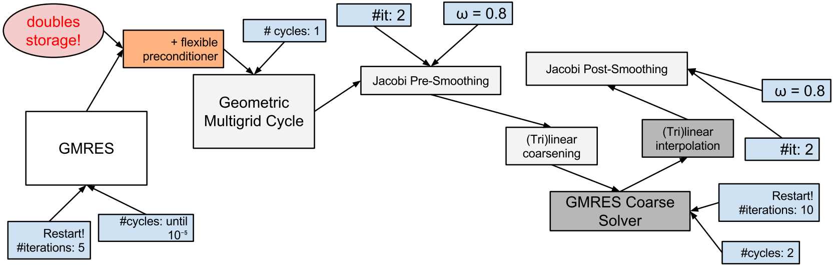

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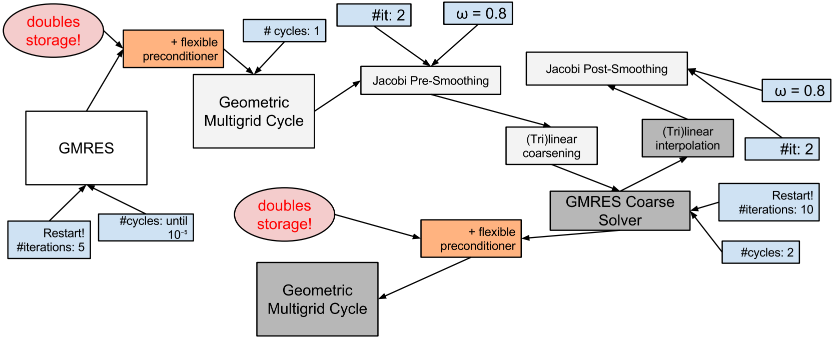

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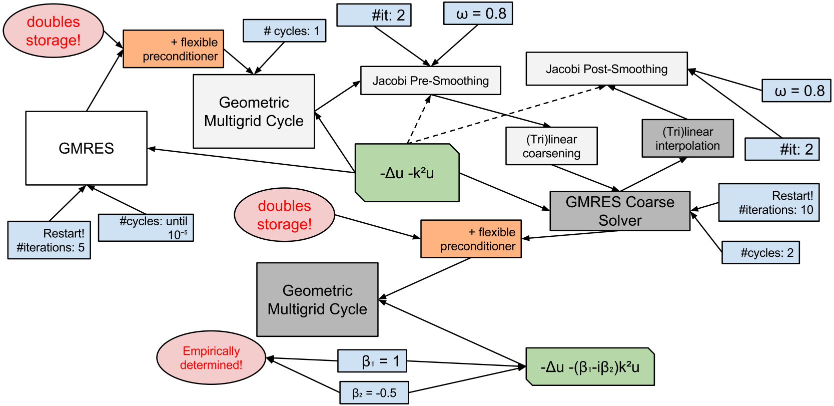

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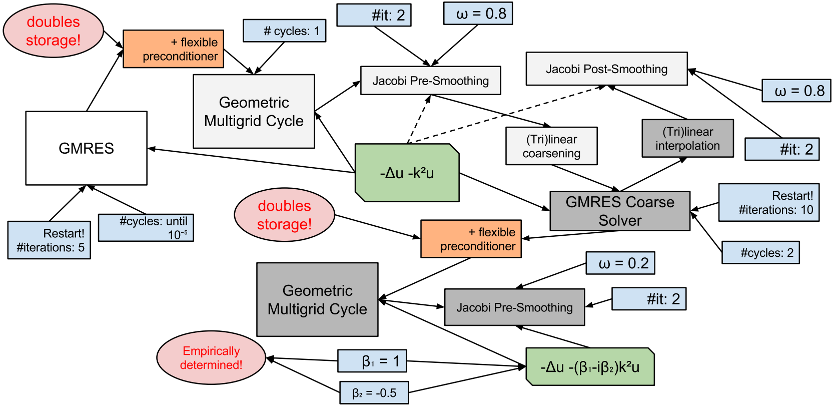
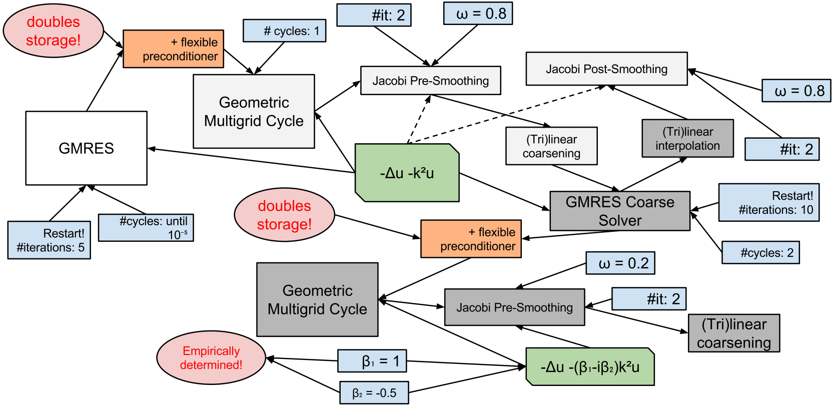
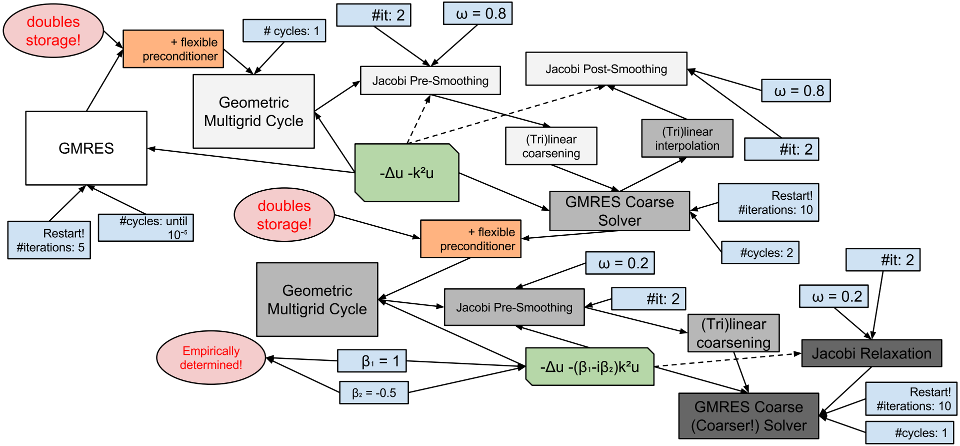



doubles storage!
+ flexible preconditioner
# cycles: 1
#it: 2
ω = 0.8
Jacobi Pre-Smoothing
Jacobi Post-Smoothing
ω = 0.8
Geometric Multigrid Cycle
GMRES
-Δu -k²u
(Tri)linear coarsening
(Tri)linear interpolation
#it: 2
GMRES Coarse Solver
Restart! #iterations: 10
Restart! #iterations: 5
#cycles: until 10⁻⁵
doubles storage!
+ flexible preconditioner
#cycles: 2
ω = 0.2
Geometric Multigrid Cycle
Jacobi Pre-Smoothing
#it: 2
(Tri)linear coarsening
Empirically determined!
β₁ = 1
β₂ = -0.5
-Δu -(β₁-iβ₂)k²u

Why so fast?

Calandra et al. 2012



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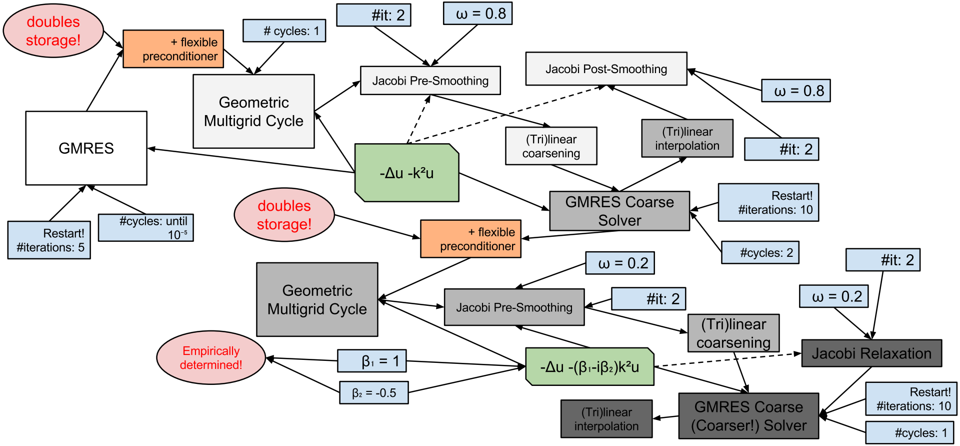

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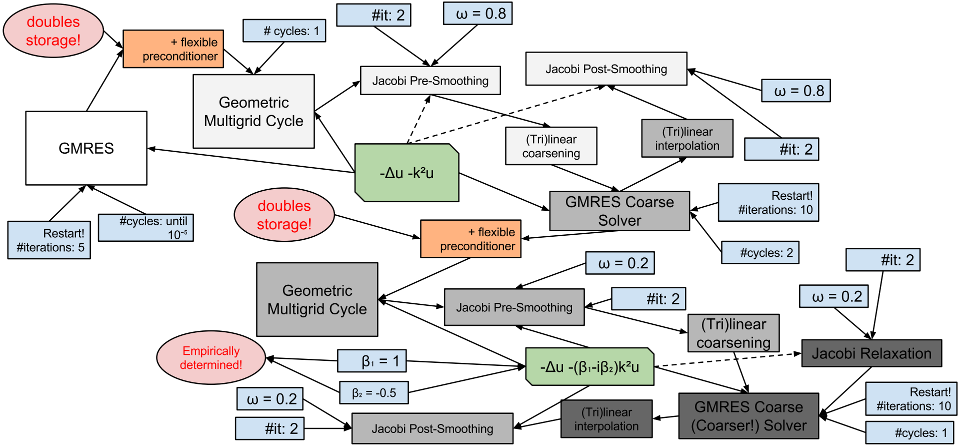
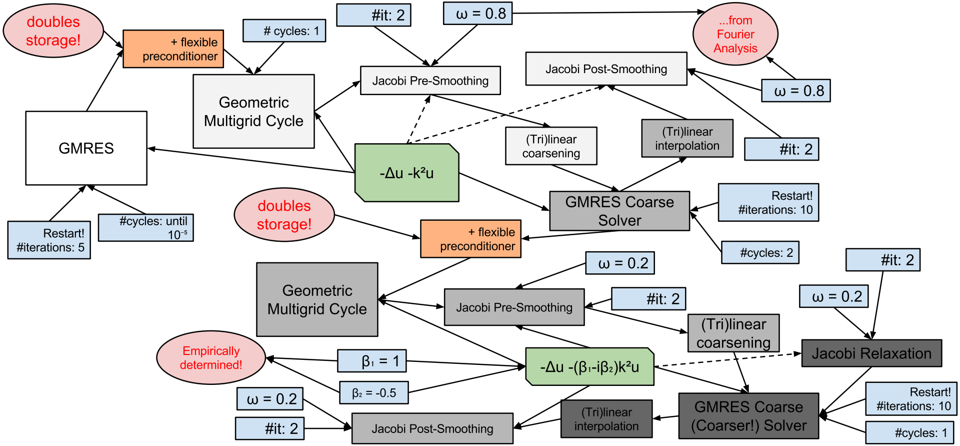
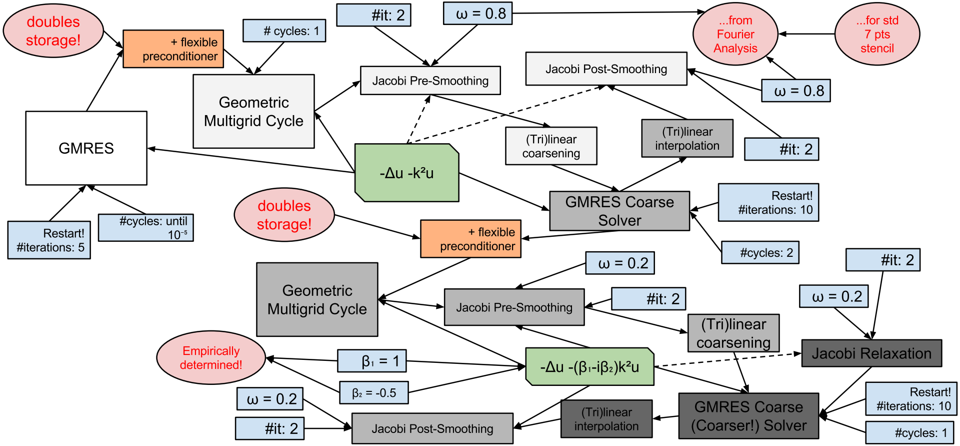

doubles storage!
+ flexible preconditioner
# cycles: 1
#it: 2
ω = 0.8
Jacobi Pre-Smoothing
Jacobi Post-Smoothing
ω = 0.8
Geometric Multigrid Cycle
GMRES
(Tri)linear coarsening
(Tri)linear interpolation
#it: 2
-Δu -k²u
GMRES Coarse Solver
Restart! #iterations: 10
Restart! #iterations: 5
#cycles: until 10⁻⁵
doubles storage!
+ flexible preconditioner
#cycles: 2
ω = 0.2
#it: 2
Geometric Multigrid Cycle
Jacobi Pre-Smoothing
#it: 2
ω = 0.2
(Tri)linear coarsening
Jacobi Relaxation
Empirically determined!
β₁ = 1
-Δu -(β₁-iβ₂)k²u
ω = 0.2
β₂ = -0.5
Restart! #iterations: 10
GMRES Coarse (Coarser!) Solver
(Tri)linear interpolation
#it: 2
Jacobi Post-Smoothing
#cycles: 1

Why so fast?


doubles storage!
+ flexible preconditioner
# cycles: 1
#it: 2
ω = 0.8
...from Fourier Analysis
Jacobi Pre-Smoothing
Jacobi Post-Smoothing
ω = 0.8
Geometric Multigrid Cycle
GMRES
(Tri)linear coarsening
(Tri)linear interpolation
#it: 2
-Δu -k²u
GMRES Coarse Solver
Restart! #iterations: 10
doubles storage!
Restart! #iterations: 5
#cycles: until 10⁻⁵
+ flexible preconditioner
#cycles: 2
ω = 0.2
#it: 2
Geometric Multigrid Cycle
Jacobi Pre-Smoothing
#it: 2
ω = 0.2
(Tri)linear coarsening
Jacobi Relaxation
Empirically determined!
β₁ = 1
-Δu -(β₁-iβ₂)k²u
β₂ = -0.5
ω = 0.2
#it: 2
Jacobi Post-Smoothing
(Tri)linear interpolation
GMRES Coarse (Coarser!) Solver
Restart! #iterations: 10
#cycles: 1

Why so fast?

Calandra et al. 2012



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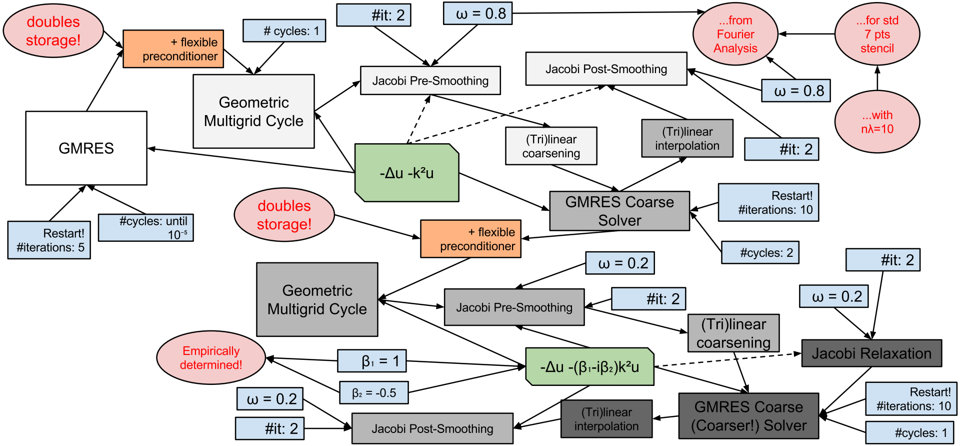

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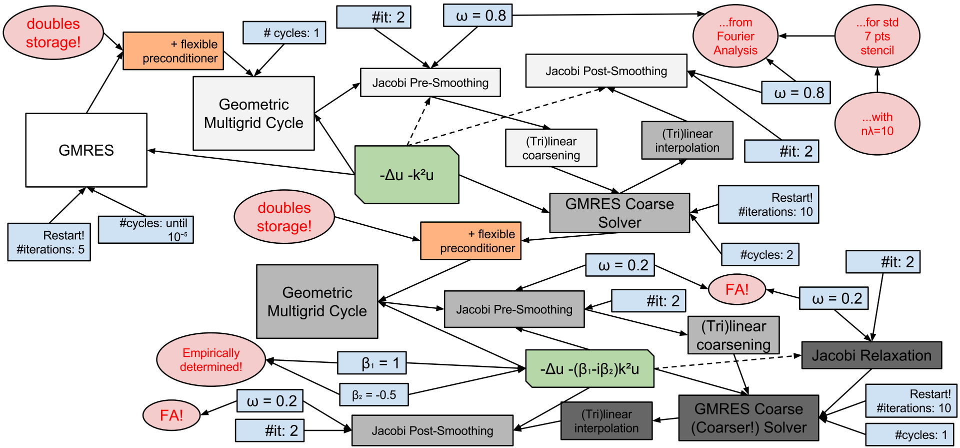

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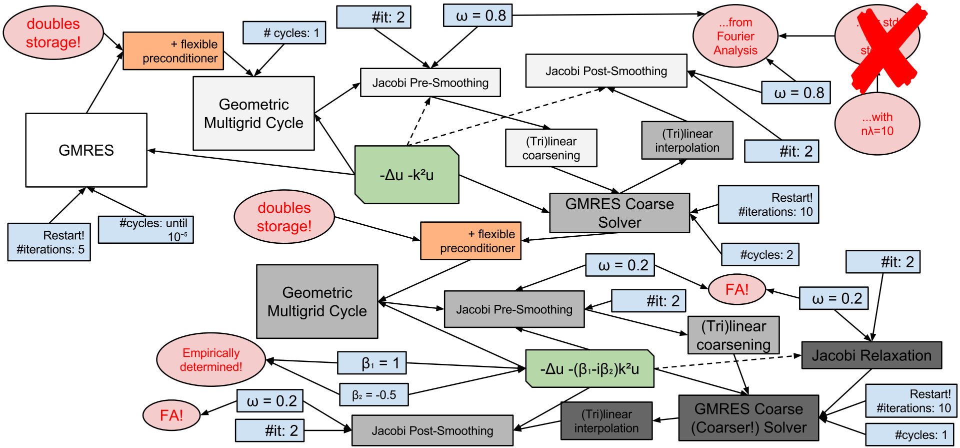
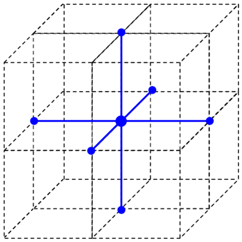
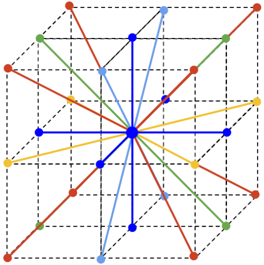


It's Just Rotated



- Stable for $n_\lambda = 10$ or more.
- 1.08×10^8 vectors $(1,661.3MB$ each)

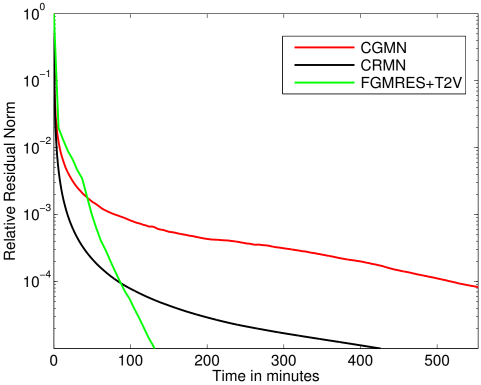


- Stable for $n_\lambda = 4$ or more.
  We choose $n_\lambda = 6$.
- 2.36×10^7 vectors $(360MB$ each)

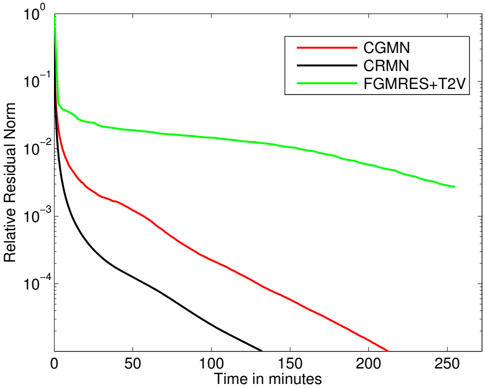
† *Stats are given for the SEG/EAGE Overthrust model discretized at the $8Hz$ for the chosen n_λ*

Still Fair Play

7-points



27-points



Still Fair Play

- CRMN with 27 points is overall **faster**

Still Fair Play

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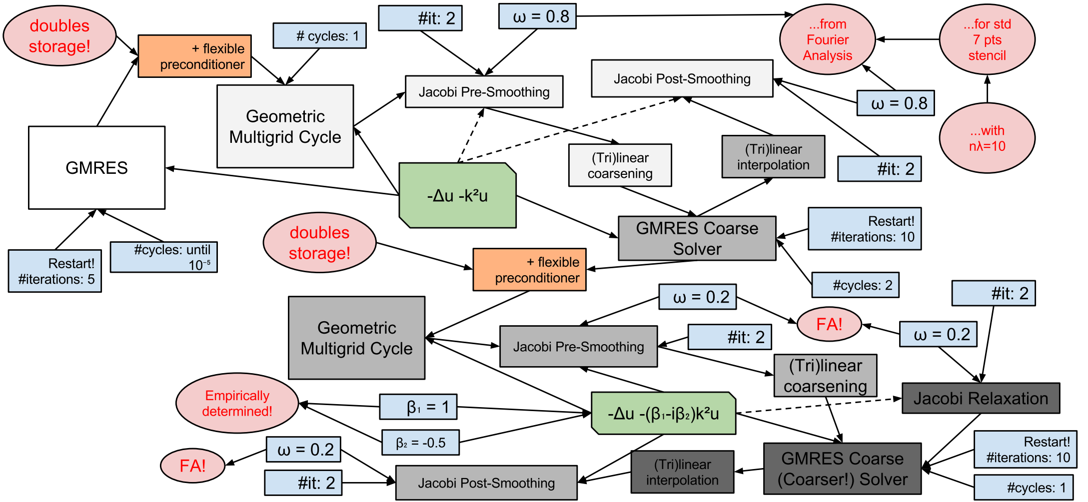
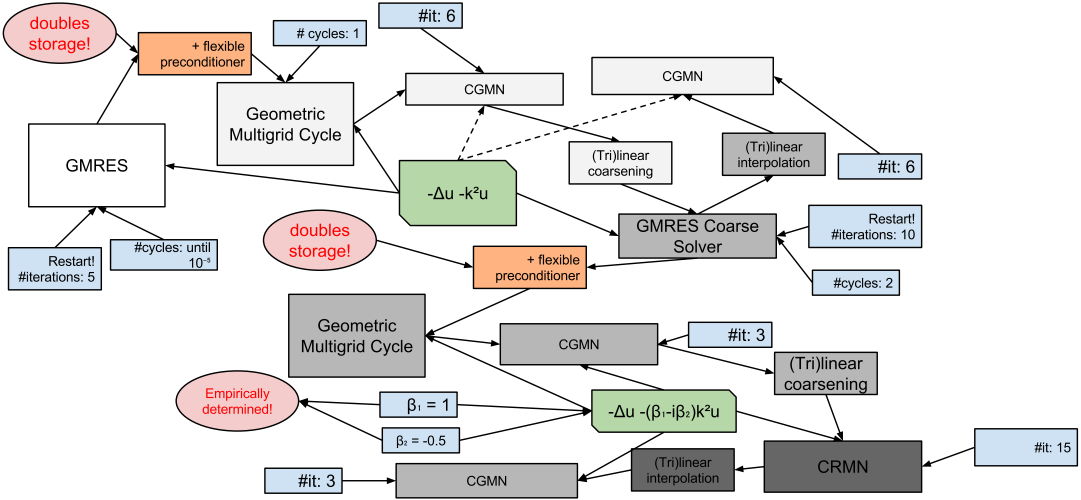
- ...but FGMRES + $\mathcal{T}_{2V}$ on 7 points is still **competitive**!

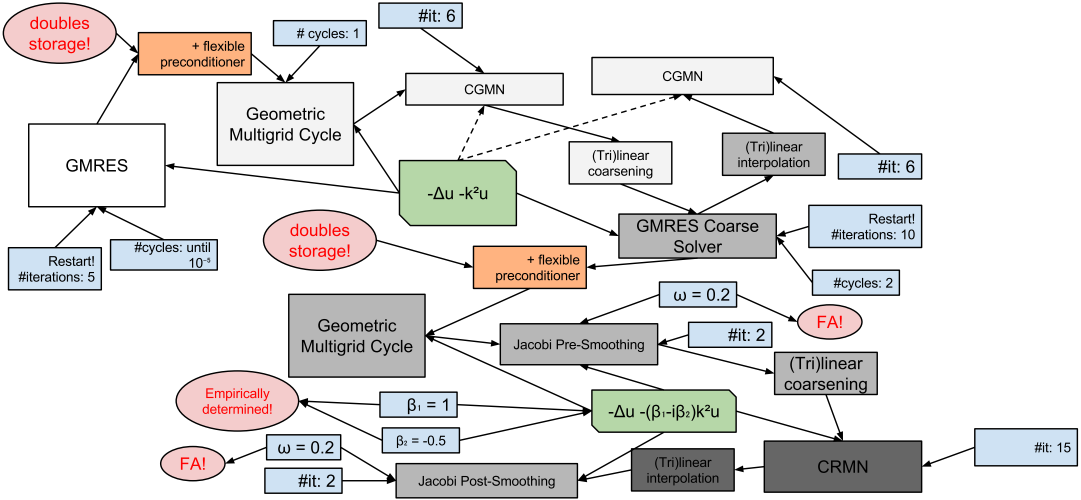
Still Fair Play

- CRMN with 27 points is overall **faster**
- ...but FGMRES + $\mathcal{T}_{2V}$ on 7 points is still **competitive**!
- ...can we try to **combine the best of both worlds**?!

The Original $\mathcal{T}_{2V}$




doubles storage!
+ flexible preconditioner
# cycles: 1
#it: 6
CGMN
CGMN
Geometric Multigrid Cycle
GMRES
(Tri)linear coarsening
(Tri)linear interpolation
#it: 6
-Δu -k²u
GMRES Coarse Solver
Restart! #iterations: 10
doubles storage!
Restart! #iterations: 5
#cycles: until 10⁻⁵
+ flexible preconditioner
#cycles: 2
Geometric Multigrid Cycle
CGMN
#it: 3
(Tri)linear coarsening
Empirically determined!
β₁ = 1
-Δu -(β₁-iβ₂)k²u
β₂ = -0.5
#it: 15
(Tri)linear interpolation
CRMN
#it: 3
CGMN


doubles storage!
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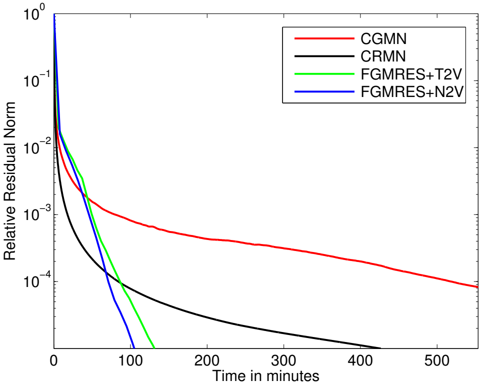
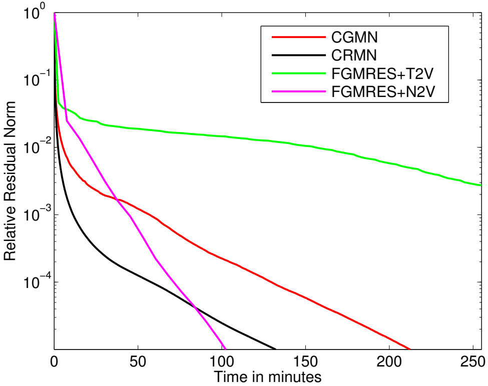
# cycles: 1
#it: 6
CGMN
CGMN
Geometric Multigrid Cycle
GMRES
-Δu -k²u
(Tri)linear coarsening
(Tri)linear interpolation
#it: 6
GMRES Coarse Solver
Restart! #iterations: 10
#cycles: 2
Restart! #iterations: 5
#cycles: until 10⁻⁵
doubles storage!
+ flexible preconditioner
Geometric Multigrid Cycle
ω = 0.2
FA!
Jacobi Pre-Smoothing
#it: 2
(Tri)linear coarsening
Empirically determined!
β₁ = 1
-Δu -(β₁-iβ₂)k²u
β₂ = -0.5
ω = 0.2
FA!
#it: 2
Jacobi Post-Smoothing
(Tri)linear interpolation
CRMN
#it: 15

Another Comparison

7-points

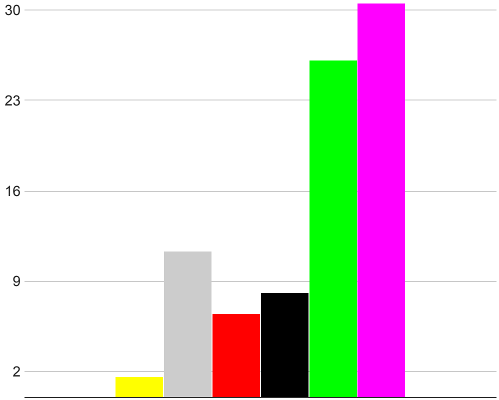


27-points

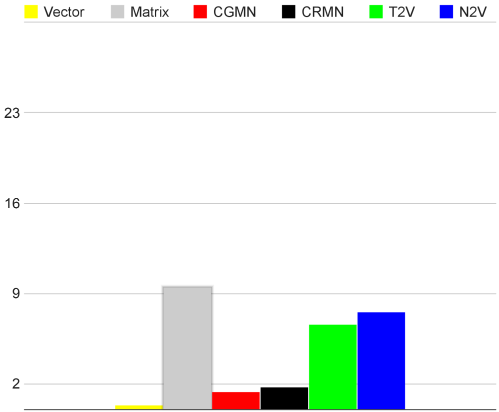


A Word on Memory Costs

7-points



27-points



Rankings

	Stencil	Time(min)	Ratio
■ $\mathcal{N}_{2V}$	27pts	113.5	100%
■ $\mathcal{N}_{2V}$	7pts	118.6	105%
■ CRMN	27pts	132.4	117%
■ $\mathcal{T}_{2V}$	7pts	141.55	125%

	Stencil	Mem(GB)	Ratio
■ CRMN	27pts	1.8	100%
■ $\mathcal{N}_{2V}$	27pts	7.6	422%
■ $\mathcal{T}_{2V}$	7pts	26.0	1444%
■ $\mathcal{N}_{2V}$	7pts	30.5	1694%

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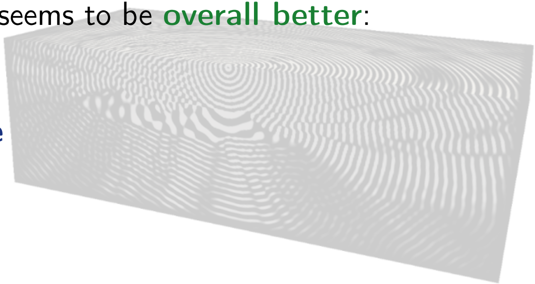
$\mathcal{N}_{2V}$	7pts	30.5	1694%

Conclusions

For Helmholz forward modelling

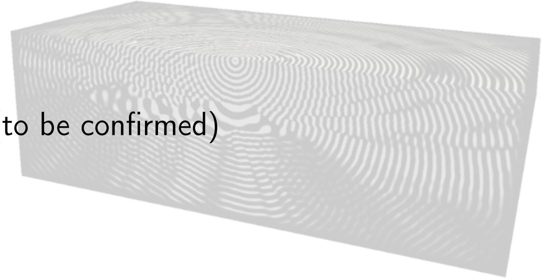
- Multigrid is very powerful - when parameters are properly chosen!
- ...but **CRMN** on 27 points stencil seems to be overall better:
  - as fast as $\mathcal{T}_{2V}$
  - remarkably low memory use
  - very simple implementation



Conclusions

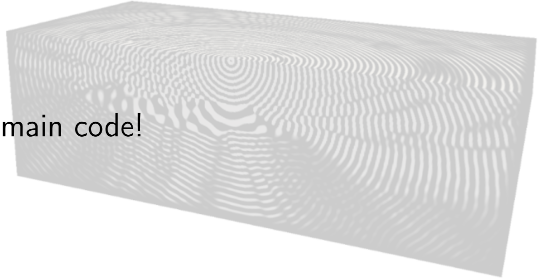
If 7 points stencil is strictly necessary:

- $\mathcal{N}_{2V}$ seems to be the best choice:
  - considerably **faster** than $\mathcal{T}_{2V}$
  - Appears to be more **robust** (to be confirmed)



Future Work

- Sotware Release and paper
- Try elastic equations
- Verify robustness of $\mathcal{N}_{2V}$
- Compare with **Xiang Li**'s time domain code!



Try Frequency Domain!

Thank you!

Calandra, H., Gratton, S., Pinel, X., and Vasseur, X. (2013).
An improved two-grid preconditioner for the solution of three-dimensional Helmholtz problems in heterogeneous media.
Numerical Linear Algebra with Applications.
to appear.

Gordon, D. and Gordon, R. (2012).
Parallel solution of high frequency Helmholtz equations using high order finite difference schemes.
Applied Mathematics and Computation, 218(21):10737–10754.

Knibbe, H., Mulder, W., Oosterlee, C. W., and Vuik, C. (2014).
Closing the performance gap between an iterative frequency-domain solver and an explicit time-domain scheme for 3D migration on parallel architectures.
Geophysics, 79:S47–S61.

Lago, R., Petrenko, A., Fang, Z., and Herrmann, F. (2014).
Fast solution of time-harmonic wave equation for full-waveform inversion.
In *EAGE 2014 Extended Abstract*.

Poulson, J., Engquist, B., Fomel, S., Li, S., and Ying, L. (2012).
A parallel sweeping preconditioner for high frequency heterogeneous 3d helmholtz equations.
CoRR, abs/1204.0111.

Wang, S., de Hoop, M. V., Xia, J., and Li, X. S. (2012).
Massively parallel structured multifrontal solver for time-harmonic elastic waves in 3-d anisotropic media.
Geophysical Journal International, 191(1):346–366.