Time-*jittered* marine acquisition
Low-rank v/s sparsity

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SLIM
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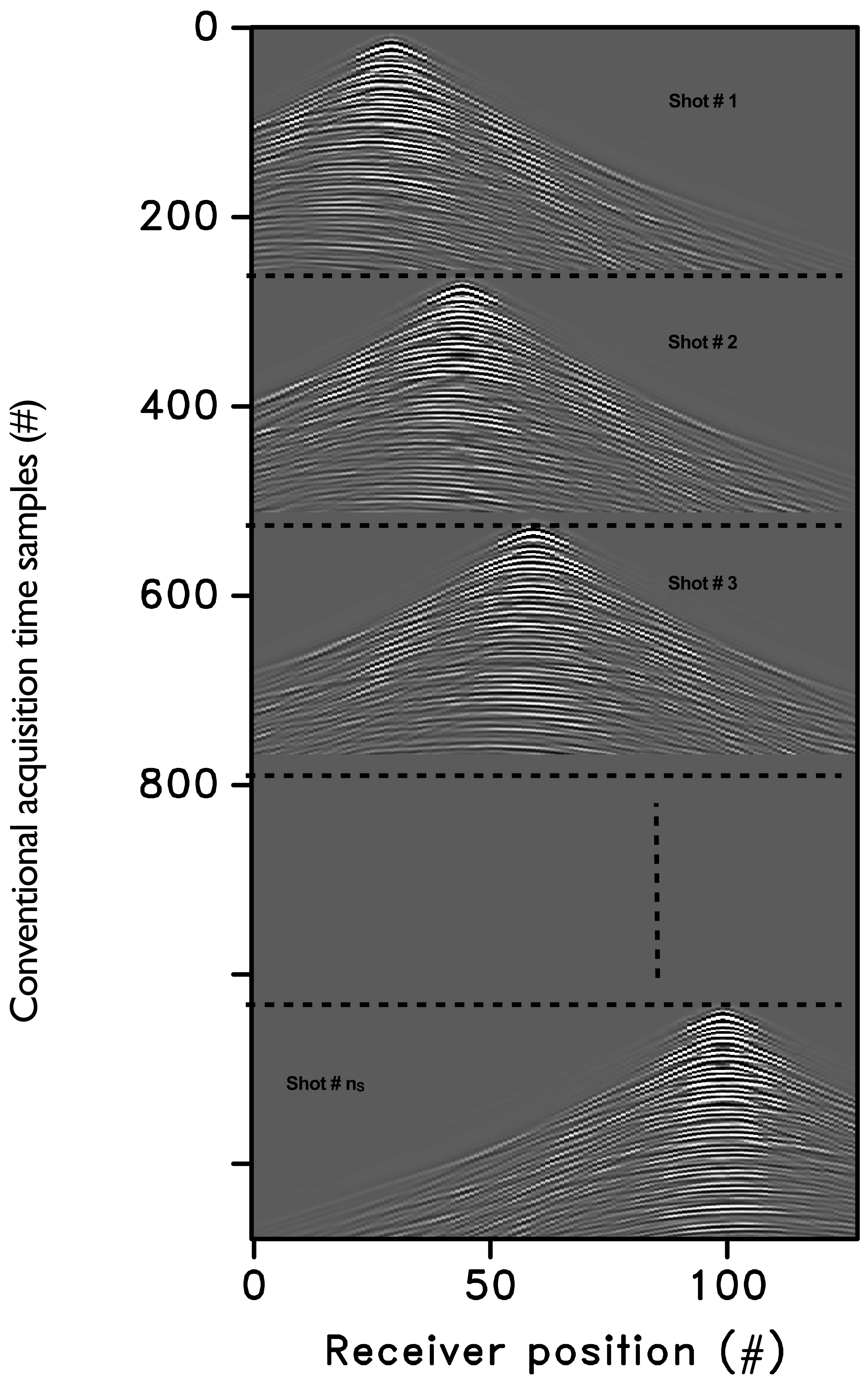
Motivation



Conventional marine acquisition

- wide-azimuth data
- expensive
- subsampled source/receivers grid
- interpolation

Want more for less ...

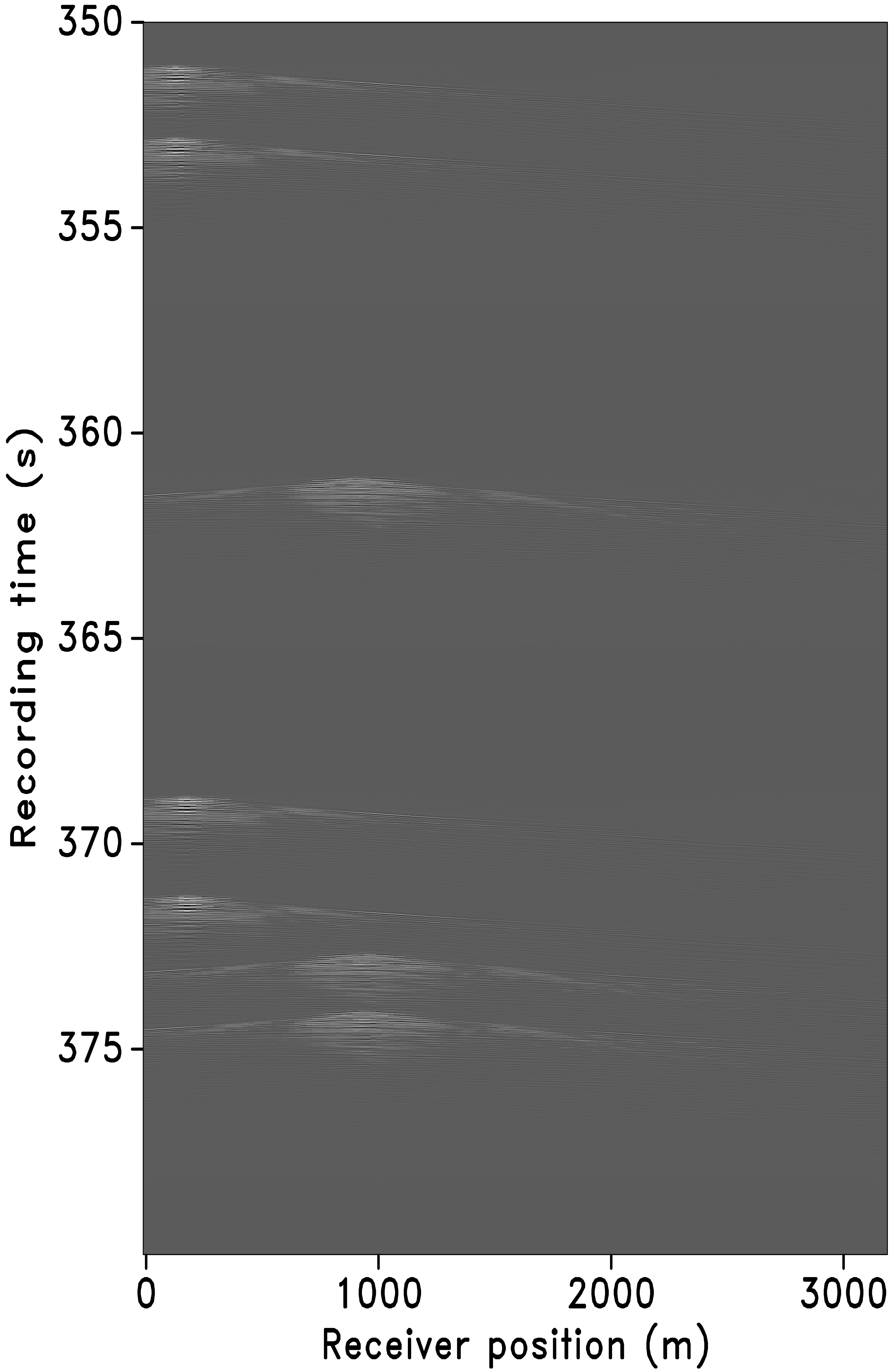
- *shorter* survey times
- *increased* spatial sampling

Motivation

Rethink marine acquisition

- sources (and receivers) at *random* locations
- as long as you know the locations afterwards... *it is fine!*

How is this possible ?



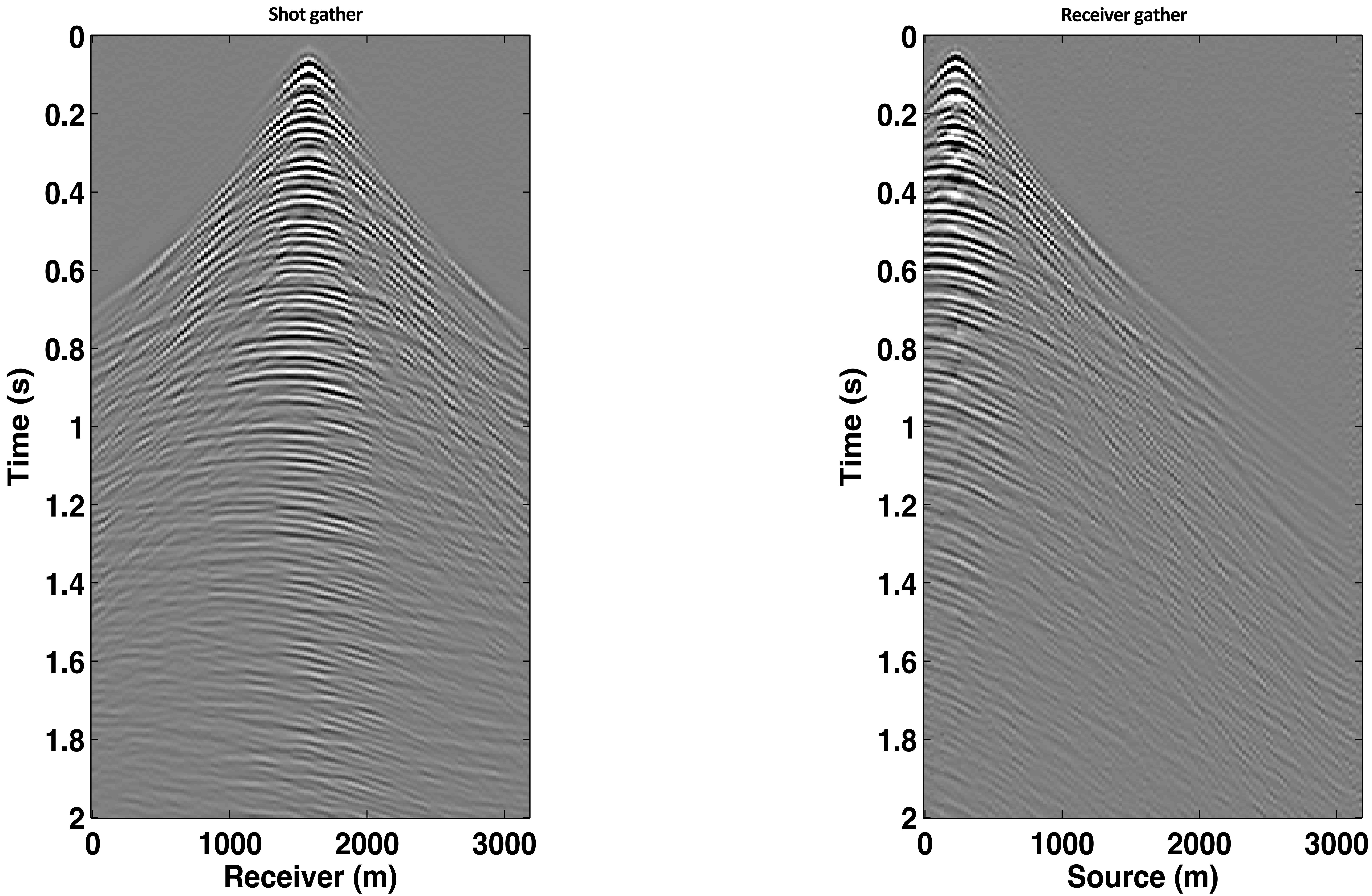
SLIM marine acquisition

- (multi-vessel, multi-airgun) acquisition w/ *jittered* sampling
- "blending" *via compressed randomized inter-shot* firing times

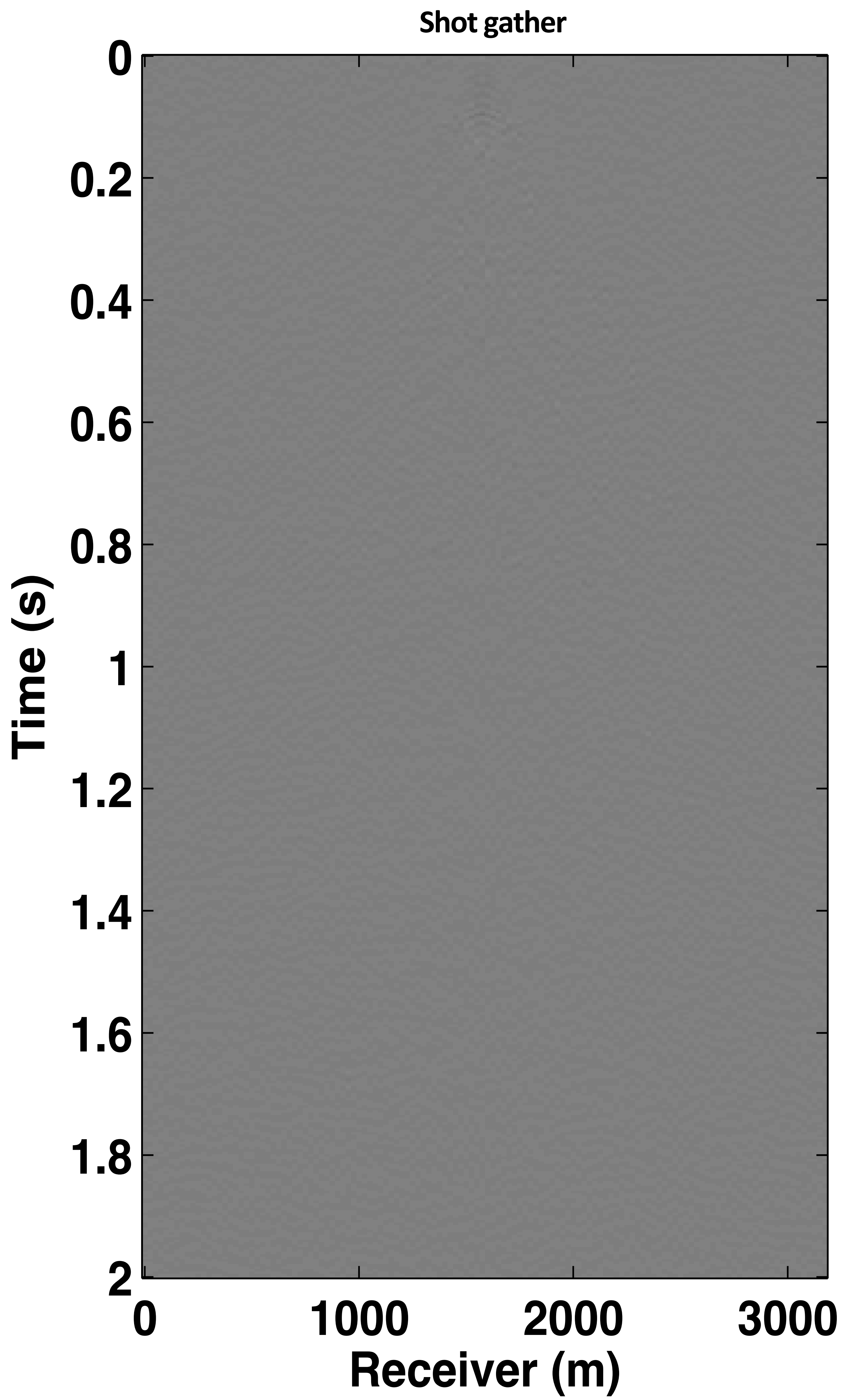
Mansour et. al., 2012 ; Wason et. al., 2013

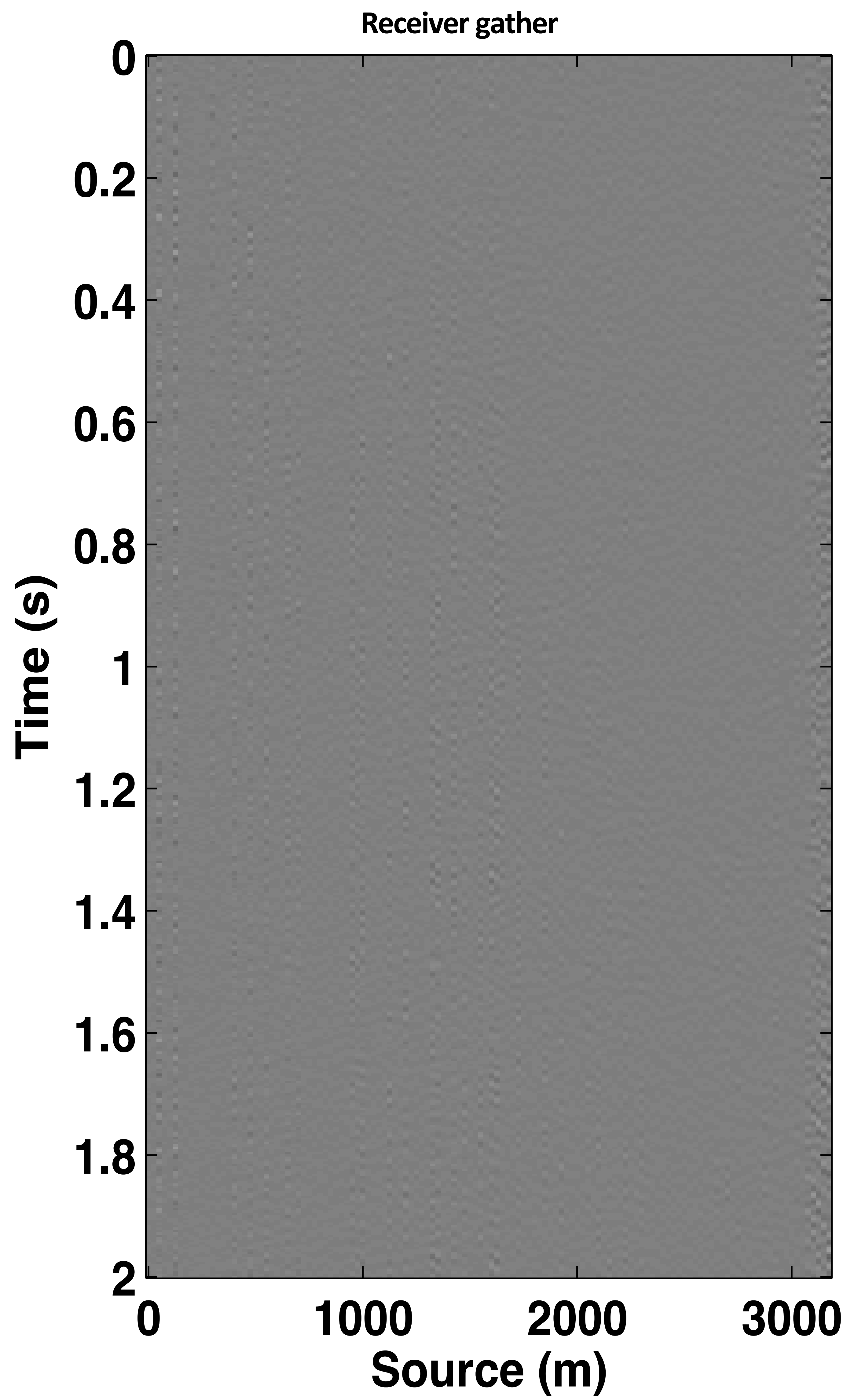
Rank minimization

source-separation, 50m grid to 25m grid,14.5 dB

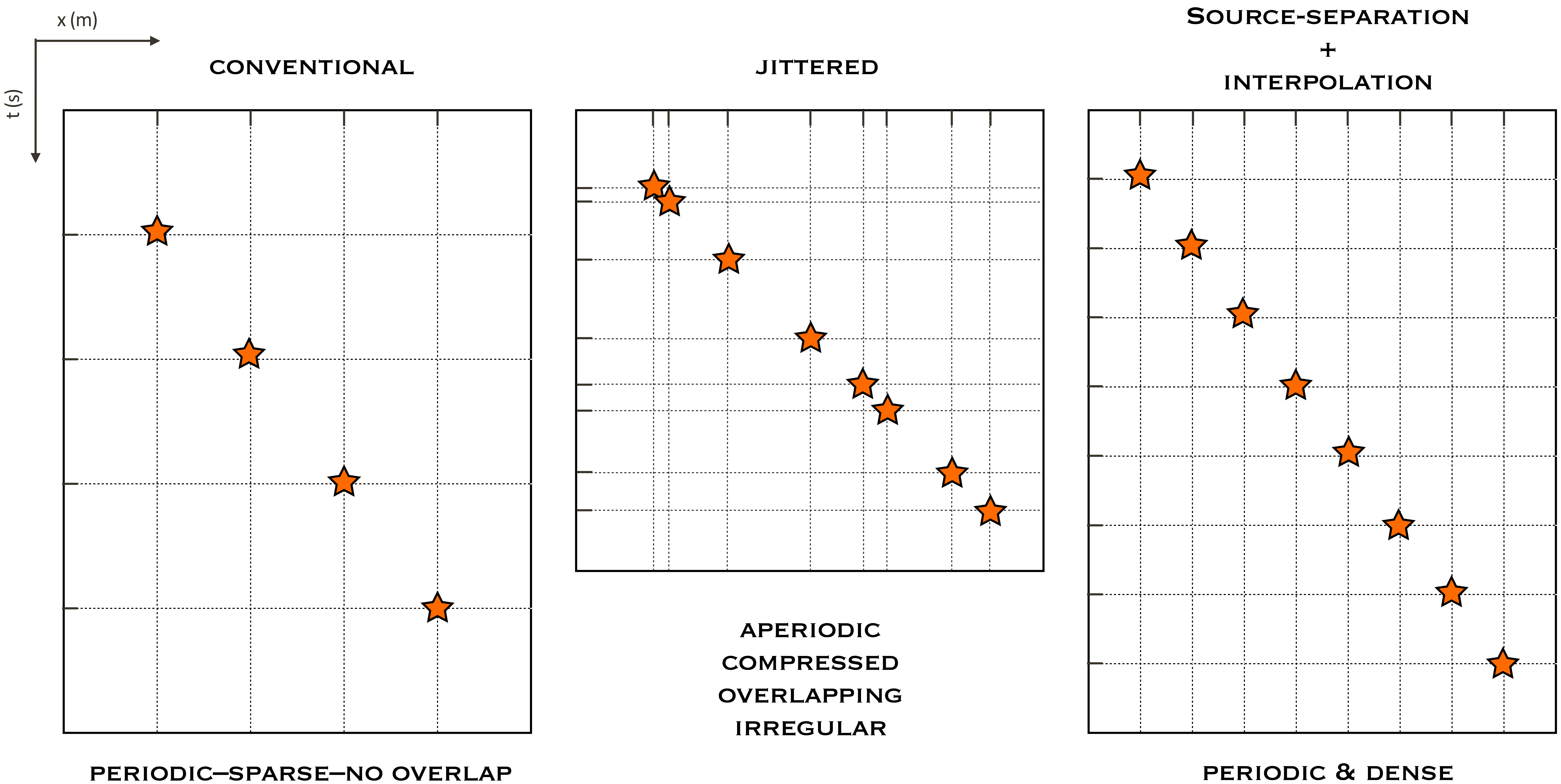


Rank minimization
difference, 50m grid to 25m grid



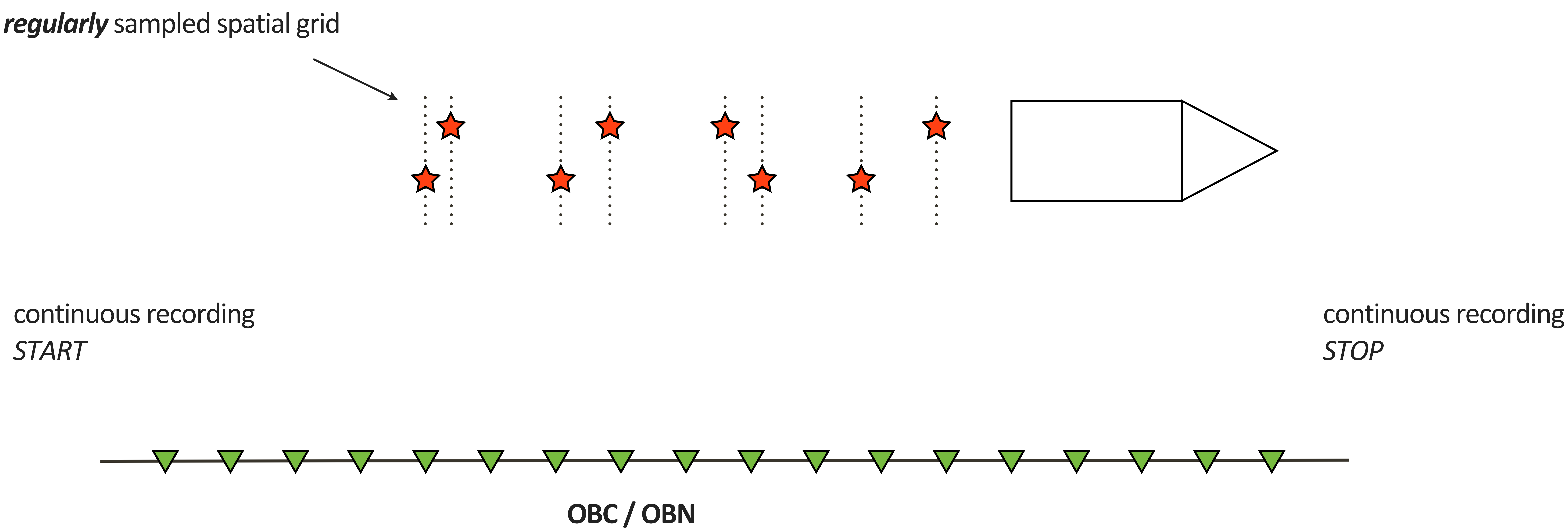


Quick recap

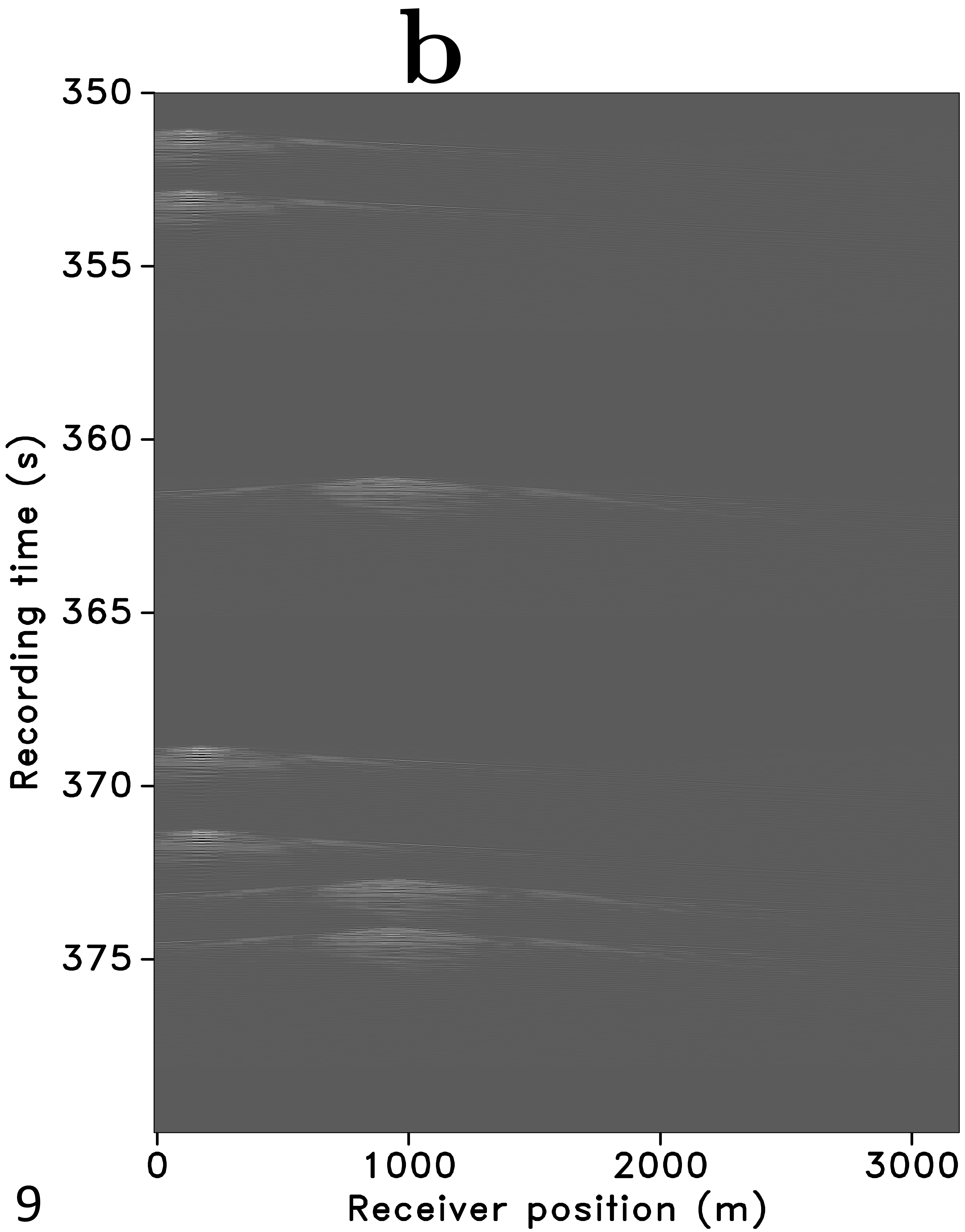


x (m)
t (s)
CONVENTIONAL
JITTERED
SOURCE-SEPARATION
+
INTERPOLATION
APERIODIC
COMPRESSED
OVERLAPPING
IRREGULAR
PERIODIC–SPARSE–NO OVERLAP
PERIODIC & DENSE

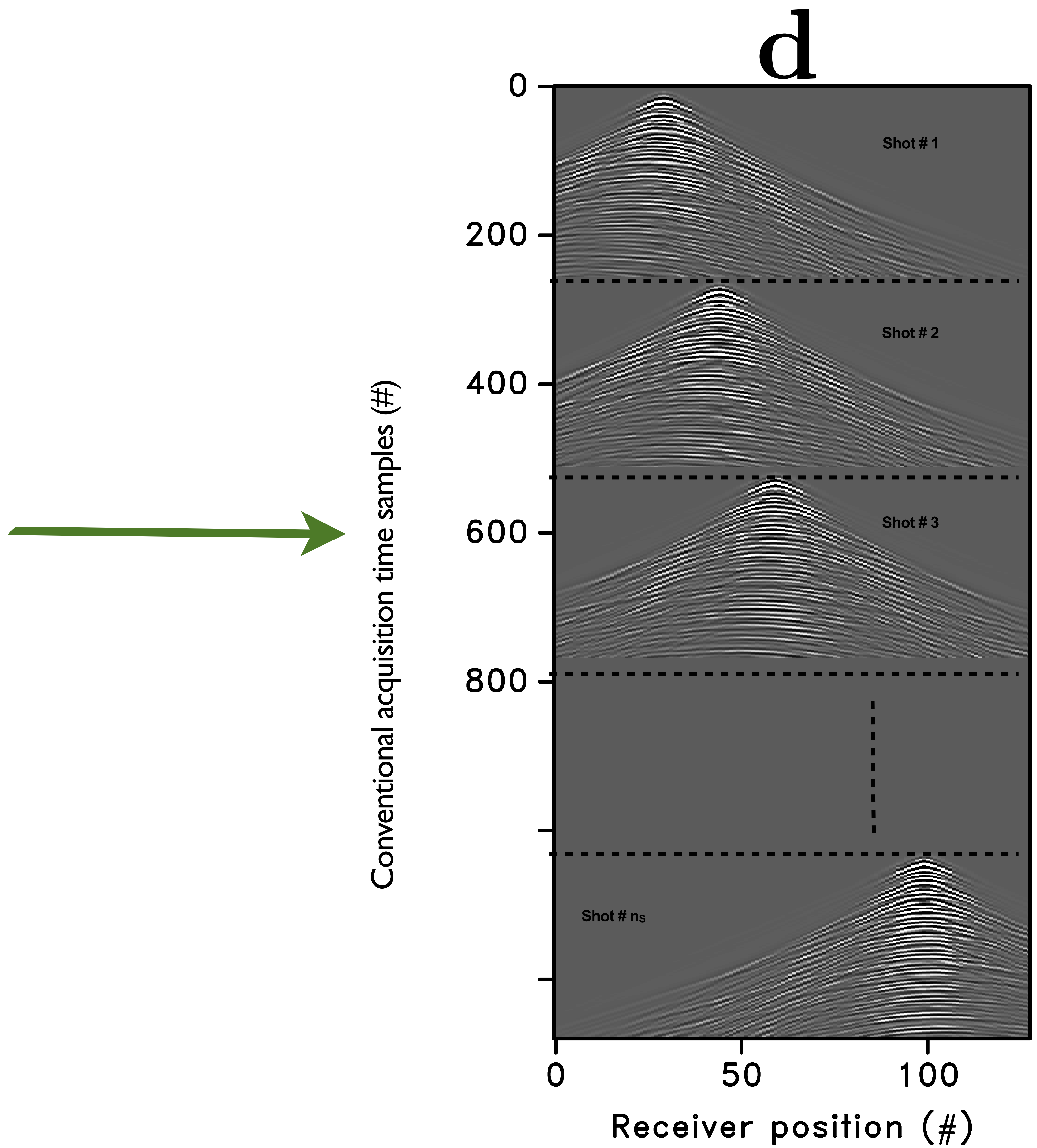
Time-jittered acquisition



Observed data



Expectations



Mansour et. al., 2012 ; Wason et. al., 2013

Source separation & Interpolation

- Signal structure
  - *Sparse/compressible*
- Sampling scheme
  - sampling make signal *less sparse* in transform domain
- Sparsity-promoting recovery using ℓ_1 constraints

Impediments (3'S)

- Speed
  - *slow* and *expensive*
- Storage
  - *redundant* transform
- Scale-up
  - challenging for *large scale* seismic data

SLIM

Matrix Completion

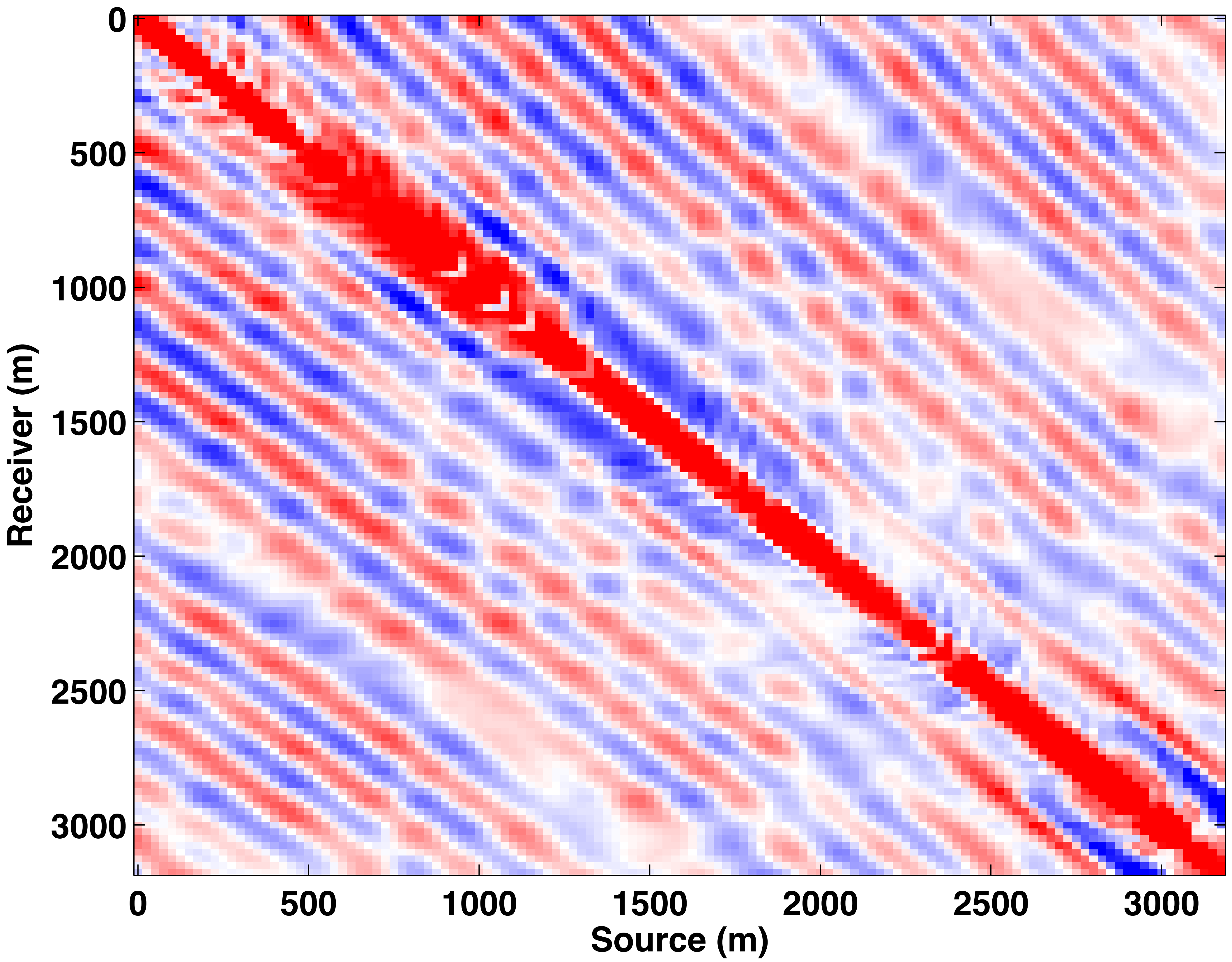
- Signal structure
  - *Low rank/fast decay* of singular values

- Sampling scheme
  - sampling data *increase* rank in a "transform domain"

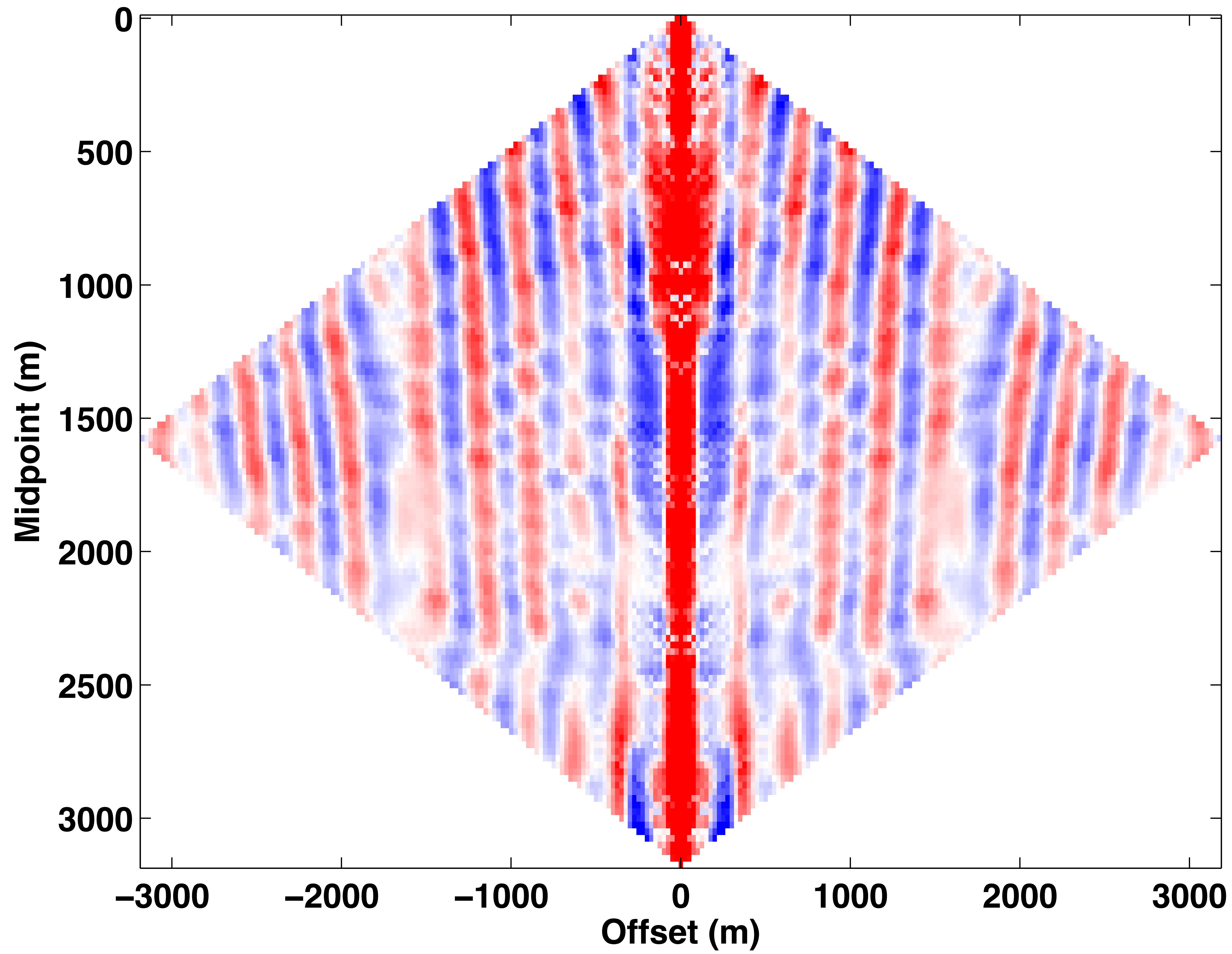
- Recovery using *rank penalization* scheme

Low-rank structure
sequential source acquisition

acquisition domain
[source-receiver]

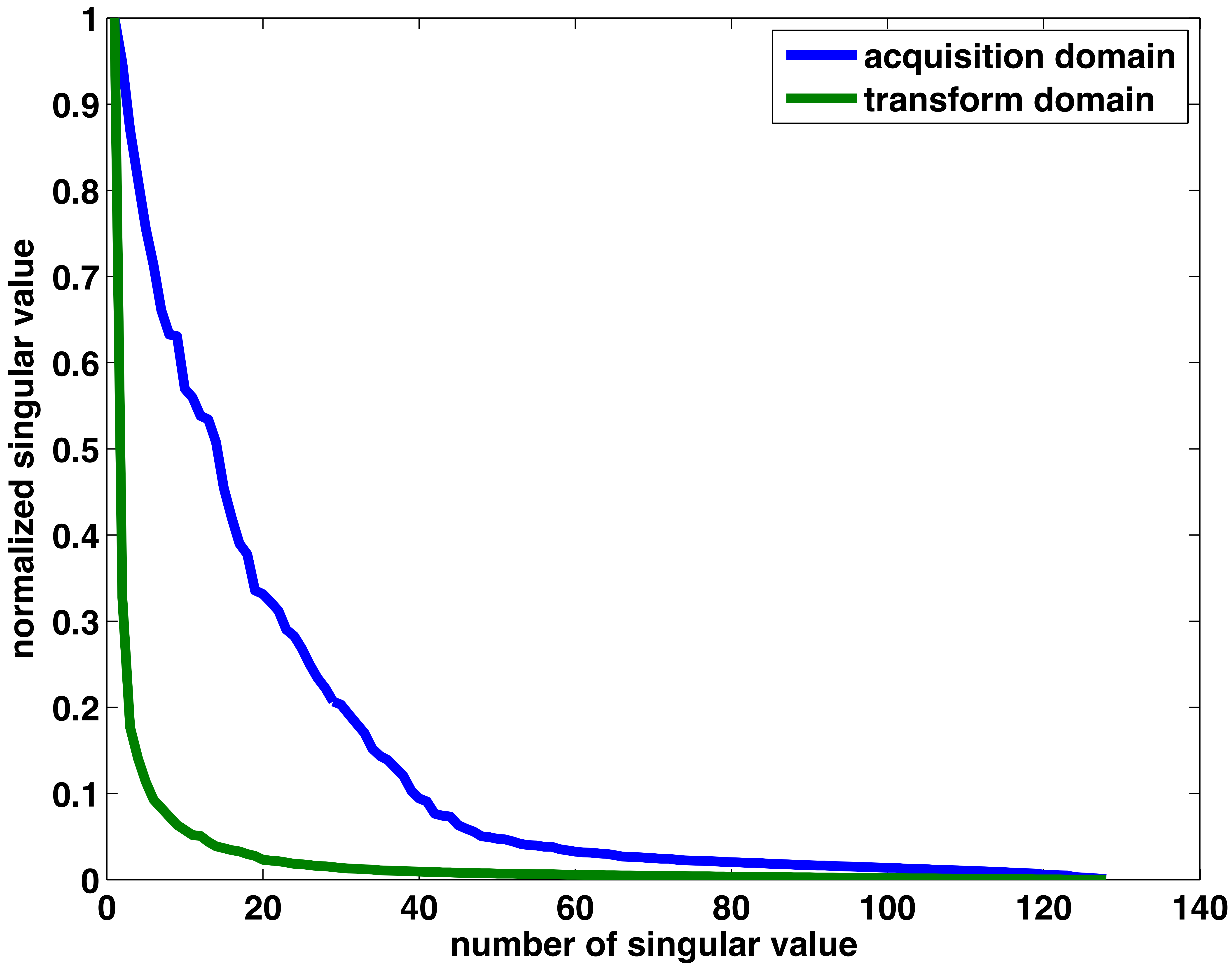


transform domain
[midpoint-offset]



Singular value decay

sequential source acquisition



Matrix Completion

- Signal structure
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- Sampling scheme
  - sampling data *increase* rank in a "transform domain"

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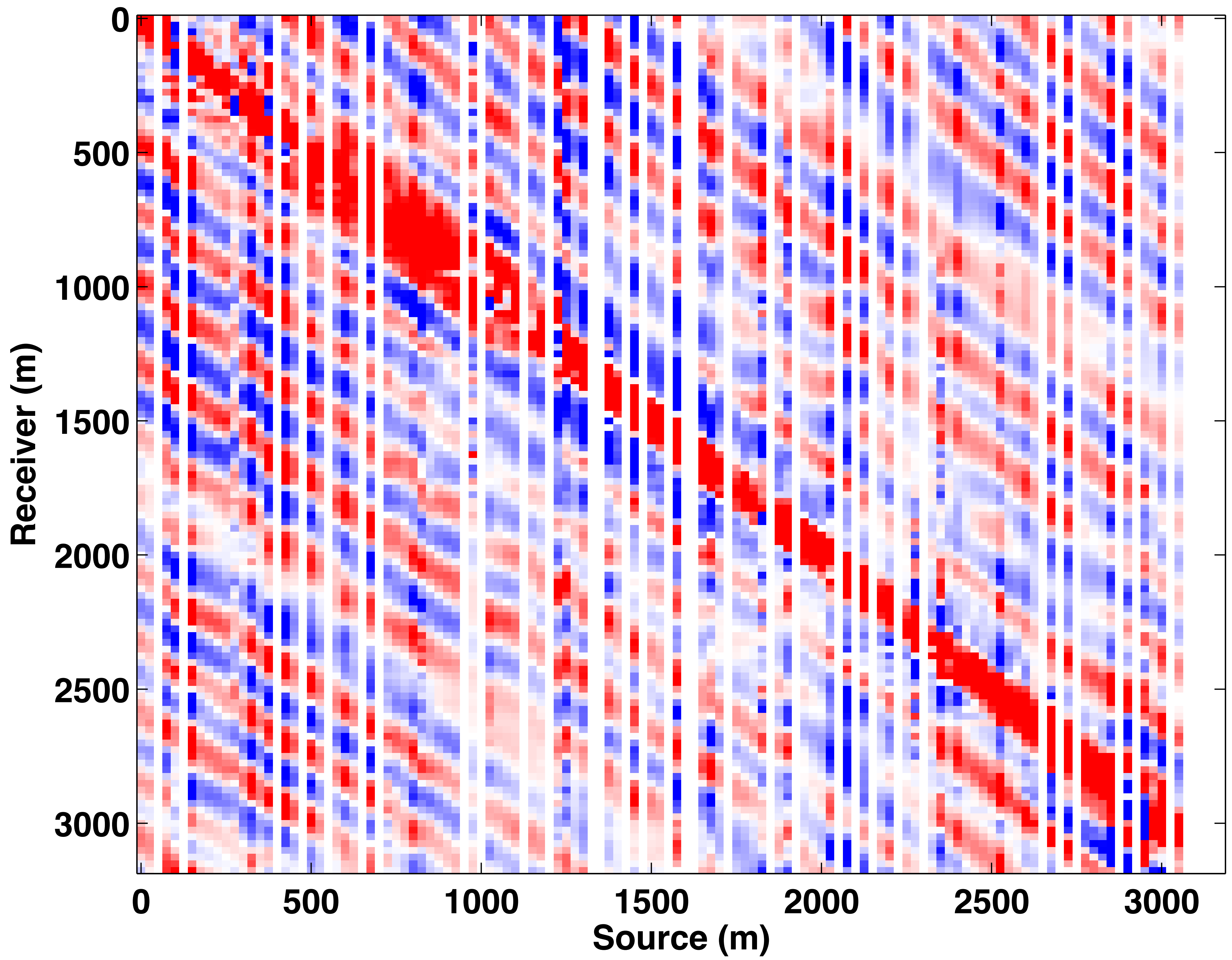
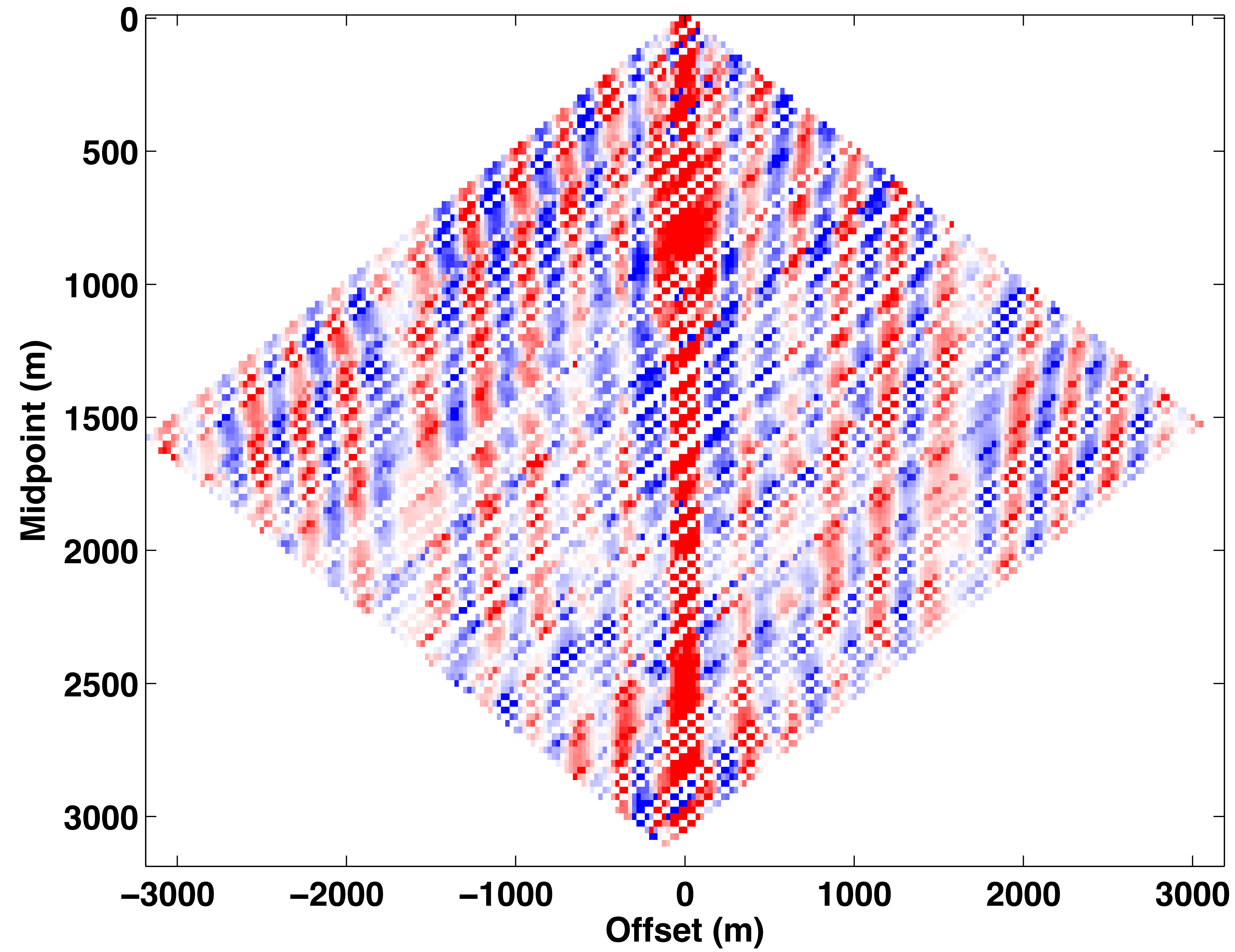
Low-rank structure
Adjoint

acquisition domain
[source-receiver]

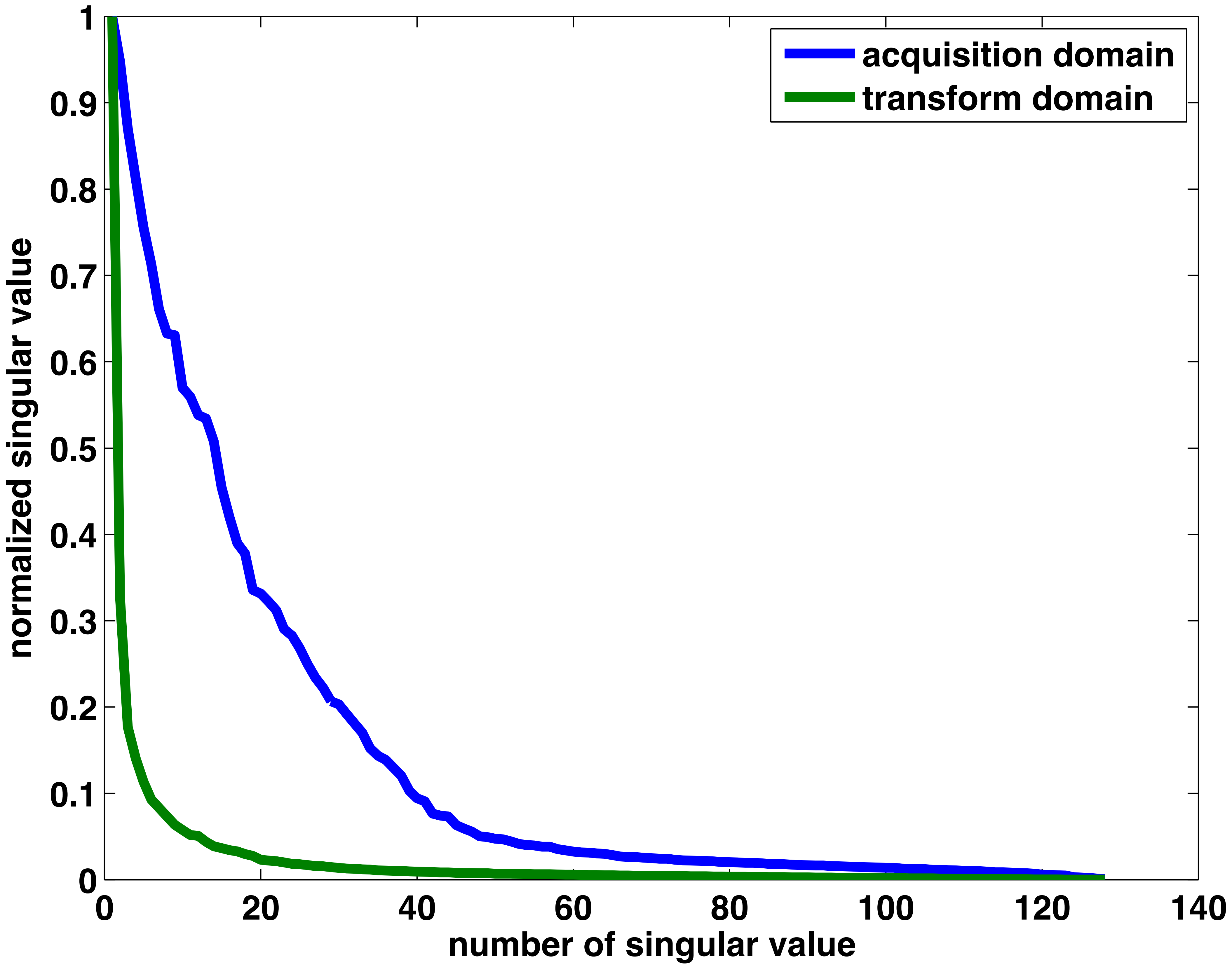


transform domain
[midpoint-offset]

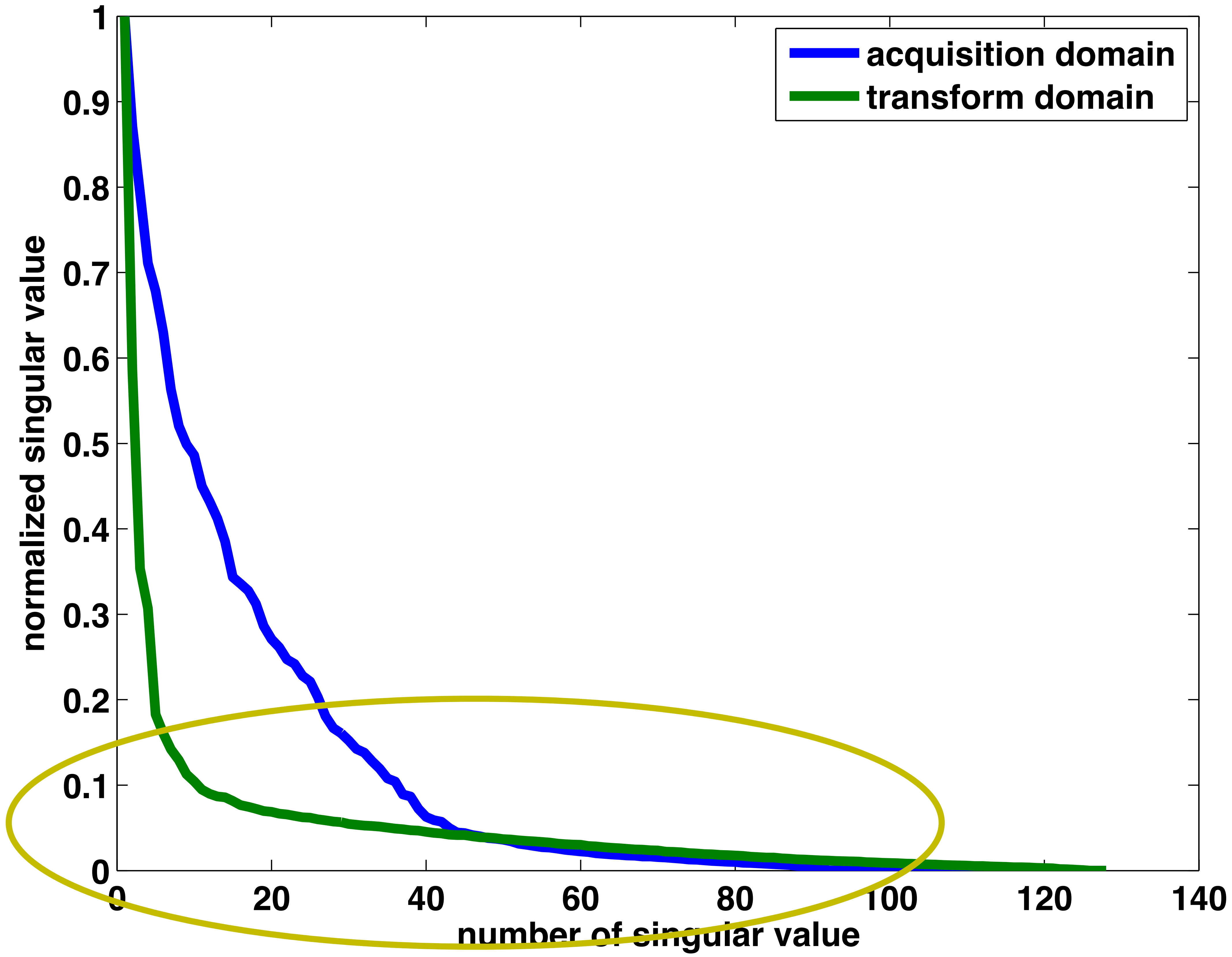


Singular value decay
Adjoint

acquisition domain
[source-receiver]



transform domain
[midpoint-offset]



Matrix Completion

- Signal structure
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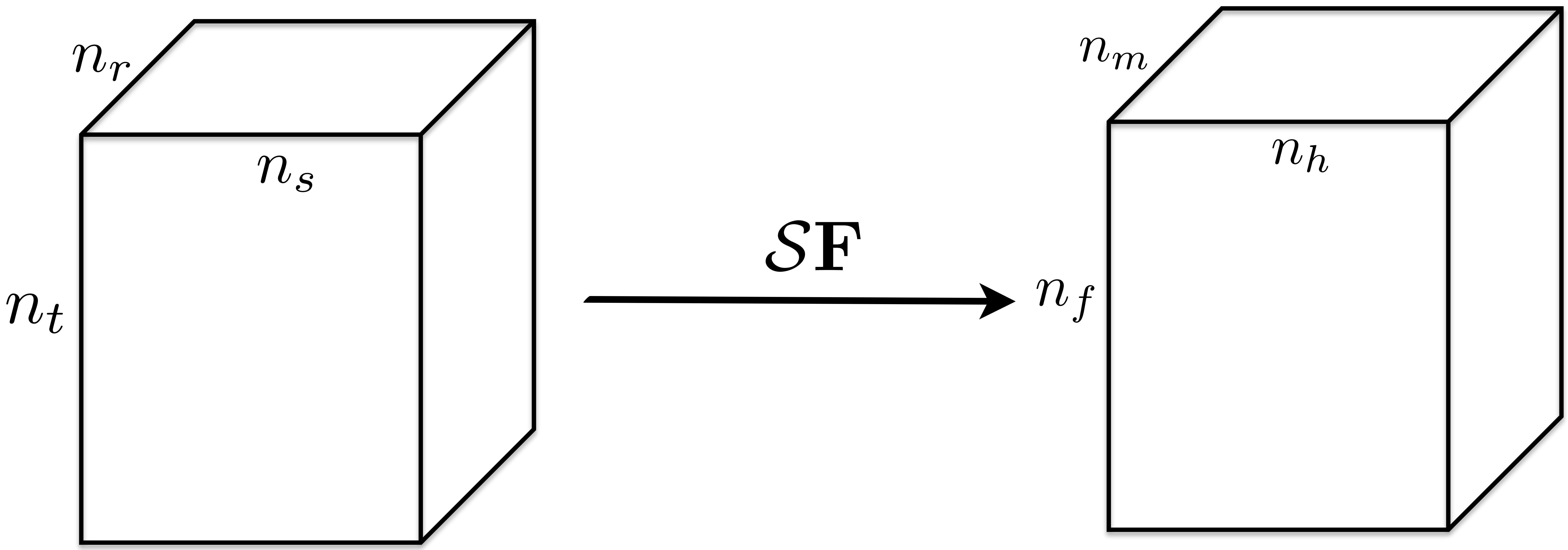
Low-rank v/s Sparsity

Unknown sparsity

Unknown low-rank



$X \in \mathbb{R}^{n_t \times n_r \times n_s}$

$X \in \mathbb{C}^{n_f \times n_m \times n_h}$

Nuclear-norm minimization

SLIM

- Given a set of measurements $\mathbf{b}$, aim is to solve

$$\min_{\mathbf{X}_f} \quad \sum_f \|\mathbf{X}_f\|_* \quad \text{s.t. } ||\mathcal{A}(\mathbf{X}_f) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}_f\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

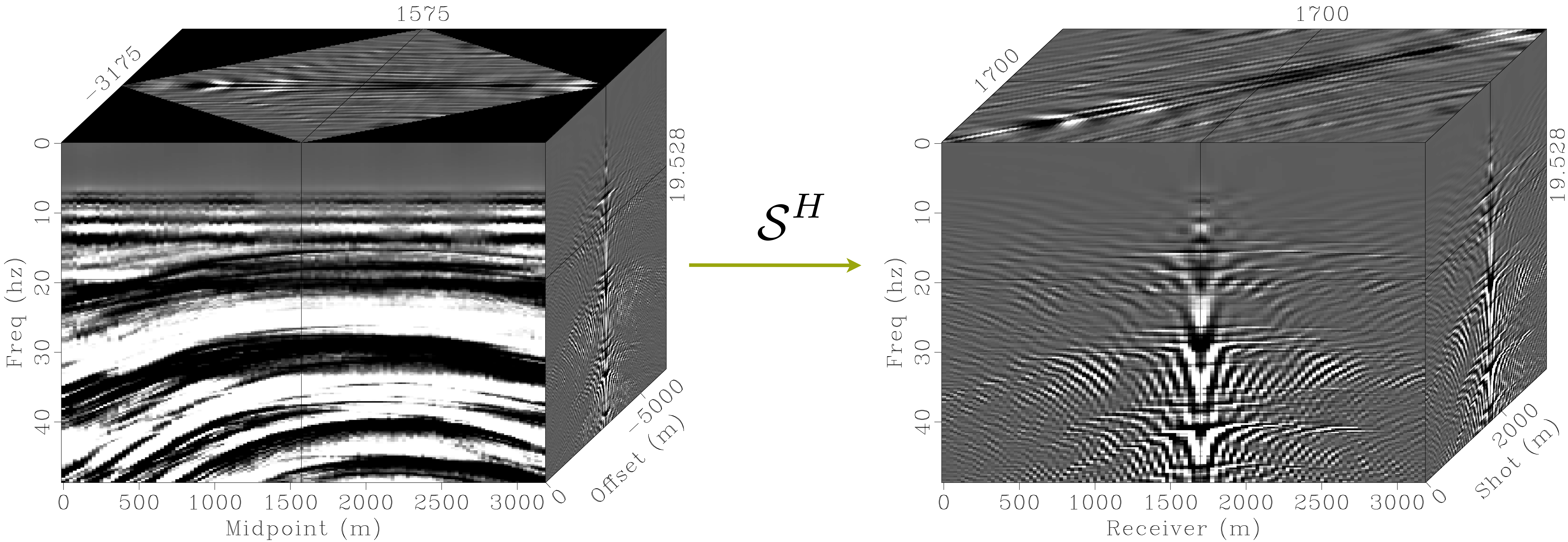
- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A}(.) = \mathbf{M}\mathbf{F}^H \mathcal{S}^H(.)$$

$\mathbf{M}$	time-jittered operator
$\mathbf{F}^H$	inverse Fourier transform along frequency axis
$\mathcal{S}^H$	transform operator

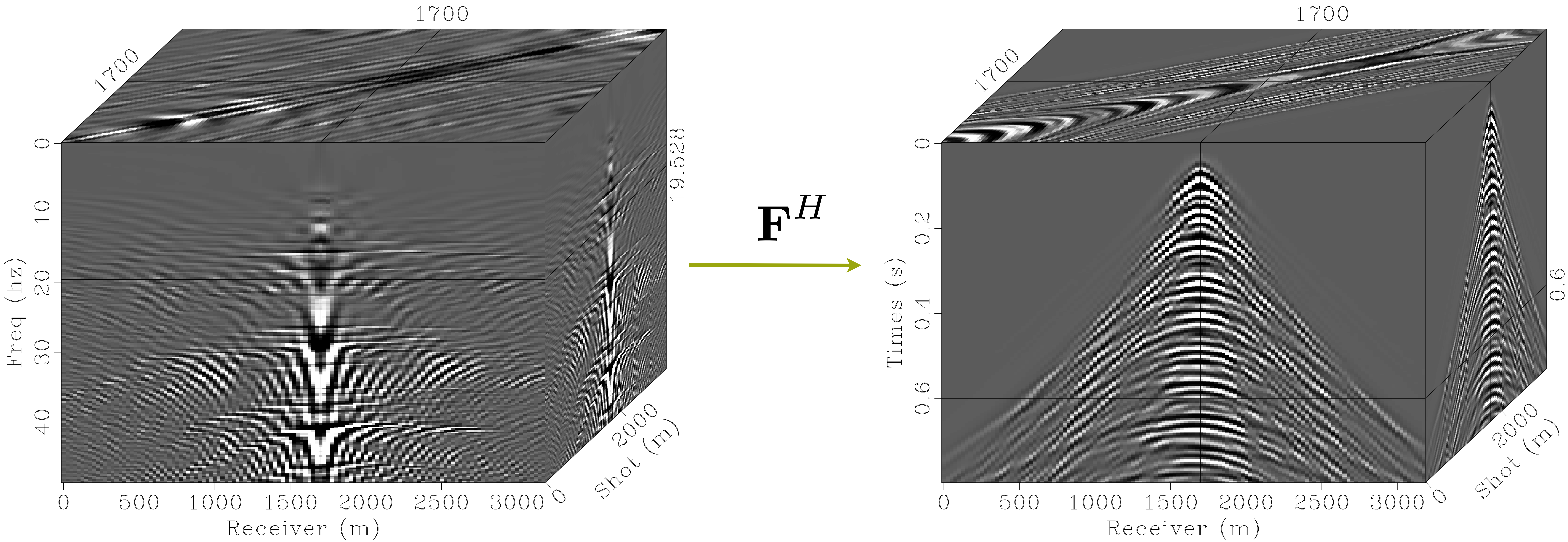
Transform-sampling operator

$$\mathcal{A}(.) = \mathbf{MF}^H \mathcal{S}^H(.)$$



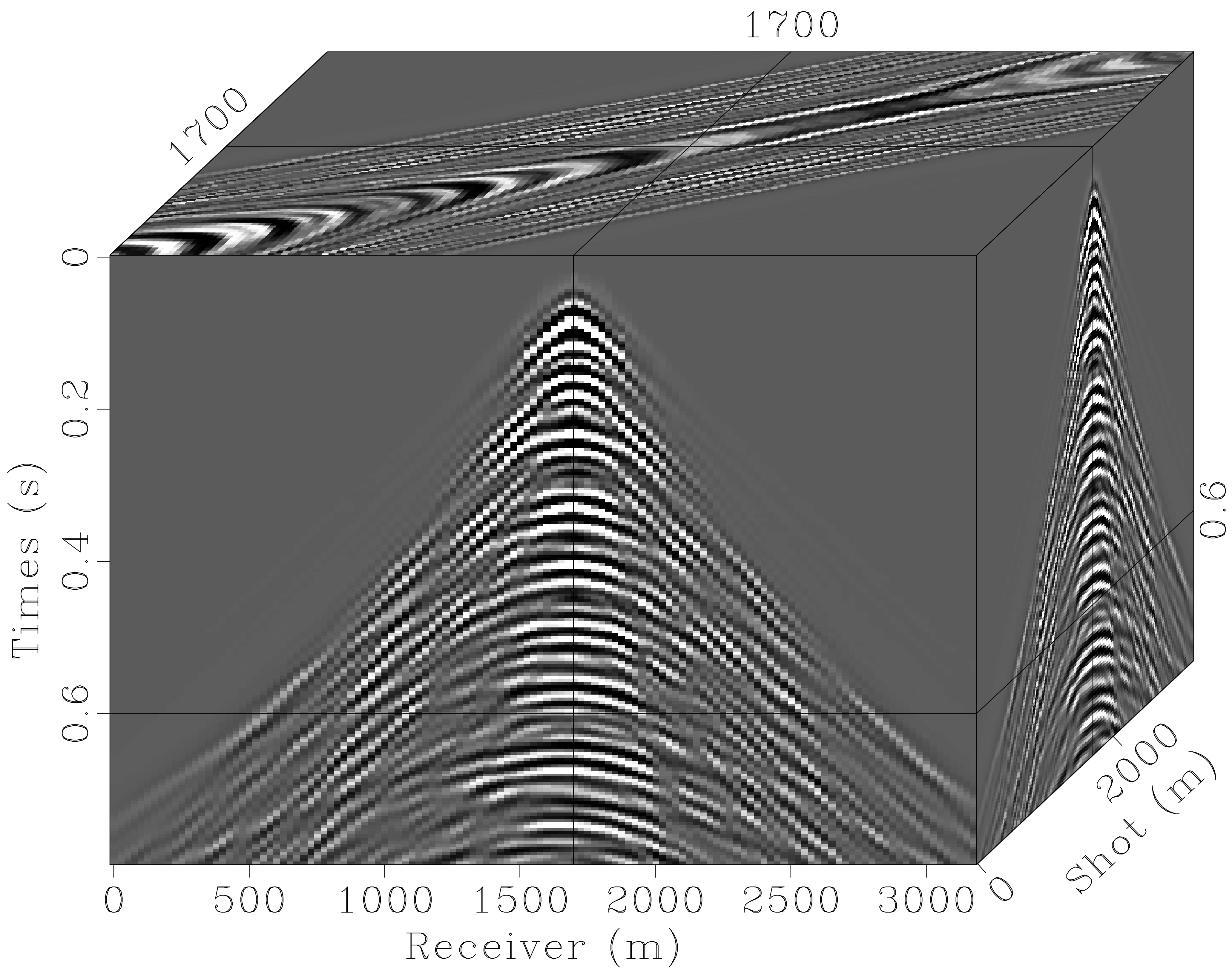
Transform-sampling operator

$$\mathcal{A}(.) = \mathbf{M}\mathbf{F}^H\mathcal{S}^H(.)$$

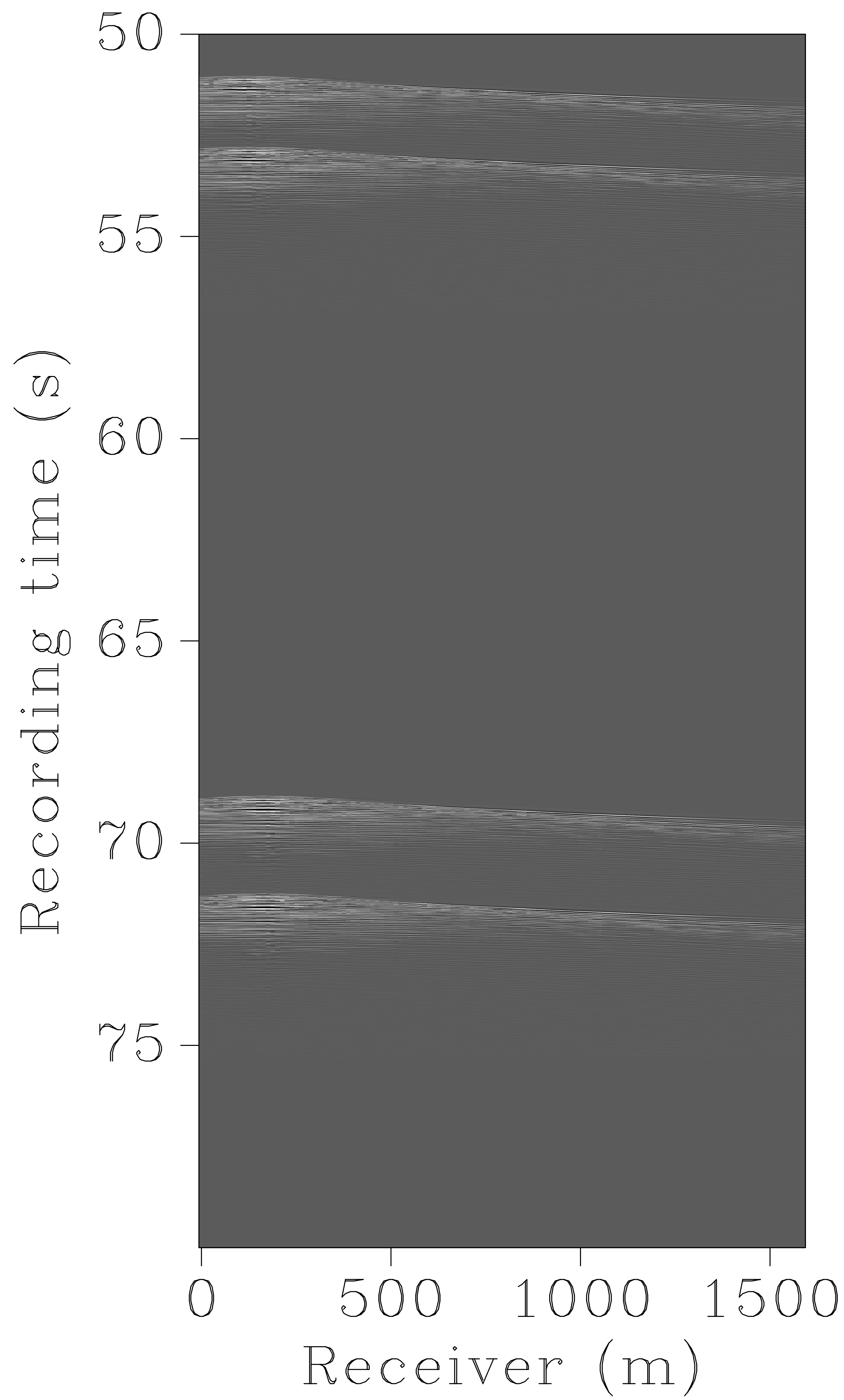


Transform-sampling operator

$$\mathcal{A}(.) = \mathbf{M}\mathbf{F}^H\mathcal{S}^H(.)$$

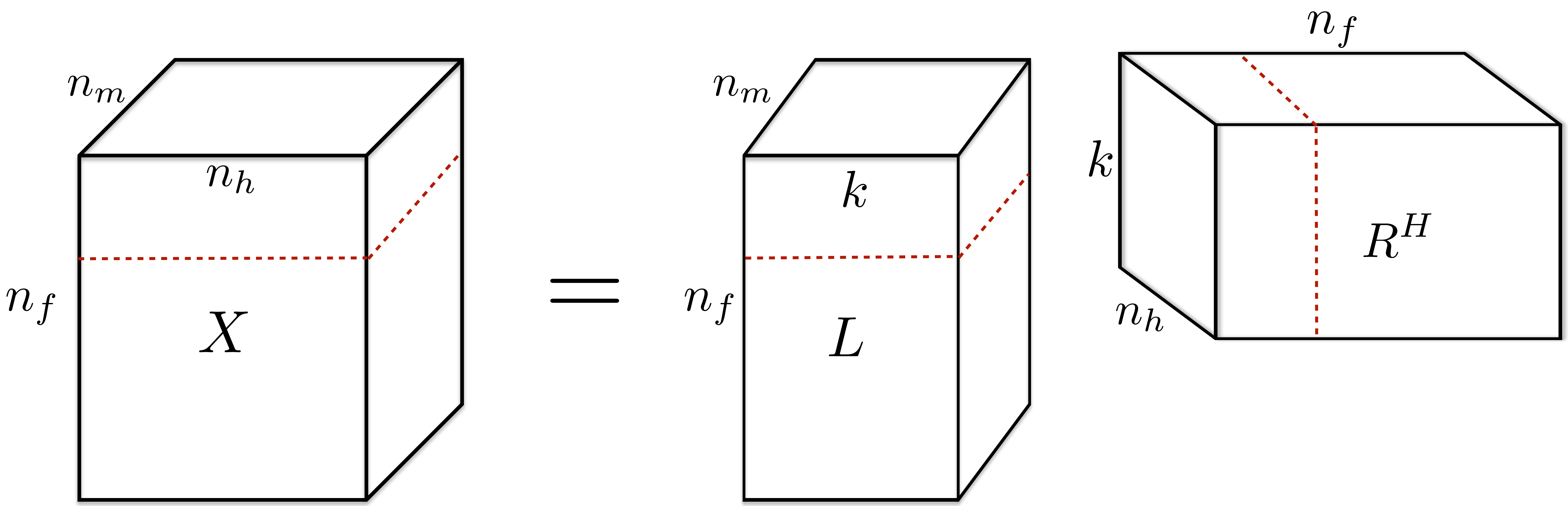


$\mathbf{M}$



Factorized formulation

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$



$X \in \mathbb{C}^{n_f \times n_m \times n_h}$ $L \in \mathbb{C}^{n_f \times n_m \times k}$ $R^H \in \mathbb{C}^{k \times n_h \times n_f}$

Berg and Friedlander 2008, Aravkin et al. 2012b

Factorized formulation

- Reformulate $BPDN_\sigma$ formulation

$$\min_{\mathbf{L}_f,\mathbf{R}_f} \quad \sum_f \|\mathbf{L}_f {\mathbf{R}_f}^H\|_* \quad \text{s.t. } ||\mathcal{A}(\mathbf{L}_f {\mathbf{R}_f}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- Approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L}_f,\mathbf{R}_f} ||\mathcal{A}(\mathbf{L}_f {\mathbf{R}_f}^H) - \mathbf{b}||_2^2 \quad \text{s.t. } \sum_f \|\mathbf{L}_f {\mathbf{R}_f}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

Factorized formulation

- Nuclear norm satisfies

$$\sum_f \|\mathbf{L}_f \mathbf{R}_f^H\|_* \leq \sum_f \frac{1}{2} \left\| \begin{bmatrix} \mathbf{L}_f \\ \mathbf{R}_f \end{bmatrix} \right\|_F^2$$

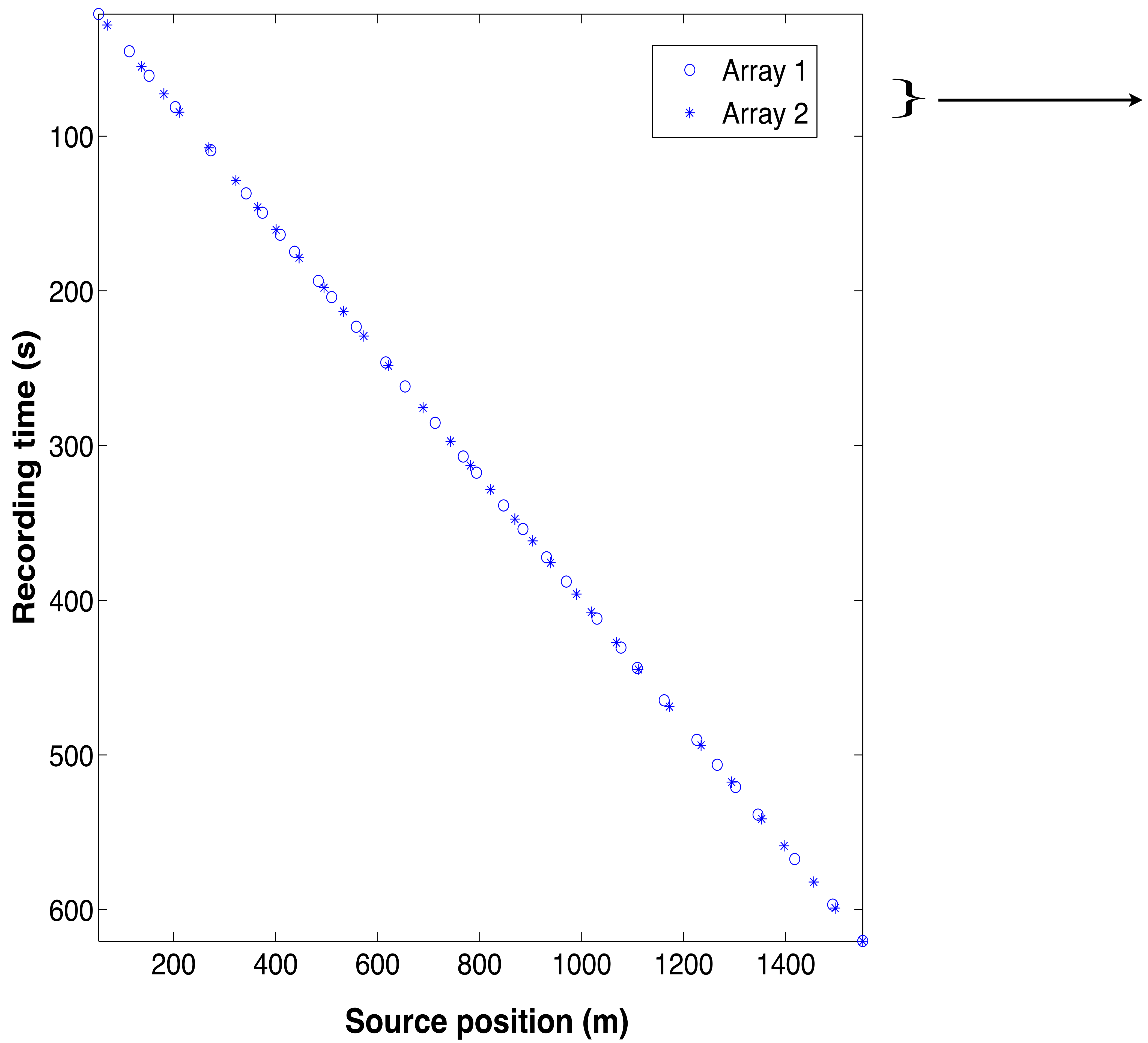
[Rennie and Srebro 2005]

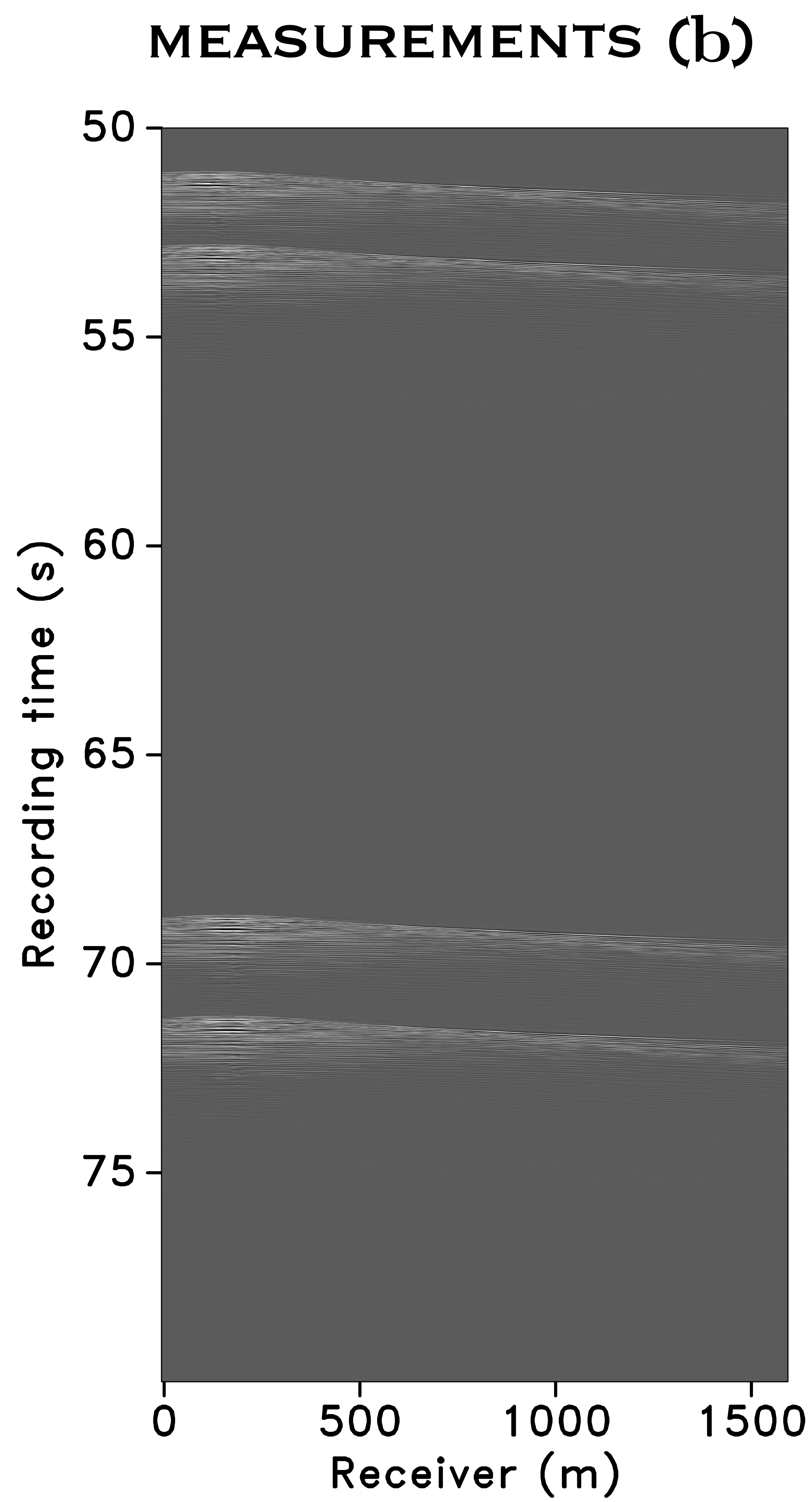
where $\|\cdot\|_F^2$ is sum of squares of all entries

- Choose rank k *explicitly* & avoid costly *SVD's*

Experiments & Results

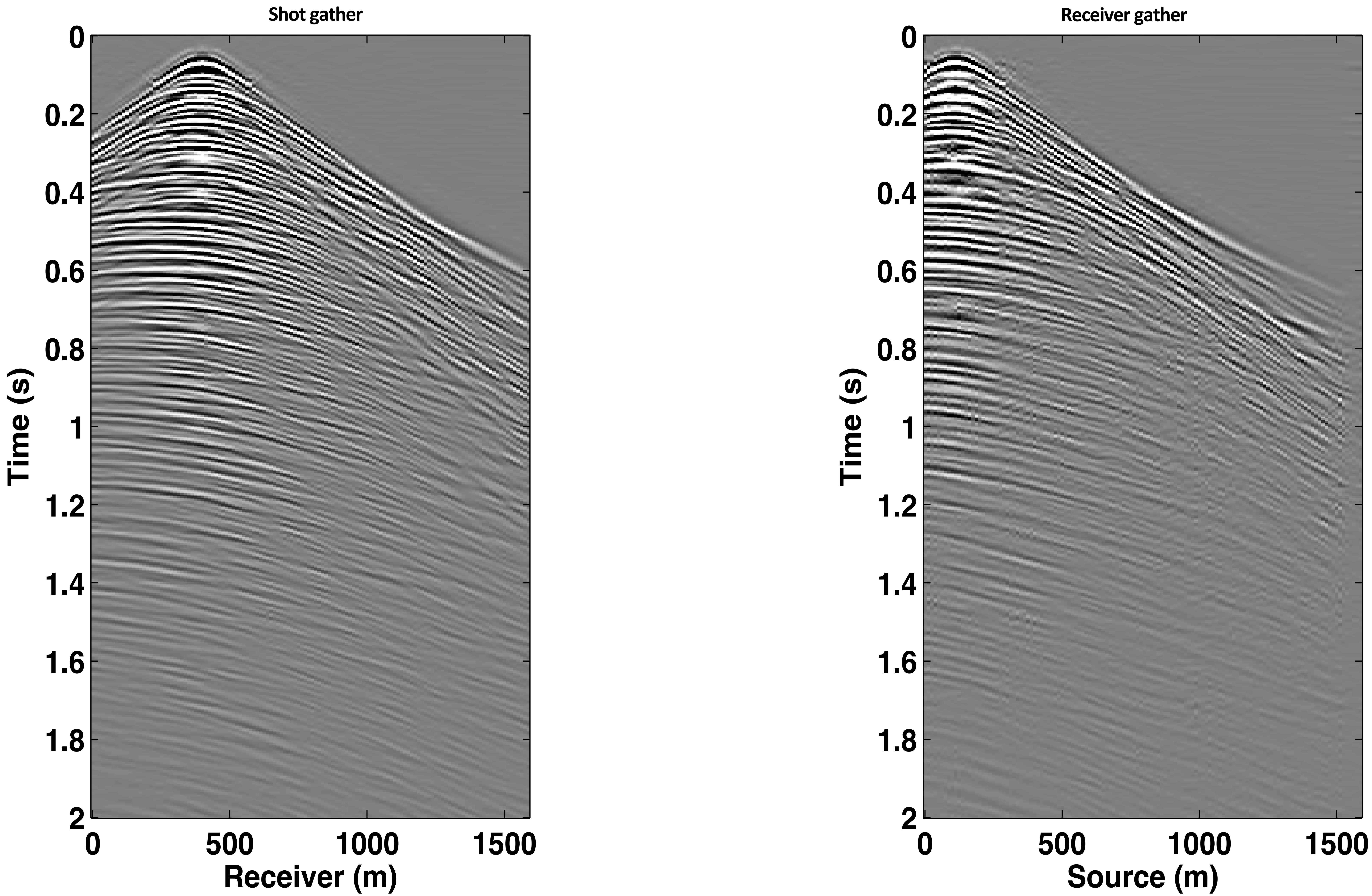
Time-jittered OBC acquisition





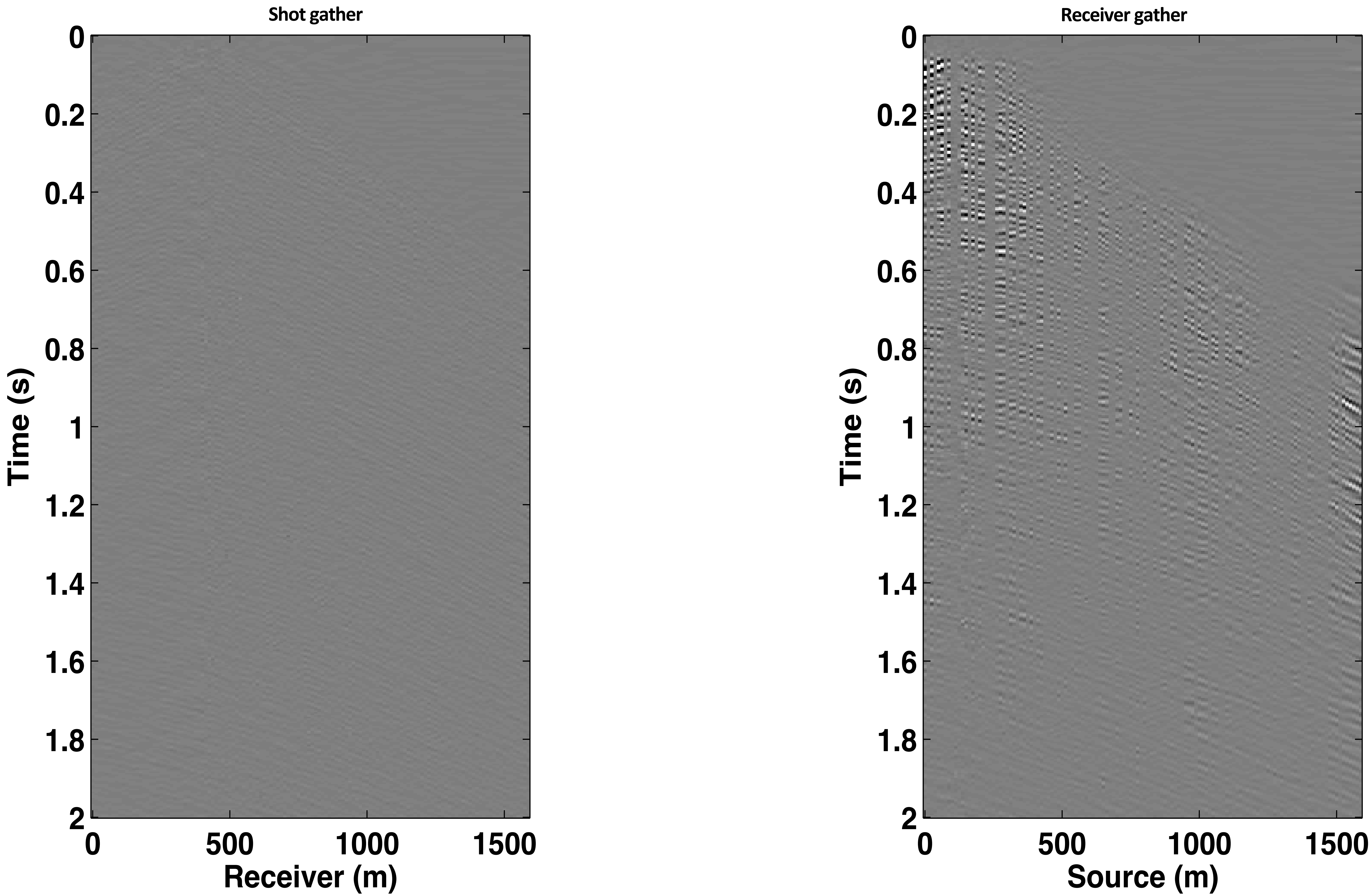
Sparsity-promoting recovery

SNR = 11.3dB



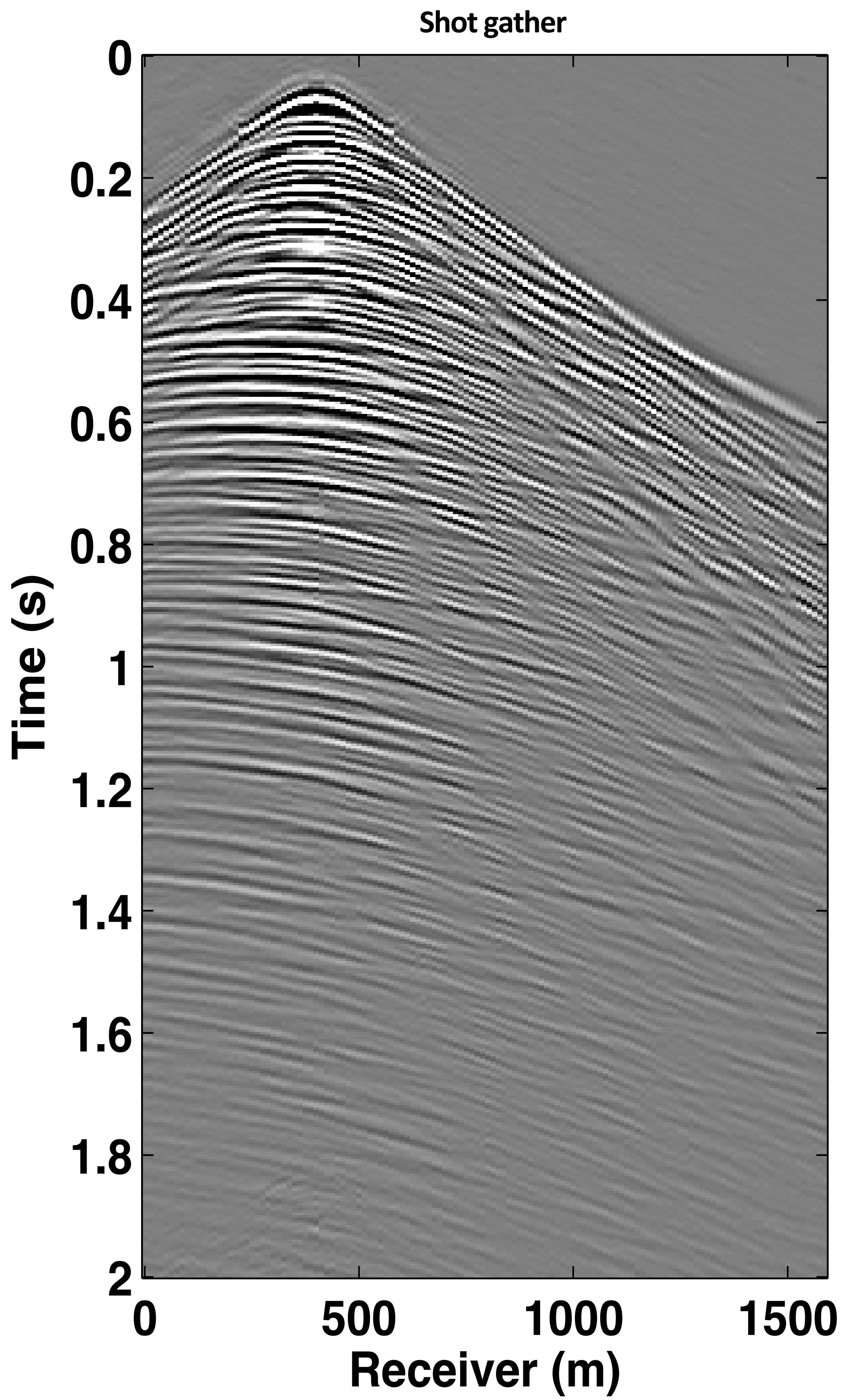
Sparsity-promoting recovery

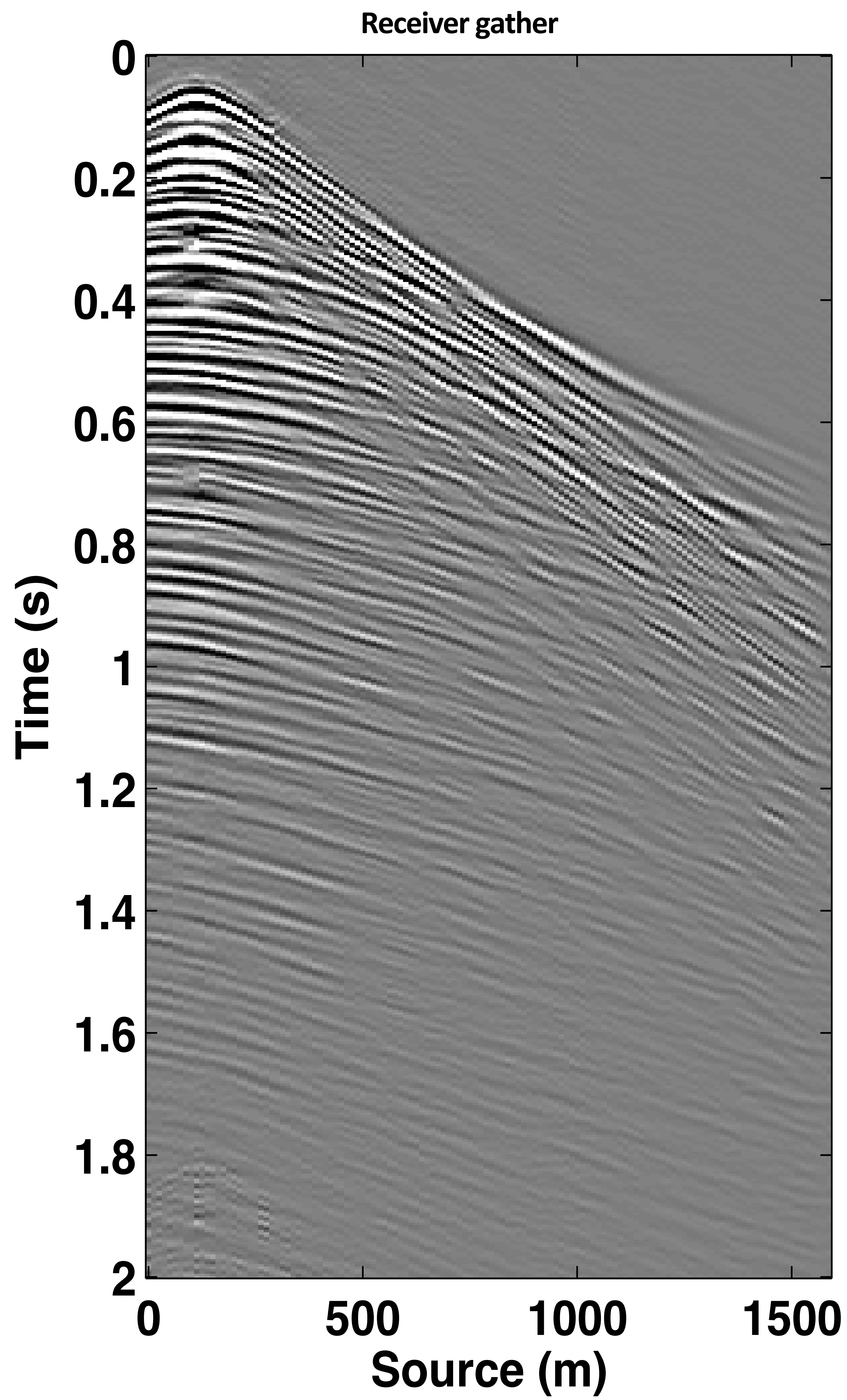
difference



Rank minimization

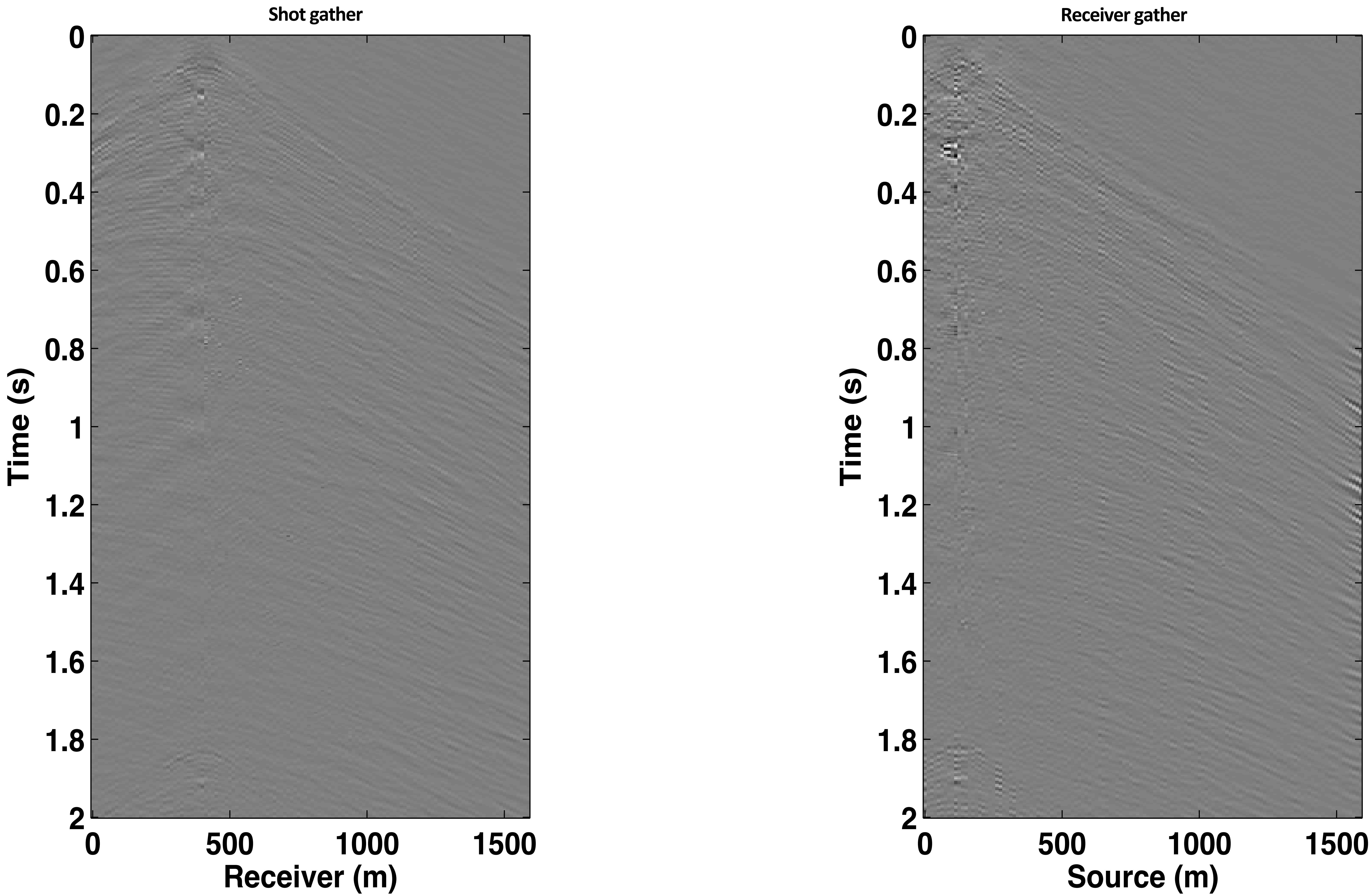
SNR = 12.6 dB





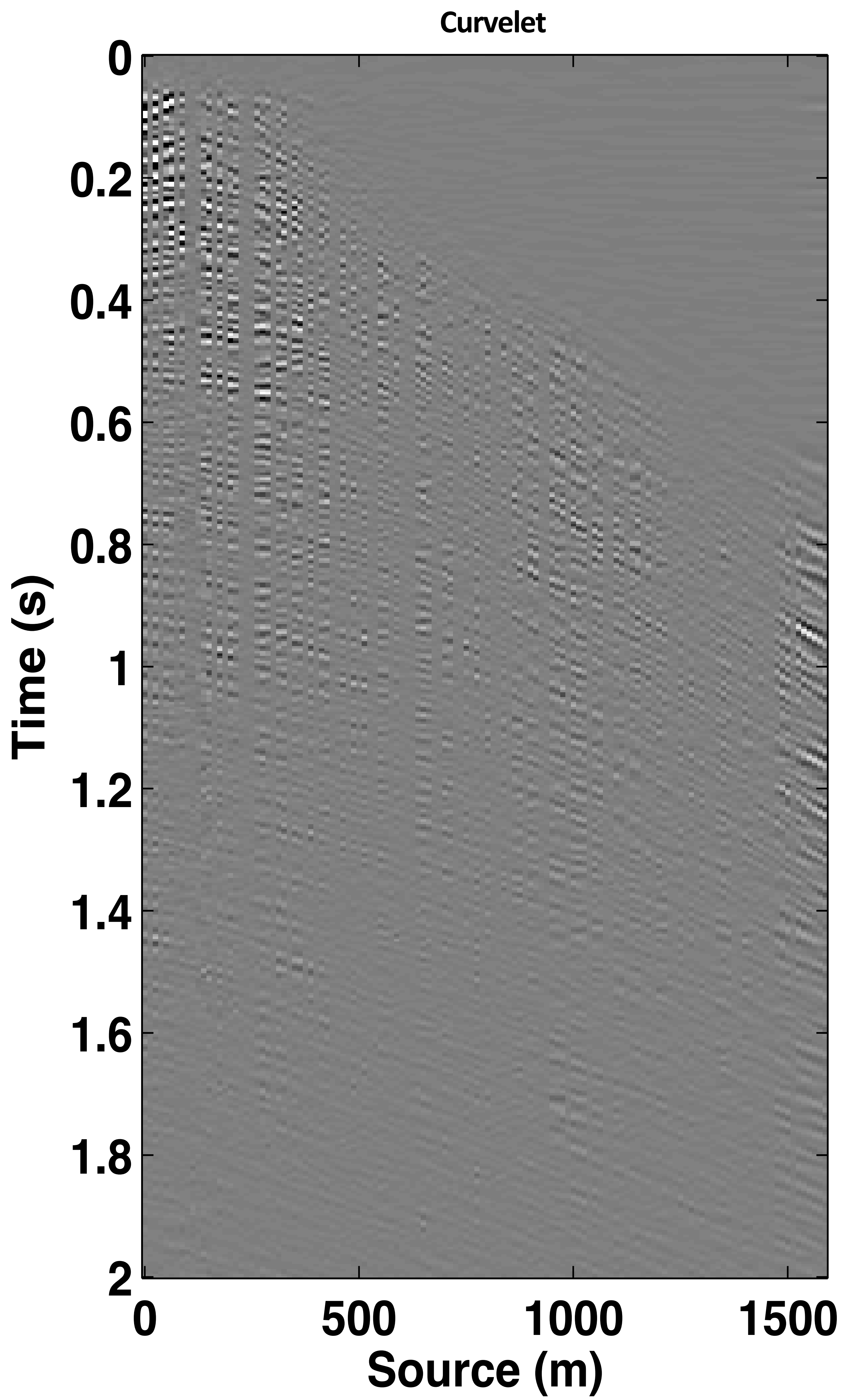
Rank minimization

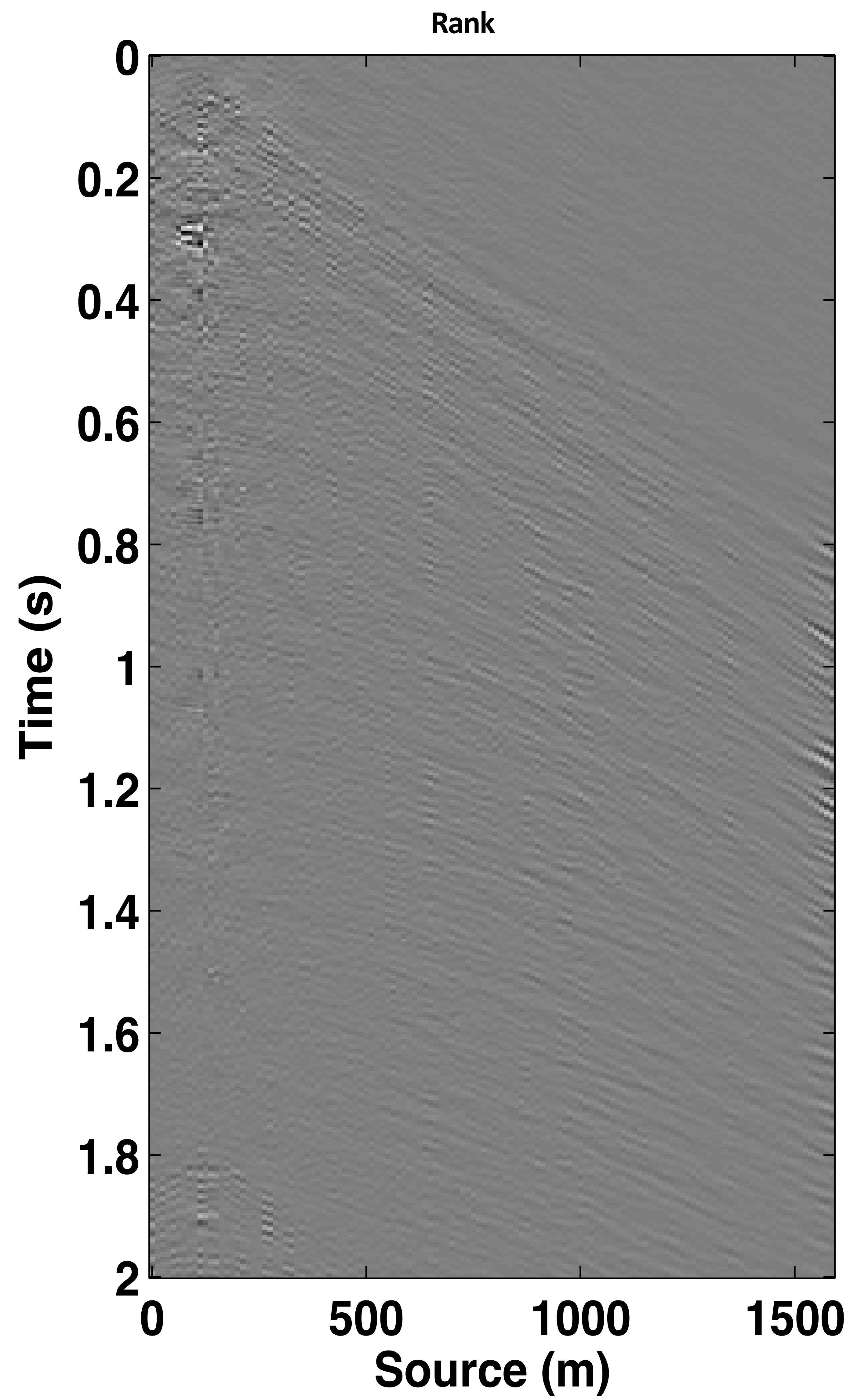
difference



Curvelet v/s Rank

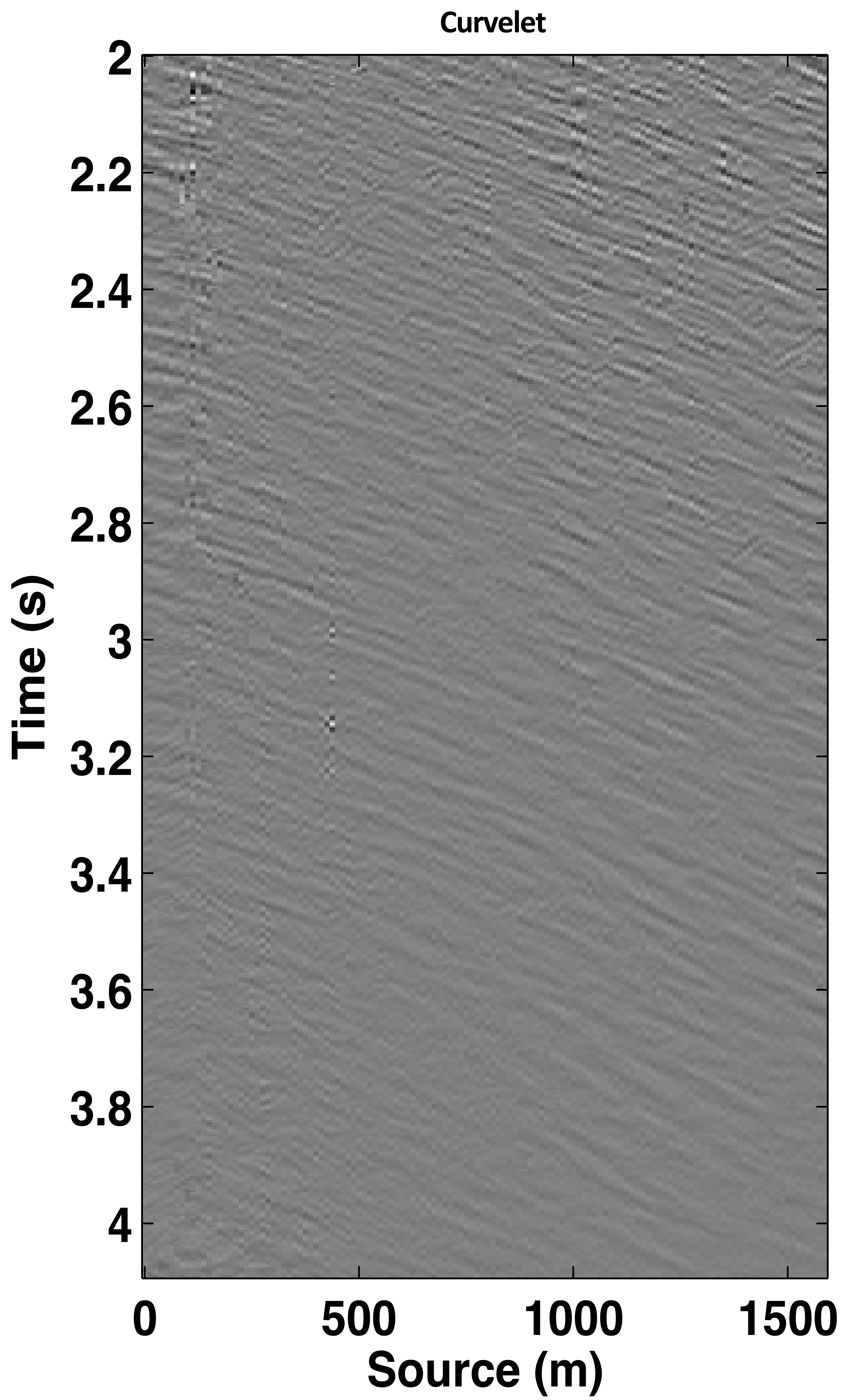
difference, shallow section

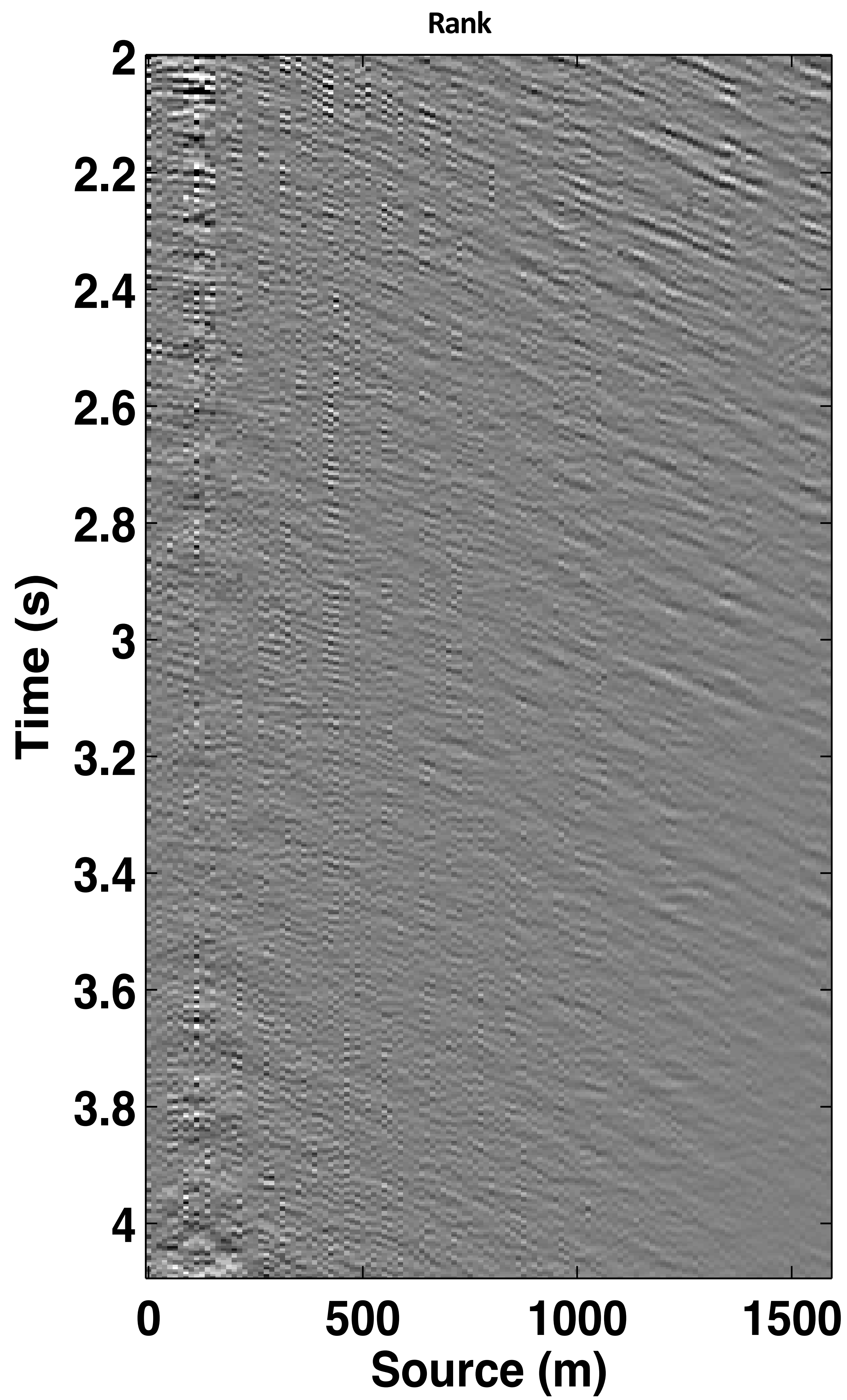




Curvelet v/s Rank

difference, deeper section





Summary

	jittered to regular (m)	***Curvelet*** [SNR (dB)]	***Low-rank*** [SNR (dB)]
1 source vessel (2 airgun arrays)	50 to 25	14.6	14.5
	50 to 12.5	11.3	12.6

Summary

- **speed up by a factor of ~ 8**

	jittered to regular (m)	***Curvelet*** [time (hr)]	***Low-rank*** [time (hr)]
1 source vessel (2 airgun arrays)	50 to 25	**14**	**1.9**
	50 to 12.5	**16**	**2.1**

Summary

- storage reduction by a factor of ~ 23 (for each copy of unknown)

	Curvelet coefficient [storage (gb)]	***Low-rank factors*** [storage (gb)]
1 source vessel (2 airgun arrays)	**2**	**0.09**

Conclusion

- Simple algorithm
- Fast, Scalable and Memory efficient
- Easily extended to 3D marine acquisition

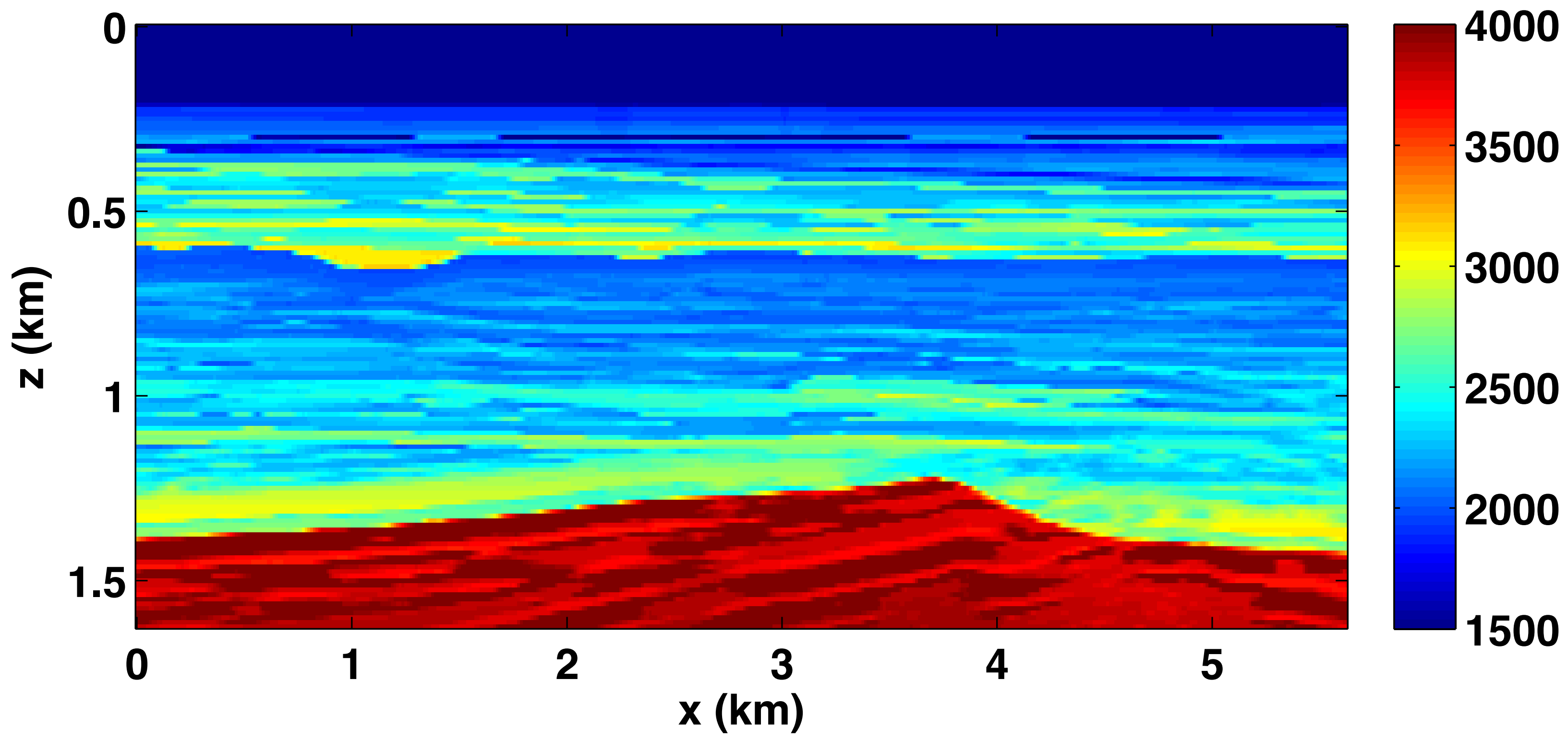
Future work

- Testing on more realistic 3D data sets
- Irregular grid
- Extension to time-lapse acquisition
- Software release

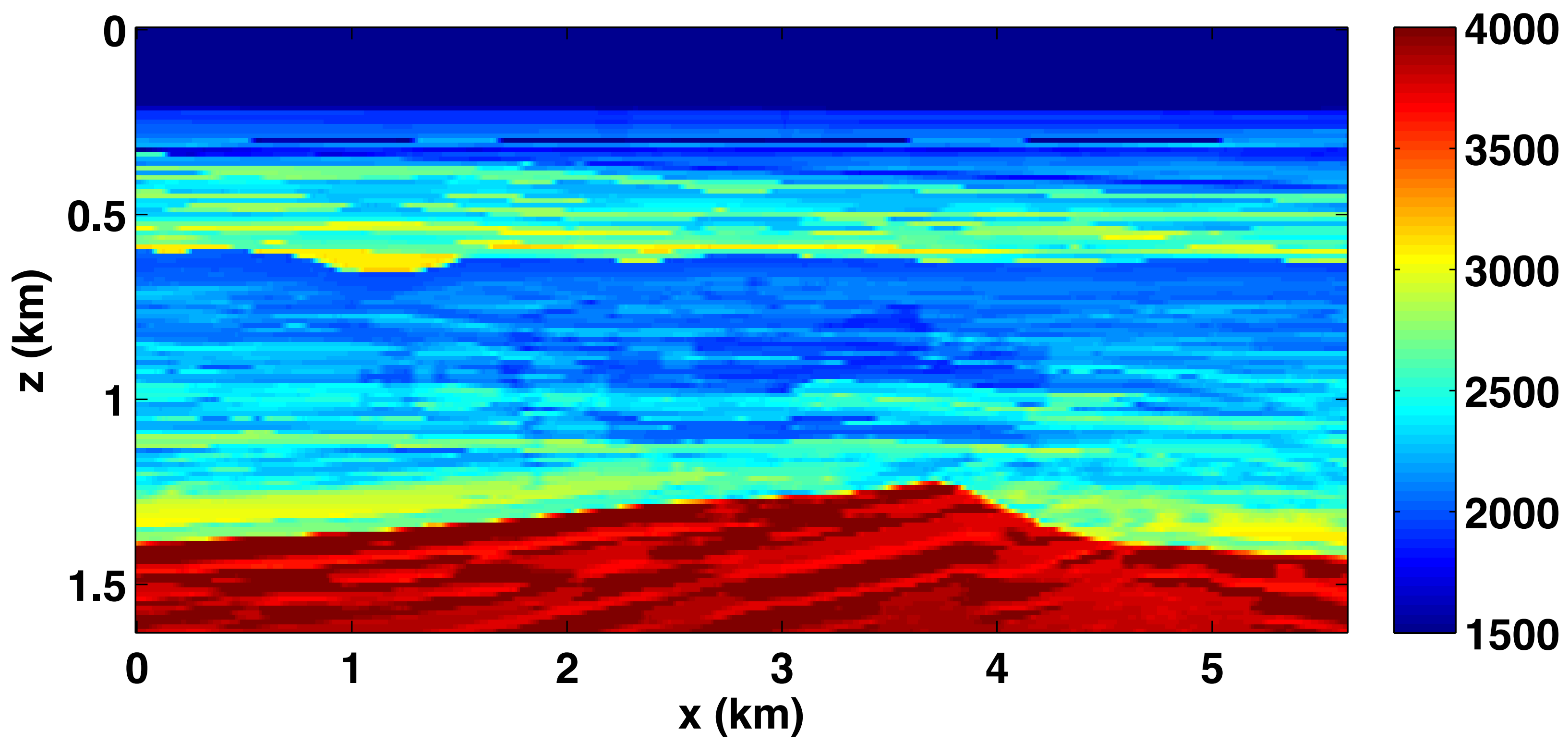
Rank minimization

BG model

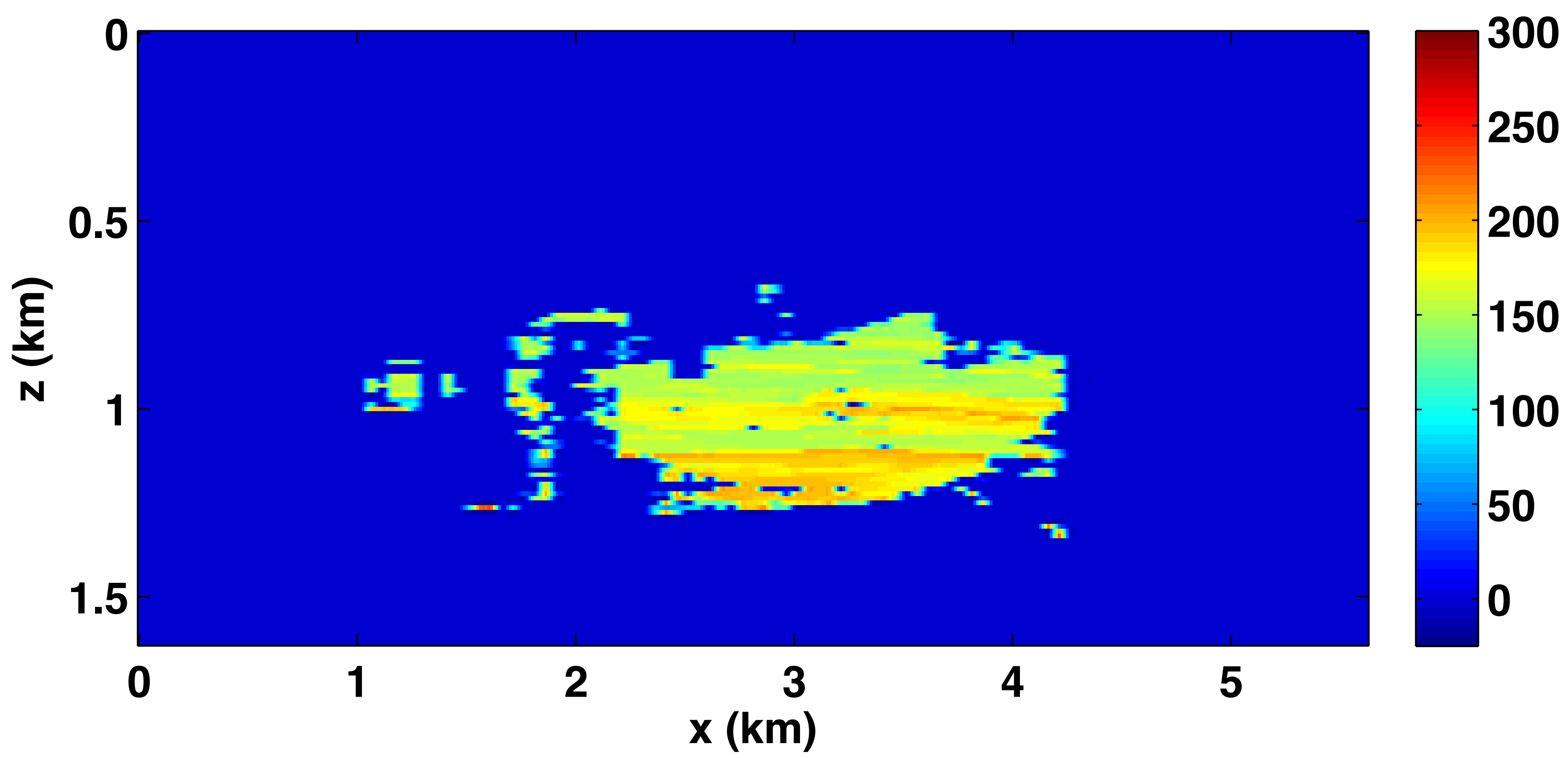
Baseline



Monitor

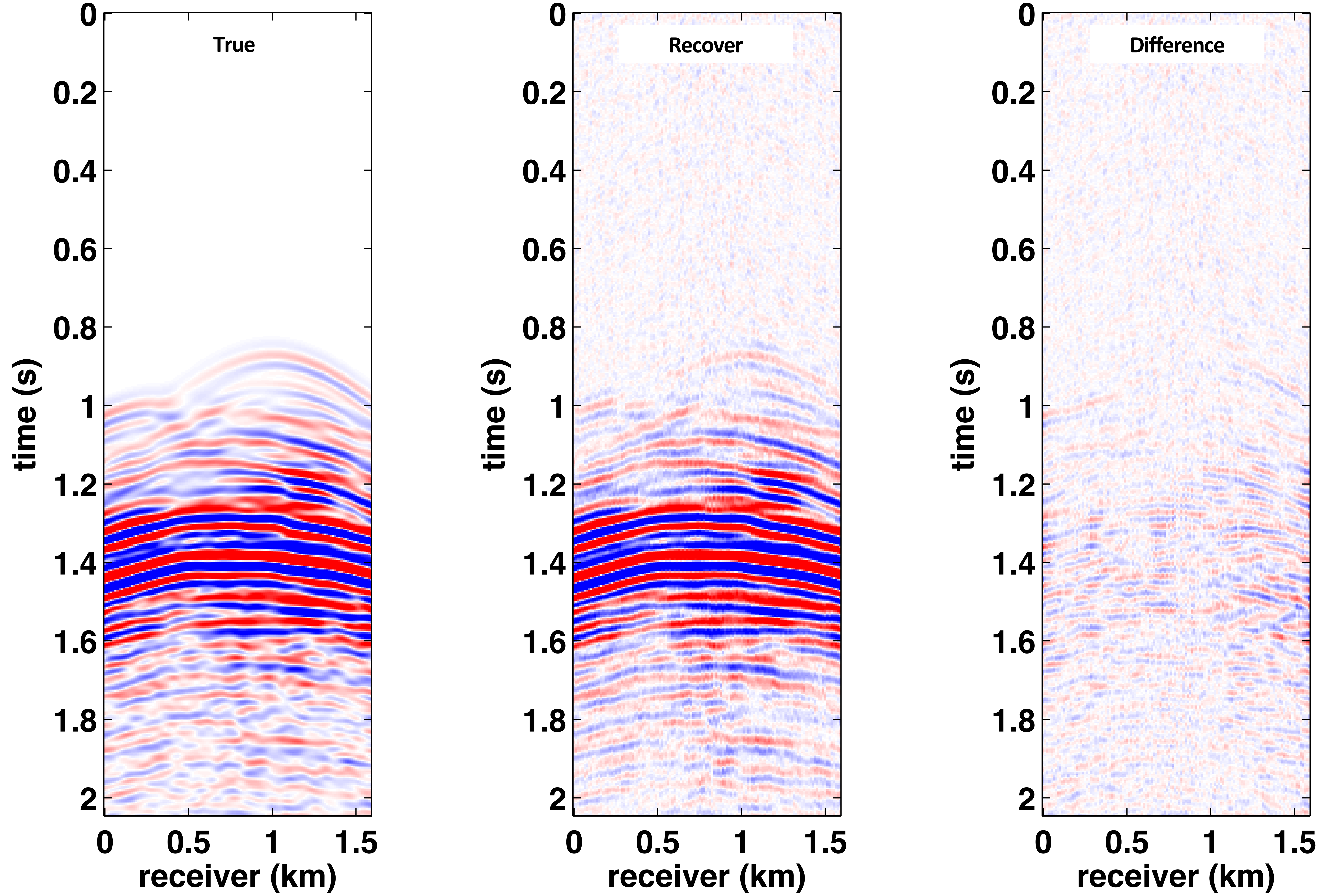


Time-lapse change



Time-lapse signal

SNR = 16.1 dB, 100 % Overlap, Joint model, 10x scale, 50 m to 12.5m grid



Acknowledgements

Thank you!

SINBAD

NSERC CRSNG

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