Wavefield reconstruction via randomized sampling

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Outline

- We do seismic trace interpolation by transforming the data to the curvelet domain.
- This problem has a lot of room for improvement.
- In this talk we consider of a few simple techniques that can improve the results significantly.

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- ℓ_1 recovery in curvelet domain

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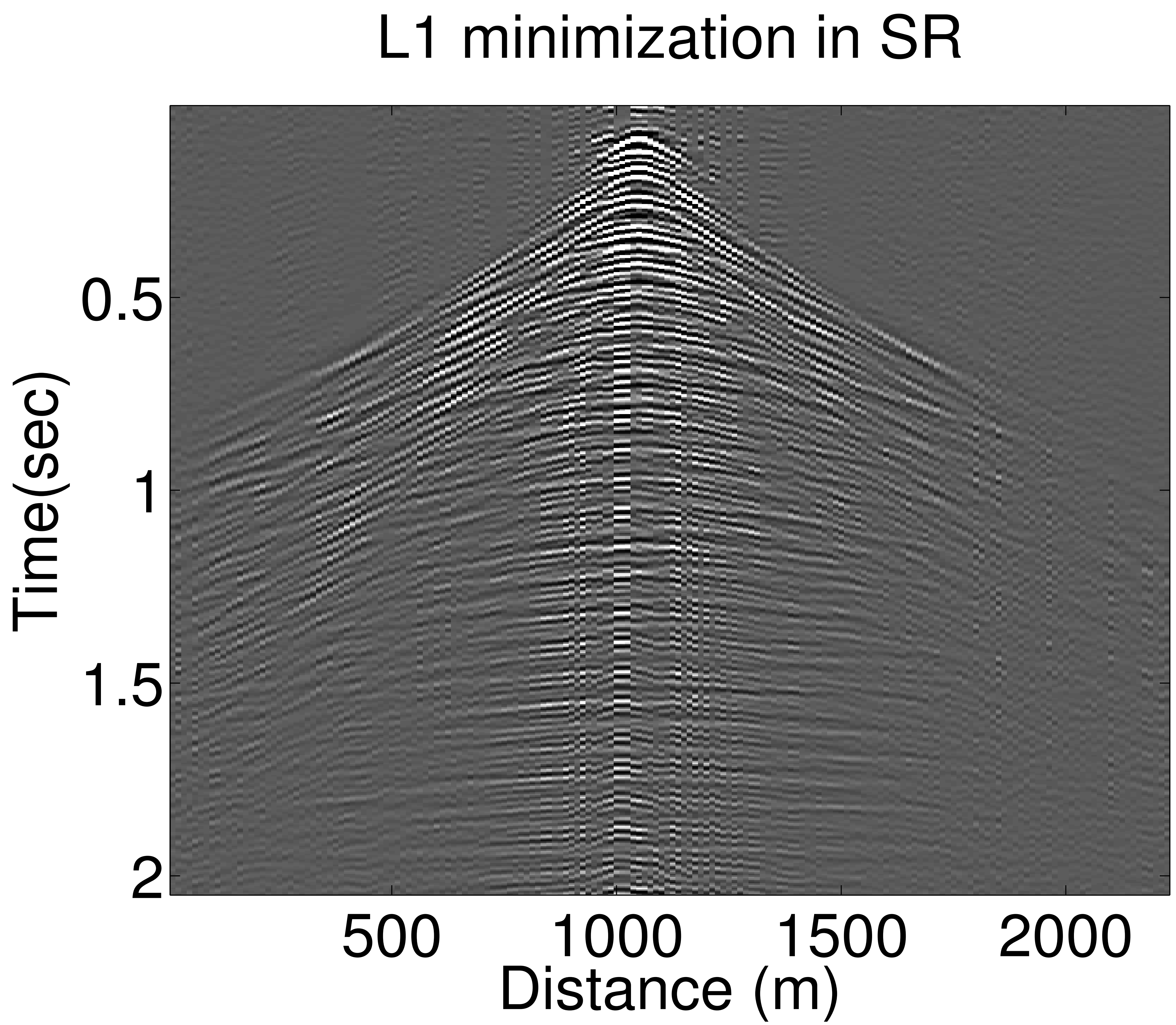
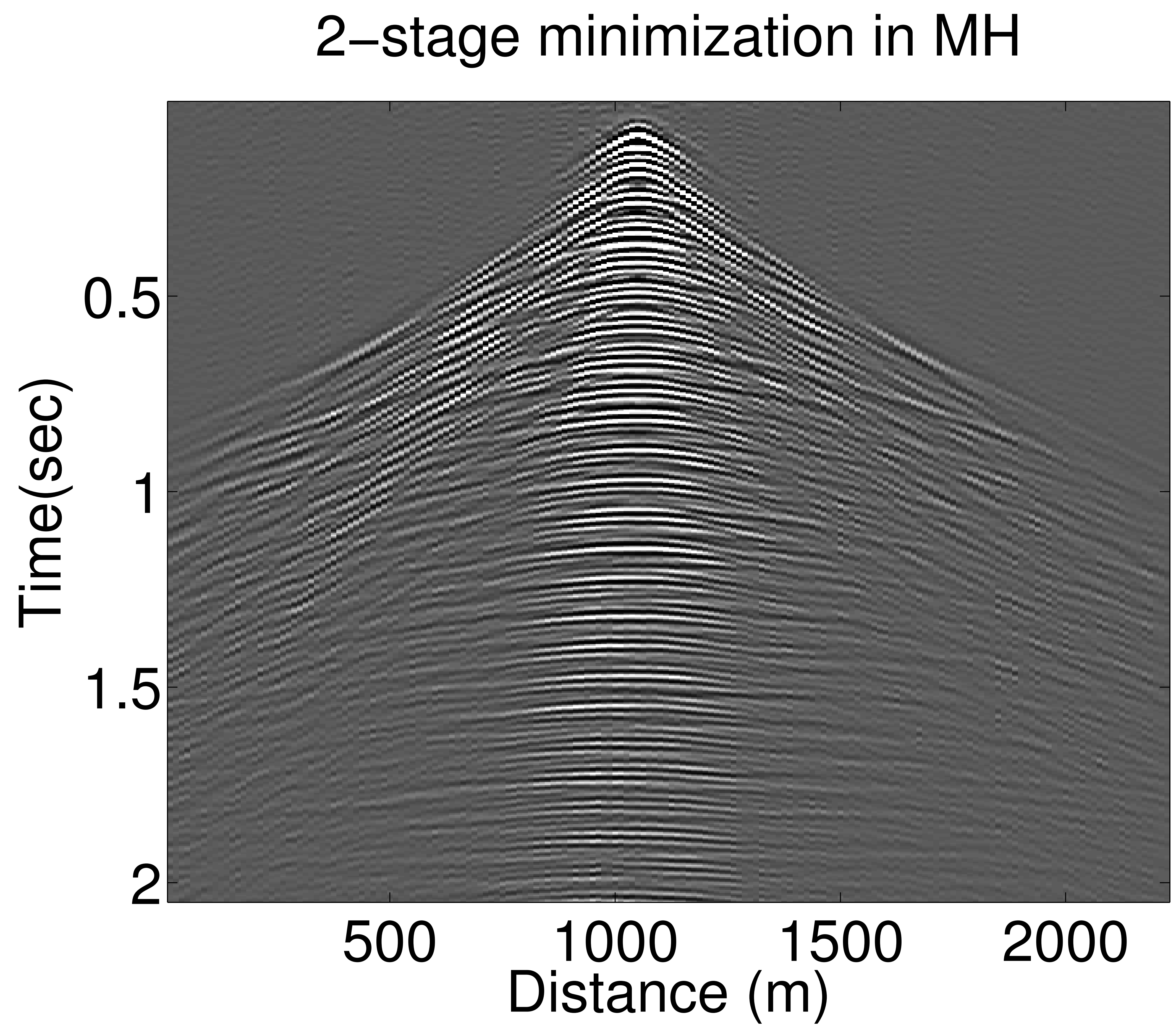
- Midpoint-offset domain
- 2-stage weighted algorithm

Main result

Improvement on an example with 70% missing receivers.



(a) SNR= 5.3 dB



(b) SNR= 14.8 dB

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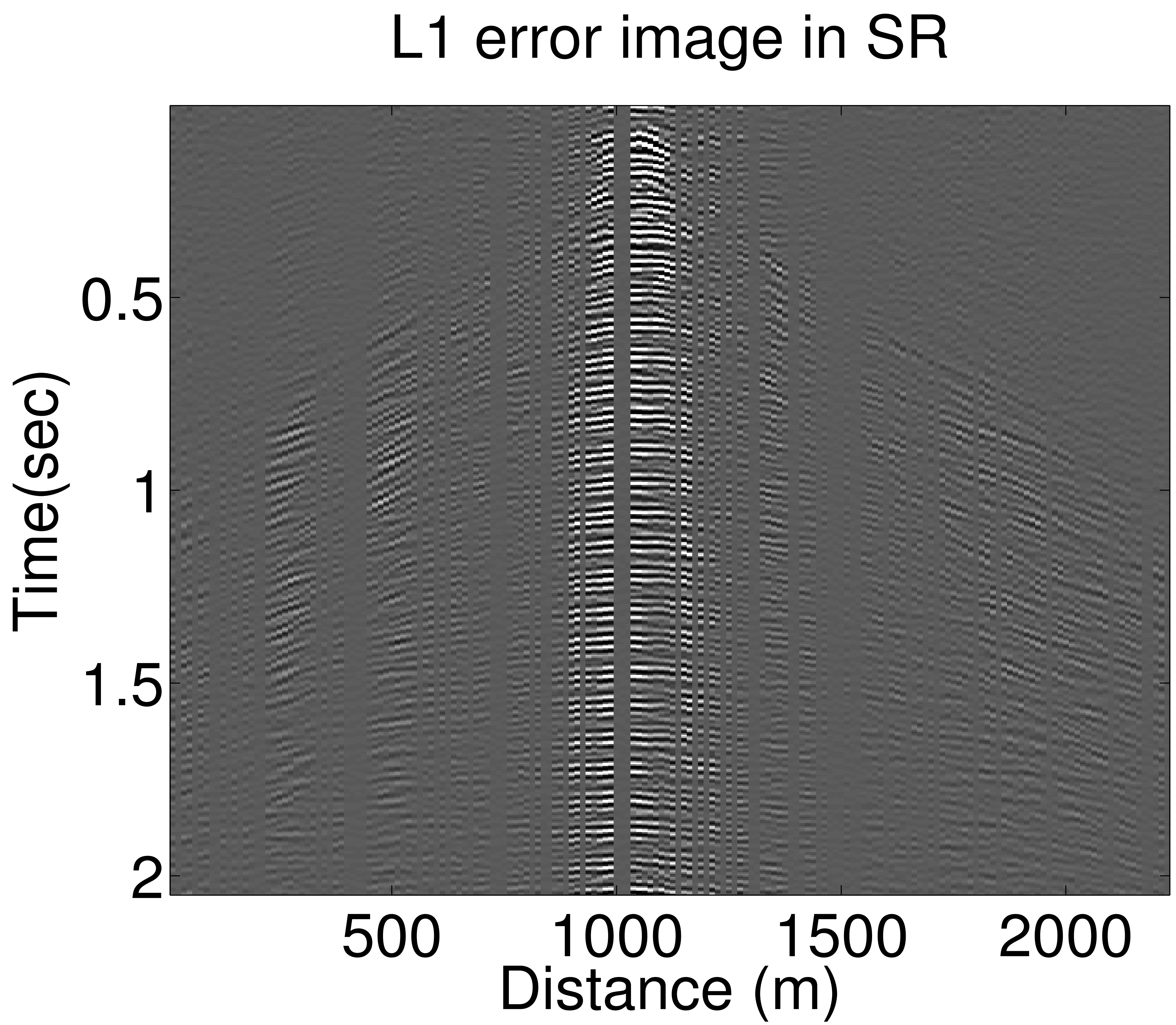
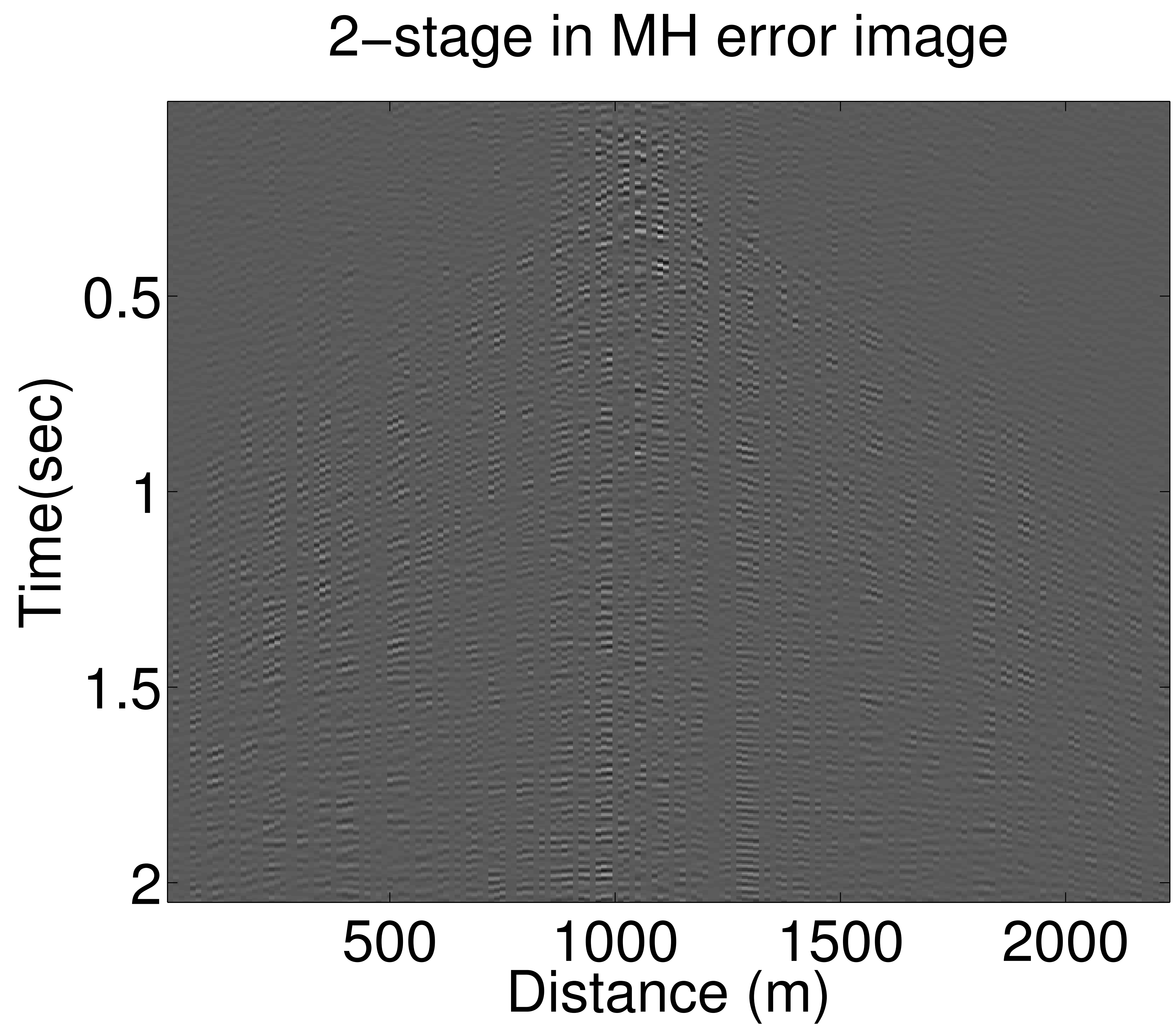


(c) SNR= 5.3 dB



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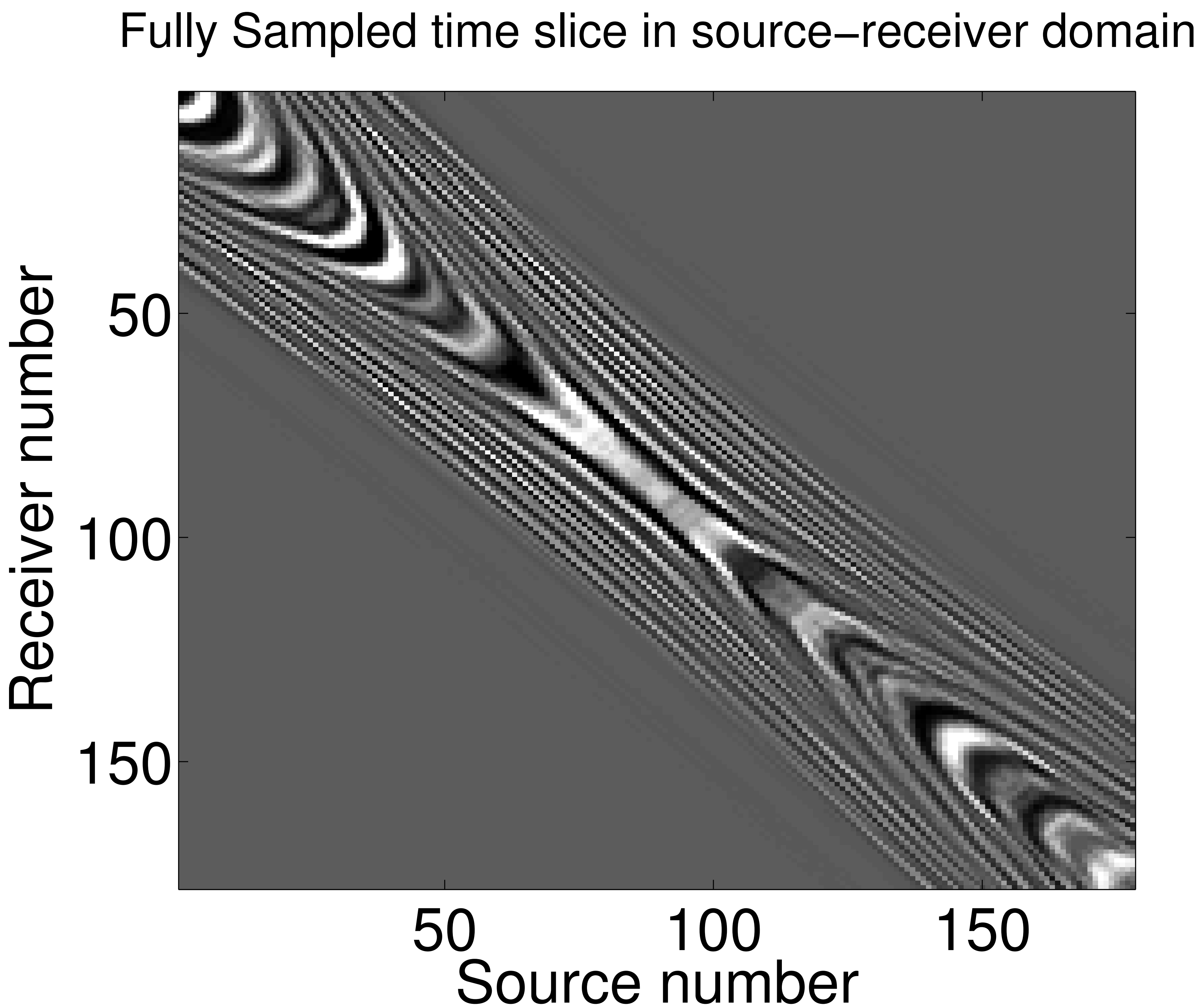
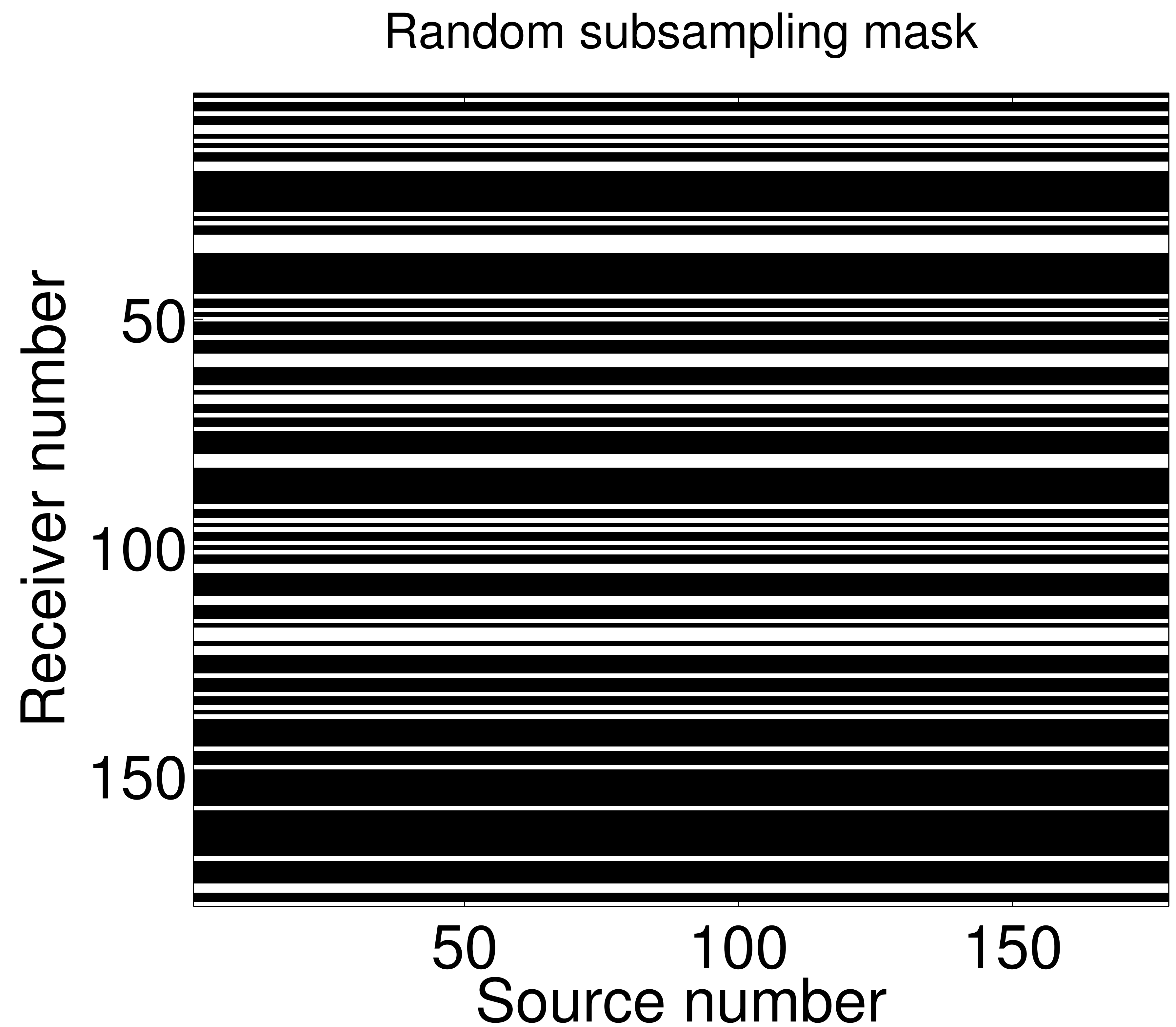
ℓ_1 for randomized acquisition of seismic lines

- Consider a seismic line with 178 sources, 178 receivers with a sample interval of 12.5m.
- 512 time samples collected in a 2s temporal window.
- 30% of the receiver spread is randomly subsampled.





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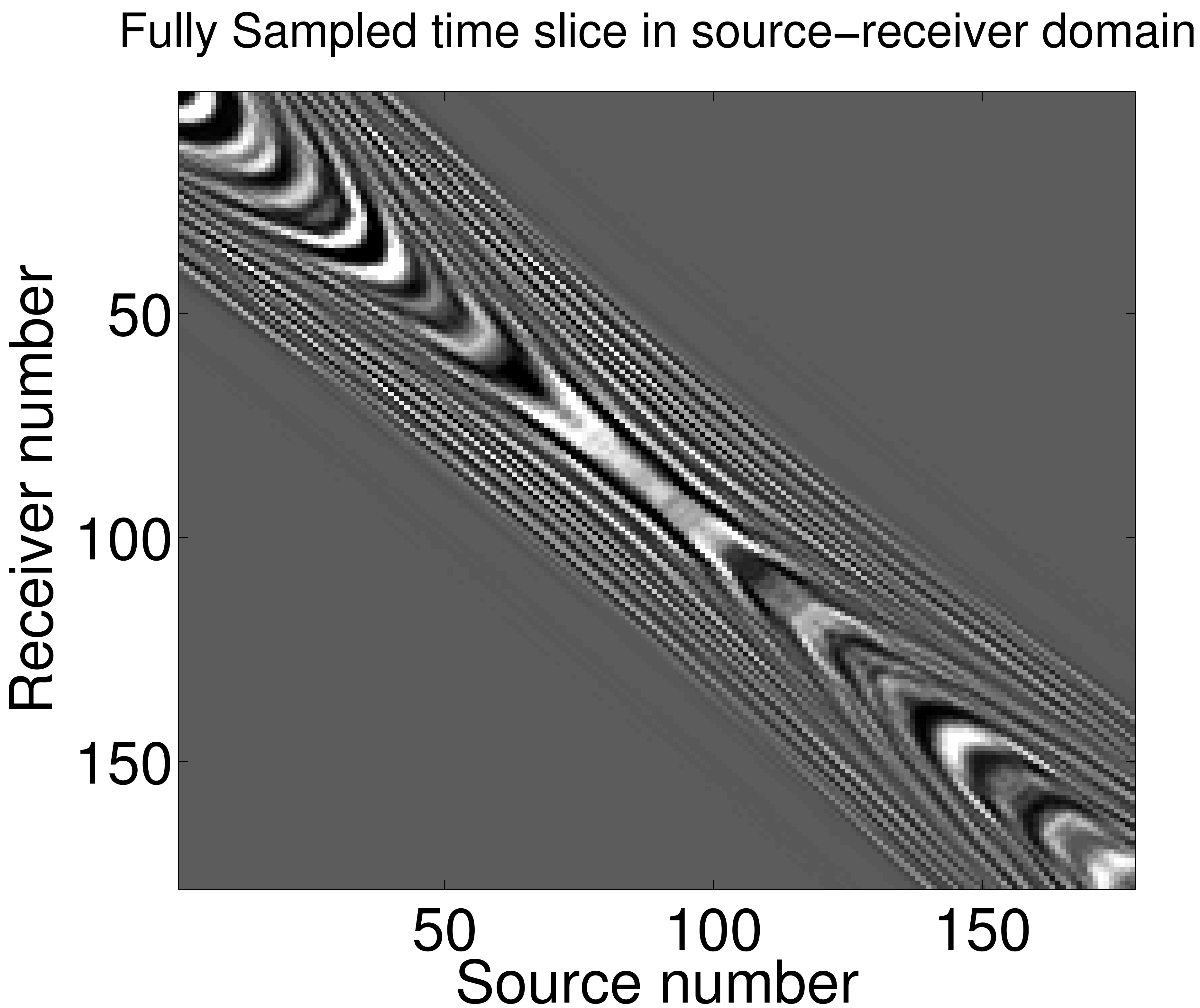

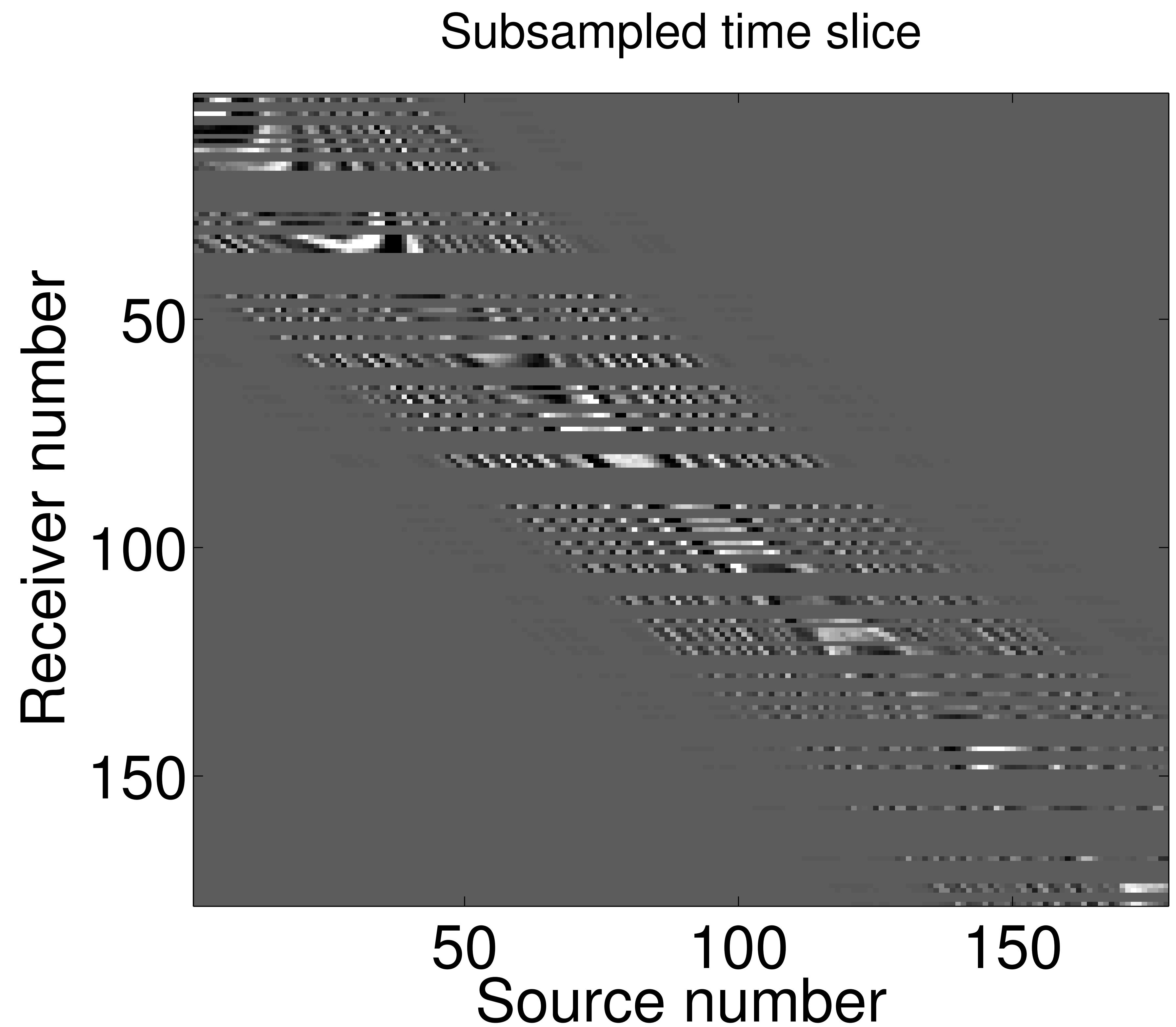

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Let $S \in \mathbb{C}^{P \times N}$ with $P > N$ be the redundant curvelet transform ($S^H S = \mathbb{I}$).

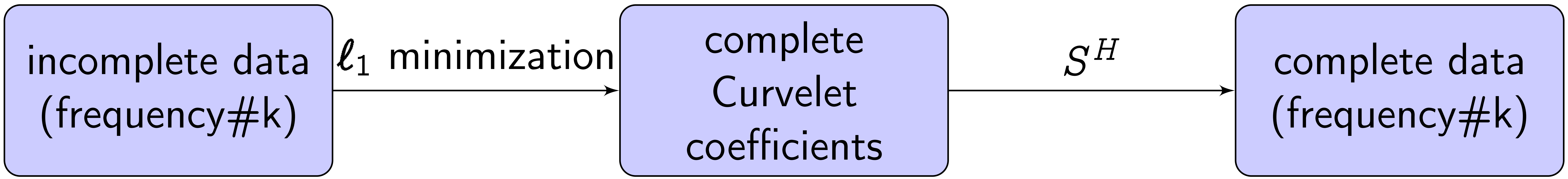
Then $b = RMS^H x$, where x can be recovered by sparse recovery algorithms like ℓ_1 minimization.

Using ℓ_1 for seismic trace interpolation

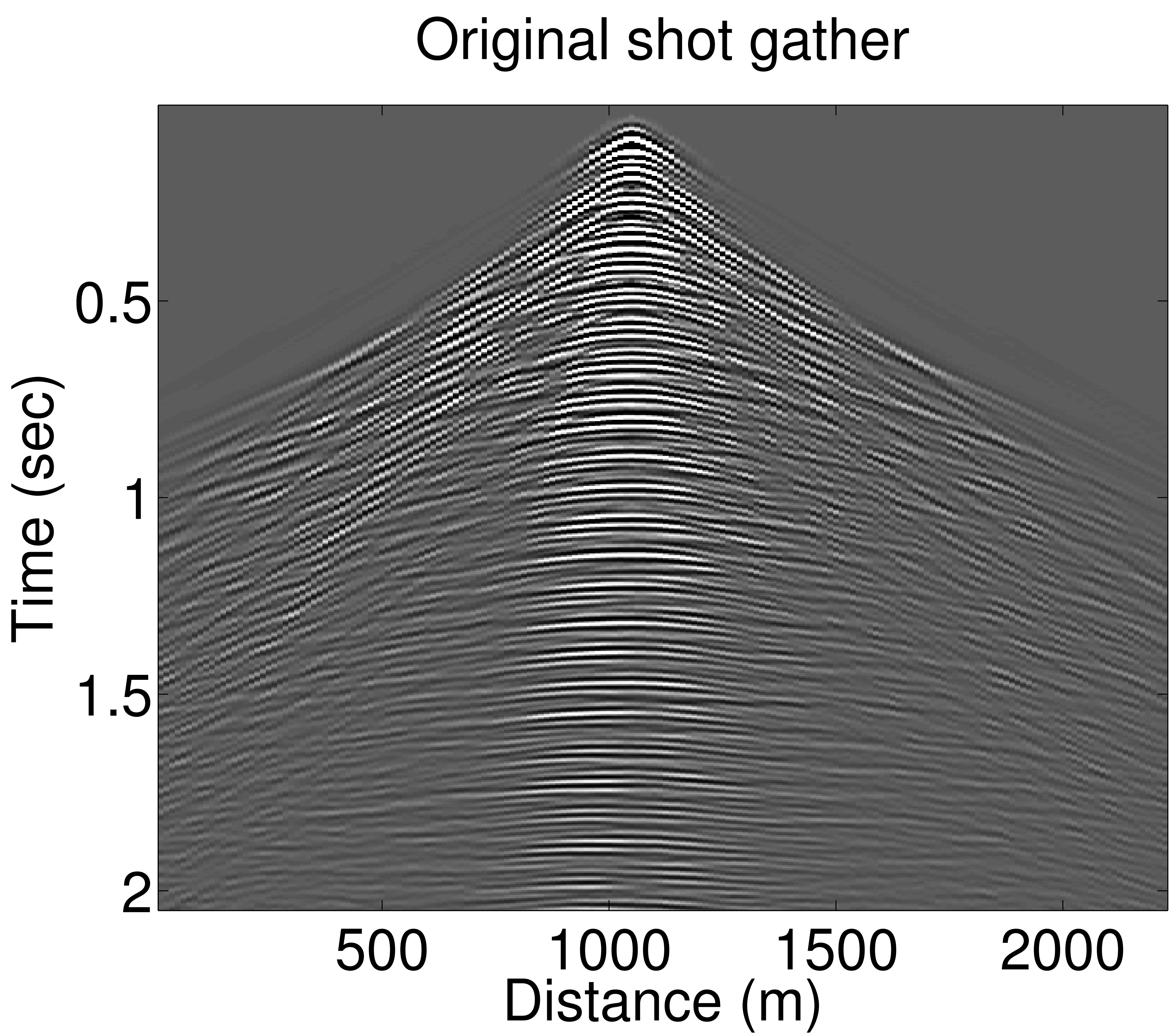
To recover f from the measurements $b = RMS^H x$, we solve the ℓ_1 minimization problem

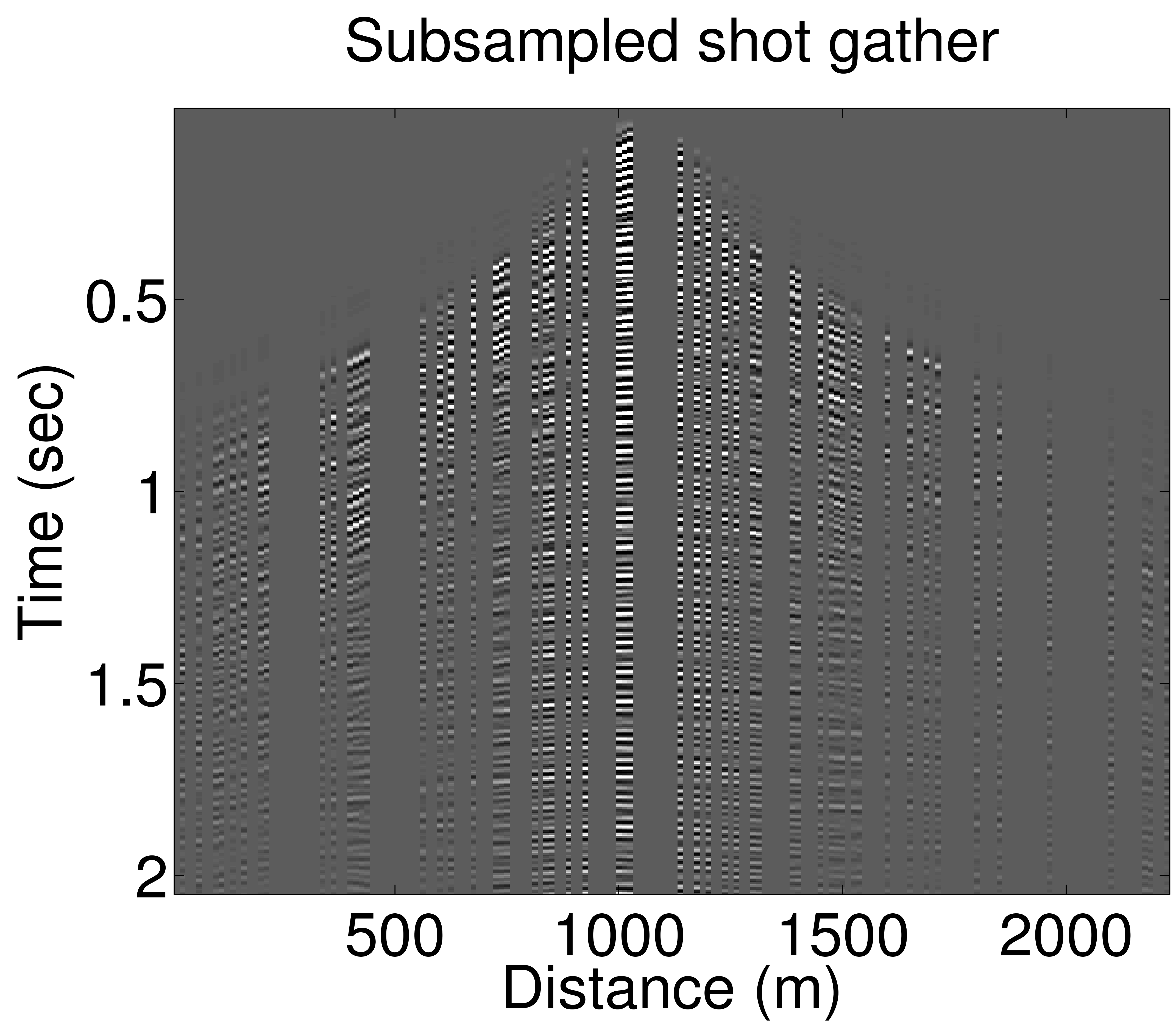
$$x^{\ell_1} := \underset{z \in R^P}{\text{minimize}} ||z||_1 \quad \text{subject to} \quad ||RMS^H z - b||_2 \leq \epsilon,$$

and approximate f by $S^H x^{\ell_1}$.

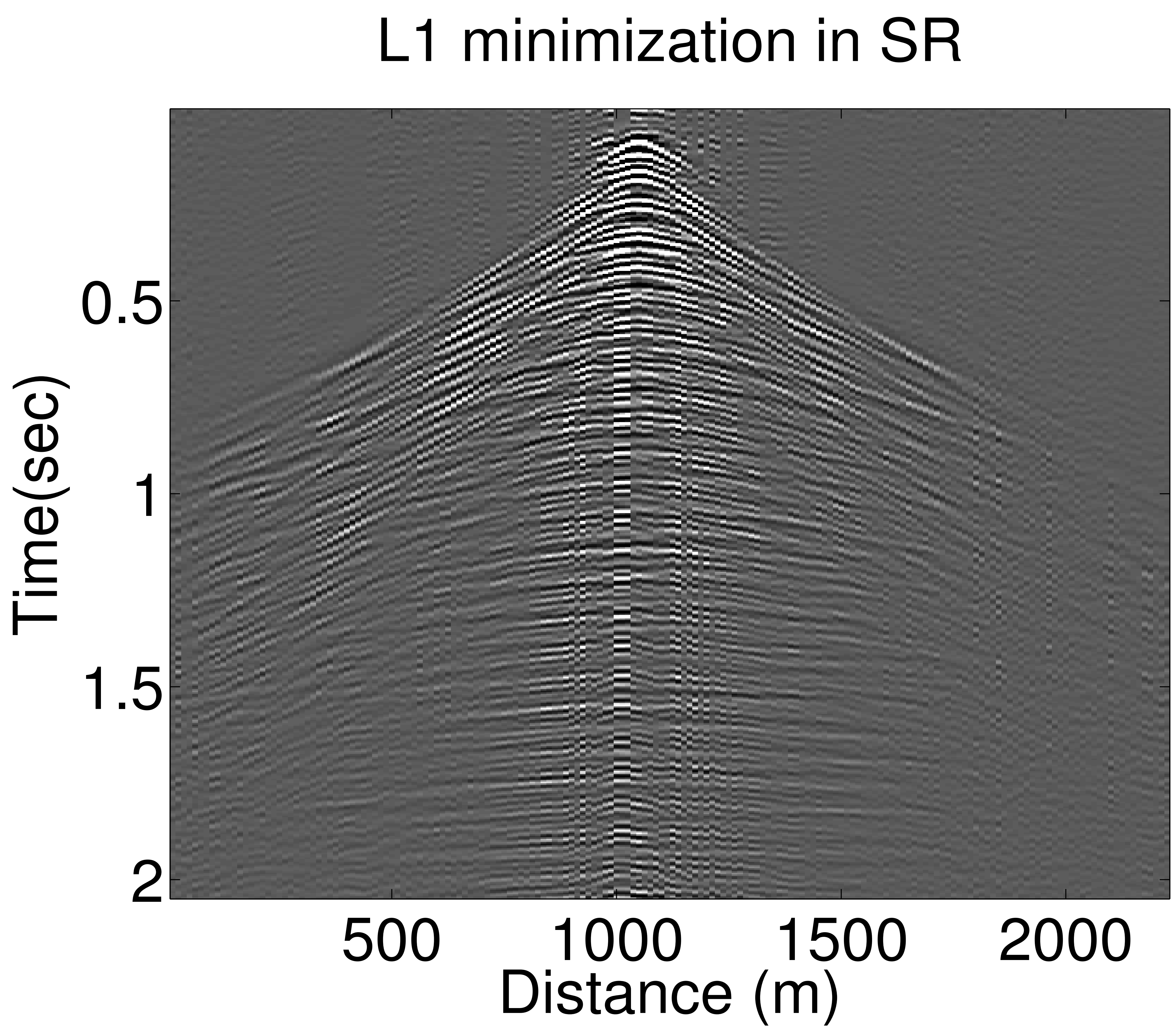


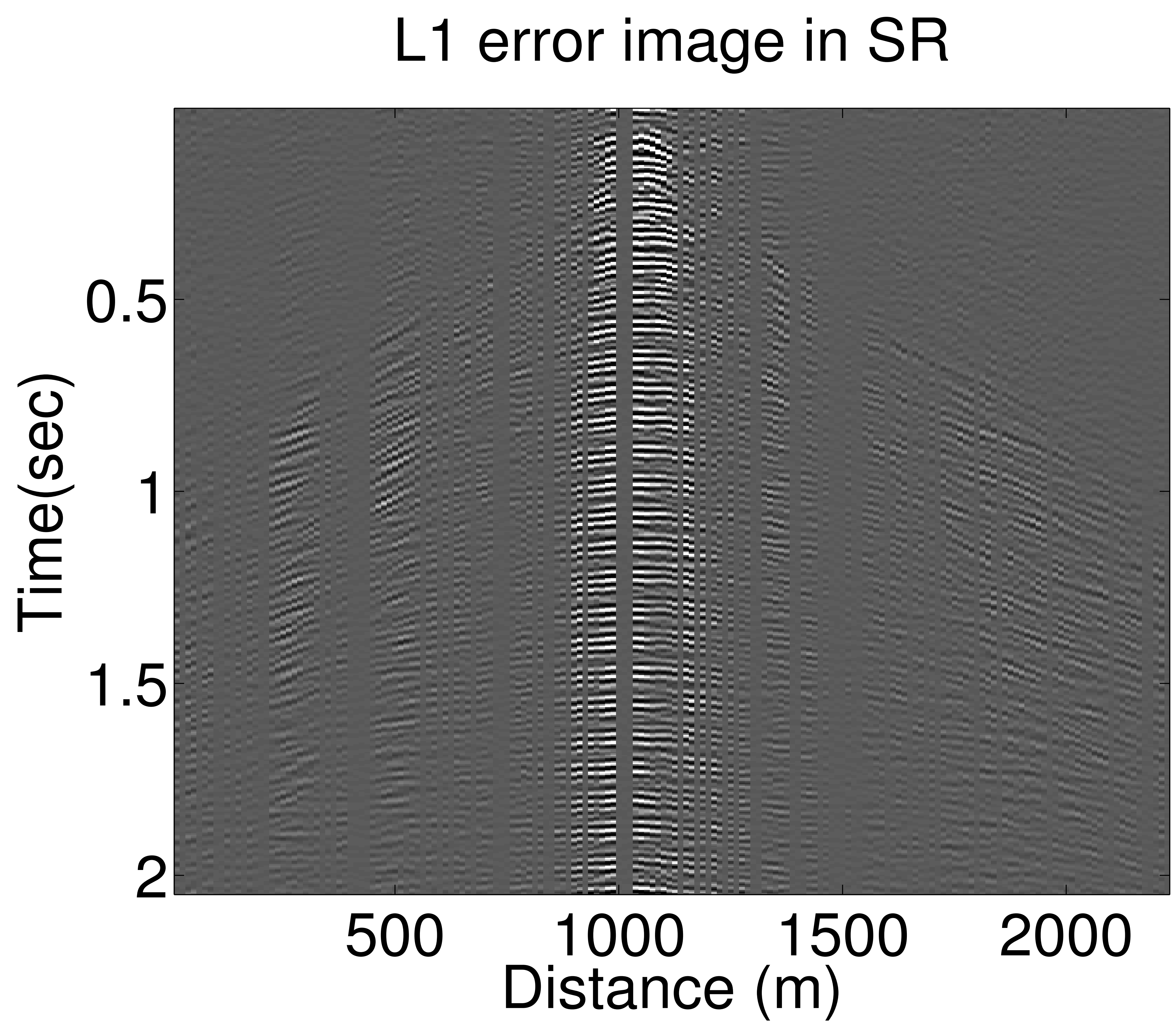
Recovery results (shotgather # 84)



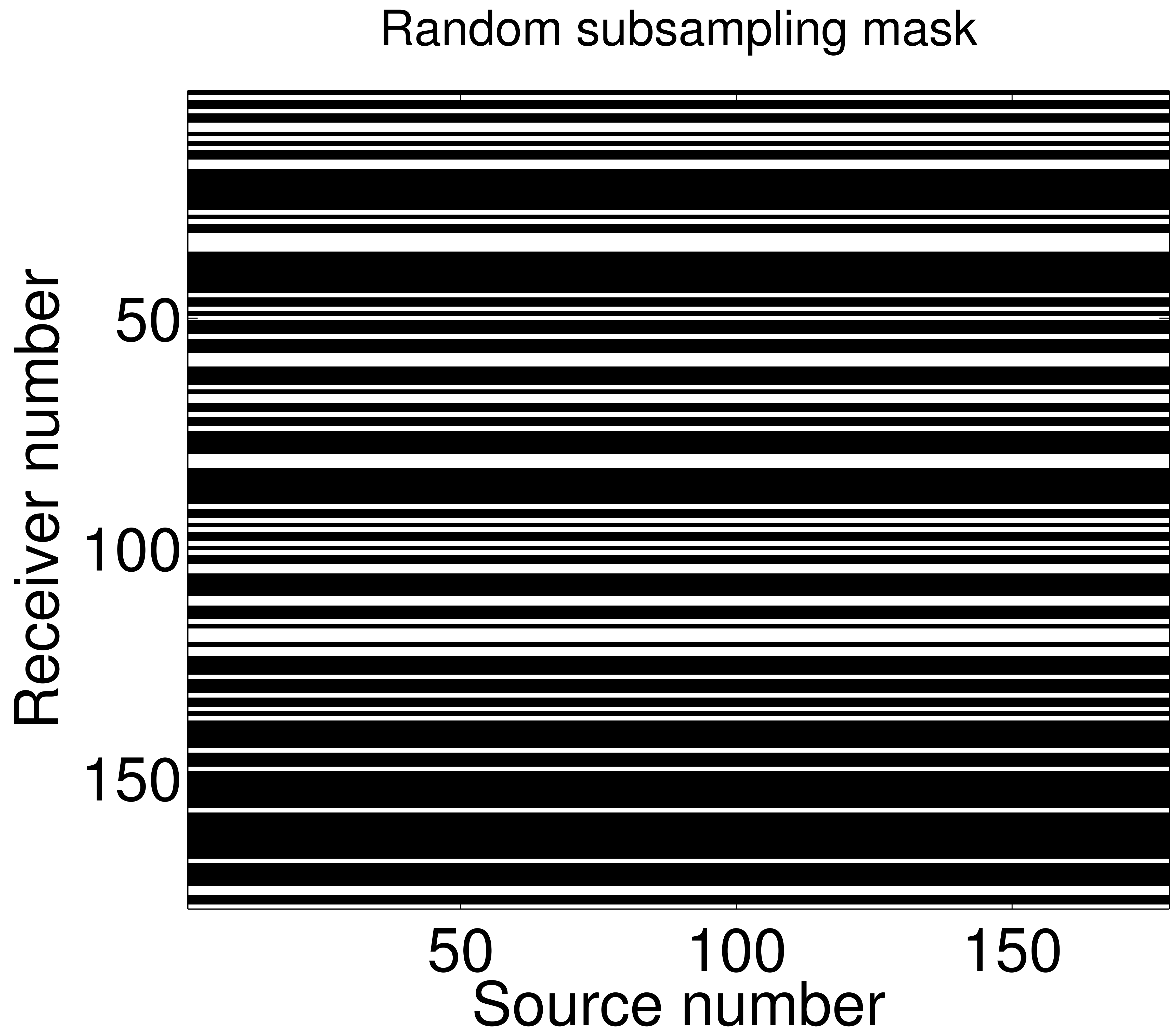


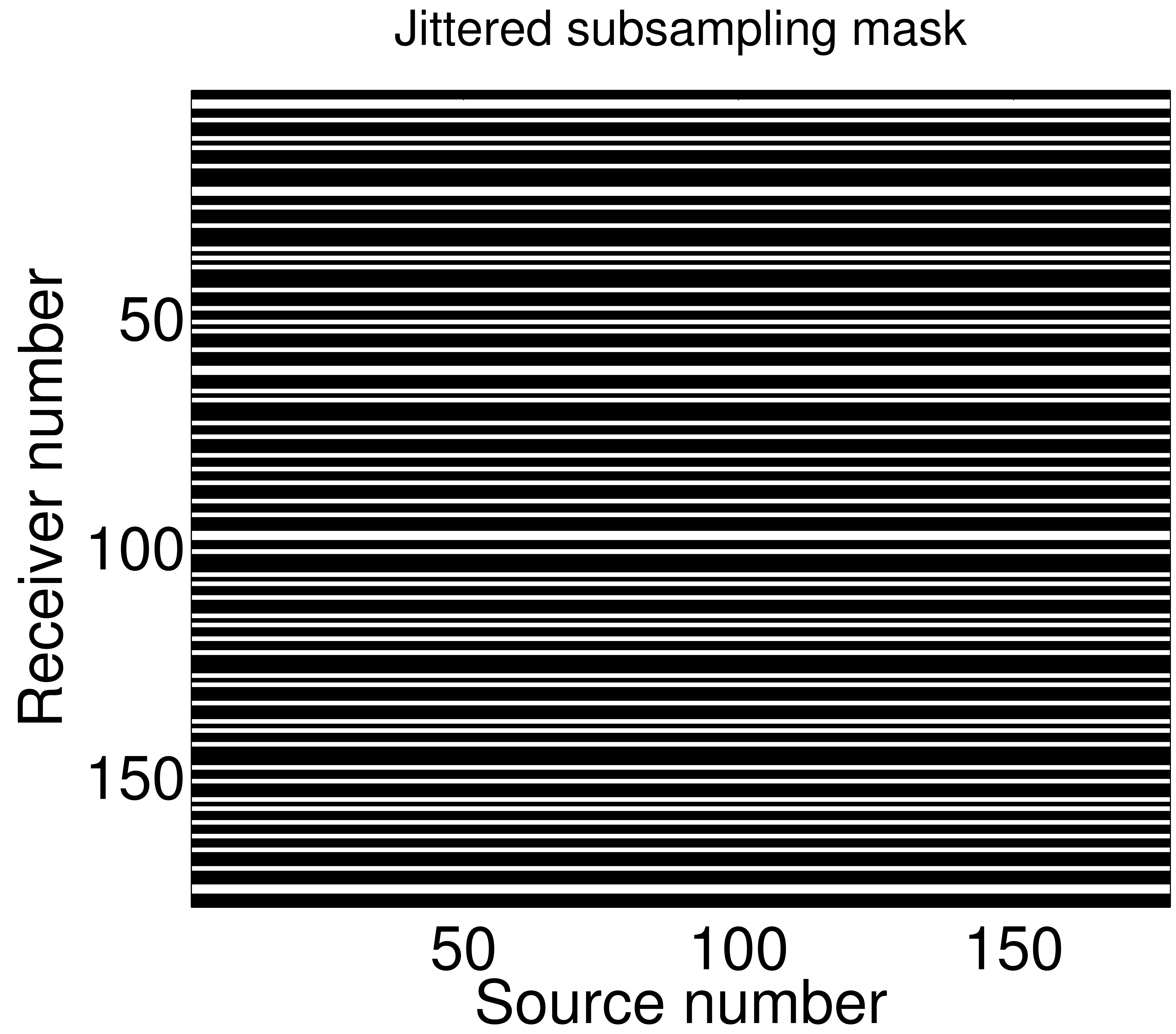
Recovery results: ℓ_1 minimization (5.3 dB)





Jittered subsampling mask





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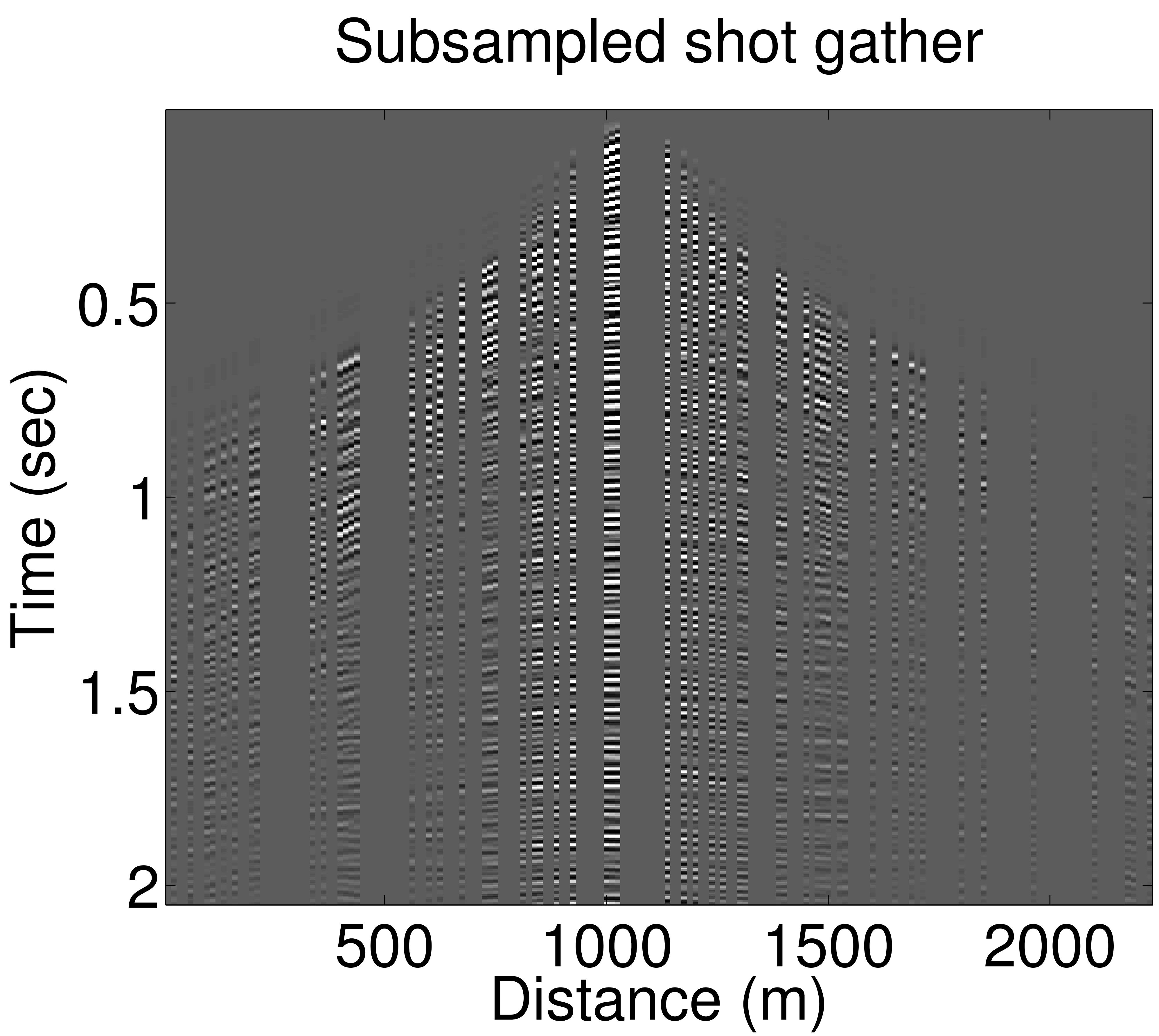
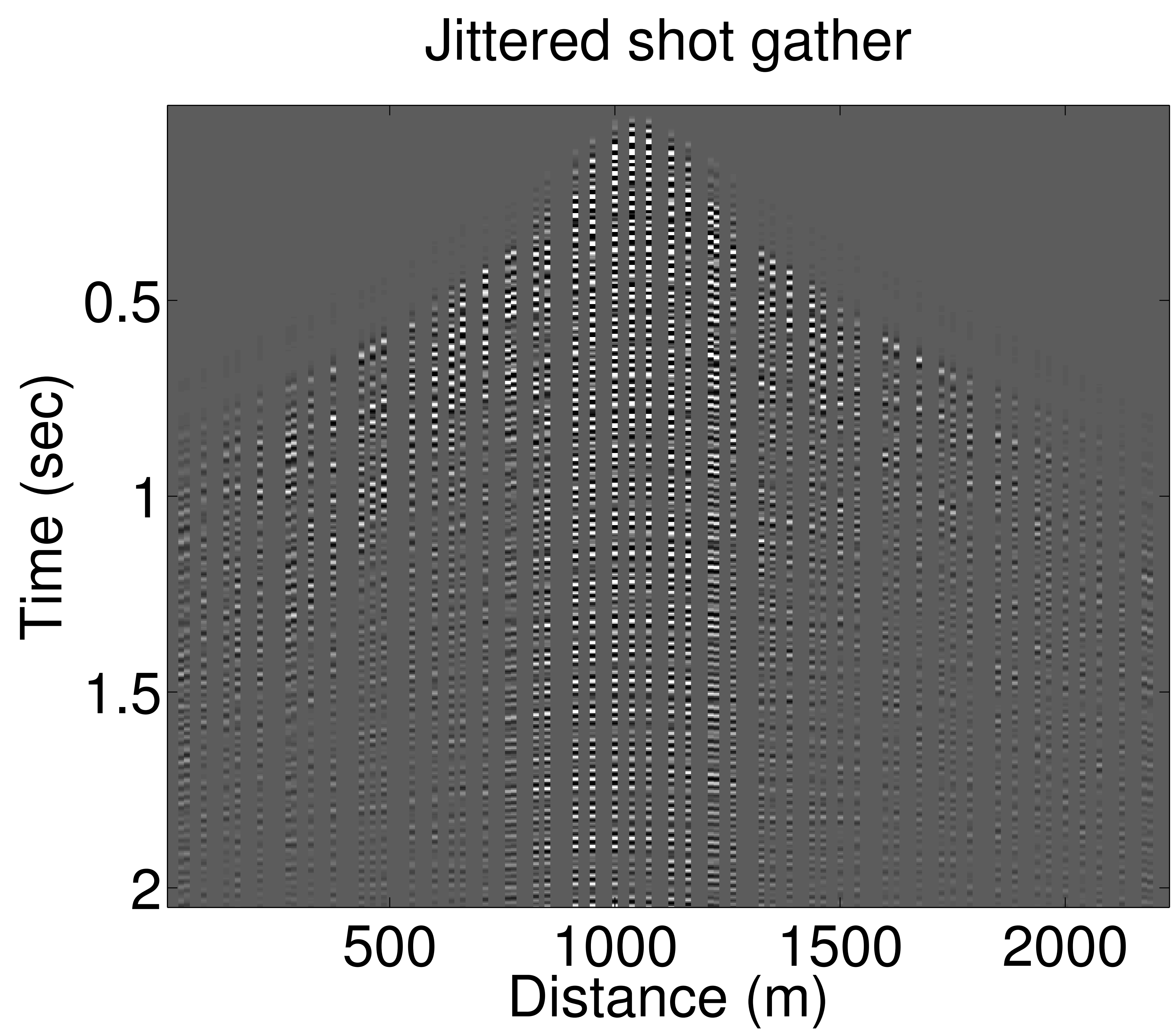
- Uniformly random subsampling might result in large gap in the data.
- Jittering is a safe alternative that doesn't allow large gaps in the data.

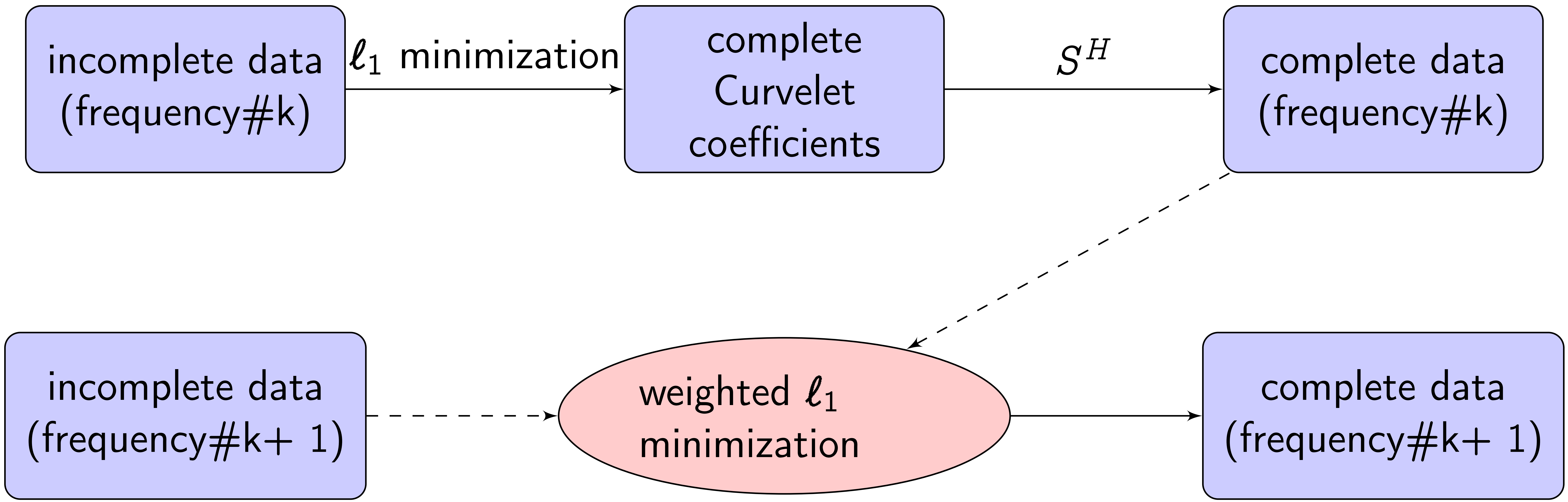
New subsampled data

Jittered sampling controls the average amount of information per row in the transform domain.





Weighted ℓ_1 for seismic trace interpolation



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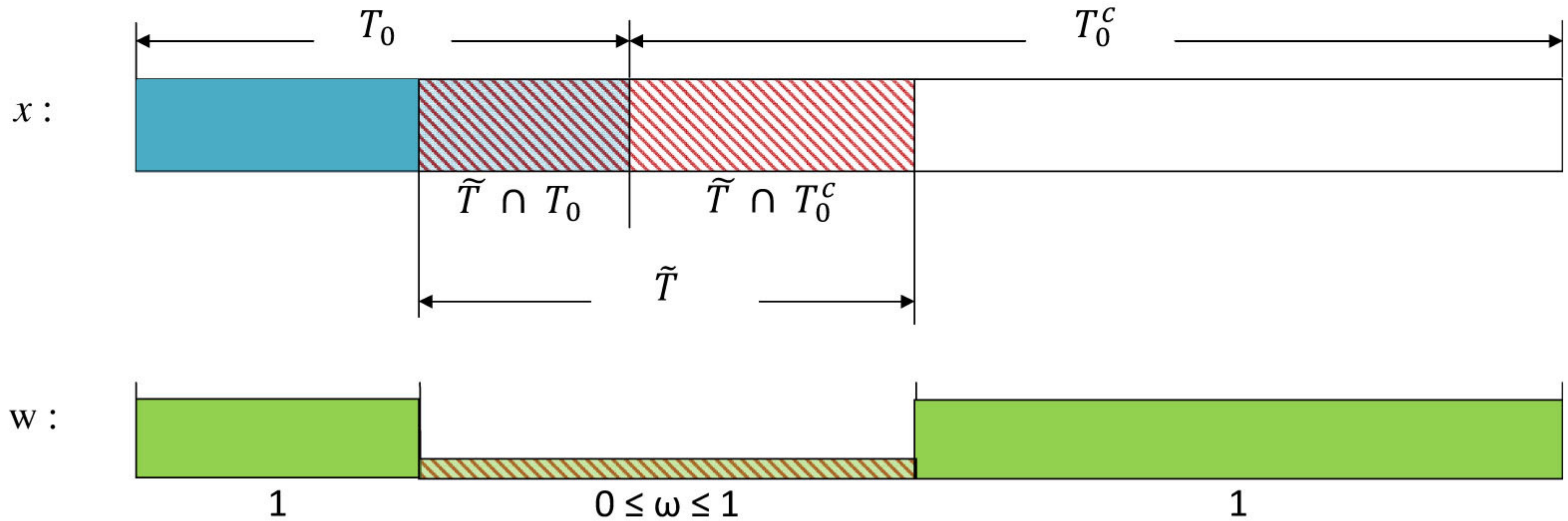
Weighted ℓ_1 for seismic trace interpolation

$$x^{w\ell_1} := \underset{z \in R^P}{\text{minimize}} ||z||_{1,\mathrm{w}} \quad \text{subject to} \quad ||RMS^H z - b||_2 \leq \epsilon,$$

- For a vector x, $||x||_{1,\mathrm{w}} := \Sigma_i \mathrm{w}_i |x_i|$ is the weighted ℓ_1 norm of x.
- $\mathrm{w}_i = \omega < 1$ if x_i is in the support estimate. Otherwise $\mathrm{w}_i = 1$.



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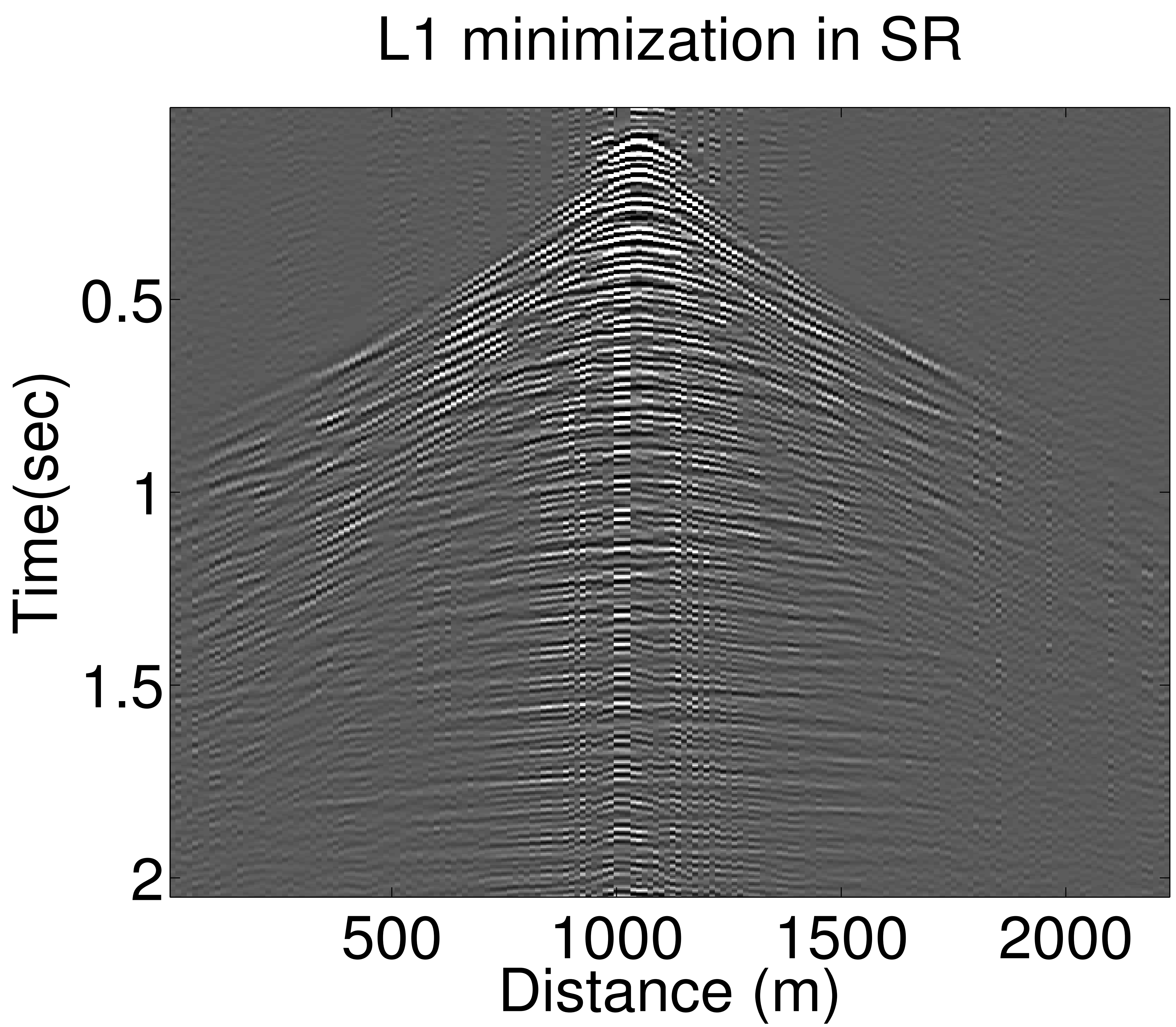
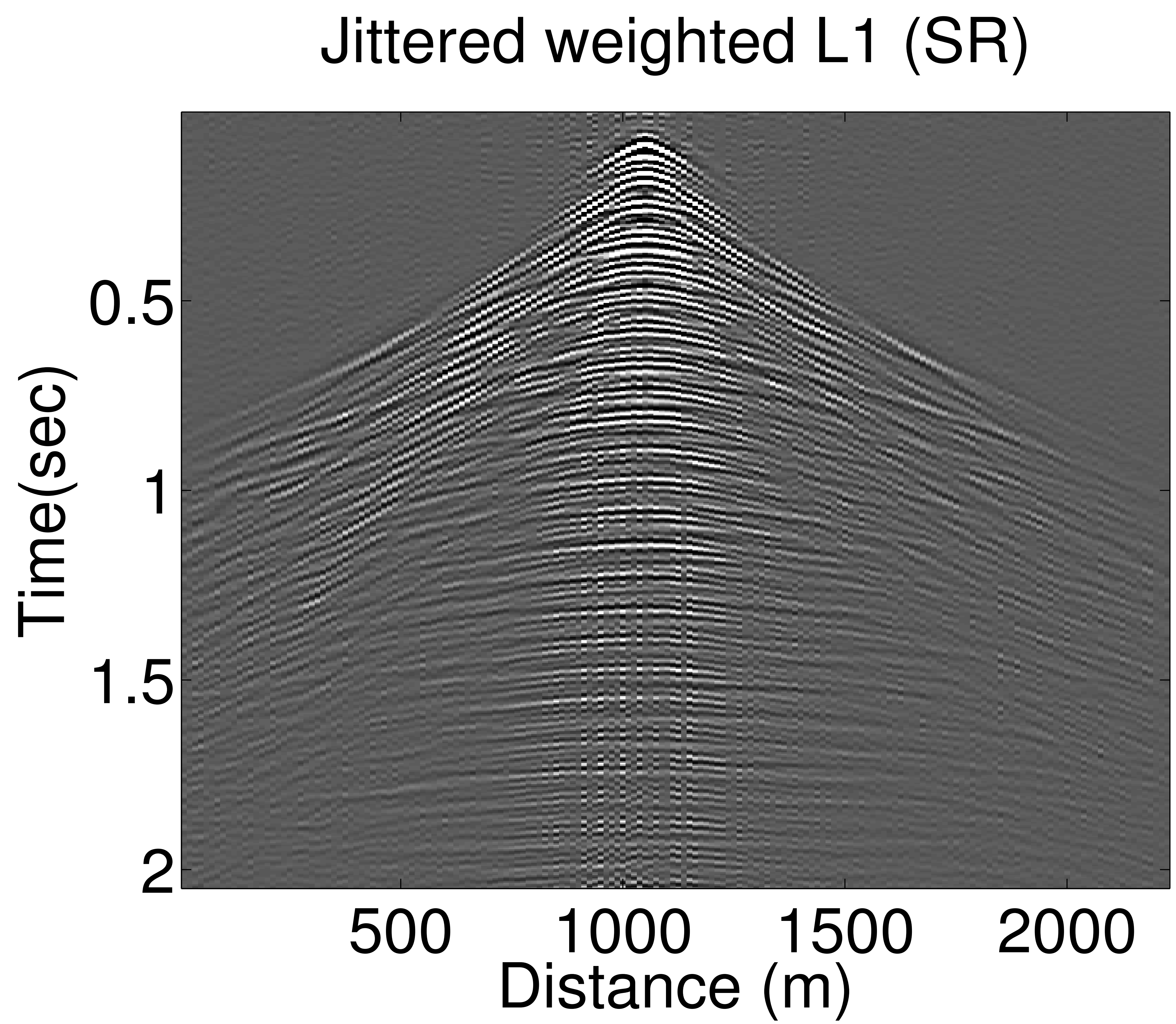
- We utilize the continuity along adjacent frequency slices by weighting.
- If the support estimate is at least 50% accurate, we get better results.

Recovery results: ℓ_1 vs weighted ℓ_1

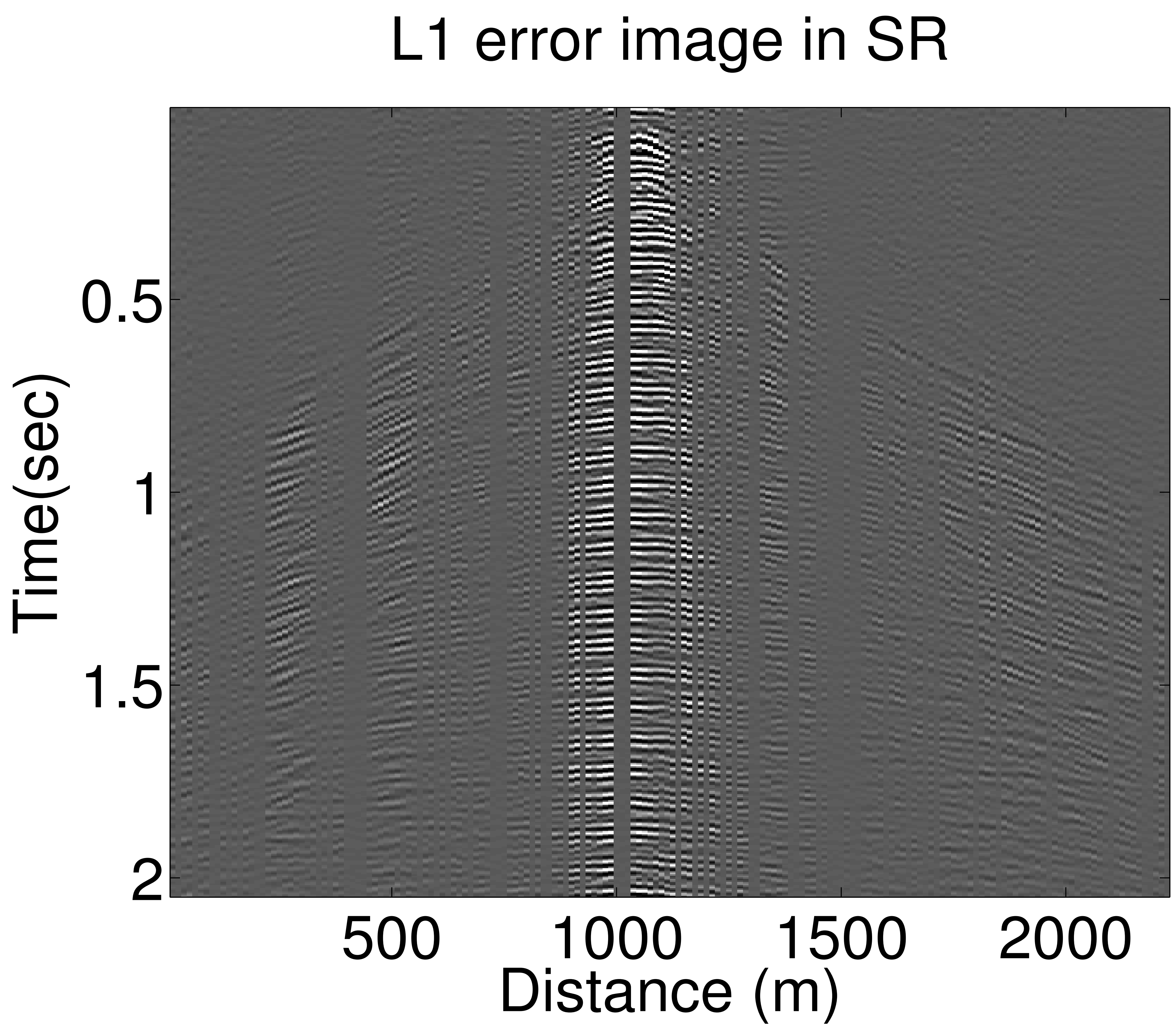


(k) SNR=5.4 dB

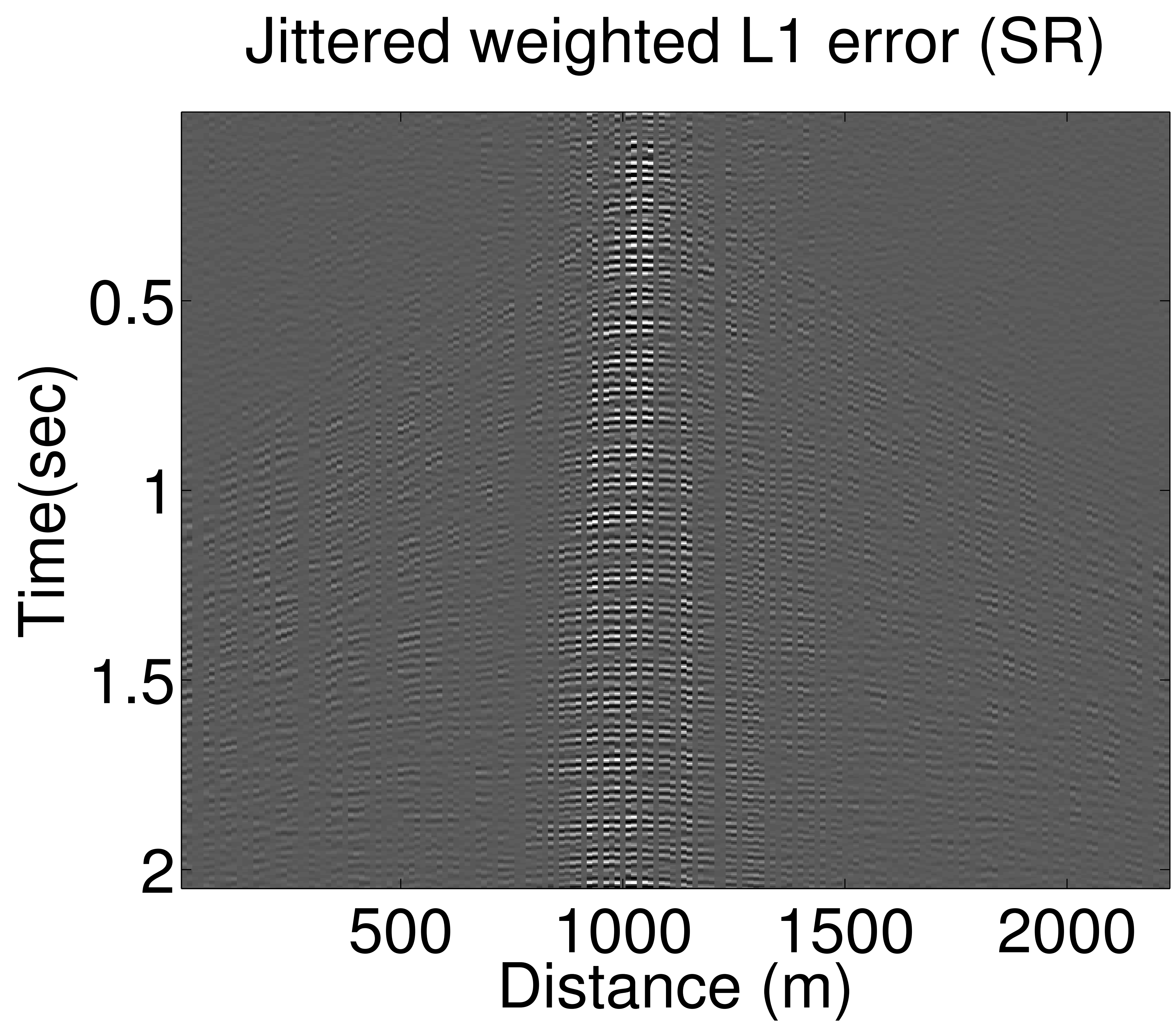


(l) SNR= 8.8 dB

Recovery error: ℓ_1 vs weighted ℓ_1

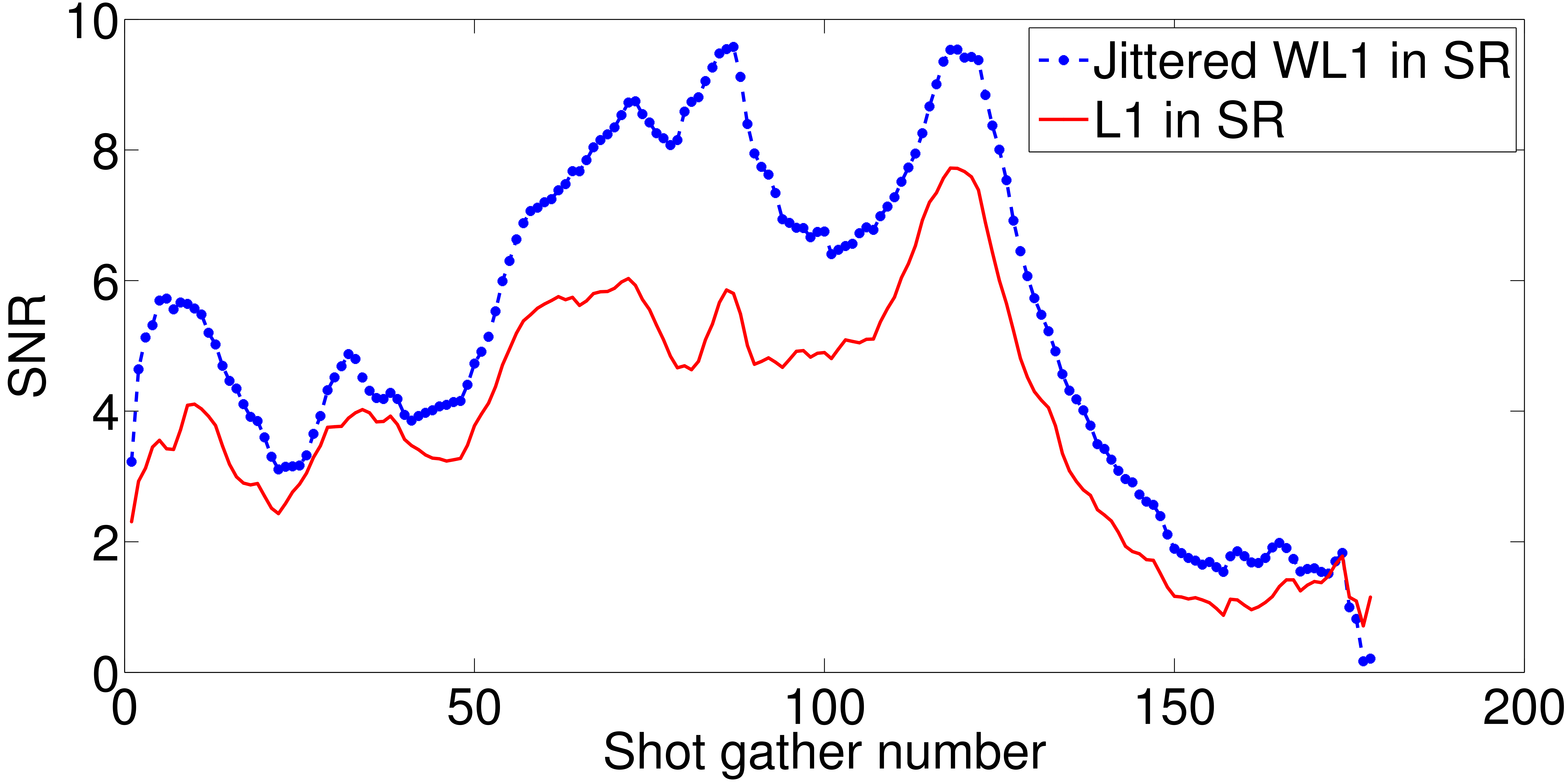


(m) SNR=5.4 dB



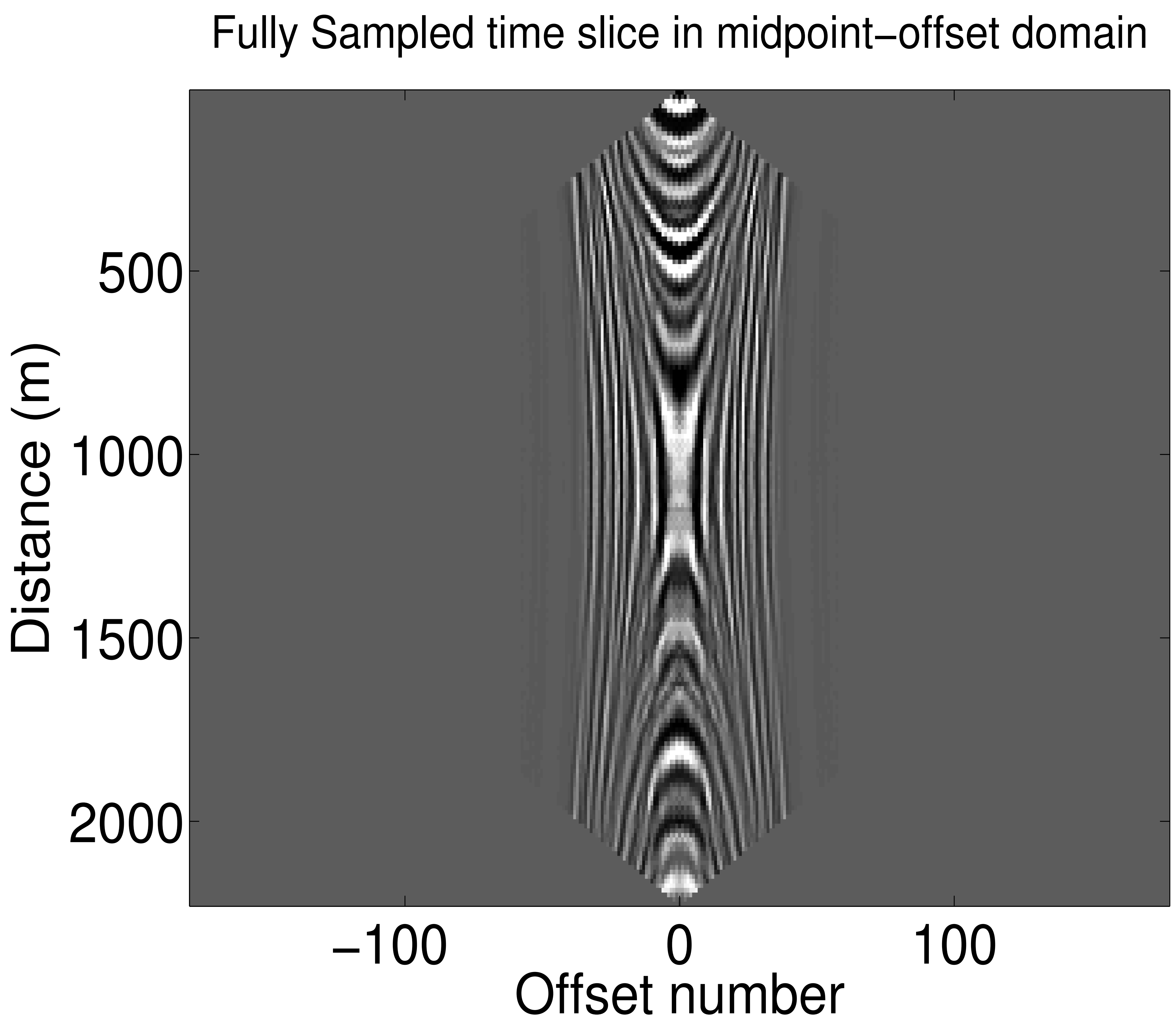
(n) SNR= 8.8 dB

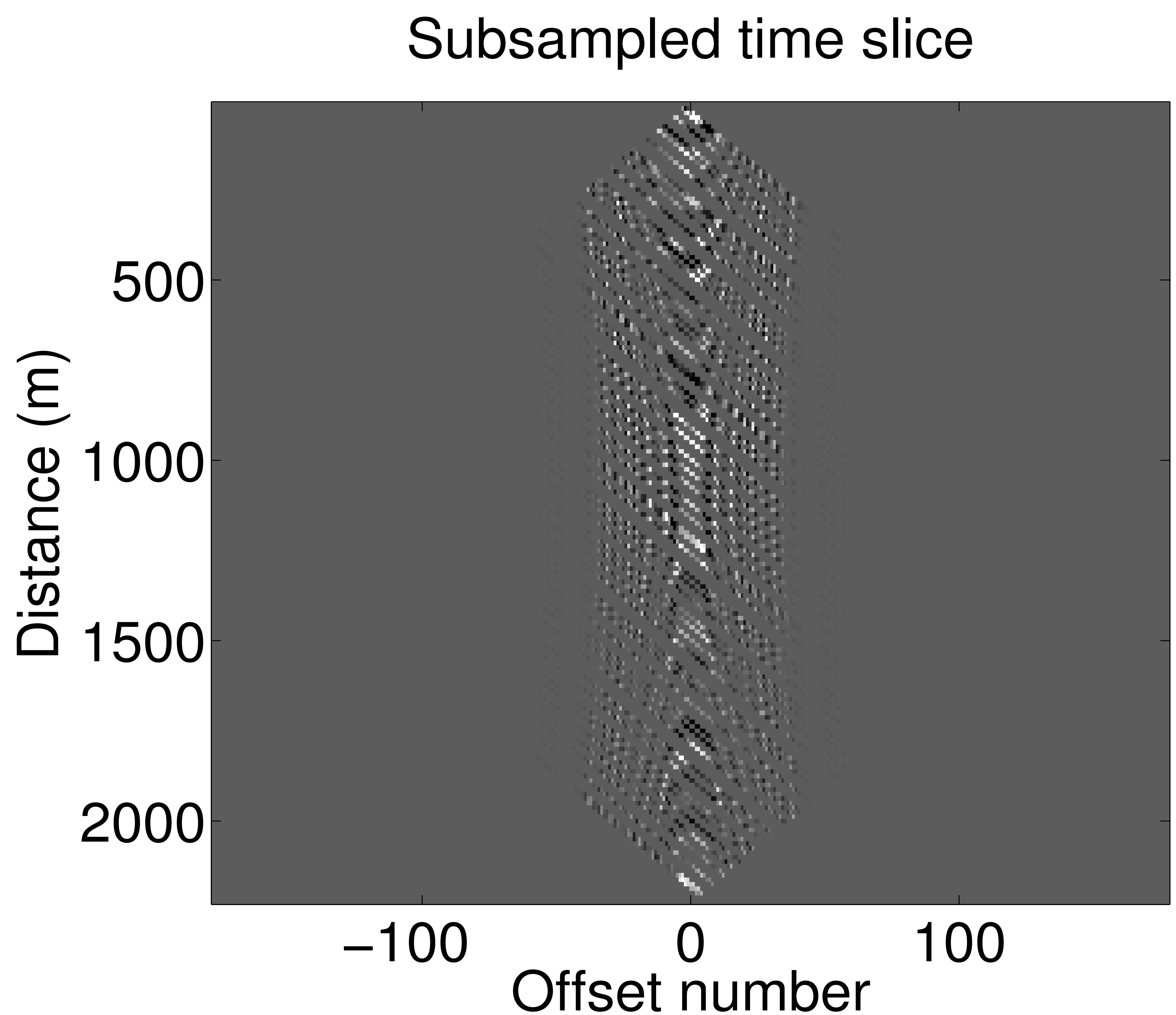
Shot gathers: ℓ_1 vs weighted ℓ_1



Recovery in the midpoint-offset domain

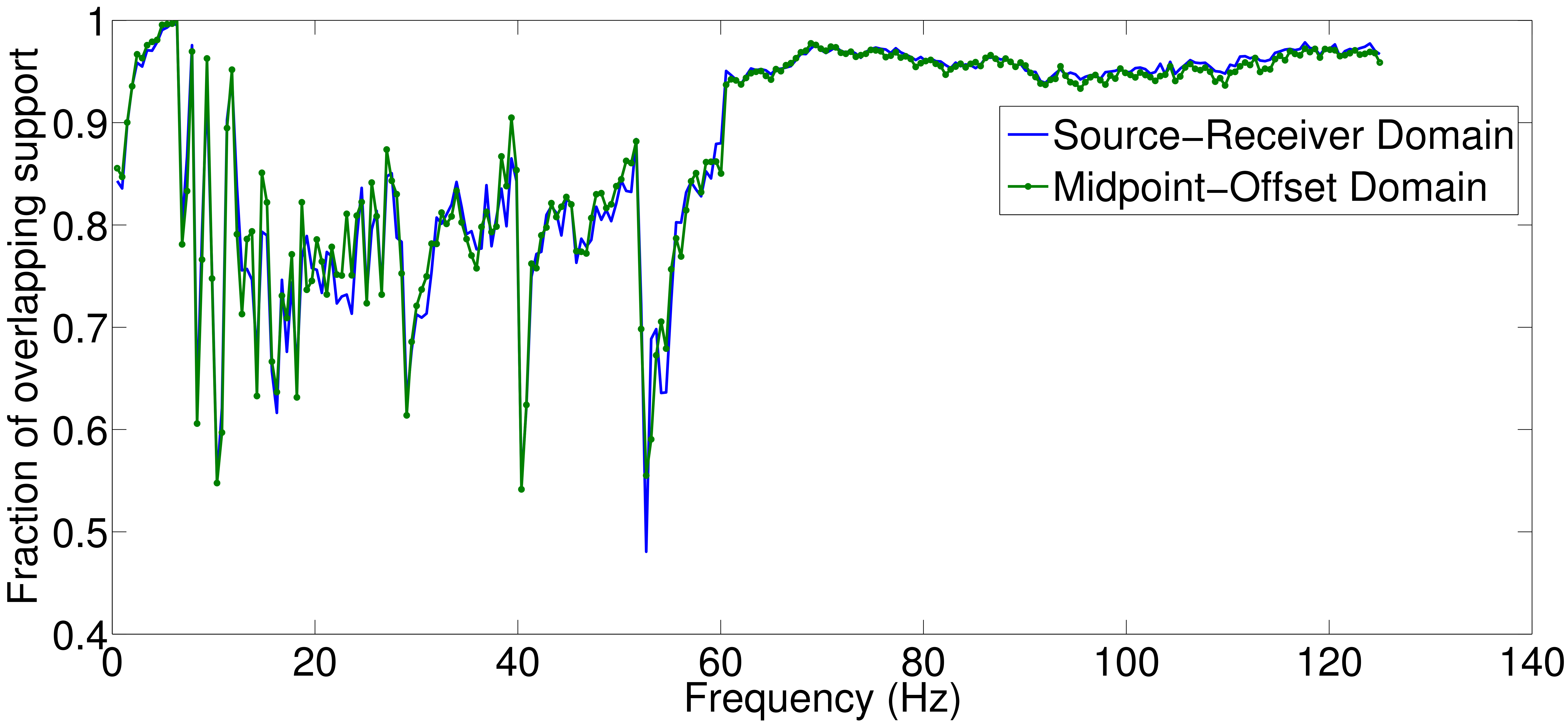
We transform the seismic line into the frequency-midpoint-offset (MH) domain.



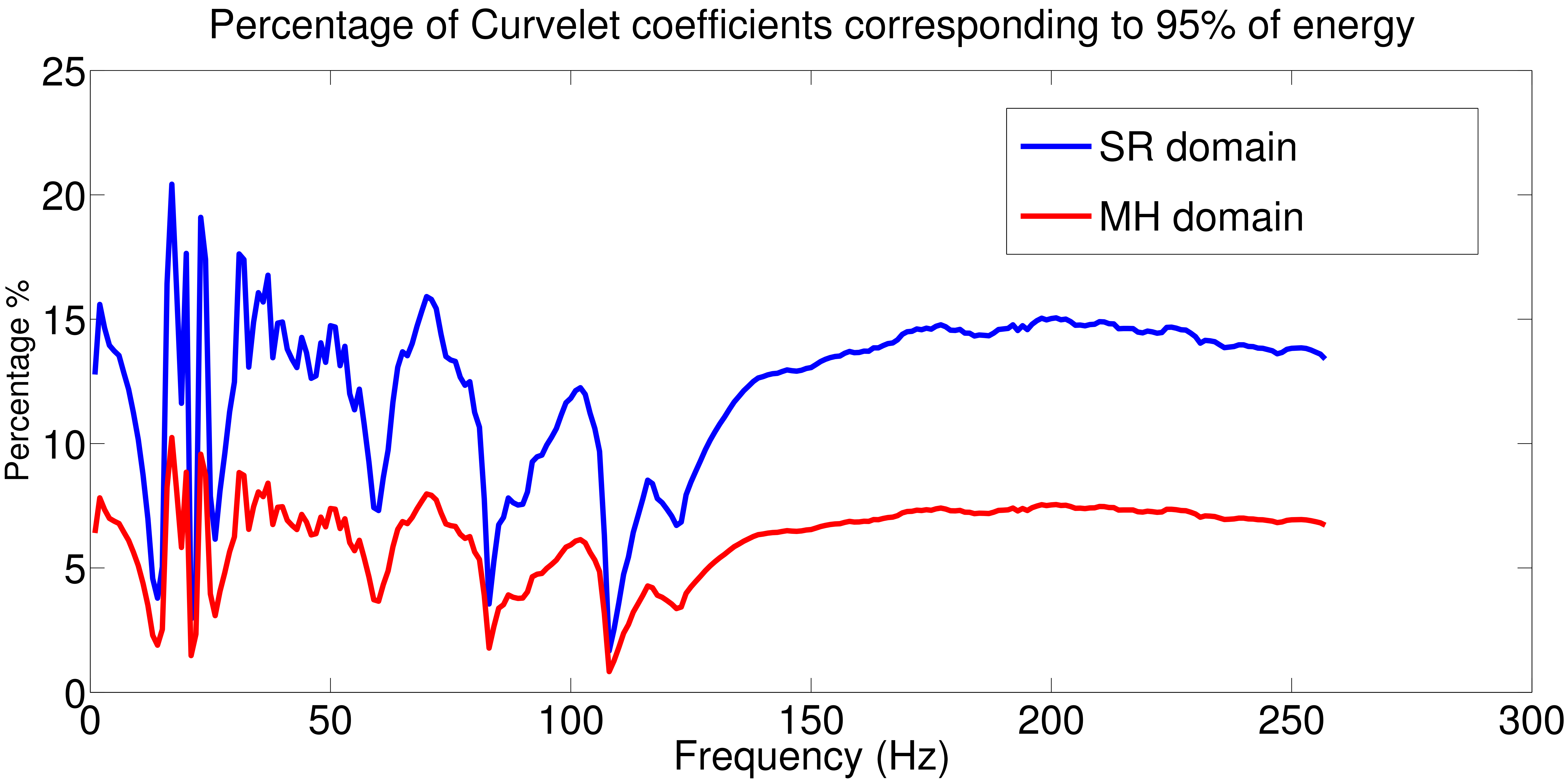


Weighting in MH and SR domain

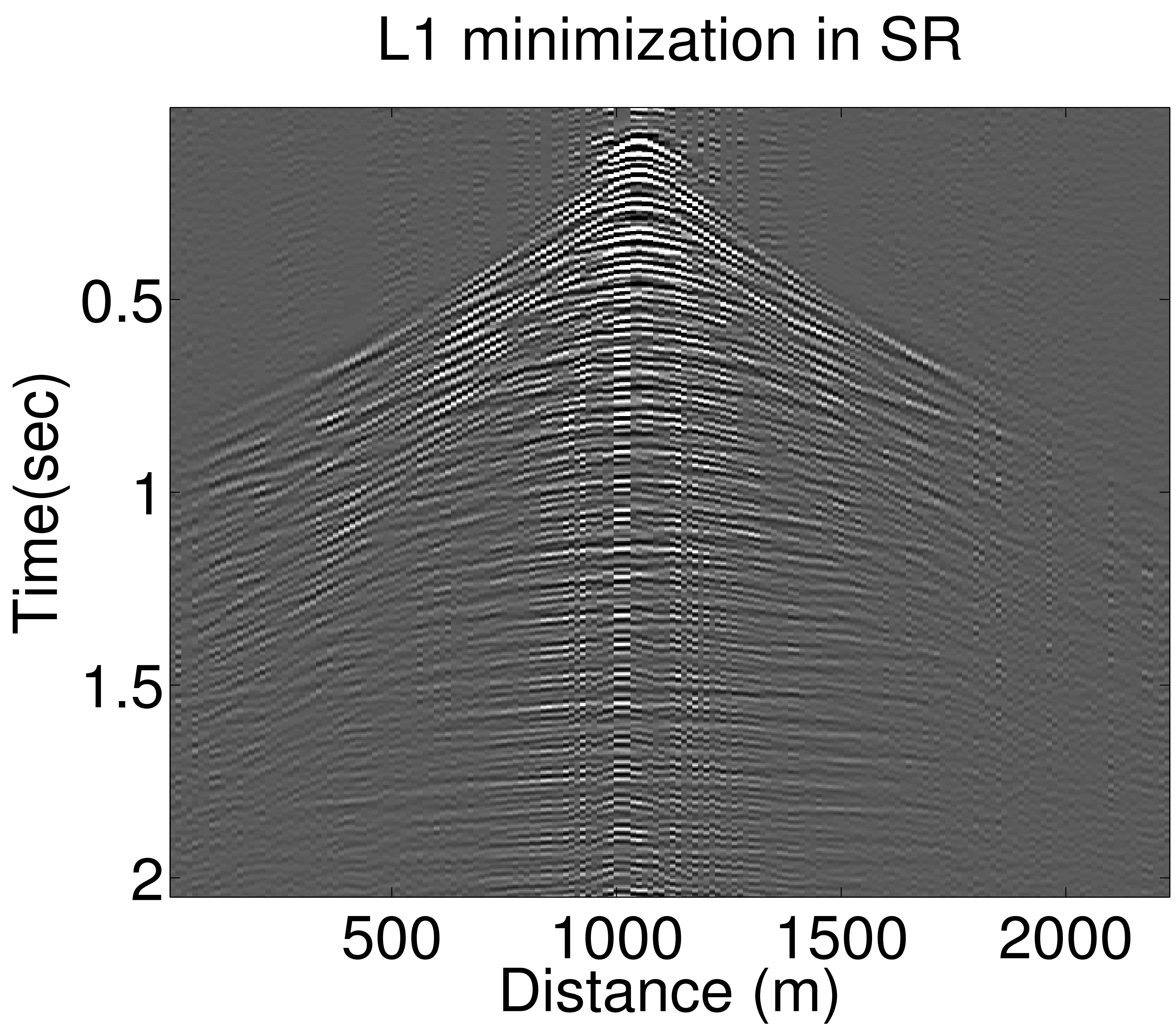
- Similar to the SR domain we do the recovery by utilizing frequency slices.
- The adjacent frequency slices have overlapping support in both cases.



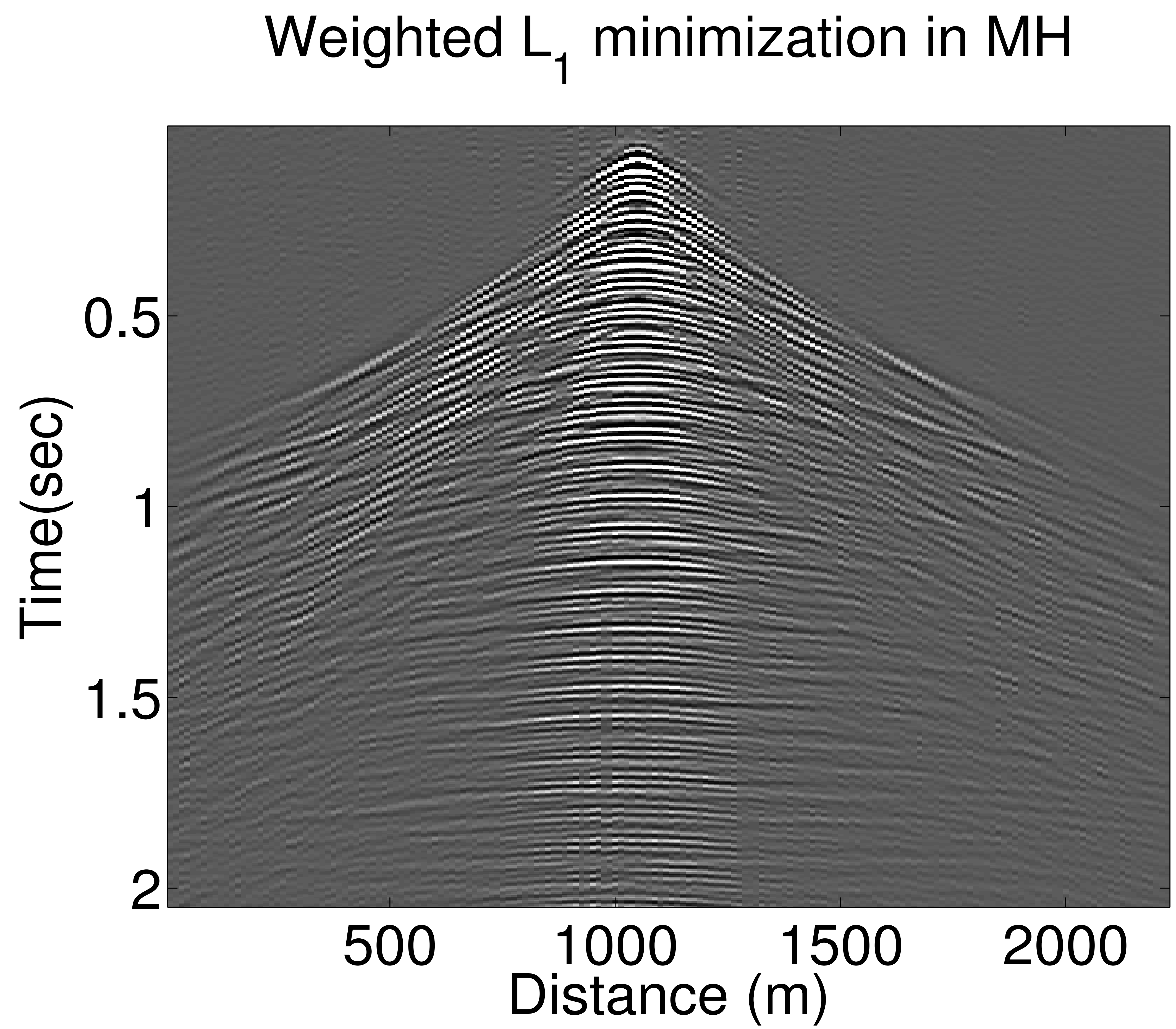
Why does transforming to MH domain help?



Recovery results: ℓ_1 in SR vs weighted ℓ_1 in MH

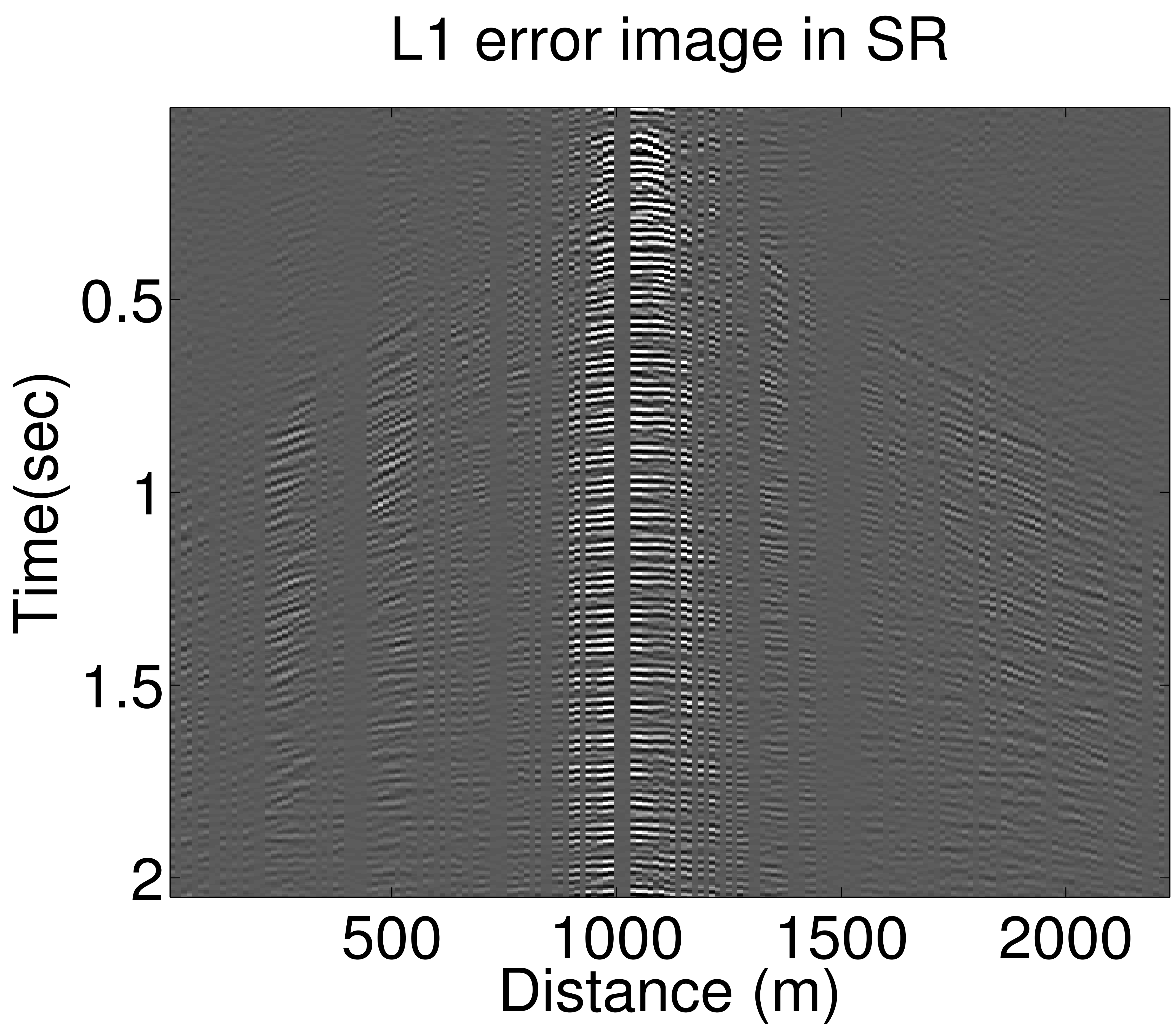


(q) SNR=5.4 dB

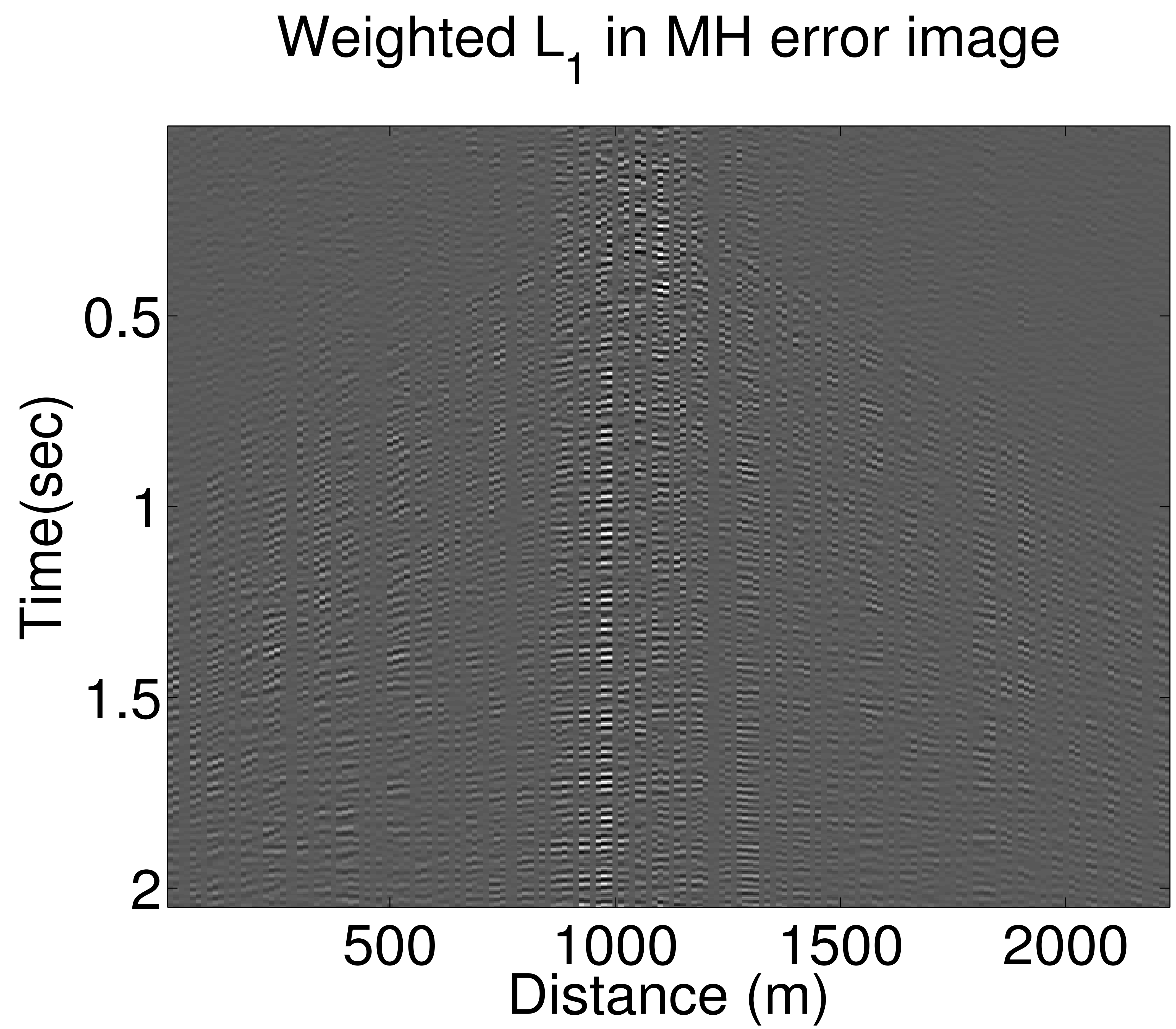


(r) SNR= 12.8 dB

Recovery error: ℓ_1 in SR vs weighted ℓ_1 in MH

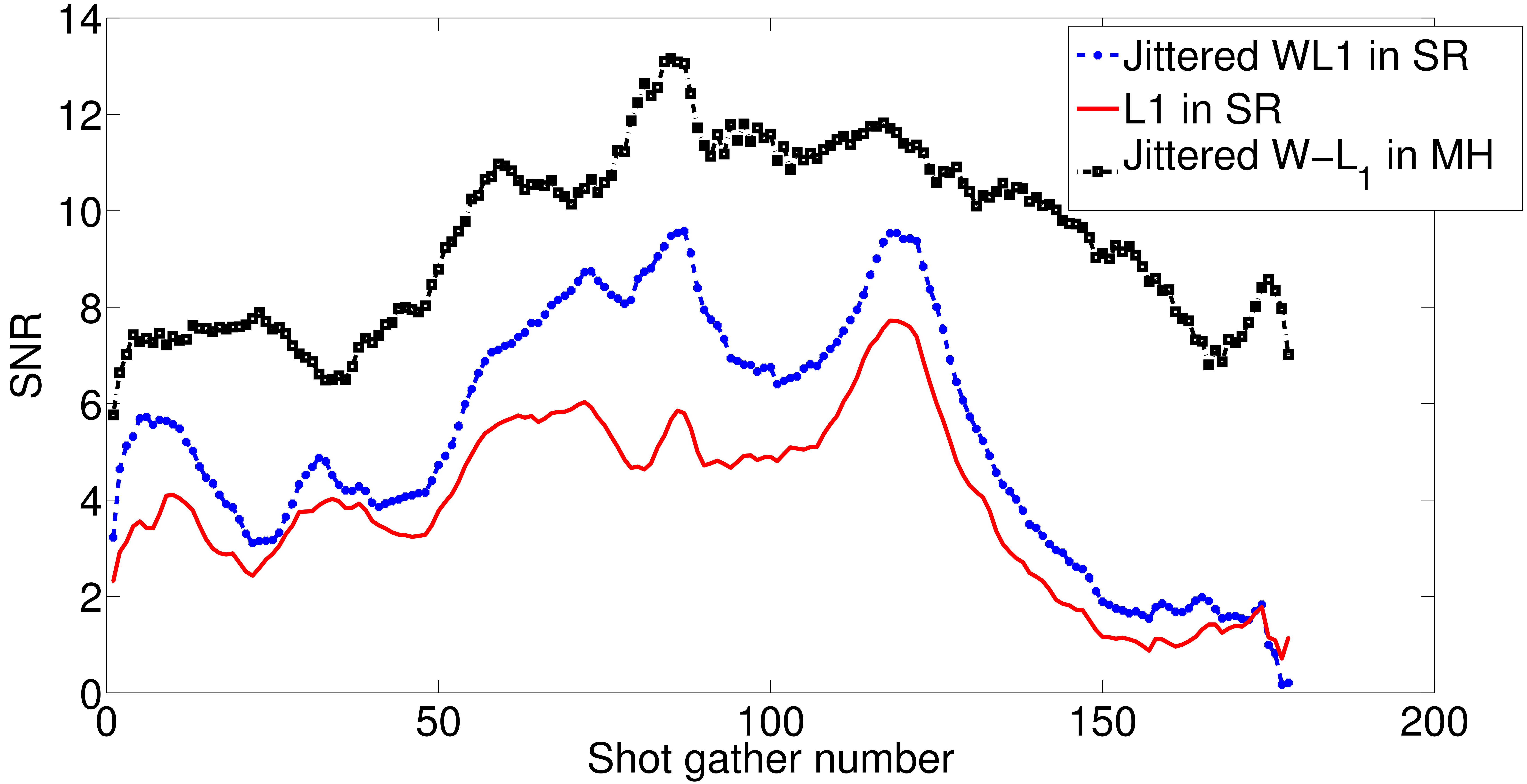


(s) SNR=5.4 dB



(t) SNR= 12.8 dB

Shot gathers results



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2-stage algorithm

- For each frequency slice first, use a fast algorithm to get a rough estimate of the data in the time domain.

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- Use the estimate in curvelet domain to improve the recovery results.

Approximate message passing (AMP)

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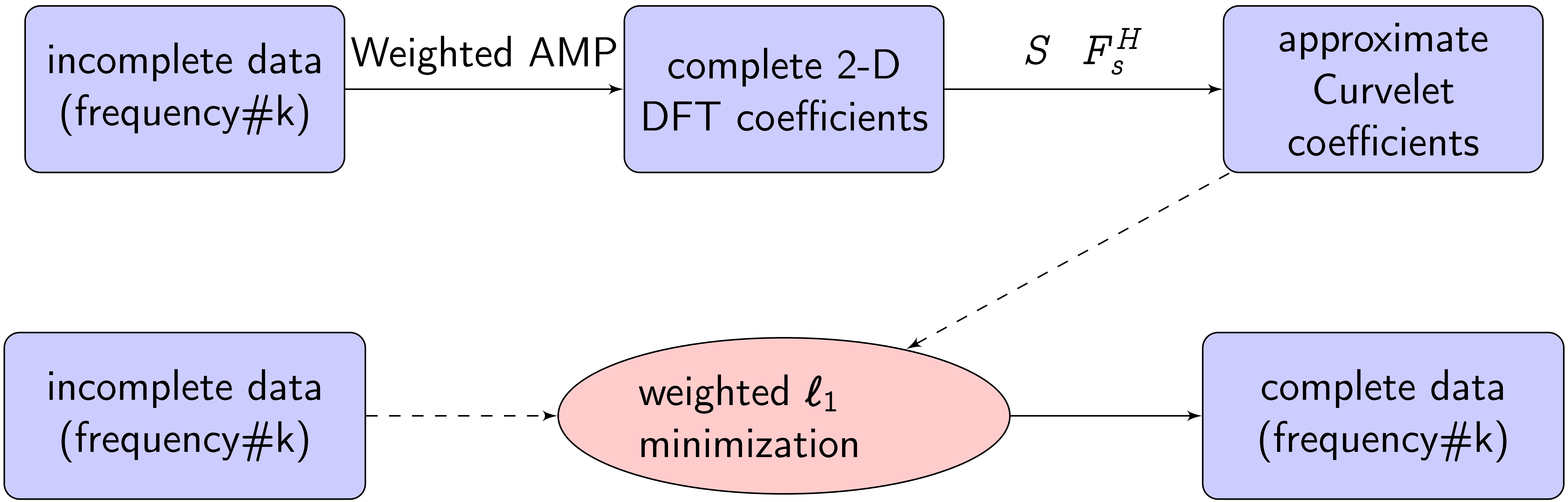
- AMP starts from an initial x^0 and iteratively goes by

$$\begin{aligned} x^{t+1} &= \eta(x^t + A^* z^t; \tau^t), \\ z^t &= y - Ax^t + \delta^{-1} z^{t-1} \left(\frac{\#\{|x^{t-1} + A^* z^{t-1}| > \tau^{t-1}\}}{N} \right). \end{aligned} \tag{1}$$

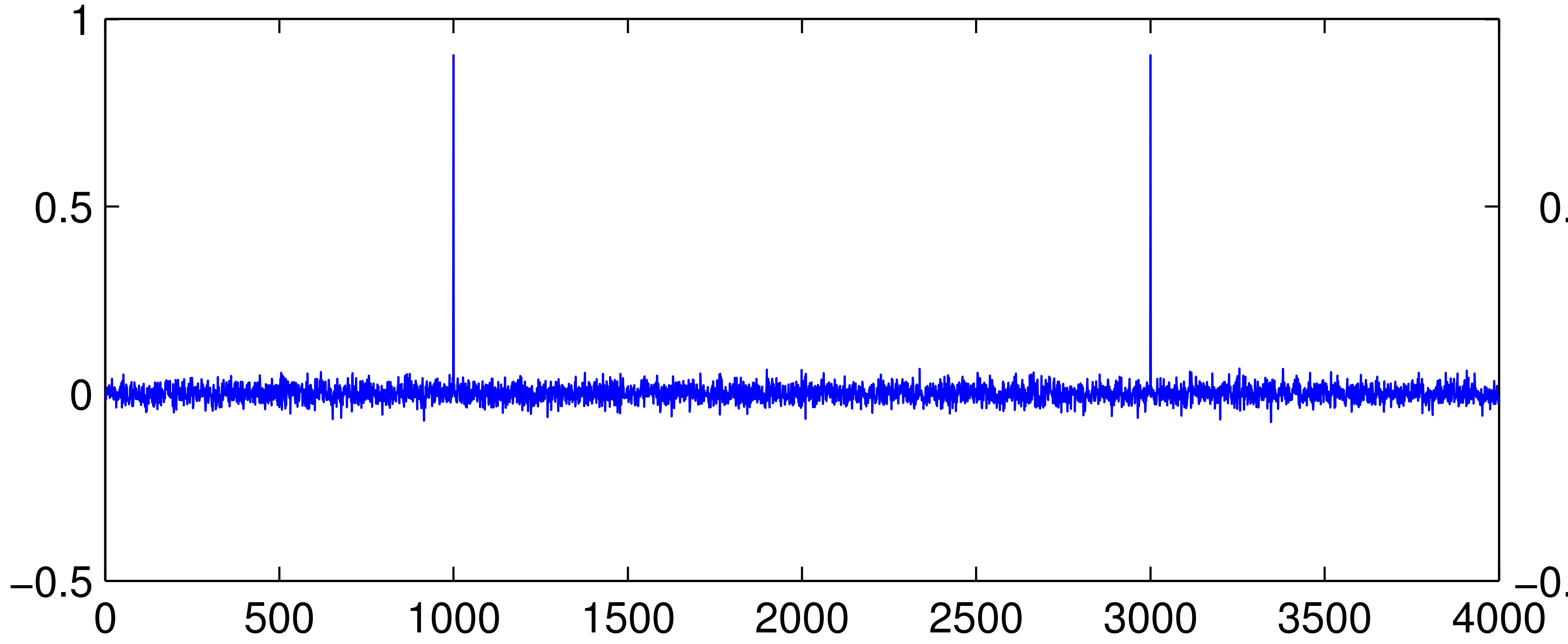
- η is the soft thresholding function $[\eta(x; s)]_j = sign(x_j)(|x_j| - s)_+$

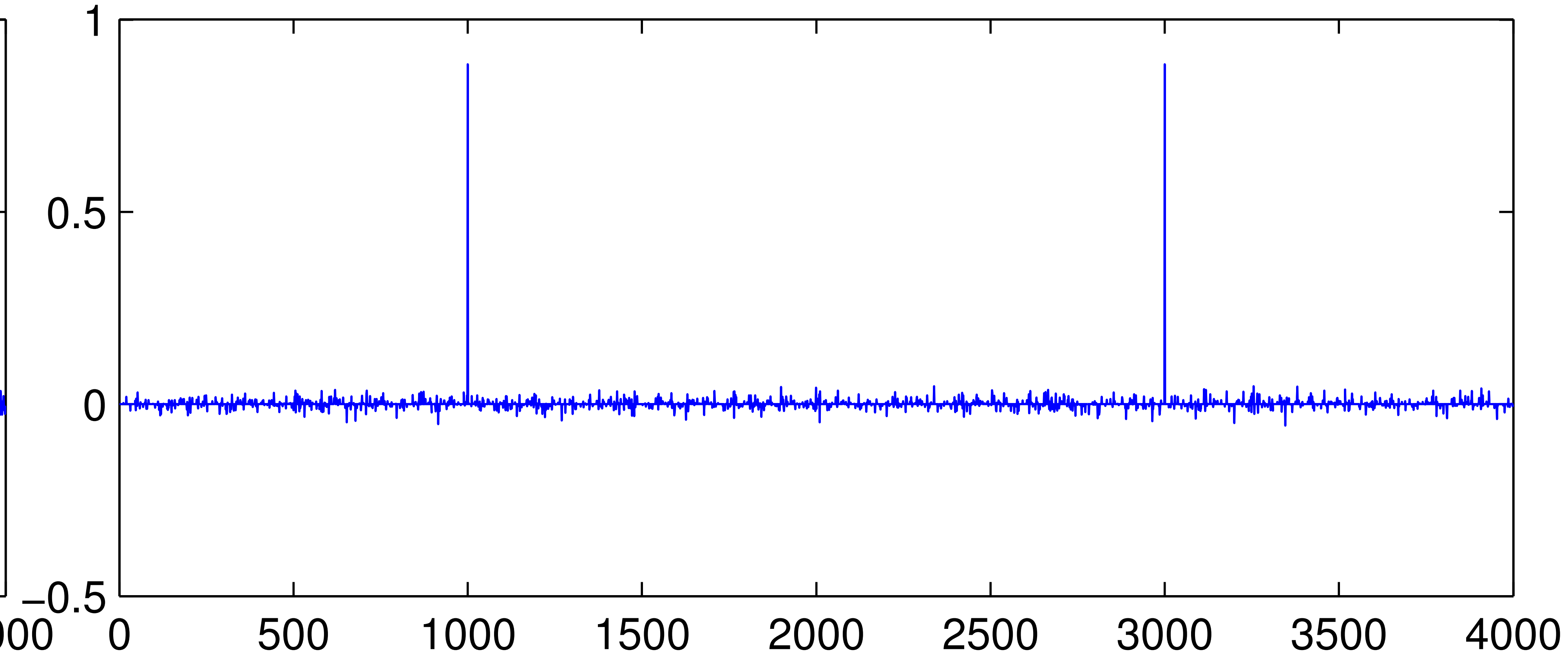
The 2-stage algorithm WAMP+weighted ℓ_1



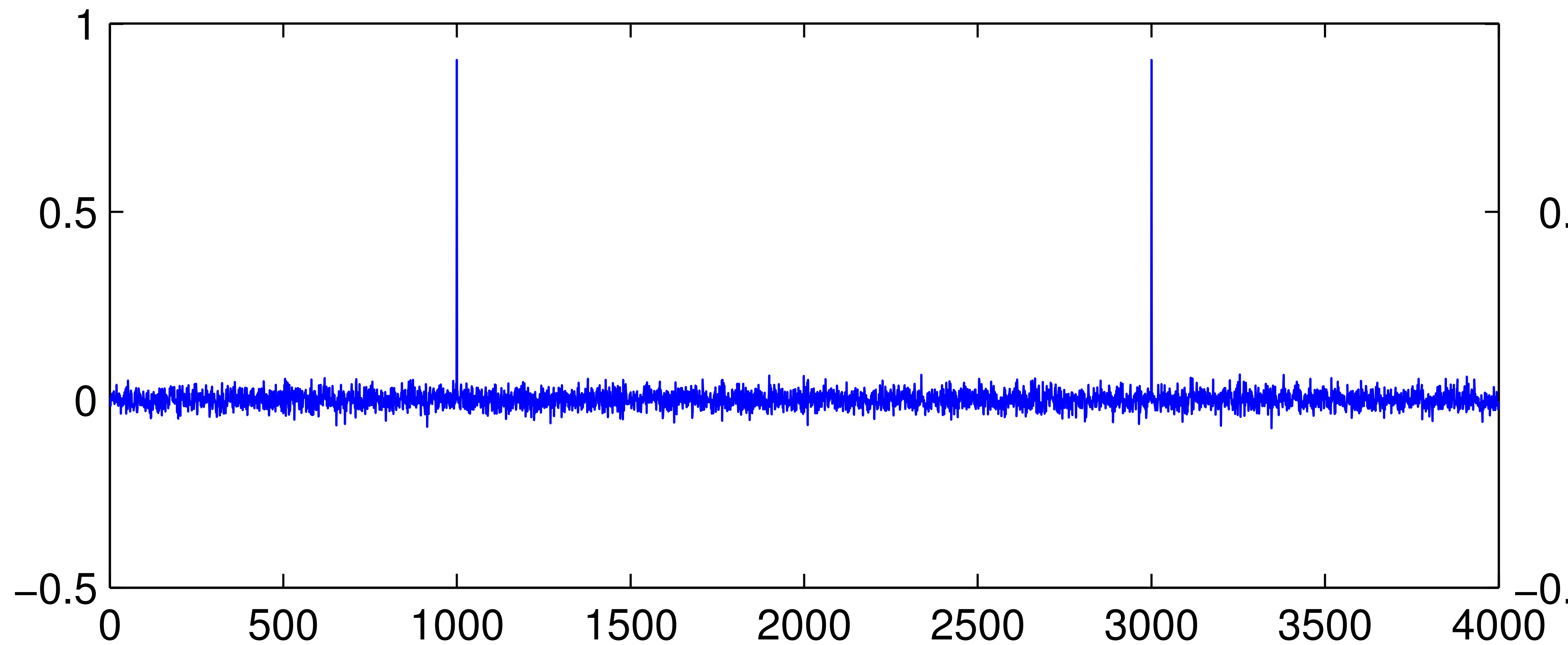
Simple example with a 2-sparse signal, iteration $t = 1$

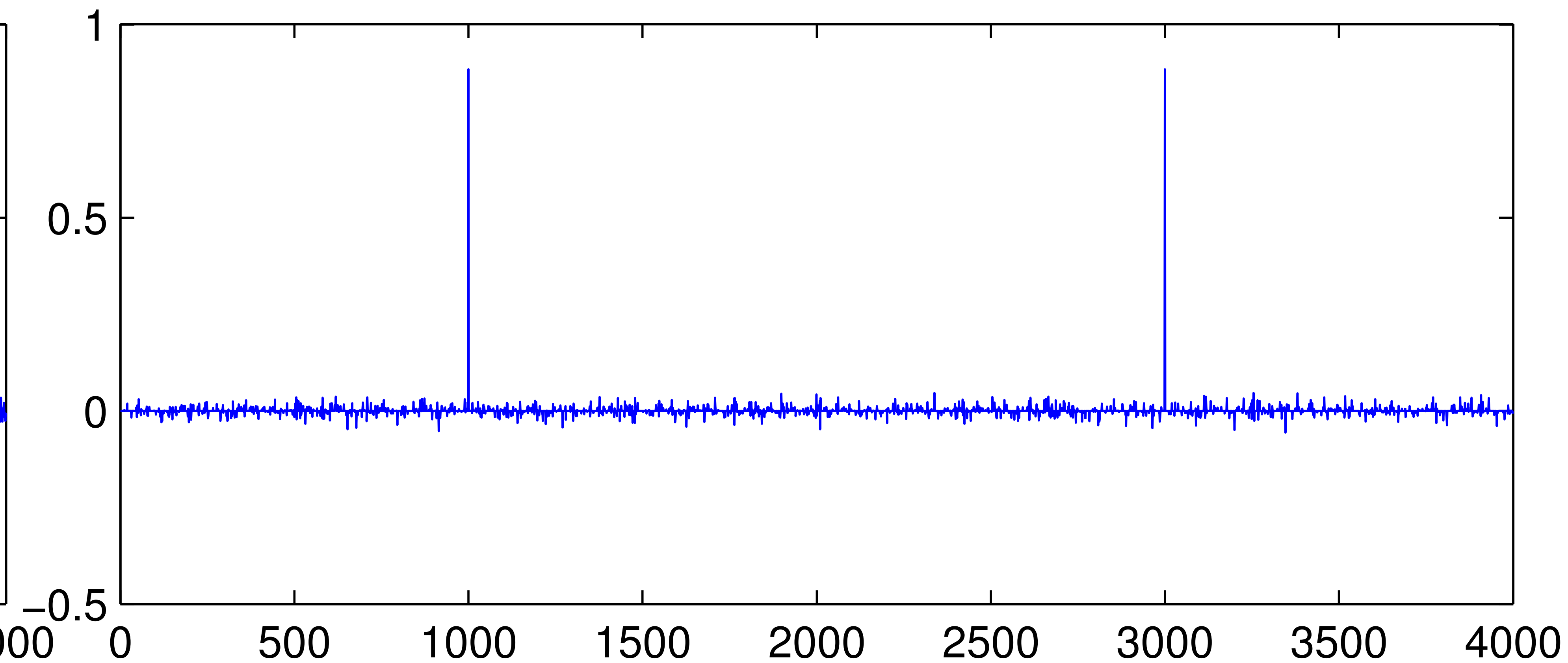
Soft thresholding: $x^t + A^* z^t$ $\qquad$ $\eta(x^t + A^* z^t; \tau^t)$





AMP: $x^t + A^*(z^t + \alpha^t z^{t-1})$ $\qquad$ $\eta(x^t + A^*(z^t + \alpha^t z^{t-1}); \tau^t)$





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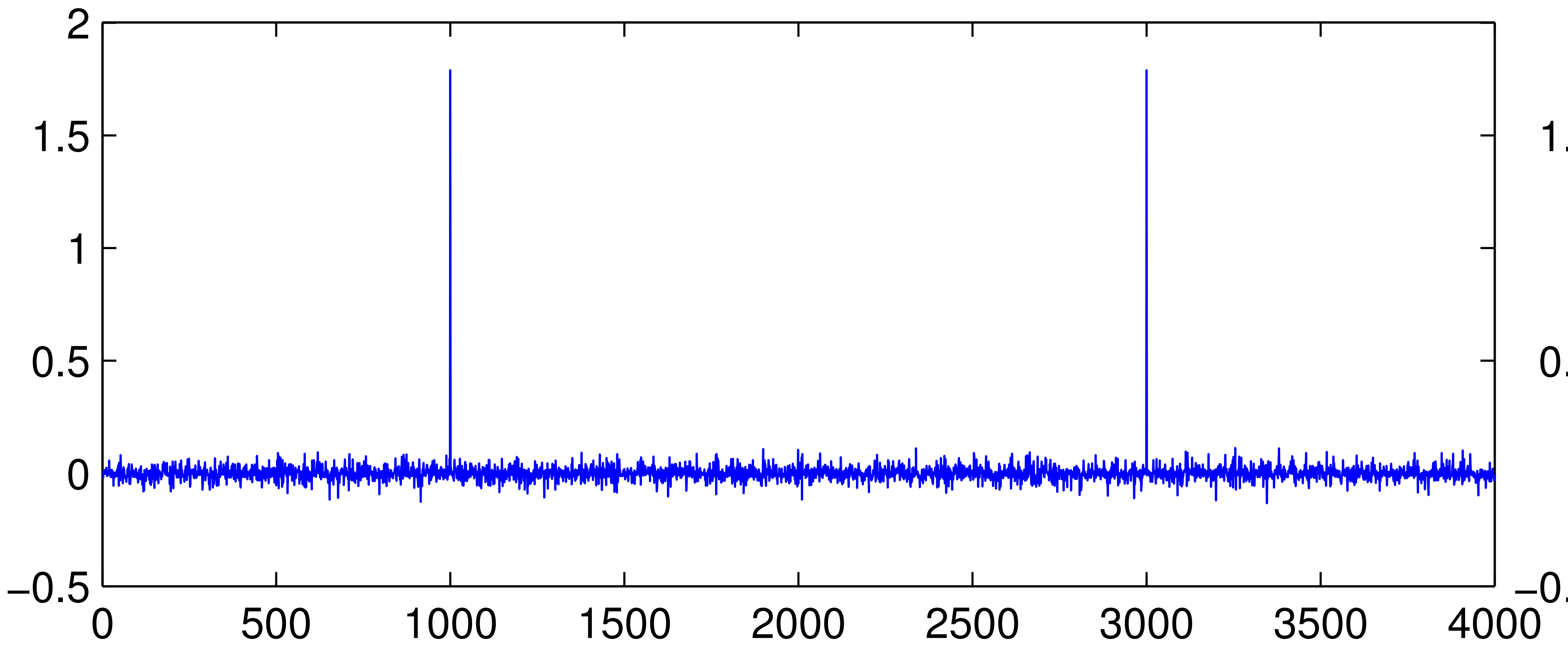
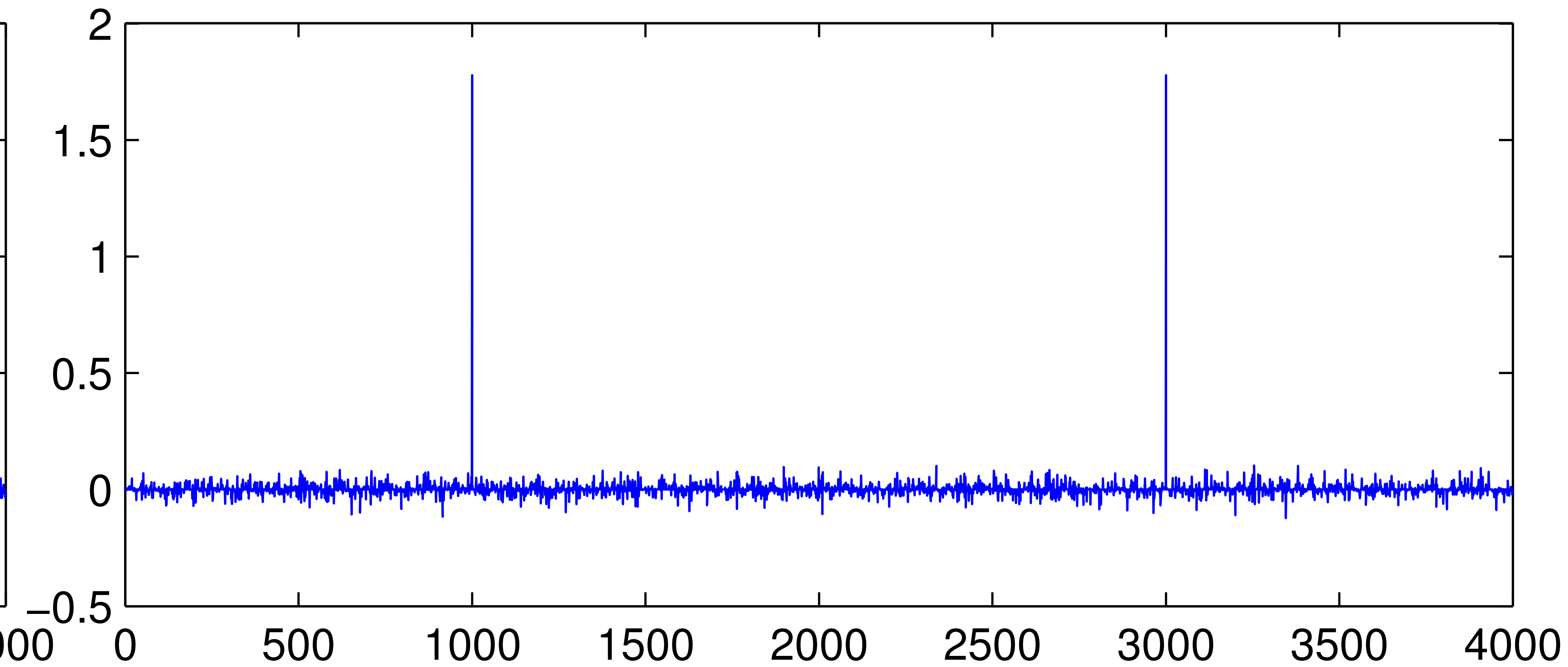
Iteration $t = 2$

Soft thresholding: $x^t + A^* z^t$ $\qquad$ $\eta(x^t + A^* z^t; \tau^t)$





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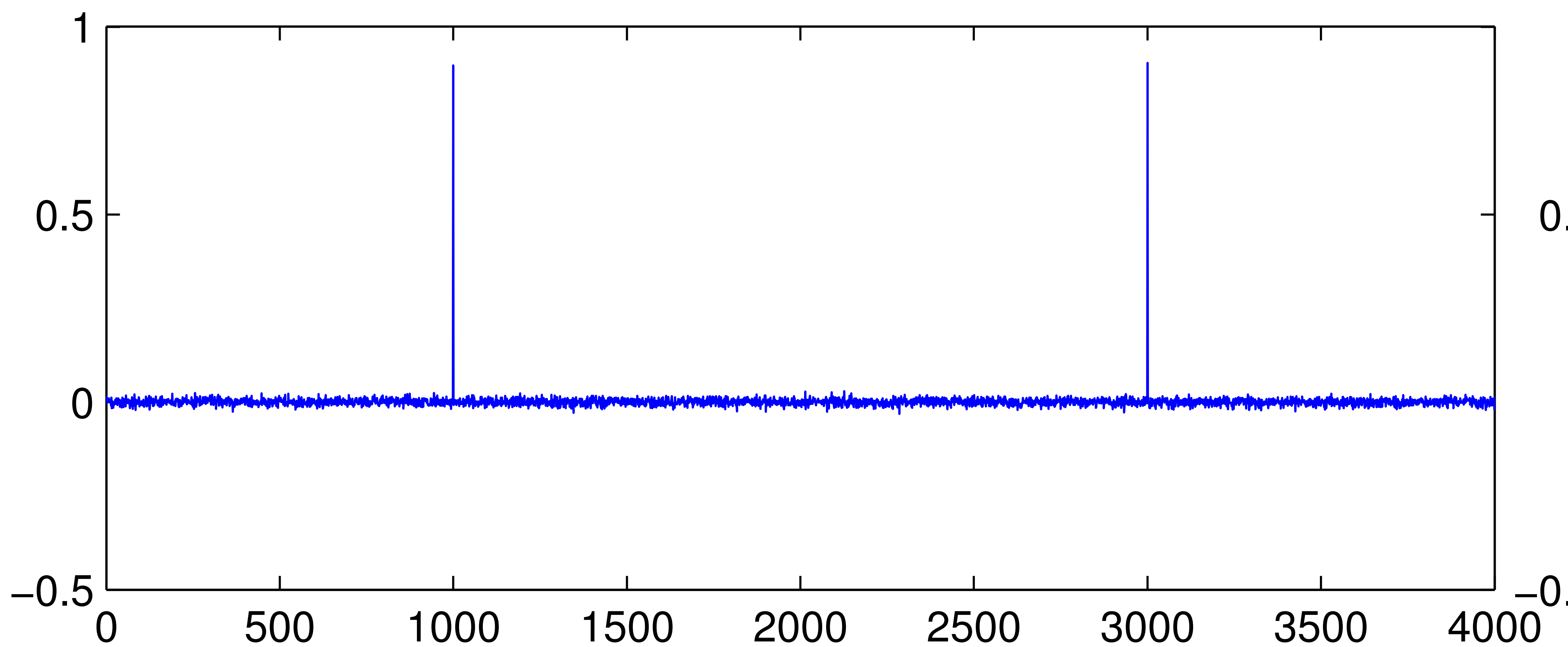
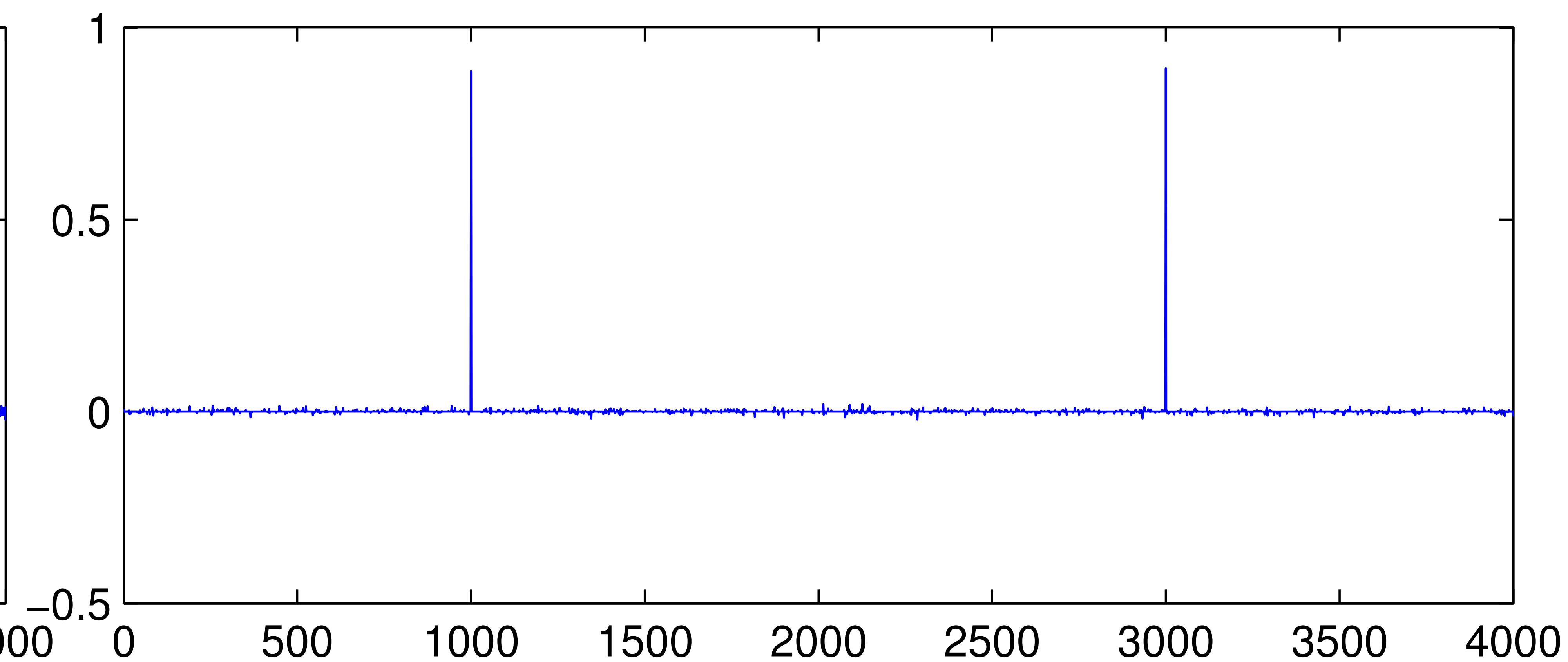
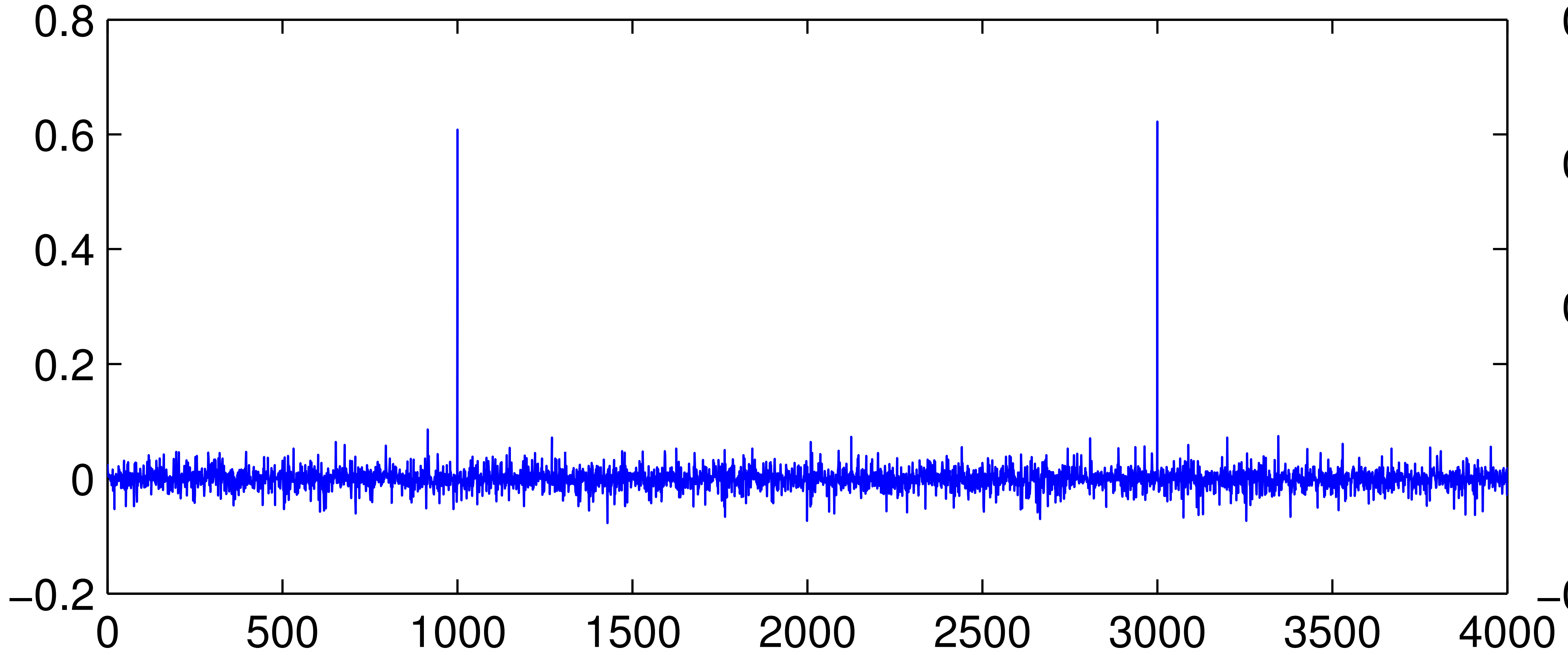
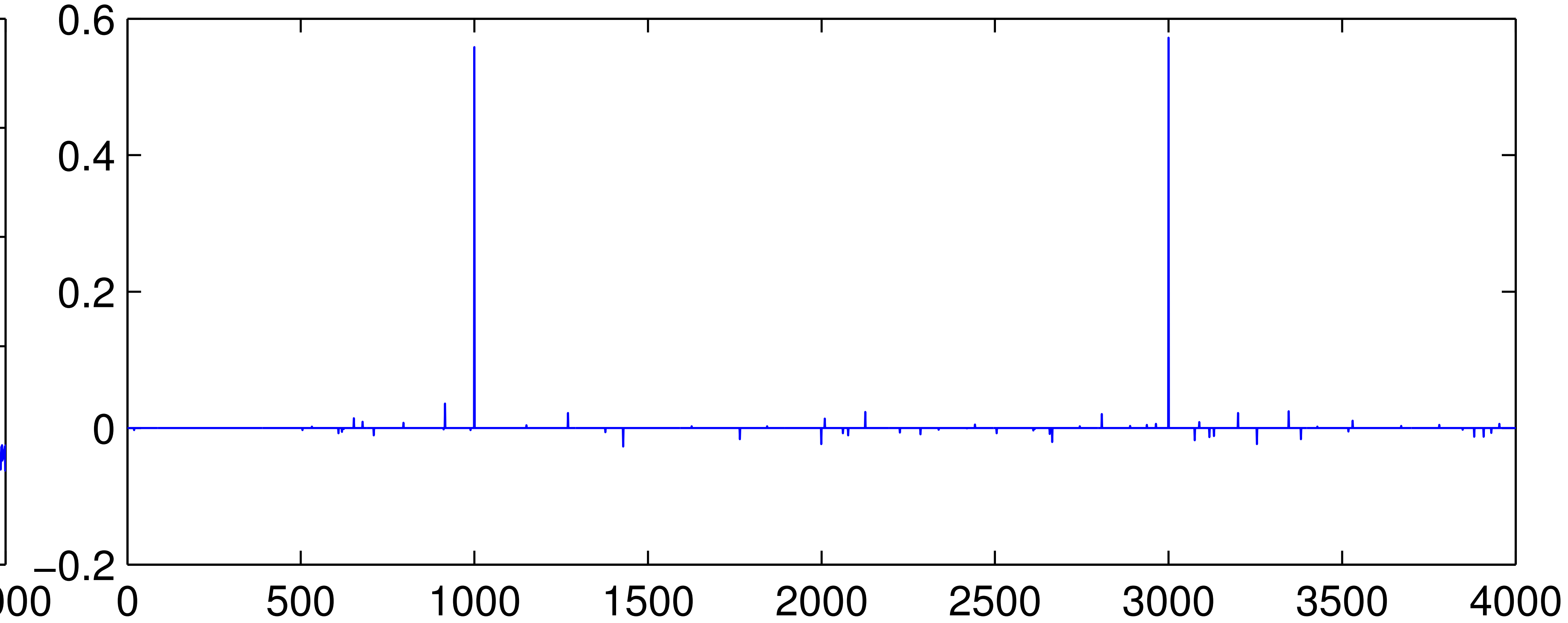




Iteration $t = 3$

Soft thresholding: $x^t + A^* z^t$ $\qquad$ $\eta(x^t + A^* z^t; \tau^t)$





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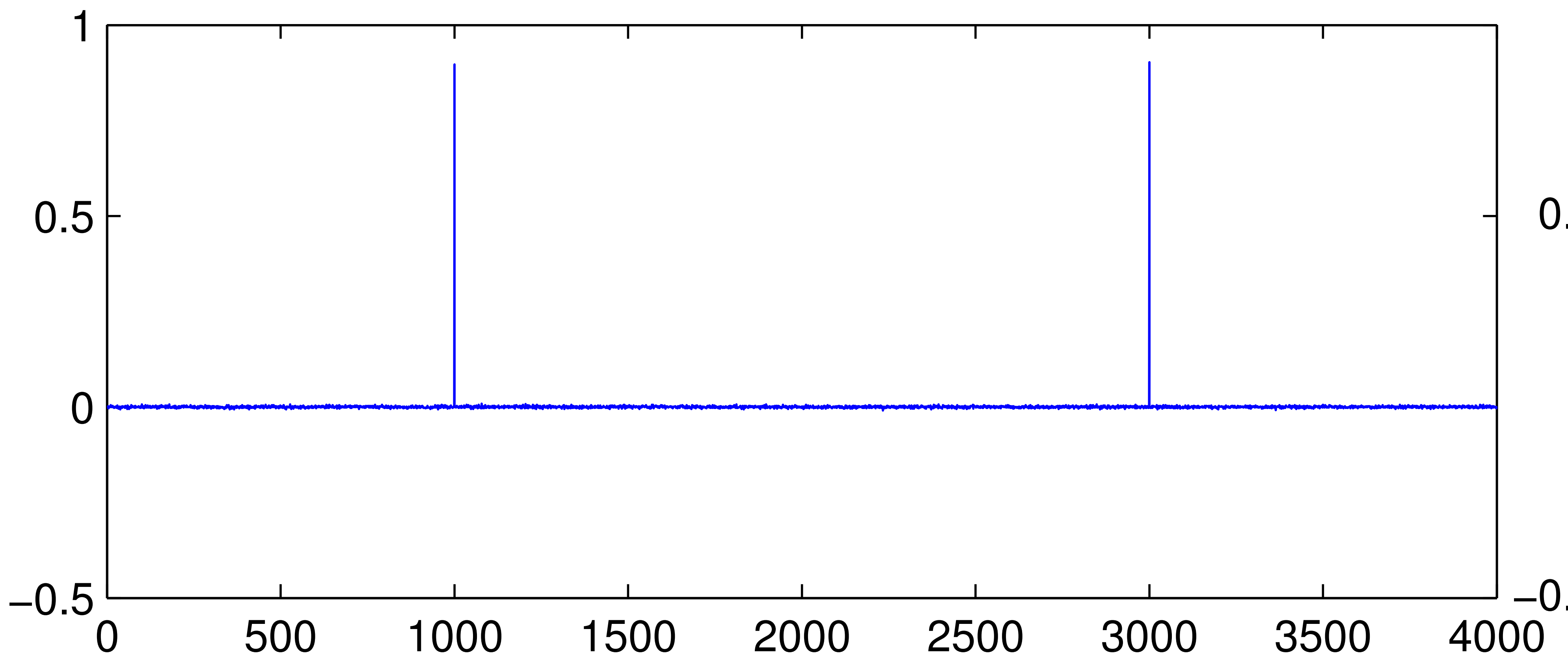
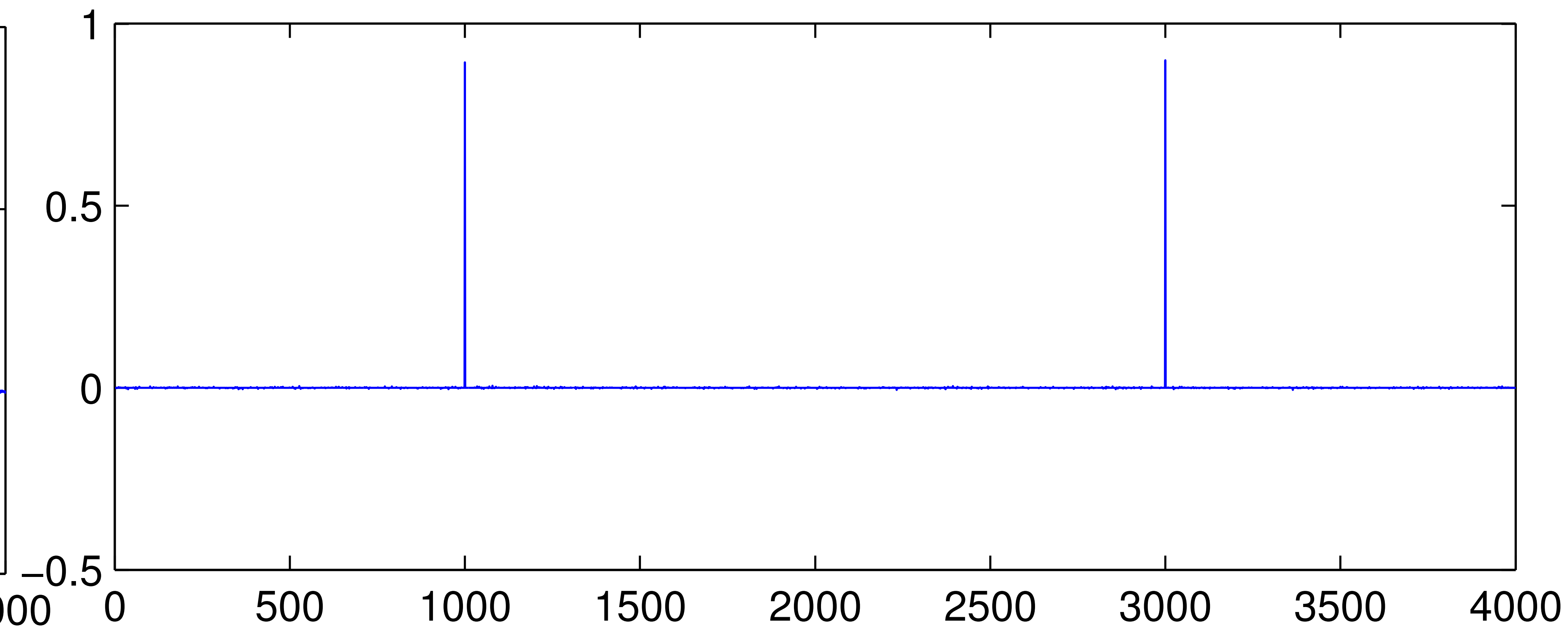
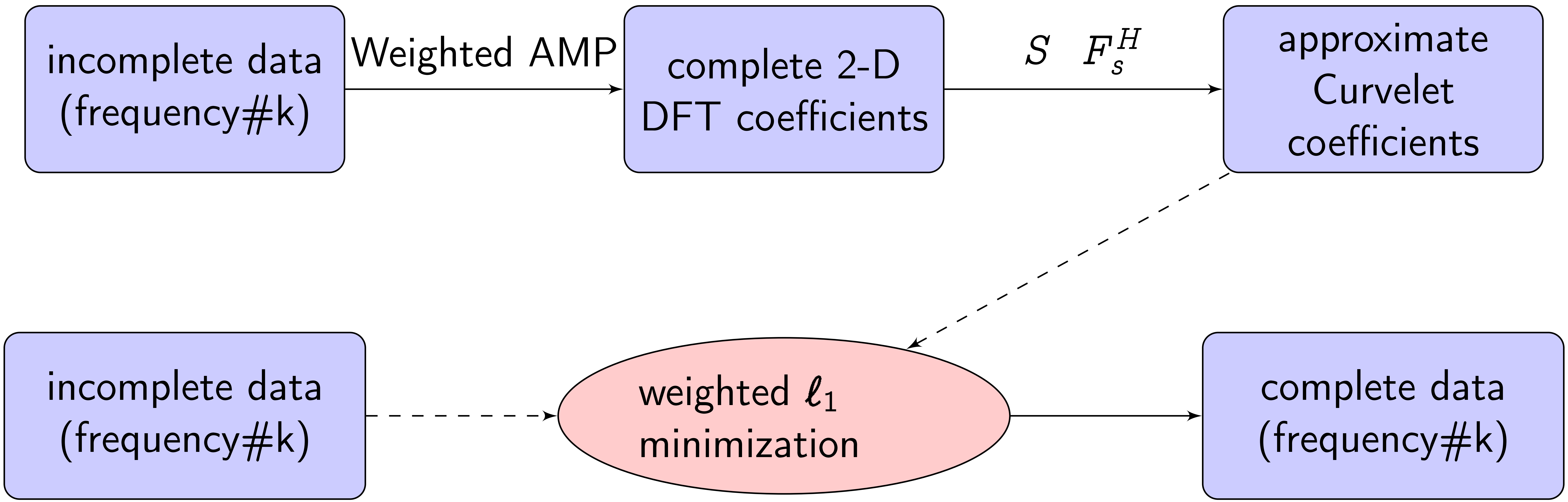




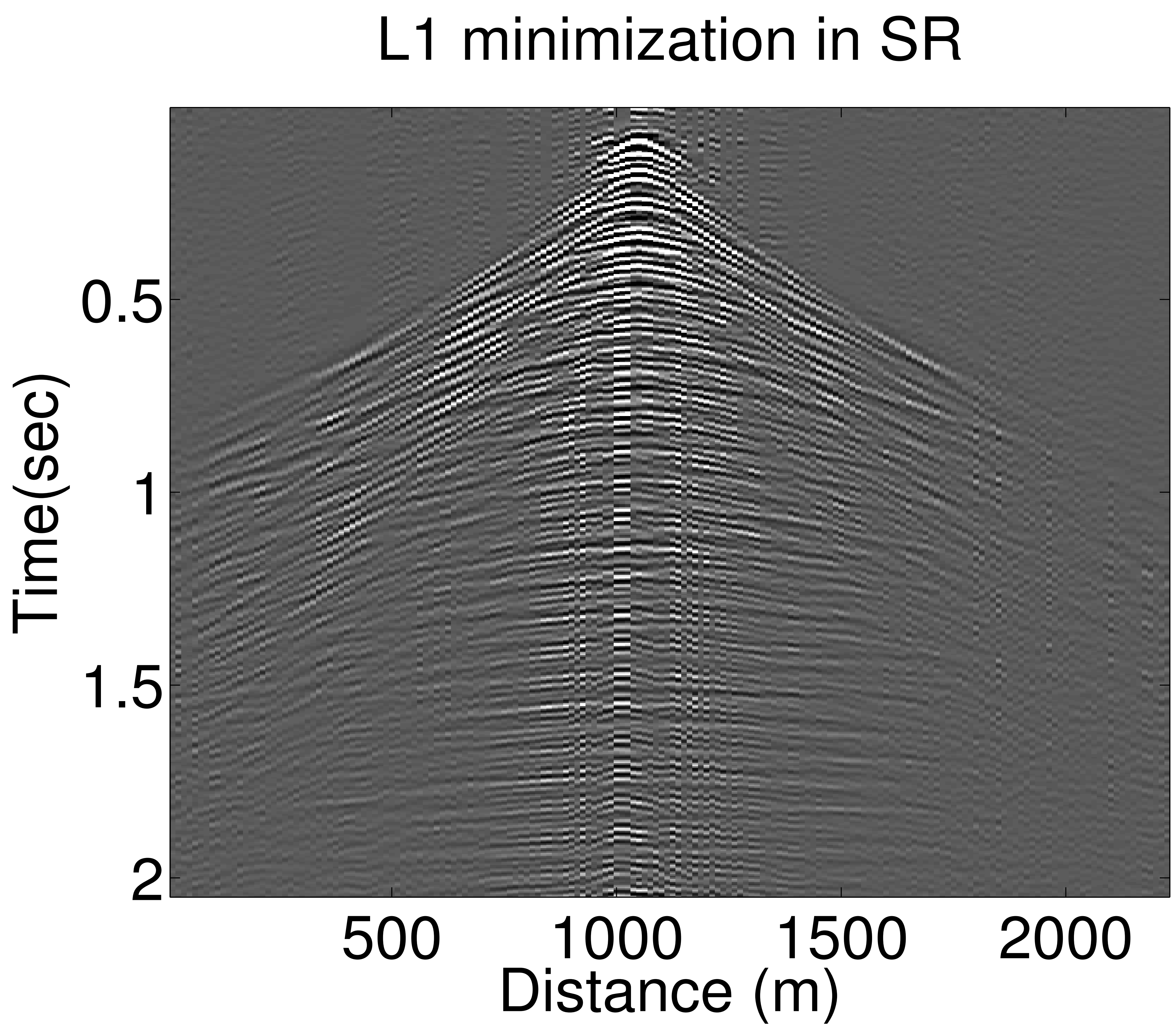
AMP and WAMP for seismic trace interpolation

- AMP is a delicate algorithm that just works for certain types of measurements.
- We can't use curvelets with AMP.
- Instead we use 2-D DFT matrix in the source-receiver domain.
- Then $b = RMF_s^H F_s f$, where F_s is a 2-D DFT matrix.

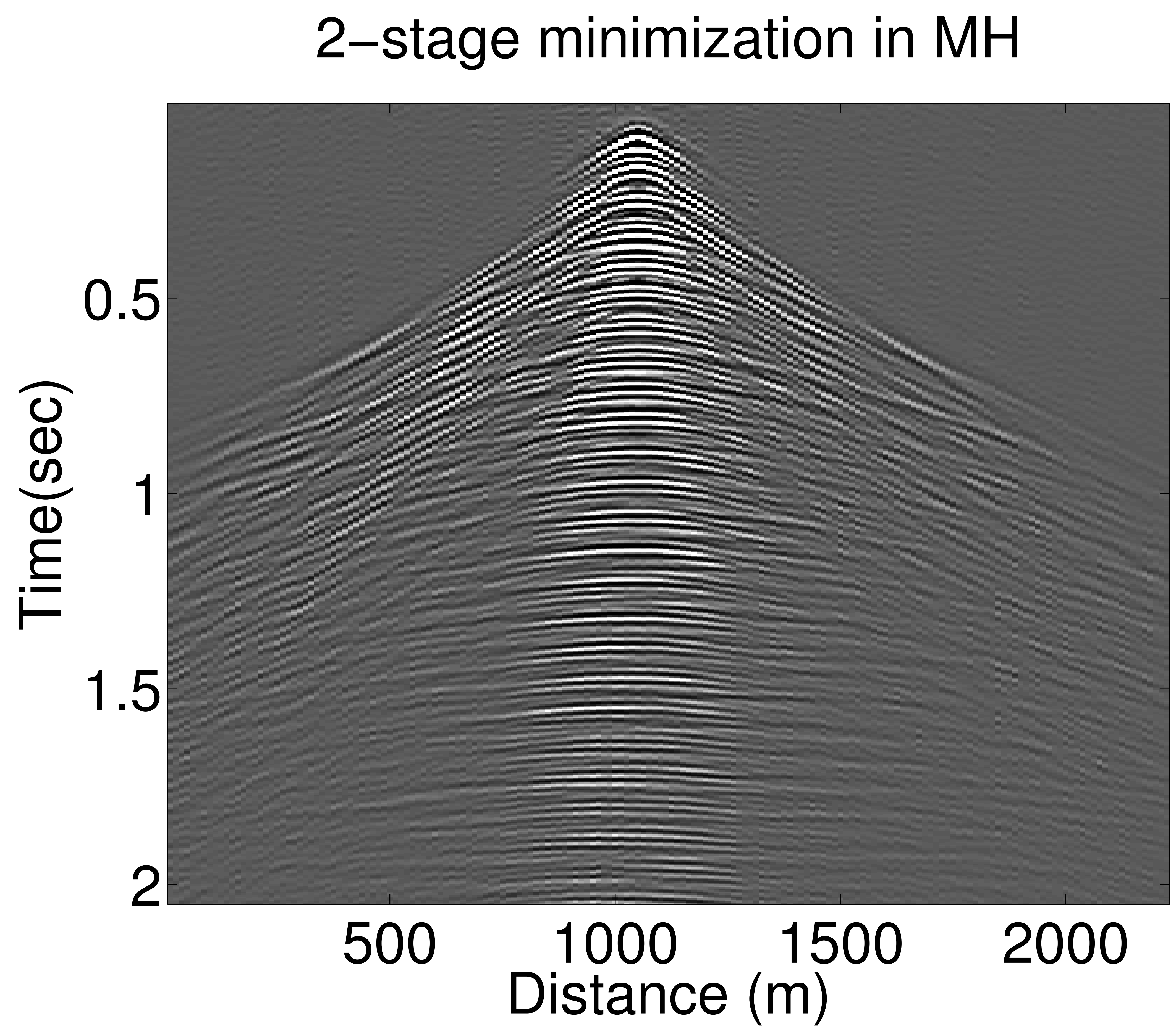
Flowchart of the 2-stage algorithm WAMP+weighted ℓ_1



ℓ_1 vs 2-stage WAMP+weighted ℓ_1

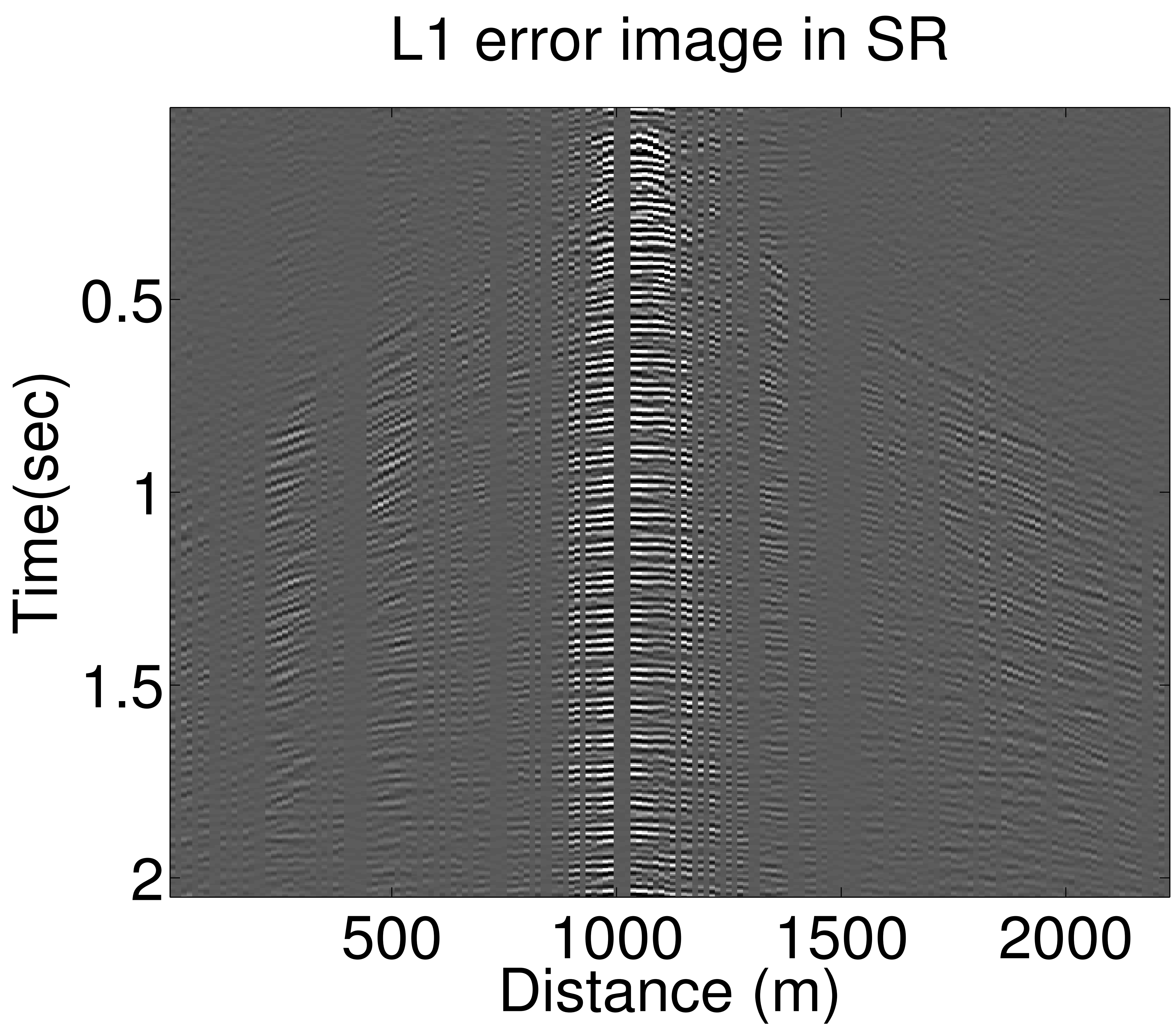


(u) SNR= 5.3 dB

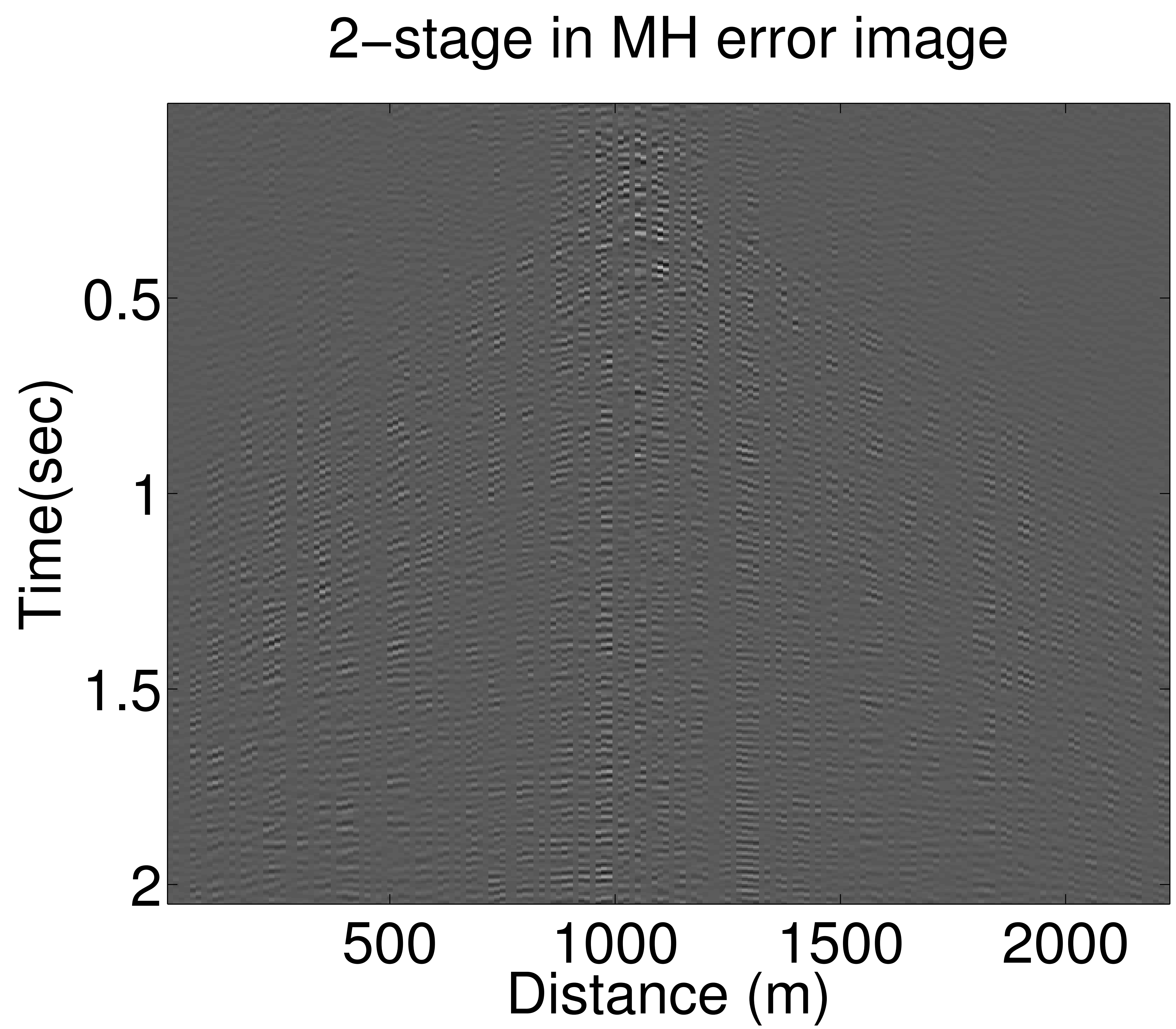


(v) SNR= 14.8 dB

Results of the 2-stage algorithm

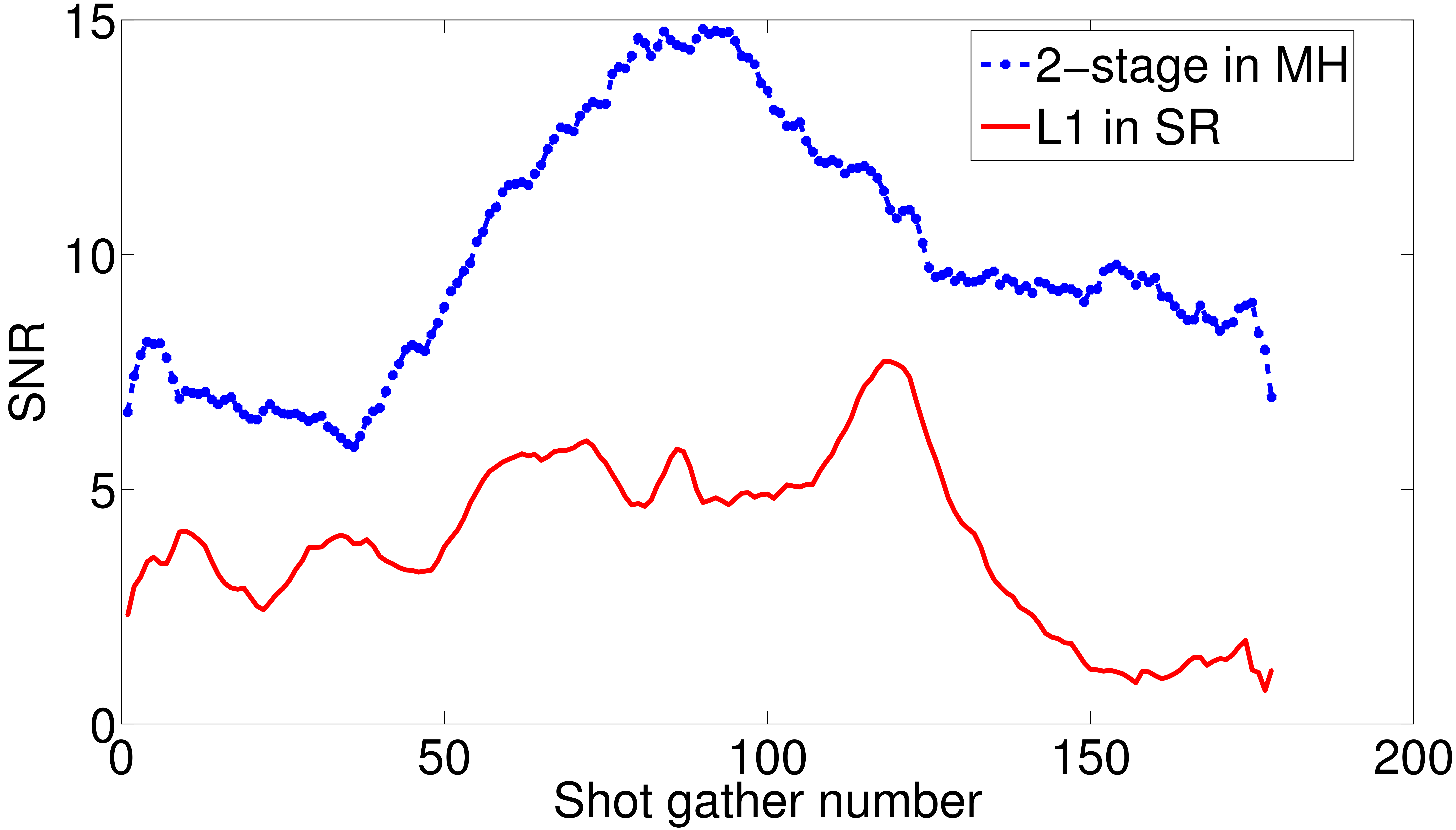


(w) SNR= 5.3 dB

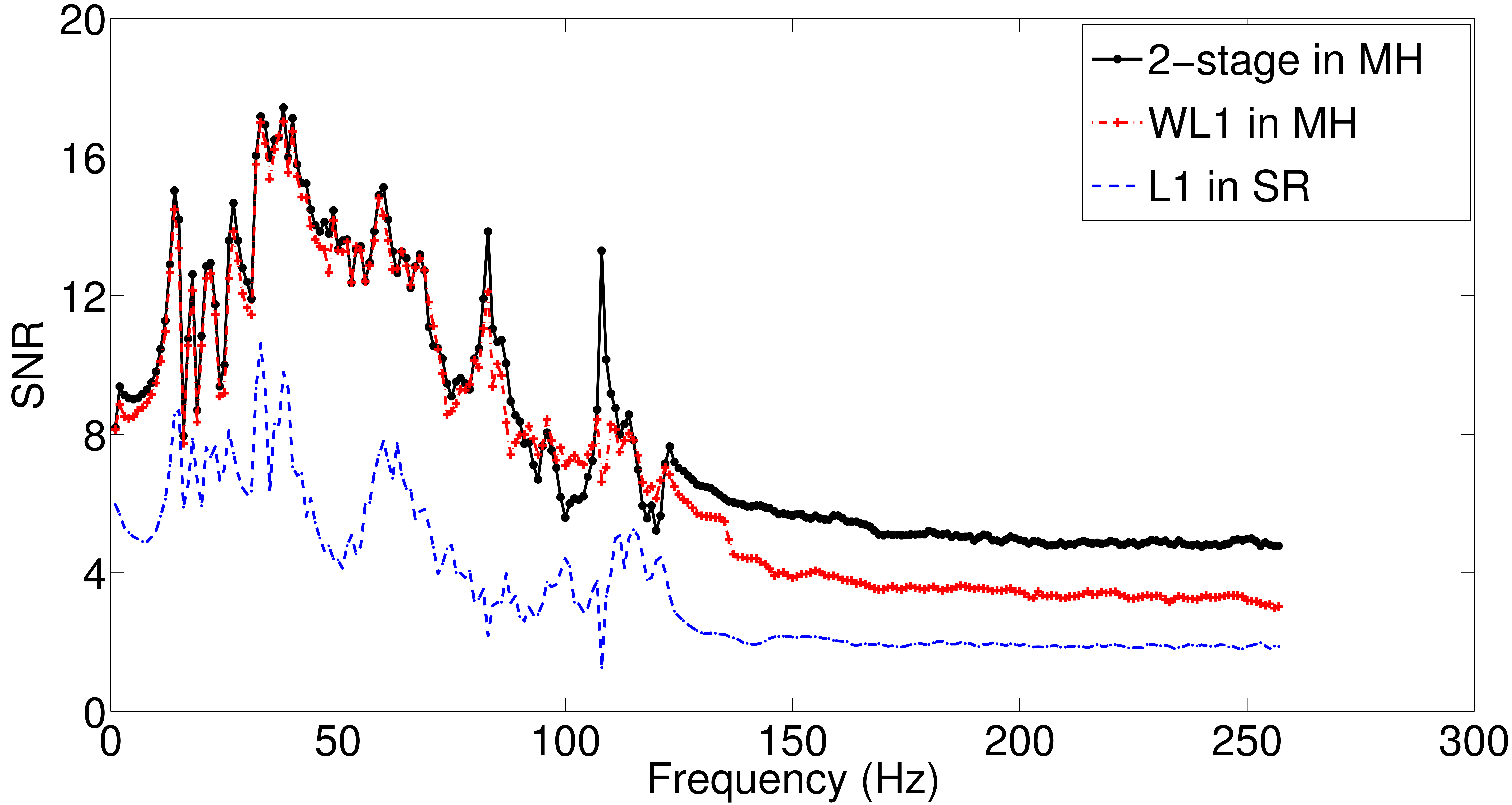


(x) SNR= 14.8 dB

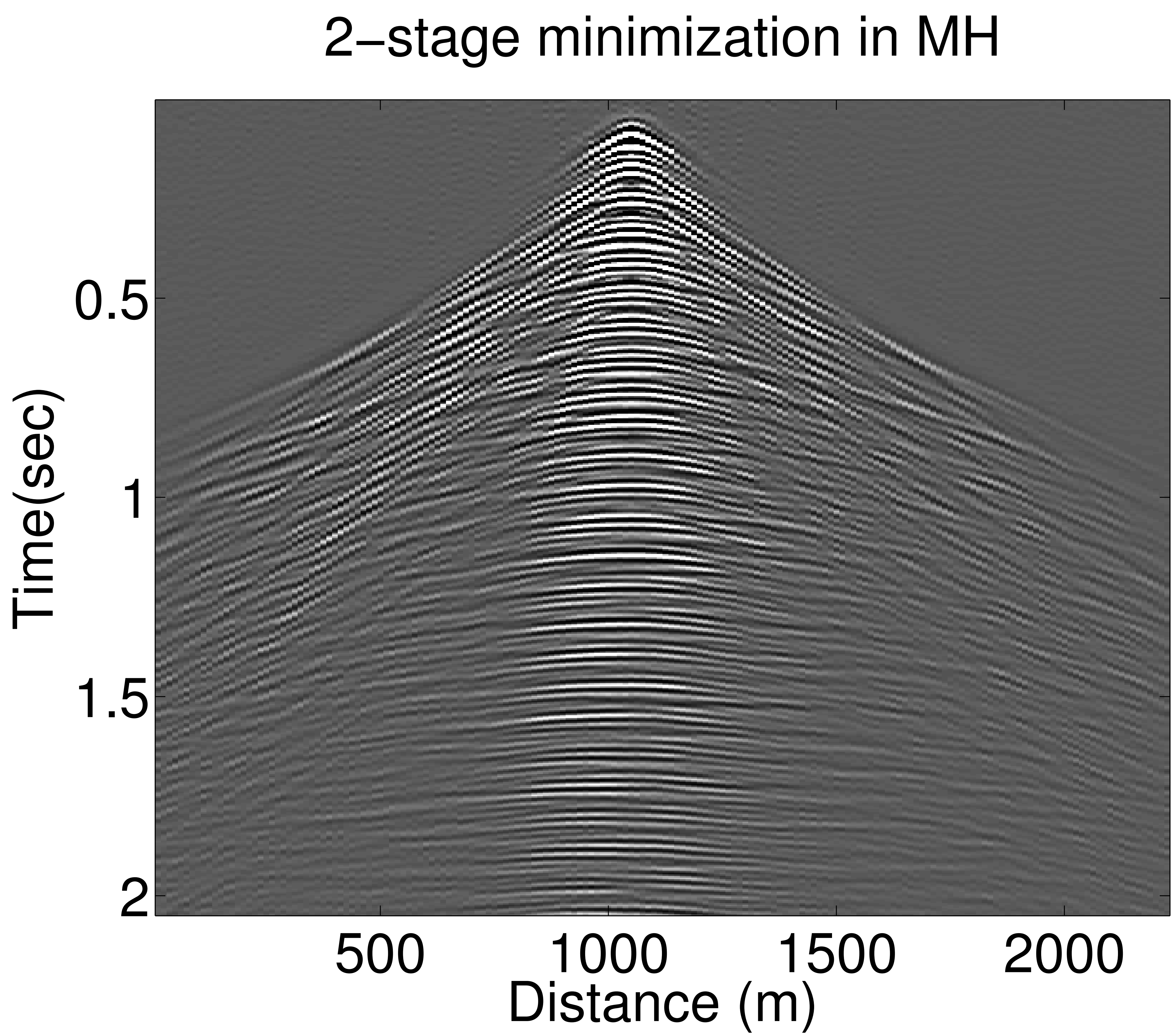
Results of the 2-stage algorithm

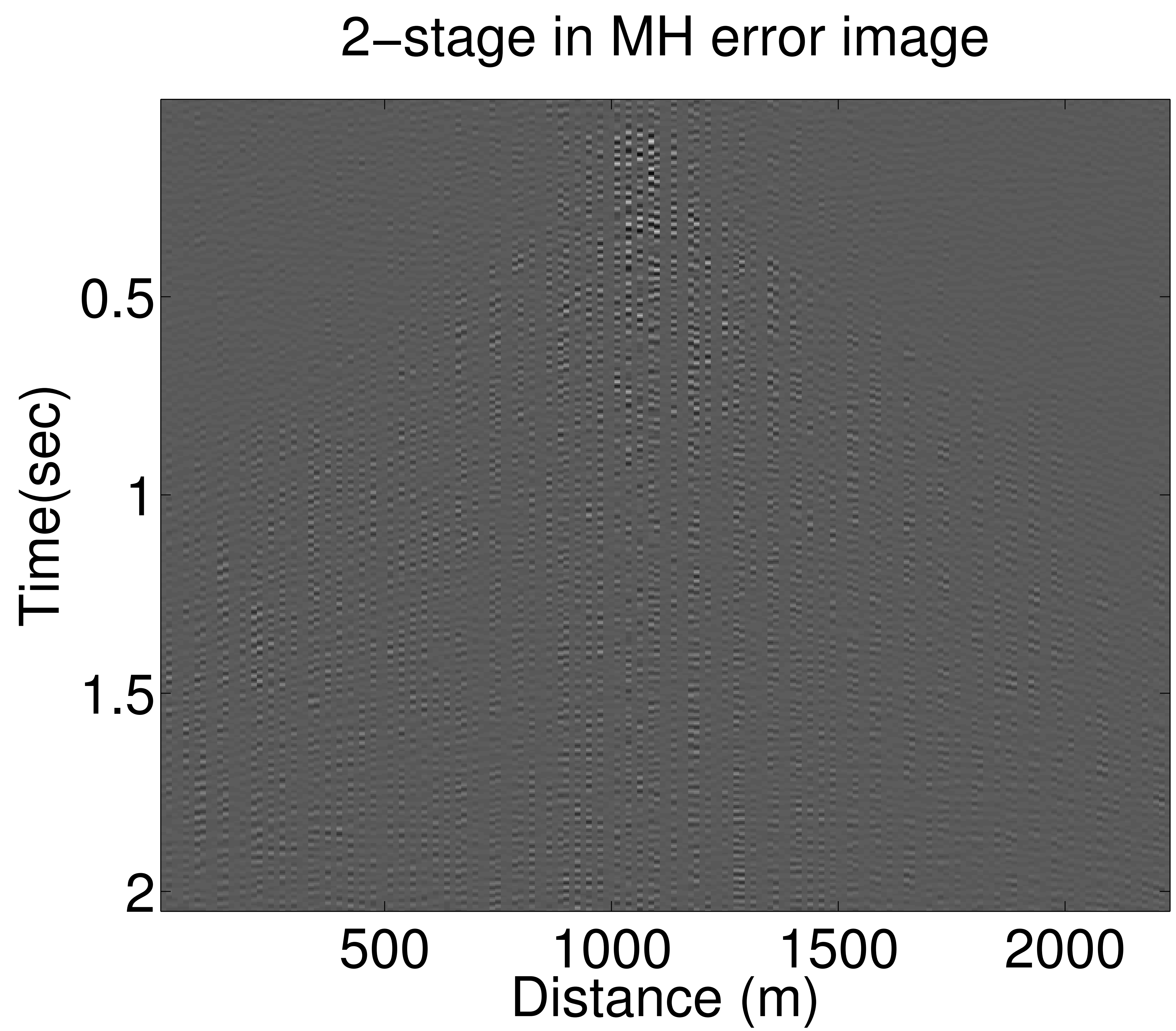


Comparison of recovery results



50% undersampling (19.6 dB)





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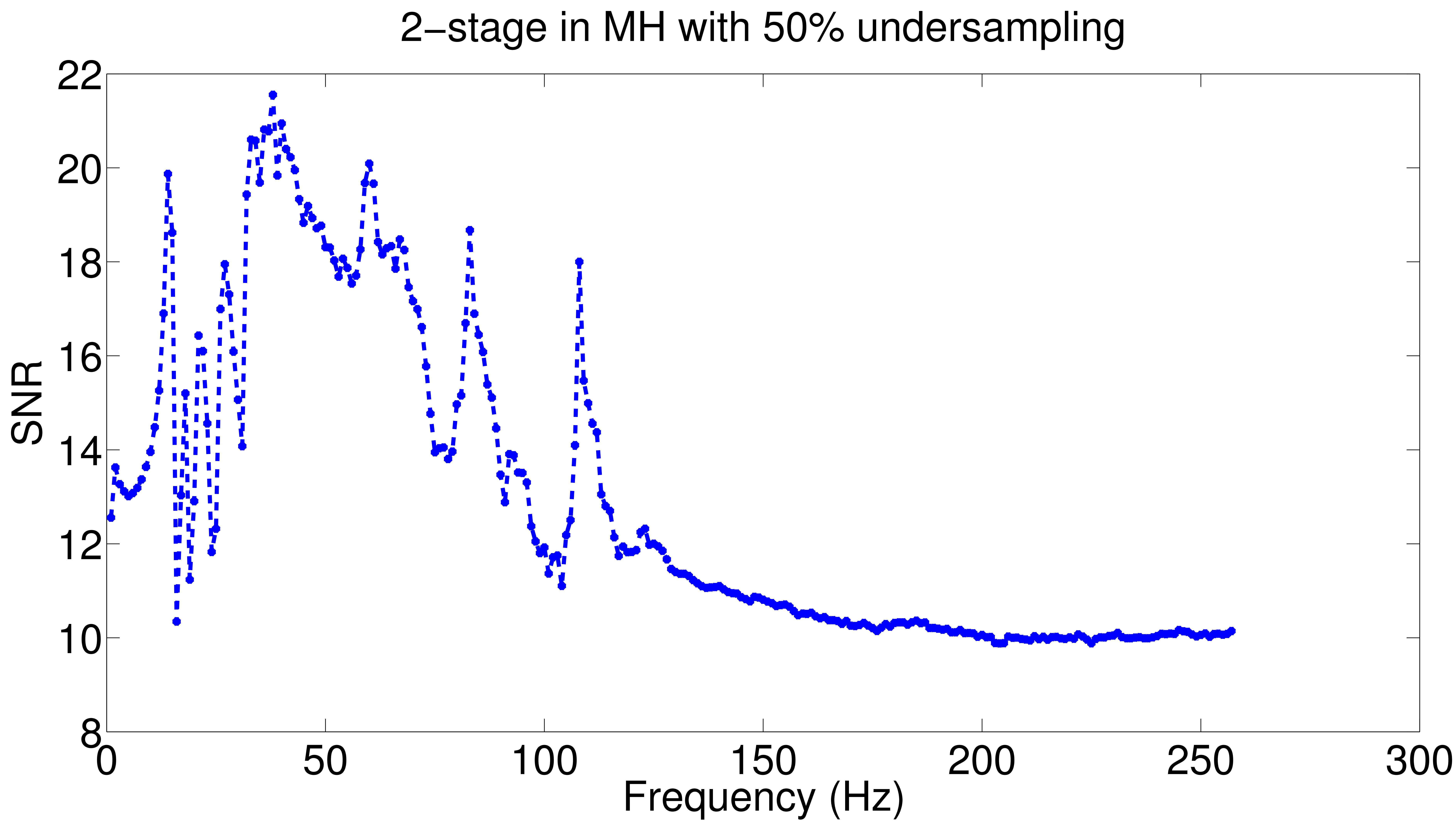

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- We get better sparse representation once we go from SR domain to MH domain.
- Finding a fast estimation of the data in the curvelet domain improves the results significantly. Especially in high frequencies.

Acknowledgement

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