Large-scale seismic data interpolation in a parallel computing environment

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Parallel matrix completion for missing source/receiver interpolation

Goals

Extend the previous implementation of spgLR to a parallel version

Handle very large scale data volumes stored across multiple nodes

Matrix completion

We use SPGLR to solve

$$\min_{\mathbf{L},\mathbf{R}} \quad \frac{1}{2}\left(\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2\right)$$
$$\text{such that} \quad \|\mathcal{A}(\mathbf{L}\mathbf{R}^T) - b\|_F \leq \sigma$$

by computing

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} \quad \|\mathcal{A}(\mathbf{L}\mathbf{R}^T) - b\|_F^2$$
$$\text{such that} \quad \frac{1}{2}\left(\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2\right) \leq \tau$$

and finding τ such that $v(\tau) = \sigma^2$

SPGLR

Basic operations of SPGLR

- compute objective, gradient
  - involves computing $\mathbf{X} = \mathbf{L}\mathbf{R}^T$, we'll come back to this

- project on to $\frac{1}{2}\left(\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2\right) \leq \tau$ ball, inner products, norms
  - trivial to parallelize

- compute $v'(\tau)$, the derivative of the value function

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} \quad \|\mathcal{A}(\mathbf{L}\mathbf{R}^T) - b\|_F^2$$

$$\text{such that} \quad \frac{1}{2}\left(\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2\right) \leq \tau$$

Computing $v'(\tau)$

Normally involves computing the largest singular value of the data matrix

- can be done iteratively, but still expensive

Instead, we use a secant approximation

$$v'(\tau) \approx (v(\tau + h) - v(\tau))/h$$

for some small h

Computing $v'(\tau)$

Each evaluation of $v(\tau)$ involves solving

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} \quad \|\mathcal{A}(\mathbf{L}\mathbf{R}^T) - b\|_F^2$$

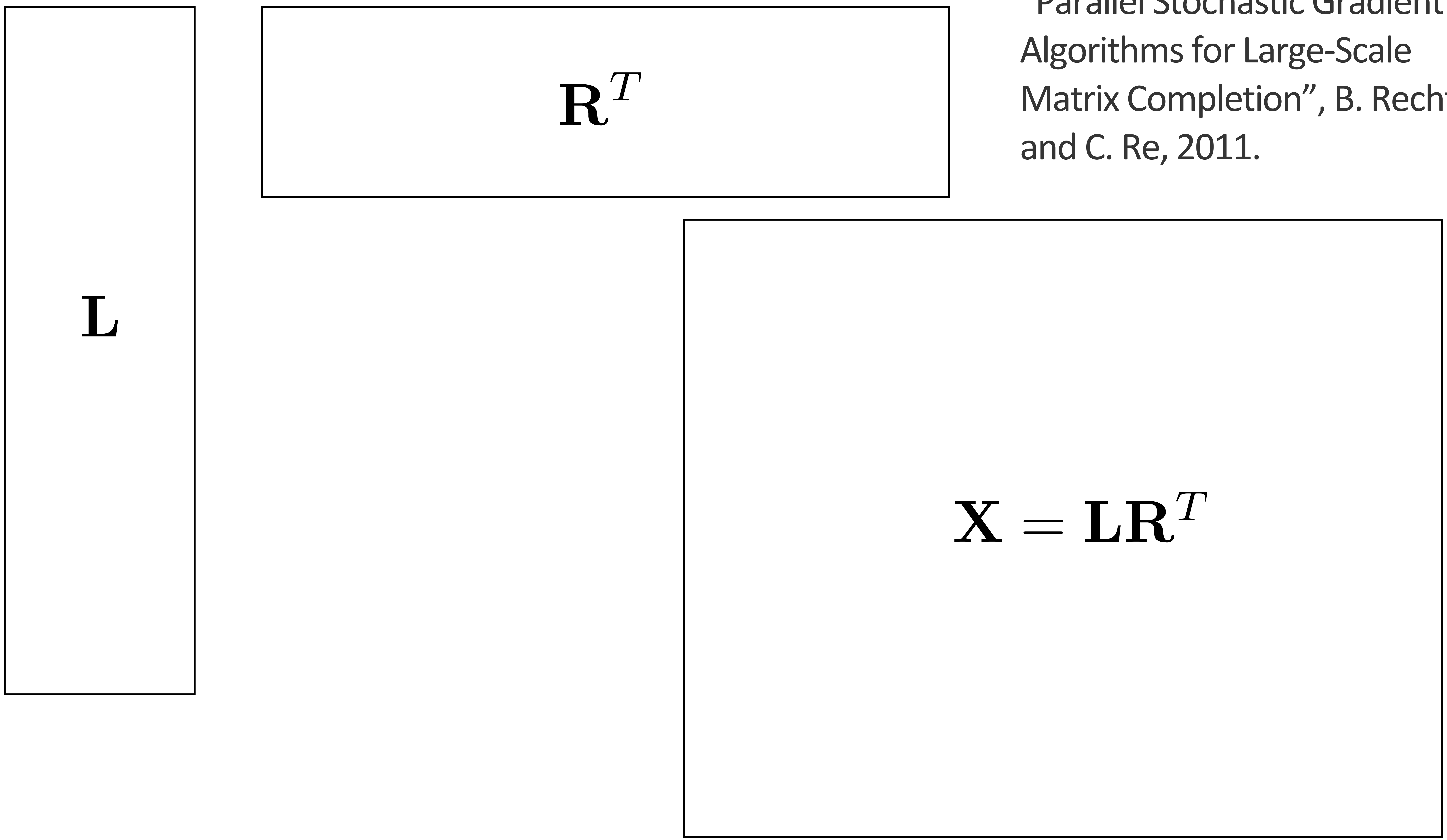
$$\text{such that} \quad \frac{1}{2}\left(\|\mathbf{L}\|_F^2 + \|\mathbf{R}\|_F^2\right) \leq \tau$$

which we can already do in a distributed environment

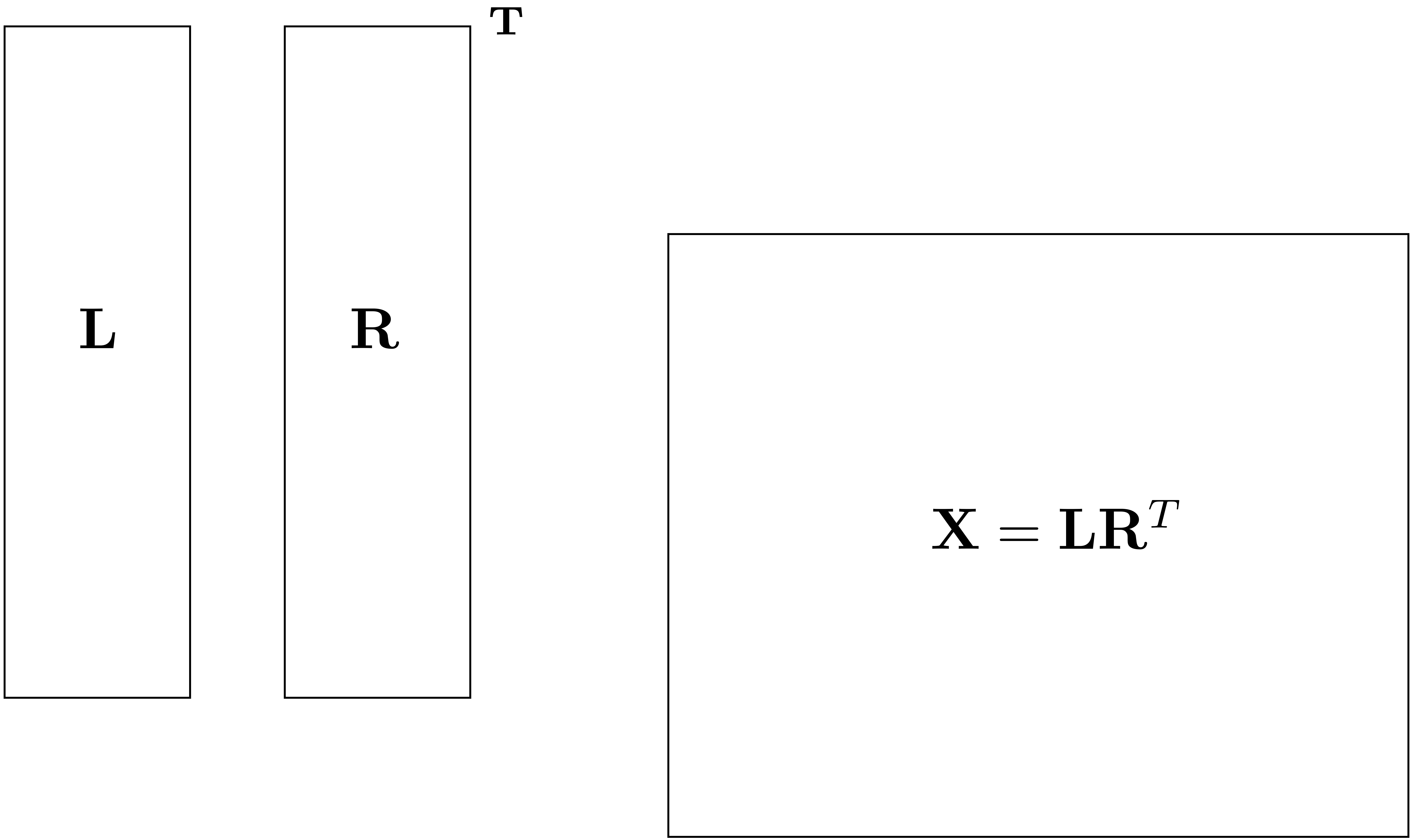
Using our approximation of $v'(\tau)$, we use Newton's method to update τ

LR parallel matrix multiplication

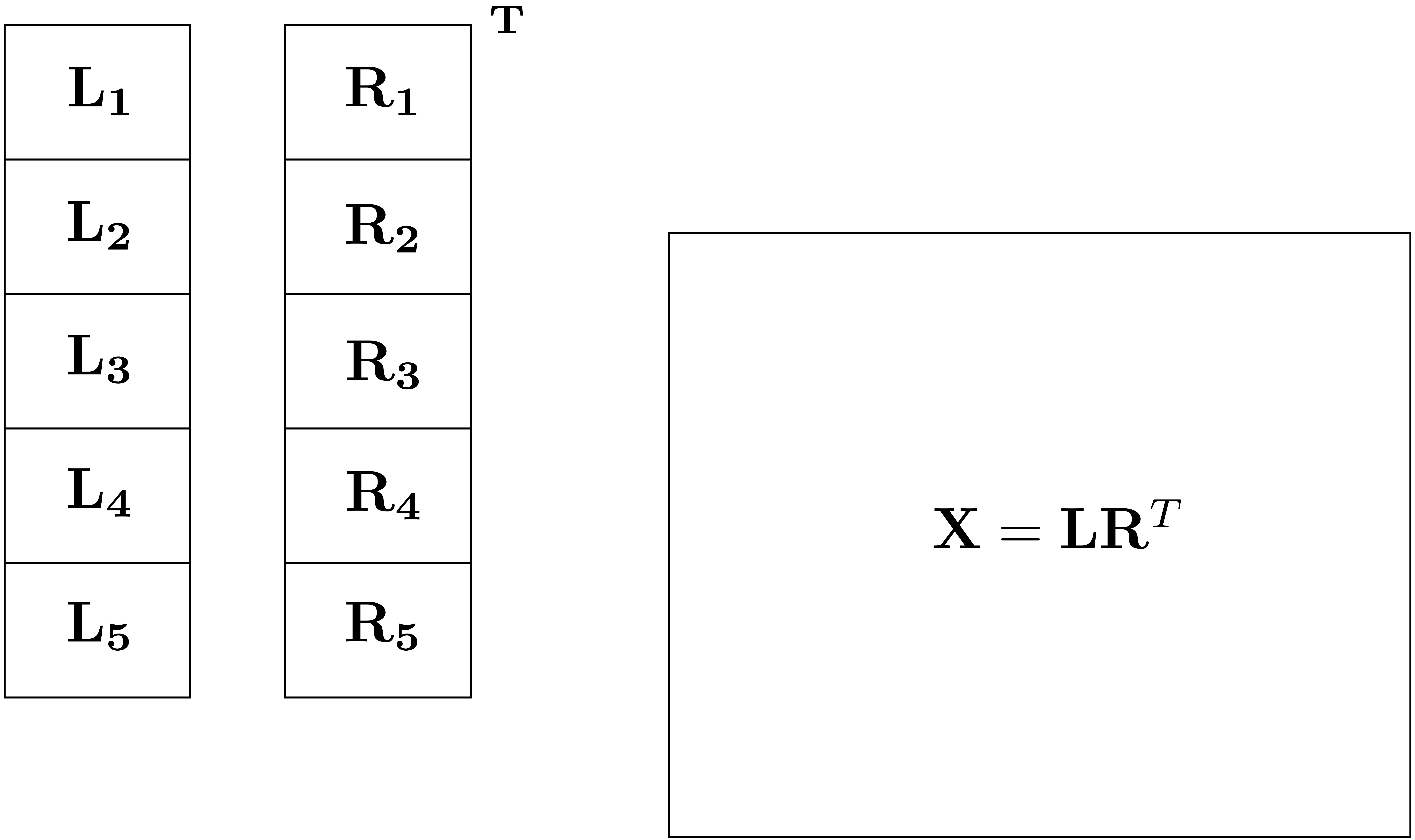
Cyclic block-permutations: “Parallel Stochastic Gradient Algorithms for Large-Scale Matrix Completion”, B. Recht and C. Re, 2011.



LR parallel matrix multiplication



LR parallel matrix multiplication



LR parallel matrix multiplication

	$\mathbf{L}$	$\mathbf{R}^{\mathbf{T}}$
Worker 1	$\mathbf{L_1}$	$\mathbf{R_1}$
Worker 2	$\mathbf{L_2}$	$\mathbf{R_2}$
Worker 3	$\mathbf{L_3}$	$\mathbf{R_3}$
Worker 4	$\mathbf{L_4}$	$\mathbf{R_4}$
Worker 5	$\mathbf{L_5}$	$\mathbf{R_5}$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^T$$

LR parallel matrix multiplication

	$\mathbf{L}$	$\mathbf{R}^{\mathbf{T}}$
Worker 1	$\mathbf{L_1}$	$\mathbf{R_1}$
Worker 2	$\mathbf{L_2}$	$\mathbf{R_2}$
Worker 3	$\mathbf{L_3}$	$\mathbf{R_3}$
Worker 4	$\mathbf{L_4}$	$\mathbf{R_4}$
Worker 5	$\mathbf{L_5}$	$\mathbf{R_5}$

$\mathbf{L_1R_1^T}$	$\mathbf{L_1R_2^T}$	$\mathbf{L_1R_3^T}$	$\mathbf{L_1R_4^T}$	$\mathbf{L_1R_5^T}$
$\mathbf{L_2R_1^T}$	$\mathbf{L_2R_2^T}$	$\mathbf{L_2R_3^T}$	$\mathbf{L_2R_4^T}$	$\mathbf{L_2R_5^T}$
$\mathbf{L_3R_1^T}$	$\mathbf{L_3R_2^T}$	$\mathbf{L_3R_3^T}$	$\mathbf{L_3R_4^T}$	$\mathbf{L_3R_5^T}$
$\mathbf{L_4R_1^T}$	$\mathbf{L_4R_2^T}$	$\mathbf{L_4R_3^T}$	$\mathbf{L_4R_4^T}$	$\mathbf{L_4R_5^T}$
$\mathbf{L_5R_1^T}$	$\mathbf{L_5R_2^T}$	$\mathbf{L_5R_3^T}$	$\mathbf{L_5R_4^T}$	$\mathbf{L_5R_5^T}$

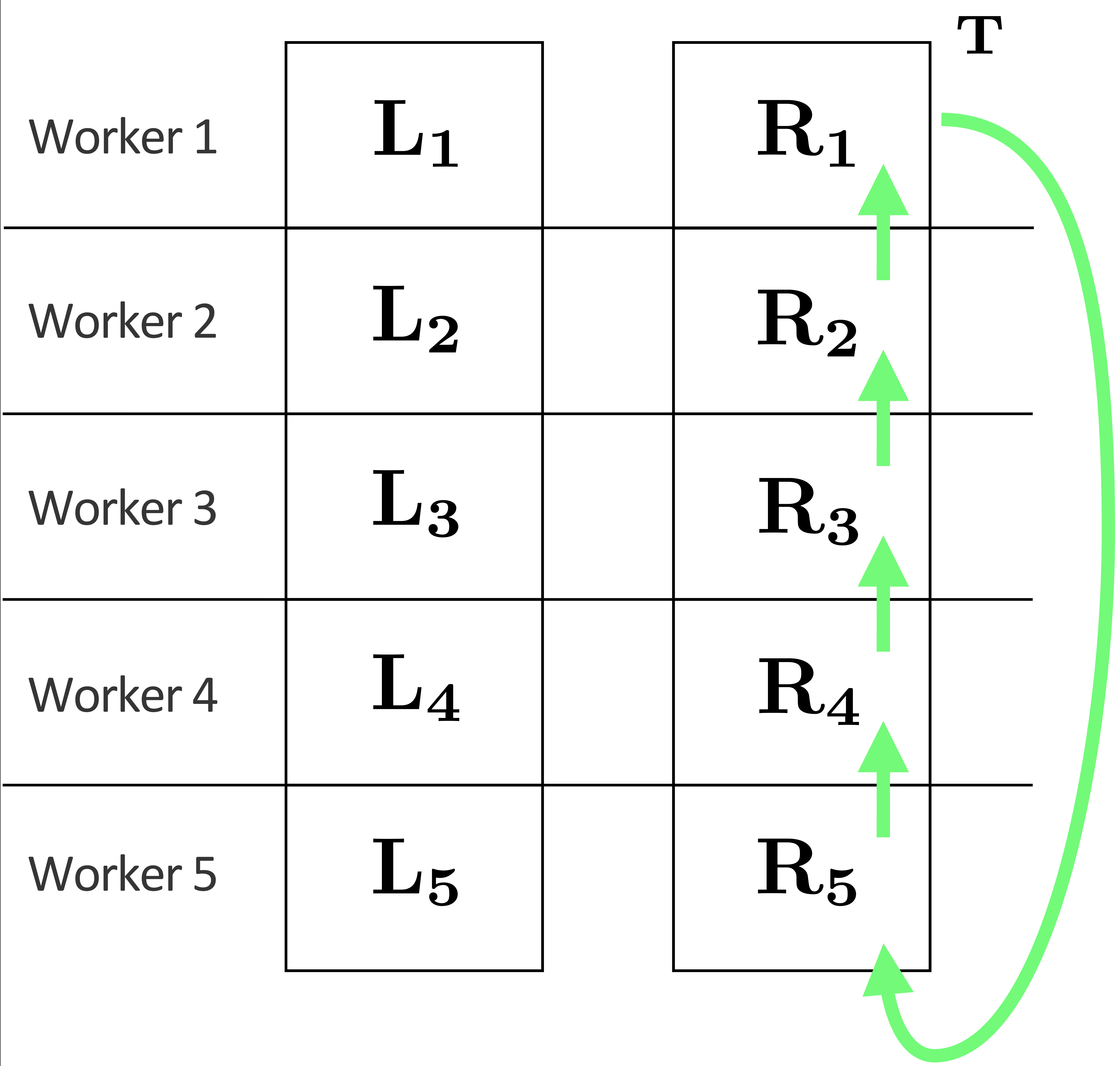
LR parallel matrix multiplication

- Process green blocks in parallel

	$\mathbf{L}$	$\mathbf{R}^{\mathbf{T}}$
Worker 1	$\mathbf{L_1}$	$\mathbf{R_1}$
Worker 2	$\mathbf{L_2}$	$\mathbf{R_2}$
Worker 3	$\mathbf{L_3}$	$\mathbf{R_3}$
Worker 4	$\mathbf{L_4}$	$\mathbf{R_4}$
Worker 5	$\mathbf{L_5}$	$\mathbf{R_5}$

$\mathbf{L_1R_1^T}$	$\mathbf{L_1R_2^T}$	$\mathbf{L_1R_3^T}$	$\mathbf{L_1R_4^T}$	$\mathbf{L_1R_5^T}$
$\mathbf{L_2R_1^T}$	$\mathbf{L_2R_2^T}$	$\mathbf{L_2R_3^T}$	$\mathbf{L_2R_4^T}$	$\mathbf{L_2R_5^T}$
$\mathbf{L_3R_1^T}$	$\mathbf{L_3R_2^T}$	$\mathbf{L_3R_3^T}$	$\mathbf{L_3R_4^T}$	$\mathbf{L_3R_5^T}$
$\mathbf{L_4R_1^T}$	$\mathbf{L_4R_2^T}$	$\mathbf{L_4R_3^T}$	$\mathbf{L_4R_4^T}$	$\mathbf{L_4R_5^T}$
$\mathbf{L_5R_1^T}$	$\mathbf{L_5R_2^T}$	$\mathbf{L_5R_3^T}$	$\mathbf{L_5R_4^T}$	$\mathbf{L_5R_5^T}$

LR parallel matrix multiplication



- Process green blocks in parallel
- Communicate R blocks to next worker

$\mathbf{L_1R_1^T}$	$\mathbf{L_1R_2^T}$	$\mathbf{L_1R_3^T}$	$\mathbf{L_1R_4^T}$	$\mathbf{L_1R_5^T}$
$\mathbf{L_2R_1^T}$	$\mathbf{L_2R_2^T}$	$\mathbf{L_2R_3^T}$	$\mathbf{L_2R_4^T}$	$\mathbf{L_2R_5^T}$
$\mathbf{L_3R_1^T}$	$\mathbf{L_3R_2^T}$	$\mathbf{L_3R_3^T}$	$\mathbf{L_3R_4^T}$	$\mathbf{L_3R_5^T}$
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$\mathbf{L_5R_1^T}$	$\mathbf{L_5R_2^T}$	$\mathbf{L_5R_3^T}$	$\mathbf{L_5R_4^T}$	$\mathbf{L_5R_5^T}$

LR parallel matrix multiplication

- Process green blocks in parallel
- Communicate R blocks to next worker

		$\mathbf{T}$
Worker 1	$\mathbf{L}_1$	$\mathbf{R}_2$
Worker 2	$\mathbf{L}_2$	$\mathbf{R}_3$
Worker 3	$\mathbf{L}_3$	$\mathbf{R}_4$
Worker 4	$\mathbf{L}_4$	$\mathbf{R}_5$
Worker 5	$\mathbf{L}_5$	$\mathbf{R}_1$

$\mathbf{L}_1\mathbf{R}_1^T$	$\mathbf{L}_1\mathbf{R}_2^T$	$\mathbf{L}_1\mathbf{R}_3^T$	$\mathbf{L}_1\mathbf{R}_4^T$	$\mathbf{L}_1\mathbf{R}_5^T$
$\mathbf{L}_2\mathbf{R}_1^T$	$\mathbf{L}_2\mathbf{R}_2^T$	$\mathbf{L}_2\mathbf{R}_3^T$	$\mathbf{L}_2\mathbf{R}_4^T$	$\mathbf{L}_2\mathbf{R}_5^T$
$\mathbf{L}_3\mathbf{R}_1^T$	$\mathbf{L}_3\mathbf{R}_2^T$	$\mathbf{L}_3\mathbf{R}_3^T$	$\mathbf{L}_3\mathbf{R}_4^T$	$\mathbf{L}_3\mathbf{R}_5^T$
$\mathbf{L}_4\mathbf{R}_1^T$	$\mathbf{L}_4\mathbf{R}_2^T$	$\mathbf{L}_4\mathbf{R}_3^T$	$\mathbf{L}_4\mathbf{R}_4^T$	$\mathbf{L}_4\mathbf{R}_5^T$
$\mathbf{L}_5\mathbf{R}_1^T$	$\mathbf{L}_5\mathbf{R}_2^T$	$\mathbf{L}_5\mathbf{R}_3^T$	$\mathbf{L}_5\mathbf{R}_4^T$	$\mathbf{L}_5\mathbf{R}_5^T$

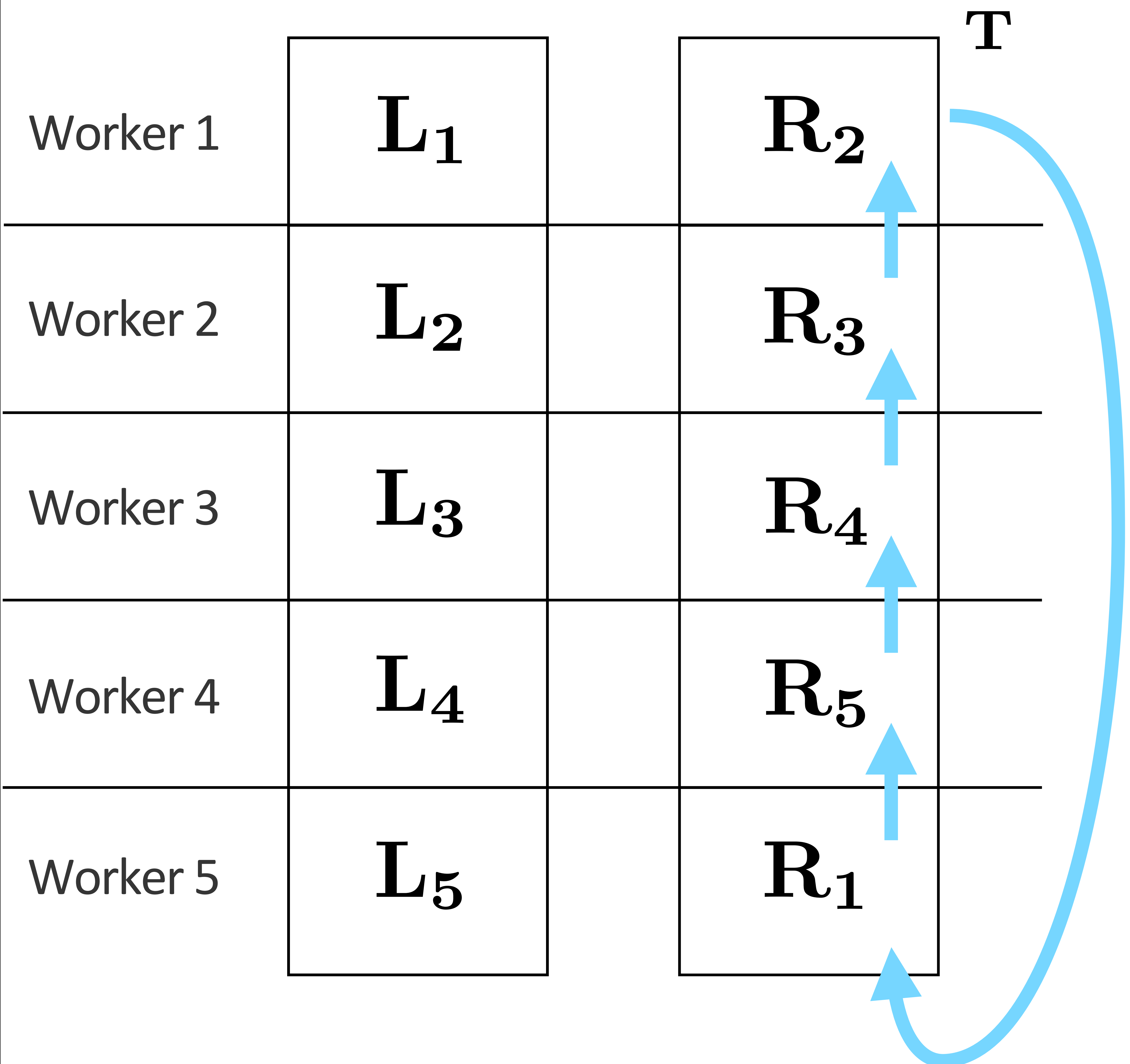
LR parallel matrix multiplication

- Process green blocks in parallel
- Communicate R blocks to next worker
- Process blue blocks in parallel

	$\mathbf{L}$	$\mathbf{R}^{\mathbf{T}}$
Worker 1	$\mathbf{L}_1$	$\mathbf{R}_2$
Worker 2	$\mathbf{L}_2$	$\mathbf{R}_3$
Worker 3	$\mathbf{L}_3$	$\mathbf{R}_4$
Worker 4	$\mathbf{L}_4$	$\mathbf{R}_5$
Worker 5	$\mathbf{L}_5$	$\mathbf{R}_1$

$\mathbf{L}_1\mathbf{R}_1^T$	$\mathbf{L}_1\mathbf{R}_2^T$	$\mathbf{L}_1\mathbf{R}_3^T$	$\mathbf{L}_1\mathbf{R}_4^T$	$\mathbf{L}_1\mathbf{R}_5^T$
$\mathbf{L}_2\mathbf{R}_1^T$	$\mathbf{L}_2\mathbf{R}_2^T$	$\mathbf{L}_2\mathbf{R}_3^T$	$\mathbf{L}_2\mathbf{R}_4^T$	$\mathbf{L}_2\mathbf{R}_5^T$
$\mathbf{L}_3\mathbf{R}_1^T$	$\mathbf{L}_3\mathbf{R}_2^T$	$\mathbf{L}_3\mathbf{R}_3^T$	$\mathbf{L}_3\mathbf{R}_4^T$	$\mathbf{L}_3\mathbf{R}_5^T$
$\mathbf{L}_4\mathbf{R}_1^T$	$\mathbf{L}_4\mathbf{R}_2^T$	$\mathbf{L}_4\mathbf{R}_3^T$	$\mathbf{L}_4\mathbf{R}_4^T$	$\mathbf{L}_4\mathbf{R}_5^T$
$\mathbf{L}_5\mathbf{R}_1^T$	$\mathbf{L}_5\mathbf{R}_2^T$	$\mathbf{L}_5\mathbf{R}_3^T$	$\mathbf{L}_5\mathbf{R}_4^T$	$\mathbf{L}_5\mathbf{R}_5^T$

LR parallel matrix multiplication



- Process green blocks in parallel
- Communicate R blocks to next worker
- Process blue blocks in parallel
- Communicate R blocks to next worker

$\mathbf{L_1R_1^T}$	$\mathbf{L_1R_2^T}$	$\mathbf{L_1R_3^T}$	$\mathbf{L_1R_4^T}$	$\mathbf{L_1R_5^T}$
$\mathbf{L_2R_1^T}$	$\mathbf{L_2R_2^T}$	$\mathbf{L_2R_3^T}$	$\mathbf{L_2R_4^T}$	$\mathbf{L_2R_5^T}$
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$\mathbf{L_5R_1^T}$	$\mathbf{L_5R_2^T}$	$\mathbf{L_5R_3^T}$	$\mathbf{L_5R_4^T}$	$\mathbf{L_5R_5^T}$

LR parallel matrix multiplication

- Process green blocks in parallel
- Communicate R blocks to next worker
- Process blue blocks in parallel
- Communicate R blocks to next worker

		$\mathbf{R}^T$
Worker 1	$\mathbf{L}_1$	$\mathbf{R}_3$
Worker 2	$\mathbf{L}_2$	$\mathbf{R}_4$
Worker 3	$\mathbf{L}_3$	$\mathbf{R}_5$
Worker 4	$\mathbf{L}_4$	$\mathbf{R}_1$
Worker 5	$\mathbf{L}_5$	$\mathbf{R}_2$

$\mathbf{L}_1\mathbf{R}_1^T$	$\mathbf{L}_1\mathbf{R}_2^T$	$\mathbf{L}_1\mathbf{R}_3^T$	$\mathbf{L}_1\mathbf{R}_4^T$	$\mathbf{L}_1\mathbf{R}_5^T$
$\mathbf{L}_2\mathbf{R}_1^T$	$\mathbf{L}_2\mathbf{R}_2^T$	$\mathbf{L}_2\mathbf{R}_3^T$	$\mathbf{L}_2\mathbf{R}_4^T$	$\mathbf{L}_2\mathbf{R}_5^T$
$\mathbf{L}_3\mathbf{R}_1^T$	$\mathbf{L}_3\mathbf{R}_2^T$	$\mathbf{L}_3\mathbf{R}_3^T$	$\mathbf{L}_3\mathbf{R}_4^T$	$\mathbf{L}_3\mathbf{R}_5^T$
$\mathbf{L}_4\mathbf{R}_1^T$	$\mathbf{L}_4\mathbf{R}_2^T$	$\mathbf{L}_4\mathbf{R}_3^T$	$\mathbf{L}_4\mathbf{R}_4^T$	$\mathbf{L}_4\mathbf{R}_5^T$
$\mathbf{L}_5\mathbf{R}_1^T$	$\mathbf{L}_5\mathbf{R}_2^T$	$\mathbf{L}_5\mathbf{R}_3^T$	$\mathbf{L}_5\mathbf{R}_4^T$	$\mathbf{L}_5\mathbf{R}_5^T$

LR parallel matrix multiplication

	L	R^T
Worker 1	L_1	R_3
Worker 2	L_2	R_4
Worker 3	L_3	R_5
Worker 4	L_4	R_1
Worker 5	L_5	R_2

- Process green blocks in parallel
- Communicate R blocks to next worker
- Process blue blocks in parallel
- Communicate R blocks to next worker
- Repeat

$L_1R_1^T$	$L_1R_2^T$	$L_1R_3^T$	$L_1R_4^T$	$L_1R_5^T$
$L_2R_1^T$	$L_2R_2^T$	$L_2R_3^T$	$L_2R_4^T$	$L_2R_5^T$
$L_3R_1^T$	$L_3R_2^T$	$L_3R_3^T$	$L_3R_4^T$	$L_3R_5^T$
$L_4R_1^T$	$L_4R_2^T$	$L_4R_3^T$	$L_4R_4^T$	$L_4R_5^T$
$L_5R_1^T$	$L_5R_2^T$	$L_5R_3^T$	$L_5R_4^T$	$L_5R_5^T$

SLIM

Costs

$\mathbf{X}$ - $m \times n$ stored on p workers, $\mathbf{L}, \mathbf{R}$ each of rank k

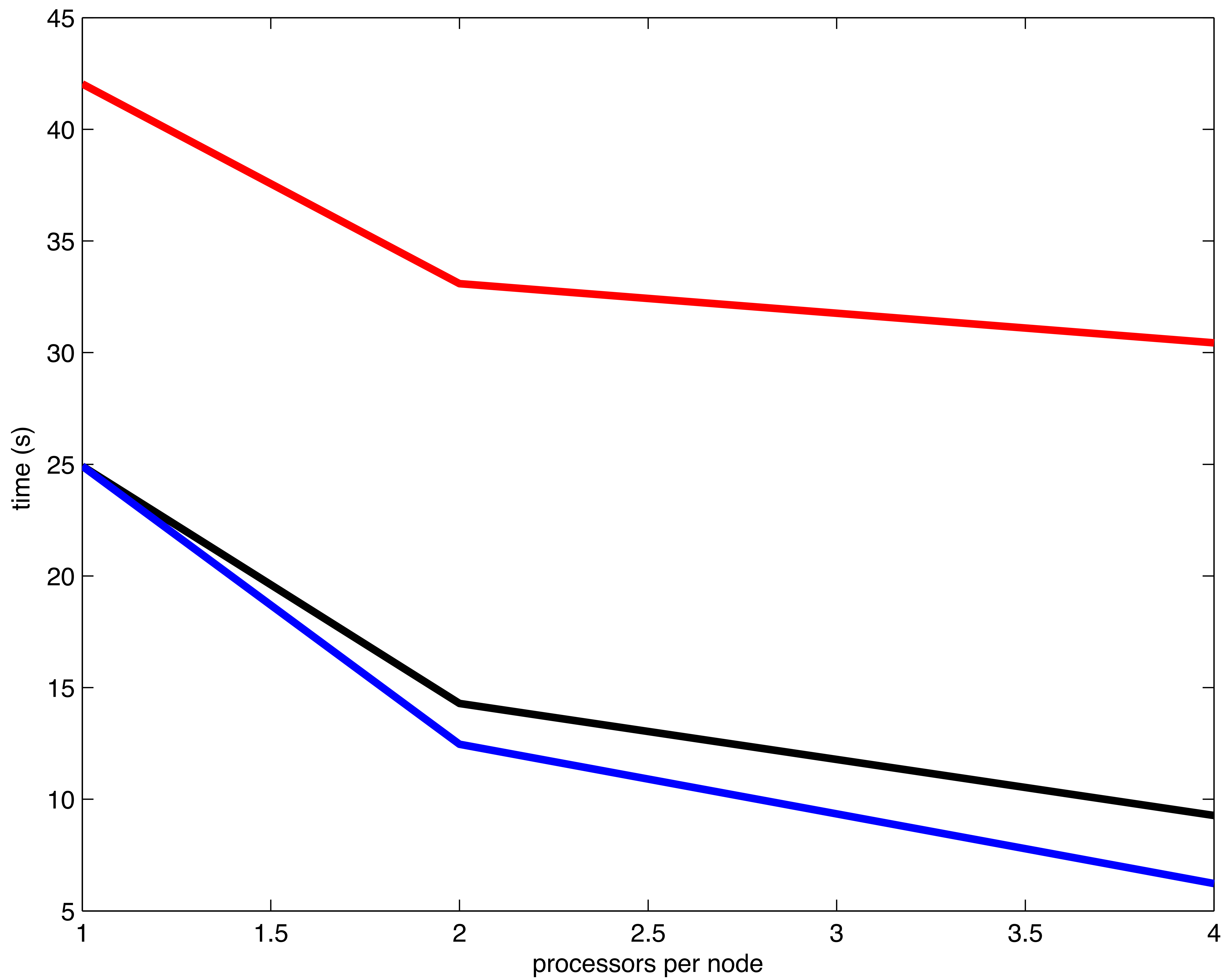
Communication per cycle : $O(nk)$ values

Total communication: $O(nkp) \ll O(mn)$ when $k, p \ll \sqrt{m}$

- much less than the full matrix

Only small submatrices in memory at any given time, *not* the full matrix

Performance scaling - evaluation of objective



Black - spgLR
Red - Naive Matlab
Blue - 1/#processors

Initial guess

Naive initialization for the algorithm

Let $\mathcal{A}^*b = \mathbf{U}\mathbf{S}\mathbf{V}^T$ be the rank k SVD of the zero-filled data

- $\mathbf{L} = \mathbf{U}\mathbf{S}^{-1/2}, \mathbf{R} = \mathbf{V}\mathbf{S}^{-1/2}$

- $\mathcal{A}^*b$ is an enormous, distributed matrix, SVD costly computationally, "overkill" as an initial guess

- can be shown to be "close" to the unknown matrix in Frobenius norm, so in some sense an optimal initial guess

Initial guess

Naive initialization for the algorithm

Let $\mathbf{L}, \mathbf{R}$ be initialized as Gaussian random matrices, appropriately scaled

- $\mathbf{L}\mathbf{R}^T$ is, in general, far from the true matrix $\mathbf{X}$ in Frobenius norm
- requires more iterations to decrease error below prescribed tolerance
- alternatively, for fixed number of iterations, error is higher using this initialization compared to one based on the data

Initial guess

Want to approximate the k-rank SVD without having to compute it exactly

- equivalent to $\min_{\mathbf{Q}\in\mathbb{R}^{m\times k}} \quad \|\mathbf{X}-\mathbf{Q}\mathbf{Q}^T\mathbf{X}\|_F^2$

$$\text{such that} \quad \mathbf{Q}^T\mathbf{Q}=\mathbf{I}$$

Approximate solution:

- Let $\mathbf{Y}=\mathbf{X}\Omega$ where Ω is a $n\times k$ Gaussian matrix

- Set $\mathbf{L}=\mathbf{Y}(\mathbf{Y}^T\mathbf{Y})^{-1/2}, \mathbf{R}=\mathbf{X}^T\mathbf{L}$

Initial guess

Approximate solution: $\mathbf{Y} = \mathbf{X}\Omega \;\; \mathbf{L} = \mathbf{Y}(\mathbf{Y}^T\mathbf{Y})^{-1/2} \;\; \mathbf{R} = \mathbf{X}^T\mathbf{L}$

When $\mathbf{X} = \mathcal{A}^* b$ is the zero-filled data, initialization only involves

- matrix-matrix multiplications - efficient in a distributed environment
- eigenvalue decomposition of a $k \times k$ matrix - easy when k is small
- much closer to the data in Frobenius norm than random noise initialization

Results

BG Data set

Single frequency slice, real part

68 x 68 source grid at 150m spacing

401 x 401 receiver grid at 50m spacing

SLIM

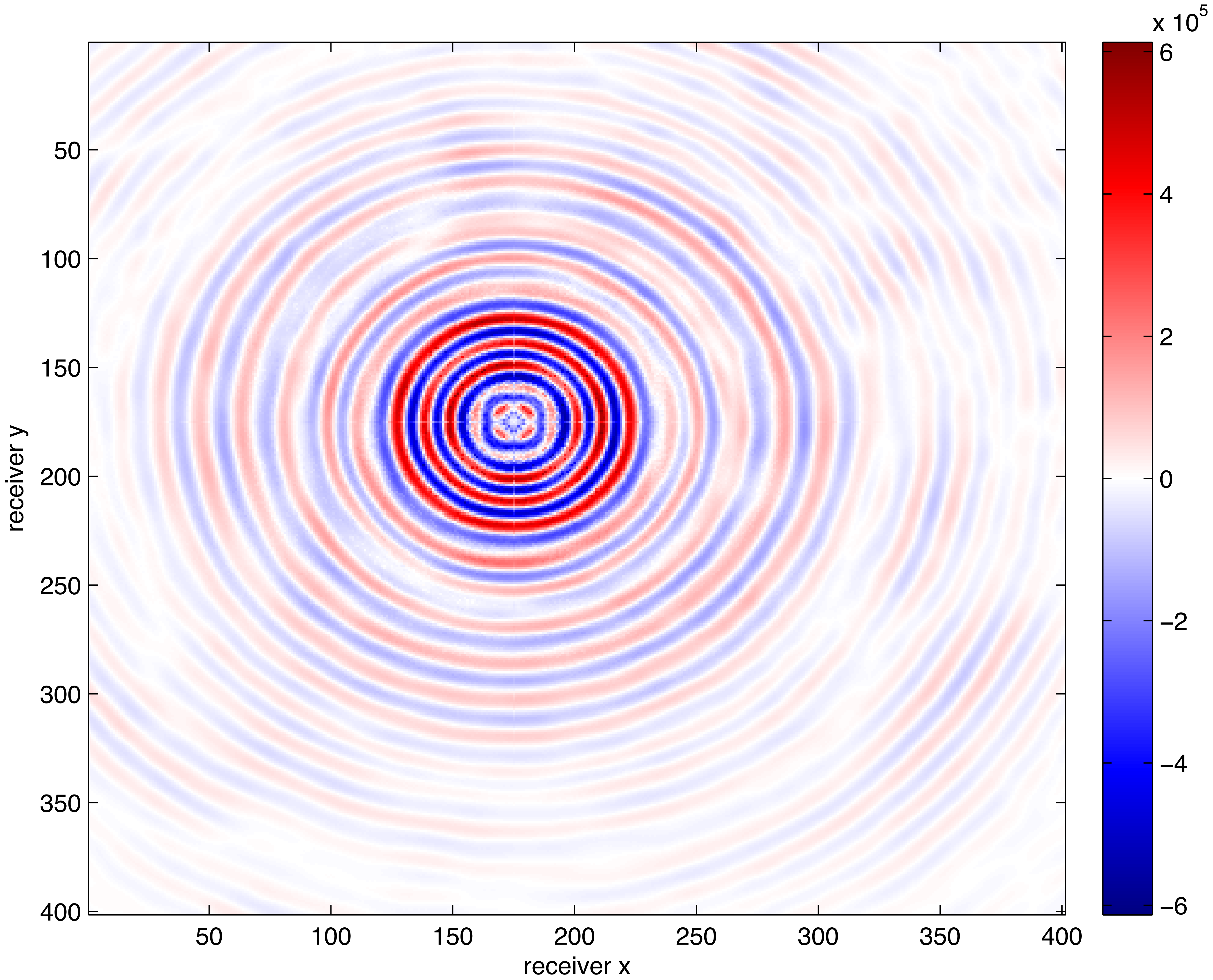
BG Data set

500 iterations of SPGLR

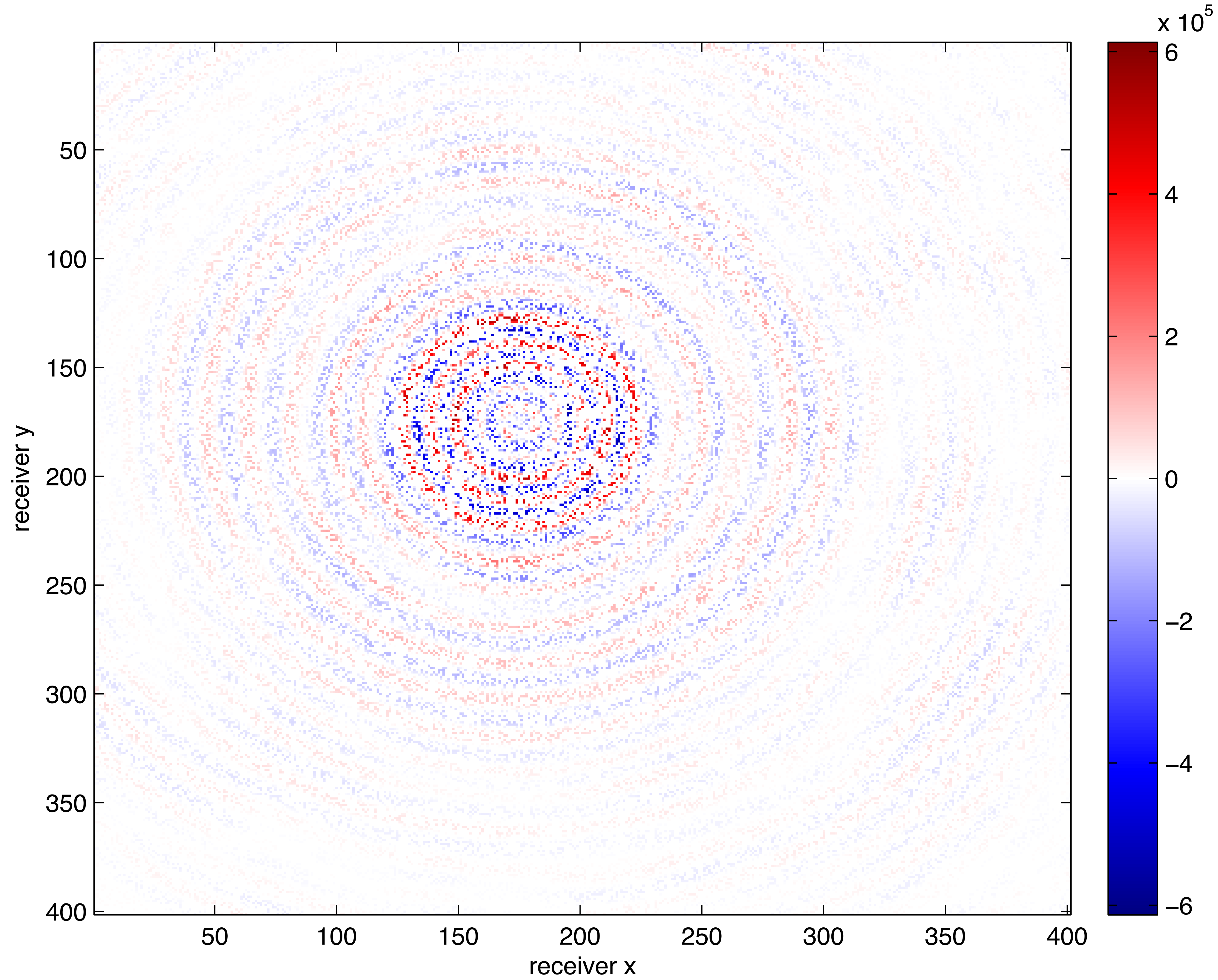
3 nodes, 4 processors per node

7.34Hz - 75% missing receivers

Common source gather



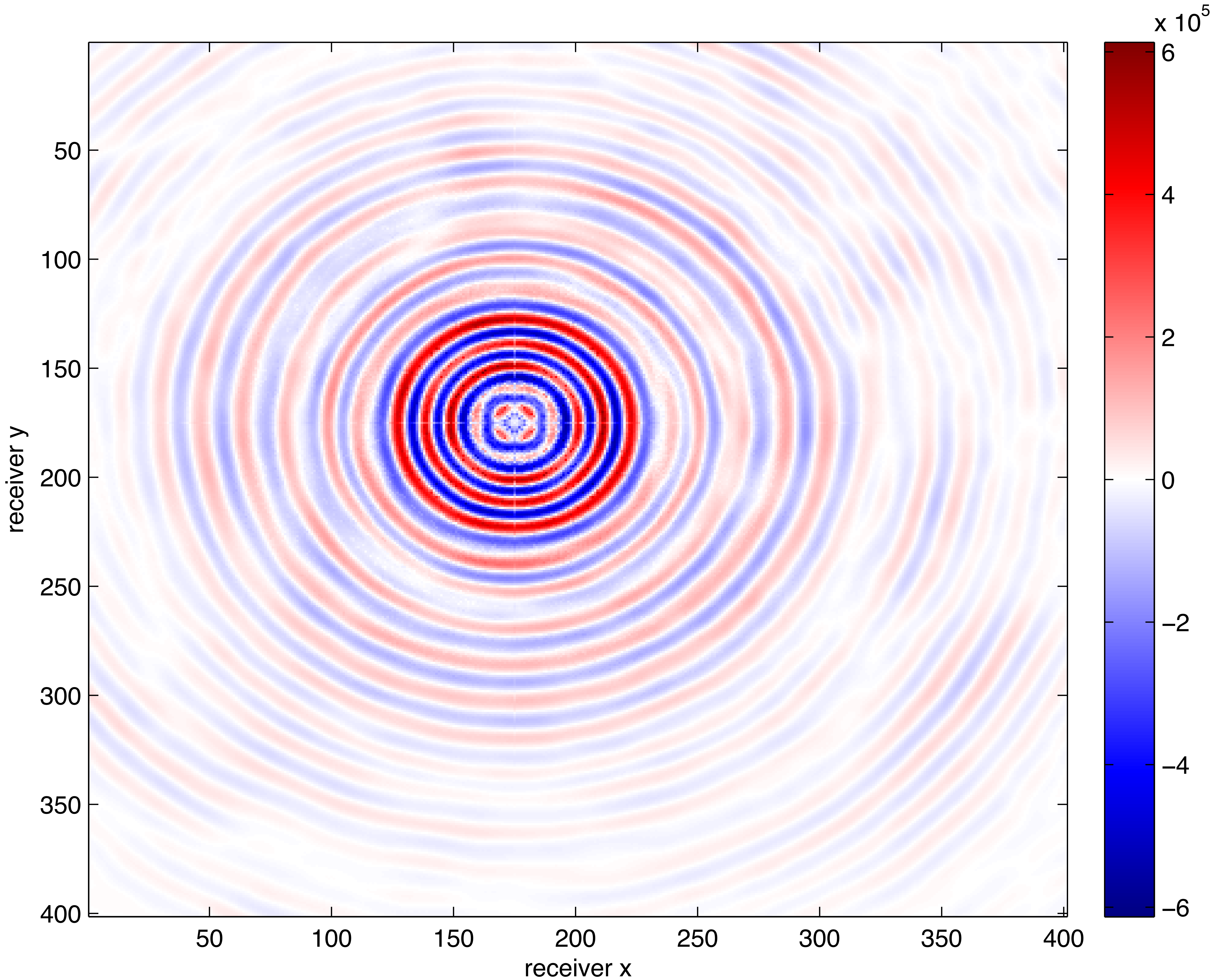
True data



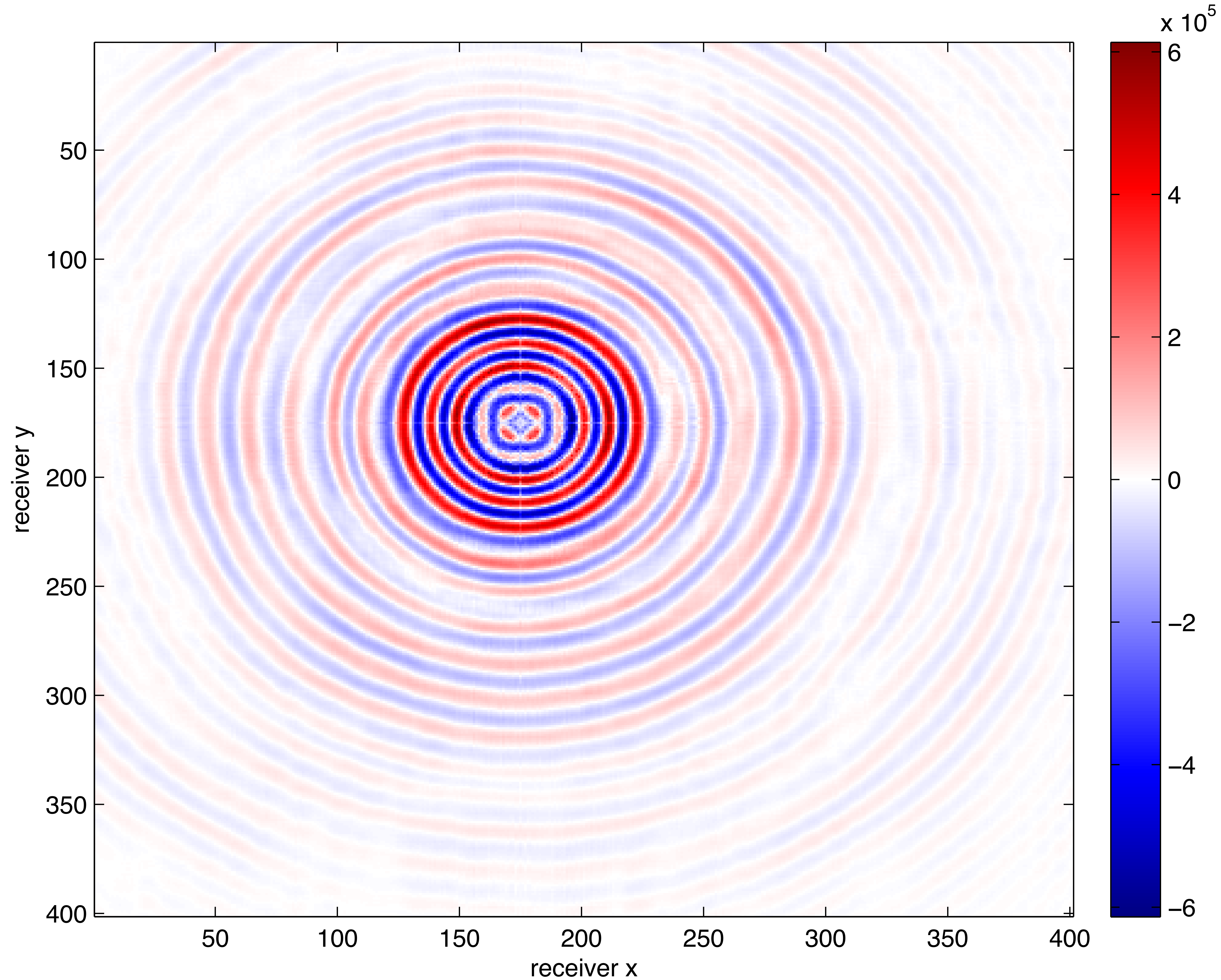
Subsampled data

7.34Hz - 75% missing receivers

Common source gather



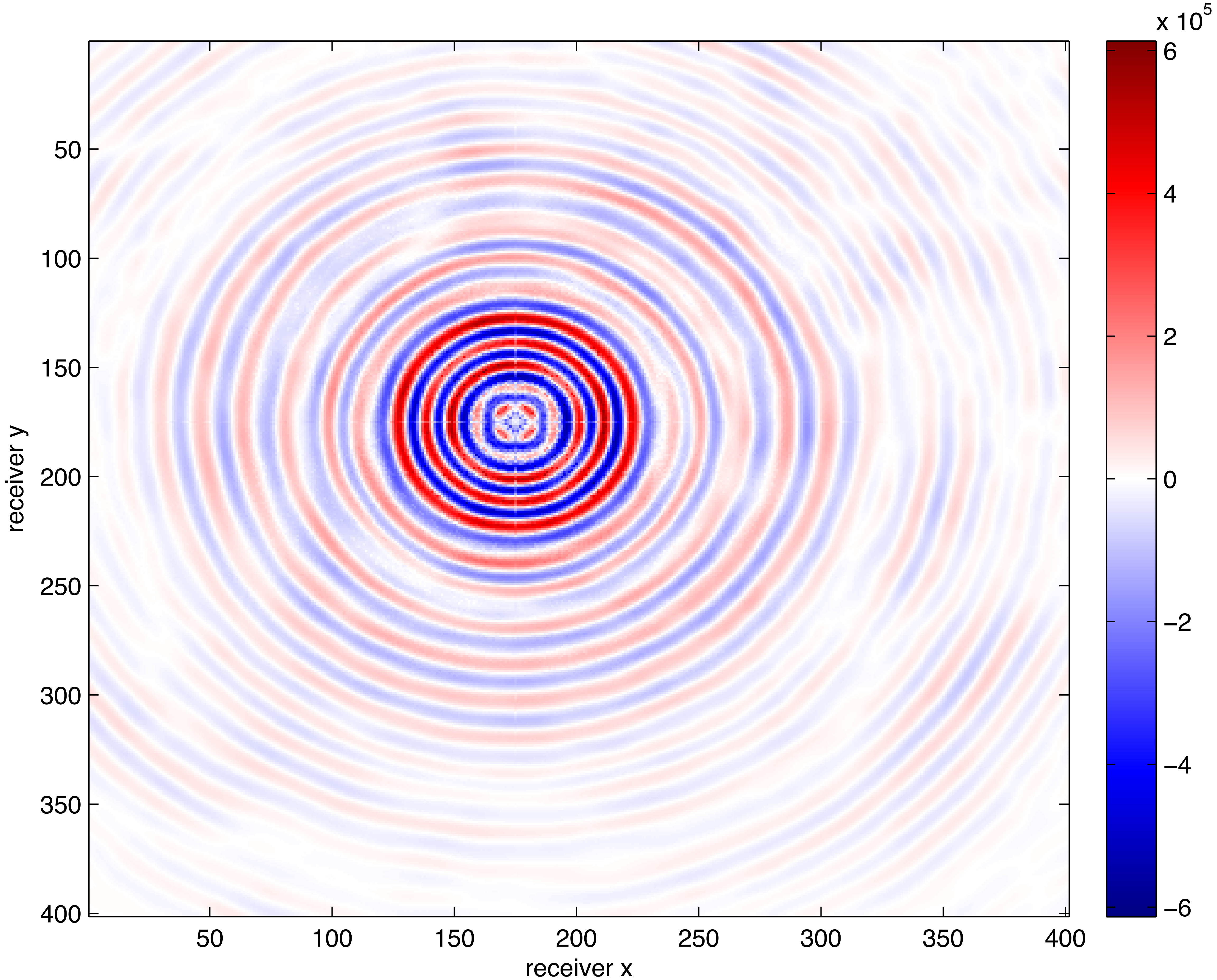
True data



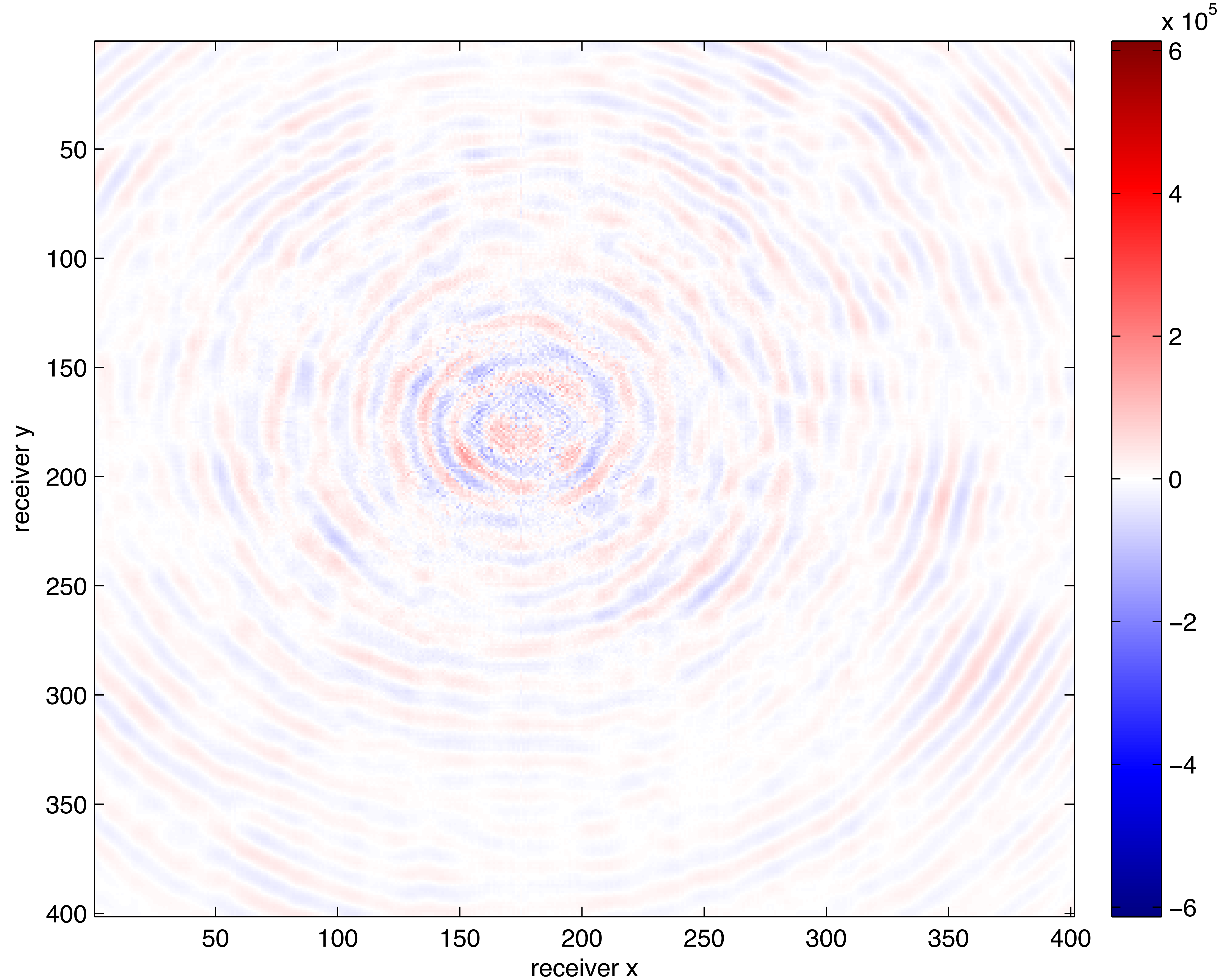
Interpolated data - SNR 14.5 dB

7.34Hz - 75% missing receivers

Common source gather



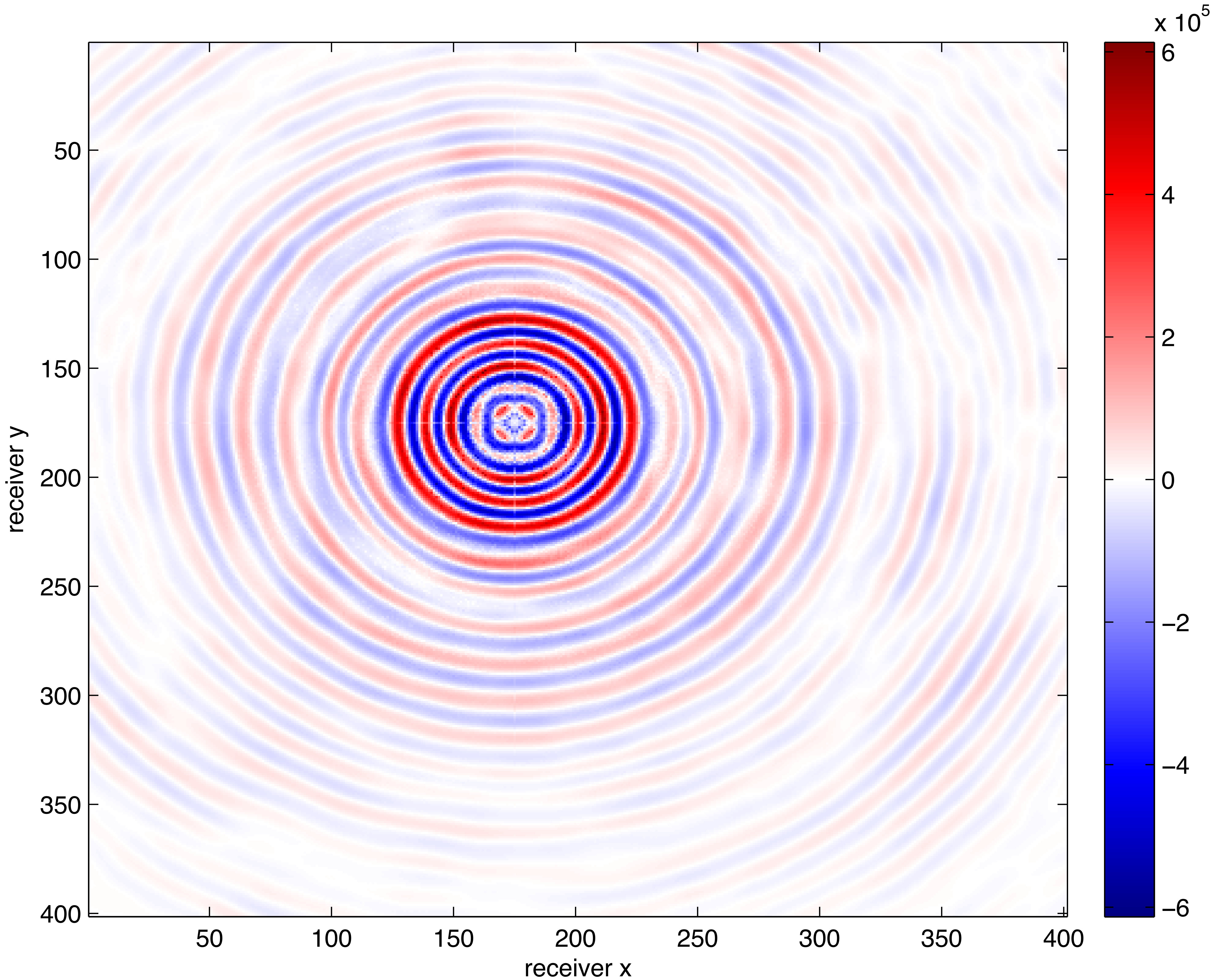
True data



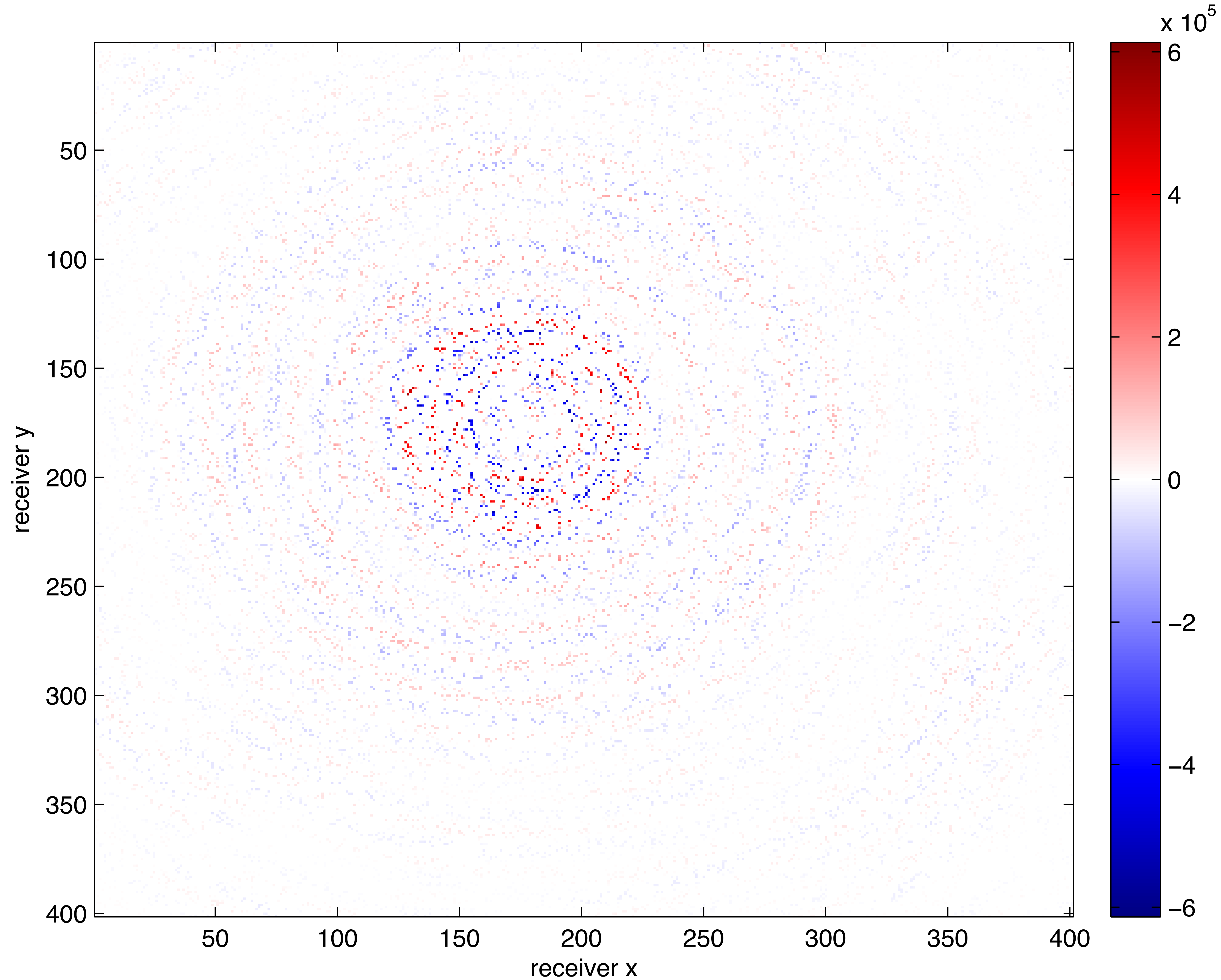
Difference

7.34Hz - 90% missing receivers

Common source gather



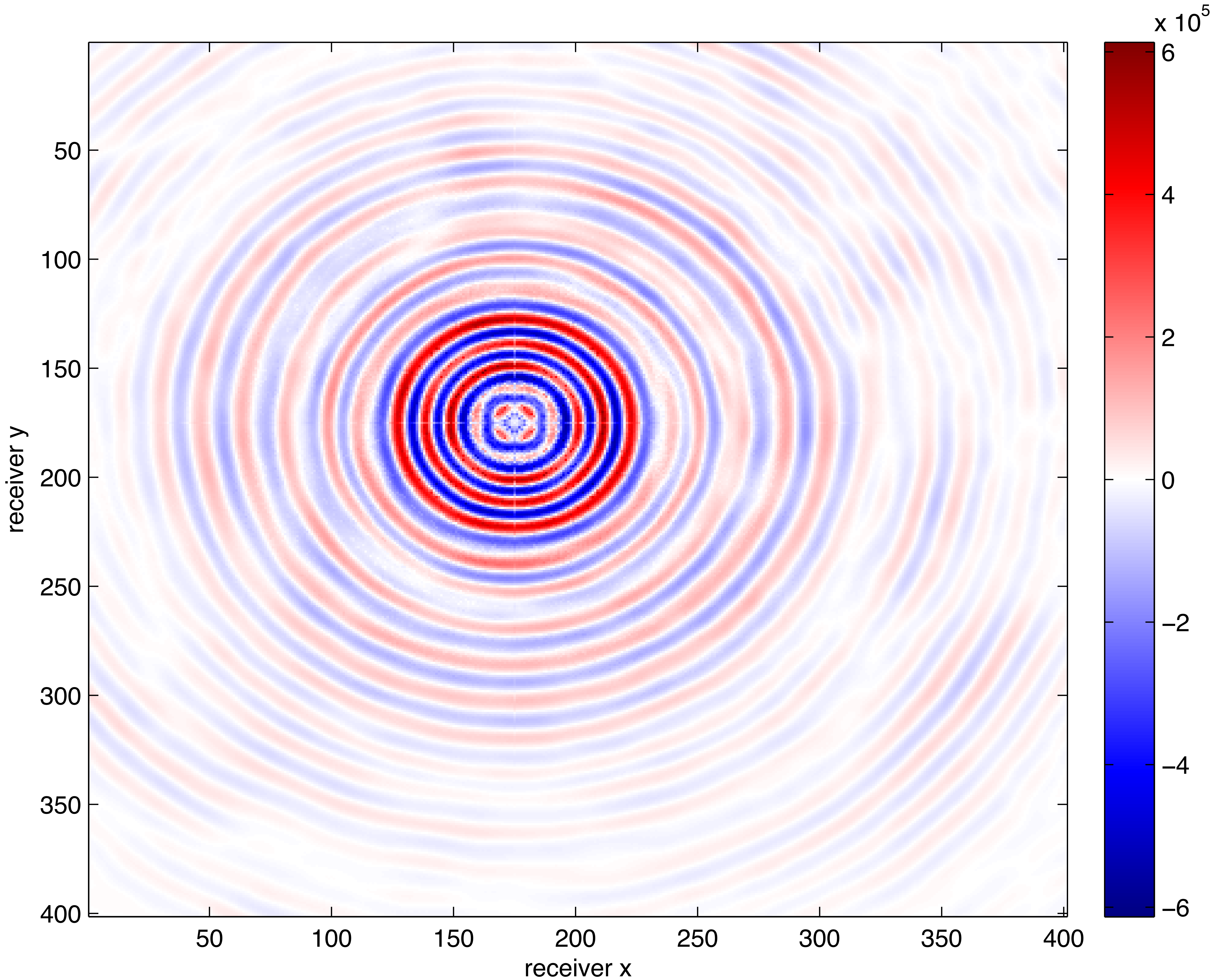
True data



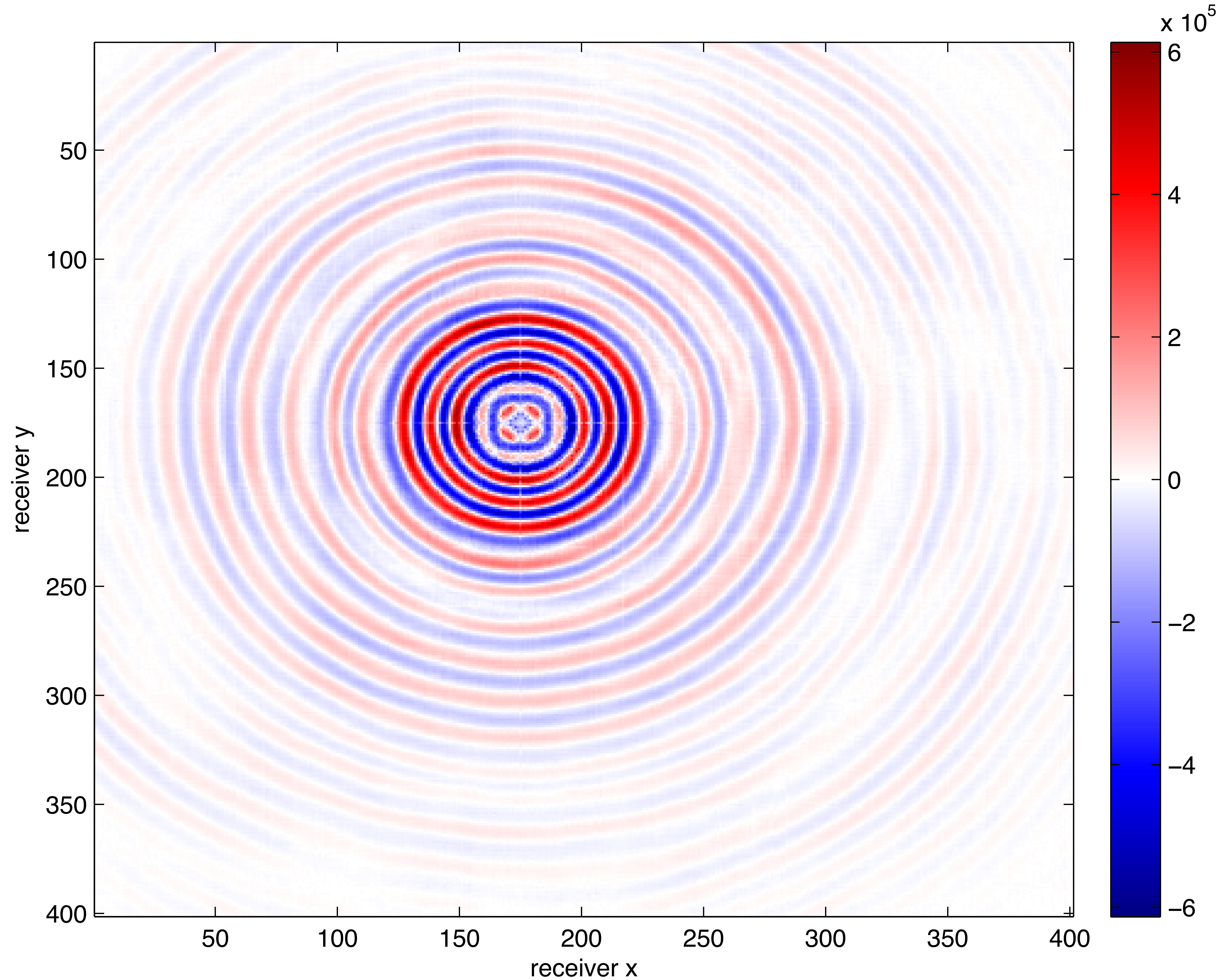
Subsampled data

7.34Hz - 90% missing receivers

Common source gather



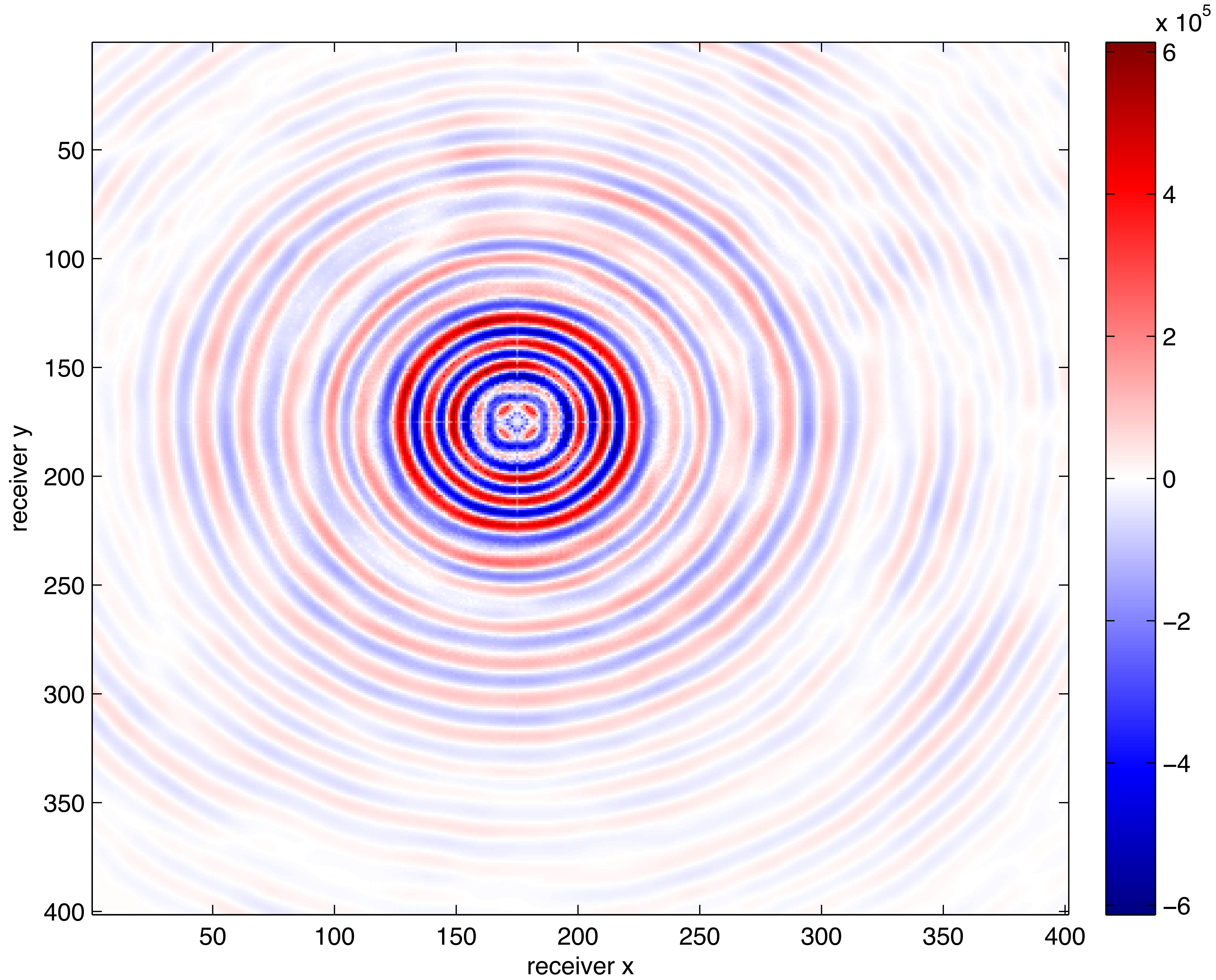
True data



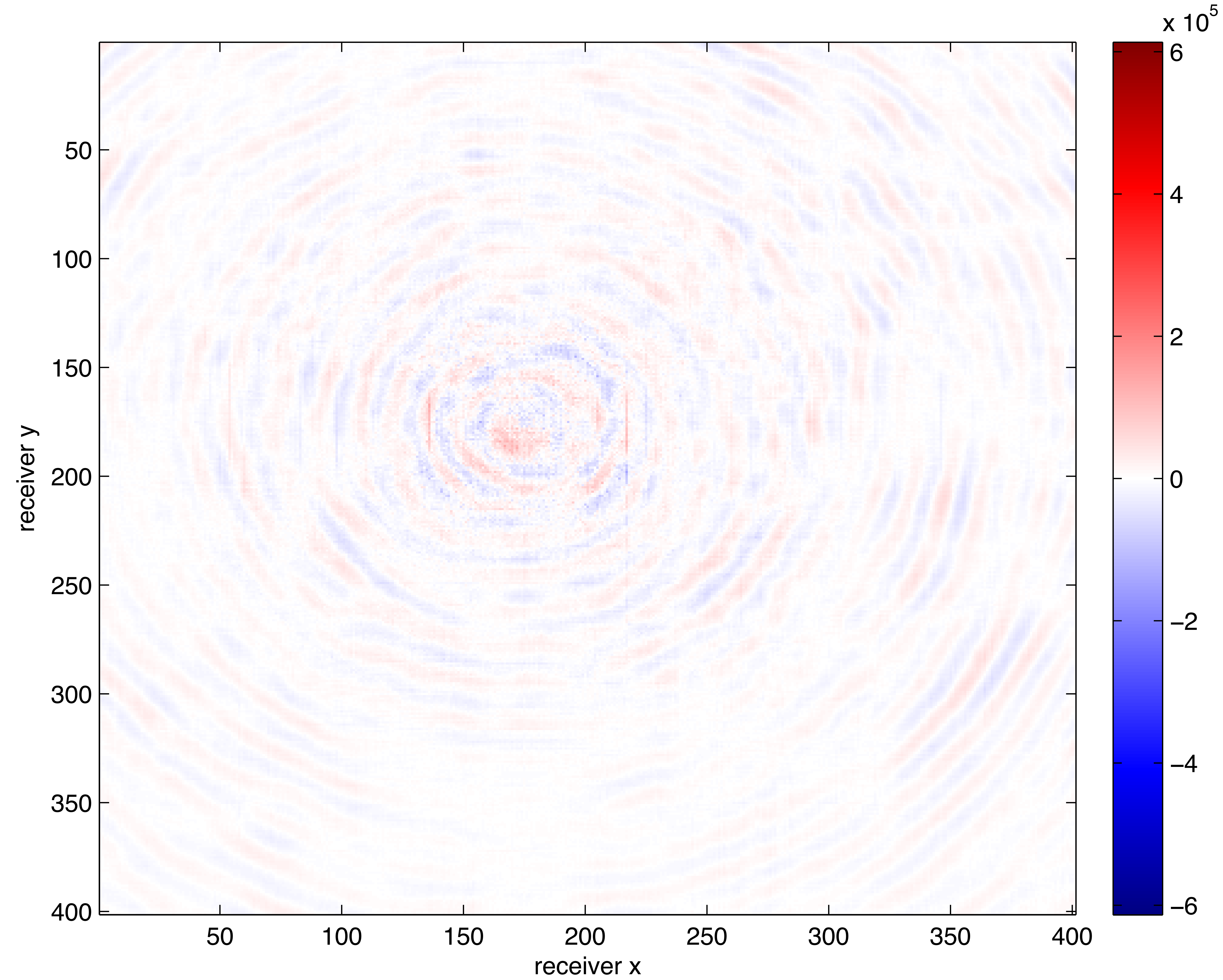
Interpolated data - SNR 15.9 dB

7.34Hz - 90% missing receivers

Common source gather



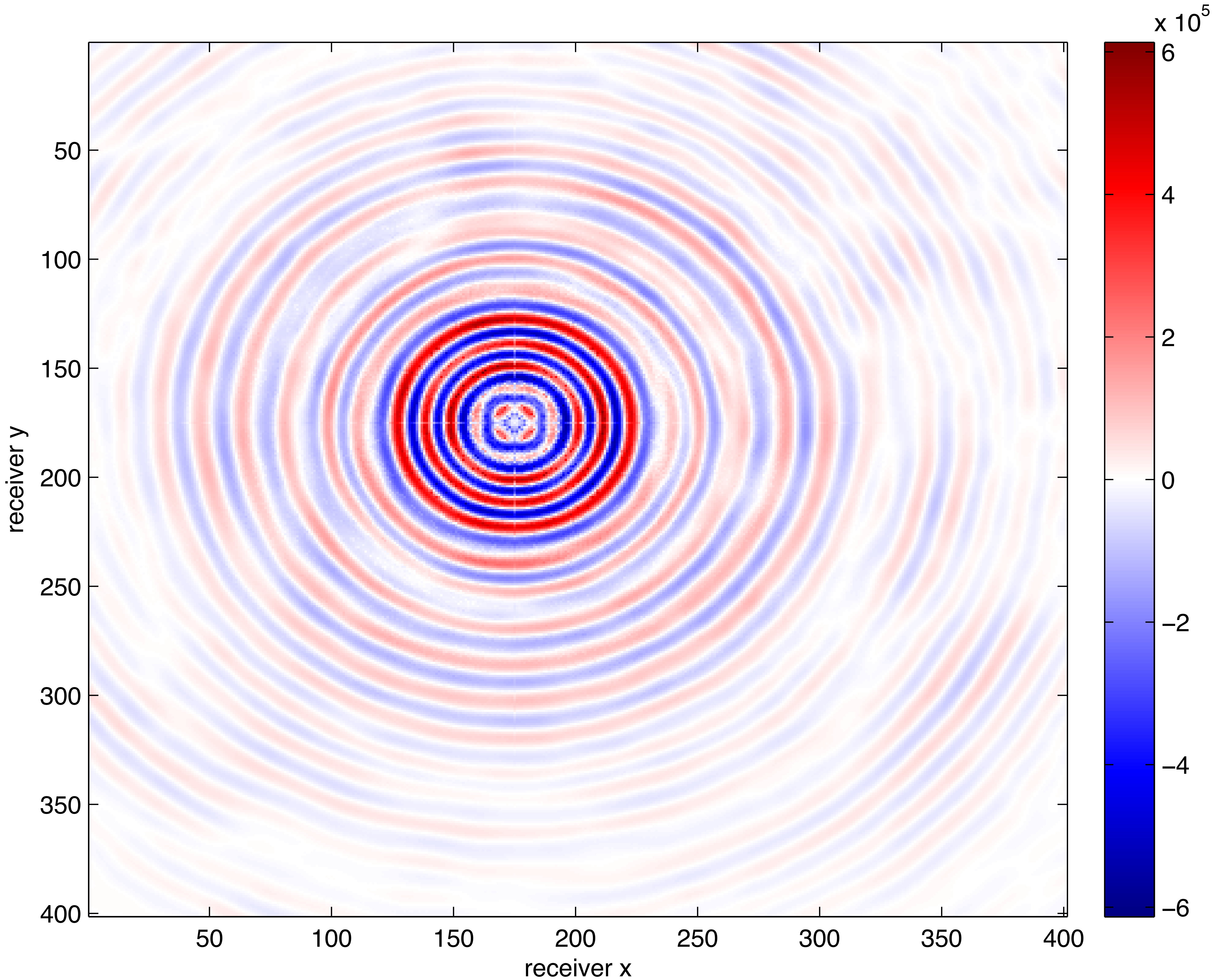
True data



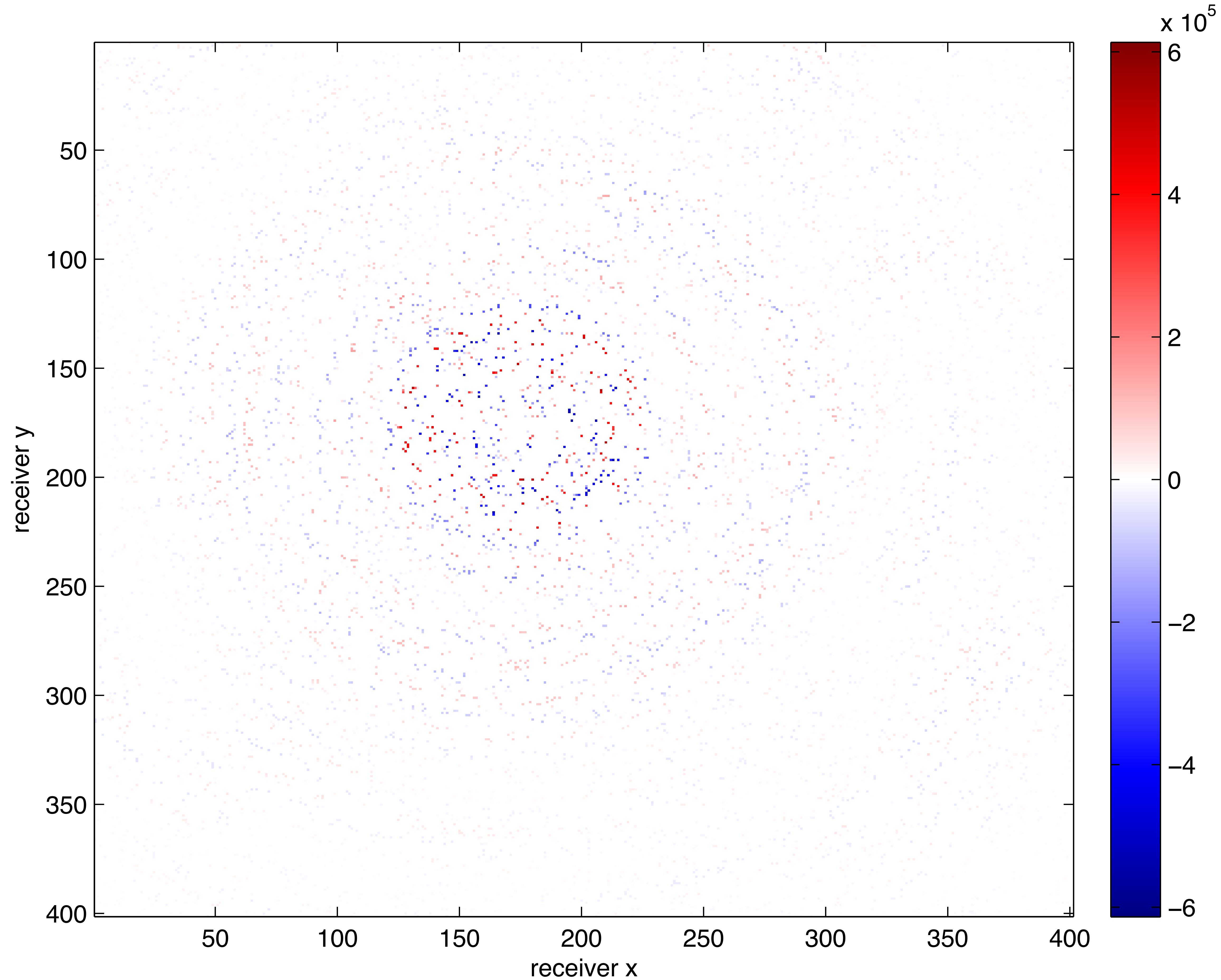
Difference

7.34Hz - 95% missing receivers

Common source gather



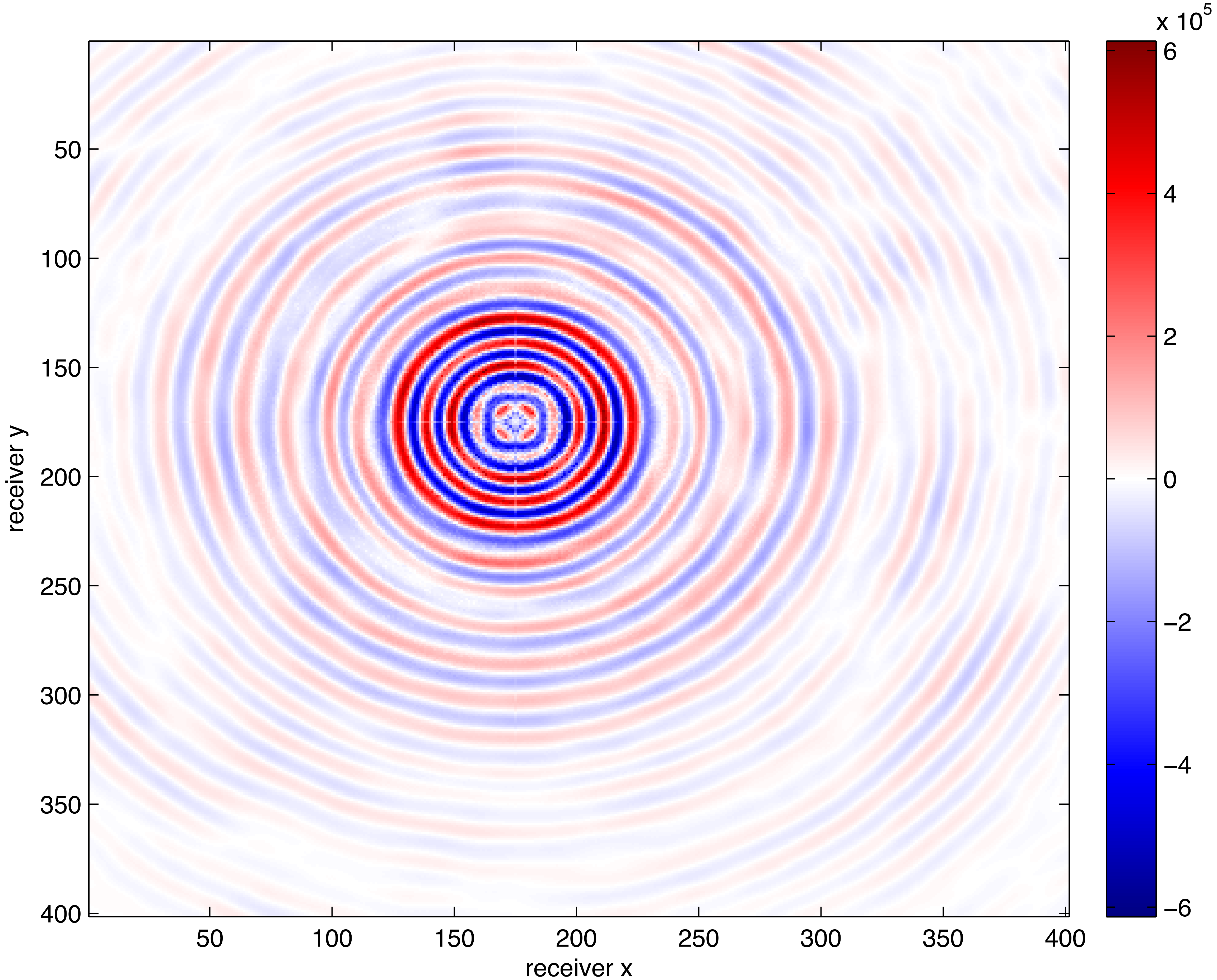
True data



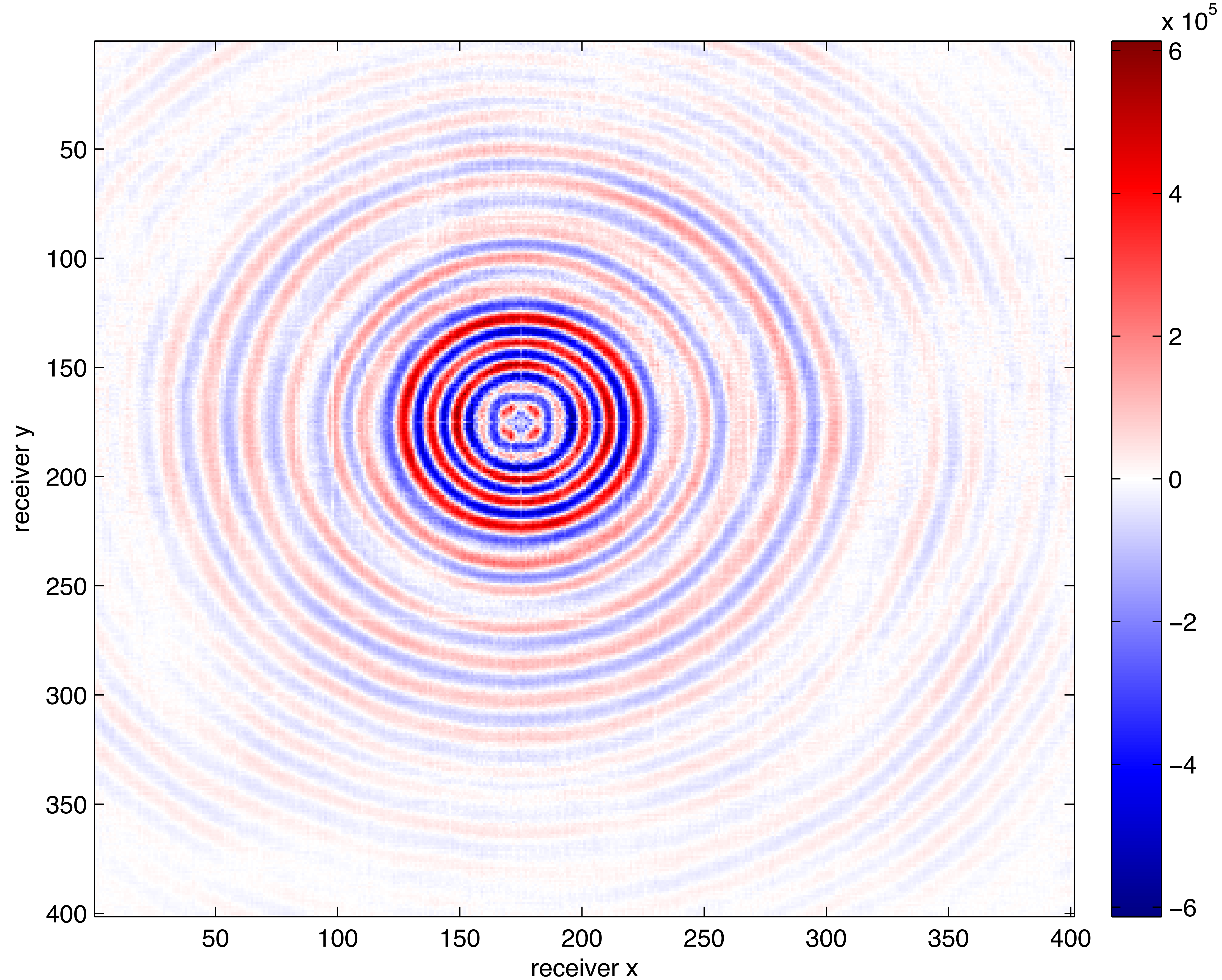
Subsampled data

7.34Hz - 95% missing receivers

Common source gather



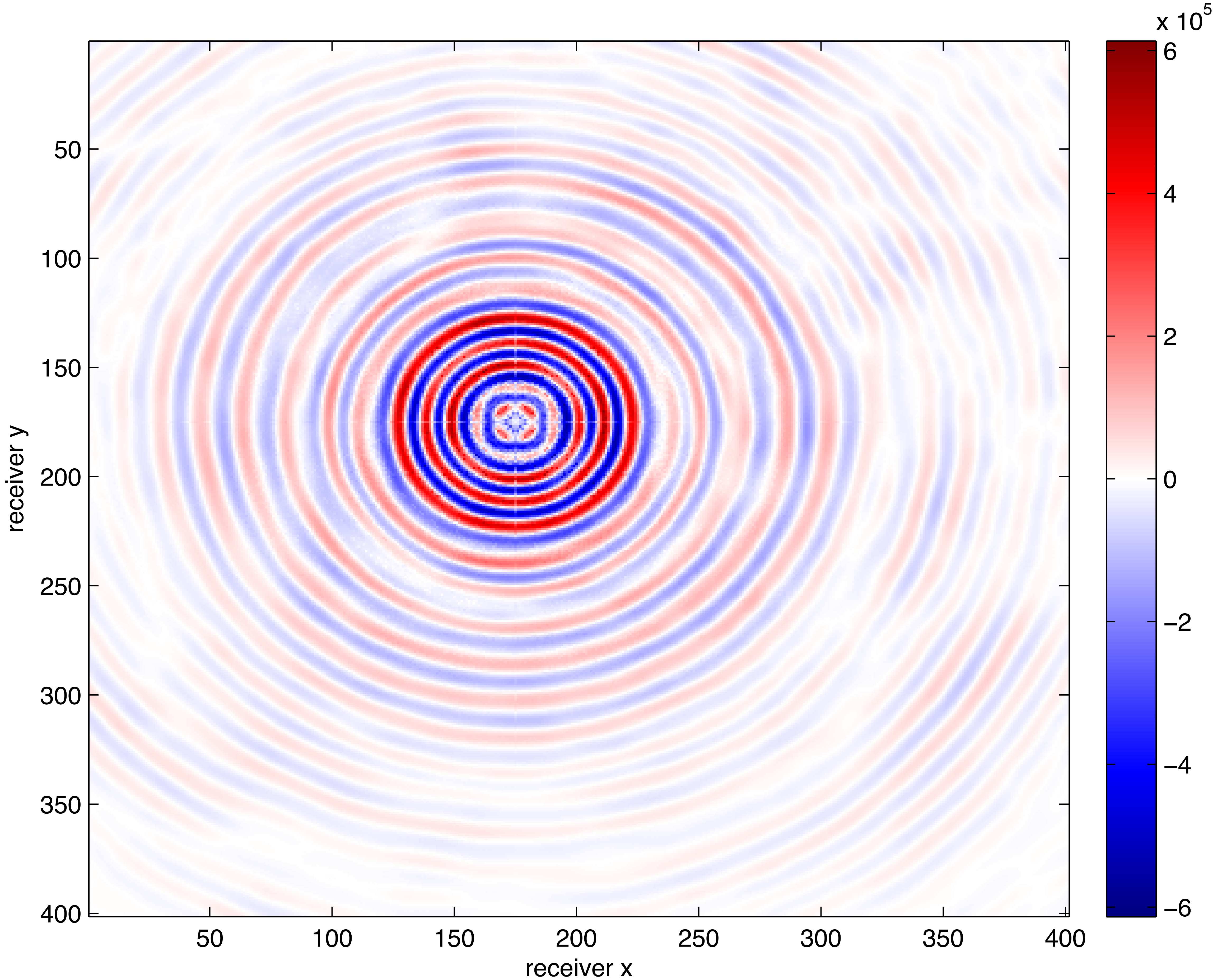
True data



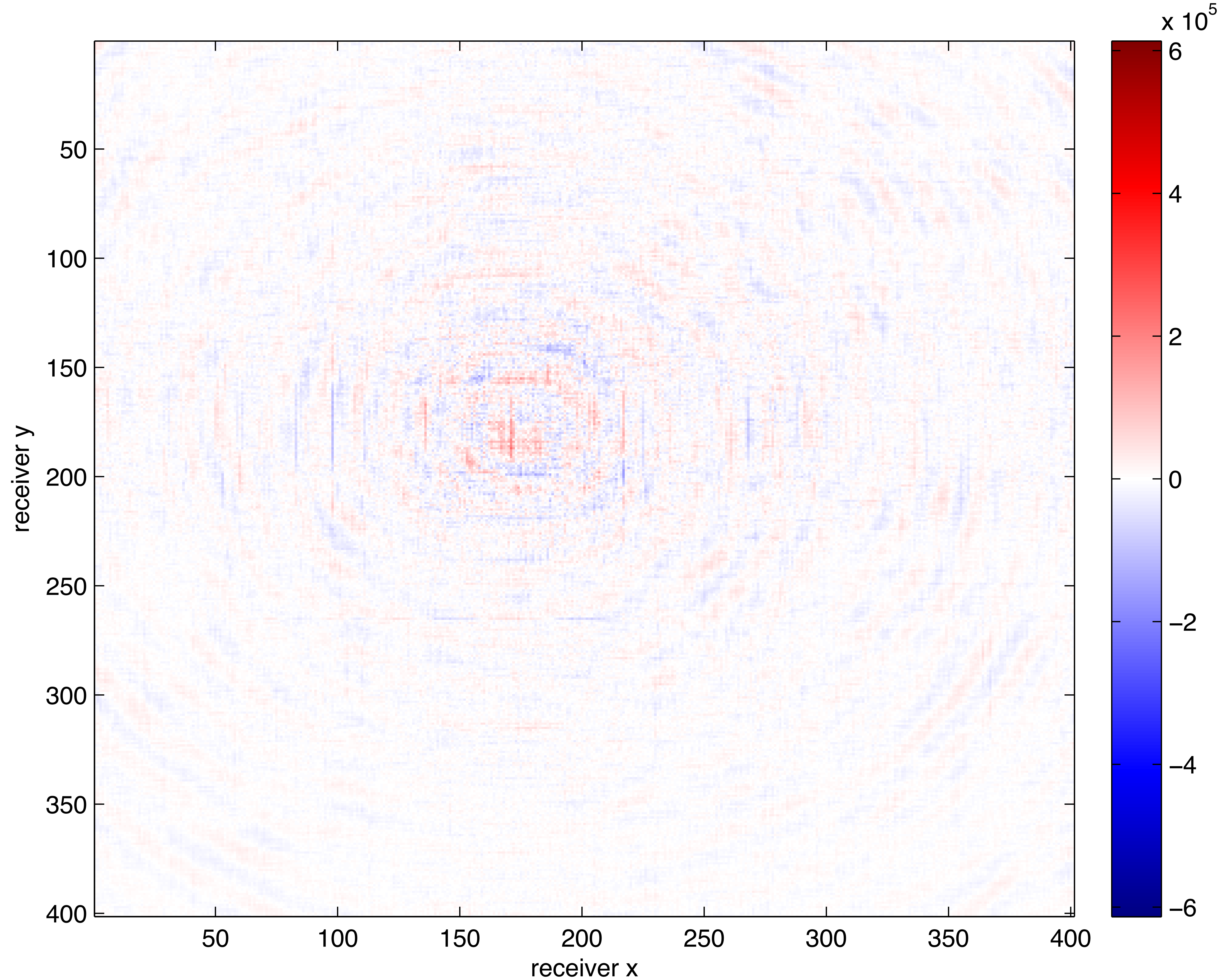
Interpolated data - SNR 14.2 dB

7.34Hz - 95% missing receivers

Common source gather



True data



Difference

BG data - 7.34Hz frequency slice

	SNR	Time (hr)	Rank
75% missing receivers	14.3	19.0	500
90% missing receivers	15.3	16.5	250
95% missing receivers	13.4	17.1	250

Straightforward extensions

Robust penalties for dealing with non-Gaussian noise

- huber, student's-t penalties, etc.

For blocks with very few data points, can exploit sparsity

Alternating LR - Next presentation by Oscar Lopez after the break

Off the grid tensor interpolation

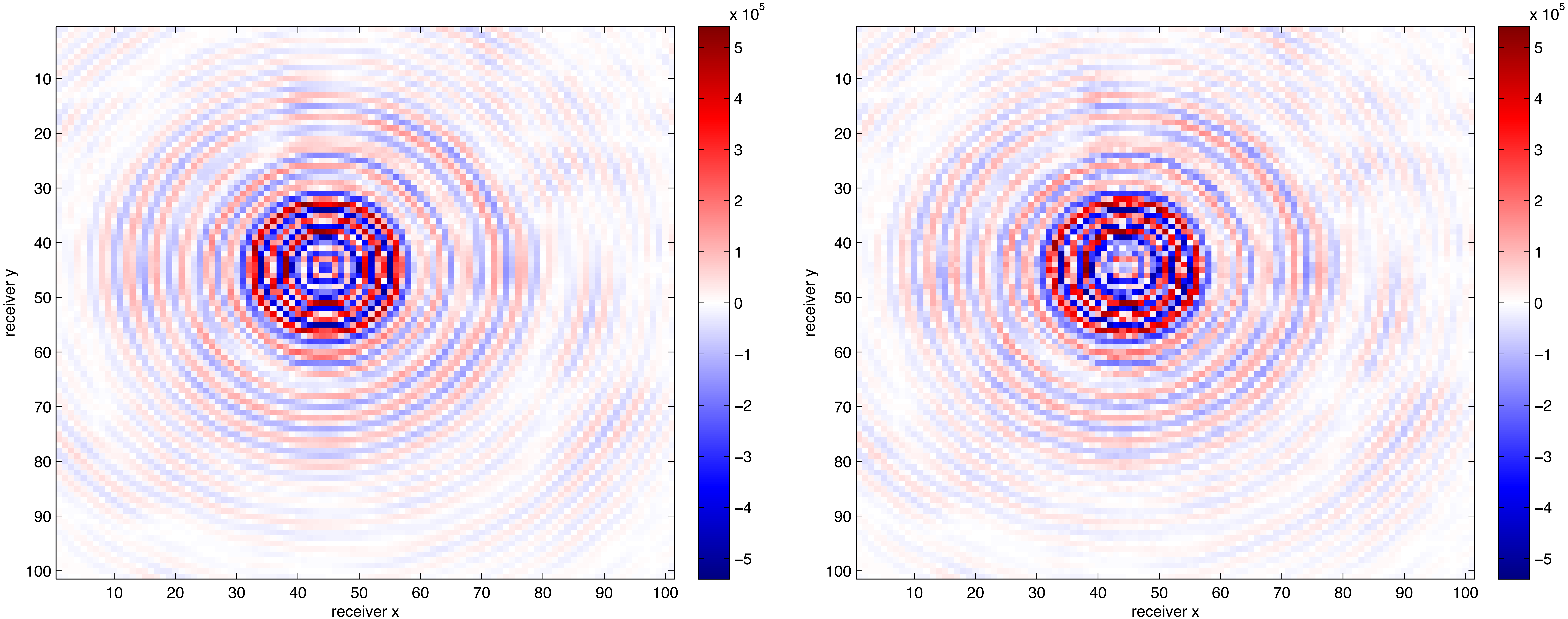
Regular vs irregular grid

Full data on a regular 401 x 401 m grid with 50m spacing

Subsampled data on an irregularly perturbed grid

- 200m spacing with 50% random 50m perturbations

Regular vs irregular grid
Common source gather



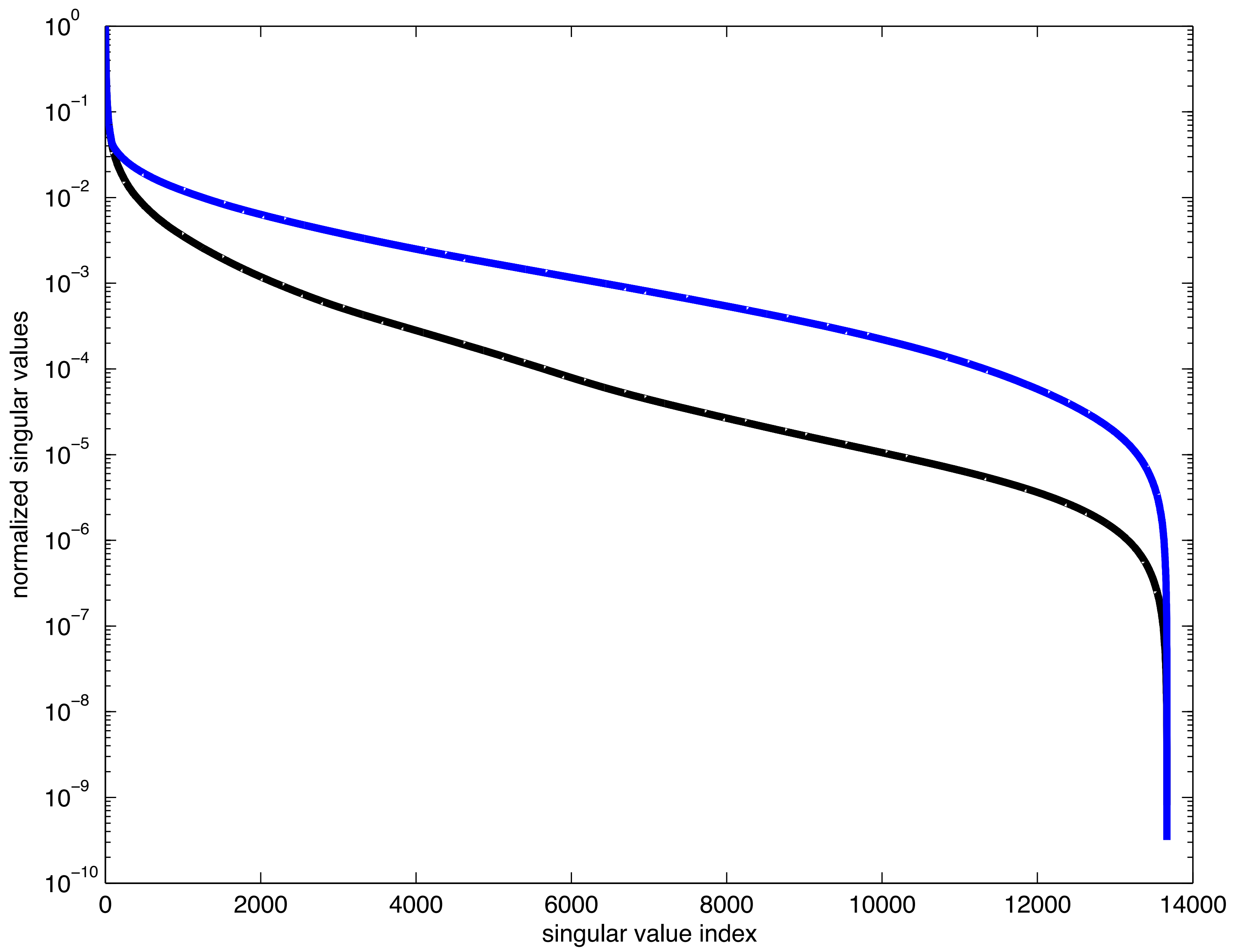
Regular grid

Irregular grid

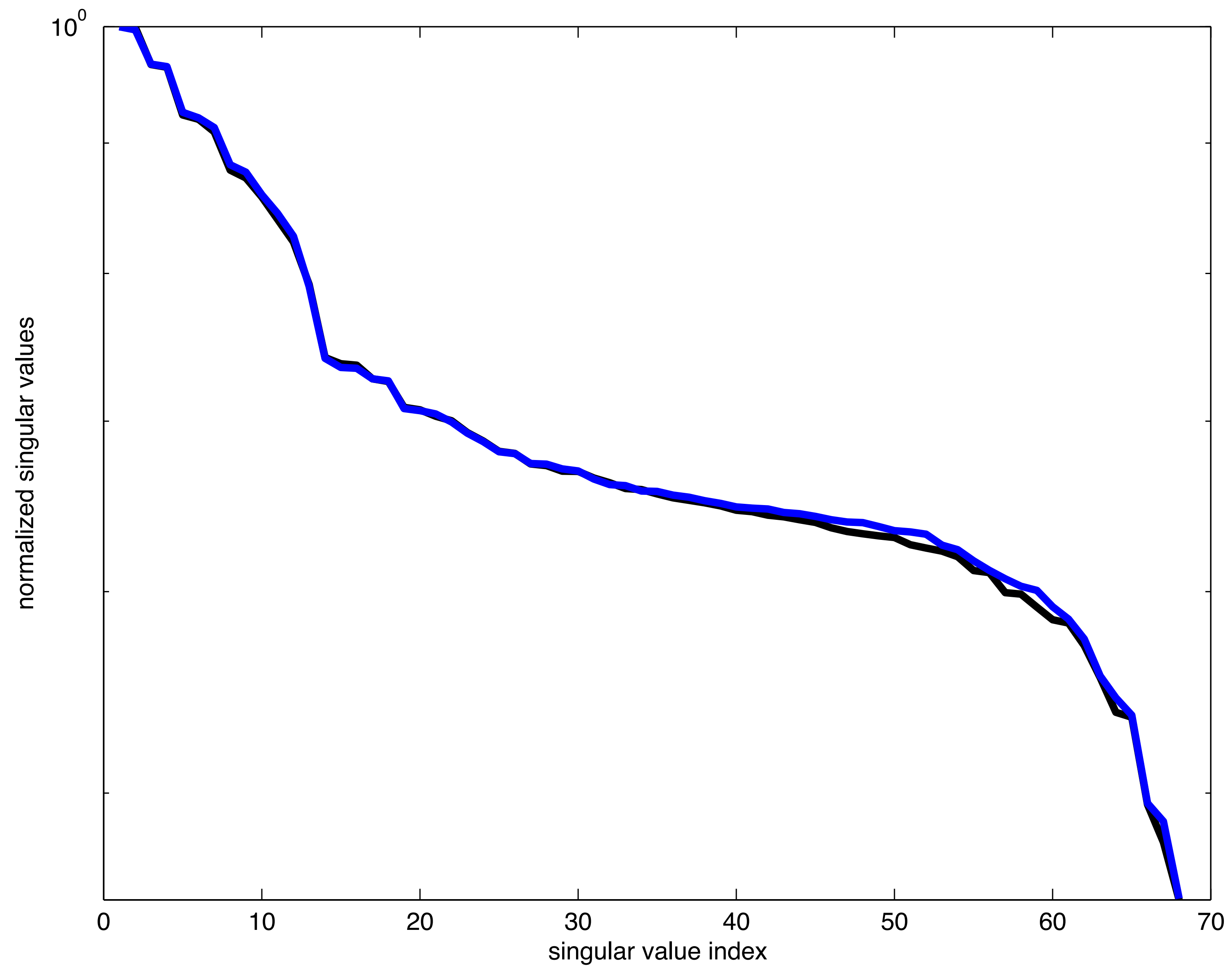
Regular vs irregular grid - singular values

Black - regular grid

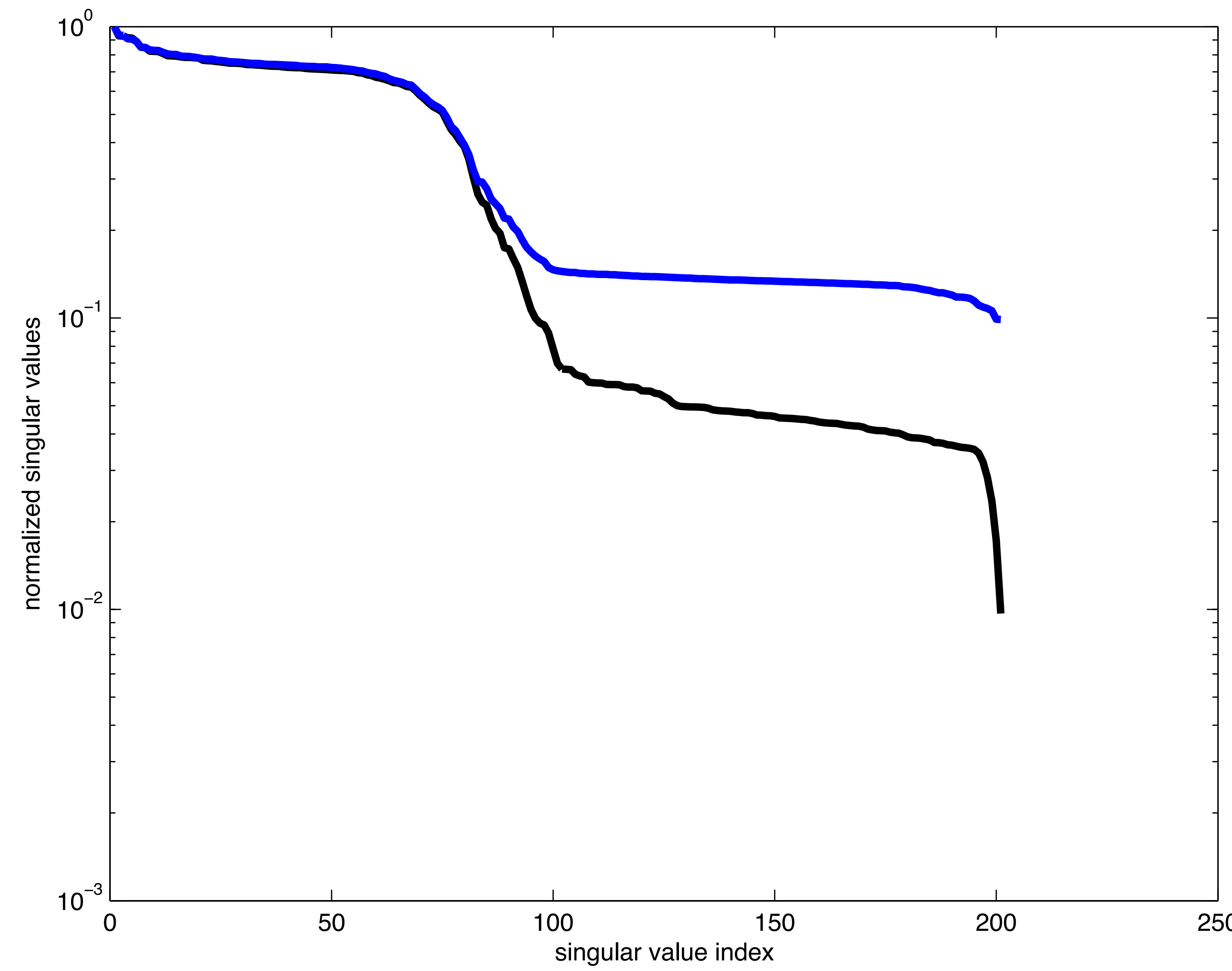
Blue - irregular grid



source x, receiver x



source x



receiver x

Off the grid tensor interpolation

The data volume is **no longer** low rank when irregularly sampled

- standard tensor completion framework won't work well

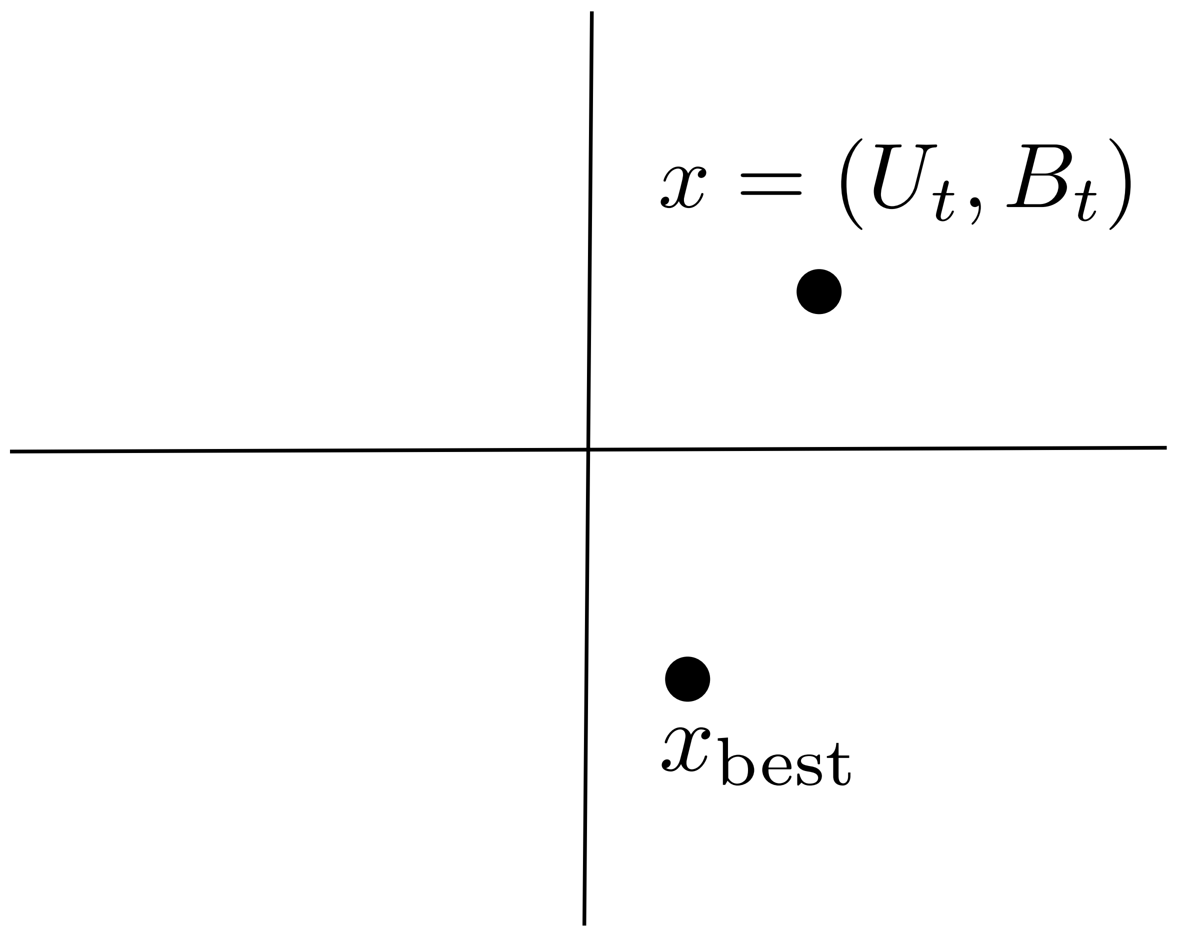
Solution

- construct a domain where the data **is** low rank
- choose an appropriate transform : low rank domain -> sampling domain
- incorporate transform in to optimization problem

Optimization program

$\mathbb{C}^D$

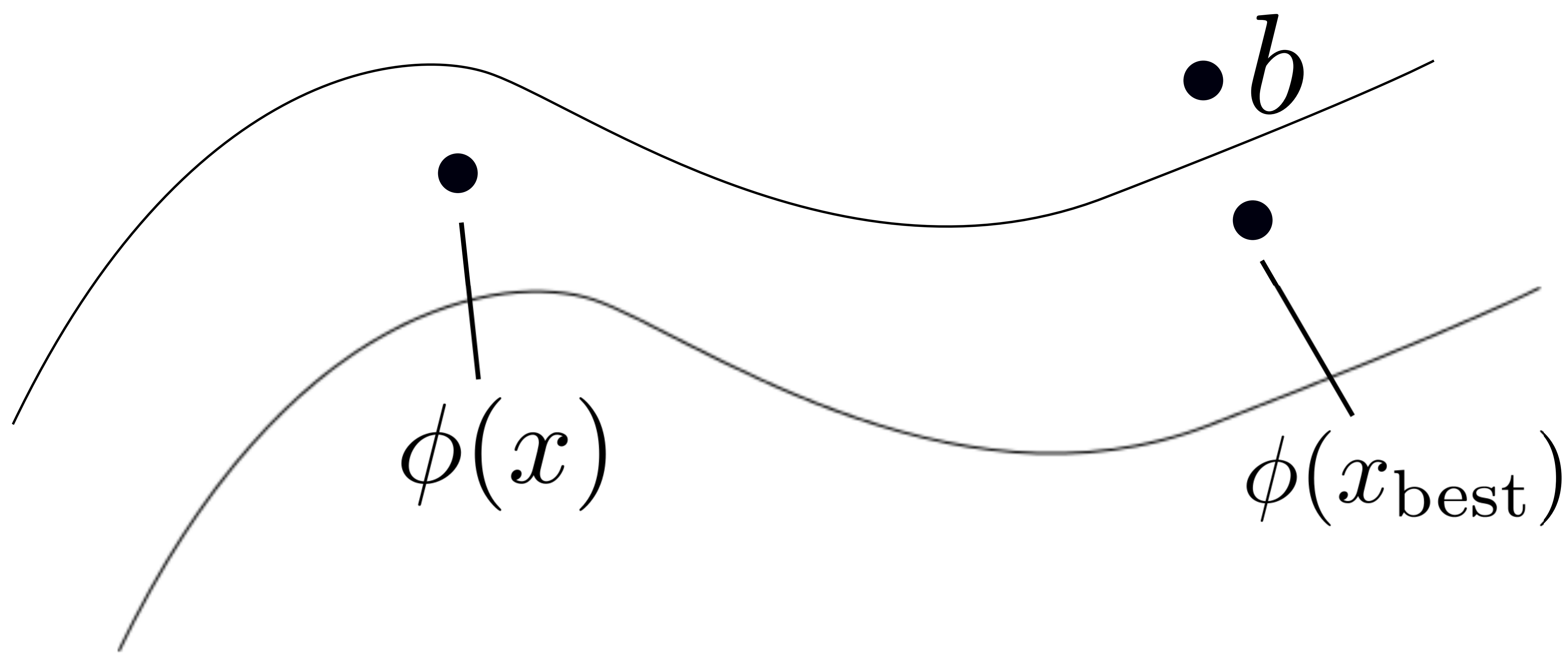
$$\phi(x)$$



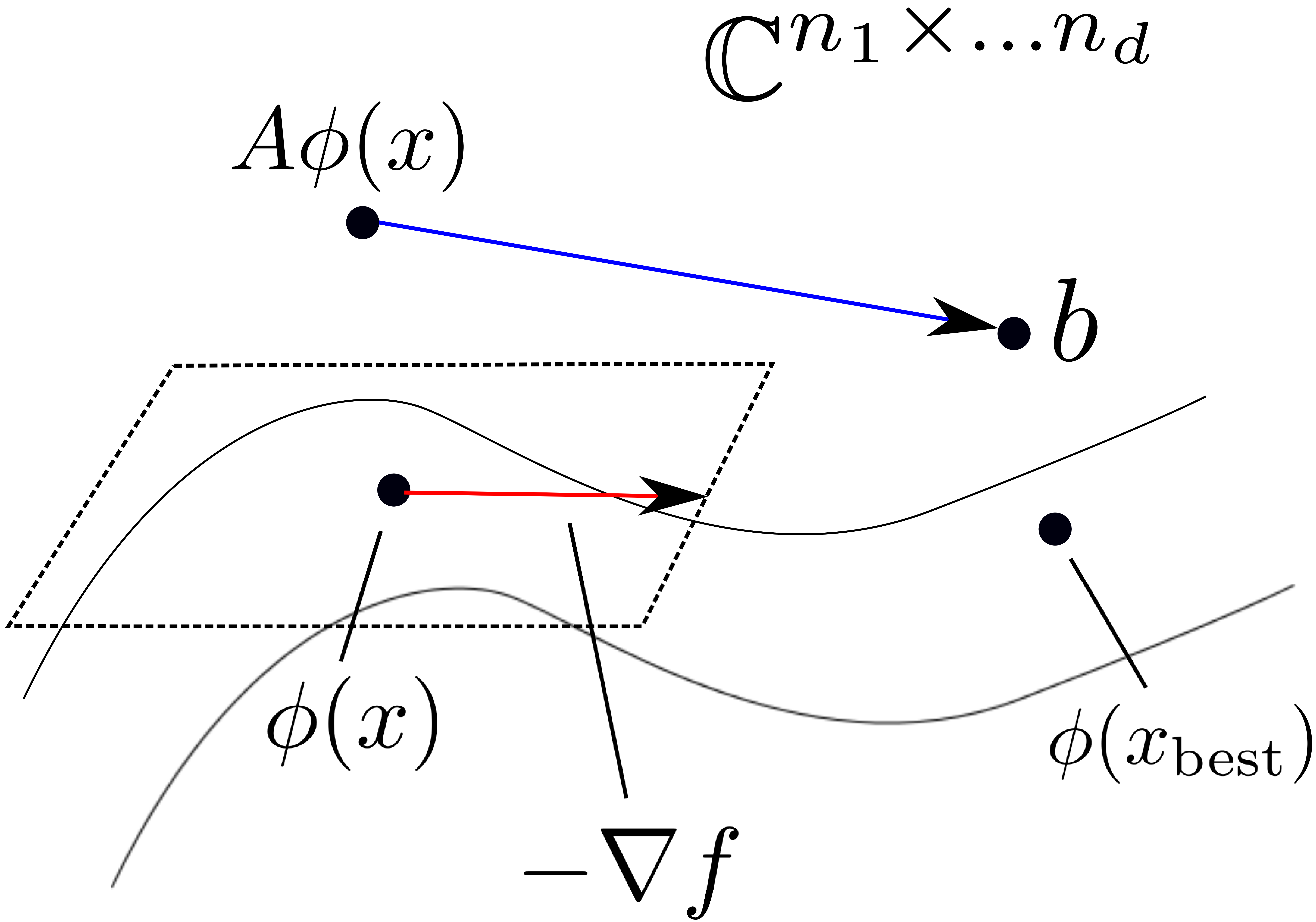
Full-tensor space

$$\mathbb{C}^{n_1 \times \dots n_d}$$

Parameter space



Optimization program



Optimization problem

The standard problem we solve is

$$\min_x \|\mathcal{A}\phi(x) - b\|_2^2$$

Our sampling operator is typically

$$\mathcal{A} = \mathcal{R}\mathcal{P}$$

where

$$\mathcal{R} : \text{regular full grid} \rightarrow \text{subsampled grid}$$

$$\mathcal{P} : (\text{src x}, \text{rec x}, \text{src y}, \text{rec y}) \rightarrow (\text{src x}, \text{src y}, \text{rec x}, \text{rec y})$$

Optimization problem

In the irregular grid case, the subsampling operator is in fact

$$\mathcal{R} : \text{irregular full grid} \rightarrow \text{subsampled grid}$$

In order to take this discrepancy in to account, we introduce an operator

$$\mathcal{F} : \text{regular full grid} \rightarrow \text{irregular full grid}$$

Optimization problem

The sequence of operators is then

$$\mathcal{R} : \text{irregular full grid} \rightarrow \text{subsampled grid}$$
$$\mathcal{F} : \text{regular full grid} \rightarrow \text{irregular full grid}$$
$$\mathcal{P} : (\text{src x}, \text{rec x}, \text{src y}, \text{rec y}) \rightarrow (\text{src x}, \text{src y}, \text{rec x}, \text{rec y})$$

We set $\mathcal{A} = \mathcal{RFP}$ and use the same optimization code as previously

In our examples, we use the non-uniform Fourier transform

Results

SLIM

Experiment setup

BG Group data

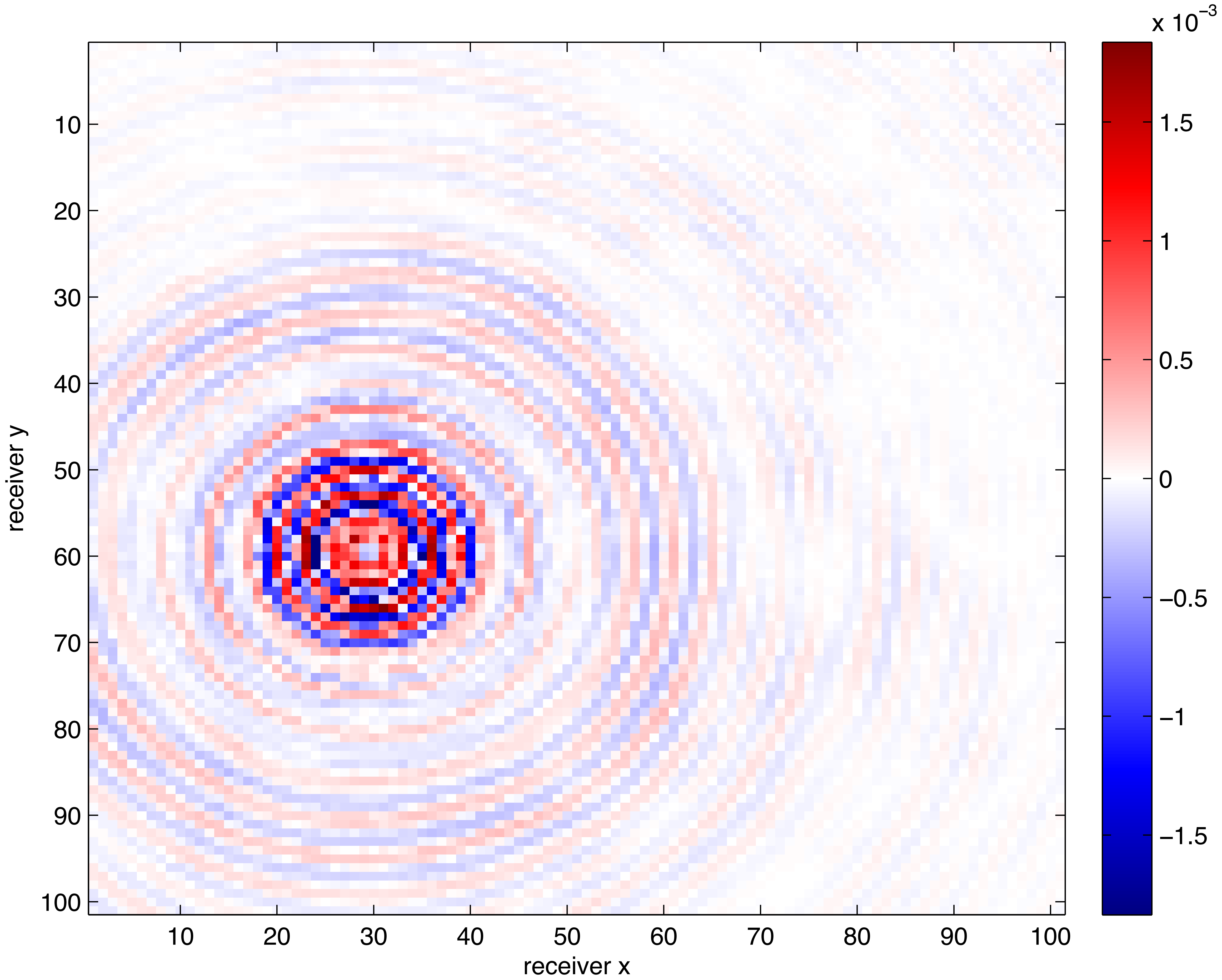
- 68 x 68 sources with 150m spacing
- 401 x 401 receivers with 50m spacing

Subsampled data on an irregularly perturbed grid

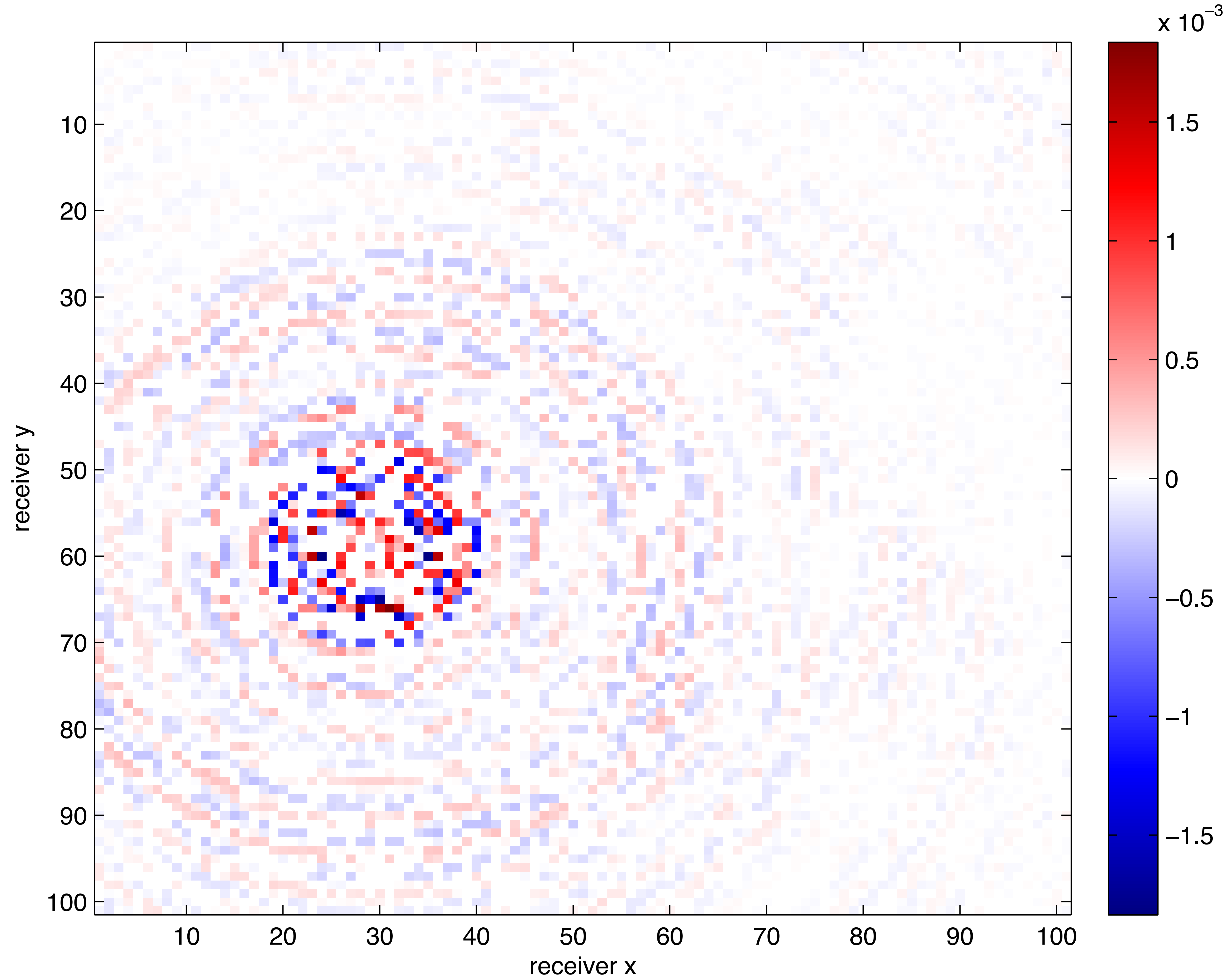
- source grid remains the same
- receiver grid subsampled to 200m spacing with 50% random 50m perturbations

Removed 50% of receivers, recovered with HT optimization

Regularized recovery - 50% missing receivers

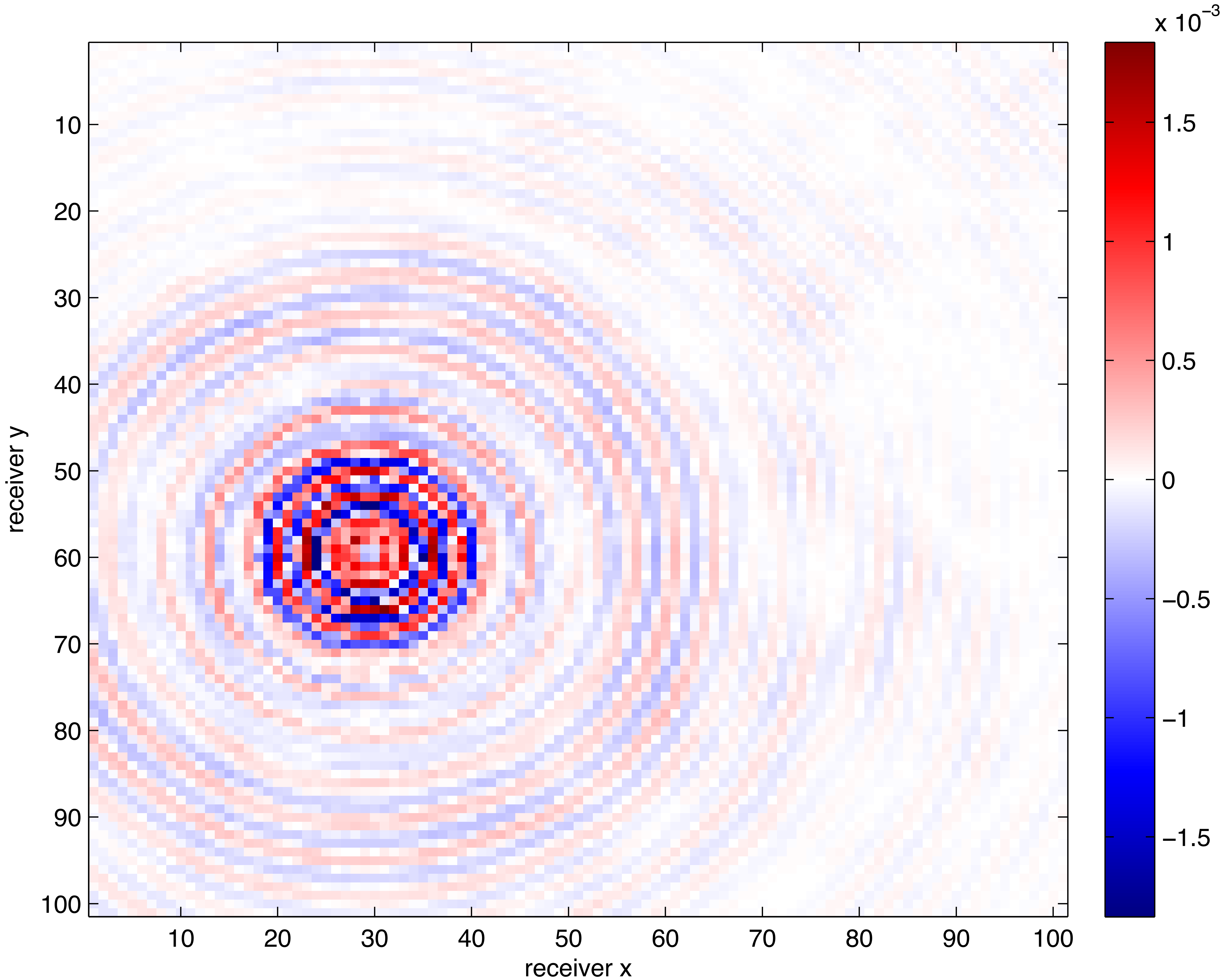


True data

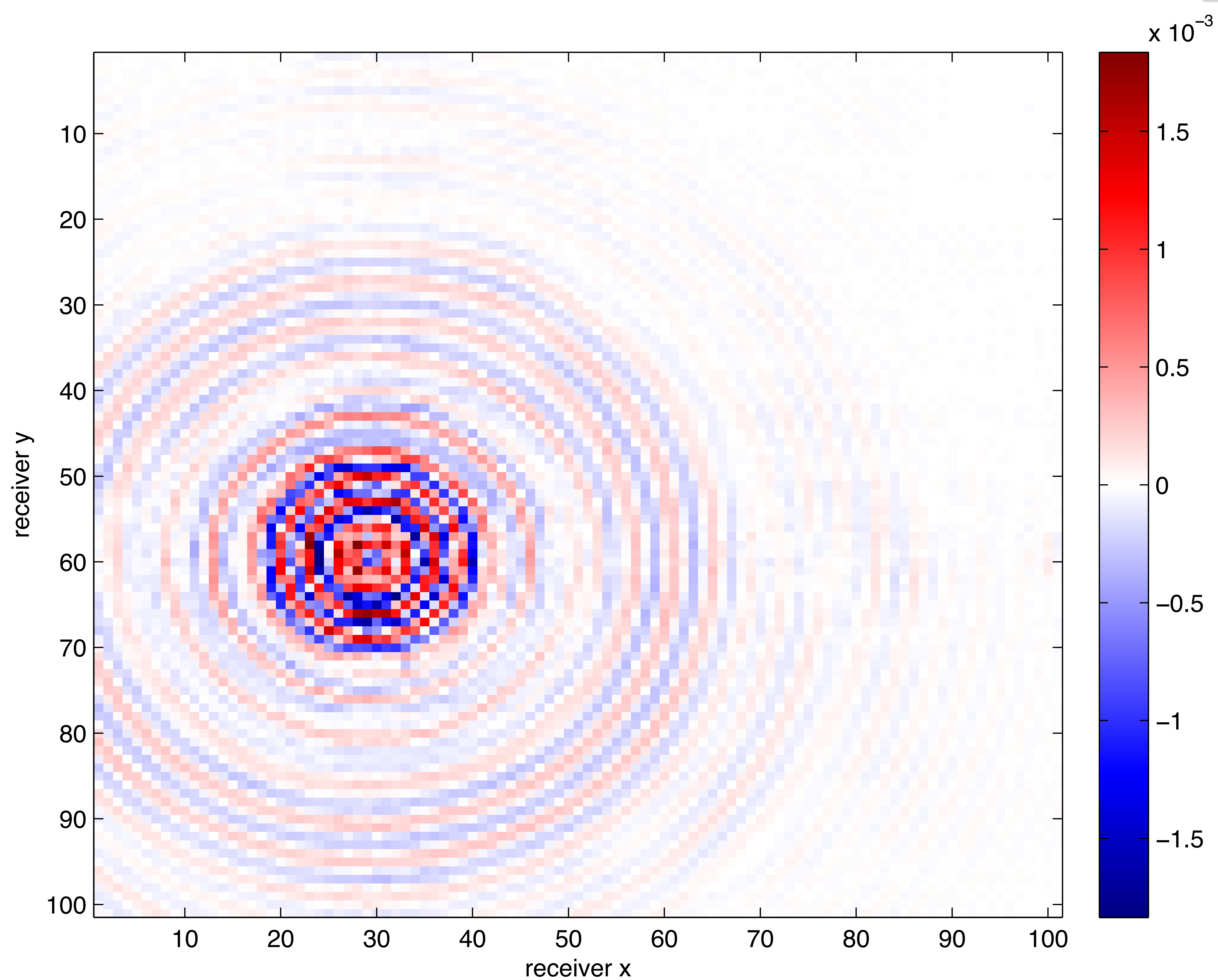


Subsampled data

Regularized recovery - 50% missing receivers

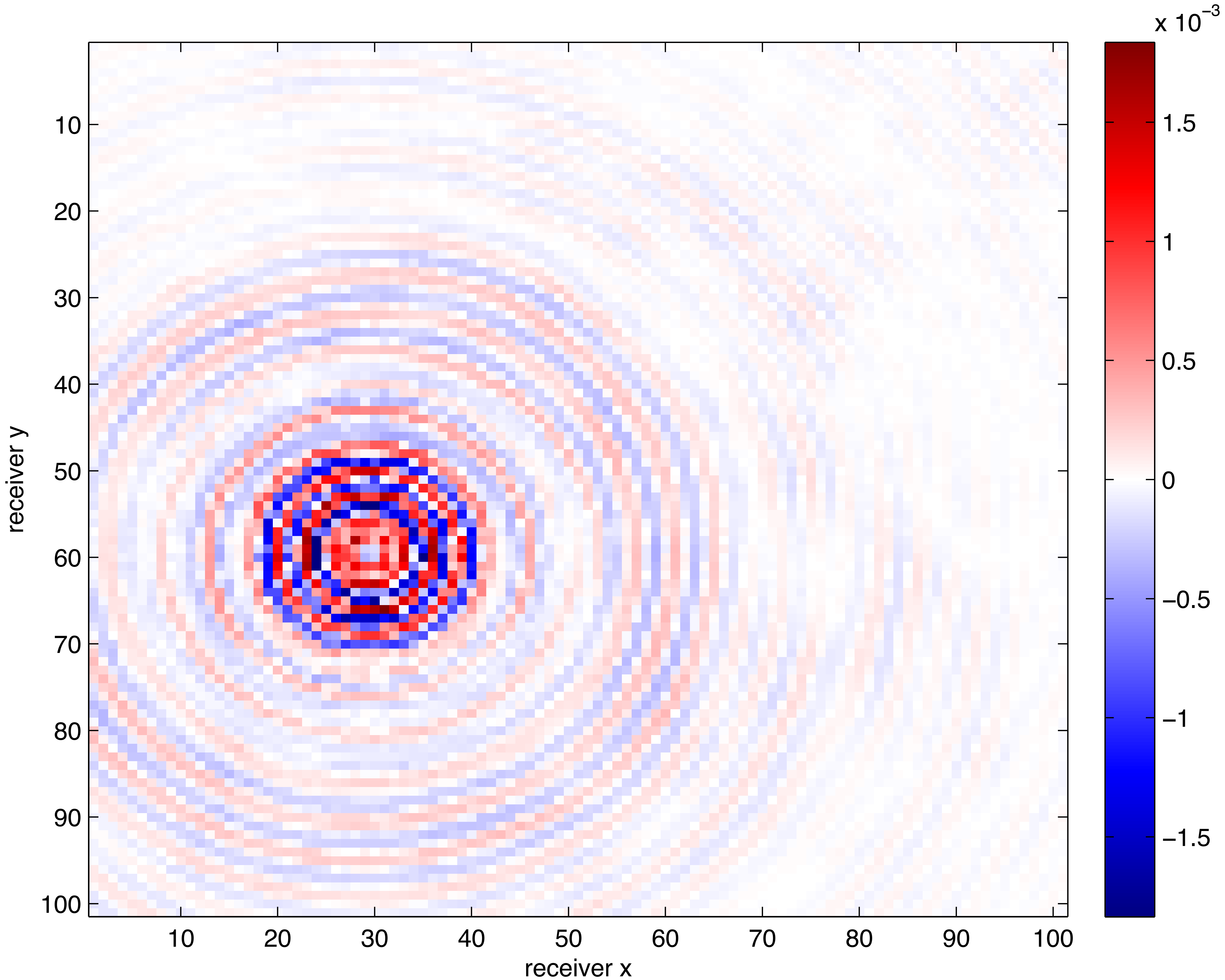


True data

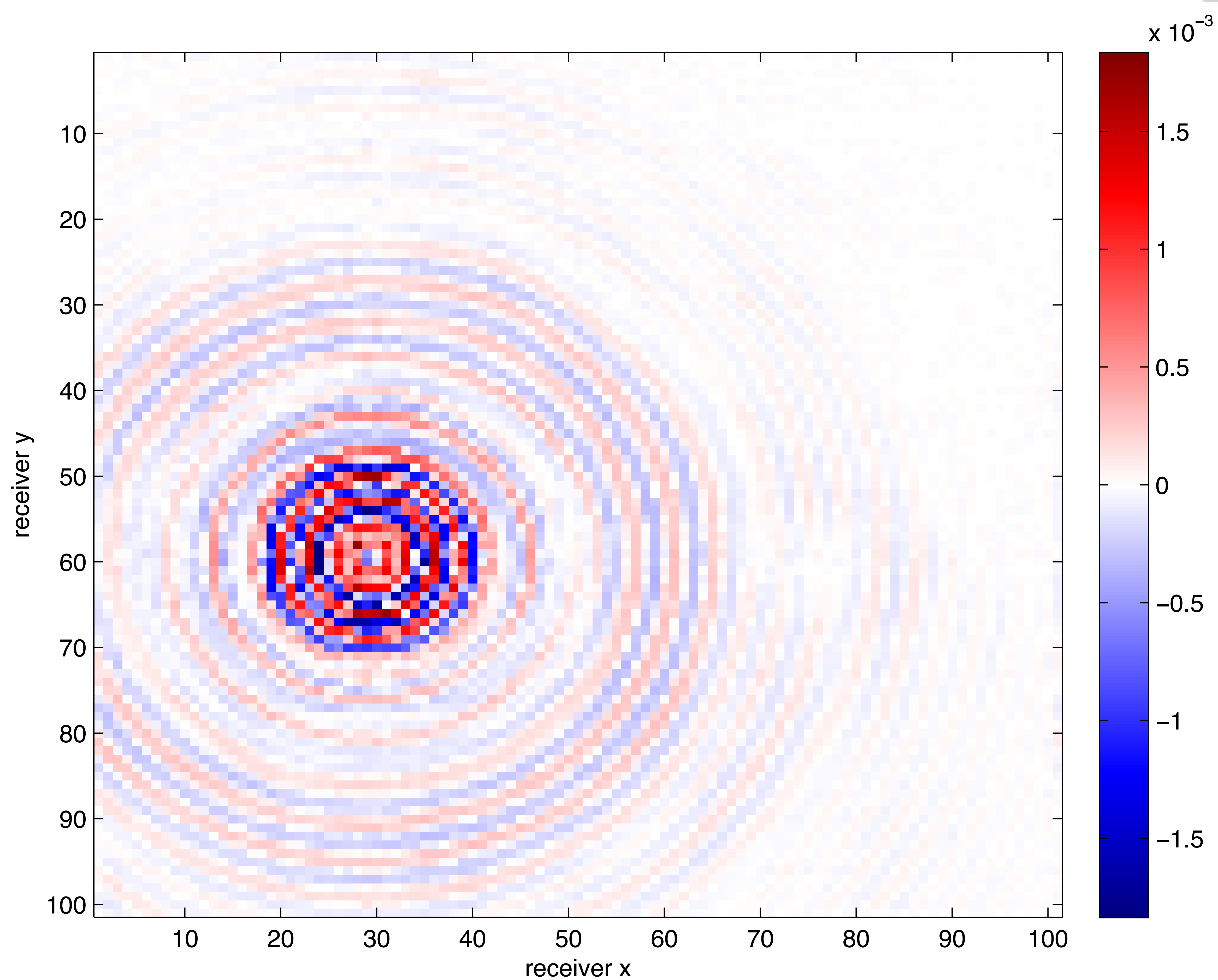


Without regularization - SNR 9.71 dB

Regularized recovery - 50% missing receivers

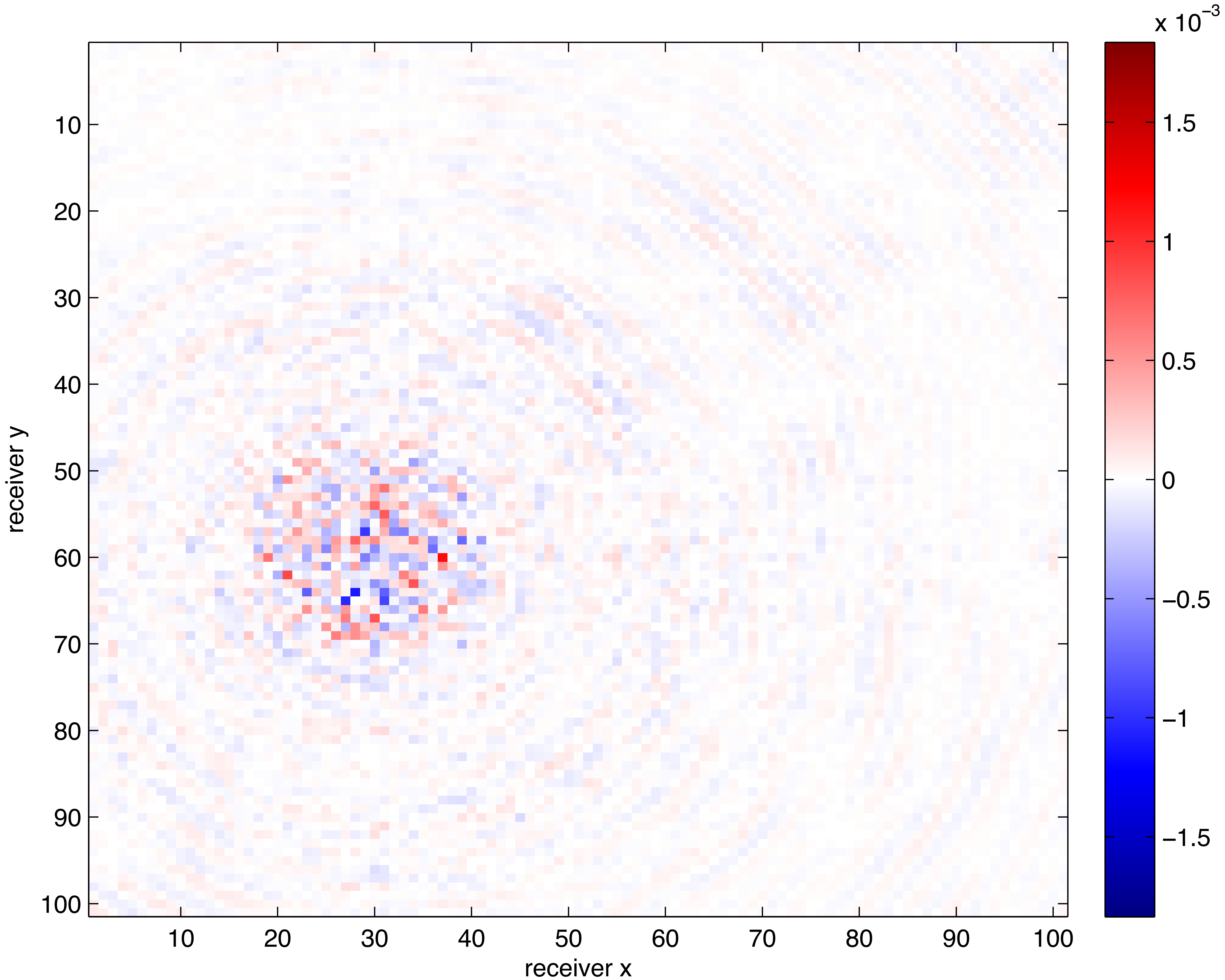


True data

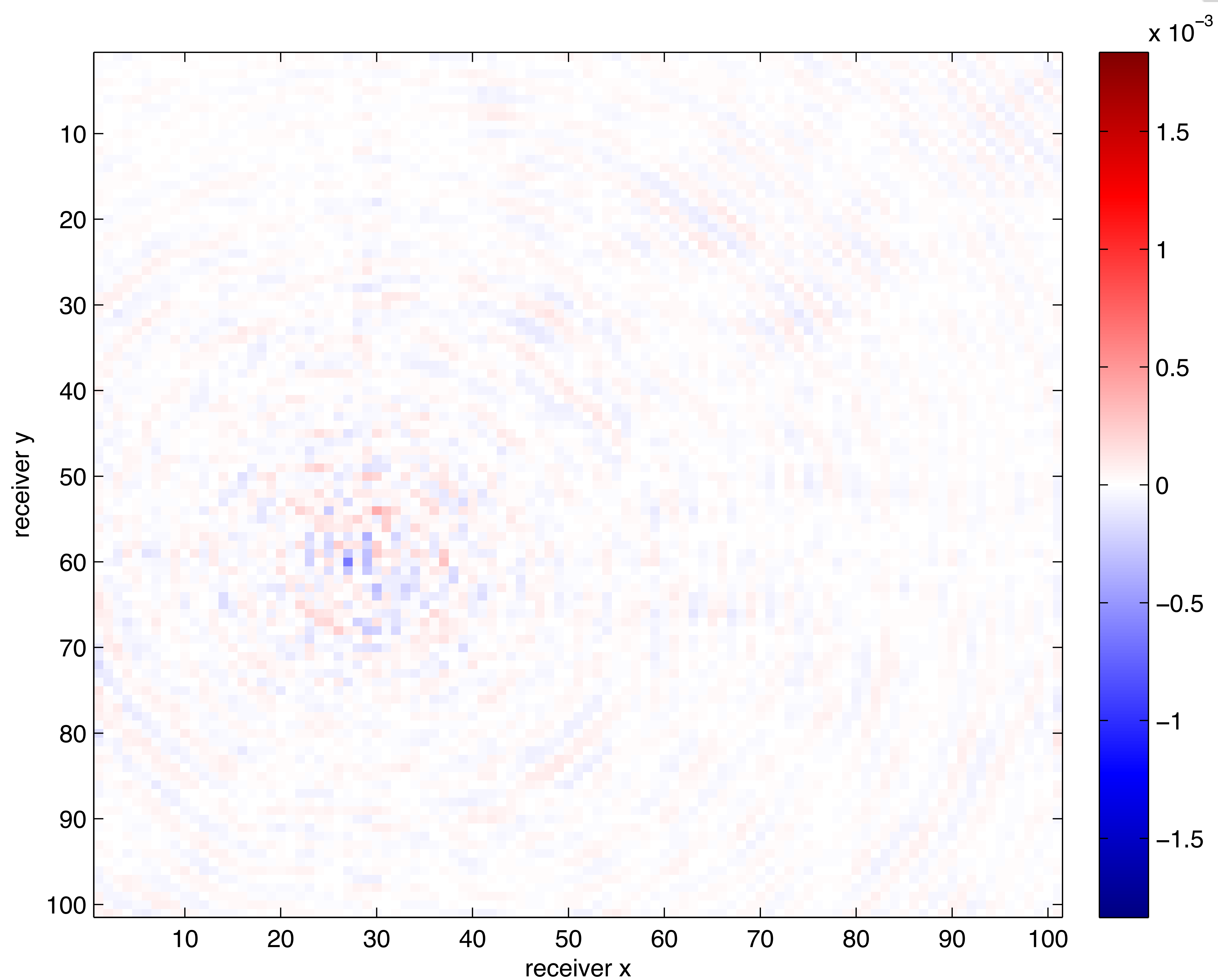


With regularization - SNR 15.3 dB

Regularized recovery - 50% missing receivers



Difference - no regularization



Difference - regularization

Summary

Irregular sampling destroys low-rank behaviour

- need an appropriate transform to operate in a low-rank domain

Regularization improves interpolation results

- can be easily incorporated in to optimization framework

Need for a fast interpolation transform

- more research needed

Summary

HTOpt

- previously released software for tensor interpolation

Software releases of SPGLR, updated HTOpt to come soon*

*graduate student clocks may not be synced to global clocks

Acknowledgements

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