Controlling linearization errors in sparse seismic inversion with **rerandomization**

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Motivation

Seismic imaging uses *linearized* modelling to predict *nonlinear* seismic data.

The formulation

$$\underset{\mathbf{x}}{\text{argmin}}\, \delta\mathbf{m} = \mathbf{C}^*\|\mathbf{x}\|_1$$

$$\text{subject to} \quad \|\nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^*\mathbf{x} - \underline{\mathbf{d}}\|_2 \leq \sigma$$

$\delta\mathbf{m}$: model perturbation

$\nabla\mathbf{F}$: linearized modelling operator

$\mathbf{m}_0$: background model

$\mathbf{q}$: source wavefield

$\mathbf{C}$: Curvelet transform

$\mathbf{d}$: observed seismic data

σ : tolerance for noise/modelling error

Rerandomization

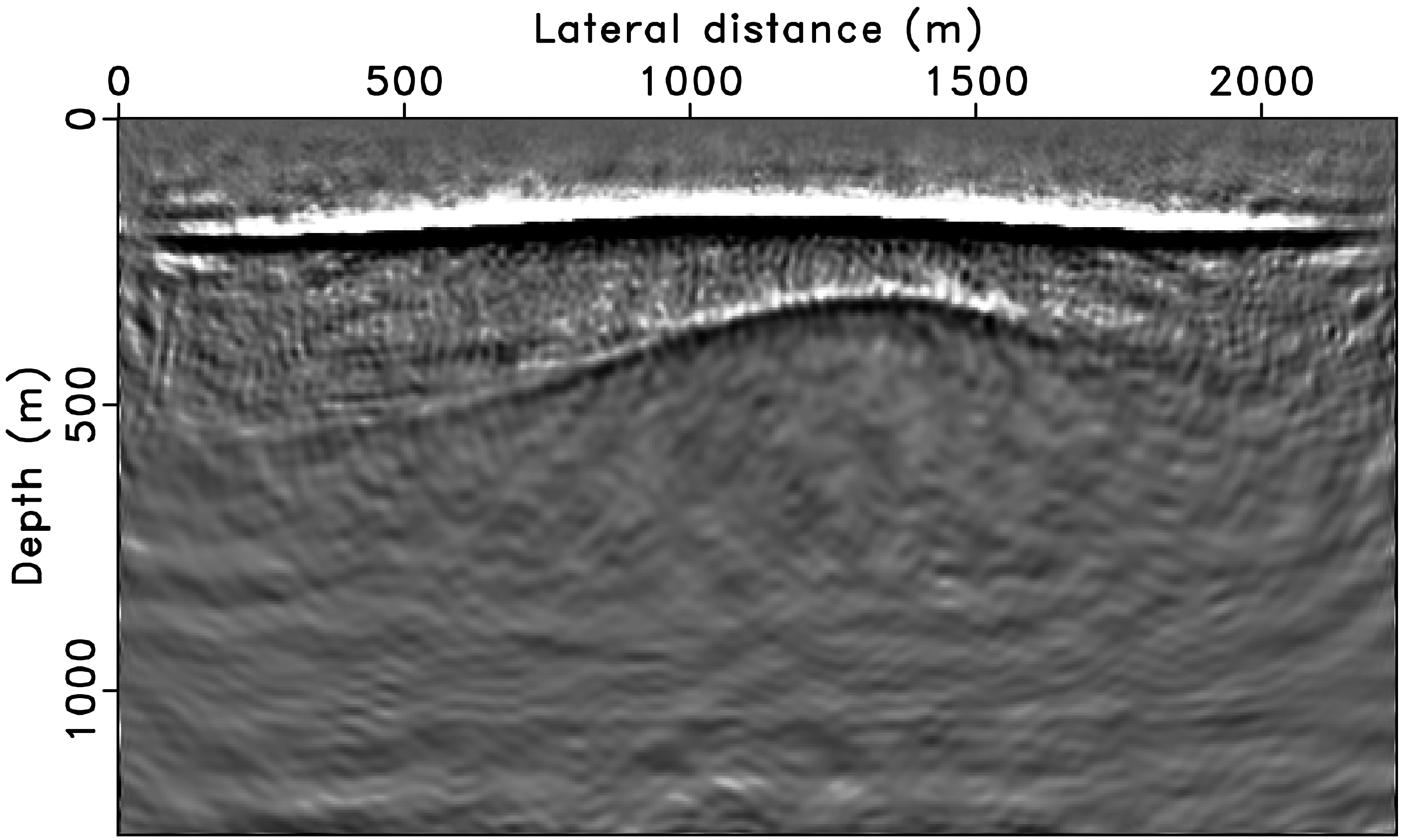
SPGl1 solves a series of LASSO subproblems for gradually relaxed τ:

$$\begin{aligned} \text{minimize} \quad & \|\nabla \mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^* \mathbf{x} - \underline{\mathbf{d}}\|_2 \\ \text{subject to} \quad & \|\mathbf{x}\|_1 \leq \tau \end{aligned}$$

Rerandomization:
For each LASSO subproblem, we draw a new subsampling operator.

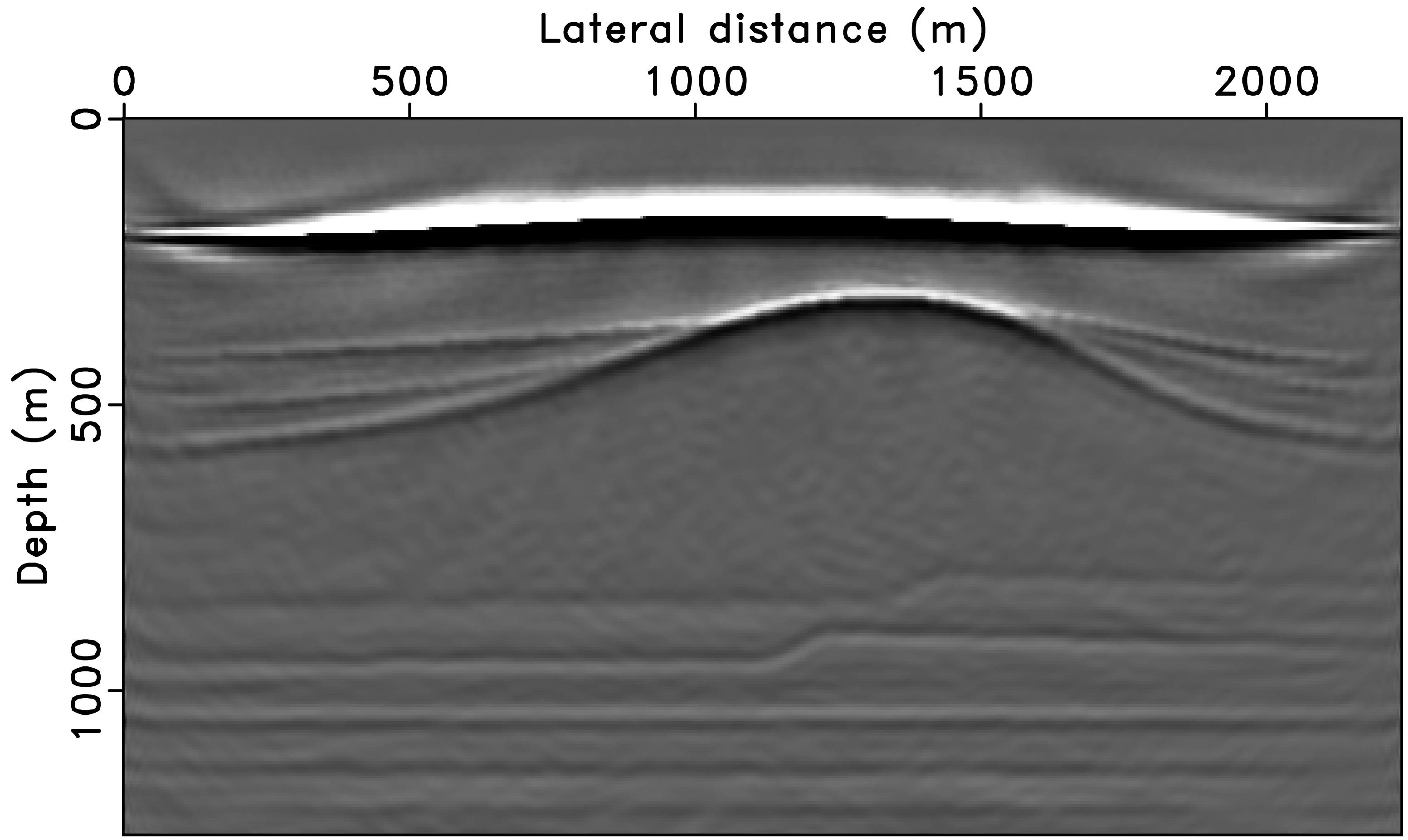
No rerandomization

[120X subsampling]



With rerandomization

[120X subsampling]



Experiment setup

We compare two datasets and the inversion results:

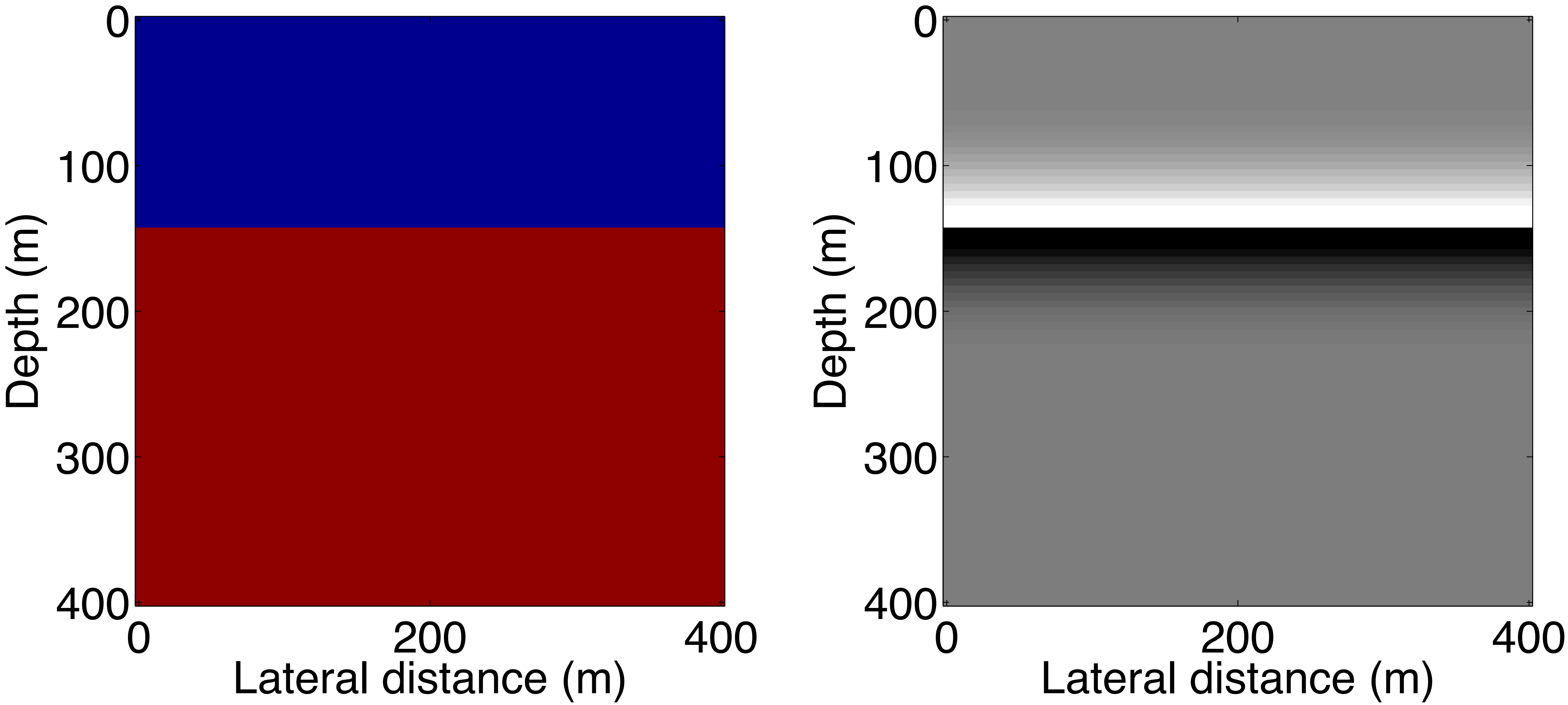
- linearized data:

$$\mathbf{d} = \nabla\mathbf{F}[\mathbf{m}_0, \mathbf{q}]\delta\mathbf{m}$$

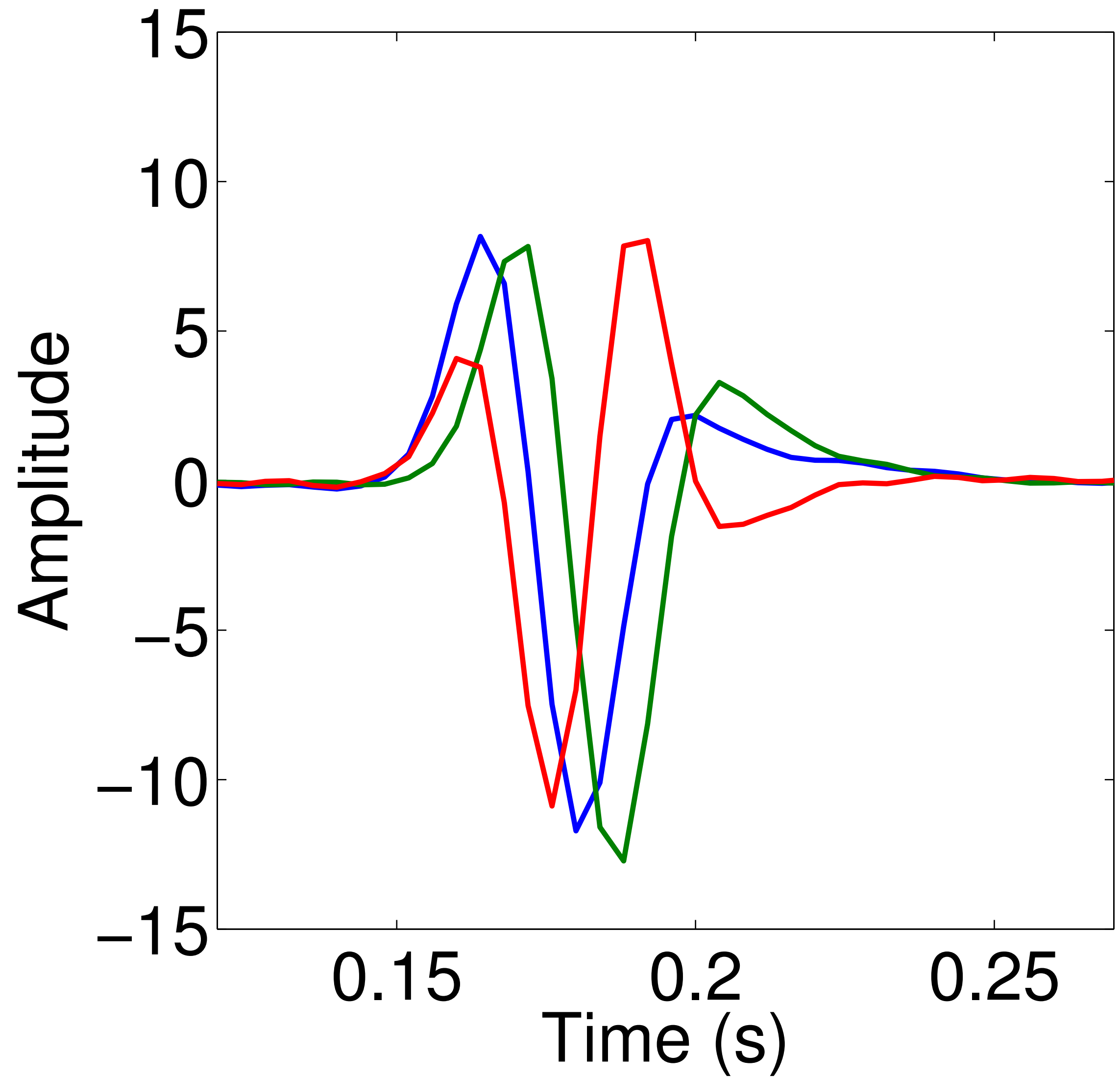
- forward modelling data:

$$\mathbf{d} = \mathbf{F}[\mathbf{m}, \mathbf{q}]$$

True model & perturbation



Data comparison



blue: linearized data

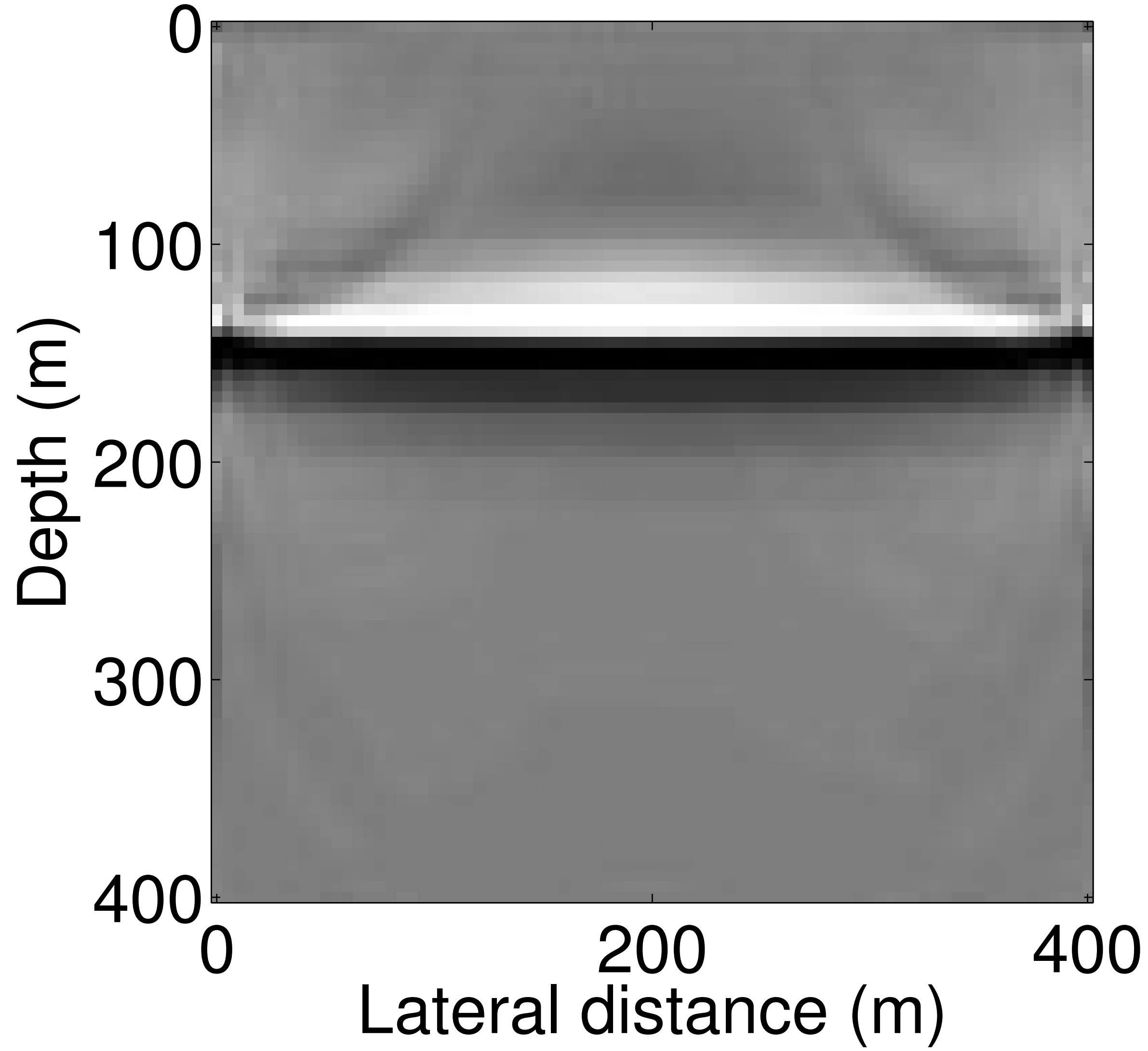
green: forward modelling data

red: difference

Full data case

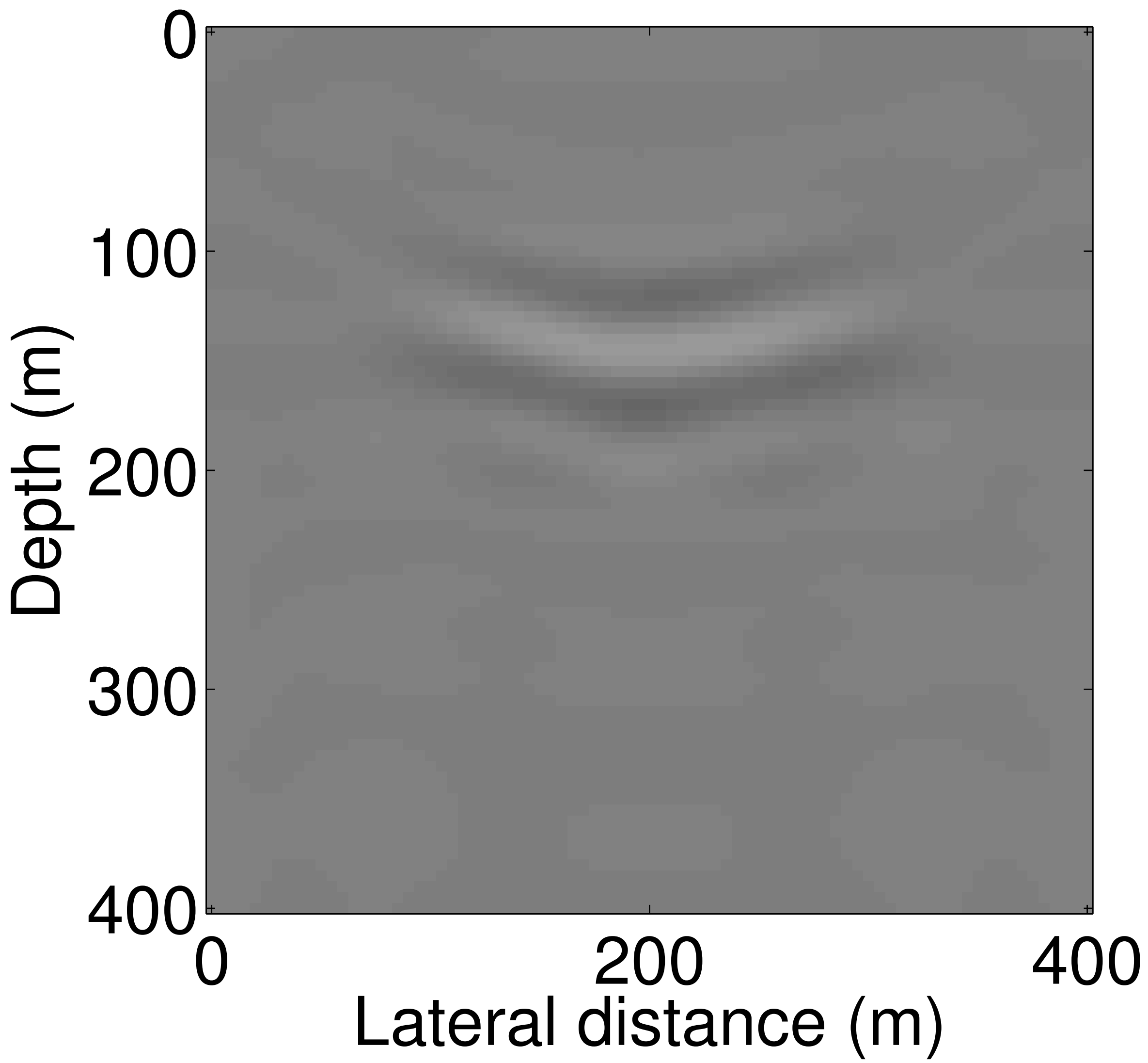
- 21 sequential sources
- 61 random frequencies
- 100 iterations

Linearized data



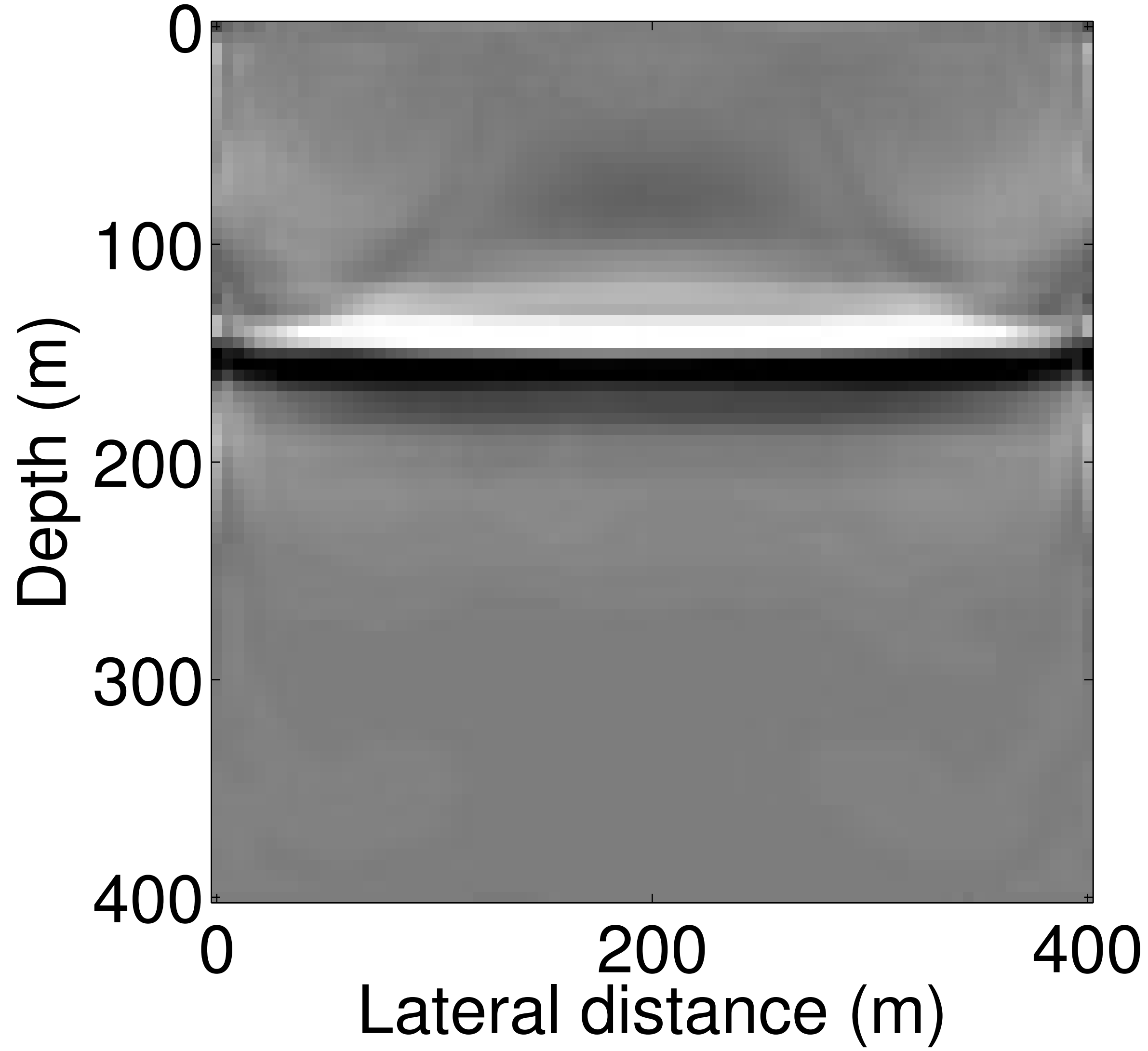
Forward modelling data

[with "true" sigma]



Forward modelling data

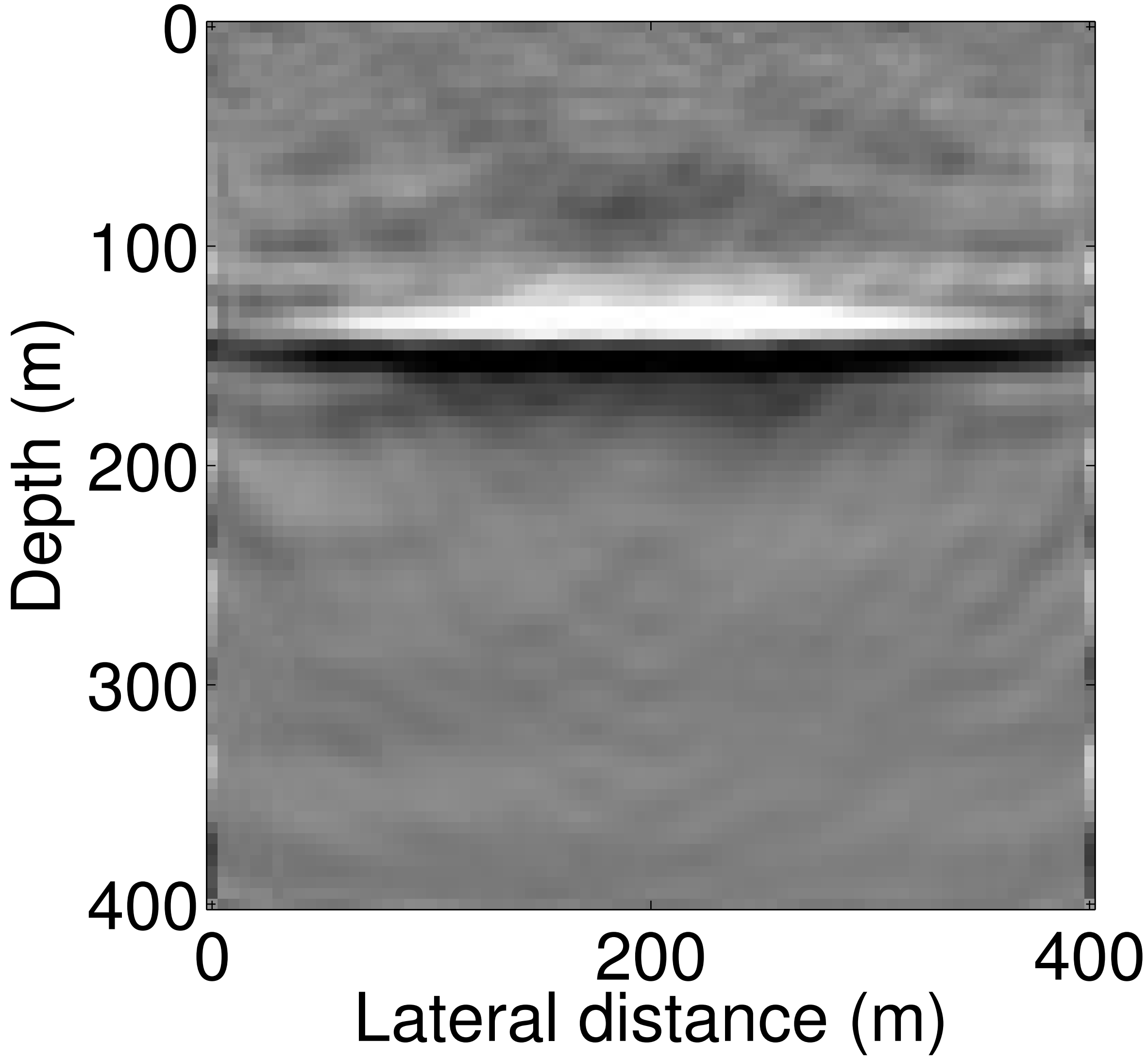
[with zero sigma]



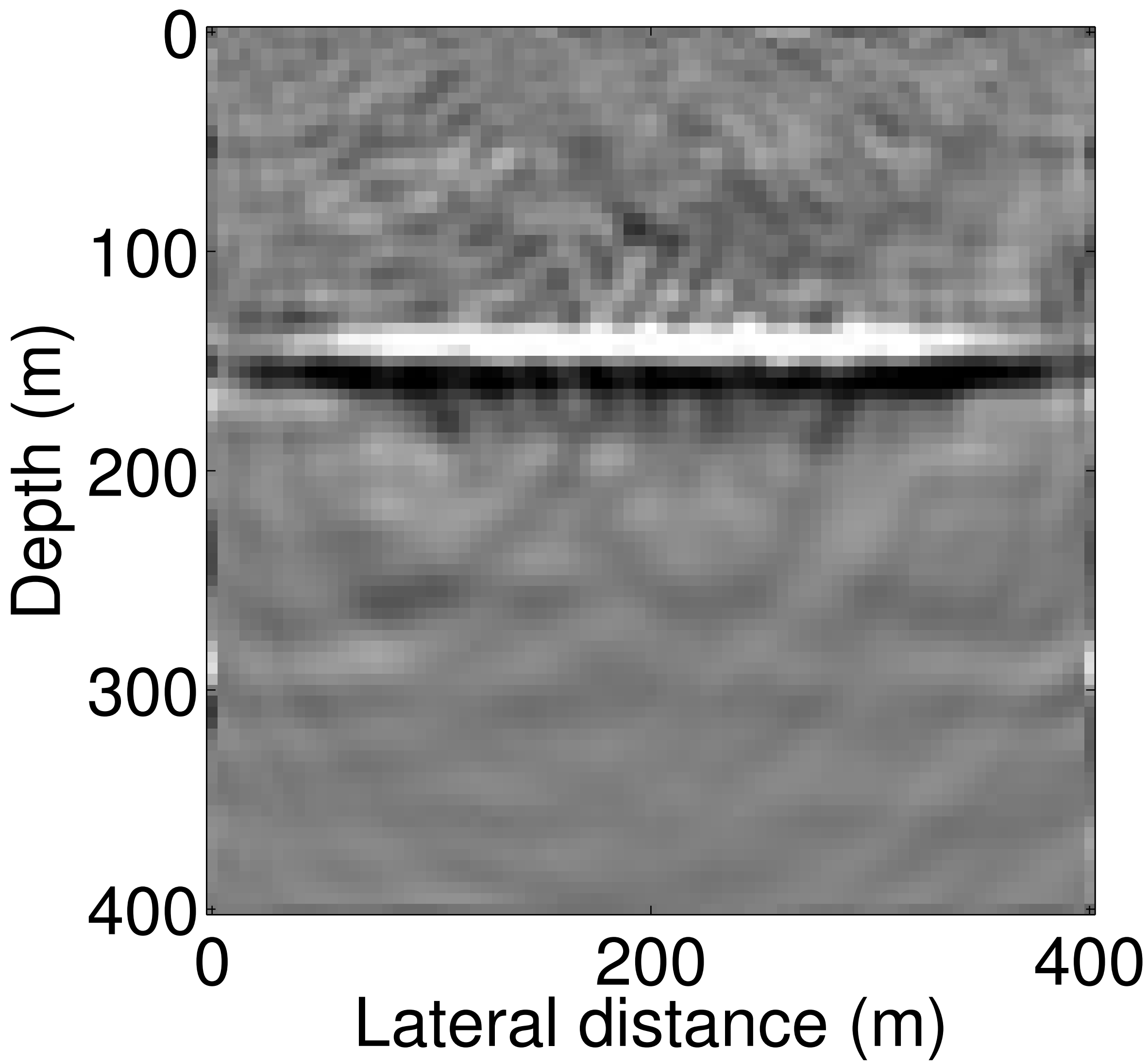
Subsampled data case

- 5 simultaneous sources
- 15 random frequencies
- 100 iterations

Linearized data

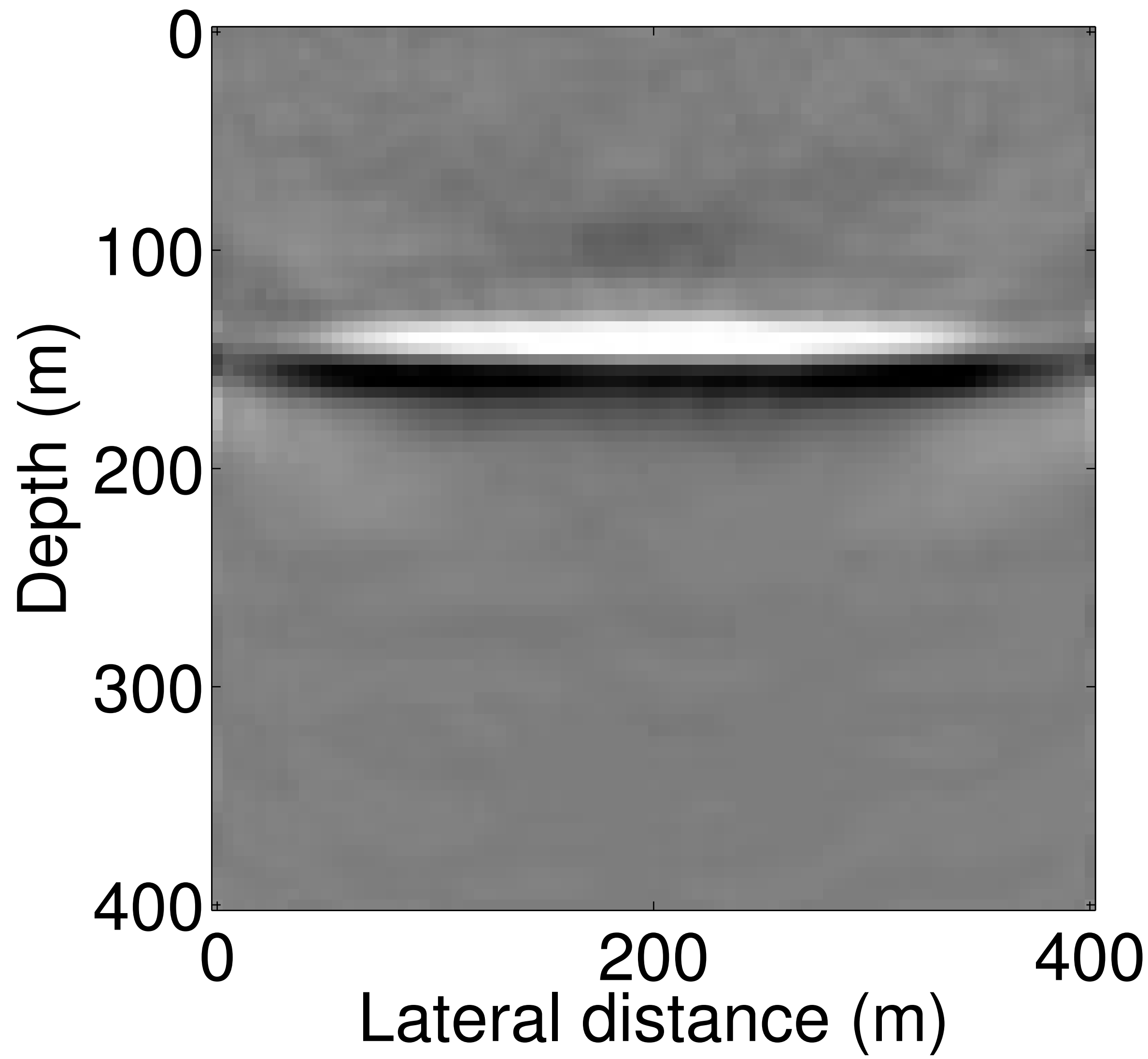


Forward modelling data
[with zero sigma]



Forward modelling data

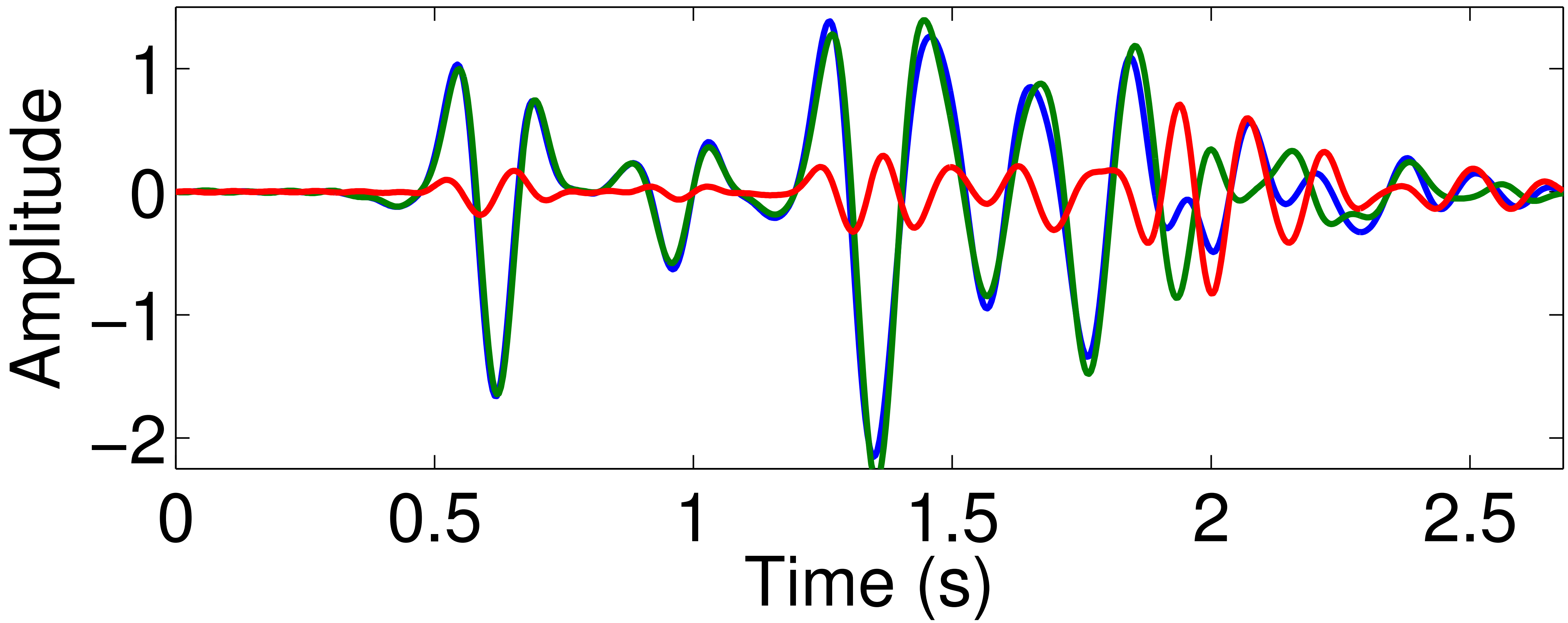
[with zero sigma, with *rerandomization*]



Case study

- a 2D slice of the SEG/EAGE salt model
- *smooth* background model
- 3.9km deep, 15.7km wide
- 80 ft grid spacing
- 5Hz Ricker wavelet, 8s recording
- 323 sources with 160 ft spacing at 80ft depth
- use *linearized* data and *forward-modelling* data
- use only 15 freq. and15 sim. sources in inversion, run for 100 iterations

Traces from the two datasets

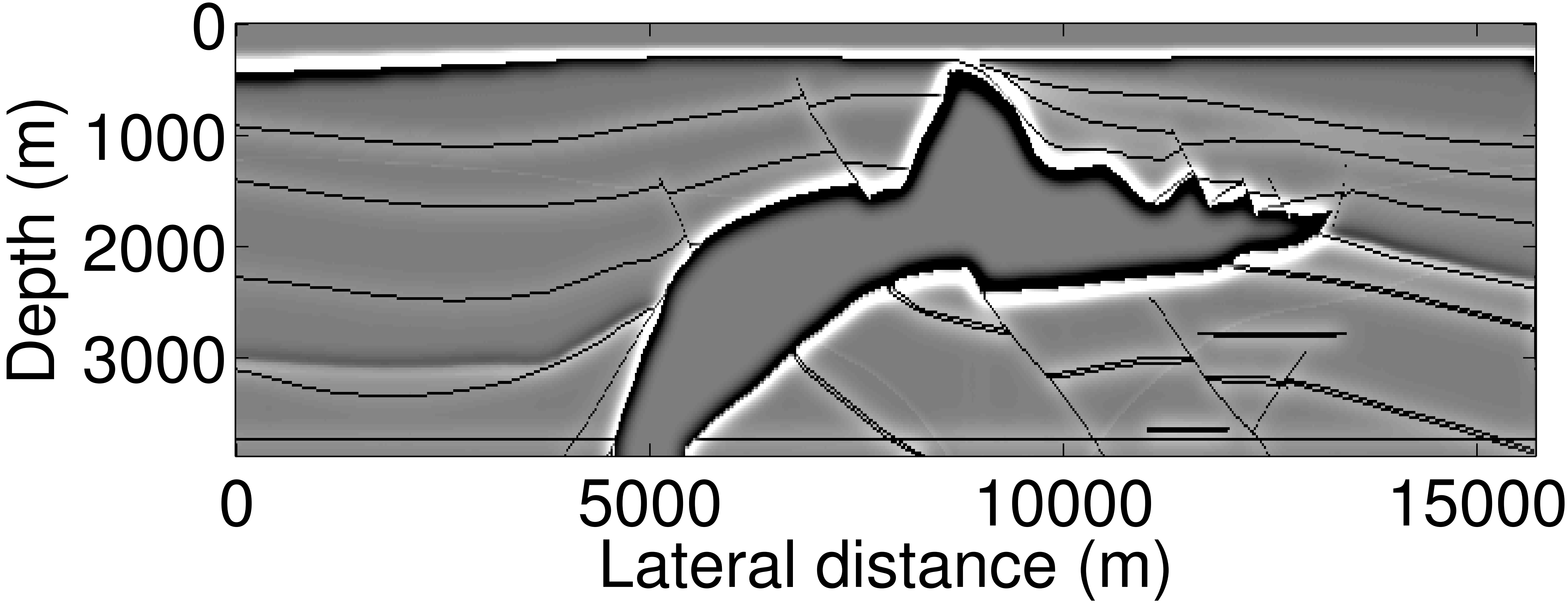


blue: linearized data

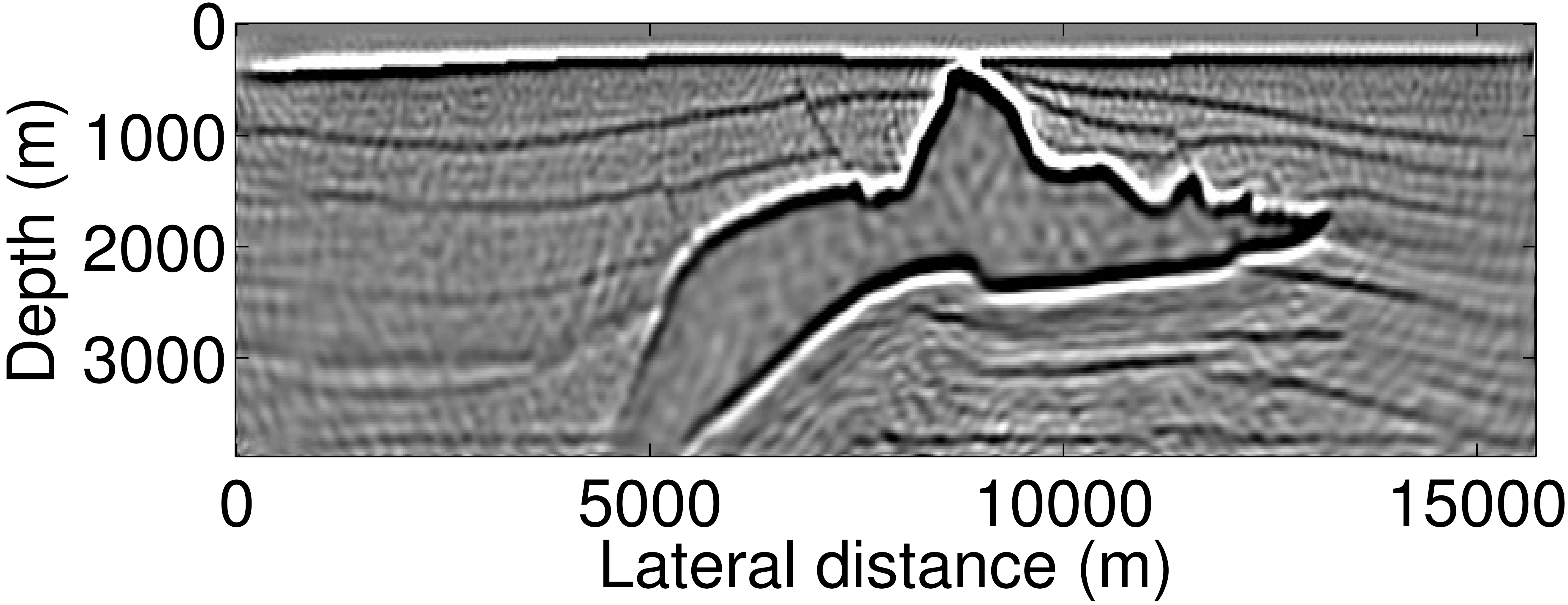
green: forward modelling data

red: difference; top salt event at ~2s

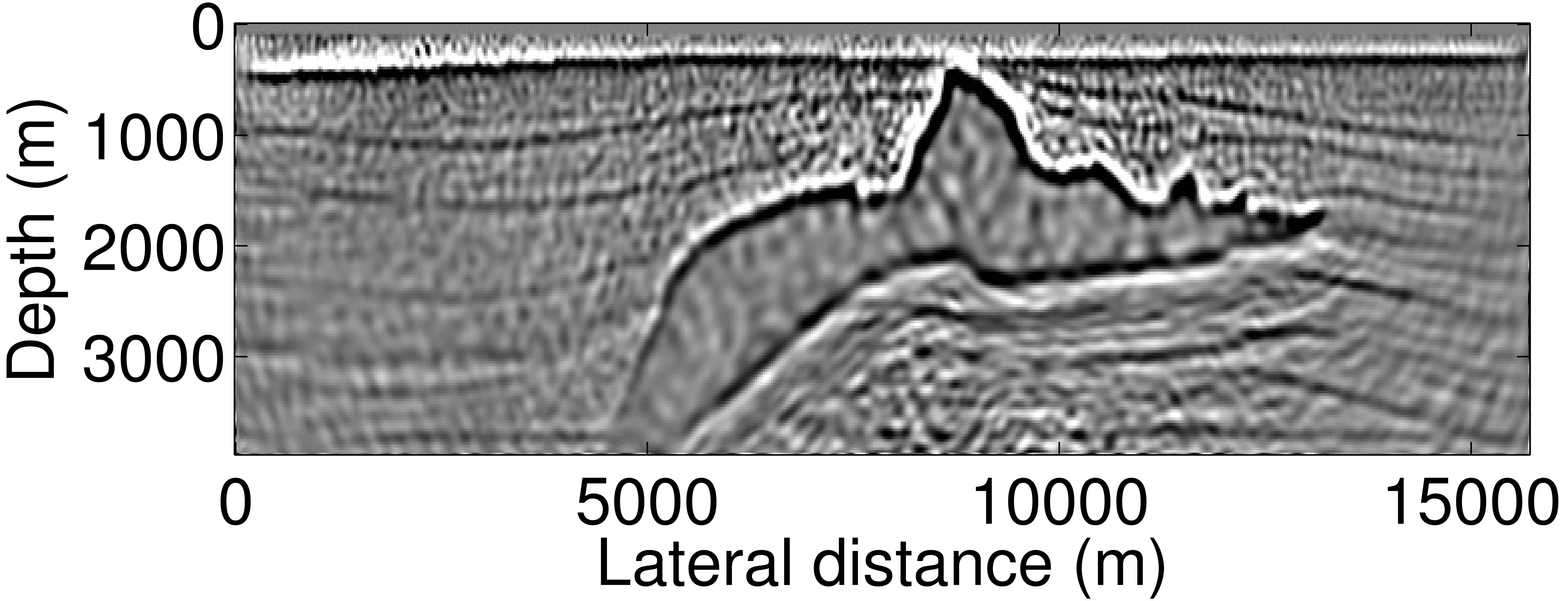
True model perturbation



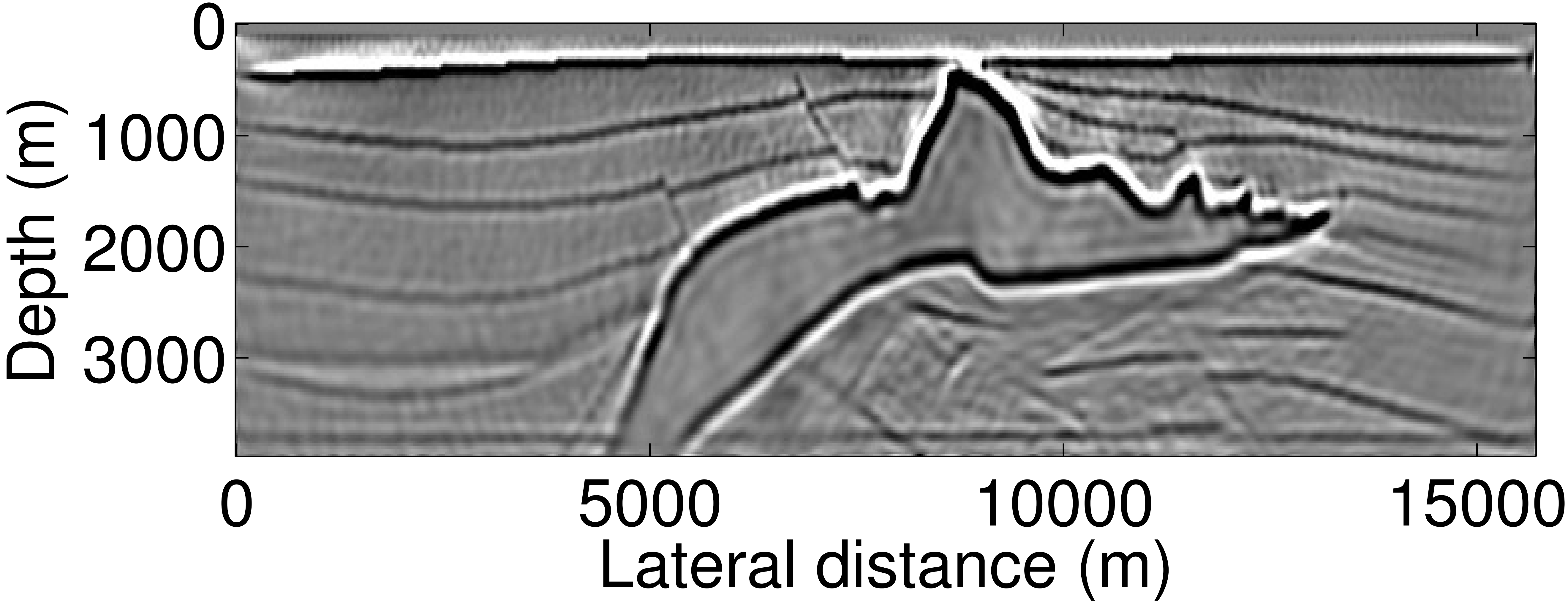
Inversion of *linearized* data



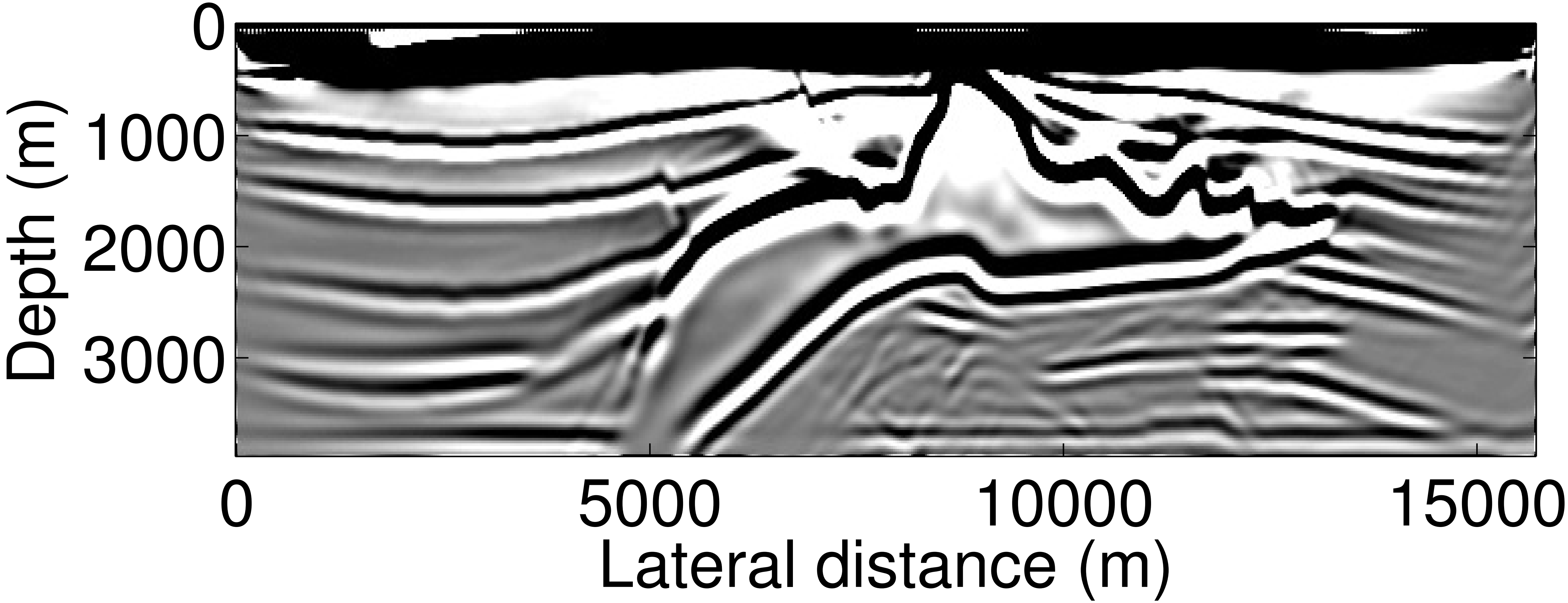
Inversion of forward modelling data, *no* rerandomization



Inversion of *forward modelling* data, *with* rerandomization

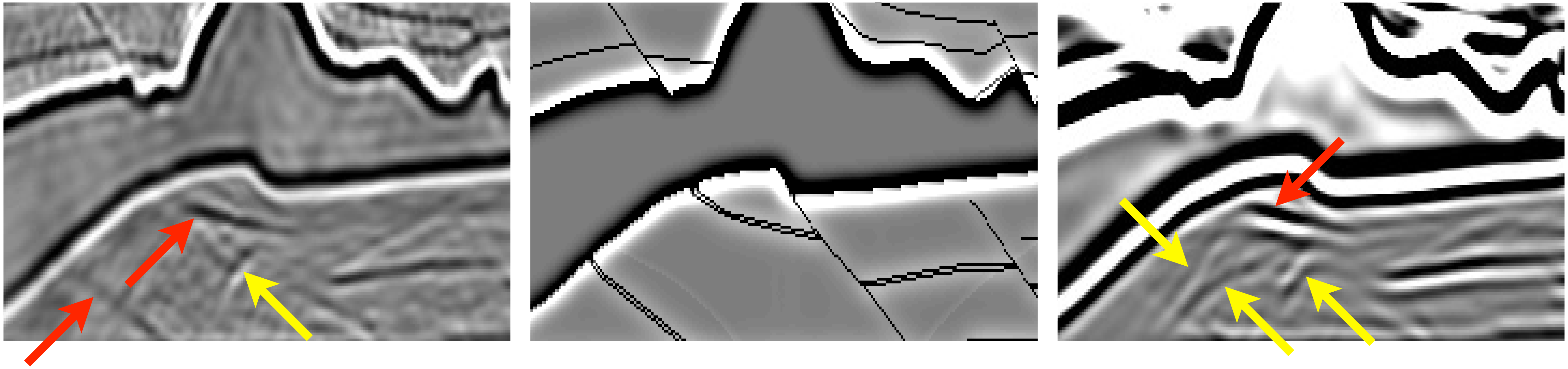


RTM of *forward modelling* data
[use *all* data]



Details

[red arrow: true reflector; yellow arrow: artifacts]



Inversion with rerandomization

RTM

Conclusions

- The linearization error is more an issue for dimensionality reduced system than the full system.
- Simply allowing a tolerance in the inversion does not address the issue.
- Rerandomization can solve the problem, apart from leading to faster convergence.
- We can potentially better resolve fine sub-salt features with sparse inversion in a computationally efficient way.