London '13

Seismic data interpolation via low-rank matrix factorization in the h(ierarchical) s(emi) s(eparable) representation

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SLIM
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Motivation

- acquisition challenges
  - missing data
  - noise

- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME

- exploit *low-rank* structure of seismic data
  - randomized sampling
  - *SVD-free* matrix factorization

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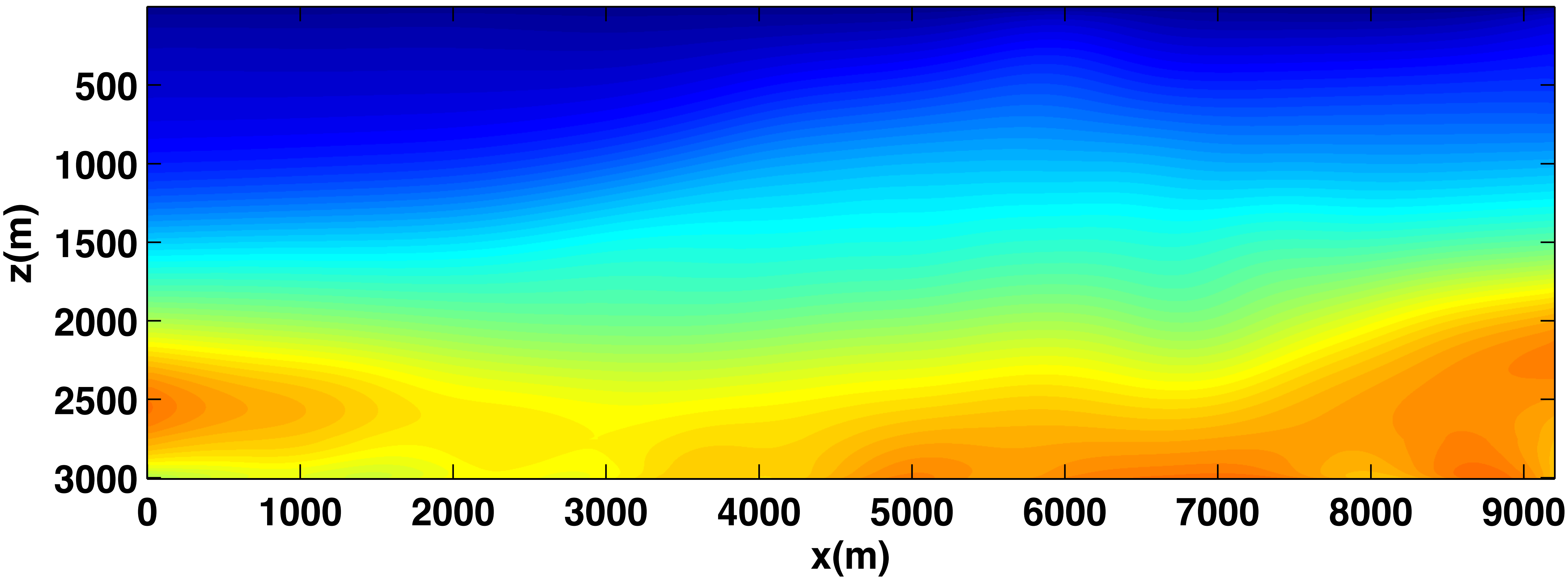
Full waveform inversion

initial model



[Li et. al. 2012, van Leeuwen and Herrmann 2012]

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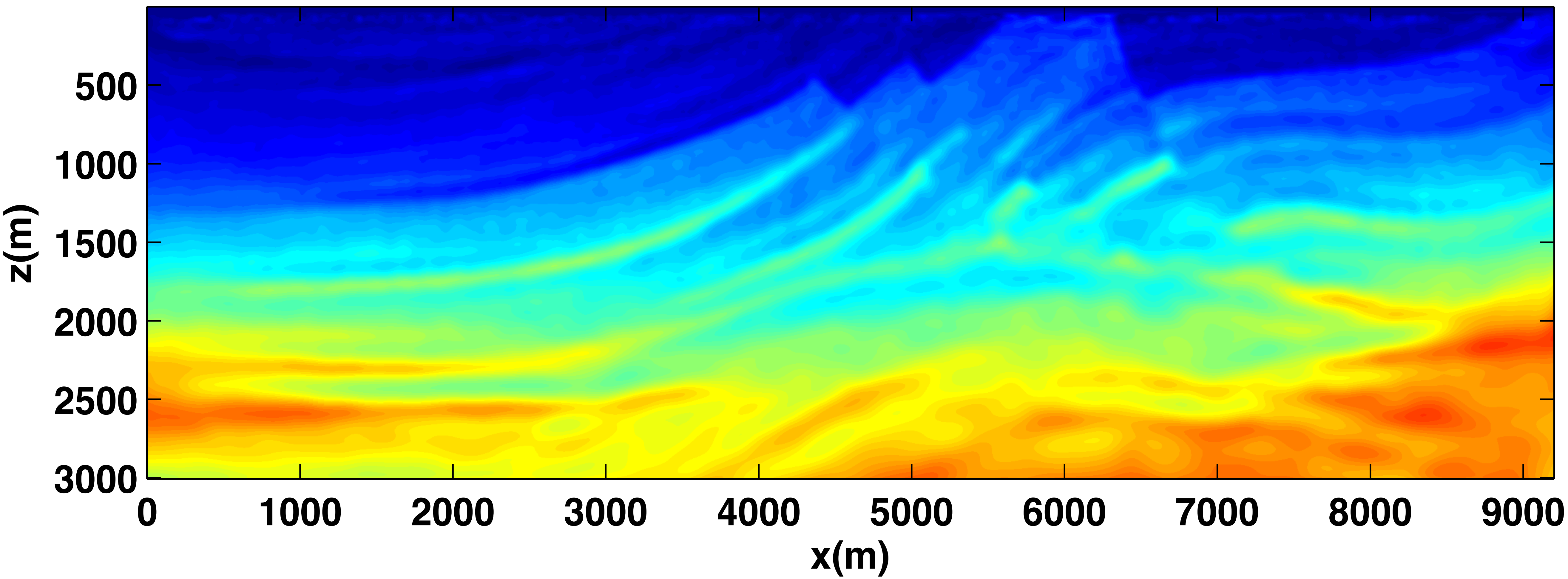
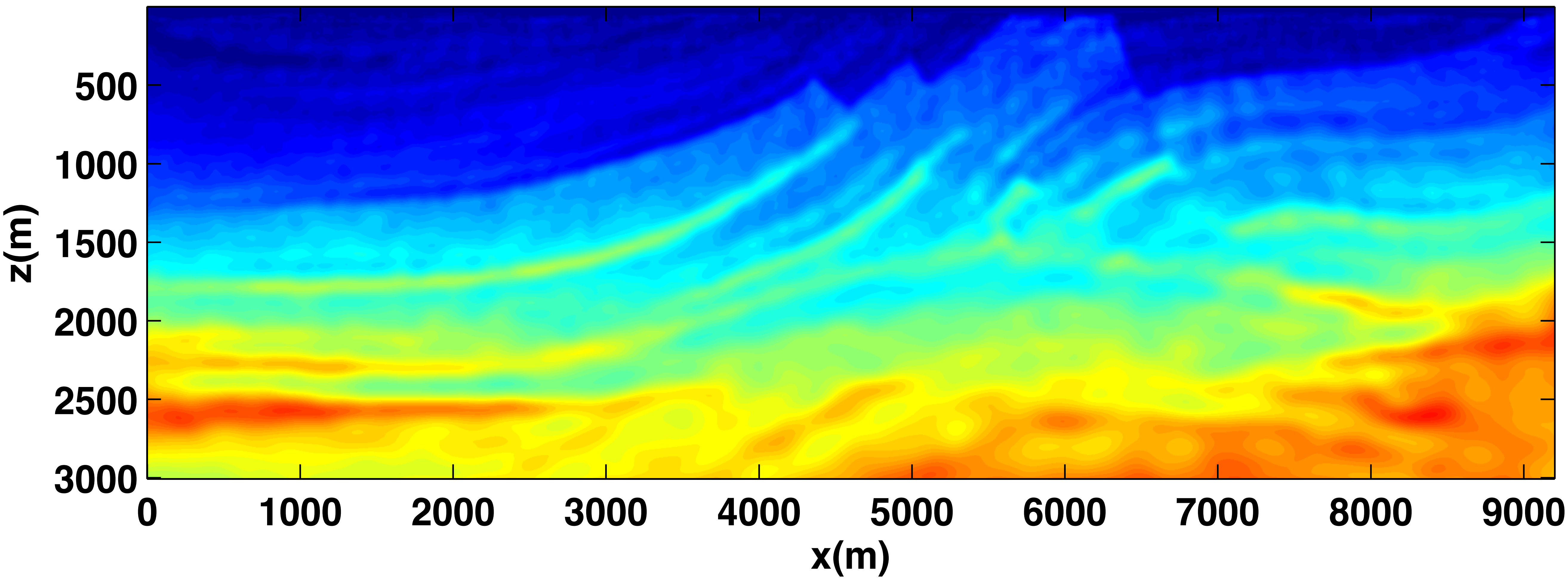
Full waveform inversion

[Ideal scenario, 60 sequential shots, redraw]



Full waveform inversion

[Ideal scenario, 20 simultaneous shots, redraw]



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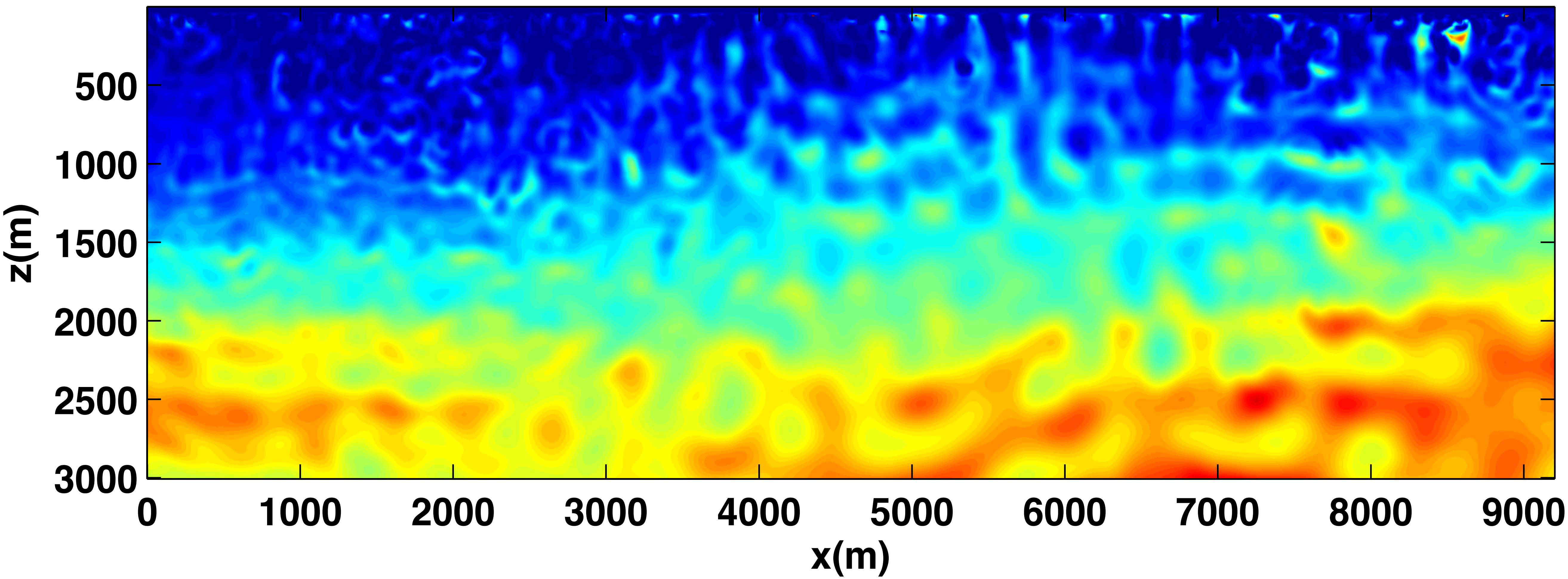
Full waveform inversion

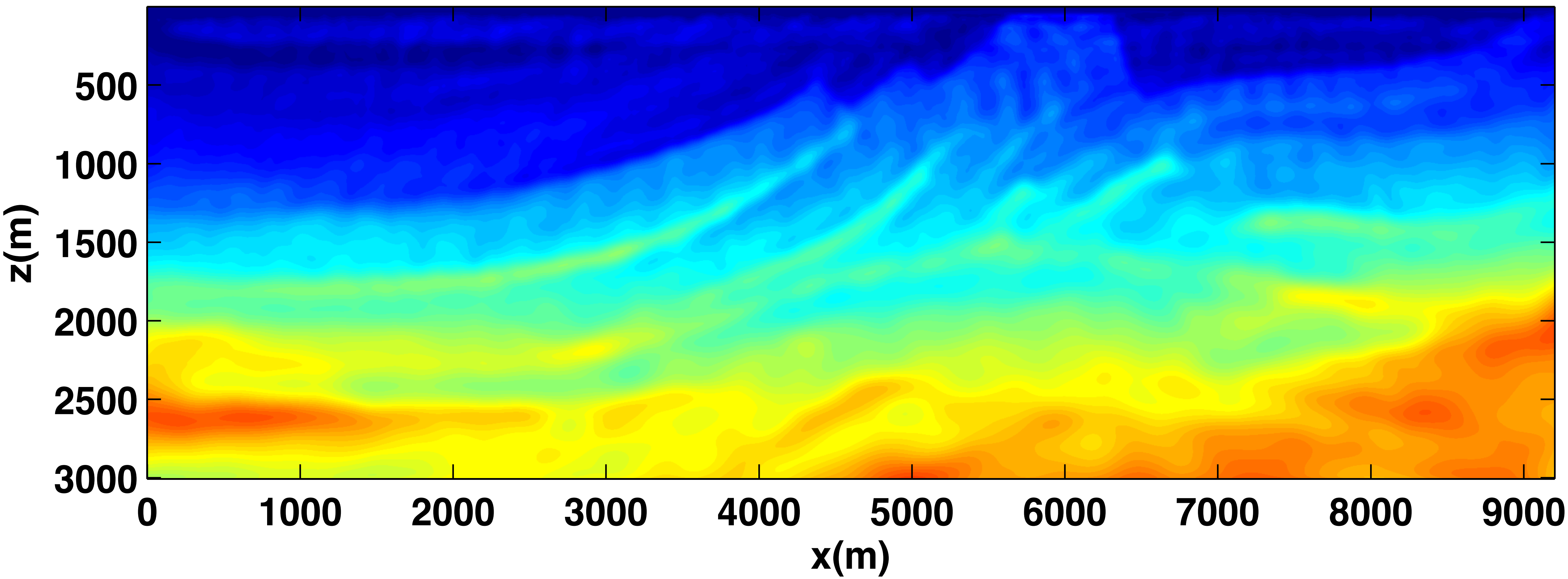
[50% random missing shots, 60 sequential shots, redraw]



SLIM

Full waveform inversion

[low-rank interpolation, 20 simultaneous shots, redraw]



[Candes and Donoho 2000, Herrmann 2008]

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Compressive sensing

- signal structure
  - *sparse/compressible*
- sampling scheme
  - random missing traces make signal *less sparse* in transform domain
- recovery using *sparsity promoting* scheme

- **is *sparsity* the only inherent structure in seismic??**

[Candes and Plan 2010, Oropeza and Sacchi 2011]

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Matrix completion

- signal structure
  - *low rank/fast decay* of singular values

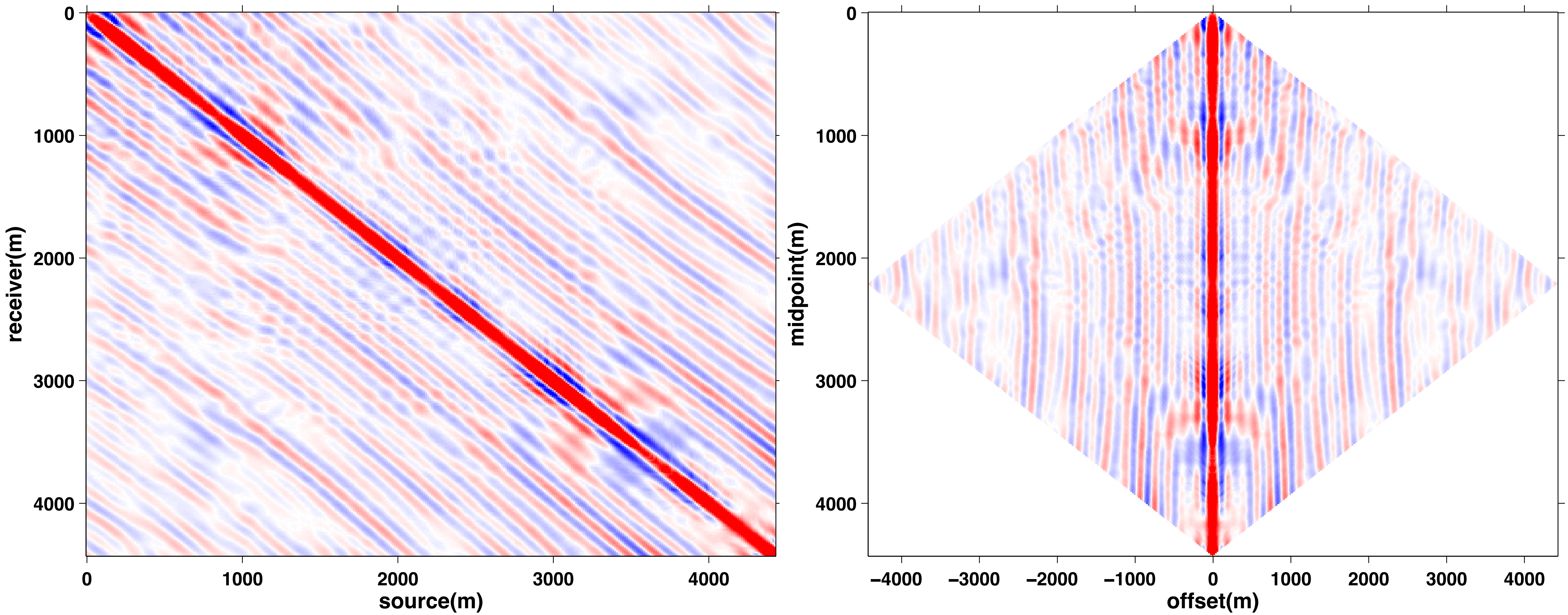
- sampling scheme
  - missing data *increase* rank in "transform domain"

- recovery using *rank penalization* scheme

Low-rank structure
2-D acquisition

acquisition domain
[source-receiver]

transform domain
[midpoint-offset]



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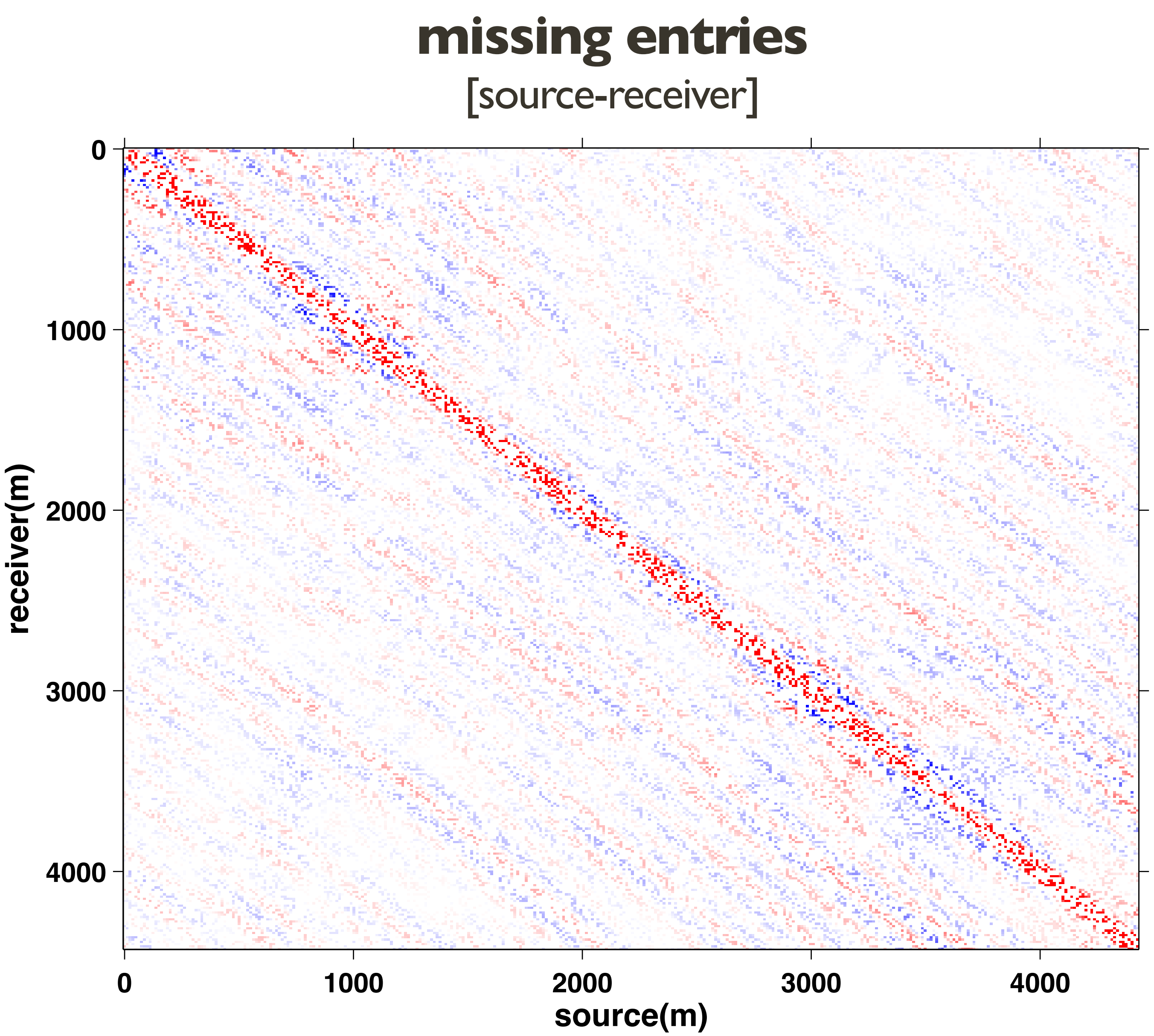
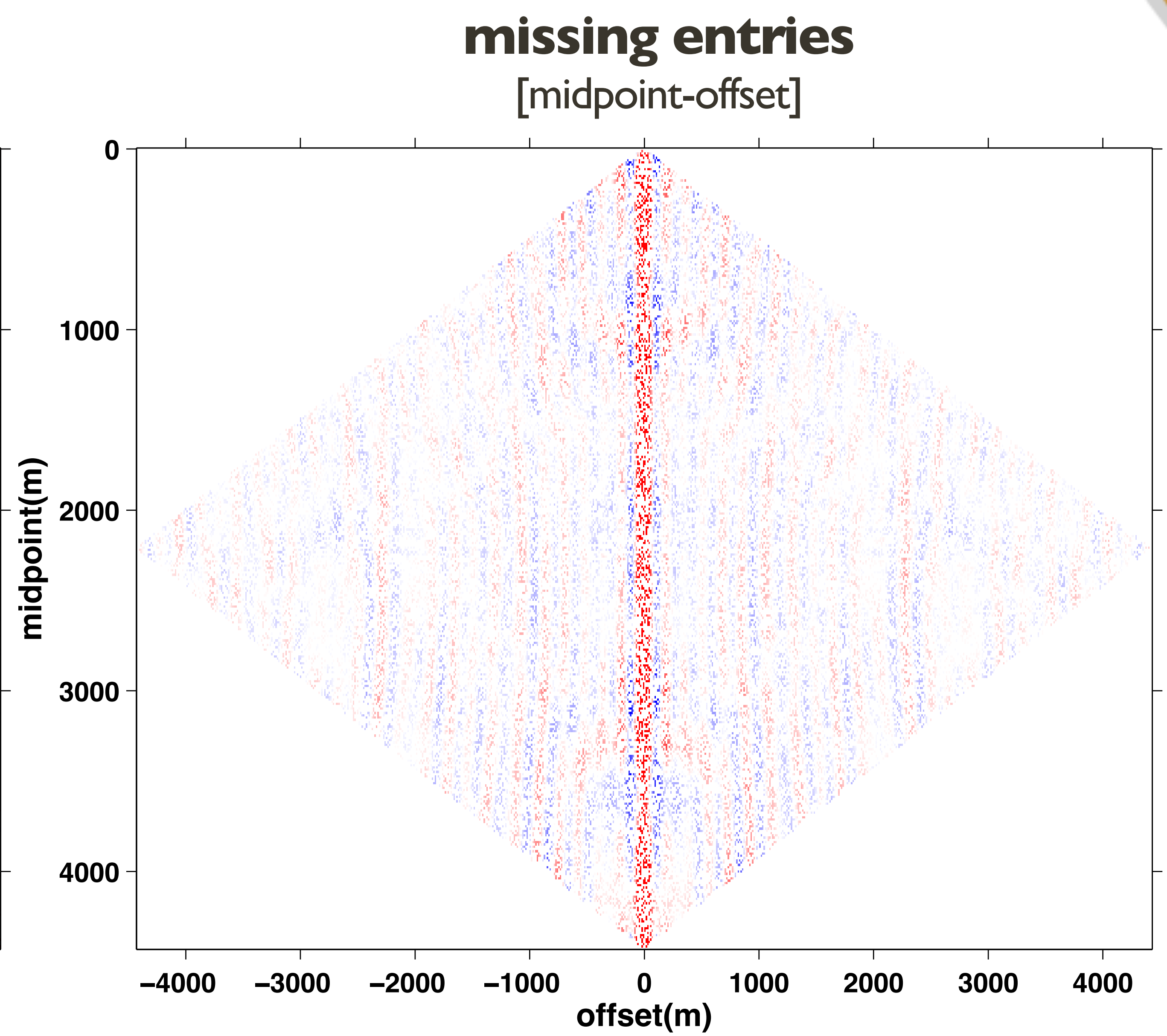
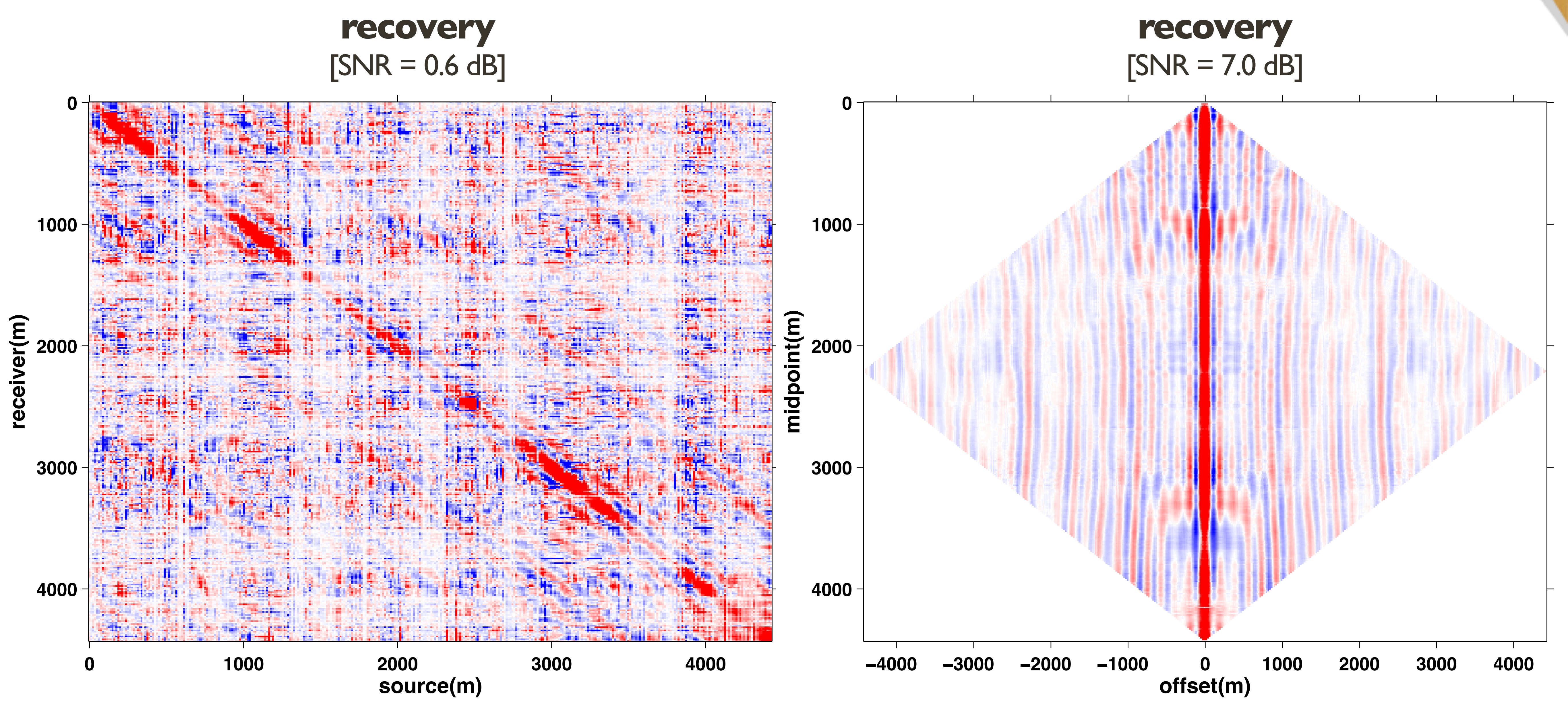
Matrix completion problem





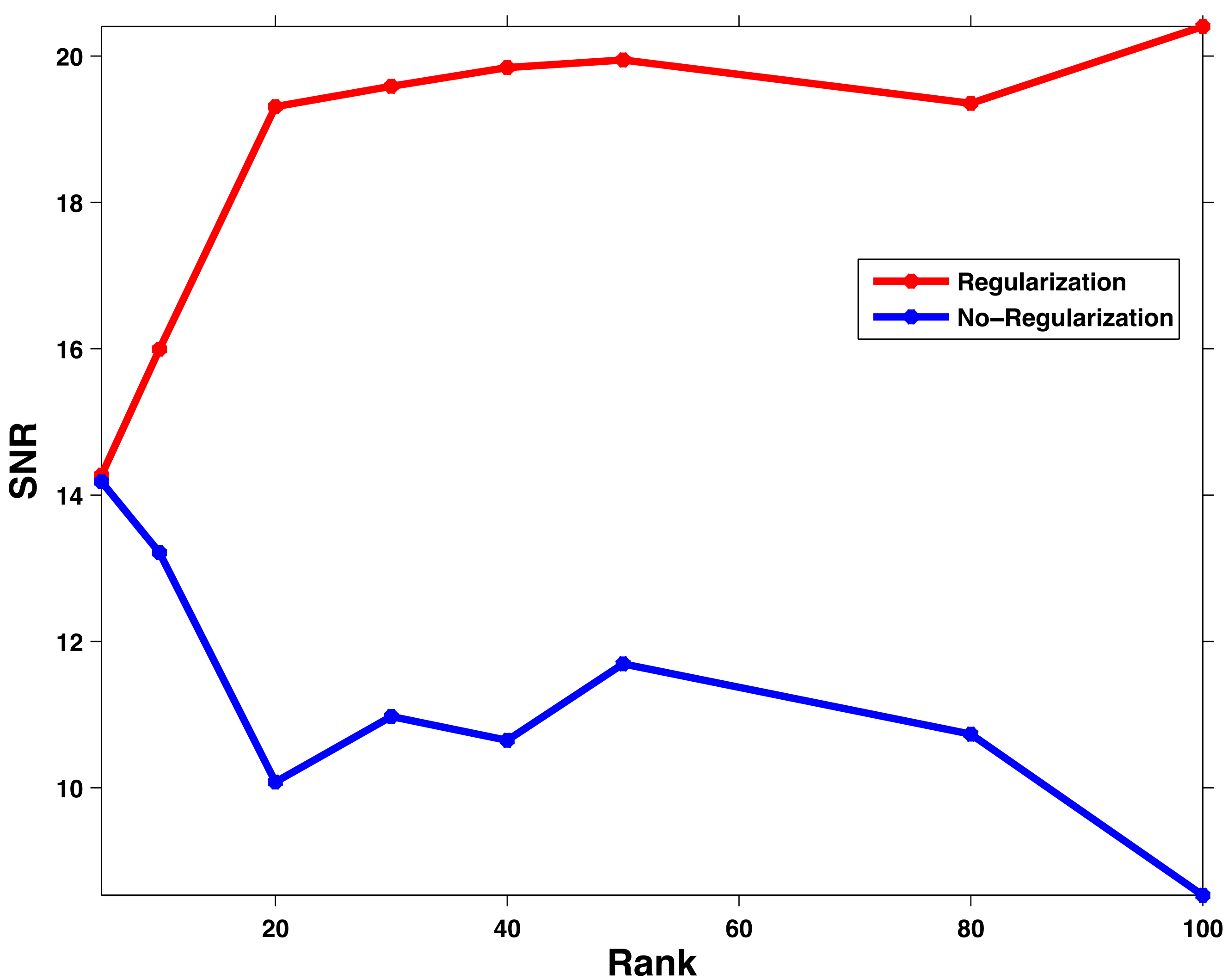
Low-rank interpolation



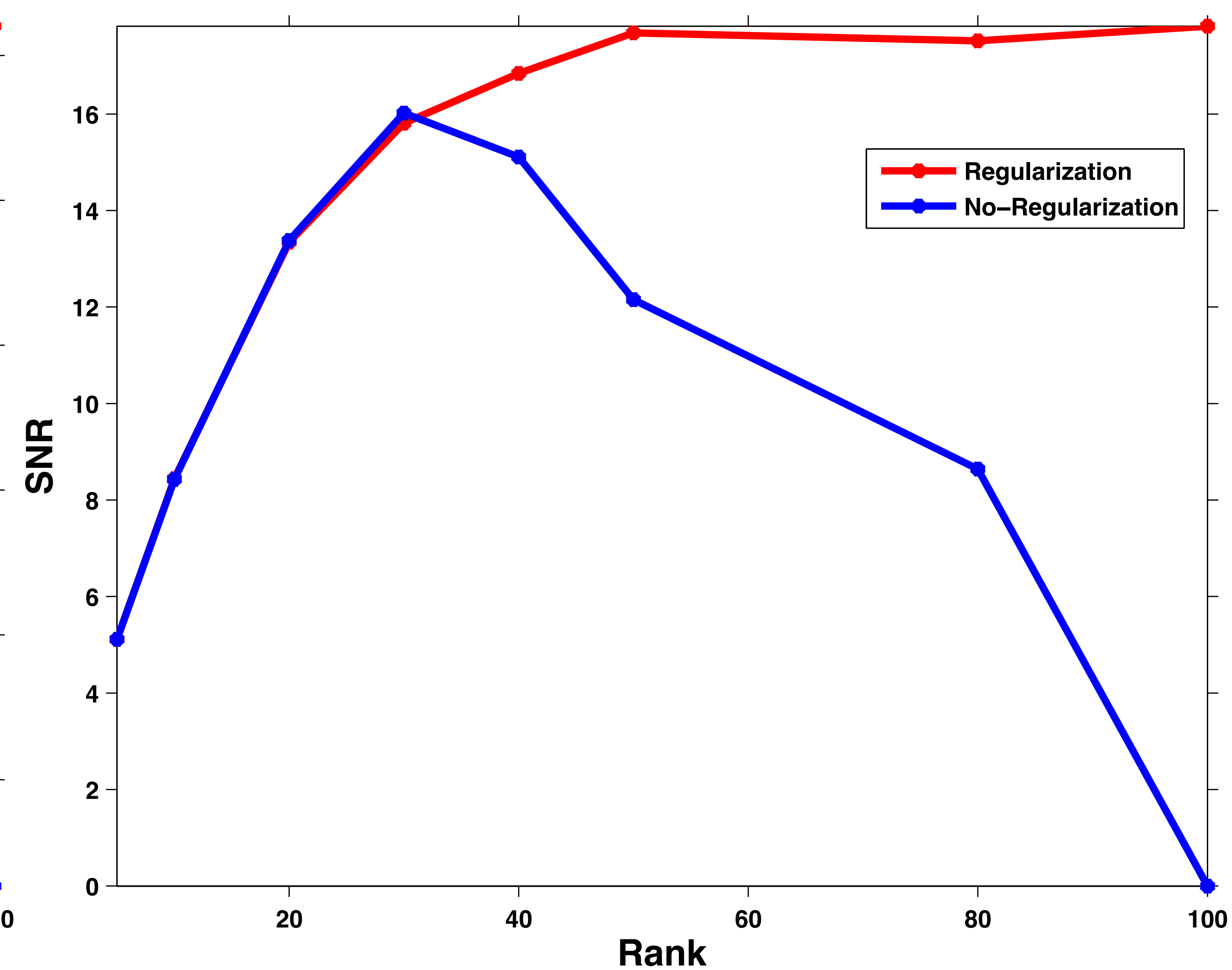
Why need regularization

$$\min_{\mathbf{X}} \; ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

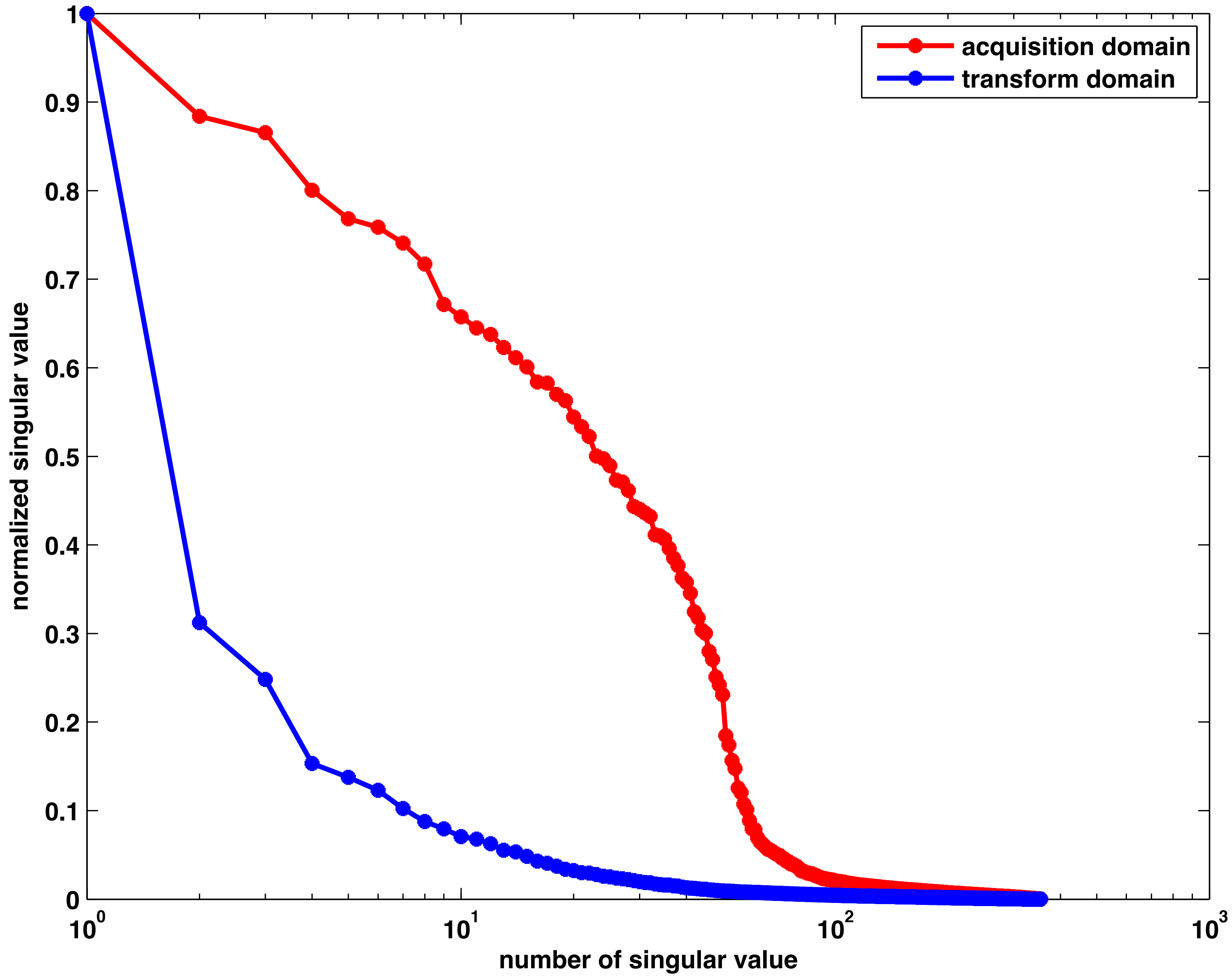
low-frequency



high-frequency

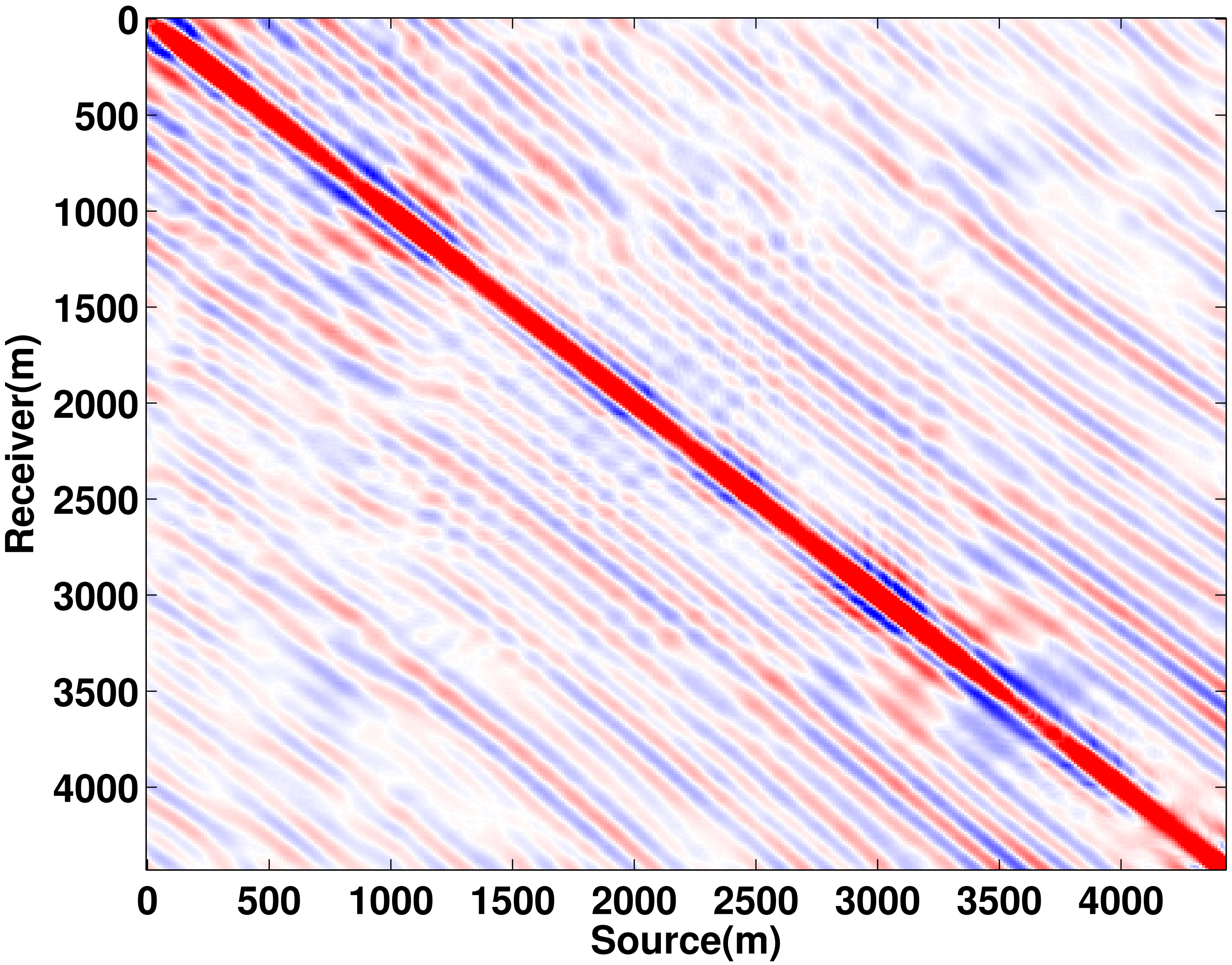


Singular value decay
2-D acquisition

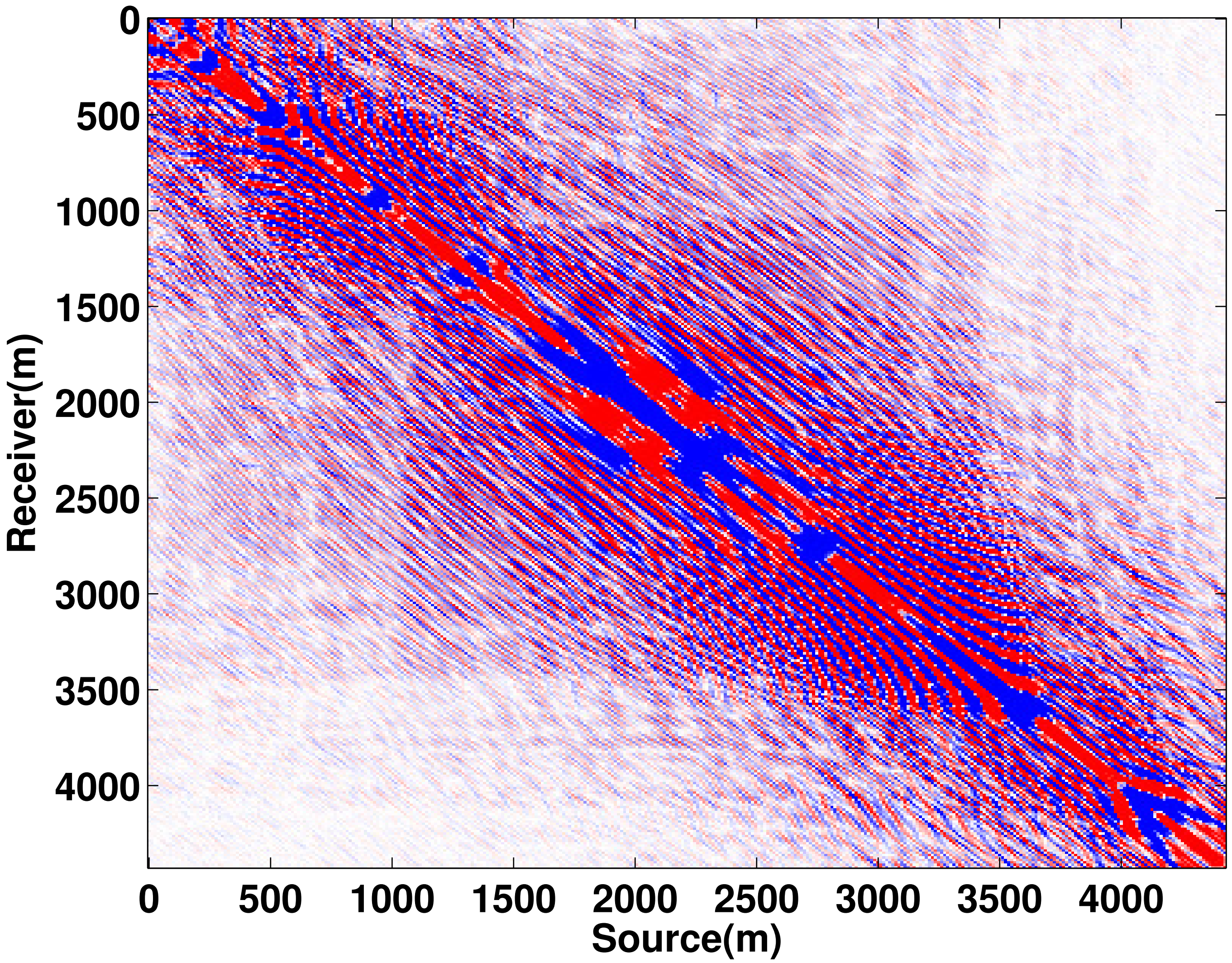


Is high frequency low-rank ?

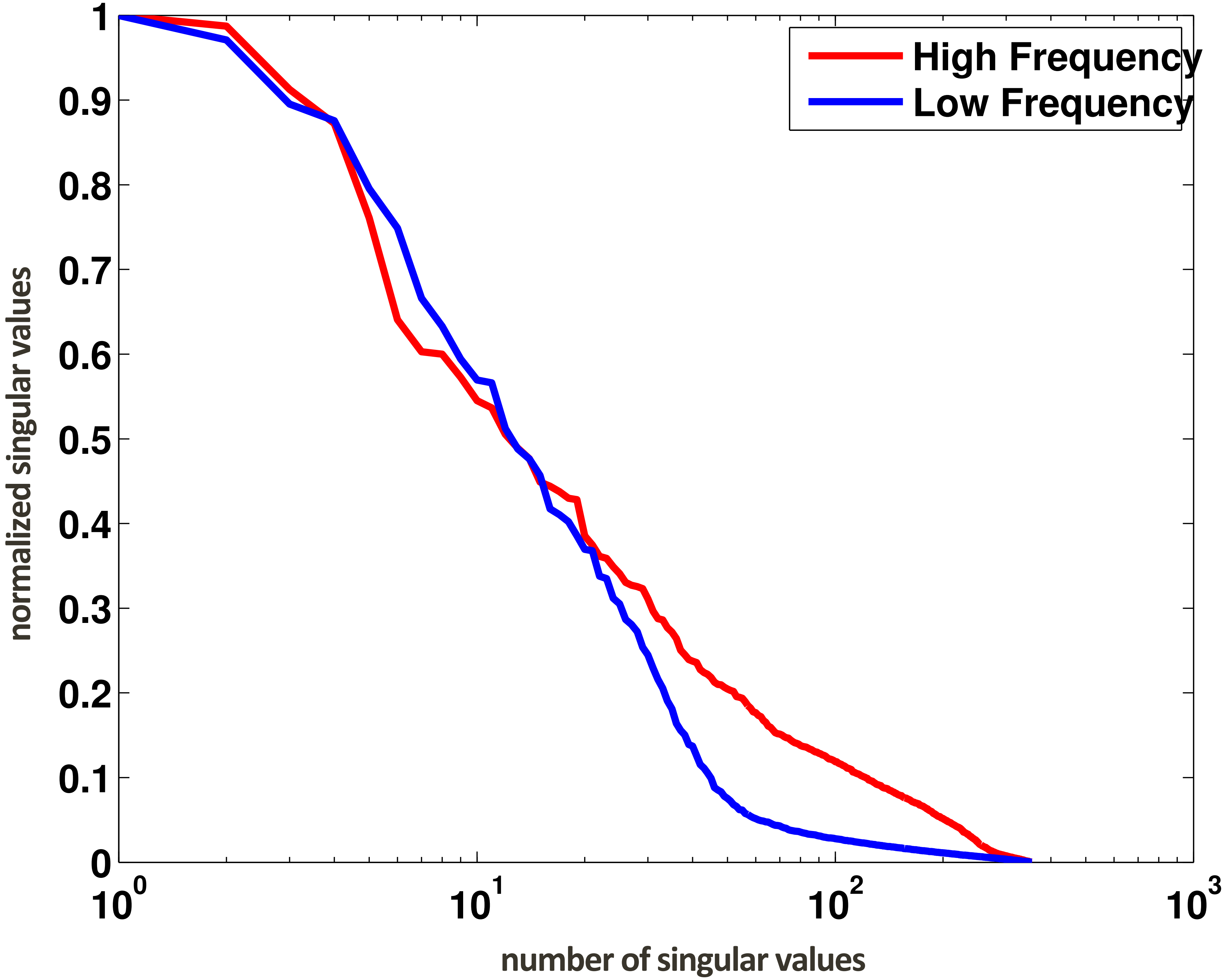
acquisition domain
[source-receiver]



acquisition domain
[source-receiver]



Singular value decay

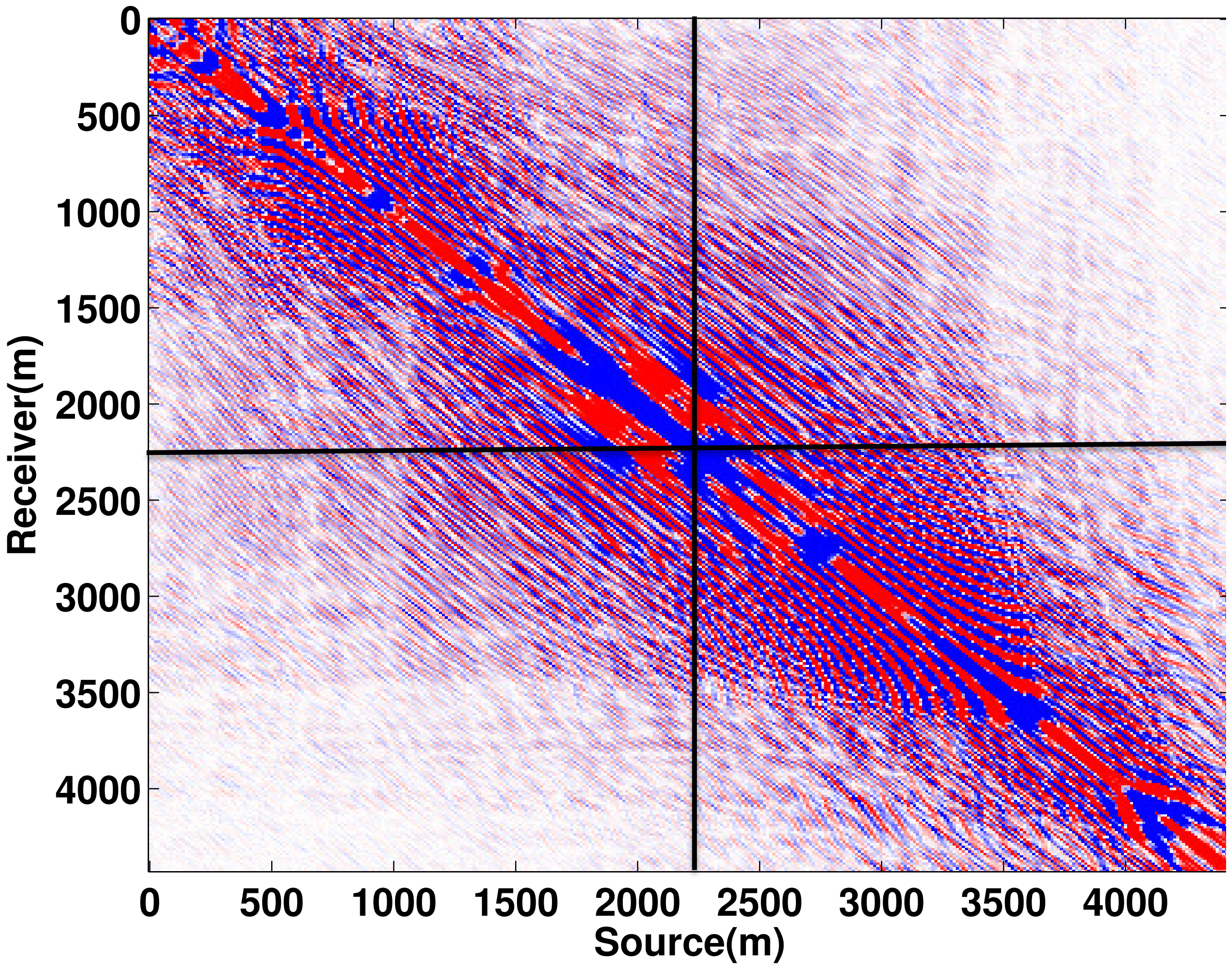


[Chandrasekaran et al. (2006)]

HSS representation

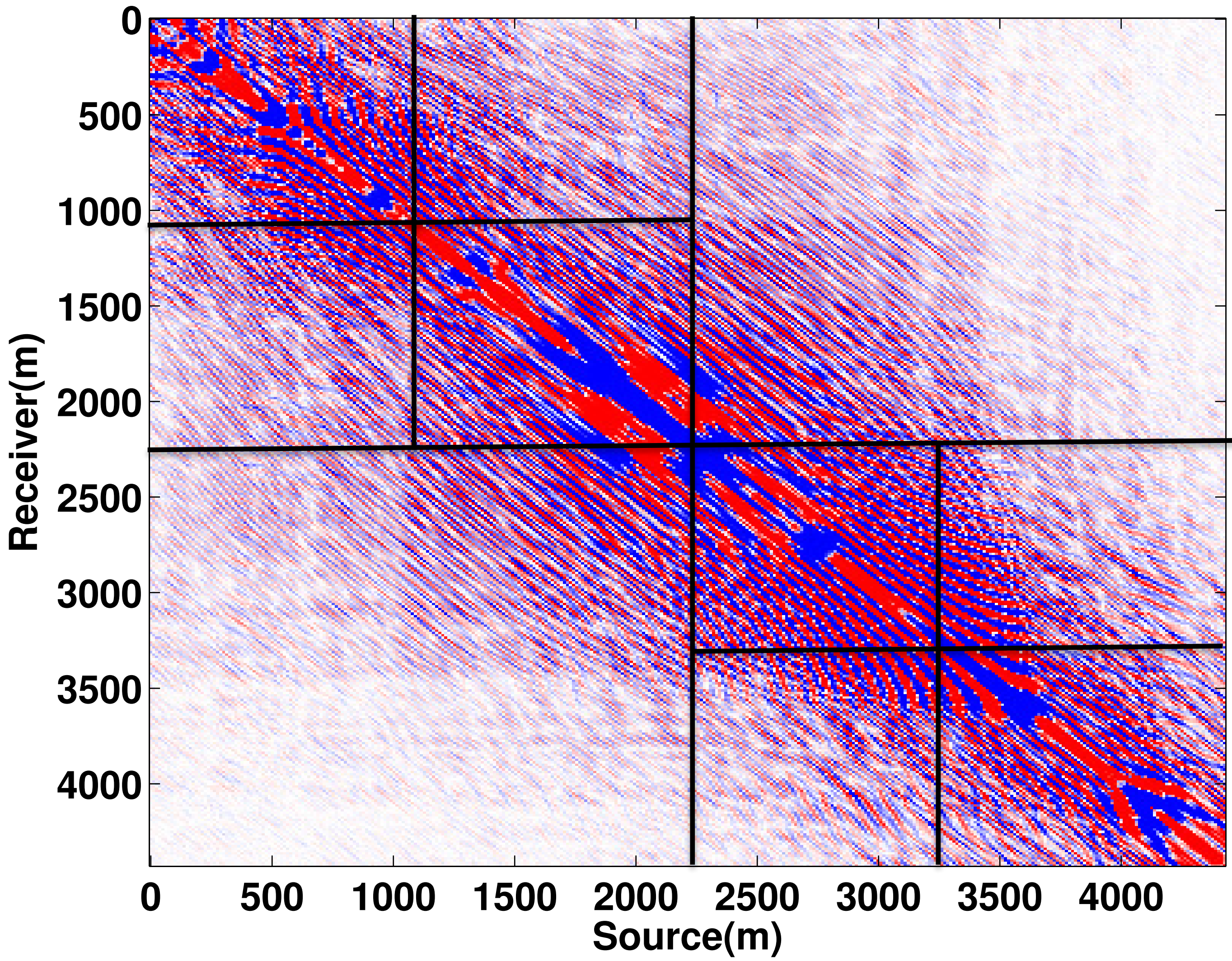
level - 1

[source-receiver]



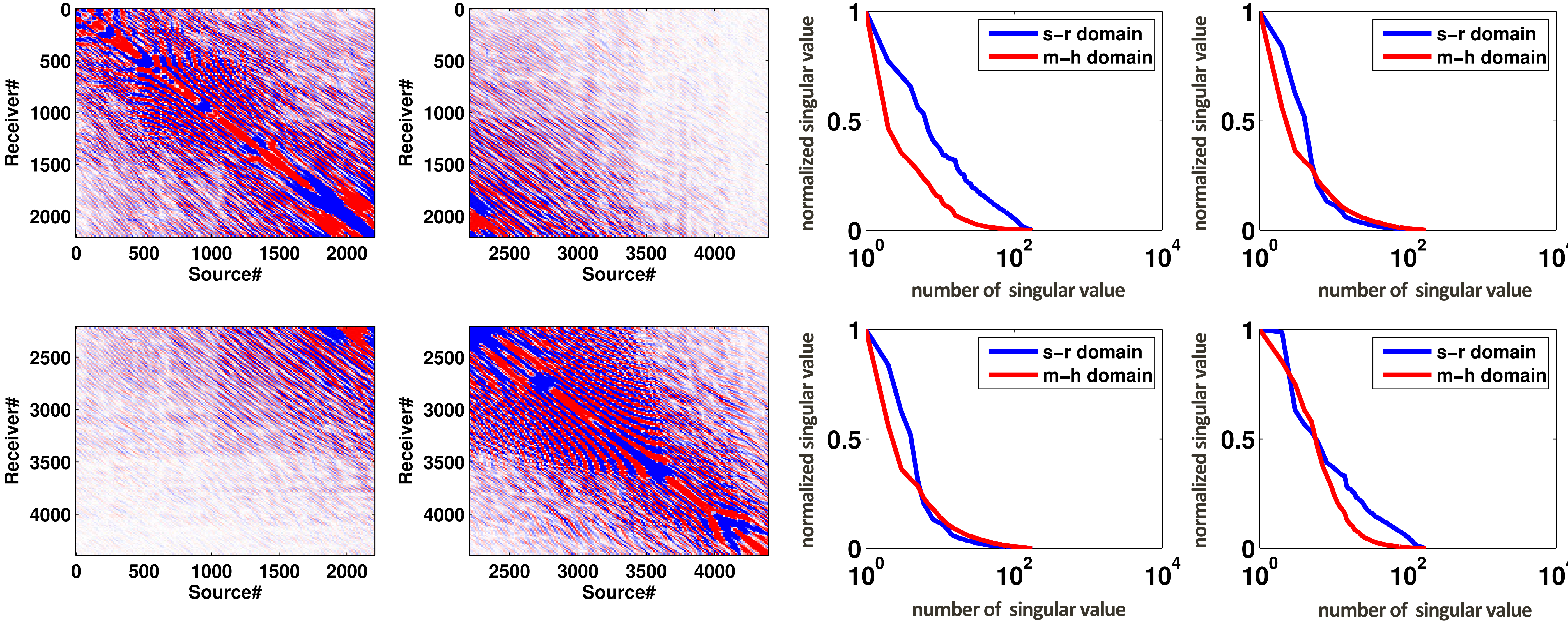
level - 2

[source-receiver]



HSS representation

[level-1]



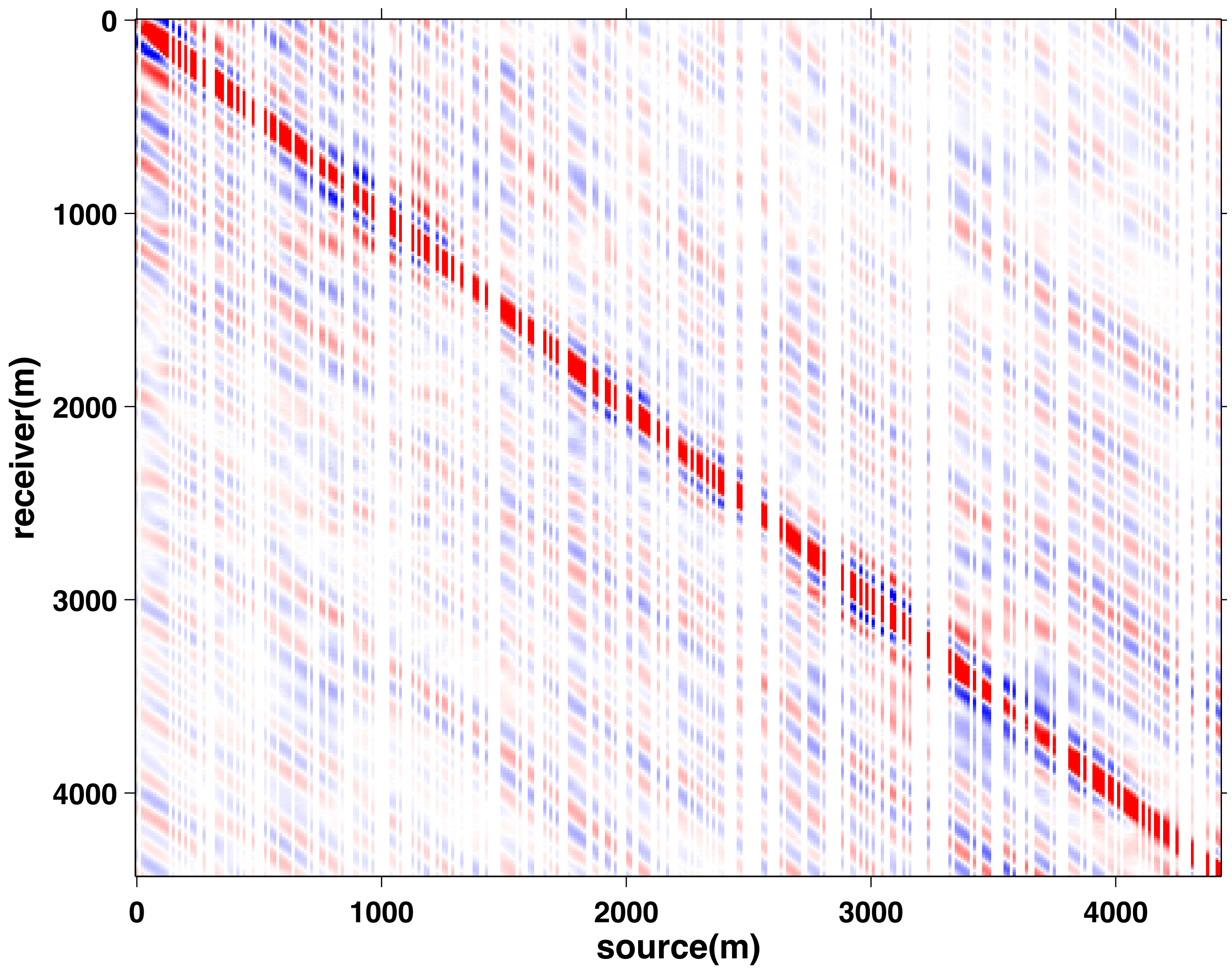
Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

2-D Acquisition
[randomized sampling]

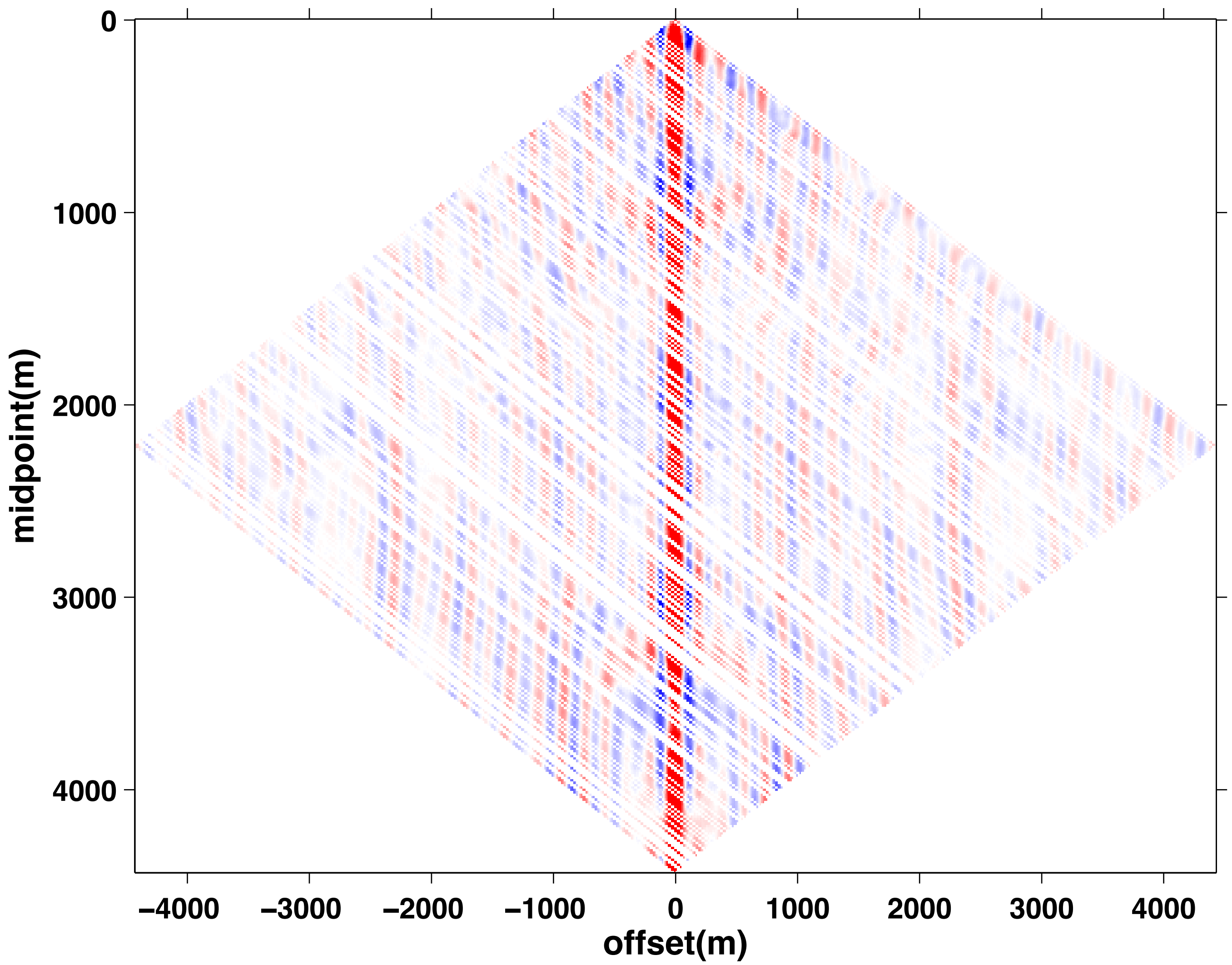
acquisition domain

missing columns *do not* increase rank



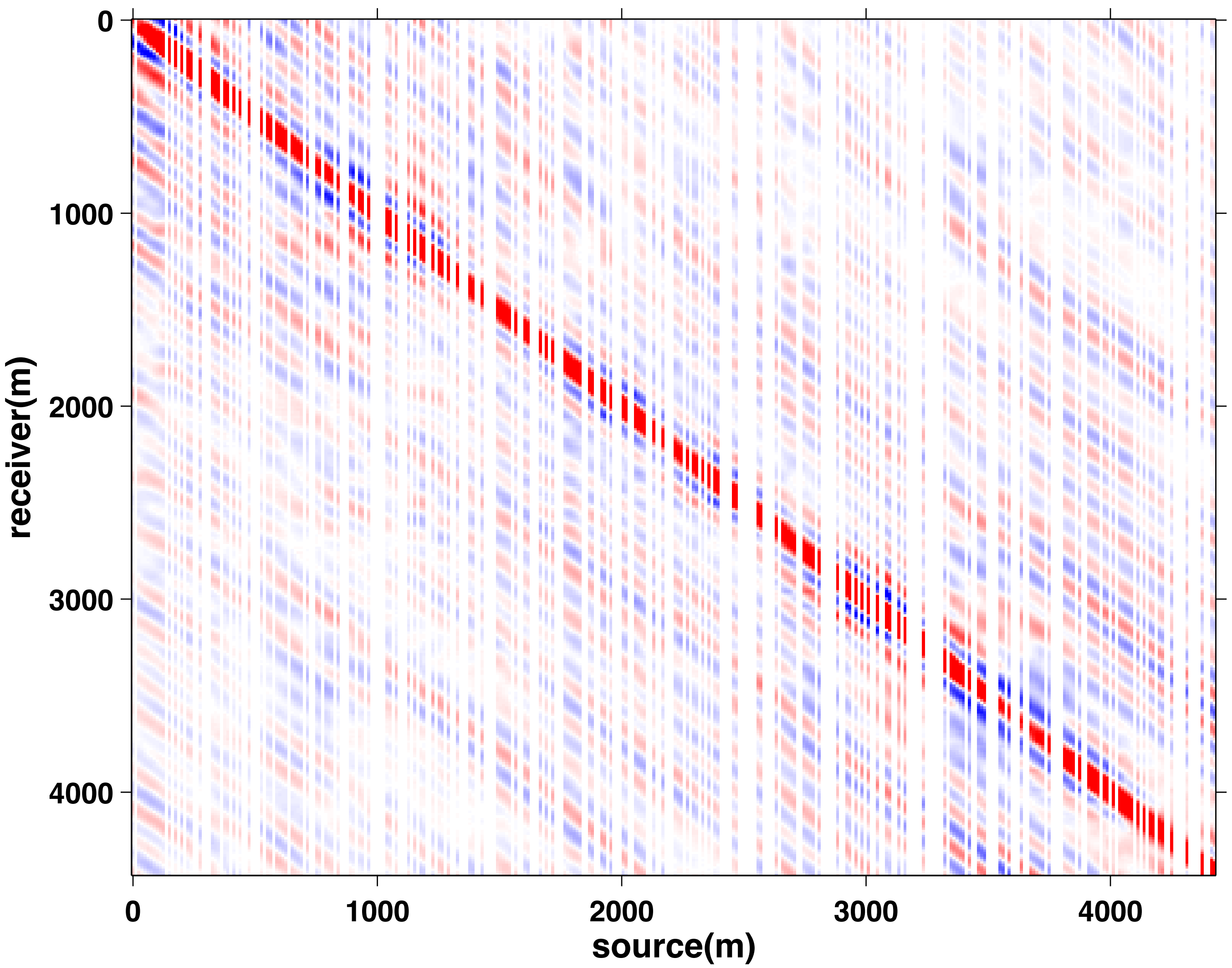
transform domain

missing columns *do* increase rank

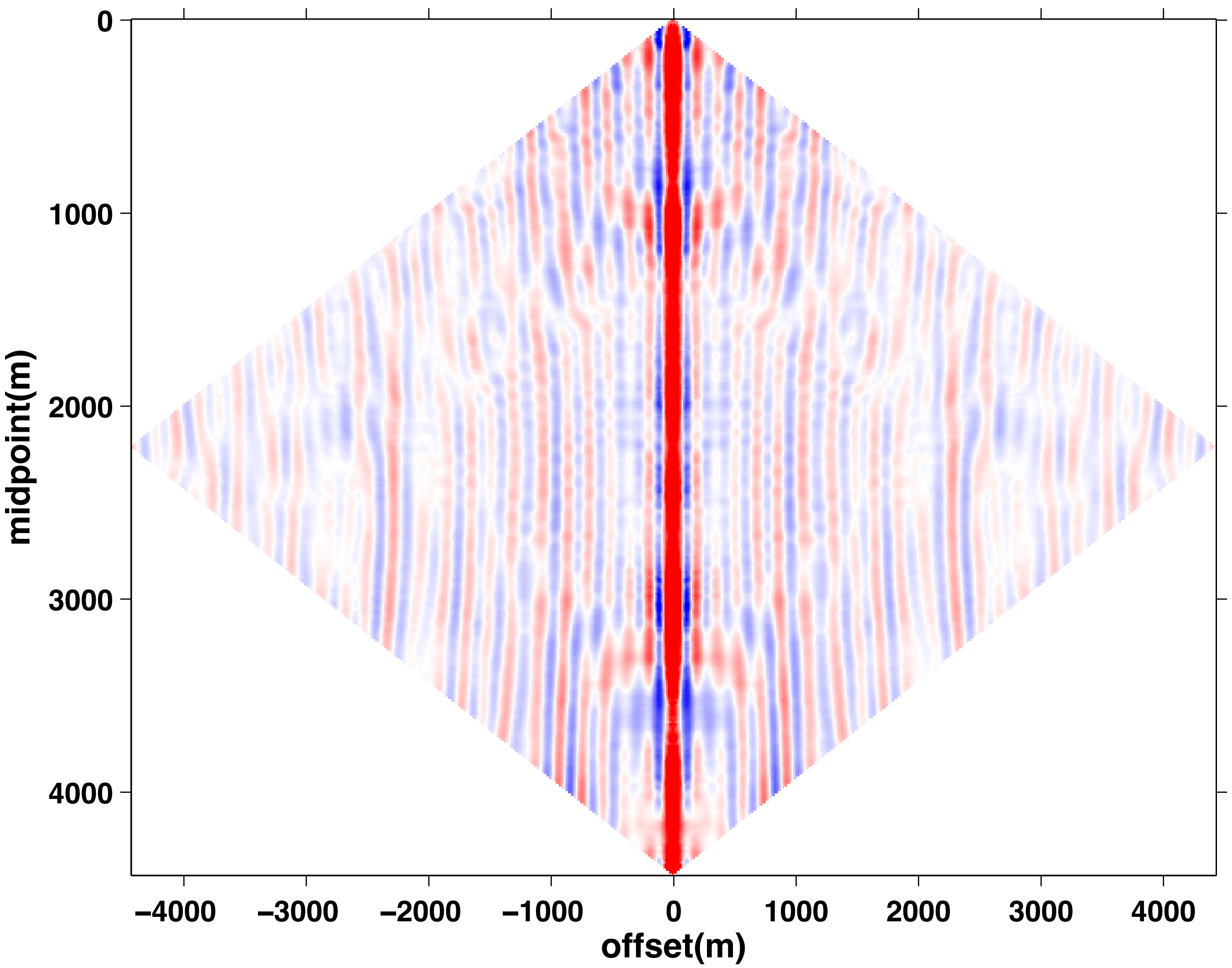


Low-rank interpolation

recovery
[SNR = 2 dB]

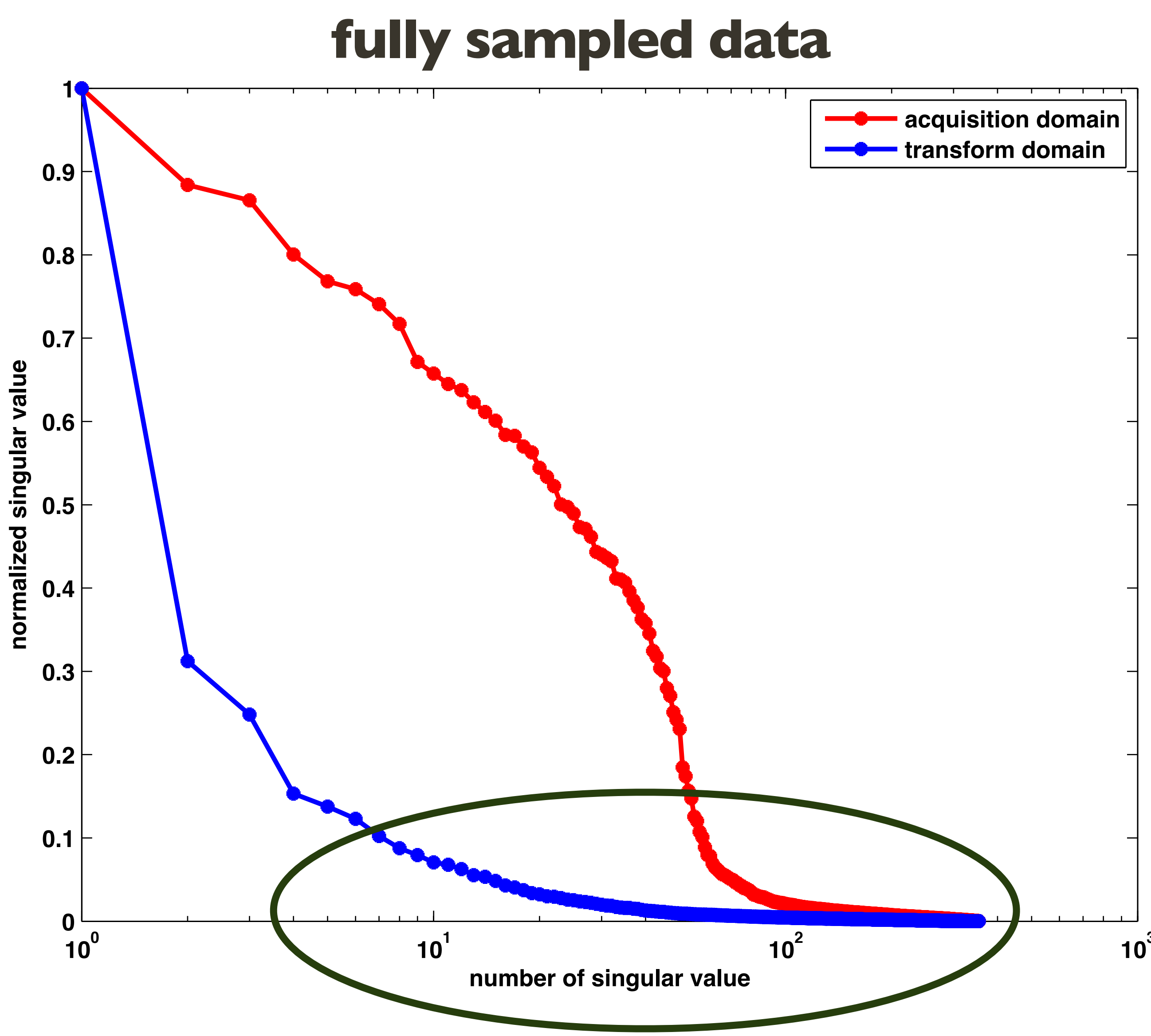


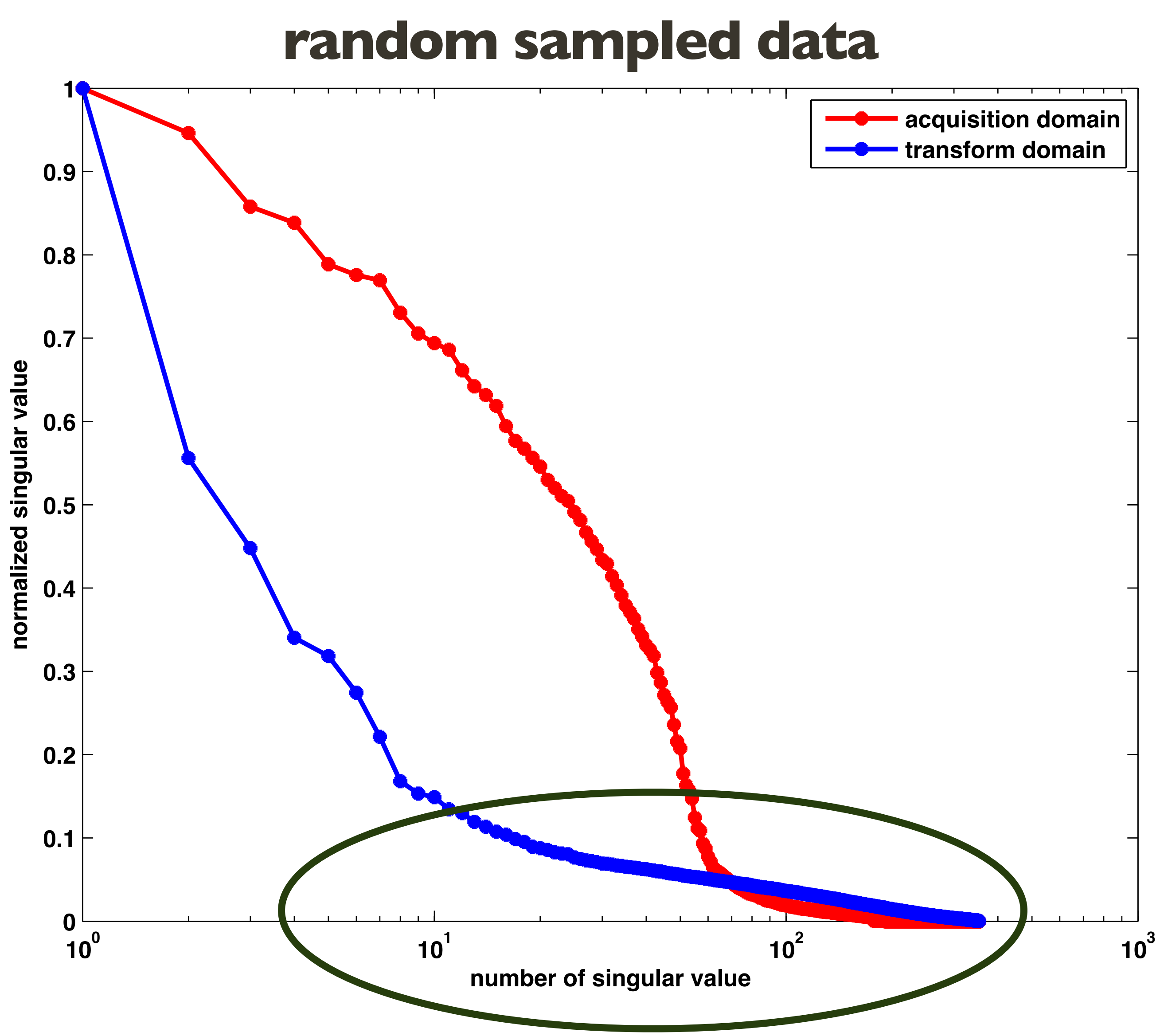
recovery
[SNR = 18.5 dB]



Randomized sampling

singular value decay

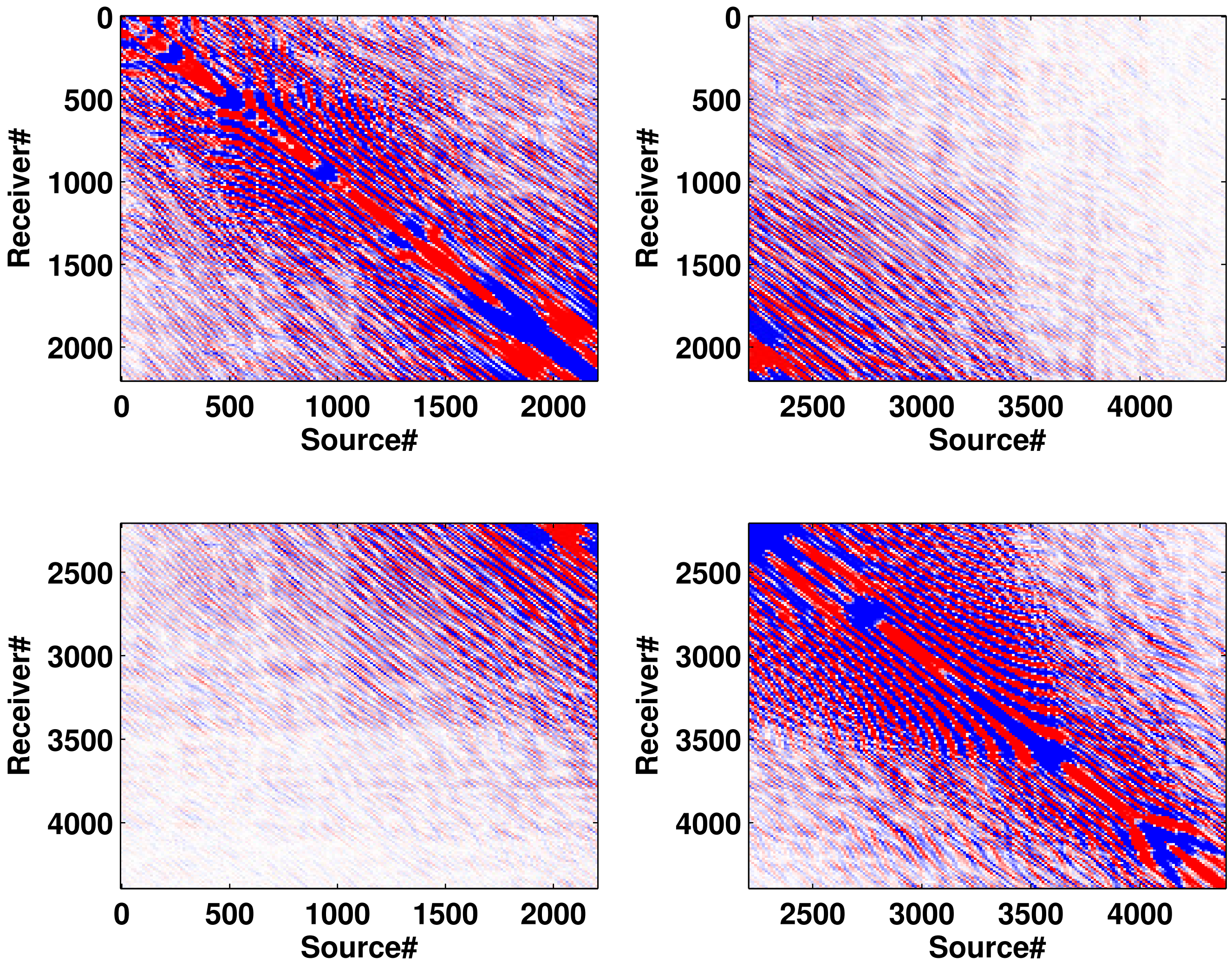




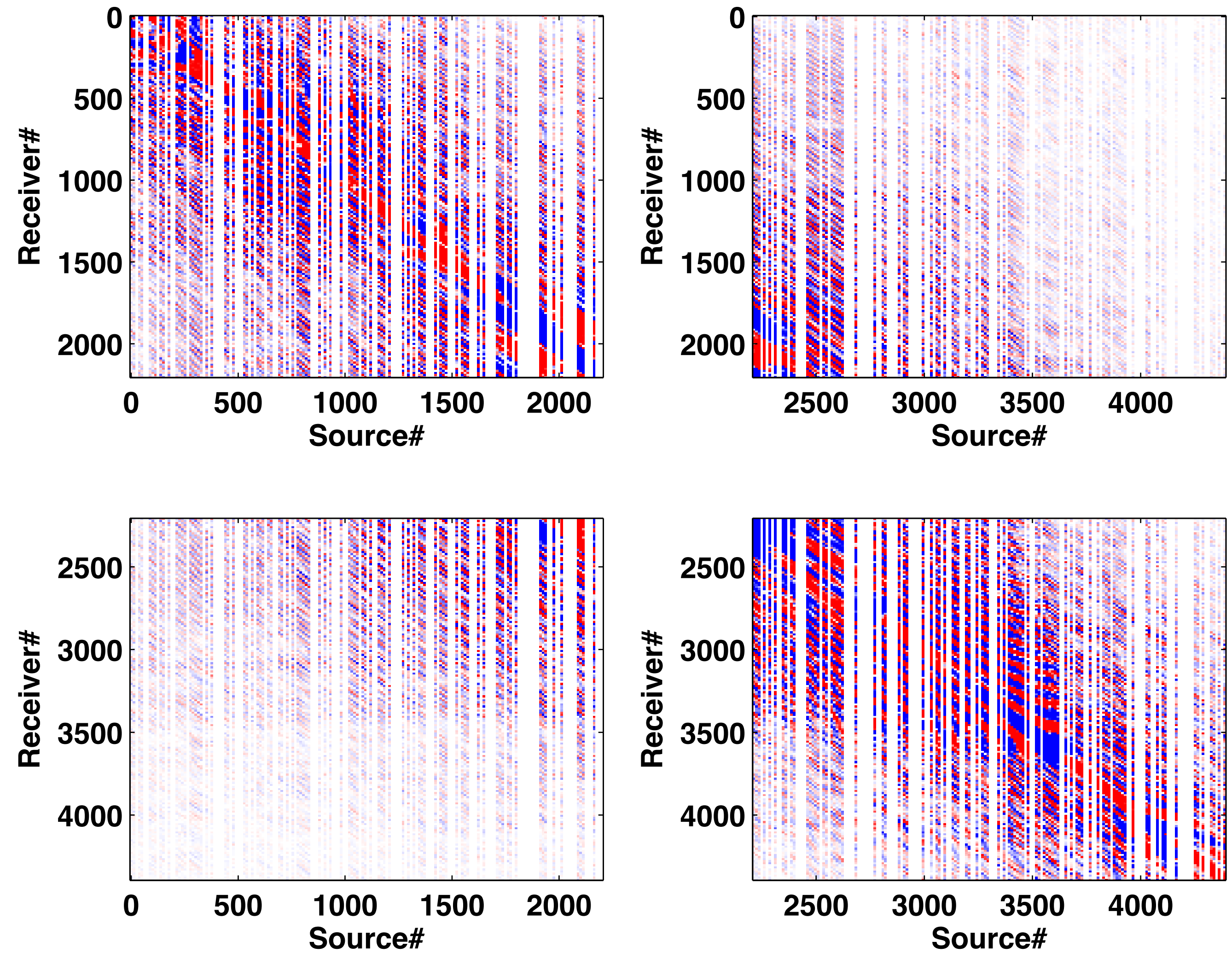
HSS representation

[level-1]

fully sampled data



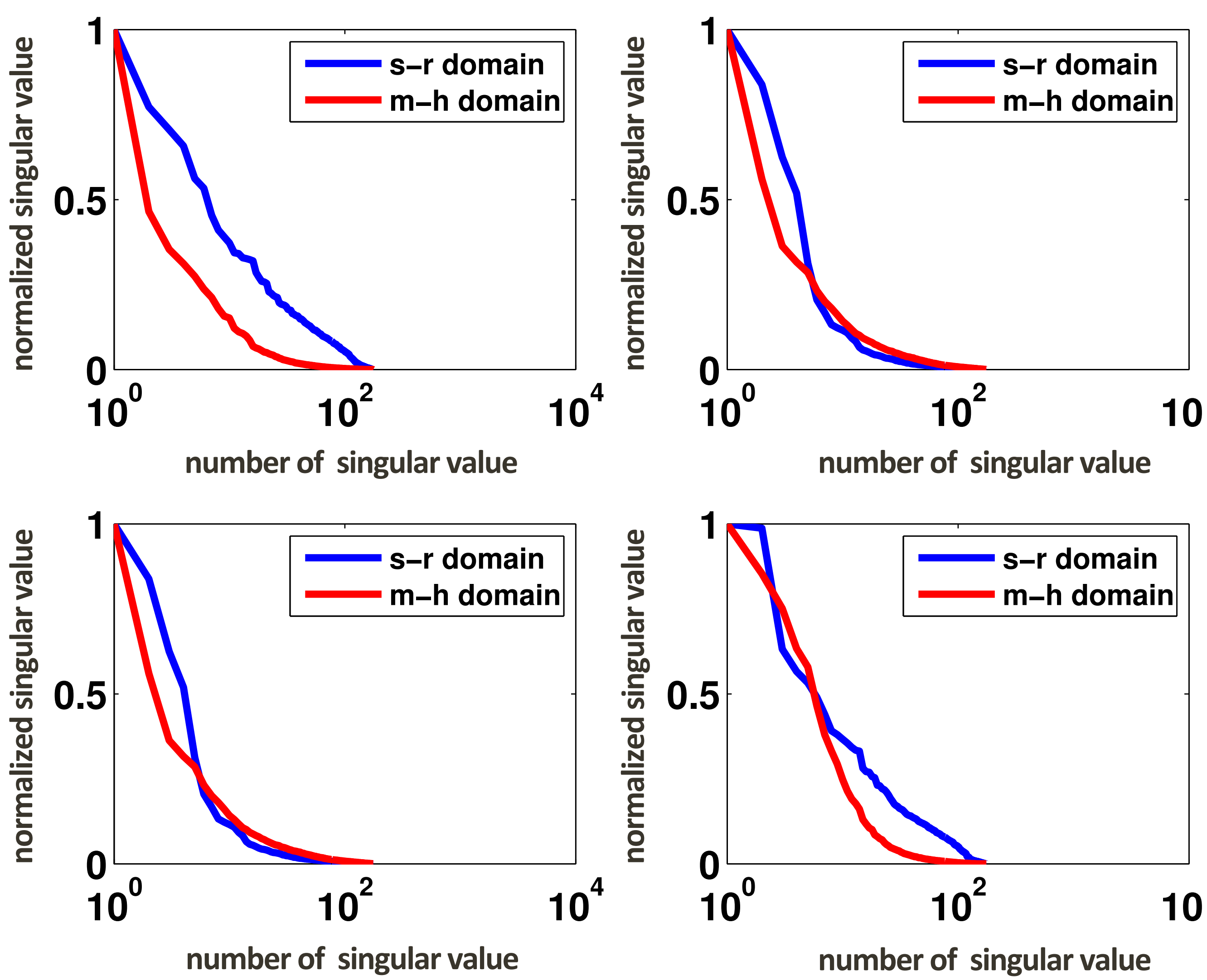
random sampled data



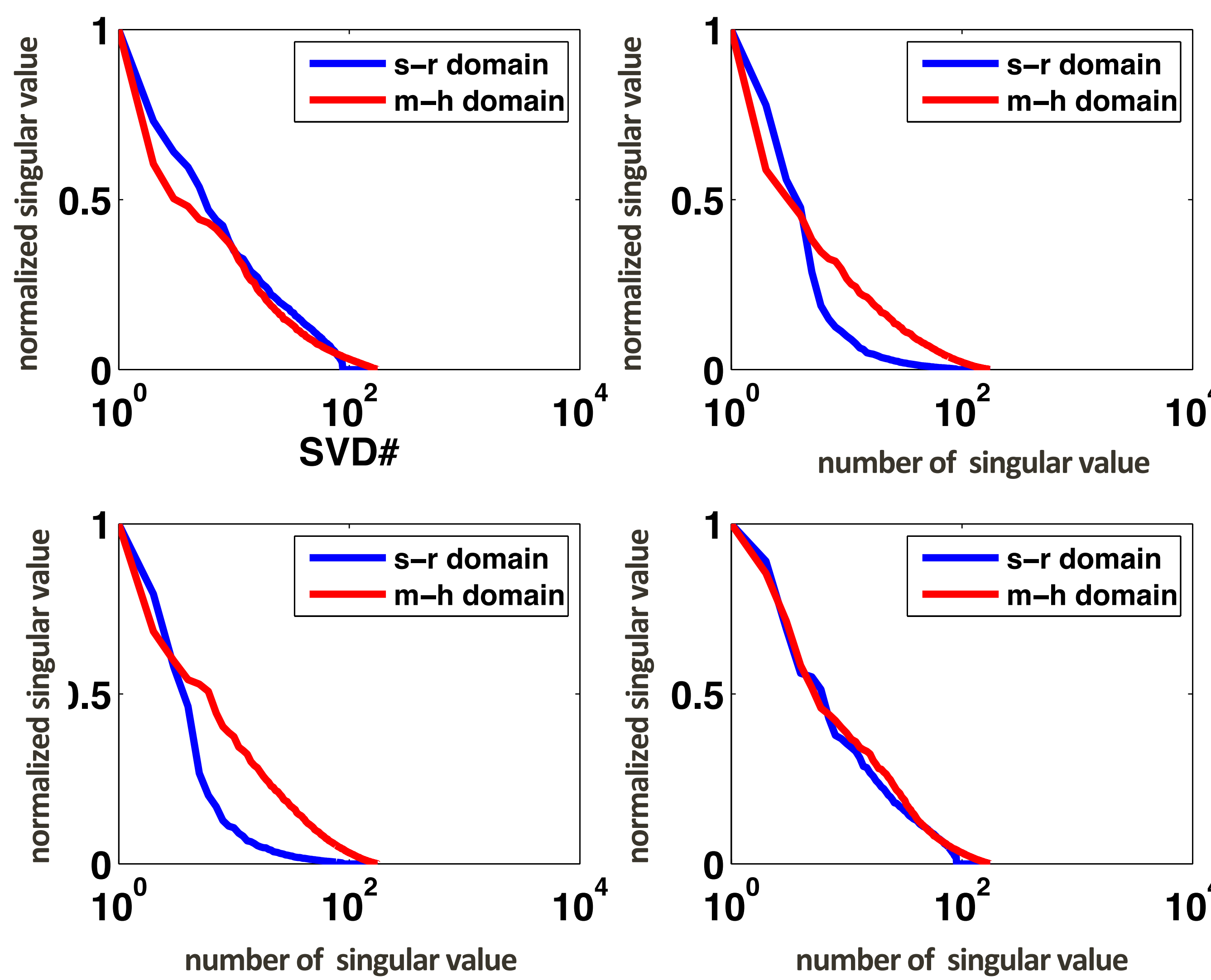
Singular value decay

[HSS, level-1]

fully sampled data



random sampled data



Low-rank domain

- 2-D acquisition
  - midpoint-offset
- 3-D acquisition
  - (source x, receiver x) and (source y, receiver y)

Observations

- sampling become *incoherent* in “transform” domain
- *slow decay* of singular values in “transform” domain

Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

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Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad rank(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$rank(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator

$\mathbf{M}$: measurement operator

$\mathcal{S}^H$: transform operator

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Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank

- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization

[Recht et. al. 2010]

[Recht et. al. 2010]

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Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

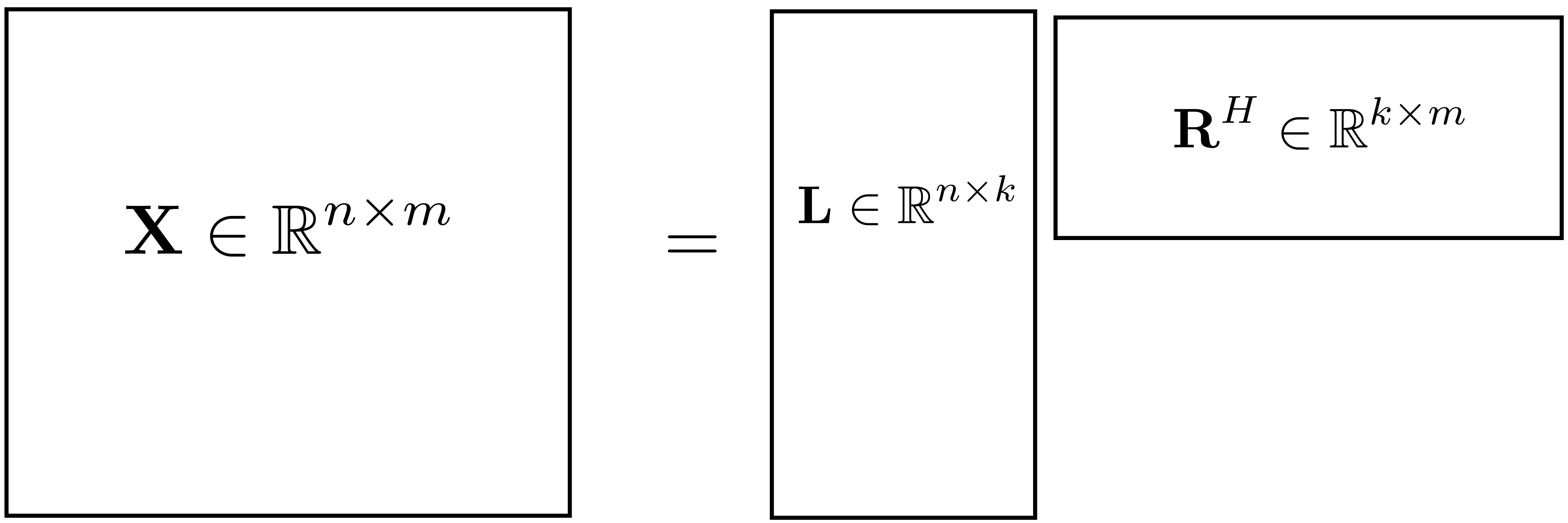
where λ_i are the *singular* values

Challenges

- requires repeated application of *SVD* for projection
- expensive to compute for large system
  - curse of dimensionality
- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, Lee et. al. 2010, Recht and Re 2011]

Factorized formulation



$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

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Factorized formulation

- reformulate $BPDN_\sigma$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \ \|\mathbf{L}\mathbf{R}^H\|_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

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Factorized formulation

- nuclear norm is then define as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

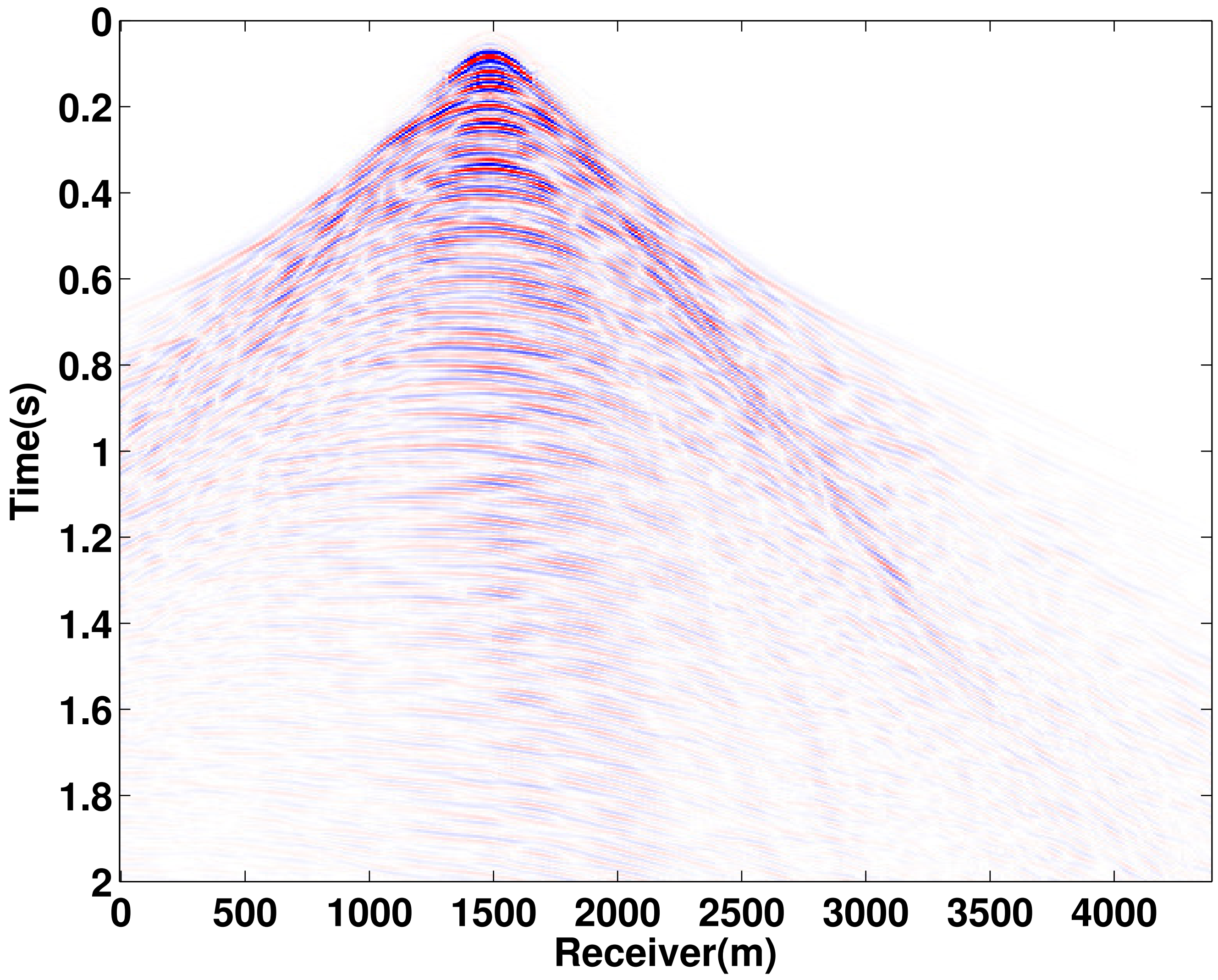
- choose rank k *explicitly* & avoid costly *SVD's*

Interpolation

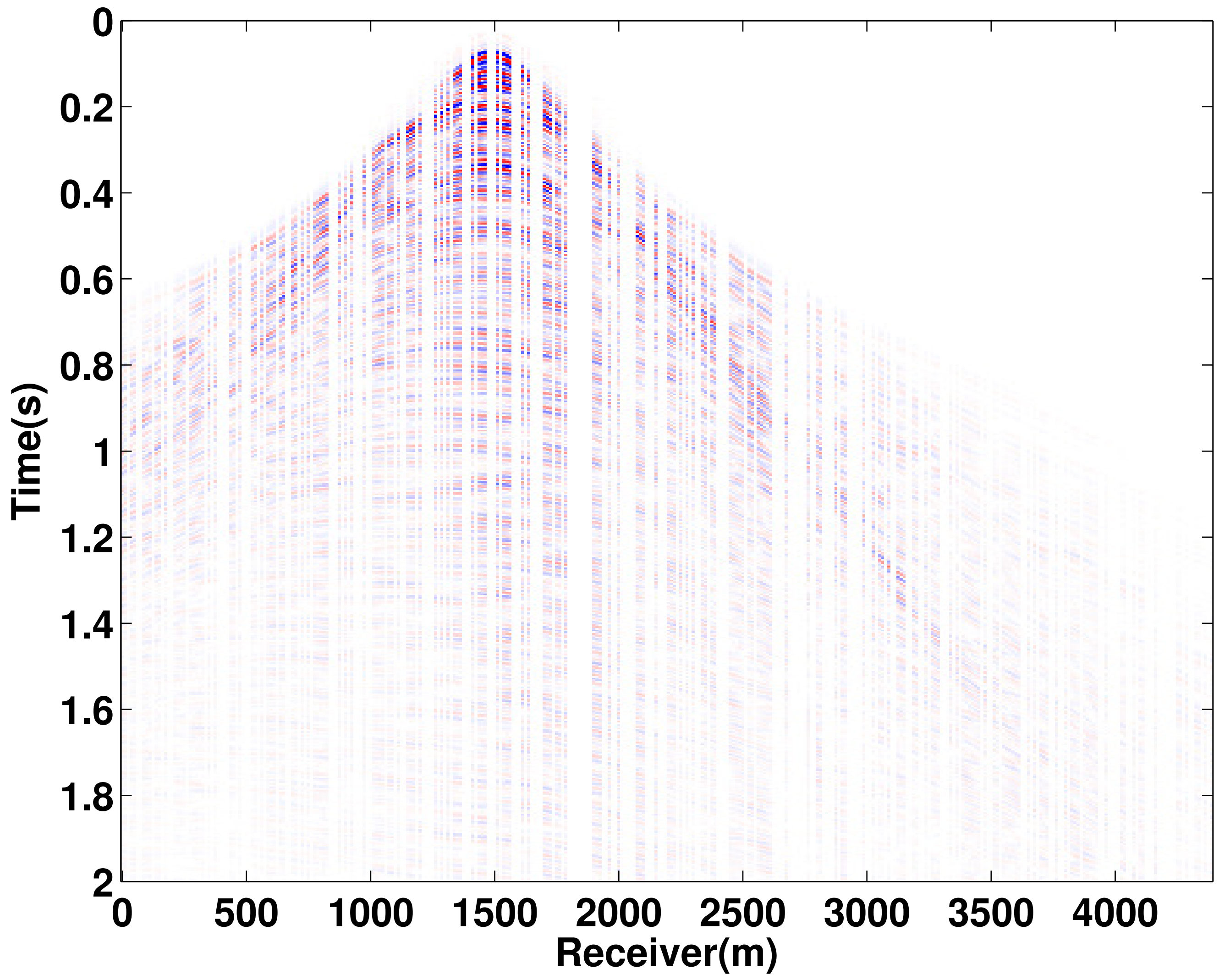
- Gulf of Suez
  - 2-D seismic line
  - 50 % missing traces
  - interpolation with HSS level 1,2,3
  - rank = 5
  - 300 iterations

Gulf of Suez

original



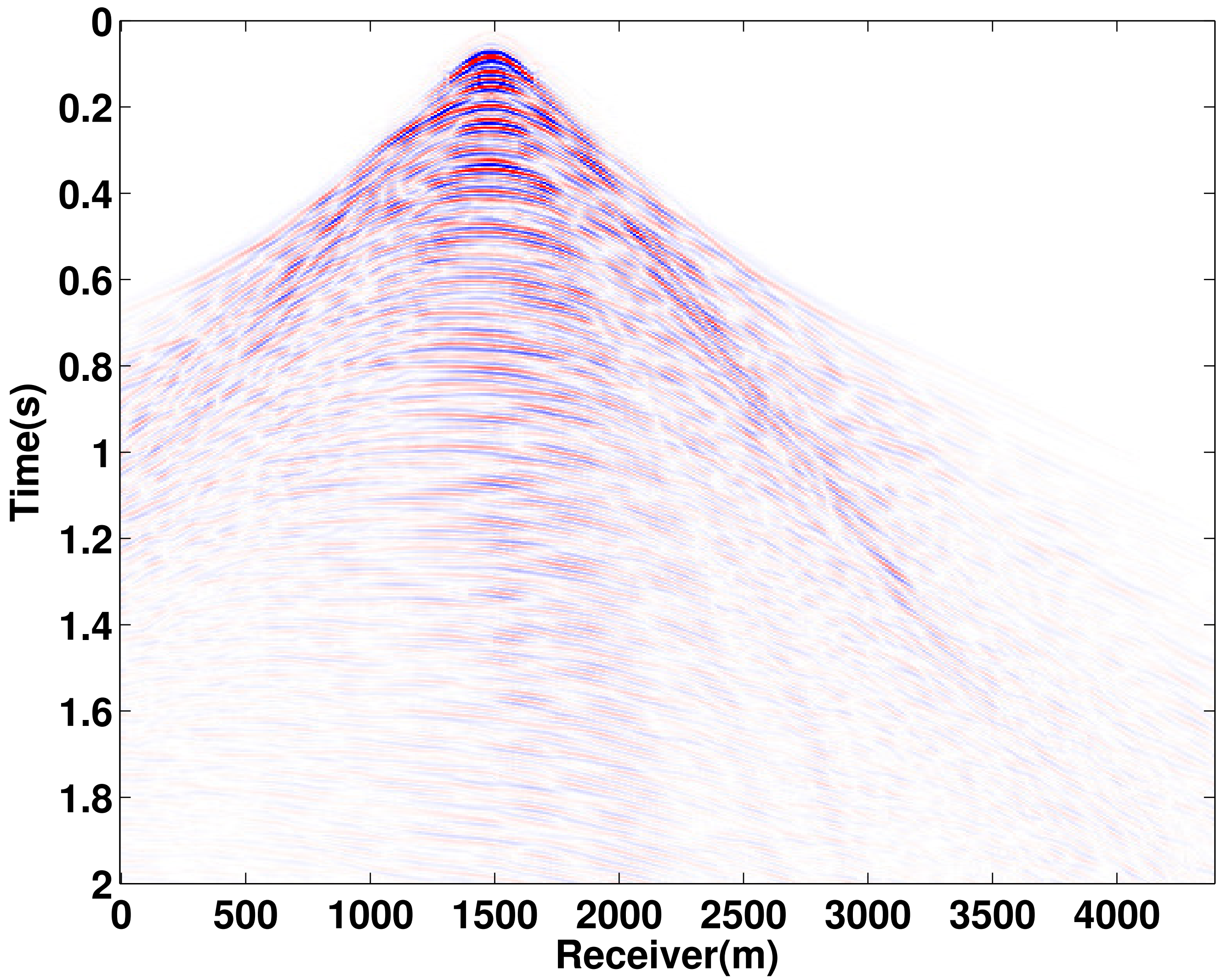
subsampled data



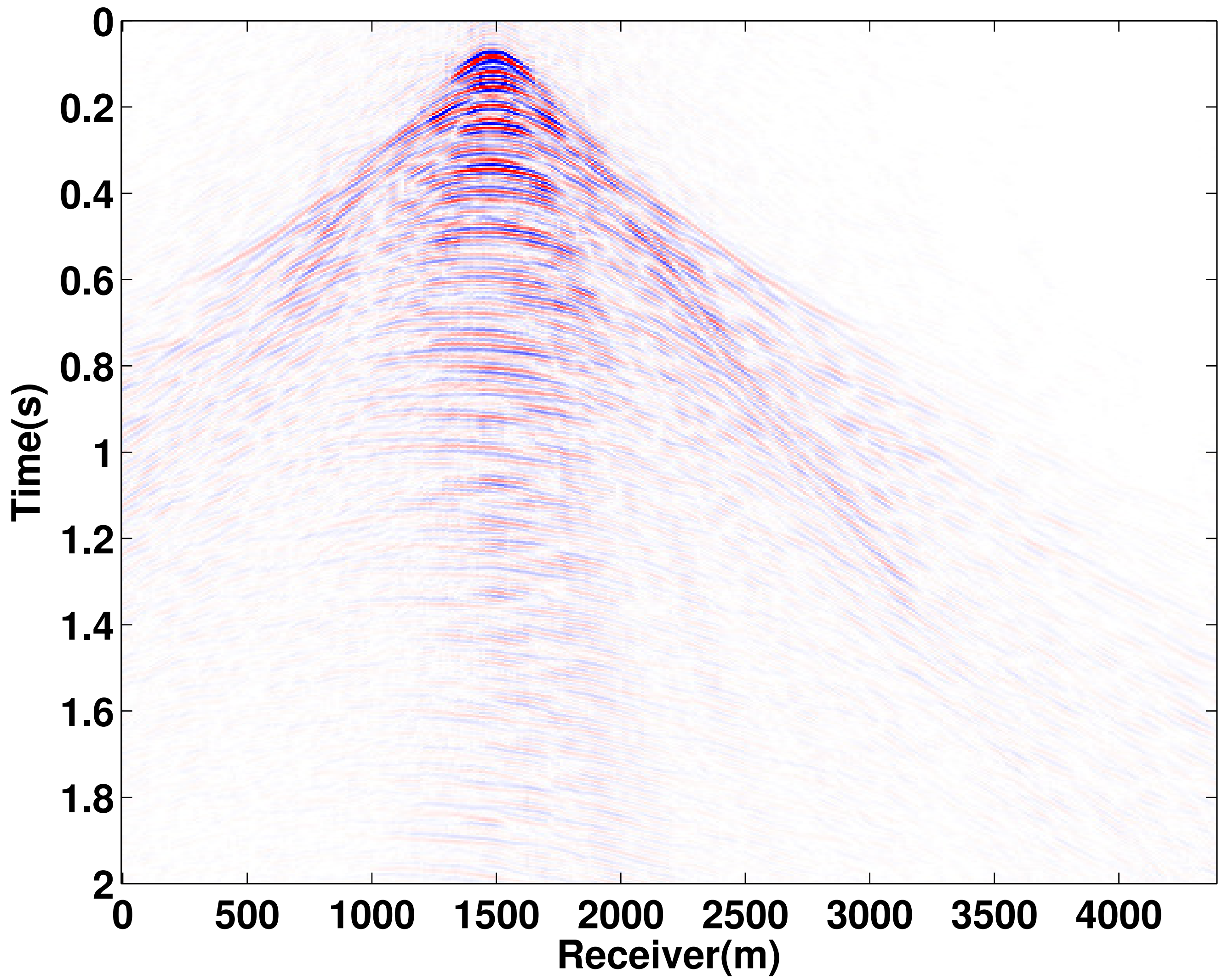
Gulf of Suez
[without HSS]

SLIM

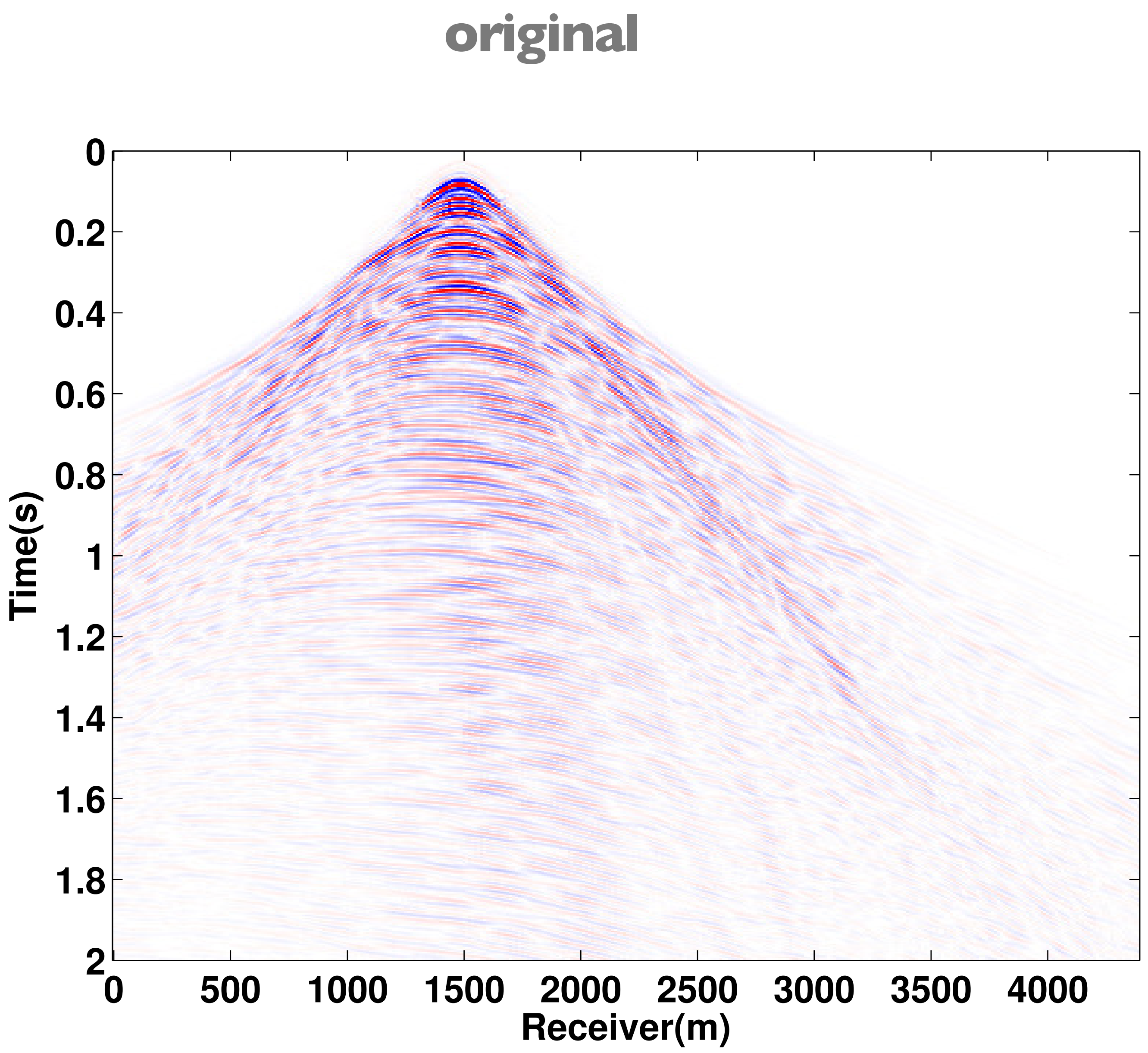
original

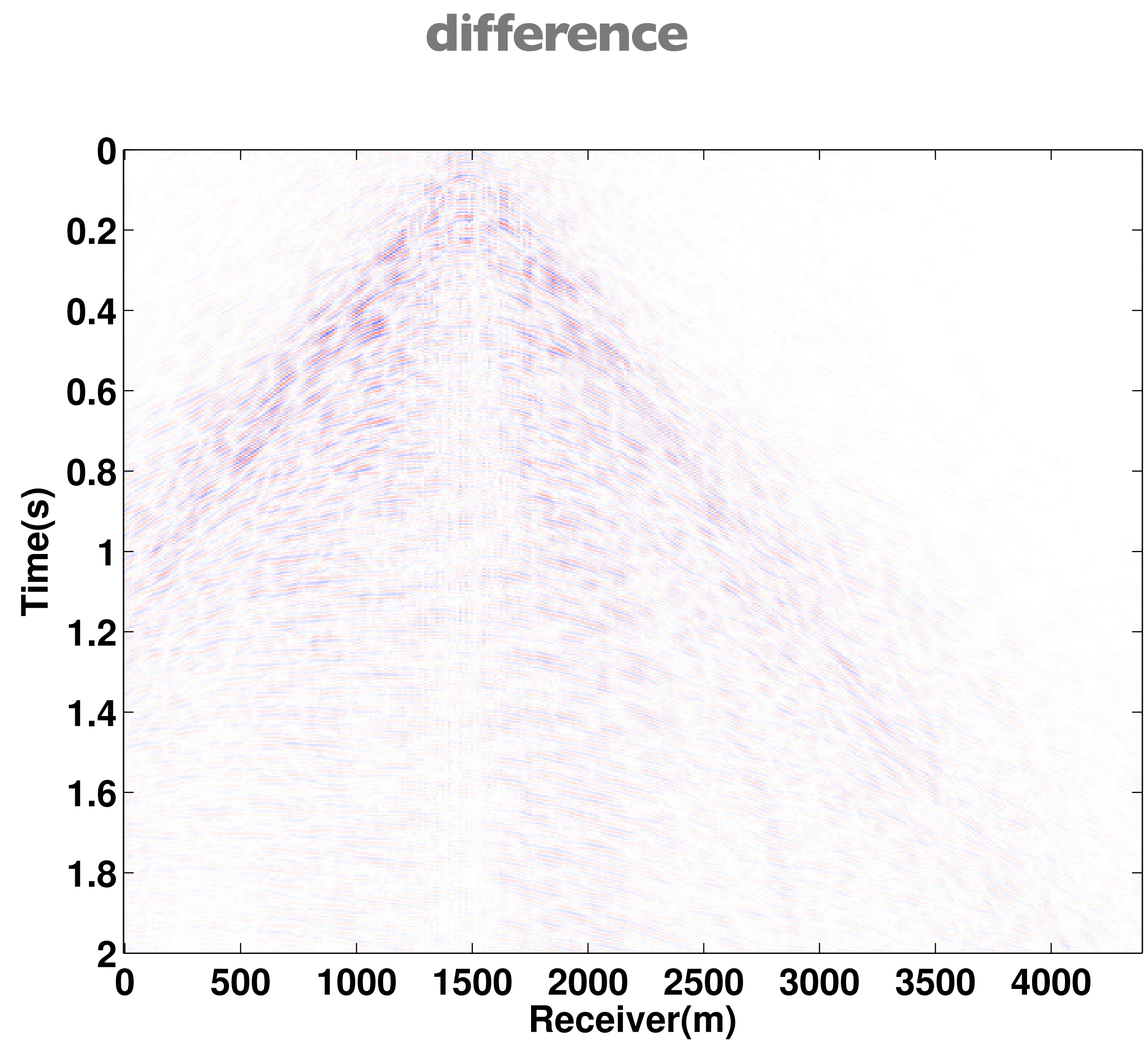


interpolation
[SNR = 7.42 db]



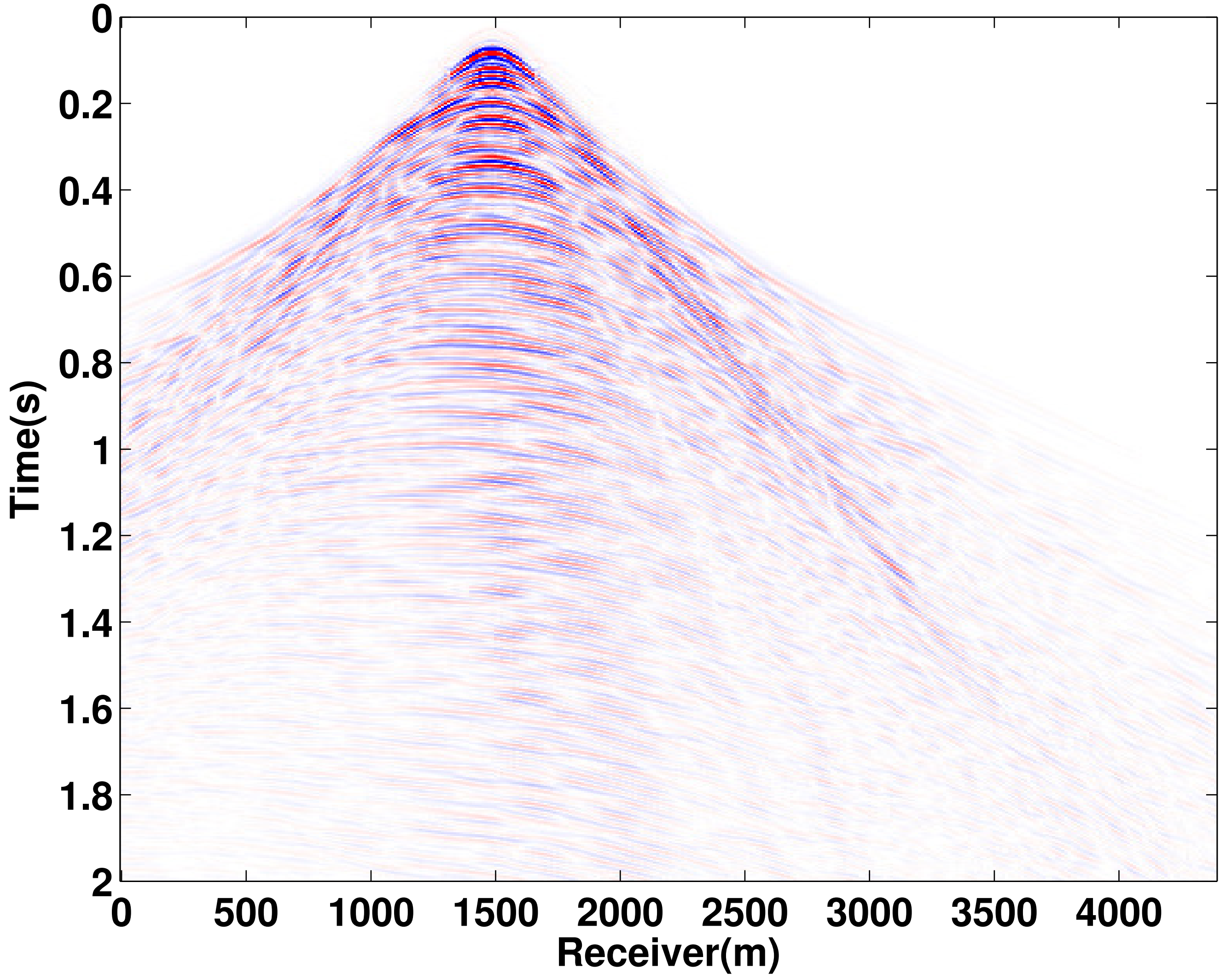
Gulf of Suez
[without HSS]



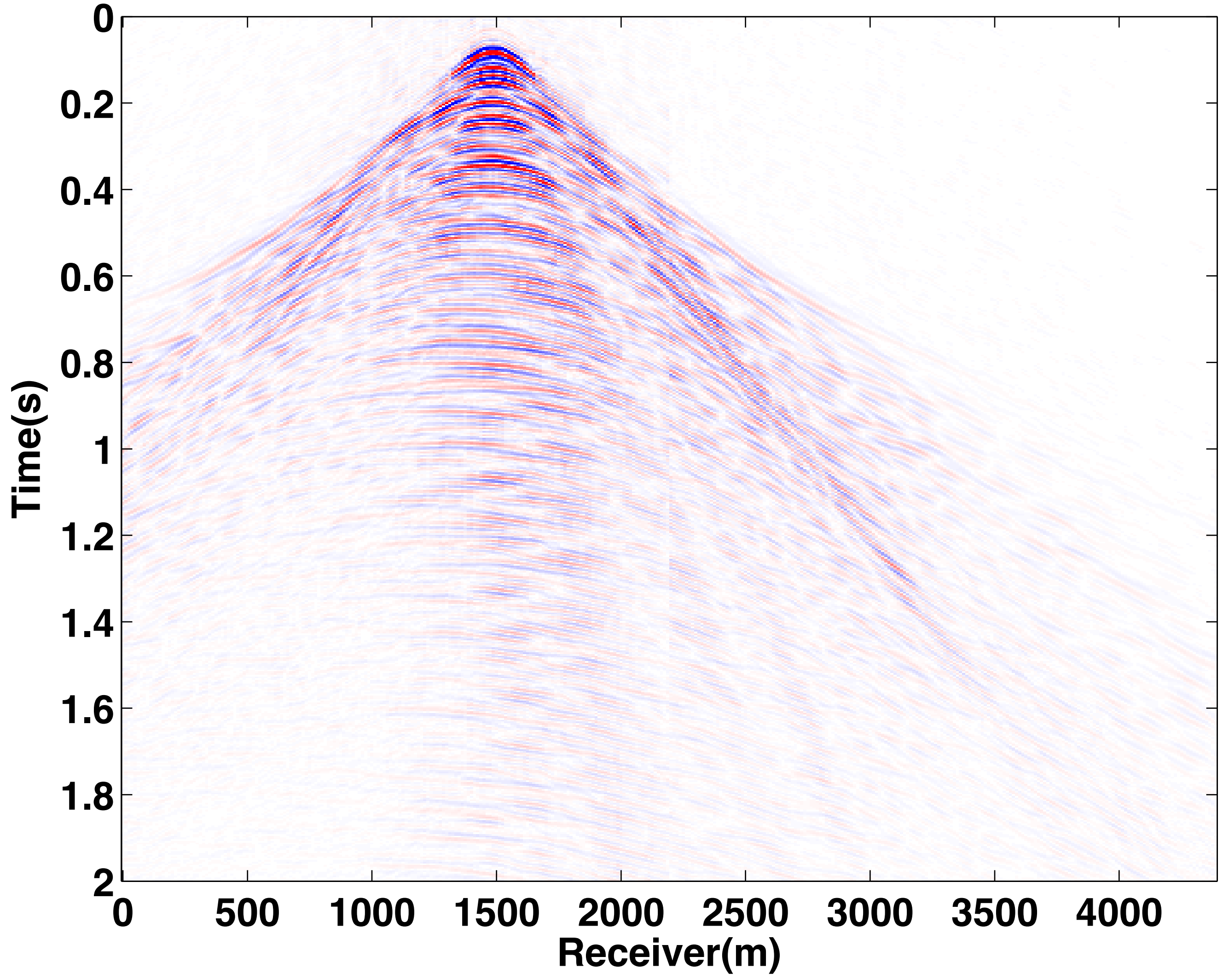


Gulf of Suez
[HSS with level-1 partitioning]

original

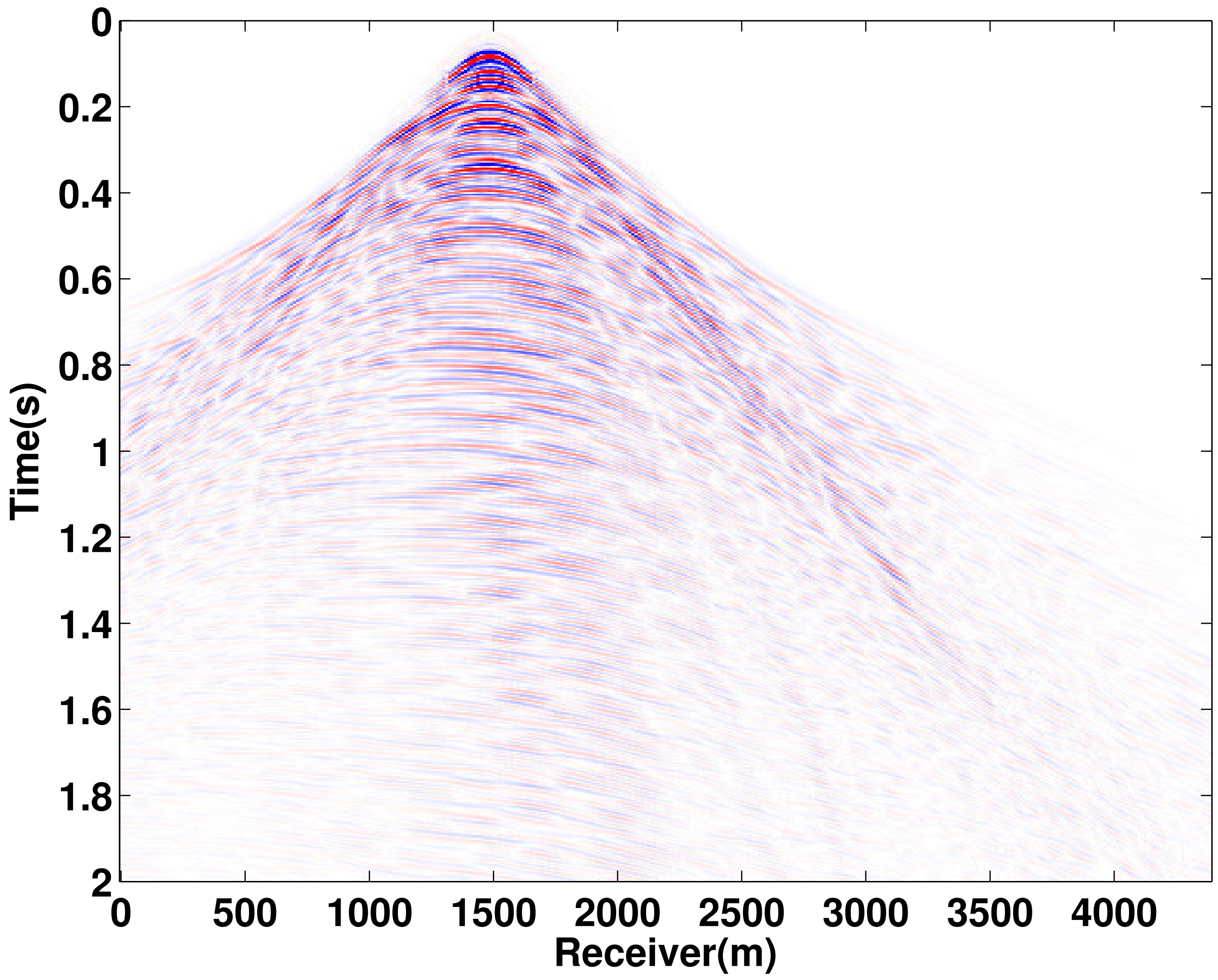


interpolation
[SNR = 10.94 db]

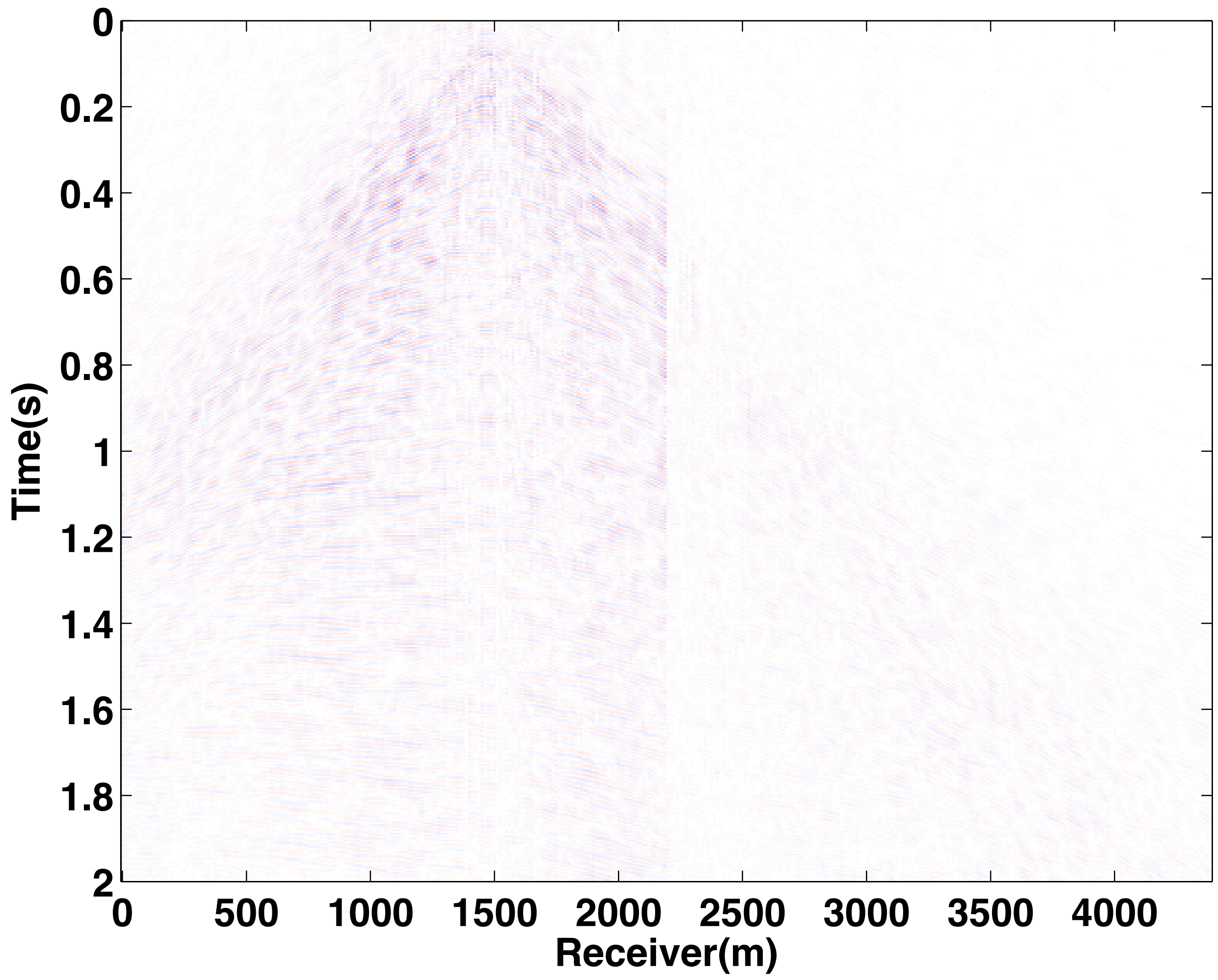


Gulf of Suez
[HSS with level-1 partitioning]

original

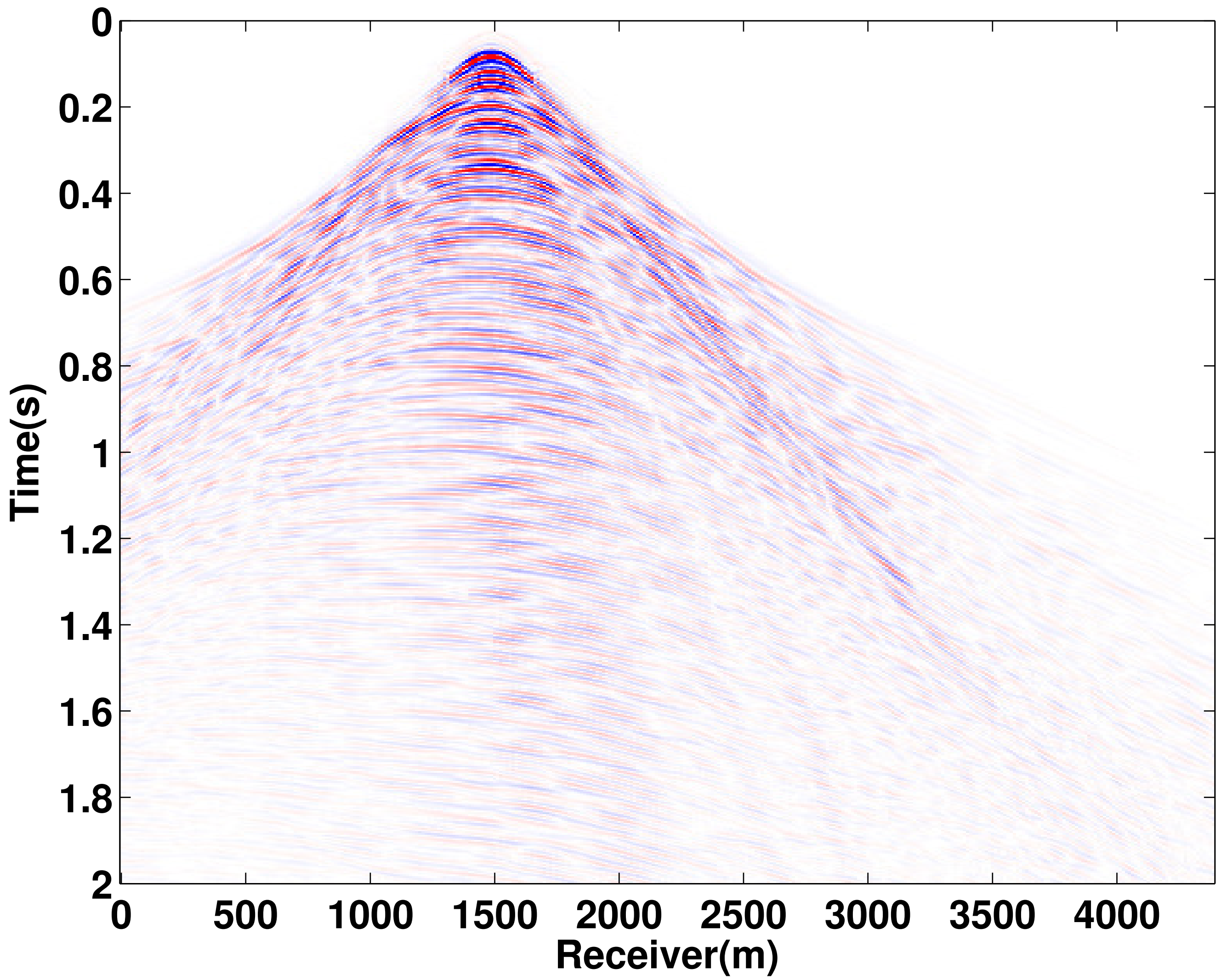


difference

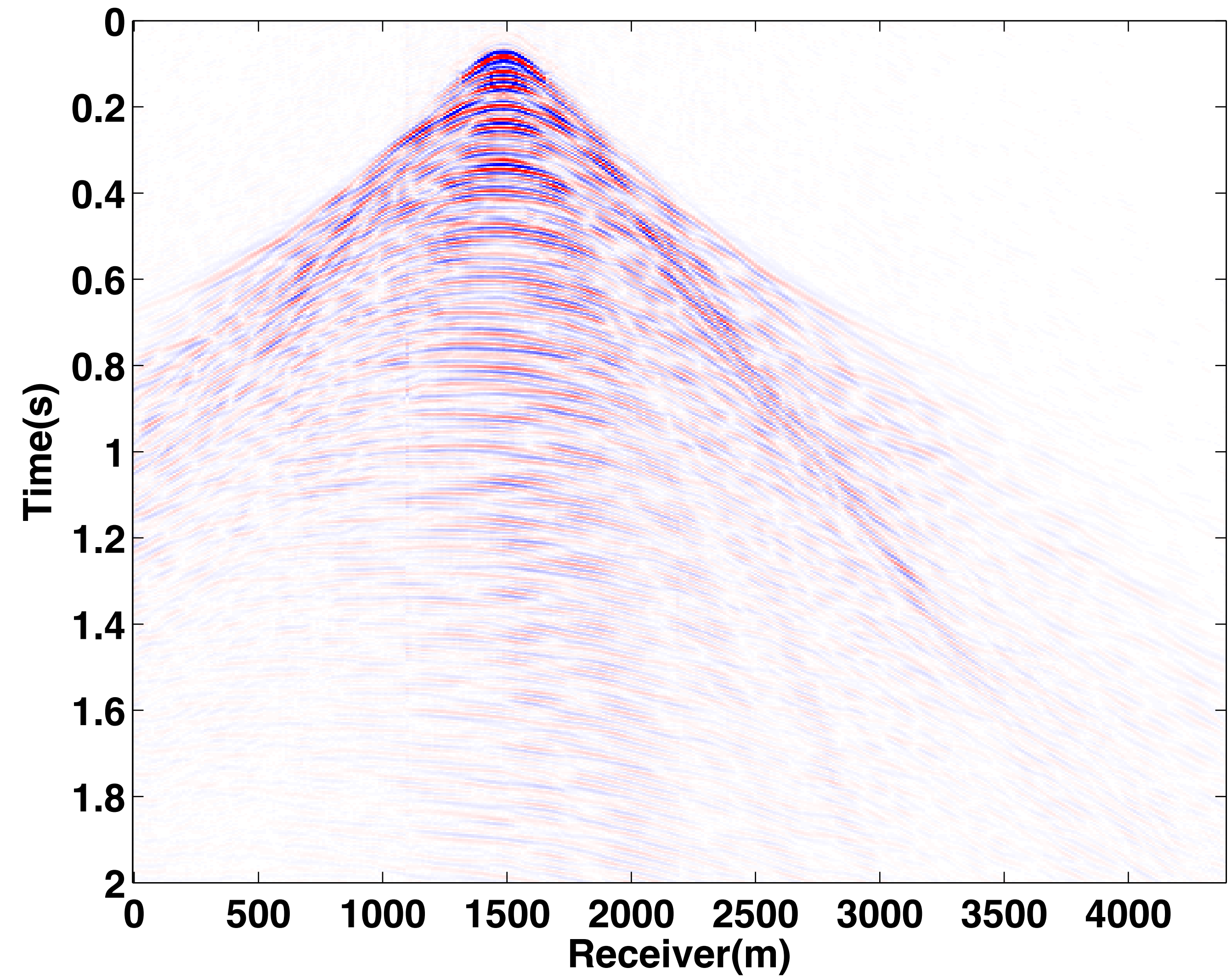


Gulf of Suez
[HSS with level-2 partitioning]

original

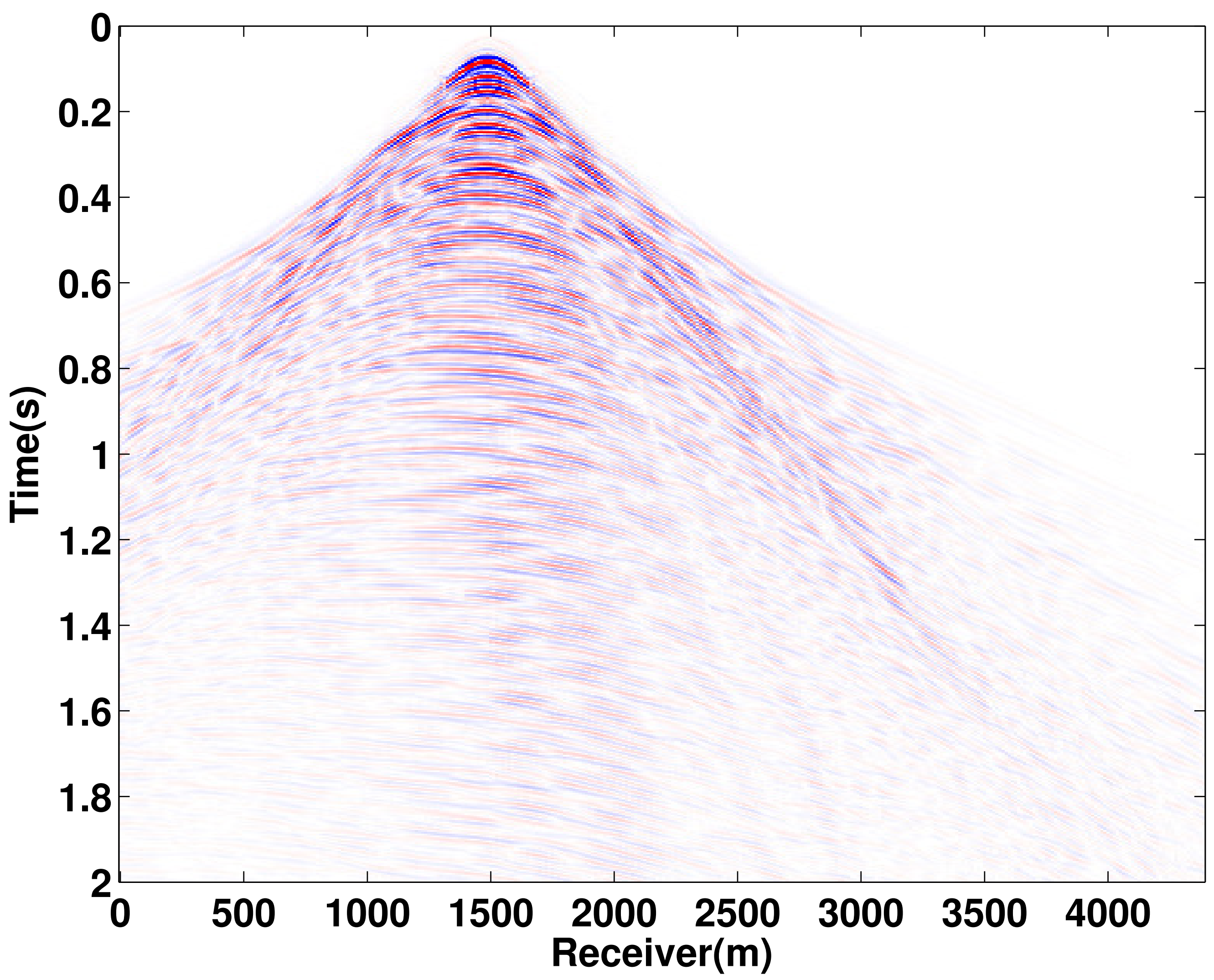


interpolation
[SNR = 15.89 db]

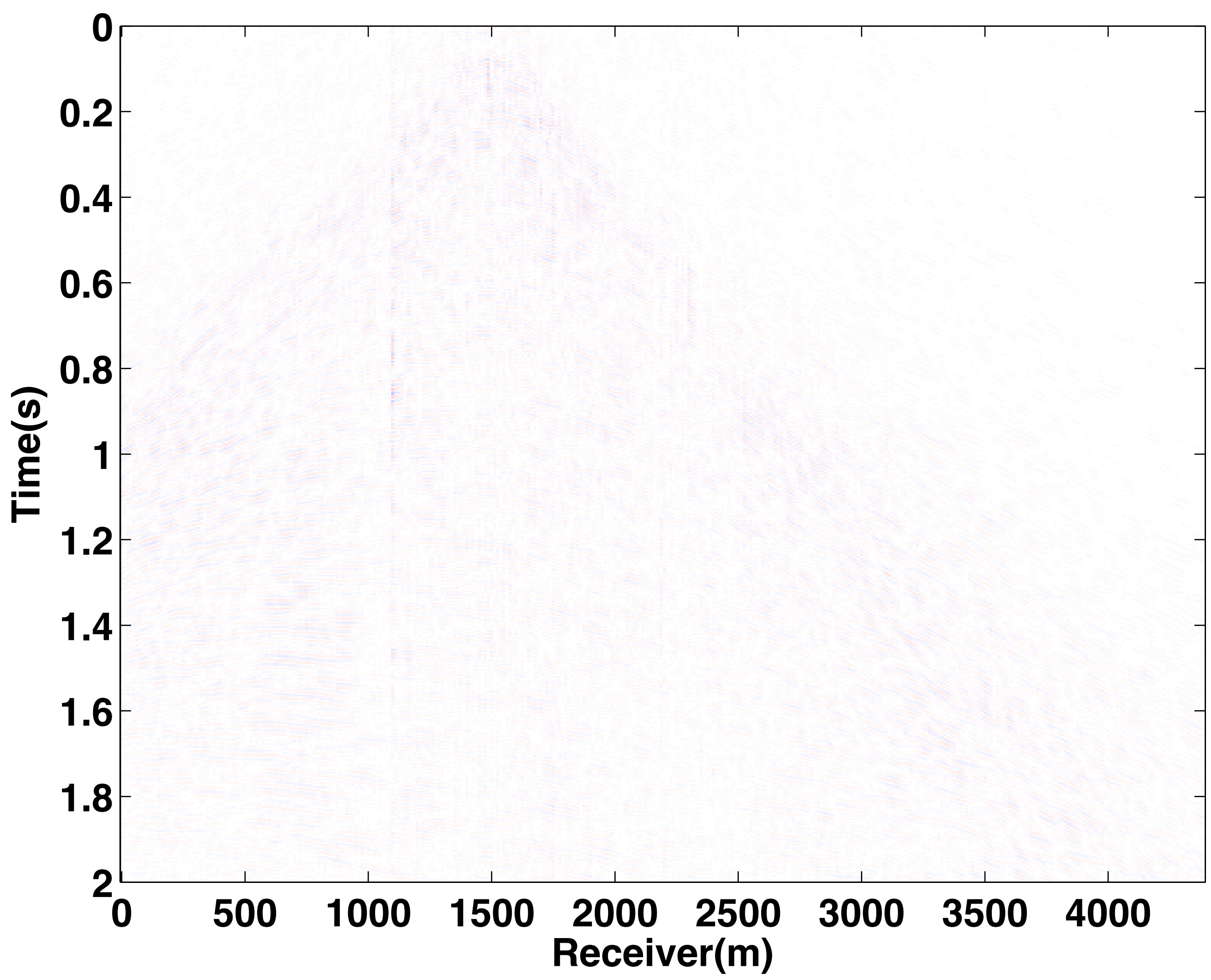


Gulf of Suez
[HSS with level-2 partitioning]

original

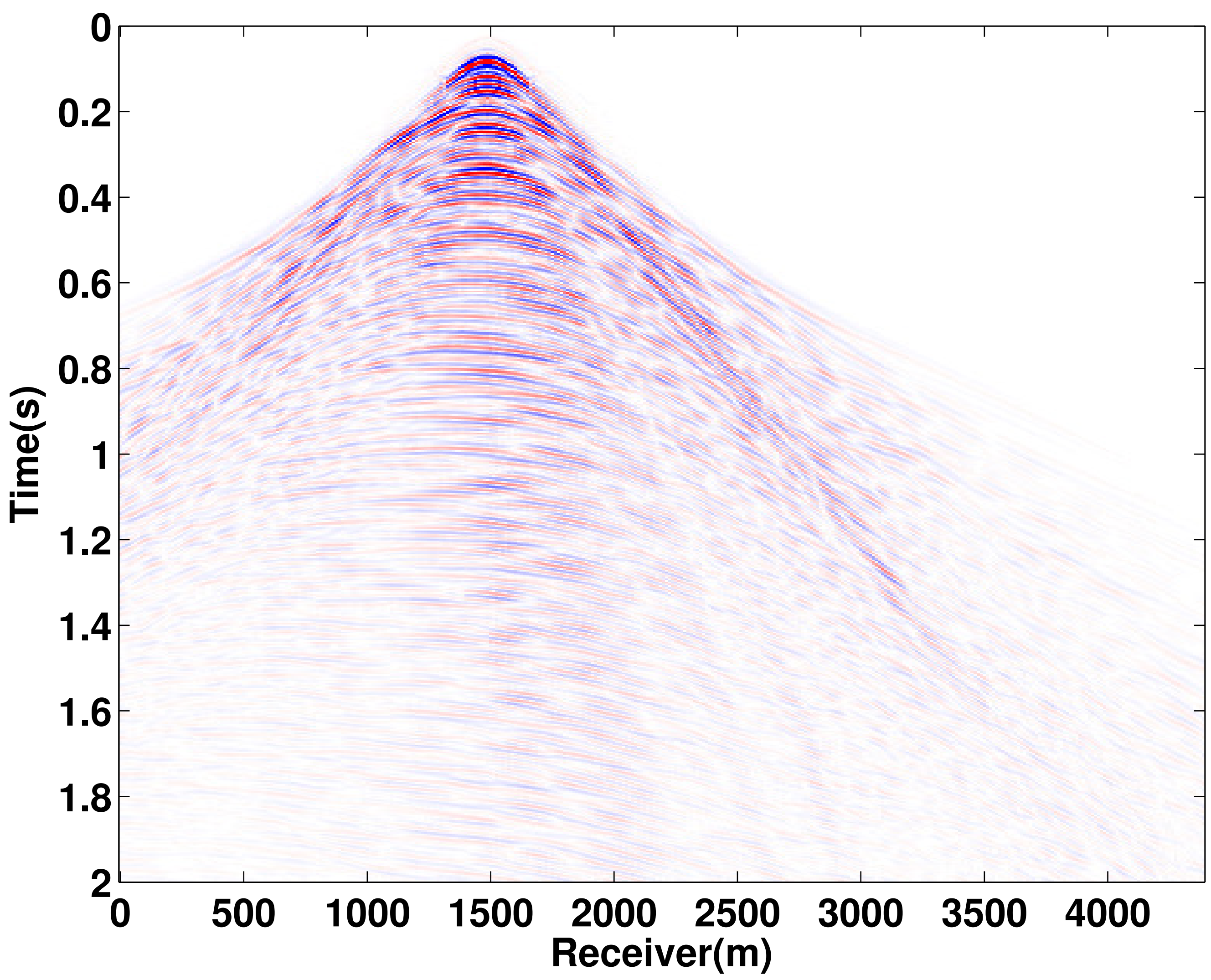


difference

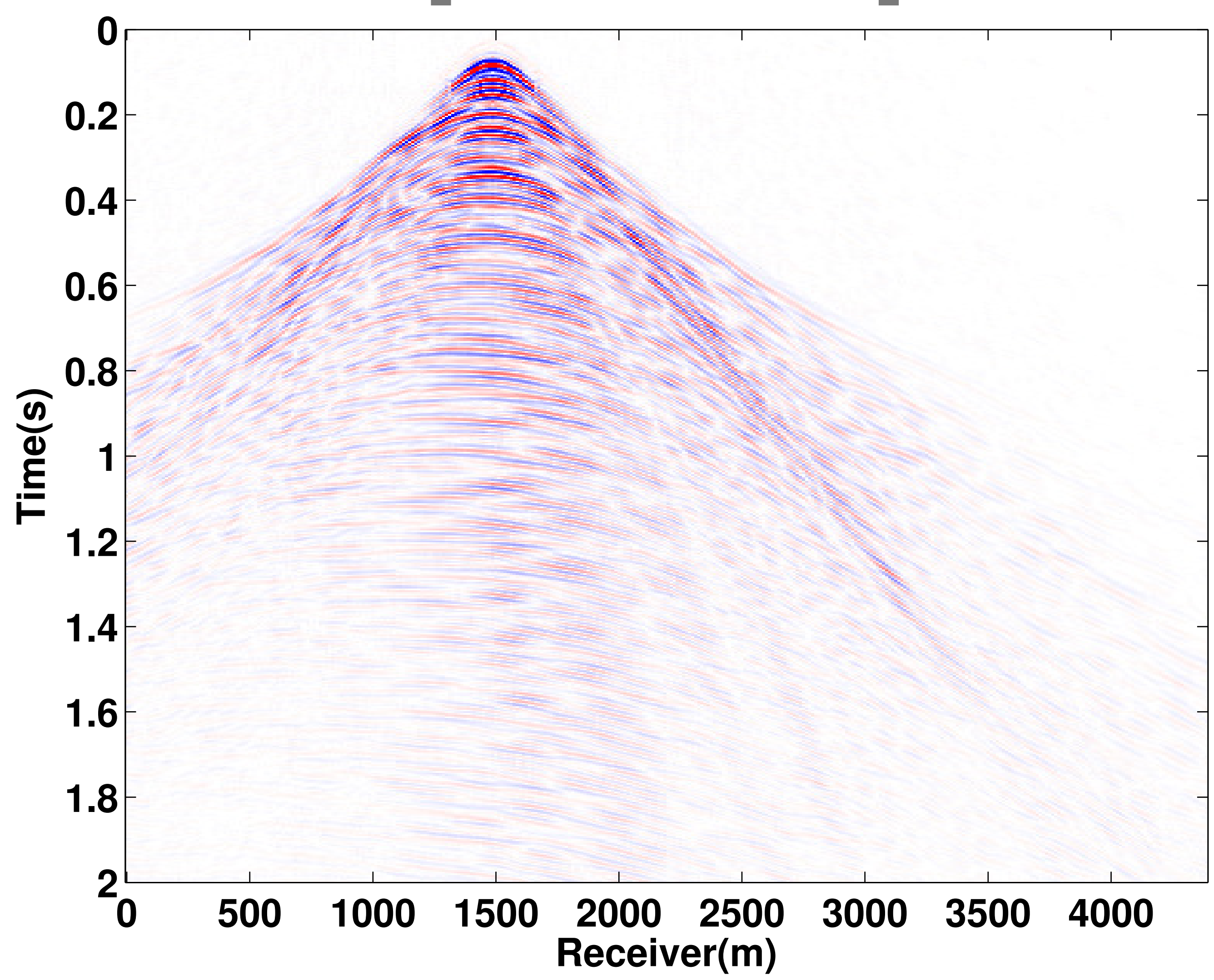


Gulf of Suez
[HSS with level-3 partitioning]

original



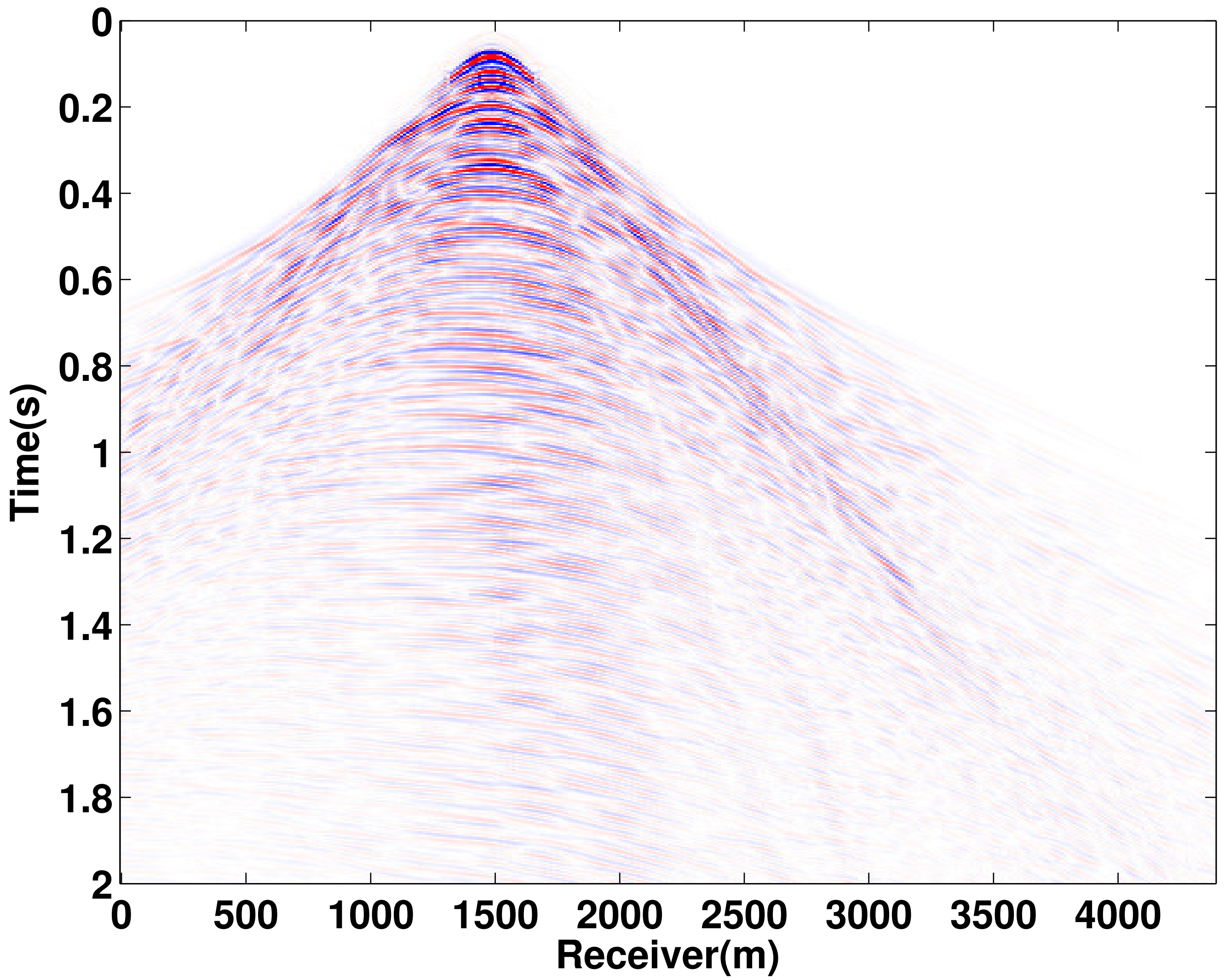
interpolation
[SNR = 17.37 db]



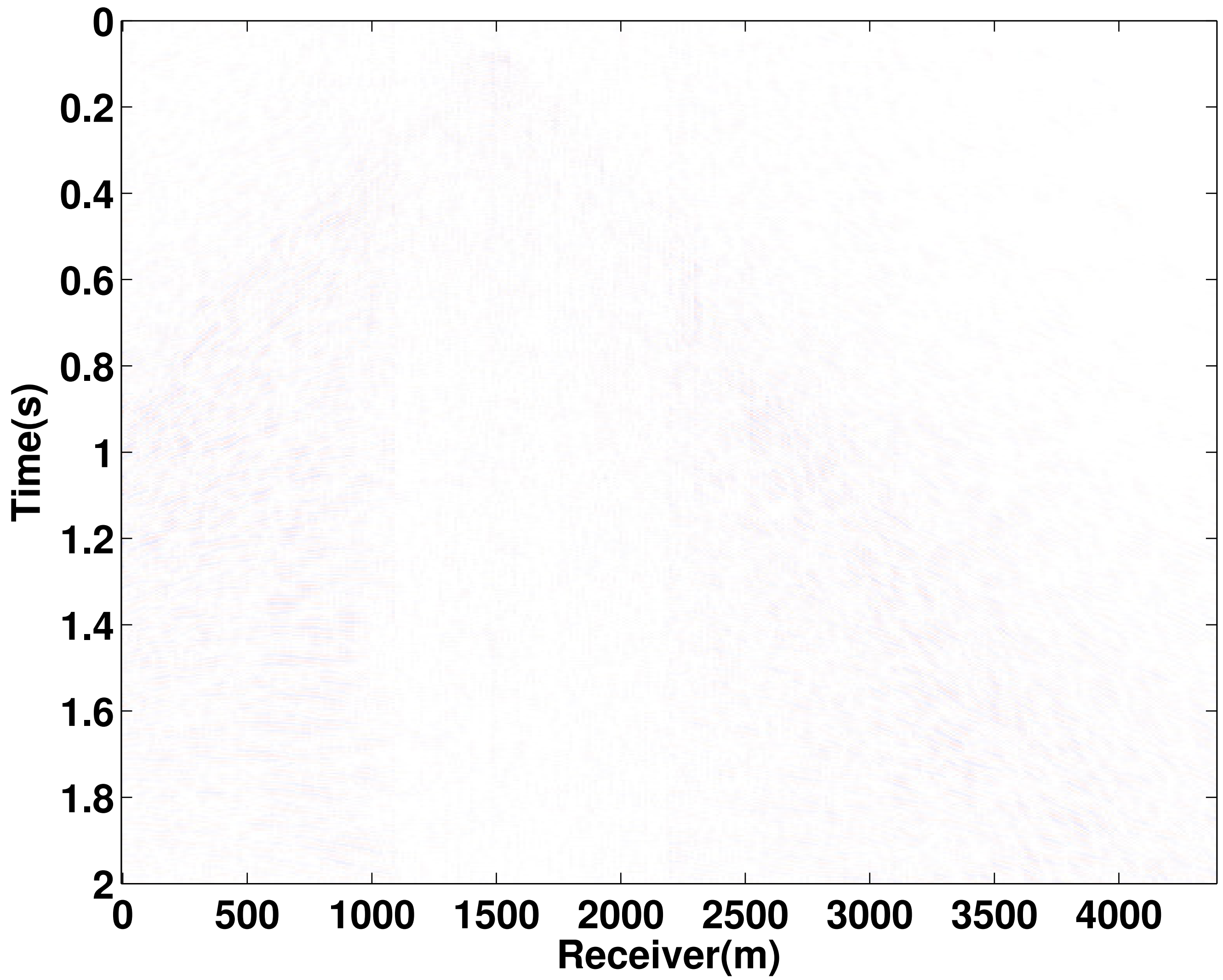
Gulf of Suez
[HSS with level-3 partitioning]

SLIM

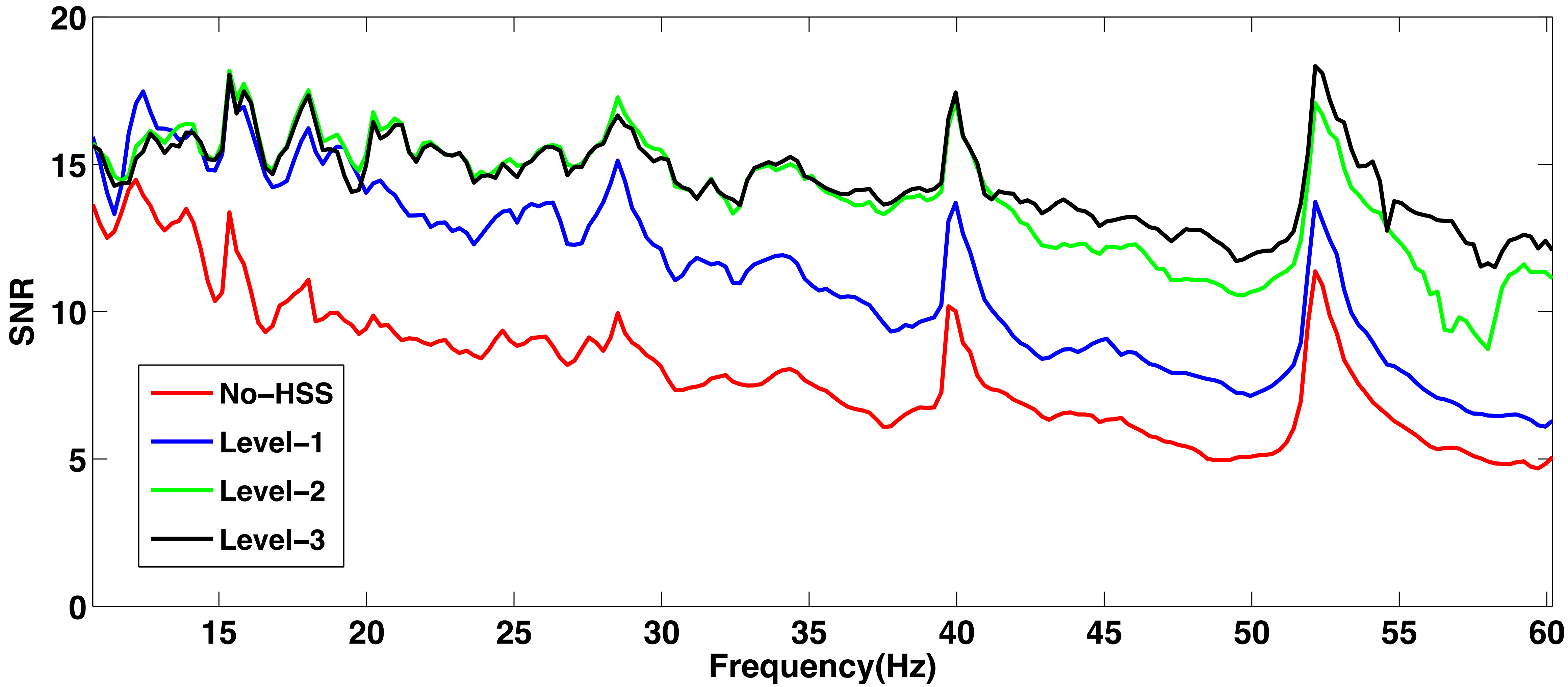
original



difference



Comparison



20
15
10
5
0
SNR
15
20
25
30
35
40
45
50
55
60
Frequency(Hz)
No–HSS
Level–1
Level–2
Level–3

Conclusion

- HSS gives advantage to attack high-rank structures using the partitioning
- matrix factorization allows *SVD-free* low-rank methods that work fast on large data
- low-rank structure holds promise for data recovery and more compact representation.

Future Work

- 3-D HSS
- HSS based denoising
- incorporate low-rank formulation in processing
- re-weighted low-rank interpolation

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